Coverage

Physics Grade 12 Advanced term 1

2024

https://t.me/h11Ad36

EXAMPLE 1.1 Net Charge

PROBLEM

If we wanted a block of iron of mass 3.25 kg to acquire a positive charge of 0.100 C, what fraction of the electrons would we have to remove?

SOLUTION

Iron has mass number 56. Therefore, the number of iron atoms in the 3.25 kg block is

$$N_{\text{atom}} = \frac{(3.25\text{ kg})(6.022\times10^{23}\text{ atoms/mole})}{0.0560\text{ kg/mole}} = 3.495\times10^{25} = 3.50\times10^{25}\text{ atoms}$$

Note that we have used Avogadro's number, 6.022×10^{23}, and the definition of the mole, which specifies that the mass of 1 mole of a substance in grams is just the mass number of the substance–in this case, 56.

Because the atomic number of iron is 26, which equals the number of protons or electrons in an iron atom, the total number of electrons in the 3.25 kg block is

$$N_e = 26N_{\text{atom}} = (26)(3.495\times10^{25}) = 9.09\times10^{26}\text{ electrons}$$

We use equation 1.5 to find the number of electrons, $N_{\Delta e}$, that we would have to remove. Because the number of electrons equals the number of protons in the original uncharged object, the difference in the number of protons and electrons is the number of removed electrons, $N_{\Delta e}$:

$$q = e\cdot N_{\Delta e} \Rightarrow N_{\Delta e} = \frac{q}{e} = \frac{0.100\text{ C}}{1.602\times10^{-19}\text{ C}} = 6.24\times10^{17}.$$

Finally, we obtain the fraction of electrons we would have to remove:

$$\frac{N_{\Delta e}}{N_e} = \frac{6.24\times10^{17}}{9.09\times10^{26}} = 6.87\times10^{-10}.$$

We would have to remove fewer than one in a billion electrons from the iron block in order to put the sizable positive charge of 0.100 C on it.

1.3 Insulators, Conductors, Semiconductors, and Superconductors

Materials that conduct electricity well are called **conductors.** Materials that do not conduct electricity are called **insulators.** (Of course, there are good and poor conductors and good and poor insulators, depending on the properties of the specific materials.)

The electronic structure of a material refers to the way in which electrons are bound to nuclei, as we'll discuss in later chapters. For now, we are interested in the relative propensity of the atoms of a material to either give up or acquire electrons. For insulators, no free movement of electrons occurs because the material has no loosely bound electrons that can escape from its atoms and thereby move freely throughout the material. Even when external charge is placed on an insulator, this external charge cannot move appreciably. Typical insulators are glass, plastic, and cloth.

On the other hand, materials that are conductors have an electronic structure that allows the free movement of some electrons. The positive charges of the atoms of a conducting material do not move, since they reside in the heavy nuclei. Typical solid conductors are metals. Copper, for example, is a very good conductor and is therefore used in electrical wiring.

Fluids and organic tissue can also serve as conductors. Pure distilled water is not a very good conductor. However, dissolving common table salt (NaCl), for example, in water improves its conductivity tremendously, because the positively charged sodium ions (Na^+) and negatively charged chlorine ions (Cl^-) can move within the water to conduct electricity. In liquids, unlike solids, positive as well as negative charge carriers are mobile. Organic tissue is not a very good conductor, but it conducts electricity well enough to make large currents dangerous to us.

Semiconductors

A class of materials called **semiconductors** can change from being an insulator to being a conductor and back to an insulator again. Semiconductors were discovered only a little more than 50 years ago but are the backbone of the entire computer and consumer electronics industries. The first widespread use of semiconductors was in transistors (Figure 1.7a); modern computer chips (Figure 1.7b) perform the functions of millions of transistors. Computers and basically all modern consumer electronics products and devices (televisions, cameras, video game players, cell phones, etc.) would be impossible without semiconductors. Gordon Moore, cofounder of Intel, famously stated that due to advancing technology, the power of the average computer's CPU (central processing unit) doubles every 18 months, which is an empirical average over the last 5 decades. This doubling phenomenon is known as *Moore's Law*. Physicists have been and will undoubtedly continue to be the driving force behind this process of scientific discovery, invention, and improvement.

Semiconductors are of two kinds: intrinsic and extrinsic. Examples of *intrinsic semiconductors* are chemically pure crystals of gallium arsenide, germanium, or, especially, silicon. Engineers produce *extrinsic semiconductors* by *doping*, which is the addition of minute amounts (typically 1 part in 10^6) of other materials that can act as electron donors or electron receptors. Semiconductors doped with electron donors are called *n-type* (*n* stands for "negative charge"). If the doping substance acts as an electron receptor, the hole left behind by an electron that attaches to a receptor can also travel through the semiconductor and acts as an effective positive charge carrier. These semiconductors are consequently called *p-type* (*p* stand for "positive charge"). Thus, unlike normal solid conductors in which only negative charges move, semiconductors have movement of negative or positive charges (which are really electron holes, that is, missing electrons).

Superconductors

Superconductors are materials that have zero resistance to the conduction of electricity, as opposed to normal conductors, which conduct electricity well but with some losses. Materials are superconducting only at very low temperatures. A typical superconductor is a niobium-titanium alloy that must be kept near the temperature of liquid helium (4.2 K) to retain its superconducting properties. During the last 20 years, new materials called *high-T_c superconductors* (T_c stands for "critical temperature," which is the maximum temperature that allows superconductivity) have been developed. These are superconducting at the temperature at which nitrogen can exist as a liquid (77.3 K). Materials that are superconductors at room temperature (300 K) have not yet been found, but they would be extremely useful. Research directed at developing such materials and theoretically explaining what physical phenomena cause high-T_c superconductivity is currently in progress.

power supply; it uses chemical reactions to create a separation between positive and negative charge. Several insulating paddles can be charged with positive or negative charge from the power supply. In addition, a conducting connection is made to the Earth. The Earth is a nearly infinite reservoir of charge, capable of effectively neutralizing electrically charged objects in contact with it. This taking away of charge is called **grounding**, and an electrical connection to the Earth is called a **ground.**

An **electroscope** is a device that gives an observable response when it is charged. You can build a relatively simple electroscope by using two strips of very thin metal foil that are attached at one end and are allowed to hang straight down adjacent to each other from an isolating frame. Kitchen aluminum foil is not suitable, because it is too thick, but hobby shops sell thinner metal foils. For the isolating frame, you can use a Styrofoam coffee cup turned sideways, for example.

FIGURE 1.8 A typical electroscope used in lecture demonstrations.

The lesson-demonstration-quality electroscope shown in Figure 1.8 has two conductors that in their neutral position are touching and oriented in a vertical direction. One of the conductors is hinged at its midpoint so that it will move away from the fixed conductor if a charge appears on the electroscope. These two conductors are in contact with a conducting ball on top of the electroscope, which allows charge to be applied or removed easily.

An uncharged electroscope is shown in Figure 1.9a. The power supply is used to give a negative charge to one of the insulating paddles. When the paddle is brought near the ball of the electroscope, as shown in Figure 1.9b, the electrons in the conducting ball of the electroscope are repelled, which produces a net negative charge on the conductors of the electroscope. This negative charge causes the movable conductor to rotate because the stationary conductor also has negative charge and repels it. Because the paddle did not touch the ball, the charge on the movable conductors is **induced.** If the charged paddle is then taken away, as illustrated in Figure 1.9c, the induced charge reduces to zero, and the movable conductor returns to its original position, because the total charge on the electroscope did not change in the process.

If the same process is carried out with a positively charged paddle, the electrons in the conductors are attracted to the paddle and flow into the conducting ball. This leaves a net positive charge on the conductors, causing the movable conducting arm to rotate again. Note that the net charge of the electroscope is zero in both cases and that the motion of the conductor indicates only that the paddle is charged. When the positively charged paddle is removed, the movable conductor again returns to its original position. It is important to note that we cannot determine the sign of this charge!

On the other hand, if a negatively charged insulating paddle *touches* the ball of the electroscope, as shown in Figure 1.10b, electrons will flow from the paddle to the conductor, producing a net negative charge. When the paddle is removed, the charge remains and the movable arm remains rotated, as shown in Figure 1.10c. Similarly, if a positively charged insulating paddle touches the ball of the uncharged electroscope, the electroscope transfers electrons to the positively charged paddle and becomes positively charged. Again, both a

Concept Check 1.2

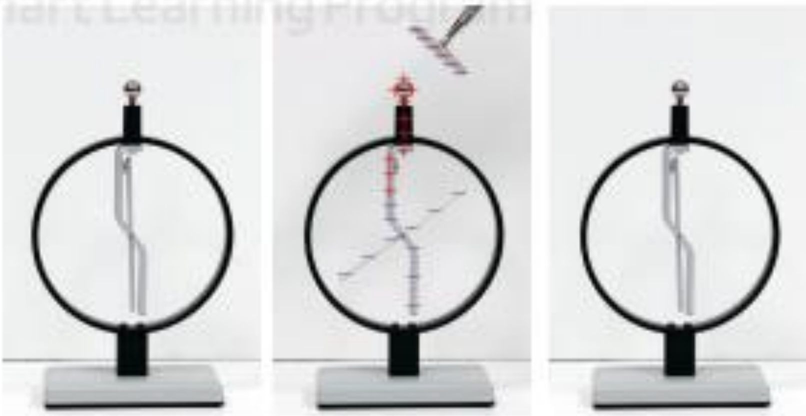
The hinged conductor moves away from the fixed conductor if a charge is applied to the electroscope, because

a) like charges repel each other.

b) like charges attract each other.

c) unlike charges attract each other.

d) unlike charges repel each other.



(a) (b) (c)

FIGURE 1.9 Inducing a charge: (a) An uncharged electroscope. (b) A negatively charged paddle is brought near the electroscope. (c) The negatively charged paddle is taken away.

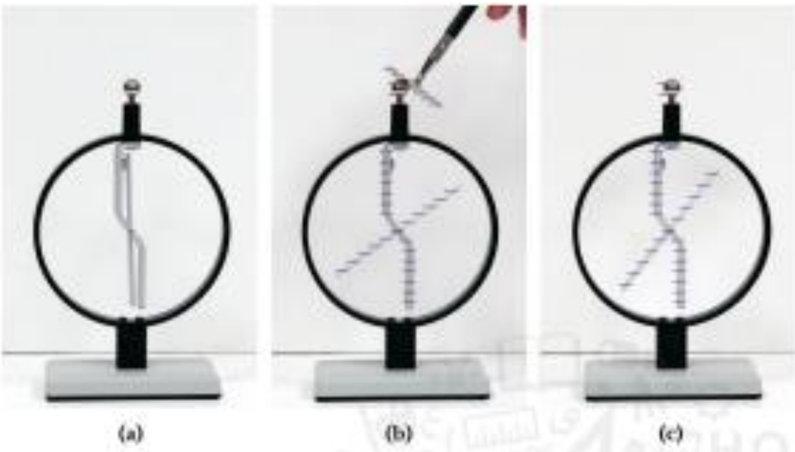


FIGURE 1.10 Charging by contact: (a) An uncharged electroscope. (b) A negatively charged paddle touches the electroscope. (c) The negatively charged paddle is removed.

positively charged paddle and a negatively charged paddle have the same effect on the electroscope, and we have no way of determining whether the paddles are positively charged or negatively charged. This process is called **charging by contact.**

The two different kinds of charge can be demonstrated by first touching a negatively charged paddle to the electroscope, producing a rotation of the movable arm, as shown in Figure 1.10. If a positively charged paddle is then brought into contact with the electroscope, the movable arm returns to the uncharged position. The charge is neutralized (assuming that both paddles originally had the same absolute value of charge). Thus, there are two kinds of charge. However, because charges are manifestations of mobile electrons, a negative charge is an excess of electrons and a positive charge is a deficit of electrons.

The electroscope can be given a charge without touching it with the charged paddle, as shown in Figure 1.11. The uncharged electroscope is shown in Figure 1.11a. A negatively charged paddle is brought close to the ball of the electroscope but not touching it, as shown in Figure 1.11b. In Figure 1.11c, the electroscope is connected to a ground. Then, while the charged paddle is still close to but not touching the ball of the electroscope, the ground connection is removed in Figure 1.11d. Next, when the paddle is moved away from the electroscope in Figure 1.11e, the electroscope is still positively charged (but with a smaller deflection than in Figure 1.11b). The same process also works with a positively charged paddle. This process is called **charging by induction** and yields an electroscope charge that has the opposite sign from the charge on the paddle.

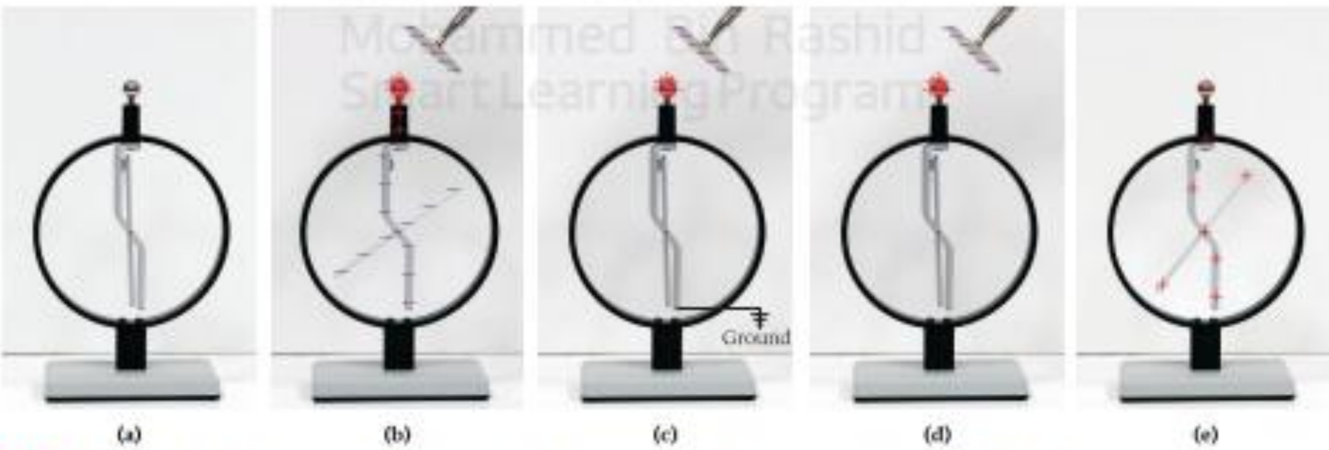


FIGURE 1.11 Charging by induction: (a) An uncharged electroscope. (b) A negatively charged paddle is brought close to the electroscope. (c) A ground is connected to the electroscope. (d) The connection to the ground is removed. (e) The negatively charged paddle is taken away, leaving the electroscope positively charged.

EXAMPLE 1.2 Electrostatic Force inside the Atom

PROBLEM 1
What is the magnitude of the electrostatic force that the two protons inside the nucleus of a helium atom exert on each other?

SOLUTION 1
The two protons and two neutrons in the nucleus of the helium atom are held together by the strong force; the electrostatic force is pushing the protons apart. The charge of each proton is $q_p = +e$. A distance of approximately $r = 2 \cdot 10^{-15}$ m separates the two protons. Using Coulomb's Law, we can find the force:

$$F = k\frac{|q_p q_p|}{r^2} = \left(8.99\times10^{9}\ \frac{\text{N m}^2}{\text{C}^2}\right)\frac{\left(+1.6\times10^{-19}\ \text{C}\right)\left(+1.6\times10^{-19}\ \text{C}\right)}{\left(2\times10^{-15}\ \text{m}\right)^2} = 58\ \text{N}.$$

Therefore, the two protons in the atomic nucleus of a helium atom are being pushed apart with a force of 58 N (approximately the weight of a small dog). Considering the size of the nucleus, this is an astonishingly large force. Why do atomic nuclei not simply explode? The answer is that an even stronger force, the aptly named strong force, keeps them together.

PROBLEM 2
What is the magnitude of the electrostatic force between a gold nucleus and an electron of the gold atom in an orbit with radius 4.88×10^{-12} m?

SOLUTION 2
The negatively charged electron and the positively charged gold nucleus attract each other with a force whose magnitude is

$$F = k\frac{|q_e q_N|}{r^2},$$

where the charge of the electron is $q_e = -e$ and the charge of the gold nucleus is $q_N = +79e$. The force between the electron and the nucleus is then

$$F = k\frac{|q_e q_N|}{r^2} = \left(8.99\times10^{9}\ \frac{\text{N m}^2}{\text{C}^2}\right)\frac{\left(1.60\times10^{-19}\ \text{C}\right)\left|(79)\left(1.60\times10^{-19}\ \text{C}\right)\right|}{\left(4.88\times10^{-12}\ \text{m}\right)^2} = 7.63\times10^{-4}\ \text{N}.$$

Thus, the magnitude of the electrostatic force exerted on an electron in a gold atom by the nucleus is about 100,000 times less than that between protons inside a nucleus.

Concept Check 1.6
Three charges are arranged on a straight line as shown in the figure. What is the direction of the electrostatic force on the *middle* charge?

a) → b) ← c) ↓ d) ↑

e) There is no force on that charge.

Concept Check 1.7
Three charges are arranged on a straight line as shown in the figure. What is the direction of the electrostatic force on the *right* charge? (Note that the left charge is double what it was in Concept Check 1.6.)

a) → b) ← c) ↓ d) ↑

e) There is no force on that charge.

EXAMPLE 1.3 Equilibrium Position

PROBLEM

Two charged particles are placed as shown in Figure 1.16: $q_1 = 0.15\ \mu C$ is located at the origin, and $q_2 = 0.35\ \mu C$ is located on the positive x-axis at $x_2 = 0.40$ m. Where should a third charged particle, q_3, be placed to be at an equilibrium point (such that the forces on it sum to zero)?

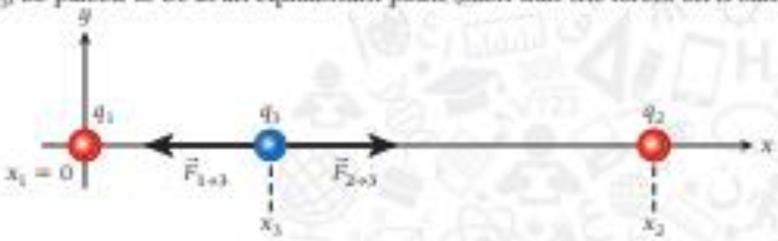


FIGURE 1.16 Placement of three charged particles. The third particle is shown as having a negative charge.

SOLUTION

Let's first determine where *not* to put the third charge. If the third charge is placed anywhere off the x-axis, there will always be a force component pointing toward or away from the x-axis. Thus, we can find an equilibrium point (a point where the forces sum to zero) only *on* the x-axis. The x-axis can be divided into three different segments: $x \leq x_1 = 0$, $x_1 < x < x_2$, and $x_2 \leq x$. For $x \leq x_1 = 0$, the force vectors from both q_1 and q_2 acting on q_3 will point in the positive direction if the charge is negative and in the negative direction if the charge is positive. Because we are looking for a location where the two forces cancel, the segment $x \leq x_1 = 0$ can be excluded. A similar argument excludes $x \geq x_2$.

In the remaining segment of the x-axis, $x_1 < x < x_2$, the forces from q_1 and q_2 on q_3 point in opposite directions. We look for the location, x_3, where the absolute magnitudes of both forces are equal and the forces thus sum to zero. We express the equality of the two forces as

$$\left|\vec{F}_{1\rightarrow 3}\right| = \left|\vec{F}_{2\rightarrow 3}\right|,$$

which we can rewrite as

$$k\frac{|q_1 q_3|}{(x_3 - x_1)^2} = k\frac{|q_3 q_2|}{(x_2 - x_3)^2}.$$

We now see that the magnitude and sign of the third charge do not matter because that charge cancels out, as does the constant k, giving us

$$\frac{q_1}{(x_3 - x_1)^2} = \frac{q_2}{(x_2 - x_3)^2}$$

or

$$q_1(x_2 - x_3)^2 = q_2(x_3 - x_1)^2. \qquad \text{(i)}$$

Taking the square root of both sides and solving for x_3, we find

$$\sqrt{q_1}(x_2 - x_3) = \sqrt{q_2}(x_3 - x_1),$$

or

$$x_3 = \frac{\sqrt{q_1}x_2 + \sqrt{q_2}x_1}{\sqrt{q_1} + \sqrt{q_2}}.$$

We can take the square root of both sides of equation (i) because $x_1 < x_3 < x_2$, and so both of the roots, $x_2 - x_3$ and $x_3 - x_1$, are assured to be positive.

Inserting the numbers given in the problem statement, we obtain

$$x_3 = \frac{\sqrt{q_1}x_2 + \sqrt{q_2}x_1}{\sqrt{q_1} + \sqrt{q_2}} = \frac{\sqrt{0.15\ \mu C}(0.4\ \text{m})}{\sqrt{0.15\ \mu C} + \sqrt{0.35\ \mu C}} = 0.16\ \text{m}.$$

This result makes sense because we expect the equilibrium point to reside closer to the smaller charge.

SOLVED PROBLEM 1.1 Charged Balls

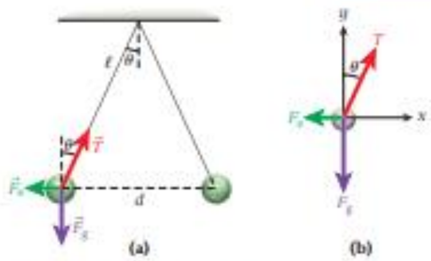


FIGURE 1.17 (a) Two charged balls hanging from the ceiling in their equilibrium position. (b) Free-body diagram for the left-hand charged ball.

PROBLEM

Two identical charged balls hang from the ceiling by insulated ropes of equal length, $\ell = 1.50$ m (Figure 1.17). A charge $q = 25.0\ \mu$C is applied to each ball. Then the two balls hang at rest, and each supporting rope has an angle of 25.0° with respect to the vertical (Figure 1.17a). What is the mass of each ball?

SOLUTION

THINK Each charged ball has three forces acting on it: the force of gravity, the repulsive electrostatic force, and the tension in the supporting rope. We can resolve the components of the three forces and set them equal to zero, allowing us to solve for the mass of the charged balls.

SKETCH A free-body diagram for the left-hand ball is shown in Figure 1.17b.

RESEARCH The condition for static equilibrium says that the sum of the x-components of the three forces acting on the ball must equal zero and the sum of y-components of these forces must equal zero. The sum of the x-components of the forces is

$$T\sin\theta - F_e = 0, \tag{i}$$

where T is the magnitude of the string tension, θ is the angle of the string relative to the vertical, and F_e is the magnitude of the electrostatic force. The sum of the y-components of the forces is

$$T\cos\theta - F_g = 0. \tag{ii}$$

The force of gravity, F_g, is just the weight of the charged ball:

$$F_g = mg, \tag{iii}$$

where m is the mass of the charged ball. The electrostatic force the two balls exert on each other is given by

$$F_e = k\frac{q^2}{d^2}, \tag{iv}$$

where d is the distance between the two balls. We can express the distance between the two balls in terms of the length of the string, ℓ, by looking at Figure 1.17a. We see that

$$\sin\theta = \frac{d/2}{\ell}.$$

We can then express the electrostatic force in terms of the angle with respect to the vertical, θ, and the length of the string, ℓ:

$$F_e = k\frac{q^2}{(2\ell\sin\theta)^2} = k\frac{q^2}{4\ell^2\sin^2\theta}. \tag{v}$$

SIMPLIFY We divide equation (i) by equation (ii):

$$\frac{T\sin\theta}{T\cos\theta} = \frac{F_e}{F_g},$$

which, after the (unknown) string tension is canceled out, becomes

$$\tan\theta = \frac{F_e}{F_g}.$$

Substituting from equations (iii) and (v) for the force of gravity and the electrostatic force, we get

$$\tan\theta = \frac{k\dfrac{q^2}{4\ell^2\sin^2\theta}}{mg} = \frac{kq^2}{4mg\ell^2\sin^2\theta}.$$

Solving for the mass of the ball, we obtain

$$m = \frac{kq^2}{4g\ell^2\sin^2\theta\tan\theta}.$$

CALCULATE Putting in the numerical values gives

$$m = \frac{(8.99\times10^9\ \mathrm{N\,m^2/C^2})(25.0\ \mu\mathrm{C})^2}{4(9.81\ \mathrm{m/s^2})(1.50\ \mathrm{m})^2(\sin^2 25.0°)(\tan 25.0°)} = 0.764116\ \mathrm{kg}.$$

ROUND We report our result to three significant figures:

$$m = 0.764\ \mathrm{kg}.$$

DOUBLE-CHECK To double-check, we make the small-angle approximations that $\sin\theta \approx \tan\theta \approx \theta$ and $\cos\theta \approx 1$. The tension in the string then approaches mg, and we can express the x-components of the forces as

$$T\sin\theta \approx mg\theta = F_e = k\frac{q^2}{d^2} \approx k\frac{q^2}{(2\ell\theta)^2}.$$

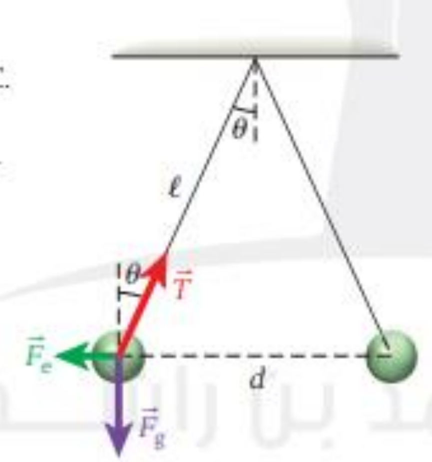
Solving for the mass of the charged ball, we get

$$m = \frac{kq^2}{4g\ell^2\theta^3} = \frac{(8.99\times10^9\ \mathrm{N\,m^2/C^2})(25.0\ \mu\mathrm{C})^2}{4(9.81\ \mathrm{m/s^2})(1.50\ \mathrm{m})^2(0.436\ \mathrm{rad})^3} = 0.768\ \mathrm{kg},$$

which is close to our answer.

1.83 Two balls have the same mass, 0.9680 kg, and the same charge, 29.59 μC. They hang from the ceiling on strings of identical length, ℓ, as shown in the figure. If the angle of the strings with respect to the vertical is 29.79°, what is the length of the strings?

1.84 Two balls have the same mass and the same charge, 15.71 μC. They hang from the ceiling on strings of identical length, $\ell = 1.223$ m, as shown in the figure. The angle of the strings with respect to the vertical is 21.07°. What is the mass of each ball?



2.5 General Charge Distributions

We have determined the electric fields of a single point charge and of two point charges (an electric dipole). What if we want to determine the electric field due to many charges? Each individual charge creates an electric field, as described by equation 2.4, and because of the superposition principle, all of these electric fields can be added to find the net field at any point in space. But we have already seen in Example 2.1 that the addition of electric field vectors can be cumbersome for a collection of only three point charges. If we had to apply this method to, say, trillions of point charges, the task would be unmanageable even if we could use a supercomputer. Since real-world applications usually involve a very large number of charges, it is clear that we need a way to simplify the calculations. This can be accomplished by using an integral, if the large number of charges are arranged in space in some regular distribution. Of particular interest are two-dimensional distributions, where charges are located on the surface of a metallic object, and one-dimensional distributions, where charges are arranged along a wire. As we will see, integration can be a surprisingly simple way to solve problems involving such charge distributions, which would be very hard to analyze by the method of direct summation.

To prepare for the integration procedure, we divide the charge into differential elements of charge, dq, and find the electric field resulting from each differential charge element as if it were a point charge. If the charge is distributed along a one-dimensional object (a line), the differential charge may be expressed in terms of a charge per unit length times a differential length, or λdx. If the charge is distributed over a surface (a two-dimensional object), dq is expressed in terms of a charge per unit area times a differential area, or σdA. And, finally, if the charge is distributed over a three-dimensional volume, then dq is written as the product of a charge per unit volume and a differential volume, or ρdV. That is,

$$\left.\begin{aligned} dq &= \lambda dx \\ dq &= \sigma dA \\ dq &= \rho dV \end{aligned}\right\} \text{ for a charge distribution } \begin{cases} \text{along a line;} \\ \text{over a surface;} \\ \text{throughout a volume.} \end{cases} \quad (2.9)$$

The magnitude of the electric field resulting from the charge distribution is then obtained from the differential charge:

$$dE = k\frac{dq}{r^2} \quad (2.10)$$

In the following example, we find the electric field due to a finite line of charge.

SOLVED PROBLEM 2.2 Electron Moving over a Charged Plate

PROBLEM

An electron with a kinetic energy of 2.00 keV ($1 \text{ eV} = 1.602 \times 10^{-19}$ J) is fired horizontally across a horizontally oriented charged conducting plate with a surface charge density of $+4.00 \times 10^{-6}$ C/m^2. Taking the positive direction to be upward (away from the plate), what is the vertical deflection of the electron after it has traveled a horizontal distance of 4.00 cm?

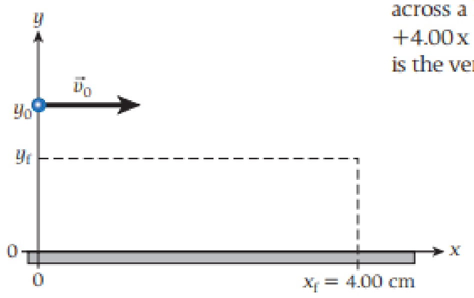


FIGURE 2.18 An electron moving to the right with initial velocity $\vec{v}_0$ over a charged conducting plate.

SOLUTION

THINK The initial velocity of the electron is horizontal. During its motion, the electron experiences a constant attractive force from the positively charged plate, which causes a constant acceleration downward. We can calculate the time it takes the electron to travel 4.00 cm in the horizontal direction and use this time to calculate the vertical deflection of the electron.

SKETCH Figure 2.18 shows the electron with initial velocity $\vec{v}_0$ in the horizontal direction. The initial position of the electron is taken to be at $x_0 = 0$ and $y = y_0$.

RESEARCH The time the electron takes to travel the given distance is

$$t = x_f / v_0 \quad \text{(i)}$$

where x_f is the final horizontal position and v_0 is the initial speed of the electron. While the electron is in motion, it experiences a force from the charged conducting plate. This force is directed downward (toward the plate) and has a magnitude given by

$$F = qE = e\frac{\sigma}{\varepsilon_0} \quad \text{(ii)}$$

where σ is the charge density on the conducting plate and e is the charge of an electron. This force causes a constant acceleration in the downward direction whose magnitude is given by $a = F/m$, where m is the mass of the electron. Using the expression for the force from equation (ii), we can express the magnitude of this acceleration as

$$a = \frac{F}{m} = \frac{e\sigma}{m\varepsilon_0} \quad \text{(iii)}$$

Note that this acceleration is constant. Thus, the vertical position of the electron as a function of time is given by

$$y_f = y_0 - \tfrac{1}{2}at^2 \Rightarrow y_f - y_0 = -\tfrac{1}{2}at^2 \quad \text{(iv)}$$

Finally, we can relate the electron's initial kinetic energy to its initial velocity through

$$K = \tfrac{1}{2}mv_0^2 \Rightarrow v_0^2 = \frac{2K}{m} \quad \text{(v)}$$

SIMPLIFY We substitute the expressions for the time and the acceleration from equations (i) and (iii) into equation (iv) and obtain

$$y_f - y_0 = -\tfrac{1}{2}at^2 = -\tfrac{1}{2}\left(\frac{e\sigma}{m\varepsilon_0}\right)\left(\frac{x_f}{v_0}\right)^2 = -\frac{e\sigma x_f^2}{2m\varepsilon_0 v_0^2} \quad \text{(vi)}$$

Now substituting the expression for the square of the initial speed from equation (v) into the right-hand side of equation (vi) gives us

$$y_f - y_0 = -\frac{e\sigma x_f^2}{2m\varepsilon_0\left(\frac{2K}{m}\right)} = -\frac{e\sigma x_f^2}{4\varepsilon_0 K} \quad \text{(vii)}$$

CALCULATE We first convert the kinetic energy of the electron from electron-volts to joules:

$$K = (2.00 \text{ keV})\frac{1.602\times10^{-19} \text{ J}}{1 \text{ eV}} = 3.204\times10^{-16} \text{ J}$$

Putting the numerical values into equation (vii), we get

$$y_f - y_0 = -\frac{e\sigma x_f^2}{4\varepsilon_0 K} = -\frac{(1.602\times10^{-19} \text{ C})(4.00\times10^{-6} \text{ C/m}^2)(0.0400 \text{ m})^2}{4(8.85\times10^{-12} \text{ C}^2/(\text{N m}^2))(3.204\times10^{-16} \text{ J})} = -0.0903955 \text{ m}$$

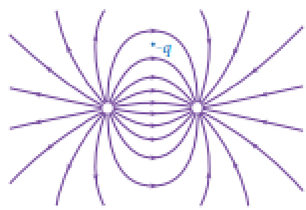
ROUND We report our result to three significant figures:

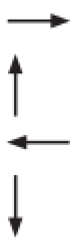
$$y_f - y_0 = -0.0904 \text{ m} = -9.04 \text{ cm}$$

DOUBLE-CHECK The vertical deflection that we calculated is about twice the distance that the electron travels in the x-direction, which seems reasonable, at least in the sense of being of the same order of magnitude. Also, equation (vii) for the deflection has several features that should be present. First, the trajectory is parabolic, which we expect for a constant force and thus constant acceleration. Second, for zero surface charge density, we obtain zero deflection. Third, for very high kinetic energy, there is negligible deflection, which is also intuitively what we expect.

Concept Check 2.7

A negative charge $-q$ is placed in a nonuniform electric field as shown in the figure. What is the direction of the electric force on this negative charge?





a)

b)

c)

d)

e) The force is zero.

Concept Check 2.5

A small positively charged object is placed at rest in a uniform electric field as shown in the figure. When the object is released, it will

a) not move.

b) begin to move with a constant speed.

c) begin to move with a constant acceleration.

d) begin to move with an increasing acceleration.

e) move back and forth in simple harmonic motion.

Planar Symmetry

Assume a flat thin, infinite, nonconducting sheet of positive charge (Figure 2.34), with uniform charge per unit area $\sigma > 0$. Let's find the electric field a distance r from the surface of this infinite plane of charge.

To do this, we choose a Gaussian surface in the form of a closed right cylinder with cross-sectional area A and length $2r$, which cuts through the plane perpendicularly, as shown in Figure 2.34. Because the plane is infinite and the charge is positive, the electric field must be perpendicular to the ends of the cylinder and parallel to the cylinder wall. Using Gauss's Law, we obtain

$$\oiint \vec{E} \bullet d\vec{A} = (EA + EA) = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

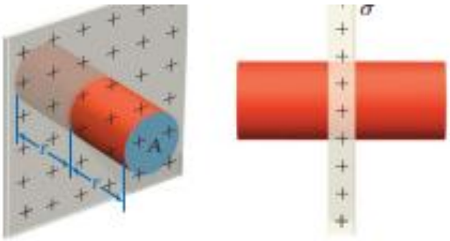


FIGURE 2.34 Infinite, flat, nonconducting sheet with charge density σ. Cutting through the plane perpendicularly is a Gaussian surface in the form of a right cylinder with cross-sectional area A parallel to the plane and height r above and below the plane.

where σA is the charge enclosed in the cylinder. Thus, the magnitude of the electric field due to an infinite plane of charge is

$$E = \frac{\sigma}{2\varepsilon_0} \tag{2.18}$$

If $\sigma < 0$, then equation 2.18 still holds, but the electric field points toward the plane instead of away from it.

For an infinite conducting sheet with charge density $\sigma > 0$ on each surface, we can find the electric field by choosing a Gaussian surface in the form of a right cylinder. However, for this case, one end of the cylinder is embedded inside the conductor (Figure 2.35). The electric field inside the conductor is zero; therefore, there is no flux through the end of the cylinder enclosed in the conductor. The electric field outside the conductor must be perpendicular to the surface and therefore parallel to the wall of the cylinder and perpendicular to the end of the cylinder that is outside the conductor. Thus, the flux through the Gaussian surface is EA. The enclosed charge is given by σA, so Gauss's Law becomes

$$\oiint \vec{E} \bullet d\vec{A} = EA = \frac{\sigma A}{\varepsilon_0}$$

Thus, the magnitude of the electric field just outside the surface of a flat charged conductor is

$$E = \frac{\sigma}{\varepsilon_0} \tag{2.19}$$

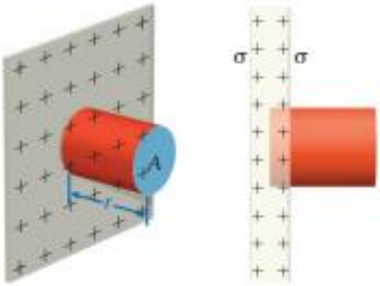


FIGURE 2.35 Infinite conducting plane with charge density σ on each surface and a Gaussian surface in the form of a right cylinder embedded in one side.

EXAMPLE 3.1 Energy Gain of a Proton

A proton is placed between two parallel conducting plates in a vacuum (Figure 3.6). The difference in electric potential between the two plates is 450 V. The proton is released from rest close to the positive plate.

PROBLEM

What is the kinetic energy of the proton when it reaches the negative plate?

SOLUTION

The difference in electric potential, ΔV, between the two plates is 450 V. We can relate this potential difference across the two plates to the change in electric potential energy, ΔU, of the proton using equation 3.7:

$$\Delta V = \frac{\Delta U}{q}$$

Because of the conservation of total energy, all the electric potential energy lost by the proton in crossing between the two plates is turned into kinetic energy due to the motion of the proton. We apply the law of conservation of energy, $\Delta K + \Delta U = 0$, where ΔU is the change in the proton's electric potential energy:

$$\Delta K = -\Delta U = -q\Delta V$$

Because the proton started from rest, we can express its final kinetic energy as $K = -q\Delta V$. Therefore, the kinetic energy of the proton after crossing the gap between the two plates is

$$K = -\left(1.602 \times 10^{-19}\ \text{C}\right)(-450\ \text{V}) = 7.21 \times 10^{-17}\ \text{J}$$

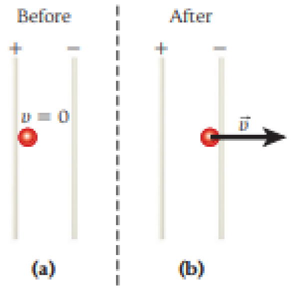


FIGURE 3.6 A proton between two charged parallel conducting plates in a vacuum. (a) The proton is released from rest. (b) The proton has moved from the positive plate to the negative plate, gaining kinetic energy.

Concept Check 3.1

An electron is positioned and then released on the *x*-axis, where the electric potential has the value −20 V. Which of the following statements describes the subsequent motion of the electron?

a) The electron will move to the left (negative *x*-direction) because it is negatively charged.

b) The electron will move to the right (positive *x*-direction) because it is negatively charged.

c) The electron will move to the left (negative *x*-direction) because the electric potential is negative.

d) The electron will move to the right (positive *x*-direction) because the electric potential is negative.

e) Not enough information is given to predict the motion of the electron.

Concept Check 3.7

Suppose an electric potential is described by $V(x, y, z) = -(5x^2 + y + z)$ in volts. Which of the following expressions describes the associated electric field, in units of volts per meter?

a) $\vec{E} = 5\hat{x} + 2\hat{y} + 2\hat{z}$

b) $\vec{E} = 10x\hat{x}$

c) $\vec{E} = 5x\hat{x} + 2\hat{y}$

d) $\vec{E} = 10x\hat{x} + \hat{y} + \hat{z}$

e) $\vec{E} = 0$

3.6 Electric Potential Energy of a System of Point Charges

Section 3.1 discussed the electric potential energy of a point charge in a given external electric field, and Section 3.4 described how to calculate the electric potential due to a system of point charges. This section combines these two pieces of information to find the electric potential energy of a system of point charges. Consider a system of charges that are infinitely far apart. To bring these charges into proximity with each other, work must be done on the charges, which changes the electric potential energy of the system. The electric potential energy of a system of point charges is defined as the work required to bring the charges together from being infinitely far apart.

As an example, let's find the electric potential energy of a system of two point charges (Figure 3.30). Assume that the two charges start at an infinite separation. We then bring point charge q_1 into the system. Because the system without charges has no electric field and no corresponding electric force, this action does not require that any work be done on the charge. Keeping this charge stationary, we bring the second point charge, q_2, from infinity to a distance r from q_1. Using equation 3.6, we can write the electric potential energy of the system as

$$U = q_2 V \tag{3.17}$$

where

$$V = \frac{kq_1}{r} \tag{3.18}$$

Thus, the electric potential energy of this system of two point charges is

$$U = \frac{kq_1 q_2}{r} \tag{3.19}$$

From the work-energy theorem, the work, W, that must be done on the particles to bring them together and keep them stationary is equal to U. If the two charges have the same sign, $W = U > 0$, positive work must be done to bring them together from infinity and keep them motionless. If the two charges have opposite signs, negative work must be done to bring them together from infinity and hold them motionless. To determine U for more than two point charges, we assemble them from infinity one charge at a time, in any order.

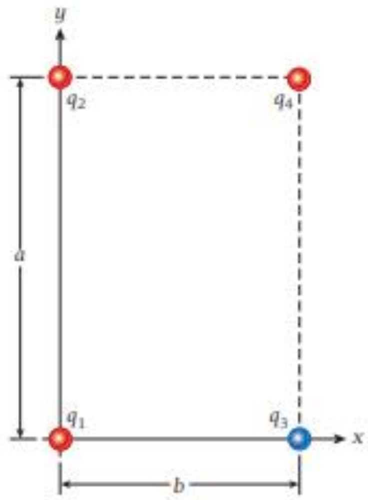


FIGURE 3.31 Calculating the potential energy of a system of four point charges.

EXAMPLE 3.7 Four Point Charges

Let's calculate $\mu$the electric potential energy of a system of four point charges, shown in Figure 3.31. The four point charges have the values $q_1 = +1.0\ \mu\text{C}$, $q_2 = +2.0\ \mu\text{C}$, $q_3 = -3.0\ \mu\text{C}$, and $q_4 = +4.0\ \mu\text{C}$. The charges are placed with $a = 6.0$ m and $b = 4.0$ m.

PROBLEM

What is the electric potential energy of this system of four point charges?

SOLUTION

We begin the calculation with the four charges infinitely far apart and assume that the electric potential energy is zero in that configuration. We bring in q_1 and position that charge at (0,0). This action does not change the electric potential energy of the system. Now we bring in q_2 and place that charge at (0,a). The electric potential energy of the system is now

$$U = \frac{kq_1q_2}{a}$$

Bringing q_3 in from an infinite distance and placing it at (b,0) changes the potential energy of the system through the interaction of q_3 with q_1 and the interaction of q_3 with q_2. The new potential energy is

$$U = \frac{kq_1q_2}{a} + \frac{kq_1q_3}{b} + \frac{kq_2q_3}{\sqrt{a^2 + b^2}}$$

Finally, bringing in q_4 and placing it at (b,a) changes the potential energy of the system through interactions with q_1, q_2, and q_3, bringing the total electric potential energy of the system to

$$U = \frac{kq_1q_2}{a} + \frac{kq_1q_3}{b} + \frac{kq_2q_3}{\sqrt{a^2 + b^2}} + \frac{kq_1q_4}{\sqrt{a^2 + b^2}} + \frac{kq_2q_4}{b} + \frac{kq_3q_4}{a}$$

Note that the order in which the charges are brought from infinity will not change this result. (You can try a different order to verify this statement.) Putting in the numerical values, we obtain

$$U = \left(3.0 \times 10^{-3}\ \text{J}\right) + \left(-6.7 \times 10^{-3}\ \text{J}\right) + \left(-7.5 \times 10^{-3}\ \text{J}\right) +$$
$$\left(5.0 \times 10^{-3}\ \text{J}\right) + \left(1.8 \times 10^{-2}\ \text{J}\right) + \left(-1.8 \times 10^{-2}\ \text{J}\right) = -6.2 \times 10^{-3}\ \text{J}$$

From the calculation in Example 3.7, we extrapolate the result to obtain a formula for the electric potential energy of a collection of point charges:

$$U = k \sum_{ij(\text{pairings})} \frac{q_i q_j}{r_{ij}} \tag{3.20}$$

where i and j label each pair of charges, the summation is over each pair ij (for all $i \neq j$), and r_{ij} is the distance between the charges in each pair. An alternative way to write this double sum is

$$U = \tfrac{1}{2}k \sum_{j=1}^{n} \sum_{i=1, i \neq j}^{n} \frac{q_i q_j}{\left|\vec{r}_i - \vec{r}_j\right|}$$

which is more explicit than the equivalent formulation of equation 3.20.

EXAMPLE 4.1 Area of a Parallel Plate Capacitor

A parallel plate capacitor has plates that are separated by 1.00 mm (Figure 4.11).

FIGURE 4.11 A parallel plate capacitor with plates separated by 1.00 mm.

PROBLEM

What is the area required to give this capacitor a capacitance of 1.00 F?

SOLUTION

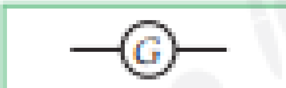
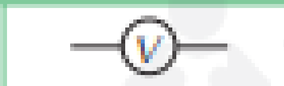
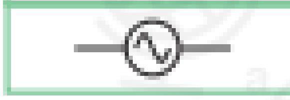
The capacitance is given by

$$C = \frac{\varepsilon_0 A}{d}. \quad \text{(i)}$$

Solving equation (i) for the area and putting in $d = 1.00 \times 10^{-3}$ m and $C = 1.00$ F, we get

$$A = \frac{dC}{\varepsilon_0} = \frac{(1.00 \times 10^{-3}\ \text{m})(1.00\ \text{F})}{(8.85 \times 10^{-12}\ \text{F/m})} = 1.13 \times 10^{8}\ \text{m}^2.$$

If these plates were square, each one would be 10.6 km by 10.6 km!
This result emphasizes that a farad is an extremely large amount of capacitance.

	Wire		Galvanometer
	Capacitor		Voltmeter
	Resistor		Ammeter
	Inductor		Battery
	Switch		AC source

https://t.me/h11Ad36

Writing :

•**1.82** In the figure, the net electrostatic force on charge Q_A is zero. If $Q_A = +1.00$ nC, determine the magnitude of Q_0.

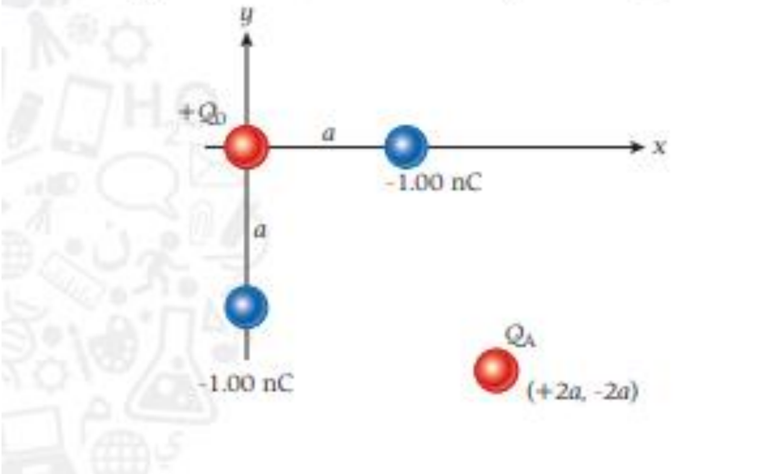


Figure 2.24 shows a nonuniform electric field, $\vec{E}$ passing through a differential area element, $d\vec{A}$ A portion of the closed surface is also shown. The angle between the electric field and the differential area element is θ.

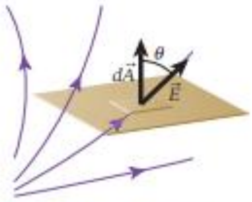


FIGURE 2.24 A nonuniform electric field, $\vec{E}$, passing through a differential area, $d\vec{A}$

EXAMPLE 2.5 Electric Flux through a Cube

Figure 2.25 shows a cube that has faces of area A in a uniform electric field, $\vec{E}$ that is perpendicular to the plane of one face of the cube.

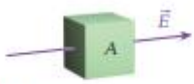


FIGURE 2.25 A cube with faces of area A in a uniform electric field, $\vec{E}$

PROBLEM

What is the net electric flux passing though the cube?

SOLUTION

The electric field in Figure 2.25 is perpendicular to the plane of one of the cube's six faces and therefore is also perpendicular to the opposite face. The area vectors of these two faces, $\vec{A}_1$ and $\vec{A}_2$ are shown in Figure 2.26a. The net electric flux passing through these two faces is

$$\Phi_{12} = \Phi_1 + \Phi_2 = \vec{E} \bullet \vec{A}_1 + \vec{E} \bullet \vec{A}_2 = -EA_1 + EA_2 = 0$$

The negative sign arises for the flux through face 1 because the electric field and the area vector, $\vec{A}_1$ are in opposite directions. The area vectors of the remaining four faces are all perpendicular to the electric field, as shown in Figure 2.26b. The net electric flux passing through these four faces is

$$\Phi_{3456} = \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6 = \vec{E} \bullet \vec{A}_3 + \vec{E} \bullet \vec{A}_4 + \vec{E} \bullet \vec{A}_5 + \vec{E} \bullet \vec{A}_6 = 0$$

All the scalar products are zero because the area vectors of these four faces are perpendicular to the electric field. Thus, the net electric flux passing through the cube is

$$\Phi = \Phi_{12} + \Phi_{3456} = 0$$

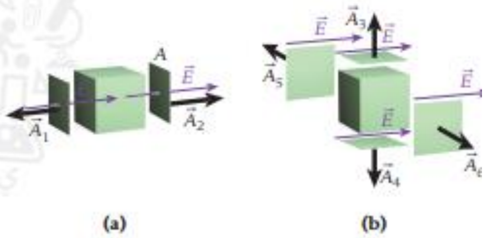


FIGURE 2.26 (a) The two faces of the cube that are perpendicular to the electric field. The area vectors are parallel and antiparallel to the electric field. (b) The four faces of the cube that are parallel to the electric field. The area vectors are perpendicular to the electric field.

FIGURE 2.28 Imaginary empty box in a uniform electric field.

In analogy with flowing water, the electric field lines seem to be flowing out of the box containing positive charge and into the box containing negative charge.

Now let's imagine an empty box in a uniform electric field (Figure 2.28). If a positive test charge is brought close to side 1, it experiences an inward force. If the charge is close to side 2, it experiences an outward force. The electric field is parallel to the other four sides, so the positive test charge does not experience any inward or outward force when brought close to those sides. Thus, in analogy with flowing water, the net amount of electric field that seems to be flowing in and out of the box is zero.

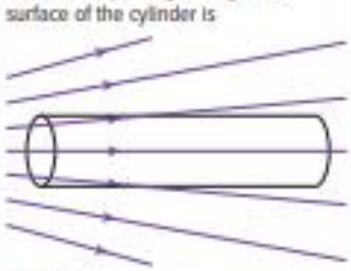
Whenever a charge is inside the box, the electric field lines seem to be flowing in or out of the box. When there is no charge in the box, the net flow of electric field lines in or out of the box is zero. These observations and the definition of electric flux, which quantifies the concept of the flow of the electric field lines, lead to **Gauss's Law:**

$$\Phi = \frac{q}{\varepsilon_0} \tag{2.15}$$

Here q is the net charge inside a closed surface, called a **Gaussian surface.** The closed surface could be a box like that we have been discussing or any arbitrarily shaped closed surface. Usually, the shape of the Gaussian surface is chosen so as to reflect the symmetries of the problem situation.

Concept Check 2.8

A cylinder made of an insulating material is placed in an electric field as shown in the figure. The net electric flux passing through the surface of the cylinder is



a) positive.

b) negative.

c) zero.

Concept Check 2.9

The lines in the figure are electric field lines, and the circle is a Gaussian surface. For which case(s) is (are) the total electric flux nonzero?

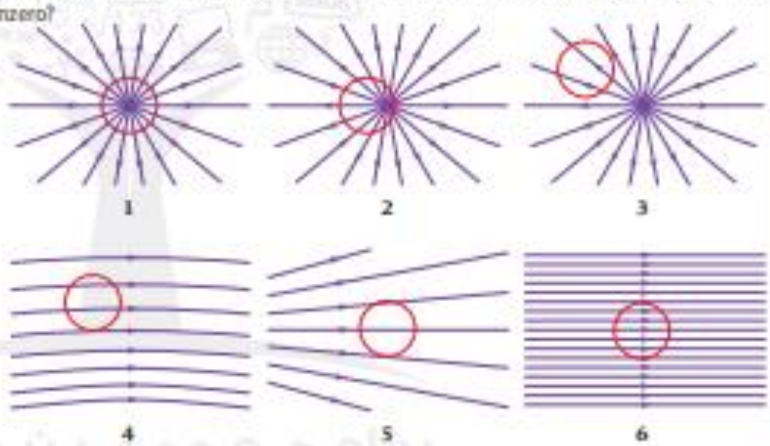
a) 1 only

b) 2 only

c) 4, 5, and 6

d) 6 only

e) 1 and 2



An alternative formulation of Gauss's Law incorporates the definition of the electric flux (equation 2.14):

$$\oiint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} \tag{2.16}$$

According to equation 2.16, Gauss's Law states that the surface integral of the electric field components perpendicular to the area times the area is proportional to the net charge within the closed surface. This expression may look daunting, but it simplifies considerably in many cases and allows us to perform very quickly calculations that would otherwise be quite complicated.

Gauss's Law and Coulomb's Law

We can derive Gauss's Law from Coulomb's Law. To do this, we start with a positive point charge, q. The electric field due to this charge is radial and pointing outward, as we saw in Section 2.3. According to Coulomb's Law (Section 1.5), the magnitude of the electric field from this charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

We now find the electric flux passing through a closed surface resulting from this point charge. For the Gaussian surface, we choose a spherical surface with radius r, with the charge at the center of the sphere, as shown in Figure 2.29. The electric field due to the positive point charge intersects each differential element of the surface of the Gaussian sphere perpendicularly. Therefore, at each point of this Gaussian surface, the electric field vector, $\vec{E}$ and the differential surface area vector, $d\vec{A}$ are parallel. The surface area vector will always point outward from the spherical Gaussian surface, but the electric field vector can point outward or inward depending on the sign of the charge. For a positive charge, the scalar product of the electric field and the surface area element is $\vec{E} \bullet d\vec{A} = E\,dA\cos 0^{\circ} = E\,dA$ The electric flux in this case, according to equation 2.14, is

$$\Phi = \oiint \vec{E} \bullet d\vec{A} = \oiint E\,dA$$

Because the electric field has the same magnitude anywhere in space at a distance r from the point charge q, we can take E outside the integral:

$$\Phi = \oiint E\,dA = E\oiint dA$$

Now what we have left to evaluate is the integral of the differential area over a spherical surface, which is given by $\oiint dA = 4\pi r^2$ Therefore, we have found from Coulomb's Law for the case of a point charge

$$\Phi = (E)\left(\oiint dA\right) = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)(4\pi r^2) = \frac{q}{\varepsilon_0}$$

which is the same as the expression for Gauss's Law in equation 2.15. We have shown that Gauss's Law can be derived from Coulomb's Law for a positive point charge, but it can also be shown that Gauss's Law holds for any distribution of charge inside a closed surface.

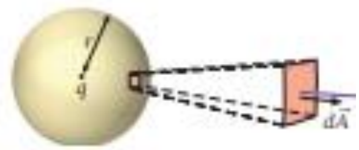


FIGURE 2.29 A spherical Gaussian surface with radius r surrounding a charge q. A closeup view of a differential surface element with area dA is shown.

Self-Test Opportunity 2.4

What changes in the preceding derivation of Gauss's Law if a negative point charge is used?

Shielding

Two important consequences of Gauss's Law are evident:

1. The electrostatic field inside any isolated conductor is always zero.
2. Cavities inside conductors are shielded from electric fields.

To examine these consequences, let's suppose a net electric field exists at some moment at some point inside an isolated conductor; see Figure 2.30a. But every conductor has free electrons inside it (blue circles in Figure 2.30b), which can move rapidly in response to any net external electric field, leaving behind positively charged ions (red circles in Figure 2.30b). The charges will move to the outer surface of the conductor, leaving no net accumulation of charge inside the volume of the conductor. These charges will in turn create an electric field inside the conductor (yellow arrows in Figure 2.30b), and they will move around until the electric field produced by them exactly cancels the external electric field. The net electric field thus becomes zero everywhere inside the conductor (Figure 2.30c).

If a cavity is scooped out of a conducting body, the net charge and thus the electric field inside this cavity is always zero, no matter how strongly the conductor is charged or how strong an external electric field acts on it. To prove this, we assume a closed Gaussian surface surrounds the cavity, completely inside the conductor. From the preceding discussion (see Figure 2.30), we know that at each point of this surface, the field is zero. Therefore, the net flux over this surface is also zero. By Gauss's Law, it then follows that this surface encloses zero net charge. If there were equal amounts of positive and negative charge on the cavity surface (and thus no net charge), this charge would not be stationary, as the positive and negative charges would be attracted to each other and would be free to move around the cavity surface to cancel each other. Therefore, any cavity inside a conductor is totally shielded from any external electric field. This effect is sometimes called **electrostatic shielding.**

A convincing demonstration of this shielding is provided by placing a plastic container filled with Styrofoam peanuts on top of a Van de Graaff generator, which serves as the source of strong electric field (Figure 2.31a). Charging the generator results in a large net charge accumulation on the dome, producing a strong electric field in the vicinity. Because

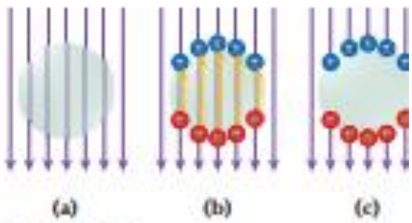
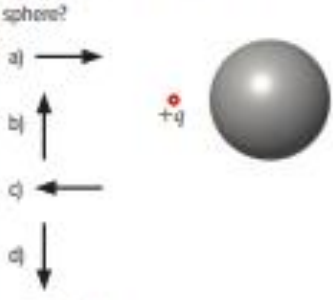


FIGURE 2.30 Shielding of an external electric field (purple vertical arrows) from the inside of a conductor.

Concept Check 2.10

A hollow, conducting sphere is initially given an evenly distributed negative charge. A positive charge $+q$ is brought near the sphere and placed at rest as shown in the figure. What is the direction of the electric field inside the hollow sphere?



e) The field is zero.

Two Point Charges of Opposite Sign

We can use the superposition principle to determine the electric field from two point charges. Figure 2.7 shows the electric field lines for two oppositely charged point charges with the same magnitude. At each point in the plane, the electric field from the positive charge and the electric field from the negative charge add as vectors to give the magnitude and the direction of the resulting electric field. (Figure 2.5 shows the same field lines in three dimensions.)

As noted earlier, the electric field lines originate on the positive charge and terminate on the negative charge. At a point very close to either charge, the field lines are similar to those for a single point charge, since the effect of the more distant charge is small. Near the charges, the electric field lines are close together, indicating that the field is stronger in those regions. The fact that the field lines between the two charges connect indicates that an attractive force exists between the two charges.

Two Point Charges with the Same Sign

We can also apply the principle of superposition to two point charges with the same sign. Figure 2.8 shows the electric field lines for two point charges with the same sign and same magnitude. If both charges are positive (as in Figure 2.8), the electric field lines originate at the charges and terminate at infinity. If both charges are negative, the field lines originate at infinity and terminate at the charges. For two charges of the same sign, the field lines do not

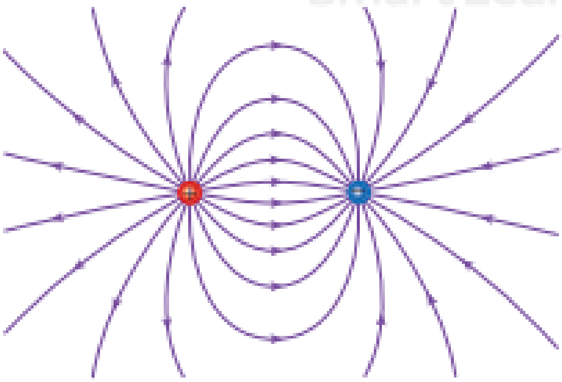


FIGURE 2.7 Electric field lines from two oppositely charged point charges. Each charge has the same magnitude.

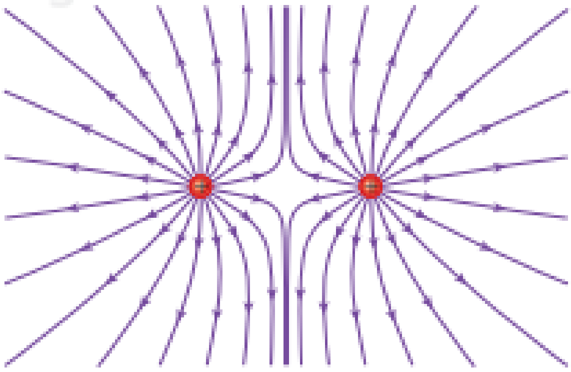
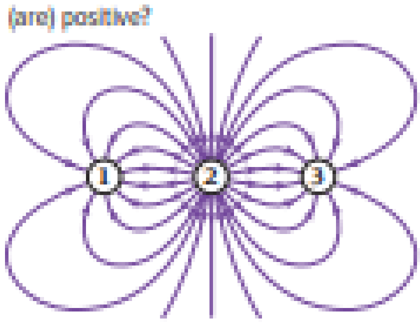


FIGURE 2.8 Electric field lines from two positive point charges with the same magnitude.

Concept Check 2.1

Which of the charges in the figure is (are) positive?



a) 1

b) 2

c) 3

d) 1 and 3

e) All three charges are positive.

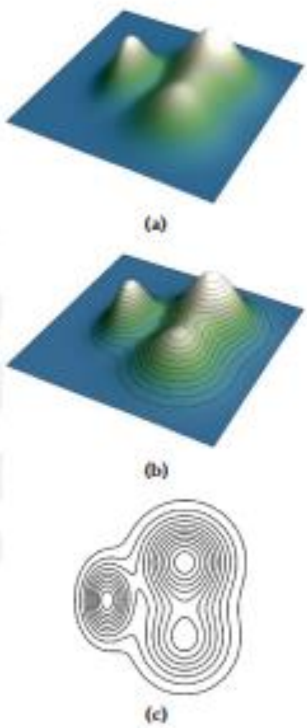


FIGURE 3.14 (a) Ski resort with three peaks; (b) the same peaks with lines of equal elevation superimposed; (c) the contour lines of equal elevation in a two-dimensional plot.

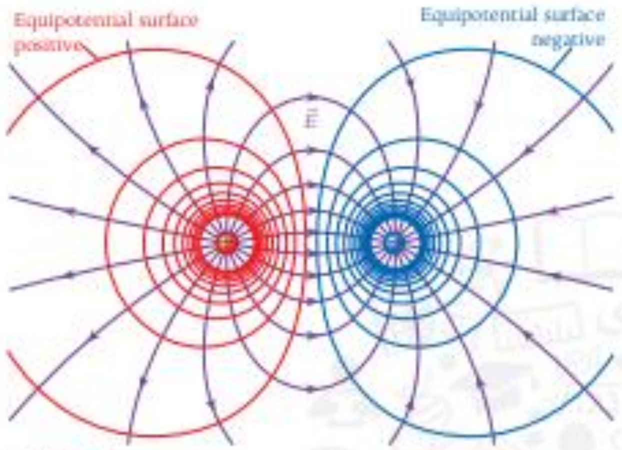


FIGURE 3.18 Equipotential surfaces created by point charges of the same magnitude but opposite sign. The red lines represent positive potential, and the blue lines represent negative potential. The purple lines with the arrowheads represent the electric field.

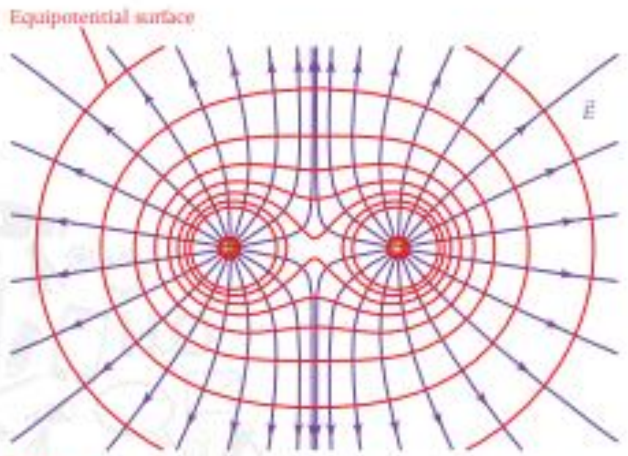


FIGURE 3.19 Equipotential surfaces (red lines) from two identical positive point charges. The purple lines with the arrowheads represent the electric field.

plane of the page cuts through equipotential spheres.) The values of the potential difference between neighboring equipotential lines are equal, producing equipotential lines that are close together near the charge and more widely spaced away from the charge. Note again that the equipotential lines are always perpendicular to the electric field lines. Equipotential surfaces do not have arrows like the field lines, because the potential is a scalar.

Two Oppositely Charged Point Charges

Figure 3.18 shows the electric field lines from two oppositely charged point charges, along with equipotential surfaces depicted as equipotential lines. An electrostatic force would attract these two point charges toward each other, but this discussion assumes that the charges are fixed in space and cannot move. The electric field lines originate at the positive charge and terminate on the negative charge. Again, the equipotential lines are always perpendicular to the electric field lines. The red lines in this figure represent positive equipotential surfaces, and the blue lines represent negative equipotential surfaces. Positive charges produce positive potential, and negative charges produce negative potential (relative to the value of the potential at infinity). Close to each charge, the resultant electric field lines and the resultant equipotential lines resemble those for a single point charge. Away from the vicinity of each charge, the electric field and the electric potential are the sums of the fields and potentials due to the two charges. The electric fields add as vectors, while the electric potentials add as scalars. Thus, the electric field is defined at all points in space in terms of a magnitude and a direction, while the electric potential is defined solely by its value at a given point in space and has no direction associated with it.

Two Identical Point Charges

Figure 3.19 shows electric field lines and equipotential surfaces resulting from two identical positive point charges. These two charges experience a repulsive electrostatic force. Because both charges are positive, the equipotential surfaces represent positive potentials. Again, the electric field and electric potential result from the sums of the fields and potentials, respectively, due to the two charges.

To determine the electric potential from the electric field, we start with the definition of the work done on a particle with charge q by a force, $\vec{F}$ over a displacement, $d\vec{s}$

$$dW = \vec{F} \bullet d\vec{s}$$

In this case, the force is given by $\vec{F} = q\vec{E}$ so

$$dW = q\vec{E} \bullet d\vec{s} \tag{3.10}$$

Integration of equation 3.10 as the particle moves in the electric field from some initial point to some final point gives

$$W = W_e = \int_i^f q\vec{E} \bullet d\vec{s} = q\int_i^f \vec{E} \bullet d\vec{s}$$

Using equation 3.8 to relate the work done to the change in electric potential, we get

$$\Delta V = V_f - V_i = -\frac{W_e}{q} = -\int_i^f \vec{E} \bullet d\vec{s}$$

As mentioned earlier, the usual convention is to set the electric potential to zero at infinity. With this convention, we can express the potential at some poin $\vec{r}$ in space as

$$V(\vec{r}) - V(\infty) \equiv V(\vec{r}) = -\int_\infty^{\vec{r}} \vec{E} \bullet d\vec{s} \tag{3.11}$$

Concept Check 3.3

In the figure, the lines represent equipotential lines. A charged object is moved from point P to point Q. How does the amount of work done on the object compare for these three cases?

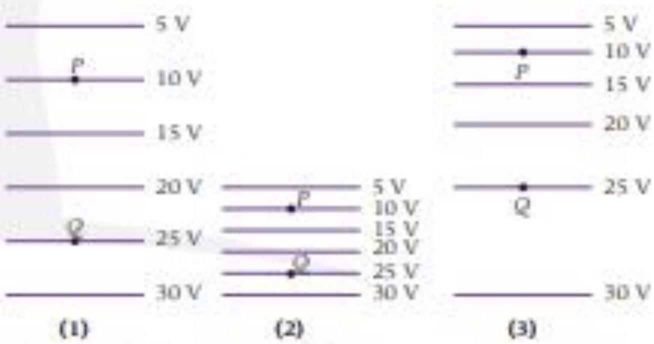
a) All three cases involve the same work.

b) The most work is done in case 1.

c) The most work is done in case 2.

d) The most work is done in case 3.

e) Cases 1 and 3 involve the same amount of work, which is more than is involved in case 2.



Point Charge

Let's use equation 3.11 to determine the electric potential due to a point charge, q. The electric field due to a point charge, q (for now, taken as positive), at a distance r from the charge is given by

$$E = \frac{kq}{r^2}$$

The direction of the electric field is radial from the point charge. Assume that the integration is carried out along a radial line from infinity to a point at a distance R from the point charge, such that $\vec{E} \bullet d\vec{s} = E dr$ Then we can use equation 3.11 to obtain

$$V(R) = -\int_\infty^R \vec{E} \bullet d\vec{s} = -\int_\infty^R \frac{kq}{r^2} dr = \left[\frac{kq}{r}\right]_\infty^R = \frac{kq}{R}$$

Thus, the electric potential due to a point charge at a distance r from the charge is given by

$$V = \frac{kq}{r} \tag{3.12}$$

SOLVED PROBLEM 3.2 Fixed and Moving Positive Charges

PROBLEM

A positive charge of 4.50 μC is fixed in place. A particle of mass 6.00 g and charge +3.00 μC is fired with an initial speed of 66.0 m/s directly toward the fixed charge from a distance of 4.20 cm away. How close does the moving charge get to the fixed charge before it comes to rest and starts moving away from the fixed charge?

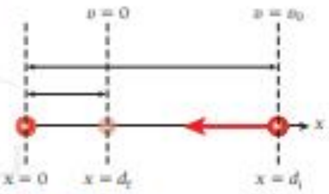


FIGURE 3.21 Two positive charges. One charge is fixed in place at $x = 0$, and the second charge begins moving with velocity $\vec{v}_0$ at $x = d_i$ and has zero velocity at $x = d_f$.

SOLUTION

THINK The moving charge will gain electric potential energy as it nears the fixed charge. The negative of the change in potential energy of the moving charge is equal to the change in kinetic energy of the moving charge because $\Delta K + \Delta U = 0$.

SKETCH We set the location of the fixed charge at $x = 0$, as shown in Figure 3.21. The moving charge starts at $x = d_i$, moves with initial speed $v = v_0$, and comes to rest at $x = d_f$.

RESEARCH The moving charge gains electric potential energy as it approaches the fixed charge and loses kinetic energy until it stops. At that point, all the original kinetic energy of the moving charge has been converted to electric potential energy. Using energy conservation, we can write this relationship as

$$\Delta K + \Delta U = 0 \Rightarrow \Delta K = -\Delta U \Rightarrow$$

$$0 - \tfrac{1}{2}mv_0^2 = -q_{\text{moving}}\Delta V \Rightarrow$$

$$\tfrac{1}{2}mv_0^2 = q_{\text{moving}}\Delta V \qquad \text{(i)}$$

The electric potential experienced by the moving charge is due to the fixed charge, so we can write the change in potential as

$$\Delta V = V_f - V_i = k\frac{q_{\text{fixed}}}{d_f} - k\frac{q_{\text{fixed}}}{d_i} = kq_{\text{fixed}}\left(\frac{1}{d_f} - \frac{1}{d_i}\right) \qquad \text{(ii)}$$

Continued

SIMPLIFY Substituting the expression for the potential difference from equation (ii) into equation (i), we find

$$\tfrac{1}{2}mv_0^2 = q_{\text{moving}}\Delta V = kq_{\text{moving}}q_{\text{fixed}}\left(\frac{1}{d_f} - \frac{1}{d_i}\right) \Rightarrow$$

$$\frac{1}{d_f} - \frac{1}{d_i} = \frac{mv_0^2}{2kq_{\text{moving}}q_{\text{fixed}}} \Rightarrow$$

$$\frac{1}{d_f} = \frac{1}{d_i} + \frac{mv_0^2}{2kq_{\text{moving}}q_{\text{fixed}}}$$

CALCULATE Putting in the numerical values, we get

$$\frac{1}{d_f} = \frac{1}{0.0420\text{ m}} + \frac{(0.00600\text{ kg})(66.0\text{ m/s})^2}{2(8.99\times10^9\text{ N m}^2/\text{C}^2)(3.00\times10^{-6}\text{ C})(4.50\times10^{-6}\text{ C})} = 131.485$$

or

$$d_f = 0.00760545\text{ m}$$

ROUND We report our result to three significant figures:

$$d_f = 0.00761\text{ m} = 0.761\text{ cm}$$

DOUBLE-CHECK The final distance of 0.761 cm is less than the initial distance of 4.20 cm. At the final distance, the electric potential energy of the moving charge is

$$U = q_{\text{moving}}V = q_{\text{moving}}\left(k\frac{q_{\text{fixed}}}{d_f}\right) = k\frac{q_{\text{moving}}q_{\text{fixed}}}{d_f}$$

$$= (8.99\times10^9\text{ N m}^2/\text{C}^2)\frac{(3.00\times10^{-6}\text{ C})(4.50\times10^{-6}\text{ C})}{0.00761\text{ m}} = 16.0\text{ J}$$

The electric potential energy at the initial distance is

$$U = q_{\text{moving}}V = q_{\text{moving}}\left(k\frac{q_{\text{fixed}}}{d_i}\right) = k\frac{q_{\text{moving}}q_{\text{fixed}}}{d_i}$$

$$= (8.99\times10^9\text{ N m}^2/\text{C}^2)\frac{(3.00\times10^{-6}\text{ C})(4.50\times10^{-6}\text{ C})}{(0.0420\text{ m})} = 2.9\text{ J}$$

The initial kinetic energy is

$$K = \tfrac{1}{2}mv^2 = \frac{(0.00600\text{ kg})(66.0\text{ m/s})^2}{2} = 13.1\text{ J}$$

We can see that the equation based on energy conservation, from which the solution process started, is satisfied:

$$\tfrac{1}{2}mv^2 = \Delta U$$

$$13.1\text{ J} = 16.0\text{ J} - 2.9\text{ J} = 13.1\text{ J}$$

This gives us confidence that our result for the final distance is correct.

3.5 Finding the Electric Field from the Electric Potential

As we mentioned earlier, we can determine the electric field starting with the electric potential. This calculation uses equations 3.8 and 3.10:

$$-q dV = q\vec{E} \bullet d\vec{s}$$

where $d\vec{s}$ is a vector from an initial point to a final point located a small (infinitesimal) distance away. The component of the electric field, E_s, along the direction of $d\vec{s}$ is given by the partial derivative

$$E_s = -\frac{\partial V}{\partial s} \quad (3.14)$$

Thus, we can find any component of the electric field by taking the partial derivative of the potential along the direction of that component. We can then write the components of the electric field in terms of partial derivatives of the potential:

$$E_x = -\frac{\partial V}{\partial x}; \quad E_y = -\frac{\partial V}{\partial y}; \quad E_z = -\frac{\partial V}{\partial z} \quad (3.15)$$

The equivalent vector calculus formulation is $\vec{E} = -\vec{\nabla}V = -(\partial V/\partial x, \partial V/\partial y, \partial V/\partial z)$, where the operator $\vec{\nabla}$ is called the **gradient.** Thus, the electric field can be determined either graphically, by measuring the negative of the change of the potential per unit distance perpendicular to an equipotential line, or analytically, by using equation 3.15.

To visually reinforce the concepts of electric fields and potentials, the following example shows how a graphical technique can be used to find the field given the potential.

Concept Check 3.7

Suppose an electric potential is described by $V(x, y, z) = -(5x^2 + y + z)$ in volts. Which of the following expressions describes the associated electric field, in units of volts per meter?

a) $\vec{E} = 5\hat{x} + 2\hat{y} + 2\hat{z}$

b) $\vec{E} = 10x\hat{x}$

c) $\vec{E} = 5x\hat{x} + 2\hat{y}$

d) $\vec{E} = 10x\hat{x} + \hat{y} + \hat{z}$

e) $\vec{E} = 0$

Concept Check 3.8

In the figure, the lines represent equipotential lines. How does the magnitude of the electric field, E, at point P compare for the three cases?

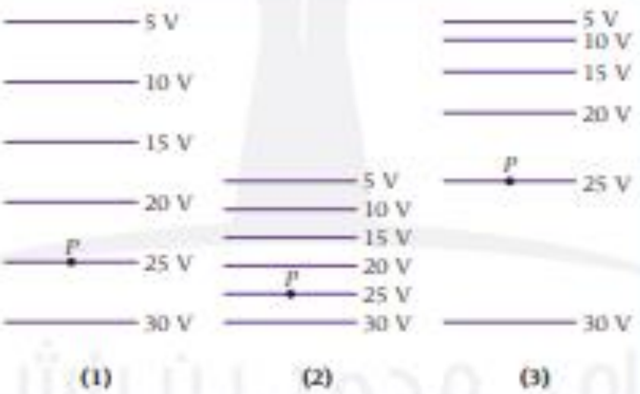
a) $E_1 = E_2 = E_3$

b) $E_1 > E_2 > E_3$

c) $E_1 < E_2 < E_3$

d) $E_3 > E_1 > E_2$

e) $E_2 < E_1 < E_3$



2.1 Definition of an Electric Field

You previously learned that the force between two or more point charges. When determining the net force exerted by other charges on a particular charge at some point in space, we obtain different directions for this force, depending on the sign of the charge that is the reference point. In addition, the net force is also proportional to the magnitude of the reference charge. The techniques used in Chapter 1 require us to redo the calculation for the net force each time we consider a different charge.

Dealing with this situation requires the concept of a **field,** which can be used to describe certain forces. An **electric field,** $E(r)$, is defined at any point in space, $\vec{r}$ as the net electric force on a charge, divided by that charge:

$$\vec{E}(\vec{r}) = \frac{\vec{F}(\vec{r})}{q} \qquad (2.1)$$

The units of the electric field are newtons per coulomb (N/C). This simple definition eliminates the cumbersome dependence of the electric force on the particular charge being used to measure the force. We can quickly determine the net force on any charge by using $\vec{F}(\vec{r}) = q\vec{E}(\vec{r})$ which is a trivial rearrangement of equation 2.1.

The electric force on a charge at a point is parallel or antiparallel, depending on the sign of the charge in question) to the electric field at that point and proportional to the magnitude of the charge. The magnitude of the force is given by $F = |q|E$ The direction of the force on a positive charge is along $\vec{E}(\vec{r})$ the direction of the force on a negative charge is in the direction opposite to $\vec{E}(\vec{r})$

If several sources of electric fields are present at the same time, such as several point charges, the electric field at any given point is determined by the superposition of the electric fields from all sources. This superposition follows directly from the superposition of forces introduced in our study of mechanics and discussed earlier for electrostatic forces. The **superposition principle** for the total electric field, $\vec{E}_t$ at any point in space with coordinate $\vec{r}$ due to n electric field sources can be stated as

$$\vec{E}_t(\vec{r}) = \vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r}) + \cdots + \vec{E}_n(\vec{r}) \qquad (2.2)$$

4.1 Capacitance

FIGURE 4.2 Some representative types of capacitors.

Figure 4.2 shows that capacitors come in a variety of sizes and shapes. In general, a **capacitor** consists of two separated conductors, which are usually called *plates* even if they are not simple planes. If we take apart one of these capacitors, we might find two sheets of metal foil separated by an insulating layer of Mylar, as shown in Figure 4.3. The sandwiched layers of metal foil and Mylar can be rolled up with another insulating layer into a compact form that does not resemble two parallel conductors, as shown in Figure 4.4. This technique produces capacitors with some of the physical formats shown in Figure 4.2. The insulating layer between the two metal foils plays a crucial role in the characteristics of the capacitor.

To study the properties of capacitors, we'll assume a convenient geometry and then generalize the results. Figure 4.5 shows a **parallel plate capacitor,** which consists of two parallel conducting plates, each with area A, separated by a distance, d, and assumed to be in a vacuum. The capacitor is charged by placing a charge of $+q$ on one plate and a charge of $-q$ on the other plate. (It is not necessary to put exactly opposite charges onto the two plates of the capacitor to charge it; any difference in charge will do. But, for practical purposes, the overall device should remain neutral, and this requires charges of equal magnitude and opposite sign on the two plates.) Because the plates are conductors, they are equipotential surfaces; thus, the electrons on the plates will distribute themselves uniformly over the surfaces.

FIGURE 4.3 Two sheets of metal foil separated by an insulating layer.

Let's apply the results obtained in Chapter 3 to determine the electric potential and electric field for the parallel plate capacitor. (In principle, we could do this by calculating the electric potential and electric field for continuous charge distributions. However, for this physical configuration we would need to use a computer to provide the solution.) Let's place the origin of the coordinate system in the middle between the two plates, with the x-axis aligned with the two plates. Figure 4.6 shows a three-dimensional plot of the electric potential, $V(x,y)$, in the xy-plane, similar to the plots in Chapter 3.

FIGURE 4.4 The metal foil and Mylar sandwich shown in Figure 4.3 can be rolled up with an insulating layer to produce a capacitor with a compact geometry.

The potential in Figure 4.6 has a very steep (and approximately linear) drop between the two plates and a more gradual drop outside the plates. This means that the electric field can be expected to be strongest between the plates and weaker outside. Figure 4.7a presents a contour plot of the electric potential shown in Figure 4.6 for the two parallel plates. Negative potential values are shaded in green, and positive values in pink. The equipotential lines, which are the lines where the three-dimensional equipotential surfaces intersect the xy-plane, displayed in Figure 4.6 are also shown in this plot, as are representations of the two plates. Note that the equipotential lines between the two plates are all parallel to each other and equally spaced.

In Figure 4.7b, the electric field lines have been added to the contour plot. The electric field is determined using $\vec{E}(\vec{r}) = -\vec{\nabla}V(\vec{r})$ introduced in Chapter 3. Far away from the two plates, the electric field looks very similar to that generated by a dipole composed of two point charges. It is easy to see that the electric field lines are perpendicular to the potential contour lines (which represent the equipotential surfaces!) everywhere in space.

But the electric field lines in Figure 4.7b do not convey adequate information about the magnitude of the electric field. Another representation of the electric field, in Figure 4.7c, displays the electric field vectors at regularly spaced grid points in the xy-plane. (The contour shading of the potential has been removed to reduce visual clutter.) In this plot, the field strength at each point of the grid is proportional to the size of the arrow at that point. You can clearly see that the electric field between the two plates is perpendicular to the plates and much larger in magnitude than the field outside the plates. The field in the space outside the plates is called the *fringe field*. If the plates are moved closer together, the electric field between the plates remains the same, while the fringe field is reduced.

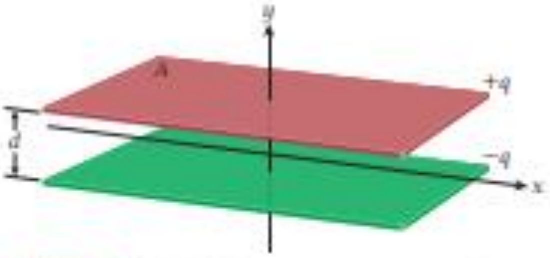


FIGURE 4.5 Parallel plate capacitor consisting of two conducting plates, each having area A, separated by a distance d.

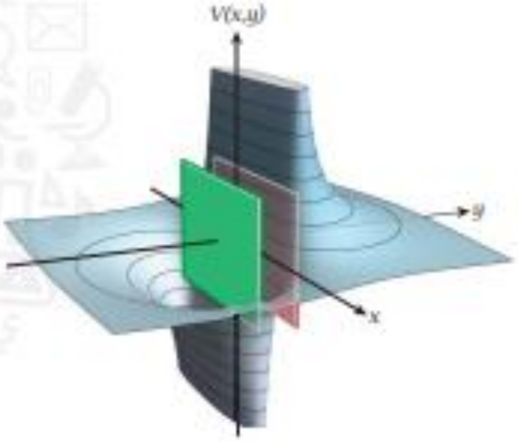


FIGURE 4.6 Electric potential in the xy-plane for the two oppositely charged parallel plates (superimposed) of Figure 4.5.

The potential difference, ΔV, between the two parallel plates of the capacitor is proportional to the amount of charge on the plates. The proportionality constant is the **capacitance,** C, of the device, defined as

$$C = \left| \frac{q}{\Delta V} \right|. \tag{4.1}$$

The capacitance of a device depends on the area of the plates and the distance between them but not on the charge or the potential difference. (This will be shown for this and other geometries in the following sections.) By definition, the capacitance is a positive number. It tells how much charge is required to produce a given potential difference between the plates. The larger the capacitance, the more charge is required to produce a given potential difference. (Note that it is a common practice to use V, not ΔV, to represent potential difference. Be sure you understand when V is being used for potential and when it is being used for potential difference.)

Equation 4.1, the definition of capacitance, can be rewritten in this commonly used form:

$$q = C\Delta V .$$

Equation 4.1 indicates that the units of capacitance are the units of charge divided by the units of potential, or coulombs per volt. A new unit was assigned to capacitance, named after British physicist Michael Faraday (1791–1867). This unit is called the **farad** (F):

$$1\ \text{F} = \frac{1\ \text{C}}{1\ \text{V}}. \tag{4.2}$$

One farad represents a very large capacitance. Typically, capacitors have a capacitance in the range from $1\ \mu\text{F} = 1 \times 10^{-6}$ F to $1\ \text{pF} = 1 \times 10^{-12}$ F.

With the definition of the farad, we can write the electric permittivity of free space, ε_0 (introduced in Chapter 1), as 8.85×10^{-12} F/m.

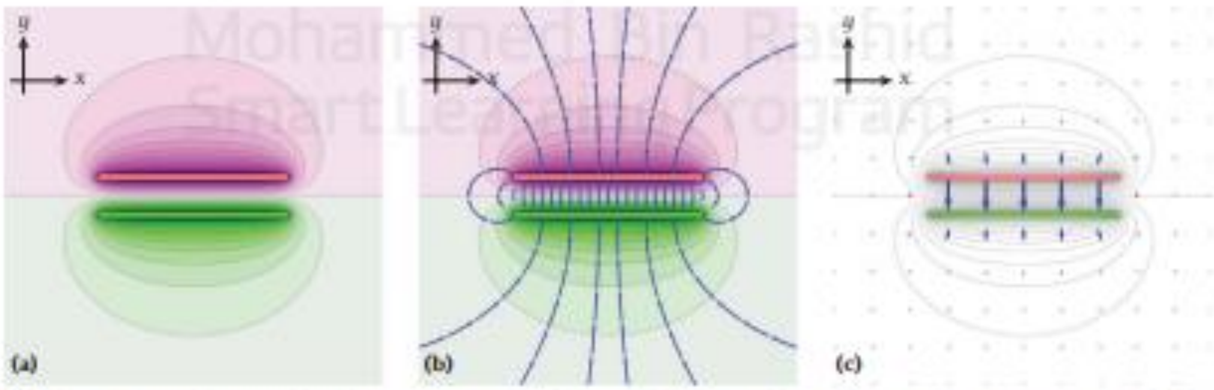


FIGURE 4.7 (a) Two-dimensional contour plot of the same potential as in Figure 4.6. (b) Contour plot with electric field lines superimposed. (c) Electric field strength at regularly spaced points in the xy-plane represented by the sizes of the arrows.