CHAPTER 2

Exponential Functions and Equations

Then

You graphed functions and transformations of functions.

Now

You will:

- Graph exponential functions.
- Solve exponential equations and inequalities.
- Solve problems involving exponential growth and decay.

Why?

SCIENCE Mathematics and science go hand in hand. Whether it is chemistry, biology, paleontology, zoology, or anthropology, you will need strong math skills. In this chapter, you will learn mathematical aspects of science such as computer viruses, populations of insects, bacteria growth, cell division, astronomy, tornados, and earthquakes.

Get Ready for the Chapter

Diagnose Readiness Take the Quick Check below to check your prerequisite skills. Refer to the Quick Review for help.

QuickCheck

Simplify. Assume that no variable equals zero.

1. $a^4a^3a^5$
2. $\left(2xy^3z^2\right)^3$
3. $\dfrac{-24x^8y^5z}{16x^2y^8z^6}$
4. $\left(\dfrac{-8r^2n}{36n^3t}\right)^2$
5. **DENSITY** The density of an object is equal to the mass divided by the volume. An object has a mass of 7.5×10^3 grams and a volume of 1.5×10^3 cubic centimeters. What is the density of the object?

QuickReview

Example 1

Simplify $\dfrac{(a^3bc^2)^2}{a^4a^2b^2bc^5c^3}$. Assume that no variable equals zero.

$$\frac{(a^3bc^2)^2}{a^4a^2b^2bc^5c^3}$$

$$= \frac{a^6b^2c^4}{a^6b^3c^8}$$ Simplify the numerator by using the Power of a Power Rule and the denominator by using the Product of Powers Rule.

$$= \frac{1}{bc^4} \text{ or } b^{-1}c^{-4}$$ Simplify by using the Quotient of Powers Rule.

QuickCheck

Find the inverse of each function. Then graph the function and its inverse.

6. $f(x) = 2x + 5$
7. $f(x) = x - 3$
8. $f(x) = -4x$
9. $f(x) = \frac{1}{4}x - 3$
10. $f(x) = \dfrac{x-1}{2}$
11. $y = \frac{1}{3}x + 4$

Determine whether each pair of functions are inverse functions.

12. $f(x) = x - 6$
    $g(x) = x + 6$
13. $f(x) = 2x + 5$
    $g(x) = 2x - 5$
14. **FOOD** A pizzeria charges AED 12 for a medium cheese pizza and AED 2 for each additional topping. If $f(x) = 2x + 12$ represents the cost of a medium pizza with x toppings, find $f^{-1}(x)$ and explain its meaning.

QuickReview

Example 2

Find the inverse of $f(x) = 3x - 1$.

Step 1 Replace $f(x)$ with y in the original equation:
$f(x) = 3x - 1 \rightarrow y = 3x - 1$.

Step 2 Interchange x and y: $x = 3y - 1$.

Step 3 Solve for y.

$x = 3y - 1$	Inverse
$x + 1 = 3y$	Add 1 to each side.
$\dfrac{x+1}{3} = y$	Divide each side by 3.
$\frac{1}{3}x + \frac{1}{3} = y$	Simplify.

Step 4 Replace y with $f^{-1}(x)$.
$y = \frac{1}{3}x + \frac{1}{3} \rightarrow f^{-1}(x) = \frac{1}{3}x + \frac{1}{3}$

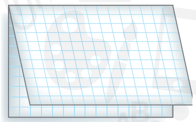
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study this chapter. To get ready, identify important terms and organize your resources.

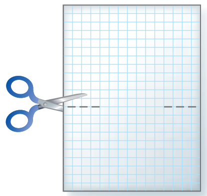
FOLDABLES StudyOrganizer

Exponential Functions and Equations
Make this Foldable to help you organize your notes about exponential functions. Begin with two sheets of grid paper.

1 **Fold** in half along the width.

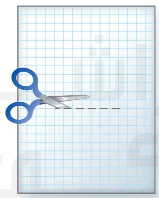


First Sheet

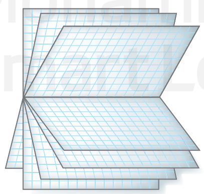


2 **On** the first sheet, cut 5 cm along the fold at the ends.

Second Sheet



3 **On** the second sheet, cut in the center, stopping 5 cm from the ends.



4 **Insert** the first sheet through the second sheet and align the folds. Label the pages with lesson numbers.

NewVocabulary

- exponential function
- exponential growth
- asymptote
- growth factor
- exponential decay
- decay factor
- exponential equation
- compound interest
- exponential inequality
- conjugate
- radical equations

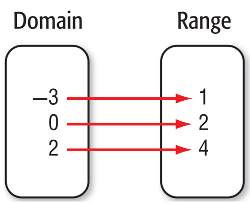
ReviewVocabulary

domain the set of all x-coordinates of the ordered pairs of a relation

function a relation in which each element of the domain is paired with exactly one element in the range

range the set of all y-coordinates of the ordered pairs of a relation

$\{(-3,1), (0, 2), (2, 4)\}$



LESSON 2-1 Exponential Functions

Then

- You evaluated numerical expressions involving exponents.

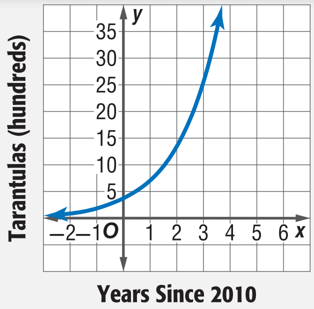
Now

1. Graph exponential functions.
2. Graph growth and decay functions.

Why?

- Tarantulas can appear scary with their large hairy bodies and legs, but they are harmless to humans. The graph shows a tarantula spider population that increases over time. Notice that the graph is not linear.

  The graph represents the function $y = 3(2)^x$. This is an example of an *exponential* function.



NewVocabulary

exponential function
exponential growth function
exponential decay function

Mathematical Practices
Make sense of problems and persevere in solving them.

1 Graph Exponential Functions

An **exponential function** is a function of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$. Notice that the base is a constant and the exponent is a variable. Exponential functions are nonlinear.

KeyConcept Exponential Function

Words An exponential function is a function that can be described by an equation of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

Examples $y = 2(3)^x$ $y = 4^x$ $y = \left(\frac{1}{2}\right)^x$

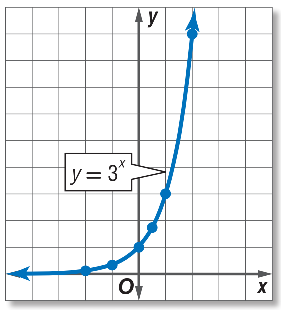
Example 1 Graph with $a > 0$ and $b > 1$

Graph $y = 3^x$. Find the y-intercept, and state the domain and range.

The graph crosses the y-axis at 1, so the y-intercept is 1. The domain is all real numbers, and the range is all positive real numbers.

Notice that the graph approaches the x-axis but there is no x-intercept. The graph is increasing on the entire domain.

x	3^x	y
-2	3^{-2}	$\frac{1}{9}$
-1	3^{-1}	$\frac{1}{3}$
0	3^0	1
$\frac{1}{2}$	$3^{\frac{1}{2}}$	≈ 1.73
1	3^1	3
2	3^2	9



GuidedPractice

1. Graph $y = 7^x$. Find the y-intercept, and state the domain and range.

Functions of the form $y = ab^x$, where $a > 0$ and $b > 1$, are called **exponential growth functions** and all have the same shape as the graph in Example 1. Functions of the form $y = ab^x$, where $a > 0$ and $0 < b < 1$ are called **exponential decay functions** and also have the same general shape.

StudyTip

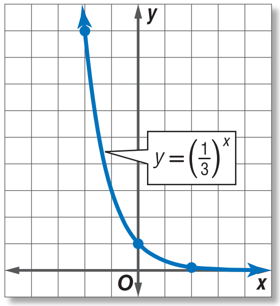
$a < 0$ If the value of a is less than 0, the graph will be reflected across the x-axis.

Example 2 Graph with $a > 0$ and $0 < b < 1$

Graph $y = \left(\frac{1}{3}\right)^x$. Find the y-intercept, and state the domain and range.

The y-intercept is 1. The domain is all real numbers, and the range is all positive real numbers. Notice that as x increases, the y-values decrease less rapidly.

x	$\left(\frac{1}{3}\right)^x$	y
-2	$\left(\frac{1}{3}\right)^{-2}$	9
0	$\left(\frac{1}{3}\right)^{0}$	1
2	$\left(\frac{1}{3}\right)^{2}$	$\frac{1}{9}$



GuidedPractice

2. Graph $y = \left(\frac{1}{2}\right)^x - 1$. Find the y-intercept, and state the domain and range.

The key features of the graphs of exponential functions can be summarized as follows.

KeyConcept Graphs of Exponential Functions

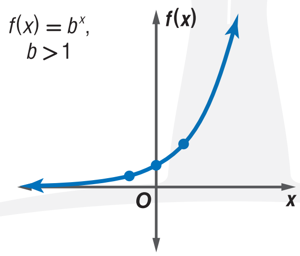
Exponential Growth Functions

Equation: $f(x) = ab^x$, $a > 0$, $b > 1$

Domain, Range: all reals; all positive reals

Intercepts: one y-intercept, no x-intercepts

End behavior: as x increases, $f(x)$ increases; as x decreases, $f(x)$ approaches 0



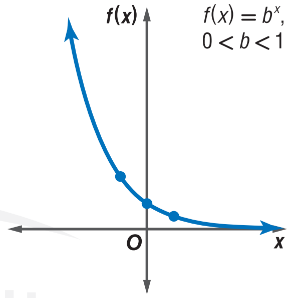
Exponential Decay Functions

Equation: $f(x) = ab^x$, $a > 0$, $0 < b < 1$

Domain, Range: all reals; all negative reals

Intercepts: one y-intercept, no x-intercepts

End behavior: as x increases, $f(x)$ approaches 0; as x decreases, $f(x)$ increases



Exponential functions occur in many real world situations.

Real-WorldLink

The United States is the largest soda consumer in the world. In a recent year, the United States accounted for one third of the world's total soda consumption.

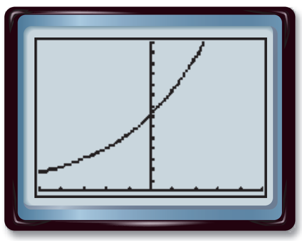
Source: Worldwatch Institute

Real-World Example 3 Use Exponential Functions to Solve Problems

SODA The function $C = 179(1.029)^t$ models the amount of soda consumed in the world, where C is the amount consumed in billions of liters and t is the number of years since 2000.

a. Graph the function. What values of C and t are meaningful in the context of the problem?

Since t represents time, $t > 0$. At $t = 0$, the consumption is 179 billion liters. Therefore, in the context of this problem, $C > 179$ is meaningful.



[−50, 50] scl: 10 by [0, 350] scl: 25

b. How much soda was consumed in 2005?

$C = 179(1.029)^t$ — Original equation

$= 179(1.029)^5$ — $t = 5$

≈ 206.5 — Use a calculator.

The world soda consumption in 2005 was approximately 206.5 billion liters.

GuidedPractice

3. **BIOLOGY** A certain bacteria population doubles every 20 minutes. Beginning with 10 cells in a culture, the population can be represented by the function $B = 10(2)^t$, where B is the number of bacteria cells and t is the time in 20 minute increments. How many will there be after 2 hours?

2 Identify Exponential Behavior

Recall from Lesson 3-3 that linear functions have a constant rate of change. Exponential functions do not have constant rates of change, but they do have constant ratios.

> **Problem-Solving Tip**
>
> **Make an Organized List** Making an organized list of x-values and corresponding y-values is helpful in graphing the function. It can also help you identify patterns in the data.

Example 4 Identify Exponential Behavior

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

x	0	5	10	15	20	25
y	64	32	16	8	4	2

Method 1 Look for a pattern.

The domain values are at regular intervals of 5. Look for a common factor among the range values.

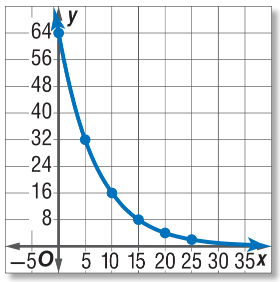
$$64 \xrightarrow{\times\frac{1}{2}} 32 \xrightarrow{\times\frac{1}{2}} 16 \xrightarrow{\times\frac{1}{2}} 8 \xrightarrow{\times\frac{1}{2}} 4 \xrightarrow{\times\frac{1}{2}} 2$$

The range values differ by the common factor of $\frac{1}{2}$.

Since the domain values are at regular intervals and the range values differ by a positive common factor, the data are probably exponential. Its equation may involve $\left(\frac{1}{2}\right)^x$.

Method 2 Graph the data.

Plot the points and connect them with a smooth curve. The graph shows a rapidly decreasing value of y as x increases. This is a characteristic of exponential behavior in which the base is between 0 and 1.



GuidedPractice

4. Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

x	0	3	6	9	12	15
y	12	16	20	24	28	32

The graph of $f(x) = b^x$ represents a parent graph of the exponential functions. The same techniques used to transform the graphs of other functions you have studied can be applied to the graphs of exponential functions.

KeyConcept Transformations of Exponential Functions

$$f(x) = ab^{x-h} + k$$

h – Horizontal Translation	k – Vertical Translation
h units right if h is positive	k units up if k is positive
$\lvert h \rvert$ units left if h is negative	$\lvert k \rvert$ units down if k is negative

a – Orientation and Shape

If $a < 0$, the graph is reflected in the x-axis.

If $\lvert a \rvert > 1$, the graph is stretched vertically.
If $0 < \lvert a \rvert < 1$, the graph is compressed vertically.

StudyTip

Precision Remember that end behavior is the action of the graph as x approaches positive infinity or negative infinity. In Example 2a, as x approaches infinity, y approaches infinity. In Example 2b, as x approaches infinity, y approaches negative infinity.

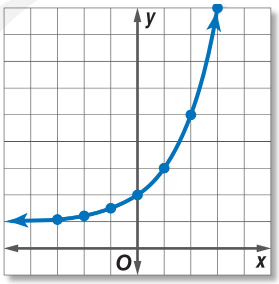
Example 2 Graph Transformations

Graph each function. State the domain and range.

a. $y = 2^x + 1$

The equation represents a translation of the graph of $y = 2^x$ one unit up.

x	$y = 2^x + 1$
-3	$2^{-3} + 1 = 1.125$
-2	$2^{-2} + 1 = 1.25$
-1	$2^{-1} + 1 = 1.5$
0	$2^0 + 1 = 2$
1	$2^1 + 1 = 3$
2	$2^2 + 1 = 5$
3	$2^3 + 1 = 9$



Domain = {all real numbers}; Range = $\{y \mid y > 1\}$

b. $y = -\frac{1}{2} \cdot 5^{x-2}$

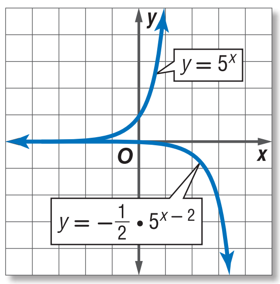
The equation represents a transformation of the graph of $y = 5^x$.

Graph $y = 5^x$ and transform the graph.

- $a = -\frac{1}{2}$: The graph is reflected in the x-axis and compressed vertically.
- $h = 2$: The graph is translated 2 units right.
- $k = 0$: The graph is not translated vertically.

Domain = {all real numbers}

Range = $\{y \mid y < 0\}$



GuidedPractice

2A. $y = 2^{x+3} - 5$

2B. $y = 0.1(6)^x - 3$

StudyTip

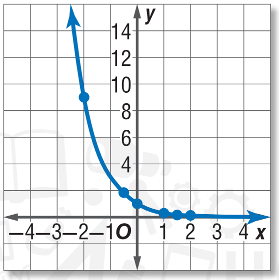
Exponential Decay Be sure not to confuse a dilation in which $|a| < 1$ with exponential decay in which $0 < b < 1$.

Example 4 Graph Exponential Decay Functions

Graph each function. State the domain and range.

a. $y = \left(\frac{1}{3}\right)^x$

x	$y = \left(\frac{1}{3}\right)^x$
-3	$\left(\frac{1}{3}\right)^{-3} = 27$
-2	$\left(\frac{1}{3}\right)^{-2} = 9$
$-\frac{1}{2}$	$\left(\frac{1}{3}\right)^{-\frac{1}{2}} = \sqrt{3}$
0	$\left(\frac{1}{3}\right)^{0} = 1$
1	$\left(\frac{1}{3}\right)^{1} = \frac{1}{3}$
$\frac{3}{2}$	$\left(\frac{1}{3}\right)^{\frac{3}{2}} = \sqrt{\frac{1}{27}}$
2	$\left(\frac{1}{3}\right)^{2} = \frac{1}{9}$



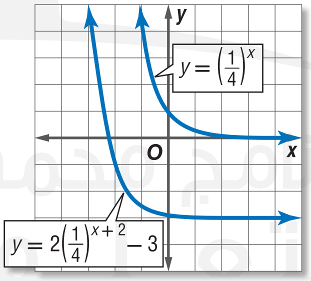
The domain is all real numbers, and the range is all positive real numbers.

b. $y = 2\left(\frac{1}{4}\right)^{x+2} - 3$

The equation represents a transformation of the graph of $y = \left(\frac{1}{4}\right)^x$.

Examine each parameter.

- $a = 2$: The graph is stretched vertically.
- $h = -2$: The graph is translated 2 units left.
- $k = -3$: The graph is translated 3 units down.



The domain is all real numbers, and the range is all real numbers greater than -3.

GuidedPractice

4A. $y = -3\left(\frac{2}{5}\right)^{x-4} + 2$

4B. $y = \frac{3}{8}\left(\frac{5}{6}\right)^{x-1} + 1$

Similar to exponential growth, you can model exponential decay with a constant percent of decrease over specific time periods using the following function.

$$A(t) = a(1 - r)^t$$

The base of the exponential expression, $1 - r$, is called the **decay factor**.

Check Your Understanding

Examples 1–2 **Graph each function. Find the y-intercept and state the domain and range.**

1. $y = 2^x$
2. $y = -5^x$
3. $y = -\left(\frac{1}{5}\right)^x$
4. $y = 3\left(\frac{1}{4}\right)^x$
5. $f(x) = 6^x + 3$
6. $f(x) = 2 - 2^x$

Example 3

7. **BIOLOGY** The function $f(t) = 100(1.05)^t$ models the growth of a fruit fly population, where $f(t)$ is the number of flies and t is time in days.

   a. What values for the domain and range are reasonable in the context of this situation? Explain.

   b. After two weeks, approximately how many flies are in this population?

Example 4 **Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.**

8.

x	1	2	3	4	5	6
y	−4	−2	0	2	4	6

9.

x	2	4	6	8	10	12
y	1	4	16	64	256	1024

Practice and Problem Solving

Examples 1–2 **Graph each function. Find the y-intercept and state the domain and range.**

10. $y = 2 \cdot 8^x$
11. $y = 2 \cdot \left(\frac{1}{6}\right)^x$
12. $y = \left(\frac{1}{12}\right)^x$
13. $y = -3 \cdot 9^x$
14. $y = -4 \cdot 10^x$
15. $y = 3 \cdot 11^x$
16. $y = 4^x + 3$
17. $y = \frac{1}{2}(2^x - 8)$
18. $y = 5(3^x) + 1$
19. $y = -2(3^x) + 5$

Example 3

20. **MODELING** A population of bacteria in a culture increases according to the model $p = 300(2.7)^{0.02t}$, where t is the number of hours and $t = 0$ corresponds to 9:00 A.M.

    a. Use this model to estimate the number of bacteria at 11 A.M.

    b. Graph the function and name the p-intercept. Describe what the p-intercept represents, and describe a reasonable domain and range for this situation.

Example 4 **Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.**

21.

x	−4	0	4	8	12
y	2	−4	8	−16	32

22.

x	−6	−3	0	3
y	5	10	15	20

23.

x	−8	−6	−4	−2
y	0.25	0.5	1	2

24.

x	20	30	40	50	60
y	1	0.4	0.16	0.064	0.0256

25. **PHOTOGRAPHY** Manal is enlarging a photograph to make a poster for school. She will enlarge the picture repeatedly at 150%. The function $P = 1.5^x$ models the new size of the picture being enlarged, where x is the number of enlargements. How many times as big is the picture after 4 enlargements?

26. **FINANCIAL LITERACY** Mohammad deposited AED 500 into a savings account and after 8 years, his investment is worth AED 807.07. The equation $A = d(1.005)^{12t}$ models the value of Mohammad's investment A after t years with an initial deposit d.

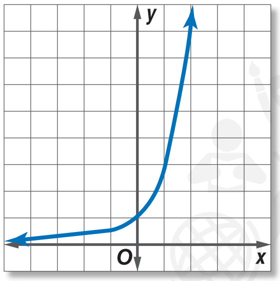
a. What would the value of Mohammad's investment be if he had deposited AED 1,000?

b. What would the value of Mohammad's investment be if he had deposited AED 250?

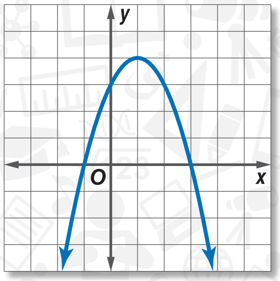
c. Interpret $d(1.005)^{12t}$ to explain how the amount of the original deposit affects the value of Mohammad's investment.

Identify each function as *linear*, *exponential*, or *neither*.

27.



28.



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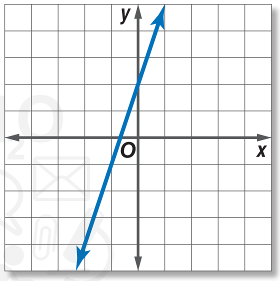


30. $y = 4^x$

31. $y = 2x(x - 1)$

32. $5x + y = 8$

33. **GRADUATION** The number of graduates at a high school has increased by a factor of 1.055 every year since 2001. In 2001, 110 students graduated. The function $N = 110(1.055)^t$ models the number of students N expected to graduate t years after 2001. How many students will graduate in 2012?

Describe the graph of each equation as a transformation of the graph of $y = 2^x$.

34. $y = 2^x + 6$

35. $y = 3(2)^x$

36. $y = -\frac{1}{4}(2)^x$

37. $y = -3 + 2^x$

38. $y = \left(\frac{1}{2}\right)^x$

39. $y = -5(2)^x$

40. **DEER** The deer population at a national park doubles every year. In 2000, there were 25 deer in the park. The function $N = 25(2)^t$ models the number of deer N in the park t years after 2000. What will the deer population be in 2015?

H.O.T. Problems Use Higher-Order Thinking Skills

41. **PERSEVERANCE** Write an exponential function for which the graph passes through the points at (0, 3) and (1, 6).

42. **REASONING** Determine whether the graph of $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$, *sometimes*, *always*, or *never* has an x-intercept. Explain your reasoning.

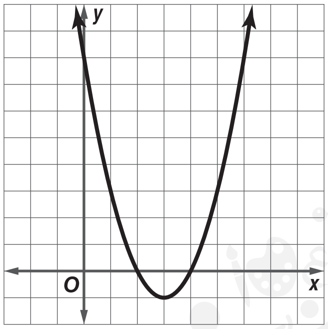
43. **OPEN ENDED** Find an exponential function that represents a real-world situation, and graph the function. Analyze the graph, and explain why the situation is modeled by an exponential function rather than a linear function.

44. **REASONING** Use tables and graphs to compare and contrast an exponential function $f(x) = ab^x + c$, where $a \neq 0$, $b > 0$, and $b \neq 1$, and a linear function $g(x) = ax + c$. Include intercepts, intervals where the functions are increasing, decreasing, positive, or negative, relative maxima and minima, symmetry, and end behavior.

45. **WRITING IN MATH** Explain how to determine whether a set of data displays exponential behavior.

Standardized Test Practice

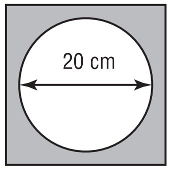
46. SHORT RESPONSE What are the x-intercepts of the function graphed below?



47. Mazen invested AED 300 into a savings account. The equation $A = 300(1.005)^{12t}$ models the amount in Mazen's account A after t years. How much will be in Mazen's account after 7 years?

A AED 25,326 **C** AED 382.01
B AED 456.11 **D** AED 301.52

48. GEOMETRY Noura placed a circular piece of paper on a square picture as shown below. If the picture extends 4 centimeter beyond the circle on each side, what is the perimeter of the square picture?



F 64 cm **H** 94 cm
G 80 cm **J** 112 cm

49. The points with coordinates $(0, -3)$ and $(2, 7)$ are on line l. Line p contains $(3, -1)$ and is perpendicular to line l. What is the x-coordinate of the point where l and p intersect?

A $\frac{1}{2}$ **B** $-\frac{2}{5}$
C $-\frac{1}{2}$ **D** -3

Spiral Review

Evaluate each product. Express the results in both scientific notation and standard form. (Lesson 8-4)

50. $(1.9 \times 10^2)(4.7 \times 10^6)$ **51.** $(4.5 \times 10^{-3})(5.6 \times 10^4)$ **52.** $(3.8 \times 10^{-4})(6.4 \times 10^{-8})$

Simplify. (Lesson 8-3)

53. $\sqrt[3]{343}$ **54.** $\sqrt[6]{729}$ **55.** $\left(\frac{1}{32}\right)^{\frac{1}{5}}$

56. $729^{\frac{5}{6}}$ **57.** $216^{\frac{5}{3}}$ **58.** $\left(\frac{1}{81}\right)^{\frac{3}{2}}$

59. DEMOLITION DERBY When a car hits an object, the damage is measured by the collision impact. For a certain car the collision impact I is given by $I = 2v^2$, where v represents the speed in kilometers per minute. What is the collision impact if the speed of the car is 4 kilometers per minute? (Lesson 8-1)

Use elimination to solve each system of equations. (Lesson 6-3)

60. $x + y = -3$
$x - y = 1$

61. $3a + b = 5$
$2a + b = 10$

62. $3x - 5y = 16$
$-3x + 2y = -10$

Skills Review

Find the next three terms of each arithmetic sequence.

63. 1, 3, 5, 7, … **64.** $-6, -4, -2, 0, \ldots$ **65.** 6.5, 9, 11.5, 14, …

66. $10, 3, -4, -11, \ldots$ **67.** $\frac{1}{2}, \frac{5}{4}, 2, \frac{11}{4}, \ldots$ **68.** $1, \frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \ldots$

EXTEND 2-1

Graphing Technology Lab
Solving Exponential Equations and Inequalities

You can use a graphing calculator to solve exponential equations and inequalities by graphing and by using tables.

Mathematical Practices
Use appropriate tools strategically.

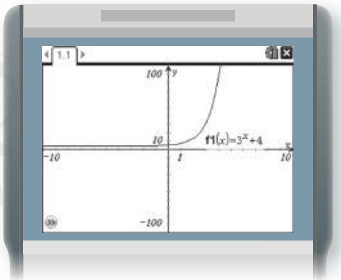
Activity 1 Graph an Exponential Equation

Graph $y = 3^x + 4$ using a graphing calculator.

Step 1 Add a new **Graphs** page.

Step 2 Enter $3^x + 4$ as **f1(*x*)**.

Step 3 Use the **Window Settings** option from the **Window/Zoom** menu to adjust the window so that x is from -10 to 10 and y is from -100 to 100. Keep the scales as **Auto**.



To solve an equation by graphing, graph both sides of the equation and locate the point(s) of intersection.

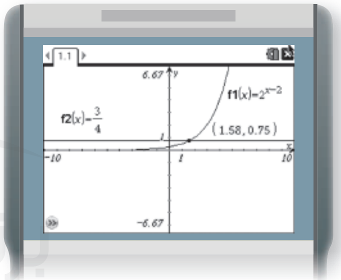
Activity 2 Solve an Exponential Equation by Graphing

Solve $2^{x-2} = \frac{3}{4}$.

Step 1 Add a new **Graphs** page.

Step 2 Enter 2^{x-2} as **f1(*x*)** and $\frac{3}{4}$ as **f2(*x*)**.

Step 3 Use the **Intersection Point(s)** tool from the **Points & Lines** menu to find the intersection of the two graphs. Select the graph of **f1(*x*) enter** and then the graph of **f2(*x*) enter**.



The graphs intersect at about (1.58, 0.75). Therefore, the solution of $2^{x-2} = \frac{3}{4}$ is 1.58.

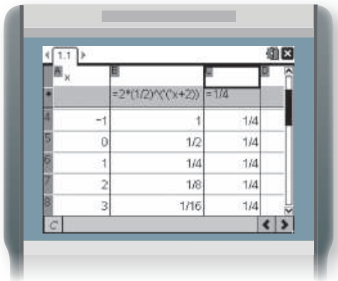
Exercises

TOOLS **Use a graphing calculator to solve each equation.**

1. $\left(\frac{1}{3}\right)^{x-1} = \frac{3}{4}$
2. $2^{2x-1} = 2x$
3. $\left(\frac{1}{2}\right)^{2x} = 2^{2x}$
4. $5^{\frac{1}{3}x+2} = -x$
5. $\left(\frac{1}{8}\right)^{2x} = -2x + 1$
6. $2^{\frac{1}{4}x-1} = 3^{x+1}$
7. $2^{3x-1} = 4^x$
8. $4^{2x-3} = 5^{-x+1}$
9. $3^{2x-4} = 2^x + 1$

Activity 3 Solve an Exponential Equation by Using a Table

Solve $2\left(\frac{1}{2}\right)^{x+2} = \frac{1}{4}$ using a table.



Step 1 Add a new **Lists & Spreadsheet** page.

Step 2 Label column A as x. Enter values from -4 to 4 in cells A1 to A9.

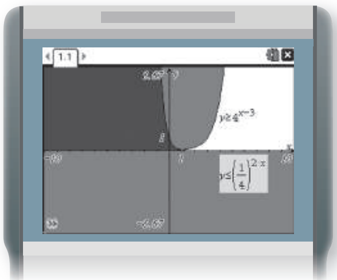
Step 3 In column B in the formula row, enter the left side of the rational equation. In column C of the formula row, enter $= \frac{1}{4}$. Specify **Variable Reference** when prompted.

Scroll until you see where the values in Columns B and C are equal. This occurs at $x = 1$. Therefore, the solution of $2\left(\frac{1}{2}\right)^{x+2} = \frac{1}{4}$ is 1.

You can also use a graphing calculator to solve exponential inequalities.

Activity 4 Solve an Exponential Inequality

Solve $4^{x-3} \leq \left(\frac{1}{4}\right)^{2x}$.



Step 1 Add a new **Graphs** page.

Step 2 Enter the left side of the inequality into **f1(*x*)**. Press **del**, select $\geq$, and enter 4^{x-3}. Enter the right side of the inequality into **f2(*x*)**. Press **tab del** $\leq$, and enter $\left(\frac{1}{4}\right)^{2x}$.

The x-values of the points in the region where the shading overlap is the solution set of the original inequality. Therefore the solution of $4^{x-3} \leq \left(\frac{1}{4}\right)^{2x}$ is $x \leq 1$.

Exercises

TOOLS Use a graphing calculator to solve each equation or inequality.

10. $\left(\frac{1}{3}\right)^{3x} = 3^x$

11. $\left(\frac{1}{6}\right)^{2x} = 4^x$

12. $3^{1-x} \leq 4^x$

13. $4^{3x} \leq 2x + 1$

14. $\left(\frac{1}{4}\right)^{x} > 2^{x+4}$

15. $\left(\frac{1}{3}\right)^{x-1} \geq 2^x$

LESSON 2-2 Analyzing Functions with Successive Differences

Then

- You graphed linear, quadratic, and exponential functions.

Now

1. Identify linear, quadratic, and exponential functions from given data.
2. Write equations that model data.

Why?

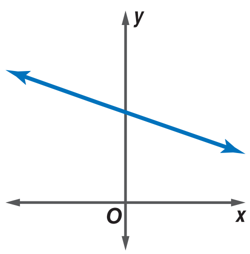
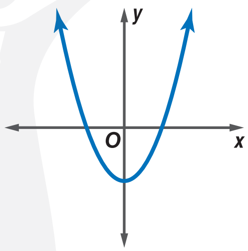
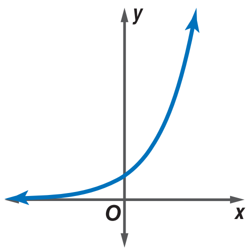
- Every year the golf team sells candy to raise money for charity. By knowing what type of function models the sales of the candy, they can determine the best price of the candy.

Mathematical Practices
Look for and make use of structure.

1 Identify Functions

You can use linear functions, quadratic functions, and exponential functions to model data. The general forms of the equations and a graph of each function type are listed below.

ConceptSummary Linear and Nonlinear Functions

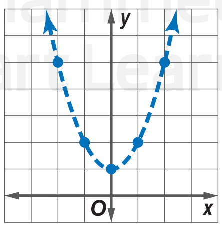
Linear Function	Quadratic Function	Exponential Function
$y = mx + b$	$y = ax^2 + bx + c$	$y = ab^x$, when $b > 0$
		

Example 1 Choose a Model Using Graphs

Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

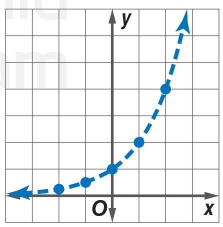
a. $\{(-2, 5), (-1, 2), (0, 1), (1, 2), (2, 5)\}$

The ordered pairs appear to represent a quadratic function.



b. $\left\{\left(-2, \frac{1}{4}\right), \left(-1, \frac{1}{2}\right), (0, 1), (1, 2), (2, 4)\right\}$

The ordered pairs appear to represent an exponential function.



GuidedPractice

1A. $(-2, -3), (-1, -1), (0, 1), (1, 3)$

1B. $(-1, 0.25), (0, 1), (1, 4), (2, 16)$

Another way to determine which model best describes data is to use patterns. The differences of successive y-values are called *first differences*. The differences of successive first differences are called *second differences*.

- If the differences of successive y-values are all equal, the data represent a linear function.
- If the second differences are all equal, but the first differences are not equal, the data represent a quadratic function.
- If the ratios of successive y-values are all equal and $r \neq 1$, the data represent an exponential function.

WatchOut!

***x*-Values** Before you check for successive differences or ratios, make sure the x-values are increasing by the same amount.

Example 2 Choose a Model Using Differences or Ratios

Look for a pattern in each table of values to determine which kind of model best describes the data.

a.

x	−2	−1	0	1	2
y	−8	−3	2	7	12

First differences: −8 −3 2 7 12

5 5 5 5

Since the first differences are all equal, the table of values represents a linear function.

b.

x	−1	0	1	2	3
y	8	4	2	1	0.5

First differences: 8 4 2 1 0.5

−4 −2 −1 −0.5

The first differences are not all equal. So, the table of values does not represent a linear function. Find the second differences and compare.

First differences: −4 −2 −1 −0.5

Second differences: 2 1 0.5

The second differences are not all equal. So, the table of values does not represent a quadratic function. Find the ratios of the y-values and compare.

8 4 2 1 0.5

Ratios: $\frac{4}{8} = \frac{1}{2}$ $\frac{2}{4} = \frac{1}{2}$ $\frac{1}{2}$ $\frac{0.5}{1} = \frac{1}{2}$

The ratios of successive y-values are equal. Therefore, the table of values can be modeled by an exponential function.

GuidedPractice

2A.

x	−3	−2	−1	0	1
y	−3	−7	−9	−9	−7

2B.

x	−2	−1	0	1	2
y	−18	−13	−8	−3	2

2 Write Equations Once you find the model that best describes the data, you can write an equation for the function. For a quadratic function in this lesson, the equation will have the form $y = ax^2$.

Example 3 Write an Equation

Determine which kind of model best describes the data. Then write an equation for the function that models the data.

Step 1 Determine which model fits the data.

x	-4	-3	-2	-1	0
y	32	18	8	2	0

32 18 8 2 0

First differences: −14 −10 −6 −2

Second differences: 4 4 4

Since the second differences are equal, a quadratic function models the data.

Step 2 Write an equation for the function that models the data.

The equation has the form $y = ax^2$. Find the value of a by choosing one of the ordered pairs from the table of values. Let's use $(-1, 2)$.

$y = ax^2$ Equation for quadratic function

$2 = a(-1)^2$ $x = -1$ and $y = 2$

$2 = a$ An equation that models the data is $y = 2x^2$.

WatchOut!

Finding a In Example 3, the point (0, 0) cannot be used to find the value of a. You will have to divide each side by 0, giving you an undefined value for a.

GuidedPractice

3A.

x	−2	−1	0	1	2
y	11	7	3	−1	−5

3B.

x	−3	−2	−1	0	1
y	0.375	0.75	1.5	3	6

Real-World Example 4 Write an Equation for a Real-World Situation

BOOK CLUB **The table shows the number of book club members for four consecutive years. Determine which model best represents the data. Then write a function that models the data.**

Understand We need to find a model for the data, and then write a function.

Time (years)	0	1	2	3	4
Members	5	10	20	40	80

Plan Find a pattern using successive differences or ratios. Then use the general form of the equation to write a function.

Solve The constant ratio is 2. This is the value of the base. An exponential function of the form $y = ab^x$ models the data.

$y = ab^x$ Equation for exponential function

$5 = a(2)^0$ $x = 0$, $y = 5$, and $b = 2$

$5 = a$ The equation that models the data is $y = 5 \cdot 2^x$.

Check You used (0, 5) to write the function. Verify that every other ordered pair satisfies the equation.

Real-WorldLink

A poll by the National Education Association found that 87% of all teens polled found reading relaxing, 85% viewed reading as rewarding, and 79% found reading exciting.

Source: *American Demographics*

GuidedPractice

4. **ADVERTISING** The table shows the cost of placing an ad in a newspaper. Determine a model that best represents the data and write a function that models the data.

No. of Lines	5	6	7	8
Total Cost (AED)	14.50	16.60	18.70	20.80

Check Your Understanding

Example 1 Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

1. $(-2, 8), (-1, 5), (0, 2), (1, -1)$
2. $(-3, 7), (-2, 3), (-1, 1), (0, 1), (1, 3)$
3. $(-3, 8), (-2, 4), (-1, 2), (0, 1), (1, 0.5)$
4. $(0, 2), (1, 2.5), (2, 3), (3, 3.5)$

Example 2 Look for a pattern in each table of values to determine which kind of model best describes the data.

5.

x	0	1	2	3	4
y	5	8	17	32	53

6.

x	−3	−2	−1	0
y	−6.75	−7.5	−8.25	−9

7.

x	−1	0	1	2	3
y	3	6	12	24	48

8.

x	3	4	5	6	7
y	−1.5	0	2.5	6	10.5

Example 3 Determine which kind of model best describes the data. Then write an equation for the function that models the data.

9.

x	−1	0	1	2	3
y	1	3	9	27	81

10.

x	−5	−4	−3	−2	−1
y	125	80	45	20	5

11.

x	−3	−2	−1	0	1
y	1	1.5	2	2.5	3

12.

x	−1	0	1	2
y	−1.25	−1	−0.75	−0.5

Example 4 13. **PLANTS** The table shows the height of a plant for four consecutive weeks. Determine which kind of function best models the height. Then write a function that models the data.

Week	0	1	2	3	4
Height (in.)	3	3.5	4	4.5	5

Practice and Problem Solving

Example 1 Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

14. $(-1, 1), (0, -2), (1, -3), (2, -2), (3, 1)$
15. $(1, 2.75), (2, 2.5), (3, 2.25), (4, 2)$
16. $(-3, 0.25), (-2, 0.5), (-1, 1), (0, 2)$
17. $(-3, -11), (-2, -5), (-1, -3), (0, -5)$
18. $(-2, 6), (-1, 1), (0, -4), (1, -9)$
19. $(-1, 8), (0, 2), (1, 0.5), (2, 0.125)$

Examples 2–3 Look for a pattern in each table of values to determine which kind of model best describes the data. Then write an equation for the function that models the data.

20.

x	−3	−2	−1	0
y	−8.8	−8.6	−8.4	−8.2

21.

x	−2	−1	0	1	2
y	10	2.5	0	2.5	10

22.

x	−1	0	1	2	3
y	0.75	3	12	48	192

23.

x	−2	−1	0	1	2
y	0.008	0.04	0.2	1	5

24.

x	0	1	2	3	4
y	0	4.2	16.8	37.8	67.2

25.

x	−3	−2	−1	0	1
y	14.75	9.75	4.75	−0.25	−5.25

Example 4

26. **WEB SITES** A company tracked the number of visitors to its Web site over 4 days. Determine which kind of model best represents the number of visitors to the Web site with respect to time. Then write a function that models the data.

Day	0	1	2	3	4
Visitors (in thousands)	0	0.9	3.6	8.1	14.4

27 **CALLING** The cost of an international call depends on the length of the call. The table shows the cost for up to 6 minutes.

Length of call (min)	1	2	3	4	5	6
Cost (AED)	0.12	0.24	0.36	0.48	0.60	0.72

a. Graph the data and determine which kind of function best models the data.

b. Write an equation for the function that models the data.

c. Use your equation to determine how much a 10-minute call would cost.

28. **DEPRECIATION** The value of a car depreciates over time. The table shows the value of a car over a period of time.

Year	0	1	2	3	4
Value (AED)	18,500	15,910	13,682.60	11,767.04	10,119.65

a. Determine which kind of function best models the data.

b. Write an equation for the function that models the data.

c. Use your equation to determine how much the car is worth after 7 years.

29. **BACTERIA** A scientist estimates that a bacteria culture with an initial population of 12 will triple every hour.

a. Make a table to show the bacteria population for the first 4 hours.

b. Which kind of model best represents the data?

c. Write a function that models the data.

d. How many bacteria will there be after 8 hours?

30. **PRINTING** A printing company charges the fees shown to print flyers. Write a function that models the total cost of the flyers, and determine how much 30 flyers would cost.

Quick 2 U Printing
Set Up Fee AED 25
15 fils each flyer

H.O.T. Problems Use Higher-Order Thinking Skills

31. **CHALLENGE** Write a function that has constant second differences, first differences that are not constant, a y-intercept of -5, and contains the point (2, 3).

32. **ARGUMENTS** What type of function will have constant third differences but not constant second differences? Explain.

33. **OPEN ENDED** Write a linear function that has a constant first difference of 4.

34. **PROOF** Write a paragraph proof to show that linear functions grow by equal differences over equal intervals, and exponential functions grow by equal factors over equal intervals. (*Hint:* Let $y = ax$ represent a linear function and let $y = a^x$ represent an exponential function.)

35. **WRITING IN MATH** How can you determine whether a given set of data should be modeled by a *linear* function, a *quadratic* function, or an *exponential* function?

Standardized Test Practice

36. SHORT RESPONSE Write an equation that models the data in the table.

x	0	1	2	3	4
y	3	6	12	24	48

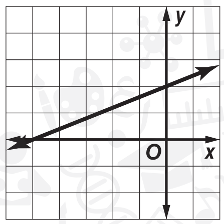
37. What is the equation of the line below?

A $y = \frac{2}{5}x + 2$

B $y = \frac{2}{5}x - 2$

C $y = \frac{5}{2}x + 2$

D $y = \frac{5}{2}x - 2$



38. The point $(r, -4)$ lies on a line with an equation of $2x + 3y = -8$. Find the value of r.

F -10 **H** 2

G 0 **J** 8

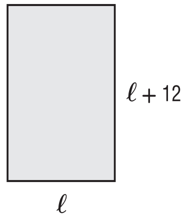
39. GEOMETRY The rectangle has an area of 220 square meters. Find the length ℓ.

A 8 meters

B 10 meters

C 22 meters

D 34 meters



Spiral Review

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 1-5)

40. $6x^2 - 3x - 30 = 0$ **41.** $4x^2 + 18x = 10$ **42.** $2x^2 + 6x = 7$

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary. (Lesson 1-4)

43. $x^2 = 25$ **44.** $x^2 + 6x + 9 = 16$ **45.** $x^2 - 14x + 49 = 15$

Look for a pattern in each table of values to determine which kind of model best describes the data. (Lesson 1-6)

46.

x	0	1	2	3	4
y	4	5	6	7	8

47.

x	1	2	3	4	5
y	2	4	8	16	32

48.

x	−3	−2	−1	0	1
y	14	9	6	5	6

49.

x	3	4	5	6	7
y	3	5	7	9	11

50. PHYSICAL SCIENCE A projectile is shot straight up from ground level. Its height h, in meters, after t seconds is given by $h = 96t - 16t^2$. Find the value(s) of t when h is 96 feet. (Lesson 1-5)

Skills Review

Evaluate each expression if $x = -3$, $y = -1$, and $z = 4$.

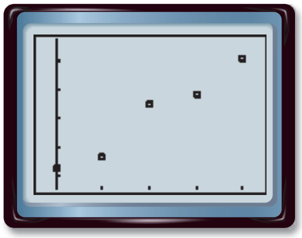
51. $|x - 4|$ **52.** $|2y + 1|$ **53.** $|4 - z|$

54. $\left|\frac{1}{2}x + 2\right|$ **55.** $|12 - 4z|$ **56.** $|2y - 3| - 6$

EXTEND 2-2 Graphing Technology Lab
Curve Fitting

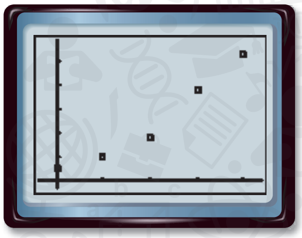
If there is a constant increase or decrease in data values, there is a linear trend. If the values are increasing or decreasing more and more rapidly, there may be a quadratic or exponential trend.

Linear Trend



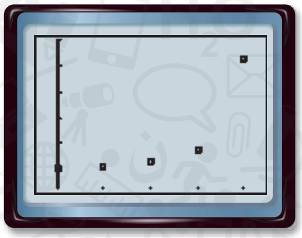
[0, 5] scl: 1 by [0, 6] scl: 1

Quadratic Trend



[0, 5] scl: 1 by [0, 6] scl: 1

Exponential Trend



[0, 5] scl: 1 by [0, 6] scl: 1

With a graphing calculator, you can find the appropriate regression equation.

Activity

CHARTER AIRLINE The table shows the average monthly number of flights made each year by a charter airline that was founded in 2000.

Year	2000	2001	2002	2003	2004	2005	2006	2007
Flights	17	20	24	28	33	38	44	50

Step 1 Make a scatter plot.

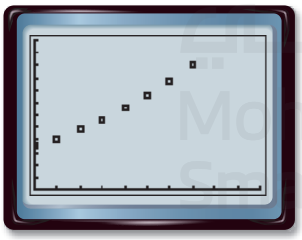
- Enter the number of years since 2000 in **L1** and the number of flights in **L2**.

KEYSTROKES: *Review entering a list on page 255.*

- Use **STAT PLOT** to graph the scatter plot.

KEYSTROKES: *Review statistical plots on page 256.*

Use ZOOM 9 to graph.



[0, 10] scl: 1 by [0, 60] scl: 5

From the scatter plot we can see that the data may have either a quadratic trend or an exponential trend.

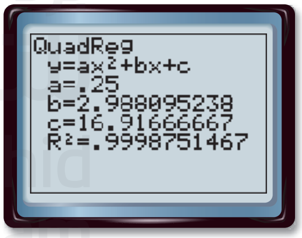
Step 2 Find the regression equation.

We will check both trends by examining their regression equations.

- Select **DiagnosticOn** from the **CATALOG**.
- Select **QuadReg** on the **STAT** menu.

KEYSTROKES: STAT ▶ 5 ENTER ENTER

The equation is in the form $y = ax^2 + bx + c$.



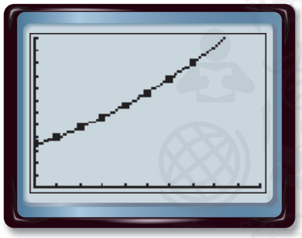
The equation is about $y = 0.25x^2 + 3x + 17$.

R^2 is the **coefficient of determination**. The closer R^2 is to 1, the better the model. To acquire the exponential equation select **ExpReg** on the **STAT** menu. To choose a quadratic or exponential model, fit both and use the one with the R^2 value closer to 1.

Step 3 Graph the quadratic regression equation.

- Copy the equation to the **Y=** list and graph.

KEYSTROKES: [Y=] [VARS] 5 [▶] [▶] 1 [ZOOM] 9

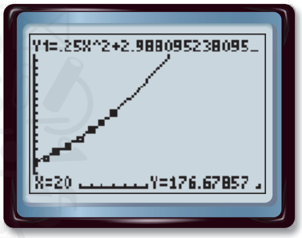


[0, 10] scl: 1 by [0, 60] scl: 5

Step 4 Predict using the equation.

If this trend continues, we can use the graph of our equation to predict the monthly number of flights the airline will make in a specific year. Let's check the year 2020. First adjust the window.

KEYSTROKES: [2nd] [CALC] 1 At x = enter 20 [ENTER].



[0, 25] scl: 1 by [0, 200] scl: 5

There will be approximately 177 flights per month if this trend continues.

Exercises

Plot each set of data points. Determine whether to use a *linear*, *quadratic* or *exponential* regression equation. State the coefficient of determination.

1.

x	y
1	30
2	40
3	50
4	55
5	50
6	40

2.

x	y
0.0	12.1
0.1	9.6
0.2	6.3
0.3	5.5
0.4	4.8
0.5	1.9

3.

x	y
0	1.1
2	3.3
4	2.9
6	5.6
8	11.9
10	19.8

4.

x	y
1	1.67
5	2.59
9	4.37
13	6.12
17	5.48
21	3.12

5. **BAKING** **Alyssa baked a cake and is waiting for it to cool so she can ice it. The table shows the temperature of the cake every 5 minutes after Alyssa took it out of the oven.**

Time (min)	Temperature (°F)
0	350
5	244
10	178
15	137
20	112
25	96
30	89

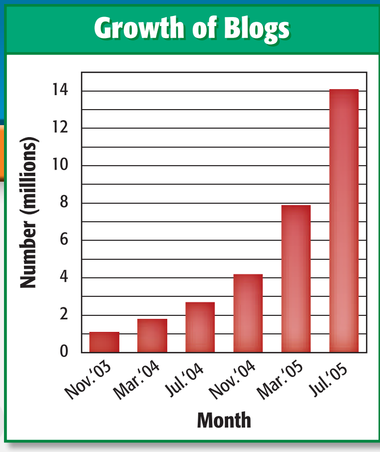
a. Make a scatter plot of the data.

b. Which regression equation has an R^2 value closest to 1? Is this the equation that best fits the context of the problem? Explain your reasoning.

c. Find an appropriate regression equation, and state the coefficient of determination. What is the domain and range?

d. Alyssa will ice the cake when it reaches room temperature (70°F). Use the regression equation to predict when she can ice her cake.

LESSON 2-3 Growth and Decay



Then

- You analyzed exponential functions.

Now

1. Solve problems involving exponential growth.
2. Solve problems involving exponential decay.

Why?

- The number of Weblogs or blogs increased at a monthly rate of about 13.7% over 21 months. The average number of blogs per month can be modeled by $y = 1.1(1 + 0.137)^t$ or $y = 1.1(1.137)^t$, where y represents the total number of blogs in millions and t is the number of months since November 2003.

NewVocabulary
compound interest

Mathematical Practices
Model with mathematics.

1 Exponential Growth

The equation for the number of blogs is in the form $y = a(1 + r)^t$. This is the general equation for exponential growth.

KeyConcept Equation for Exponential Growth

a is the initial amount. t is time.

$$y = a(1 + r)^t$$

y is the final amount. r is the rate of change expressed as a decimal, $r > 0$.

Real-World Example 1 Exponential Growth

CONTEST The prize for a radio station contest begins with a AED 100 gift card. Once a day, a name is announced. The person has 15 minutes to call or the prize increases by 2.5% for the next day.

a. Write an equation to represent the amount of the gift card in dirhams after t days with no winners.

$y = a(1 + r)^t$	Equation for exponential growth
$y = 100(1 + 0.025)^t$	$a = 100$ and $r = 2.5\%$ or 0.025
$y = 100(1.025)^t$	Simplify.

In the equation $y = 100(1.025)^t$, y is the amount of the gift card and t is the number of days since the contest began.

b. How much will the gift card be worth if no one wins after 10 days?

$y = 100(1.025)^t$	Equation for amount of gift card
$= 100(1.025)^{10}$	$t = 10$
≈ 128.01	Use a calculator.

In 10 days, the gift card will be worth AED 128.01.

GuidedPractice

1. **TUITION** A college's tuition has risen 5% each year since 2000. If the tuition in 2000 was AED 10,850, write an equation for the amount of the tuition t years after 2000. Predict the cost of tuition for this college in 2015.

Compound interest is interest earned or paid on both the initial investment and previously earned interest. It is an application of exponential growth.

KeyConcept Equation for Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A is the current amount.

n is the number of times the interest is compounded each year, and *t* is time in years.

P is the principal or initial amount.

r is the annual interest rate expressed as a decimal, $r > 0$.

Real-WorldCareer

Financial Advisor Financial advisors help people plan their financial futures. A good financial advisor has mathematical, problem-solving, and communication skills. A bachelor's degree is strongly preferred but not required.

Real-World Example 2 Compound Interest

FINANCE Huda's parents invested AED 14,000 at 6% per year compounded monthly. How much money will there be in the account after 10 years?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$	Compound interest equation
$= 14{,}000\left(1 + \frac{0.06}{12}\right)^{12(10)}$	$P = 14{,}000$, $r = 6\%$ or 0.06, $n = 12$, and $t = 10$
$= 14{,}000(1.005)^{120}$	Simplify.
$\approx 25{,}471.55$	Use a calculator.

There will be about AED 25,471.55 in 10 years.

GuidedPractice

2. **FINANCE** Determine the amount of an investment if AED 300 is invested at an interest rate of 3.5% compounded monthly for 22 years.

2 Exponential Decay

In exponential decay, the original amount decreases by the same percent over a period of time. A variation of the growth equation can be used as the general equation for exponential decay.

StudyTip

Growth and Decay Since *r* is added to 1, the value inside the parentheses will be greater than 1 for exponential growth functions. For exponential decay functions, this value will be less than 1 since *r* is subtracted from 1.

KeyConcept Equation for Exponential Decay

$$y = a(1 - r)^t$$

a is the initial amount.

t is time.

y is the final amount.

r is the rate of decay expressed as a decimal, $0 < r < 1$.

Real-World Example 3 Exponential Decay

SWIMMING A fully inflated child's raft for a pool is losing 6.6% of its air every day. The raft originally contained 74,000 cubic centimeters of air.

a. Write an equation to represent the loss of air.

$y = a(1 - r)^t$	Equation for exponential decay
$= 74000(1 - 0.066)^t$	$a = 74000$ and $r = 6.6\%$ or 0.066
$= 74000(0.934)^t$	Simplify.

$y = 74000(0.934)^t$, where *y* is the air in the raft in cubic centimeters after *t* days.

Estimate the amount of air in the raft after 7 days.

$y = 74000(0.934)^t$	Equation for air loss
$= 74000(0.934)^7$	$t = 7$
≈ 45880	Use a calculator.

The amount of air in the raft after 7 days will be about 45,880 cubic centmeters.

GuidedPractice

3. **POPULATION** The population of Campbell County, Kentucky, has been decreasing at an average rate of about 0.3% per year. In 2000, its population was 88,647. Write an equation to represent the population since 2000. If the trend continues, predict the population in 2010.

Check Your Understanding

Example 1

1. **SALARY** Ms. Hidaya received a job as a teacher with a starting salary of AED 125,000. According to her contract, she will receive a 1.5% increase in her salary every year. How much will Ms. Hidaya earn in 7 years?

Example 2

2. **MONEY** Yousif invested AED 400 into an account with a 5.5% interest rate compounded monthly. How much will Yousif's investment be worth in 8 years?

Example 3

3. **ENROLLMENT** In 2000, 2200 students attended Polaris High School. The enrollment has been declining 2% annually.
   a. Write an equation for the enrollment of Polaris High School t years after 2000.
   b. If this trend continues, how many students will be enrolled in 2015?

Practice and Problem Solving

Example 1

4. **MEMBERSHIPS** The Work-Out Gym sold 550 memberships in 2001. Since then the number of memberships sold has increased 3% annually.
   a. Write an equation for the number of memberships sold at Work-Out Gym t years after 2001.
   b. If this trend continues, predict how many memberships the gym will sell in 2020.

5. **COMPUTERS** The number of people who own computers has increased 23.2% annually since 1990. If half a million people owned a computer in 1990, predict how many people will own a computer in 2015.

6. **COINS** Majed purchased a rare coin from a dealer for AED 300. The value of the coin increases 5% each year. Determine the value of the coin in 5 years.

Example 2

7. **INVESTMENTS** Mahmoud invested AED 6,600 at an interest rate of 4.5% compounded monthly. Determine the value of his investment in 4 years.

8. **COMPOUND INTEREST** Nisreen invested AED 1,200 at an interest rate of 5.75% compounded quarterly. Determine the value of her investment in 7 years.

9. **PRECISION** Najla is saving money for a trip to the Bahamas that costs AED 1,087.76. She puts AED 550 into a savings account that pays 7.25% interest compounded quarterly. Will she have enough money in the account after 4 years? Explain.

Example 3

10. **INVESTMENTS** Ali's investment of AED 4,500 has been losing its value at a rate of 2.5% each year. What will his investment be worth in 5 years?

11. **POPULATION** In the years from 2010 to 2015, the population of Washington DC is expected decrease about 0.9% annually. In 2010, the population was about 530,000. What is the population of Washington DC expected to be in 2015?

12. **CARS** Faris purchases a car for AED 18,995. The car depreciates at a rate of 18% annually. After 6 years, Faleh offers to buy the car for AED 4,500. Should Faris sell the car? Explain.

13. **HOUSING** The median house price increased an average of 1.4% each year between 2005 and 2007. Assume that this pattern continues.

a. Write an equation for the median house price for t years after 2007.

b. Predict the median house price in 2018.

Median House Price	
2005	AED 885,000
2006	AED 905,800
2007	AED 910,000

Source: *Real Estate Journal*

14. **ELEMENTS** A radioactive element's half-life is the time it takes for one half of the element's quantity to decay. The half-life of Plutonium-241 is 14.4 years. The number of grams A of Plutonium-241 left after t years can be modeled by $A = p(0.5)^{\frac{t}{14.4}}$, where p is the original amount of the element.

a. How much of a 0.2-gram sample remains after 72 years?

b. How much of a 5.4-gram sample remains after 1095 days?

15. **COMBINING FUNCTIONS** A swimming pool holds a maximum of 77, 600 liters of water. It evaporates at a rate of 0.5% per hour. The pool currently contains 71,900 liters of water.

a. Write an exponential function $w(t)$ to express the amount of water remaining in the pool after time t where t is the number of hours after the pool has reached 71,900 liters.

b. At this same time, a hose is turned on to refill the pool at a rate of 1,100 liters per hour. Write a function $p(t)$, where t is the time in hours the hose is running, to express the amount of water that is pumped into the pool.

c. Find $C(t) = p(t) + w(t)$. What does this new function represent?

d. Use the graph of $C(t)$ to determine how long the hose must run to fill the pool to its maximum capacity.

H.O.T. Problems Use Higher-Order Thinking Skills

16. **REASONING** Determine the growth rate (as a percent) of a population that quadruples every year. Explain.

17. **PRECISION** Mansour invested AED 1,200 into an account with an interest rate of 8% compounded monthly. Use a calculator to approximate how long it will take for Mansour's investment to reach AED 2,500.

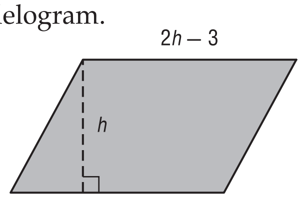
18. **REASONING** The amount of water in a container doubles every minute. After 8 minutes, the container is full. After how many minutes was the container half full? Explain.

19. **WRITING IN MATH** What should you consider when using exponential models to make decisions?

20. **WRITING IN MATH** Compare and contrast the exponential growth formula and the exponential decay formula.

Standardized Test Practice

21. GEOMETRY The parallelogram has an area of 35 square centimeters. Find the height h of the parallelogram.



A 3.5 centimeters
B 4 centimeters
C 5 centimeters
D 7 centimeters

22. Which is greater than $64^{\frac{1}{3}}$?

F 2^2
G $64^{\frac{1}{6}}$
H $64^{\frac{1}{2}}$
J 64^{-3}

23. Eissa purchased a car for AED 22,900. The car depreciated at an annual rate of 16%. Which of the following equations models the value of Eissa's car after 5 years?

A $A = 22{,}900(1.16)^5$
B $A = 22{,}900(0.16)^5$
C $A = 16(22{,}900)^5$
D $A = 22{,}900(0.84)^5$

24. GRIDDED RESPONSE A deck measures 4 meters by 6 meters. If a painter charges AED 9.75 per square meter including tax, how much will it cost in dirhams to have the deck painted?

Spiral Review

Graph each function. Find the y-intercept and state the domain and range. (Lesson 8-5)

25. $y = 3^x$
26. $y = \left(\frac{1}{2}\right)^x$
27. $y = 6^x$

Evaluate each product. Express the results in both scientific notation and standard form. (Lesson 8-4)

28. $(4.2 \times 10^3)(3.1 \times 10^{10})$
29. $(6.02 \times 10^{23})(5 \times 10^{-14})$
30. $(7 \times 10^5)^2$
31. $(1.1 \times 10^{-2})^2$
32. $(9.1 \times 10^{-2})(4.2 \times 10^{-7})$
33. $(3.14 \times 10^2)(6.1 \times 10^{-3})$

34. EVENT PLANNING A hall does not charge a rental fee as long as at least AED 4,000 is spent on food. For the graduation, the hall charges AED 28.95 per person for a buffet. How many people must attend the graduation to avoid a rental fee for the hall?

Determine whether the graphs of each pair of equations are *parallel*, *perpendicular*, or *neither*.

35. $y = -2x + 11$
$y + 2x = 23$

36. $3y = 2x + 14$
$-3x - 2y = 2$

37. $y = -5x$
$y = 5x - 18$

38. AGES The table shows equivalent ages for horses and humans. Write an equation that relates human age to horse age and find the equivalent horse age for a human who is 16 years old.

Horse age (x)	0	1	2	3	4	5
Human age (y)	0	3	6	9	12	15

Find the total price of each item.

39. umbrella: AED 14.00
tax: 5.5%

40. sandals: AED 29.99
tax: 5.75%

41. backpack: AED 35.00
tax: 7%

Skills Review

Graph each set of ordered pairs.

42. (3, 0), (0, 1), (−4, −6)
43. (0, −2), (−1, −6), (3, 4)
44. (2, 2), (−2, −3), (−3, −6)

Algebra Lab
Transforming Exponential Expressions

You can use the properties of rational exponents to transform exponential functions into other forms in order to solve real-world problems.

Activity Write Equivalent Exponential Expressions

Nabila is trying to decide between two savings account plans. Plan A offers a monthly compounding interest rate of 0.25%, while Plan B offers 2.5% interest compounded annually. Which is the better plan? Explain.

In order to compare the plans, we must compare rates with the same compounding frequency. One way to do this is to compare the approximate monthly interest rates of each plan, also called the *effective* monthly interest rate. While you can use the compound interest formula to find this rate, you can also use the properties of exponents.

Write a function to represent the amount A Nabila would earn after t years with Plan B. For convenience, let the initial amount of Nabila's investment be AED 1.

$y = a(1 + r)^t$	Equation for exponential growth
$A(t) = 1(1 + 0.025)^t$	$y = A(t)$, $a = 1$, $r = 2.5\%$ or 0.025
$= 1.025^t$	Simplify.

Now write a function equivalent to $A(t)$ that represents 12 compoundings per year, with a power of $12t$, instead of 1 per year, with a power of $1t$.

$A(t) = 1.025^{1t}$	Original function
$= 1.025^{\left(\frac{1}{12} \cdot 12\right)t}$	$1 = \frac{1}{12} \cdot 12$
$= \left(1.025^{\frac{1}{12}}\right)^{12t}$	Power of a Power
$\approx 1.0021^{12t}$	$(1.025)^{\frac{1}{12}} = \sqrt[12]{1.025}$ or about 1.0021

From this equivalent function, we can determine that the effective monthly interest by Plan B is about 0.0021 or about 0.21% per month. This rate is less than the monthly interest rate of 0.25% per month offered by Plan A, so Plan A is the better plan.

Model and Analyze

1. Use the compound interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to determine the effective monthly interest rate for Plan B. How does this rate compare to the rate calculated using the method in the Activity above?
2. Write a function to represent the amount A Nabila would earn after t months by Plan A. Then use the properties of exponents to write a function equivalent to $A(t)$ that represents the amount earned after t years.
3. From the expression you wrote in Exercise 2, identify the effective annual interest rate by Plan A. Use this rate to explain why Plan A is the better plan.
4. Suppose Plan A offered a quarterly compounded interest rate of 1.5%. Use the properties of exponents to explain which is the better plan.

LESSON 2-4 Geometric Sequences as Exponential Functions

Then

- You related arithmetic sequences to linear functions.

Now

1. Identify and generate geometric sequences.
2. Relate geometric sequences to exponential functions.

Why?

- You send a chain email to a friend who forwards the email to five more people. Each of these five people forwards the email to five more people. The number of new email generated forms a geometric sequence.

NewVocabulary
geometric sequence
common ratio

Mathematical Practices
Look for and make use of structure.

1 Recognize Geometric Sequences

The first person generates 5 emails. If each of these people sends the email to 5 more people, 25 emails are generated. If each of the 25 people sends 5 emails, 125 emails are generated. The sequence of emails generated, 1, 5, 25, 125, ... is an example of a **geometric sequence**.

In a geometric sequence, the first term is nonzero and each term after the first is found by multiplying the previous term by a nonzero constant r called the **common ratio**. The common ratio can be found by dividing any term by its previous term.

Example 1 Identify Geometric Sequences

Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.

a. 256, 128, 64, 32, ...

Find the ratios of consecutive terms.

256 128 64 32

$\frac{128}{256} = \frac{1}{2}$ $\frac{64}{128} = \frac{1}{2}$ $\frac{32}{64} = \frac{1}{2}$

Since the ratios are constant, the sequence is geometric. The common ratio is $\frac{1}{2}$.

b. 4, 9, 12, 18, ...

Find the ratios of consecutive terms.

4 9 12 18

$\frac{9}{4} = 2\frac{1}{4}$ $\frac{12}{9} = 1\frac{1}{3}$ $\frac{18}{12} = 1\frac{1}{2}$

The ratios are not constant, so the sequence is not geometric.

Find the differences of consecutive terms.

4 9 12 18

$9 - 4 = 5$ $12 - 9 = 3$ $18 - 12 = 6$

There is no common difference, so the sequence is not arithmetic.
Thus, the sequence is neither geometric nor arithmetic.

GuidedPractice

1A. 1, 3, 9, 27, ... **1B.** −20, −15, −10, −5, ... **1C.** 2, 8, 14, 22, ...

Once the common ratio is known, more terms of a sequence can be generated. The formula can be rewritten as $a_n = a_1 r^{n-1}$, where n is a counting number and r is the common ratio.

StudyTip

Structure If the terms of a geometric sequence alternate between positive and negative terms or vice versa, the common ratio is negative.

Example 2 Find Terms of Geometric Sequences

Find the next three terms in each geometric sequence.

a. $1, -4, 16, -64, \ldots$

Step 1 Find the common ratio.

$$1 \quad -4 \quad 16 \quad -64$$

$$\frac{-4}{1} = -4 \qquad \frac{16}{-4} = -4 \qquad \frac{-64}{16} = -4$$

Step 2 Multiply each term by the common ratio to find the next three terms.

$$-64 \xrightarrow{\times(-4)} 256 \xrightarrow{\times(-4)} -1024 \xrightarrow{\times(-4)} 4096$$

The next three terms are 256, −1024, and 4096.

b. $9, 3, 1, \frac{1}{3} \ldots$

Step 1 Find the common ratio.

$$9 \quad 3 \quad 1 \quad \frac{1}{3}$$

$$\frac{3}{9} = \frac{1}{3} \qquad \frac{1}{3} = \frac{1}{3} \qquad \frac{\frac{1}{3}}{3} = \frac{1}{3}$$

The value of r is $\frac{1}{3}$.

Step 2 Multiply each term by the common ratio to find the next three terms.

$$\frac{1}{3} \xrightarrow{\times\frac{1}{3}} \frac{1}{9} \xrightarrow{\times\frac{1}{3}} \frac{1}{27} \xrightarrow{\times\frac{1}{3}} \frac{1}{81}$$

The next three terms are $\frac{1}{9}$, $\frac{1}{27}$, and $\frac{1}{81}$.

GuidedPractice

2A. $-3, 15, -75, 375, \ldots$

2B. $24, 36, 54, 81, \ldots$

Math HistoryLink

Thomas Robert Malthus (1766–1834) Malthus studied populations and had pessimistic views about the future population of the world. In his work, he stated: "Population increases in a geometric ratio, while the means of subsistence increases in an arithmetic ratio."

2 Geometric Sequences and Functions

Finding the nth term of a geometric sequence would be tedious if we used the above method. The table below shows a rule for finding the nth term of a geometric sequence.

Position, n	1	2	3	4	...	n
Term, a_n	a_1	$a_1 r$	$a_1 r^2$	$a_1 r^3$	...	$a_1 r^{n-1}$

Notice that the common ratio between the terms is r. The table shows that to get the nth term, you multiply the first term by the common ratio r raised to the power $n - 1$. A geometric sequence can be defined by an exponential function in which n is the independent variable, a_n is the dependent variable, and r is the base. The domain is the counting numbers.

KeyConcept ***n*th term of a Geometric Sequence**

The nth term a_n of a geometric sequence with first term a_1 and common ratio r is given by the following formula, where n is any positive integer and $a_1, r \neq 0$.

$$a_n = a_1 r^{n-1}$$

WatchOut!

Negative Common Ratio If the common ratio is negative, as in Example 3, make sure to enclose the common ratio in parentheses. $(-2)^8 \neq -2^8$

Example 3 **Find the *n*th Term of a Geometric Sequence**

a. Write an equation for the *n*th term of the sequence −6, 12, −24, 48, … .

The first term of the sequence is −6. So, $a_1 = -6$. Now find the common ratio.

$$-6 \quad 12 \quad -24 \quad 48$$

$$\frac{12}{-6} = -2 \qquad \frac{-24}{12} = -2 \qquad \frac{48}{-24} = -2$$

The common ratio is −2.

$a_n = a_1 r^{n-1}$ Formula for nth term

$a_n = -6(-2)^{n-1}$ $a_1 = -6$ and $r = 2$

b. Find the ninth term of this sequence.

$a_n = a_1 r^{n-1}$ Formula for nth term

$a_9 = -6(-2)^{9-1}$ For the nth term, $n = 9$.

$= -6(-2)^8$ Simplify.

$= -6(256)$ $(-2)^8 = 256$

$= -1536$

GuidedPractice

3. Write an equation for the nth term of the geometric sequence 96, 48, 24, 12, … . Then find the tenth term of the sequence.

Real-WorldLink

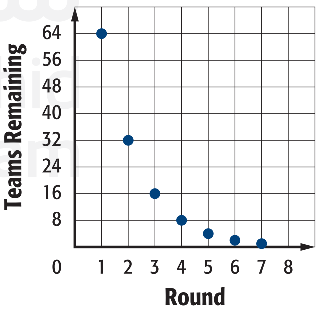
The first NCAA Division I women's basketball tournament was held in 1982. The University of Tennessee has won the most national titles with 8 titles as of 2010.

Source: NCAA Sports

Real-World Example 4 **Graph a Geometric Sequence**

BASKETBALL **The NCAA women's basketball tournament begins with 64 teams. In each round, one half of the teams are left to compete, until only one team remains. Draw a graph to represent how many teams are left in each round.**

Compared to the previous rounds, one half of the teams remain. So, $r = \frac{1}{2}$. Therefore, the geometric sequence that models this situation is 64, 32, 16, 8, 4, 2, 1. So in round two, 32 teams compete, in round three 16 teams compete and so forth. Use this information to draw a graph.



GuidedPractice

4. **TENNIS** A tennis ball is dropped from a height of 12 meters. Each time the ball bounces back to 80% of the height from which it fell. Draw a graph to represent the height of the ball after each bounce.

Check Your Understanding

Example 1 **Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.**

1. 200, 40, 8, … **2.** 2, 4, 16, … **3.** −6, −3, 0, 3, … **4.** 1, −1, 1, −1, …

Example 2 **Find the next three terms in each geometric sequence.**

5. 10, 20, 40, 80, … **6.** 100, 50, 25, … **7.** 4, −1, $\frac{1}{4}$, … **8.** −7, 21, −63, …

Example 3 **Write an equation for the *n*th term of each geometric sequence, and find the indicated term.**

9. the fifth term of −6, −24, −96, …

10. the seventh term of −1, 5, −25, …

11. the tenth term of 72, 48, 32, …

12. the ninth term of 112, 84, 63, …

Example 4 **13.** **EXPERIMENT** In a physics class experiment, Lamis drops a ball from a height of 16 meters. Each bounce has 70% the height of the previous bounce. Draw a graph to represent the height of the ball after each bounce.

Practice and Problem Solving

Example 1 **Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.**

14. 4, 1, 2, … **15.** 10, 20, 30, 40, … **16.** 4, 20, 100, …

17. 212, 106, 53, … **18.** −10, −8, −6, −4, … **19.** 5, −10, 20, 40, …

Example 2 **Find the next three terms in each geometric sequence.**

20. 2, −10, 50, … **21** 36, 12, 4, … **22.** 4, 12, 36, …

23. 400, 100, 25, … **24.** −6, −42, −294, … **25.** 1024, −128, 16, …

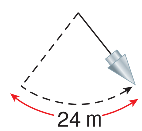
Example 3 **26.** The first term of a geometric series is 1 and the common ratio is 9. What is the 8th term of the sequence?

27. The first term of a geometric series is 2 and the common ratio is 4. What is the 14th term of the sequence?

28. What is the 15th term of the geometric sequence −9, 27, −81, …?

29. What is the 10th term of the geometric sequence 6, −24, 96, …?

Example 4 **30.** **PENDULUM** The first swing of a pendulum is shown. On each swing after that, the arc length is 60% of the length of the previous swing. Draw a graph that represents the arc length after each swing.



31. Find the eighth term of a geometric sequence for which $a_3 = 81$ and $r = 3$.

32. **REASONING** At an online mapping site, Mr. Adnan notices that when he clicks a spot on the map, the map zooms in on that spot. The magnification increases by 20% each time.

a. Write a formula for the *n*th term of the geometric sequence that represents the magnification of each zoom level. (*Hint:* The common ratio is not just 0.2.)

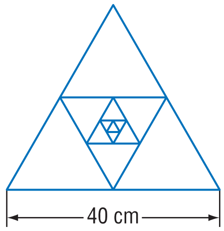
b. What is the fourth term of this sequence? What does it represent?

33 **ALLOWANCE** Laila's parents have offered her two different options to earn her allowance for a 9-week period over the summer. She can either get paid AED 30 each week or AED 1 the first week, AED 2 for the second week, AED 4 for the third week, and so on.

a. Does the second option form a geometric sequence? Explain.

b. Which option should Laila choose? Explain.

34. **SIERPINSKI'S TRIANGLE** Consider the inscribed equilateral triangles at the right. The perimeter of each triangle is one half of the perimeter of the next larger triangle. What is the perimeter of the smallest triangle?



35. If the second term of a geometric sequence is 3 and the third term is 1, find the first and fourth terms of the sequence.

36. If the third term of a geometric sequence is −12 and the fourth term is 24, find the first and fifth terms of the sequence.

37. **EARTHQUAKES** The Richter scale is used to measure the force of an earthquake. The table shows the increase in magnitude for the values on the Richter scale.

Richter Number (x)	Increase in Magnitude (y)	Rate of Change (slope)
1	1	—
2	10	9
3	100	
4	1000	
5	10,000	

a. Copy and complete the table. Remember that the rate of change is the change in y divided by the change in x.

b. Plot the ordered pairs (Richter number, increase in magnitude).

c. Describe the graph that you made of the Richter scale data. Is the rate of change between any two points the same?

d. Write an exponential equation that represents the Richter scale.

H.O.T. Problems Use Higher-Order Thinking Skills

38. **CHALLENGE** Write a sequence that is both geometric and arithmetic. Explain your answer.

39. **CRITIQUE** Ibrahim and Ahmed are finding the ninth term of the geometric sequence −5, 10, −20, … . Is either of them correct? Explain your reasoning.

Ibrahim

$r = \frac{10}{-5}$ or -2

$a_9 = -5(-2)^{9-1}$

$= -5(512)$

$= -2560$

Ahmed

$r = \frac{10}{-5}$ or -2

$a_9 = -5 \cdot (-2)^{9-1}$

$= -5 \cdot -256$

$= 1280$

40. **REASONING** Write a sequence of numbers that form a pattern but are neither arithmetic nor geometric. Explain the pattern.

41. **WRITING IN MATH** How are graphs of geometric sequences and exponential functions similar? different?

42. **WRITING IN MATH** Summarize how to find a specific term of a geometric sequence.

Standardized Test Practice

43. Find the eleventh term of the sequence 3, −6, 12, −24, … .

A 6144
B 3072
C 33
D −6144

44. What is the total amount of the investment shown in the table below if interest is compounded monthly?

Principal	AED 500
Length of Investment	4 years
Annual Interest Rate	5.25%

F AED 613.56
G AED 616.00
H AED 616.56
J AED 718.75

45. **SHORT RESPONSE** Asma has AED 6.50 in 25 fils coins and 10 fils coins. If she has 35 coins in total, how many of each coin does she have?

46. What are the domain and range of the function $y = 4(3^x) - 2$?

A D = {all real numbers}, R = $\{y \mid y > -2\}$
B D = {all real numbers}, R = $\{y \mid y > 0\}$
C D = {all integers}, R = $\{y \mid y > -2\}$
D D = {all integers}, R = $\{y \mid y > 0\}$

Spiral Review

Find the next three terms in each geometric sequence. (Lesson 1-6)

47. 2, 6, 18, 54, …

48. −5, −10, −20, −40, …

49. $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$

50. −3, 1.5, −0.75, 0.375, …

51. 1, 0.6, 0.36, 0.216, …

52. 4, 6, 9, 13.5, …

Graph each function. Find the *y*-intercept and state the domain and range. (Lesson 1-5)

53. $y = \left(\frac{1}{4}\right)^x - 5$

54. $y = 2(4)^x$

55. $y = \frac{1}{2}(3^x)$

56. **LANDSCAPING** A blue spruce grows an average of 15 centimeters per year. A hemlock grows an average of 10 centimeters per year. If a blue spruce is 120 centimeters tall and a hemlock is 180 centimeters tall, write a system of equations to represent their growth. Find and interpret the solution in the context of the situation.

57. **MONEY** City Bank requires a minimum balance of AED 1,500 to maintain free checking services. If Mr. Ismail is going to write checks for the amounts listed in the table, how much money should he start with in order to have free checking?

Check	Amount
750	AED 1,300
751	AED 947

Write an equation in slope-intercept form of the line with the given slope and *y*-intercept.

58. slope: 4, *y*-intercept: 2

59. slope: −3, *y*-intercept: $-\frac{2}{3}$

60. slope: $-\frac{1}{4}$, *y*-intercept: −5

61. slope: $\frac{1}{2}$, *y*-intercept: −9

62. slope: $-\frac{2}{5}$, *y*-intercept: $\frac{3}{4}$

63. slope: −6, *y*-intercept: −7

Skills Review

Simplify each expression. If not possible, write *simplified*.

64. $3u + 10u$

65. $5a - 2 + 6a$

66. $6m^2 - 8m$

67. $4w^2 + w + 15w^2$

68. $13(5 + 4a)$

69. $(4t - 6)16$

EXTEND 2-4

Algebra Lab
Average Rate of Change of Exponential Functions

You know that the rate of change of a linear function is the same for any two points on the graph. The rate of change of an exponential function is not constant.

Activity Evaluating Investment Plans

Ali has AED 2,000 to invest in one of two plans. Plan 1 offers to increase his principal by AED 75 each year, while Plan 2 offers to pay 3.6% interest compounded monthly. The dirham value of each investment after t years is given by $A_1 = 2000 + 75t$ and $A_2 = 2000(1.003)^{12t}$, respectively. Use the function values, the average rate of change, and the graphs of the equations to interpret and compare the plans.

Step 1 Copy and complete the table below by finding the missing values for A_1 and A_2.

t	0	1	2	3	4	5
A_1						
A_2						

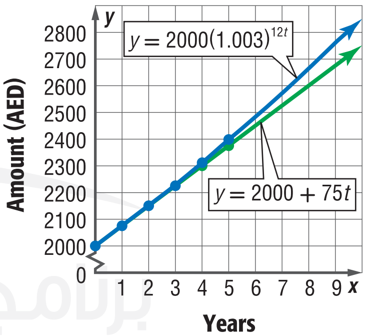
Step 2 Find the average rate of change for each plan from $t = 0$ to 1, $t = 3$ to 4, and $t = 0$ to 5.

Plan 1: $\frac{2075 - 2000}{1 - 0}$ or 75 $\quad$ $\frac{2300 - 2225}{4 - 3}$ or 75 $\quad$ $\frac{2375 - 2000}{5 - 0}$ or 75

Plan 2: $\frac{2073.2 - 2000}{1 - 0}$ or 73.2 $\quad$ $\frac{2309.27 - 2227.74}{4 - 3}$ or about 82 $\quad$ $\frac{2393.79 - 2000}{5 - 0}$ or about 79

Step 3 Graph the ordered pairs for each function. Connect each set of points with a smooth curve.

Step 4 Use the graph and the rates of change to compare the plans. Both graphs have a rate of change for the first year of about AED 75 per year. From year 3 to 4, Plan 1 continues to increase at AED 75 per year, but Plan 2 grows at a rate of more than AED 81 per year. The average rate of change over the first five years for Plan 1 is AED 75 per year and for Plan 2 is over AED 78 per year. This indicates that as the number of years increases, the investment in Plan 2 grows at an increasingly faster pace. This is supported by the widening gap between their graphs.



Exercises

The value of a company's piece of equipment decreases over time due to depreciation. The function $y = 16{,}000(0.985)^{2t}$ represents the value after t years.

1. What is the average rate of change over the first five years?
2. What is the average rate of change of the value from year 5 to year 10?
3. What conclusion about the value can we make based on these average rates of change?
4. **REGULARITY** Copy and complete the table for $y = x^4$.

x	−3	−2	−1	0	1	2	3
y							

Compare and interpret the average rate of change for $x = -3$ to 0 and for $x = 0$ to 3.

LESSON 2-5 Recursive Formulas

Then

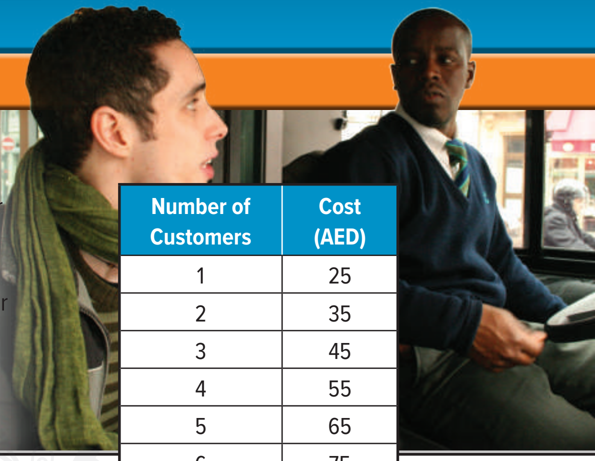
- You wrote explicit formulas to represent arithmetic and geometric sequences.

Now

1. Use a recursive formula to list terms in a sequence.
2. Write recursive formulas for arithmetic and geometric sequences.

Why?

- Clients of a shuttle service get picked up from their homes and driven to premium outlet stores for shopping. The total cost of the service depends on the total number of customers. The costs for the first six customers are shown.



Number of Customers	Cost (AED)
1	25
2	35
3	45
4	55
5	65
6	75

NewVocabulary
recursive formula

Mathematical Practices
Construct viable arguments and critique the reasoning of others.

1 Using Recursive Formulas

An explicit formula allows you to find any term a_n of a sequence by using a formula written in terms of n. For example, $a_n = 2n$ can be used to find the fifth term of the sequence 2, 4, 6, 8, ...: $a_5 = 2(5)$ or 10.

A **recursive formula** allows you to find the nth term of a sequence by performing operations to one or more of the preceding terms. Since each term in the sequence above is 2 greater than the term that preceded it, we can add 2 to the fourth term to find that the fifth term is 8 + 2 or 10. We can then write a recursive formula for a_n.

$$\begin{aligned}
a_1 &= & &= 2\\
a_2 &= a_1 + 2 \text{ or } 2 + 2 & &= 4\\
a_3 &= a_2 + 2 \text{ or } 4 + 2 & &= 6\\
a_4 &= a_3 + 2 \text{ or } 6 + 2 & &= 8\\
&\vdots \qquad \vdots & &\\
a_n &= a_{n-1} + 2 & &
\end{aligned}$$

A recursive formula for the sequence above is $a_1 = 2$, $a_n = a_{n-1} + 2$, for $n \geq 2$ where n is an integer. The term denoted a_{n-1} represents the term immediately before a_n. Notice that the first term a_1 is given, along with the domain for n.

Example 1 Use a Recursive Formula

Find the first five terms of the sequence in which $a_1 = 7$ and $a_n = 3a_{n-1} - 12$, if $n \geq 2$.

Use $a_1 = 7$ and the recursive formula to find the next four terms.

$a_2 = 3a_{2-1} - 12$ $n = 2$
$= 3a_1 - 12$ Simplify.
$= 3(7) - 12$ or 9 $a_1 = 7$

$a_3 = 3a_{3-1} - 12$ $n = 3$
$= 3a_2 - 12$ Simplify.
$= 3(9) - 12$ or 15 $a_2 = 9$

$a_4 = 3a_{4-1} - 12$ $n = 4$
$= 3a_3 - 12$ Simplify.
$= 3(15) - 12$ or 33 $a_3 = 15$

$a_5 = 3a_{5-1} - 12$ $n = 5$
$= 3a_4 - 12$ Simplify.
$= 3(33) - 12$ or 87 $a_4 = 33$

The first five terms are 7, 9, 15, 33, and 87.

GuidedPractice

1. Find the first five terms of the sequence in which $a_1 = -2$ and $a_n = (-3)a_{n-1} + 4$, if $n \geq 2$.

2 Writing Recursive Formulas

To write a recursive formula for an arithmetic or geometric sequence, complete the following steps.

KeyConcept Writing Recursive Formulas

Step 1 Determine if the sequence is arithmetic or geometric by finding a common difference or a common ratio.

Step 2 Write a recursive formula.

Arithmetic Sequences	$a_n = a_{n-1} + d$, where d is the common difference
Geometric Sequences	$a_n = r \cdot a_{n-1}$, where r is the common ratio

Step 3 State the first term and domain for n.

StudyTip

Defining n For the nth term of a sequence, the value of n must be a positive integer. Although we must still state the domain of n, from this point forward, we will assume that n is an integer.

Example 2 Write Recursive Formulas

Write a recursive formula for each sequence.

a. 17, 13, 9, 5, ...

Step 1 First subtract each term from the term that follows it.

$13 - 17 = -4 \qquad 9 - 13 = -4 \qquad 5 - 9 = -4$

There is a common difference of -4. The sequence is arithmetic.

Step 2 Use the formula for an arithmetic sequence.

$a_n = a_{n-1} + d$ — Recursive formula for arithmetic sequence

$a_n = a_{n-1} + (-4)$ — $d = -4$

Step 3 The first term a_1 is 17, and $n \geq 2$.

A recursive formula for the sequence is $a_1 = 17$, $a_n = a_{n-1} - 4$, $n \geq 2$.

StudyTip

Domain The domain for n is decided by the given terms. Since the first term is already given, it makes sense that the first term to which the formula would apply is the 2nd term of the sequence, or when $n = 2$.

b. 6, 24, 96, 384, ...

Step 1 First subtract each term from the term that follows it.

$24 - 6 = 18 \qquad 96 - 24 = 72 \qquad 384 - 96 = 288$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$\frac{24}{6} = 4 \qquad \frac{96}{24} = 4 \qquad \frac{384}{96} = 4$

There is a common ratio of 4. The sequence is geometric.

Step 2 Use the formula for a geometric sequence.

$a_n = r \cdot a_{n-1}$ — Recursive formula for geometric sequence

$a_n = 4a_{n-1}$ — $r = 4$

Step 3 The first term a_1 is 6, and $n \geq 2$.

A recursive formula for the sequence is $a_1 = 6$, $a_n = 4a_{n-1}$, $n \geq 2$.

GuidedPractice

2A. 4, 10, 25, 62.5, ...

2B. 9, 36, 63, 90, ...

Real-World Career

Transportation The number of jobs in the transportation industry is expected to grow by an estimated 1.1 million between 2004 and 2014. The specific fields dictate the educational requirements, which include a high school diploma and some form of specialized training.

Source: United States Department of Labor

A sequence can be represented by both an explicit formula and a recursive formula.

Example 3 Write Recursive and Explicit Formulas

COST **Refer to the beginning of the lesson. Let n be the number of customers.**

a. Write a recursive formula for the sequence.

Steps 1 and 2 First subtract each term from the term that follows it.

$35 - 25 = 10 \qquad 45 - 35 = 10 \qquad 55 - 45 = 10$

There is a common difference of 10. The sequence is arithmetic.

Step 3 Use the formula for an arithmetic sequence.

$a_n = a_{n-1} + d$ — Recursive formula for arithmetic sequence

$a_n = a_{n-1} + 10$ — $d = 10$

Step 4 The first term a_1 is 25, and $n \geq 2$.

A recursive formula for the sequence is $a_1 = 25$, $a_n = a_{n-1} + 10$, $n \geq 2$.

b. Write an explicit formula for the sequence.

Step 1 The common difference is 10.

Step 2 Use the formula for the nth term of an arithmetic sequence.

$$
\begin{aligned}
a_n &= a_1 + (n-1)d && \text{Formula for the } n\text{th term} \\
&= 25 + (n-1)10 && a_1 = 25 \text{ and } d = 10 \\
&= 25 + 10n - 10 && \text{Distributive Property} \\
&= 10n + 15 && \text{Simplify.}
\end{aligned}
$$

An explicit formula for the sequence is $a_n = 10n + 15$.

Guided Practice

3. **SAVINGS** The money that Badr has in his savings account earns interest each year. He does not make any withdrawals or additional deposits. The account balance at the beginning of each year is AED 10,000, AED 10,300, AED 10,609, AED 10,927.27, and so on. Write a recursive formula and an explicit formula for the sequence.

If several successive terms of a sequence are needed, a recursive formula may be useful, whereas if just the nth term of a sequence is needed, an explicit formula may be useful. Thus, it is sometimes beneficial to translate between the two forms.

Example 4 Translate between Recursive and Explicit Formulas

a. Write a recursive formula for $a_n = 6n + 3$.

$a_n = 6n + 3$ is an explicit formula for an arithmetic sequence with $d = 6$ and $a_1 = 6(1) + 3$ or 9. Therefore, a recursive formula for a_n is $a_1 = 9$, $a_n = a_{n-1} + 6$, $n \geq 2$.

b. Write an explicit formula for $a_1 = 120$, $a_n = 0.8a_{n-1}$, $n \geq 2$.

$a_n = 0.8a_{n-1}$ is a recursive formula for a geometric sequence with $a_1 = 120$ and $r = 0.8$. Therefore, an explicit formula for a_n is $a_n = 120(0.8)^{n-1}$.

Study Tip

Geometric Sequence Recall that the formula for the nth term of a geometric sequence is $a_n = a_1 r^{n-1}$.

Guided Practice

4A. Write a recursive formula for $a_n = 4(3)^{n-1}$.

4B. Write an explicit formula for $a_1 = -16$, $a_n = a_{n-1} - 7$, $n \geq 2$.

Check Your Understanding

Example 1 **Find the first five terms of each sequence.**

1. $a_1 = 16,\ a_n = a_{n-1} - 3,\ n \geq 2$

2. $a_1 = -5,\ a_n = 4a_{n-1} + 10,\ n \geq 2$

Example 2 **Write a recursive formula for each sequence.**

3. 1, 6, 11, 16, ...

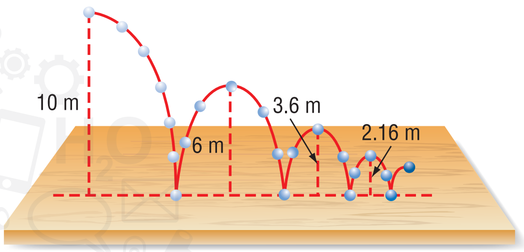
4. 4, 12, 36, 108, ...

Example 3

5. **BALL** A ball is dropped from an initial height of 10 meters. The maximum heights the ball reaches on the first three bounces are shown.

a. Write a recursive formula for the sequence.

b. Write an explicit formula for the sequence.



Example 4 **For each recursive formula, write an explicit formula. For each explicit formula, write a recursive formula.**

6. $a_1 = 4,\ a_n = a_{n-1} + 16,\ n \geq 2$

7 $a_n = 5n + 8$

8. $a_n = 15(2)^{n-1}$

9. $a_1 = 22,\ a_n = 4a_{n-1},\ n \geq 2$

Practice and Problem Solving

Example 1 **Find the first five terms of each sequence.**

10. $a_1 = 23,\ a_n = a_{n-1} + 7,\ n \geq 2$

11. $a_1 = 48,\ a_n = -0.5a_{n-1} + 8,\ n \geq 2$

12. $a_1 = 8,\ a_n = 2.5a_{n-1},\ n \geq 2$

13. $a_1 = 12,\ a_n = 3a_{n-1} - 21,\ n \geq 2$

14. $a_1 = 13,\ a_n = -2a_{n-1} - 3,\ n \geq 2$

15. $a_1 = \frac{1}{2},\ a_n = a_{n-1} + \frac{3}{2},\ n \geq 2$

Example 2 **Write a recursive formula for each sequence.**

16. 12, −1, −14, −27, ...

17. 27, 41, 55, 69, ...

18. 2, 11, 20, 29, ...

19. 100, 80, 64, 51.2, ...

20. 40, −60, 90, −135, ...

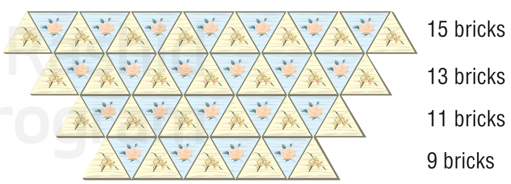
21. 81, 27, 9, 3, ...

Example 3

22. **MODELING** A landscaper is building a brick patio. Part of the patio includes a pattern constructed from triangles. The first four rows of the pattern are shown.

a. Write a recursive formula for the sequence.

b. Write an explicit formula for the sequence.



Example 4 **For each recursive formula, write an explicit formula. For each explicit formula, write a recursive formula.**

23. $a_n = 3(4)^{n-1}$

24. $a_1 = -2,\ a_n = a_{n-1} - 12,\ n \geq 2$

25. $a_1 = 38,\ a_n = \frac{1}{2}a_{n-1},\ n \geq 2$

26. $a_n = -7n + 52$

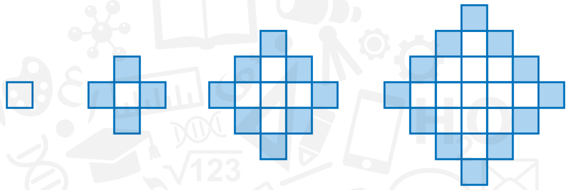
27 **TEXTING** Amani received a chain text that she forwarded to five of her friends. Each of her friends forwarded the text to five more friends, and so on.

a. Find the first five terms of the sequence representing the number of people who receive the text in the nth round.

b. Write a recursive formula for the sequence.

c. If Amani represents a_1, find a_8.

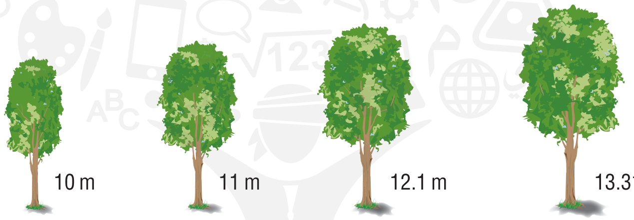
28. **GEOMETRY** Consider the pattern below. The number of blue boxes increases according to a specific pattern.



a. Write a recursive formula for the sequence of the number of blue boxes in each figure.

b. If the first box represents a_1, find the number of blue boxes in a_8.

29. **TREE** The growth of a certain type of tree slows as the tree continues to age. The heights of the tree over the past four years are shown.



a. Write a recursive formula for the height of the tree.

b. If the pattern continues, how tall will the tree be in two more years? Round your answer to the nearest tenth of a meter.

30. **MULTIPLE REPRESENTATIONS** The Fibonacci sequence is neither arithmetic nor geometric and can be defined by a recursive formula. The first terms are 1, 1, 2, 3, 5, 8, ...

a. **Logical** Determine the relationship between the terms of the sequence. What are the next five terms in the sequence?

b. **Algebraic** Write a formula for the nth term if $a_1 = 1$, $a_2 = 1$, and $n \geq 3$.

c. **Algebraic** Find the 15th term.

d. **Analytical** Explain why the Fibonacci sequence is not an arithmetic sequence.

H.O.T. Problems Use Higher-Order Thinking Skills

31. **ERROR ANALYSIS** Bilal and Jassim are working on a math problem that involves the sequence 2, −2, 2, −2, 2, ... Bilal thinks that the sequence can be written as a recursive formula. Jassim believes that the sequence can be written as an explicit formula. Is either of them correct? Explain.

32. **CHALLENGE** Find a_1 for the sequence in which $a_4 = 1104$ and $a_n = 4a_{n-1} + 16$.

33. **ARGUMENTS** Determine whether the following statement is *true* or *false*. Justify your reasoning.

There is only one recursive formula for every sequence.

34. **CHALLENGE** Find a recursive formula for 4, 9, 19, 39, 79, ...

35. **WRITING IN MATH** Explain the difference between an explicit formula and a recursive formula.

Standardized Test Practice

36. Find a recursive formula for the sequence 12, 24, 36, 48, ...

A $a_1 = 12, a_n = 2a_{n-1}, n \geq 2.$

B $a_1 = 12, a_n = 4a_{n-1} - 24, n \geq 2.$

C $a_1 = 12, a_n = a_{n-1} + 12, n \geq 2.$

D $a_1 = 12, a_n = 12a_{n-1} + 12, n \geq 2.$

37. GEOMETRY The area of a rectangle is $36m^4n^6$ square meters. The length of the rectangle is $6m^3n^3$ meters. What is the width of the rectangle?

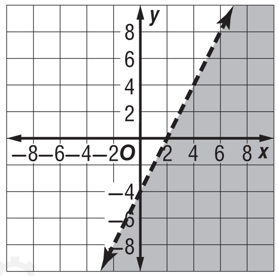
F $216m^7n^9$ m

G $6mn^3$ m

H $42m^7n^3$ m

J $30mn^3$ m

38. Find an inequality for the graph shown.



A $y > 2x - 4$ **C** $y < 2x - 4$

B $y \geq 2x - 4$ **D** $y \leq 2x - 4$

39. Write an equation of the line that passes through (−2, −20) and (4, 58).

F $y = 13x + 6$ **H** $y = 19x + 18$

G $y = 19x - 18$ **J** $y = 13x - 6$

Spiral Review

Find the next three terms in each geometric sequence. (Lesson 2-4)

40. 675, 225, 75, ...

41. 16, −24, 36, ...

42. 6, 18, 54, ...

43. 512, −256, 128, ...

44. 125, 25, 5, ...

45. 12, 60, 300, ...

46. INVESTMENT Ahmed invested AED 2,000 with a 5.75% interest rate compounded monthly. How much money will Ahmed have after 5 years? (Lesson 2-6)

47. TOURS Rashid's family and Saleh's family are traveling together on a trip to visit a candy factory. The number of people in each family and the total cost are shown in the table below. Find the adult and children's admission prices. (Lesson 6-3)

Family	Number of Adults	Number of Children	Total Cost
Rashid	2	3	AED 58
Saleh	2	1	AED 38

Write each equation in standard form. (Lesson 4-3)

48. $y + 6 = -3(x + 2)$

49. $y - 12 = 4(x - 7)$

50. $y + 9 = 5(x - 3)$

51. $y - 1 = \frac{1}{3}(x + 15)$

52. $y + 10 = \frac{2}{5}(x - 6)$

53. $y - 4 = -\frac{2}{7}(x + 1)$

Skills Review

Simplify each expression. If not possible, write *simplified*.

54. $8x + 3y^2 + 7x - 2y$

55. $4(x - 16) + 6x$

56. $4n - 3m + 9m - n$

57. $6r^2 + 7r$

58. $-2(4g - 5h) - 6g$

59. $9x^2 - 7x + 16y^2$

EXPLORE

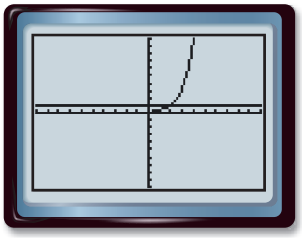
2-6 Graphing Technology Lab
Solving Exponential Equations and Inequalities

You can use a graphing calculator to solve exponential equations by graphing or by using the table feature. To do this, you will write the equations as systems of equations.

Activity 1

Solve $3^{x-4} = \frac{1}{9}$.

Step 1 Graph each side of the equation as a separate function. Enter 3^{x-4} as **Y1**. Be sure to include parentheses around the exponent. Enter $\frac{1}{9}$ as **Y2**. Then graph the two equations.



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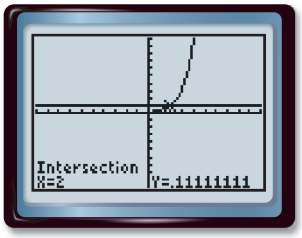
Step 2 Use the **intersect** feature.

You can use the **intersect** feature on the **CALC** menu to approximate the ordered pair of the point at which the graphs cross.

The calculator screen shows that the x-coordinate of the point at which the curves cross is 2. Therefore, the solution of the equation is 2.



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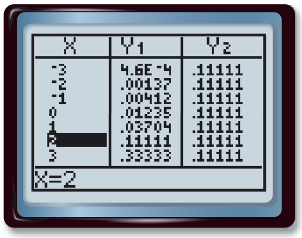
Step 3 Use the **TABLE** feature.

You can also use the **TABLE** feature to locate the point at which the curves intersect.

The table displays x-values and corresponding y-values for each graph. Examine the table to find the x-value for which the y-values of the graphs are equal.

At $x = 2$, both functions have a y-value of $0.\overline{1}$ or $\frac{1}{9}$. Thus, the solution of the equation is 2.



CHECK Substitute 2 for x in the original equation.

$3^{x-4} \stackrel{?}{=} \frac{1}{9}$ Original equation

$3^{2-4} \stackrel{?}{=} \frac{1}{9}$ Substitute 2 for x.

$3^{-2} \stackrel{?}{=} \frac{1}{9}$ Simplify.

$\frac{1}{9} = \frac{1}{9}$ ✓ The solution checks.

A similar procedure can be used to solve exponential inequalities.

(continued on the next page)

Solving Exponential Equations and Inequalities *Continued*

Activity 2 Description

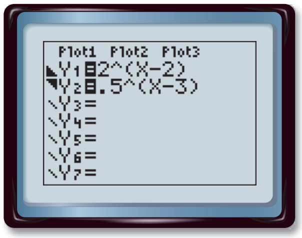
Solve $2^{x-2} \geq 0.5^{x-3}$.

Step 1 Enter the related inequalities.

Rewrite the problem as a system of inequalities.

The first inequality is $2^{x-2} \geq y$ or $y \leq 2^{x-2}$. Since this inequality includes the *less than or equal to* symbol, shade below the curve.

First enter the boundary, and then use the arrow and [ENTER] keys to choose the shade below icon, ◣.



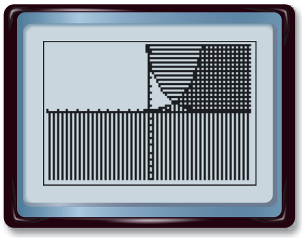
The second inequality is $y \geq 0.5^{x-3}$. Shade above the curve since this inequality contains *greater than or equal to*.

KEYSTROKES: [Y=] [◀] [◀] [ENTER] [ENTER] [ENTER] [▶] [▶] 2 [^] [(] [X,T,θ,n] [−] 2 [)] [ENTER] [◀] [◀] [ENTER] [ENTER] [▶] [▶] .5 [^] [(] [X,T,θ,n] [−] 3 [)]

Step 2 Graph the system.

KEYSTROKES: [GRAPH]

The x-values of the points in the region where the shadings overlap is the solution set of the original inequality. Using the **intersect** feature, you can conclude that the solution set is $\{x \mid x \geq 2.5\}$.



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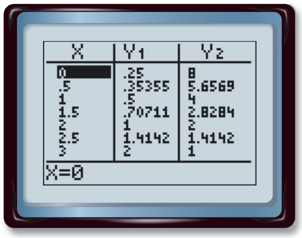
Step 3 Use the **TABLE** feature.

Verify using the **TABLE** feature. Set up the table to show x-values in increments of 0.5.

KEYSTROKES: [2nd] [TBLSET] 0 [ENTER] .5 [ENTER] [2nd] [TABLE]



Notice that for x-values greater than $x = 2.5$, **Y1** > **Y2**. This confirms that the solution of the inequality is $\{x \mid x \geq 2.5\}$.

Exercises

Solve each equation or inequality.

1. $9^{x-1} = \frac{1}{81}$
2. $4^{x+3} = 2^{5x}$
3. $5^{x-1} = 2^{x}$
4. $3.5^{x+2} = 1.75^{x+3}$
5. $-3^{x+4} = -0.5^{2x+3}$
6. $6^{2-x} - 4 < -0.25^{x-2.5}$
7. $16^{x-1} > 2^{2x+2}$
8. $3^{x} - 4 \leq 5^{\frac{x}{2}}$
9. $5^{x+3} \leq 2^{x+4}$
10. **WRITING IN MATH** Explain why this technique of graphing a system of equations or inequalities works to solve exponential equations and inequalities.

LESSON 2-6 Solving Exponential Equations and Inequalities

Then

- You graphed exponential functions.

Now

1. Solve exponential equations.
2. Solve exponential inequalities.

Why?

- Membership on Internet social networking sites tends to increase exponentially. The membership growth of one Web site can be modeled by the equation $y = 2.2(1.37)^x$, where x is the number of years since 2004 and y is the number of members in millions.

  You can use $y = 2.2(1.37)^x$ to determine how many members there will be in a given year, or to determine the year in which membership was at a certain level.

NewVocabulary
exponential equation
compound morabaha
exponential inequality

Mathematical Practices
Reason abstractly and quantitatively.

1 Solve Exponential Equations

In an **exponential equation**, variables occur as exponents.

KeyConcept Property of Equality for Exponential Functions

Words	Let $b > 0$ and $b \neq 1$. Then $b^x = b^y$ if and only if $x = y$.
Example	If $3^x = 3^5$, then $x = 5$. If $x = 5$, then $3^x = 3^5$.

The Property of Equality can be used to solve exponential equations.

Example 1 Solve Exponential Equations

Solve each equation.

a. $2^x = 8^3$

$2^x = 8^3$	Original equation
$2^x = (2^3)^3$	Rewrite 8 as 2^3.
$2^x = 2^9$	Power of a Power
$x = 9$	Property of Equality for Exponential Functions

b. $9^{2x-1} = 3^{6x}$

$9^{2x-1} = 3^{6x}$	Original equation
$(3^2)^{2x-1} = 3^{6x}$	Rewrite 9 as 3^2.
$3^{4x-2} = 3^{6x}$	Power of a Power
$4x - 2 = 6x$	Property of Equality for Exponential Functions
$-2 = 2x$	Subtract $4x$ from each side.
$-1 = x$	Divide each side by 2.

GuidedPractice

1A. $4^{2n-1} = 64$

1B. $5^{5x} = 125^{x+2}$

You can use information about growth or decay to write the equation of an exponential function.

Real-World Example 2 Write an Exponential Function

SCIENCE **Huda starts an experiment with 7500 bacteria cells. After 4 hours, there are 23,000 cells.**

a. Write an exponential function that could be used to model the number of bacteria after x hours if the number of bacteria changes at the same rate.

At the beginning of the experiment, the time is 0 hours and there are 7500 bacteria cells. Thus, the y-intercept, and the value of a, is 7500.

When $x = 4$, the number of bacteria cells is 23,000. Substitute these values into an exponential function to determine the value of b.

$y = ab^x$	Exponential function
$23{,}000 = 7500 \cdot b^4$	Replace x with 4, y with 23,000, and a with 7500.
$3.067 \approx b^4$	Divide each side by 7500.
$\sqrt[4]{3.067} \approx b$	Take the 4th root of each side.
$1.323 \approx b$	Use a calculator.

An equation that models the number of bacteria is $y \approx 7500(1.323)^x$.

b. How many bacteria cells can be expected in the sample after 12 hours?

$y \approx 7500(1.323)^x$	Modeling equation
$\approx 7500(1.323)^{12}$	Replace x with 12.
$\approx 215{,}665$	Use a calculator.

There will be approximately 215,665 bacteria cells after 12 hours.

Real-WorldLink

In 2008, the U.S. recycling rate for metals of 35% prevented the release of approximately 25 million metric tons of carbon into the air—roughly the amount emitted annually by 4.5 million cars.

Source: Environmental Protection Agency

GuidedPractice

2. **RECYCLING** A manufacturer distributed 3.2 million aluminum cans in 2005.

A. In 2010, the manufacturer distributed 420,000 cans made from the recycled cans it had previously distributed. Assuming that the recycling rate continues, write an equation to model the distribution each year of cans that are made from recycled aluminum.

B. How many cans made from recycled aluminum can be expected in the year 2050?

Exponential functions are used in situations involving compound morabaha. **Compound morabaha** is morabaha paid on the principal of an investment and any previously earned morabaha.

KeyConcept Compound Morabaha

You can calculate compound morabaha using the following formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt},$$

where A is the amount in the account after t years, P is the principal amount invested, r is the annual morabaha rate, and n is the number of compounding periods each year.

Example 3 Compound Morabaha

An investment account pays 4.2% annual interest compounded monthly. If AED 2500 is invested in this account, what will be the balance after 15 years?

Understand Find the total amount in the account after 15 years.

Plan Use the compound interest formula.

$P = 2500$, $r = 0.042$, $n = 12$, and $t = 15$

Solve

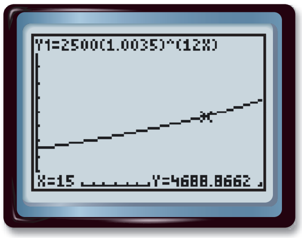
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$ Compound interest Formula

$$= 2500\left(1 + \frac{0.042}{12}\right)^{12 \cdot 15}$$ $P = 2500, r = 0.042, n = 12, t = 15$

$$\approx 4688.87$$ Use a calculator.

Check Graph the corresponding equation $y = 2500(1.0035)^{12t}$. Use **CALC: value** to find y when $x = 15$.

The y-value 4688.8662 is very close to 4688.87, so the answer is reasonable.



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WatchOut!

Percents Remember to convert all percents to decimal form; 4.2% is 0.042.

GuidedPractice

3. Find the account balance after 20 years if AED 100 is placed in an account that pays 1.2% interest compounded twice a month.

2 Solve Exponential Inequalities

An **exponential inequality** is an inequality involving exponential functions.

KeyConcept Property of Inequality for Exponential Functions

Words	Let $b > 1$. Then $b^x > b^y$ if and only if $x > y$, and $b^x < b^y$ if and only if $x < y$.
Example	If $2^x > 2^6$, then $x > 6$. If $x > 6$, then $2^x > 2^6$.

This property also holds true for $\leq$ and $\geq$.

Example 4 Solve Exponential Inequalities

Solve $16^{2x-3} < 8$.

$16^{2x-3} < 8$ Original inequality

$(2^4)^{2x-3} < 2^3$ Rewrite 16 as 2^4 and 8 as 2^3.

$2^{8x-12} < 2^3$ Power of a Power

$8x - 12 < 3$ Property of Inequality for Exponential Functions

$8x < 15$ Add 12 to each side.

$x < \frac{15}{8}$ Divide each side by 8.

GuidedPractice

Solve each inequality.

4A. $3^{2x-1} \geq \frac{1}{243}$

4B. $2^{x+2} > \frac{1}{32}$

Check Your Understanding

Example 1 **Solve each equation.**

1. $3^{5x} = 27^{2x-4}$
2. $16^{2y-3} = 4^{y+1}$
3. $2^{6x} = 32^{x-2}$
4. $49^{x+5} = 7^{8x-6}$

Example 2

5. **SCIENCE** Mitosis is a process in which one cell divides into two. The *Escherichia coli* is one of the fastest growing bacteria. It can reproduce itself in 15 minutes.
   a. Write an exponential function to represent the number of cells c after t minutes.
   b. If you begin with one *Escherichia coli* cell, how many cells will there be in one hour?

Example 3

6. A certificate of deposit (CD) pays 2.25% annual interest compounded biweekly. If you deposit AED 500 into this CD, what will the balance be after 6 years?

Example 4 **Solve each inequality.**

7. $4^{2x+6} \leq 64^{2x-4}$
8. $25^{y-3} \leq \left(\frac{1}{125}\right)^{y+2}$

Practice and Problem Solving

Example 1 **Solve each equation.**

9. $8^{4x+2} = 64$
10. $5^{x-6} = 125$
11. $81^{a+2} = 3^{3a+1}$
12. $256^{b+2} = 4^{2-2b}$
13. $9^{3c+1} = 27^{3c-1}$
14. $8^{2y+4} = 16^{y+1}$

Example 2

15. **MODELING** In 2009, Reham received AED 10,000 from her grandmother. Her parents invested all of the money, and by 2021, the amount will have grown to AED 16,960.
    a. Write an exponential function that could be used to model the money y. Write the function in terms of x, the number of years since 2009.
    b. Assume that the amount of money continues to grow at the same rate. What would be the balance in the account in 2031?

Write an exponential function for the graph that passes through the given points.

16. (0, 6.4) and (3, 100)
17. (0, 256) and (4, 81)
18. (0, 128) and (5, 371,293)
19. (0, 144), and (4, 21,609)

Example 3

20. Find the balance of an account after 7 years if AED 700 is deposited into an account paying 4.3% interest compounded monthly.
21. Determine how much is in a retirement account after 20 years if AED 5000 was invested at 6.05% interest compounded weekly.
22. A savings account offers 0.7% interest compounded bimonthly. If AED 110 is deposited in this account, what will the balance be after 15 years?
23. A college savings account pays 13.2% annual interest compounded semiannually. What is the balance of an account after 12 years if AED 21,000 was initially deposited?

Example 4 **Solve each inequality.**

24. $625 \geq 5^{a+8}$
25. $10^{5b+2} > 1000$
26. $\left(\frac{1}{64}\right)^{c-2} < 32^{2c}$
27. $\left(\frac{1}{27}\right)^{2d-2} \leq 81^{d+4}$
28. $\left(\frac{1}{9}\right)^{3t+5} \geq \left(\frac{1}{243}\right)^{t-6}$
29. $\left(\frac{1}{36}\right)^{w+2} < \left(\frac{1}{216}\right)^{4w}$

30. **SCIENCE** A mug of hot chocolate is 90°C at time $t = 0$. It is surrounded by air at a constant temperature of 20°C. If stirred steadily, its temperature in Celsius after t minutes will be $y(t) = 20 + 70(1.071)^{-t}$.

a. Find the temperature of the hot chocolate after 15 minutes.

b. Find the temperature of the hot chocolate after 30 minutes.

c. The optimum drinking temperature is 60°C. Will the mug of hot chocolate be at or below this temperature after 10 minutes?

31 **ANIMALS** Studies show that an animal will defend a territory, with area in square yards, that is directly proportional to the 1.31 power of the animal's weight in pounds.

a. If a 45-pound beaver will defend 170 square yards, write an equation for the area a defended by a beaver weighing w pounds.

b. Scientists believe that thousands of years ago, the beaver's ancestors were 11 feet long and weighed 430 pounds. Use your equation to determine the area defended by these animals.

Solve each equation.

32. $\left(\frac{1}{2}\right)^{4x+1} = 8^{2x+1}$

33. $\left(\frac{1}{5}\right)^{x-5} = 25^{3x+2}$

34. $216 = \left(\frac{1}{6}\right)^{x+3}$

35. $\left(\frac{1}{8}\right)^{3x+4} = \left(\frac{1}{4}\right)^{-2x+4}$

36. $\left(\frac{2}{3}\right)^{5x+1} = \left(\frac{27}{8}\right)^{x-4}$

37. $\left(\frac{25}{81}\right)^{2x+1} = \left(\frac{729}{125}\right)^{-3x+1}$

38. **MODELING** In 1950, the world population was about 2.556 billion. By 1980, it had increased to about 4.458 billion.

a. Write an exponential function of the form $y = abx$ that could be used to model the world population y in billions for 1950 to 1980. Write the equation in terms of x, the number of years since 1950. (Round the value of b to the nearest ten-thousandth.)

b. Suppose the population continued to grow at that rate. Estimate the population in 2000.

c. In 2000, the population of the world was about 6.08 billion. Compare your estimate to the actual population.

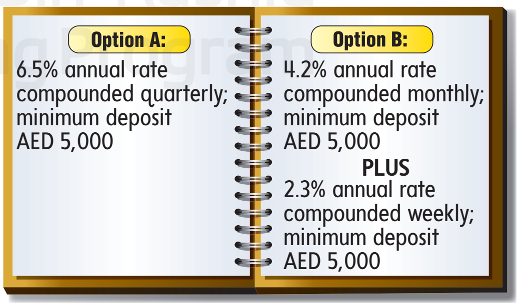
d. Use the equation you wrote in Part **a** to estimate the world population in the year 2020. How accurate do you think the estimate is? Explain your reasoning.

39. **TREES** The diameter of the base of a tree trunk in centimeters varies directly with the $\frac{3}{2}$ power of its height in meters.

a. A young sequoia tree is 6 meters tall, and the diameter of its base is 19.1 centimeters. Use this information to write an equation for the diameter d of the base of a sequoia tree if its height is h meters high.

b. The General Sherman Tree in Sequoia National Park, California, is approximately 84 meters tall. Find the diameter of the General Sherman Tree at its base.

40. **FINANCIAL LITERACY** Mrs. Amna has two different retirement investment plans from which to choose.



a. Write equations for Option A and Option B given the minimum deposits.

b. Draw a graph to show the balances for each investment option after t years.

c. Explain whether Option A or Option B is the better investment choice.

41. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the rapid increase of an exponential function. A large sheet of paper is cut in half, and one of the resulting pieces is placed on top of the other. Then the pieces in the stack are cut in half and placed on top of each other. Suppose this procedure is repeated several times.

a. **Concrete** Perform this activity and count the number of sheets in the stack after the first cut. How many pieces will there be after the second cut? How many pieces after the third cut? How many pieces after the fourth cut?

b. **Tabular** Record your results in a table.

c. **Symbolic** Use the pattern in the table to write an equation for the number of pieces in the stack after x cuts.

d. **Analytical** The thickness of ordinary paper is about 0.003 cm. Write an equation for the thickness of the stack of paper after x cuts.

e. **Analytical** How thick will the stack of paper be after 30 cuts?

H.O.T. Problems Use Higher-Order Thinking Skills

42. **WRITING IN MATH** In a problem about compound interest, describe what happens as the compounding period becomes more frequent while the principal and overall time remain the same.

43. **ERROR ANALYSIS** Amna and Badreya are solving $6^{x-3} > 36^{-x-1}$. Is either of them correct? Explain your reasoning.

Amna	Badreya
$6^{x-3} > 36^{-x-1}$	$6^{x-3} > 36^{-x-1}$
$6^{x-3} > (6^2)^{-x-1}$	$6^{x-3} > (6^2)^{-x-1}$
$6^{x-3} > 6^{-2x-2}$	$6^{x-3} > 6^{-x+1}$
$x-3 > -2x-2$	$x-3 > -x+1$
$3x > 1$	$2x > 4$
$x > \frac{1}{3}$	$x > 2$

44. **CHALLENGE** Solve for x: $16^{18} + 16^{18} + 16^{18} + 16^{18} + 16^{18} = 4^x$.

45. **OPEN ENDED** What would be a more beneficial change to a 5-year loan at 8% morabaha compounded monthly: reducing the term to 4 years or reducing the morabaha rate to 6.5%?

46. **ARGUMENTS** Determine whether the following statements are *sometimes*, *always*, or *never* true. Explain your reasoning.

a. $2^x > -8^{20x}$ for all values of x.

b. The graph of an exponential growth equation is increasing.

c. The graph of an exponential decay equation is increasing.

47. **OPEN ENDED** Write an exponential inequality with a solution of $x \leq 2$.

48. **PROOF** Show that $27^{2x} \cdot 81^{x+1} = 3^{2x+2} \cdot 9^{4x+1}$.

49. **WRITING IN MATH** If you were given the initial and final amounts of a radioactive substance and the amount of time that passes, how would you determine the rate at which the amount was increasing or decreasing in order to write an equation?

Standardized Test Practice

50. $3 \times 10^{-4} =$

A 0.003 **C** 0.00003
B 0.0003 **D** 0.000003

51. Which of the following could *not* be a solution to $5 - 3x < -3$?

F 2.5 **H** 3.5
G 3 **J** 4

52. GRIDDED RESPONSE The three angles of a triangle are $3x$, $x + 10$, and $2x - 40$. Find the measure of the smallest angle in the triangle.

53. SAT/ACT Which of the following is equivalent to $(x)(x)(x)(x)$ for all x?

A $x + 4$ **D** $4x^2$
B $4x$ **E** x^4
C $2x^2$

Spiral Review

Graph each function. (Lesson 3-1)

54. $y = 2(3)^x$ **55.** $y = 5(2)^x$ **56.** $y = 4\left(\frac{1}{3}\right)^x$

Use the Distributive Property to factor each polynomial. (Lesson 1-5)

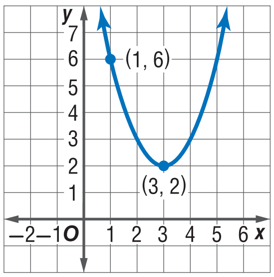
57. $4m^3n^2 + 16m^2n^3 - 8m^3n^4$ **58.** $12j^4k^4 + 36j^3k^2 - 3j^2k^5$

Factor each polynomial. (Lesson 0-3)

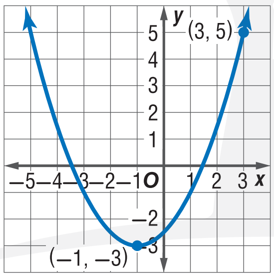
59. $x^2 - 4x + 3xy - 12y$ **60.** $4a - 10ab + 6b - 15b^2$

Write an equation in vertex form for each parabola. (Lesson 3-7)

61.



62.



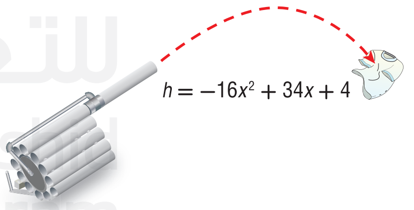
Graph each function. State the domain and range. (Lesson 3-1)

63. $f(x) = \frac{2}{3}(2^x)$ **64.** $f(x) = 4^x + 3$ **65.** $f(x) = 2\left(\frac{1}{3}\right)^x - 1$

66. PRIZES A machine is used to throw T-shirts into the crowd at basketball games. (Lesson 1-7)

a. What is the initial height of the T-shirt?

b. If the T-shirt is caught after 2 seconds, what is the height?



Skills Review

Find $[g \circ h](x)$ and $[h \circ g](x)$.

66. $h(x) = 2x - 1$
$g(x) = 3x + 4$

67. $h(x) = x^2 + 2$
$g(x) = x - 3$

68. $h(x) = x^2 + 1$
$g(x) = -2x + 1$

69. $h(x) = -5x$
$g(x) = 3x - 5$

70. $h(x) = x^3$
$g(x) = x - 2$

71. $h(x) = x + 4$
$g(x) = |x|$

EXTEND 2-6

Algebra Lab: Transforming Exponential Expressions

You can use the properties of rational exponents to transform exponential functions into other forms in order to solve real-world problems.

Activity Write Equivalent Exponential Expressions

Ghaya is trying to decide between two savings account plans. Plan A offers a monthly compounding interest rate of 0.25%, while Plan B offers 2.5% interest compounded annually. Which is the better plan? Explain.

In order to compare the plans, we must compare rates with the same compounding frequency. One way to do this is to compare the approximate monthly interest rates of each plan, also called the *effective* monthly interest rate. While you can use the compound interest formula to find this rate, you can also use the properties of exponents.

Write a function to represent the amount A Ghaya would earn after t years with Plan B. For convenience, let the initial amount of Ghaya's investment be AED 1.

$y = a(1 + r)^t$	Equation for exponential growth
$A(t) = 1(1 + 0.025)^t$	$y = A(t)$, $a = 1$, $r = 2.5\%$ or 0.025
$= 1.025^t$	Simplify.

Now write a function equivalent to $A(t)$ that represents 12 compoundings per year, with a power of $12t$, instead of 1 per year, with a power of $1t$.

$A(t) = 1.025^{1t}$	Original function
$= 1.025^{\left(\frac{1}{12} \cdot 12\right)t}$	$1 = \frac{1}{12} \cdot 12$
$= \left(1.025^{\frac{1}{12}}\right)^{12t}$	Power of a Power
$\approx 1.0021^{12t}$	$(1.025)^{\frac{1}{12}} = \sqrt[12]{1.025}$ or about 1.0021

From this equivalent function, we can determine that the effective monthly interest by Plan B is about 0.0021 or about 0.21% per month. This rate is less than the monthly interest rate of 0.25% per month offered by Plan A, so Plan A is the better plan.

Model and Analyze

1. Use the compound interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to determine the effective monthly interest rate for Plan B. How does this rate compare to the rate calculated using the method in the Activity above?
2. Write a function to represent the amount A Ghaya would earn after t months by Plan A. Then use the properties of exponents to write a function equivalent to $A(t)$ that represents the amount earned after t years.
3. From the expression you wrote in Exercise 2, identify the effective annual interest rate by Plan A. Use this rate to explain why Plan A is the better plan.
4. Suppose Plan A offered a quarterly compounded interest rate of 1.5%. Use the properties of exponents to explain which is the better plan.

CHAPTER 2

Study Guide and Review

Study Guide

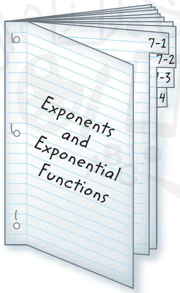
KeyConcepts

Eponential Functions

- The equation for exponential growth is $y = a(1 + r)^t$, where $r > 0$. The equation for exponential decay is $y = a(1 - r)^t$, where $0 < r < 1$. y is the final amount, a is the initial amount, r is the rate of change, and t is the time in years.

FOLDABLES® StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

common ratio	monomial
compound interest	negative exponent
constant	order of magnitude
exponential decay	rational exponent
exponential equation	recursive formula
exponential function	scientific notation
exponential growth	zero exponent
geometric sequence	

VocabularyCheck

Choose the word or term that best completes each sentence.

1. $7xy^4$ is an example of a(n) ____________.

2. The ____________ of 95,234 is 10^5.

3. 2 is a(n) ____________ of 8.

4. The rules for operations with exponents can be extended to apply to expressions with a(n) ____________ such as $7^{\frac{2}{3}}$.

5. A number written in ____________ is of the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer.

6. $f(x) = 3^x$ is an example of a(n) ____________.

7. $a_1 = 4$ and $a_n = 3a_{n-1} - 12$, if $n \geq 2$, is a(n) ____________ for the sequence 4, −8, −20, −32, ...

8. $2^{3x-1} = 16$ is an example of a(n) ____________.

9. The equation for ____________ is $y = C(1 - r)^t$.

10. If $a^n = b$ for a positive integer n, then a is a(n) ____________ of b.

Lesson-by-Lesson Review

2-1 Exponential Functions

Graph each function. Find the *y*-intercept, and state the domain and range.

42. $y = 2^x$

43. $y = 3^x + 1$

44. $y = 4^x + 2$

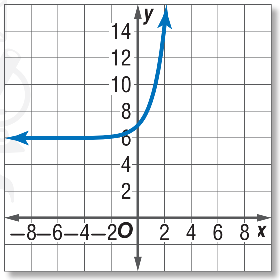
45. $y = 2^x - 3$

46. **BIOLOGY** The population of bacteria in a petri dish increases according to the model $p = 550(2.7)^{0.008t}$, where t is the number of hours and $t = 0$ corresponds to 1:00 P.M. Use this model to estimate the number of bacteria in the dish at 5:00 P.M.

Example 1

Graph $y = 3^x + 6$. Find the *y*-intercept, and state the domain and range.

x	$3^x + 6$	y
−3	$3^{-3} + 6$	6.04
−2	$3^{-2} + 6$	6.11
−1	$3^{-1} + 6$	6.33
0	$3^0 + 6$	7
1	$3^1 + 6$	9



The *y*-intercept is (0, 7). The domain is all real numbers, and the range is all real numbers greater than 6.

2-2 Analyzing Functions with Successive Differences

Look for a pattern in each table of values to determine which kind of model best describes the data. Then write an equation for the function that models the data.

45.

x	0	1	2	3	4
y	0	3	12	27	48

46.

x	0	1	2	3	4
y	1	2	4	8	16

47.

x	0	1	2	3	4
y	0	−1	−4	−9	−16

Example 2

Determine the model that best describes the data. Then write an equation for the function that models the data.

x	0	1	2	3	4
y	3	4	5	6	7

Step 1 First differences: 3 4 5 6 7

1 1 1 1

A linear function models the data.

Step 2 The slope is 1 and the *y*-intercept is 3, so the equation is $y = x + 3$.

2-3 Growth and Decay

47. Find the final value of AED 2,500 invested at an interest rate of 2% compounded monthly for 10 years.

48. **COMPUTERS** Alia's computer is depreciating at a rate of 3% per year. She bought the computer for AED 1,200.
  a. Write an equation to represent this situation.
  b. What will the computer's value be after 5 years?

Example 3

Find the final value of AED 2,000 invested at an interest rate of 3% compounded quarterly for 8 years.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest equation

$= 2000\left(1 + \frac{0.03}{4}\right)^{4(8)}$ $P = 2000$, $r = 0.03$, $n = 4$, and $t = 8$

$\approx$ AED 2,540.22 Use a calculator.

2-4 Geometric Sequences as Exponential Functions

Find the next three terms in each geometric sequence.

49. $-1, 1, -1, 1, \ldots$

50. $3, 9, 27, \ldots$

51. $256, 128, 64, \ldots$

Write the equation for the *n*th term of each geometric sequence.

52. $-1, 1, -1, 1, \ldots$

53. $3, 9, 27, \ldots$

54. $256, 128, 64, \ldots$

55. **SPORTS** A basketball is dropped from a height of 20 meters. It bounces to $\frac{1}{2}$ its height after each bounce. Draw a graph to represent the situation.

Example 4

Find the next three terms in the geometric sequence 2, 6, 18, ...

Step 1 Find the common ratio. Each number is 3 times the previous number, so $r = 3$.

Step 2 Multiply each term by the common ratio to find the next three terms.

$18 \times 3 = 54$, $54 \times 3 = 162$, $162 \times 3 = 486$

The next three terms are 54, 162, and 486.

Example 5

Write the equation for the *n*th term of the geometric sequence $-3, 12, -48, \ldots$.

The common ratio is -4. So $r = -4$.

$a_n = a_1 r^{n-1}$ Formula for the *n*th term

$a_n = -3(-4)^{n-1}$ $a_1 = -3$ and $r = -4$

2-5 Recursive Formulas

Find the first five terms of each sequence.

56. $a_1 = 11, a_n = a_{n-1} - 4, n \geq 2$

57. $a_1 = 3,\ a_n = 2a_{n-1} + 6, n \geq 2$

Write a recursive formula for each sequence.

58. 2, 7, 12, 17, ...

59. 32, 16, 8, 4, ...

60. 2, 5, 11, 23, ...

Example 6

Write a recursive formula for 3, 1, −1, −3, ...

Step 1 First subtract each term from the term that follows it.

$1 - 3 = -2, -1 - 1 = -2, -3 - (-1) = -2$

There is a common difference of −2. The sequence is arithmetic.

Step 2 Use the formula for an arithmetic sequence.

$a_n = a_{n-1} + d$ Recursive formula

$a_n = a_{n-1} + (-2)$ $d = -2$

Step 3 The first term a_1 is 3, and $n \geq 2$.

A recursive formula is $a_1 = 3, a_n = a_{n-1} - 2, n \geq 2$.

2-6 Solving Exponential Equations and Inequalities

Solve each equation or inequality.

12. $16^x = \frac{1}{64}$

13. $3^{4x} = 9^{3x+7}$

14. $64^{3n} = 8^{2n-3}$

15. $8^{3-3y} = 256^{4y}$

16. $9^{x-2} > \left(\frac{1}{81}\right)^{x+2}$

17. $27^{3x} \leq 9^{2x-1}$

18. BACTERIA A bacteria population started with 5000 bacteria. After 8 hours there were 28,000 in the sample.

a. Write an exponential function that could be used to model the number of bacteria after x hours if the number of bacteria changes at the same rate.

Example 7

Solve $4^{3x} = 32^{x-1}$ for x.

$4^{3x} = 32^{x-1}$ Original equation

$(2^2)^{3x} = (2^5)^{x-1}$ Rewrite so each side has the same base.

$2^{6x} = 2^{5x-5}$ Power of a Power

$6x = 5x - 5$ Property of Equality for Exponential Functions

$x = -5$ Subtract $5x$ from each side.

The solution is −5.

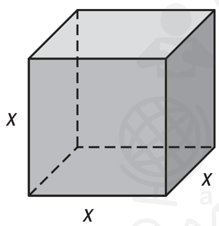
CHAPTER 2 Practice Test

Simplify each expression.

1. $(x^2)(7x^8)$

2. $(5a^7bc^2)(-6a^2bc^5)$

3. MULTIPLE CHOICE Express the volume of the solid as a monomial.



A x^3 **C** $6x^3$

B $6x$ **D** x^6

Simplify each expression. Assume that no denominator equals 0.

4. $\dfrac{x^6y^8}{x^2}$

5. $\left(\dfrac{2a^4b^3}{c^6}\right)^0$

6. $\dfrac{2xy^{-7}}{8x}$

Simplify.

7. $\sqrt[3]{1000}$

8. $\sqrt[5]{3125}$

9. $1728^{\frac{1}{3}}$

10. $\left(\dfrac{16}{81}\right)^{\frac{1}{2}}$

11. $27^{\frac{2}{3}}$

12. $10{,}000^{\frac{3}{4}}$

13. $27^{\frac{5}{3}}$

14. $\left(\dfrac{1}{121}\right)^{\frac{3}{2}}$

Solve each equation.

15. $12^x = 1728$

16. $7^{x-1} = 2401$

17. $9^{x-3} = 729$

Express each number in scientific notation.

18. 0.00021

19. 58,000

Express each number in standard form.

20. 2.9×10^{-5}

21. 9.1×10^6

Evaluate each product or quotient. Express the results in scientific notation.

22. $(2.5 \times 10^3)(3 \times 10^4)$

23. $\dfrac{8.8 \times 10^2}{4 \times 10^{-4}}$

24. ASTRONOMY The average distance from Mercury to the Sun is 57,910,000 km. Express this distance in scientific notation.

Graph each function. Find the *y*-intercept, and state the domain and range.

25. $y = 2(5)^x$

26. $y = -3(11)^x$

27. $y = 3^x + 2$

Find the next three terms in each geometric sequence.

28. 2, −6, 18, ...

29. 1000, 500, 250, ...

30. 32, 8, 2, ...

31. MULTIPLE CHOICE Shaima invested AED 500 into an account with a 6.5% interest rate compounded monthly. How much will Shaima's investment be worth in 10 years?

F AED 600.00

G AED 938.57

H AED 956.09

J AED 957.02

32. INVESTMENTS Suha's investment of AED 3,000 has been losing value at a rate of 3% each year. What will her investment be worth in 6 years?

Find the first five terms of each sequence.

33. $a_1 = 18$, $a_n = a_{n-1} - 4$, $n \geq 2$

34. $a_1 = -2$, $a_n = 4a_{n-1} + 5$, $n \geq 2$

CHAPTER 2 Preparing for Standardized Tests

Using a Scientific or Graphing Calculator

Scientific and graphing calculators are powerful problem-solving tools. There are times when a calculator can be used to make computations faster and easier, such as computations with very large numbers. However, there are times when using a calculator is necessary, like the estimation of irrational numbers.

Strategies for Using a Scientific or Graphing Calculator

Step 1

Familiarize yourself with the various functions of a scientific or graphing calculator as well as when they should be used:

- **Exponents** scientific notation, calculating with large or small numbers
- **Pi** solving circle problems, like circumference and area
- **Square roots** distance on a coordinate plane, Pythagorean theorem
- **Graphs** analyzing paired data in a scatter plot, graphing functions, finding roots of equations

Step 2

Use your scientific or graphing calculator to solve the problem.

- Remember to work as efficiently as possible. Some steps may be done mentally or by hand, while others should be completed using your calculator.
- If time permits, check your answer.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

The distance from the Sun to Jupiter is approximately 7.786×10^{11} meters. If the speed of light is about 3×10^{8} meters per second, how long does it take for light from the Sun to reach Jupiter? Round to the nearest minute.

A about 43 minutes

B about 51 minutes

C about 1876 minutes

D about 2595 minutes

Read the problem carefully. You are given the approximate distance from the Sun to Jupiter as well as the speed of light. Both quantities are given in scientific notation. You are asked to find how many minutes it takes for light from the Sun to reach Jupiter. Use the relationship distance = rate × time to find the amount of time.

$d = r \times t$

$\frac{d}{r} = t$

To find the amount of time, divide the distance by the rate. Notice, however, that the units for time will be seconds.

$\frac{7.786 \times 10^{11}\text{ m}}{3 \times 10^{8}\text{ m/s}} = t$ seconds

Use a scientific calculator to quickly find the quotient. On most scientific calculators, the EE key is used to enter numbers in scientific notation.

KEYSTROKES: (7.786 2nd [EE] 11) ÷ (3 2nd [EE] 8) ENTER

The result is 2595.33333333 seconds. To convert this number to minutes, use your calculator to divide the result by 60. This gives an answer of about 43.2555 minutes. The answer is A.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

1. Since its creation 5 years ago, approximately 2.504×10^7 items have been sold or traded on a popular online website. What is the average daily number of items sold or traded over the 5-year period?

   A about 9640 items per day

   B about 13,720 items per day

   C about 1,025,000 items per day

   D about 5,008,000 items per day

2. Evaluate $\sqrt{ab}$ if $a = 121$ and $b = 23$.

   F about 5.26

   G about 9.90

   H about 12

   J about 52.75

3. The population of the United States is about 3.034×10^8 people. The land area of the country is about 3.54 to 9.17 square kilometer. What is the average *population density* (number of people per square kilometer) of the United States?

   A about 136.3 people per square kilometer

   B about 30.2 people per square kilometer

   C about 94.3 people per square kilometer

   D about 33.1 people per square kilometer

4. Ghaya is making a cover for the marching band's drum. The drum has a diameter of 20 centimeters. Estimate the area of the face of the bass drum.

   F 31.41 square centimeters

   G 62.83 square centimeters

   H 78.54 square centimeters

   J 314.16 square centimeters

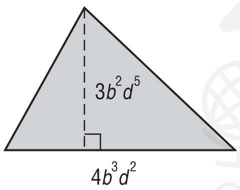
CHAPTER 2 Standardized Test Practice

Cumulative, Chapters 1 through 2

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Express the area of the triangle below as a monomial.



A $12b^5d^7$

B $12b^6d^{10}$

C $6b^6d^{10}$

D $6b^5d^7$

2. Simplify the following expression.

$$\left(\frac{2w^2z^5}{3y^4}\right)^3$$

F $\frac{2w^5z^8}{3y^7}$

G $\frac{8w^6z^{15}}{27y^{12}}$

H $\frac{8w^5z^8}{27y^7}$

J $\frac{2w^6z^{15}}{3y^{12}}$

3. Which equation of a line is perpendicular to $y = \frac{3}{5}x - 3$?

A $y = -\frac{5}{3}x + 2$

B $y = -\frac{3}{5}x + 2$

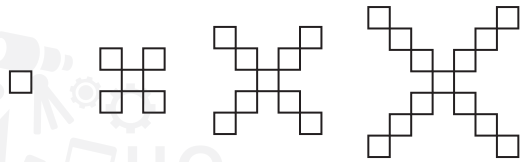
C $y = \frac{5}{3}x - 2$

D $y = \frac{3}{5}x - 2$

Test-Taking Tip

Question 2 Use the laws of exponents to simplify the expression. Remember, to find the power of a power, multiply the exponents.

4. Write a recursive formula for the sequence of the number of squares in each figure.



F $a_1 = 1, a_n = 4a_{n-1} - 3, n \geq 1$

G $a_1 = 1, a_n = 4a_{n-1}, n \geq 2$

H $a_1 = 1, a_n = a_{n-1} + 4, n \geq 2$

J $a_1 = 1, a_n = 4a_{n-1} + 4, n \geq 2$

5. Evaluate $(4.2 \times 10^6)(5.7 \times 10^8)$.

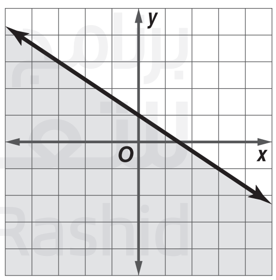
A 2.394×10^{15}

B 23.94×10^{14}

C 9.9×10^{14}

D 2.394×10^{48}

6. Which inequality is shown in the graph?



F $y \leq -\frac{2}{3}x - 1$

G $y \leq -\frac{3}{4}x - 1$

H $y \leq -\frac{2}{3}x + 1$

J $y \leq -\frac{3}{4}x + 1$

Short Response/Gridded Response

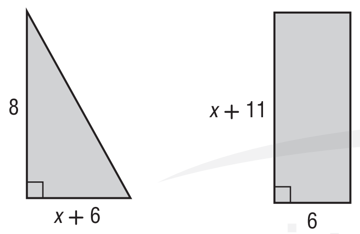
7. Saeed created a Web site for the Science Olympiad team. The total number of hits the site has received is shown.

Day	Total Hits	Day	Total Hits
3	5	17	27
6	7	21	33
10	12	26	40
13	17	34	55

a. Find an equation for the regression line.

b. Predict the number of total hits that the Web site will have received on day 46.

8. Find the value of x so that the figures have the same area.



9. What is the solution to the following system of equations? Show your work.

$$\begin{cases} y = 6x - 1 \\ y = 6x + 4 \end{cases}$$

10. **GRIDDED RESPONSE** At a family fun center, Amer's and Abdalla's families each bought video game tokens and batting cage tokens as shown in the table.

Family	Amer	Abdalla
Number of Video Game Tokens	25	30
Number of Batting Cage Tokens	8	6
Total Cost	AED 26.50	AED 25.50

What is the cost in dirhams of a batting cage token at the family fun center?

Extended Response

Record your answers on a sheet of paper. Show your work.

11. The table below shows the distances from the Sun to Mercury, Earth, Mars, and Saturn. Use the data to answer each question.

Planet	Distance from Sun (km)
Mercury	5.79×10^7
Earth	1.50×10^8
Mars	2.28×10^8
Saturn	1.43×10^9

a. Of the planets listed, which one is the closest to the Sun?

b. About how many times as far from the Sun is Mars as Earth?