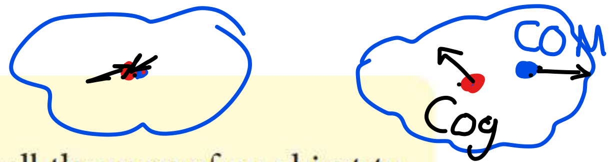
8.1 Center of Mass and Center of Gravity



Definition

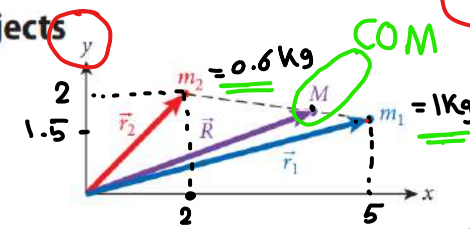
The **center of mass** is the point at which we can imagine all the mass of an object to be concentrated.

Thus, the center of mass is also the point at which we can imagine the force of gravity acting on the entire object to be concentrated. If we can imagine all of the mass to be concentrated at this point when calculating the force due to gravity, it is legitimate to call this point the *center of gravity*, a term that can often be used interchangeably with *center of mass*. (To be precise, we should note that these two terms are only equivalent in situations where the gravitational force is constant everywhere throughout the object. In Chapter 12, we will see that this is not the case for very large objects.)

Combined Center of Mass for Two Objects

$$\vec{R} = \frac{\vec{r}_1 m_1 + \vec{r}_2 m_2}{m_1 + m_2}.$$

$$X = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2},\quad Y = \frac{y_1 m_1 + y_2 m_2}{m_1 + m_2},\quad Z = \frac{z_1 m_1 + z_2 m_2}{m_1 + m_2}.$$



$x = \frac{5 \times 1 + 2 \times 0.6}{1 + 0.6}$

$\boxed{x = \quad m}$

$y = \frac{1.5 \times 1 + 2 \times 0.6}{1 + 0.6} =$

Concept Check 8.1

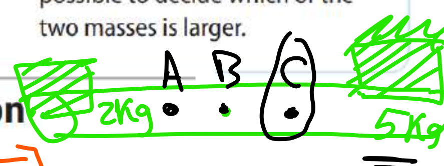
In the case shown in Figure 8.2, what are the relative magnitudes of the two masses m_1 and m_2?

a) $m_1 < m_2$

b) $m_1 > m_2$

c) $m_1 = m_2$

d) Based solely on the information given in the figure, it is not possible to decide which of the two masses is larger.

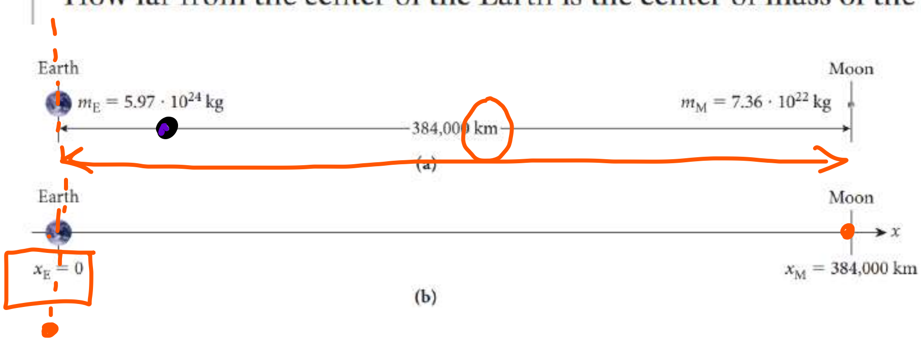


SOLVED PROBLEM 8.1 Center of Mass of Earth and Moon

The Earth has a mass of $5.97 \cdot 10^{24}$ kg, and the Moon has a mass of $7.36 \cdot 10^{22}$ kg. The Moon orbits the Earth at a distance of 384,000 km; that is, the center of the Moon is a distance of 384,000 km from the center of Earth, as shown in Figure 8.3a.

PROBLEM

How far from the center of the Earth is the center of mass of the Earth-Moon system?



$X = \frac{r_E \times M_E + r_M \times M_M}{M_E + M_M}$

$= \frac{0 + 384000 \times 1000 \times 7.36 \times 10^{22}}{(\quad) + (\quad)}$

Combined Center of Mass for Several Objects

$$X = \frac{x_1 m_1 + x_2 m_2 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

where M represents the combined mass of all n objects:

$$M = \sum_{i=1}^{n} m_i.$$

Writing equation 8.3 in Cartesian components, we obtain

$$X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i,\quad Y = \frac{1}{M}\sum_{i=1}^{n} y_i m_i,\quad Z = \frac{1}{M}\sum_{i=1}^{n} z_i m_i.$$

EXAMPLE 8.1 Shipping Containers

Large freight containers, which can be transported by truck, railroad, or ship, come in standard sizes. One of the most common sizes is the ISO 20' container, which has a length of 6.1 m, L W a width of 2.4 m, and a height of 2.6 m. This container is allowed to have a mass (including its contents, of course) of up to 30,400 kg. h

PROBLEM

mo 2mo

The five freight containers shown in Figure 8.4 sit on the deck of a container ship. Each one has a mass of 9,000 kg, except for the red one, which has a mass of 18,000 kg. Assume that each of the containers has an individual center of mass at its geometric center. What are the x-coordinate and the y-coordinate of the containers' combined center of mass? Use the coordinate system shown in the figure to describe the location of this center of mass.

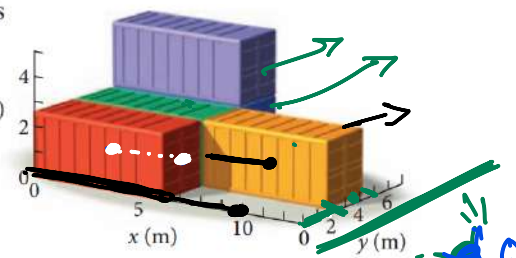


FIGURE 8.4 Freight containers arranged on the deck of a container ship.

SOLUTION

$$X = \frac{\frac{L}{2}\times 2m_0 + \frac{L}{2}\times m_0 + \frac{L}{2}\times m_0 + \frac{L}{2}\times m_0 + \frac{3L}{2}\times m_0}{2m_0 + m_0 + m_0 + m_0 + m_0} \quad *$$

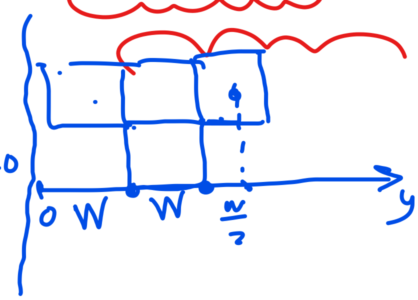
$$= \frac{\cancel{m_0}\left(L + \frac{L}{2} + \frac{L}{2} + \frac{L}{2} + \frac{3L}{2}\right)}{\cancel{m_0}(2+1+1+1+1)} = \ldots\ldots \text{ m}$$

$M_{red} = 18000\text{ kg}$

$m_0 = 9000$

$m_{red} = 2m_0$

$$Y = \frac{\frac{W}{2}\times 2m_0 + \frac{3W}{2}\times m_0 + \frac{3W}{2}\times m_0 + \frac{5W}{2}\times m_0 + \frac{5W}{2}\times m_0}{2m_0 + m_0 + m_0 + m_0 + m_0}$$



$r = \sqrt{x^2+y^2}$ $\quad \{\theta = \tan^{-1}\left(\frac{y}{x}\right)\}$

9.1 Polar Coordinates

Trigonometry provides the relationship between the Cartesian coordinates x and y and the polar coordinates θ and r:

$$\boxed{r=\sqrt{x^2+y^2}} \tag{9.1}$$

$$\boxed{\theta=\tan^{-1}(y/x).} \tag{9.2}$$

The inverse transformation from polar to Cartesian coordinates is given by

$$x = r\cos\theta \tag{9.3}$$

$$y = r\sin\theta. \tag{9.4}$$

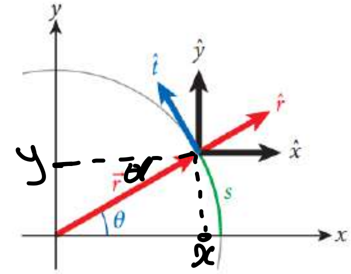


FIGURE 9.3 Polar coordinate system for circular motion.

→ radial unit vector

$$\hat{r}=\frac{x}{r}\hat{x}+\frac{y}{r}\hat{y}=(\cos\theta)\hat{x}+(\sin\theta)\hat{y}\equiv(\cos\theta,\sin\theta).$$

→ tangential unit vector

$$\hat{t}=\frac{-y}{r}\hat{x}+\frac{x}{r}\hat{y}=(-\sin\theta)\hat{x}+(\cos\theta)\hat{y}\equiv(-\sin\theta,\cos\theta).$$

$\theta = 30^\circ \Rightarrow \vec{r} = ?\ (\ ,\ )$

$r = (\cos 30, \sin 30)$

$t = (-\sin 30, \cos 30)$

9.2 Angular Coordinates and Angular Displacement

$$\Delta\theta=\theta_2-\theta_1.$$

$$\theta\ (\text{degrees})\frac{\pi}{180}=\theta\ (\text{radians})\Leftrightarrow\theta\ (\text{radians})\frac{180}{\pi}=\theta\ (\text{degrees})$$

$$1\ \text{rad}=\frac{180^\circ}{\pi}\approx 57.3^\circ.$$

$\boxed{2\pi\ \text{rad} = 360^\circ}$

$2\pi = 360^\circ$

$\theta = 45^\circ$

Ex $\theta = 45^\circ \Rightarrow \theta_{rad} = ?$

$\theta \times \cancel{360} = 2\pi \times 45^\circ$ $\quad \frac{2\pi \times 45}{360} = \ldots$ (rad)

EXAMPLE 9.1 Locating a Point with Cartesian and Polar Coordinates

A point has a location given in Cartesian coordinates as (4,3), as shown in Figure 9.5.

x, y

☐ (5, 0.34 rad)

☑ (5, 0.64 rad)

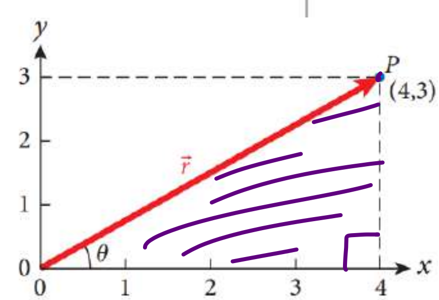
PROBLEM

How do we represent the position of this point in polar coordinates?

(r, θ)

$r = \sqrt{4^2+3^2} = 5$ m

$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.86^\circ$



2π rad $= 360^\circ$

$\theta_{rad} = 36.86^\circ$

$\theta = \frac{36.86 \times 2\pi}{360} = 0.64$ rad

(deg حاسبة الآلة)

2π rad = 360°
0.5 rad = θ
$\boxed{\theta = 28.6°}$ →

given $r = 6$ m , $\theta = 0.5$ rad
$x = ?$
$y = ?$

$x = r\cos\theta = 6\cos 28.6 = \ldots$ m
$y = 6\sin 28.6 = \ldots$ m

Arc Length $s = r\theta.$

10 m

2 m

EXAMPLE 9.2 **CD Track**

$\times 10^{-3}$ or $(\div 1000)$

The track on a compact disc (CD) is represented in Figure 9.6. The track is a spiral, originating at an inner radius of $\boxed{r_1 = 25 \text{ mm}}$ and terminating at an outer radius of $\boxed{r_2 = 58 \text{ mm}}$. The spacing between successive loops of the track is a constant, $\boxed{\Delta r = 1.6\ \mu\text{m}}$.

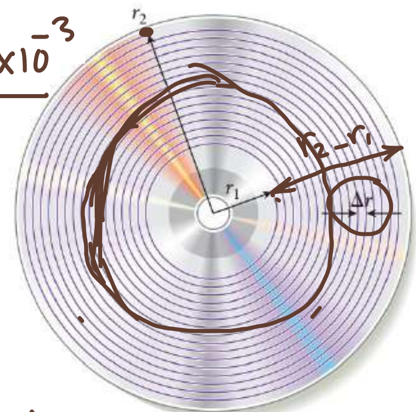
→ $\times 10^{-6}$ $(\div 1000000)$

PROBLEM
What is the total length of this track?

$L = ?$

$$n = \frac{r_2 - r_1}{\Delta r} = \frac{58\times10^{-3} - 25\times10^{-3}}{1.6\times10^{-6}}$$

$$\boxed{n = 20625}$$



average

$$r_{avg} = \frac{r_2 + r_1}{2} = \frac{58\times10^{-3} + 25\times10^{-3}}{2} = 0.0415 \text{ m}$$

$$\bar{S}_{avg} = 2\pi \bar{r}_{avg} = 2\pi(0.0415) = 0.2607 \text{ m}$$

$L = 20625 \times 0.2607$
$L = \text{-----}$ m

9.3 Angular Velocity, Angular Frequency, and Period

$v = \frac{x_f - x_i}{t_2 - t_1}$

$$\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}.$$

$t_1 =$
$t_2 =$

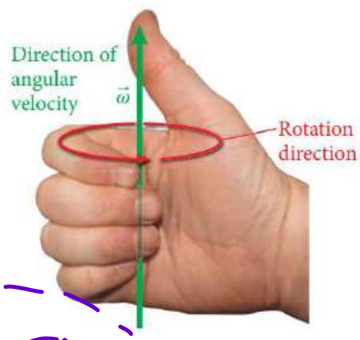
$$\omega = \lim_{\Delta t \to 0} \bar{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \equiv \frac{d\theta}{dt}.$$

$t = \ldots$

$x \to \theta$
$v \to \omega$

$$\boxed{f = \frac{\omega}{2\pi} \Leftrightarrow \omega = 2\pi f.}$$

تردد زاوي



period of rotation, T, is defined as the inverse of the frequency:

$$\boxed{T = \frac{1}{f}.}$$

$T = 365$ days

$T = 24$ h

= 1 day

Angular Velocity and Linear Velocity

ω v

$$v = r\omega.$$

m/s m rad/s

v
r ω

$$\boxed{\omega = \frac{v}{r}}$$

Concept Check 9.1

A bicycle's wheels have a radius R. The bicycle is traveling with speed v. Which one of the following expressions describes the angular speed of the front tire?

a) $\omega = \frac{1}{2}Rv^2$
b) $\omega = \frac{1}{2}vR^2$
c) $\omega = R/v$
d) $\omega = Rv$
e) $\boxed{\omega = v/R}$

EXAMPLE 9.3 Revolution and Rotation of the Earth

PROBLEM

The Earth orbits around the Sun and also rotates on its pole-to-pole axis. What are the angular velocities, frequencies, and linear speeds of these motions?

$\omega = ?$ $f = ?$ $v = ?$

1 day → 24 hour → 60 min → 60 sec

$$f_E = \frac{1}{T_E} = \frac{1}{1 \times 24 \times 60 \times 60} = 1.157 \times 10^{-5}\ (Hz)$$

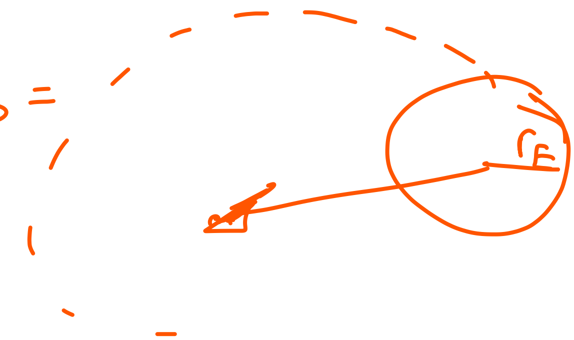
$$f_S = \frac{1}{T_S} = \frac{1}{365 \times 24 \times 60 \times 60} = 3.17 \times 10^{-8}\ (Hz)$$

$$\omega_E = 2\pi f_E = \ldots$$

$$\omega_S = 2\pi f_S = \ldots$$

$$v = r_E \omega_S =$$

$$v = r_S \cdot \omega_S =$$



9.4 Angular and Centripetal Acceleration

$$\bar{\alpha} = \frac{\Delta \omega}{\Delta t}.$$

$$a = \frac{\Delta v}{\Delta t}$$

$$\alpha = \lim_{\Delta t \to 0} \bar{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} \equiv \boxed{\frac{d\omega}{dt}} = \boxed{\frac{d^2\theta}{dt^2}}$$

m

The magnitude of the centripetal acceleration is

$$a_c = \frac{v^2}{r} =$$

$$a_c = [redacted] \frac{v^2}{r} = \omega^2 r.$$

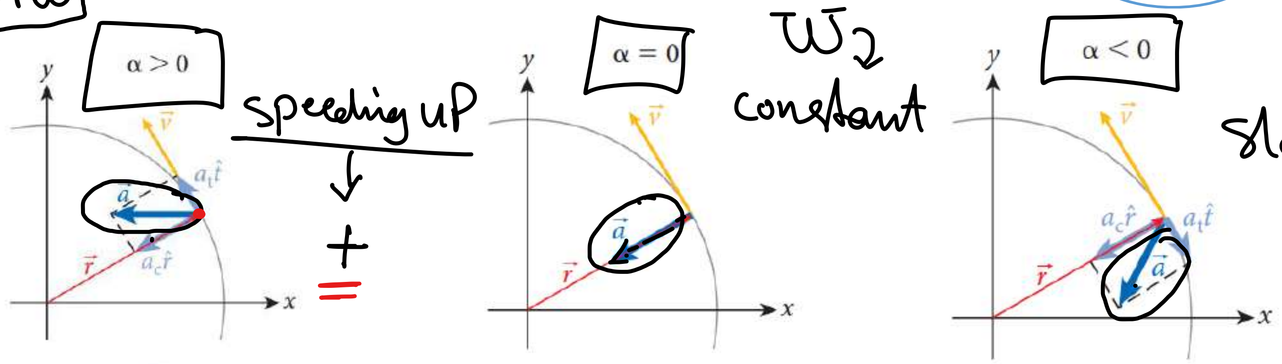
the tangential acceleration

$$\boxed{a_t = r\alpha}$$

$$\boxed{v = r\omega}$$

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}.$$

tot



(a) (b) C

speeding up → +

ω_2 constant

slowing down

or ÷100

$R \times 10^{-2}$

PROBLEM

$a_c = 840\,000 \times 9.8 = 8232000 \ \frac{m}{s}$

If you want to generate 840,000*g* of centripetal acceleration in a sample rotating at a distance of 23.5 cm from the ultracentrifuge's rotation axis, what is the frequency you have to enter into the controls? What is the linear speed with which the sample is then moving?

$$a_c = \frac{V^2}{r} = \omega^2 \cdot r$$

$$8232000 = \omega^2 \times 23.5 \times 10^{-2}$$

$$\boxed{\omega = 5918.6 \ \frac{rad}{s}}$$

$$\omega = 2\pi f$$

$$5918.6 = 2\pi \cdot f$$

$$f = 941.97 \ (Hz)$$

$a_c \ (\frac{m}{s^2})$

$\alpha \ (\frac{rad}{s^2})$

$$V = r \cdot \omega$$

$$= \ldots \ \frac{m}{s}$$

9.40 What is the centripetal acceleration of the Moon? The period of the Moon's orbit about the Earth is 27.3 days, measured with respect to the fixed stars. The radius of the Moon's orbit is $R_M = 3.85 \cdot 10^8$ m.

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\left(f = \frac{1}{T}\right)$$

$a_c = ?$ $\quad T = 27.3$ days

$$a_c = \frac{v^2}{r} = \omega^2 \cdot r$$

$$a_c = \ldots \left(\frac{m}{s^2}\right)$$

$$\omega = \frac{2\pi}{27.3 \times 24 \times 60 \times 60}$$

$$\omega = \ldots \ \frac{rad}{s}$$

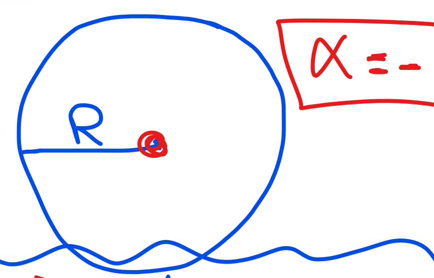
9.41 You are holding the axle of a bicycle wheel with radius 35.0 cm and mass 1.00 kg. You get the wheel spinning at a rate of 75.0 rpm and then stop it by pressing the tire against the pavement. You notice that it takes 1.20 s for the wheel to come to a complete stop. What is the angular acceleration of the wheel?

$\times 10^{-2}$

$m =$

$\alpha = ?$

$\alpha = -$



$$\omega_i = 75 \ rpm = 75 \times \frac{2\pi}{60} = \frac{5\pi}{2} \ rad/s$$

$$\omega_f = 0 \quad , \quad t = 1.2 \ (s)$$

$$\left(\begin{array}{l} \omega = \omega_0 + \alpha t \\ \omega_f = \omega_i + \alpha t \end{array}\right)$$

$$\omega_f = \omega_i + \alpha t$$

$$0 = \frac{5\pi}{2} + \alpha \times 1.2$$

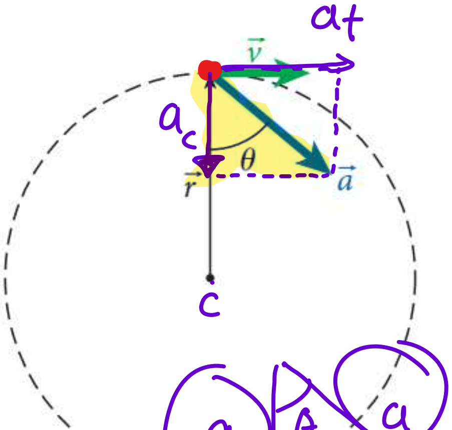
$$\boxed{\alpha = -6.5 \ \frac{rad}{s^2}}$$

a_c , a_t

a_{tot}

9.46 A particle is moving clockwise in a circle of radius (R) 1.00 m. At a certain instant, the magnitude of its acceleration is $a = |\vec{a}| = 25.0$ m/s^2, and the acceleration vector has an angle of $\theta = 50.0°$ with the position vector, as shown in the figure. At this instant, find the speed, $v = |\vec{v}|$, of this particle. ?

tot

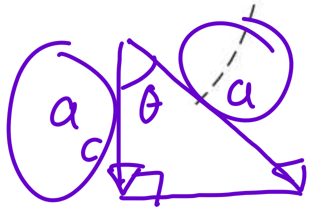


$\cos\theta = \dfrac{a_c}{a}$

$a_c = a\cos\theta$

$= 25 \times \cos 50$

$\boxed{a_c = 16.07 \text{ m/s}^2}$



$a_c = \dfrac{v^2}{r}$

$16.07 = \dfrac{v^2}{1} \rightarrow v \simeq 4$ m/s

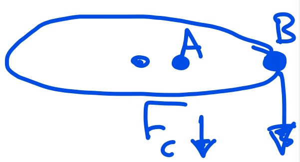
$v^2 = 16 \rightarrow v = \pm 4$

9.5 Centripetal Force

$$\boxed{F_c = ma_c} = mv\omega = m\frac{v^2}{r} = m\omega^2 r.$$

$a_c = \dfrac{v^2}{r} = r\omega^2$

$F_c = ma_c$



Concept Check 9.3

You are sitting on a carousel, which is in motion. Where should you sit so that the largest possible centripetal force is acting on you?

a) close to the outer edge

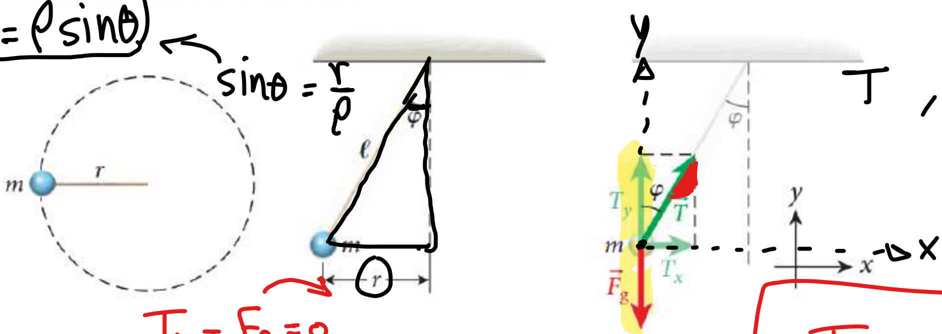
b) close to the center

c) in the middle

d) The force is the same everywhere.

Conical Pendulum

$\boxed{r = \ell\sin\theta}$ ← $\sin\theta = \dfrac{r}{\ell}$



(a) (b) (c)

T , mg

$T_y - F_g = 0$

$T_y = F_g$

y → $\boxed{T\cos\varphi = mg}$ *

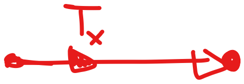
x

$$T\sin\varphi = mr\omega^2.$$

$$r = \ell\sin\varphi.$$

$\boxed{* \; T_x = T\sin\theta \quad * \; T_y = T\cos\theta}$

$$\omega = \sqrt{\frac{g}{\ell\cos\varphi}}.\ *$$



$F_c = ma_c$

$T_x = mr\omega^2 \Rightarrow \boxed{T\sin\theta = mr\omega^2}$ #

Concept Check 9.4

A certain angular velocity, ω_0, of a conical pendulum results in an angle φ_0. If this conical pendulum were taken to the Moon, where the gravitational acceleration is a sixth of that on Earth, how would one have to adjust the angular velocity to obtain the same angle φ_0?

a) $\omega_{Moon} = 6\omega_0$

b) $\omega_{Moon} = \sqrt{6}\,\omega_0$ *

c) $\omega_{Moon} = \omega_0$

d) $\boxed{\omega_{Moon} = \omega_0/\sqrt{6}}$ *

e) $\omega_{Moon} = \omega_0/6$

$g_M = \frac{1}{6} g_E$

$\omega_M = \frac{1}{\sqrt{6}}\omega_E$

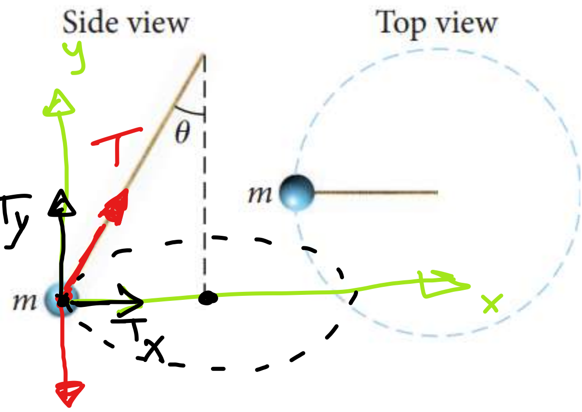
•9.56 A ball of mass $m = 0.200$ kg is attached to a (massless) string of length $L = 1.00$ m and is undergoing circular motion in the horizontal plane, as shown in the figure.

~~a)~~ Draw a free-body diagram for the ball.

~~b)~~ Which force plays the role of the centripetal force? T_x

~~c)~~ What should the $\boxed{\text{speed}}$ of v the mass be for θ to be 45.0°?

d) What is the tension in the string?



$$T_y - mg = 0$$

$$T\cos\theta = mg \quad (1)$$

$$T_x = m\frac{v^2}{r}$$

$$T\sin\theta = m\frac{v^2}{\ell\sin\theta} \quad (2)$$

$$\frac{\cancel{T}\cos\theta}{\cancel{T}\sin\theta} = \frac{\cancel{m}g}{\cancel{m}r\omega^2}$$

$$\frac{\cos\theta}{\cancel{\sin\theta}} = \frac{g}{\ell\cancel{\sin\theta}\,\omega^2}$$

$$\cos\theta = \frac{g}{\ell\omega^2} \; *$$

$$T_x = T\sin\theta$$
$$T_y = T\cos\theta$$

$$a_c = \frac{v^2}{r} = r\omega^2$$

$$r = \ell\,.\sin\theta$$

$$\frac{\cancel{T}\cos\theta}{\cancel{T}\sin\theta} = \frac{\cancel{m}g}{\cancel{m}\,\frac{v^2}{\ell\sin\theta}}$$

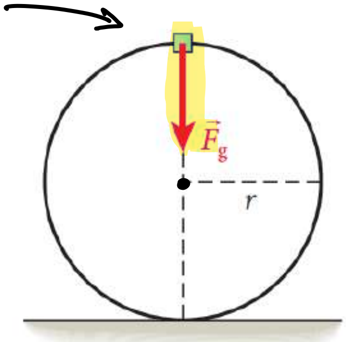
$$\frac{\cos\theta}{\sin\theta} = \frac{\ell\sin\theta\, g}{v^2} \rightarrow \frac{\cos 45}{\sin 45} = \frac{1\times 9.8\times\sin 45}{v^2}$$

$$\boxed{v = 2.6\ \text{m/s}}$$

PROBLEM

Suppose the vertical loop has a radius of 5.00 m (r). What does the linear speed (v) of the roller coaster have to be at the top of the loop for the passengers to feel weightless? (Assume that friction between roller coaster and rails can be neglected.)

$N = 0$



$$F_c = m a_c$$

$$\cancel{m}g = \cancel{m}\frac{v^2}{r}$$

$$v^2 = g.r \longrightarrow v = \sqrt{g.r}$$

9.6 Circular and Linear Motion

Table 9.1 Comparison of Kinematical Variables for Circular Motion

Quantity	Linear	Angular	Relationship
Displacement	s	θ	$s = r\theta$
Velocity	v	ω	$v = r\omega$
Acceleration	a	α	$a_t = r\alpha$
			$a_c = r\omega^2$

(i) 1 $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ (i) $x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

(ii) 2 $\theta = \theta_0 + \bar{\omega} t$ (ii) $x = x_0 + \bar{v}_x t$

(iii) 3 $\omega = \omega_0 + \alpha t$ (iii) $v_x = v_{x0} + a_x t$

(iv) 4 $\bar{\omega} = \frac{1}{2}(\omega + \omega_0)$ (iv) $\bar{v}_x = \frac{1}{2}(v_x + v_{x0})$

(v) 5 $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ (v) $v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$\omega = 2\pi f = \frac{2\pi}{T}$

Concept Check 9.5

When you go through a vertical loop on a high-speed roller coaster, what keeps you in your seat?

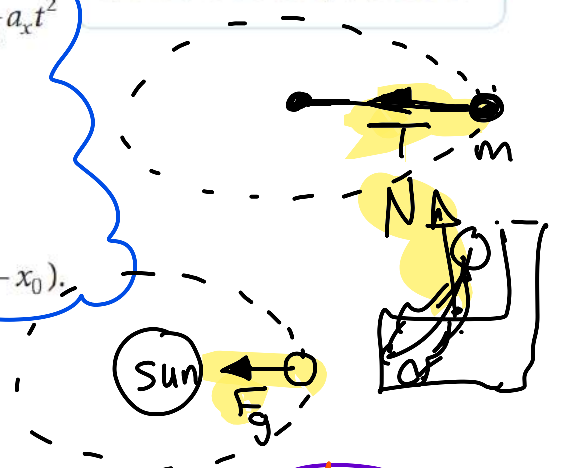
a) centrifugal force

b) the normal force from the track

c) the force of gravity

d) the force of friction

e) the force exerted by the seat belt



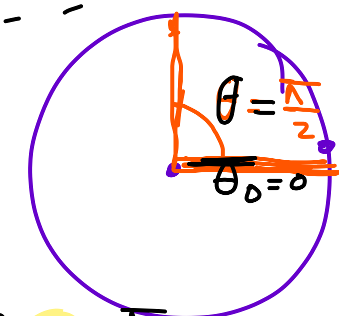
9.61 A boy is on a Ferris wheel, which takes him in a vertical circle of radius 9.00 m once every 12.0 s.

r T_ω

a) What is the angular speed of the Ferris wheel?

b) Suppose the wheel comes to a stop at a uniform rate during one quarter of a revolution. What is the angular acceleration of the wheel during this time?

c) Calculate the tangential acceleration of the boy during the time interval described in part (b).



(a) $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{12} = 0.52$ rad/s

(b) $\omega = 0$, $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \rightarrow 0^2 = 0.52^2 + 2\alpha \times \frac{\pi}{2} \rightarrow \alpha = -0.08$ rad/s

(c) $a_t = r.\alpha = 9 \times (-0.08) = -0.774$ m/s²

9.62 Consider a 53.0-cm-long lawn mower blade rotating about its center at 3400. rpm.

ω ÷100

$\omega = 3400 \times \frac{2\pi}{60} = 356$ rad/s

a) Calculate the linear speed of the tip of the blade.

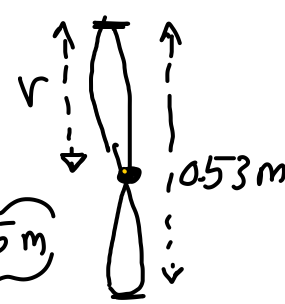
b) If safety regulations require that the blade be stoppable within 3.00 s, what minimum angular acceleration will accomplish this? Assume that the angular acceleration is constant.

t α

(a) $V = r.\omega = 0.265 \times 3400 \times \frac{2\pi}{60} = 94.35$ m/s

$r = \frac{0.53}{2}$

$r = 0.265$ m



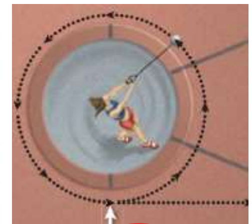
(b) $\omega_f = 0$

$\omega = \omega_0 + \alpha t$

$0 = 356 + \alpha \times 3 \Rightarrow \alpha = -118.6$ rad/s²

EXAMPLE 9.7 Hammer Throw

One of the most interesting events in track-and-field competitions is the hammer throw. The task is to throw the "hammer," a 12-cm-diameter iron ball attached to a grip by a steel cable, a maximum distance. The hammer's total length is 121.5 cm, and its total mass is 7.26 kg. The athlete has to accomplish the throw from within a circle of radius 2.13 m (7 ft), and the best way to throw the hammer is for the athlete to spin, allowing the hammer to move in a circle around him, before releasing it. At the 1988 Olympic Games in Seoul, the Russian thrower Sergey Litvinov won the gold medal with an Olympic record distance of 84.80 m. He took seven turns before releasing the hammer, and the period to complete each turn was obtained from examining the video recording frame by frame: 1.52 s, 1.08 s, 0.72 s, 0.56 s, 0.44 s, 0.40 s, and 0.36 s.



PROBLEM 1

What was the average angular acceleration during the seven turns? Assume constant angular acceleration for the solution, and then check whether this assumption is justified.

$$\theta = \cancel{\theta_0} + \cancel{\omega_0} t + \frac{1}{2}\alpha t^2$$

$$t_{all} = \cdots + \cdots + \cdots$$

$$\theta_{all} = 2\pi \times 7 = 14\pi \text{ rad}$$

0 0

$$\theta = \frac{1}{2}\alpha t^2 \rightarrow \alpha = 3.41 \text{ rad/s}^2$$

PROBLEM 2

r

Assuming that the radius of the circle on which the hammer moves is 1.67 m (the length of the hammer plus the arms of the athlete), what is the linear speed with which the hammer is released?

$v = ?$

$$V = r.\omega$$

$$\omega = \omega_0 + \alpha t$$

$$\omega = 0 + 3.41 \times 5.08$$

$$\omega = 17.3 \frac{\text{rad}}{\text{s}}$$

$$V = 1.67 \times 17.3 = 28.9 \text{ m/s}$$

PROBLEM 3

What is the centripetal force that the hammer thrower has to exert on the hammer right before he releases it?

F_c

$$F_c = m a_c = m \frac{V^2}{r}$$

$$F_c = 7.26 \times \frac{28.9^2}{1.67} = 3630.9 \text{ (N)}$$

$x = v \cdot t$

$v = \frac{x}{t}$

SOLVED PROBLEM 9.3 Flywheel

α → 25.9 → 59.5 (s)

0 ① ω ② ω, $\omega = \text{const.}$

PROBLEM

$\omega = 0$

The flywheel of a steam engine starts to rotate from rest with a constant angular acceleration of $\alpha = 1.43\ \text{rad/s}^2$. The flywheel undergoes this constant angular acceleration for $t = 25.9$ s and then continues to rotate at a constant angular velocity, ω. After the flywheel has been rotating for 59.5 s, what is the total angle through which it has rotated since it started?

$\theta_{tot} = ?$

$\theta_1 = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta_1 = 0 + 0 + \frac{1}{2}(1.43)(25.9)^2$

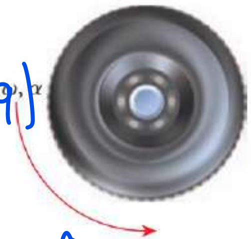
$\theta_1 = 479.6$ rad

$\omega = \omega_0 + \alpha t = 0 + 1.43 \times 25.9 = 37.03$ rad/s

$\theta_2 = \omega \cdot t$

$= 37.03 \times (59.5 - 25.9)$

$\theta_2 = 1244.2$ rad



$\theta_{tot} = \theta_1 + \theta_2$

9.7 More Examples for Circular Motion

•**9.54** A race car is making a U-turn at constant speed. The coefficient of friction between the tires and the track is $\mu_s = 1.20$. If the radius of the curve is 10.0 m, what is the maximum speed at which the car can turn without sliding? Assume that the car is undergoing uniform circular motion.

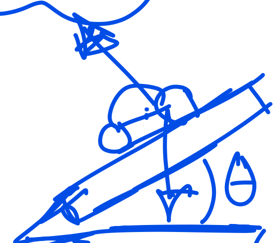
$T,\ mg,\ F_f$

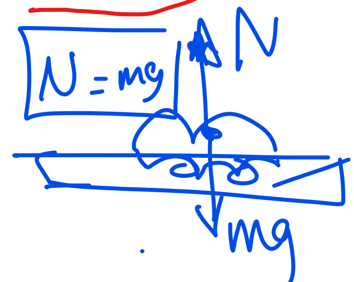
$F_c = m a_c$

$F_k = m \frac{v^2}{r}$

$\mu \cdot N = m \frac{v^2}{r}$

$F_k = \mu \cdot N$





$\boxed{N = mg}$

$\boxed{N = mg\cos\theta}$

$\mu \cancel{m} g = \cancel{m} \frac{v^2}{r}$

$1.2 \times 9.8 = \frac{v^2}{10}$ → $v = 10.8$ m/s

$x^2 = 25$

$x = \pm 5$

$(+5)^2$

$(-5)^2$

SOLVED PROBLEM 9.4 NASCAR Racing

As a NASCAR racer moves through a banked curve, the banking helps the driver achieve higher speeds. Let's see how. Figure 9.26 shows a race car on a banked curve.

PROBLEM

If the coefficient of static friction between the track surface and the car's tires is $\mu_s = 0.620$ and the radius of the turn is $R = 110.$ m, what is the maximum speed with which a driver can take a curve banked at $\theta = 21.1°$? (This is a fairly typical banking angle for NASCAR tracks. Indianapolis has only 9° banking, but there are some tracks with banking angles over 30°, including Daytona (31°), Talladega (33°), and Bristol (36°).)

$V_{max} = ?$

$F_s = \mu_s \cdot N$

$N_x = N\sin\theta$, $N_y = N\cos\theta$

$f_{sx} = f_s\cos\theta$, $f_{sy} = f_s\sin\theta$

$\boxed{x}$

$N_x + f_{sx} = m a_c$

$N\sin\theta + f_s\cos\theta = m a_c$

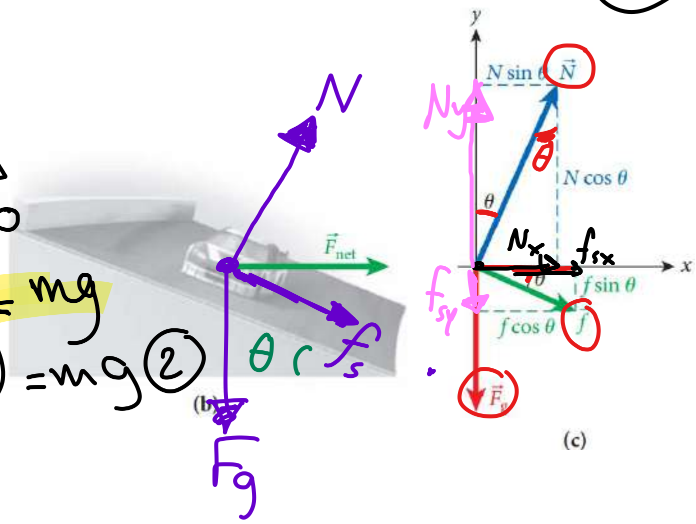
$N\sin\theta + \mu \cdot N\cos\theta = m\frac{v^2}{r}$

$N(\sin\theta + \mu\cos\theta) = m\frac{v^2}{r}$ (1)

$\boxed{y}$ $N_y - F_{sy} - mg = 0$

$N\cos\theta - \mu_s N\sin\theta = mg$

$N(\cos\theta - \mu\sin\theta) = mg$ (2)



$$\frac{(1)}{(2)} \Rightarrow \frac{\cancel{N}(\sin\theta + \mu\cos\theta)}{\cancel{N}(\cos\theta - \mu\sin\theta)} = \frac{\cancel{m}\frac{v^2}{r}}{\cancel{m}g}$$

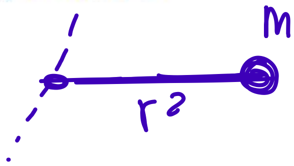
$$\frac{\sin 21.1 + 0.62 \times \cos 21.1}{\cos 21.1 - 0.62 \times \sin 21.1} = \frac{v^2}{9.8 \times 110} \rightarrow V = 37.8 \text{ m/s}$$

10.1 Kinetic Energy of Rotation

$$\boxed{K = \frac{1}{2}mv^2}$$

$$\boxed{K = \frac{1}{2}I\omega^2.}$$

kinetic energy of rotation



$$I = \sum_{i=1}^{n} m_i r_i^2.$$

moment of inertia

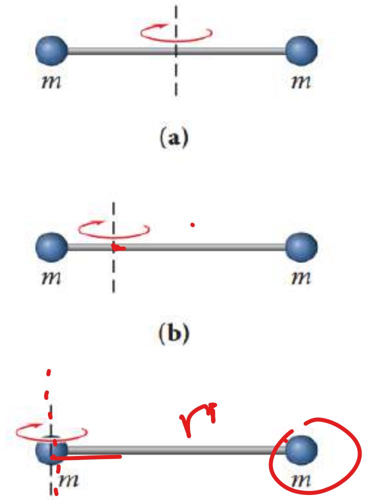
$\boxed{I = cMR^2,}$ with $0 < c \le 1.$

$\rightarrow I = MR^2$

$I = \frac{1}{2}MR^2$

Concept Check 10.1

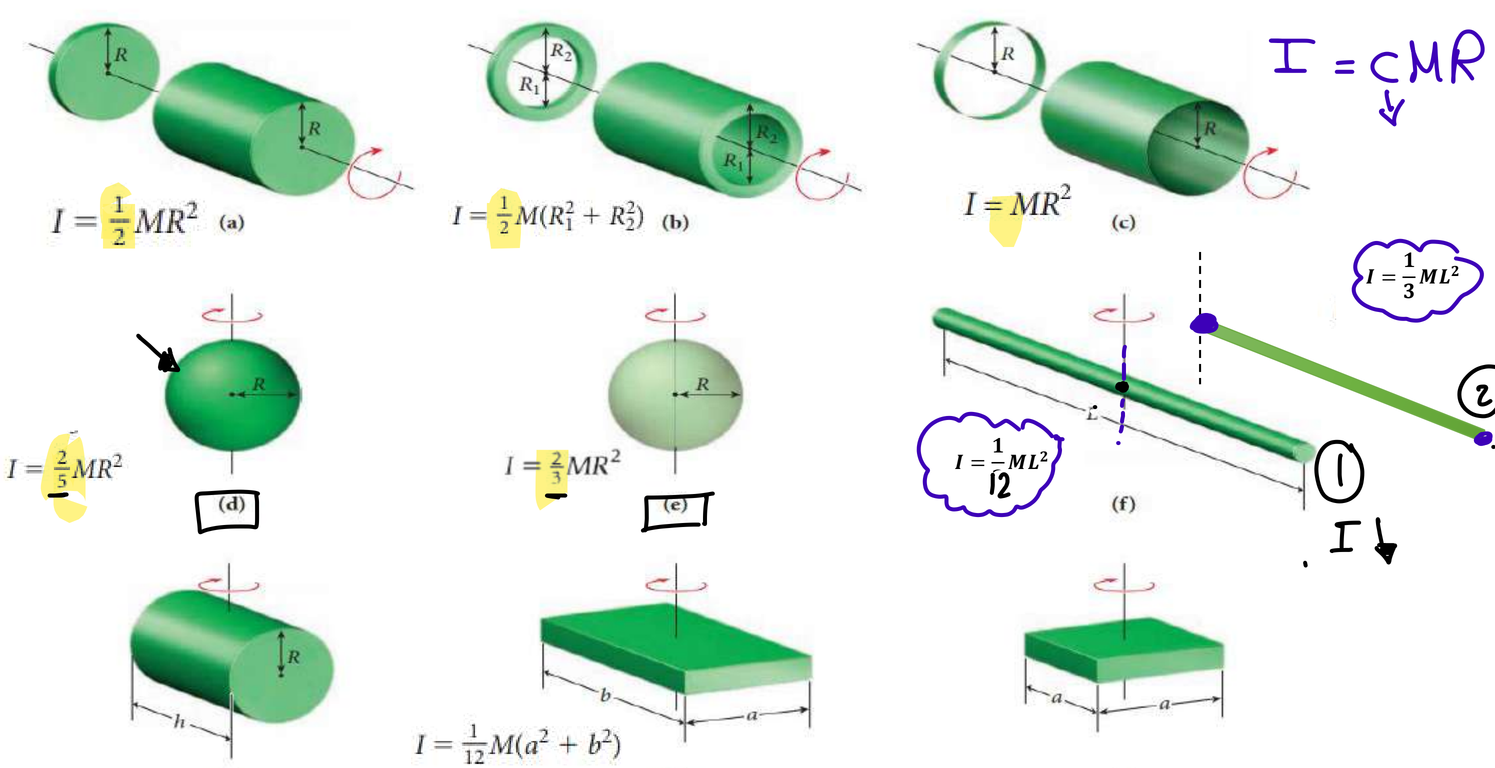
Consider two equal masses, m, connected by a thin, massless rod. As shown in the figures, the two masses spin in a horizontal plane around a vertical axis represented by the dashed line. Which system has the highest moment of inertia?



(c)

10.2 Calculation of Moment of Inertia

$I = cMR^2$



$I = \frac{1}{2}MR^2$ (a)

$I = \frac{1}{2}M(R_1^2 + R_2^2)$ (b)

$I = MR^2$ (c)

$I = \frac{1}{3}ML^2$

$I = \frac{2}{5}MR^2$ $\boxed{\text{(d)}}$

$I = \frac{2}{3}MR^2$ $\boxed{\text{(e)}}$

$I = \frac{1}{12}ML^2$ (f)

I ↓

(g)

$I = \frac{1}{12}M(a^2 + b^2)$ (h)

(i)

EXAMPLE 10.1 Rotational Kinetic Energy of Earth

Assume that the Earth is a solid sphere of constant density, with mass $5.98 \cdot 10^{24}$ kg and radius 6370 km

$R = 6370\,\text{km} \times 1000$

(حفظ)

$$\boxed{I = \tfrac{2}{5}MR^2}$$

PROBLEM

What is the moment of inertia of the Earth with respect to rotation about its axis, and what is the kinetic energy of this rotation?

$$I = \frac{2}{5}(5.98\times10^{24})(6370000)^2 = \text{---}\ (\text{kg}\cdot\text{m}^2)$$

10.3 Rolling without Slipping

$$K = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}(cR^2m)\left(\frac{v}{R}\right)^2$$

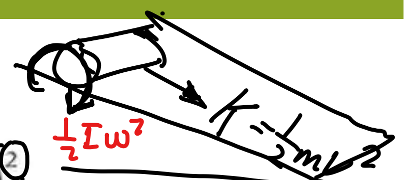
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2c \Rightarrow$$

$$\boxed{K = (1+c)\tfrac{1}{2}mv^2}$$

$v = \omega r$

$\omega = \frac{v}{r}$

$\omega^2 = \frac{v^2}{r^2}$



$I = cMR^2$

$\frac{1}{2}c\cancel{R^2}m\frac{v^2}{\cancel{R^2}}$

$I_{ss} = \frac{2}{5}MR^2$, $I_{sc} = \frac{1}{2}MR^2$, $I_{hc} = MR^2$

Concept Check 10.2

A solid sphere, a solid cylinder, and a hollow cylinder have the same mass and radius and are rolling with the same speed. Which one of the following statements is true?

a) The solid sphere has the highest kinetic energy.

b) The solid cylinder has the highest kinetic energy.

c) The hollow cylinder has the highest kinetic energy.

d) All three objects have the same kinetic energy.

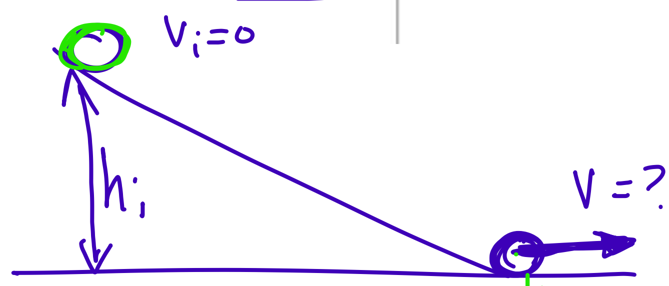
SOLVED PROBLEM 10.1 Sphere Rolling Down an Inclined Plane

PROBLEM

A solid sphere with a mass of 5.15 kg and a radius of 0.340 m starts from rest at a height of 2.10 m above the base of an inclined plane and rolls down without sliding under the influence of gravity. What is the linear speed of the center of mass of the sphere just as it leaves the incline and rolls onto a horizontal surface?

$v_i = 0$ h_i

$$\boxed{I = \tfrac{2}{5}MR^2}$$ c



$$K_i = K_f$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2(1+c)$$

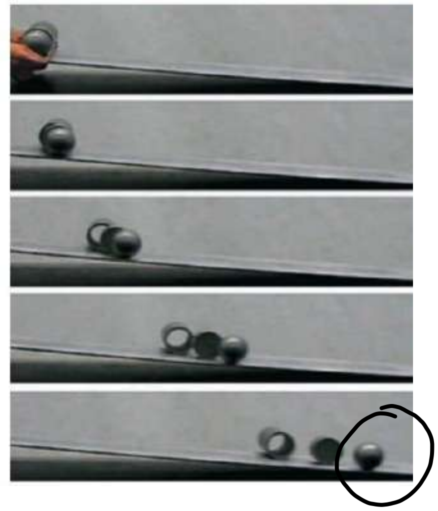
$$9.8 \times 2.1 = \frac{1}{2}v^2\left(1 + \frac{2}{5}\right)$$

$$\boxed{v = 8.3\ \text{m/s}}$$

EXAMPLE 10.2 Race Down an Incline

PROBLEM

A solid sphere, a solid cylinder, and a hollow cylinder (a tube), all of the same mass m and the same outer radius R, are released from rest at the top of an incline and start rolling without sliding. In which order do they arrive at the bottom of the incline?



$$\boxed{I = \tfrac{2}{5}MR^2 \qquad I = \tfrac{1}{2}MR^2 \qquad I = MR^2}$$

$$\cancel{m}gh = \tfrac{1}{2}\cancel{m}v^2(1+c)$$

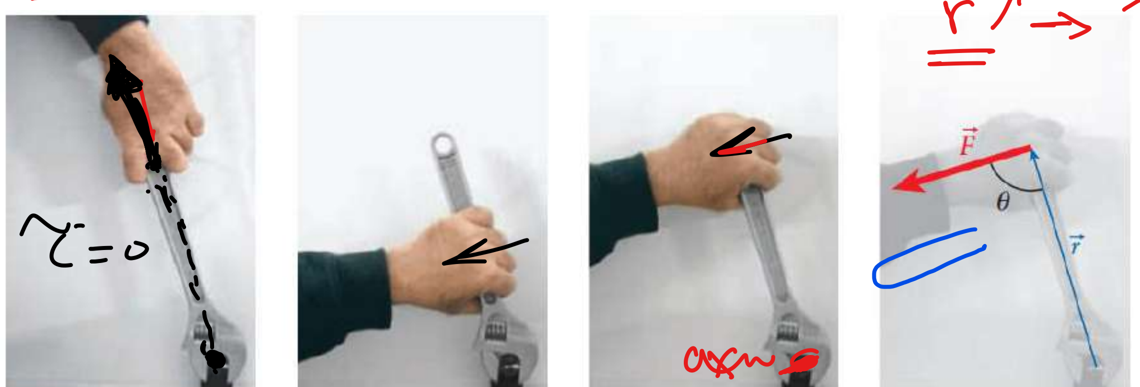
$$\frac{2gh}{1+c} = \frac{v^2\cancel{(1+c)}}{\cancel{(1+c)}} \Rightarrow \boxed{V = \sqrt{\frac{2gh}{1+c}}}$$

The solid sphere has less C so it's the greater velocity

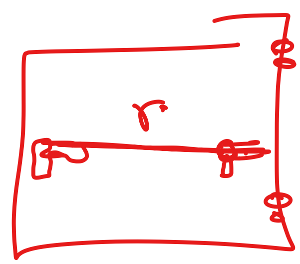
10.4 Torque

Moment Arm The perpendicular distance from the line of action of the force to the axis of rotation

$r\uparrow \rightarrow \tau\uparrow \rightarrow$ easy to move

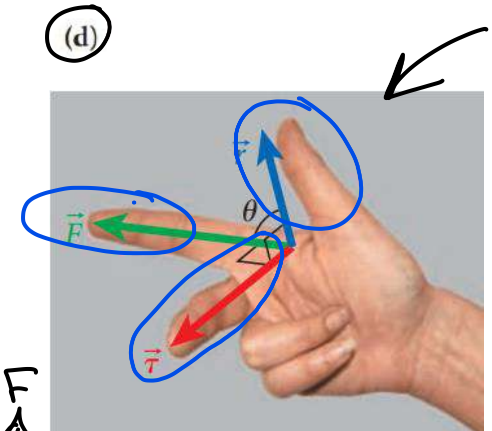


(a) (b) (c) (d)

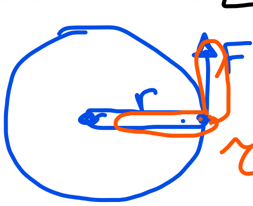


$$\vec{\tau} = \vec{r} \times \vec{F}.$$

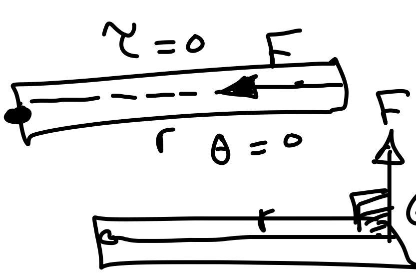
$$\boxed{\tau = rF\sin\theta}$$



$$[\tau] = [F]\cdot[r] = \boxed{\mathrm{N\,m}}$$



τ (inside)



$\theta = 90 \rightarrow \tau_{max}$

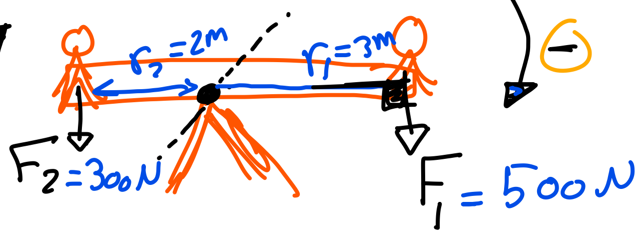
The **net torque**

$\tau_{net} = ?$

$\boxed{\tau = r . F \sin\theta}$

$$\tau_{net} = \sum_i \tau_{counterclockwise,i} - \sum_j \tau_{clockwise,j}$$

$+$ $-$



with clockwise

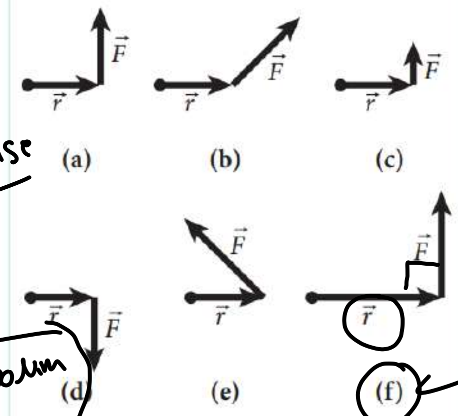
$-\tau_1 + \tau_2 = \tau_{net}$

$-r_1 F_1 \sin\theta + r_2 F_2 \sin\theta = \tau_{net}$

$-3 \times 500 \sin 90 + 2 \times 300 \times \sin 90 = \tau_{net} \Rightarrow \boxed{\tau_{net} = -900 Nm}$

Concept Check 10.4

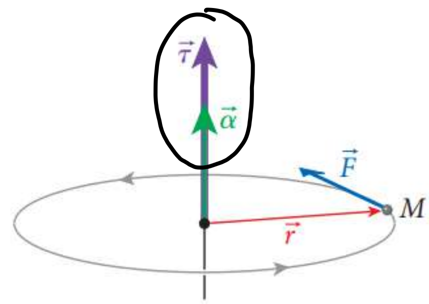
Choose the combination of position vector, $\vec{r}$, and force vector, $\vec{F}$, that produces the torque of highest magnitude around the point indicated by the black dot.



10.5 Newton's Second Law for Rotation

Newton's Second Law for rotational motion: $\tau = I\alpha$

$\boxed{F = ma}$

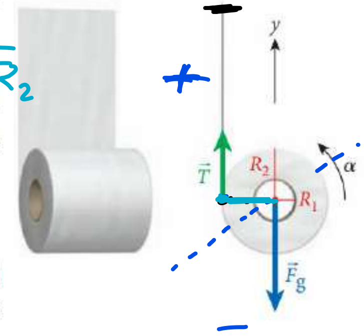


Axis of rotation

$a = R_2\alpha \rightarrow \alpha = \frac{a}{R_2}$

EXAMPLE 10.3 Toilet Paper

This may have happened to you: You are trying to put a new roll of toilet paper into its holder. However, you drop the roll, managing to hold onto just the first sheet. On its way to the floor, the toilet paper roll unwinds, as Figure 10.19a shows.



(a) (b)

PROBLEM

How long does it take the roll of toilet paper to hit the floor, if it was released from a height of 0.73 m? The roll has an inner radius $R_1 = 2.7$ cm, an outer radius $R_2 = 6.1$ cm, and a mass of 274 g.

$T - mg = ma$

$T = mg + ma$ (1)

$\tau_{net} = I\alpha$

$-T R_2 = \frac{1}{2}m(R_1^2 + R_2^2) \times \frac{a}{R_2}$

$-T = \frac{1}{2}m(R_1^2 + R_2^2) \times \frac{a}{R_2^2}$ (2)

(1) + (2) $T - T = mg + ma + \frac{1}{2}m(R_1^2 + R_2^2) \times \frac{a}{R_2^2}$ (3)

$0 = \ldots\ldots$

Atwood Machine

$F = m_1 a$

$\tau = I \alpha$

$$-T_1 + m_1 g = m_1 a$$

$$T_2 - m_2 g = m_2 a.$$

$T_1 - m_1 g = -m_1 a$ (1)

$\tau_1 - \tau_2 = \frac{1}{2} m_p R^2 \left(\frac{a}{R}\right)$

$T_2 - m_2 g = m_2 a$ (2)

$(T_1 - T_2) - m_1 g + m_2 g = -m_1 a - m_2 a$

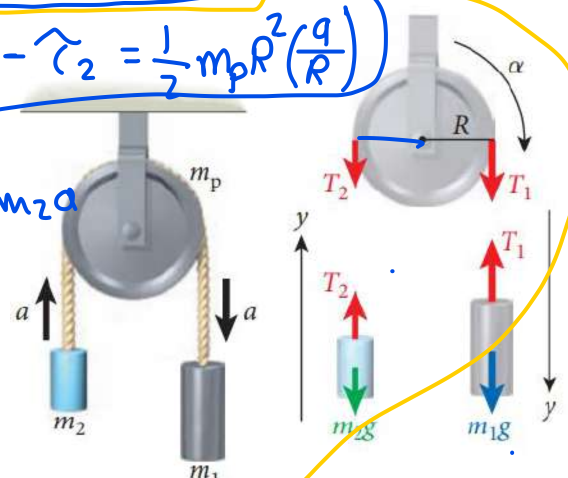
$$\tau = \tau_1 - \tau_2 = RT_1 \sin 90° - RT_2 \sin 90° = R(T_1 - T_2).$$

$$R(T_1 - T_2) = \tau = \left(\tfrac{1}{2} m_p R^2\right)\left(\frac{a}{R}\right) \Rightarrow$$

$$T_1 - T_2 = \tfrac{1}{2} m_p a.$$

$$m_1 g - m_2 g = (m_1 + m_2 + \tfrac{1}{2} m_p) a \Rightarrow$$

$$a = \frac{m_1 - m_2}{m_1 + m_2 + \frac{1}{2} m_p} g.$$

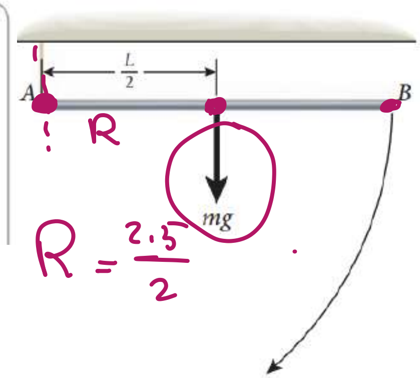


(a) (b)

$(T_1 - T_2)R = \frac{1}{2} m_p R^2 \left(\frac{a}{R}\right)$

SOLVED PROBLEM 10.3 Falling Horizontal Rod

A thin rod of length $L = 2.50$ m and mass $m = 3.50$ kg is suspended horizontally by a pair of vertical strings attached to the ends (Figure 10.22). The string supporting end B is then cut.



$R = \frac{2.5}{2}$

PROBLEM

What is the linear acceleration of end B of the rod just after the string is cut?

$a = ?$

$$\tau = I\alpha$$

$$\cancel{M}g \cdot R = \left(\frac{1}{3}\cancel{M}L^2\right)\frac{a}{R}$$

$$\boxed{a = \quad}$$