Chapter 1
Ratios and Rates

Essential Question

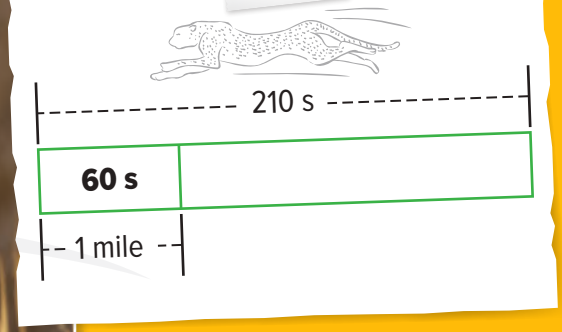
HOW do you use equivalent rates in the real world?

MP Mathematical Practices
1, 3, 4, 5, 6, 7, 8

Math in the Real World

Cheetahs are the fastest land animals. They can chase prey by running at speeds of 60 miles per hour.

A cheetah can only maintain top speeds for a short time. If a cheetah runs 1 mile in 60 seconds, fill in the diagram to show how far the cheetah will run in 210 seconds.



FOLDABLES® Study Organizer

1. Cut out the Foldable in the back of this book.
2. Place your Foldable on page 82.
3. Use the Foldable throughout this chapter to help you learn about ratios and rates.

What Tools Do You Need?

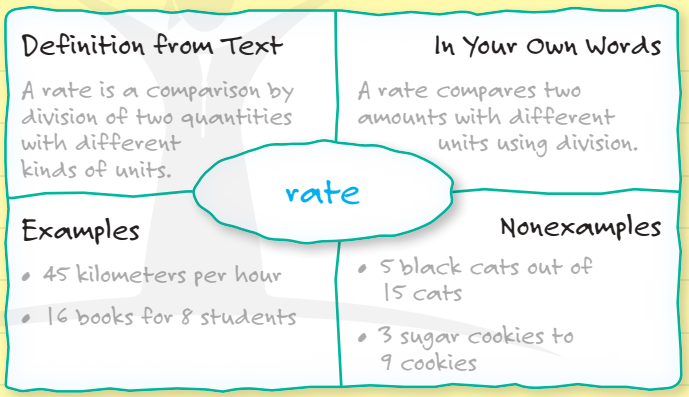
Vocabulary

coordinate plane	origin	unit price
equivalent ratio	prime factorization	unit rate
graph	rate	*x*-axis
greatest common factor	ratio	*x*-coordinate
least common multiple	ratio table	*y*-axis
ordered pair	scaling	*y*-coordinate

Study Skill: Studying Math

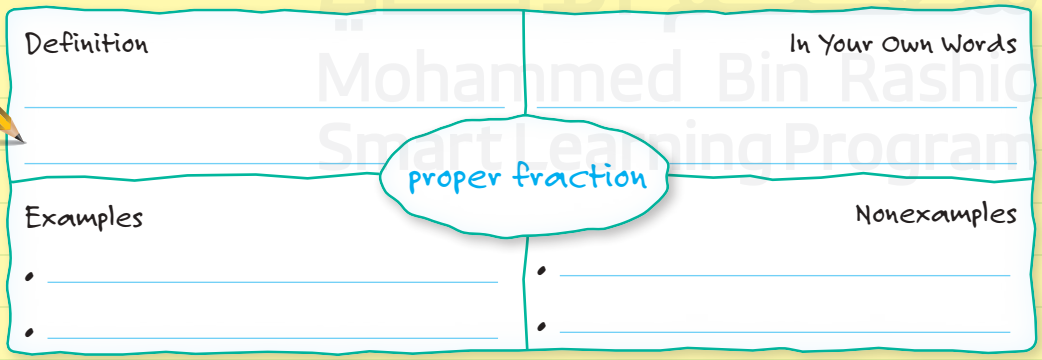
New Vocabulary New vocabulary terms are clues about important concepts. Learning new vocabulary words is more than just memorizing the definition. Whenever you see a new vocabulary word, ask yourself:

- How does this fit with what I already know?
- How is this alike or different from something I learned earlier?



Organize your answers in a word map like the one shown.

Make a word map for *proper fraction*.



What Do You Already Know?

Place a checkmark below the face that expresses how much you know about each concept. Then scan the chapter to find a definition or example of it.

I have no clue. I've heard of it. I know it!

Ratios and Rates				
Concept				Definition or Example
greatest common factor				
ordered pairs				
prime factorization				
ratios				
the coordinate plane				
unit rates				

When Will You Use This?

Here is an example of how unit rates are used in the real world.

Activity Use a newspaper to find an ad for two different grocery stores. See if you can find an offer for the same item. How do the prices compare? Which one do you think is offering a better deal? Explain your reasoning.

Are You Ready?

Try the Quick Check below. Check

Quick Review

Example 1

Find $6\overline{)348}$.

$$\begin{array}{r} 58 \\ 6\overline{)348} \\ -30 \\ \hline 48 \\ -48 \\ \hline 0 \end{array}$$

Divide each place-value position from left to right.

Since $48 - 48 = 0$, there is no remainder.

Example 2

Write $\frac{40}{64}$ in simplest form.

$$\frac{40}{64} \overset{\div 8}{=} \frac{5}{8}$$

Divide the numerator and denominator by the greatest common factor (GCF), 8.

Since the GCF of 5 and 8 is 1, the fraction $\frac{5}{8}$ is in simplest form.

Quick Check

Divide Whole Numbers **Find each quotient.**

1. $3\overline{)87}$

Show your work.

2. $8\overline{)584}$

3. $52\overline{)312}$

Simplify Fractions **Write each fraction in simplest form.**

4. $\frac{32}{48} =$ _____

5. $\frac{7}{28} =$ _____

6. $\frac{15}{25} =$ _____

7. An airplane has flown 260 kilometers out of a total trip of 500 kilometers. What fraction, in simplest form, of the trip has been completed?

_____

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7

Lesson 1

Factors and Multiples

Vocabulary Start-Up

A *common factor* is a number that is a factor of two or more numbers. The greatest of the common factors of two or more numbers is called the **greatest common factor** (GCF).

The least nonzero number that is a multiple of two or more whole numbers is the **least common multiple** (LCM) of the numbers.

Fill in the charts below.

GCF	LCM
• stands for:	• stands for:
Define:	Define:
• Greatest	• Least
• Common	• Common
• Factor	• Multiple

Real-World Link

Abdalla is making balloon arrangements. He has 8 blue and 12 green balloons. What is the greatest amount of arrangements he can make if he wants them to be identical? _____

Essential Question

HOW do you use equivalent rates in the real world?

Vocabulary

greatest common factor
least common multiple

MP Mathematical Practices
1, 3, 4, 8

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Prime Numbers

Remember that a prime number is a whole number that has exactly two factors, 1 and the number itself.

Find the Greatest Common Factor

You can use common factors or prime factors to find the GCF.

Example

1. **There are one-slice servings of three types of cake on a table. Each row has an equal number of servings and only one type of cake. What is the greatest number of servings in each row?**

Cakes	
Type	**Number of Servings**
marble	10
red velvet	15
chocolate	20

To solve this problem, use common factors.

factors of 10: **1**, 2, **5**, 10

factors of 15: **1**, 3, **5**, 15

factors of 20: **1**, 2, 4, **5**, 10, 20 The common factors are 1 and 5.

The GCF of 10, 15, and 20 is 5. So, the greatest number of pieces of cake that can be placed in each row is 5.

Got it? **Do this problem to find out.**

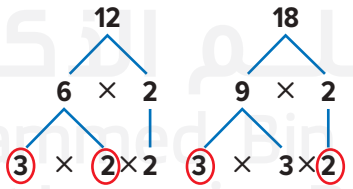
Show your work.

a. Mariam earned AED 49 on Sunday, AED 42 on Monday, and AED 21 on Tuesday selling bracelets. She sold each bracelet for the same amount. What is the most she could have charged for each bracelet?

a. _____

Example

2. **Find the GCF of 12 and 18.**



2 and 3 are the common prime factors.

So, the GCF of 12 and 18 is 2 × 3, or 6.

Got it? **Do these problems to find out.**

Find the greatest common factor of each set of numbers.

b. 12, 66 **c.** 18, 30 **d.** 32, 48

b. _____

c. _____

d. _____

Find the Least Common Multiple

You can find the least common multiple (LCM) by using a number line, making a list, or by using prime factors.

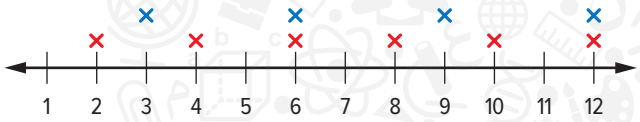
Multiples
A multiple of a number is the product of the number and any whole number (0, 1, 2, 3, ...).

Examples

3. **Find the LCM of 2 and 3.**

Method 1 **Use a number line.**

Put a red **X** above each nonzero multiple of 2 and a blue **X** above each nonzero multiple of 3.



The least number with both a red and a blue X is 6.

So, 6 is the least common multiple of 2 and 3.

Method 2 **Use an organized list.**

List the nonzero multiples of 2 and 3.

multiples of 2: 2, 4, **6**, 8, 10, **12**,... $1 \times 2, 2 \times 2, 3 \times 2, \ldots$

multiples of 3: 3, **6**, 9, **12**, 15,... $1 \times 3, 2 \times 3, 3 \times 3, \ldots$

Notice that 6 and 12 are common multiples.

So, the least common multiple of 2 and 3 is 6.

4. **Find the LCM of 14 and 21 using prime factorization.**

Write the prime factorization of each number.

7 is the only common prime factor.

Multiply using each common prime factor only once.

So, the LCM is $7 \times 2 \times 3$ or 42.

Got it? **Do these problems to find out.**

Find the least common multiple of each set of numbers.

e. 2, 6 **f.** 4, 5, 10 **g.** 3, 5, 7

Show your work.

e. _____

f. _____

g. _____

Example

5. **Hamdan has painting class every 2 weeks. Saif has a pottery class every 5 weeks. Hamdan and Saif met at the art building for class this week. How many weeks will it be until they see each other again?**

multiples of 2: 2, 4, 6, 8, **10**, 12, 14,...

multiples of 5: 5, **10**, 15, 20, 25, 30,...

The least common multiple of 2 and 5 is 10. So, Hamdan and Saif will see each other again in 10 weeks.

Guided Practice

Check

Find the greatest common factor of each set of numbers. (Example 1 and 2)

1. 8, 32 _____

Show your work.

2. 24, 60 _____

3. 3, 12, 18 _____

Find the least common multiple of each set of numbers. (Examples 3 and 4)

4. 7, 9 _____

5. 6, 15 _____

6. 9, 12, 15 _____

7. The Movie House gives away a AED 5 coupon for every 4 movies purchased. They give away a bag of popcorn for every 3 movies purchased. How many movies would you have to purchase in all before receiving both a AED 5 coupon and a bag of popcorn at the same purchase? (Example 5)

_____

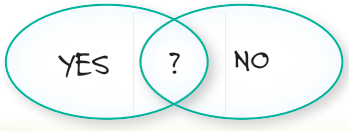
8. **Building on the Essential Question** How does finding the greatest common factor help you to solve real-world problems? _____

_____

_____

Rate Yourself!

Are you ready to move on? Shade the section that applies.



Name _____ My Homework _____

Independent Practice

Find the greatest common factor of each set of numbers. (Example 2)

1. 8, 14 _____

Show your work.

2. 21, 24, 27 _____

3. 21, 35, 49 _____

4. 12, 18, 26 _____

Find the least common multiple of each set of numbers. (Examples 3 and 4)

5. 5 and 6 _____

6. 6 and 9 _____

7. 6, 12, and 15 _____

8. 3, 9, and 15 _____

9. A gardener has 27 pansies and 36 daisies. He plants an equal number of each type of flower in each row. What is the greatest possible number of pansies in each row? (Example 1)

10. Fourteen boys and 21 girls will be equally divided into groups. Find the greatest number of groups that can be created if no one is left out. (Example 1)

11. Latifa waters her plants every two days. She trims them every 15 days. She did both today. When will she do both again? (Example 5) _____

12. MP **Identify Repeated Reasoning** An airport offers two shuttles that run on different schedules. If both shuttles leave the airport at 4:00 P.M., at what time will they next leave the airport together?

Shuttle Schedule	
Shuttle	**Departs**
A	every 6 minutes
B	every 9 minutes

_____

_____

H.O.T. Problems Higher Order Thinking

13. MP **Model with Mathematics** Write and solve a real-world problem that can be solved using the greatest common factor of two numbers.

_____

_____

_____

14. MP **Identify Repeated Reasoning** How can you use number patterns to find the least common multiple of 120 and 360?

_____

_____

_____

15. MP **Persevere with Problems** If the GCF of two numbers is 1, they are called *relatively prime*. Find three sets of relatively prime numbers.

_____

16. MP **Use a Counterexample** Determine whether each statement is *true* or *false*. If *true*, explain why. If *false*, give a counterexample.

a. The GCF of any two even numbers is always even.

_____

_____

b. The GCF of any two odd numbers is always odd.

_____

_____

c. The GCF of an odd number and an even number is always even.

_____

_____

Name _____ My Homework _____

Extra Practice

Find the greatest common factor of each set of numbers.

17. 15, 20 5

Homework Help

factors of 15: 1, 3, 5, 15

factors of 20: 1, 2, 4, 5, 10, 20

The common factors are 1 and 5.

The GCF is 5.

18. 30, 48, 60 _____

19. 24, 30, 42 _____

20. 24, 40, 56 _____

Find the least common multiple of each set of numbers.

21. 3 and 5 15

multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

multiples of 5: 5, 10, 15, 20, 25, 30

The common multiples are 15 and 30.

The LCM is 15.

22. 12 and 18 _____

23. 5, 10, and 15 _____

24. 9, 12, and 18 _____

25. A grocery store clerk has 16 oranges, 20 apples, and 24 pears. The clerk needs to put an equal number of apples, oranges, and pears into each basket. What is the greatest number of baskets that can be made so that no fruit is left?

_____

_____

26. MP **Identify Repeated Reasoning** The science department buys the equipment shown in the table. They bought all three items this year. In how many years will they have to buy all three items again?

_____

Item	Time Bought
Microscopes	every 5 years
Safety goggles	every 4 years
Test tubes	every 2 years

Power Up! Test Practice

27. Amani replaces the light bulb in the hall closet every 9 months and replaces the air filter every 6 months. She just replaced both items this month. After how many months will she replace both the light bulb and the air filter? Select all that apply.

☐ 12 months ☐ 18 months ☐ 36 months ☐ 48 months

28. Ayesha is painting a design that contains two repeating patterns. One pattern repeats every 8 centimeters. The other repeats every 12 centimeters. The design is 500 centimeters long. Both patterns begin at the same place. Use the pattern pieces to create a sample of the design. Use the sample to determine the number of times the patterns begin in the same place.

8-centimeter pattern:

12-centimeter pattern:

Spiral Review

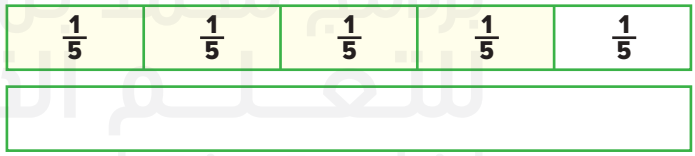
Write each fraction in simplest form.

29. $\frac{9}{18}$ = _____

30. $\frac{21}{35}$ = _____

31. $\frac{36}{48}$ = _____

32. Heba ran $\frac{4}{5}$ kilometer. How many tenths are equal to $\frac{4}{5}$ kilometer? Use bar diagrams to find the answer.



33. Pizza Palace cuts a medium pizza into 8 slices. The same size pizza at Pizza Pioneers is cut into 16 slices. Yasmin ate 4 slices of a medium pizza from Pizza Pioneers. What fraction of the pizza from Pizza Palace is equal to $\frac{4}{16}$? Explain.

Inquiry Lab 1
Ratios

MP Mathematical Practices 1, 3, 4

Inquiry **HOW can you use tables to relate quantities?**

Faisal has 3 fiction books and 6 nonfiction books to donate to the community center. He wants to package them so that there are an equal number of fiction and nonfiction books in each group. He also wants to have as many packages as possible. How many books are in each group?

What do you know? _____

_____

_____

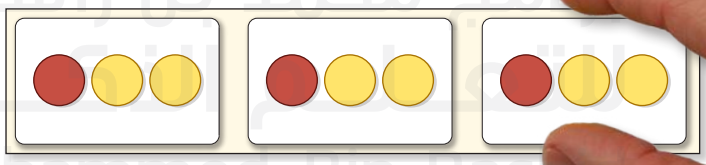
What do you need to find? _____

_____

Hands-On Activity 1

Step 1 Use 3 red counters to represent the fiction books. Use 6 yellow counters to represent the nonfiction books.

Step 2 Determine the smallest possible equal-size groups. Use mats to divide the counters into the groups.



Each group has an equal number of fiction books and an equal number of nonfiction books.

Each group has ☐ fiction book and ☐ nonfiction books.

Hands-On Activity 2

HInd is also collecting books. She wants to make packages that have 3 fiction books and 4 nonfiction books. She already has 9 fiction books. How many nonfiction books will she need?

Use a multiplication table to compare the numbers.

Step 1 Complete the rows for 3 and 4 on a multiplication table.

fiction →	3	6							
nonfiction →	4	8							

Step 2 Read across the top until you reach 9. Find the corresponding number in the bottom row and circle the 2 numbers.

Hind needs ____ nonfiction books.

Hands-On Activity 3

Khalid has 27 jerseys. Divide them into two groups so that for every 4 red jerseys, there are 5 blue jerseys.

Step 1 Complete the rows for 4 and 5 on a multiplication table.

red →	4	8							
blue →	5	10							

Step 2 Read across both rows until you find two numbers with a sum of 27.

There are ____ red jerseys and ____ blue jerseys.

Check Draw a picture to check your answer.

Investigate

Collaborate

Work with a partner. Determine the number of pieces of fruit that should be put in each group. Make as many equal-size groups as possible using all the fruit. Use and draw counters to represent the fruit.

1. 3 apples and 9 pears

2. 4 peaches and 6 oranges

Show your work.

3. 4 plums and 7 bananas

4. 6 apricots and 9 mangos

Work with a partner. Use a multiplication table to solve the following problems.

5. Dana wants groups of 3 notebooks and 5 pens. She already has 12 notebooks. How many pens will she need?

notebooks →										
pens →										

6. Hussein wants groups of 6 daisies and 8 tulips for flower arrangements. He already has 24 daisies. How many tulips will he need?

daisies →										
tulips →										

7. Selma has 77 strawberries. Divide them into two groups so that for every 4 strawberries in Group 1 there are 7 strawberries in Group 2.

Group 1 →										
Group 2 →										

Analyze and Reflect

Collaborate

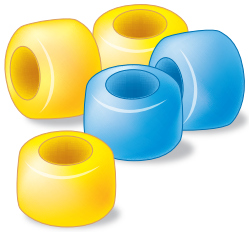
8. MP **Identify Repeated Reasoning** Describe the patterns used in the tables in Activities 2 and 3.

9. MP **Reason Inductively** How would finding the least common multiple help you when dividing items into equal groups?

Create

On Your Own

10. MP **Model with Mathematics** Write and solve a word problem in which there are 3 yellow beads for every 2 blue beads.



11. MP **Model with Mathematics** Write and solve a real-world word problem in which there are 3 tables for every 8 chairs.

12. MP **Model with Mathematics** Write and solve a real-world word problem in which there are 3 pancakes for every person.

13. Inquiry HOW can you use tables to relate quantities?

Lesson 2

Ratios

Real-World Link

Cats Mrs. Shamsa owns 2 large cats and 8 smaller cats.

Compare the number of smaller cats to the larger ones. Use yellow counters to represent the large cats. Use red counters to represent the small cats. Draw the counters in the box.

1. $2 + \square = 8$ There are $\square$ *more* small cats than large cats.

2. $2 \times \square = 8$ There are $\square$ *times* as many small cats as large cats.

3. $8 - \square = 2$ There are $\square$ *fewer* large cats than small cats.

4. $8 \div \square = 2$ The number of large cats is $\frac{\square}{\square}$ the number of small cats.

Essential Question

HOW do you use equivalent rates in the real world?

Vocabulary

ratio

Mathematical Practices

1, 3, 4, 5

Which Mathematical Practices did you use? Shade the circle(s) that applies.

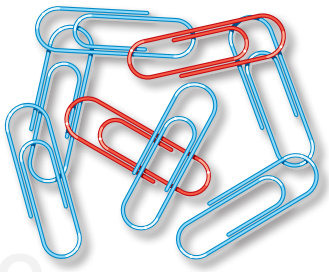
1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Write a Ratio in Simplest Form

There are many different ways to compare amounts or *quantities*. A **ratio** is a comparison of two quantities by division. A ratio of 2 red paper clips to 6 blue paper clips can be written in three ways.

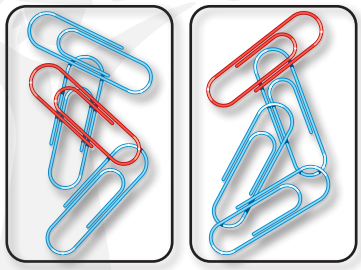
2 to 6 **2:6** $\frac{2}{6}$



As with fractions, ratios are often expressed in simplest form.

Example

1. **Write the ratio in simplest form that compares the number of red paper clips to the number of blue paper clips. Then explain its meaning.**



Write the ratio as a fraction. Then simplify.

red paper clips
blue paper clips
$\frac{2}{6} = \frac{1}{3}$ (÷2 above, ÷2 below)
The GCF of 2 and 6 is 2.

The ratio of red to blue paper clips is $\frac{1}{3}$, 1 to 3, or 1:3. This means that for every 1 red paper clip there are 3 blue paper clips.

Show your work.

Got it? **Do this problem to find out.**

a. Write the ratio in simplest form that compares the number of suns to the number of moons. Then explain its meaning.

a. _____

Use Ratios to Compare Categorical Data

Each piece of categorical data can only be assigned to one group. Bar diagrams (or tape diagrams) and frequency tables can be used to represent categorical data. Ratios can be used to compare the data.

Examples

2. **Several students named their favorite flavor of gum. Write the ratio that compares the number who chose fruit to the total number of students.**

Favorite Flavor of Gum	
Flavor	**Number of Responses**
Peppermint	9
Cinnamon	8
Fruit	3
Spearmint	1

Fruit: 3

Total: 9 + 8 + 3 + 1, or 21

fruit flavor responses → $\frac{3}{21} = \frac{1}{7}$ ← The GCF of 3 and 21 is 3.
total responses →

(÷3 above, ÷3 below)

The ratio is $\frac{1}{7}$, 1 to 7, or 1:7.

So, 1 out of every 7 students preferred fruit-flavored gum.

3. **Monday's yogurt sales are recorded in the table. Write the ratio that compares the sales of strawberry yogurt to the total sales. Then explain its meaning.**

Flavor	Number Sold
Peach	3
Blueberry	6
Vanilla	7
Strawberry	8

Strawberry: ☐

Total: ☐ + ☐ + ☐ + ☐, or ☐

strawberry yogurt sold → $\frac{\square}{\square} \div \frac{\square}{\square} = \frac{\square}{\square}$ or ☐ to ☐
total sold →

So, ☐ out of every ☐ yogurt cups sold were strawberry.

Got it? **Do this problem to find out.**

b. A pet store sold the animals listed in the table in one week. Write the ratio of cats to pets sold that week. Then explain its meaning.

Pet	Number Sold
Birds	10
Turtles	14
Cats	8

Accuracy

It is important to read the entire problem so that an accurate answer can be determined.

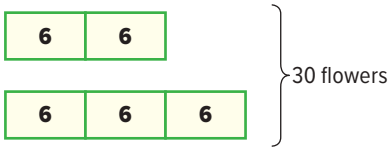
Show your work.

b. _____

Example

4. **Samira wants to divide her 30 flowers into two groups, so that the ratio is 2 to 3.**

Step 1 Use a bar diagram to show a ratio of 2 to 3.



Step 2 There are 5 equal sections. So, each section represents 30 ÷ 5 or 6 flowers.

There are 12 flowers in one group and 18 in the other.

Guided Practice

Check

Write each ratio as a fraction in simplest form. Then explain its meaning. (Example 1)

1. Show your work.

pens to pencils

2.

fils : dirhams

3. Last month, Adham ate 9 apples, 5 bananas, 4 peaches, and 7 oranges. Find the ratio of bananas to the total number of fruit. Then explain its meaning. (Examples 2 and 3)

4. Divide 28 cans of soda into two groups so the ratio is 3 to 4. (Example 4)

5. **Building on the Essential Question** How can you use mental math to determine if a ratio is simplified?

Rate Yourself!

How confident are you about ratios? Shade the ring on the target.

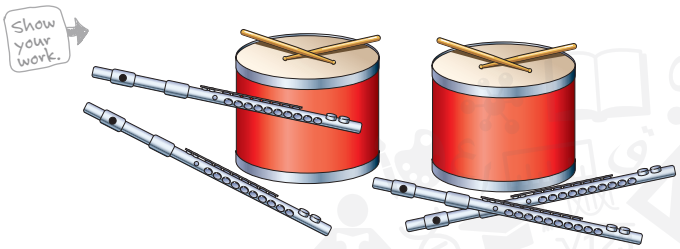
FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

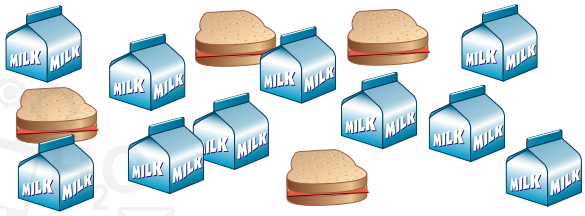
Write each ratio as a fraction in simplest form. Then explain its meaning. (Example 1)

1. _____



flutes : drums

2. _____



sandwiches to milk cartons

3. A class has 6 boys and 15 girls. What is the ratio of boys to girls?

(Example 2) _____

4. The table shows the number of books Sami has read. Find the ratio of mystery books to the total. Explain its meaning. (Example 3)

_____

Type	Number of Books
Mystery	10
Nonfiction	7
Science Fiction	5
Western	2

5. Divide 33 photos into two groups so the ratio is 4 to 7. (Example 4)

_____

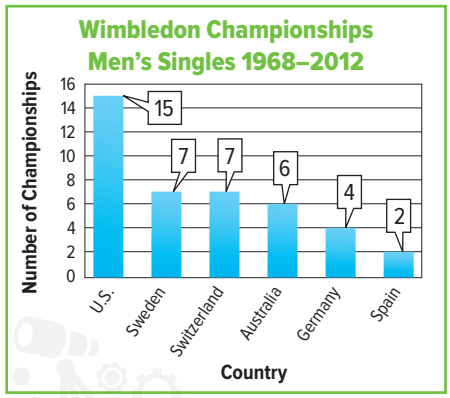
6. MP **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.

a. For each store, what is the ratio of the number of cans to the price?

_____

b. What would be the ratio of the number of cans to the price at Super Saver and Price Busters if a coupon for AED 1 off the total purchase is used? _____

7. MP **Use Math Tools** The graph shows the number of Wimbledon championships of several countries.



a. Write the ratio that compares the championships won by Australia to the total number won by the United States in simplest form. Then explain its meaning.

b. Write the ratio that compares the championships won by Australia to the total number of championships. Then explain its meaning.

H.O.T. Problems Higher Order Thinking

8. MP **Model with Mathematics** Create three different drawings showing a number of rectangles and circles in which the ratio of rectangles to circles is 3:1.

Show your work.

9. MP **Persevere with Problems** Find the missing number in the following pattern. Explain your reasoning.

12, 24, 72, 288, ____

10. MP **Persevere with Problems** The table shows how Zaid spends his time at the gym. Over the course of a week, he wants to spend 600 minutes at the gym. How much more time will he spend lifting weights than on the treadmill? Explain your reasoning.

Activity	Time (min)
Treadmill	25
Lifting weights	35

Name _____ My Homework _____

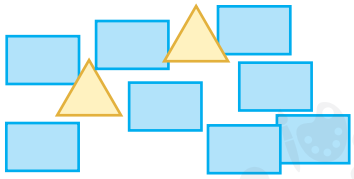
Extra Practice

Write each ratio as a fraction in simplest form. Then explain its meaning.

11. $\frac{1}{4}$; for every 1 triangle there are 4 rectangles.

Homework Help

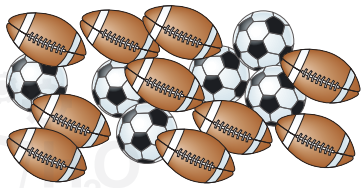
12. _____



triangles to rectangles

There are 2 triangles and 8 rectangles.

The ratio is $\frac{2}{8}$. $\frac{2}{8} \div \frac{2}{2} = \frac{1}{4}$



soccer balls : footballs

13. An animal shelter has 36 kittens and 12 bunnies available for adoption. What is the ratio of bunnies to kittens?

_____

14. Find the ratio of black cell phone covers sold to the total number of cell phone covers sold last week. Then explain its meaning.

Color	Number of Cell Phone Covers Sold
Green	5
Silver	6
Red	3
Black	4

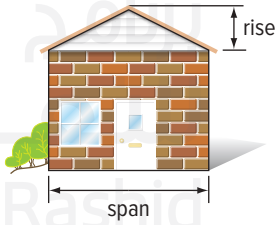
_____

15. On the first day of the food drive, Mrs. Lamis' classes brought in 6 cans of fruit, 4 cans of beans, 7 boxes of noodles, and 4 cans of soup. Find the ratio of cans of fruit to the total number of food items collected. Then explain its meaning.

_____

_____

16. The rise and span for a roof are shown. The pitch of a roof is the ratio of the rise to the half-span. If the rise is 8 feet and the span is 30 feet, what is the pitch in simplest form?



_____

_____

17. MP **Justify Conclusions** Dalia found that 6 of the 24 students in her class own a cell phone. What is the ratio of students that own a cell phone to students that do not? Explain your reasoning to a classmate.

_____

_____

Power Up! Test Practice

18. At a putt-putt course there are 50 yellow golf balls, 45 red golf balls, 65 blue golf balls, 40 orange golf balls, and 60 green golf balls. Select the correct ratio to complete the table.

8:13	4:5
5:6	3:4
10:9	12:13

Comparison	Ratio
yellow to red	
yellow to green	
green to blue	

Comparison	Ratio
red to green	
orange to yellow	
orange to blue	

19. The table shows the number of each type of sports card that Yasmin has collected.

baseball	basketball	football	soccer
45	14	20	21

Write a ratio in simplest form that compares the number of basketball cards to the total number of cards. ☐

Spiral Review

Find the equivalent fraction.

20. $\frac{3}{7} = \frac{\square}{21}$

21. $\frac{1}{6} = \frac{\square}{24}$

22. $\frac{4}{5} = \frac{28}{\square}$

23. Sultan's family is going on vacation. If they drive for 3 hours at the posted speed, how many kilometers will they travel?

_____

24. Islam made $\frac{3}{5}$ of the baskets he shot. Suppose he shot 60 baskets. How many did he make? _____

25. There are 36 students in Mrs. Amal's sixth grade class. If $\frac{5}{12}$ of her students are girls, how many girls are in the class? _____

Inquiry Lab 2
Unit Rates

Inquiry **HOW can you use bar diagrams to compare quantities in real-world situations?**

MP **Mathematical Practices** 1, 3, 8

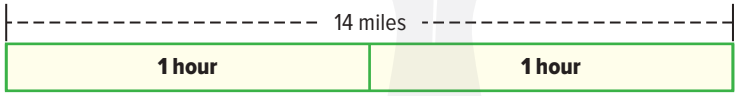
Ahmad and Amjad were rollerblading. They skated 14 miles in 2 hours. If they skated at a constant rate, how many miles did they skate in 1 hour?

What do you know? _____

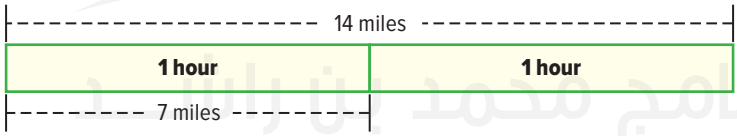
What do you need to find? _____

Hands-On Activity 1

Step 1 Use a bar diagram to represent 14 miles. The box is separated into two equal sections to represent 2 hours.



Step 2 Each section represents one hour. Determine the number of miles skated in one hour.



So, they skated ☐ miles in one hour.

Hands-On Activity 2

A package of 5 crackers contains 205 Calories. How many Calories are in one cracker?

Step 1 Draw a bar diagram to represent 205 Calories. Divide the bar diagram into 5 equal sections to represent 5 crackers.

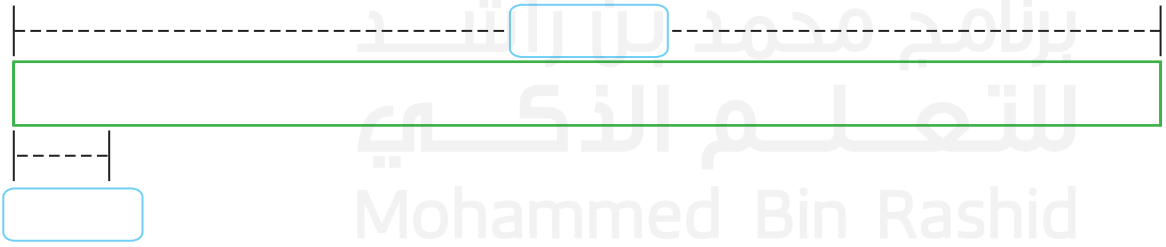
Step 2 Label the first section "1 cracker." Determine the number of Calories in 1 cracker.

So, one cracker contains ____ Calories.

Hands-On Activity 3

A bottle of body wash costs AED 28.80 and contains 12 liters. How much does it cost per liter?

Step 1 Draw a bar diagram to represent _____. Divide the bar diagram into ____ equal sections to represent ____ liters.



Step 2 Label the first section "_____." Determine the cost for 1 liter of body wash.

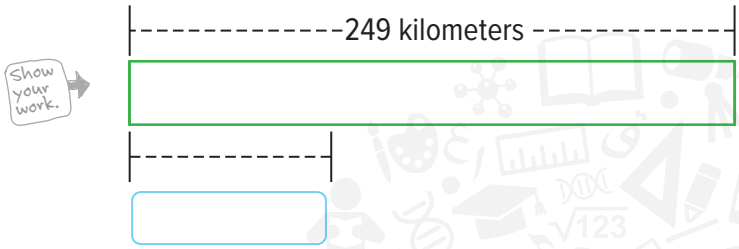
So, one liter of body wash costs AED ____.

Investigate

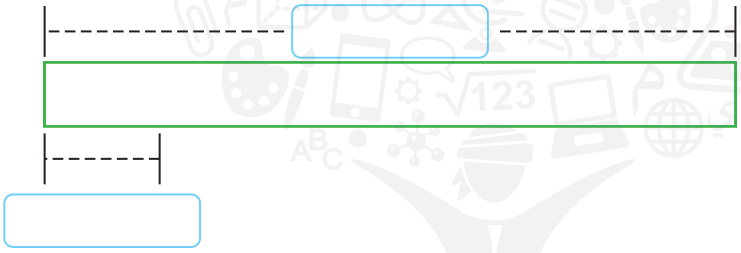
Collaborate

Work with a partner to solve. Use a bar diagram.

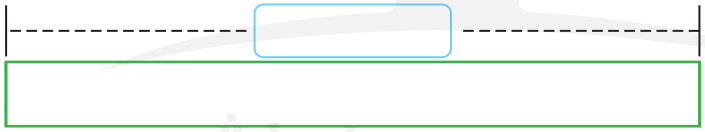
1. Omar drove 249 kilometers in 3 hours. He drove at a constant speed. How many kilometers did he drive in 1 hour? _____



2. Six oranges cost AED 5.34. How much does 1 orange cost? _____

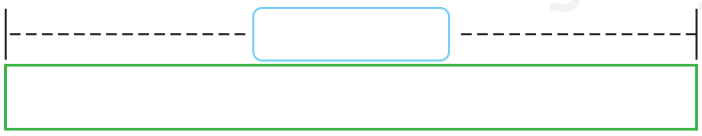


3. Fahd read 231 pages in 7 hours. He read the same number of pages each hour. How many pages did he read in 1 hour? _____



☐ pages

4. Badria has 72 flowers in 4 vases. She put the same number of flowers in each vase. How many flowers are in 1 vase? _____



☐ flowers

Analyze and Reflect

Collaborate

Work with a partner to complete the problem.

5. In the bakery, a container of cookies is AED 12.75 and contains 3 servings. The coins below equal AED 12.75. Divide the coins into 3 equal groups to determine the cost per serving. Circle each group.

6. MP **Reason Inductively** How does dividing the coins into equal groups help solve the problem?

7. MP **Justify Conclusions** The comparison of kilometers to hours in Activity 1 is 14:2, which can be reduced to 7:1. How is simplifying similar to division?

Create

On Your Own

8. MP **Identify Repeated Reasoning** Write a rule for how to compare two quantities so that the second quantity has a value of 1 without using a diagram.

9. MP **Model with Mathematics** Write a real-world word problem in which the unit rate is 6 miles per hour.

10. Inquiry HOW can you use bar diagrams to compare quantities in real-world situations?

Lesson 3

Rates

Vocabulary Start-Up

Use your glossary, which starts on page GL1, to complete the definitions of the vocabulary words in the table.

Definition	Examples
fraction: A number that represents part of a _____ or part of a _____.	$\frac{1}{2}, \frac{3}{4}, \frac{9}{12}, \frac{45}{3}$
ratio: A comparison of two _____ by _____.	2 out of 3, 2 to 3, 2:3, $\frac{2}{3}$
rate: A _____ comparing two _____ with different kinds of _____.	$\frac{36 \text{ kilometers}}{3 \text{ hours}}$ 36 kilometers for every 3 hours AED 26 for 5 bags 19 songs in 5 minutes
unit rate: A _____ that is _____ so that it has a denominator of _____.	$\frac{12 \text{ kilometers}}{1 \text{ hour}}$, 12 kilometers per hour AED 5.20 for 1 bag 3.8 songs in 1 minute

Real-World Link

Dina typed a 15-character text message in 5 seconds.

1. Write the rate Dina typed as a fraction. $\frac{\square \text{ characters}}{\square \text{ seconds}}$

2. What operation would you use to write the fraction in simplest form? _____

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

(1) Persevere with Problems	(5) Use Math Tools
(2) Reason Abstractly	(6) Attend to Precision
(3) Construct an Argument	(7) Make Use of Structure
(4) Model with Mathematics	(8) Use Repeated Reasoning

Essential Question

HOW do you use equivalent rates in the real world?

Vocabulary

rate
unit rate
unit price

MP Mathematical Practices
1, 3, 4

Work Zone

Find a Unit Rate

A **rate** is a ratio comparing two quantities of different kinds of units. A **unit rate** has a denominator of 1 unit when the rate is written as a fraction. To write a rate as a unit rate, divide the numerator and the denominator of the rate by the denominator.

Ratio	Rate	Unit Rate
$15:5 = \frac{15 \text{ characters}}{5 \text{ seconds}}$	$= \frac{3 \text{ characters}}{1 \text{ second}}$	

Examples

1. **Sana picked 45 oranges in 5 minutes. Write this rate as a unit rate.**

Write the rate as a fraction. Compare the number of oranges to the number of minutes. Then divide.

$$\frac{45 \text{ oranges}}{5 \text{ minutes}} \overset{\div 5}{=} \frac{9 \text{ oranges}}{1 \text{ minute}}$$

So, the unit rate is $\frac{9 \text{ oranges}}{1 \text{ minute}}$, or 9 oranges per minute.

2. **The Australian dragonfly can travel 18 miles in 30 minutes. How far can the dragonfly travel in 1 minute?**

Write the rate as a fraction. Compare the distance to the number of minutes. Then divide.

$$\frac{18 \text{ miles}}{30 \text{ minutes}} \overset{\div 6}{=} \frac{3 \text{ miles}}{5 \text{ minutes}}$$

The ratio 3 to 5 cannot be simplified to a whole number rate. It can be written as $\frac{3 \text{ miles}}{5 \text{ minutes}}$ or as a unit rate of $\frac{3}{5}$ mile to 1 minute.

The dragonfly can travel $\frac{3}{5}$ mile every minute.

Simplifying Ratios

The least common factor of 3 and 5 is 1. To find the unit rate of the ratio $\frac{3 \text{ miles}}{5 \text{ minutes}}$, divide both the numerator and denominator by 5. So, the unit rate in fraction form is $\frac{3}{5}$ mile per minute.

Show your work.

Got it? **Do these problems to find out.**

a. _____

b. _____

a. Amna downloaded 35 songs in 5 minutes. How many songs did she download per minute?

b. Kareem is baking several loaves of bread to sell in his bakery. He used 9 cups of water and 12 cups of whole wheat flour. How much water was used per cup of flour?

Example

3. **An adult's heart beats about 2,100 times every 30 minutes. A baby's heart beats about 2,600 times every 20 minutes. How many more beats does a baby's heart beat in 60 minutes than an adult's heart?**

Step 1 Find the unit rates.

Adult: $\frac{2{,}100 \text{ beats}}{30 \text{ minutes}}$ or $\frac{70 \text{ beats}}{1 \text{ minute}}$

Baby: $\frac{2{,}600 \text{ beats}}{20 \text{ minutes}}$ or $\frac{130 \text{ beats}}{1 \text{ minute}}$

Step 2 Using the unit rate for each, determine the number of beats in 60 minutes.

Adult: $70 \times 60 = 4{,}200$ beats

Baby: $130 \times 60 = 7{,}800$ beats

Step 3 Find the difference.

$7{,}800 - 4{,}200 = 3{,}600$

So, a baby's heart beats 3,600 more times in 60 minutes than an adult's heart.

Got it? **Do this problem to find out.**

c. A hummingbird's heart rate while resting is about 7,500 beats every 30 minutes. How many more beats does a hummingbird's heart beat in 60 minutes than a human baby's heart?

Show your work.

c. _____

Key Phrases

Key phrases such as *per*, *in*, *and* for every are often used to describe unit rates.

Find a Unit Price

You can use what you know about unit rates to find a unit price. The **unit price** is the cost per unit. To write a price as a unit price, divide the numerator and the denominator of the rate by the denominator.

$$\frac{\text{AED } 36}{4 \text{ tickets}} = \frac{\text{AED } 9}{1 \text{ ticket}}$$

(÷4 above the left fraction; ÷4 below the left fraction)

For example, it costs AED 36 for 4 movie tickets. So, the cost per unit, or per ticket, is AED 9.

Example

4. **Financial Literacy** **Four potted plants cost AED 88. What is the price per plant?**

Write the rate as a fraction. Compare the total cost to the number of plants. Then divide.

$$\frac{\text{AED } 88}{4 \text{ plants}} \overset{\div 4}{=} \frac{\text{AED } 22}{1 \text{ plant}}$$

So, the price per potted plant is AED 22.00.

Guided Practice

Write each rate as a unit rate. (Examples 1 and 2)

1. 44 points in 4 quarters = _____

Show your work.

2. 125 feet in 5 seconds = _____

3. 360 kilometers traveled on 12 liter of petrol = _____

4. 12 meters in 28 seconds = _____

5. Rahaf shot 20 baskets in 4 minutes. Ahmed shot 42 baskets in 6 minutes. How many more baskets did Ahmed shoot per minute? (Example 3) _____

6. For Marwa's graduation, her mom took her and 4 friends to a water park. Marwa's mom paid AED 400 for 5 student tickets. What was the price for one student ticket? (Example 4)

_____

7. **Building on the Essential Question** How are rates and ratios related? _____

Rate Yourself!

☐ I understand how to find a unit rate.

Great! You're ready to move on!

☐ I still have some questions about rates.

Name _____ My Homework _____

Independent Practice

Write each rate as a unit rate. (Examples 1 and 2)

1. 72 ounces in 6 steaks = _____

2. 162 water bottles in 9 cases = _____

Show your work.

3. Munira divided 40.8 liters of paint among 8 containers. How much paint is in each container? (Example 1) _____

4. Central Subs made 27 sandwiches using 12 pounds of turkey. How much turkey was used per sandwich? (Example 2) _____

5. The results of a car race are shown. Determine who drove the fastest. Explain. (Example 3) _____

Drivers' Times		
Driver	**Laps**	**Time (min)**
Kamal	35	84
Ibrahim	42	96.6
Mustafa	38	102.6

6. Hamza's mom bought an eight-pack of juice boxes at the store for AED 4. Find the unit rate for the juice boxes. (Example 4) _____

7. Hassan's cousin pledged AED 12 for a charity walk. If Hassan walked 3 kilometers, how much did his cousin pay per kilometer? (Example 4) _____

8. MP **Justify Conclusions** The Lemon Company sells a 4-gallon jug of lemonade for AED 24. The Sweet and Sour Company sells an eight-pack of 1-quart bottles of lemonade for AED 16.00. Which company has a higher unit price? Explain your answer. _____

9. The Shanghai Maglev Train is one of the fastest trains in the world, traveling about 2,144 miles in 8 hours.

a. How many miles does it travel in one hour? _____

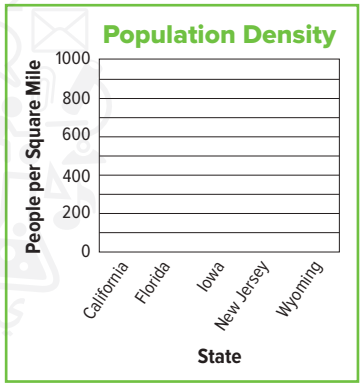
b. The distance between Columbus, Ohio, and New York City is about 560 miles. How many hours would it take the train to travel between the cities? _____

10. MP **Multiple Representations** The table shows the approximate population and areas of five states. *Population density* is the number of people per square unit of an area.

State	Population Estimate (as of July 2007)	Area (square miles)
California	36,500,000	163,707
Florida	18,300,000	65,758
Iowa	2,990,000	56,276
New Jersey	8,690,000	8,722
Wyoming	522,000	97,818

a. **Numbers** Find the population density of each state. Round to the nearest tenth.

b. **Graph** Make a bar graph of the five population densities.



c. **Words** Connecticut has about the same population as Iowa, but its area is 4,875 square miles. Without calculating, compare Connecticut's population density to Iowa's. Justify your answer.

H.O.T. Problems Higher Order Thinking

11. MP **Find the Error** Jamila wrote the rate AED 108 in 6 weeks as a unit rate. Find her mistake and correct it.

12. MP **Persevere with Problems** The ratio of red jelly beans to yellow jelly beans in a dish is 3:4. If Akram eats 3 red jelly beans and 6 yellow ones, the ratio is 4:5. How many yellow jelly beans were originally in the dish?

13. MP **Justify Conclusions** If you travel at a rate of 45 kilometers per hour, how many minutes will it take you to travel 1 kilometer? Justify your response.

Name _____ My Homework _____

Extra Practice

Write each rate as a unit rate.

14. Yousef printed 24 photos in 8 minutes. How many photos did he print per minute?

3 photos per minute

Homework Help

$\frac{24 \text{ photos}}{8 \text{ minutes}} \overset{\div 8}{=} \frac{3 \text{ photos}}{1 \text{ minute}}$

15. Safiya planted 48 tulips in 12 minutes. How many tulips did she plant per minute?

_____

16. Shaikha decorated 72 cookies in 36 minutes. How many cookies did she decorate per minute?

_____

17. Safaa biked 45 kilometers in 3 hours. How many kilometers did she bike per hour?

_____

18. A Ruby Throated Hummingbird beats its wings 159 times in 3 seconds. How many times does the Ruby Throated Hummingbird beat its wings per second? _____

19. Abdulrahman's family bought four concert tickets for AED 252. What was the price per ticket? _____

20. An adult blinks about 450 times in 30 minutes. A 12-year-old blinks about 150 times in 15 minutes. How many more times does an adult blink in 60 minutes than a 12-year-old? _____

21. Find the number of meters each record holder ran in one second of each event. Round to the nearest tenth.

a. 200 meters, 19.30 seconds, Usain Bolt, Jamaica _____

b. 400 meters, 43.18 seconds, Michael Johnson, USA _____

c. 100 meters, 9.69 seconds, Usain Bolt, Jamaica _____

22. MP **Justify Conclusions** The 24 students in Mr. Ismail's homeroom sold 72 magazine subscriptions. The 28 students in Mrs. Nuha's homeroom sold 98 magazine subscriptions. Whose homeroom sold more magazine subscriptions per student? Explain your reasoning. _____

_____

_____

Power Up! Test Practice

23. Boxes of fruit snacks are on sale at the grocery. The boxes are the same size. A family needs to purchase 24 boxes.

Which snacks should they purchase if they want to spend a lesser amount?

How much money will they save?

24. A runner is training for a half marathon. Her training schedule is shown in the table.

Graph and label the unit rates for each day on the number line.

Day	Distance (km)	Time (min)
Tuesday	4	38
Wednesday	6	72
Thursday	4	37
Saturday	3	31.5
Sunday	12	138

Which day did she run the fastest?

Spiral Review

Simplify each fraction.

25. $\frac{16}{80} = \frac{\square}{\square}$

26. $\frac{4}{10} = \frac{\square}{\square}$

27. $\frac{48}{200} = \frac{\square}{\square}$

28. Nahla wants to put a wallpaper border around the ceiling of her room. The dimensions are shown at the right. How many feet of border does she need?

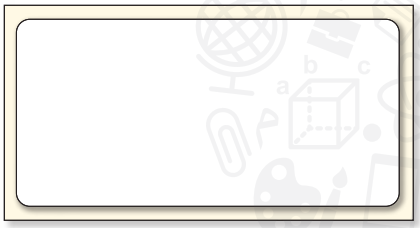
Lesson 4

Ratio Tables

Real-World Link

Refreshments A punch recipe uses one container of soda and three containers of juice to make one batch of punch.

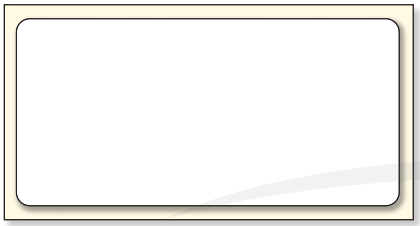
1. Draw red counters to show the number of containers of soda and draw yellow counters to show the number of containers of juice needed to make 2 batches of punch.



soda → ☐

juice → ☐

2. Draw red counters to show the number of containers of soda and draw yellow counters to show the number of containers of juice needed to make 3 batches of punch.



soda → ☐

juice → ☐

3. Find the ratio in simplest form of soda to juice needed for 1, 2, and 3 batches. What do you notice?

_____

Essential Question

HOW do you use equivalent rates in the real world?

Vocabulary

ratio table
equivalent ratios
scaling

MP Mathematical Practices
1, 3, 4, 7, 8

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Equivalent Ratios

The quantities in the opening activity can be organized into a table. This table is called a **ratio table** because the columns are filled with pairs of numbers that have the same ratio.

Soda	1	2	3
Juice	3	6	9

The ratios $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{3}{9}$ are equivalent, since each simplifies to a ratio of $\frac{1}{3}$.

Equivalent ratios express the same relationship between quantities.

Examples

1. **To make yellow icing, you mix 6 drops of yellow food coloring with 1 cup of white icing. How much yellow food coloring should you mix with 5 cups of white icing to get the same shade?**

Use a ratio table. Since $1 \times 5 = 5$, multiply each quantity by 5.

So, add 30 drops of yellow food coloring to 5 cups of icing.

		×5
Drops of Yellow	6	30
Cups of Icing	1	5
		×5

2. **In a recent year, Sultan won a sandwich eating contest by eating nearly 66 sandwiches in 12 minutes. If he ate at a constant rate, determine about how many sandwiches he ate every 2 minutes.**

Divide each quantity by one or more common factors until you reach a quantity of 2 minutes.

So, he ate about 11 sandwiches every 2 minutes.

	÷2	÷2	÷3
Sandwiches	66	33	11
Time (min)	12	6	2
	÷2	÷2	÷3

Check for Accuracy

To check your answer for Example 2, check to see if the ratio of the two new quantities is equivalent to the ratio of the original quantities.

$\frac{11}{2} \times \frac{6}{6} = \frac{66}{12}$

Got it? **Do these problems to find out.**

Show your work.

a. A patient receives 1 liter of IV fluids every 8 hours. At that rate, find how many hours it will take to receive 4 liters of IV fluids.

IV Fluids (L)	1	4
Time (h)	8	

a. _____

b. To make cranberry jam, you need 12 cups of sugar for every 16 cups of cranberries. Find the amount of sugar needed for 4 cups of cranberries.

Sugar (c)	12		
Cranberries (c)	16		4

b. _____

Use Scaling

Multiplying or dividing two related quantities by the same number is called **scaling**. Sometimes you may need to *scale back* and then *scale forward* to find an equivalent ratio.

Examples

3. **Cans of corn are on sale at 10 for AED 20. Find the cost of 15 cans.**

Cans of Corn	10		15
Cost in Dirhams	20		■

There is no whole number by which you can multiply 10 to get 15. So, scale back to 5 and then scale forward to 15.

	÷2	×3
Cans of Corn	10	5	15
Cost in Dirhams	20	10	6
	÷2	×3

Divide each quantity by a common factor, 2.

Then, since $5 \times 3 = 15$, multiply each quantity by 3.

So, 15 cans of corn would cost AED 30.

4. **Salim mows lawns during his summer vacation to earn money. He took 14 hours last week to mow 8 lawns. At this rate, how many lawns could he mow in 49 hours?**

Is there a whole number by which you can multiply 14 to get 49? _____

Scale back to _____, and then scale forward to _____.

	÷2	×7
Number of Hours	14	7	49
Number of Lawns	8	4	28
	÷2	×7

So, Salim can mow _____ lawns in 49 hours.

Got it? **Do this problem to find out.**

c. A child's height measures 105 centimeters. Estimate her height in inches.

Height (cm)	25		105
Height (in.)	10		

Show your work.

c. _____

Example

5. **On her vacation, Muna exchanged AED 50 and received SAR 60. Use a ratio table to find how many Saudi Riyals she would receive for AED 20.**

Set up a ratio table. Use scaling to find the desired quantity.

		÷10	×4
Saudi Riyals	60	6	24
Emirates Dirhams	50	5	20

Divide each quantity by a common factor, 10.

Then, since $5 \times 4 = 20$, multiply each quantity by 4.

Muna would receive SAR 24 for AED 20.

Guided Practice

Check

Complete each ratio table to solve each problem.

1. Khalid receives an allowance of AED 35 every week. How much total does he receive after 4 weeks? (Example 1)

Allowance (AED)	35			
Number of Weeks	1			4

2. Salman runs 8 kilometers in 60 minutes. At this rate, how long would it take him to run 2 kilometers? (Example 2)

Distance Run (km)	8		2
Time (min)	60		

3. Suad buys 12 packs of juice boxes that are on sale and pays a total of AED 48. Use a ratio table to determine how much Suad will pay to buy 8 more packs of juice boxes at the same store. (Example 5)

Number of Juice Boxes			
Price (AED)			

4. **Building on the Essential Question** How can you determine if two ratios are equivalent?

Rate Yourself!

How well do you understand ratio tables? Circle the image that applies.

Clear | Somewhat Clear | Not So Clear

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Complete each ratio table to solve each problem.

1. To make 5 apple pies, you need about 2 pounds of apples. How many pounds of apples do you need to make 20 apple pies? (Example 1)

Number of Pies	5		20
Pounds of Apples	2		

2. Four balls of wool will make 8 knitted caps. How many balls of wool will Amal need if she wants to make 6 caps? (Examples 3 and 4)

Balls of Wool	4		
Number of Caps	8		6

3. Before leaving to visit Mexico, Hussein traded 270 American dollars and received 3,000 Mexican pesos. When he returned from Mexico, he had 100 pesos left. How much will he receive when he exchanges these pesos for dollars? (Example 2)

American Dollars	270		
Mexican Pesos	3000		100

4. On a bike trip across the United Arab Emirates, Rashid notes that he covers about 190 Kilometers every 4 days. If he continues at this rate, use a ratio table to determine about how many Kilometers he could bike in 6 days. (Example 5)

Miles Biked			
Days			

5. MP **Identify Repeated Reasoning** A punch recipe that serves 24 people calls for 4 liters of lemon-lime soda, 2 pints of sherbet, and 6 cups of ice.

People Served	
Liters of Soda	
Pints of Sherbet	
Cups of Ice	

a. Complete a ratio table to represent this situation.

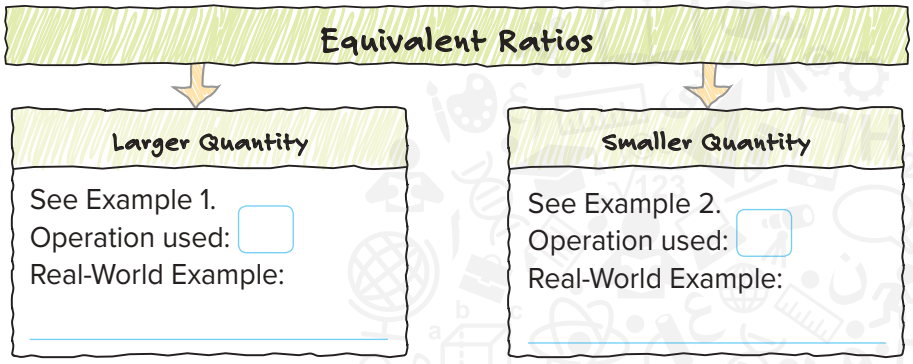
b. How much of each ingredient would you need to make an identical recipe that serves 12 people? 36 people?

c. How much of each ingredient would you need to make an identical recipe that serves 18 people? Explain your reasoning.

6. On a typical day, flights at a local airport arrive at a rate of 10 every 15 minutes. At this rate, how many flights would you expect to arrive in 1 hour?

Number of Flights			
Minutes			

7. MP **Identify Structure** Complete the graphic organizer to explain how equivalent ratios are used to find larger quantities and smaller quantities.



H.O.T. Problems Higher Order Thinking

8. MP **Find the Error** Rana used the ratio table at the right to find the number of people served with 15 pounds of ground turkey. Find her error and correct it.

Pounds of Ground Turkey	2	1	15
People Served	6	5	19

9. MP **Justify Conclusions** There are 18 bulls and 45 cows on a farm. If 4 more bulls and 4 more cows were added, will the ratio of bulls to cows remain the same? Justify your answer using a ratio table.

Bulls			
Cows			

10. MP **Use Math Tools** Complete the ratio table to illustrate a real-world relationship among two quantities in which the scale factor is 4.

Name _____ My Homework _____

Extra Practice

Complete each ratio table to solve each problem.

11. A zoo requires that 1 adult accompany every 7 students that visit the zoo. How many adults must accompany 28 students? 4 adults

Homework Help

Number of Adults	1	2	3	4
Number of Students	7	14	21	28

12. Huda purchased 200 beads for AED 48 to make necklaces. If she needs to buy 25 more beads, how much will she pay if she is charged the same rate?

Number of Beads	200		25
Cost in Dirhams	48		

13. If a hummingbird were to get all of its food from a feeder, then a 16-Grams nectar feeder could feed about 80 hummingbirds a day. How many hummingbirds would you expect to be able to feed with a 12-Grams feeder?

Grams of Nectar	16		12
Number of Birds Fed	80		

14. When a photo is reduced or enlarged, its length to width ratio usually remains the same. Asma wants to enlarge a 8-centimeter by 12-centimeter photo so that it has a width of 15 centimeters. Use a ratio table to determine the new length of the photo.

Length (cm)	8			
Width (cm)	12			

15. Shawqi owns a hybrid SUV that can travel 400 kilometers on a 15-liter tank of petrol. Determine how many kilometers he can travel on 6 liters.

16. MP **Justify Conclusions** A veterinarian needs to know an animal's weight in kilograms. If 20 pounds is about 9 kilograms and a dog weighs 30 pounds, use a ratio table to find the dog's weight in kilograms. Explain your reasoning.

Pounds			
Kilograms			

Power Up! Test Practice

17. Nuhad is making biscuits using the recipe.

How many cups of flour will he need to make 30 biscuits? ____

Whole Wheat Biscuits

2 c	Whole wheat flour
4 tsp	Baking powder
$\frac{1}{2}$ tsp	Salt
2 tbsp	Shortening
1 c	Milk
1	Small egg

Makes 8 biscuits

18. Ruba walked 3 blocks in 15 minutes. Based on this rate, determine if each statement is true or false.

a. Ruba walks 9 blocks in 45 minutes. ☐ True ☐ False

b. Ruba walks 4 blocks in 25 minutes. ☐ True ☐ False

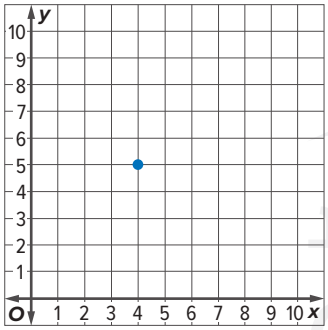
c. Ruba walks 8 blocks in 40 minutes. ☐ True ☐ False

d. Ruba walks 12 blocks in 60 minutes. ☐ True ☐ False

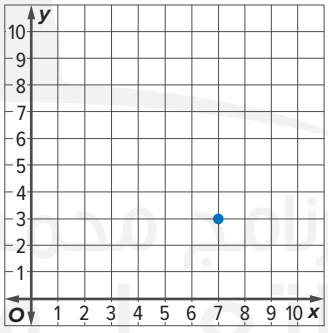
Spiral Review

Identify each point shown on the graph.

19. (_____, _____)



20. (_____, _____)



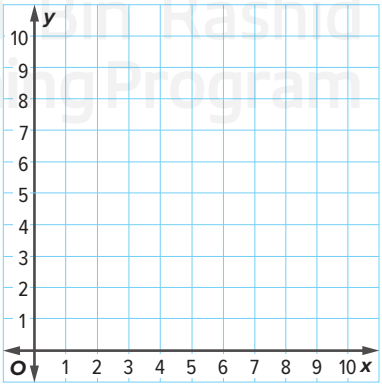
21. Tamir is drawing a map. He needs to plot four points to identify four places on his map. Plot and label the following points.

a. the library at (3, 2)

b. the school at (6, 4)

c. the park at (8, 1)

d. Tamir's house at (2, 8)



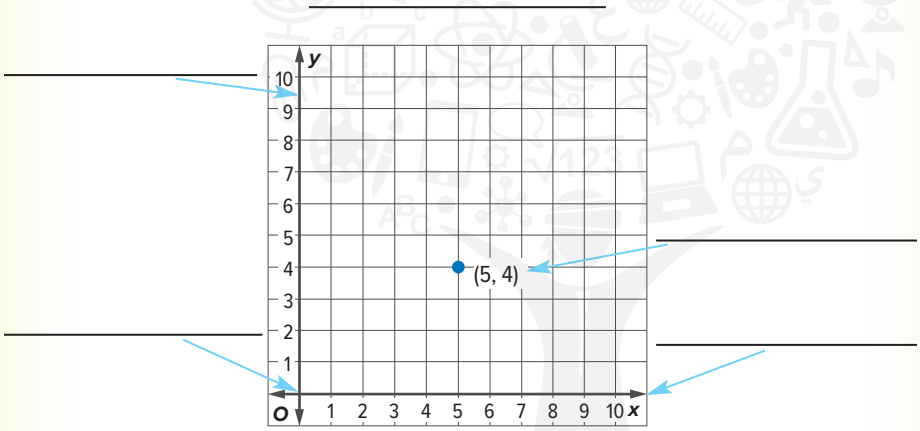
Lesson 5

Graph Ratio Tables

Vocabulary Start-Up

The **coordinate plane** is formed when two perpendicular number lines intersect at their zero points. This point is called the **origin**. The horizontal number line is called the ***x*-axis** and the vertical number line is called the ***y*-axis**. An **ordered pair**, such as (2, 3), is a pair of numbers used to locate a point on the coordinate plane.

Fill in the blanks with the hightlighted words from above.



Real-World Link

In 3 minutes, a North American wood turtle can travel about 17 yards. If the *x*-axis represents minutes and the *y*-axis represents yards, write an ordered pair to represent this situation.

(,)

minutes yards

Essential Question

HOW do you use equivalent rates in the real world?

Vocabulary

coordinate plane
origin
x-axis
y-axis
ordered pair
x-coordinate
y-coordinate
graph

MP Mathematical Practices
1, 3, 4

Which **MP Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Graph Ordered Pairs

You can use an ordered pair to name any point on the coordinate plane. The first number in an ordered pair is the ***x*-coordinate**, and the second number is the ***y*-coordinate**.

The *x*-coordinate corresponds to a number on the *x*-axis. → **(3, 6)** ← The *y*-coordinate corresponds to a number on the *y*-axis.

You can express information in a table as a set of ordered pairs. To see patterns, **graph** the ordered pairs on the coordinate plane.

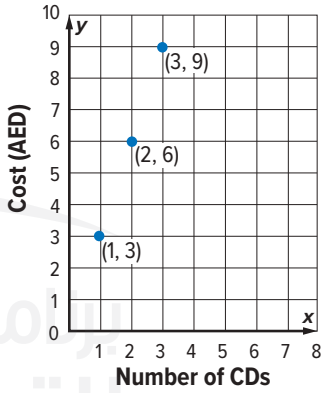
Examples

The table shows the cost in dirhams to create CDs of digital photos at a photo shop. The table also shows this information as ordered pairs (number of CDs, cost in dirhams).

Cost to Create CDs		
Number of CDs, *x*	Cost in Dirhams, *y*	Ordered Pair (*x*, *y*)
1	3	(1, 3)
2	6	(2, 6)
3	9	(3, 9)

1. Graph the ordered pairs.

Start at the origin. Use the *x*-coordinate and move along the *x*-axis. Then use the *y*-coordinate and move along the *y*-axis. Draw a dot at each point.



2. Describe the pattern in the graph.

The points appear in a line. Each point is one unit to the right and three units up from the previous point.

So, the cost increases by AED 3 for every CD created.

Got it? Do these problems to find out.

The table shows Hadil's earnings for 1, 2, and 3 hours. The table also lists this information as ordered pairs (hours, earnings).

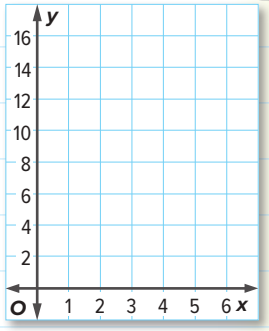
Hadil's Earnings		
Hours, *x*	Dirhams Earned, *y*	Ordered Pair (*x*, *y*)
1	5	(1, 5)
2	10	(2, 10)
3	15	(3, 15)

a. Graph the ordered pairs.

b. Describe the pattern in the graph.

Show your work.

a.



b. _____

Compare Ratios

You can use tables and graphs to compare ratios. The greater the ratio, the steeper the line will appear.

Examples

Two friends are making scrapbooks. Dunia places 4 photos on each page of her scrapbook. Sumaya places 6 photos on each page of her scrapbook.

3. Make a table for each scrapbook that shows the total number of photos placed, if each book has 1, 2, 3, or 4 pages. List the information as ordered pairs (pages, photos).

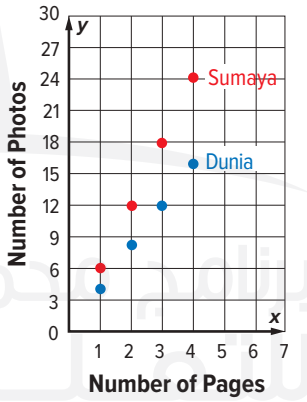
Dunia's Scrapbook		
Pages, x	Photos, y	(x, y)
1	4	(1, 4)
2	8	(2, 8)
3	12	(3, 12)
4	16	(4, 16)

Sumaya's Scrapbook		
Pages, x	Photos, y	(x, y)
1	6	(1, 6)
2	12	(2, 12)
3	18	(3, 18)
4	24	(4, 24)

4. Graph the ordered pairs for each friend on the same coordinate plane.

Graph the ordered pairs for Dunia's scrapbook in blue.

Graph the ordered pairs for Sumaya's scrapbook in red.



5. How does the ratio of photos to each page compare for each person? How is this shown on the graph?

The ratio of photos to pages for Dunia's scrapbook is 4:1 while the ratio for Sumaya's scrapbook is 6:1. On the graph, both sets of points appear to be in a straight line, but the line for Sumaya is steeper than the line for Dunia.

STOP and Reflect

Mariam is also making a scrapbook. She places 5 photos on each page. How does the ratio of photos to each page compare for her book, Dunia's book, and Sumaya's book?

Guided Practice

Two friends are each saving money in their bank accounts. Abdul Rahman saves AED 10 each week while Ali saves AED 15 each week. (Examples 1–5)

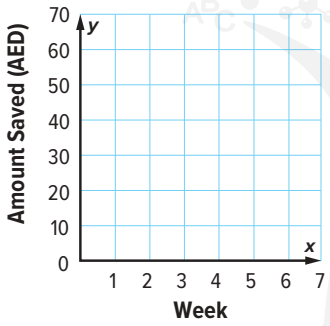
1. Make a table for each friend that shows the total amount saved for 1, 2, 3, and 4 weeks. List the information as ordered pairs (weeks, total dirhams saved).

Show your work.

Abdul Rahman		
Weeks, x	Total Saved (AED), y	(x, y)
1		
2		
3		
4		

Ali		
Weeks, x	Total Saved (AED), y	(x, y)
1		
2		
3		
4		

2. Graph the ordered pairs for each friend on the same coordinate plane.



3. How do the ratios of Abdul Rahman savings and Ali's savings compare? How is this shown on the graph?

4. **Building on the Essential Question** How can graphing help solve a problem involving ratios?

Rate Yourself!

How confident are you about graphing ratios? Check the box that applies.

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

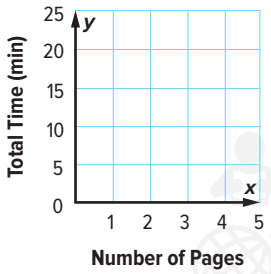
Independent Practice

The table shows the total time it took Samir to read 0, 1, 2, and 3 pages of the book. The table also lists this information as ordered pairs (number of pages, total minutes). (Examples 1–2)

Samir's Reading		
Number of Pages, x	Total Minutes, y	Ordered Pair (x, y)
0	0	(0, 0)
1	4	(1, 4)
2	8	(2, 8)
3	12	(3, 12)

1. Graph the ordered pairs.

Show your work.



2. Describe the pattern in the graph.

_____

_____

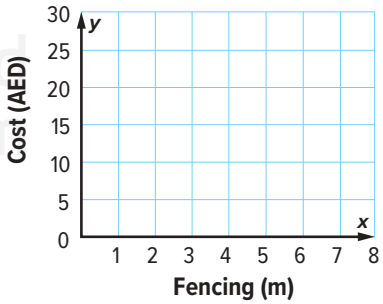
Zaid's Home Supply charges AED 5 for each meter of fencing. Ya Hala's Warehouse charges AED 6 for each meter of fencing. (Examples 3–5)

3. Make a table for each store that shows the total cost for 1, 2, 3, or 4 meter of fencing. List the information as ordered pairs (meter of fencing, total cost).

Zaid's Home Supply		
Fencing (m), x	Cost (AED), y	(x, y)
1		
2		
3		
4		

Ya Hala's Warehouse		
Fencing (m), x	Cost (AED), y	(x, y)
1		
2		
3		
4		

4. Graph the ordered pairs for each store on the same coordinate plane.



5. Using the tables and graphs, write a few sentences comparing the ratios of amount charged per meter of fencing for each store. How is this shown on the graph?

_____

_____

_____

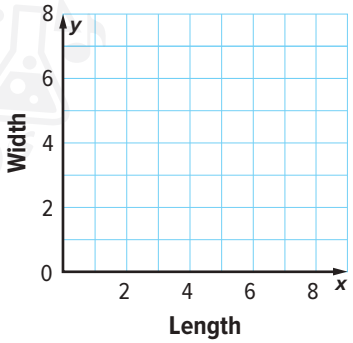
6. MP **Justify Conclusions** Bassam's Pies made 2 peach pies using 10 cups of peaches. They made 3 pies using 15 cups of peaches and 4 pies using 20 cups of peaches. Predict how many cups of peaches would be needed to make 9 peach pies. Explain. _____

7. MP **Multiple Representations** The *golden rectangle* is a rectangle in which the ratio of the length to the width is approximately 1.618 to 1. This ratio is called the *golden ratio*.

Length, x	Width, y	(x, y)

a. **Table** Make a ratio table to show the approximate lengths of golden rectangles given widths that are 1, 2, 3, and 4 units. List the information as ordered pairs (length, width).

b. **Graph** Graph the ordered pairs on the coordinate plane.



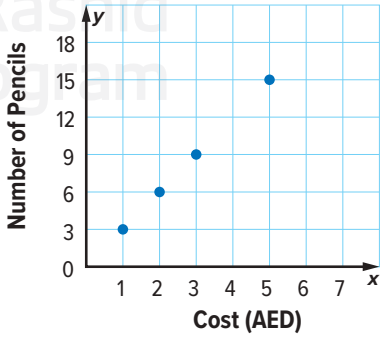
c. **Analyze** How does the area of each rectangle change as the dimensions change?

H.O.T. Problems Higher Order Thinking

8. MP **Model with Mathematics** Write a real-world problem using ratios or rates that could be represented on the coordinate plane. _____

9. MP **Persevere with Problems** Give the coordinates of the point located halfway between (2, 1) and (2, 4). _____

10. MP **Persevere with Problems** The graph shows the cost of purchasing pencils from the school office. The graph is missing a point to indicate the cost of 12 pencils. Complete the graph by plotting the missing information. Explain your answer.

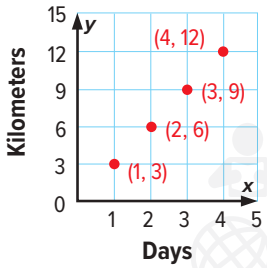


Extra Practice

The table shows the total number of kilometers Ahmad runs for several days. The table also lists this information as ordered pairs (number of days, total kilometers).

11. Graph the ordered pairs.

Homework Help



Ahmad's Running Record		
Days, *x*	**Kilometers, *y***	**(*x*, *y*)**
1	3	(1, 3)
2	6	(2, 6)
3	9	(3, 9)
4	12	(4, 12)

12. Describe the pattern in the graph. The graph shows that as the number of days increases by 1, the number of kilometers run increases by 3.

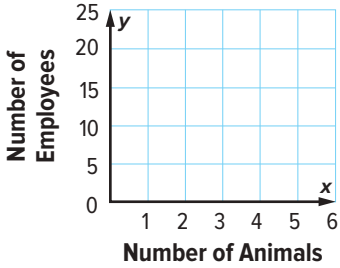
There are two employees for every tiger in the tiger exhibit at a local zoo. For every elephant in the elephant exhibit, there are four employees.

13. Make a table for each animal that shows the total number of employees for 1, 2, 3, or 4 animals. List the information as ordered pairs (number of animals, number of employees).

Tiger Exhibit		
Animals, *x*	**Employees, *y***	**(*x*, *y*)**
1		
2		
3		
4		

Elephant Exhibit		
Animals, *x*	**Employees, *y***	**(*x*, *y*)**
1		
2		
3		
4		

14. Graph the ordered pairs for each exhibit on the same coordinate plane.



15. MP **Justify Conclusions** Using the tables and graphs, write a few sentences comparing the ratios of the number of employees per animal. How is this shown on the graph?

Power Up! Test Practice

16. The table gives the ratio of teachers to students at Al Nahda Middle School.

At Al Salam Middle School, the ratio of teachers to students is 12 to 312. Which statement corretly compares the ratio of the teachers to students at the two schools?

Al Nahda Middle School	
Students, x	**Teachers, y**
24	1
48	2
72	3
96	4

- ☐ There are more students per teacher at Al Salam Middle School than at Al Nahda Middle School.
- ☐ Both schools have an equivalent ratio of students to teachers.
- ☐ There are more students at Al Salam Middle School than at Al Nahda Middle School.
- ☐ There are more students per teacher at Al Nahda Middle School than at Al Salam Middle School.

17. Nihal earns AED 15 for each yard she mows. She wants to buy a dress that costs AED 109. How many yards will she need to mow to earn the money for the dress? Explain.

Spiral Review

Simplify each fraction.

18. $\frac{13}{78} = \frac{\square}{\square}$

19. $\frac{26}{130} = \frac{\square}{\square}$

20. $\frac{20}{240} = \frac{\square}{\square}$

21. There are 270 sixth grade students and 45 chaperones going on a field trip. How many students will be with each chaperone if the groups are divided equally? _____

22. Several students were surveyed about their favorite class. The results are shown in the table. What fraction of the students chose music as their favorite subject? Write the fraction in simplest form.

_____

Favorite Class	
Art	26
English	19
Math	21
Music	16
Science	32

MP Problem-Solving Investigation

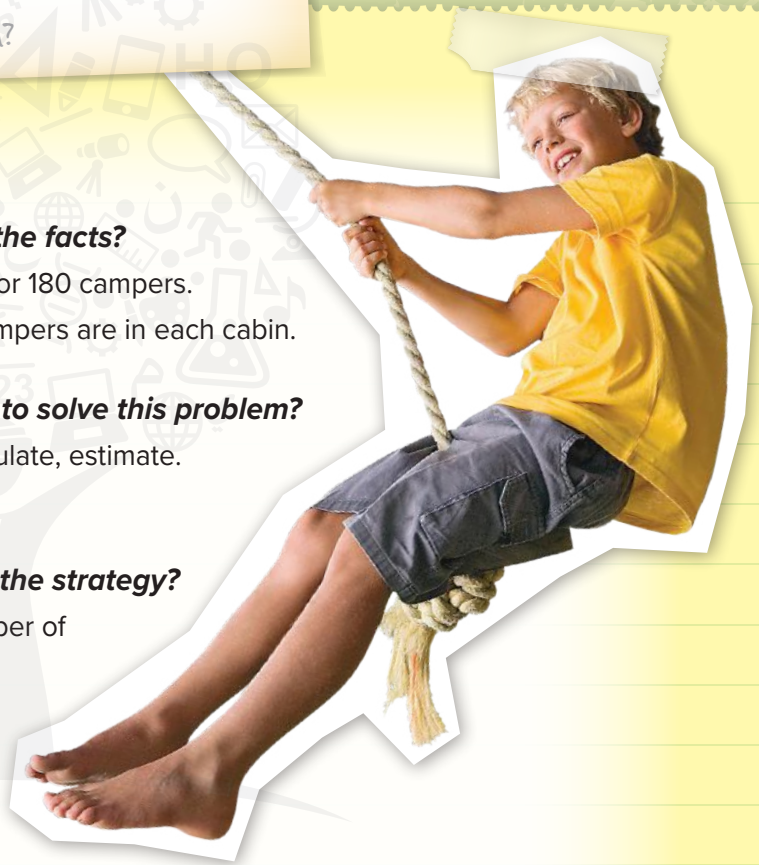
The Four-Step Plan

MP Mathematical Practices
1, 3, 4

Case #1 Cabin Fever

At a summer camp, the ratio of cabins to campers is 15 to 180. An equal number of campers are staying in each cabin.

How many campers are in each cabin?



1 Understand *What are the facts?*

- You know there are 15 cabins for 180 campers.
- You need to find how many campers are in each cabin.

2 Plan *What is your strategy to solve this problem?*

Divide 180 by 15. Before you calculate, estimate.

Estimate $200 \div 20 =$ ☐

3 Solve *How can you apply the strategy?*

Use long division to find the number of campers in each cabin.

$$\begin{array}{r} \square \\ 15\overline{)180} \\ -\square \\ \hline 30 \\ -30 \\ \hline 0 \end{array}$$

There are ☐ campers in each cabin.

4 Check *Does the answer make sense?*

Check by multiplying. Since $12 \times 15 =$ ☐, the answer is correct.

Analyze the Strategy

MP **Justify Conclusions** How many campers would be in each cabin if the ratio of cabins to campers was 15 to 225? Explain.

Case #2 Show Me the Money

The table shows Karima's weekly allowance.

Age	10	11	12	13
Weekly Allowance (AED)	2	4	6	■

If the pattern continues, how much allowance will Karima earn when she is 13 years old?

1 Understand

Read the problem. What are you being asked to find?

I need to find _____

Underline key words and values in the problem. What information do you know?

The top row shows an increase of ☐ year. The bottom row shows an increase of AED ☐ per year.

2 Plan

Choose an operation.

I will use _____ to solve this problem.

3 Solve

Describe the pattern in the table. Then complete it using your problem-solving strategy.

_____

Age	10	11	12	13
Weekly Allowance (AED)	2	4	6	

+1 +1 +1

+2 +2 ☐

6 + ☐ = ☐ So, Karima will earn AED ☐ when she is 13 years old.

4 Check

Use information from the problem to check your answer.

Use subtraction to check your answer. ☐ − ☐ = 6

Collaborate

Work with a small group to solve the following cases.
Show your work on a separate piece of paper.

Case #3 Walking

Khuloud uses a pedometer to find how many steps she takes each school day. She took 32,410 steps over the course of 5 days. She took the same number of steps each day and each step is 28 inches.

How many miles did she walk on Monday? Round to the nearest hundredth. (Hint: There are 72,913 inches in one mile.)

Case #4 Savings

Ayman is earning money to buy a AED 100 bicycle. For each dirham Ayman earns, his mother has agreed to give him AED 1. So far, he has earned AED 14 mowing lawns and AED 7 washing cars.

How much more must Ayman earn in order to buy the bicycle?

Case #5 Money

Mrs. Zahra is buying a new big-screen television. She made an initial payment of AED 50 and paid a total of AED 890 over 12 months.

How much did she pay each month?

Use any strategy!

Case #6 Sports Equipment

Mrs. Badreya has AED 130 to buy basketballs for Emirates Middle School.

How many can she buy at AED 15 each? Interpret the remainder.

Mid-Chapter Check

Vocabulary Check

1. Fill in the blank in the sentence below with the correct term. (Lesson 2)

A _____ is a comparison of two quantities by division.

Skills Check and Problem Solving

Find the greatest common factor or least common multiple of each set of numbers. (Lesson 1)

2. 24 and 18

GCF = _____

3. 12 and 20

LCM = _____

4. 16 and 32

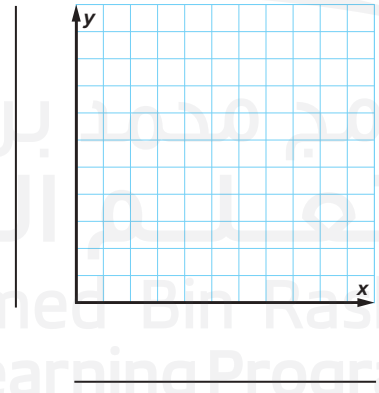
GCF = _____

5. Write *15 cookies to 40 brownies* as a ratio in simplest form. (Lesson 2) _____

6. Write 171 kilometers in 3 hours as a unit rate. (Lesson 3) _____

7. MP **Use Math Tools** The table below shows the amount in Khaled's account each week. List the information as ordered pairs and then graph the ordered pairs. Describe the pattern in the graph. (Lesson 5)

Khaled's Savings		
Week, x	Savings (AED), y	Ordered Pair (x, y)
1	5	
2	10	
3	15	
4	20	
5	25	



8. MP **Persevere with Problems** An artist is using three different colors in a mosaic. The ratio of green to blue to yellow color tiles in the mosaic is 4:6:9. She has 42 blue tiles to use. How many green and yellow tiles does she need? (Lesson 4)

Lesson 6

Equivalent Ratios

Real-World Link

Photography Amani spent AED 2 to make 10 prints from a photo booth. Later, she spent AED 6 to make 30 prints.

Number of Prints	Cost (AED)
10	2
30	6

1. Express the relationship between the number of prints she made and the total cost for each situation as a rate in fraction form.

$\frac{\square}{\square}$ prints and $\frac{\square}{\square}$ prints

2. Compare the relationship between the numerators of each rate in Exercise 1. Compare the relationship between the denominators of these rates.

3. What is the unit rate for 10 prints? _____

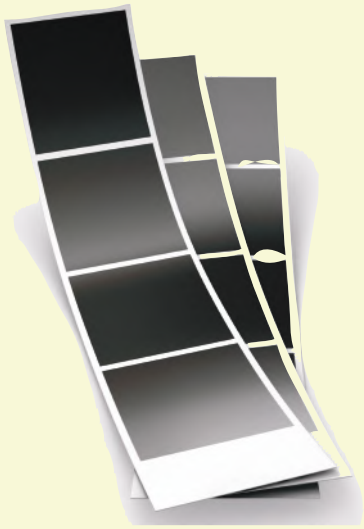
4. What is the unit rate for 30 prints? _____

5. Are the rates in Exercise 1 equivalent? Explain.

Essential Question

HOW do you use equivalent rates in the real world?

MP Mathematical Practices
1, 3, 4, 6, 7



Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Use Unit Rates

There are different ways to determine if two ratios or rates are equivalent. One way is by examining unit rates. By comparing quantities as rates in simplest form, you can determine if the relationship between the two quantities stays the same.

$$\frac{10 \text{ prints}}{\text{AED } 2} \overset{\div 2}{=} \frac{5 \text{ prints}}{\text{AED } 1} \quad \text{and} \quad \frac{30 \text{ prints}}{\text{AED } 6} \overset{\div 6}{=} \frac{5 \text{ prints}}{\text{AED } 1}$$

Since the rates have the same unit rate, they are equivalent ratios.

Examples

Determine if each pair of rates is equivalent. Explain your reasoning.

1. **20 kilometers in 5 hours; 45 kilometers in 9 hours**

Write each rate as a fraction. Then find its unit rate.

$$\frac{20 \text{ km}}{5 \text{ hours}} \overset{\div 5}{=} \frac{4 \text{ km}}{1 \text{ hour}} \qquad \frac{45 \text{ km}}{9 \text{ hours}} \overset{\div 9}{=} \frac{5 \text{ km}}{1 \text{ hour}}$$

Since the rates do not have the same unit rate, they are not equivalent.

Unit Rates

The unit rate in Example 2, $\frac{\text{AED } 15}{1 \text{ T-shirt}}$, is called the unit price since it gives the cost per unit.

2. **3 T-shirts for AED 45; 5 T-shirts for AED 75**

$$\frac{\text{AED } 45}{3 \text{ T-shirts}} \overset{\div 3}{=} \frac{\text{AED } 15}{1 \text{ T-shirt}} \qquad \frac{\text{AED } 75}{5 \text{ T-shirts}} \overset{\div 5}{=} \frac{\text{AED } 15}{1 \text{ T-shirt}}$$

Since the rates have the same unit rate, they are equivalent.

Got it? **Do these problems to find out.**

Determine if each pair of rates is equivalent. Explain your reasoning.

a. 36 T-shirts in 3 boxes; 60 T-shirts in 6 boxes

b. 42 flowers in 7 vases; 54 flowers in 9 vases

a. _____

Show your work.

b. _____

Example

3. **Fatima read the first 60 pages of a book in 3 days. She read the last 90 pages in 6 days. Are these reading rates equivalent? Explain your reasoning.**

$$\frac{60 \text{ pages}}{3 \text{ days}} \overset{\div 3}{=} \frac{20 \text{ pages}}{1 \text{ day}} \qquad \frac{90 \text{ pages}}{6 \text{ days}} \overset{\div 6}{=} \frac{15 \text{ pages}}{1 \text{ day}}$$

Since the rates do not have the same unit rate, they are not equivalent. So, Fatima's reading rates are not equivalent.

Got it? **Do these problems to find out.**

c. Laila made 10 bracelets for 5 friends. Jana made 12 bracelets for 4 friends. Are these rates equivalent? Explain your reasoning.

d. Club A raised AED 504 by washing 42 cars. Club B raised AED 456 by washing 38 cars. Are these fundraising rates equivalent? Explain your reasoning.

Proportion

A proportion is an equation stating that two ratios or rates are equivalent.

c. _____

Show your work.

d. _____

Use Equivalent Fractions

If a unit rate is not easily found, use equivalent fractions to decide whether the ratios or rates are equivalent.

Examples

Determine if the pair of ratios or rates is equivalent. Explain your reasoning.

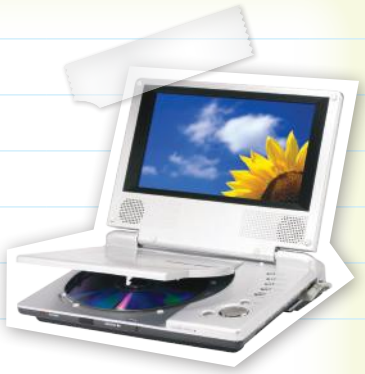
4. **3 free throws made out of 7 attempts;**
9 free throws made out of 14 attempts

Write each ratio as a fraction.

$$\frac{3 \text{ free throws}}{7 \text{ attempts}} \overset{?}{=} \frac{9 \text{ free throws}}{14 \text{ attempts}} \qquad (\times 3 \text{ on the left, } \times 2 \text{ on the right})$$

The numerator and the denominator are not multiplied by the same number. So, the fractions are not equivalent.

Since the fractions are *not* equivalent, the ratios are not equivalent.



5. **Shaikha is comparing the cost of two packages of DVDs. A package of 6 DVDs costs AED 90 and a package of 3 DVDs costs AED 45. Are the rates equivalent? Explain your reasoning.**

$$\frac{6 \text{ DVDs}}{\text{AED } 90} \overset{\div 2}{=} \frac{3 \text{ DVDs}}{\text{AED } 45} \quad (\div 2)$$

The numerator and the denominator are divided by the same number. So, the fractions are equivalent.

Since the fractions are equivalent, the ratios are equivalent.

Got it? **Do this problem to find out.**

e. Mrs. Muna has 12 girls out of 16 students on the Student Council. The Earth Day Committee has 4 girls out of 8 students. Are the ratios equivalent? Explain your reasoning.

Show your work.

e. _____

Guided Practice

Check

Determine if each pair of ratios or rates is equivalent. Explain your reasoning.

1. AED 24 saved after 3 weeks; AED 52 saved after 7 weeks (Examples 1 and 2)

Show your work.

2. 270 Calories in 3 servings; 450 Calories in 5 servings (Examples 1 and 2)

3. Marwan can do 75 push-ups in 3 minutes. Ehab can do 130 push-ups in 5 minutes. Are these rates equivalent?

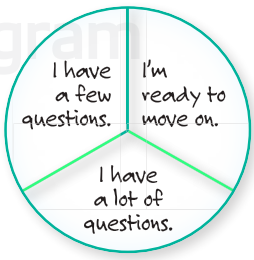
Explain. (Example 3) a

4. A human adult takes about 16 breaths in 60 seconds. A puppy takes about 8 breaths in 15 seconds. Are these rates equivalent? Explain your reasoning. (Examples 4 and 5)

5. **Building on the Essential Question** How can you determine if two ratios are equivalent?

Rate Yourself!

Are you ready to move on? Shade the section that applies.



FOLDABLES Time to update your Foldable!

Independent Practice

Determine if each pair of ratios or rates is equivalent. Explain your reasoning. (Examples 1–2, 4–5)

1. AED 3 for 6 bagels; AED 8 for 20 bagels

2. AED 12 for 3 paperback books; AED 28 for 7 paperback books

3. 3 hours worked for AED 12; 9 hours worked for AED 36

4. 12 minutes to drive 30 laps; 48 minutes to drive 120 laps

5. Reem is comparing the cost of two packages of socks. One package has 8 pairs of socks for AED 12. Another package has 3 pairs of socks for AED 6. Are the rates equivalent? Explain your reasoning.

6. Jamal enlarged the photograph at the right to a poster. The size of the poster is 60 centimeters by 100 centimeters. Is the ratio of the poster's length and width equivalent to the ratio of the photograph's length and width? Explain your reasoning. (Example 3)

7. MP **Justify Conclusions** On a math test, it took Abeer 30 minutes to do 6 problems. Hala finished 18 problems in 40 minutes. Did the students work at the same rate? Explain your reasoning.

8. MP **Be Precise** Refer to the graphic novel frame below for Exercises a–b.

a. What is the unit price for the cans of lemonade at each of the stores?

b. From which store should Amer purchase the cans of lemonade? Explain.

H.O.T. Problems Higher Order Thinking

9. MP **Which One Doesn't Belong?** Identify the rate that does not belong with the other three. Justify your response.

4.5 meters per second	112.5 meters in 25 seconds	86.4 meters in 18 seconds	54 meters in 12 seconds

10. MP **Identify Structure** Write two ratios that are equivalent to $\frac{5}{7}$.

11. MP **Persevere with Problems** The ratio of girls to boys in the junior high band is 5 to 7. At the beginning of the year, there were 72 students in the band. By the end of the year, the ratio of girls to boys was 3 to 4. If there are now 48 boys in the band, how many girls joined the band during the school year?

Name _____ My Homework _____

Extra Practice

Determine if each pair of ratios or rates is equivalent. Explain your reasoning.

12. 16 points scored in 4 games; 48 points scored in 8 games

Homework Help

No; $\frac{16 \text{ points}}{4 \text{ games}} = \frac{4 \text{ points}}{1 \text{ game}}$ and $\frac{48 \text{ points}}{8 \text{ games}} = \frac{6 \text{ points}}{1 \text{ game}}$; Since the unit rates are not the same, the rates are not equivalent.

13. 96 words typed in 3 minutes; 160 words typed in 5 minutes

14. 15 computers for 45 students; 45 computers for 135 students

15. 16 out of 28 students own pets; 240 out of 560 students own pets

16. 288 miles on 12 gallons of fuel; 240 miles on 10 gallons of fuel

17. Fahd is building a model of a living room. The model sofa is 16 inches long and 7 inches deep. The real sofa's dimensions are 80 inches long and 35 inches deep. Is the ratio of the model's dimensions equivalent to the ratio of the real sofa's dimensions? Explain your reasoning.

18. Store A sells 12 juice bottles for AED 4 and store B sells 18 juice bottles for AED 6. Are the rates equivalent? Explain your reasoning.

19. MP **Justify Conclusions** Raghda saved AED 35 in 5 weeks. Her sister saved AED 56 in 56 days. Are the rates at which each sister saved equivalent? Explain your reasoning.

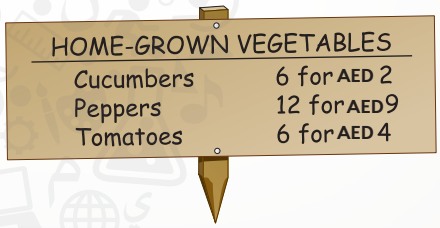
Power Up! Test Practice

20. Sort the ratios listed at the right into bins so that equivalent ratios are grouped together.

40:64	$\frac{5}{75}$	14:35	24:64
$\frac{6}{15}$	$\frac{65}{104}$	$\frac{66}{176}$	$\frac{12}{30}$
15:225	$\frac{6}{16}$	15:24	48:128

Equivalent Ratios

21. A chef buys 9 cucumbers, 18 peppers, and 21 tomatoes at the farmer's market. How much does she spend?



Spiral Review

Write an equivalent fraction.

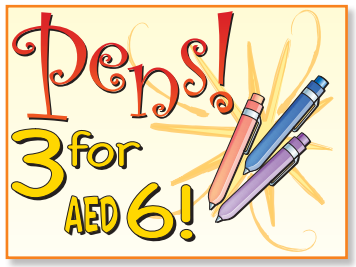
22. $\frac{11}{50} = \frac{33}{\square}$

23. $\frac{4}{5} = \frac{\square}{80}$

24. $\frac{2}{9} = \frac{28}{\square}$

25. Socks are on sale 4 pairs for AED 5. How much would you pay for 8 pairs of socks?

26. Salma bought 3 pens. Ayat bought 1 pen. How much more did Salma spend than Ayat?



Inquiry Lab 3
Ratio and Rate Problems

Inquiry **HOW can you use unit rates and multiplication to solve for missing measures in equivalent ratio problems?**

MP **Mathematical Practices** 1, 3, 4, 5, 8

Anas and Sammy are racing go-karts. Anas completed 6 laps in 12 minutes. If Sammy raced at the same rate, how many minutes did it take him to complete 3 laps?

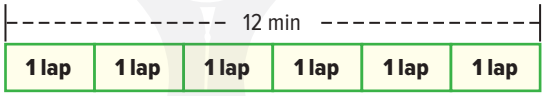
What do you know? _____

What do you need to find? _____

Hands-On Activity 1

Step 1 Use a bar diagram to represent the number of laps Sammy completed. The time to travel 6 laps is 12 minutes.

Anas' race

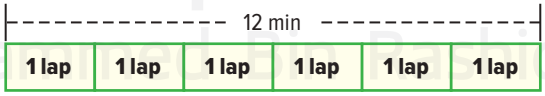


Step 2 Each section represents 1 lap. Determine the number of minutes it took Anas to complete one lap.

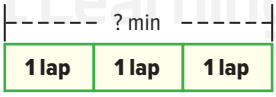
Anas completed each lap in 12 ÷ 6, or ☐ minutes.

Step 3 Determine the number of minutes it took Sammy to complete 3 laps.

Anas' race



Sammy's race



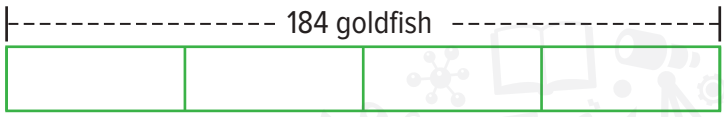
Each lap was completed in ☐ minutes.

So, Sammy's time was 3 × ☐, or ☐ minutes.

Hands-On Activity 2

There are 184 goldfish at a pet store. The goldfish are in 4 tanks, each with the same number of fish. Determine how many fish are in 3 tanks.

Step 1 Use a bar diagram to represent the total number of goldfish.



☐ goldfish

Step 2 Label each section "1 tank." There are 184 ÷ 4, or ☐ goldfish in each tank.

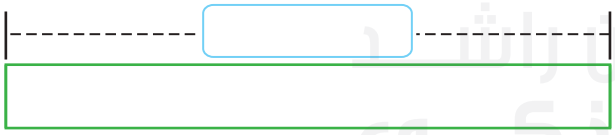
So, there are 46 × ☐, or ☐ goldfish in three tanks.

Hands-On Activity 3

Karim drove 171 kilometers in 3 hours. Hashim drove 177 kilometers in 3 hours. At these rates, how many more kilometers can Hashim drive in 7 hours than Karim?

Step 1 Use bar diagrams to represent the number of kilometers Karim and Hashim drove.

Karim



☐ kilometers

Hashim

☐ kilometers

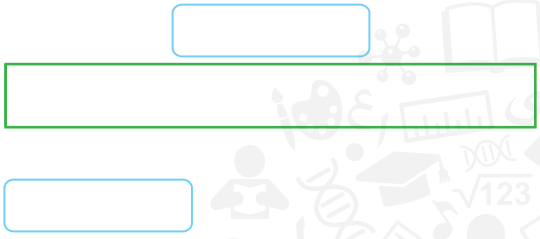
Step 2 Label each section "1 hour". In one hour, Karim drove 171 ÷ 3, or ☐ kilometers and Hashim drove 177 ÷ 3, or ☐ kilometers.

Karim will drive 7 × ☐, or ☐ kilometers in 7 hours. Hashim will drive 7 × ☐, or ☐ kilometers in 7 hours. So, Hashim will drive ☐ − ☐ or ☐ more kilometers in 7 hours than Karim.

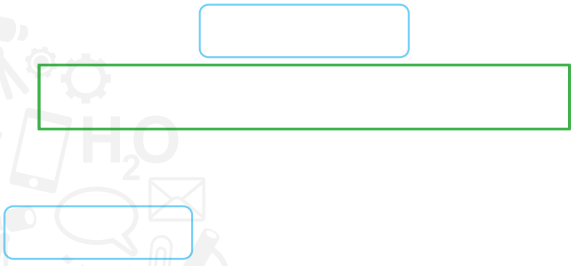
Investigate

Work with a partner. Use a bar diagram to help solve each problem.

1. Determine the kilometers traveled in 5 hours at a rate of 189 kilometers in 3 hours.



2. Determine the number of ice cubes in 32 glasses at a rate of 20 ice cubes in 5 glasses.



3. Determine the cost of 5 kilograms of bananas if 2 kilograms cost AED 11.60.

4. Determine the time needed to deliver 72 papers at a rate of 9 papers in 18 minutes.

5. Predict the number of blue squares in a quilt with 11 green squares if there are 4 green squares in a quilt with 68 blue squares.

6. Predict the number of roses in a garden with 16 sunflowers if there are 3 sunflowers in a garden with 81 roses.

7. How does using a bar diagram help you in predicting the solution to ratio and rate problems?

8. How can you use a bar diagram to check the accuracy of the solution to a ratio or rate problem?

Analyze and Reflect

Collaborate

Work with a partner. Refer to Exercise 4 on the previous page.

9. Suppose Rashid delivers papers at a rate of 9 papers in 18 minutes. How much longer would it take him to deliver 100 papers than 72 papers? Justify your response.

10. How can you determine the time it takes to deliver one paper without drawing a bar diagram?

11. Without using a bar diagram, explain how you would solve the following comparison problem. Then solve the problem. *Rami delivers papers at the rate of 6 papers in 24 minutes. How much longer would it take him to deliver 56 papers than 41 papers?*

Create

On Your Own

12. MP **Model with Mathematics** Reham can read at a rate of 1,100 words in 5 minutes. Write and solve a word problem that uses this information.

13. MP **Model with Mathematics** Maha uses 42 liters of water for a 10-minute shower. Write and solve a prediction problem that uses this information.

14. Inquiry HOW can you use unit rates and multiplication to solve for missing measures in equivalent ratio problems?

Lesson 7

Ratio and Rate Problems

Real-World Link

Games An arcade sells game tokens individually or in packages. They are having a sale on token packages, as shown below.

Number of Packages	Price (AED)
1	5
2	10
3	15

1. How many token packages can you buy with AED 20? ☐ AED 25? ☐ Explain.

2. What is the unit price?

3. How much would it cost to buy 6 token packages?

4. The arcade sells individual tokens for AED 0.25 each. If a token package contains 25 tokens, how much would you save by buying a package of 25 tokens instead of 25 individual tokens? Explain.

Essential Question

HOW do you use equivalent rates in the real world?

MP Mathematical Practices
1, 3, 4, 5, 7

Corbis RF/Alamy

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Solve Ratio Problems

You can use bar diagrams or equations with equivalent ratios to solve ratio and rate problems.

Examples

Equivalent Ratios

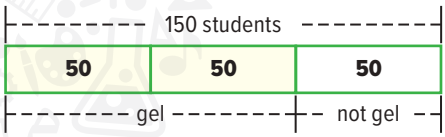
Notice that the numerators of both fractions in Method 2 refer to the number of students who like gel toothpaste. The denominators of both fractions refer to the total number of students being referenced.

1. **Emirates Primary School has 150 students. Two out of three students in Mrs. Munira's class prefer gel toothpaste. Use this ratio to predict how many students in the entire middle school prefer gel toothpaste.**

Method 1 **Use a bar diagram.**

Step 1 Draw a bar diagram.

Step 2 Determine how many students are in each section.



Method 2 **Use equivalent fractions.**

Write an equivalent ratio.

$$\frac{\text{likes gel}}{\text{total}} \rightarrow \frac{2}{3} = \frac{\blacksquare}{150} \leftarrow \frac{\text{likes gel}}{\text{total}}$$

$$\frac{2}{3} \overset{\times 50}{=} \frac{100}{150}$$

Since $3 \times 50 = 150$, multiply 2 by 50.

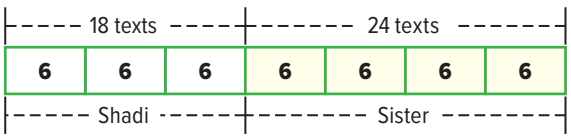
So, 100 students would prefer gel toothpaste.

2. **The ratio of the number of text messages sent by Shadi to the number of text messages sent by his sister is 3 to 4. Shadi sent 18 text messages. How many text messages did his sister send?**

Method 1 **Use a bar diagram.**

Step 1 Draw a bar diagram.

Step 2 Determine how many text messages are in each section.



Method 2 **Use equivalent fractions.**

Write an equivalent ratio.

Shadi → $\frac{3}{4} = \frac{18}{\square}$ ← Shadi
his sister → ← his sister

$\frac{3}{4} \overset{\times 6}{=} \frac{18}{24}$ Since $6 \times 3 = 18$, multiply 4 by 6.

So, Shadi's sister sent 24 text messages.

Got it? **Do these problems to find out.**

a. In a survey, four out of five people preferred creamy over chunky peanut butter. There are 120 people shopping at the grocery store. Use the survey to predict how many people in the store would prefer creamy peanut butter.

b. A survey found that 12 out of every 15 people in the United States prefer eating at a restaurant over cooking at home. If 400 people selected eating at a restaurant on the survey, how many people took the survey?

STOP and Reflect

What is the relationship between ratios and fractions?

Show your work.

a. _____

b. _____

Solve Rate Problems

You can use double number lines or equations to solve rate problems.

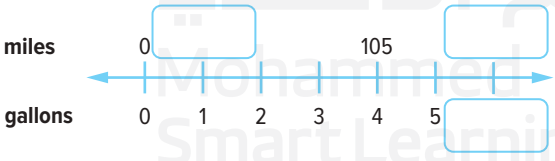
Example

3. **Mahmoud's family drove 105 miles on 4 gallons of gas. At this rate, how many miles can they drive on 6 gallons of gas?**

Draw a double number line.

$105 \div 4 = 26.25$ Find the unit rate.

$26.25 \times 6 = 157.5$ Multiply.



So, Mahmoud's family can drive 157.5 miles on 6 gallons of gas.

Got it? **Do this problem to find out.**

c. There are 810 Calories in 3 scoops of vanilla ice cream. How many Calories are there in 7 scoops of ice cream?

c. _____

Example

4. **Othman drove his motorcycle 120 kilometers in 3 hours. At this rate, how many kilometers can he drive in 5 hours? At what rate did he drive his motorcycle?**

$\frac{120 \text{ kilometers}}{3 \text{ hours}} = \frac{\blacksquare \text{ kilometers}}{1 \text{ hour}}$ (÷3) $\quad \frac{120 \text{ kilometers}}{3 \text{ hours}} = \frac{40 \text{ kilometers}}{1 \text{ hour}}$ Find the unit rate.

$\frac{40 \text{ kilometers}}{1 \text{ hour}} \times 5 \text{ hours} = 200 \text{ kilometers}$ (÷3) Multiply.

So, Othman can drive 200 kilometers in 5 hours driving at a rate of 40 kilometers per hour.

Got it? **Do this problem to find out.**

d. STEM While resting, a human takes in about 5 liters of air in 30 seconds. At this rate, how many liters of air does he take in during 150 seconds?

Show your work.

d. _____

Guided Practice

Check

1. Out of 30 students surveyed, 17 have a cat. Based on these results, predict how many of the 300 students in the school have a cat? (Example 1)

Show your work.

_____

2. If one out of 12 students at a school share a locker, how many share a locker in a school of 456 students? (Example 2) _____

_____

3. Shayma jogged 2 kilometers in 30 minutes. At this rate, how far would she jog in 90 minutes? At what rate did she jog each hour? (Examples 3 and 4)

_____

4. **Building on the Essential Question** How can you use diagrams and equations to solve ratio and rate problems?

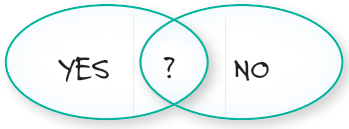
_____

_____

_____

Rate Yourself!

Are you ready to move on? Shade the section that applies.



FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

1. If 45 cookies will serve 15 students, how many cookies are needed for 30 students? (Examples 1 and 2)

Show your work.

2. Four students spent AED 12 on school lunch. At this rate, find the amount 10 students would spend on the same school lunch. (Example 3)

3. A Clydesdale drinks about 120 gallons of water every 4 days. At this rate, about how many gallons of water does a Clydesdale drink in 28 days? (Example 3)

4. STEM In 10 minutes, a heart can beat 700 times. At this rate, in how many minutes will a heart beat 140 times? At what rate can a heart beat? (Example 4)

5. MP **Make a Prediction** The table shows which school subjects are favored by a group of students. Predict the number of students out of 400 that would pick science as their favorite subject.

Favorite Subject	
Subject	**Number of Responses**
Math	6
Science	3
English	4
History	7

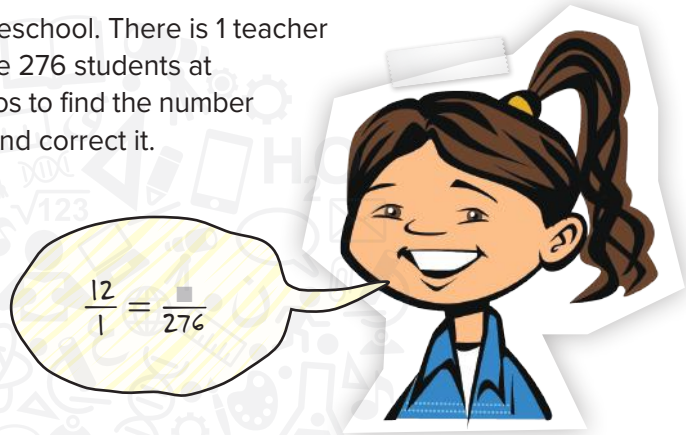
6. Seham takes 4 breaths per 10 seconds during yoga. At this rate, about how many breaths would Seham take in 2 minutes of yoga?

7. MP **Use Math Tools** Find a report in a newspaper or magazine, or on the Internet that uses results from a survey. Evaluate how the survey uses ratios to reach conclusions.

H.O.T. Problems Higher Order Thinking

8. MP **Identify Structure** One rate of an equivalent ratio is $\frac{9}{n}$. Select two other rates, one that can be solved using equivalent fractions and the other that can be solved with unit rates.

9. MP **Find the Error** Lamis' mom teaches at a preschool. There is 1 teacher for every 12 students at the preschool. There are 276 students at the preschool. Lamis is setting up equivalent ratios to find the number of teachers at the preschool. Find her mistake and correct it.



10. MP **Reason Inductively** Tell whether the following statement is *always, sometimes,* or *never* true for numbers greater than zero. Explain.

In equivalent ratios, if the numerator of the first ratio is greater than the denominator of the first ratio, then the numerator of the second ratio is greater than the denominator of the second ratio.

11. MP **Persevere with Problems** Suppose 25 out of 175 people said they like to play disc golf and 5 out of every 12 of the players have a personalized flying disc. At the same rates, in a group of 252 people, predict how many you would expect to have a personalized flying disc. Explain.

12. MP **Persevere with Problems** A car traveling at a certain speed will travel 76 feet per second. How many miles will the car travel in 3.1 hours if it maintains the same speed? Round to the nearest tenth. *(Hint: There are 5,280 feet in one mile.)*

Name _____ My Homework _____

Extra Practice

13. A survey reported that out of 50 teenagers, 9 said they get their news from a newspaper. At this rate, how many out of 300 teenagers would you expect to get their news from a newspaper? 54 teenagers

Homework Help

$\frac{50}{9} = \frac{300}{54}$ (×6 on each side)

14. Nata spent AED 28 on 2 DVDs. At this rate, how much would 5 DVDs cost? At what rate did she spend her money? _____

15. If 15 baseballs weigh 75 ounces, how many baseballs weigh 15 ounces?

_____

16. MP **Make a Prediction** Suppose 8 out of every 20 students are absent from school less than five days a year. Predict how many students would be absent from school less than five days a year out of 40,000 students.

_____

17. For a store contest, 4 out of every 65 people who visit the store will receive a free DVD. If 455 people visit the store, how many DVDs were given away? _____

18. There were 340,000 cattle placed on feed. Write an equivalent ratio that could be used to find how many of these cattle were between 700 and 799 kilograms. How many of the 340,000 cattle placed on feed were between 700 and 799 kilograms?

_____

Cattle Placed on Feed	
Weight Group	**Fraction of Total Cattle**
Less than 600 kilograms	$\frac{1}{5}$
600–699 kilograms	$\frac{11}{50}$
700–799 kilograms	$\frac{2}{5}$
800 kilograms or more	$\frac{9}{50}$

Power Up! Test Practice

19. At a bus station, buses depart at a rate of 3 every 10 minutes. At this rate, _____ buses would depart in one hour.

20. Student Council sells bottled water at the football competition as shown in the table. Determine if each statement is true or false.

Cases Sold	3	6
Time (min)	20	40

a. They sell 27 cases in 3 hours. ☐ True ☐ False

b. They sell 12 cases in 1 hour 20 minutes. ☐ True ☐ False

c. They sell 18 cases in 2 hours. ☐ True ☐ False

d. They sell 24 cases in 2 hours 40 minutes. ☐ True ☐ False

e. They sell 36 cases in 3 hours 20 minutes. ☐ True ☐ False

Spiral Review

Write each fraction as a unit fraction.

21. $\frac{12}{84} =$ _____

22. $\frac{13}{143} =$ _____

23. $\frac{23}{138} =$ _____

24. Mazen gained 64 yards on 16 carries during a recent football game. Find the ratio of yards per carry.

_____

25. The drama club is washing cars for a fundraiser. If the rate continues, how many cars will they wash in 4 hours?

_____

_____

Hours	Cars Washed
1	8
2	16
3	24

26. Follow the rule to find the next three numbers in the pattern. Describe the pattern using the terms *even* and *odd*.

Add 5: 1, 6, 11, _____ , _____ , _____ ...

_____

21ST CENTURY CAREER in Chemistry

Cosmetic Chemist

Are you naturally curious and analytical? Do you like discovering new things? If so, a career as a cosmetics chemist might be a good choice for you. Cosmetics chemists spend time researching, mixing, and testing new formulas that will make cosmetic products both effective and safe. A cosmetics chemist explained, "When you're developing a product, you play with chemicals and balance ratios to get it to feel right. Basically, it's trial and error."

Is This the Career for You?

Are you interested in a career as a cosmetics chemist? Take some of the following courses in high school.

- Algebra
- Biology
- Chemical Science
- Chemistry
- Statistics

Find out how math relates to a career in Chemisty.

MP Beauty is Only Science Deep

Use the information in the recipes below to solve each problem.

1. Using the soap recipe, write a ratio comparing the amount of palm kernel oil to the amount of rose hydrosol as a fraction in simplest form.

2. Write a ratio to compare the amount of jojoba oil to the total amount of the ingredients in the lip balm recipe.

3. The lip balm costs about AED 16 to make. What is the cost per ounce?

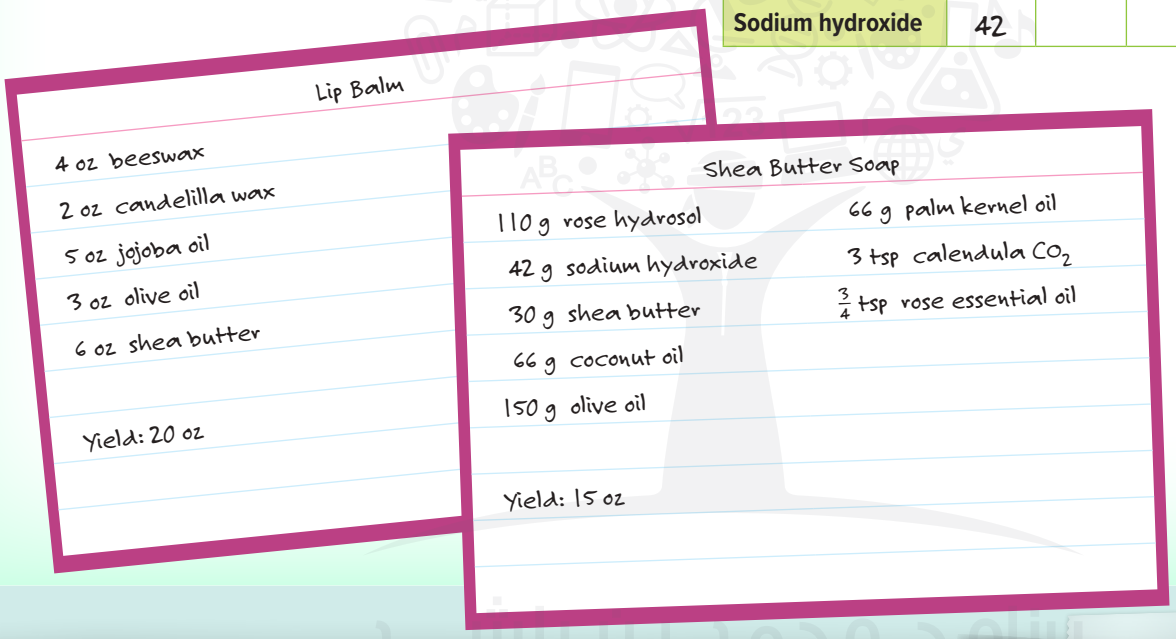
4. The soap recipe makes 4 bars of soap. What is the weight per bar?

5. The lip balm recipe is increased so that 10 ounces of candelilla wax is needed. Complete the ratio table to find the amount of shea butter that is needed.

Candelilla wax	2				10
Shea butter	6				

6. The soap recipe is increased so that 75 grams of shea butter are needed. Complete the ratio table to find the amount of sodium hydroxide that is needed.

Shea butter	30		75
Sodium hydroxide	42		



MP Career Project

It's time to update your career portfolio! There are many different types of jobs in cosmetic chemistry. Research one of these jobs and write a two- or three-sentence job description.

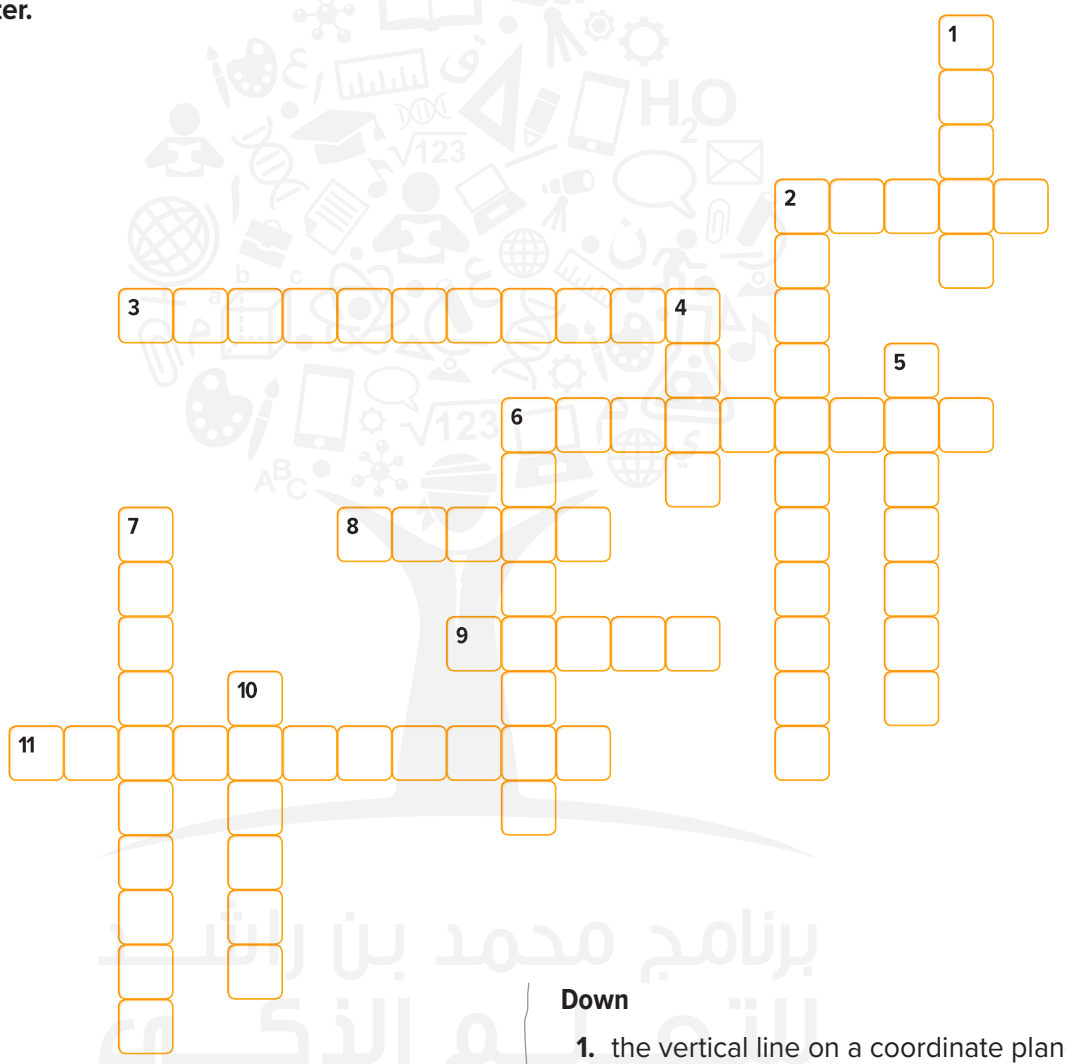
List other careers that someone with an interest in chemistry could pursue.

-
-
-
-
-

Chapter Review

Vocabulary Check

Complete the crossword puzzle using the vocabulary list at the beginning of the chapter.



Across

2. the horizontal line on a coordinate plane

3. used to locate a point on the coordinate plane

6. the cost per unit

8. a comparison of two quantities by division

9. to place a dot a the point named by an ordered pair

11. the second number of an ordered pair

Down

1. the vertical line on a coordinate plane

2. the first number of an ordered pair

4. a ratio comparing two quantities with different kinds of units

5. multiply or divide two quantities by the same number

6. a rate simplified so that it has a denominator of 1

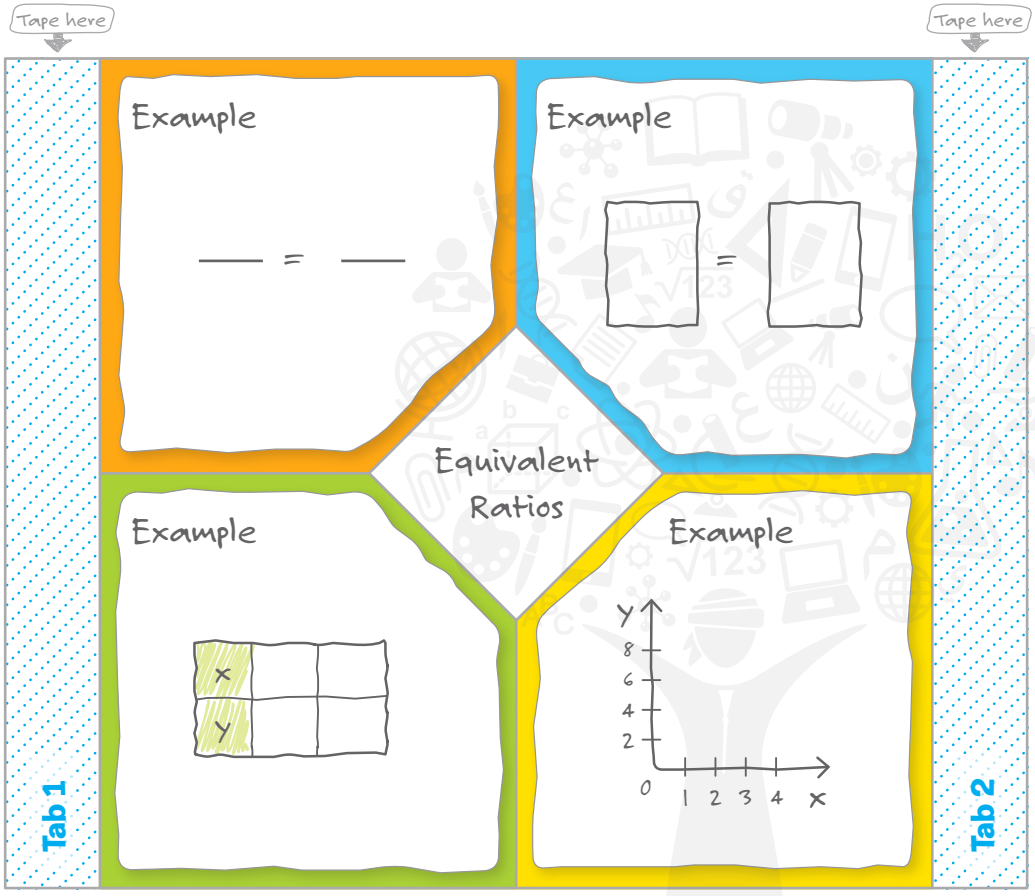
7. columns filled with pairs of numbers that have the same ratio

10. (0, 0)

Key Concept Check

Use Your FOLDABLES

Use your Foldable to help review the chapter.



Got it?

Match each ratio with an equivalent ratio.

1. 65:390
2. $\frac{64}{256}$
3. 156:390
4. $\frac{204}{306}$
5. 56:84
6. $\frac{87}{174}$

a. $\frac{2}{5}$

b. $\frac{2}{3}$

c. $\frac{1}{3}$

d. $\frac{1}{6}$

e. $\frac{1}{4}$

f. $\frac{1}{2}$

Power Up! Performance Task

Community Party

A local club is hosting a party at the school gym. Seeking to establish a connection with the community, the club will be providing a fun event and a meal for the children. The doors open at 6:00 P.M. and the games begin at 6:30 P.M.

Ages	Number of Children That Arrived at 6:00 P.M.	Number of Children That Arrived at 6:30 P.M.
6–10	18	6
11–14	12	4

Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A

As the assistant to the party director, you are instructed to split the children into groups so there is an equal number of children in each group. Each group must also have the same number of children from each age range. At 6:00 P.M., what is the greatest number of groups that can be created? How many children of each age range are represented in each group?

Part B

Additional children arrive at 6:30 P.M. What is the greatest number of groups that can be created using the same guidelines in Part A?

Part C

The students begin to play games. Each game takes 10 minutes to complete. There are five game stations, so five games can be played at once. Based on your answer to Part B, how long will it take all the groups to play every game? Explain your answer.

Part D

Each child receives a meal, which consists of a beefburger, a bag of pretzels, and a drink for the children ages 6–10 and a beefburger, two bags of pretzels, and a drink for the children that are older than 10. It costs the club AED 0.80 for each beefburger, AED 0.35 for each bag of pretzels, and AED 0.60 for each drink. Calculate the cost to feed each age group, as well as the total cost to feed all the children.

Reflect

Answering the Essential Question

Use what you learned about ratios and rates to complete the graphic organizer.

Essential Question

HOW do you use equivalent rates in the real world?

Ratio
What is it?
Examples
Nonexamples

Rate
What is it?
Examples
Nonexamples

How are rates and ratios the same?

How are rates and ratios different?

Answer the Essential Question. HOW do you use equivalent rates in the real world?