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### Series E - Multiplication and Division

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## Series E – Multiplication and Division

#### Contents

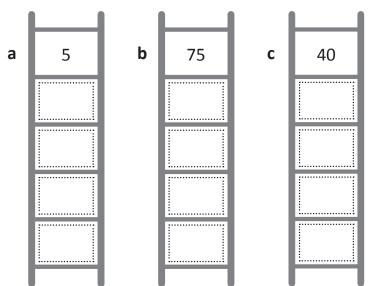
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### Multiplication facts – 5 and 10 times tables

The 5 and 10 times tables are easier if you learn them together.

Answer the 5 times table:

**2** Count in 5s down the ladders:



Fill in the missing number for each times table fact:

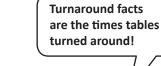
d 
$$\times$$
 5 = 50





 $5 \times 3 = 15$ 

4 Complete the 5 times table turnarounds.





REMEMBER

### Multiplication facts – 5 and 10 times tables

Answer the 10 times table:

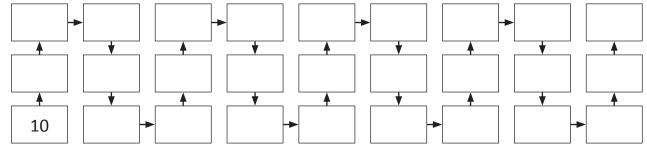
Write the missing numbers for these 5 times table facts:

$$\mathbf{a} \qquad \times \quad 5 \quad = \quad 35$$

$$c \times 5 = 30$$

Write the missing numbers for these 10 times table facts:

Follow the arrows by counting up in 10s:



Multiply each number in the top row by 5 and then by 10:

×	2	11	1	4	5	9	12	6	8	7	10	3
5												
10												

What do you notice?

2



### Multiplication facts – 2 and 4 times tables

The 2 and 4 times tables are good facts to learn together.

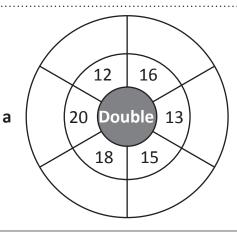
1 Complete the skip counting pattern of 2:



2 Answer the 2 times table. One is in order, the other is mixed up.

It is useful to be able to multiply numbers above 10 by 2. Try these:

4 Complete these doubling wheels as quickly as you can. Multiplying by 2 is the same as doubling.



b 19 Double 100 17 24

#### Multiplication facts – 2 and 4 times tables

Now for the 4 times table. The 4 times table is just double the 2 times table. This is handy to remember if you forget a 4 times table fact.

The 2 times table should be easier, so complete it first. Then double each of the 2 times table facts to get the 4 times table facts:

Write the missing numbers for these 4 times table facts:

d 
$$\times 4 = 24$$

$$g \times 4 = 20$$

Use the hint to get the answer. Then fill in the missing digit to make the 4 times table fact complete:

8 Look at the numbers in the grid and circle 3 numbers that would make a multiplication fact. Look for  $\times$  2 and  $\times$  4 facts. They are either left to right or top to bottom. The first one has been done for you. There are 10 to find.

4	3	12	4	8	32
4	1	3	2	7	1
16	5	3	8	2	9
3	4	6	24	14	4
2	8	16	7	9	36
9	2	18	10	2	20

### Multiplication facts – 8 times table

Here is the 8 times table. You can double the 4 times table to get the 8 times table.

Complete the 4 times table as quickly as you can. Then after you have checked them, double them to complete the 8 times table facts:

2 Use double, double and double again for these problems:



If you get stuck on the 8s, think double, double and double again.
For example, 3 × 8
Think: double 3 is 6
double 6 is 12
double 12 is 24

On Mia's calculator, the 8 key is broken. Show her the steps she could follow to find the answer to 16 × 8. Use a calculator to test the steps.

### Multiplication facts – 3 and 6 times tables

Here are the 3 times and 6 times tables together. Can you think of why it's better to learn these facts together?



Use the picture of the dice above to complete both the 3 times table and the 6 times table:

## Now try these mixed up:

#### Fill in the missing digits to make these times table facts complete:

### Multiplication facts – 3 and 6 times tables

4 Match the answers to the questions. Each answer has two matching questions.

4 × 6

16 × 3

3 × 8

3 × 10

8 × 6

30









3 × 4

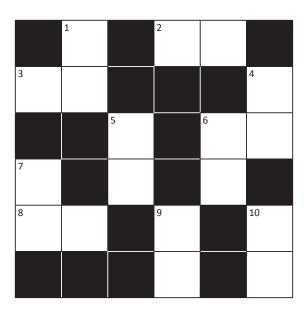
2 × 3

5 × 6

6 × 2

1 × 6

**5** Complete the cross number puzzle:



**Across** 

2. 9 × 3

3. 3 × 6

6. 5 × 6

8. 7 × 6

Down

1. 8 × 6

4.  $10 \times 6$ 

5. 9 × 6

6. 6 × 6

7.  $4 \times 6$ 

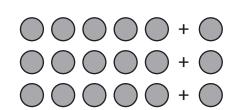
9. 6 × 3

10. 7 × 3

What number am I? I am in the 3 times table, 4 times table and 6 times table. I'm not 12.

I am

#### Multiplication facts - 6 times table



You know more times tables facts than you realise. For example, knowing your  $\times$  5 can help with your  $\times$  6.

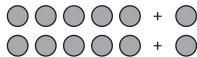
The array shows 3 rows of 5. If we add another dot to each row we can change 3 rows of 5 to 3 rows of 6. This is called building up.

$$3 \times 5 = 15 + 3 \longrightarrow 3 \times 6 = 18$$

1 Change these × 5 arrays into × 6 arrays.

a

b



Complete this table to show how to change a × 5 array to a × 6 array by building up. The first one has been done for you.

	× 5	Number to add	× 6
а	3 × 5 = 15	3	3 × 6 = 18
b	2 × 5 = 10		
С	7 × 5 = 35		
d	4 × 5 = 20		
е	6 × 5 = 30		
f	9 × 5 = 45		



### Multiplication facts – 7 times table

Practise your 7 times table.

## Use this array to complete the 7 times table:

)(	)(	)(	)(	)(	)(	
)(	)(	)(	)(	)(	)(	
)(	)(	)(	)(	)(	)(	
)(	)(	)(	)(	)(	)(	
)(	)(	)(	)(	)(	)(	
)(	)(	)(	)(	)(	)(	
)(	)(	)(	)(	)	)(	
)(	)(	)(	)(	)(	)(	<u> </u>
)(	)(	)(	)(	)(	)(	

## Fill in the missing numbers:

d 
$$\times$$
 7 = 28

#### **3** Complete these × 7 facts. Look out for turnarounds.

#### 4 Solve these problems.

- a Boxes of oranges hold 8 oranges each. If I have7 boxes, how many oranges do I have altogether?
- b Our hockey team scored 3 goals in each of our7 games. How many goals did we score in total?
- **c** There are 35 frogs in the glass cases at the zoo. Each case hold 7 frogs. How many cases are there?

### Multiplication facts – 7 times table

If you get stuck on a 7 times table fact, remember the 8 times table fact and build down.

#### 5 Think of the × 8 table fact to get the × 7 table fact.

× 8 table	Number to subtract	× 7 table
1 × 8 = 8	1	1 × 7 =
2 × 8 = 16	2	2 × 7 =
3 × 8 = 24	3	3 × 7 =
4 × 8 = 32		4 × 7 =
5 × 8 = 40		5 × 7 =
6 × 8 = 48		6 × 7 =
7 × 8 = 56		7 × 7 =
8 × 8 = 64		8 × 7 =
9 × 8 = 72		9 × 7 =
10 × 8 = 80		10 × 7 =
11 × 8 = 88		11 × 7 =
12 × 8 = 96		12 × 7 =

#### 6 Add the missing numbers to each fact:

d 
$$\times$$
 7 = 42

#### Use the $\times$ 8 to complete the $\times$ 7:

×	11	4	2	6	1	12	9	5	3	7	8
8											
7											



### Multiplication facts – 9 times table

Practise your 9 times table.

## Use this array to complete the 9 times table:

		)(	
$\bigcirc$		)(	
		)(	
$\bigcirc$		)(	)
$\bigcirc$		)(	)
$\bigcirc$		)(	)
		$\bigcirc$	

#### **2** Complete these × 9 facts. Look out for turnarounds.

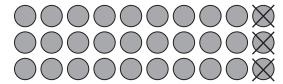
#### 3 Find the cost of these items:





#### Multiplication facts - 9 times table

If you get stuck on a 9 times table fact, you can use the 10 times table facts and then build down.



$$3 \times 10 = 30 - 3 \longrightarrow So, 3 \times 9 = 27$$

If you want to check whether a number is in the 9 times table add its digits together. If the answer is 9, then it is! For example, if you add the digits of 27 together, you get 9 (2 + 7 = 9), so you know that 27 is in the 9 times table.



4 Think of the × 10 facts and build down to get the × 9 facts. The first one is done for you.

× 10 table	Number to subtract	× 9 table
1 × 10 = 10	1	1 × 9 = 9
2 × 10 = 20		
3 × 10 = 30		
4 × 10 = 40		
5 × 10 = 50		
6 × 10 = 60		
7 × 10 = 70		
8 × 10 = 80		
9 × 10 = 90		
10 × 10 = 100		
11 × 10 = 110		
12 × 10 = 120		

Can you see a pattern in the numbers in the 9 times table? As the numbers get larger the tens digit goes up one and the ones digit goes down one.



DISCOVER

Complete the × 9:

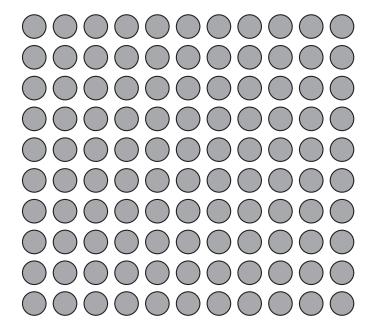
×	2	6	4	8	12	3	9	10	5	7	11
9											



### Multiplication facts – 11 times table

Practise your 11 times table. Can you see the pattern?

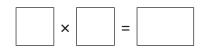
#### Use this array to complete the 11 times table:



#### Complete these × 11 facts. Look out for turnarounds.

#### Solve these problems.

- **a** There are 11 players in a football team and 10 teams in the league. How many players are there in total?
- **b** On each of our 6 class tables is a pot containing 11 pencils. How many pencils are there altogether?
- **c** On our school trip we split our class of 33 into groups of 11. How many children were in each group?

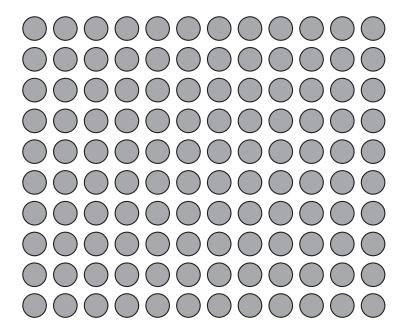


13

### Multiplication facts – 12 times table

Practise your 12 times table.

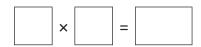
#### Use this array to complete the 12 times table:



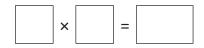
#### **2** Complete these × 12 facts. Look out for turnarounds.

#### 3 Solve these problems.

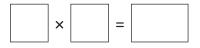
**a** I make 3 batches of 12 cookies. How many cookies is this altogether?



**b** A florist is selling bunches of 12 roses. She sells 6 bunches. How many roses is this?



**c** Eggs cost £3 for a dozen? If I spend £15 on eggs, how many eggs have I bought in total?





### Using known facts – factors and multiples

When 2 numbers are multipled together, the answer is called a multiple. The first 3 multiples of 2 are 2, 4, 6.

$$1 \times 2 = 2 \qquad \qquad 2 \times 2 = 4$$

$$2 \times 2 = 4$$

$$3 \times 2 = 6$$

5, 10, 15, 20, 25, 30, 35, 40, 45, 50 are the first 10 multiples of 5.

List the first 12 multiples of each number:

а	6	6						
b	2	2						
С	10							
d	3							
e	4							

Write these numbers in the correct spots on the Venn diagram:

8

4

9

Multiples

of 2

6

Multiples

of 3

12

3

The space in the diagram where the circles overlap is where you put numbers that are both multiples of 2 and 3.

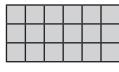


THINK

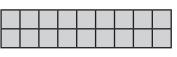
Can you think of any other numbers up to 60 that could go into the overlapping space in the Venn diagram above?

### Using known facts – factors and multiples

Factors are numbers that you multiply together to give a multiple.



$$3 \times 6 = 18$$

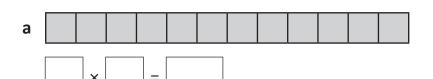


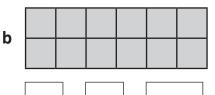
$$2 \times 9 = 18$$

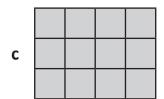
These arrays show some of the factors of 18: 3, 6, 2 and 9.

Can you think of any other factors of 18?

4 Complete the number sentence for each set of arrays and then list the factors.

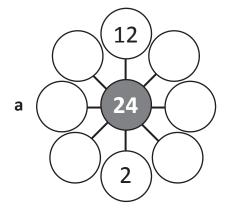


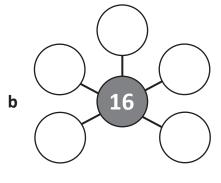


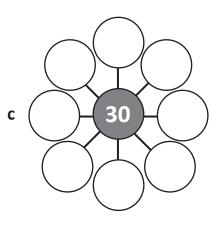


**d** The factors of 12 are:

5 Complete each diagram to show the factors of the number in the middle circle:







### Mental multiplication strategies – multiplying by 10 and 100

When we multiply any whole number by 10, the number is getting 10 times bigger. This means that each digit moves one place value column to the left and we use 0 as a place holder in the ones column.

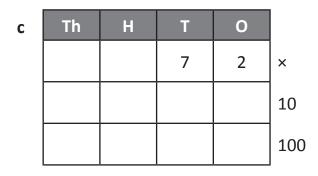
When we multiply any whole number by 100 the number gets 100 times bigger. This means that each digit moves two place value columns to the left and we use 0 as a place holder in the ones and tens columns.

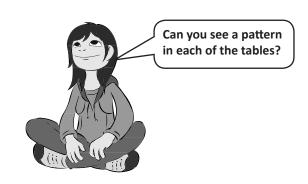
Thousands	Hundreds	Tens	Units	
		4	5	×
	4	5	0	10
4	5	0	0	100

1 Use the place value tables to multiply these numbers by 10 and 100:

а	Th	Н	Т	0	
			1	5	×
					10
					100

b	Th	Н	Т	0	
			4	8	×
					10
					100





2 Use patterns to solve these:

### Mental multiplication strategies – multiplying by 10 and 100

How do you multiply by other multiples of 10? Let's look at  $8 \times 20$ . We can use known times tables facts and write this as place value amounts:

$$8 \times 2 \text{ tens} = 16 \text{ tens So}, 8 \times 20 = 160$$

Draw lines from the numbers written as place value amounts to the times tables facts:

10 tens

14 tens

36 tens

27 tens

12 tens

16 tens

 $3 \times 4$  tens

 $4 \times 4$  tens

 $5 \times 2$  tens

 $7 \times 2$  tens

 $6 \times 6$  tens

9 × 3 tens

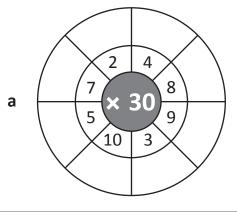
Write the digit that represents each place value amount:

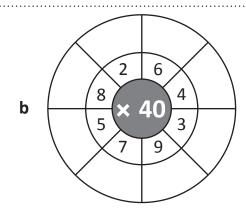
First complete the hints and then use them to write the facts:

Hints:

 $\mathbf{a}$  4 × 6 tens = tens Facts:

Complete the number wheels:





# Mental multiplication strategies – multiplying/dividing by 0 and 1

If you multiply by 0 the answer will always be 0.

 $5 \times 0$  means '5 lots of 0', which is nothing.

The answer is not going to change, whether you have 5 or 35 or 3,005 lots of nothing. The answer will always be zero.

Multiplying by 1 is also very simple.

$$8 \times 1$$
 means '8 lots of 1'.

$$73 \times 1$$
 means '73 lots of 1', which is 73.

So if you multiply any number by 1 the answer will always be the number with which you started.

**Dividing by 1** is straightforward too. If we divide a number, we are working out how many equal groups can be made from that number. So,  $10 \div 1$  means 'we have 10 and we want to make one group with it'. How many will be in that one group? The answer is 10. So, as with multiplying by 1, you always end up with your starting number when you divide by 1.

$$3 \div 1 = 3$$

$$333 \div 1 = 333$$

$$33\,333 \div 1 = 33\,333$$

(In case you are wondering, **you can't divide by 0**. We can't split, say, a bag of sweets into groups of nothing – it doesn't make any sense to divide a number by zero. It can't be done. We say that dividing by 0 is **undefined**.)

#### Solve these calculations:

$$\mathbf{k} \quad 0 \times 0 \quad = \quad$$

$$m 74 \div 0 =$$

$$3 \times 1 = 3$$
$$3 \div 1 = 3$$
$$3 \times 0 = 0$$
$$3 \div 0 = impossible!$$



### Mental multiplication strategies – multiplying 3 numbers

There is a law in maths called the **Commutative Law**. This states that for certain types of calculation, the order of the numbers doesn't matter. The answer will be the same. It is true for addition.

$$3 + 4 = 7$$

$$3 + 4 = 7$$
  $4 + 3 = 7$ 

$$62 + 19 = 71$$

$$62 + 19 = 71$$
  $19 + 62 = 71$ 

The same is true for multiplication.

$$5 \times 2 = 10$$

$$5 \times 2 = 10$$
  $2 \times 5 = 10$ 

$$8 \times 7 = 56$$

$$8 \times 7 = 56$$
  $7 \times 8 = 56$ 

If you are multiplying more than two numbers, the Commutative Law still applies.

$$3 \times 2 \times 6 = 36$$

$$6 \times 2 \times 3 = 36$$

$$6 \times 2 \times 3 = 36$$
  $2 \times 6 \times 3 = 36$ 

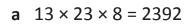
$$2 \times 3 \times 6 = 36$$

$$6 \times 3 \times 2 = 36$$

$$2 \times 3 \times 6 = 36$$
  $6 \times 3 \times 2 = 36$   $3 \times 6 \times 2 = 36$ 

#### Solve these multiplications:

#### 2 Using the Commutative Law, create two different correct multiplications using the same numbers:



**b** 
$$7 \times 14 \times 26 = 2548$$

**Does the Commutative** Law work for subtraction and division too?



THINK



#### Mental multiplication strategies – doubling strategy

There are many double facts that you should know.

This includes numbers outside the times tables we have been working on.

b

Here are 2 double facts that are handy to know:

double 15 is 30

a

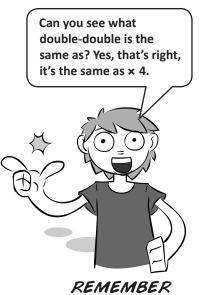
double 50 is 100

Can you think of more?

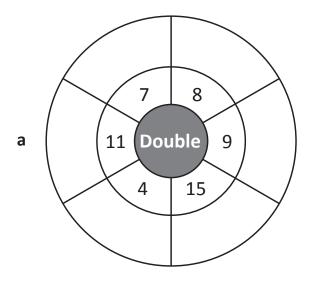
#### Complete these function machines:

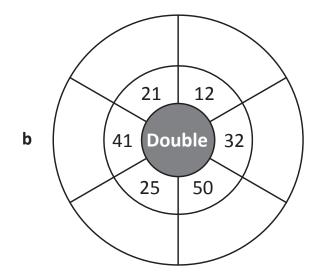
Double						
IN	OUT					
15	30					
24						
30						
45						
18						

Double-double						
IN	OUT					
15	60					
24						
30						
45						
50						



#### Complete these doubling wheels:





### Mental multiplication strategies – doubling strategy

We also use doubling when we multiply by 4 and by 8.

To multiply a number by 4, double it twice.

10 × 4 = 40	
Double 10 once	20
Double 10 twice	40

To multiply a number by 8, double it 3 times.

11 × 8 = 88					
Double 11 once	22				
Double 11 twice	44				
Double 11 three times	88				

3 Keep doubling to get the × 4 and × 8 facts. Here are some tables to help you. The first one has been done for you.

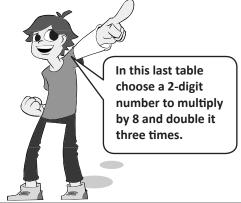
a  $12 \times 4 = 48$ Double 12 once 24Double 12 twice 48

C 18 × 4 = Double 18 once Double 18 twice

Double 16 once

Double 16 twice

Double 16 three times



Double 15 once

Double 15 twice

Double 22 once

Double 22 twice

Double 35 once

Double 35 twice

Double 35 three times

Double
once

Double
twice

Double
three times

e

g

### Mental multiplication strategies – split strategy

The split strategy is when we multiply numbers in 2 pairs and then add the parts. Let's use the split strategy for 26  $\times$  4.

- Split 26 into 20 and 6.
- Multiply each part.
- Add the answers together.

$$26 \times 4 \longrightarrow 20 \times 4 + 6 \times 4$$
  
 $80 + 24 = 104$   
So,  $26 \times 4 = 104$ 

Use the split strategy to answer these:

a 
$$34 \times 3 \longrightarrow 30 \times 3 + 4 \times 3$$

$$90 + \boxed{\phantom{0}} = \boxed{\phantom{0}}$$

$$50, 34 \times 3 = \boxed{\phantom{0}}$$

### Mental multiplication strategies – compensation strategy

Use the compensation strategy to make it easier to multiply 2-digit numbers that are close to a ten.

Look at  $4 \times 19$ .

19 is close to 20, so we can multiply by the next multiple of ten which is 20. Then we build down because we have an extra group of 4.

$$4 \times 19 \longrightarrow 4 \times 20 = 80 - 4$$
  
So,  $19 \times 4 = 76$ 

1 Use the compensation strategy to answer these:

a 
$$5 \times 29 \longrightarrow 5 \times \boxed{\phantom{0}} = \boxed{\phantom{0}} - \boxed{\phantom{0}}$$
So,  $5 \times 29 = \boxed{\phantom{0}}$ 

**b** 
$$3 \times 49 \longrightarrow 3 \times \boxed{\phantom{0}} = \boxed{\phantom{0}} - \boxed{\phantom{0}}$$

c 
$$4 \times 39 \longrightarrow 4 \times \boxed{\phantom{0}} = \boxed{\phantom{0}} - \boxed{\phantom{0}}$$
So,  $4 \times 39 = \boxed{\phantom{0}}$ 

Use the compensation strategy to answer these questions. This time you need to look for more than one extra group to subtract:

We have rounded up to 20. So instead of 4 × 18 we have 4 × 20. This is 2 more groups of 4. So we subtract 8.



THINK



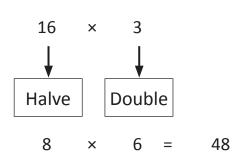
### Mental multiplication strategies – choose a strategy

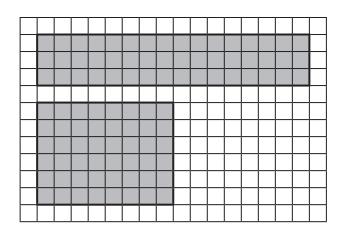
Roll a die to get the missing number, then use either the split or compensation strategy to get the answer. You can place the numbers rolled on the die in any question.



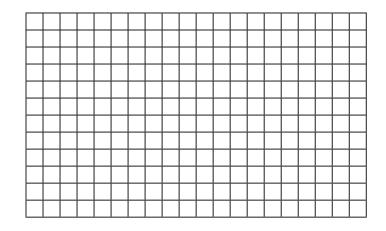
### Mental multiplication strategies – doubling and halving

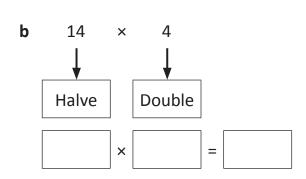
We can change the factors of a multiplication question to make it easier. Look at  $16 \times 3$ . If we halve the larger factor and double the smaller factor, we make an array on the grid that is the same size. Both arrays have the same amount of squares. Count the squares, are they equal to  $8 \times 6$ ?

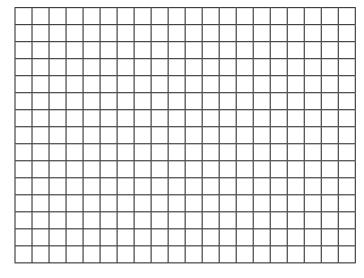




1 Make these problems easier by using doubling and halving. Shade an array for each:







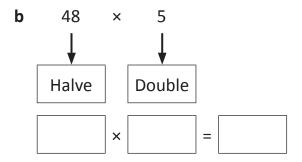
### Mental multiplication strategies – doubling and halving

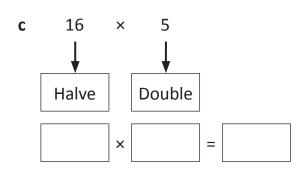
#### 2 Use the doubling and halving strategy to solve these:

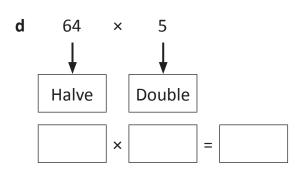
a 14 × 3

Halve Double

× =

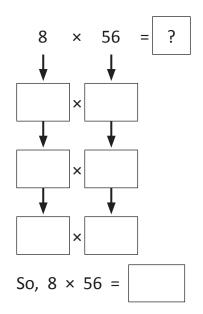




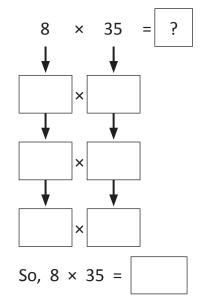


#### 3 Follow this doubling and halving trail through to the bottom:

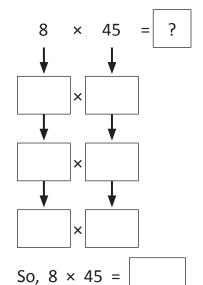
**a** Halve Double



**b** Halve Double



**c** Halve Double



**d** What do you notice?

### Mental multiplication strategies - word problems

When you are faced by a word problem, read it carefully. Ask yourself...

What are the important numbers?

Which key words give clues to the correct operation?

Jim makes boxes of biscuits for his 5 friends. There are 16 biscuits in each box. How many biscuit does he make altogether?

**Important numbers:** 5 friends 16 biscuits in each box

**Key words/operations:** 'altogether' suggests multiplication  $5 \times 16$ 

**Strategy:** split

$$5 \times 16 = 5 \times 10 \text{ and } 5 \times 6$$

$$5 \times 10 = 50$$

$$5 \times 6 = 30$$

$$50 + 30 = 80$$

If I buy 4 packets of sweets and each packet contain 6 sweets, how many sweets will I have altogether?

Every minute I complete one length of the swimming pool.

### Mental multiplication strategies – word problems

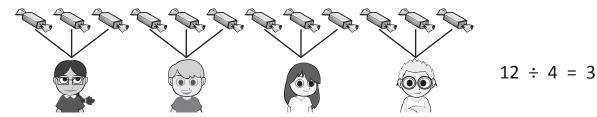
3	Jimmy lines up his soldiers in lines of 9. If he has 8 lines, how many soldiers does he have?	
4	There are 15 toys in a tin. Lily has 8 tins. How many toys does she have altogether?	
5	Eggs come in boxes of a dozen. Our local shop has 14 boxes on its shelves. How many eggs is this?	Read carefully! What are the important numbers? What are the key words?
6	Mike and I are playing darts. On my first throw I score 3 double 15s. Mike's score is twice mine. How many do we score between us?	What operations do I need? What is the best strategy?

REMEMBER

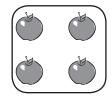
#### Division – division is sharing and grouping

Division can mean sharing *or* grouping.

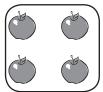
There are 12 lollies shared between 4 kids. How many are in each share?

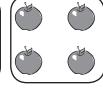


There are 16 apples and 4 go into each basket. How many baskets do I need?









$$16 \div 4 = 4$$

- Solve these sharing and grouping questions:
  - **a** There are 9 cupcakes and 3 kids are sharing. How many are in each share?

























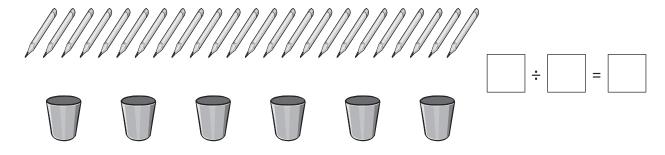


÷		=	
---	--	---	--

**b** 10 lollies are shared between a group of kids so they each get 2. How many kids are sharing?



c There are 24 pencils and 6 pencil pots. How many pencils go into each pencil pot?





Division – division is sharing and grouping

2 Draw pictures to show these division questions. Then write the division fact and decide whether it is a sharing or a grouping question. If you need to find out how many items there are in each share, it's a sharing question. If you need to find out the number of equal shares, it's a grouping question.



CHECK

a Divide 16 lollies between 4 girls. How many does each girl get?

sharing / grouping

**b** From a packet of 24 pencils, each person will get 6. How many people are sharing the pencils?

sharing / grouping

c 48 eggs are laid by 6 hens. If they all laid the same amount, how many did each hen lay?

sharing / grouping

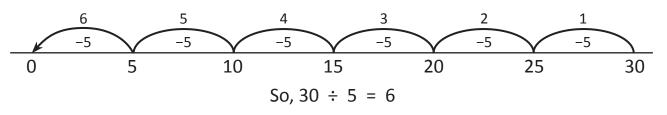
31

### Division – division is repeated subtraction

Division can also be thought of as repeated subtraction.

Look at  $30 \div 5 = ?$  This question is asking how many groups of 5 there are in 30.

Jump in 5s along the number line and then count the jumps.



- Show these division facts as repeated subtraction. First label the number lines and then show the jumps.
  - **a** 36 ÷ 6 =

0					36
					1

**b** 21 ÷ 3 =



Write a division fact to match these number lines. Show the jumps.

## Division – linking multiplication and division facts

Knowing multiplication facts will help with division facts. This is because they are opposites. Look at how we can describe this array:

- $6 \times 4 = 24$
- 6 groups of 4 is 24.

- $4 \times 6 = 24$
- 4 groups of 6 is 24.

- $24 \div 4 = 6$
- 24 divided into 4 shares is 6.

- $24 \div 6 = 4$
- 24 divided into 6 shares is 4.
- Describe each of these arrays using two multiplication and two division facts:

a

- ×
- b

× =



- X

X





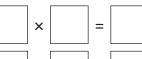
X









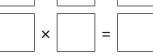






- Draw an array of 6 rows of 3 then describe it with multiplication and division facts.

×





REMEMBER



# Division – linking multiplication and division facts

Write a fact family for each set of numbers in the triangle. The first one has been done for you.

5 35 a

> 35 7 5

35

35 5

35 ÷ 5

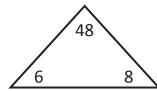
b ×

×

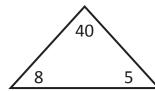
9 3

C ×

×



d X



- For these problems, think of a multiplication fact to help write the division fact:
  - a £25 is shared between 5 people. How much does each person get?

×

**b** 45 people get into 9 cars. How many people are in each car?

X

## Mental division strategies – dividing by 10 and 100

When we divide any number by 10, we move the number one place value space to the right because the number is getting 10 times smaller.

When we divide any number by 100, we move the number two place value spaces to the right because the number is getting 100 times smaller.

Thousands	Hundreds	Tens	Ones	
6	7	0	0	
	6	7	0	÷ 10
		6	7	÷ 100

Use the place value tables to divide these numbers by 10 and 100.

а	Th	Н	Т	0	
	5	3	0	0	
					÷ 10
					÷ 100

d

b

	0	Т	Н	Th
	0	0	1	4
÷ 10				
÷ 100				

Th	Н	Т	0	
8	4	0	0	
				÷ 10
				÷ 100

Use patterns to solve these:

C

Use a calculator to solve these:

**Multiplication and Division** 

### Mental division strategies – halving strategy

When you halve numbers you are dividing them by 2. In this function machine, numbers go IN, have the rule applied and come OUT again.

IN

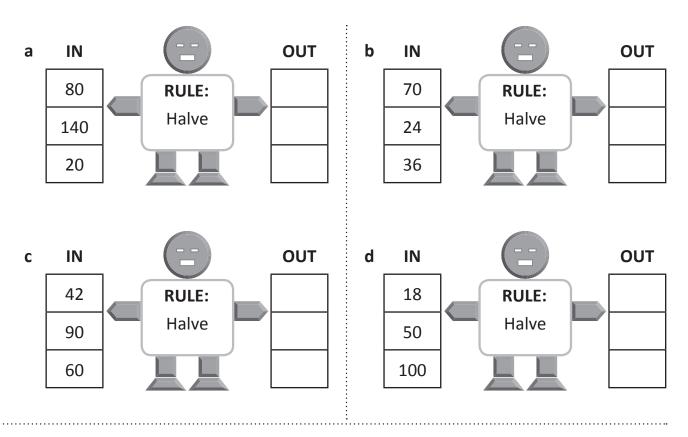
RULE:

4

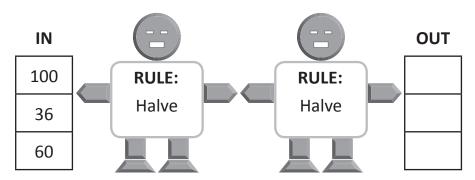
6

2

1 Complete the halving function machines. Halve the number going IN the machine and write the answer in the OUT column:



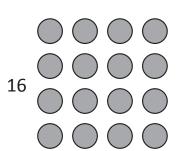
Below is a halving-halving function machine. The number goes IN and is halved and then halved again and comes OUT.

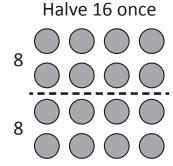


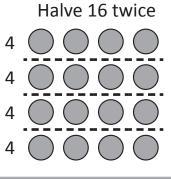


# Mental division strategies – halving strategy

We also use halving-halving to divide by 4. Look at these diagrams:







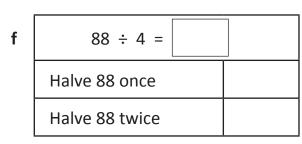
Use the tables for halving-halving to divide by 4:

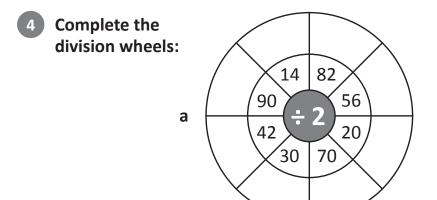
a	80 ÷ 4 =	
	Halve 80 once	
	Halve 80 twice	

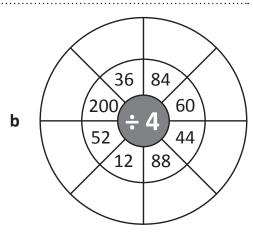
b	48 ÷ 4 =	
	Halve 48 once	
	Halve 48 twice	

С	64 ÷ 4 =	
	Halve 64 once	
	Halve 64 twice	

е	244 ÷ 4 =	
	Halve 244 once	
	Halve 244 twice	







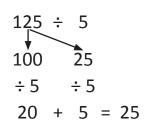
## Mental division strategies – split strategy

Division problems can be much easier to solve if you split the number.

Look at 125 ÷ 5.

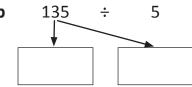
Can we split the number into two multiples of 5?

Yes, we can split 125 into 100 and 25.



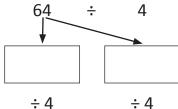
We divide each part by 5 and then add the two answers together.

Use the split strategy to divide these by 5:

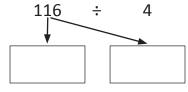


Use the split strategy to divide these by 4:

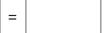
a



=

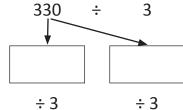


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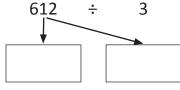


Use the split strategy to divide these by 3:

a



b



+



## Mental division strategies – word problem

Review your division strategies.

- Use either the halving strategy or the split strategy to complete the tables. The first one has been done for you.
  - **a** Use the split strategy:

$$30 \div 3 = 10$$
 and  $18 \div 3 = 6$ 

$$10 + 6 = 16$$

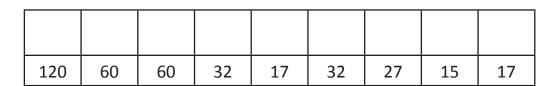
**b** Use the halving strategy:

**c** Use the split strategy:

**d** Use the halving strategy:

2 Solve this riddle by matching the letter to the answer. Use a mental division strategy for each problem.

What is it that the more you take, the more you leave behind?



### Mental division strategies – word problem

Remember the steps and questions to ask yourself when you are trying to solve a word problem.

Four friends have a party. They share out all the food equally. There are 164 blueberries in total. How many do they get each?

**Important numbers:** 4 friends 164 blueberries

**Key words/operations:** 'share' = multiplication  $164 \div 4$ 

Strategy: halving

 $164 \div 2 = 82$ 

 $82 \div 2 = 41$ 

Tom, Milo and Xav have been trick and treating. They agree to share their sweets out equally between them. They have 33 sweets in total. How many do they get each?

Read carefully!

What are the important numbers?

What are the key words?

What operations do I need?

What is the best strategy?



4 Lillies are sold in bunches of 7. A florist has 42 lillies. How many bunches can she make?



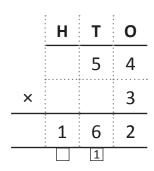
REMEMBER



# Mental division strategies – word problem

5	Jon needs to buy some files. They cost £9 each. He has £72. How many files can he buy?	
6	Andy loves astronomy. He's worked out that he can see about 32 000 stars with his new telescope. If there are about 100 stars visible in any one galaxy. How many galaxies can he see?	
7	Kate has been planting trees. She has planted a total of 155 trees in rows of 5. How many rows has she planted?	
8	Charles is saving up to buy a new bike. The bike costs £170. He gets £74 for his birthday, and £4 pocket money a week. How many weeks will he have to save until he can get the bike?	

# Written methods – short multiplication



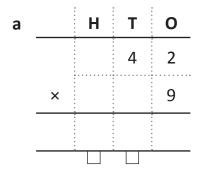
Start with the ones.  $4 \times 3 = 12$  ones.

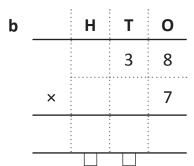
Rename this as 1 ten and 2 ones. Put the 2 in the ones column and regroup the 1 to the tens column.

 $3 \times 5$  plus the regrouped 1 is 16 tens.

Rename this as 1 hundred and 6 tens.

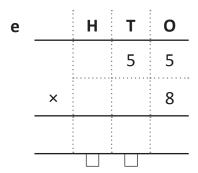
### 1 Practise these problems:

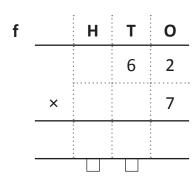




С		Н	Т	0
		•	2	5
	×			4
		•	•	

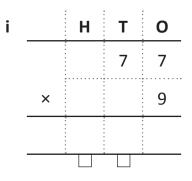
d		Н	Т	0
			2	6
	×			4
			•	•





g		Н	Т	0
			8	6
	×			6
			•	0 0 0 0 0 0

h		н	Т	0
			9	3
	×			5



## Written methods – short multiplication

### 2 Solve these multiplications:

a		Th	Н	Т	0
			1	2	3
	×				4
			$\Box$		

b		Th	Н	Т	0
			2	5	6
	×				6
				•	

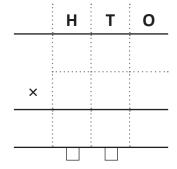
С		Th	Н	Т	O
		0 0 0 0 0	1	8	7
	×				8
		6 0 0 0 0			

d		Th	Н	T	0
			3	4	2
	×				7
				•	
			$\top$		

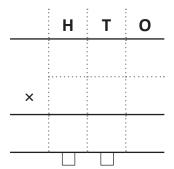
f		Th	Н	Т	0
		0 0 0 0	6	7	8
	×				9
		•			• • • • •

### 3 Use short multiplication to solve these word problems:

**a** On a farm, 6 lambs were born every day over 25 days. How many lambs were born in total?



**b** For my school fete day, I baked 9 trays of cupcakes. If there are 14 cupcakes on each tray, how many did I bake in total?



### Written methods - short division

Another way to represent division is with the division symbol.

- т о
- This is the same as  $36 \div 6 = 6$
- 6
- 6 3 6
- If the answer is a single digit, it should go in the ones column.

### 1 Solve these division problems using the division symbol:

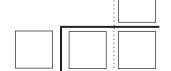
- **a** 5 3 5
- **b** 4 2 8
- **c** 9 1 8

- **d** 6 5 4
- e 2 1 4
- **f** 4 1 6

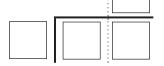
- g 5 2 5
- **h** 7 4 9
- i 8 4 8

### 2 Use the division symbol to solve each problem:

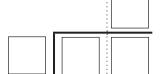
**a** 42 cupcakes were iced by 7 kids. If they each iced the same amount, how many did they ice each?



**b** How many pots were used if 6 seeds were planted in each pot from a packet of 54?



c I run the same distance each day. Over 9 days the total distance is 72 km. How far did I run each day?



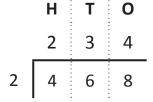


# Written methods - short division with 3-digit numbers

In short division with 3-digit numbers we split the number:

468 is 400 + 60 + 8

400 divided by 2 is 200, so we put a 2 in the hundreds place. 60 divided by 2 is 30, so we put a 3 in the tens place. 8 is divided by 2 is 4, so we put a 4 in the ones place.



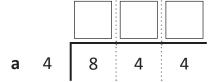
### 1 Practise splitting these:

- **a** 368 is \_\_\_\_\_ + \_\_\_\_ + \_\_\_\_
- **b** 445 is \_\_\_\_\_ + \_\_\_\_ + \_\_\_\_
- **c** 567 is \_\_\_\_\_ + \_\_\_\_ + \_\_\_\_
- **d** 235 is \_\_\_\_\_ + \_\_\_\_ + \_\_\_\_

### Now put these split numbers back together:

- **a** 500 + 70 + 8 is \_\_\_\_\_
- **b** 700 + 90 + 4 is \_\_\_\_\_
- **c** 200 + 40 + 6 is \_\_\_\_\_
- **d** 800 + 50 + 5 is \_\_\_\_\_

### Solve these division problems with 3-digit numbers:

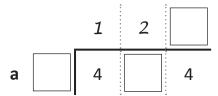


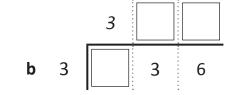
**b** 3 6 9 3

c 2 8 4 2

d 2 4 8 8

# 4 Here are two division problems with missing numbers in the questions. Find out the missing numbers by using the numbers that are part of the answer as clues.





## Written methods – short division with 3-digit numbers

Sometimes we need to split the number a different way,

for example: 515 = 500 + 15

500 divided by 5 is 100, so we put a 1 in the hundreds place.

15 divided by 5 is 3, so we put a 3 in the ones place.

What goes in the tens place?

A zero does. The zero has the very important job of

keeping the other numbers in their place!

	Н	Т	0
	1	0	3
5	5	1	5

Practise these problems. We have put the zero in to remind you:

6 Practise these problems. This time, you need to remember the zero!

### Patterns and algebra – skip counting

Using a 100 square can help us to identify skip counting patterns.

- 1 Colour the counting pattern on each 100 square:
  - a Count in 6s.

_									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**b** Count in 7s.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

c Count in 9s.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**d** Count in 3s and 6s. Shade the 3s and circle the 6s.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**e** Look at the completed number square in question d. Describe the pattern that you see. What is the relationship between counting in 3s and 6s? Explain your answer.

\_\_\_\_\_

### Patterns and algebra – skip counting

Complete these number patterns by looking for skip counting patterns.

a

b

C

Colour the skip counting pattern for 3s up to 30. If you kept going on a complete hundred grid, would 52 be coloured in?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

How can you tell without using a whole hundred grid?

Only 3 numbers are shaded in each of the skip counting patterns below. Work out the pattern and complete the shading:

b

a

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

This shows a skip counting pattern of:

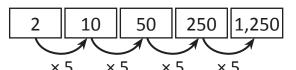
This shows a skip counting pattern o

of:	

# Patterns and algebra – completing and describing patterns

This is a pattern involving multiplication.

The pattern begins at 2.



The rule is: multiply by 5.

Figure out the missing numbers in each pattern and write the rule.

a

7	21
-	

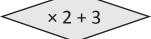
52

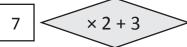
Rule: \_\_\_\_\_

Rule:

Some number patterns can be formed with two operations each time. For example:









37

The rule is to multiply by 2 and add 3 each time.

Complete these number patterns, by following the rules written in the diamond shapes. Describe the rule underneath.







The rule is

Roll a die to make the starting number. Continue the sequence by following the rule:

Rule:  $+4 \times 2$ a



Rule:  $+1 \times 3$ b

Rule:  $+3 \times 2$ 

### Patterns and algebra – predicting repeating patterns

When we use number patterns in tables, it can help us to predict what comes next. Look at the table below and how we can use it to predict the total number of sweets needed for any number of children at a party.

This table shows us that 1 sweet bag contains 8 sweets and 2 bags contain 16 sweets. We can see that the rule for the pattern is to multiply the top row by 8 to get the bottom row each time.

Number of sweet bags	1	2	3	4	5	10	
Number of sweets	8	16	24	32	40	80	$  \  $

To find out how many sweets are in 10 bags, we don't need to extend the table, we can just apply the rule.

 $10 \times 8 = 80$ . So, 10 bags contain 80 sweets. This helps us plan how many sweets are needed for a party.

### Complete the table for each problem:

**a** Tom receives £5 a week pocket money as long as he does all his chores. How much pocket money does Tom get after 10 weeks?

Weeks	1	2	3	4	5	10
Pocket money	5	10				

**b** A flower has 7 petals. How many petals are there in a bunch of 10 flowers?

Flowers	1	2	3	4	5	10
Number of petals	7	14				

c A flag has 6 stars. How many stars are there on 10 flags?

Flags	1	2	3	4	5	10
Number of stars	6	12				

**d** At a pizza party, each person eats 3 pieces of pizza. How many pieces of pizza do 10 people eat?

Guests	1	2	3	4	5	10
Pizza pieces			9	12		



### Patterns and algebra – predicting repeating patterns

Each of these kids wrote the first 3 numbers of a skip counting pattern of 6, starting at different numbers. Each kid's sequence goes down the column. Imagine the sequence continues.

Mel	Brianna	Brad	Gen	Jo	Kate
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18

)
---

- **b** Who had the number 50 in their column? \_\_\_\_\_
- 3 Look at each pattern of shapes and complete the table below:



Repeat section	1	2	3	4	5	10
Number of circles	2	4	6	8	10	20
Number of triangles	1	2	3	4	5	10

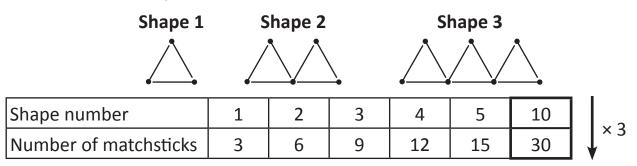
Show what this entire sequence would look like with 10 repeat sections:

Look for the section that repeats. What is it made up of? This is the rule.



### Patterns and algebra – predicting growing patterns

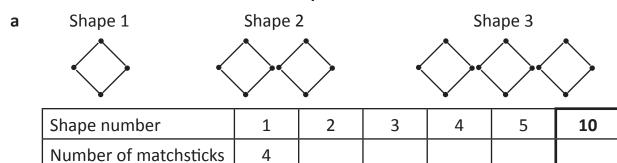
Number patterns in tables can help us with problems like this. Mia is making this sequence of shapes with matchsticks and wants to know how many she will need for 10 shapes.

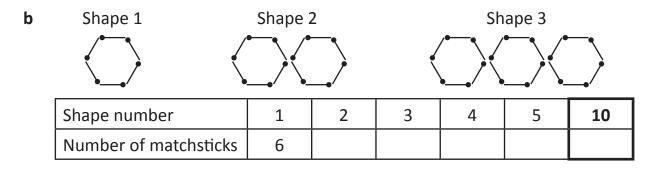


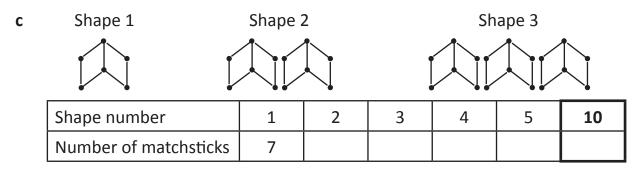
To find out how many matchsticks are needed for 10 triangles, we don't need to extend the table, we can just apply the function rule:

Number of matchsticks = Shape number  $\times$  3

1 Complete the table for each sequence of matchstick shapes and find the number of matchsticks needed for the 10th shape.



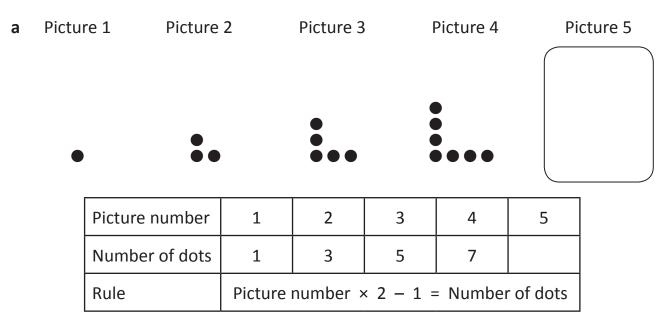


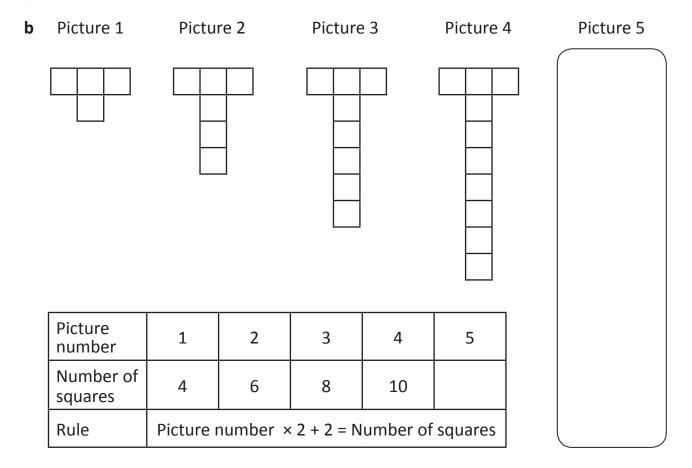




## Patterns and algebra – predicting growing patterns

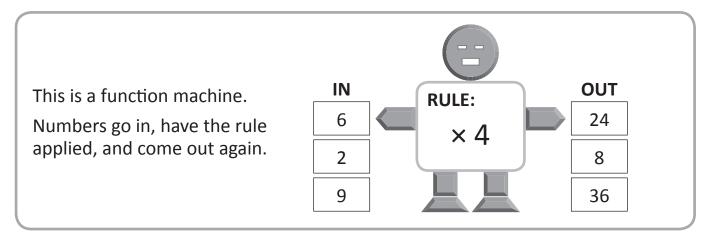
Look at these growing patterns. Complete the table and follow the rule to draw Picture 5:



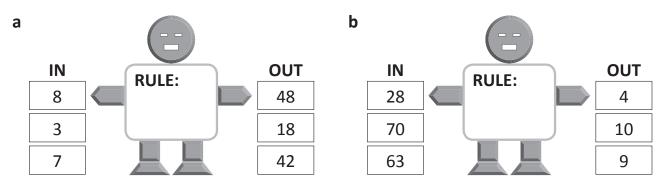


How many squares will Picture 8 have?

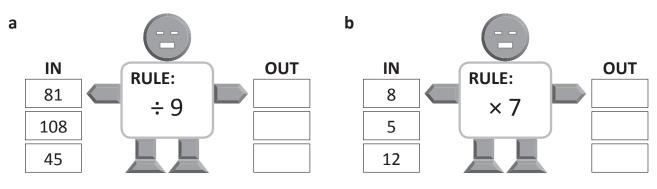
# Patterns and algebra – function machines



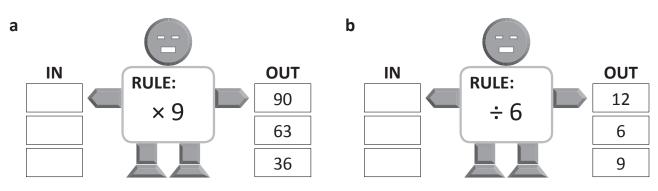
Look carefully at the numbers going *in* these function machines and the numbers coming out. What is the rule?



2 What numbers will come out of these function machines?



**3** What numbers go in to these number function machines?

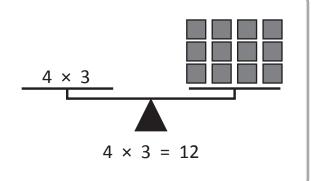


## Patterns and algebra – understanding equivalence

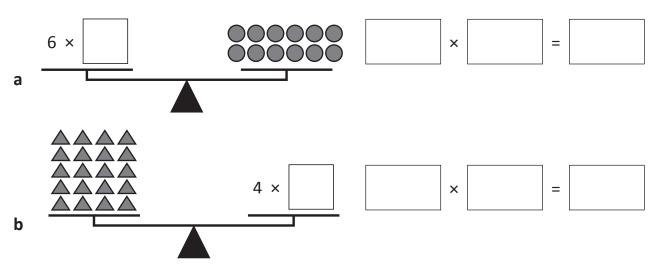
Look at these balanced scales.

On one side there is the sum  $4 \times 3$  and on the other side there is a total of 12 triangles. This makes sense because it shows the equation  $4 \times 3 = 12$ .

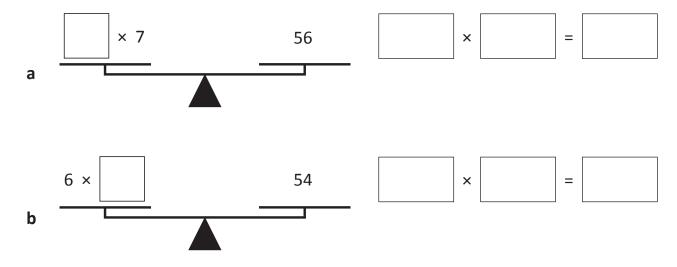
Equation is another word for a sum. With equations, both sides must be equal.



Balance each set of scales by writing a number in the box that is equivalent to the total number of shapes. Then write the matching equation.

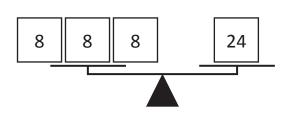


2 Balance each set of scales by writing a number in the box. Then write the matching equation.

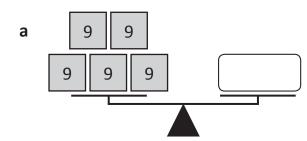


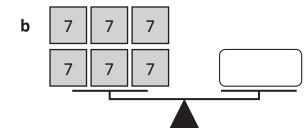
# Patterns and algebra – balanced equations using + and $\times$

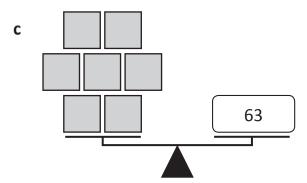
There are 2 different equations we could write for one set of balanced scales.

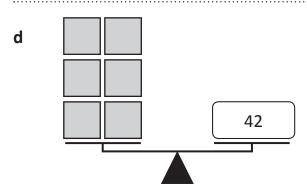


### 1 Work out the values of the symbols in each problem:





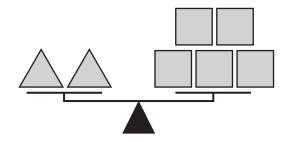




# Patterns and algebra – balanced equations using + and $\times$

### 2 Find the values of these symbols:

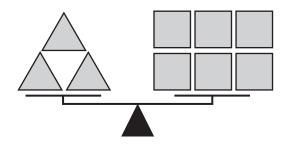
a If is 5, what is the value of



2 × 5 = 5 ×



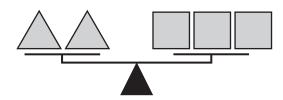
**b** If is 8, what is the value of



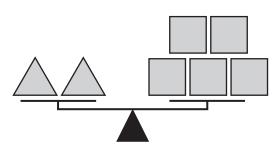
3 × 8 = 6 ×

### 3 Find the values of both symbols from the clues:

a If both sides are equal to 36, what is the value of each symbol?



- 2 x = 3 x
- **b** If both sides are equal to 10, what is the value of each symbol?



- 2 × 5 = 5 ×

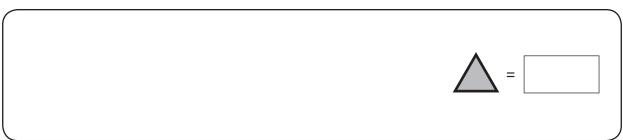
# Patterns and algebra – using symbols for unknowns

- Write an equation for these word problems. Write an equation using a for the unknown number.
  - **a** Bec collects stickers. She has 48 bumper stickers, 12 glitter stickers and 15 smiley face stickers. How many stickers does Bec have in her collection?

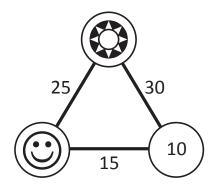


**b** Charlie saved £5 a week of his pocket money over 8 weeks but then spent £15. How much did Charlie have at the end of 8 weeks?

**c** 5,000 people are spectators at a football match. 2,700 are there to support Team A while the rest are there to support Team B. How many spectators support Team B?



2 In this triangle, the numbers on the sides are the totals.



Work out the value of the other symbols:





This is a game for 2 players. You will need a copy of this page, 6 counters each and 3 dice.





Player 1 rolls all 3 dice and chooses 2 of the numbers to multiply. If the player can see the answer in the grid, they claim this number by placing a counter over the number. Then Player 2 has a turn. The winner is the first to place all 6 counters on the grid.

20	15	12	2	8
6	12	6	16	6
36	20	18	8	10
12	10	6	12	4
10	12	15	24	25



This is a game for three players. Each player needs a copy of this page. The caller needs a pile of the numbers from 1 to 9.





Each multiplication grid contains all the answers, while the factors are missing. Remember factors are the numbers that you multiply to get the answer.

The aim of the game is to be the first player to fill their grid with the factors. One hint is provided in each grid to start you off. Choose one person to be the caller and the other two play the round. The caller picks a number without looking and reads it out to the players. The players write it on the grids, if it fits as a factor. The first to fill in one of the grids completely is the winner.

1	
2	7
3	8

×			
6	42	24	18
	63	36	27
	35	20	15

×			
	8	40	64
3	3	15	24
	9	45	72

×	3		
	12	20	28
	18	30	42
	27	45	63

×			
	8	40	64
3	3	15	24
	9	45	72

×			9
	4	14	18
	2	7	9
	12	42	54



This is a game for two players. You will need a copy of page 63, a die and a pencil to write down your scores. You may like to make extra copies of page 63 to play again later. COPY





The aim of this game is to score the highest number of points each time without going over 20. Roll the dice and choose which strategy you will use. From the Strategy column, circle 1 for double, 2 for double-double or 3 for double-double. For example, Player 2 has rolled a 5 and has chosen strategy 3 double-double-double. This makes a score of 40 but because it is over 20 it doesn't count. Look at the rest of the sample game to see how the game turned out.

Strategy 1	Strategy 2	Strategy 3
Double	Double Double	Double Double Double

#### Sample game

Player 1			
Die	Strategy	Score	
6	2 3	12	
2	1 2 3	16	
4	1 2 3	16	
6	1 3	24	
3	1 2 3	12	
	Total	56	

Player 2			
Die	Strategy	Score	
5	1 3	40	
3	1 2 3	12	
1	1 2 3	8	
4	1 2 3	16	
2	1 2 3	16	
	Total	52	

Strategy 1	Strategy 2	Strategy 3
Double	Double Double	Double Double Double

Player 1		
Die	Strategy	Score
	1	
	2	
	3	
	1	
	2	
	3	
	1	
	2	
	3	
	1	
	2	
	3	
	1	
	2	
	3	
Total		

Player 2		
Die	Strategy	Score
	1	
	2	
	3	
	1	
	2	
	3	
	1	
	2	
	3	
	1	
	2	
	3	
	1	
	2	
	3	
Total		



**Symbols** solve



Can you work out the value of each symbol?

The values are 2, 3, 4, 6, 8, 9 and 12. Remember, the same symbol means that it's the same number.

$$\times =$$

$$\times \times \times =$$

$$\times \times = \Leftrightarrow$$

$$\nabla \times$$
 =  $\bigcirc$ 

$$\times$$
  $\times$  =