9-3 Polar and Rectangular Forms of Equations

Find the rectangular coordinates for each point with the given polar coordinates. Round to the nearest hundredth, if necessary.

1. $\left(2, \frac{\pi}{4}\right)$

SOLUTION:

For $\left(2, \frac{\pi}{4}\right)$, $r = 2$ and $\theta = \frac{\pi}{4}$.

$$x = r\cos\theta \qquad y = r\sin\theta$$
$$= 2\cos\frac{\pi}{4} \qquad = 2\sin\frac{\pi}{4}$$
$$= 2\left(\frac{\sqrt{2}}{2}\right) \qquad = 2\left(\frac{\sqrt{2}}{2}\right)$$
$$= \sqrt{2} \qquad = \sqrt{2}$$

The rectangular coordinates of $\left(2, \frac{\pi}{4}\right)$ are $\left(\sqrt{2}, \sqrt{2}\right)$.

ANSWER:

$\left(\sqrt{2}, \sqrt{2}\right)$

2. $\left(\frac{1}{4}, \frac{\pi}{2}\right)$

SOLUTION:

For $\left(\frac{1}{4}, \frac{\pi}{2}\right)$, $r = \frac{1}{4}$ and $\theta = \frac{\pi}{2}$.

$$x = r\cos\theta \qquad y = r\sin\theta$$
$$= \frac{1}{4}\cos\frac{\pi}{2} \qquad = \frac{1}{4}\sin\frac{\pi}{2}$$
$$= \frac{1}{4}(0) \qquad = \frac{1}{4}(1)$$
$$= 0 \qquad = \frac{1}{4}$$

The rectangular coordinates of $\left(\frac{1}{4}, \frac{\pi}{2}\right)$ are $\left(0, \frac{1}{4}\right)$.

ANSWER:

$\left(0, \frac{1}{4}\right)$

3. (5, 240°)

SOLUTION:

For (5, 240°), $r = 5$ and $\theta = 240°$.

$$x = r\cos\theta \qquad y = r\sin\theta$$
$$= 5\cos 240 \qquad = 5\sin 240$$
$$= 5\left(-\frac{1}{2}\right) \qquad = 5\left(-\frac{\sqrt{3}}{2}\right)$$
$$= -\frac{5}{2} \qquad = -\frac{5\sqrt{3}}{2}$$

The rectangular coordinates of (5, 240°) are $\left(-\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)$.

ANSWER:

$\left(-\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)$

4. (2.5, 250°)

SOLUTION:

For (2.5, 250°), $r = 2.5$ and $\theta = 250°$.

$$x = r\cos\theta \qquad y = r\sin\theta$$
$$= 2.5\cos 250 \qquad = 2.5\sin 250$$
$$\approx 2.5(-0.34) \qquad \approx 2.5(-0.94)$$
$$\approx -0.86 \qquad \approx -2.35$$

The rectangular coordinates of (2.5, 250°) are about (−0.86, −2.35).

ANSWER:

(−0.86, −2.35)

5. $\left(-2, \frac{4\pi}{3}\right)$

SOLUTION:

For $\left(-2, \frac{4\pi}{3}\right)$, $r = -2$ and $\theta = \frac{4\pi}{3}$.

$$\begin{aligned} x &= r\cos\theta & y &= r\sin\theta \\ &= -2\cos\frac{4\pi}{3} & &= -2\sin\frac{4\pi}{3} \\ &= -2\left(-\frac{1}{2}\right) & &= -2\left(-\frac{\sqrt{3}}{2}\right) \\ &= 1 & &= \sqrt{3} \end{aligned}$$

The rectangular coordinates of $\left(-2, \frac{4\pi}{3}\right)$ are $\left(1, \sqrt{3}\right)$.

ANSWER:

$\left(1, \sqrt{3}\right)$

6. (−13, −70°)

SOLUTION:

For (−13, −70°), $r = -13$ and $\theta = -70°$.

$$\begin{aligned} x &= r\cos\theta & y &= r\sin\theta \\ &= -13\cos(-70°) & &= -13\sin(-70°) \\ &\approx -13(0.34) & &\approx -13(-0.94) \\ &\approx -4.45 & &\approx 12.22 \end{aligned}$$

The rectangular coordinates of $(-13, -70°)$ are about (−4.45, 12.22).

ANSWER:

(−4.45, 12.22)

7. $\left(3, \frac{\pi}{2}\right)$

SOLUTION:

For $\left(3, \frac{\pi}{2}\right)$, $r = 3$ and $\theta = \frac{\pi}{2}$.

$$\begin{aligned} x &= r\cos\theta & y &= r\sin\theta \\ &= 3\cos\frac{\pi}{2} & &= 3\sin\frac{\pi}{2} \\ &= 3(0) & &= 3(1) \\ &= 0 & &= 3 \end{aligned}$$

The rectangular coordinates of $\left(3, \frac{\pi}{2}\right)$ are $(0, 3)$.

ANSWER:

(0, 3)

8. $\left(\frac{1}{2}, \frac{3\pi}{4}\right)$

SOLUTION:

For $\left(\frac{1}{2}, \frac{3\pi}{4}\right)$, $r = \frac{1}{2}$ and $\theta = \frac{3\pi}{4}$.

$$\begin{aligned} x &= r\cos\theta & y &= r\sin\theta \\ &= \frac{1}{2}\cos\frac{3\pi}{4} & &= \frac{1}{2}\sin\frac{3\pi}{4} \\ &= \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right) & &= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) \\ &= -\frac{\sqrt{2}}{4} & &= \frac{\sqrt{2}}{4} \end{aligned}$$

The rectangular coordinates of $\left(\frac{1}{2}, \frac{3\pi}{4}\right)$ are $\left(-\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}\right)$.

ANSWER:

$\left(-\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}\right)$

9. $(-2, 270°)$

SOLUTION:

For $(-2, 270°)$, $r = -2$ and $\theta = 270°$.

$$\begin{aligned} x &= r\cos\theta \\ &= -2\cos(270°) \\ &= -2(0) \\ &= 0 \end{aligned} \qquad \begin{aligned} y &= r\sin\theta \\ &= -2\sin(270°) \\ &= -2(-1) \\ &= 2 \end{aligned}$$

The rectangular coordinates of $(-2, 270°)$ are $(0, 2)$.

ANSWER:

$(0, 2)$

10. $(4, 210°)$

SOLUTION:

For $(4, 210°)$, $r = 4$ and $\theta = 210°$.

$$\begin{aligned} x &= r\cos\theta \\ &= 4\cos(210°) \\ &= 4\left(-\frac{\sqrt{3}}{2}\right) \\ &= -2\sqrt{3} \end{aligned} \qquad \begin{aligned} y &= r\sin\theta \\ &= 4\sin(210°) \\ &= 4\left(-\frac{1}{2}\right) \\ &= -2 \end{aligned}$$

The rectangular coordinates of $(4, 210°)$ are $(-2\sqrt{3}, -2)$.

ANSWER:

$\left(-2\sqrt{3}, -2\right)$

11. $\left(-1, -\frac{\pi}{6}\right)$

SOLUTION:

For $\left(-1, -\frac{\pi}{6}\right)$, $r = -1$ and $\theta = -\frac{\pi}{6}$.

$$\begin{aligned} x &= r\cos\theta \\ &= -1\cos\left(-\frac{\pi}{6}\right) \\ &= -1\left(\frac{\sqrt{3}}{2}\right) \\ &= -\frac{\sqrt{3}}{2} \end{aligned} \qquad \begin{aligned} y &= r\sin\theta \\ &= -1\sin\left(-\frac{\pi}{6}\right) \\ &= -1\left(-\frac{1}{2}\right) \\ &= \frac{1}{2} \end{aligned}$$

The rectangular coordinates of $\left(-1, -\frac{\pi}{6}\right)$ are $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

ANSWER:

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

12. $\left(5, \frac{\pi}{3}\right)$

SOLUTION:

For $\left(5, \frac{\pi}{3}\right)$, $r = 5$ and $\theta = \frac{\pi}{3}$.

$$\begin{aligned} x &= r\cos\theta & y &= r\sin\theta \\ &= 5\cos\left(\frac{\pi}{3}\right) & &= 5\sin\left(\frac{\pi}{3}\right) \\ &= 5\left(\frac{1}{2}\right) & &= 5\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{5}{2} & &= \frac{5\sqrt{3}}{2} \end{aligned}$$

The rectangular coordinates of $\left(5, \frac{\pi}{3}\right)$ are $\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$.

ANSWER:

$\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

Find two pairs of polar coordinates for each point with the given rectangular coordinates if $0 \le \theta \le 2\pi$. Round to the nearest hundredth, if necessary.

13. (7, 10)

SOLUTION:

For (7, 10), $x = 7$ and $y = 10$.

Since $x > 0$, use $\tan^{-1}\frac{y}{x}$ to find θ.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{7^2 + 10^2} \\ &= \sqrt{149} \text{ or about } 12.21 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\frac{y}{x} \\ &= \tan^{-1}\frac{10}{7} \\ &\approx 0.96 \end{aligned}$$

One set of polar coordinates is (12.21, 0.96). Another representation that uses a negative r-value is $(-12.21, 0.96 + \pi)$ or (−12.21, 4.10).

ANSWER:

(12.21, 0.96) and (−12.21, 4.10)

14. (−13, 4)

SOLUTION:

For (−13, 4), $x = -13$ and $y = 4$.
Since $x < 0$, use $\tan^{-1}\frac{y}{x} + \pi$ to find θ.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-13)^2 + 4^2} \\ &= \sqrt{185} \text{ or about } 13.60 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\frac{y}{x} + \pi \\ &= \tan^{-1}\frac{4}{-13} + \pi \\ &\approx 2.84 \end{aligned}$$

One set of polar coordinates is (13.60, 2.84). Another representation that uses a negative r-value is (−13.60, 2.84 + π) or (−13.60, 5.98).

ANSWER:

(13.60, 2.84) and (−13.60, 5.98)

15. (−6, −12)

SOLUTION:

For (−6, −12), $x = -6$ and $y = -12$.
Since $x < 0$, use $\tan^{-1}\frac{y}{x} + \pi$ to find θ.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-6)^2 + (-12)^2} \\ &= \sqrt{180} \text{ or about } 13.42 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\frac{y}{x} + \pi \\ &= \tan^{-1}\frac{-12}{-6} + \pi \\ &\approx 4.25 \end{aligned}$$

One set of polar coordinates is (13.42, 4.25). Another representation that uses a negative r-value is (−13.42, 4.25 − π) or (−13.42, 1.11).

ANSWER:

(13.42, 4.25) and (−13.42, 1.11)

16. (4, −12)

SOLUTION:

For (4, −12), $x = 4$ and $y = −12$.

Since $x > 0$, use $\tan^{-1} \frac{y}{x}$ to find θ.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{4^2 + (-12)^2} \\ &= \sqrt{160} \text{ or about } 12.65 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\frac{y}{x} \\ &= \tan^{-1}\frac{-12}{4} \\ &\approx -1.25 \end{aligned}$$

One set of polar coordinates is (12.65, −1.25). Since this set is not in the required domain, two more sets have to be found. A representation that uses a positive r-value is (12.65, −1.25 + 2π) or (12.65, 5.03). A representation that uses a negative r-value is (−12.65, −1.25 + π) or (−12.65, 1.89).

ANSWER:

(12.65, 5.03) and (−12.65, 1.89)

17. (2, −3)

SOLUTION:

For (2, −3), $x = 2$ and $y = −3$.

Since $x > 0$, use $\tan^{-1} \frac{y}{x}$ to find θ.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{2^2 + (-3)^2} \\ &= \sqrt{13} \text{ or about } 3.61 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\frac{y}{x} \\ &= \tan^{-1}\frac{-3}{2} \\ &\approx -0.98 \end{aligned}$$

One set of polar coordinates is (3.61, −0.98). Since this set is not in the required domain, two more sets have to be found. A representation that uses a positive r-value is (3.61, −0.98 + 2π) or (3.61, 5.30). A representation that uses a negative r-value is (−3.61, −0.98 + π) or (−3.61, 2.16).

ANSWER:

(3.61, 5.30) and (−3.61, 2.16)

18. (0, −173)

SOLUTION:

For (0, −173), $x = 0$ and $y = −173$.

Since (0, −173) is on the negative y-axis, $\theta = \frac{3\pi}{2}$.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{0^2 + (-173)^2} \\ &= 173 \end{aligned}$$

One set of polar coordinates is $\left(173, \frac{3\pi}{2}\right)$. Another representation that uses a negative r-value is $\left(-173, \frac{3\pi}{2} - \pi\right)$ or $\left(-173, \frac{\pi}{2}\right)$.

ANSWER:

$\left(173, \frac{3\pi}{2}\right)$ and $\left(-173, \frac{\pi}{2}\right)$

19. $(a, 3a), a > 0$

SOLUTION:

For $(a, 3a)$, $x = a$ and $y = 3a$.
Since $a > 0$, use $\tan^{-1}\frac{y}{x}$ to find θ.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{a^2 + (3a)^2} \\ &= \sqrt{10a^2} \text{ or about } 3.16a \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\frac{y}{x} \\ &= \tan^{-1}\frac{3a}{a} \\ &\approx 1.25 \end{aligned}$$

One set of polar coordinates is $(3.16a, 1.25)$. Another representation that uses a negative r-value is $(-3.16a, 1.25 + \pi)$ or $(-3.16a, 4.39)$.

ANSWER:

$(3.16a, 1.25)$ and $(-3.16a, 4.39)$

20. $(-14, 14)$

SOLUTION:

For $(-14, 14)$, $x = -14$ and $y = 14$.
Since $x < 0$, use $\tan^{-1}\frac{y}{x} + \pi$ to find θ.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-14)^2 + 14^2} \\ &= \sqrt{392} \text{ or } 14\sqrt{2} \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\frac{y}{x} + \pi \\ &= \tan^{-1}\frac{14}{-14} + \pi \\ &= \frac{3\pi}{4} \end{aligned}$$

One set of polar coordinates is $\left(14\sqrt{2}, \frac{3\pi}{4}\right)$.

Another representation that uses a negative r-value is $\left(-14\sqrt{2}, \frac{3\pi}{4} + \pi\right)$ or $\left(-14\sqrt{2}, \frac{7\pi}{4}\right)$.

ANSWER:

$\left(14\sqrt{2}, \frac{3\pi}{4}\right)$ and $\left(-14\sqrt{2}, \frac{7\pi}{4}\right)$

21. (52, −31)

SOLUTION:

For (52, −31), $x = 52$ and $y = -31$.

Since $x > 0$, use $\tan^{-1}\frac{y}{x}$ to find θ.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{52^2 + (-31)^2} \\ &= \sqrt{3665} \text{ or about } 60.54 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\frac{y}{x} \\ &= \tan^{-1}\frac{-31}{52} \\ &\approx -0.54 \end{aligned}$$

One set of polar coordinates is (60.54, −0.54). Since this set is not in the required domain, two more sets have to be found. A representation that uses a positive r-value is (60.54, −0.54 + 2π) or (60.54, 5.74). A representation that uses a negative r-value is (−60.54, −0.54 + π) or (−60.54, 2.60).

ANSWER:

(60.54, 5.74) and (−60.54, 2.60)

22. (3*b*, −4*b*), $b > 0$

SOLUTION:

For (3*b*, −4*b*), $x = 3b$ and $y = -4b$.

Since $b > 0$, use $\tan^{-1}\frac{y}{x}$ to find θ.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(3b)^2 + (-4b)^2} \\ &= \sqrt{25b^2} \text{ or } 5b \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\frac{y}{x} \\ &= \tan^{-1}\frac{-4b}{3b} \\ &\approx -0.93 \end{aligned}$$

One set of polar coordinates is (5*b*, −0.93). Since this set is not in the required domain, two more sets have to be found. A representation that uses a positive r-value is (5*b*, −0.93 + 2π) or (5*b*, 5.35). A representation that uses a negative r-value is (−5*b*, −0.93 + π) or (−5*b*, 2.21).

ANSWER:

(5*b*, 5.35) and (−5*b*, 2.21)

23. (1, −1)

SOLUTION:

For (1, −1), $x = 1$ and $y = -1$.

Since $x > 0$, use $\tan^{-1}\frac{y}{x}$ to find θ.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\frac{y}{x} \\ &= \tan^{-1}\frac{-1}{1} \\ &= -\frac{\pi}{4} \end{aligned}$$

One set of polar coordinates is $\left(\sqrt{2}, -\frac{\pi}{4}\right)$. Since this set is not in the required domain, two more sets have to be found. A representation that uses a positive *r*-value is $\left(\sqrt{2}, -\frac{\pi}{4} + 2\pi\right)$ or $\left(\sqrt{2}, \frac{7\pi}{4}\right)$. A representation that uses a negative *r*-value is $\left(-\sqrt{2}, -\frac{\pi}{4} + \pi\right)$ or $\left(-\sqrt{2}, \frac{3\pi}{4}\right)$.

ANSWER:

$\left(\sqrt{2}, \frac{7\pi}{4}\right)$ and $\left(-\sqrt{2}, \frac{3\pi}{4}\right)$

24. (2, $\sqrt{2}$)

SOLUTION:

For $\left(2, \sqrt{2}\right)$, $x = 2$ and $y = \sqrt{2}$.

Since $x > 0$, use $\tan^{-1}\frac{y}{x}$ to find θ.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{2^2 + \left(\sqrt{2}\right)^2} \\ &= \sqrt{6} \text{ or about } 2.45 \end{aligned}$$

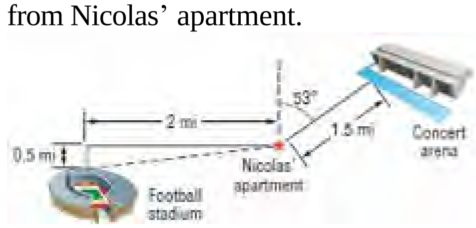
$$\begin{aligned} \theta &= \tan^{-1}\frac{y}{x} \\ &= \tan^{-1}\frac{\sqrt{2}}{2} \\ &\approx 0.62 \end{aligned}$$

One set of polar coordinates is (2.45, 0.62). Another representation that uses a negative *r*-value is (−2.45, 0.62 + π) or (−2.45, 3.76).

ANSWER:

(2.45, 0.62) and (−2.45, 3.76)

25. **DISTANCE** Standing on top of his apartment building, Nicolas notices that a concert arena is 53° east of north. Suppose the arena is exactly 1.5 miles from Nicolas' apartment.



a. How many miles north and east will Nicolas have to travel to reach the arena?

b. If a football stadium is 2 miles west and 0.5 mile south of Nicolas' apartment, what are the polar coordinates of the stadium if Nicolas' apartment is at the pole?

SOLUTION:

a. Let Nicolas' apartment represent the pole and due east represent the polar axis. Then the concert arena is at a 37° angle with the polar axis and has the polar coordinates (1.5, 37°). To calculate how many miles north and east Nicolas will have to travel to reach the arena, find the rectangular coordinates that represent the arena.

$$\begin{aligned} x &= r\cos\theta & y &= r\sin\theta \\ &= 1.5\cos 37^\circ & &= 1.5\sin 37^\circ \\ &\approx 1.20 & &\approx 0.90 \end{aligned}$$

The x-component represents the distance traveled east and the y-component represents the distance traveled north. Thus, Nicolas will have to travel about 0.90 miles north and about 1.20 miles east to reach the arena.

b. The rectangular coordinates of the football stadium are (−2, −0.5). For (−2, −0.5), $x = -2$ and $y = -0.5$.

Since $x < 0$, use $\tan^{-1}\frac{y}{x} + \pi$ to find θ.

$r = \sqrt{x^2 + y^2}$	Conversion formula	$\theta = \tan^{-1}\frac{y}{x} + 180$
$= \sqrt{(-2)^2 + (-0.5)^2}$	$x = -2$ and $y = -0.5$	$= \tan^{-1}\frac{-0.5}{-2} + 180$
$= \sqrt{4.25}$ or about 2.06	Simplify.	≈ 194.04

A set of polar coordinates that represents the football stadium is (2.06, 194.04°).

ANSWER:

a. about 0.90 mi north and about 1.20 mi east
b. Sample answer: (2.06, 194.04°)

Identify the graph of each rectangular equation. Then write the equation in polar form. Support your answer by graphing the polar form of the equation.

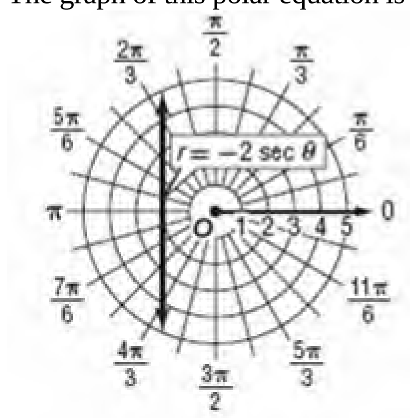
26. $x = -2$

SOLUTION:

The graph of $x = -2$ is a line. To find the polar form of this equation, replace x with $r\cos\theta$. Then simplify.

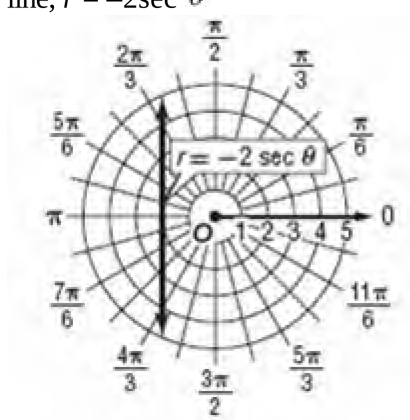
$$\begin{aligned} x &= -2 \\ r\cos\theta &= -2 \\ r &= \frac{-2}{\cos\theta} \\ r &= -2\sec\theta \end{aligned}$$

Evaluate the function for several θ-values in its domain and use these points to graph the function. The graph of this polar equation is a line.



ANSWER:

line; $r = -2\sec\theta$



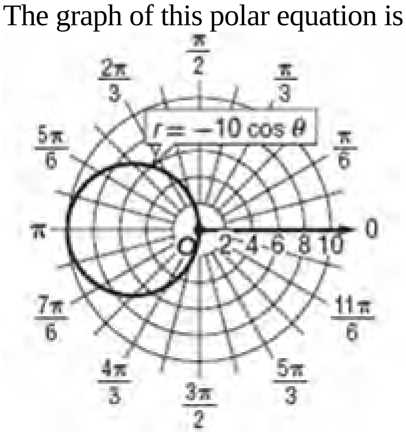
27. $(x + 5)^2 + y^2 = 25$

SOLUTION:

The graph of $(x + 5)^2 + y^2 = 25$ is a circle with radius 5 centered at (–5, 0). To find the polar form of this equation, replace x with $r \cos \theta$ and y with $r \sin \theta$. Then simplify.

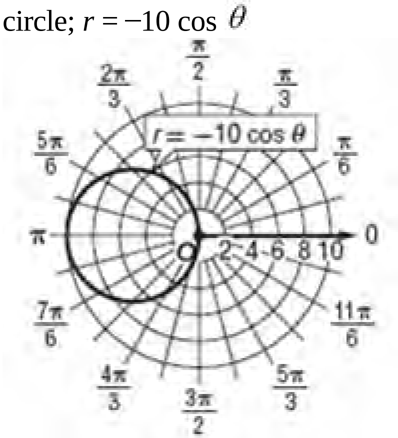
$$\begin{aligned} (x+5)^2 + y^2 &= 25 \\ (r\cos\theta + 5)^2 + (r\sin\theta)^2 &= 25 \\ r^2\cos^2\theta + 10r\cos\theta + 25 + r^2\sin^2\theta &= 25 \\ r^2\cos^2\theta + 10r\cos\theta + r^2\sin^2\theta &= 0 \\ r^2\cos^2\theta + r^2\sin^2\theta &= -10r\cos\theta \\ r^2(\cos^2\theta + \sin^2\theta) &= -10r\cos\theta \\ r^2(1) &= -10r\cos\theta \\ r &= -10\cos\theta \end{aligned}$$

Evaluate the function for several θ-values in its domain and use these points to graph the function. The graph of this polar equation is a circle.



ANSWER:

circle; $r = -10 \cos \theta$



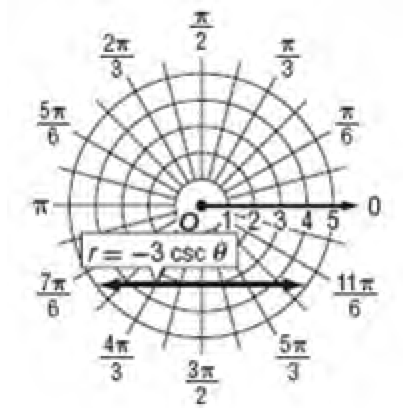
28. $y = -3$

SOLUTION:

The graph of $y = -3$ is a line. To find the polar form of this equation, replace y with $r \sin \theta$. Then simplify.

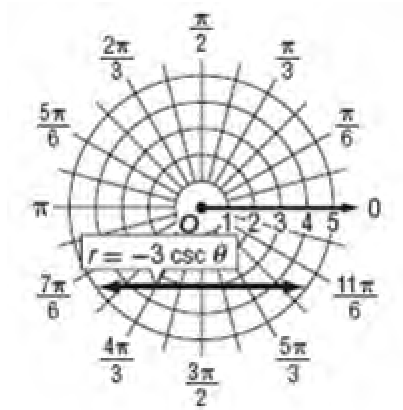
$$\begin{aligned} y &= -3 \\ r\sin\theta &= -3 \\ r &= \frac{-3}{\sin\theta} \\ r &= -3\csc\theta \end{aligned}$$

Evaluate the function for several θ-values in its domain and use these points to graph the function. The graph of this polar equation is a line.



ANSWER:

line; $r = -3 \csc \theta$



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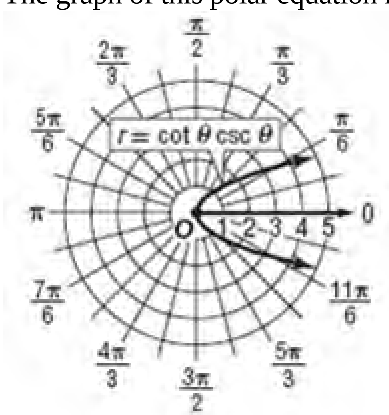
29. $x = y^2$

SOLUTION:

The graph of $x = y^2$ is a parabola. To find the polar form of this equation, replace x with $r\cos\theta$ and y with $r\sin\theta$. Then simplify.

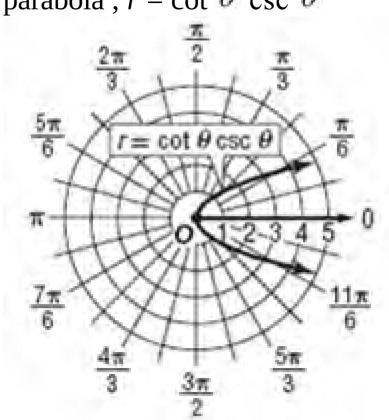
$$
\begin{aligned}
x &= y^2 \\
r\cos\theta &= (r\sin\theta)^2 \\
r\cos\theta &= r^2\sin^2\theta \\
\frac{\cos\theta}{\sin^2\theta} &= r \\
r &= \frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\sin\theta} \\
r &= \cot\theta\csc\theta
\end{aligned}
$$

Evaluate the function for several θ-values in its domain and use these points to graph the function. The graph of this polar equation is a parabola.



ANSWER:

parabola ; $r = \cot\theta\csc\theta$



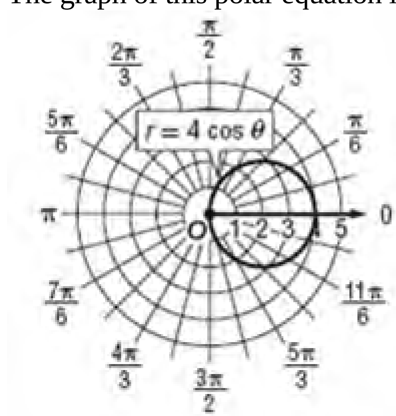
30. $(x - 2)^2 + y^2 = 4$

SOLUTION:

The graph of $(x - 2)^2 + y^2 = 4$ is a circle with radius 2 centered at (2, 0). To find the polar form of this equation, replace x with $r\cos\theta$ and y with $r\sin\theta$. Then simplify.

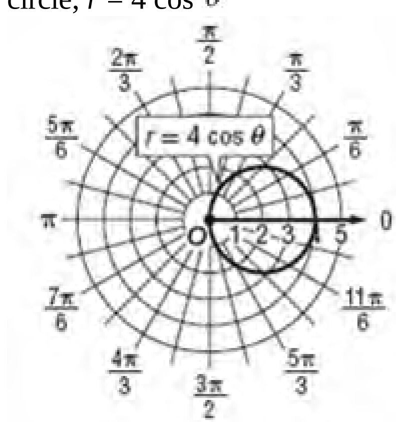
$$
\begin{aligned}
(x-2)^2 + y^2 &= 4 \\
(r\cos\theta - 2)^2 + (r\sin\theta)^2 &= 4 \\
r^2\cos^2\theta - 4r\cos\theta + 4 + r^2\sin^2\theta &= 4 \\
r^2\cos^2\theta - 4r\cos\theta + r^2\sin^2\theta &= 0 \\
r^2\cos^2\theta + r^2\sin^2\theta &= 4r\cos\theta \\
r^2(\cos^2\theta + \sin^2\theta) &= 4r\cos\theta \\
r^2(1) &= 4r\cos\theta \\
r &= 4\cos\theta
\end{aligned}
$$

Evaluate the function for several θ-values in its domain and use these points to graph the function. The graph of this polar equation is a circle.



ANSWER:

circle; $r = 4\cos\theta$



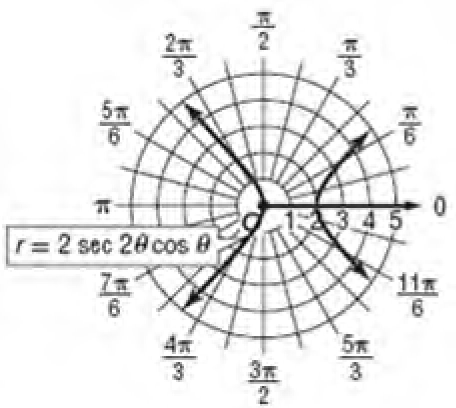
31. $(x - 1)^2 - y^2 = 1$

SOLUTION:

The graph of $(x - 1)^2 - y^2 = 1$ is a hyperbola. To find the polar form of this equation, replace x with $r\cos\theta$ and y with $r\sin\theta$. Then simplify.

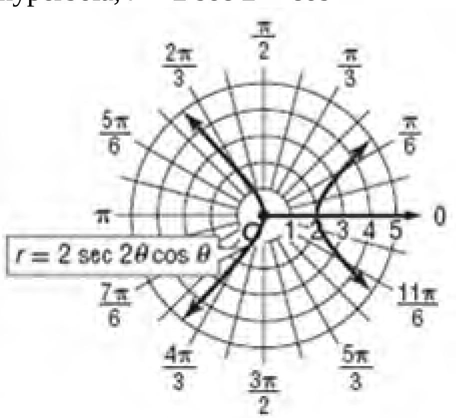
$$\begin{aligned}(x-1)^2 - y^2 &= 1\\ (r\cos\theta - 1)^2 - (r\sin\theta)^2 &= 1\\ r^2\cos^2\theta - 2r\cos\theta + 1 - r^2\sin^2\theta &= 1\\ r^2\cos^2\theta - r^2\sin^2\theta &= 2r\cos\theta\\ r^2(\cos^2\theta - \sin^2\theta) &= 2r\cos\theta\\ r^2(\cos 2\theta) &= 2r\cos\theta\\ r &= \frac{2\cos\theta}{\cos 2\theta}\\ r &= 2\sec 2\theta\cos\theta\end{aligned}$$

Evaluate the function for several θ-values in its domain and use these points to graph the function. The graph of this polar equation is a hyperbola.



ANSWER:

hyperbola; $r = 2\sec 2\theta\cos\theta$



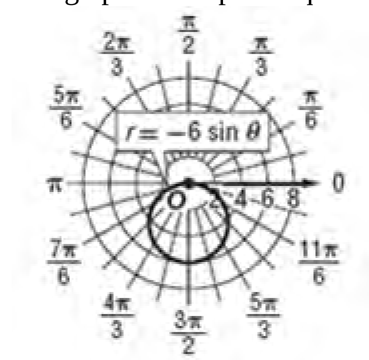
32. $x^2 + (y + 3)^2 = 9$

SOLUTION:

The graph of $x^2 + (y + 3)^2 = 9$ is a circle with radius 3 centered at (0, −3). To find the polar form of this equation, replace x with $r\cos\theta$ and y with $r\sin\theta$. Then simplify.

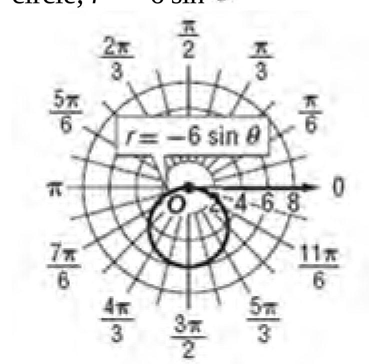
$$\begin{aligned}x^2 + (y+3)^2 &= 9\\ (r\cos\theta)^2 + (r\sin\theta + 3)^2 &= 9\\ r^2\cos^2\theta + r^2\sin^2\theta + 6r\sin\theta + 9 &= 9\\ r^2\cos^2\theta + r^2\sin^2\theta &= -6r\sin\theta\\ r^2(\cos^2\theta + \sin^2\theta) &= -6r\sin\theta\\ r^2(1) &= -6r\sin\theta\\ r &= -6\sin\theta\end{aligned}$$

Evaluate the function for several θ-values in its domain and use these points to graph the function. The graph of this polar equation is a circle.



ANSWER:

circle; $r = -6\sin\theta$



9-3 Polar and Rectangular Forms of Equations

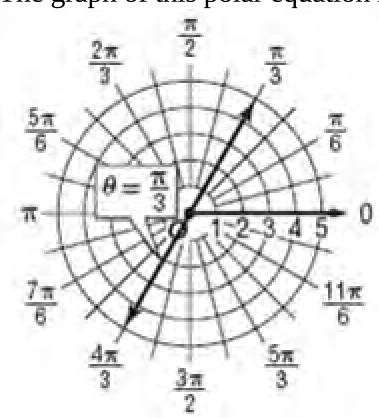
33. $y = \sqrt{3}\,x$

SOLUTION:

The graph of $y = \sqrt{3}\,x$ is a line. To find the polar form of this equation, replace y with $r \sin \theta$ and x with $r \cos \theta$. Then simplify.

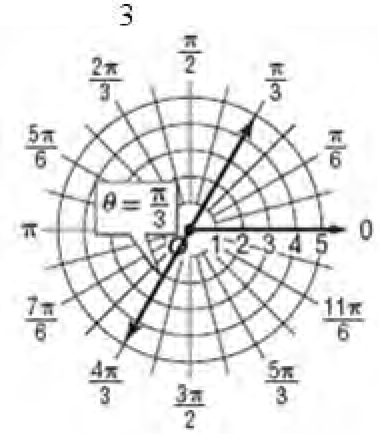
$$\begin{aligned} y &= \sqrt{3}x \\ r\sin\theta &= \sqrt{3}(r\cos\theta) \\ \frac{r\sin\theta}{r\cos\theta} &= \sqrt{3} \\ \tan\theta &= \sqrt{3} \\ \theta &= \tan^{-1}\sqrt{3} \\ \theta &= \frac{\pi}{3} \end{aligned}$$

Evaluate the function for several θ-values in its domain and use these points to graph the function. The graph of this polar equation is a line.



ANSWER:

line; $\theta = \frac{\pi}{3}$



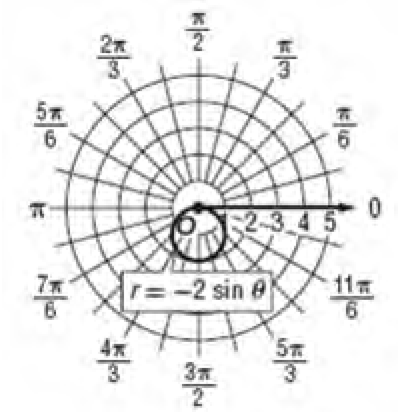
34. $x^2 + (y + 1)^2 = 1$

SOLUTION:

The graph of $x^2 + (y + 1)^2 = 1$ is a circle with radius 1 centered at (0, −1). To find the polar form of this equation, replace x with $r \cos \theta$ and y with $r \sin \theta$. Then simplify.

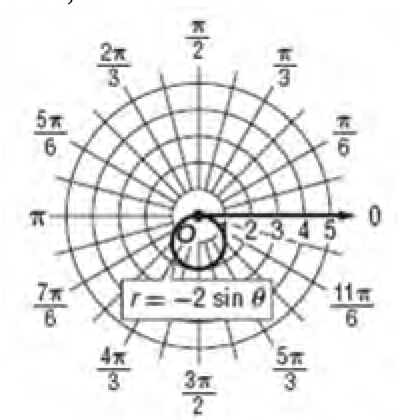
$$\begin{aligned} x^2 + (y+1)^2 &= 1 \\ (r\cos\theta)^2 + (r\sin\theta + 1)^2 &= 1 \\ r^2\cos^2\theta + r^2\sin^2\theta + 2r\sin\theta + 1 &= 1 \\ r^2\cos^2\theta + r^2\sin^2\theta &= -2r\sin\theta \\ r^2(\cos^2\theta + \sin^2\theta) &= -2r\sin\theta \\ r^2(1) &= -2r\sin\theta \\ r &= -2\sin\theta \end{aligned}$$

Evaluate the function for several θ-values in its domain and use these points to graph the function. The graph of this polar equation is a circle.



ANSWER:

circle; $r = -2 \sin \theta$



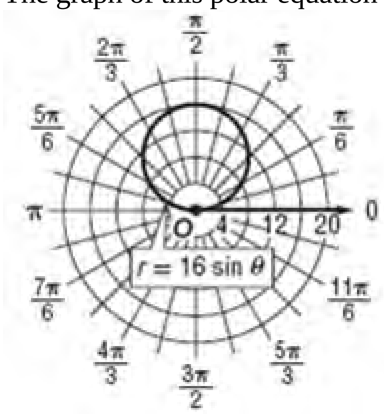
35. $x^2 + (y - 8)^2 = 64$

SOLUTION:

The graph of $x^2 + (y - 8)^2 = 64$ is a circle with radius 8 centered at (0, 8). To find the polar form of this equation, replace x with $r \cos \theta$ and y with $r \sin \theta$. Then simplify.

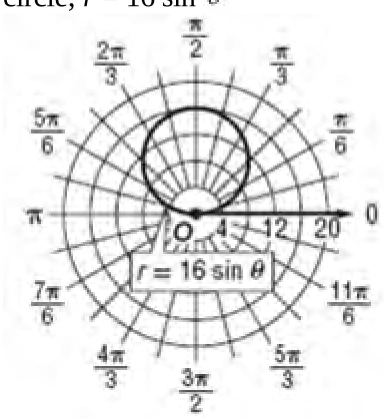
$$\begin{aligned} x^2 + (y-8)^2 &= 64 \\ (r\cos\theta)^2 + (r\sin\theta - 8)^2 &= 64 \\ r^2\cos^2\theta + r^2\sin^2\theta - 16r\sin\theta + 64 &= 64 \\ r^2\cos^2\theta + r^2\sin^2\theta &= 16r\sin\theta \\ r^2(\cos^2\theta + \sin^2\theta) &= 16r\sin\theta \\ r^2(1) &= 16r\sin\theta \\ r &= 16\sin\theta \end{aligned}$$

Evaluate the function for several θ-values in its domain and use these points to graph the function. The graph of this polar equation is a circle.



ANSWER:

circle; $r = 16 \sin \theta$



Write each equation in rectangular form, and then identify its graph. Support your answer by graphing the polar form of the equation.

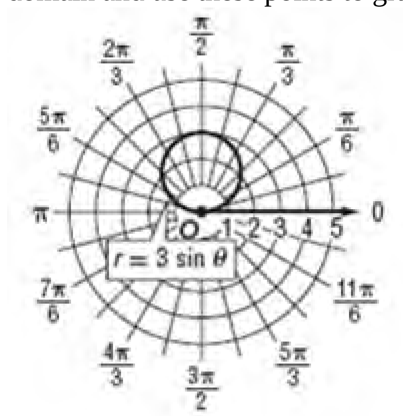
36. $r = 3 \sin \theta$

SOLUTION:

$$\begin{aligned} r &= 3\sin\theta \\ r^2 &= 3r\sin\theta \\ x^2 + y^2 &= 3y \\ x^2 + y^2 - 3y &= 0 \\ x^2 + y^2 - 3y + 1.5^2 &= 1.5^2 \\ x^2 + (y - 1.5)^2 &= 2.25 \end{aligned}$$

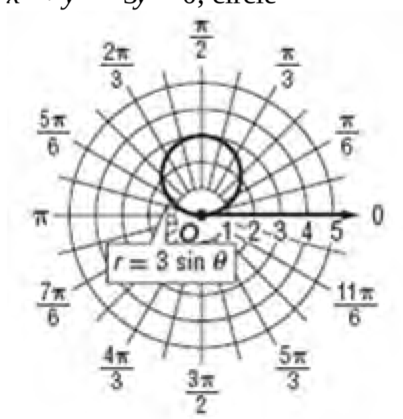
The graph of this equation is a circle centered at (0, 1.5) with radius 1.5.

Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$x^2 + y^2 - 3y = 0$; circle

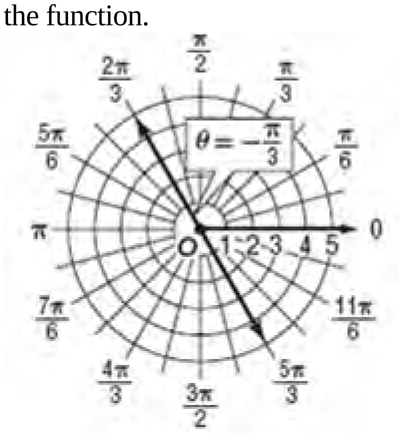


37. $\theta = -\frac{\pi}{3}$

SOLUTION:

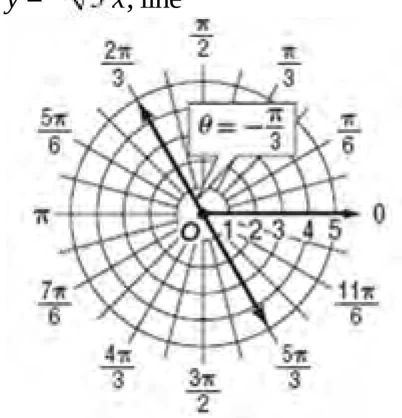
$$\theta = -\frac{\pi}{3}$$
$$\tan\theta = \tan\left(-\frac{\pi}{3}\right)$$
$$\frac{y}{x} = -\sqrt{3}$$
$$y = -\sqrt{3}x$$

The graph of this equation is a line through the origin with slope $-\sqrt{3}$. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$y = -\sqrt{3}\,x$; line

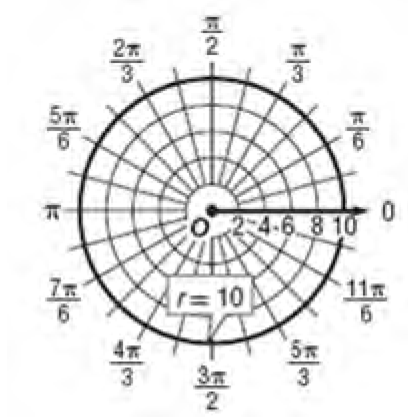


38. $r = 10$

SOLUTION:

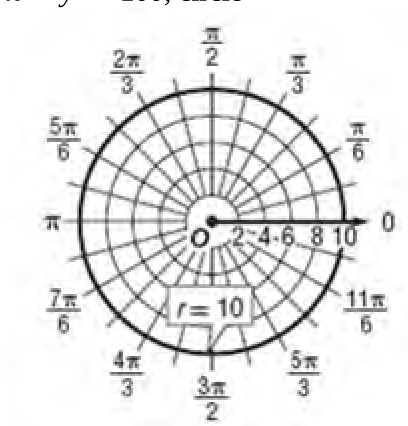
$$r = 10$$
$$r^2 = 100$$
$$x^2 + y^2 = 100$$

The graph of this equation is a circle with a center at the origin and radius 10. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$x^2 + y^2 = 100$; circle



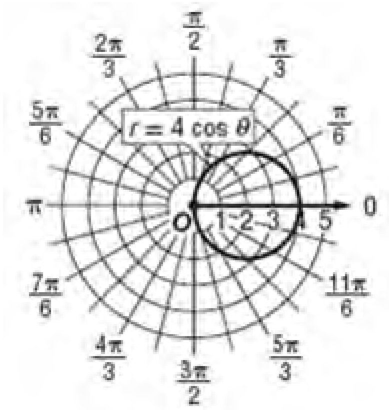
9-3 Polar and Rectangular Forms of Equations

39. $r = 4 \cos \theta$

SOLUTION:

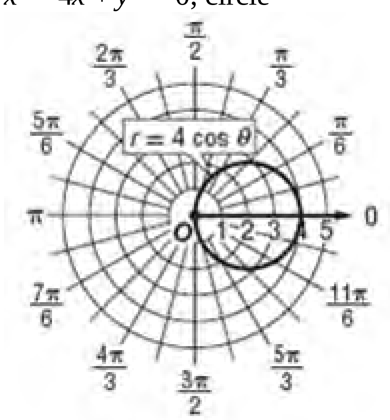
$$\begin{aligned} r &= 4\cos\theta \\ r^2 &= 4r\cos\theta \\ x^2 + y^2 &= 4x \\ x^2 - 4x + y^2 &= 0 \\ x^2 - 4x + 4 + y^2 &= 4 \\ (x-2)^2 + y^2 &= 4 \end{aligned}$$

The graph of this equation as a circle centered at (2, 0) with radius 2. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$x^2 - 4x + y^2 = 0$; circle

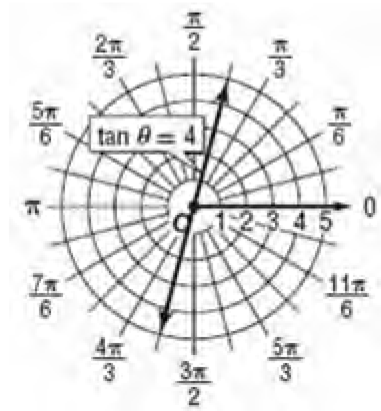


40. $\tan \theta = 4$

SOLUTION:

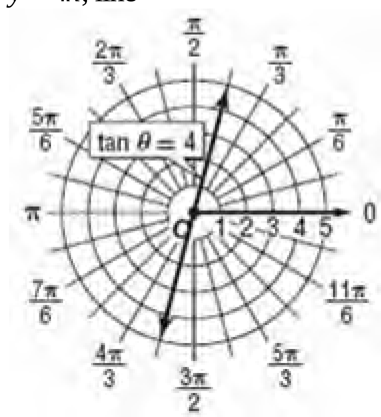
$$\begin{aligned} \tan\theta &= 4 \\ \frac{y}{x} &= 4 \\ y &= 4x \end{aligned}$$

The graph of this equation is a line through the origin with slope 4. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$y = 4x$; line

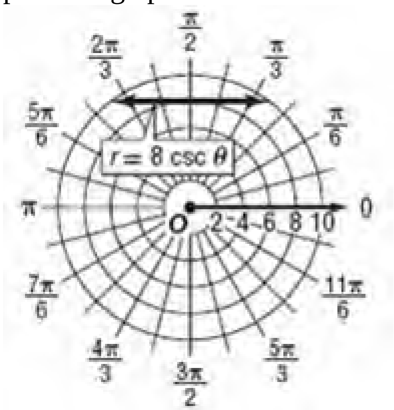


41. $r = 8 \csc \theta$

SOLUTION:

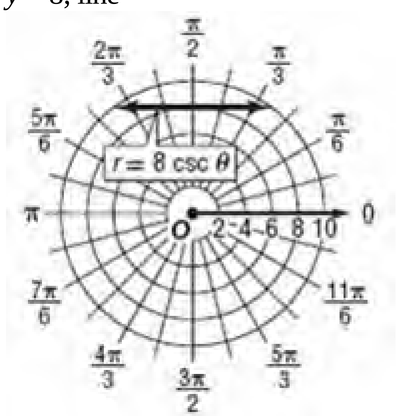
$$r = 8\csc\theta$$
$$r = \frac{8}{\sin\theta}$$
$$r\sin\theta = 8$$
$$y = 8$$

The graph of this equation is a horizontal line through the y-intercept 8 with slope 0. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$y = 8$; line

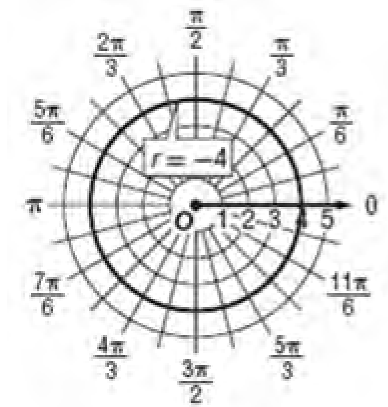


42. $r = -4$

SOLUTION:

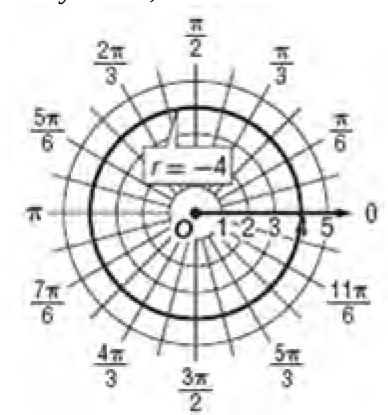
$$r = -4$$
$$r^2 = 16$$
$$x^2 + y^2 = 16$$

The graph of this equation is a circle with a center at the origin and radius 4. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$x^2 + y^2 = 16$; circle



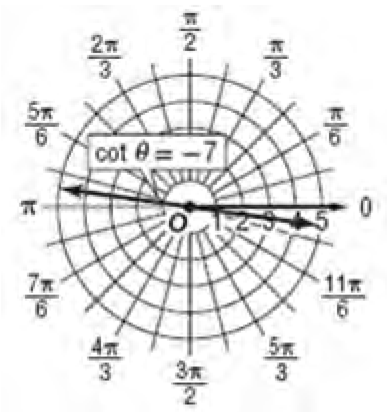
9-3 Polar and Rectangular Forms of Equations

43. $\cot \theta = -7$

SOLUTION:

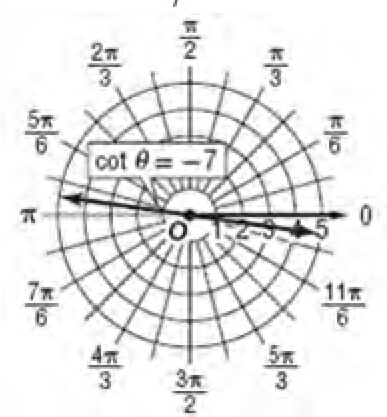
$$\cot\theta = -7$$
$$\frac{x}{y} = -7$$
$$x = -7y$$
$$y = -\frac{1}{7}x$$

The graph of this equation is a line through the origin with slope $-\frac{1}{7}$. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$x = -7y$ or $-\frac{1}{7}x = y$; line

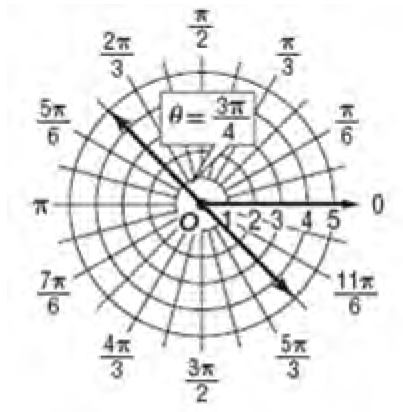


44. $\theta = \frac{3\pi}{4}$

SOLUTION:

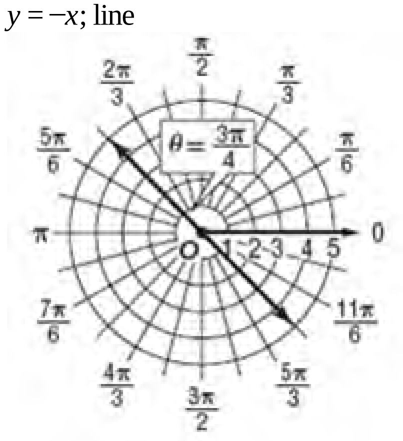
$$\theta = \frac{3\pi}{4}$$
$$\tan\theta = \tan\frac{3\pi}{4}$$
$$\frac{y}{x} = -1$$
$$y = -x$$

The graph of this equation is a line through the origin with slope −1. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$y = -x$; line

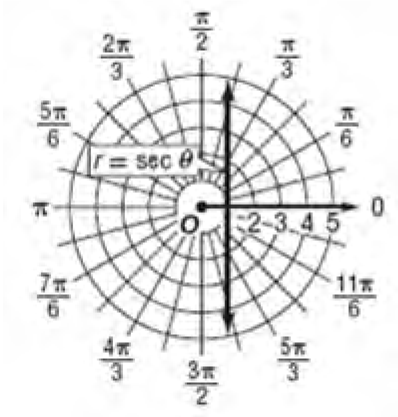


45. $r = \sec\,\theta$

SOLUTION:

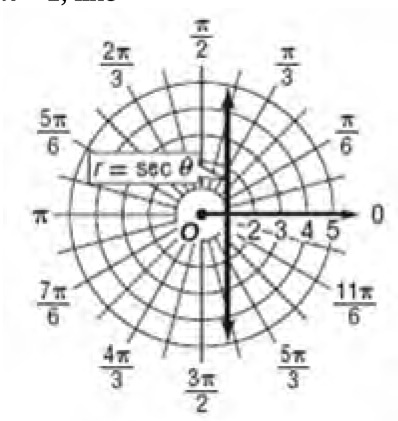
$$r = \sec\theta$$
$$r = \frac{1}{\cos\theta}$$
$$r\cos\theta = 1$$
$$x = 1$$

The graph of this equation is a vertical line through the x-intercept 1 with an undefined slope. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$x = 1$; line



46. **EARTHQUAKE** An equation to model the seismic waves of an earthquake is $r = 12.6\,\sin\,\theta$, where r is measured in miles.

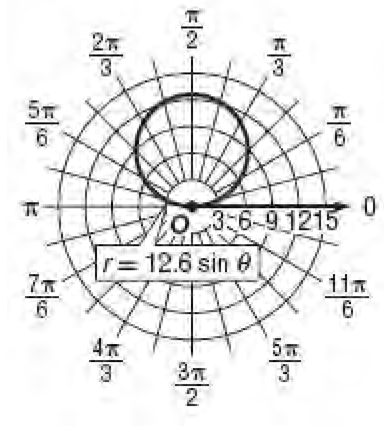
a. Graph the polar pattern of the earthquake.

b. Write an equation in rectangular form to model the seismic waves.

c. Find the rectangular coordinates of the epicenter of the earthquake, and describe the area that is affected by the earthquake.

SOLUTION:

a. Evaluate the function for several θ-values in its domain and use these points to graph the function.



b.

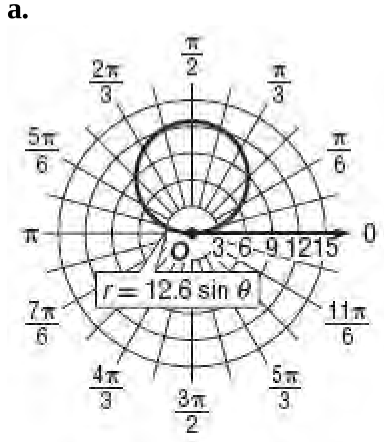
$$r = 12.6\sin\theta$$
$$r^2 = 12.6r\sin\theta$$
$$x^2 + y^2 = 12.6y$$
$$x^2 + y^2 - 12.6y = 0$$

An equation in rectangular form to model the seismic waves is $x^2 + y^2 - 12.6y = 0$.

c. The epicenter is the center of the circle formed by $r = 12.6\,\sin\,\theta$ or $x^2 + y^2 - 12.6y = 0$. A circle in the form $r = a\,\sin\,\theta$ has a diameter of a. Thus, the diameter of $r = 12.6\,\sin\,\theta$ is 12.6, which means it has a radius of 6.3. So, the center of the circle is at the rectangular coordinates (0, 6.3). People within a 6.3-mile radius of the epicenter felt the effects of the earthquake.

ANSWER:

a.



b. $x^2 + y^2 - 12.6y = 0$

c. (0, 6.3); Sample answer: People within a 6.3-mile radius of the epicenter felt the effects of the earthquake.

47. **MICROPHONE** The polar pattern for a directional microphone at a football game is given by $r = 2 + 2\cos\theta$.

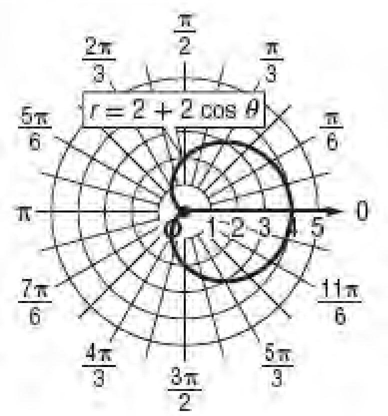
a. Graph the polar pattern.

b. Will the microphone detect a sound that originates from the point with rectangular coordinates (−2, 0)? Explain.

SOLUTION:

a. This graph is symmetric with respect to the polar axis, so you can find points on the interval [0, π] and then use polar axis symmetry to complete the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
r	4	3.7	3.4	3	2	1	0.6	0.3	0



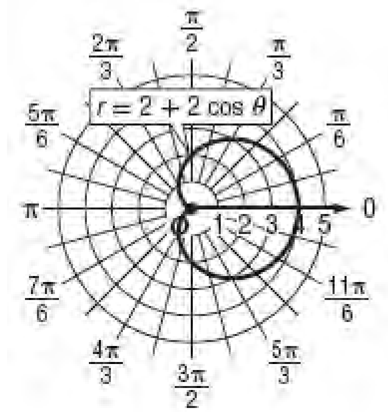
b. Convert the rectangular coordinates (−2, 0) to polar coordinates.

$r = \sqrt{x^2 + y^2}$	Conversion formula	$\theta = \tan^{-1}\frac{y}{x} + \pi$
$= \sqrt{(-2)^2 + 0^2}$	$x = -2$ and $y = 0$	$= \tan^{-1}\frac{0}{-2} + \pi$
$= 2$	Simplify.	$= \pi$

The sound originates from the point with polar coordinates (2, π). This point does not lie within the polar region that is graphed. Thus, the microphone will not detect the sound.

ANSWER:

a.



b. No; sample answer: The given point is on the negative *x*-axis, directly behind the microphone. The polar pattern indicates that the microphone does not pick up any sound from this direction.

9-3 Polar and Rectangular Forms of Equations

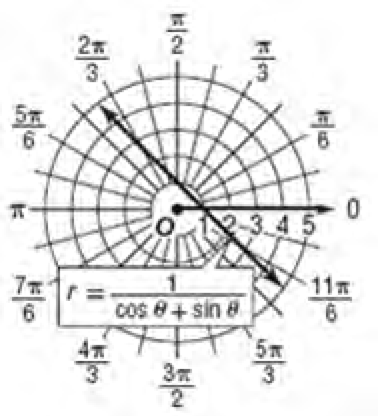
Write each equation in rectangular form, and then identify its graph. Support your answer by graphing the polar form of the equation.

48. $r = \dfrac{1}{\cos\theta + \sin\theta}$

SOLUTION:

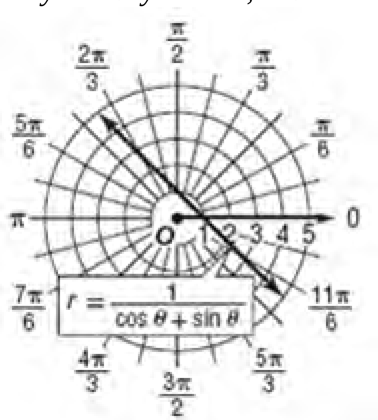
$$r = \frac{1}{\cos\theta + \sin\theta}$$
$$r(\cos\theta + \sin\theta) = 1$$
$$r\cos\theta + r\sin\theta = 1$$
$$x + y = 1$$
$$y = 1 - x$$

The graph of this equation is a line through the point (0, 1) with slope −1. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$x + y = 1$ or $y = 1 - x$; line

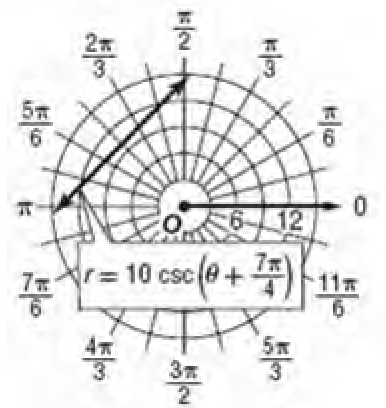


49. $r = 10\csc\left(\theta + \dfrac{7\pi}{4}\right)$

SOLUTION:

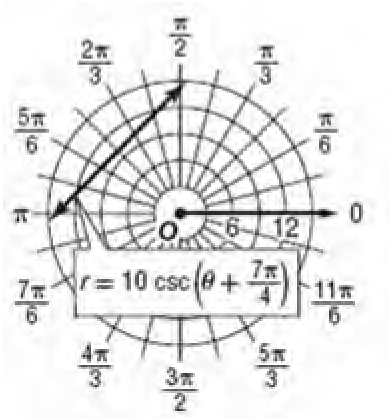
$$r = 10\csc\left(\theta + \frac{7\pi}{4}\right)$$
$$r = \frac{10}{\sin\left(\theta + \frac{7\pi}{4}\right)}$$
$$r\left[\sin\left(\theta + \frac{7\pi}{4}\right)\right] = 10$$
$$r\left(\sin\theta\cos\frac{7\pi}{4} + \cos\theta\sin\frac{7\pi}{4}\right) = 10$$
$$r\left[\sin\theta\left(\frac{\sqrt{2}}{2}\right) + \cos\theta\left(-\frac{\sqrt{2}}{2}\right)\right] = 10$$
$$\frac{\sqrt{2}}{2}r\sin\theta - \frac{\sqrt{2}}{2}r\cos\theta = 10$$
$$\frac{\sqrt{2}}{2}y - \frac{\sqrt{2}}{2}x = 10$$
$$\frac{\sqrt{2}}{2}y = \frac{\sqrt{2}}{2}x + 10$$
$$y = x + 10\sqrt{2}$$

The graph of this equation is a line through the point $(0, 10\sqrt{2})$ or $(0, 14.14)$ with slope 1. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$\frac{\sqrt{2}}{2}y - \frac{\sqrt{2}}{2}x = 10$ or $y = x + 10\sqrt{2}$; line

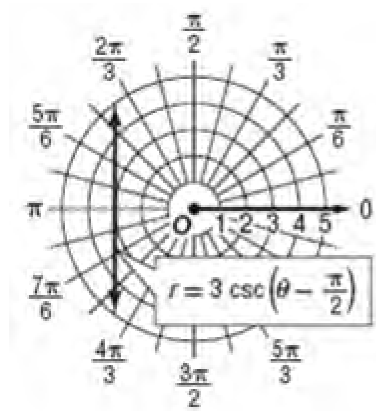


50. $r = 3\csc\left(\theta - \frac{\pi}{2}\right)$

SOLUTION:

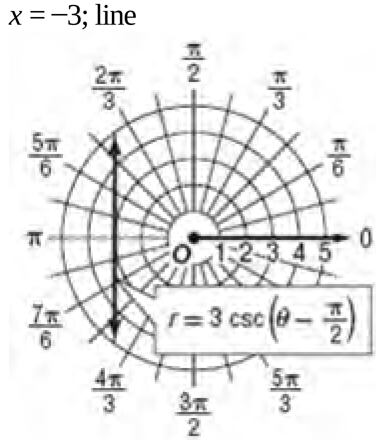
$$
\begin{aligned}
r &= 3\csc\left(\theta - \frac{\pi}{2}\right) \\
r &= \frac{3}{\sin\left(\theta - \frac{\pi}{2}\right)} \\
r\left[\sin\left(\theta - \frac{\pi}{2}\right)\right] &= 3 \\
r\left(\sin\theta\cos\frac{\pi}{2} - \cos\theta\sin\frac{\pi}{2}\right) &= 3 \\
r(\sin\theta \cdot 0 - \cos\theta \cdot 1) &= 3 \\
-r\cos\theta &= 3 \\
-x &= 3 \\
x &= -3
\end{aligned}
$$

The graph of this equation is a vertical line through the point (−3, 0). Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$x = -3$; line

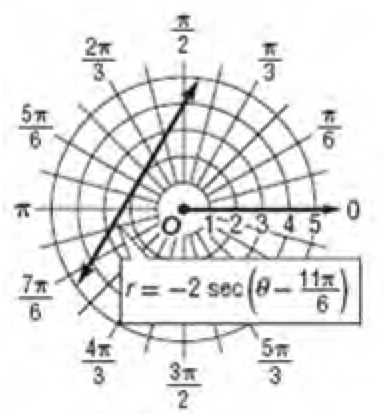


51. $r = -2\sec\left(\theta - \frac{11\pi}{6}\right)$

SOLUTION:

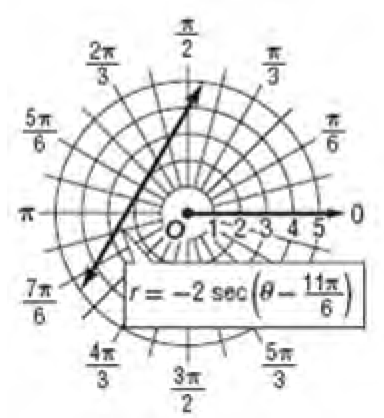
$$\begin{aligned} r &= -2\sec\left(\theta - \frac{11\pi}{6}\right) \\ r &= \frac{-2}{\cos\left(\theta - \frac{11\pi}{6}\right)} \\ r\cos\left(\theta - \frac{11\pi}{6}\right) &= -2 \\ r\left(\cos\theta\cos\frac{11\pi}{6} + \sin\theta\sin\frac{11\pi}{6}\right) &= -2 \\ r\left[\cos\theta\left(\frac{\sqrt{3}}{2}\right) + \sin\theta\left(-\frac{1}{2}\right)\right] &= -2 \\ \frac{\sqrt{3}}{2}r\cos\theta - \frac{1}{2}r\sin\theta &= -2 \\ \frac{\sqrt{3}}{2}x - \frac{1}{2}y &= -2 \\ \frac{\sqrt{3}}{2}x + 2 &= \frac{1}{2}y \\ \sqrt{3}x + 4 &= y \end{aligned}$$

The graph of this equation is a line through the point (0, 4) with slope $\sqrt{3}$. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$\frac{\sqrt{3}}{2}x - \frac{1}{2}y = -2$ or $y = \sqrt{3}x + 4$; line

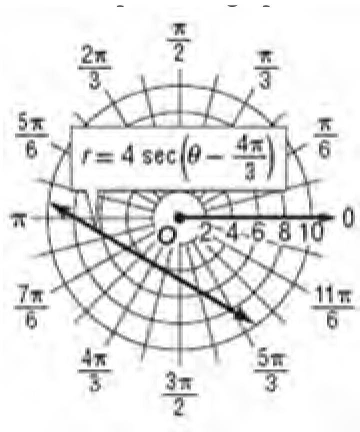


52. $r = 4\sec\left(\theta - \frac{4\pi}{3}\right)$

SOLUTION:

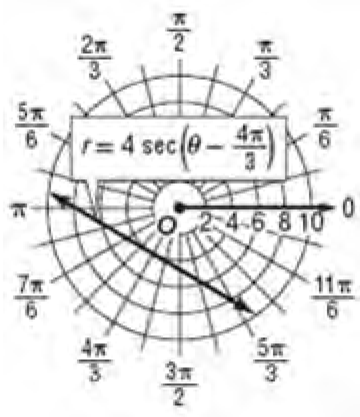
$$\begin{aligned} r &= 4\sec\left(\theta - \frac{4\pi}{3}\right) \\ r &= \frac{4}{\cos\left(\theta - \frac{4\pi}{3}\right)} \\ r\cos\left(\theta - \frac{4\pi}{3}\right) &= 4 \\ r\left(\cos\theta\cos\frac{4\pi}{3} + \sin\theta\sin\frac{4\pi}{3}\right) &= 4 \\ r\left[\cos\theta\left(-\frac{1}{2}\right) + \sin\theta\left(-\frac{\sqrt{3}}{2}\right)\right] &= 4 \\ -\frac{1}{2}r\cos\theta - \frac{\sqrt{3}}{2}r\sin\theta &= 4 \\ -\frac{1}{2}x - \frac{\sqrt{3}}{2}y &= 4 \\ -\frac{1}{2}x - 4 &= \frac{\sqrt{3}}{2}y \\ -x - 8 &= \sqrt{3}y \\ -\frac{x}{\sqrt{3}} - \frac{8}{\sqrt{3}} &= y \\ -\frac{\sqrt{3}}{3}x - \frac{8\sqrt{3}}{3} &= y \end{aligned}$$

The graph of this equation is a line through the point $\left(0, -\frac{8\sqrt{3}}{3}\right)$ or $(0, -4.62)$ with slope $-\frac{\sqrt{3}}{3}$. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$-\frac{1}{2}x - \frac{\sqrt{3}}{2}y = 4$ or $y = -\frac{\sqrt{3}}{3}x - \frac{8\sqrt{3}}{3}$; line

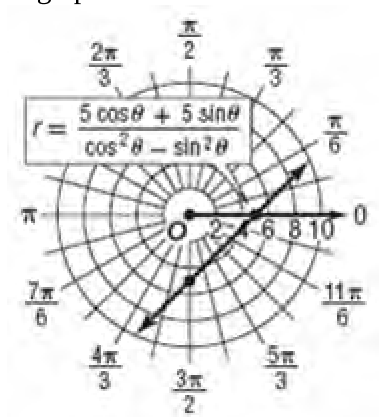


53. $r = \frac{5\cos\theta + 5\sin\theta}{\cos^2\theta - \sin^2\theta}$

SOLUTION:

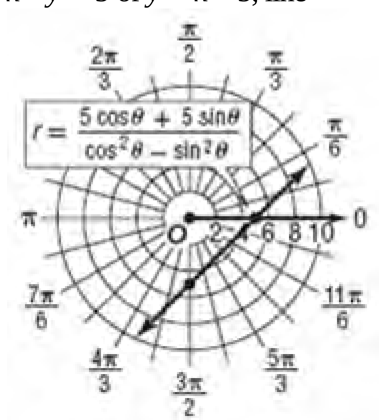
$$r = \frac{5\cos\theta + 5\sin\theta}{\cos^2\theta - \sin^2\theta}$$
$$r = \frac{5(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$$
$$r = \frac{5}{\cos\theta - \sin\theta}$$
$$r(\cos\theta - \sin\theta) = 5$$
$$r\cos\theta - r\sin\theta = 5$$
$$x - y = 5$$
$$x - 5 = y$$

The graph of this equation is a line through the point (0, −5) with slope 1. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$x - y = 5$ or $y = x - 5$; line

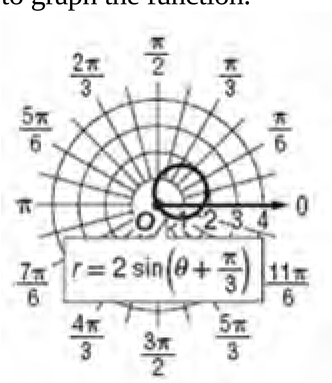


54. $r = 2\sin\left(\theta + \frac{\pi}{3}\right)$

SOLUTION:

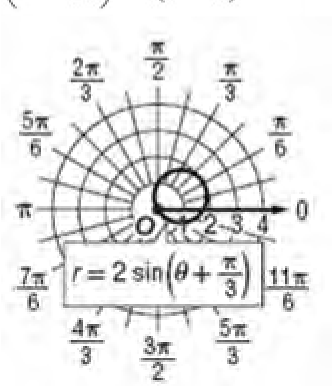
$$r = 2\sin\left(\theta + \frac{\pi}{3}\right)$$
$$r = 2\left(\sin\theta\cos\frac{\pi}{3} + \cos\theta\sin\frac{\pi}{3}\right)$$
$$r = 2\left[\sin\theta\left(\frac{1}{2}\right) + \cos\theta\left(\frac{\sqrt{3}}{2}\right)\right]$$
$$r = \sin\theta + \sqrt{3}\cos\theta$$
$$r^2 = r\sin\theta + \sqrt{3}r\cos\theta$$
$$x^2 + y^2 = y + \sqrt{3}x$$
$$x^2 + y^2 - \sqrt{3}x - y = 0$$
$$x^2 - \sqrt{3}x + y^2 - y = 0$$
$$x^2 - \sqrt{3}x + \frac{3}{4} + y^2 - y + \frac{1}{4} = \frac{3}{4} + \frac{1}{4}$$
$$\left(x - \frac{\sqrt{3}}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = 1$$

The graph of this equation is a circle with a center at $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and radius 1. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$x^2 + y^2 - \sqrt{3}\,x - y = 0$ or $\left(x - \frac{\sqrt{3}}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = 1$; circle

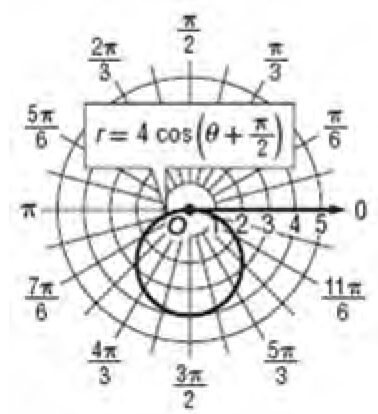


55. $r = 4\cos\left(\theta + \frac{\pi}{2}\right)$

SOLUTION:

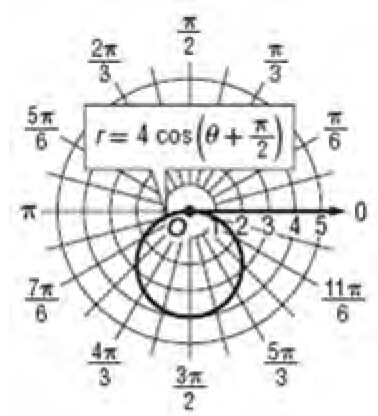
$$r = 4\cos\left(\theta + \frac{\pi}{2}\right)$$
$$r = 4\left(\cos\theta\cos\frac{\pi}{2} - \sin\theta\sin\frac{\pi}{2}\right)$$
$$r = 4(\cos\theta \cdot 0 - \sin\theta \cdot 1)$$
$$r = -4\sin\theta$$
$$r^2 = -4r\sin\theta$$
$$x^2 + y^2 = -4y$$
$$x^2 + y^2 + 4y = 0$$
$$x^2 + y^2 + 4y + 4 = 4$$
$$x^2 + (y + 2)^2 = 4$$

The graph of this equation is a circle with a center at (0, −2) and radius 2. Evaluate the function for several θ-values in its domain and use these points to graph the function.



ANSWER:

$x^2 + y^2 + 4y = 0$ or $x^2 + (y + 2)^2 = 4$; circle



56. **ASTRONOMY** Polar equations are used to model

the paths of various satellites in space. Suppose the path of a satellite is modeled by $r = \frac{4}{4 + 3\sin\theta}$, where r is measured in tens of thousands of miles, with Earth at the pole.

a. Sketch a graph of the path of the satellite.

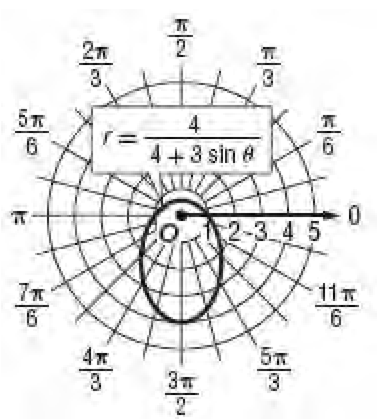
b. Determine the minimum and maximum distances the satellite is from Earth at any time.

c. Suppose a second satellite passes through a point with rectangular coordinates (1.5, −3). Are the two satellites at risk of ever colliding at this point? Explain.

SOLUTION:

a. Evaluate the function for several θ-values in its domain and use these points to graph the function.

θ	r
$\frac{\pi}{6}$	0.727
$\frac{\pi}{4}$	0.653
$\frac{\pi}{3}$	0.606
$\frac{\pi}{2}$	0.571
$\frac{3\pi}{4}$	0.653
π	1
$\frac{5\pi}{4}$	2.129
$\frac{3\pi}{2}$	4
$\frac{7\pi}{4}$	2.129
2π	1



b. The minimum distance the satellite will be from the Earth occurs when $\theta = \frac{\pi}{2}$ and the maximum distance the satellite will be from Earth occurs when $\theta = \frac{3\pi}{2}$. Evaluate the function for these two values of θ.

$$r = \frac{4}{4+3\sin\theta} = \frac{4}{4+3\sin\frac{\pi}{2}} = \frac{4}{4+3(1)} = \frac{4}{7}$$

When $\theta = \frac{\pi}{2}$, $r = \frac{4}{7}$. Since r is measured in tens of thousands of miles, $r = \frac{4}{7} \cdot 10{,}000$ or about 5714 miles.

$$r = \frac{4}{4+3\sin\theta} = \frac{4}{4+3\sin\frac{3\pi}{2}} = \frac{4}{4+3(-1)} = 4$$

When $\theta = \frac{3\pi}{2}$, $r = 4$. Since r is measured in tens of thousands of miles, $r = 4 \cdot 10{,}000$ or 40,000 miles. So, the minimum distance that the satellite is from Earth is about 5714 miles and the maximum distance that the satellite is from Earth is 40,000 miles.

c. Convert the rectangular coordinates (1.5, −3) to polar coordinates.

$$r = \sqrt{x^2 + y^2} = \sqrt{1.5^2 + (-3)^2} = \sqrt{11.25} \text{ or about } 3.35$$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-3}{1.5} \approx -1.11$$

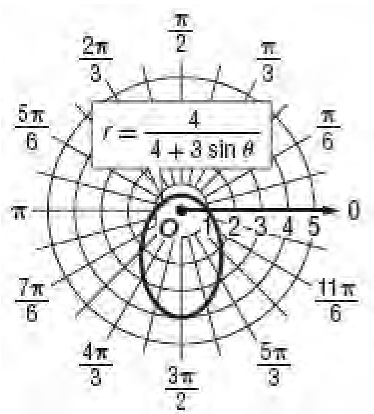
The second satellite passes through the point with polar coordinates (3.35, −1.11). Find the location of the first satellite for this value of θ.

$$\begin{aligned} r &= \frac{4}{4+3\sin\theta} \\ &= \frac{4}{4+3\sin(-1.11)} \\ &\approx 3.05 \end{aligned}$$

Since $r \approx 3.05$ for the first satellite and $r \approx 3.35$ for the second satellite when $\theta \approx -1.11$, the two satellites are 3.35 − 3.05 or 0.3 apart. Since *r* is measured in tens of thousands of miles, the two satellites are $0.3 \cdot 10{,}000$ or 3,000 miles apart.

ANSWER:

a.



b. The minimum distance from Earth is approximately 5714 miles. The maximum distance from Earth is 40,000 miles.

c. No; sample answer: The second satellite has polar coordinates of (3.35, −1.11). For this value of θ, the first satellite will have coordinates of (3.05, −1.11). The two satellites will be 3000 miles apart.

Identify the graph of each rectangular equation. Then write the equation in polar form. Support your answer by graphing the polar form of the equation.

57. $6x - 3y = 4$

SOLUTION:

Rewrite $6x - 3y = 4$ in slope intercept form.

$$\begin{aligned} 6x - 3y &= 4 \\ 6x - 3y - 6x &= -6x + 4 \\ \frac{-3y}{-3} &= \frac{-6x}{-3} + \frac{4}{-3} \\ y &= 2x - \frac{4}{3} \end{aligned}$$

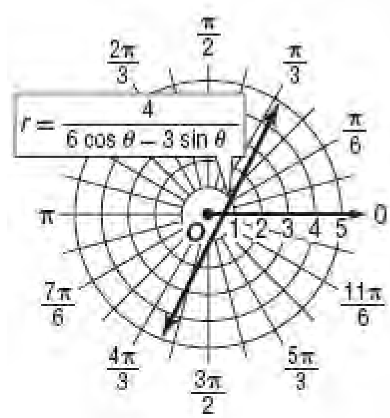
$$\begin{aligned} 6x - 3y &= 4 \\ 6x - 3y - 6x &= -6 \\ \frac{-3y}{-3} &= \frac{-6}{-3} \\ y &= 2x - \end{aligned}$$

The graph of $y = 2x - \frac{4}{3}$ is a line with point $\left(0, -\frac{4}{3}\right)$ slope 2. To find the polar form of the equation, repla *r* cos θ and *y* with *r* sin θ in the original equation. T simplify.

$$\begin{aligned} 6x - 3y &= 4 \\ 6(r\cos\theta) - 3(r\sin\theta) &= 4 \\ 6r\cos\theta - 3r\sin\theta &= 4 \\ r(6\cos\theta - 3\sin\theta) &= 4 \\ r &= \frac{4}{6\cos\theta - 3\sin\theta} \end{aligned}$$

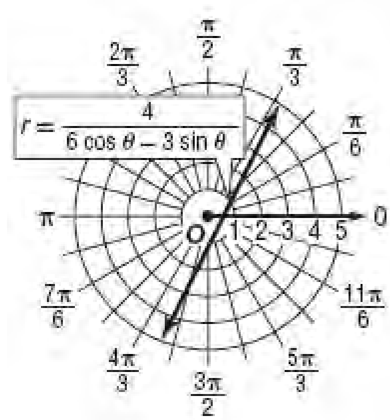
Evaluate the function for several θ-values in its dom use these points to graph the function.

Note that this graph will be similar to $y = 2x - \frac{4}{3}$.



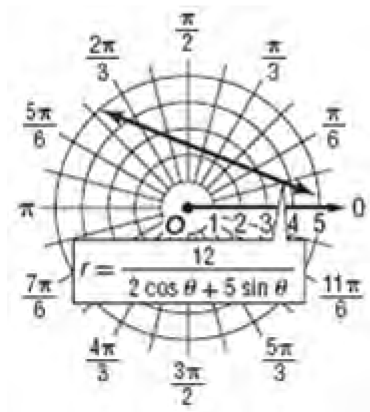
ANSWER:

line; $r = \dfrac{4}{6\cos\theta - 3\sin\theta}$



9-3 Polar and Rectangular Forms of Equations

58. $2x + 5y = 12$



SOLUTION:

Rewrite $2x + 5y = 12$ in slope intercept form.

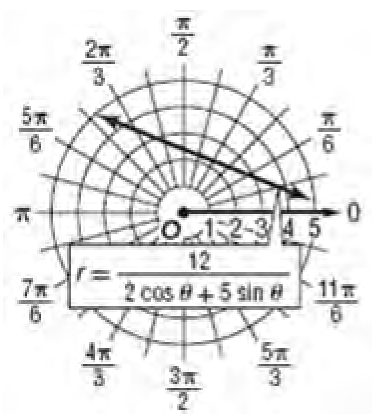
$$\begin{aligned} 2x + 5y &= 12 \\ 2x + 5y - 2x &= -2x + 12 \\ \frac{5y}{5} &= \frac{-2x}{5} + \frac{12}{5} \\ y &= -\frac{2}{5}x + \frac{12}{5} \end{aligned}$$

The graph of $y = -\frac{2}{5}x + \frac{12}{5}$ is a line with point (0, 2.4) and slope $-\frac{2}{5}$. To find the polar form of the equation, replace x with $r\cos\theta$ and y with $r\sin\theta$ in the original equation. Then simplify.

$$\begin{aligned} 2x + 5y &= 12 \\ 2(r\cos\theta) + 5(r\sin\theta) &= 12 \\ 2r\cos\theta + 5r\sin\theta &= 12 \\ r(2\cos\theta + 5\sin\theta) &= 12 \\ r &= \frac{12}{2\cos\theta + 5\sin\theta} \end{aligned}$$

Evaluate the function for several θ-values in its domain and use these points to graph the function.

Note that this graph will be similar to $y = -\frac{2}{5}x + \frac{12}{5}$.


$\frac{\pi}{2}$
$\frac{2\pi}{3}$
$\frac{\pi}{3}$
$\frac{5\pi}{6}$
$\frac{\pi}{6}$
π
0
O 1 2 3 4 5
$r = \frac{12}{2\cos\theta + 5\sin\theta}$
$\frac{7\pi}{6}$
$\frac{11\pi}{6}$
$\frac{4\pi}{3}$
$\frac{5\pi}{3}$
$\frac{3\pi}{2}$

ANSWER:

line; $r = \frac{12}{2\cos\theta + 5\sin\theta}$

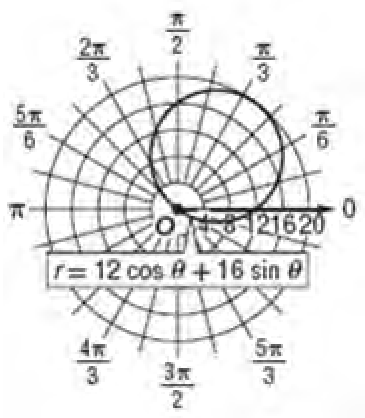
59. $(x - 6)^2 + (y - 8)^2 = 100$

SOLUTION:

The graph of $(x - 6)^2 + (y - 8)^2 = 100$ is a circle with radius 10 centered at (6, 8). To find the polar form of this equation, replace x with $r \cos \theta$ and y with $r \sin \theta$. Then simplify.

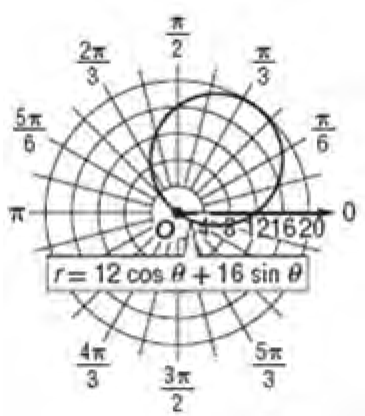
$$\begin{aligned}
(x-6)^2 + (y-8)^2 &= 100 \\
(r\cos\theta - 6)^2 + (r\sin\theta - 8)^2 &= 100 \\
r^2\cos^2\theta - 12r\cos\theta + 36 + r\sin^2\theta - 16r\sin\theta + 64 &= 100 \\
r^2\cos^2\theta - 12r\cos\theta + r\sin^2\theta - 16r\sin\theta &= 0 \\
r^2\cos^2\theta + r^2\sin^2\theta &= 12r\cos\theta + 16r\sin\theta \\
r^2(\cos^2\theta + \sin^2\theta) &= 12r\cos\theta + 16r\sin\theta \\
r^2(1) &= 12r\cos\theta + 16r\sin\theta \\
r &= 12\cos\theta + 16\sin\theta
\end{aligned}$$

Evaluate the function for several θ-values in its domain and use these points to graph the function. The graph of this polar equation is a circle.



ANSWER:

circle; $r = 12 \cos \theta + 16 \sin \theta$



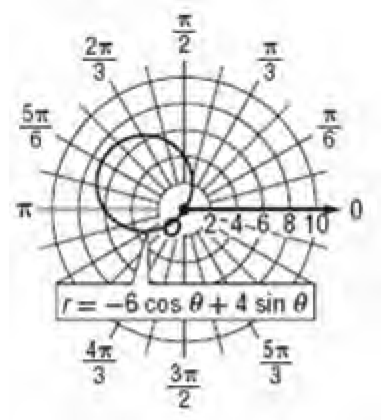
60. $(x + 3)^2 + (y - 2)^2 = 13$

SOLUTION:

The graph of $(x + 3)^2 + (y - 2)^2 = 13$ is a circle with radius $\sqrt{13}$ or about 3.61 centered at (−3, 2). To find the polar form of this equation, replace x with $r \cos \theta$ and y with $r \sin \theta$. Then simplify.

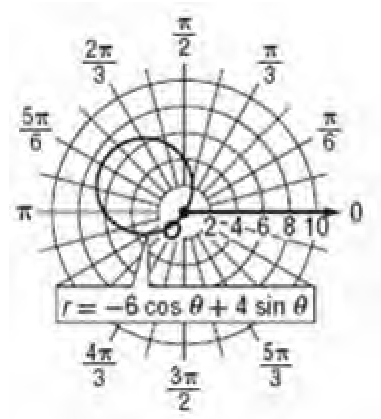
$$\begin{aligned}
(x+3)^2 + (y-2)^2 &= 13 \\
(r\cos\theta + 3)^2 + (r\sin\theta - 2)^2 &= 13 \\
r^2\cos^2\theta + 6r\cos\theta + 9 + r\sin^2\theta - 4r\sin\theta + 4 &= 13 \\
r^2\cos^2\theta + 6r\cos\theta + r\sin^2\theta - 4r\sin\theta &= 0 \\
r^2\cos^2\theta + r^2\sin^2\theta &= -6r\cos\theta + 4r\sin\theta \\
r^2(\cos^2\theta + \sin^2\theta) &= -6r\cos\theta + 4r\sin\theta \\
r^2(1) &= -6r\cos\theta + 4r\sin\theta \\
r &= -6\cos\theta + 4\sin\theta
\end{aligned}$$

Evaluate the function for several θ-values in its domain and use these points to graph the function. The graph of this polar equation is a circle.



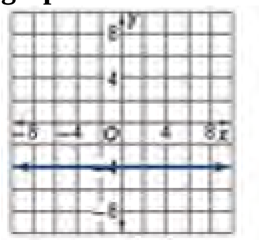
ANSWER:

circle; $r = -6 \cos \theta + 4 \sin \theta$



9-3 Polar and Rectangular Forms of Equations

Write rectangular and polar equations for each graph.

61. 

SOLUTION:

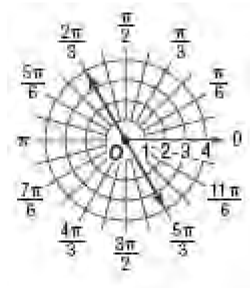
Sample answer: The graph is of the line $y = -4$. To find the polar form of this equation, replace y with $r \sin \theta$. Then simplify.

$$y = -4$$
$$r \sin \theta = -4$$
$$r = \frac{-4}{\sin \theta}$$
$$r = -4 \csc \theta$$

The graph has a rectangular equation $y = -4$ and a polar equation $r = -4 \csc \theta$.

ANSWER:

Sample answer: $y = -4$; $r = -4 \csc \theta$

62. 

SOLUTION:

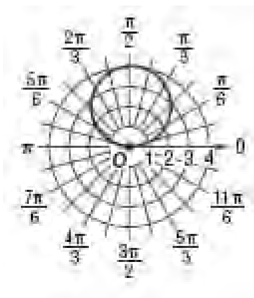
Sample answer: The graph is of the line $\theta = \frac{2\pi}{3}$.

$$\theta = \frac{2\pi}{3}$$
$$\tan \theta = -\sqrt{3}$$
$$\frac{y}{x} = -\sqrt{3}$$
$$y = -\sqrt{3}x$$

The graph has a rectangular equation $y = -\sqrt{3}x$ and a polar equation $\theta = \frac{2\pi}{3}$.

ANSWER:

Sample answer: $y = -\sqrt{3}x$; $\theta = \frac{2\pi}{3}$

63. 

SOLUTION:

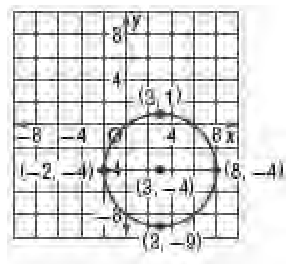
Sample answer: Given the orientation of the circle and the length of a, the graph is of the circle $r = 4 \sin \theta$.

$$
\begin{aligned}
r &= 4\sin\theta \\
r^2 &= 4r\sin\theta \\
x^2 + y^2 &= 4y \\
x^2 + y^2 - 4y &= 0 \\
x^2 + y^2 - 4y + 4 &= 4 \\
x^2 + (y-2)^2 &= 4
\end{aligned}
$$

The graph has a rectangular equation $x^2 + (y - 2)^2 = 4$ and a polar equation $r = 4 \sin \theta$.

ANSWER:

Sample answer: $x^2 + (y - 2)^2 = 4$; $r = 4 \sin \theta$

64. 

SOLUTION:

Sample answer: The graph is of a circle with a center at (3, −4) and radius 5. Thus, it has a rectangular equation $(x - 3)^2 + (y + 4)^2 = 25$. To find the polar form of the equation, replace x with $r \cos \theta$ and y with $r \sin \theta$. Then simplify.

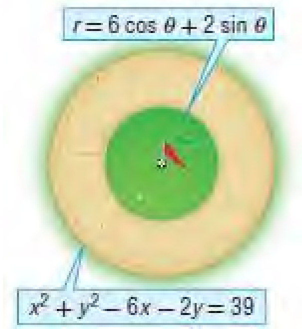
$$
\begin{aligned}
(x-3)^2 + (y+4)^2 &= 25 \\
(r\cos\theta - 3)^2 + (r\sin\theta + 4)^2 &= 25 \\
r^2\cos^2\theta - 6r\cos\theta + 9 + r^2\sin^2\theta + 8r\sin\theta + 16 &= 25 \\
r^2\cos^2\theta - 6r\cos\theta + r^2\sin^2\theta + 8r\sin\theta &= 0 \\
r^2\cos^2\theta + r^2\sin^2\theta &= 6r\cos\theta - 8r\sin\theta \\
r^2(\cos^2\theta + \sin^2\theta) &= 6r\cos\theta - 8r\sin\theta \\
r^2(1) &= 6r\cos\theta - 8r\sin\theta \\
r &= 6\cos\theta - 8\sin\theta
\end{aligned}
$$

The graph has a rectangular equation $(x - 3)^2 + (y + 4)^2 = 25$ and a polar equation $r = 6 \cos \theta - 8 \sin \theta$.

ANSWER:

Sample answer: $(x - 3)^2 + (y + 4)^2 = 25$; $r = 6 \cos \theta - 8 \sin \theta$

65. **GOLF** On the 18th hole at Hilly Pines Golf Course, the circular green is surrounded by a ring of sand as shown in the figure. Find the area of the region covered by sand assuming the hole acts as the pole for both equations and units are given in yards.



SOLUTION:

The area of the region covered by sand is equal to the area of the circle formed by $x^2 + y^2 - 6x - 2y = 39$ minus the area of the circle formed by $r = 6\cos\theta + 2\sin\theta$. Find the area of each circle.

$$x^2 + y^2 - 6x - 2y = 39$$
$$x^2 - 6x + 9 + y^2 - 2y + 1 = 39 + 9 + 1$$
$$(x-3)^2 + (y-1)^2 = 49$$

The radius of the large circle is 7. Thus, the area of the large circle is 49π.

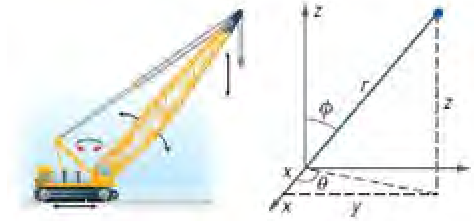
$$r = 6\cos\theta + 2\sin\theta$$
$$r^2 = 6r\cos\theta + 2r\sin\theta$$
$$x^2 + y^2 = 6x + 2y$$
$$x^2 - 6x + y^2 - 2y = 0$$
$$x^2 - 6x + 9 + y^2 - 2y + 1 = 9 + 1$$
$$(x-3)^2 + (y-1)^2 = 10$$

The radius of the small circle is $\sqrt{10}$. Thus, the area of the small circle is 10π. The area of the region covered by sand is 49π − 10π or 39π square yards, which is approximately 122.52 square yards.

ANSWER:

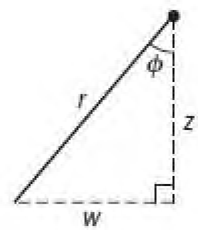
39π yd^2 or about 122.52 yd^2

66. **CONSTRUCTION** Boom cranes operate on three-dimensional counterparts of polar coordinates called *spherical coordinates*. A point in space has spherical coordinates (r, θ, ϕ), where r represents the distance from the pole, θ represents the angle of rotation about the vertical axis, and ϕ represents the polar angle from the positive vertical axis. Given a point in spherical coordinates (r, θ, ϕ) find the rectangular coordinates (x, y, z) in terms of r, θ, and ϕ.



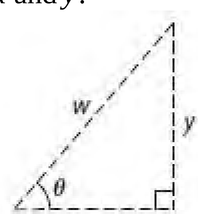
SOLUTION:

Let the dashed line that extends from z to the pole be w. The angle created by z and r is ϕ because they are alternate interior angles. Sketch the right triangle formed by z, r, and w and use trigonometric ratios to solve for z and w.



$$\cos\phi = \frac{z}{r} \qquad \sin\phi = \frac{w}{r}$$
$$r\cos\phi = z \qquad r\sin\phi = w$$

w is the hypotenuse of the right triangle formed with x and y.



Use trigonometric rations and $w = r\sin\phi$ to solve for x and y.

$$\cos\theta = \frac{x}{r\sin\phi} \qquad \sin\theta = \frac{y}{r\sin\phi}$$
$$r\sin\phi\cos\theta = x \qquad r\sin\phi\sin\theta = y$$

The rectangular coordinates of a point in spherical coordinates (r, θ, ϕ) are $(x, y, z) = (r\sin\phi\cos\theta, r\sin\phi\sin\theta, r\cos\phi)$.

ANSWER:

$(x, y, z) = (r\sin\phi\cos\theta, r\sin\phi\sin\theta, r\cos\phi)$

67. **MULTIPLE REPRESENTATIONS** In this

problem, you will investigate the relationship between complex numbers and polar coordinates.

a. GRAPHICAL The complex number $a + b\boldsymbol{i}$ can be plotted on a complex plane using the ordered pair (a, b), where the x-axis is the real axis R and the y-axis is the imaginary axis $\boldsymbol{i}$. Graph the complex number $6 + 8\boldsymbol{i}$.

b. GRAPHICAL Find polar coordinates for the complex number using the rectangular coordinates plotted in part **a** if $0 < \theta < 360°$. Graph the coordinates on a polar grid.

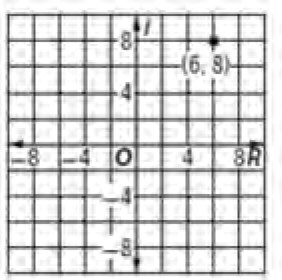
c. GRAPHICAL Graph the complex number $-3 + 3\boldsymbol{i}$ on a rectangular coordinate system.

d. GRAPHICAL Find polar coordinates for the complex number using the rectangular coordinates plotted in part **c** if $0 < \theta < 360°$. Graph the coordinates on a polar grid.

e. ANALYTICAL For a complex number $a + b\boldsymbol{i}$, find an expression for converting to polar coordinates.

SOLUTION:

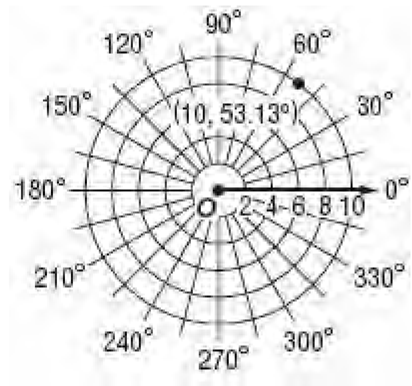
a. For $6 + 8\boldsymbol{i}$, $(a, b) = (6, 8)$. Plot the point (6, 8).



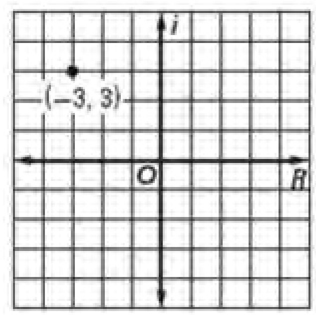
b. For (6, 8), $x = 6$ and $y = 8$. Since $x > 0$, use $\tan^{-1}\frac{y}{x}$ to find θ.

$$r = \sqrt{x^2 + y^2} \quad \text{Conversion formula} \quad \theta = \tan^{-1}\frac{y}{x}$$
$$= \sqrt{6^2 + 8^2} \quad x = 6 \text{ and } y = 8 \quad = \tan^{-1}\frac{8}{6}$$
$$= 10 \quad \text{Simplify} \quad \approx 53.13°$$

Polar coordinates for the point (6, 8) are (10, 53.13°).
Graph a point 10 units from the pole at a 53.13° angle with the polar axis.



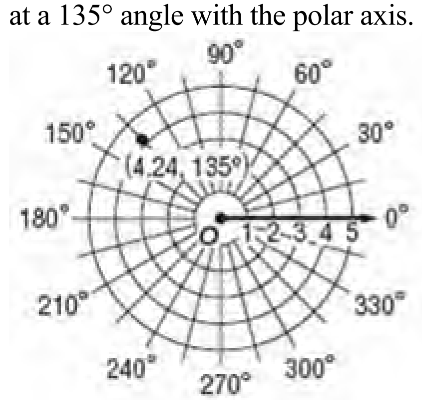
c. For $-3 + 3\boldsymbol{i}$, $(a, b) = (-3, 3)$. Plot the point $(-3, 3)$.



d. For $(-3, 3)$, $x = -3$ and $y = 3$. Since $x < 0$, use $\tan^{-1}\frac{y}{x} + 180°$ to find θ.

$$r = \sqrt{x^2 + y^2} \quad \text{Conversion formula} \quad \theta = \tan^{-1}\frac{y}{x} + 180°$$
$$= \sqrt{(-3)^2 + 3^2} \quad x = -3 \text{ and } y = 3 \quad = \tan^{-1}\frac{3}{-3} + 180°$$
$$= 3\sqrt{2} \text{ or about } 4.24 \quad \text{Simplify} \quad = 135°$$

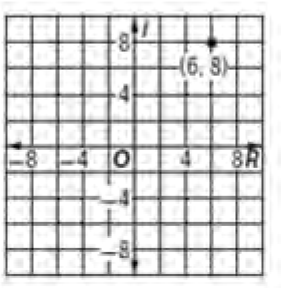
Polar coordinates for the point $(-3, 3)$ are (4.24, 135°). Graph a point about 4.24 units from the pole at a 135° angle with the polar axis.



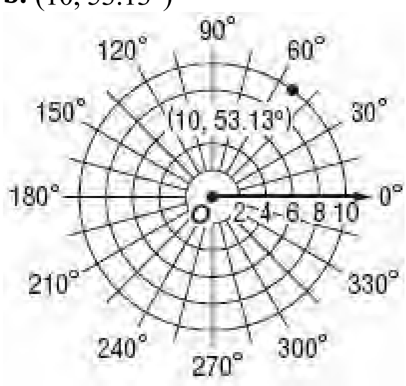
e. The complex number $a + b\boldsymbol{i}$ can be represented by the rectangular coordinates (x, y), where $a = x$ and $b = y$. To convert rectangular coordinates to polar coordinates, $r = \sqrt{x^2 + y^2}$, and $\theta = \tan^{-1}\frac{y}{x}$ when x is positive and $\theta = \tan^{-1}\frac{y}{x} + 180°$ when x is negative. Since $a = x$ and $b = y$, polar coordinates for the complex number $a + bi$ can be found using the expressions $r = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\frac{b}{a}$ when a is positive and $\theta = \tan^{-1}\frac{b}{a} + 180°$ when a is negative.

ANSWER:

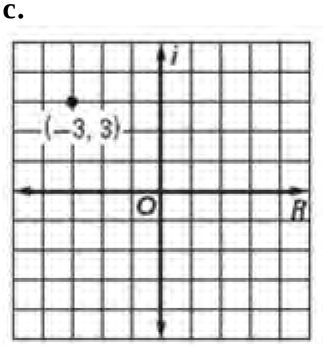
a.



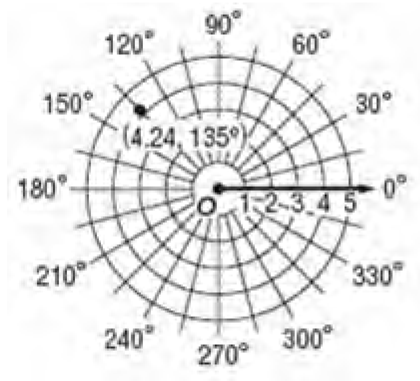
b. (10, 53.13°)



c.



d. (4.24, 135°)



e. $r = \sqrt{a^2 + b^2}$, $\theta = \tan^{-1}\frac{b}{a}$, when a is positive; $\theta = \tan^{-1}\frac{b}{a} + 180°$, when a is negative.

68. **ERROR ANALYSIS** Becky and Terrell are writing the polar equation $r = \sin\theta$ in rectangular form. Terrell believes that the answer is $x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$. Becky believes that the answer is simply $y = \sin x$. Is either of them correct? Explain your reasoning.

SOLUTION:

Convert the polar equation $r = \sin\theta$ to a rectangular equation.

$$r = \sin\theta$$
$$r^2 = r\sin\theta$$
$$x^2 + y^2 = y$$
$$x^2 + y^2 - y = 0$$
$$x^2 + y^2 - y + \frac{1}{4} = 0 + \frac{1}{4}$$
$$x^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{4}$$

Terrell is correct. Sample answer: Terrell used the proper substitutions, and the graph of his equation matches the original equation. Becky's answer is the sine function, which is not the same as the circle represented by the original polar function.

ANSWER:

Terrell; sample answer: Terrell used the proper substitutions. Becky's answer is the sine function, which is not the same as the circle represented by the original polar function.

69. **CHALLENGE** The equation for a circle is $r = 2a \cos \theta$. Write this equation in rectangular form. Find the center and radius of the circle.

SOLUTION:

$$\begin{aligned} r &= 2a\cos\theta \\ r^2 &= 2ar\cos\theta \\ x^2 + y^2 &= 2ax \\ x^2 - 2ax + y^2 &= 0 \\ x^2 - 2ax + \left(\frac{2a}{2}\right)^2 + y^2 &= 0 + \left(\frac{2a}{2}\right)^2 \\ (x-a)^2 + y^2 &= a^2 \end{aligned}$$

The rectangular form of $r = 2a \cos \theta$ is $(x - a)^2 + y^2 = a^2$. The circle has a radius of a and a center at $(a, 0)$.

ANSWER:

$(x - a)^2 + y^2 = a^2$, radius = a, center = $(a, 0)$

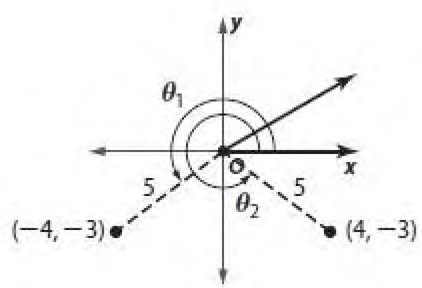
70. **REASONING** Given a set of rectangular coordinates (x, y) and a value for r, write expressions for finding θ in terms of sine and in terms of cosine. (*Hint*: You may have to write multiple expressions for each function, similar to the expressions given in this lesson using tangent.)

SOLUTION:

Start with $x = r \cos \theta$ and $y = r \sin \theta$ and solve for θ.

$$\begin{aligned} x &= r\cos\theta & y &= r\sin\theta \\ \frac{x}{r} &= \cos\theta & \frac{y}{r} &= \sin\theta \\ \cos^{-1}\frac{x}{r} &= \theta & \sin^{-1}\frac{y}{r} &= \theta \end{aligned}$$

Since the inverse cosine function is only defined on the interval [0, 180°], a second expression is needed for when y is negative. Consider the points $(-4, -3)$ and $(4, -3)$ with $r = 5$.

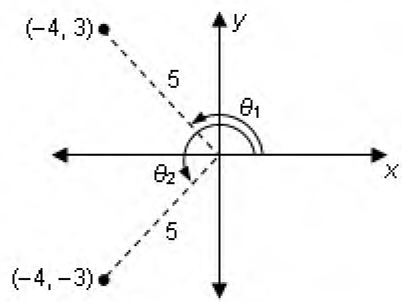


$$\begin{aligned} \theta_1 &= \cos^{-1}\frac{x}{r} & \theta_2 &= \cos^{-1}\frac{x}{r} \\ &= \cos^{-1}\frac{-4}{5} & &= \cos^{-1}\frac{4}{5} \\ &\approx 143 & &\approx 37 \end{aligned}$$

For $(-4, -3)$, $\theta_1 \approx 143°$. However, θ_1 is located in Quadrant III. To obtain the correct directed angle, subtract θ_1 from 360° to obtain 217°.

For $(4, -3)$, $\theta_2 \approx 37°$. However, θ_2 is located in Quadrant IV. To obtain the correct directed angle, subtract θ_2 from 360° to obtain 323°. Thus, $\theta = \cos^{-1}\frac{x}{r}$ when y is positive and $\theta = 2\pi - \cos^{-1}\frac{x}{r}$ or $\theta = 360° - \cos^{-1}\frac{x}{r}$ when y is negative.

Since the inverse sine function is only defined on the interval [90°, −90°], a second expression is needed for when x is negative. Consider the points $(-4, 3)$ and $(-4, -3)$ with $r = 5$.



$$\begin{aligned} \theta_1 &= \sin^{-1}\frac{y}{r} & \theta_2 &= \sin^{-1}\frac{y}{r} \\ &= \sin^{-1}\frac{3}{5} & &= \sin^{-1}\frac{-3}{5} \\ &\approx 37 & &\approx -37 \end{aligned}$$

For $(-4, 3)$, $\theta_1 \approx 37°$. However, θ_1 is located in Quadrant II. To obtain the correct directed angle, subtract θ_1 from 180° to obtain 143°. For $(-4, -3)$, $\theta_2 \approx -37°$. However, θ_2 is located in Quadrant III. To obtain the correct directed angle, subtract θ_2 from 180° to obtain 217°. Thus, $\theta = \sin^{-1}\frac{y}{r}$ when x is positive and $\theta = \pi - \sin^{-1}\frac{y}{r}$ or $\theta = 180° -$

$\sin^{-1}\frac{y}{r}$ when x is negative.

ANSWER:

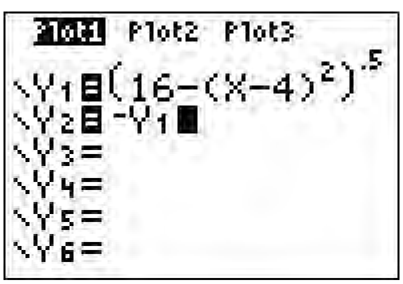
$\theta = \sin^{-1}\frac{y}{r}$, when x is positive; $\theta = \pi - \sin^{-1}\frac{y}{r}$ or $\theta = 180° - \sin^{-1}\frac{y}{r}$, when x is negative; $\theta = \cos^{-1}\frac{x}{r}$, when y is positive; $\theta = 2\pi - \cos^{-1}\frac{x}{r}$ or $\theta = 360° - \cos^{-1}\frac{x}{r}$, when y is negative.

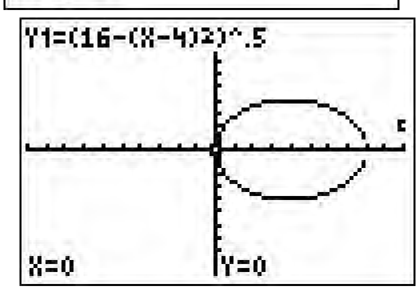
71. **WRITING IN MATH** Make a conjecture about when graphing an equation is made easier by representing the equation in polar form rather than rectangular form and vice versa.

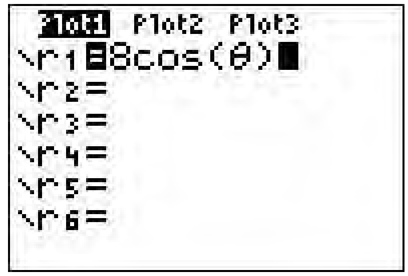
SOLUTION:

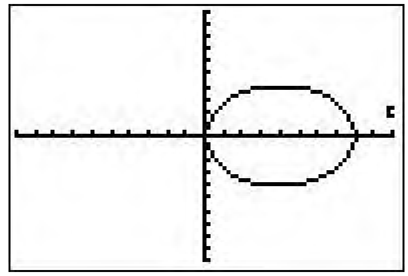
Rectangular equations that are not functions, such as equations representing ellipses or circles, are easier to graph in polar form. Equations that represent functions, such as linear functions, are easier to graph in rectangular form.

Consider the conic represented by $(x-4)^2 + y^2 = 16$ and $r = 8\cos\theta$. It is much easier to graph the equation in polar form than rectangular.

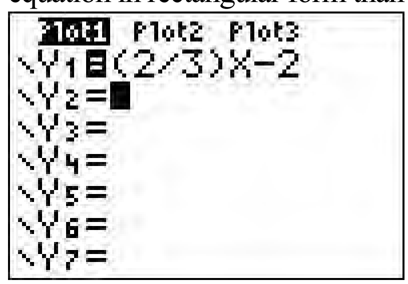


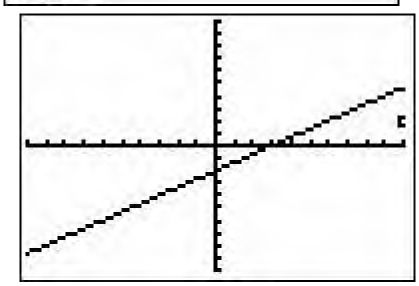


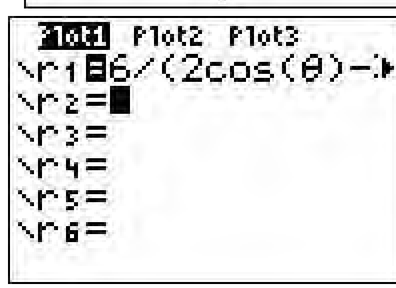


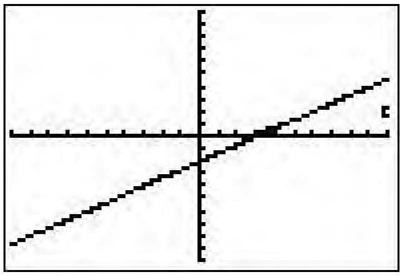


Consider the conic represented by $2x - 3y = 6$ and $r = \frac{6}{2\cos\theta - 3\sin\theta}$. It is much easier to graph the equation in rectangular form than polar.









ANSWER:

Sample answer: Rectangular equations that are not functions, such as equations representing ellipses or circles, are easier to graph in polar form. Equations that represent functions, such as linear functions, are easier to graph in rectangular form.

72. **PROOF** Use $x = r\cos\theta$ and $y = r\sin\theta$ to prove t $= x\sec\theta$ and $r = y\csc\theta$.

SOLUTION:

$x = r\cos\theta$	Conversion formula
$\frac{x}{\cos\theta} = r$	Divide each side by $\cos\theta$.
$x\sec\theta = r$	Reciprocal Identity
$y = r\sin\theta$	Conversion formula
$\frac{y}{\sin\theta} = r$	Divide each side by $\sin\theta$.
$y\csc\theta = r$	Reciprocal Identity

ANSWER:

$$\begin{aligned} x &= r\cos\theta \\ \frac{x}{\cos\theta} &= r \\ x\cdot\frac{1}{\cos\theta} &= r \\ x\sec\theta &= r \end{aligned}$$

$$\begin{aligned} y &= r\,\text{si} \\ \frac{y}{\sin\theta} &= r \\ y\cdot\frac{1}{\sin\theta} &= r \\ y\csc\theta &= r \end{aligned}$$

73. **CHALLENGE** Write $r^2(4\cos^2\theta + 3\sin^2\theta) + r(-8a\cos\theta + 6b\sin\theta) = 12 - 4a^2 - 3b^2$ in rectangular form. (*Hint*: Distribute before substituting values for r^2 and r. The rectangular equation should be a conic.)

SOLUTION:

$$\begin{aligned} r^2(4\cos^2\theta + 3\sin^2\theta) + r(-8a\cos\theta + 6b\sin\theta) &= 12 - 4a^2 - 3b^2 \\ 4r^2\cos^2\theta + 3r^2\sin^2\theta - 8ar\cos\theta + 6br\sin\theta &= 12 - 4a^2 - 3b^2 \\ 4(r\cos\theta)^2 + 3(r\sin\theta)^2 - 8a(r\cos\theta) + 6b(r\sin\theta) &= 12 - 4a^2 - 3b^2 \\ 4x^2 + 3y^2 - 8ax + 6by &= 12 - 4a^2 - 3b^2 \\ 4x^2 - 8ax + 4a^2 + 3y^2 + 6by + 3b^2 &= 12 \\ 4(x^2 - 2ax + a^2) + 3(y^2 + 2by + b^2) &= 12 \\ 4(x-a)^2 + 3(y+b)^2 &= 12 \\ \frac{(x-a)^2}{3} + \frac{(y+b)^2}{4} &= 1 \end{aligned}$$

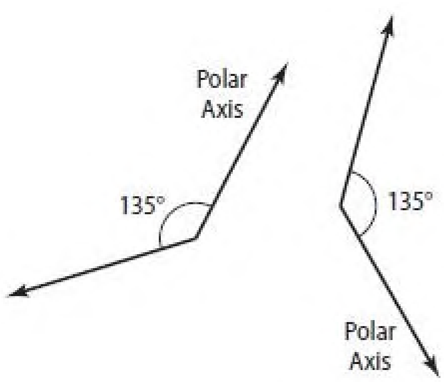
ANSWER:

$$\frac{(x-a)^2}{3} + \frac{(y+b)^2}{4} = 1$$

74. **WRITING IN MATH** Use the definition of a polar axis given in Lesson 9-1 to explain why it was necessary to state that the robot in Example 3 was facing due east. How can the use of quadrant bearings help to eliminate this?

SOLUTION:

Sample answer: When given an angle θ with polar coordinates, it is necessary to know the position of the polar axis. While the polar axis is usually a horizontal line drawn to the right, or due east, it can be drawn in any direction. Thus, an angle of 135° drawn relative to the polar axis can face any direction if the polar axis is not specified, as shown below.



This can then lead to an error if polar coordinates are to be converted to rectangular coordinates and the wrong polar axis is referenced. Since quadrant bearings are determined in relation to the directions north and south, they are universally understood. For example, 45° west of north will always be in the same position.

ANSWER:

Sample answer: When given an angle θ with polar coordinates, it is necessary to know the position of the polar axis. While the polar axis is usually a horizontal line drawn to the right, or due east, it can be drawn in any direction. Thus, an angle of 135° drawn relative to the polar axis can face any direction if the polar axis is not specified. This can then lead to an error if polar coordinates are to be converted to rectangular coordinates and the wrong polar axis is referenced. Since quadrant bearings are determined in relation to the directions north and south, they are universally understood. For example, 45° west of north will always be in the same position.

Use symmetry to graph each equation.

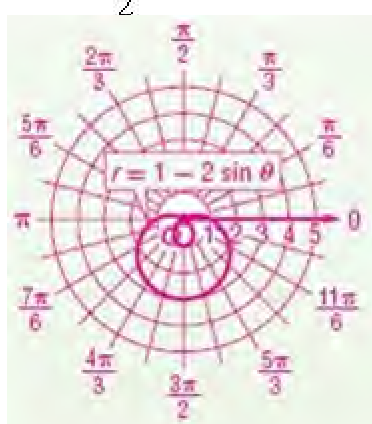
75. $r = 1 - 2 \sin \theta$

SOLUTION:

Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$. Therefore, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = 1 - 2 \sin \theta$
$-\frac{\pi}{2}$	−3
$-\frac{\pi}{3}$	2.74
$-\frac{\pi}{4}$	2.41
$-\frac{\pi}{6}$	3
0	1
$\frac{\pi}{6}$	0
$\frac{\pi}{4}$	−0.41
$\frac{\pi}{3}$	−0.73
$\frac{\pi}{2}$	−1

Use these points and symmetry with respect to the line $\theta = \frac{\pi}{2}$ to graph the function.



ANSWER:

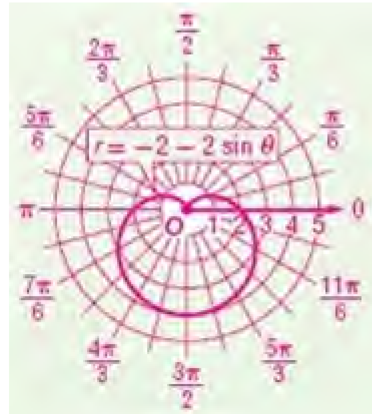
76. $r = -2 - 2 \sin \theta$

SOLUTION:

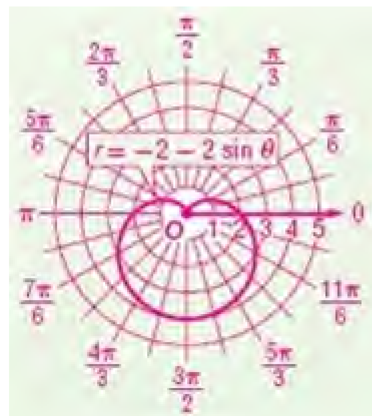
Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$. Therefore, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = -2 - 2 \sin \theta$
$-\frac{\pi}{2}$	0
$-\frac{\pi}{3}$	−0.27
$-\frac{\pi}{4}$	−0.59
$-\frac{\pi}{6}$	−1
0	−2
$\frac{\pi}{6}$	−3
$\frac{\pi}{4}$	−3.41
$\frac{\pi}{3}$	−3.72
$\frac{\pi}{2}$	−4

Use these points and symmetry with respect to the line $\theta = \frac{\pi}{2}$ to graph the function.



ANSWER:



77. $r = 2 \sin 3\theta$

SOLUTION:

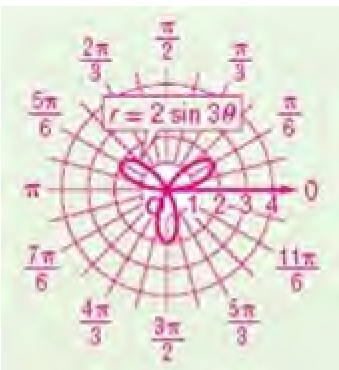
Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$. Therefore, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = 2 \sin 3\theta$
$-\frac{\pi}{2}$	2
$-\frac{\pi}{3}$	0
$-\frac{\pi}{4}$	−1.41
$-\frac{\pi}{6}$	−2
0	0
$\frac{\pi}{6}$	2
$\frac{\pi}{4}$	1.41
$\frac{\pi}{3}$	0

$\frac{\pi}{2}$	−2

Use these points and symmetry with respect to the line $\theta = \frac{\pi}{2}$ to graph the function.

ANSWER:



Find three different pairs of polar coordinates that name the given point if $-360° < \theta \le 360°$ or $-2\pi < \theta \le 2\pi$.

78. $T(1.5, 180°)$

SOLUTION:

For the point (1.5, 180°), the other three representations are as follows.

$$(1.5, 180°) = (1.5, 180° - 360°)$$
$$= (1.5, -180°)$$
$$(1.5, 180°) = (-1.5, 180° + 180°)$$
$$= (-1.5, 360°)$$
$$(1.5, 180°) = (-1.5, 180° - 180°)$$
$$= (-1.5, 0°)$$

ANSWER:

(1.5, −180°), (−1.5, 0°), (−1.5, 360°)

79. $U\left(-1, \frac{\pi}{3}\right)$

SOLUTION:

For the point $U\left(-1, \frac{\pi}{3}\right)$, the other three representations are as follows.

$$\left(-1, \frac{\pi}{3}\right) = \left(1, \frac{\pi}{3} - \pi\right)$$
$$= \left(1, -\frac{2\pi}{3}\right)$$
$$\left(-1, \frac{\pi}{3}\right) = \left(1, \frac{\pi}{3} + \pi\right)$$
$$= \left(1, \frac{4\pi}{3}\right)$$
$$\left(-1, \frac{\pi}{3}\right) = \left(-1, \frac{\pi}{3} - 2\pi\right)$$
$$= \left(-1, -\frac{5\pi}{3}\right)$$

ANSWER:

$\left(1, -\frac{2\pi}{3}\right), \left(1, \frac{4\pi}{3}\right), \left(-1, -\frac{5\pi}{3}\right)$

80. $V(4, 315°)$

SOLUTION:

For the point $V(4, 315°)$, the other three representations are as follows.

$$(4, 315°) = (4, 315° - 360°)$$
$$= (4, -45°)$$
$$(4, 315°) = (-4, 315° - 180°)$$
$$= (-4, 135°)$$
$$(4, 315°) = (-4, 315° - 540°)$$
$$= (-4, -225°)$$

ANSWER:

(4, −45°), (−4, 135°), (−4, −225°)

Find the angle θ between u and v to the nearest tenth of a degree.

81. $\mathbf{u} = \langle 6, -4\rangle$, $\mathbf{v} = \langle -5, -7\rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle 6, -4\rangle \cdot \langle -5, -7\rangle}{|\langle 6, -4\rangle||\langle -5, -7\rangle|}$$

$$\cos\theta = \frac{6(-5) + (-4)(-7)}{\sqrt{6^2 + (-4)^2}\sqrt{(-5)^2 + (-7)^2}}$$

$$\cos\theta = \frac{-30 + 28}{\sqrt{36 + 16}\sqrt{25 + 49}}$$

$$\cos\theta = \frac{-2}{\sqrt{52}\sqrt{74}}$$

$$\cos\theta = \frac{-2}{4\sqrt{962}}$$

$$\cos\theta = -\frac{1}{2\sqrt{962}}$$

$$\theta = \cos^{-1} - \frac{1}{2\sqrt{962}}$$

$$\theta \approx 91.8°$$

91.8°, not orthogonal

ANSWER:

91.8°, not orthogonal

82. $\mathbf{u} = \langle 2, 3\rangle$, $\mathbf{v} = \langle -9, 6\rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle 2, 3\rangle \cdot \langle -9, 6\rangle}{|\langle 2, 3\rangle||\langle -9, 6\rangle|}$$

$$\cos\theta = \frac{2(-9) + 3(6)}{\sqrt{2^2 + 3^2}\sqrt{(-9)^2 + 6^2}}$$

$$\cos\theta = \frac{-18 + 18}{\sqrt{4 + 9}\sqrt{81 + 36}}$$

$$\cos\theta = \frac{0}{\sqrt{13}\sqrt{127}}$$

$$\cos\theta = \frac{0}{\sqrt{1651}}$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1} 0$$

$$\theta = 90°$$

90°, orthogonal

ANSWER:

90°, orthogonal

83. **u** = $\langle 1, 10 \rangle$, **v** = $\langle 8, -2 \rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle 1, 10 \rangle \cdot \langle 8, -2 \rangle}{|\langle 1, 10 \rangle||\langle 8, -2 \rangle|}$$

$$\cos\theta = \frac{1(8) + 10(-2)}{\sqrt{1^2 + 10^2}\sqrt{8^2 + \left(-2^2\right)}}$$

$$\cos\theta = \frac{8 + (-20)}{\sqrt{1 + 100}\sqrt{64 + 4}}$$

$$\cos\theta = \frac{-12}{\sqrt{101}\sqrt{68}}$$

$$\cos\theta = \frac{-12}{2\sqrt{1717}}$$

$$\cos\theta = -\frac{6}{\sqrt{1717}}$$

$$\theta = \cos^{-1} - \frac{6}{\sqrt{1717}}$$

$$\theta \approx 98.3^\circ$$

98.3°, not orthogonal

ANSWER:

98.3°, not orthogonal

Write each pair of parametric equations in rectangular form. Then graph and state any restrictions on the domain.

84. $y = t + 6$ and $x = \sqrt{t}$

SOLUTION:

Solve for t in the parametric equation for y.

$$y = t + 6$$
$$y - 6 = t$$

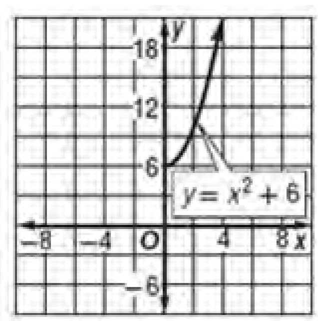
Substitute for t in the parametric equation for x.

$$x = \sqrt{t}$$
$$x = \sqrt{y - 6}$$
$$x^2 = y - 6$$
$$x^2 + 6 = y$$

Make a table of values to graph y with $x \geq 0$.

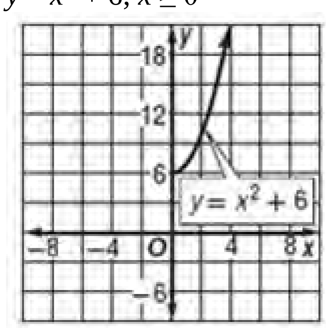
x	y
0	6
1	7
2	10
3	15
4	22

Plot the (x, y) coordinates and connect the points to form a smooth curve.

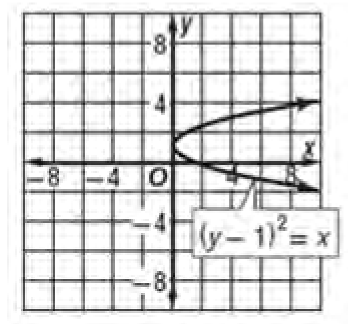


ANSWER:

$y = x^2 + 6$; $x \geq 0$



85. $y = \frac{t}{2} + 1$ and $x = \frac{t^2}{4}$



SOLUTION:

Solve for t in the parametric equation for y.

$$y = \frac{t}{2} + 1$$
$$y - 1 = \frac{t}{2}$$
$$2y - 2 = t$$

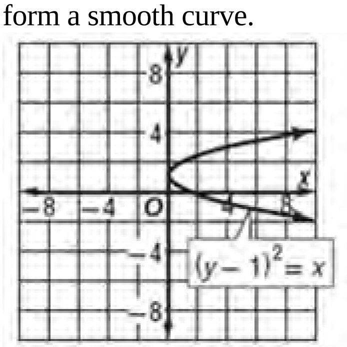
Substitute for t in the parametric equation for x.

$$x = \frac{t^2}{4}$$
$$x = \frac{(2y-2)^2}{4}$$
$$x = \frac{4y^2 - 8y - 4}{4}$$
$$x = y^2 - 2y - 1$$
$$x = (y-1)^2$$

Make a table of values to graph x.

x	y
4	–1
2.25	–0.5
1	0
0.25	0.5
0	1

Plot the (x, y) coordinates and connect the points to form a smooth curve.



ANSWER:

$(y - 1)^2 = x$

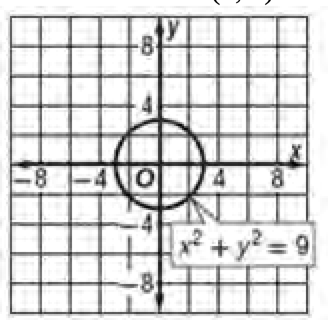
86. $y = -3 \sin t$ and $x = 3 \cos t$

SOLUTION:

Solve the equations for sin *t* and cos *t*. Then use a trigonometric identity.

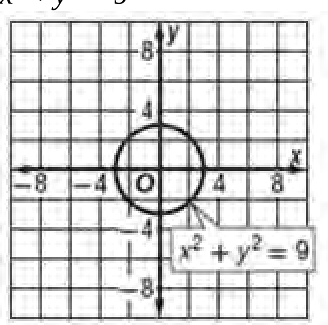
$$y = -3\sin t \qquad x = 3\cos t$$
$$-\frac{y}{3} = \sin t \qquad \frac{x}{3} = \cos t$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\left(-\frac{y}{3}\right)^2 + \left(\frac{x}{3}\right)^2 = 1$$
$$\frac{y^2}{9} + \frac{x^2}{9} = 1$$
$$y^2 + x^2 = 9$$

The parametric equations represent the graph of a circle with center (0, 0) and radius 3.

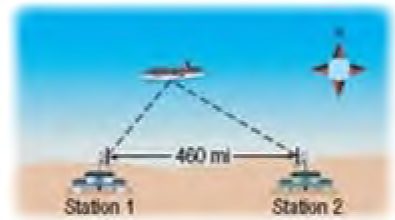


ANSWER:

$x^2 + y^2 = 9$



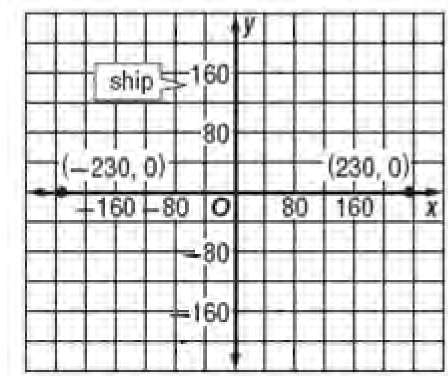
87. **NAVIGATION** Two LORAN broadcasting stations are located 460 miles apart. A ship receives signals from both stations and determines that it is 108 miles farther from Station 2 than Station 1.



a. Determine the equation of the hyperbola centered at the origin on which the ship is located.
b. Graph the equation, indicating on which branch of the hyperbola the ship is located.
c. Find the coordinates of the location of the ship on the coordinate grid if it is 110 miles from the *x*-axis.

SOLUTION:

a. First, place the two stations on a coordinate grid so that the origin is the midpoint of the segment between Station 1 and Station 2. The ship is located 108 miles farther from Station 2 than Station 1, and from the picture, the ship is located above the *x*-axis. Thus, the ship is located in the 2nd quadrant.



The two stations are located at the foci of the hyperbola, so *c* is 230. Recall that the absolute value of the difference of the distances from any point on a hyperbola to the foci is 2*a*. Because the ship is 108 miles farther from Station 2 than Station 1, 2*a* = 108 and *a* is 54. Use these values of *a* and *c* to find b^2.

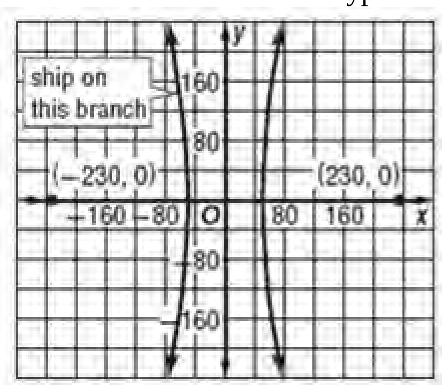
$$c^2 = a^2 + b^2$$
$$c^2 - a^2 = b^2$$
$$230^2 - 54^2 = b^2$$
$$49{,}984 = b^2$$

The transverse axis is horizontal and the center of the hyperbola is located at the origin, so the equation will be of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Substituting the values of a^2 and b^2, the equation for the hyperbola is $\frac{x^2}{2916} - \frac{y^2}{49,984} = 1$.

b. For $\frac{x^2}{2916} - \frac{y^2}{49,984} = 1$, $a = 54$, $c = 230$, $h = 0$, and $k = 0$.

center: $(h, k) = (0, 0)$
vertices: $(h \pm a, k) = (54, 0)$ and $(-54, 0)$
foci: $(h \pm c, k) = (230, 0)$ and $(-230, 0)$

Graph the center, vertices, and foci. Then make a table of values to sketch the hyperbola.



c. When the ship is 110 miles from the x-axis, then $y = 110$. Substitute $y = 110$ into the equation from part **a** and solve for x.

$$\frac{x^2}{2916} - \frac{y^2}{49,984} = 1$$

$$\frac{x^2}{2916} - \frac{110^2}{49,984} = 1$$

$$\frac{x^2}{2916} = 1 + \frac{110^2}{49,984}$$

$$x^2 = 2916\left(1 + \frac{110^2}{49,984}\right)$$

$$x = \pm\sqrt{2916\left(1 + \frac{110^2}{49,984}\right)}$$

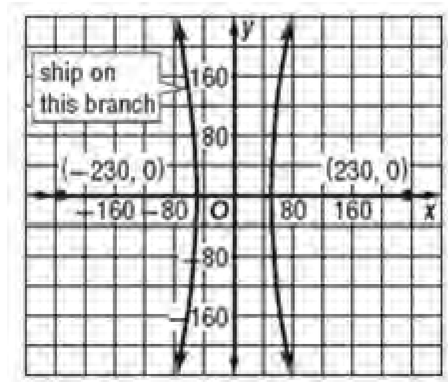
$$x \approx \pm 60.2$$

Since the ship is in the 2nd quadrant, the coordinates of the ship when it is 110 miles from the x-axis are $(-60.2, 110)$.

ANSWER:

a. $\frac{x^2}{2916} - \frac{y^2}{49,984} = 1$

b.



c. $(-60.2, 110)$

88. **BICYCLES** Woodland Bicycles makes two models of off-road bicycles: the Adventure, which sells for \$250, and the Grande Venture, which sells for \$350. Both models use the same frame. The painting and assembly time required for the Adventure is 2 hours, while the time is 3 hours for the Grande Venture. If there are 175 frames and 450 hours of labor available for production, how many of each model should be produced to maximize revenue? What is the maximum revenue?

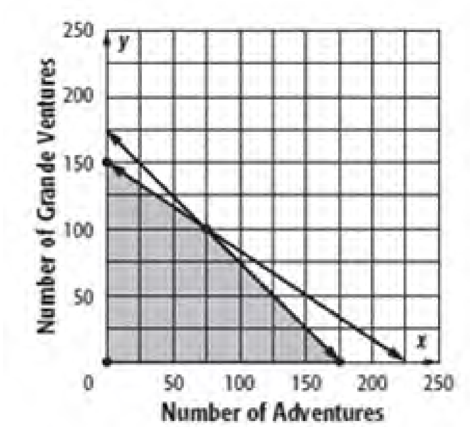
SOLUTION:

Let x represent the number of Adventures produced and y the number of Grande Ventures produced. The objective function is then given by $f(x, y) = 250x + 350y$.

The constraints are given by the following.
$x + y \le 175$ Frame constraint
$2x + 3y \le 450$ Production constraint

Because x and y cannot be negative, additional constraint are that $x \ge 0$ and $y \ge 0$.

Sketch a graph of the region determined by the constraints to find how many of each product should be produced to maximize revenue.



The shaded polygonal region has four vertex points at (0, 0), (0, 150), (75, 100), and (175, 0). Find the value of $f(x, y) = 250x + 350y$ at each of the four vertices.

$f(0, 0) = 250(0) + 350(0)$ or 0
$f(0, 150) = 250(0) + 350(150)$ or 52,500
$f(75, 100) = 250(75) + 350(100)$ or 53,750
$f(175, 0) = 250(175) + 350(0)$ or 43,750

Because f is greatest at (75, 100), 75 Adventures and 100 Grande Ventures should be produced. The maximum revenue is $53,750.

ANSWER:

75 Adventures, 100 Grande Ventures; $53,750

Solve each system of equations using Gauss-Jordan elimination.

89. $3x + 9y + 6z = 21$
$4x - 10y + 3z = 15$
$-5x + 12y - 2z = -6$

SOLUTION:

Write the augmented matrix.

$$\left[\begin{array}{ccc|c} 3 & 9 & 6 & 21 \\ 4 & -10 & 3 & 15 \\ -5 & 12 & -2 & -6 \end{array}\right]$$

Apply elementary row operations to obtain reduced row-echelon form.

$$\left[\begin{array}{ccc|c} 3 & 9 & 6 & 21 \\ 4 & -10 & 3 & 15 \\ -5 & 12 & -2 & -6 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 7 \\ 4 & -10 & 3 & 15 \\ -5 & 12 & -2 & -6 \end{array}\right] \leftarrow R_1 \div (3)$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 7 \\ 4 & -10 & 3 & 15 \\ 0 & 27 & 8 & 29 \end{array}\right] \leftarrow R_3 + 5R_1$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 7 \\ 0 & -22 & -5 & -13 \\ 0 & 27 & 8 & 29 \end{array}\right] \leftarrow R_2 - 4R_1$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 7 \\ 0 & 1 & \frac{5}{22} & \frac{13}{22} \\ 0 & 27 & 8 & 29 \end{array}\right] \leftarrow R_2 \div (-22)$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 7 \\ 0 & 1 & \frac{5}{22} & \frac{13}{22} \\ 0 & 0 & \frac{41}{22} & \frac{287}{22} \end{array}\right] \leftarrow R_3 - 27R_2$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 7 \\ 0 & 1 & \frac{5}{22} & \frac{13}{22} \\ 0 & 0 & 1 & 7 \end{array}\right] \leftarrow R_3 \div (41/22)$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 7 \end{array}\right] \leftarrow R_2 - (5/22)R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 10 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 7 \end{array}\right] \leftarrow R_1 - 3R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 7 \end{array}\right] \leftarrow R_1 - 2R_3$$

ANSWER:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 7 \end{array}\right]$$

90. $x + 5y - 3z = -14$
$2x - 4y + 5z = 18$
$-7x - 6y - 2z = 1$

SOLUTION:

Write the augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 5 & -3 & -14 \\ 2 & -4 & 5 & 18 \\ -7 & -6 & -2 & 1 \end{array}\right]$$

Apply elementary row operations to obtain reduced row-echelon form.

$$\left[\begin{array}{ccc|c} 1 & 5 & -3 & -14 \\ 2 & -4 & 5 & 18 \\ -7 & -6 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 5 & -3 & -14 \\ 0 & -14 & 11 & 46 \\ -7 & -6 & -2 & 1 \end{array}\right] \leftarrow R_2 - 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & 5 & -3 & -14 \\ 0 & -14 & 11 & 46 \\ 0 & 29 & -23 & -97 \end{array}\right] \leftarrow R_3 + 7R_1$$

$$\left[\begin{array}{ccc|c} 1 & 5 & -3 & -14 \\ 0 & 1 & -\frac{11}{14} & \frac{23}{7} \\ 0 & 29 & -23 & -97 \end{array}\right] \leftarrow R_2 \div (-14)$$

$$\left[\begin{array}{ccc|c} 1 & 5 & -3 & -14 \\ 0 & 1 & -\frac{11}{14} & \frac{23}{7} \\ 0 & 0 & -\frac{3}{14} & -\frac{12}{7} \end{array}\right] \leftarrow R_3 - 29R_2$$

$$\left[\begin{array}{ccc|c} 1 & 5 & -3 & -14 \\ 0 & 1 & -\frac{11}{14} & \frac{23}{7} \\ 0 & 0 & 1 & 8 \end{array}\right] \leftarrow R_3 \div (-3/14)$$

$$\left[\begin{array}{ccc|c} 1 & 5 & -3 & -14 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 8 \end{array}\right] \leftarrow R_2 + (11/14)R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -29 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 8 \end{array}\right] \leftarrow R_1 - 5R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 8 \end{array}\right] \leftarrow R_1 + 3R_3$$

ANSWER:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 8 \end{array}\right]$$

91. $2x - 4y + z = 20$
$5x + 2y - 2z = -4$
$6x + 3y + 5z = 23$

9-3 Polar and Rectangular Forms of Equations

SOLUTION:

Write the augmented matrix.

$$\left[\begin{array}{ccc|c} 2 & -4 & 1 & 20 \\ 5 & 2 & -2 & -4 \\ 6 & 3 & 5 & 23 \end{array}\right]$$

Apply elementary row operations to obtain reduced row-echelon form.

$$\left[\begin{array}{ccc|c} 2 & -4 & 1 & 20 \\ 5 & 2 & -2 & -4 \\ 6 & 3 & 5 & 23 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0.5 & 10 \\ 5 & 2 & -2 & -4 \\ 6 & 3 & 5 & 23 \end{array}\right] \leftarrow R_1 \div (2)$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0.5 & 10 \\ 0 & 12 & -4.5 & -54 \\ 6 & 3 & 5 & 23 \end{array}\right] \leftarrow R_2 - 5R_1$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0.5 & 10 \\ 0 & 12 & -4.5 & -54 \\ 0 & 15 & 2 & -37 \end{array}\right] \leftarrow R_3 - 6R_1$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0.5 & 10 \\ 0 & 1 & -\frac{3}{8} & -\frac{9}{2} \\ 0 & 15 & 2 & -37 \end{array}\right] \leftarrow R_2 \div (12)$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0.5 & 10 \\ 0 & 1 & -\frac{3}{8} & -\frac{9}{2} \\ 0 & 0 & \frac{61}{8} & \frac{61}{2} \end{array}\right] \leftarrow R_3 - 15R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0.5 & 10 \\ 0 & 1 & -\frac{3}{8} & -\frac{9}{2} \\ 0 & 0 & 1 & 4 \end{array}\right] \leftarrow R_3 \div (61/8)$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0.5 & 10 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right] \leftarrow R_2 + (3/8)R_3$$

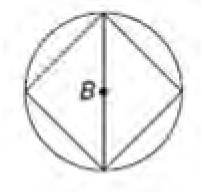
$$\left[\begin{array}{ccc|c} 1 & 0 & 0.5 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right] \leftarrow R_1 + 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right] \leftarrow R_1 - 0.5R_3$$

ANSWER:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

92. **SAT/ACT** A square is inscribed in circle *B*. If the circumference of the circle is 50π, what is the length of the diagonal of the square?



A $10\sqrt{2}$
B 25
C $25\sqrt{2}$
D 50
E $50\sqrt{2}$

SOLUTION:

The circumference of a circle is π*d*, where *d* is the circle's diameter.

$\pi d = 50\pi$

$d = 50$

The diagonal is the diameter of the circle or 50 units. Choice D is correct.

ANSWER:

D

93. **REVIEW** Which of the following could be an equation for a rose with three petals?
F $r = 3\sin\theta$
G $r = \sin 3\theta$
H $r = 6\sin\theta$
J $r = \sin 6\theta$

SOLUTION:

The general form of a rose is $r = a\cos n\theta$ or $r = a\sin n\theta$ where $n \geq 2$ is an integer. For an odd number of x petals, $n = x$. Therefore, for a rose with 3 petals, $n = 3$.

In Choice F, $r = 3\sin\theta$ has the form $r = a\sin n\theta$, but $n = 1$.

In Choice G, $r = \sin 3\theta$ has the form $r = a\sin n\theta$, and $n = 3$.

In Choice H, $r = 6\sin\theta$ has the form $r = a\sin n\theta$, but $n = 1$.

In Choice J, $r = \sin 6\theta$ has the form $r = a\sin n\theta$, but $n = 6$.

Choice G is correct.

ANSWER:

G

94. What is the polar form of $x^2 + (y - 2)^2 = 4$?

A $r = \sin\theta$
B $r = 2\sin\theta$
C $r = 4\sin\theta$
D $r = 8\sin\theta$

SOLUTION:

The graph of $(x - 2)^2 + y^2 = 4$ is a circle with radius 2 centered at (0, 2). To find the polar form of this equation, replace x with $r\cos\theta$ and y with $r\sin\theta$. Then simplify.

$$x^2 + (y-2)^2 = 4$$
$$(r\cos\theta)^2 + (r\sin\theta - 2)^2 = 4$$
$$r^2\cos^2\theta + r^2\sin^2\theta - 4r\sin\theta + 4 = 4$$
$$r^2\cos^2\theta + r^2\sin^2\theta - 4r\sin\theta = 0$$
$$r^2\cos^2\theta + r^2\sin^2\theta = 4r\sin\theta$$
$$r^2(\cos^2\theta + \sin^2\theta) = 4r\sin\theta$$
$$r^2(1) = 4r\sin\theta$$
$$r = 4\sin\theta$$

Choice C is correct.

ANSWER:

C

95. **REVIEW** Which of the following could be an equation for a spiral of Archimedes that passes through $A\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$?

F $r = \frac{\sqrt{2}\pi}{2}\cos\theta$
G $r = \theta$
H $r = \frac{3}{4}$
J $r = \frac{\theta}{2}$

SOLUTION:

The general form of a spiral of Archimedes is $r = a\theta + b$. If $b = 0$, the $r = a\theta$. If $A\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ is a point on this spiral, then Substitute theses values for r and θ in the general form of the equation and solve for a.

$$r = a\theta$$
$$\frac{\pi}{4} = a\left(\frac{\pi}{2}\right)$$
$$\frac{\pi}{4}\cdot\frac{2}{\pi} = a$$
$$\frac{1}{2} = a$$

Therefore, a spiral of Archimedes that contains the point $A\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ is $r = \frac{1}{2}\theta$ or $r = \frac{\theta}{2}$. Choice J is correct.

ANSWER:

J