13-2 Probability with Permutations and Combinations

1. **GEOMETRY** Five students are asked to randomly select and name a polygon from the group shown below. What is the probability that the first two students choose the triangle and quadrilateral, in that order?

SOLUTION:

Find the total number of outcomes. There are 5 polygons. Therefore, 5 polygons can be chosen in 5! or 120 ways.

Find the number of favorable outcomes. This is the number of permutations of the other polygons if the triangle is chosen first and the quadrilateral is chosen second: (5 – 2)! or 3!.

Calculate the probability.

$$P(\text{triangle, then quadrilateral}) = \frac{(5-2)!}{5!} = \frac{3!}{5!} = \frac{6}{120} = \frac{1}{20}$$

The probability that the first two students choose the triangle and then the quadrilateral is $\frac{1}{20}$.

ANSWER:

$\frac{1}{20}$

2. **PLAYS** A high school performs a production of *A Raisin in the Sun* with each freshman English class of 18 students. If the three members of the crew are decided at random, what is the probability that Chase is selected for lighting, Jaden is selected for props, and Emelina for spotlighting?

SOLUTION:

Since choosing students for the particular roles is a way of ranking the members, order in this situation is important. The number of possible outcomes in the sample space is the number of permutations of 18 people taken 3 at a time, ${}_{15}P_3$.

$${}_{18}P_3 = \frac{18!}{(18-3)!} = \frac{18 \cdot 17 \cdot 16 \cdot 15!}{15!} = 4896$$

Among these, there is only one particular arrangement in which Chase is selected for lighting, Jaden is selected for props, and Emelina for spotlighting. Therefore, the probability is $\frac{1}{4896}$.

ANSWER:

$\frac{1}{4896}$

3. **DRIVING** What is the probability that a license plate using the letters C, F, and F and numbers 3, 3, 3, and 1 will be CFF3133?

SOLUTION:

There are 3 letters and 4 digits This is a permutation with repetition.

There are a total of 7 objects in which ***F*** appears twice and ***3*** appears three times.

The number of distinguishable permutations is

$$\frac{7!}{2! \cdot 3!} = 420.$$

The total number of possible outcomes is 420 and there is only one favorable outcome which is CFF3133. Therefore, the probability is $\frac{1}{420}$.

ANSWER:

$\frac{1}{420}$

4. **CHEMISTRY** In chemistry lab, you need to test six samples that are randomly arranged on a circular tray.

a. What is the probability that the arrangement shown is produced?
b. What is the probability that test tube 2 will be in the top middle position?

SOLUTION:

a. The number of distinguishable permutations of n objects arranged in a circle with no fixed reference point is $\frac{n!}{n} = (n-1)!$. The 6 samples can be arranged in 5! = 120 different ways. There is only one favorable outcome as shown. Therefore, the probability is $\frac{1}{120}$.

b. Since the samples are arranged with a fixed reference point (the top middle position), this is a linear permutation. So there are 6! or 720 ways in which the samples can be arranged. The number of favorable outcomes is the number of permutations of the other 5 samples given that the test tube 2 will be in the top middle position, 5! or 120. Therefore, the probability is $\frac{120}{720} = \frac{1}{6}$.

ANSWER:

a. $\frac{1}{120}$

b. $\frac{1}{6}$

5. Five hundred boys, including Josh and Sokka, entered a drawing for two football game tickets. What is the probability that the tickets were won by Josh and Sokka?

SOLUTION:

Two students are chosen out of 500. The order in which the two students are chosen does not matter, so the total number of outcomes is:

$${}_{500}C_2 = \frac{500!}{(500-2)!2!}$$
$$= 124,750$$

There is only 1 favorable outcome, so the probability is

$\frac{1}{124,750}$

ANSWER:

$\frac{1}{124,750}$

6. **CONCERTS** Nia and Chad are going to a concert with their high school's key club. If they choose a seat on the row below at random, what is the probability that Chad will be in seat C11 and Nia will be in C12?

SOLUTION:

Since choosing seats is a way of arranging the students, order in this situation is important. There are 12 seats. The number of possible outcomes in the sample space is the number of permutations of 12 seats taken 2 at a time, ${}_{12}P_2$.

$${}_{12}P_2 = \frac{12!}{(12-2)!} = \frac{12 \cdot 11 \cdot 10!}{10!} = 132$$

Among these, there is only one particular arrangement in which Chad will be in seat C11 and Nia will be in C12. Therefore, the probability is $\frac{1}{132}$.

ANSWER:

$\frac{1}{132}$

7. **FAIRS** Alfonso and Colin each bought one raffle ticket at the state fair. If 50 tickets were randomly sold, what is the probability that Alfonso got ticket 14 and Colin got ticket 23?

SOLUTION:

Since buying tickets is a way of arranging the people, order in this situation is important. There are 50 tickets. The number of possible outcomes in the sample space is the number of permutations of 50 seats taken 2 at a time, ${}_{50}P_2$.

$$ {}_{50}P_2 = \frac{50!}{(50-2)!} = \frac{50 \cdot 49 \cdot 48!}{48!} = 2450 $$

Among these, there is only one particular arrangement in which Alfonso gets ticket 14 and Colin gets ticket 23. Therefore, the probability is $\frac{1}{2450}$.

ANSWER:

$\frac{1}{2450}$

8. **CCSS MODELING** The table shows the finalists for a floor exercises competition. The order in which they will perform will be chosen randomly.

Floor Exercises Finalists
Eliza Hernandez
Kimi Kanazawa
Cecilia Long
Annie Montgomery
Shenice Malone
Caroline Smith
Jessica Watson

a. What is the probability that Cecilia, Annie, and Kimi are the first 3 gymnasts to perform, in any order?

b. What is the probability that Cecilia is first, Annie is second, and Kimi is third?

SOLUTION:

a. The number of possible outcomes is the number of arrangements of 7 performers taken 7 at a time. So, the number of possible outcomes is 7! = 5040.

The first 3 finalists can be arranged in 3! = 6 ways. The rest of the finalists can be arranged 4! ways.

Therefore, the number of favorable outcomes is(4!)(3!)= (24)(6) = 144.

The probability is $\frac{144}{5040} = \frac{1}{35}$.

b. The number of possible outcomes is 5040. The number of favorable outcomes with Cecilia in first place, Annie second, and Kimi in third place is 1. After the first three places are set, the rest of the finalists can be arranged 4! ways.

Therefore, the probability is $\frac{24}{5040} = \frac{1}{210}$.

ANSWER:

a. $\frac{1}{35}$

b. $\frac{1}{210}$

9. **JOBS** A store randomly assigns their employees work identification numbers to track productivity. Each number consists of 5 digits ranging from 1–9. If the digits cannot repeat, find the probability that a randomly generated number is 25938.

SOLUTION:

The number of possible outcomes is the number of arrangements of 9 digits taken 5 at a time, ${}_9P_5$.

$${}_9P_5 = \frac{9!}{(9-5)!} = \frac{9\cdot8\cdot7\cdot6\cdot5\cdot4!}{4!} = 15,120$$

The only favorable outcome is the arrangement 25398. Therefore, the probability is $\frac{1}{15,120}$.

ANSWER:

$\frac{1}{15,120}$

10. **GROUPS** Two people are chosen randomly from a group of ten. What is the probability that Jimmy was selected first and George second?

SOLUTION:

Since choosing people is a way of ranking the members, order in this situation is important. The number of possible outcomes in the sample space is the number of permutations of 10 people taken 2 at a time, ${}_{10}P_2$.

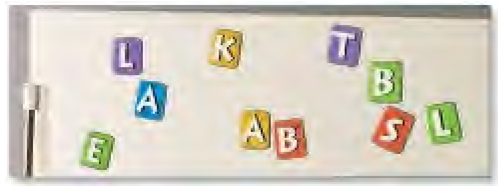
$${}_{18}P_3 = \frac{10!}{(10-2)!} = \frac{10\cdot9\cdot8!}{8!} = 90$$

Among these, there is only one particular arrangement in which Jimmy is selected first and George second. Therefore, the probability is $\frac{1}{90}$.

ANSWER:

$\frac{1}{90}$ or about 1%

11. **MAGNETS** Santiago bought some letter magnets that he can arrange to form words on his fridge. If he randomly selected a permutation of the letters shown below, what is the probability that they would form the word BASKETBALL?



SOLUTION:

There are 10 letters in which A, B, and L each appear twice.

The number of distinguishable permutations is

$$\frac{10!}{2!\cdot2!\cdot2!} = 453,600.$$

The total number of possible outcomes is 453,600 and there is only one favorable outcome which is BASKETBALL. Therefore, the probability is

$\frac{1}{453,600}$.

ANSWER:

$\frac{1}{453,600}$

12. **ZIP CODES** What is the probability that a zip code randomly generated from among the digits 3, 7, 3, 9, 5, 7, 2, and 3 is the number 39372?

SOLUTION:

There are 8 digits in which 3 appears three times and 7 appears twice.

The number of distinguishable permutations is

$$\frac{8!}{2!\cdot3!} = 3360.$$

The total number of possible outcomes is 3360 and there is only one favorable outcome which is 39372. Therefore, the probability is $\frac{1}{3360}$.

ANSWER:

$\frac{1}{3360}$

13. **GROUPS** Keith is randomly arranging desks into circles for group activities. If there are 7 desks in his circle, what is the probability that Keith will be in the desk closest to the door?

SOLUTION:

Since the people are seated with a fixed reference point, this is a linear permutation. So there are 7! or 5040 ways in which the people can be seated. The number of favorable outcomes is the number of permutations of the other 6 people given that Keith will be in the desk closest to the door, 6! or 720.

Therefore, the probability is $\frac{720}{5040} = \frac{1}{7}$.

ANSWER:

$\frac{1}{7}$

14. **AMUSEMENT PARKS** Sylvie is at an amusement park with her friends. They go on a ride that has bucket seats in a circle. If there are 8 seats, what is the probability that Sylvie will be in the seat farthest from the entrance to the ride?

SOLUTION:

Since the people are seated with a fixed reference point, this is a linear permutation. So there are 8! or 40,320 ways in which the people can be seated. The number of favorable outcomes is the number of permutations of the other 7 people given that Sylvie will be in the seat farthest from the entrance to the ride, 7! or 5040.

Therefore, the probability is $\frac{5040}{40,320} = \frac{1}{8}$.

ANSWER:

$\frac{1}{8}$

15. **PHOTOGRAPHY** If you are randomly placing 24 photos in a photo album and you can place four photos on the first page, what is the probability that you choose the photos shown?

SOLUTION:

We are choosing 4 photos from a set of 24 photos, and the 4 chosen photos can be placed in any order. So, the number of possible outcomes is ${}_{24}C_4$.

$${}_{24}C_4 = \frac{24!}{(24-4)!(4!)} = \frac{24 \cdot 23 \cdot 22 \cdot 21 \cdot 20!}{20!(4 \cdot 3 \cdot 2 \cdot 1)} = 10,626$$

There is only one favorable outcome. Therefore, the probability is $\frac{1}{10,626}$.

ANSWER:

$\frac{1}{10,626}$

16. **ROAD TRIPS** Rita is going on a road trip across the U.S. She needs to choose from 15 cities where she will stay for one night. If she randomly pulls 3 city brochures from a pile of 15, what is the probability that she chooses Austin, Cheyenne, and Savannah?

SOLUTION:

We are choosing 3 cities from a set of 15 cities and order is not important. So, the number of possible outcomes is ${}_{15}C_3$.

$${}_{15}C_3 = \frac{15!}{(15-3)!(3!)} = \frac{15 \cdot 14 \cdot 13 \cdot 12!}{12!(3 \cdot 2 \cdot 1)} = 455$$

The number of favorable outcomes is only one of choosing the cities Austin, Cheyenne, and Savannah.

Therefore, the probability is $\frac{1}{455}$.

ANSWER:

$\frac{1}{455}$

17. **CCSS SENSE-MAKING** Use the figure below.

Assume that the balls are aligned at random.

a. What is the probability that in a row of 8 pool balls, the solid 2 and striped 11 would be first and second from the left?

b. What is the probability that if the 8 pool balls were mixed up at random, they would end up in the order shown?

c. What is the probability that in a row of seven balls, with three 8 balls, three 9 balls, and one 6 ball, the three 8 balls would be to the left of the 6 ball and the three 9 balls would be on the right?

d. If the balls were randomly rearranged and formed a circle, what is the probability that the 6 ball is next to the 7 ball?

SOLUTION:

a. The number of possible outcomes in the sample space is the number of permutations of 8 balls taken 8 at a time, 8! = 40,320.

The number of favorable outcomes is the number of permutations of the remaining 6 balls after fixing the solid 2 and striped 11 at the first and second from the left. 6! = 720

Therefore, the probability is $\frac{720}{40320} = \frac{1}{56}$.

b. The number of possible outcomes is 40,320 and the number of favorable outcome is the only one as shown. Therefore, the probability is $\frac{1}{40,320}$.

c. The required probability is to the probability of getting an arrangement of (8886999) from three 8 balls, three 9 balls and one 6 ball. So, the number of distinguishable permutations is $\frac{7!}{3!\cdot 3!} = 140$.

There is only one favorable outcome which is (8886999). Therefore, the probability is $\frac{1}{140}$.

d. Eight balls are arranged in the form of a circle with one fixed ball (7 ball). So, the circular permutation of arranging 8 balls with 1 fixed ball is 7!.

The 6 ball can be arranged either to the left of ball 7 or right of ball 7. So the circular permutation of arranging 8 balls with 2 fixed balls is $2 \cdot 6!$.

$$P(\text{6 ball is next to 7 ball}) = \frac{2 \cdot 6!}{7!} = \frac{2}{7}$$

ANSWER:

a. $\frac{1}{56}$

b. $\frac{1}{40,320}$

c. $\frac{1}{140}$

d. $\frac{2}{7}$

18. How many lines are determined by 10 randomly selected points, no 3 of which are collinear? Explain your calculation.

SOLUTION:

Two points determine a line. If no 3 points are collinear, then each line will consist of *exactly* 2 points. Since there are 10 total points, the number of lines is the combination of 10 points taken 2 at a time, which is $\frac{10!}{8!2!}$ or 45.

ANSWER:

45; Sample answer: The number of lines is the combination of 10 objects taken 2 at a time, which is $\frac{10!}{8!2!}$ or 45.

19. Suppose 7 points on a circle are chosen at random.

a. Using the letters A through E, how many ways can the points on the circle be named?
b. If one point on the circle is fixed, how many arrangements are possible?

SOLUTION:

a. The number of distinguishable permutations of *n* objects arranged in a circle with no fixed reference point is $\frac{n!}{n} = (n-1)!$. The 7 points can be named in a circular way in 6! = 720 different ways.
b. Since the points are named with a fixed reference point, this is a linear permutation. There are 7! or 5040 ways in which the points can be named.

ANSWER:

a. 720
b. 5040

20. **RIDES** A carousel has 7 horses and one bench seat that will hold two people. One of the horses does not move up or down.

a. How many ways can the seats on the carousel be randomly filled by 9 people?
b. If the carousel is filled randomly, what is the probability that you and your friend will end up in the bench seat?
c. If 6 of the 9 people randomly filling the carousel are under the age of 8, what is the probability that a person under the age of 8 will end up on the one horse that does not move up or down?

SOLUTION:

a. The number of ways in which the 9 seats can be filled is the permutation of 9 people taken 9 at a time. So, the number of ways is 9! = 362,880.

b. The total number of possible outcomes is 362,880. Since the carousel is filled with a fixed reference point, this is a linear permutation. So there are 7! or 5040 ways to arrange the remaining 7 people after you and your friend are placed in the bench seat. Also, for each arrangement there is an alternate arrangement by interchanging the position of you and your friend. Therefore, the probability is $\frac{2(5040)}{362,880} = \frac{1}{36}$.

c. The total number of arrangements of the 9 people is $\frac{9!}{6!}$ or 504. The number of favorable outcomes is the number of distinguishable permutations of the other eight places if a child under 8 is on the horse that does not move up or down: $\frac{8!}{5!}$ or 336.
Calculate the probability.
$P(\text{child under 8 on broken horse}) = \frac{336}{504}$ or $\frac{2}{3}$

ANSWER:

a. 362,880
b. $\frac{1}{36}$
c. $\frac{2}{3}$

13-2 Probability with Permutations and Combinations

21. **LICENSES** A camera positioned above a traffic light photographs cars that fail to stop at a red light. In one unclear photograph, an officer could see that the first letter of the license plate was a Q, the second letter was an M or an N and the third letter was a B, P, or D. The first number was a 0, but the last two numbers were illegible. How many possible license plates fit this description?

SOLUTION:

By the Fundamental Counting Principle the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

The first letter of the license plate is Q.
There are 2 choices for the second letter.
There are 3 choices for the third letter.
The first number is 0
There are 10 choices for the second digit.
There are 10 choices for the third digit.

Therefore, the total number of choices is $1 \times 2 \times 3 \times 1 \times 10 \times 10 = 600$.

ANSWER:

600

22. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate permutations.

a. NUMERICAL Randomly select three digits from 0 to 9. Find the possible permutations of the three integers.

b. TABULAR Repeat part a for four additional sets of three integers. You will use some digits more than once. Copy and complete the table below.

Integers	Permutations	Average of Permutations	Average of Permutations ÷ 37
1, 4, 7	147, 174, 417, 471, 714, 741	444	12

c. VERBAL Make a conjecture about the value of the average of the permutations of three digits between 0 and 9.

d. SYMBOLIC If the three digits are x, y, and z, is it possible to write an equation for the average A of the permutations of the digits? If so, write the equation. If not, explain why not.

SOLUTION:

a. Sample answer: 1, 4, 7; permutations 147, 174, 417, 471, 714, 741 (each permutation will be a unique listing of the numbers.)

b. Sample Answer:
There are 6 permutations for each set of numbers. Find the sum of each set and then divide by 6 to find the average of the permutations. Finally, divide by 37.

Integers	Permutations	Average of Permutations	Average of Permutations ÷ 37
1, 4, 7	147, 174, 417, 471, 714, 741	444	12
2, 3, 5	235, 253, 325, 352, 523, 532	370	10
6, 8, 9	689, 698, 869, 896, 968, 986	851	23
1, 3, 5	135, 153, 315, 351, 513, 531	333	9
0, 4, 6	046, 064, 406, 460, 604, 640	370	10

c. Analyze the table to see if there are any patterns. The last column is the strange column. Why are we asking to divide by 37? Look at those numbers more closely.

$2 + 3 + 5 = 10$
$6 + 8 + 9 = 23$
$1 + 3 + 5 = 9$
$0 + 4 + 6 = 10$

It seems that the sum of the values in each set of numbers multiplied by 37 is equal to the average of all of the possible permutations.

d. The average of the permutations of three digits between 0 and 9 is the sum of the digits multiplied by 37. So, the average $A = 37(x + y + z)$.

ANSWER:

a. Sample answer: 1, 4, 7; permutations 147, 174, 417, 471, 714, 741

b. Sample Answer:

Integers	Permutations	Average of Permutations	Average of Permutations ÷ 37
1, 4, 7	147, 174, 417, 471, 714, 741	444	12
2, 3, 5	235, 253, 325, 352, 523, 532	370	10
6, 8, 9	689, 698, 869, 896, 968, 986	851	23
1, 3, 5	135, 153, 315, 351, 513, 531	333	9
0, 4, 6	046, 064, 406, 460, 604, 640	370	10

c. Sample answer: The average of the permutations of three digits between 0 and 9 is the sum of the digits multiplied by 37.

d. yes; $A = 37(x + y + z)$

23. **CHALLENGE** Fifteen boys and fifteen girls entered a drawing for four free movie tickets. What is the probability that all four tickets were won by girls?

SOLUTION:

Four people can be chosen from 30 people in ${}_{30}C_4$ ways and 4 girls can be chosen from 15 girls in ${}_{15}C_4$ ways.

$${}_{30}C_4 = \frac{30!}{(30-4)!4!} = \frac{30\cdot 29\cdot 28\cdot 27\cdot 26!}{26!\cdot 4\cdot 3\cdot 2\cdot 1} = 27,405$$

$${}_{15}C_4 = \frac{15!}{(15-4)!4!} = \frac{15\cdot 14\cdot 13\cdot 12\cdot 11!}{11!\cdot 4\cdot 3\cdot 2\cdot 1} = 1365$$

The number of possible outcomes is 27,405 and the number of favorable outcomes is 1365. Therefore, the probability is $\frac{1365}{27405} = \frac{13}{261} \approx 5\%$.

ANSWER:

$\frac{13}{261}$ or about 5%

24. **CHALLENGE** A student claimed that permutations and combinations were related by $r!\cdot {}_nC_r = {}_nP_r$ Use algebra to show that this is true. Then explain why ${}_nC_r$ *and* ${}_nP_r$ differ by the factor *r*!.

SOLUTION:

Sample answer:

$$\begin{aligned} r!\cdot {}_nC_r &= r!\cdot \frac{n!}{(n-r)!r!} \\ &= \frac{n!\cancel{r!}}{(n-r)!\cancel{r!}} \\ &= \frac{n!}{(n-r)!} \\ &= {}_nP_r \end{aligned}$$

${}_nC_r$ and ${}_nP_r$ differ by the factor *r*! because there are always *r*! ways to order the groups that are selected. Therefore, there are *r*! permutations of each combination.

For example, there are 6 combinations of 4 letters taken 2 at a time (AB, AC, AD, BC, BD, CD). In this case $r = 2$ because that is how many objects are taken at a time.

There are also 2! permutations of each combination (AB/BA, AC/CA, AD/DA, BC/CB, BD/DB, and CD/DC).

ANSWER:

Sample answer:

$$\begin{aligned} r!\cdot {}_nC_r &= r!\cdot \frac{n!}{(n-r)!r!} \\ &= \frac{n!\cancel{r!}}{(n-r)!\cancel{r!}} \\ &= \frac{n!}{(n-r)!} \\ &= {}_nP_r \end{aligned}$$

${}_nC_r$ and ${}_nP_r$ differ by the factor *r*! because there are always *r*! ways to order the groups that are selected. Therefore, there are *r*! permutations of each combination.

25. **OPEN ENDED** Describe a situation in which the probability is given by $\frac{1}{{}_7C_3}$.

SOLUTION:

We have 7 objects taken 3 at a time and order is not important because it is a combination.

Sample answer: A bag contains seven marbles that are red, orange, yellow, green, blue, purple, and black. The probability that the orange, blue, and black marbles will be chosen if three marbles are drawn at random can be calculated using a combination.

ANSWER:

Sample answer: A bag contains seven marbles that are red, orange, yellow, green, blue, purple, and black. The probability that the orange, blue, and black marbles will be chosen if three marbles are drawn at random can be calculated using a combination.

26. **CCSS ARGUMENTS** Is the following statement *sometimes, always,* or *never* true? Explain.
${}_nP_r = {}_nC_r$

SOLUTION:

$$\begin{aligned} {}_nP_r &= {}_nC_r \\ \frac{n!}{(n-r)!} &= \frac{n!}{(n-r)!r!} \\ \frac{n!(n-r)!}{(n-r)!} &= \frac{n!(n-r)!}{(n-r)!r!} \\ n! &= \frac{n!}{r!} \\ n!r! &= n! \\ r! &= 1 \end{aligned}$$

$r! = 1$ when $r = 0$ or 1. However, objects cannot be taken 0 at a time.

The statement is true when r is 1. Therefore, the statement is *sometimes* true.

ANSWER:

Sometimes; sample answer: The statement is true when r is 1.

27. **PROOF** Prove that ${}_nC_{n-r} = {}_nC_r$.

SOLUTION:

Set the two sides equal to each other. Develop each side using the formulas in this lesson and simplify. Working with the left side will eventually reduce it to the right side.

$$\begin{aligned} C(n, n-r) &\stackrel{?}{=} C(n, r) \\ \frac{n!}{[n-(n-r)]!(n-r)!} &\stackrel{?}{=} \frac{n!}{(n-r)!r!} \\ \frac{n!}{r!(n-r)!} &\stackrel{?}{=} \frac{n!}{(n-r)!r!} \\ \frac{n!}{(n-r)!r!} &= \frac{n!}{(n-r)!r!} \checkmark \end{aligned}$$

ANSWER:

$$\begin{aligned} C(n, n-r) &\stackrel{?}{=} C(n, r) \\ \frac{n!}{[n-(n-r)]!(n-r)!} &\stackrel{?}{=} \frac{n!}{(n-r)!r!} \\ \frac{n!}{r!(n-r)!} &\stackrel{?}{=} \frac{n!}{(n-r)!r!} \\ \frac{n!}{(n-r)!r!} &= \frac{n!}{(n-r)!r!} \checkmark \end{aligned}$$

28. **WRITING IN MATH** Compare and contrast permutations and combinations.

SOLUTION:

Sample answer: Both permutations and combinations are used to find the number of possible arrangements of a group of objects. The order of the objects is important in permutations, but not in combinations.

For example, randomly choosing 5 songs to play on a playlist in which you do not care about the order in which the songs are played is an example of a combination. If you *do* care about the order in which the songs are played, then it is an example of a permutation.

ANSWER:

Sample answer: Both permutations and combinations are used to find the number of possible arrangements of a group of objects. The order of the objects is important in permutations, but not in combinations.

29. **PROBABILITY** Four members of the pep band, two girls and two boys, always stand in a row when they play. What is the probability that a girl will be at each end of the row if they line up in random order?

A. $\frac{1}{24}$

B. $\frac{1}{12}$

C. $\frac{1}{6}$

D. $\frac{1}{2}$

SOLUTION:

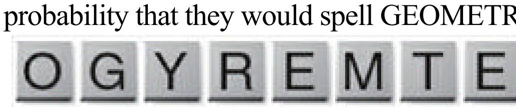
There are ${}_4P_4$ = 24 ways to arrange the band. Out of these, there are 2! or 2 ways to arrange the 2 boys in the middle of the two girls at the end and for each arrangement there is an alternate arrangement by interchanging the position of the two girls.

Therefore, the probability is $\frac{4}{24}=\frac{1}{6}$. The correct choice is C.

ANSWER:

C

30. **SHORT RESPONSE** If you randomly select a permutation of the letters shown below, what is the probability that they would spell GEOMETRY?



SOLUTION:

There are 8 letters in which only E appears twice and the others once each. So, the number of distinguishable permutations is $\frac{8!}{7(1!)\cdot 2!}=20160.$

The total number of possible outcomes is 20160 and there is only one favorable outcome which is GEOMETRY. Therefore, the probability is $\frac{1}{20160}$.

ANSWER:

$\frac{1}{20160}$

13-2 Probability with Permutations and Combinations

31. **ALGEBRA** Student Council sells soft drinks at basketball games and makes \$1.50 from each. If they pay \$75 to rent the concession stand, how many soft drinks would they have to sell to make \$250 profit?
F 116
G 117
H 167
J 217

SOLUTION:
Let x be the number of soft drinks that they have to sell to make a profit of \$250.

$$1.50x = 250 + 75$$
$$1.50x = 325$$
$$x \approx 217$$

The correct choice is J.

ANSWER:
J

32. **SAT/ACT** The ratio of 12 : 9 is equal to the ratio of $\frac{1}{3}$ to __
A $\frac{1}{4}$
B 1
C $\frac{5}{4}$
D 2
E 4

SOLUTION:
Use the ratio to write and solve a proportion.

$$\frac{12}{9} = \frac{\frac{1}{3}}{x}$$
$$12x = 3$$
$$x = \frac{1}{4}$$

Therefore, the correct choice is A.

ANSWER:
A

33. **SHOPPING** A women's coat comes in sizes 4, 6, 8, or 10 in black, brown, ivory, and cinnamon. How many different coats could be selected?

SOLUTION:
By the Fundamental Counting Principle the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

There are 4 choices for the size and four for the color. Therefore, we can select 4(4) = 16 different coats.

ANSWER:
16

34. Two similar prisms have surface areas of 256 square inches and 324 square inches. What is the ratio of the height of the small prism to the height of the large prism?

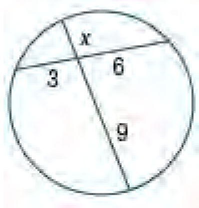
SOLUTION:
If two similar solids have a scale factor of a: b, then the surface areas have a ratio of $a^2 : b^2$.

The areas are in the ratio 256 : 324.

The sides will be in the ratio $\sqrt{256} : \sqrt{324} = 16:18 = 8:9$.

ANSWER:
8:9

Find *x*. Round to the nearest tenth, if necessary

35. 

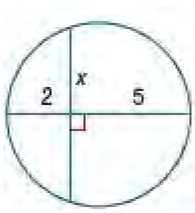
SOLUTION:

If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.

$$\begin{aligned} x(9) &= 3(6) \\ 9x &= 18 \\ x &= 2 \end{aligned}$$

ANSWER:

2

36. 

SOLUTION:

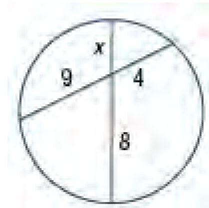
If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.

The diameter of length 7 units bisects the chord, so the length of the chord is 2*x*. If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.

$$\begin{aligned} x(x) &= 2(5) \\ x^2 &= 10 \\ x &= \sqrt{10} \\ x &\approx 3.2 \end{aligned}$$

ANSWER:

3.2

37. 

SOLUTION:

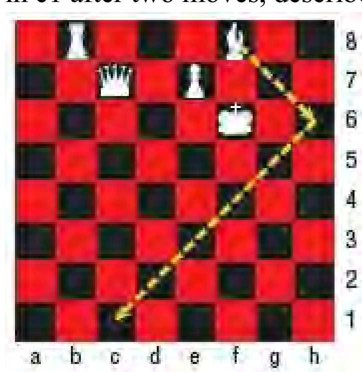
If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.

$$\begin{aligned} x(8) &= 4(9) \\ 8x &= 36 \\ x &= 4.5 \end{aligned}$$

ANSWER:

4.5

38. **CHESS** The bishop shown in square f8 can only move diagonally along dark squares. If the bishop is in c1 after two moves, describe the translation.



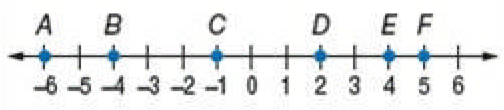
SOLUTION:

On the first move, the bishop moved 2 squares right and 2 squares down. On the second move, the bishop moved 5 squares left and 5 squares down. This is a final translation of left 3 squares and then down 7 squares from the bishop's original position.

ANSWER:

left 3 squares and down 7 squares

Use the number line to find each measure.



39. *DF*

SOLUTION:

The coordinates of *D* and *F* are 2 and 5.

$DF = |5 - 2| = 3$

ANSWER:

3

40. *AE*

SOLUTION:

The coordinates of *A* and *E* are –6 and 4.

$AE = |4 - (-6)| = 10$

ANSWER:

10

41. *EF*

SOLUTION:

The coordinates of *E* and *F* are 4 and 5.

$EF = |5 - 4| = 1$

ANSWER:

1

42. *BD*

SOLUTION:

The coordinates of *B* and *D* are –4 and 2.

$BD = |2 - (-4)| = 6$

ANSWER:

6

43. *AC*

SOLUTION:

The coordinates of *A* and *C* are –6 and –1.

$AC = |-6 - (-1)| = 5$

ANSWER:

5

44. *CF*

SOLUTION:

The coordinates of *C* and *F* are –1 and 5.

$CF = |5 - (-1)| = 6$

ANSWER:

6