







# **Teacher Edition**

# **McGraw-Hill Education**

# Integrated Math

# **United Arab Emirates Edition**

GRADE 8 · VOLUME 2



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- 5. Triangles and Pythagorean Theorem, from Glencoe Math Course 3 Vol 2 Chapter 05 © 2015
- 6. Transformations, from Glencoe Math Course 3 Vol 2 Chapter 06 © 2015

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"Extensive knowledge and modern science must be acquired. The educational process we see today is in an ongoing and escalating challenge which requires hard work.

We succeeded in entering the third millennium, while we are more confident in ourselves."

# H.H. Sheikh Khalifa Bin Zayed Al Nahyan

President of the United Arab Emirates

# **CONTENTS IN BRIEF**

# Units organized by domain

This book is organized into units based on groups called domains.

Mathematical Practices are embedded throughout the course.



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Chapter 8 Volume and Surface Area

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# **Teacher Notes**

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# **Mathematical Practices**

| Mathematical Practices   | Student Edition   |
|--|---|
| MP1 Make sense of problems and persevere in solving them.  Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.  Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. | A strong problem-solving strand is present throughout the textbook with an emphasis on strategies in the Problem-Solving Investigation lessons. Look for the Persevere with Problems head in the exercises. |
| MP2 Reason abstractly and quantitatively.  Mathematically proficient students make sense of quantities and their relationships in problem situations.  They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.  | Students are routinely asked to write an equation or an expression in order to solve a real-world problem. Exercises that emphasize this practice are labeled as Reason Abstractly.                         |

| Mathematical Practices  | Student Edition  |
|---|--|
| MP3 Construct viable arguments and critique the reasoning of others.  Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. | Students are required to justify their reasoning in problems and to find the errors in samples of other's work. Look for these heads in the exercises:  Justify Conclusions Reason Inductively Make a Conjecture Use a Counterexample Find the Error Which One Doesn't Belong Make a Prediction Multiple Representations Construct an Argument |
| MP4 Model with mathematics.  Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.   | Real-world applications in problem solving are woven throughout every lesson. In addition to the real-world examples in each lesson, look for Model with Mathematics heads in the exercises.   |
| MP5 Use appropriate tools strategically.  Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.  | In addition to the traditional mathematical tools like estimating, using mental math, or measuring, students are encouraged to use software and the Internet in problem solving. Exercises utilizing this strategy are labeled with Use Math Tools.  |

| Mathematical Practices  | Student Edition   |
|---|---|
| MP6 Attend to precision.  Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.   | Solutions are not just numbers, but include measurements to give the solution meaning. Look for Be Precise heads in the exercises.  |
| MP7 Look for and make use of structure.  Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$ , older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as $5$ minus a positive number times a square and use that to realize that its value cannot be more than $5$ for any real numbers $x$ and $y$ . | Emphasizing the structure of mathematics is present through use of classifying, explaining, giving examples as well as nonexamples. Exercises that emphasize this practice are labeled with Identify Structure. |
| MP8 Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y-2)/(x-1)=3$ . Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$ , $(x-1)(x^2+x+1)$ , and $(x-1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.   | Patterns in reasoning are demonstrated throughout leading students to sound mathematical conclusions. Exercises with Identify Repeated Reasoning heads exemplify this practice.                                 |

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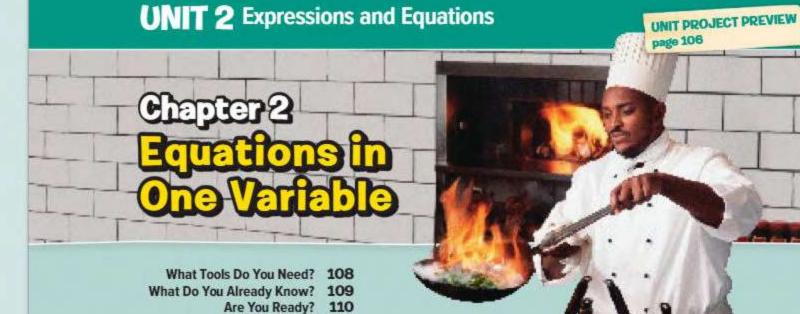
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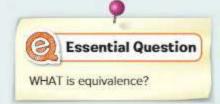
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**Essential Question** 

WHY are graphs helpful?

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#### **Essential Question**

HOW can we model relationships between quantities?

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**UNIT 4** Geometry

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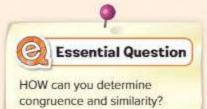
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21st Century Career

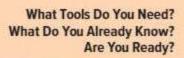
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Chapter Review Performance Task Reflect



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Inquiry Lab: Three-Dimensional Figures Lesson 1 Volume of Cylinders

Lesson 2 Volume of Cones

Lesson 3 Volume of Spheres

N Problem-Solving Investigation: Solve a Simpler Problem

**Essential Question** WHY are formulas important in

math and science?

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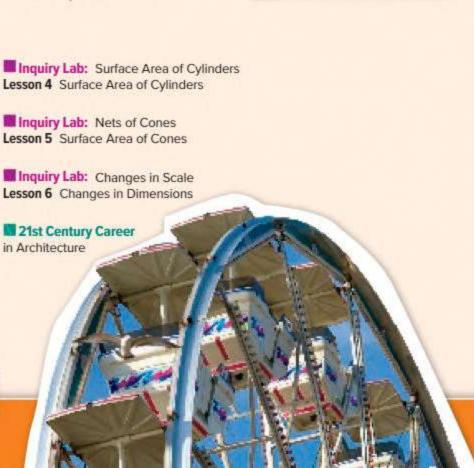


**Chapter Review** Performance Task Reflect

# **UNIT PROJECT**

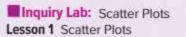
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**UNIT 5** Statistics and Probability

What Tools Do You Need? What Do You Already Know? Are You Ready?



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Inquiry Lab: Graphing Technology: Linear and Nonlinear Association

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Lesson 5 Measures of Variation

Lesson 6 Analyze Data Distributions

21st Century Career in Sports Marketing

**Chapter Review** Performance Task Reflect

# **UNIT PROJECT**

**Olympic Games** 



Glossary Work Mats **Foldables** 

GL1 WM1 FL1

UNIT PROJECT PREVIEW

**Essential Question** 

HOW are patterns used when

comparing two quantities?



This book focuses on three critical areas: (1) applying equations in one and two variables;

- (2) understanding the concept of a function and using functions to describe quantitative relationships;
- (3) applying the Pythagorean Theorem and the concepts of similarity and congruence.

# Content

## The Number System

Know that there are numbers that are not rational, and approximate them by rational numbers.

# **Expressions and Equations**

- · Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous equations.

#### **Functions**

- · Define, evaluate, and compare functions.
- · Use functions to model relationships between quantities.

## Geometry

- · Understand congruence and similarity using physical models, transparencies, or geometry software.
- · Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

# Statistics and Probability

· Investigate patterns of association in bivariate data.

# Mathematical Practices

- 1 Make sense of problems and persevere in solving them.
- 2 Reason abstractly and quantitatively.
- 3 Construct viable arguments and critique the reasoning of others.
- 4 Model with mathematics.
- 5 Use appropriate tools strategically.
- 6 Attend to precision.
- 7 Look for and make use of structure.
- 8 Look for and express regularity in repeated reasoning.

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**Mathematical Practices** 

# Mathematical **Practices** Handbook



## Essential Question

WHAT practices help me develop and demonstrate mathematical understanding?



#### Mathematical **Practices**

The standards for mathematical practice will help you become a successful problem solver and to use math effectively in your daily life.



# What is the Mathematical Practices Handbook?

Use the Mathematical Practices Handbook to introduce students to the Mathematical Practices.

The Standards for Mathematical Practice describe how students should approach mathematics. The goal of the practice standards is to instill in all students the abilities to be mathematically literate and to create a positive disposition for the importance of using math effectively.

Included in this handbook are activities and exercises that allow students to become familiar with the Mathematical Practices below. Throughout this text, students will see 12 to remind them that they are using these Mathematical Practices.

- Persevere with Problems
- Reason Abstractly and Quantitatively
- Construct an Argument
- Model with Mathematics
- Use Math Tools
- Attend to Precision
- Make Use of Structure
- Use Repeated Reasoning

# **Building on the Essential Question**

At the end of the Mathematical Practices Handbook, students should be able to answer "WHAT practices help me develop and demonstrate mathematical understanding?"

Throughout this text, refer to the following icons to find differentiated strategies to meet the needs of all learners.

- Approaching-Level Learners
- On-Level Learners
- Bl Beyond-Level Learners
- Language Acquisition

|   |              |          | ill learn about each of these mathematical<br>I in the chapters and lessons of this book. |
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| 1) Focus on Mathematic                        |              | •        | 5 Focus on Mathematical Practice Use Math Tools   |
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| 3) Focus on Mathematic<br>Construct an Argume | cal Practice |          | (7) Focus on Mathematical Practice Make Use of Structure                                  |
| Focus on Mathematic                           | cal Practice | e        | Focus on Mathematical Practice     Use Repeated Reasoning                                 |
| I have no clue.                               |              | I VC III | eard of it. UI know it!   |
| U have no clue.                               |              |          | eard of it. UI brow it! thematical Practices  |
| Mathematical<br>Practice                      | 0            |          | thematical Practices  |
| Nathematical                                  | 0            | Hat      | thematical Practices  |
| Mathematical<br>Practice                      | 8            | Hat      | thematical Practices  |
| nathematical<br>Practice                      | 8            | Hat      | thematical Practices  |
| nathematical<br>Practice ①                    | 8            | Hat      | thematical Practices  |
| hathematical<br>Practice ① ② ③                | 8            | Hat      | thematical Practices  |
| Mathematical<br>Practice ① ② ③ ④              | 0            | Hat      | thematical Practices  |
| nathematical Practice  ① ② ③ ③ ⑤              | 8            | Hat      | thematical Practices  |

#### Focus on Mathematical Practice 1

# **Persevere with Problems**

# How do I make sense of a problem?

Making and using a step-by-step plan to solve a problem is like using directions to build a piece of furniture. If you follow the directions correctly, there is a good chance you will end up with a solid piece of furniture. Once you understand the meaning of the problem, you can decide what strategy will work best to solve it. You might try several strategies and then ask yourself, "Does this make sense?"

You have already used the four-step problem-solving plan in previous courses. Complete the graphic organizer that shows the four steps to solve the given problem.

Of the 480 students at Lincoln Middle School, one third have traveled overseas. Of these, 15% have been to Australia. How many students have not been to Australia?

## Step 1. Understand What are the facts?

Step 2. Plan

What strategy will you use to solve the problem above?

## Step 3. Solve Solve the problem. Show your steps below.

Step 4. Check

How do you know your answer is reasonable?

Mathematical Practice 1

Make sense of problems and persevere in solving them.

# Focus narrowing the scope

Objective Persevere with problems.

# Coherence connecting within and across grades

# Previous

Students found percent of a number.

#### Now

Students make sense of problems and persevere in solving them.

## Next

Students will reason abstractly and quantitatively.

# Rigor pursuing concepts, fluency, and applications

Mathematically proficient students use a logical process to make sense of problems, understand that there may be more than one way to solve a problem, and alter the process if needed.

ENGAGE EXPLORE EXPLAIN BLABORATE EVALUATE

# Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

# Ideas for Use

Paired Heads Together Have students complete the graphic organizer individually. Then have students pair up with a partner and share their answers. If either answer is incorrect, have the students alternate to go back through the steps to check their answers. 21, 3, 5, 6, 7

# Alternate Strategy

Before solving the problem on page MP3, have students make a list of all of the problem-solving strategies they have used in previous math classes. Then randomly call on students to share their strategies. 31, 6

# 2 Practice and Apply

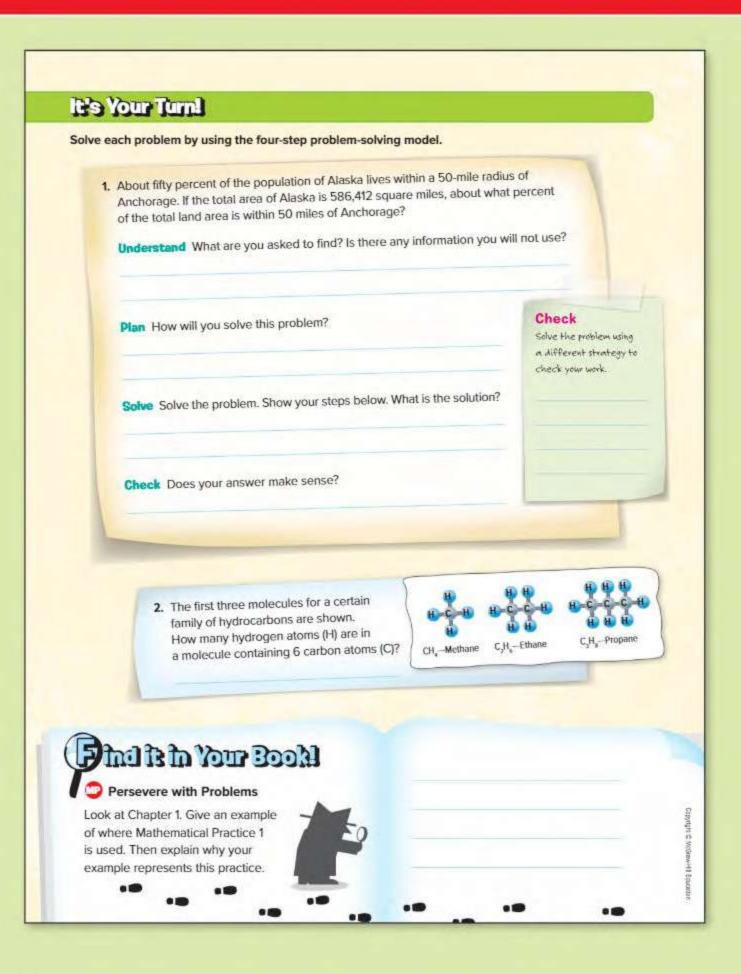
## **Ideas for Use**

circle the Sage Poll the class to see which students have a solid understanding of how to solve the problem presented in Exercise 1. These students (the sages) spread around the room. Have the rest of the class divide into small groups. Have each group member report to a different sage, if possible. The sages lead the discussion in how to solve the problem. Then have all students return to their groups to compare what was discussed with each sage.

# **Alternate Strategies**

Review the formula for finding the area of a circle,  $A = \pi r^2$ . Have students describe the formula using multiple representations, such as verbal descriptions, models, tables, and graphs of ordered pairs.  $\bigcirc$  1, 2, 4, 6, 7

BL (IA) Have students research a hydrocarbon molecule not shown in Exercise 2. Students should draw the molecular diagram, write the chemical formula, and include at least one other characteristic. (21, 2, 5, 6, 8)



#### Focus on Mathematical Practice 2

# **Reason Abstractly and Quantitatively**

# What does it mean to reason abstractly and quantitatively?

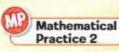
In math, we solve real-world problems where numbers and variables in an equation represent concrete objects. This involves thinking quantitatively.

Suppose you are given a AED 25 gift card to an online music store. Each song costs AED 1.95 to purchase and download. How many songs can you buy?

- What values in the problem do we already know?
- 2. What are we trying to find?
- 3. What symbol can we use to represent the unknown value?

Now that the problem is broken down into known and unknown values, we can manipulate the symbols in order to solve the problem. This is thinking abstractly.

- 4. Write an equation to solve the problem. Explain what each quantity or symbol represents.
- 5. Use your equation to solve the problem and label your solution. Explain the meaning of the solution.



Reason abstractly and quantitatively.

# Focus narrowing the scope

Objective Reason abstractly and quantitatively.

# Coherence connecting within and across grades

# Previous

Students made sense of problems and persevered in solving them.

#### Now

Students reason abstractly and quantitatively.

## Next

Students will construct viable arguments and critique the reasoning of others.

# Rigor pursuing concepts, fluency, and applications

Mathematically proficient students can start with a concrete or real-world context and then represent it with abstract numbers or symbols (decontextualize), find a solution, then refer back to the context to check that the solution makes sense (contextualize).

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **Launch the Lesson**

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

## Ideas for Use

Team-Pair-Solo Have students work in small groups to complete Exercises 1–3. Then have them divide into pairs to complete Exercise 4. Have students work individually to complete Exercise 5. Have them regroup to discuss their responses to Exercises 4 and 5. 21, 2, 4, 6

# Alternate Strategy

Tell students that there is a one-time \$5 fee to create an account before you can purchase and download any songs. Have them write a new equation given this information and find how many songs they can purchase after creating an account.

1, 2, 4, 7

# **Practice and Apply**

## Ideas for Use

Rally Coach Have students work in pairs to complete Exercises 6 and 7. Have one partner complete Exercise 6 while the second partner watches, listens, coaches, and praises. Have them trade turns for Exercise 7. 1, 2, 3, 4, 6, 8

# **Alternate Strategies**

If students need help completing the table, have them make a three column table (input, equation, and output) to organize their work. Remind students that by using an equation, they are using symbols to communicate mathematical ideas. 1, 2, 4, 6, 7

Have students determine how many pounds of fuel the car will need in order to finish the race. 21, 5, 6

# It's Your Turn!

Write and solve an equation for each of the following.

- 6. You are in the pit crew for a driver at a car race. The gas weighs 5.92 pounds per gallon. Your driver uses 0.25 gallons per lap. With 42 laps to go, you put 60 pounds of fuel in the tank of the car. Will your driver finish the race at the same rate without more gas?
  - a. What values do we already know? What are we trying to find?
  - b. Write an equation to find the number of gallons in 60 pounds of fuel.
  - c. Use the equation to solve the problem and explain the meaning of the solution.
  - 7. A class trip is scheduled for an amusement park. Group admission prices are AED 31 per student. Parking is AED 18 per bus.
    - a. Complete the table to show the total cost of 10, 20, 30, and 40 students and two buses.
    - b. Write an equation to show the total cost c if two buses transport s students to the park.
    - There are a total of 78 students attending on two buses. What is the total cost? Label your solution and explain its meaning.





Reason Abstractly

Look at Chapter 2. Give an example of where Mathematical Practice 2 is used. Then explain why your example represents this practice.



## Focus on Mathematical Practice 3

# **Construct an Argument**

# How do I construct a viable argument in math class?

Suppose your friend told you that his rectangular flatscreen T.V. has congruent diagonals, simply because it was rectangular. How could you ask your friend to justify his argument? You could use inductive reasoning or deductive reasoning. Inductive reasoning uses examples to draw conclusions, while deductive reasoning uses definitions, rules, or facts.

1. How could you use inductive reasoning to justify why the following statement is true?

All rectangles have diagonals that are congruent.

2. How could you use deductive reasoning to justify why the following statement is false?

Each angle of an equilateral triangle measures 90°.

3. Complete the graphic organizer to show that the statement below is sometimes true.

> The product of two integers is positive.

Example when it is true:

Example when it is false:

#### Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

# Focus narrowing the scope

Objective Construct viable arguments and critique the reasoning of others.

# Coherence connecting within and across grades

## Previous

abstractly and quantitatively.

#### Now

Students construct viable arguments and critique the reasoning of others.

## Next

Students will model with mathematics.

# Rigor pursuing concepts, fluency, and applications

Mathematically proficient students can clearly communicate their thoughts and defend them using sound mathematical arguments.

ENGAGE EXPLORE EXPLAIN BLABORATE EVALUATE

# **Launch the Lesson**

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

# Ideas for Use

Numbered Heads Together Have students work in groups of 3-4 and work as a team to solve each problem, making sure that everyone understands. Then call on a random student to explain the solution.

1. 3. 5. 6. 7

# Alternate Strategy

Provide students with several real-world examples in which either inductive or deductive reasoning were used. Have them classify each example as inductive reasoning or deductive reasoning. 21,6

# 2

# **Practice and Apply**

## Ideas for Use

Group-Pair-Share Have students work in groups to complete Exercises 4 and 5, ensuring that each member understands. Then have the groups divide into pairs to complete Exercise 6. Have them reconvene into the larger group to share responses and resolve any discrepancies. 1, 3, 6, 7

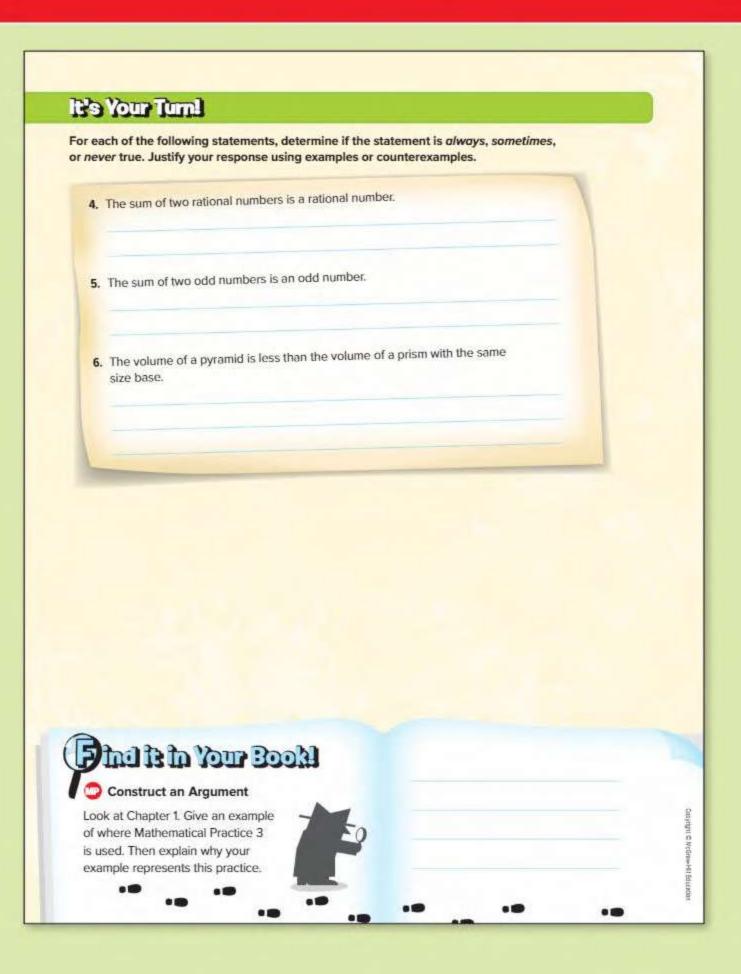
# **Alternate Strategies**

Create several statements similar to those in Exercises 4–6. Have the students determine if the statement is *always true*, *sometimes true*, or *never true*, and justify their choice. Guide students in providing convincing justifications. For example, if a statement is sometimes true, they should be able to give an example showing it is true and a counterexample showing it is false. 

1, 3, 6, 7

Gallery Walk Have students work with a partner to create a problem similar to those in Exercises 4–6. Post the problems around the room. Students walk around the room and select a problem, not their own. Working with their partner, they determine the solution and write a convincing argument supporting their solution. Have them locate the pair of students who wrote the problem to share their solution and discuss and resolve any errors. 

1, 3, 4, 6



## Focus on Mathematical Practice 4

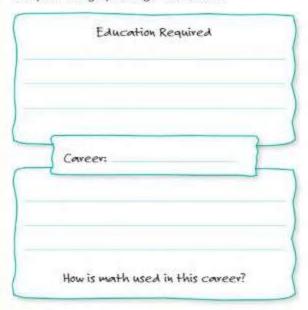
# **Model with Mathematics**

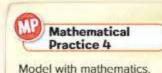
# How does math fit into your future?

No matter what career path you choose, you are sure to use math in your job or career. Graphic organizers arrange ideas so that you can make informed decisions. Using and understanding models such as graphs, tables, and diagrams helps you to simplify a complicated situation and to identify important quantities in a real-life situation.

Suppose you are a doctor or a nurse. A prescription directs a patient to take 2.5 cc (cubic centimeters) of a medicine per 50 pounds of body weight.

- 1. What skill(s) would you use to see how much medicine you should give to a 125 pound person?
- 2. How much medicine would the 125 pound patient need?
- 3. What career path interests you? Research that career and complete the graphic organizer below.





# Focus narrowing the scope

Objective Model with mathematics.

# Coherence connecting within and across grades

# Previous

Students constructed viable arguments and critiqued the reasoning of others.

## Now

Students model with mathematics.

# Next

Students will use math tools.

# Rigor pursuing concepts, fluency, and applications

Mathematically proficient students explain their thinking or search for patterns using models such as diagrams, drawings, classroom objects, and manipulatives, or geometric, graphical, algebraic, tabular, and statistical models.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **Launch the Lesson**

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

# Ideas for Use

Pairs Discussion Have students work in pairs to complete Exercises 1 and 2. Have them trade their solutions with another pair of students to discuss and resolve any differences. 31, 4, 6, 7

# **Alternate Strategy**

If students have difficulty deciding on a career, have them first take a career interest survey before completing Exercise 3. 21, 5

# Practice and Apply

## **Ideas for Use**

Numbered Heads Together Assign students to a 3- or 4-person learning team. Each member is assigned a number from 1 to 4. Each team completes Exercises 4–6, making sure every team member understands the solution. Call on a specific number from one team to present the team's solution to the class. 

1, 2, 4, 6, 7

# **Alternate Strategies**

Have students practice finding the percent of a number using either the percent proportion or the percent equation.

1, 2, 6, 8

Have students use the Internet, or another source, to research city, county, and state sales tax rates for their area. Have them recalculate the total cost of the party in Exercise 6 based on their findings. 21, 5, 6

# It's Your Turn!

Use the given tools to solve each problem.

- 4. You are saving money to buy a new game system. You received AED 50 as a graduation gift from your grandparents. You want to save AED 25 a week from mowing lawns.
  - a. Tables Complete the table to show the total amount saved after 1, 2, 3, 4, and 5 weeks.
  - Symbols Write an equation to show the total amount saved s after w weeks.
  - c. Algebra Use the equation to determine the total amount saved after 17 weeks.

Use the table for Exercises 5 and 6.

5. Mrs. Fatma hired a party planner to plan Noha's dinner party. There will be 125 guests and she wants to offer appetizers and a buffet dinner. What is the cost, before tax, for the party?

|                    | Polly's Per    | fect Parties                           |                     |
|--------------------|----------------|--|---------------------|
| Cost of            | Food<br>erson) | Cost of E                              | In the state of     |
| Appetizers         | AED 9.20       | Hall                                   | AED 250             |
| Buffet             | AED<br>18.30   | Linens                                 | AED 15<br>per table |
| Sit-down<br>Dinner | AED<br>25.75   | Table and<br>Chair Rental<br>(seats 8) | AED 60<br>per table |

Week, w Total Saved, s (\$)

6. There is a 7<sup>1</sup>/<sub>2</sub>% sales tax added to the party bill. Mrs. Fatma also wants to add an 18% tip for the servers. This will be figured before tax is added. What will be the total cost of the party?



Model with Mathematics

Look at Chapter 1. Give an example of where Mathematical Practice 4 is used. Then explain why your example represents this practice.



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## Focus on Mathematical Practice 5

# **Use Math Tools**

# How do I use tools and strategies in math class?

Sometimes using math tools and strategies helps make solving problems easier if you know which tool to use in a given situation. Math tools are physical objects you use when solving problems. Paper and pencil, technology, or calculators are examples of tools.

|  | 1. | List three | other | tools | you | can | use | to | solve | math | problems |
|--|----|------------|-------|-------|-----|-----|-----|----|-------|------|----------|
|--|----|------------|-------|-------|-----|-----|-----|----|-------|------|----------|

Math strategies are more like skills or the ability to apply your math knowledge. Some math strategies are mental math, number sense, estimation, drawing a diagram, or solving a simpler problem.

- 2. List three other strategies you can use to solve math problems.
- 3. Complete the graphic organizer.

| Problem   | Tool | Strategy |
|---|------|----------|
| You want to leave a 20% tip for your server.  |      |          |
| You want to determine how long<br>it will take to drive from Abu<br>Dhabi to Fujira |      |          |
| You are stuck while in the middle of solving an equation.                           |      |          |

# Mathematical Practice 5

Use appropriate tools strategically.

## Focus narrowing the scope

Objective Select tools and strategies to solve problems.

# Coherence connecting within and across grades

## Previous

Students modeled with mathematics.

#### Now

Students use math tools and strategies.

#### Next

Students will attend to precision.

# Rigor pursuing concepts, fluency, and applications

Mathematically proficient students understand the benefits and limitations of using mathematics tools, including estimation and virtual tools, and use them appropriately.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **Launch the Lesson**

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

# Ideas for Use

Rally Robin In groups, assign one student as the Rally Robin Leader, who poses questions to help complete Exercise 3. The rest of the group takes turns responding orally to each question. 21, 5

# Alternate Strategy

Mave students work in small groups to create a graphic organizer that lists as many tools and strategies they can brainstorm. 21, 5

# Practice and Apply

## **Ideas for Use**

of three to complete Exercises 4–6. Each student is given a number, 4–6, which represents the exercise discussion they are leading. Teammates discuss Exercise 4 with Student 4 leading the discussion. All members of the team contribute, but all have to agree upon one answer. Continue by rotating the leader role until all the exercises are completed. 21, 3, 4, 5, 6

# **Alternate Strategies**

If students have difficulty solving the problems, have them make a template of the four-step problem-solving plan to use for each exercise. 1,5

Pairs Discussion Have students choose one of the exercises and write an extension of that problem. For example, for Exercise 5, students may choose to determine how long it will take Natalie to walk the  $7\frac{1}{2}$  miles for a chosen rate. Have them discuss with a partner which tools or strategies can be used to solve their extension problem.  $\bigcirc$  1, 3, 4, 5

# It's Your Turn!

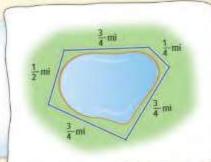
List the tools or strategies you would use to solve each problem. Then solve the problem.

- A pre-election survey was taken in Ms. Noha's homeroom.
   The results for class president are shown in the table.
  - a. Based on the survey, if there are 850 students in the 8th grade, how many votes will Asmaa get?

| b. | A candidate needs to receive at least 51% of the votes to |
|----|---|
|    | win the election. If every student votes, how many more   |
|    | votes would Asmaa need to win?                            |

| Class Pr | esident |
|----------|---------|
| Marwa    | 10      |
| Karami   | 8       |
| Asmaa    | 20      |
| Sara     | 12      |

5. A walking path around a lake is in the shape of a pentagon like the one shown. If Natalie wants to walk 4½ miles, how many times does she need to walk around the lake?



6. Write a word problem that requires the use of a protractor, a calculator, and one strategy, like mental math or estimation. Find the solution to your problem and explain how you used the tools to solve it.





Use Math Tools

Look at Chapter 1. Give an example of where Mathematical Practice 5 is used. Then explain why your example represents this practice.



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#### Focus on Mathematical Practice 6

# **Attend to Precision**

# What does it mean to be precise?

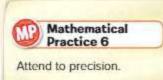
Communication is important to our daily life, whether it's in school, sports, at home, or hanging out with friends. If you can't clearly express your thoughts, no one will understand what you mean! Math also requires clear and precise communication by using labels, appropriate symbols, and clear definitions.

Suppose you and your brother want to paint two walls in your bedroom a new color. Your bedroom is 12 feet 5 inches long, 14 feet 8 inches wide, and has an 8-foot ceiling height.

- 1. What skill(s) would you use to see how much paint you need?
- 2. What information do you need to know in order to make your calculations?

You are painting two walls that are perpendicular to each other. They do not have doors or windows on them. A gallon of paint covers about 350 square feet.

- 3. What is the area of wall space you will be painting? Label your answer.
- 4. How precise does the area need to be to determine how much paint you will need? Round the area and explain why you rounded to the place value you chose.
- 5. How many gallons of paint do you need? Round to an appropriate place value and label your answer. Explain your rounding.



# Focus narrowing the scope

Objective Attend to precision.

# Coherence connecting within and across grades

# Previous

Students used math tools and strategies.

# Now

Students attend to precision.

### Next

Students will look for and make use of structure.

# Rigor pursuing concepts, fluency, and applications

Mathematically proficient students communicate the language of mathematics precisely, as well as calculate efficiently and accurately.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **Launch the Lesson**

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

# Ideas for Use

Pairs Discussion Have students work in pairs to complete Exercises 1 and 2. Then have them work individually to complete Exercises 3-5. Have them reconvene with their partner to check their answers and resolve any differences. 31, 3, 4, 6

# **Alternate Strategy**

Review the formula for area of a rectangle and ask students which measurements they need to know to determine how many gallons of paint they need. 21, 6, 7

# Practice and Apply

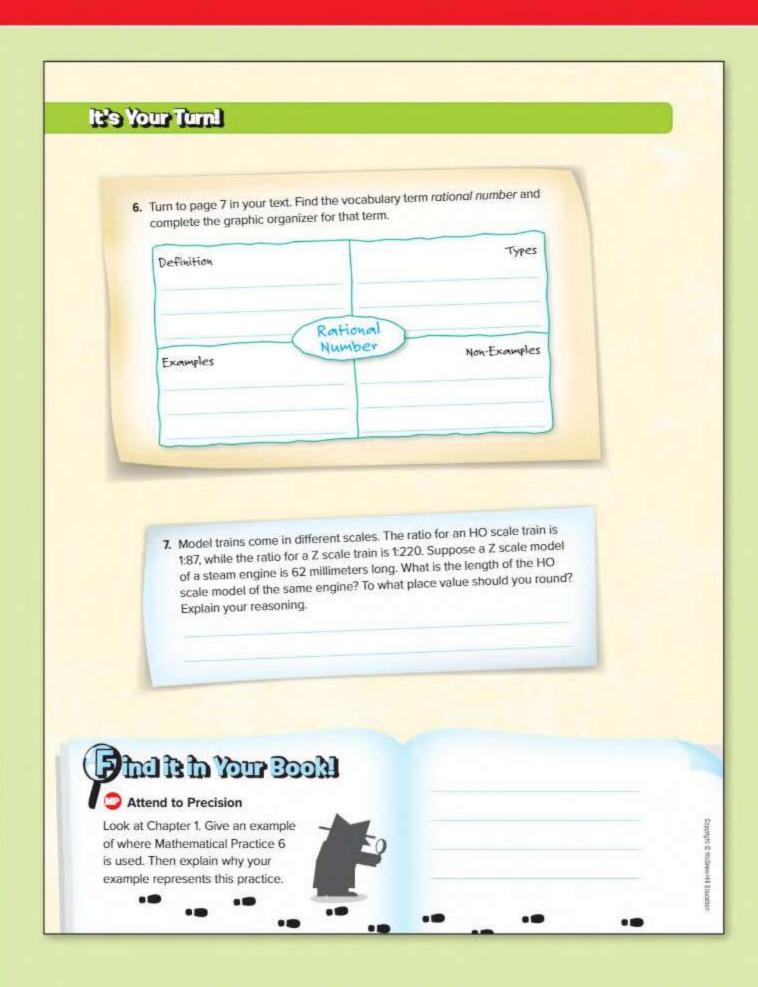
### **Ideas for Use**

Think-Pair-Share Give students one to two minutes to think through their responses to Exercises 6 and 7. Then have them share their responses with their partner first, then with the entire class. 1,3

# **Alternate Strategies**

Before students complete Exercise 6, complete a similar graphic organizer for another concept, such as equations, as a whole group. 1, 5, 6

BL (IA) Find the Fib Have students work with a partner to write two facts and one fib for Exercise 7. Then have them exchange facts and fibs with another pair of students. Each pair identifies the other pair's facts and fib. (2) 1, 3, 7

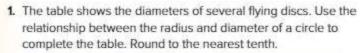


#### Focus on Mathematical Practice 7

# **Make Use of Structure**

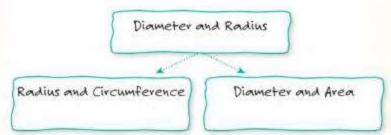
# What does it mean to use structure in math?

When you use structure in math, you might apply properties to solve equations or you might examine patterns in tables and graphs to describe relationships.



| Diameter (cm) | Radius (cm) | Circumference<br>(cm) | Area (cm²) |   |
|---------------|-------------|-----------------------|------------|---|
| 20            |             |                       |            | 6 |
| 22            |             |                       |            |   |
| 25            |             |                       |            | 1 |

- 2. Describe the relationship between the diameter and radius of a circle.
- 3. Describe the relationship between the circumference and diameter of a circle.
- 4. Complete the graphic organizer by writing a formula in each box that shows the relationship between each term.





Mathematical Practice 7

Look for and make use of structure.

# Focus narrowing the scope

Objective Look for and make use of structure.

# Coherence connecting within and across grades

### Previous

Students attended to precision.

#### Now

Students look for and make use of structure.

#### Next

Students will look for and express regularity in repeated reasoning.

# Rigor pursuing concepts, fluency, and applications

Mathematically proficient students look for structure to find easier ways to solve problems.

ENGAGE EXPLORE EXPLAIN BAHORATE EVALUATE

# **Launch the Lesson**

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

# Ideas for Use

Talking Chips Have students work in small groups to complete Exercises 1–4. Give each student 5 chips. Students must place a chip in the center of the table each time they contribute to the discussion. After they have used all of their chips, they may no longer contribute to the discussion. All students must use all of their chips.

1, 2, 4, 5, 6, 7, 8

# Alternate Strategy

Review the formulas for the circumference and area of a circle with students before completing Exercises 1-4.

21, 2, 6

# 2 Practice and Apply

### **Ideas for Use**

Rally Coach Have students work in pairs on Exercises 5-8. Have Partner A work through Exercise 5 while Partner B listens, coaches, and praises. Partners switch roles for Exercise 6. Continue this process with Exercises 7 and 8. 1, 2, 4, 6, 7

# **Alternate Strategies**

Before students complete Exercise 5, review rate of change and what the rate means in the context of the situation.
1,6

Have students research the average time that runners completed marathons in three different cities and compare the averages to their answer for Exercise 8. Have them explain if their answer is reasonable. 21, 3, 5, 6

# It's Your Turn! Suppose you are training for a marathon. A marathon is 26.2 miles long. You can run 3 miles in 16 minutes. 5. At this rate, how many miles can you run in one hour? Complete the table and plot the points to make a line graph. **Marathon Training** 30 25 20 Write an equation that shows the relationship between distance and time. Estimate how long it will take to complete the marathon. Find is in Your Book! Make Use of Structure Look at Chapter 1. Give an example

of where Mathematical Practice 7 is used. Then explain why your example represents this practice.

#### Focus on Mathematical Practice 8

# **Use Repeated Reasoning**

# What does it mean to look for repeated reasoning?

Problems can often be solved by finding patterns or repeated processes. Sometimes you can even create shortcuts to solve a problem once you understand the pattern. For example, multiplication is a shortcut for repeating the same addition over and over.

Suppose you have a garden with a length of 6 feet and a width of 4 feet and you want to increase its size. Before making any changes, do some math!

A AMBOUT IN ALL OF THE PARTY OF THE PROPERTY OF

| 1. | what is the perimeter of tr  | ie garden?                         |
|----|------------------------------|------------------------------------|
|    | the area?                    |                                    |
| 2. | If you double the dimension  | ons of the garden, what is the new |
|    | perimeter?                   | new area?                          |
| 3. | What number can you mul      | tiply the original perimeter by to |
|    | find the new perimeter?      | What number can you                |
|    | multiply the original area b | by to find the new area?           |

Oh no, the increased size of the garden is too big! Using the original dimensions of the garden, you increase the length to 9 feet and the width to 6 feet.

| 4. What is the new perimeter: | 4. | What is the new | perimeter? | new area |
|-------------------------------|----|-----------------|------------|----------|
|-------------------------------|----|-----------------|------------|----------|

- 5. What number can you multiply the original perimeter by to find the new perimeter? What number can you multiply the original area by to find the new area?
- 6. Try other changes in the dimensions of the garden to find the new perimeter and area of the garden.



#### Mathematical Practice 8

Look for and express regularity in repeated reasoning.

# Focus narrowing the scope

Objective Look for and express regularity in repeated reasoning.

# Coherence connecting within and across grades

#### Previous

Students looked for and made use of structure.

#### Now

Students look for and express regularity in repeated reasoning.

#### Next

Students will write rational numbers as fractions, mixed numbers, and decimals.

# Rigor pursuing concepts, fluency, and applications

Mathematically proficient students recognize and use patterns that can lead to results more quickly and efficiently.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **Launch the Lesson**

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

# Ideas for Use

Pairs Discussion Have students work in pairs to complete Exercises 1-3. Then have them work individually to complete Exercises 4-6. Have them reconvene with their partner to check their answers and resolve any differences. 21, 4, 5, 6, 7, 8

# Alternate Strategy

Review how to find perimeter and area. Then complete Exercises 1 and 2 as a whole group. 31, 4, 5, 6, 7

# 2 Practice and Apply

### **Ideas for Use**

students have a good understanding of how to solve Exercises 7 and 8. These students (the sages) spread around the room. Have the rest of the class divide into small groups. Have each group member report to a different sage if possible. The sages lead the discussion in how to solve the problems. Then have the students return to their groups to compare what was discussed with each sage. 

(2) 1, 3, 4, 6, 7, 8

# **Alternate Strategies**

Act it Out In small groups or as a class, have students act out pouring and combining cups of orange juice and apple juice for Exercise 7. Then have them continue this individually to fill in the table. 1, 4, 6, 8

For Exercise 8, have students make an "Option C" table that is better than Options A and B. Have them explain the pattern and why it is the best choice. 1, 3, 4

# It's Your Turn!

7. Ahmed is mixing orange juice and apple juice in a ratio of 3 to 4 to make a fruit punch. He wants to make 35 cups of the punch. To determine how many cups of each juice he needs, he started making a table. Complete the table to find how many cups of each juice he will need. Then explain a shortcut you could use to solve the problem.

| Orange<br>Juice | Apple<br>Juice | Total<br>Cups |
|-----------------|----------------|---------------|
| 3               | 4              | 7             |
| 6               | 8              | 14            |
| 9               | 12             | 21            |

Amina's parents are going to pay her for doing chores 6 days a week and they offer her two payment plans.

|       |       |       | Option A |       |       |       |
|-------|-------|-------|----------|-------|-------|-------|
| Day 1 | Day 2 | Day 3 | Day 4    | Day 5 | Day 6 | Total |
| AED 3 | AED 6 | AED 9 |          |       |       |       |

|          |          |          | Option B |       |       |       |
|----------|----------|----------|----------|-------|-------|-------|
| Day 1    | Day 2    | Day 3    | Day 4    | Day 5 | Day 6 | Total |
| AED 0.75 | AED 1.50 | AED 3.00 |          |       |       | _     |

Complete the table to determine which is the better option for Savannah to choose. Explain the pattern for each option.



Use Repeated Reasoning

Look at Chapter 1. Give an example of where Mathematical Practice 8 is used. Then explain why your example represents this practice.



2 MoStawid II Education

#### Mathematical Practices Handbook

# Review

# **Use the Mathematical Practices**

#### Solve.

You are boxing and wrapping gifts for a club fundraiser. The charge to wrap a gift in the shape of a rectangular prism is shown in the table.

| Total Surface Area       | Cost   |
|--------------------------|--------|
| up to 35 in <sup>2</sup> | AED 5  |
| 36-54 in <sup>2</sup>    | AED 8  |
| over 55 in <sup>2</sup>  | AED 12 |

a. Mariam wrapped three different boxes with measurements shown in the table. Complete the table with the cost per box and the cost per square inch. Which box has the least cost per square inch?

| Вох | height in. | width in. | length in. | Cost to Wrap | Cost per Square Inch |
|-----|------------|-----------|------------|--------------|----------------------|
| Α   | 2          | 4         | 3          |              |                      |
| В   | 2          | 5         | 6          |              |                      |
| C   | 2          | 3         | 2          |              |                      |

b. Which of those boxes has the least cost per cubic inch? Explain.

Determine which mathematical practices you used to determine the solution. Shade the circles that apply.

| •                        |
|--------------------------|
| Practices did you use?   |
| e(s) that applies.       |
| (5) Use Math Tools       |
| Attend to Precision      |
| Make Use of Structure    |
| ® Use Repeated Reasoning |
|                          |

# **Use the Mathematical Practices**

Use the next two pages to review each of the Mathematical Practices and how students will use them to engage in the content they will encounter throughout this text.

### Ideas for Use

 Round Table Consensus Have students work in small groups to complete the problem. Each student is responsible for contributing their individual response to the group for each step. Group members show agreement (thumbs up) or disagreement (thumbs down). If there is any disagreement, group members discuss to resolve it.

1, 3, 4, 6



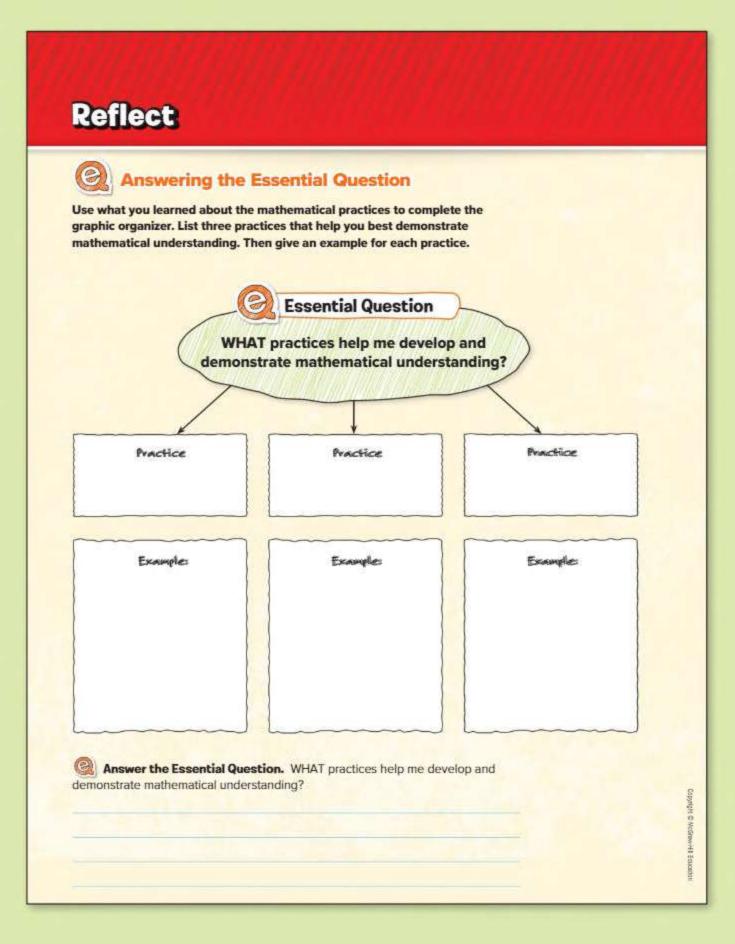
### **Answering the Essential Question**

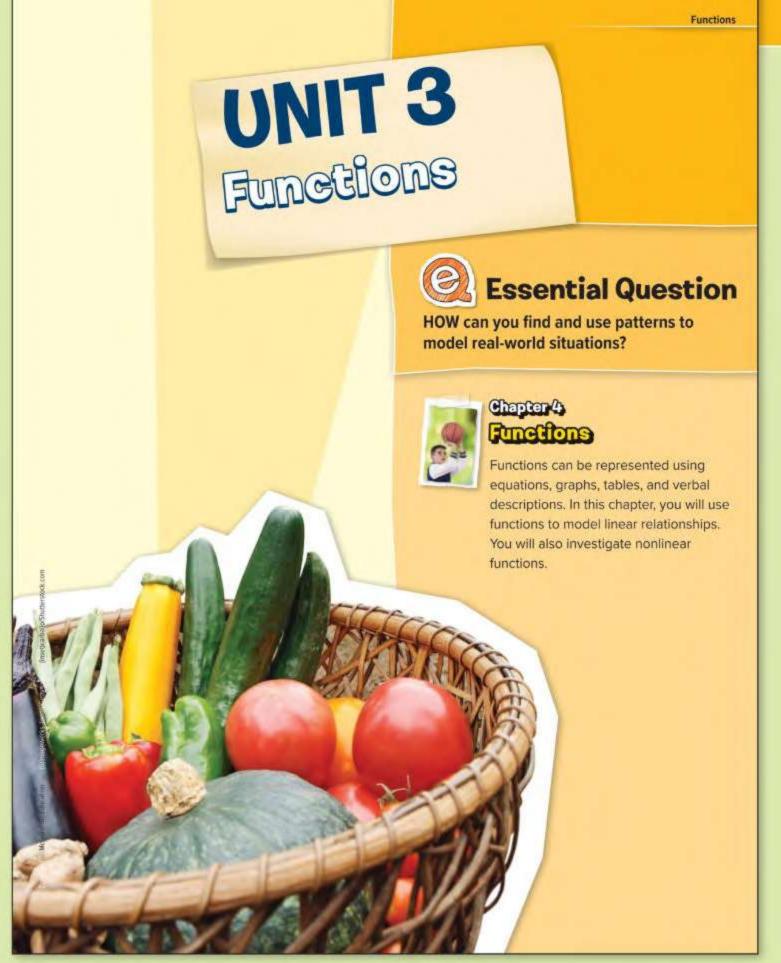
At the end of the Mathematical Practices Handbook, students should be able to answer "WHAT practices help me develop and demonstrate mathematical understanding?"

### Ideas for Use

Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their classmate before they complete the graphic organizer.

1, 5, 6







# **Essential Question**

At the end of this unit, students should be able to answer "How can you find and use patterns to model real-world situations?"

Chapter 4 explores an essential question that assists students in answering the unit question. The lessons in this chapter include exercises that lead students to various aspects of the essential question.

#### Define, evaluate, and compare functions.

- 1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- 2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- **3.** Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

continued on page 262

# Use functions to model relationships between quantities.

- 4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- 5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.



# **Unit Project Preview**

Ask students what they know about gardening.

Remind students to label their costs by unit or by Kilogram.

The Unit Project can be found on pages 361-362.



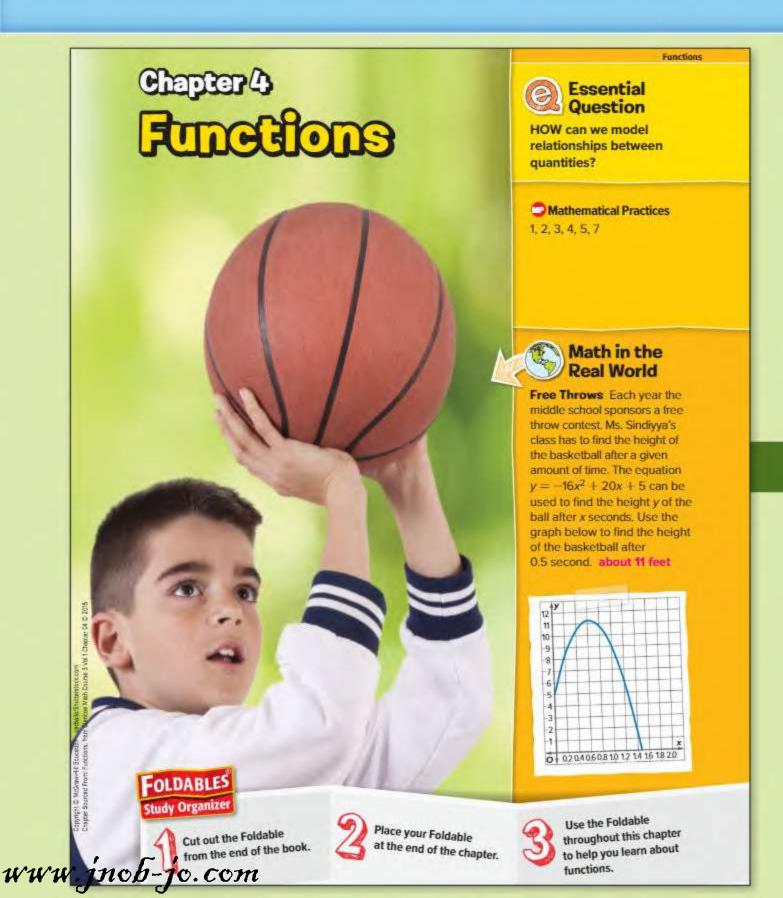
# **Unit Project Preview**

**Green Thumb** Do you have a green thumb for gardening? A community garden is a great way to meet your neighbors, beautify your neighborhood, and strengthen the sense of community.

There are many types of community gardens. Food pantry gardens donate the produce they grow to local food pantries. School gardens help to educate students in science and math, while entrepreneurial gardens generate income by selling the produce.

At the end of Chapter 4, you'll complete a project to discover the costs involved in creating a community garden. But for now, it's time to do an activity in your book. Complete the table shown by estimating the cost of selling various vegetables and fruits.





# Focus narrowing the scope

This chapter focuses on content from the Functions (F) domain. Mathematical Background

# Coherence connecting within and across grades

### Previous

Students solved and graphed equations in one and two variables.

#### Now

Students use words. tables, equations, and graphs to represent linear and nonlinear functions.

#### Next

Students will apply expressions, equations, and functions to geometric concepts.

# Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

# **Launch the Chapter**



#### Math in the Real World

Free Throws Students can find the actual height of the basketball after 0.5 second if they replace x with 0.5 in the equation and simplify.

# What Tools Do You Need?

# **Vocabulary Activity**

As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

**Define:** A function is a relation in which each member of the domain (input value) is paired with exactly one member of the range (output value).

#### Example:

| Input | Output |
|-------|--------|
| 1     | 3      |
| 2     | 5      |
| 3     | 7      |

#### Ask:

What are the domain and range of the function? D: [1, 2, 3];
 R: (3, 5, 7)

# **Reading Math**

A Have et

Have students read the Reading Math section.

#### Ask:

- How does rewriting the problem in fewer words help in solving it? Sample answer: It helps to sort out what you know and what you need to determine.
- Have students read Exercise 1. Then ask, what are the important words and numbers in the problem? Sample answer: costs AED125, saved AED80, additional AED5 each week, in how many weeks will he have enough money for the scooter
- Have students read Exercise 2. Then ask, what are the important words and numbers in the problem? Sample answer: some DVDs are AED10 each, plus a CD that costs AED15, AED75 to spend

# What Tools Do You Need?



#### Vocabulary

continuous data function table
dependent variable independent variable
discrete data linear equation
domain linear function

function nonlinear function

# Study Skill: Reading Math

One way to make a word problem easier to understand is to rewrite it using fewer words.

Step 1 Read the problem and identify the important words and numbers.

There are a great deal of cell phone plans available for students.

With Suha's plan, she pays AED 150 per month for 200 minutes, plus AED 0.30 per minute once she talks for more than 200 minutes.

Suppose Suha can spend AED 200 each month for her cell phone. How many more minutes can she talk?

quadratic function

qualitative graphs

range

relation

Step 2 Simplify the problem. Keep all of the important words and numbers, but use fewer of them.

The total monthly cost is AED 150 for 200 minutes, plus AED 0.30 times the number of minutes over 200. How many minutes can she talk for AED 200?

Step 3 Simplify it again. Use a variable for the unknown.

The cost of m minutes at AED 0.30 per minute plus AED 150 is AED 200.

#### Rewrite each problem using the method above.

- 1. Hamdan is saving money to buy a scooter that costs AED 1,250. He has already saved AED 800 and plans to save an additional AED 50 each week. In how many weeks will he have enough money for the scooter?
  - Sample answer: Saving AED 50 each week for x weeks plus AED 800 is AED 1,250.
- Humaid wants to buy some DVDs that are each on sale for AED 10 plus a CD that costs AED 15. How many DVDs can he buy if he has AED 75 to spend?

Sample answer: The total cost of x DVDs at AED 10 each plus AED 15 is AED 75.

AND A NORMALIA STANS

# What Do You Aiready Know

Place a checkmark below the face that expresses how much you know about each concept. Then scan the chapter to find a definition or example of it. See students' work.

😃 I know it.

| 11 | I've | heard | 90 | it |
|----|------|-------|----|----|
| •  |      | Links | •• | ** |

| w | 1 | have | 40 | du |
|---|---|------|----|----|
|   |   |      |    |    |

| lutegers   |  |   |   |                       |
|--|--|---|---|-----------------------|
| Concept  |  | 2 | 0 | Definition or Example |
| domain and range                                   |  |   |   |                       |
| functions  |  |   |   |                       |
| independent and<br>dependent variables             |  |   |   |                       |
| multiple<br>representations of<br>linear functions |  |   |   |                       |
| nonlinear functions                                |  |   |   |                       |
| relations  |  |   |   |                       |

Here is an example of how functions are used in the real world.

Activity Use the Internet to research the cost of printing and shipping photos from two different photo printing services. Which company offers the better deal?

See students' work.

# **What Do You Already**

In this activity students assess their prior knowledge choosing a face to represent their knowledge about concepts in the chapter.

After completing the chapter, have students return to this page and have them reevaluate their knowledge level about the content.

# When Will You Use This

# **Activity**

Students may not realize how functions help to represent realworld situations and reveal comparisons.

Use this page to determine if students have skills that are needed for the chapter.



#### **Quick Review**

Students with strong math backgrounds may opt to go directly to the Quick Check.

| REVIEW  |                                |
|---------|--------------------------------|
| Example | Skill                          |
| 1       | Graph on the coordinate plane. |
| 2       | Evaluate expressions.          |



#### **Quick Check**

If students have difficulty with the exercises, present an additional example to clarify any misconceptions they may have.

#### Exercises 1-6

Refer to Example 1 on the student page. Name the ordered pair for point R. (2, 0)

#### Exercises 7-11

Evaluate 6x + 2 if x = -2. -10

# Track Your Progress

Prior to beginning this chapter, have your students rate their knowledge of the objectives it addresses. At the end of the chapter, have your students return to rate their knowledge again. They should see that their knowledge of the key ideas increased.

# Are You Read

# Try the Quick Check below



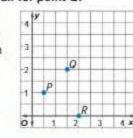
#### Quick Review

Review

# Example 1

Name the ordered pair for point Q.

Start at the origin. Move right along the x-axis until you reach 1.5. Then move up until you reach the y-coordinate, 2. Point Q is located at (1.5, 2).



# Example 2

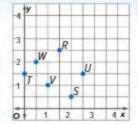
Evaluate 6x + 1 if x = -4.

$$6x + 1 = 6(-4) + 1$$
 Replace x with  $-4$   
=  $-24 + 1$  Multiply 6 by  $-4$ .  
=  $-23$  Add.

# Quick Check

Coordinate Graphing Name the ordered pair for each point.

- 1. R (1.5, 2.5)
- 4. (2.5, 1.5)
- 2. 5 (2, 0.5)
- 5. V (1, 1)
- 3, 7 (0, 1.5)
- 6. W (0.5, 2)



Evaluate Expressions Evaluate each expression if x = -6.

- 7. 3x -18
- 8. 4x + 9 -15
- 9.  $\frac{x}{2}$  -3
- 10. 3x -2



11. The weekly profit of a certain company is 48x - 875, where x represents the number of units sold. Find the weekly profit if the company sells 37 units. \$901

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.















Lesson 1

Functions

**Essential Question** 

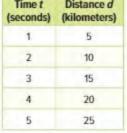
HOW can we model

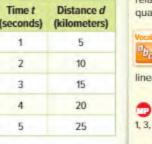
# Representing Relationships

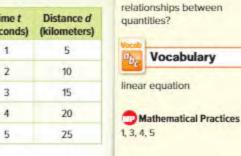
# Real-World Link

Space To achieve orbit, the space shuttle must travel at a rate of about 5 kilometers per second. The table shows the total distance d that the craft covers in certain periods of time t.

| 1. | Write an algebraic expression for the    |
|----|--|
|    | distance in kilometers for any number of |
|    | seconds t 5t                             |



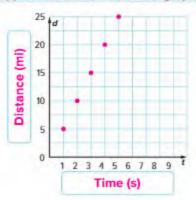




Describe the relationship in words. The distance is 5 times the number of seconds

3. Graph the ordered pairs. Describe the shape of

The points appear to be in a line, so the graph is linear.



Which Mathematical Practices did you use? Shade the circle(s) that applies.

- 1) Persevere with Problems
- 2 Reason Abstractly
- 3 Construct an Argument
- Model with Mathematics
- (5) Use Math Tools 6 Attend to Precision (7) Make Use of Structure

8 Use Repeated Reasoning

# Focus narrowing the scope

Objective Translate tables and graphs into linear equations.

# Coherence connecting within and across grades

### Previous

Students wrote an equation in the form y = mx + b to model linear relationships.

#### Now

Students will translate tables and graphs into linear equations.

#### Next

Students will use tables and graphs to represent relations.

# Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 273.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **Launch the Lesson**

#### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

 Small Group Grocery Introduce the lesson by having small groups act out a grocery store

purchase. Set up a produce section with potatoes or another item that is purchased by the kilogram Have one student play the role of the grocer and choose several students to play customers. Have the first student ask to buy 3 kilograms of potatoes. The grocer says that will cost AED6. The next student asks to buy 5 kilograms at a cost of AED10. The last student asks for 1 kilogram at a cost of AED2. Ask the class if they see a relationship between the number of pounds and the cost. Have them make a table of values. When the activity is complete, have students work in groups to complete Exercises 1-3. 21, 2, 4

# **Teach the Concept**

Ask the scaffolded questions for each example to differentiate instruction.

# **Examples**

- Translate a table into an equation.
- How do the values for quarts change? They increase by 1.
  - How do the values for liters change? They increase by
- · Is the rate of change constant? yes
- What is another word for the constant rate of change?
- 2. Use an equation to solve a real-world problem.
- How can you verify that the equation from Example 1 is correct? Substitute the value for each number of guarts from the table for q in the equation and determine the value for liters. Check that the solution matches the corresponding value for the number of liters in the table.
- How would you find the number of liters in 8 quarts? Replace q with 8 in the equation and multiply.
  - About how many liters are in 8 quarts? about 7.6 liters
- Is the solution for the number of liters in 8 quarts reasonable? How do you know? yes; Sample answer: The number of liters in 1 quart is 0.95, which is a little less than 1. So, the number of liters in 8 quarts should be a little less than 8.

### Need Another Example?

The table shows the relationship between miles and kilometers. Write an equation to

| Kilometers, k | 1    | 2    | 3    | 4    |
|---------------|------|------|------|------|
| Miles, m      | 0.62 | 1.24 | 1.86 | 2.48 |

find the number of miles in any number of kilometers. Describe the relationship in words then use the equation to find the number of miles in 20 kilometers. m = 0.62k; There is 0.62 mile in one kilometer; 12.4 mi

#### Work Zone

Variables

Recall you can use any

letter to represent the

using those letters.

independent and dependent

variables. If you graph the equation, label your axes

# **Tables, Graphs, and Equations**

Recall that an equation is a mathematical sentence stating that two quantities are equal. A linear equation is an equation with a graph that is a straight line. Some equations contain more than one variable.



# Examples

The table shows the number of liters in quarts of liquid.

 Write an equation to find the number of liters in any number of quarts. Describe the relationship in words.

> The rate of change is the rate that describes how one quantity changes in relation to another quantity. The rate of change of quarts to liters

is 
$$\frac{1.9 - 0.95}{2 - 1} = \frac{0.95}{1}$$
 or 0.95 liter in

every quart.

Let  $\ell$  represent the liters and q represent the quarts. The equation is  $\ell = 0.95q$ .

2. About how many liters are in 8 quarts?

 $\ell = 0.95a$ 

Write the equation.

 $\ell = 0.95(8)$ 

Replace q with 8.

 $\ell = 7.6$ 

There are about 7.6 liters in 8 quarts.

#### Got it? Do these problems to find out.

The total cost of tickets to the school play is shown in the table.

- Write an equation to find the total cost of any number of tickets. Describe the relationship in words.
- b. Use the equation to find the cost of 15 tickets.

| Number of<br>Tickets, t | Total<br>Cost (AED), c |  |
|-------------------------|------------------------|--|
| 1                       | 4.50                   |  |
| 2                       | 9.00                   |  |
| 3                       | 13.50                  |  |
| 4                       | 18.00                  |  |

Quarts, q Liters, &

0.95

1.9

2.85

3.8

4.75

+0.95

+0.95

+0.95

+0.95

1

2

3

4

5

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c = 4.5t; Each

ticket costs

AED 4.50.

AED 67.50

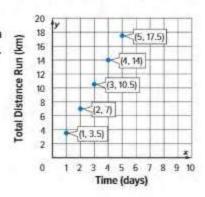
Functions



# Examples

The total distance Khalifa ran in one week is shown in the graph.

3. Write an equation to find the number of kilometers ran y after any number of days x.



Find the rate of change or the slope of the line.

Step 1 
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
 Definition of slope

$$m = \frac{14-7}{4-2}$$
  $(x_1, y_1) = (2, 7); (x_2, y_2) = (4, 14)$ 

$$m = \frac{7}{2}$$
 or 3.5 Simplif

To find the y-intercept, use the slope and the coordinates of a point to write the equation of the line in slopeintercept form.

$$y = mx + b$$
 Slope-intercept form

$$y = 3.5x + b$$
 Replace in with the slope, 3.5.

$$7 = 3.5(2) + b$$
 Use the point (2, 7).  $x = 2, y = 7$ 

$$0 = b$$
 Solve for  $b$ .

The slope is 3.5 and the y-intercept is 0. So, the equation of the line is y = 3.5x + 0 or y = 3.5x.

4. How many kilometers will Khalifa run after 2 weeks?

y = 3.5x

y = 3.5(14)There are 14 days in 2 weeks, Replace x with 14.

v = 49

Khalifa will run 49 kilometers in 2 weeks.

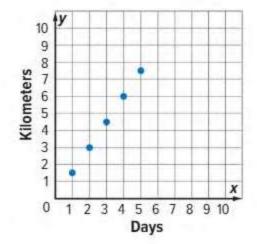
# **Examples**

- 3-4. Translate a graph into an equation. Use an equation to solve a real-world problem.
- Example 3: Is there a constant rate of change? Explain. yes; Sample answer: The rise over run is the same for any two points.
  - Example 4: How many days are in two weeks? 14 days
- Example 3: How would you find the slope of the line that runs through the points? Choose two points on the change in y line and find change in x\*
  - Example 3: Using the slope, how could you write an equation for the line? What is the equation? Sample answer: Use the slope and a point on the line to find the y-intercept. Then write the equation in slope-intercept form; v = 3.5x
  - Example 3: What is the y-intercept? 0
  - Example 4: How would you find the number of kilometers ran after two weeks? Replace x with 14 in the equation and simplify.
- Bl Is this relationship proportional or non-proportional? Explain. proportional; Sample answer: The equation can be

written in the form y = mx.

### Need Another Example?

The graph shows the total number of kilometers Hala hiked. Write an equation to find the number of kilometers hiked after any number of days. Then use the equation to find the number of kilometers Hala will hike after 1 week. y = 1.5x; 10.5 km



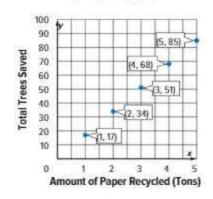
# Watch Out!

Common Error Students may have trouble writing an equation given a graph. Remind them to find the rate of change between points and substitute that number for m in y = mx or y = mx + b form.

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Got it? Do these problems to find out.

The number of trees saved by recycling paper is shown.



- c. Write an equation to find the total number of trees y that can be saved for any number of tons of paper x.
- d. Use the equation to find how many trees could be saved if 500 tons of paper are recycled.

Equation

Graph

Key Concept

c. y = 17x

a. 8,500 trees

# Multiple Representations of Linear Equations

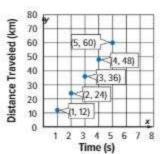
Distance traveled is equal to 12 kilometers per second times the number of seconds.

d = 12s

Table

Words

| Time<br>(seconds) | Distance<br>(kilometers) |  |
|-------------------|--------------------------|--|
| 1                 | 12                       |  |
| 2                 | 24                       |  |
| 3                 | 36                       |  |
| 4                 | 48                       |  |
| 5                 | 60                       |  |



Words, equations, tables, and graphs can be used to represent linear relationships.

# Examples

Shaikha competes in jump rope competitions. Her average rate is 225 jumps per minute.

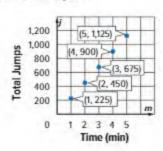
Write an equation to find the number of jumps in any number

Let j represent the number of jumps and m represent the minutes.

The equation is j = 225m.

6. Make a table to find the number of jumps in 1, 2, 3, 4, or 5 minutes. Then graph the ordered pairs.

| m | 225m   | j     |
|---|--------|-------|
| 1 | 225(1) | 225   |
| 2 | 225(2) | 450   |
| 3 | 225(3) | 675   |
| 4 | 225(4) | 900   |
| 5 | 225(5) | 1,125 |

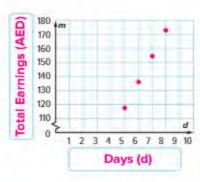


Got it? Do these problems to find out.

Financial Literacy Khamis earns AED 25 for grooming a horse plus AED 18.50 per day for boarding the same horse.

- e. Write an equation to find the amount of money Khamis earned m for grooming a horse once and boarding it for any number of days d.
- f. Make a table to find his earnings for 5, 6, 7, or 8 days. Then graph the ordered pairs.

| ď | 25 + 18.5d   | m &    |
|---|--------------|--------|
| 5 | 25 + 18.5(5) | 117.50 |
| 6 | 25 + 18.5(6) | 136.00 |
| 7 | 25 + 18.5(7) | 154.50 |
| 8 | 25 + 18.5(8) | 173.00 |



**Functions** 



#### and Reflect

A gym charges an annual membership fee of AED 10 but you must pay AED 9.50 for each visit. What equation could be used to represent this real-world situation?

$$y = 9.50x + 10$$



$$e. m = 25 + 18.5d$$

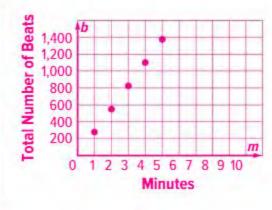
# **Examples**

- Represent a relationship with an equation.
- What does the variable i represent? the number of jumps *m*? minutes
- What is the equation that represents this situation? j = 225m
- Why can we use j instead of y and m instead of x? Sample answer: The letter we choose for our variable does not matter. In this case, it makes sense to use j for jumps and m for minutes.
- 6. Represent a relationship with a table and graph.
- How many jumps will she have in two minutes? 450 three? 675 four? 900 five? 1.125
- What ordered pairs could be used to represent the number of jumps in 1, 2, 3, 4, and 5 minutes? (1, 225), (2, 450), (3, 675), (4, 900), (5, 1,125)
- Describe two different ways you can complete the j-values in the table. Sample answer: Start at 225 and add 225 for each successive row or multiply each number of minutes by 225.

#### Need Another Example?

The average heart rate of a chicken is 275 beats per minute. Write an equation to find the number of beats in any number of minutes. Make a table to find the number of beats in 1, 2, 3, 4, or 5 minutes. Then graph the ordered pairs. b = 275m

| m | 275m   | b     |
|---|--------|-------|
| 1 | 275(1) | 275   |
| 2 | 275(2) | 550   |
| 3 | 275(3) | 825   |
| 4 | 275(4) | 1,100 |
| 5 | 275(5) | 1,375 |



# **Guided Practice**

**Formative Assessment** Use these exercises to assess students' understanding of the concepts in this lesson.



If some of your students are not ready for assignments, use the differentiated activities below.

in a group of 4 students. Each student is responsible to ensure that every other group member understands how to write an equation in y = mx form from a table of values and how to use that equation to determine an unknown value. Then have the group split into pairs to complete Exercise 2. Students should ask for support, if needed, prior to having students work individually to complete Exercises 3 and 4. Then have them share their responses with a partner.  $\bigcirc$  1, 2, 4

problem similar to Exercise 3. Have them trade their problems with a partner. Each partner should create a table of values, create a graph, and write an equation that represents the relationship. Have students check each other's work, and discuss and resolve any differences. 21, 2, 4

# **Guided Practice**



100 150

- The table shows the total number of text messages that Omar sent over 4 days. (Examples 1 and 2)
- a. Write an equation to find the total number of messages sent in any number of days. Describe the relationship in words.

m = 50d; He sends an averages of 50 messages each day.

- b. Use the equation to find how many text messages
   Omar would send in 30 days. 1,500 messages
- 2. Financial Literacy The graph shows the amount of money Rasheed's family budgets for food each month. Write an equation to find the total amount of money c budgeted in any number of months m. Use the equation to determine how much money Rasheed's family should budget for

12 months. (Examples 3 and 4) c = 625m; AED 7,500

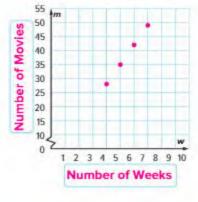
- A store receives an average of 7 new movies per week. (Examples 5 and 6)
  - a. Write an equation to find the number of new movies m in any number of weeks w. m = 7w
  - b. Make a table to find the number of new movies received in 4, 5, 6, or 7 weeks. Then graph the ordered pairs.

| w | 7w   | m  | 1 |
|---|------|----|---|
| 4 | 7(4) | 28 |   |
| 5 | 7(5) | 35 |   |
| 6 | 7(6) | 42 |   |
| 7 | 7(7) | 49 |   |

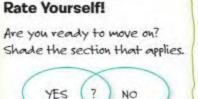
2,800 2,400 2,400 1,600 1,200 800 400 1, 625) 0 1 2 3 4 5 6 7 Number of Months

Number of Days, d

Total Messages, m 50



4. Q Building on the Essential Question How can you use a graph to write an equation? Sample answer: Choose two points on the graph and find the slope. Then use the slope and one point in the slope-intercept form of an equation to find the y-intercept. Then write the equation.



CODYNST & MONTHWEST EDUC

My Hamework

# Independent Practice

The number of baskets a company produces each day is shown in the table. (Examples 1 and 2)



 Write an equation to find the total number of baskets crafted in any number of days. Describe the relationship in words.

b = 45d; Forty-five baskets are produced every day.

b. Use the equation to determine how many baskets the company makes in one non-leap year. 16,425 baskets

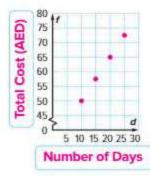
| Number<br>of Days, d | Total<br>Baskets, b |
|----------------------|---------------------|
|                      | 45                  |
| 2                    | 90                  |
| 3                    | 135                 |
| 4                    | 180                 |

- 2. A type of dragonfly is the fastest insect. The graph shows how far the dragonfly can travel. (Examples 3 and 4)
  - a. Write an equation to find how far the dragonfly can travel d in any number of seconds s. d = 23s
  - b. Use the equation to determine how far the dragonfly can travel in one minute. 1,380 ft

| +d |          |
|----|----------|
| 20 | (5, 115) |
| 0  | (4, 92)  |
|    | (3, 69)  |
|    | (2, 46)  |
|    | (1, 23)  |

- A library charges a late return fee of AED 35 plus AED 1.50 per day that a book is returned late. (Examples 5 and 6)
  - a. Write an equation to find the total late fee f for any number of days late d f = 3.5 + 0.15d
  - b. Make a table to find the total fee if a book is 10, 15, 20, or 25 days late. Then graph the ordered pairs.

| 7 | f    | 35 + 1.5d    | d  |
|---|------|--------------|----|
| 4 | 50   | 35 + 1.5(10) | 10 |
|   | 57.5 | 35 + 1.5(15) | 15 |
|   | 65   | 35 + 1.5(20) | 20 |
|   | 72.5 | 35 + 1.5(25) | 25 |



ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

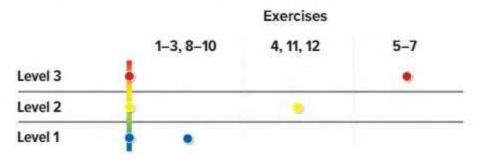
# **Practice and Apply**

#### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

#### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



### Suggested Assignments

You can use the table below that includes exercises of all complexity levles to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                   |  |
|---------------------------------|-------------------|-------------------|--|
| AL                              | Approaching Level | 1–3, 5, 7, 11, 12 |  |
| 01                              | On Level          | 1, 3–5, 7, 11, 12 |  |
| BL                              | Beyond Level      | 4–7, 11, 12       |  |

| MATHEMATICAL PRACTICES  |             |  |
|---|-------------|--|
| Emphasis On   | Exercise(s) |  |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 6           |  |
| 3 Construct viable arguments and critique the reasoning of others.            | 4           |  |
| 4 Model with mathematics.   | 5, 7        |  |
| 5 Use appropriate tools strategically.  | 10          |  |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

# TICKET Out the Door

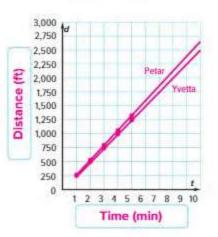
There are 1,000 meters in every kilometer. Have students write an equation to find the number of meters m in any number of kilometers k. Then have them find the number of feet in 2.5 kilometers. m = 1,000k; 2,500 m

- Multiple Representations The two fastest times for swimming the English Channel belong to Petar Stoychev and Yvetta Hlaváčová. Petar's average speed was 80 meters per minute. Yvetta's average speed was 75 meters per minute.
- a. Tables Complete the table of ordered pairs in which the x-coordinate represents the time and the y-coordinate represents the total distance swum in 1, 2, 3, 4, and 5 minutes.

| b. | Graphs   | Graph each set of ordered pairs on the |
|----|----------|--|
|    | coordina | ate plane.                             |

- c. Algebra Write an equation for each swimmer to find the number of meters swum d in any number of minutes t. Petar: d = 265t; Yvetta: d = 249t
- d. Numbers If Petar Stoychev swam the Channel in 6 hours, 57 minutes, and 50 seconds, approximately how wide in kilometers is the English Channel? (Hint: 1 km = 1,000 m) about 34 kilometers

| Time (min) | Petar | Yvetta |
|------------|-------|--------|
| 1          | 80    | 75     |
| 2          | 530   | 498    |
| 3          | 795   | 747    |
| 4          | 1,060 | 996    |
| 5          | 1,325 | 1,245  |



# H.O.

# H.O.T. Problems Higher Order Thinking

 Model with Mathematics Write an equation with two variables that represents a real-world situation. Sample answer: d = 60t;

A car is traveling at a rate of 60 kilometers per hour.

Persevere with Problems The table shows the areas of circles with radii from 1 through 3 m.

| Radius (m), r | 1 | 2  | 3  |
|---------------|---|----|----|
| Area (m²), A  | π | 4π | 9π |

Recall that  $\pi$  has a value of about 3. Write an equation in two variables to represent the relation in the table.  $\mathbf{A} = \pi \cdot \mathbf{r} \cdot \mathbf{r}$  or  $\mathbf{A} = \pi \mathbf{r}^2$ 

 Model with Mathematics Write about a real-world situation that can be represented by the equation y = 4x.

See students' work.

Name

My Homework

# **Extra Practice**

- 8. The table shows the number of square inches per square foot.
  - a. Write an equation to find the number of square inches i in any number of square feet f. Describe the relationship in words.

i = 144f; There are 144 square inches in every square foot. Homework The rate of change is  $\frac{288-144}{2-1}$  or 144 square inches for every square foot. So the equation is i = 144f.

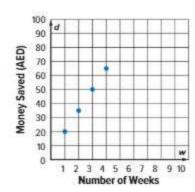
| Square<br>Feet, f | Square<br>Inches, i |
|-------------------|---------------------|
| 1                 | 144                 |
| 2                 | 288                 |
| 3                 | 432                 |
| 4                 | 576                 |



b. Use the equation to determine how many square inches are in

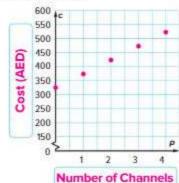
15 square feet. 2,140 square inches Use the equation i = 144f. i = 144(15) 1=2,160

- 9. Financial Literacy Shaima is saving money for a school trip. The graph shows how much money she has saved over 4 weeks.
  - a. Write an equation to find how much money d Shaima can save over w weeks. d = 15w + 5
  - b. Use the equation to determine how much money Shaima can save in 24 weeks. AED 365



- 10. Dse Math Tools Clearview Cable charges AED 325 a month for basic cable television. Each premium channel selected costs an additional AED 49.50 per month.
  - a. Write an equation to find the total monthly cost c for any number of premium channels p. c = 325 + 49.5p
  - b. Make a table to show the monthly cost for 0, 1, 2, 3, and 4 premium channels. Then graph the ordered pairs.

| P | 325 + 49.5p   | c      |
|---|---------------|--------|
| 0 | 325 + 495(0)  | 325.00 |
| 1 | 325 + 49.5(1) | 374.50 |
| 2 | 325 + 49.5(2) | 424.00 |
| 3 | 325 + 49.5(3) | 473.50 |
| 4 | 325 + 49.5(4) | 523.00 |



# Power Up! Test Practice

Exercises 11 and 12 prepare students for more rigorous thinking needed for assessment.

- 11. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

  Depth of Knowledge DOK1

  Mathematical Practice MP1

  Scoring Rubric

  1 point Students correctly answer the question.
- 12. This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.

  Depth of Knowledge DOK3

  Mathematical Practice MP1, MP4

  Scoring Rubric

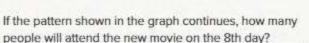
  2 points Students correctly graph the points and state the number of attendees.

  1 point Students correctly graph the points OR state the number of attendees.

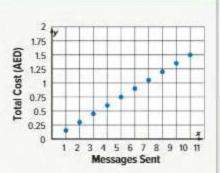
# Power Up! Test Practice

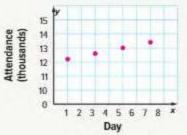
- The graph represents the total cost to send text messages.
   Based on the graph, which of the following costs are correct? Select all that apply.
  - 32 text messages cost AED 4.80
  - 50 text messages cost AED 7.50
  - 60 text messages cost AED 9.50
  - 70 text messages cost AED 10.50
- 12. The table shows the number of people who attended a new movie over the course of a week. Graph the relationship on the coordinate plane.

| Day        | 1      | 3      | 5      | 7      |
|------------|--------|--------|--------|--------|
| Attendance | 12,200 | 12,600 | 13,000 | 13,400 |



13,600 people





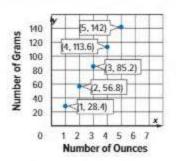
# **Spiral Review**

- The graph at the right shows the approximate number of grams in one ounce.
  - Write an algebraic equation to represent the data in the graph.

y = 28.4x

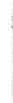
b. Use the expression to find the number of grams in 150 ounces.

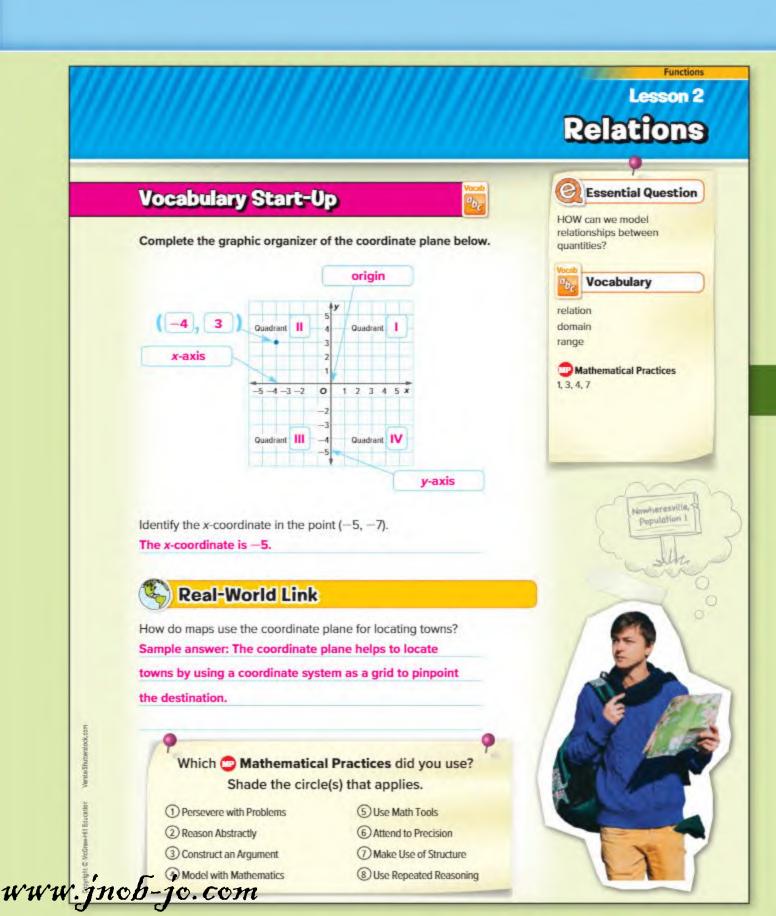
4,260 g



 Write an algebraic expression to represent the phrase the difference between two times b and eleven.

2b - 11





### Focus narrowing the scope

Objective Represent relations using tables and graphs.

# Coherence connecting within and across grades

### Previous

Students translated tables and graphs into linear equations.

#### Now

Students will represent relations using tables and graphs.

#### Next

Students will determine whether a relation is a function.

# Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 281.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **Launch the Lesson**

#### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

 Find the Fib Have students work in groups of 3–4 to label a copy of the coordinate plane with at least one error. Then have students trade papers with another group. Each group should determine the error(s). Have students correct the errors. 21, 2, 3, 4

# **Alternate Strategies**

 Provide students with a word bank of terms: origin, x-axis, y-axis, and quadrants I, II, III, and IV.

B Have students determine whether the sign of the x- and y-coordinates of any point will be positive, negative, or zero, depending upon its location in the coordinate plane.

# **2** Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

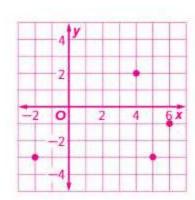
# **Examples**

- 1. Represent a relation as a table and a graph.
- In the ordered pairs, which value is the x-coordinate? the first value
  - In the ordered pairs, which value is the y-coordinate? the second value
- Which values make up the domain of the relation? the x-values
  - Which values make up the range of the relation? the y-values
- Which representation makes it easier for you to see the relationship between the x- and y-coordinates?
   See students' preferences.

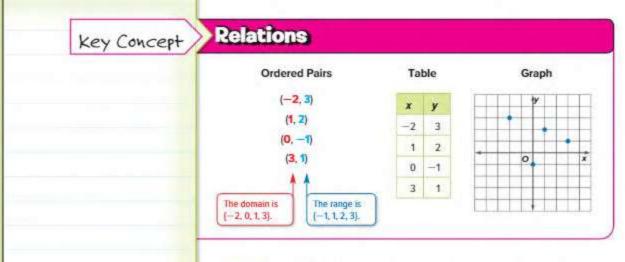
#### **Need Another Example?**

Express the relation  $\{(4, 2), (6, -1), (5, -3), (-2, -3)\}$  as a table and a graph. Then state the domain and range.

| х  | у  |
|----|----|
| 4  | 2  |
| 6  | -1 |
| 5  | -3 |
| -2 | -3 |



D: {-2, 4, 5, 6}; R: {-3, -1, 2}



A **relation** is any set of ordered pairs. Relations can be represented as a table and as a graph. The **domain** of the relation is the set of x-coordinates. The **range** of the relation is the set of y-coordinates.

# Example

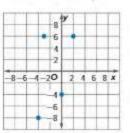
**Domain and Range** 

If a term in the domain or range appears more than once, only write it one time. In Example 1, the value 6 appears twice in the range.  Express the relation {(2, 6), (-4, -8), (-3, 6), (0, -4)} as a table and a graph. Then state the domain and range.

Place the ordered pairs in a table with x-coordinates in the first column and the y-coordinates in the second column.

| x  | y  |
|----|----|
| 2  | 6  |
| -4 | -8 |
| -3 | 6  |
| 0  | -4 |

Graph the ordered pairs on a coordinate plane.

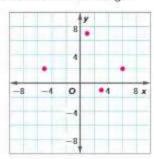


The domain is [-4, -3, 0, 2]. The range is [-8, -4, 6].

Got it? Do this problem to find out.

a. Express the relation [(-5, 2), (3, -1), (6, 2), (1, 7)] as a table and a graph. Then state the domain and range.

| ×  | у   | 1 |
|----|-----|---|
| -5 | 2 / | / |
| 3  | -1  |   |
| 6  | 2   |   |
| 1  | 7   |   |



D: (-5, 1, 3, 6); a. R: [-1, 2, 7]

SATION WINEY

**Functions** 



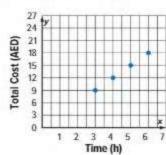
### Example

It costs AED 3 per hour to park at the Amusement Park.

in which the x-coordinate represents the hours and the y-coordinate represents the total cost for 3, 4, 5, and 6 hours.

| x | y  |
|---|----|
| 3 | 9  |
| 4 | 12 |
| 5 | 15 |
| 6 | 18 |

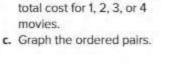
a. Make a table of ordered pairs | b. Graph the ordered pairs.



Got it? Do these problems to find out.

A movie rental store charges AED 39.50 per movie rental.

b. Make a table of ordered pairs in which the x-coordinate represents the number of movies rented and the y-coordinate represents the total cost for 1, 2, 3, or 4 movies.



180 160 140 Cost (AED) 120 100 Total 40 2 **Number of Movies Rented** 

| x | У     |
|---|-------|
| 1 | 39.5  |
| 2 | 79    |
| 3 | 118.5 |
| 4 | 158   |

# **Examples**

2. Represent a relation using a table and a graph.



· How can you complete the table? Multiply the number of hours by AED3.



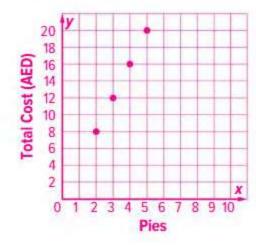
- What equation could be used to show the total cost y to park for any number of hours x? y = 3x
  - Using the equation, how much would it cost to park for 3, 4, 5, and 6 hours? AED9, AED12, AED15, AED18
  - · What ordered pairs represent these values? (3, 9), (4, 12), (5, 15), (6, 18)
- BD How much would it cost to park for a full day? AED72 Need Another Example?

Mohammad is buying pies for an event. The price for one pie is AED4.

a. Make a table of ordered pairs in which the x-coordinate represents the number of pies and the y-coordinate represents the total cost for 2, 3, 4, and 5 pies.

| Pies | Total Cost (AED) |
|------|------------------|
| 2    | 8                |
| 3    | 12               |
| 4    | 16               |
| 5    | 20               |

b. Graph the ordered pairs.



# **Guided Practice**

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.



If some of your students are not ready for assignments, use the differentiated activities below.

Gallery Walk Have students work in small groups to create a table, set of ordered pairs, and a graph for one of Exercises 1–3. Each representation should be created using a separate piece of paper. Students should not label the representation with the exercise number. Have them post the representations around the room. Students should walk around the room and determine which representation matches which exercise. 21, 2, 4

another source to research a real-world situation in which a relationship was represented using a table or a graph. Have them represent the relationship using another form (table, graph, or set of ordered pairs). 

1, 2, 5

# Watch Out!

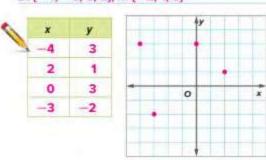
**Common Error** Students may think that because the graph of the ordered pairs does not form a line there is no relation. Remind them that a relation is any set of ordered pairs and it does not have to form a line.

# **Guided Practice**

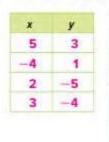


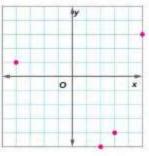
Express each relation as a table and a graph. Then state the domain and range. (Example 1)

1. {(-4, 3), (2, 1), (0, 3), (-3, -2)} D: {-4, -3, 0, 2}; R: {-2, 1, 3}



2. ((5, 3), (-4, 1), (2, -5), (3, -4)) D: {-4, 2, 3, 5); R: {-5, -4, 1, 3}

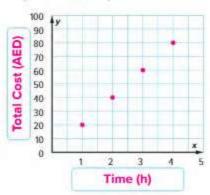




- At a vacation resort, you can rent a personal watercraft for AED 20 per hour. (Example 2)
  - a. Make a table of ordered pairs in which the x-coordinate represents the number of hours and the y-coordinate represents the total cost for 1, 2, 3, or 4 hours.

| x | y  |
|---|----|
| 1 | 20 |
| 2 | 40 |
| 3 | 60 |
| 4 | 80 |

b. Graph the ordered pairs.



4. Building on the Essential Question How do tables and graphs represent relations?

Sample answer: A set of ordered pairs is a relation.

They can be represented by a table with a column for the x-values and a column for the y-values. The ordered pairs can also be graphed on a coordinate plane.

#### Rate Yourself!

How confident are you about relations? Check the box that applies.



OLDABLES Time to update your Foldable!

Suču z posawa sem

Name

My Hamework

# **Independent Practice**

Express each relation as a table and a graph. Then state the domain and range. (Example 1)

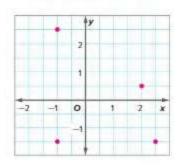
**1** ((8, 5), (-6, -9), (2, 5), (0, -8))

2.  $\left\{ \left(2\frac{1}{2}, -1\frac{1}{2}\right), \left(2, \frac{1}{2}\right), \left(-1, 2\frac{1}{2}\right), \left(-1, -1\frac{1}{2}\right) \right\}$ D:  $\left\{ -1, 2, 2\frac{1}{2} \right\}$ ; R:  $\left\{ -1\frac{1}{2}, \frac{1}{2}, 2\frac{1}{2} \right\}$ 

D: {-6, 0, 2, 8}; R: {-9, -8, 5}

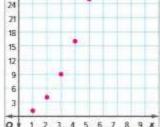
| 1 | x  | у  |    |    | 8  |   |     |
|---|----|----|----|----|----|---|-----|
| 1 | 8  | 5  |    | -  |    |   |     |
|   | -6 | -9 |    |    | 4  |   |     |
|   | 2  | 5  | -R | -4 | 0  | 4 | 8 x |
|   | 0  | -8 |    |    |    | H |     |
|   |    |    |    | -  | -9 |   |     |
|   |    |    |    |    | -8 | - | -   |

| x    | y               |
|------|-----------------|
| 21/2 | $-1\frac{1}{2}$ |
| 2    | 1 2             |
| -1   | 2 1 2           |
| -1   | $-1\frac{1}{2}$ |



Copy and Solve Draw the table and graph on a separate sheet of paper. A company can manufacture 825 small cars per day. (Example 2) 3–4. See Answer Appendix.

- Make a table of ordered pairs in which the x-coordinate represents the number of days and the y-coordinate represents the total number of cars produced in 1, 2, 3, 4, and 5 days.
- Graph the ordered pairs.
- (a) Multiple Representations Refer to the table at the right.
  - a. Words Describe the pattern, if any, in the table. To get the y-value, the x-value was multiplied by itself.
  - Numbers Write the ordered pairs (x, y).
     (1, 1), (2, 4), (3, 9), (4, 16), (5, 25)
- x y
  1 1
  2 4
  3 9
  4 16
  5 25



- c. Graphs Graph the ordered pairs on a coordinate plane.
- d. Words Describe the graph. How is it different from the other real-world graphs in this lesson?

Sample answer: This graph curves upward. The points in all of the other graphs in the lesson lie in a straight line.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

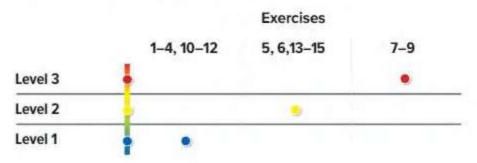
# Practice and Apply

#### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

#### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



#### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                      |
|---------------------------------|-------------------|----------------------|
| AL                              | Approaching Level | 1–5, 8, 9, 14, 15    |
| OL)                             | On Level          | 1, 3–6, 8, 9, 14, 15 |
| BL                              | Beyond Level      | 5–9, 14, 15          |

| MATHEMATICAL PRACTICES  |             |
|---|-------------|
| Emphasis On   | Exercise(s) |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 7           |
| 3 Construct viable arguments and critique the reasoning of others.            | 5, 9        |
| 4 Model with mathematics.   | 6, 8        |
| 7 Look for and make use of structure.   | 13          |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

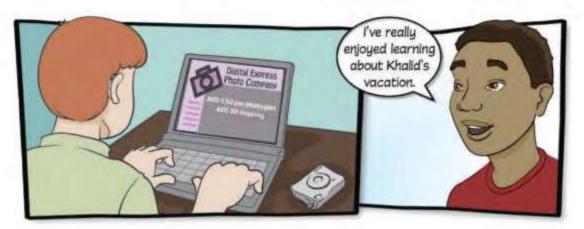
#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

# TICKET Out the Door

Have students find the domain and range of the relation  $\{(3, 7), (-2, 5), (-3, -3), (4, -1)\}$ . D:  $\{-3, -2, 3, 4\}$ ; R:  $\{-3, -1, 5, 7\}$ 

 Model with Mathematics Refer to the graphic novel frame below for Exercises a-c. Show your work on a separate sheet of paper. a-c. See Answer Appendix.



- a. Make a table to find the cost to print 10, 20, 30, 40 pictures.
- b. Graph the ordered pairs.
- c. How much would it cost for Khalid to print and ship 75 pictures? 100?

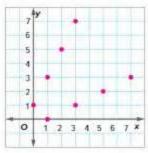


### H.O.T. Problems Higher Order Thinking

- Persevere with Problems Refer to the table at the right.
  - a. Graph the ordered pairs.
  - Reverse the y-coordinates and x-coordinates in each ordered pair.

| (1, 0), ( | 3, 1), | (5, 2), | (7, 3) |
|-----------|--------|---------|--------|
|-----------|--------|---------|--------|

| x | y |
|---|---|
| 0 | 1 |
| 1 | 3 |
| 2 | 5 |
| 3 | 7 |
|   |   |



- c. Graph the new ordered pairs on the same coordinate plane in part a.
- d. Describe the relationship between the two sets of ordered pairs.
  Sample answer: The distance between each point in the original table and the x-axis is the same as the distance between the points with the reversed ordered pairs and the y-axis.
- Model with Mathematics Describe a real-world situation that can be represented using a table and a graph. Sample answer: The number of movie tickets bought and the total cost of the tickets can be represented using a table and graph.
- Find the Error Ayesha says that the domain of the relation {(2, 3), (-4, 2), (0, -4), (1, 5)} is {-4, 2, 3, 5}. Find her mistake and correct it. Sample answer: The domain is the set of x-coordinates. Ayesha listed the set of y-coordinates; {-4, 0, 1, 2}

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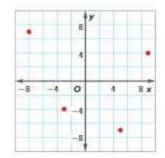
Name

My Homework

# **Extra Practice**

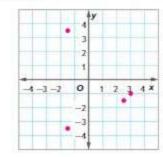
Express each relation as a table and a graph. Then state the domain and range.

| X  | у  |
|----|----|
| 9  | 4  |
| 5  | -7 |
| -3 | -4 |
| -8 | 7  |



- **11.** {(-1.5, 3.5), (2.5, -1.5), (3, -1), (-1.5, -3.5)
  - D: [-1.5, 2.5, 3];
  - R: (-3.5, -1.5, -1, 3.5)

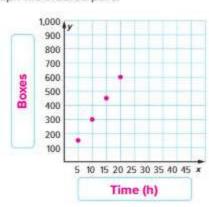
| x    | у    |
|------|------|
| -1.5 | 3.5  |
| 2.5  | -1.5 |
| 3    | -1   |
| -1.5 | -3.5 |



- 12. A candy company produces 30 boxes of candy per hour.
  - a. Make a table of ordered pairs in which the x-coordinate represents the number of hours and the y-coordinate represent the number of boxes of candy in 5, 10, 15, and 20 hours.

| ×  | у   |
|----|-----|
| 5  | 150 |
| 10 | 300 |
| 15 | 450 |
| 20 | 600 |

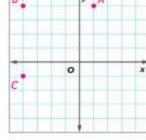
b. Graph the ordered pairs.



13. Coldentify Structure Graph the points in the table on a coordinate plane. Label the points A, B, and C. What are the coordinates of point D if points A, B, C, and D form a square?



4 -44 -4 -1



# Power Up! Test Practice

Exercises 14 and 15 prepare students for more rigorous thinking needed for assessment.

This test item requires students to analyze and solve complex realworld problems through the use of mathematical tools and models.

|                       | the use of mathematical tools and models.   |  |
|-----------------------|---|--|
| Depth of Knowledge    | DOK3  |  |
| Mathematical Practice | MP1, MP4  |  |
| Scoring Rubric        |   |  |
| 2 points              | Students correctly complete the table, the graph, and find the earnings.  |  |
| 1 point               | Students correctly complete the table and find the earnings but have errors in the graph OR students incorrectly complete the table, but graph the points listed and find the earnings based on the incorrect table OR students correctly complete the table and the graph but fail to find the earnings. |  |

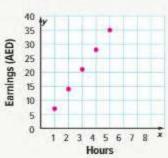
15. This test item requires students to reason abstractly and

| quantitatively when pro | blem solving.                        |
|-------------------------|--------------------------------------|
| Depth of Knowledge      | DOK1                                 |
| Mathematical Practice   | MP1                                  |
| Scoring Rubric          |                                      |
| 1 point                 | Students correctly answer each part. |

# Power Up! Test Practice

14. Ibrahim earns AED 7 an hour for washing cars as a summer job. Complete the table of ordered pairs to show his total earnings for several hours. Then express the relation as a graph.

| Hours Worked | Total Earned |
|--------------|--------------|
| 1            | AED 7        |
| 2            | AED 14       |
| 3            | AED 21       |
| 4            | AED 28       |
| 5            | AED 35       |



How much would Ibrahim earn for 12 hours of washing cars? **AED 84** 

- 15. Determine if each statement about the relation ((3, 7), (5, 1), (6, 4), (2, 5)) is true or false.
  - a. The domain of the relation is (2, 3, 5, 6).
- True False
- b. The range of the relation is {1, 4, 5, 7}.
- True False
- c. The value 5 is a member of both the domain and range.
- True False

### **Spiral Review**

Name the ordered pair for each point. 16.  $P(\frac{1}{2}, -1)$  17.  $Q(\frac{3}{4}, \frac{1}{2})$ 

16. 
$$P\left(\frac{1}{2}, -1\right)$$

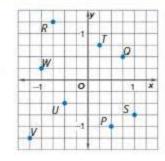
17. 
$$Q^{(\frac{3}{4},\frac{1}{2})}$$

18. 
$$R\left(-\frac{3}{4}, 1\frac{1}{4}\right)$$
 19.  $S\left(1, -\frac{3}{4}\right)$ 

20. 
$$T\left(\frac{1}{4}, \frac{3}{4}\right)$$

21. 
$$U\left(-\frac{1}{2}, -\frac{1}{2}\right)$$

20. 
$$T = \begin{pmatrix} \frac{1}{4}, \frac{3}{4} \end{pmatrix}$$
 21.  $U = \begin{pmatrix} -\frac{1}{2}, -\frac{1}{2} \end{pmatrix}$  22.  $V = \begin{pmatrix} -\frac{1}{4}, -\frac{1}{4} \end{pmatrix}$  23.  $W = \begin{pmatrix} -\frac{1}{4}, \frac{1}{4} \end{pmatrix}$ 



# **Inquiry Lab**

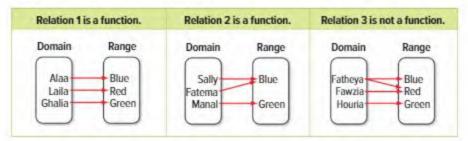
**Relations and Functions** 



#### HOW can I determine if a relation is a function?

Mathematical Practices 1.3.4

Mrs. Abeer asked three members of her class their favorite color. The mapping diagrams below show some possible results.



A function is a special relation in which each member of the domain is paired with exactly one member in the range. In the mapping above, Relation 3 is not a function because Fatheya chose two favorite colors, blue and red.

# **Hands-On Activity**

Mr. Saleh asked his students how many pets they have. Some of the student responses are shown in the table.

| Student Number | 1 | 3 | 6 |
|----------------|---|---|---|
| Number of Pets | 2 | 5 | 7 |

Domain

Complete the mapping diagram shown.

Is the relation a function? Explain.

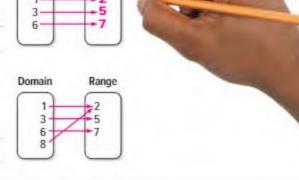
yes; Each member of the domain is

paired with only member of the range.

Suppose Student 8 has 2 pets. Make a mapping diagram of this situation. Is this relation a function? Explain.

Yes; it is a function because each member of

be domain is paired with exactly one



#### Focus narrowing the scope

Objective Determine whether a relation is a function.

# Coherence connecting within and across grades

#### Now

Students will use mapping diagrams to identify functions.

Students will find function values and complete function tables.

# Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 286.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# Launch the Lab

The activity is intended to be used as whole-group activity.

# **Hands-On Activity**

Think-Pair-Draw Have students create reference mappings to display around the room for the following scenarios. Then have them determine if each relation is a function in each instance. Display the mappings with their identification as a function or not a function in the classroom so students can refer to them. 21, 2, 4, 6

- Each element of the domain is paired with exactly one element of the range. (function)
- Some elements of the domain are paired with more than one element of the range. (not a function)
- Bl Have students survey the class on a topic of their choice. Have them create their own domain and range and then determine if the relation is a function. Have them display their mappings around the room.

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# 2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

#### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

# Exercises 1 2 3-4 Level 3 Level 2 Level 1



Think-Pair-Share Give pairs of students one minute to respond to Exercise 2, and then have them share their response with their partner. Then call on one pair to share with the class. 1,3

#### Ask:

 If the domain and range were reversed, would the relation now be a function? Draw a mapping diagram. no; Some elements of the domain are still paired with more than one element of the range; (-2, 0) and (-2, 1); See students' mappings.



Trade-a-Problem For Exercise 3, have students trade their problem with a partner. The partner should check to make sure the relation is not a function. Have them justify their response using the mapping and their own words. 1,3

determine if a relation is a function?" Check for student understanding and provide guidance, if needed.



# Investigate

 Model with Mathematics Students were asked about the number of cell phone minutes they use. Some of the responses are shown in the table. Make a mapping diagram for the relation.

| Student              | Eiman | Tarek | Ahmed | Lamya |
|----------------------|-------|-------|-------|-------|
| Number of<br>Minutes | 275   | 220   | 350   | 275   |



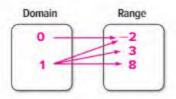
Is this relation a function? Explain. yes; Each member of the domain is paired with exactly one member of the range.



# **Analyze and Reflect**

 Model with Mathematics Make a table and a mapping diagram for the relation ((0, -2), (1, -2), (1, 3), (1, 8)).

| Domain | 0  | 1  | 1 | 1 |
|--------|----|----|---|---|
| Range  | -2 | -2 | 3 | 8 |



Is this relation a function? Explain. no; Some members of the domain are not paired with exactly one member of the range. The x-coordinate of 1 has 3 different corresponding y-coordinates.



### Create

 Use Math Tools Think of a real-world situation that is not a function. Complete the table and mapping diagram for your situation. Sample answers are given.

| Pet<br>Names | Tiny   | Rover<br>Betty | Mimi    |
|--------------|--------|----------------|---------|
| Student      | Hidaya | Huda           | Mahmoud |
|              |        |                |         |



Explain why your situation is not a function. Each member of the domain is not paired with exactly one member of the range. Huda has two pet names.

4. (nquir) HOW can I determine if a relation is a function?

Sample answer: If you make a mapping diagram you can tell if one or more arrows point from the domain to the range.

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# Lesson 3 **Functions**

**Essential Question** 

HOW can we model

quantities?

function function table

1, 2, 3, 4

relationships between

independent variable dependent variable

Mathematical Practices

Vocabulary

# Vocabulary Start-Up



A function is a relation in which every member of the domain (input value) is paired with exactly one member of the range (output value). An example of a function is m = 20n, where m represents the amount of money earned and n represents the number of lawns mowed. In this example, n is the independent variable and m is the dependent variable.

Independent Variable What I think it means Sample answer: The variable that can change.

Dependent Variable What I think it means Sample answer: The variable that is affected when the independent variable changes.

For each situation, determine which unknown is the dependent variable and which one is the independent variable.

| Independent<br>Variable | Equation  | Dependent Variable   |
|-------------------------|---|----------------------|
| number of<br>downloads  | The equation $c = 0.99 \text{ n}$<br>represents the total cost $c$ for<br>n music downloads.                  | cost                 |
| number of hours         | The equation d = 4.5h represents the number of kilometers d Alia can run in h hours.                          | number of kilometers |
| number of goals         | The equation $s = g + 3$<br>represents the final score of<br>the game s after g goals in<br>the final period. | final score          |

Which Mathematical Practices did you use? Shade the circle(s) that applies.

- 1) Persevere with Problems
- 2 Reason Abstractly
- 3 Construct an Argument
- Model with Mathematics www.jnob-jo.com
- (5) Use Math Tools
- (6) Attend to Precision
- (7) Make Use of Structure
- (8) Use Repeated Reasoning

Objective Find function values and complete function tables.

# Coherence connecting within and across grades

### Previous

Students represented relationships using multiple representations.

Focus narrowing the scope

#### Now

Students use function tables to determine the domain and range of a function.

#### Next

Students will use multiple representations to represent linear functions.

# Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 291.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **Launch the Lesson**

#### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BD (IA) Find a Fib Students work in pairs. Divide the pairs into two groups, one to write two facts and one fib about the dependent variable and one to write two facts and one fib about the independent variable. Pairs then trade their facts and fib with another pair in a different group. Each pair of students identifies the facts and fibs. Then the two pairs of students come together as a team to resolve any differences. 21, 3

# Alternate Strategy

Tell students that the dependent variable is related to the output value and the independent variable is related to the input value, which may help differentiate between independent and dependent variables.

Lesson 3 Functions 287

# **Teach the Concept**

**Ask** the scaffolded questions for each example to differentiate instruction.

# **Examples**

- 1. Find function values.
- What does f(−3) mean? find the value of the function when x = −3
- What value do you need to substitute for x? —3
  - How would you find f(−3)? replace x with −3 in the expression 2x + 1 and then simplify
- What does f(x) mean? Sample answer: the value of the function that corresponds to the number x.

### **Need Another Example?**

Find f(-6) if f(x) = 3x + 4. -14

- 2. Make a function table.
- Which variable represents the input value in the function table? x
  - What is used to represent the output value in the function table? f(x)
- How would you find each output value for the input values of -2, -1, 0, and 1? Replace x with each value in the expression x + 5 and simplify.
  - Does the set of input values or the set of output values give the domain of the function? input values
- What represents the independent variable? x the dependent variable? f(x)
  - Why do you think the values of -2, -1, 0, and 1 were chosen? Sample answer: The numbers are in order and represent negative numbers, zero, and a positive number.

### **Need Another Example?**

Choose four values for x to make a function table for f(x) = 4x - 1. Then state the domain and range of the function. See Answer Appendix.

Work Zone

### **Functions**

To find the value of a function for a certain number, substitute the number for the variable x.

$$f(x) = 15x$$

I(x) is read the function of x, or t of x.
It is the output or range.

The input x is any real number. It is the domain.

### Example

1. Find f(-3) if f(x) = 2x + 1.

$$f(x) = 2x + 1$$

Write the function.

$$f(-3) = 2(-3) + 1$$

Substitute -3 for x into the function rule.

$$f(-3) = -6 + 1 \text{ or } -5$$
 Simplify.

So, 
$$f(-3) = -5$$
.

Got it? Do these problems to find out.

Find each function value.

a. 
$$f(2)$$
 if  $f(x) = x - 4$ 

**b.** 
$$f(11)$$
 if  $f(x) = \frac{1}{2}x + 5$ 

# **Function Tables**

You can organize the input, rule, and output into a function table. The variable for the domain is called the independent variable because it can be any number. The variable for the range is called the dependent variable because it depends on the domain.

### Example

2. Choose four values for x to make a function table for f(x) = x + 5. Then state the domain and range of the function.

Substitute each domain value x into the function rule. Then simplify to find the range value. The domain is  $\{-2, -1, 0, 1\}$ . The range is  $\{3, 4, 5, 6\}$ .

| Domain | Rule         | Range |
|--------|--------------|-------|
| x      | f(x) = x + 5 | f(x)  |
| -2     | -2+5         | 3     |
| -1     | -1+5         | 4     |
| 0      | 0+5          | 5     |
| 1      | 1+5          | 6     |

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**Functions** 

#### Got it? Do this problem to find out.

c. Choose four values for x to complete the function table for the function f(x) = x - 7. Then state the domain and range of the function.



### Examples

There are approximately 770 peanuts in a jar of peanut butter. The total number of peanuts p(j) is a function of the number of jars of peanut butter purchased j.

Identify the independent and dependent variables.

Since the total number of peanuts depends on the number of jars of peanut butter, the number of peanuts p(j) is the dependent variable and the jars of peanut butter j is the independent variable.

4. What values of the domain and range make sense for this situation? Explain.

Only whole numbers make sense for the domain because you cannot buy a fraction of a jar. The range values depend on the domain values, so the range will be multiples of 770.

Write a function to represent the total number of peanuts. Then determine the number of peanuts in 7 jars of peanut butter.

| Words    | The number<br>of peanuts | equals | 770 times | the number<br>of jars |
|----------|--------------------------|--------|-----------|-----------------------|
| Function | $\rho(j)$                | =      | 770 -     | j                     |

The function p(j) = 770j represents the situation.

To find the number of peanuts in 7 jars of peanut butter, substitute 7 for j.

$$p(j) = 770j$$
 Write the function.  
 $p(j) = 770(7)$  or 5,390 Substitute 7 for j.

There are 5,390 peanuts in 7 jars of peanut butter.

### Sample answer:

| X  | f(x) = x - 7 | 1(x) | 1 |
|----|--------------|------|---|
| -1 | -1-7         | -8,  |   |
| 0  | 0 - 7        | -7   |   |
| 1  | 1-7          | -6   |   |
| 2  | 2-7          | -5   |   |



What are the similarities and difference among the terms domain, range, independent variable, and dependent variable? Explain below.

Sample answer: The domain is the set of all input values for the independent variable. The value of the independent variable influences the value of the dependent variable. The range is the set of all output values of the dependent variable.

### **Examples**

#### 3-5. Write a function.



- How can you determine the independent variable? The independent variable is the input variable.
  - Do negative values make sense for the independent variable? Explain. No; you cannot have a negative number of jars of peanut butter.
  - · What does j represent? the number of jars of peanut
  - What does p(i) represent? the number of peanuts in i jars of peanut butter



- What is the independent variable? the number of jars of peanut butter the dependent variable? the number of
  - What function is used to represent the situation? p(i) = 770i
  - How can you find the number of peanuts in 7 jars of peanut butter? Replace j with 7 and simplify.



- Would fractions ever make sense for this function? Explain. Sample answer: Yes, you could have a fraction of a jar of peanut butter at your house.
  - How many peanuts are in (x + 5) jars of peanut butter? 770x + 3,850

### Need Another Example?

Fatema buys a can of tuna fish that weighs 120 grams. The total weight w(c) is a function of the number of cans of tuna fish c.

- a. Identify the independent and dependent variables. The number of cans c is the independent variable. The total weight w is the dependent variable.
- Explain what values of the domain and range make sense for this situation. Only whole numbers make sense for the domain because you cannot buy a fraction of a can of tuna fish. The range will be multiples of 120.
- c. Write a function to represent the total weight. Then determine the ounces in 8 cans of tuna fish. w(c) = 120c; 960 q

### **Guided Practice**

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.



If some of your students are not ready for assignments, use the differentiated activities below.

Think-Pair-Share Have students work in pairs. Give students one minute to think through their responses to each part of Exercise 3. Have them exchange thoughts on how to answer each part, then record their collaborative solutions. Have them trade their solutions with another pair of students and discuss any differences. 01, 3, 6

 Trade-a-Problem Have students create their own problem, similar to Exercise 3. Challenge students to create a problem where a negative number or fractions make up the domain and/or range. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. 21,3 The number of

stamps n is the independent variable and the total sales f(n) is the dependent

variable.

Only whole e. numbers make sense for the domain because you cannot buy a fraction of a stamp. The range will be multiples of 4.95.

f(n) = 4.95n; AED 24.75

Got it? Do these problems to find out.

A scrapbooking store is selling rubber stamps for AED 4.95 each. The total sales f(n) is a function of the number of rubber stamps n sold.

- Identify the independent and dependent variables.
- e. What values of the domain and range make sense for this situation? Explain.
- f. Write a function to represent the total sales. Then determine the total sales for 5 stamps.

### **Guided Practice**



- **1.** Find f(4) if f(x) = x 6. (Example 1) -2
- 2. Choose four values for x to make a function table for f(x) = 8 x.

Then state the domain and range of the function. (Example 2)

Sample answer: D: {-3, -1, 2, 4}; R: [11, 9, 6, 4]

| X  | 8-x      | f(x) |
|----|----------|------|
| -3 | 8 - (-3) | 11   |
| -1 | 8 - (-1) | 9    |
| 2  | 8-2      | 6    |
| 4  | 8 - 4    | 4    |

- 3. A hot air balloon can hold 2,500 cubic meters of air. It is being inflated at a rate of 170 cubic meters per minute. The total cubic meters of air a(t) is a function of the time in minutes t. (Examples 3-5)
  - a. Identify the independent and dependent variables.

The air a(t) is the dependent variable and the time t is the independent variable.

b. What values of the domain and range make sense for

this situation? Explain. The domain can be any positive number. Since the

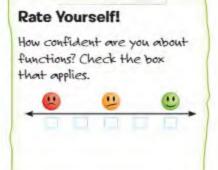
balloon only holds 2,500 cubic meters, the range values can be any rational number greater than 0 but less than 2,500.

c. Write a function to represent the total amount of air. Then determine the total amount of air in 6 minutes. a(t) = 170t; 1,020 m<sup>3</sup>

4. Q Building on the Essential Question How does the domain affect the range in a function?

Sample answer: Any number can be used for the domain.

So, the range depends on the domain.



My Hamework

### Independent Practice

Find each function value. (Example 1)

1. 
$$f(7)$$
 if  $f(x) = 5x^{35}$ 

2. 
$$f(9)$$
 if  $f(x) = x + 13$  22

2. 
$$f(9)$$
 if  $f(x) = x + 13$  22  $f(4)$  if  $f(x) = 3x - 1$  11



Choose four values for x to make a function table for each function. Then state the domain and range of the function. (Example 2)

| x  | 6x-4      | f(x) |
|----|-----------|------|
| -5 | 6(-5) - 4 | -34  |
| -1 | 6(-1)-4   | -10  |
| 2  | 6(2) - 4  | 8    |
| 7  | 6(7) - 4  | 38   |

$$f(x) = 5 - 2x$$

| X  | 5 – 2x    | f(x) |
|----|-----------|------|
| -2 | 5 - 2(-2) | 9    |
| 0  | 5 - 2(0)  | 5    |
| 3  | 5 - 2(3)  | -1   |
| 5  | 5 - 2(5)  | -5   |

| x  | 7 + 3x    | f(x) |
|----|-----------|------|
| -3 | 7 + 3(-3) | -2   |
| -2 | 7 + 3(-2) | -1   |
| 1  | 7 + 3(1)  | 10   |
| 6  | 7 + 3(6)  | 25   |

- 7. In a recent 82-game season, a professional basketball player averaged 20.7 points per game. His approximate total points scored p(g) is a function of the number of games played g. (Examples 3-5)
  - a. Identify the independent and dependent variables. The total points p(g) is the dependent variable and the number of games g is the independent variable.
  - b. What values of the domain and range make sense for this situation? Only whole numbers between and including 0 and 82 make sense for the

domain because you do not want data for a partial game and there are only 82 games in a season. The range will be multiples of 20.7.

c. Write a function to represent the total points scored. Then determine the number of points scored in 9 games.

$$p(g) = 20.7g$$
; 186.3 points

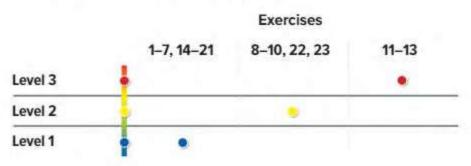
# **Practice and Apply**

### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                       |
|---------------------------------|-------------------|-----------------------|
| <b>AD</b>                       | Approaching Level | 1–7, 9, 11, 22, 23    |
| 01                              | On Level          | 1–7 odd, 8–11, 22, 23 |
| <b>(B)</b>                      | Beyond Level      | 8-13, 22, 23          |

| Emphasis On  | Exercise(s) |
|--|-------------|
| Make sense of problems and persevere in<br>solving them. | 12, 13      |
| 2 Reason abstractly and quantitatively.                  | 11, 21      |
| 4 Model with mathematics.                                | 8           |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.



Have students explain how to find a function value given a rule and a value in the domain. Substitute the value in the function and evaluate. See students' work.

8. Model with Mathematics Refer to the graphic novel frame below for Exercises a-c.



- a. Write a function to represent the total cost c of printing and shipping any number of pictures p. c(p) = 1.5p + 30
- b. Make a function table on a separate piece of paper to find the total cost of printing and shipping 25, 50, 75, and 100 pictures.
- c. On a separate piece of paper, graph the ordered pairs on a coordinate plane. Can you determine how many pictures Khalid can ship for AED 250?

Copy and Solve Find each function value. Show your work on a separate piece of paper.

**9.** 
$$f\left(\frac{5}{6}\right)$$
 if  $f(x) = 2x + \frac{1}{3}$  **2**

**10.** 
$$f\left(\frac{5}{8}\right)$$
 if  $f(x) = 4x - \frac{1}{4}$   $2\frac{1}{4}$ 



### H.O.T. Problems Higher Order Thinking

Peason Abstractly If f(-3) = -8, write a function rule and find the function values for zero, a negative, and a positive value of x.

Sample answer: 
$$f(x) = 2x - 2$$
;  $f(0) = -2$ ,  $f(-4) = -10$ ,  $f(3) = 4$ 

12. Persevere with Problems Write the function rule for each function.

| 3. | ×  | f(x) |
|----|----|------|
|    | -3 | -30  |
|    | -1 | -10  |
|    | 2  | 20   |
|    | 6  | 60   |

y = 2x + 1

$$\begin{array}{c|cccc}
x & y \\
-2 & -5 \\
1 & 1 \\
3 & 5 \\
5 & 9
\end{array}$$

$$y = 2x - 1$$

Persevere with Problems If f(x) = 4x - 3 and g(x) = 8x + 2, find each function value.

f(x) = x - 4

a. f[g(3)] 101

f(x) = 10x

**b.** g[f(5)] 138

**c.** g[f[g(-4)]] -982

My Homework

### **Extra Practice**

Find each function value.

14. 
$$f(-12)$$
 if  $f(x) = 2x + 15$   
 $f(x) = 2x + 15$   
 $f(-12) = 2(-12) + 15$   
 $f(-12) = -24 + 15$ 

**15.** 
$$f(-7)$$
 if  $f(x) = 8x + 15$ 

**16.** 
$$f(9)$$
 if  $f(x) = 5x - 16$  **29**

Choose four values for x to make a function table for each function. Then state the domain and range of the function.

17. 
$$f(x) = x - 9$$
  
Sample answer:

| x  | x-9    | f(x) |
|----|--------|------|
| -2 | -2-9   | -11  |
| -1 | -1-9   | -10  |
| 7  | 7-9    | -2   |
| 12 | 12 - 9 | 3    |

f(-12) = -9

**18.** f(x) = 7xSample answer:

| ×  | 7x    | f(x) |
|----|-------|------|
| -5 | 7(-5) | -35  |
| -3 | 7(-3) | -21  |
| 2  | 7(2)  | 14   |
| 6  | 7(6)  | 42   |

**19.** f(x) = 4x + 3Sample answer:

| x  | 4x + 3    | f(x) |
|----|-----------|------|
| -4 | 4(-4) + 3 | -13  |
| -2 | 4(-2) + 3 | -5   |
| 3  | 4(3) + 3  | 15   |
| 5  | 4(5) + 3  | 23   |

- 20. A photographer takes an average of 15 pictures per session. The total number of pictures p(s) is a function of the number of sessions s.
  - a. Identify the independent and dependent variables.

The number of pictures p(s) is the dependent variable and the number

of sessions s is the independent variable.

b. What values of the domain and range make sense for this situation? Explain. Only whole numbers make sense for the domain because you cannot

have a fraction of a session. The range values will be multiples of 15.

c. Write a function to represent the total number of pictures taken. Then determine the number of pictures taken in 22 sessions.

p(s) = 15s; 330 pictures

21. Reason Abstractly Amer belongs to a music club that charges a monthly fee of AED 5, plus AED 0.50 per song that he downloads. Write a function to represent the amount of money m(s) he would pay in one month to download s songs. What is the cost if he downloads 30 songs?

$$m(s) = 5 + 0.50s$$
; AED 20

### Power Up! Test Practice

Exercises 22 and 23 prepare students for more rigorous thinking needed for assessment.

- 22. This test item requires students to analyze and solve complex realworld problems through the use of mathematical tools and models. Depth of Knowledge DOK2 Mathematical Practice MP1, MP4 Scoring Rubric Students correctly place all inputs with the 2 points corresponding outputs. Students correctly place 2-3 inputs with 1 point the corresponding outputs.
- 23. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure. Depth of Knowledge DOK2 Mathematical Practice Scoring Rubric 2 points Students complete the table correctly. Students correctly type 3-4 of the 1 point 5 values.

### Power Up! Test Practice

22. A store is having a 25% off sale, and Abdulrahman has a coupon good for AED 10 off his total purchase. The function f(x) = 0.75x - 10 represents the final cost of an item that costs x dirhams after the discount and coupon are applied.

| AED 5  | AED 50 |
|--------|--------|
| AED 20 | AED 60 |
| AED 35 | AED 75 |
| AED 40 | AED 80 |

Select values to complete the function machine for the regular prices and sale prices of items A, B, C, and D.

| Input A: AED 60 |                   | Output A: | AED 35 |
|-----------------|-------------------|-----------|--------|
| Input B: AED 40 |                   | Output B: | AED 20 |
| Input C: AED 20 | f(x) = 0.75x - 10 | Output C: | AED 5  |
| Input D: AED 80 |                   | Output D: | AED 50 |

23. Lamis received a AED 25 gift card to an online music store. The cost of purchasing one song is AED 0.95. Complete the table to show the balance remaining on Lamis' gift card n(s) after purchasing s songs.

| Number of<br>Songs s | Balance<br>Remaining o(s) |
|----------------------|---------------------------|
| 2                    | AED 23.10                 |
| 4                    | AED 21.20                 |
| 5                    | AED 20.25                 |
| 8                    | AED 17.40                 |
| 10                   | AED 15.50                 |

### **Spiral Review**

- 24. Laila is training for a marathon. She runs about 136 kilometers per week.
  - a. Write an equation to find the total kilometers k run in any number of weeks w, k = 136w
  - b. Complete the table to find the total kilometers run in 3, 4, 5, or 6 weeks.

| w | 85w   | k   |
|---|-------|-----|
| 3 | 85(3) | 255 |
| 4 | 85(4) | 340 |
| 5 | 85(5) | 425 |
| 6 | 85(6) | 510 |

Evaluate each expression if p = 5 and q = 12.

25. 
$$\frac{3p-6}{8-p}$$
 3

**26.** 
$$\frac{4q}{q+2(p+1)}$$
 **2 27.**  $\frac{q \cdot q}{4p-2}$  **8**

27. 
$$\frac{q \cdot q}{4p-2}$$
 8

Lesson 4

### **Linear Functions**



### Real-World Link

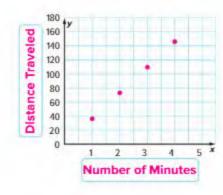
Up, Up, and Away The Lockheed SR-71 Blackbird has a top speed of 36.6 kilometers per minute. If x represents the minutes traveled at this speed, the function rule for the distance traveled is y = 36.6x.

Complete the function table.

| Input              | ×      | 1        | 2         | 3          | 4          |
|--------------------|--------|----------|-----------|------------|------------|
| Rule               | 36.6x  | 36.6(1)  | 36.6(2)   | 36.6(3)    | 36.6(4)    |
| output             | У      | 36.6     | 73.2      | 109.8      | 146.4      |
| (Input,<br>Output) | (x, y) | (1,36.6) | (2, 73.2) | (3, 109.8) | (4, 146.4) |

2. Graph the ordered pairs (x, y) on the coordinate plane provided. What do you notice about the graph?

The points seem to be in a straight line.



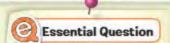
Which Mathematical Practices did you use? Shade the circle(s) that applies.

- 1) Persevere with Problems
- 2 Reason Abstractly
- 3 Construct an Argument
- Model with Mathematics

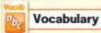
(7) Make Use of Structure 8 Use Repeated Reasoning

(5) Use Math Tools

6 Attend to Precision



HOW can we model relationships between quantities?



linear function continuous data discrete data

Mathematical Practices 1.3.4.7

### Focus narrowing the scope

Objective Represent linear functions using tables and graphs.

### Coherence connecting within and across grades

### Previous

tables to determine the domain and range of a function.

#### Now

Students will represent linear functions using function tables and graphs.

### Next

Students will compare functions represented in different forms.

### RIGOT pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 301.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

### **Launch the Lesson**

### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Circle the Sage Poll students to determine who has a solid understanding of functions. Have those students (the sages) spread out around the room. Place the remaining students into teams. Have team members spread out to the sages, with no two team members going to the same sage. Have groups complete the exercises with sages leading the discussion. Once the exercises are complete, send students back to their original teams to discuss the exercises and any differences in how the sages explained the work. 21, 3

### Alternate Strategy

Mean Have students line up the points on the graph using a ruler to verify that the points fall in a straight line.

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# **Teach the Concept**

Ask the scaffolded questions for each example to differentiate instruction.

### **Examples**

### Graph a function.



- What values of x would be reasonable to choose in this scenario? 0, 1, 2, 3
  - How can we determine each corresponding value of y in the table? Substitute each value of x into the expression 5 - 2x.



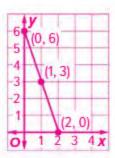
- What do you notice about the points graphed in the coordinate plane? Sample answer: The points fall on a straight line.
  - What do you notice about the constant rate of change of the graph? Sample answer: The constant rate of change is negative.



• Why would you not choose a negative input value for x? Sample answer: She cannot buy a negative number of book covers, so a negative input value does not make sense in this situation.

### Need Another Example?

During a clearance sale, a store is selling DVDs for AED3 and CDs for AED1. Graph the function y = 6 - 3x to find all possible values of DVDs x and CDs y that Mansour can buy with AED6.



0 DVDs and 6 CDs; 1 DVD and 3 CDs; 2 DVDs and 0 CDs

Work Zone

**Function Notation** 

The equation y = 5 - 3x

Sample answer:

Abdulraheem

can purchase

10 kilograms of

of oranges and

2 kilograms of

of oranges and

4 kilograms of

apples.

oranges, 6 kilograms

apples, 2 kilograms

can also be written in

function notation as

f(x)=5-3x.

### **Graph a Function**

Sometimes functions are written using two variables. One variable, usually x, represents the domain and the other, usually y, represents the range. When a function is written in this form it is an equation.

Like equations, functions can be represented in words, in a table, with a graph, and as ordered pairs. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

### Example

The school store sells book covers for AED 2 each and notebooks for AED 1. Manal has AED 5 to spend. The function y = 5 - 2xrepresents the number of book covers x and notebooks y she can buy. Graph the function. Interpret the points graphed.

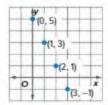
Step 1

Choose values for x and substitute them in the function to find v.

| x | 5 - 2x   | y  |
|---|----------|----|
| 0 | 5 - 2(0) | 5  |
| 1 | 5 - 2(1) | 3  |
| 2 | 5 - 2(2) | 1  |
| 3 | 5 - 2(3) | -1 |

Graph the ordered pairs (x, y).

She cannot buy negative amounts. So she can buy 0 covers and 5 notebooks, 1 cover and 3 notebooks, or 2 covers and 1 notebook.



### Got it? Do this problem to find out.

a. The farmer's market sells apples for AED 2 per kilogram and oranges for AED 1 per kilogram. Abdulraheem has AED 10 to spend. The function y = 10 - 2xrepresents the number of apples x and oranges y Abdulraheem can purchase. Graph the function and interpret the points graphed.



**Functions** 

Solutions

The solutions of an equation

are ordered pairs that make

an equation representing

the function true.

### Example

### 2. Graph y = x + 2.

Step 1

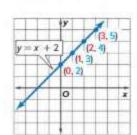
Make a function table. Select any four values for the domain x. Substitute these values for x to find the value of y, and write the corresponding ordered pairs.

| x | x+2 | y | (x, y) |
|---|-----|---|--------|
| 0 | 0+2 | 2 | (0, 2) |
| 1 | 1+2 | 3 | (1, 3) |
| 2 | 2+2 | 4 | (2, 4) |
| 3 | 3+2 | 5 | (3, 5) |

Step 2

Graph each ordered pair. Draw a line that passes through each point.

The line is the complete graph of the function. The ordered pair corresponding to any point on the line is a solution of the equation y = x + 2.



Check It appears that (-2, 0) is also a solution. Check this by substitution.

| _ | X | 1 |
|---|---|---|

Write the function.

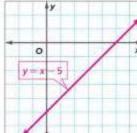
0 = -2 + 2

Replace x with -2 and y with 0.

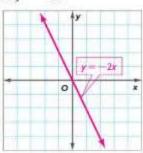
0 = 0 🗸

### Got it? Do these problems to find out.

**b.** 
$$y = x - 5$$



c. 
$$y = -2x$$



### **Examples**

### 2. Graph a function.



- What method could we use to help graph y = x + 2? Sample answer: make a function table to find the x- and y-values
  - · What input values could we use for the x-values? Sample answer: 0, 1, 2, 3



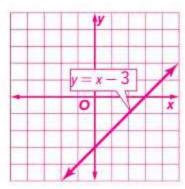
- How can we check that our graph is correct? Sample answer: Check a point on the line to see that it makes a true statement when the x- and y-coordinates are substituted into the equation.
  - How many solutions are there for the function y = x + 2? Explain. There are an infinite number of solutions because there are an infinite number of points on the line.



 Is there another method you could use to graph the equation? Sample answer: I could use the slope and y-intercept. The y-intercept is 2. Plot 2 on the y-axis. From there, use the slope, 1, to go up 1 and to the right 1 to plot the next point. Connect the points with a straight line.

### Need Another Example?

Graph y = x - 3.



### **Examples**

- 3. Write a function for a real-world problem.
- How much is the coupon worth? AED5
  - If 6 people entered the store, what is the total value of coupons given out? How can you determine this value? AED30; Multiply 6 by AED5.
- What are the dependent and independent variables? Sample answer: Because the total value of the coupons depends on the number of people, the value of the coupons is the dependent variable and the number of people is the independent variable.
  - What function can be used to represent the situation? v = 5x
- What is the y-intercept? What does this mean in the context of the problem? The y-intercept is 0; If 0 people enter the store, the total value of the coupons given out is AEDO.
- 4. Make a function table for a real-world problem.
- How can we determine the y-values? Multiply each x-value by 5.
  - What are the y-values? 25, 50, 75, and 100
- How does the table show the constant rate of change? The change in y-values divided by the change in x-values is 5.
  - · Is this relationship proportional? Explain. yes; Sample answer: The quantities are in a constant ratio.
- Determine the total value of the coupons given out to c customers. AED5c

### Key Concept

### **Representing Functions**

Words The value of y is one less than the corresponding value of x.

Ordered Pairs (0, -1), (1, 0), (2, 1), (3, 2) Equation y = x - 1

Graph

Table

#### Continuous and Discrete

If the domain of a function is integers, this is an example of a discrete function. If the domain is all real numbers. this is an example of a continuous function.

A linear function is a function in which the graph of the solutions forms a straight line. Therefore, an equation of the form y = mx + b is a linear function.

A function can be considered continuous or discrete. Continuous data can take on any value, so there is no space between data values for a given domain. Discrete data have space between possible data values. Graphs of continuous data are represented by solid lines and graphs of discrete data are represented by dots.

| Continuous Data                      | Discrete Data                          |
|--------------------------------------|--|
| the number of milliliters in a glass | the number of glasses in a cupboard    |
| the weight of each chocolate<br>chip | the number of chocolate chips in a bag |

You can determine if data that model real-world situations are discrete or continuous by considering whether all numbers are reasonable as part of the domain.



### Examples

Each person that enters a store receives a coupon for AED 5 off his or her entire purchase.

3. Write a function to represent the total value of the coupons

Let y represent the total value of the coupons and x represent the number of people. The function is y = 5x.

**Functions** 

and Reflect

Explain below how a

function table can be

used to graph a function.

Make ordered pairs

its corresponding

y-value. Then graph

the ordered pairs on

a coordinate plane.

Draw a line that the

Sample answer:

don't have to buy

the nuts in whole

increments. You

can buy any amount of nuts.

p This situation is

continuous

number

because you

points suggest.

a, c = 5.95k

Aoone And

using the x-value and

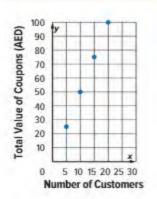
4. Make a function table to find the total value of the coupons given out to 5, 10, 15, and 20 customers.

| x  | 5x    | y   |
|----|-------|-----|
| 5  | 5(5)  | 25  |
| 10 | 5(10) | 50  |
| 15 | 5(15) | 75  |
| 20 | 5(20) | 100 |

Graph the function. Is the function continuous or discrete? Explain.

Use the ordered pairs from the function table to graph the function.

There can only be a whole number amount of customers. The function is discrete. So, the points are not connected.

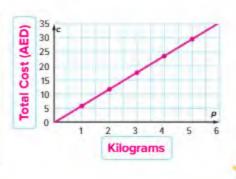


Got it? Do these problems to find out.

A store sells assorted nuts for AED 5.95 per kilogram.

- d. Write a function to represent the total cost of any number of kilograms of nuts.
- e. Complete the function table below to find the total cost of 1, 2, 3, 4, or 5 kilograms of nuts.
- f. Graph the function. Is the function continuous or discrete? Explain.

| p | 5.95p   | c     |
|---|---------|-------|
| 1 | 5.95(1) | 5.95  |
| 2 | 5.95(2) | 11.90 |
| 3 | 5.95(3) | 17.85 |
| 4 | 5.95(4) | 23.80 |
| 5 | 5.95(5) | 29.75 |



Examples

- Graph a function and determine continuous or discrete data.
   How would you write the relationship as a set of
- How would you write the relationship as a set of ordered pairs? (5, 25), (10, 50), (15, 75), and (20, 100)
  - Can you have a number of customers that is not a whole number? no
- Since you cannot have part of a customer, is this function continuous or discrete? discrete
  - Should we connect the points? Explain. no; Sample answer: The function is discrete, so we leave the points unconnected.
- Does the ordered pair (2.5, 12.5) make sense in the context of the problem? Explain. no; Sample answer: There cannot be 2.5 customers.

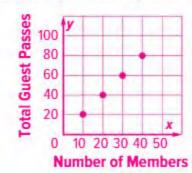
### **Need Another Example?**

Each member of a health club receives two free guest passes.

- a. Write a function to represent the situation. y = 2x
- b. Make a function table to show the number of guest passes given out to 10, 20, 30, and 40 members.

| x  | 2x    | y  |
|----|-------|----|
| 10 | 2(10) | 20 |
| 20 | 2(20) | 40 |
| 30 | 2(30) | 60 |
| 40 | 2(40) | 80 |

c. Graph the function. Is the function continuous or discrete? Explain. discrete; Sample answer: The number of health club members can only be represented by whole numbers.



### **Guided Practice**

**Formative Assessment** Use these exercises to assess students' understanding of the concepts in this lesson.



If some of your students are not ready for assignments, use the differentiated activities below.

Pairs Create Have students work in pairs to create function machines out of construction paper or draw a function machine on paper. Have them use their machines to generate a table of values for Exercises 1 and 2. 21,5

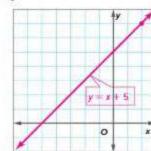
Roundrobin For each exercise, have students create a different representation than the one given. For example, in Exercise 1, students may create a table of values, set of ordered pairs, or a mapping diagram. 1, 2, 4

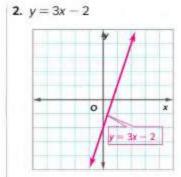
### **Guided Practice**



Graph each function. (Example 2)

1. 
$$y = x + 5$$

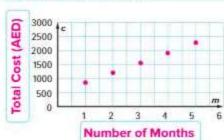




- A satellite cable company charges an installation fee of AED 500 plus an additional AED 359.50 per month for service.
  - a. Write a function to represent the the total cost of any number of months of service. c = 500 + 359.5m
  - b. Make a function table to find the total cost for 1, 2, 3, 4, or 5 months.
  - c. Graph the function. Is the function continuous or discrete? Explain.

This situation is discrete because you cannot pay for

a partial month of service,



d. Interpret the points graphed. One month costs AED 859.50,

2 months cost AED 1,219.00, 3 months cost AED 1,578.50,

- 4 months cost AED 1938.00, and 5 months cost AED 2297.50.

in which the data is discrete or continuous.

| m | 500 + 359.5m                     | c              |
|---|----------------------------------|----------------|
| 1 | 500 + 359.5(1)                   | 859.50         |
| 2 | 500 + 359.5(2)                   | 1,219.00       |
| 3 | 500 + 359.5(3)                   | 1,578.50       |
| 4 | 500 + 359.5(4)                   | 1,938.00       |
| 5 | 500 + 359.5(5)                   | 2,297.50       |
| 3 | 500 + 359.5(3)<br>500 + 359.5(4) | 1,578<br>1,938 |

#### Rate Yourself!

How well do you understand linear functions? Circle the image that applies.







ear Somewh

Not So

FOLDABLES - time to update your Foldable!

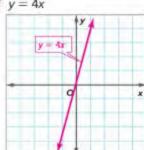
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My Homework

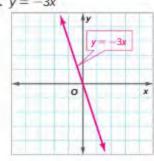
### **Independent Practice**

Graph each function. (Example 2)









3. y = x - 3

m

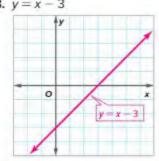
2

4

6

8

10



250 + 50m

250 + 50(2)

250 + 50(4)

250 + 50(6)

250 + 50(8)

250 + 50(10)

Abdul's Savings

5

350

450

550

650

750

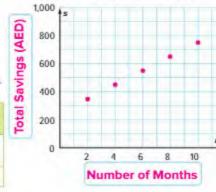
- 4. Financial Literacy Abdul is saving money for college. He already has AED 250. He plans to save another AED 50 per month. (Examples 1, 3-5)
  - a. Write a function to represent his savings for any number of months. s = 250 + 50m
  - b. Make a function table to find his total savings for 2, 4, 6, 8 and 10 months.
  - c. Graph the function. Is the function continuous or discrete? Explain. discrete; You cannot find Abdul's total savings for part of a month.
  - d. Interpret the points graphed. Abdul saved a total of

AED 350 in 2 months, AED 450 in 4 months, AED 550 in

6 months, AED 650 in 8 months, and AED 750 in 10 months.

- 5. Copy and Solve The table shows the cost to rent different items.
  - a. Write a function to represent each situation.

| Item             | Deposit<br>(AED) | Cost per<br>Hour (AED) |  |
|------------------|------------------|------------------------|--|
| Mountain<br>bike | 150              | 42.50                  |  |
| Scooter          | 250              | 25.00                  |  |



bike: 
$$c = 150 + 4.25h$$
: scooter:  $c = 250 + 2.5h$ 

- b. On a separate piece of paper, make a function table to find the total cost to rent each item for 2, 3, 4, or 5 hours. See Answer Appendix.
- c. On a separate piece of grid paper, graph the functions on the same coordinate plane. Are the functions continuous or discrete? Explain. See Answer Appendix for graph. Both situations are discrete because you cannot rent either piece of equipment
- d. Will the mountain bike or the scooter cost more to rent for 8 hours? for a partial hour. mountain bike
- e. How much is the cost to rent the mountain bike for 8 hours? AED 490

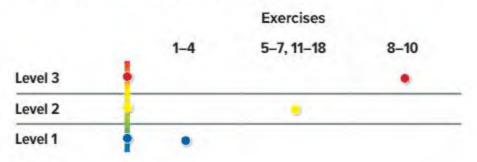
### **Practice and Apply**

### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                          |  |  |
|---------------------------------|-------------------|--------------------------|--|--|
| AL                              | Approaching Level | 1–5, 7, 8, 10, 17, 18    |  |  |
| OL.                             | On Level          | 1–5 odd, 6–8, 10, 17, 18 |  |  |
| <b>®</b>                        | Beyond Level      | 5–10, 17, 18             |  |  |

### Watch Out!

Common Error Some students may find that not all of the ordered pairs in their tables lie on a straight line. Suggest that they check each of their output calculations and pay special attention to negative signs in equations.

| MATHEMATICAL PRACTICES  |             |  |  |
|---|-------------|--|--|
| Emphasis On   | Exercise(s) |  |  |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 9           |  |  |
| 4 Model with mathematics.   | 6, 10, 16   |  |  |
| 7 Look for and make use of structure.   | 8           |  |  |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

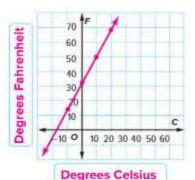
#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

### TICKET Out the Door

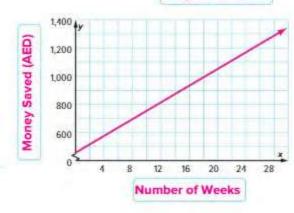
Ask students to write about how what they learned yesterday about functions helped them understand this lesson's topic of representing functions with tables, graphs, and equations. See students' work.

Model with Mathematics The formula F = 1.8C + 32 compares temperatures in degrees Celsius C to temperatures in degrees Fahrenheit F. Find four ordered pairs (C, F) that are solutions of the equation. Then graph the function.
 Sample answer: (0, 32), (-10, 14), (10, 50), (20, 68)



Abdulaziz is saving money to buy a new computer for AED 1,200. He already has AED 450 and plans to save AED 30 a week. The function y = 30x + 450 represents the amount Abdulaziz has saved after x weeks. Graph the function to determine the number of weeks it will take Abdulaziz to save enough money to buy the computer.

25 weeks





### H.O.T. Problems Higher Order Thinking

- 8. Identify Structure Explain why a linear function that is continuous has an infinite number of solutions. Then determine which of the following representations shows all the solutions of the function: a table, a graph, or an equation. Explain. Sample answer: Since the function is continuous, an infinite number of values can be substituted for the domain. A table shows a finite number of solutions. An equation or a graph represent all the solutions of a function.
- Persevere with Problems Name the coordinates of four points that satisfy the linear function shown. Then give the function rule.
   Sample answer: (-2, -4), (0, -2), (2, 0), (4, 2); y = x 2
- . . . .
- Model with Mathematics Write a set of four ordered pairs that represents a linear function. Then give the function rule.

Sample answer: (1, 6), (2, 11), (3, 16), (4, 21); y = 5x + 1

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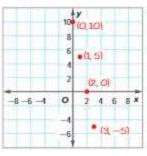
Name

My Homework

### **Extra Practice**

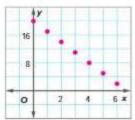
11. A store sells T-shirts x in packs of 5 and regular shirts y individually. Graph the function y = 10 - 5x to determine the number of each type of shirt Muna can have if she buys 10 shirts.





She cannot buy negative amounts. So, she can buy OT-shirt packs and 10 shirts individually, 1 T-shirt pack and 5 shirts individually, or 2 T-shirt packs and 0 shirts individually.

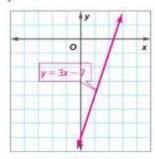
12. Fancy goldfish x cost AED 3 each and common goldfish y cost AED 1 each. Graph the function y = 20 - 3x to determine how many of each type of goldfish Yasmin can buy for AED 20.



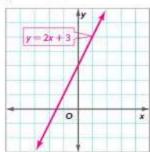
She can buy 20 common and 0 fancy, 17 common and 1 fancy, 14 common and 2 fancy, 11 common and 3 fancy, 8 common and 4 fancy, 5 common and 5 fancy or 2 common and 6 fancy.

Graph each function.

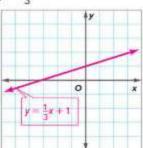
**13.** 
$$y = 3x - 7$$



14. y = 2x + 3



**15.**  $y = \frac{1}{3}x + 1$ 

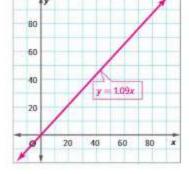


- 16. Model with Mathematics The equation y = 1.09x describes the approximate number of yards y in x meters.
  - a. Would negative values of x have any meaning in this situation? Explain.

No; you cannot have a negative distance.

- b. Graph the function.
- c. About how many meters is a 40-yard race?

36.7 m



### Power Up! Test Practice

Exercises 17 and 18 prepare students for more rigorous thinking needed for assessment

- This test item requires students to reason abstractly and quantitatively when problem solving.

  Depth of Knowledge DOK1

  Mathematical Practice MP1

  Scoring Rubric

  2 points Students correctly complete each statement.

  1 point Students correctly complete 3 of the 4 statements.
- 18. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

  Depth of Knowledge DOK1

  Mathematical Practice MP1

  Scoring Rubric

  1 point Students correctly assign each graph to the corresponding table.

### Power Up! Test Practice

- A rental company charges AED 80 plus AED 55 per hour to rent a canoe.
   Fill in each box below to make true statements.
  - a. A function that can be used to find the total cost y of renting a canoe for x hours is y = 55x + 80
  - b. The domain of the function represents the number of hours
  - c. The range of the function represents the total cost
  - d. Maha rents a canoe for 4 hours. The amount she pays is

    AED 300
- Select the correct graph of the linear function shown in each table below.

| Graph   | Function |    |    |   |  |
|---------|----------|----|----|---|--|
|         | 4        | 0  | -4 | × |  |
| Graph B | 0        | -1 | -2 | y |  |
| Graph C | 1        | 0  | -1 | * |  |
| Graph C | 3        | -1 | -5 | y |  |
|         | 4        | 0  | -4 | × |  |
| Graph A | 2        | 1  | 0  | y |  |

| Graph A  | Graph B | Graph C |
|----------|---------|---------|
| 2 7      | 2 7     | 2 7     |
| 4 -2 O x | 0 2 4 x | -2 O 2x |

### **Spiral Review**

- 19. The table shows the rate a hotel charges per day for a room.
  - Write an equation to find the total cost to stay in the hotel for any number of days. c = 650d

| b. | Use the | equation to determine how much it would cost to stay for |
|----|---------|--|
|    | 9 days  | AED 5,850  |

| Numb<br>of Day |       |
|----------------|-------|
| 1              | 650   |
| 2              | 1,300 |
| 3              | 1,950 |
| 4              | 2 600 |

 Write an expression that can be used to find the nth term of the arithmetic sequence 15, 30, 45, 60, .... Then write the next three terms.

15n; 75, 90, 105

### Problem-Solving Investigation

### Make a Table

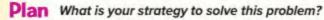
### Case #1 Play Catch Up

Abdul's family is going on vacation. His mom and sister leave at 7:00 in the morning, driving an average of 45 kilometers per hour. Abdul and his dad leave at 8:00. His dad drives an average of 60 kilometers per hour.

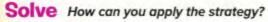
Will Abdul and his dad catch up to his mom and sister?

#### Understand What are the facts?

You know the times they left and their rates. You need to know if Abdul and his dad will catch up to his mom and sister.



Make a table that shows how many miles each driver has driven.



| Hours Since | Distance Traveled (km) |             |
|-------------|------------------------|-------------|
| 7:00        | Abdul's Mom            | Abdul's Dad |
| 0           | 0                      | 0           |
| 1           | 45                     | 0           |
| 2           | 90                     | 60          |
| 3           | 135                    | 120         |
| 4           | 180                    | 180         |

At 11:00 , Abdul and his dad will catch up to his mom and sister.

### Check Does the answer make sense?

60 kmph x 3 h = 180 km 45 kmph x 4 h = 180 km

The distances are equal. <

### **Analyze the Strategy**

Reason Abstractly Suppose Abdul's mom drives at an average speed of 50 kilometers per hour. At what time will Abdul and his dad catch up to her?

Mathematical Practices



### Focus narrowing the scope

Objective Solve problems by making a table. This lesson emphasizes Mathematical Practice 2 Reason Abstractly.

Make a Table Students use the make a table strategy as a way to organize data given in a problem. By recording values that represent the relationships between changing quantities, students use tables to solve problems involving rates of change.

### Coherence connecting within and across grades

### Now

Students apply the content standard to solve non-routine problems.

Students will make tables to solve realworld problems.

### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 307.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

### **Launch the Lesson**

The problems on pages 305 and 306 are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance. The problem on page 305 walks students through the solution, while the problem on page 306 asks students to come up with their own solutions.

### Case #1 Play Catch-Up

(BI) Have students extend the problem by having them answer the question below.

#### Ask:

 How many kilometers ahead of Abdulkarim's mom will Abdulkarim and his dad be at 1 P.M. if they both stay at their same average rates? 30 kilometers ahead

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### Case #2 Karaoke Kid

 Roundrobin Have students work in pairs to create a table for Step 3. Have them alternate completing the rows in the table. 2 1,4

(B) (IA) Pairs Discussion Have students work in pairs to answer the following extension question. 11

#### Ask:

 Sally's Songs has a daily rate of AED2.50 and no deposit. How many days would Moza need to rent from Sally's Songs for the price to be the same as at Karaoke Korner? 4 days

### Need Another Example?

Hana and Hessa are both riding their bikes on a trail that is 30 kilometers long. Hana rides an average of 3 kilometers per hour with stops and Hessa rides an average of 5 kilometers per hour with stops. If Hana starts at 9:00 A.M. and Hessa starts at 11:00 A.M., will Hessa catch up to Hana?

| Hours              | Distance Traveled (km) |       |  |
|--------------------|------------------------|-------|--|
| Since 9:00<br>A.M. | Hana                   | Hessa |  |
| 0                  | 0                      | 0     |  |
| 1                  | 3                      | 0     |  |
| 2                  | 6                      | 0     |  |
| 3                  | 9                      | 5     |  |
| 4                  | 12                     | 10    |  |
| 5                  | 15                     | 15    |  |

Hessa will catch up to Hana 5 hours after 9:00 a.m., or 2:00 p.m.

### Case #2 Amplifier for Kids

Moza wants to rent a amplifier for a family reunion. The prices to rent the machine

from two different companies are shown. For how many days must she rent the machine

for the cost from each place to be the same?

### Understand

Read the problem. What are you being asked to find?

I need to find time resulting in equal costs

Underline key words and values. What information do you know?

I know that Company A has a deposit of AED 5 and charges AED 1.25 a day.

Company B deposit is AED 4 and they charge AED 1.50 a day.

### Plan

Choose a problem-solving strategy.

I will use the make a table

strategy.

Deposit

AED 5

AED 4

Company

Company A

Company B

Cost Per Day

AED 1.25

AED 1.50

### Solve

Use your problem-solving strategy to solve the problem.

|           | Dayl     | Day 2    | Day3     | Day 4     |
|-----------|----------|----------|----------|-----------|
| Company A | AED 6.25 | AED 7.50 | AED 8.75 | AED 10.00 |
| Company B | AED 5.50 | AED 7.00 | AED 8.50 | AED 10.00 |



So, the cost at both companies is the same at 4 days

### Check

Use information from the problem to check your answer.

Company A charges AED 5.00 + AED 1.25 or AED 6.25 for the first day.

Each day adds another AED 1.25

Company B charges AED 4.00 + AED 1.50 or AED 5.50 for the first day.

Each day adds another AED 1.50

At 4 days, both companies charge AED 10.00

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Work with a small group to solve the following cases. Show your work on a separate piece of paper.

#### Case #3 Plants

The table shows the height of a giant bamboo plant for Days 5-9. The bamboo grew at a steady rate each day, starting on Day 5.

From Day 5 to Day 9, the bamboo plant grew about what percent of its final height?

Sample answer: about 80%





**Functions** 

### Case #4 Financial Literacy

Maysa and her brother Abdalla each open a bank account with an initial deposit of AED 50 each. Abdalla planned to save 30% of his earnings from his after-school job. The job pays AED 8 per hour, and he works 25 hours each week. For four weeks, Maysa saved AED 45 per week. After that, she saved an additional AED 30 per week.

During which week will they have the same amount of money in

Week 8

### Case #5 Fitness

Maysoun created her own fitness plan to increase the amount of time she spends exercising each week. The total number of minutes in her exercise plan for each week is shown.

If she continues this pattern, for how many hours will she exercise in the 8th week?

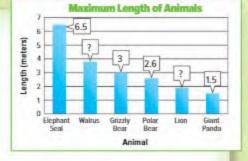
77 hours

| Week | Exercise<br>(min) |
|------|-------------------|
| 1    | 35                |
| 2    | 50                |
| 3    | 80                |
| 4    | 125               |
| 5    | 185               |

# strategy.

### Case #6 Animals

The graph shows the maximum length of several animals. The maximum length of a walrus is twice the maximum length of a lion, which is 0.4 meter longer than the maximum length of a giant panda.



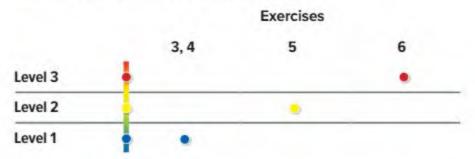
Find the maximum length of a walrus. 3.8 m

Collaborate

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE



(I) Think-Pair-Share Have students work in pairs. Give them one minute to think through their responses to Case 3. Then have them discuss their thoughts with a partner. Have them respond to the questions below. Upon completing Case 3, have them share their results with another pair. Repeat this process for Cases 4-6. 21,3

#### Ask:

- What do you need to determine? the percent of growth from Day 5 to Day 9
- What information do you need to solve the problem? the final height on day 9 and the height on day 5

(B) (IA) Trade-a-Problem Have students create their own real-world problem using the information in the table in Case 3. Students trade their problems, solve, and compare solutions. If the solutions do not agree, students work together to find any errors. 21,4

### Mid-Chapter Check

If students have trouble with Exercises 1-7, they may need help with the following concepts.

| Concept                       | Exercise(s) |
|-------------------------------|-------------|
| linear equations (Lesson 1)   | 1           |
| graph relations (Lesson 2)    | 3           |
| evaluate functions (Lesson 3) | 4, 5, 6     |
| graph functions (Lesson 4)    | 2,7         |
| domain and range (Lesson 2)   | 7           |

### **Vocabulary Activity**

(IA) Rally Coach Have students work in pairs to complete Exercise 1. Have Student 1 speak aloud about what a linear equation means, while Student 2 listens, coaches, and encourages. If students are having trouble remembering the definition of a linear equation, have them discuss how the word line might help them. 21,3

### **Alternate Strategies**

Write four equations on the board, three of which are linear. Have students take turns coming to the board and circling the equations that are linear. When all equations are circled, have students define linear equation in their own words.

 Give each student a number. Then have students write equations for which their number is a solution.

### Mid-Chapter Check

### Vocabulary Check



 Be Precise Define linear equation. Give an example of a linear equation. (Lesson 1)

A linear equation is an equation with a graph that is a straight line.

Sample answer: y = 8x

2. Describe the difference between the graph of a set of discrete data and the graph of a set of continuous data. (Lesson 4) Sample answer: The graph of discrete data has space between the points while the graph of continuous data does not.

### Skills Check and Problem Solving

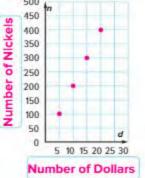
- 3. There are 20 5-fils coins in one dirham. (Lesson 2)
  - a. Write an equation to find the number of 5-fils coins n in any



number of dirhams d. n = 20d

b. Make a table to find the number of 5-fils coins in 5, 10, 15, or 20 dirhams. Then graph the ordered pairs.

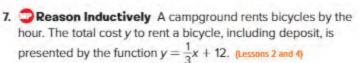
| d  | 20d   | n   |
|----|-------|-----|
| 5  | 20(5) | 100 |
| 10 | 20(5) | 200 |
| 15 | 20(5) | 300 |
| 20 | 20(5) | 400 |



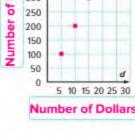
Find each function value. (Lesson 3)

**4.** 
$$f(8)$$
 if  $f(x) = 15x$ 

**5.** 
$$f(2)$$
 if  $f(x) = 2x - 5$ 



- a. Graph the function.
- b. What do the domain and range of the function represent? Sample answer: The domain is the hours that a bicycle is rented and the range is the total cost.
- c. Is the function continuous or discrete? discrete



**6.** f(4) if f(x) = -3x + 15



Lesson 5

### **Compare Properties of Functions**



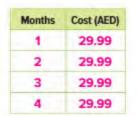
### Real-World Link

Science Museum Obaid's annual membership to the science museum can be represented by the function c = 29.99, where c represents the cost in dirhams. The cost for Lamis to pay per visit is show

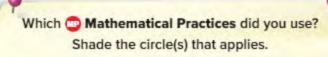
| own in the table.                 | 3 |
|-----------------------------------|---|
| Make a table to represent Obaid's | 4 |
| membership                        |   |

| Visits | Cost (AED) |
|--------|------------|
| 1      | 5          |
| 2      | 10         |
| 3      | 15         |
| 4      | 20         |
| 5      | 25         |

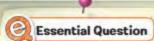
| ı |   | 3  |
|---|---|----|
|   | 2 | 10 |
|   | 3 | 15 |
|   | 4 | 20 |
|   | 5 | 25 |
|   |   |    |
|   |   |    |



- Describe the rate of change for each function. Sample answer: Obaid's membership has a rate of change of 0. The rate of change for Lamis' membership is AED 5 per visit.
- 3. Who pays more for two visits? Explain. Obaid; Sample answer: Lamis will pay AED 10 for two visits, and Obaid will pay AED 29.99.
- 4. Who pays more for six visits? Explain. Lamis; Sample answer: Lamis will pay 6 × AED 5 or AED 30 and Obaid will pay AED 29.99.



- 1) Persevere with Problems
- (2) Reason Abstractly
- 3 Construct an Argument
- Model with Mathematics www.jnob-jo.com
- (5) Use Math Tools
- (6) Attend to Precision
- (7) Make Use of Structure
- (8) Use Repeated Reasoning



HOW can we model relationships between quantities?

Mathematical Practices 1.2.3.4

### Focus narrowing the scope

Objective Compare properties of functions represented in different ways.

### Coherence connecting within and across grades

### Previous

Students represented relationships using multiple representations.

#### Now

Students use different representations of two functions to compare the functions.

### Next

Students will use multiple representations to construct functions.

### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 315.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

### Launch the Lesson

### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Trade-a-Problem Have students create their own problem similar to the Real-World Link.

Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. 21,4

### Alternate Strategy

Explain that since there is no independent variable in the equation that represents Obaid's membership, the dependent value does not change. Ask students to give examples of other situations where the dependent value is a constant.

### Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

### Example

### Compare two functions.



- What is the lion's rate of speed? 16 meters per second
  - How is the speed of the zebra represented? by a graph
  - . Choose two points on the graph. What is the rate of change between the two points?
  - What is the zebra's speed? 18 meters per second



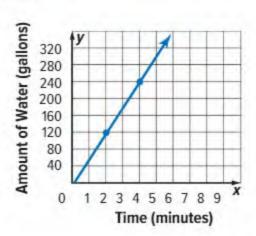
- In what forms are the functions in this Example given? words and a graph
  - What is the rate of change of the lion's speed? 16 meters per second
  - How would you find the rate of change of the zebra's speed? Use two points on the line and find the rate of change between them.
  - What is the rate of change of the zebra's speed? 18 meters per second



• How do the rates for the two animals compare? Interpret your answer in the context of the problem. The rate of change for the zebra's speed is greater than the rate of change for the lion's speed. The zebra is faster than the lion.

### Need Another Example?

The flow rate of water in a water garden is 52 liters per minute. The graph shows the flow rate of water in a koi pond. Compare the functions by comparing their rates of change. See Answer Appendix.



Work Zone

### **Compare Two Functions**

Functions can be represented by a table, graph, equation, or words. You can compare two functions represented in different forms.



### Example

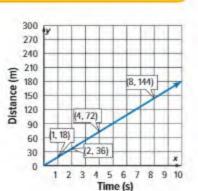
1. A zebra's main predator is a lion. Lions can run at a speed of 16 meters per second over short distances. The graph at the right shows the speed of a zebra. Compare their speeds.

> To compare their speeds, compare the rates of change.

A lion can travel at a rate of 16 meters per second.

To find the rate of change for a zebra, choose two points on the line and find the rate of change between them.

$$\frac{\text{Change in distance}}{\text{Change in time}} = \frac{36 - 18}{2 - 1} \text{ or } \frac{18}{1}$$

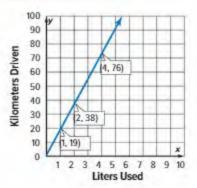


A zebra can travel at a rate of 18 meters per second. Since 18 > 16, the speed of a zebra is greater than the speed of a lion.

a. The car has a gas kilometrage of 22 km/l and the sport utility vehicle has a gas kilometrage of 19 mi/gal. The car has a greater gas kilometrage.

### Got it? Do this problem to find out.

a. A certain car has a gas kilometrage of 22 kilometers per liter. The gas kilometrage of a certain sport utility vehicle is represented by the function shown. Compare their gas kilometrage.



### Example

The function k = 225h, where k is the kilometers traveled in h hours, represents the distance traveled of the first Japanese high speed train. The distance traveled of a high speed train operating today in China is shown in the table. Assume the relationship between the two quantities is linear.

| Train Rate in China |            |
|---------------------|------------|
| Hours               | Kilometers |
| 1                   | 350        |
| 2                   | 700        |
| 3                   | 1,050      |

a. Compare the functions' y-intercepts and rates of change.

Compare the y-intercepts.

At 0 hours, no distance has been covered. So, the y-intercepts are

Compare the rates of change.

The speed of the Japanese train is 225 kilometers per hour.

Use the table to find the speed of the Chinese train.

The speed of the Chinese train is 350 kilometers or 1 hour

350 kilometers per hour.

Since 350 > 225, the function representing the

Chinese high speed train has a greater rate of change than the function representing the Japanese high speed train.

#### b. If you ride each train for 5 hours, how far will you travel on each?

Find the distance on the Japanese train.

k = 225hk = 225(5)

Write the function Replace h with 5.

k = 1,125

Simplify.

You will travel 1,125 kilometers in 5 hours on the Japanese train.

Find the distance on the Chinese train by extending

the table.

You will travel 1.750 kilometers in 5 hours on the Chinese train.

|     | Train Ra | ite in China |       |
|-----|----------|--------------|-------|
|     | Hours    | Kilometers   |       |
| +1( | 1        | 350          | )+350 |
| +1  | 2        | 700          | +350  |
| >   | 3        | 1,050        | +350  |
| T'> | 4        | 1,400        | K     |
| +1  | 5        | 1,750        | +350  |

Train Rate in China

Kilometers

700

1.050

+350

+350

Hours

### Example

### 2. Compare functions.



**Functions** 

- In what forms are the functions in this Example given? table and equation
  - What is a y-intercept? where a function crosses the y-axis
  - If you ride the Japanese train for 5 hours, how far will you travel? 1,125 km
  - If you ride the Chinese train for 5 hours, how far will you travel? 1,750 km



- How could you find the y-intercepts of the functions? Since the functions represent distances, there is no distance covered at 0 hours. You could also replace h with 0 in the equation and simplify. Either way, the y-intercept is 0 for both functions.
  - What is the rate of change for the Japanese train? 225 kmph
  - What is the rate of change for the Chinese train? 350 kmph



 What is the value of the independent variable in the y-intercept? 0

### Need Another Example?

A bowling alley offers different event packages. Package A is represented by the function c = 7p + 5, where c is the total cost and p is the number of people. Package B is represented in the table below.

| Package B        |                  |
|------------------|------------------|
| Number of People | Total Cost (AED) |
| 1                | 9                |
| 2                | 18               |
| 3                | 27               |
| 4                | 36               |

- a. Compare the functions by comparing their y-intercepts and rates of change. See Answer Appendix.
- b. How much more will Package B cost than Package A if there are 12 people at the event? AED19 more

### Example

### Compare functions.



- In what forms are the functions in this Example given? equation and graph
  - What is the y-intercept for Nabila's bill?
  - What is the y-intercept for Asma's bill? 60
  - Compare what the two y-intercepts represent. Asma has a greater initial cost in his plan.



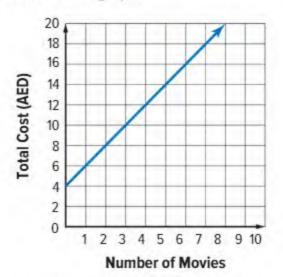
- What is the rate of change for Nabila's bill? 0.15
  - What does it represent? the cost per minute
  - What is the rate of change for Asma's bill? 0.10
  - Compare what the two rates of change represent. Asma has a lower cost per minute than Nabila.



 After how many minutes would the plans cost the same? 220

### Need Another Example?

The total cost c to rent any number of movies m from an online movie rental company is represented by the function c = 1.5m + 5. The cost to rent movies from a different company is shown in the graph.



- a. Compare the y-intercepts and rates of change. See Answer Appendix.
- b. What will be the cost from each company if 15 movies are rented in one month? first company: AED27.50; second company: AED34

The function for b. the movies has a y-intercept of 2 and the function for the games has a y-intercept of 0. The store receives 7 movies per week and

> 3 games per week. The rate of change for

the movies is greater.

Got it? Do these problems to find out.

The number of new movies a store receives can be represented by the function m = 7w + 2, where m represents the number of movies and w represents the number of weeks. The number of games the same store receives is shown in the table.

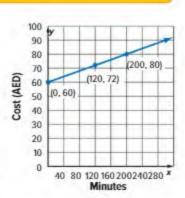
| Week | Number of<br>New Games |
|------|------------------------|
| 1    | 3                      |
| 2    | 6                      |
| 3    | 9                      |

- Compare the functions' y-intercepts and rates of change.
- c. How many new movies and games will the store have in Week 6?
- c. 44 new movies and 18 new games



### Example

3. Financial Literacy Nabila and Asma each have a monthly cell phone bill. Nabila's monthly cell phone bill is represented by the function y = 0.15x + 49, where x represents the minutes and y represents the cost. Asma's monthly cost is shown in the graph.



a. Compare the y-intercepts and rates of change.

The function for Nabila's bill has a y-intercept of 49 You cam see from the graph that the function for Asma's bill has a y-intercept of 60. So, Asma has a greater initial cost.

The rate of change for Nabila's monthly bill is AED 0.15 per minute. Find the rate of change for Asma's bill.

$$\frac{\text{change in cost}}{\text{change in minutes}} = \frac{80 - 60}{200 - 0} \text{ or } 0.10$$

The rate of change for Asma's bill is AED 0.10 per minute. So, Nabila pays more per minute than Asma.

b. What will be the monthly cost for Nabila and Asma for 200 minutes?

Nabila's monthly cost is represented by y = 0.15x + 49. At 200 minutes, Nabila will pay 0.15(200) + 49 or AED 79.

Use the graph to find Asma's cost. At 200 minutes, Asma will pay AED 80.

Got it? Do these problems to find out.

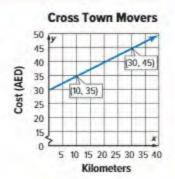
Financial Literacy Najat and Eiman each have a membership to the gym. Najat's membership is represented by the function y = 30x + 290, where x represents the hours with a trainer and y represents the cost. The cost of Eiman's membership is shown in the graph.



- Compare the y-intercepts and rates of change.
- e. What will be the total cost for Najat and Eiman if they each have 4 hours with a trainer?

### Example

4. Financial Literacy Najla's mother needs to rent a truck to move some furniture. The cost to rent a truck from two different companies is shown in the table and graph. Which company should she use to rent the truck for 40 kilometers?



Find the cost of renting a truck from Rafi's Rentals by extending the table.

After 40 kilometers, the cost will be AED 75 + AED 25 or AED 100.

|       | Rafi's Rentals |               |     |
|-------|----------------|---------------|-----|
|       | Kilometers     | Cost<br>(AED) |     |
|       | 10             | 25            |     |
| +10 ( | 20             | 50            | +25 |
|       | 30             | 75            |     |

Find the cost of renting a truck from Cross Town Movers by analyzing the graph. The y-intercept of the graph is 30. The slope or rate of change is  $\frac{45-35}{30-10}$  or 0.5. The equation y = 0.5x + 30 where y represents the total cost and x represents the kilometers driven can be used to find the total cost of renting the truck. After 40 kilometers, the cost will be 0.5(40) + 30 or AED 50. So. Cross Town Movers would cost less for 40 kilometers.

**Functions** 

a. The function for Najat's membership has a y-intercept of 290. The function for Eiman's membership has a y-intercept of 390. Eiman has a greater initial cost. The rate of change for Najat's membership is AED 30. The rate of change for Eiman's membership is AED 30. Najat and Eiman pay the same amount per hour.

Najat: AED 410; e. Eiman: AED 510

Multiple Representations

You can find the two costs of truck ventals by extending the table, extending the line on the graph, or writing an equation. The method you use will depend on the information that you are given.

### Example

### 4. Compare functions.



- In what forms are the functions in this Example given? table and graph
  - · What are you being asked to find? which company is less expensive for 40 kilometers
  - Which company costs less to rent for 10 kilometers? Ron's Rentals



- How could you find the total cost to rent a truck for 40 kilometers from Ron's Rentals? You can extend the pattern in the table or you could write and evaluate an equation.
  - How could you find the total cost to rent a truck for 40 kilometers from Cross Town Movers? Extend the line on the graph to find the corresponding y value when the x value is 40.



BD . When is it cheaper to use Ron's Rentals? when you rent the truck for less than 15 kilometers

### Need Another Example?

The eighth grade class is selling pizzas and subs for a fundraiser. The amount of money they earn selling pizzas is shown in the table. The amount of money they earn selling sub sandwiches can be represented by the function m = 4s, where m is the total amount of money earned and s is the number of sub sandwiches sold. Which food will the students earn more money selling if they sell 100 of each item?

| Selling Pizzas |                       |
|----------------|-----------------------|
| Number Sold    | Total Earned<br>(AED) |
| 20             | 100                   |
| 40             | 200                   |
| 60             | 300                   |
| 80             | 400                   |

pizzas; The students will receive 4(100) or AED400 for selling 100 subs, but will earn AED500 for selling 100 pizzas.

### **Guided Practice**

**Formative Assessment** Use these exercises to assess students' understanding of the concepts in this lesson.



If some of your students are not ready for assignments, use the differentiated activities below.

All Numbered Heads Together Assign students to 3-person teams. Each team completes Exercises 1–4 with a different team member leading each exercise and making sure each team member understands the problem before moving on. Call on a specific person from each team to present the team's solution to the class. 21,3

Pairs Discussion Have students work in pairs to create a graphic organizer that answers Exercise 4. Have them extend the question by adding tables and words to the types of representations. Then have them display their graphic organizers around the room. 1, 2, 4

### **Guided Practice**



A tiger in captivity is fed 13.5 kilograms of food a day.
 The graph shows the kilograms of food an elephant in captivity eats per day. Compare the functions by comparing their rates of change. (Example 1)

A tiger is fed 13.5 kilograms per day. An elephant is fed 125



kilograms per day. Since 125 > 13.5, the function for the elephant has a greater rate of change.

| 0 700<br>600      | (5, 625)                     |
|-------------------|------------------------------|
| 5 500             | (4, 500)                     |
| 300<br>200<br>200 | (3, 375)                     |
| 100               | (1, 125)                     |
| 0                 | 1 2 3 4 5 6 7<br>Time (days) |

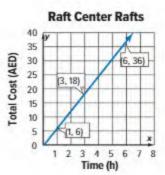
Nisreen's profit at a craft fair is represented by the function
 p = 5b - 15, where p is the profit and b is the number of bracelets she sells. Nahla's profit is shown in the table. (Examples 2 and 3)

a. Compare the y-intercepts and rates of change.
 Nisreen makes AED 5 per bracelet, and Nahla makes AED 5 per bracelet.
 At 0 bracelets, Nisreen's profit is —AED 15 and Nahla's profit is AED 0.

| Bracelets<br>Sold | Profit<br>(AED) |  |
|-------------------|-----------------|--|
| 1                 | 5               |  |
| 2                 | 10              |  |
| 3                 | 15              |  |
| 4                 | 20              |  |

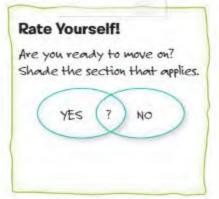
b. How much will each girl make if she sells 30 bracelets?
Nisreen: AED 135: Nahla: AED 150

The cost to rent a raft from two different companies is shown. Which company should you use if you rent the raft for 9 hours?
 (Example 4) Original Rafts; The cost to rent from Original Rafts is AED 33 and the cost to rent from Raft Center Rentals is AED 54.



| Original Rafts |                     |  |
|----------------|---------------------|--|
| Time (h)       | Total<br>Cost (AED) |  |
| 1              | 15.00               |  |
| 2              | 17.25               |  |
| 3              | 19.50               |  |
| 4              | 21.75               |  |
| 5              | 24.00               |  |

4. Question What are the advantages and disadvantages to representing a function as an equation instead of a graph? Sample answer: It is easy to see the rate of change for a function shown as an equation. In a graph, to find the rate of change, you have to determine the slope. A function shown as a graph displays the ordered pairs so you can easily see the relationship, but the relationship is not as easily shown in an equation.



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Name

My Homework

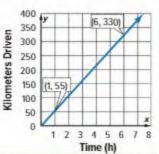
### **Independent Practice**

For the first leg of Adnan's family's trip, their speed averages
68 kilometers per hour. The second leg is shown in the graph. Compare the speeds for each part of their trip. (Example 1)

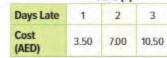


First part: 68 kilometers per hour; Second part: 55 kilometers per

hour. The speed for the first leg is greater by 13 kilometers per hour.



The late fees for a school library are represented by the function c = 2.5d, where c is the total cost and d is the number of days a book is late. The fees charged by a city library are shown in the table. (Examples 2 and 3)



a. Compare the functions' y-intercepts and rates of change.
Both have the same y-intercept of 0. The rates of change are different. Rates of change: school library: AED 2.50 per day; city library: AED 3.50 per day

Wafa checks out one book at each library and returns both books
 3 days late. What are the late fees for each library? school library:
 AED 7.50; city library: AED 10.50

Ali and Omar purchase baseball cards each week. The amount of cards they each have in their collection is shown in the graph and table. Who will have more cards in Week 20? Justify your response. (Example 4)

| Omar's Collection |                    |  |
|-------------------|--------------------|--|
| Week              | Number<br>of Cards |  |
| 1                 | 4                  |  |
| 2                 | 8                  |  |
| 3                 | 12                 |  |

50 (9, 38)

12 (9, 38)

(9, 38)

(9, 38)

(1, 22)

(1, 22)

(1, 22)

(1, 22)

(1, 22)

(1, 22)

(2, 30)

Matt's Collection

Omar; Omar will have 4(20) or 80 cards and Ali will have 2(20) + 20 or 60 cards.

4. Eissa's family is building a patio. One person can place the flagstone at a rate of 4.5 per hour. The equation s = 11h represents the number of stones s that two people can place in h hours. How many more flagstones can 2 people place in 3 hours than one person? Explain.

19.5 stones; Sample answer: One person can lay 4.5(3) or 13.5 stones.

Two people can lay 11(3) or 33 stones. 33 - 13.5 = 19.5

### ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **3** Practice and Apply

### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                       |  |  |
|---------------------------------|-------------------|-----------------------|--|--|
| AL                              | Approaching Level | 1–3, 5, 6, 8, 14, 15  |  |  |
| OL)                             | On Level          | 1, 3, 5, 6, 8, 14, 15 |  |  |
| <b>®</b>                        | Beyond Level      | 4–8, 14, 15           |  |  |

| MATHEMATICAL PRACTICES  |             |  |
|---|-------------|--|
| Emphasis On   | Exercise(s) |  |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 7           |  |
| 2 Reason abstractly and quantitatively.                                       | 5           |  |
| 3 Construct viable arguments and critique the reasoning of others.            | 8, 11       |  |
| 4 Model with mathematics.   | 6           |  |
|   |             |  |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

### TICKET Out the Door

Have students create a table of data that has a constant rate of change. Then write a linear function on the board. Have students compare their functions with the function on the board by comparing the rates of change. See students' work.

Reason Abstractly Refer to the conversions in the tables below.

| Cups | Ounces |  |
|------|--------|--|
| 1    | 8      |  |
| 2    | 16     |  |
| 3    | 24     |  |
| 4    | 32     |  |

| Pints | Ounces |
|-------|--------|
| 1     | 16     |
| 2     | 32     |
| 3     | 48     |
| 4     | 64     |

| Quarts | Ounces |  |
|--------|--------|--|
| 1      | 32     |  |
| 2      | 64     |  |
| 3      | 96     |  |
| 4      | 128    |  |

a. Write a function for each table.

$$z = 8c; z = 16p; z = 32q$$

b. If you graph the points, the graph for which function would have the steepest slope? Justify your response.

the quart equation; Sample answer: The greater the rate of change,

the steeper the slope of the graph.

c. Which function has the least rate of change? Explain.

The first function has the least rate of change because 8 is less than 16 and 32.



### H.O.T. Problems Higher Order Thinking

Model with Mathematics Write a real-world problem where you would want to compare rates of change for two different functions.

See students' work.

- 7. Persevere with Problems Explain why the graph of the function y = 3x + 40 will never intersect the graph of the function y = 3x + 35.
  Sample answer: Both functions have the same rate of change but because they have different y-intercepts, they are parallel lines and parallel lines will never intersect.
- 8. Reason Inductively The exchange rate to convert U.S. dollars to UAE dirhams is represented by the function e = 3.67d where e is the amount in dirhams and d is the number of dollars. One U.S. dollar can also be exchanged for 0.94 European euro. If you exchange \$250 for dirhams and \$250 for euros,

which of the following is true?

- You will receive AED 917.50 pounds and 235 euros.
- II You will receive about AED 410 and about 362 euros.
- III You will receive 250 dollars.
- IV You will receive the same amount of dirhams and euros.

appropriate a constant and a constant

Name

My Homework

### **Extra Practice**

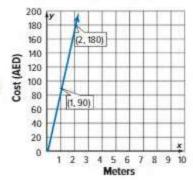
9. A fabric store sells cotton for AED 20.00 a meter. The price of special occasion fabric is shown in the graph. Compare the functions' rates of change.



Cotton fabric: AED 20.00 per meter

Special occasion fabric:  $\frac{180-90}{2-1} = \frac{90}{1}$  or AED 90.00 per weter.

The special occasion fabric has the greater rate of change.



- 10. Two players played a game. The first player's score is represented by the function p = 5c - 3, where p is the number of points scored and c is the number of correct answers. The second player's score is shown in the
  - a. Compare the functions by comparing their y-intercepts and rates of change. Player 1: 5 points per question; Player 2: 5 points per question. Both have the same rate of change, but the function for Player 1 has a y-intercept of -3 and the function for Player 2 has a y-intercept of 0.
- Questions Score Answered 5 2 10 15 3 4 20
- b. How many points will the first player have if he or she correctly answers 30 questions? 147 points
- Dustify Conclusions Usama and Faris each open savings accounts. The amounts in Usama's account are shown in the table. Faris saves AED 50 per week. Who will have more saved in 8 weeks? Explain.

Faris; Sample answer: In 8 weeks Faris will have 50(8) or AED 400. Usama will have saved AED 370.

12. Canada Olympic Park features sports training and entertainment facilities. The Monster zip line produces average speeds of 120 kilometers per hour. A smaller line produces speeds represented by the function d = 50h where d is the distance in kilometers after h hours. How much farther could you travel on the Monster zip line in 0.25 hours?



17.5 km

13. Faleh gets a 1.5 kilometers head start and runs at a rate of 4.5 kilometers per hour. Fahd's progress is represented by a graph that goes through the points (1, 10), (2, 20), and (3, 30). How long will Fahd need to run to catch up with Faleh? 11 h

### Power Up! Test Practice

Exercises 14 and 15 prepare students for more rigorous thinking needed for assessment.

This test item requires students to support their previous reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.

| constructing arguments | •  |  |  |
|------------------------|--|--|--|
| Depth of Kowledge      | DOK3   |  |  |
| Mathematical Practice  | MP1, MP3, MP4  |  |  |
| Scoring Rubric         |  |  |  |
| 2 points               | Students correctly graph both functions, compare the rates of change and y-intercepts, and explain what they represent.  |  |  |
| 1 point                | Students correctly graph both functions but fail to correctly answer the question OR students correctly answer the question but fail to correctly graph either function. |  |  |

15. This test item requires students to support their previous reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.

| Depth of Knowledge    | DOK3   |  |
|-----------------------|--|--|
| Mathematical Practice | MP1, MP3   |  |
| Scoring Rubric        |  |  |
| 2 points              | Students correctly answer the question and justify their response.       |  |
| 1 point               | Students correctly answer the question b fail to justify their response. |  |

### Power Up! Test Practice

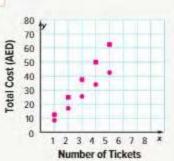
14. A museum charges AED 12.50 per adult ticket. The price of a student ticket is represented in the table.

| Student Ticket Price |      |       |       |  |
|----------------------|------|-------|-------|--|
| Tickets              | 1    | 2     | 3     |  |
| Price<br>(AED)       | 8.50 | 17.00 | 25.50 |  |

Graph both functions on the coordinate plane. Use circles to represent student ticket prices and use squares to represent adult ticket prices.

Compare the rates of change and y-intercepts of the linear functions. Explain what each of these represent.

Sample answer: The rates of change represent the cost per ticket for adult and student tickets. The adult ticket function has a higher rate of change because the cost per ticket is higher. Both functions have a y-intercept at the origin because buying 0 tickets costs AED 0.



15. Noura swam, biked, and ran in a 35.6 kilometer triathlon. She completed the race in 2.15 hours. The function k = 22.1h represents the kilometers k Noura biked in h hours. Was her average speed biking less than or greater than her average speed for the entire race? Justify your answer. Round to the nearest tenth if necessary.

Sample answer: Her average speed biking was 13.8 kmph, and her average speed for the entire race was about 22.2 ÷ 2.15 or about 10.3 kmph. So, her average speed biking was greater than her average speed for the entire race.

### **Spiral Review**

Write an equation in slope-intercept form for each table of values.

| ò. | x | -1 | 0  | 1 | 2 |
|----|---|----|----|---|---|
|    | y | -7 | -3 | 1 | 5 |

$$y = 4x - 3$$

| 17. | x | -3 | -1 | 1 | 3 |
|-----|---|----|----|---|---|
|     | y | 7  | 5  | 3 | 1 |

$$y = -x + 4$$

Functions

Lesson 6

### **Construct Functions**



### Real-World Link

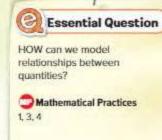
Parties Majed is planning to have a celebration at a skating rink. The rink charges a group fee plus an additional charge for each quest.

 Choose two points from the table and find the rate of change.

Sample answer: (3, 59) and (5, 65).

The rate of change is 3.

| Number of Guests, x | Total Cost<br>(AED), y |  |
|---------------------|------------------------|--|
| 1                   | 53                     |  |
| 2                   | 56                     |  |
| 3                   | 59                     |  |
| 4                   | 62                     |  |
| 5                   | 65                     |  |
| 6                   | 68                     |  |



2. Write a function to represent this situation.

v = 50 + 30

- Graph the ordered pairs. Then extend the line of the graph until it crosses the y-axis.
- Use the function to find the amount the skating rink charges for the group fee.

AED 50



Which Mathematical Practices did you use?

Shade the circle(s) that applies.

- 1) Persevere with Problems
  - ere with Problems
- ② Reason Abstractly
- 3 Construct an Argument
- www.jnob-jo.com
- © Use Math Tools

  © Attend to Precision
- Make Use of Structure

  8 Use Repeated Reasoning



### Focus narrowing the scope

**Objective** Find and interpret the rate of change and initial value of a function.

### Coherence connecting within and across grades

### Previous

Students used different representations of two functions to compare the functions.

### Now

Students use multiple representations to construct functions.

### Next

Students will use multiple representations to identify linear and nonlinear functions.

### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 323.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

### 1 Launch the Lesson

### **Ideas for Use**

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Pairs Discussion Have students work in pairs to complete Exercises 1–4. Ask them how they can determine the *y*-intercept from the table and the graph. Then ask them to interpret the *y*-intercept in the context of the real-world problem.

### **Alternate Strategy**

Have students use the rate of change to count backward in the table to find the value of y when x = 0. Have them use the graph in Exercise 3 to verify that their function is correct.

# **2** Teach the Concept

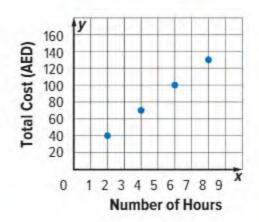
**Ask** the scaffolded questions for each example to differentiate instruction.

### Example

- Find and interpret the rate of change and the initial value.
- How can you find the rate of change? Determine the slope of the line by finding rise/run.
  - What two points can you choose? Sample answer: (2, 60) and (4, 90)
- What is the rate of change? What does it mean in the context of the problem? 15; The amount of points earned per pair of shoes is 15.
  - How can you determine the y-intercept? What is the y-intercept? Extend the line so that it crosses the y-axis.
     The y-intercept is the y-coordinate of the point where the line intersects the y-axis. The y-intercept is 30.
  - What does the y-intercept mean in the context of the problem? The initial number of points earned is 30.
- How is the slope different than the y-intercept? The slope is the rate of how the y-values change as the x-values change. The y-intercept is the initial value.

### **Need Another Example?**

A pottery studio charges a certain amount per hour, plus a firing fee to fire the pottery. The graph shows the total cost of using the studio for different amounts of time. Find and interpret the rate of change and initial value. The firing fee is AED10. The hourly fee is AED15.



Work Zone

### **Analyze Graphs, Words, and Tables**

The *initial value of a function* is the corresponding *y*-value when *x* equals 0. You can find the initial value of a function from graphs, words, and tables.



### Example

 A shoe store offers free points when you sign up for their rewards card. Then, for each pair of shoes purchased, you earn an additional number of points. The graph shows the total points earned for several pairs of shoes. Find and interpret the rate of change and initial value.



To find the rate of change, choose two points from the graph.

$$\frac{\text{change in points}}{\text{change in pairs}} = \frac{(90 - 60) \text{ points}}{(4 - 2) \text{ pairs}}$$
$$= \frac{15 \text{ points}}{1 \text{ pair}}$$

The rate of change is 15, so the number of points earned per pair of shoes is 15.

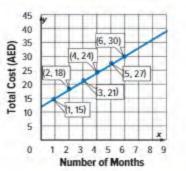
Next find the initial value or the *y*-value when x = 0. Recall this value is called the *y*-intercept. Extend the line so it intersects the *y*-axis. The value for *y* when x = 0 is 30. So, the initial number of points earned is 30.



AED 3. The yearly fee is AED 12.

### Got it? Do this problem to find out.

a. Music Inc. charges a yearly subscription fee plus a monthly fee. The total cost for different numbers of months, including the yearly fee, is shown in the graph. Find and interpret the rate of change and initial value.



apyright © Woorewell Equ

Functions

The hourly cost is AED 20.

The rental fee is AED 30.



### Example

 Amani has some photos in her photo album. Each week she plans to add 12 photos. Amani had 120 photos after 8 weeks. Assume the relationship is linear. Find and interpret the rate of change and initial value.

Since each week Amani adds 12 photos to her photo album the rate of change is 12. To find the initial value, use slope-intercept form to find the *y*-intercept.

y = mx + b

Slope-intercept form

y = 12x + b

Replace in with the rate of change, 12.

120 = 12(8) + b

Replace y with 120 and x with 8

24 = b

Solve for b.

The y-intercept is 24. So, the initial number of photos is 24.

#### Got it? Do this problem to find out.

b. A zoo charges a rental fee plus AED 20 per hour for strollers. The total cost of 5 hours is AED 130. Assume the relationship is linear. Find and interpret the rate of change and initial value.



### Example

 The table shows how much money Amal has saved. Assume the relationship between the two quantities is linear. Find and interpret the rate of change and initial value.

| Number<br>of<br>Months, x | Money<br>Saved<br>(AED), y |
|---------------------------|----------------------------|
| 3                         | 110                        |
| 4                         | 130                        |
| 5                         | 150                        |
| 6                         | 170                        |

Choose any two points from the table to find the rate of change. The rate of change is

 $\frac{150-110}{5-3}$  or 20, so Amal saves AED 20 each

month. To find the initial value, use the slope-intercept form to find the y-intercept.

y = mx + b

Slope-intercept form

y = 20x + b

Replace in with the rate of change, 20.

110 = 20(3) + b

Use the point (3, 110), x = 3, y = 110

50 = b

Solve for b.

The y-intercept is 50, so Amal had initially saved AED 50.

### **Examples**

- Find and interpret the rate of change and the initial value.
- How many photos were added each week? 12
  - How many photos were in the album after 8 weeks? 120
- How can you determine the initial number of photos in the album? You know that when the number of weeks x is 8, there are 120 photos y. You can use these values in the slope-intercept form equation to determine the initial value.
- What is the equation of the line in slope-intercept form? y = 12x + 24

### Need Another Example?

Each week, Hiyam works the same number of hours. Her first week included an orientation. The total number of hours that she worked the 2nd, 3rd, and 4th weeks was 20 hours, 28 hours, and 36 hours. Assume the relationship is linear. Find and interpret the rate of change and initial value. She works 8 hours each week. The orientation was 4 hours long.

- 3. Find and interpret the rate of change and the initial value.
- What two values do we need to determine? slope and y-intercept
- How can you use the information in the table to find the initial amount? You know one point on the line. You can find the slope using two points and then use the slopeintercept form equation to find the initial value.
- If the equation was y = 20x + 45, how much would Ava had initially saved? AED45

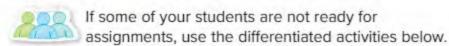
### **Need Another Example?**

The table shows the weight of a kitten in weeks 4, 5, 6, and 7. Assume the relationship between the two quantities is linear. Find and interpret the rate of change and initial value. The kitten gains 3 kilograms per week. At birth, the kitten weighed 5 kilograms.

| Number of Weeks, x | 4  | 5  | 6  | 7  |
|--------------------|----|----|----|----|
| Weight (kg), y     | 17 | 20 | 23 | 26 |

### **Guided Practice**

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.



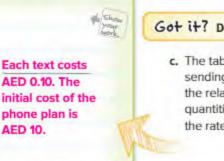
Think-Pair-Share Have students work in pairs. Give them one minute to think through their response to Exercise 1. Then have them share their responses with their partner and discuss any differences in their solutions. Have them repeat this process for Exercises 2-4. Call on one pair of students to share their responses with the class. 21,3

III UA Trade-a-Problem Have students create a realworld scenario similar to Exercises 2-3. Have them make a table of values or create a graph on the coordinate plane. Then have students trade their problem and write an equation for each other's graph or table. Discuss and resolve any differences in their solutions. 2, 3, 4

### Watch Out!

Common Error Students may use the first value given in the graph or table as the initial value. Remind them to extend the graph to an x-value of 0 to find the initial value.

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Got it? Do this problem to find out.

c. The table shows the monthly cost of sending text messages. Assume the relationship between the two quantities is linear. Find and interpret the rate of change and intial value.

| Number of<br>Messages, x | Cost (AED), y |
|--------------------------|---------------|
| 5                        | 10.50         |
| 6                        | 10.60         |
| 7                        | 10.70         |

### **Guided Practice**

**AED 10.** 

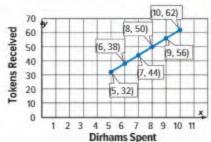


1. As part of a grand opening, a funfair gave out free tokens to the first 100 customers. The graph shows the number of tokens customers received for each dirham spent at the funfair. Find and interpret rate of change and the initial value. (Example 1)



Each dirham buys 6 tokens. The initial number of

tokens given out is 2.



2. A historic museum charges a rental fee plus AED 20 per hour for an audio tour guide. The total cost for 4 hours is AED 120. Find and interpret the rate of change and initial value. (Example 2)

The hourly cost is AED 20. The rental fee is AED 40.

3. A science center charges an initial membership fee. The total cost of the membership depends on the number of people on the membership as shown in the table. Assume the relationship between the two quantities is linear. Find and interpret the rate of change and the initial value. (Example 3) Each person pays an additional AED 15. The initial fee is AED 35.

Number of People, x 3 4 Additional Cost (AED), y 65

4. Building on the Essential Question How is the initial value of a function represented in a table and in a graph? Sample answer: In a table, the initial value of a function is the corresponding y-value when x = 0. In a

graph, the initial value is the y-intercept.

Rate Yourself!

I understand how to construct functions.

Great! You're ready to move on!

1 still have questions about constructing functions.

My Hamework

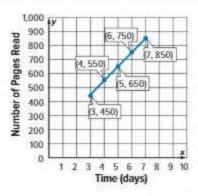
### **Independent Practice**

A teacher read part of a book to a class. The graph shows the number of pages read by the teacher over the next several days. Find and interpret the rate of change and the initial value. (Example 1)

The teacher read 100 pages per day. The teacher initially

read 150 pages before the graph begins.





2. A water park charges a rental fee plus AED 15 per hour to rent inflatable rafts. The total cost to rent a raft for 6 hours is AED 150. Assume the relationship is linear. Find and interpret the rate of change and the initial

value. (Example 2) The hourly cost is AED 15. The rental fee is AED 60.

A teacher already had a certain number of canned goods for the food drive. Each day of the food drive, the class plans to bring in 10 cans. The total number of canned goods for day 10 is 205. Assume the relationship is linear. Find and interpret the rate of change and the initial value. (Example 2)

The class brings in 10 cans per day. The teacher initially had 105 cans.

4. Hidaya frosted some cupcakes in the morning for a celebration. The table shows the total number of cupcakes frosted after she starts up after lunch. Assume the relationship between the two quantities is linear. Find and interpret the rate of change and the initial value. (Example 3)

| Time (min), x         | 5  | 10 | 15 | 20 |
|-----------------------|----|----|----|----|
| Number of Cupcakes, y | 28 | 32 | 36 | 40 |

She can frost 0.8 cupcake per minute. She frosted 24 cupcakes in the morning.

Ayman has a certain number of DVDs in his collection. The table shows the total number of DVDs in his collection over several months. Assume the relationship between the two quantities is linear. Find and interpret the rate of change and the initial value. (Example 3)

| Month, x          | 3  | 6  | 9  | 12 |
|-------------------|----|----|----|----|
| Number of DVDs, y | 18 | 27 | 36 | 45 |

Each month Ayman adds 3 DVDs. He started with 9 DVDs.

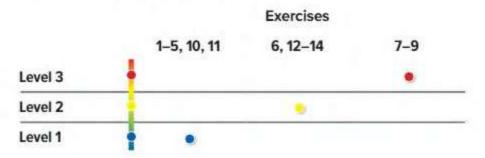
# **Practice and Apply**

### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                          |
|---------------------------------|-------------------|--------------------------|
| AL                              | Approaching Level | 1–5, 8, 9, 13, 14        |
| OL)                             | On Level          | 1–5 odd, 6, 8, 9, 13, 14 |
| BL                              | Beyond Level      | 6–9, 13, 14              |

| MATHEMATICAL PRACTICES  |             |  |
|---|-------------|--|
| Emphasis On   | Exercise(s) |  |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 7           |  |
| 3 Construct viable arguments and critique the reasoning of others.            | 6, 9, 12    |  |
| 4 Model with mathematics.   | 8           |  |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

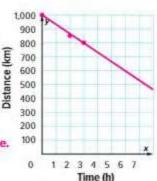
### TICKET Out the Door

Have students describe a function that has an initial value of 30 by making a table or drawing a graph. Tell them to include the rate of change of their function. See students' work.

- Multiple Representations Badr's family is driving from Boston to Chicago. The total distance of the trip is 1,600 kilometers and each hour they will drive 104 kilometers.
  - a. Algebra Write an equation to represent the number of remaining kilometers y after driving any number of hours x.

$$y = 1,600 - 104x$$

- Graphs Graph the equation from part a on a coordinate plane.
- c. Numbers What is the rate of change and y-intercept of the line? -104 kmph;1,600
- d. Words Explain why the line slopes down by 104 for each hour. Sample answer: The line slopes down by 104 because each hour they travel they have 104 less kilometers to drive.
- e. Words Why does the line cross the y-axis at 1,600?
  They began the trip with 1,600 kilometers to drive.



### OL,

### H.O.T. Problems Higher Order Thinking

- 7. Persevere with Problems Explain why a horizontal line has a rate of change of zero. Sample answer: The rate of change is represented by the ratio change in y change in x. For a horizontal line, x can increase or decrease, but y does not change. The numerator will be 0, so the rate of change is 0.
- Model with Mathematics Write and solve a real-world problem in which you need to find the initial value of a function. Then explain to a classmate how you solved your problem.

See students' work.

They are all correct; Sample answer: The properties of operations show

that these four equations are equivalent.

Name

My Homework

### **Extra Practice**

10. A snowboard instructor charges an initial fee plus AED 40 per hour for private snowboarding lessons. Amna paid AED 265 for six hours of instruction. Assume the relationship is linear. Find and interpret the rate of change and initial value.

The instructor charges AED 40 per hour. The initial fee is AED 25.

Since the instructor charges AED 40 per hour, the rate of change is 40. To find the initial value, use slope-intercept form to find the y-intercept.



y= mx+b y=40x+6

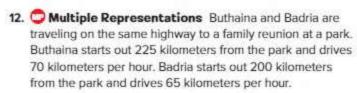
265 = 40(6) + 6

25 = b

The y-intercept is 25. So, the initial fee is AED 25.

11. A family drove to their grandmother's house. After that they averaged 200 kilometers per day for 8 days. They drove a total of 1,880 kilometers over the eight days. Assume the relationship is linear. Find and interpret the rate of change and initial value.

The family drove 200 kilometers per day. They drove 280 kilometers to their grandmother's house.

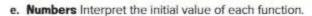


a. Algebra Write an equation for Buthaina's trip where y is the total distance from the park after x hours.

$$y = 225 - 70x$$

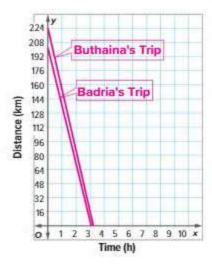
- b. Algebra Write an equation for Badria's trip where y is the total distance from the park after x hours. y = 200 - 65x
- c. Graphs Graph both equations on the same coordinate plane.
- d. Words Will Buthaina's and Badria's trip overlap before they reach the park? Explain your reasoning.

No: the lines do not intersect.



Buthaina begins her trip 225 kilometers from the park. Badria begins

her trip 200 kilometers from the park.



# Power Up! Test Practice

Exercises 13 and 14 prepare students for more rigorous thinking needed for assessment.

13. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge DOK1

Mathematical Practice MP1

Scoring Rubric

1 point Students correctly answer each part of the

question.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments. Depth of Knowledge DOK3 Mathematical Practice MP1, MP4 **Scoring Rubric** 2 points Students correctly model the slope, y-intercept, and linear function and explain what the slope and y-intercept represent. Students correctly model all three, but fail 1 point to explain what the slope and y-intercept represent OR students correctly model 2 of the 3 and may or may not explain what the slope and y-intercept represent OR students correctly model 1 and explain what slope and y-intercept represent.

# Power Up! Test Practice

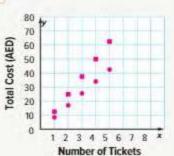
 A museum charges AED 12.50 per adult ticket. The price of a student ticket is represented in the table.

|                | Student Ticket Price |       |       |  |  |  |
|----------------|----------------------|-------|-------|--|--|--|
| Tickets        | 1                    | -2    | 3     |  |  |  |
| Price<br>(AED) | 8.50                 | 17.00 | 25.50 |  |  |  |

Graph both functions on the coordinate plane. Use circles to represent student ticket prices and use squares to represent adult ticket prices.

Compare the rates of change and y-intercepts of the linear functions. Explain what each of these represent.

Sample answer: The rates of change represent the cost per ticket for adult and student tickets. The adult ticket function has a higher rate of change because the cost per ticket is higher. Both functions have a y-intercept at the origin because buying 0 tickets costs AED 0.



15. Noura swam, biked, and ran in a 35.6 kilometer triathlon. She completed the race in 2.15 hours. The function k = 22.1h represents the kilometers k Noura biked in h hours. Was her average speed biking less than or greater than her average speed for the entire race? Justify your answer. Round to the nearest tenth if necessary.

Sample answer: Her average speed biking was 13.8 kmph, and her average speed for the entire race was about 22.2 ÷ 2.15 or about 10.3 kmph. So, her average speed biking was greater than her average speed for the entire race.

### **Spiral Review**

Write an equation in slope-intercept form for each table of values.

16. x -1 0 1 2 y -7 -3 1 5

y = 4x - 3



Functions

Lesson 7

# **Linear and Nonlinear Functions**

Height

0

6.2

9.7

10.5

8.7

4.2

0.0

0.5

1.0

1.5

2.0

2.5

Distance

0

13

26

39

52

65



### Real-World Link

Football The table shows the approximate height and horizontal distance traveled by a football kicked at an angle of 30° with an initial velocity of 30 meters per second.

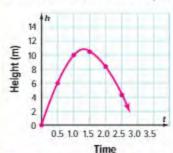
1. Is the rate of change for the height of the football constant? Explain.

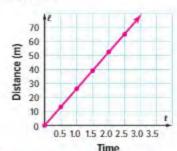
no; Sample answer: In the first half second, the ball's change in height was 6.2 meters. In the next halfsecond, the ball's change in height was 9.7 - 6.2, or 3.5 meters.

2. Is the rate of change for the distance traveled constant? Explain.

yes; Sample answer: The football travels a distance of 13 meters each half-second.

3. Graph the ordered pairs (time, height) and (time, distance) on separate grids. Connect the points with a straight line or smooth curve. Then compare the graphs.





Sample answer: The graph for the height of the football is a curve and the graph for the distance is linear.

Which Mathematical Practices did you use? Shade the circle(s) that applies.

- 1) Persevere with Problems
- 2 Reason Abstractly
- (3) Construct an Argument

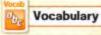
Model with Mathematics

(5) Use Math Tools Attend to Precision

(8) Use Repeated Reasoning

Make Use of Structure

**Essential Question** HOW can we model relationships between quantities?



nonlinear function

Mathematical Practices

### Focus narrowing the scope

Objective Determine whether a function is linear or nonlinear.

### Coherence connecting within and across grades

### Previous

Students used multiple representations to construct functions.

#### Now

Students use multiple representations to identify linear and nonlinear functions.

### Next

Students will graph and analyze quadratic functions.

### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 331.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **Launch the Lesson**

### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BD Research-Pair-Share Have students work in pairs. Give students five minutes to research quadratic equations. Then have them share what they found with their partner and determine the maximum and minimum values for the height of the football over time. Then call on one student to share what they found within a small or large group. 1, 3, 5

Alternate Strategy

Tell students the graph of the height of the football should look like a curve. The graph of the distance traveled should be a straight line. Ask students how the rate of change differs in the two graphs.

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# **7** Teach the Concept

**Ask** the scaffolded questions for each example to differentiate instruction.

## Example

- 1. Identify linear or nonlinear functions using a table.
- What is the first step to find the rates of change for the values in the table? find the change between the x-coordinates and the change between the y-coordinates
- Does the function have a constant rate of change? yes
  - · Does this indicate a linear or nonlinear function? linear
- Is this function increasing or decreasing? Explain. decreasing, as x increases, y decreases

### **Need Another Example?**

Determine whether the table represents a *linear* or *nonlinear* function. Explain. Nonlinear; the rate of change is not constant.

| x | 2 | 4  | 6  | 8   |
|---|---|----|----|-----|
| у | 2 | 20 | 54 | 104 |

- 2. Identify linear or nonlinear functions using a table.
- Does the function have a constant rate of change? no
- Is the function linear or nonlinear? Explain. Nonlinear because the rate of change is not constant.
- Did you need to find the rate of change for the entire table to determine if the function is linear or nonlinear? Explain. No, once you find the rate of change was different for the first few values, you know the function is nonlinear.

### **Need Another Example?**

Determine whether the table represents a *linear* or *nonlinear* function. Explain. Linear; the rate of change is constant, as *x* increases by 3, *y* increases by 9.

| x | 1 | 4 | 7  | 10 |
|---|---|---|----|----|
| у | 0 | 9 | 18 | 27 |

Work Zone

Increasing and

If vincreases as xincreases,

decreases as xincreases, the

a. Linear; the rate of

change is constant:

as x increases by 5,

y decreases by 4.

increases by 2, y increases by a greater amount each time.

b. Nonlinear; as x

the function is called an

Increasing function. If y

function is called a

decreasing function.

Decreasing

**Functions** 

### **Identify Linear and Nonlinear Functions**

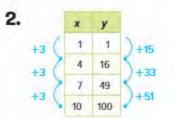
In a previous lesson, you learned that linear functions have graphs that are straight lines. This is because the rate of change between any two data points is a constant. **Nonlinear functions** are functions whose rates of change are not constant. Therefore, their graphs are not straight lines.

# Examples

Determine whether each table represents a linear or nonlinear function. Explain.

| 1. |      | x | у  |      |
|----|------|---|----|------|
|    | +2 ( | 2 | 50 | )-15 |
|    | +2 ( | 4 | 35 | 5-15 |
|    | >    | 6 | 20 | 1    |
|    | +2   | 8 | 5  | 1-15 |

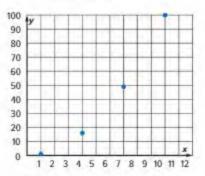
As x increases by 2, y decreases by 15 each time. The rate of change is constant, so this function is linear.



As x increases by 3, y increases by a greater amount each time. The rate of change is not a constant, so this function is nonlinear.

#### Check

Graph the points on a coordinate plane.



The points do not fall in a line. The function is nonlinear. ✓

Got it? Do these problems to find out.

Determine whether each table represents a linear or nonlinear function. Explain.

| a. | x | 0  | 5 | 10 | 15 |   |
|----|---|----|---|----|----|---|
|    |   | 20 | - | -  | -  | ١ |

| b. | x | 0 | 2 | 4 | 6  |  |
|----|---|---|---|---|----|--|
|    | y | 0 | 2 | 8 | 18 |  |

Functions



### Example

Use the table to determine whether the minimum number of Calories a tiger cub should eat is a linear function of its age in weeks.

| Use the table to find the rates of change. | Use | the | table | to | find | the | rates | of | change. |
|--|-----|-----|-------|----|------|-----|-------|----|---------|
|--|-----|-----|-------|----|------|-----|-------|----|---------|

1.000 - 825 = 175

1.185 - 1.000 = 185

1,320 - 1,185 = 135

1.420 - 1.320 = 100

The rates of change are not the same. Therefore, this function is nonlinear.

Check Graph the data to verify the ordered pairs do not lie on a straight line.

#### Got it? Do this problem to find out.

c. Tickets to the school dinner cost AED 5 per student. Are the ticket sales a linear function of the number of tickets sold? Explain.

| Number of<br>Tickets Sold | 1     | 2      | 3      |
|---------------------------|-------|--------|--------|
| Ticket Sales              | AED 5 | AED 10 | AED 15 |

Minimum

Calorie

Intake

825

1.000

1,185

1,320

1,420

Shou

C. Yes; the rate of

tickets sold

change is constant;

as the number of

increases by 1, the

increases by AED 5.

total ticket sales

(weeks)

2

3

4

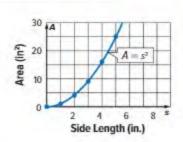
### Example

4. A square has a side length of s centimeters. The area of the square is a function of the side length. Does this situation represent a linear or nonlinear function? Explain.

Make a table to show the area of the square for side lengths of 1, 2, 3, 4, and 5 centimeters.

| Side Length (cm) | 1 | 2 | 3 | 4  | 5  |
|------------------|---|---|---|----|----|
| Area (cm²)       | 1 | 4 | 9 | 16 | 25 |

Graph the function. The function is not linear because the points (1, 1), (2, 4), (3, 9), (4, 16), and (5, 25) are not on a straight line.



### Identify linear and nonlinear functions.



• What is the rate of change from week 2 to week 3?



Is the function linear or nonlinear? nonlinear



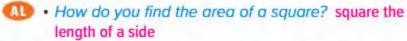
#### Need Another Example?

**Examples** 

Use the table to determine whether or not the number of revolutions per hour of a second hand on a clock is a linear function of the number of hours that pass. Linear; the rate of change is constant, as the number of hours increased by 1, the number of second hand revolutions increases by 60.

| Hour                          | 1  | 2   | 3   | 4   | 5   |
|-------------------------------|----|-----|-----|-----|-----|
| Second<br>Hand<br>Revolutions | 60 | 120 | 180 | 240 | 300 |

### 4. Identify linear and nonlinear functions.



 What is the area of a square with a side length of 1 centimeter? 2 centimeters? 3 centimeters? 1cm<sup>2</sup>; 4 cm2; 9 cm2

What points will you graph? (1, 1), (2, 4), (3, 9), (4, 16),

Is there a constant rate of change? no

Is the function linear or nonlinear? nonlinear

BD • Can you look at the equation A = s² and determine if the function is linear or nonlinear? Explain. Yes, the equation has a variable to a power other than 1 in it, so it is nonlinear.

### Need Another Example?

At the first level of a maze, there are three possible paths that can be chosen. At the next level, each of those three paths have three more possible paths. Does this situation represent a linear or nonlinear function? Explain. Nonlinear; if you graph the function, the points do not lie on a straight line.

## **Guided Practice**

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.



If some of your students are not ready for assignments, use the differentiated activities below.

Team-Pair-Solo Have students work in teams of four to complete Exercise 1. Then each team splits into two pairs to complete Exercises 2 and 3. Finally, students work by themselves to solve Exercise 4. The team regroups to compare solutions and discuss any differences. 01,3

Find the Fib Students write down two facts and one fib about linear functions or nonlinear functions. Students then form teams of three. The job of the team is to identify the fib in each group of statements. 21,3

Sample answer:

A If you graph the function, the ordered pairs (side length, perimeter) lie on a straight line.

Got it? Do this problem to find out.

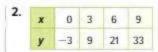
d. A square has a side length of s centimeters. The perimeter of the square is a function of the side length. Does this situation represent a linear or nonlinear function? Explain.

### **Guided Practice**



Determine whether each table represents a linear or nonlinear function. Explain. (Examples 1 and 2)

x 0 1 2 3 y 1 3 5 10



Nonlinear; as x increases by 1, y increases by a greater amount of time.

Linear; the rate of change is constant; as x increases by 3, y increases by 12.

3. The table shows the measures of the sides of several rectangles. Are the widths of the rectangles a linear function of the lengths? Explain. (Example 3) No; the rate of change is not constant.

| Length (cm) | 1  | 4  | 8 | 10  |
|-------------|----|----|---|-----|
| Width (cm)  | 64 | 16 | 8 | 6.4 |

- 4. A cube has a side length of s meters. The volume of the cube is represented by the expression s3. The volume of the cube is a function of the side length. Does this situation represent a linear or nonlinear function? Explain. (Example 4) Nonlinear; sample answer: If you graph the function, the ordered pairs (side length, volume) do not lie on a straight line.
- 5. Q Building on the Essential Question How can you use a table or a graph to determine if a function is Sample answer: The table values indicates a constant

rate of change between the x- and y-values in a linear function; the graph of a linear function is a straight line that is not vertical.



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My Homework

# Independent Practice

Determine whether each table represents a linear or nonlinear function.

Explain. (Examples 1 and 2)

Linear; rate of change is constant; as x increases by 2, y increases by 1.

Nonlinear; rate of change is not

20 43

Linear; rate of change is constant; as x increases by 5, y increases by 15.

5 -18 -50

Nonlinear; rate of change is not constant.

IS Jassim's family drove from Sharjah to Ruwais. Use the table to determine whether the distance driven is a linear function of the hours

| Time (h)      | 1  | 2   | 3   | 4   |
|---------------|----|-----|-----|-----|
| Distance (km) | 65 | 130 | 195 | 260 |

traveled. Explain. (Example 3) Yes; the rate of change is constant; as the time increases by 1 hour, the distance increases by 65 kilometers.

6. The table shows the height of several buildings in Sharjah. Use the table to determine whether the height of the building is a linear function of the number of stories. Explain. (Example 3)

No; the rate of change is not constant.

| Building         | Stories | Height (m) |
|------------------|---------|------------|
| United Bank      | 35      | 170        |
| The Blue Tower   | 40      | 172        |
| Department Store | 45      | 182        |
| Etisalat         | 50      | 194        |
| Health Center    | 55      | 184        |

- 7. There are 3.600 seconds in one hour. The total seconds is a function of the hours. Does this situation represent a linear or nonlinear function? Explain. (Example 4) Linear; sample answer: If you graph the function, the ordered pairs (hours, seconds) lie on a straight line.
- 8. A football is placed on the ground for a free kick. The height of the ball is a function of the time in seconds. Does the path the football follows after being kicked represent a linear or nonlinear function? Explain. (Example 4) Nonlinear; sample answer: After being kicked, the ball will reach a maximum height and come back to the ground.



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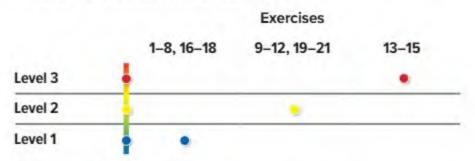
# **Practice and Apply**

### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                               |
|---------------------------------|-------------------|-------------------------------|
| AL                              | Approaching Level | 1–9, 11, 14, 15, 20, 21       |
| 0                               | On Level          | 1–7 odd, 9–12, 14, 15, 20, 21 |
| <b>B</b>                        | Beyond Level      | 9–15, 20, 21                  |

### Watch Out!

Common Error Since the graphs of some nonlinear functions may appear linear because of the scales on the axes, students may incorrectly identify those functions as linear. Advise students to use a table to check that the rate of change is constant.

| MATHEMATICAL PRACTICES  |             |  |
|---|-------------|--|
| Emphasis On   | Exercise(s) |  |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 13          |  |
| 3 Construct viable arguments and critique the reasoning of others.            | 15, 19      |  |
| 4 Model with mathematics.   | 14          |  |
| 7 Look for and make use of structure.   | 12          |  |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

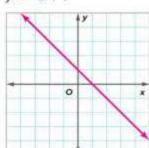
Use this activity as a closing formative assessment before dismissing students from your class.



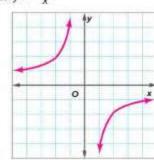
Have students explain how to tell whether a table of x- and y-values describes a function. See students' work.

Graph each function by making a table of ordered pairs. Determine whether each function is *linear* or *nonlinear*. Explain.

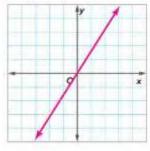
9. 
$$y = -x + 1$$



10.  $y = \frac{-4}{x}$ 



**11.**  $y = \frac{3x}{2}$ 



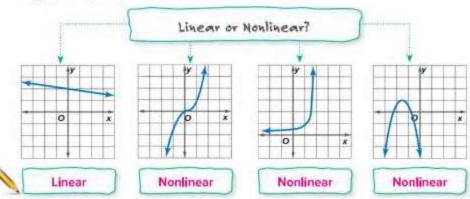
Linear; sample answer: The points lie on a straight line.

Nonlinear; sample answer:

The graph is a curve.

Linear; sample answer: The points lie on a straight line.

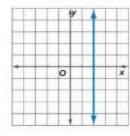
 Gldentify Structure Complete the graphic organizer by determining if the graphs represent linear or nonlinear functions.





### H.O.T. Problems Higher Order Thinking

13. Persevere with Problems Does the graph at the right represent a linear function? Explain. No; sample answer: the graphs of vertical lines are not functions because there is more than one value of y that corresponds to x = 2.



- Model with Mathematics Give an example of a situation that can be represented by a nonlinear function. Sample answer: Every hour the number of bacteria in a petri dish doubles.
- 15. Reason Inductively Explain how you can use different representations to determine whether a function is linear. Sample answer: A non-vertical graph that is a straight line is linear. An equation that can be written in the form y = mx + b is linear. If a table of values shows a constant rate of change, it is linear.

absular a problem-41 condition

Name

My Hamework

### **Extra Practice**

Determine whether each table represents a linear or nonlinear function. Explain.

16.

|      | X. | y  |     |
|------|----|----|-----|
| 126  | 2  | 10 | 3.0 |
| +2 ( | 4  | 12 | 5   |
| +2 ( | 6  | 16 | 2+4 |
| +2(  | 8  | 24 | )+8 |



Moulinear; rate of change is not constant. As x increases by 2, y increases by a greater amount each time. The rate of change is not constant, so this function is nonlinear.

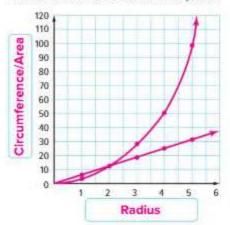
| ž | X  | y  |
|---|----|----|
|   | 4  | 3  |
|   | 8  | 0  |
|   | 12 | -3 |
|   | 16 | -6 |

Linear; rate of change is constant; as x increases by 4, y decreases by 3.

- 18. Copy and Solve The area of a square is a function of its perimeter. Graph the function on a separate sheet of grid paper. Explain whether the function is linear and if the graph is increasing or decreasing. See margin.
- 19. Multiple Representations Recall that the circumference of a circle is equal to two times  $\pi$  times its radius and that the area of a circle is equal to  $\pi$  times the square of the radius.
  - a. Tables Complete the table showing the circumference and area of circles with radius r.

| Radius<br>r | Circumference<br>2 • π • r          | Area<br>πr <sup>2</sup>       |
|-------------|-------------------------------------|-------------------------------|
| V           | 2 • π •   ≈ 6.28                    | $\pi \cdot 1^2 \approx 3.14$  |
| 2           | 2 • π • 2 ≈ 12.57                   | $\pi \cdot 2^2 \approx 12.57$ |
| 3           | 2 • π • 3 ≈ 18.85                   | $\pi * 3^2 \approx 28.27$     |
| 4           | $2 \cdot \pi \cdot 4 \approx 25.13$ | $\pi \cdot 4^2 \approx 50.27$ |
| 5           | 2 • π • 5 ≈ 31.42                   | $\pi \cdot 5^2 \approx 78.54$ |

b. Graphs Graph the ordered pairs (radius, circumference) and (radius, area) for each function on the same coordinate plane.



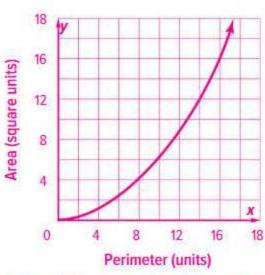
c. Words Is the circumference of a circle a linear or nonlinear function of its radius? the area?

Explain your reasoning. Circumference: linear; sample answer: When the ordered pairs are graphed, the points fall in a line. Area: nonlinear; sample answer:

When the ordered pairs are graphed, the points do not fall in a line.

### **Additional Answer**

18.



Sample answer: The function is nonlinear because the graph of the function is not a straight line and the line is increasing.

# Power Up! Test Practice

Exercises 20 and 21 prepare students for more rigorous thinking needed for assessment.

20. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure. Depth of Knowledge DOK1 MP1 Mathematical Practice **Scoring Rubric** Students correctly answer each part of 1 point the question.

| 21. | This test item requires s<br>quantitatively when pro | tudents to reason abstractly and<br>blem solving.    |
|-----|--|--|
|     | Depth of Knowledge                                   | DOK1   |
|     | Mathematical Practice                                | MP1  |
|     | Scoring Rubric                                       |  |
|     | 1 point  | Students correctly answer each part of the question. |

# Power Up! Test Practice

20. Determine if each table represents a linear or nonlinear function.

| *    | y    |
|------|------|
| 9    | 1.25 |
| 11.5 | 2    |
| 14   | 2.75 |
| 16.5 | 3.5  |

| ×   |    |
|-----|----|
| -3  | -6 |
| -7  | -1 |
| -11 | 4  |
| -15 | 9  |

| *  | y  |
|----|----|
| 10 | 5  |
| 13 | 7  |
| 16 | 10 |
| 19 | 14 |

| 1 | li | in | e  | a | r |   |   |
|---|----|----|----|---|---|---|---|
| п | 0  | n  | li | n | e | а | r |

21. Jamal has AED 20,000 in a safe. Each month, he adds another AED 1,000 to the safe. Hareb opens a savings account with a AED 20,000 deposit and earns 0.25% profit each month on the total amount of money in the bank. Determine if each statement is true or false.

a. The function representing Jamal's savings is nonlinear.

True False

b. The function representing Hareb's savings is linear.

True False True False

2 15

20

25

30

linear

c. After 1 year, Jamal will have saved AED 32,000.

# **Spiral Review**

Find each function value.

**22.** 
$$f(5)$$
 if  $f(x) = 3x + 4$  19

**23.** 
$$f(-3)$$
 if  $f(x) = 2x - 8$  -14 **24.**  $f(7)$  if  $f(x) = 9x - 24$  39

**24.** 
$$f(7)$$
 if  $f(x) = 9x - 24$  **39**

25. The table shows the average number of phone calls Husam makes per day. 8.F.4

a. Write an equation to find the total number of phone calls made in any number of days. Describe the relationship in words.

c = 5d; Husam makes an average of 5 phone calls per day.

b. Use the equation to determine how many phone calls Husam would make in 1 week.

35 phone calls

| Number of Days, d | Total Phone<br>Calls, c |
|-------------------|-------------------------|
| 1                 | 5                       |
| 2                 | 10                      |
| 3                 | 15                      |
| 4                 | 20                      |

### Lesson 8 **Quadratic Functions Essential Question** Vocabulary Start-Up HOW can we model relationships between In the previous lesson, you learned about nonlinear functions. quantitites? A special type of a nonlinear function is a quadratic function. A quadratic function is a function in which the greatest power of the Vocabulary variable is 2. Its graph is U-shaped, opening upward or downward. quadratic function Complete the graphic organizer by determining if the facts about quadratic functions are true or false. If false, give the true fact. Mathematical Practices A function in which the 1, 3, 4, 7 greatest power of the variable is 2. true The graph of a The graph is a Quadratic straight line. quadratic function Functions False, the graph sometimes opens is U-shaped. upward, true The graph always opens downward. False, the graph can open upward or downward. Real-World Link Hassan kicked a soccer ball straight into the air. The height h in meters of the ball after t seconds is found using the equation $h = -8t^2 + 16t + 2$ . What is the height of the ball after 1.5 seconds? 8 meters Which Mathematical Practices did you use? Shade the circle(s) that applies.

(5) Use Math Tools

(6) Attend to Precision (7) Make Use of Structure

8 Use Repeated Reasoning

### Focus narrowing the scope

Objective Graph quadratic functions.

### Coherence connecting within and across grades

### Previous

Students identified functions as linear or nonlinear.

#### Now

Students graph quadratic functions and analyze the graphs of quadratic functions.

### Next

Students will use technology to analyze families of quadratic functions.

### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 339.

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# **Launch the Lesson**

### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Pairs Consult Have students work in pairs to complete the graphic organizer, and write a definition of quadratic function in their own words. Have them trade their definition with another pair and discuss any differences. 21, 2, 4

### Alternate Strategy

B Have students graph the functions  $y = x^2 + 1$ ,  $y = x^2 + 2$ , and  $y = x^2 - 1$ . Then have them answer the following:

- Compare and contrast the graphs. Sample answer: The graphs have the same shape but different minimum points.
- Without graphing, describe the graph of y = x<sup>2</sup> − 2. Sample answer: The shape is the same; the minimum is at (0, -2).

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1) Persevere with Problems

3 Construct an Argument Model with Mathematics

2 Reason Abstractly

# **2** Teach the Concept

**Ask** the scaffolded questions for each example to differentiate instruction.

# **Examples**

- Graph quadratic functions.
- What method could you use to graph the function y = x<sup>2</sup>? Make a function table, and then plot the ordered pairs on a coordinate plane.
  - · How do you square a number? multiply it by itself
  - What is (-2)<sup>2</sup>? 4
- Without graphing, does the graph of the function open upward or downward? Explain. Upward; the coefficient of x is positive.
  - · What is the minimum point of the graph? (0, 0)
- Why is it important to use varied numbers for x?
   Sample answer: to get a good picture of the graph and when the graph changes direction

### **Need Another Example?**

Graph  $y = 5x^2$ . See Answer Appendix.

- 2. Graph quadratic functions.
- Why is it helpful to use a table of values to graph the equation? Sample answer: It helps to organize the input and output values into ordered pairs for graphing.
  - What is the value of  $-x^2$  when x = -2? -4
- Does the graph of the function open upward or downward? Explain. Downward; the coefficient of x is negative
- Is there a minimum value or a maximum value for the graph? Explain. Since the graph opens down, there is a maximum value for the graph.
  - · What is the maximum value for the graph? (0, 4)

Need Another Example?

Graph  $y = -x^2 - 2$ . See Answer Appendix.

Work Zone

### **Quadratic Functions**

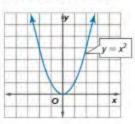
A quadratic function can be written in the form  $y = ax^2 + bx + c$ , where  $a \neq 0$ . The graph of a quadratic function is called a parabola. The graph opens upward if the coefficient of the variable that is squared is positive, downward if it is negative.

### **Examples**

**1.** Graph  $y = x^2$ .

To graph a quadratic function, make a table of values, plot the ordered pairs, and connect the points with a smooth curve.

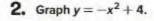
| x  | x <sup>2</sup> | y | (x, y)  |
|----|----------------|---|---------|
| -2 | $(-2)^2 = 4$   | 4 | (-2, 4) |
| -1 | $(-1)^2 = 1$   | 1 | (-1, 1) |
| 0  | $(0)^2 = 0$    | 0 | (0, 0)  |
| 1  | $(1)^2 = 1$    | 1 | (1, 1)  |
| 2  | $(2)^2 = 4$    | 4 | (2, 4)  |



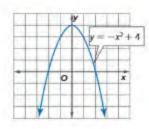


Is it possible for a function to be both increasing and decreasing? Explain below.

Yes; Sample answer: The graph of a quadratic function both increases and decreases.

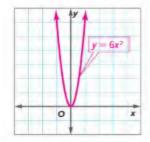


| x  | $-x^2 + 4$        | y | (x, y)  |
|----|-------------------|---|---------|
| -2 | $-(-2)^2+4=0$     | 0 | (-2, 0) |
| -1 | $-(-1)^2 + 4 = 3$ | 3 | (-1, 3) |
| 0  | $-(0)^2+4=4$      | 4 | (0, 4)  |
| 1  | $-(1)^2 + 4 = 3$  | 3 | (1, 3)  |
| 2  | $-(2)^2+4=0$      | 0 | (2, 0)  |



#### Got it? Do this problem to find out.

a. Graph  $y = 6x^2$ .

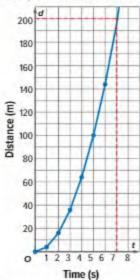


### Examples

3. The function  $d = 4t^2$  represents the distance d in meters that a race car will travel over t seconds with a constant acceleration of 8 meters per second. Graph the function. Then use the graph to find how much time it will take for the race car to travel 200 meters.

Time cannot be negative, so only use positive values of t.

| $d=4t^2$       | (t, d)   |
|----------------|--|
| $4(0)^2 = 0$   | (0, 0)   |
| $4(1)^2 = 4$   | (1, 4)   |
| $4(2)^2 = 16$  | (2, 16)  |
| $4(3)^2 = 36$  | (3, 36)  |
| $4(4)^2 = 64$  | (4, 64)  |
| $4(5)^2 = 100$ | (5, 100)   |
| $4(6)^2 = 144$ | (6, 144)   |
|                | $4(0)^{2} = 0$ $4(1)^{2} = 4$ $4(2)^{2} = 16$ $4(3)^{2} = 36$ $4(4)^{2} = 64$ $4(5)^{2} = 100$ |



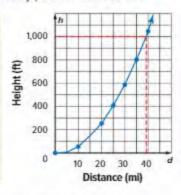
Locate 200 on the vertical axis. Move over to the graph and locate the corresponding value for the time.

The car will travel 200 meters after about 7 seconds.

4. The function  $h = 0.66d^2$  represents the distance d in miles you can see from a height of h feet. Graph this function. Then use the graph to estimate how far you could see from a hot air balloon 1,000 feet in the air.

Distance cannot be negative, so use only positive values of d.

| d  | $h=0.66d^2$          | (d, h)      |
|----|----------------------|-------------|
| 0  | $0.66(0)^2 = 0$      | (0, 0)      |
| 10 | $0.66(10)^2 = 66$    | (10, 66)    |
| 20 | $0.66(20)^2 = 264$   | (20, 264)   |
| 25 | $0.66(25)^2 = 412.5$ | (25, 412.5) |
| 30 | $0.66(30)^2 = 594$   | (30, 594)   |
| 35 | $0.66(35)^2 = 808.5$ | (35, 808.5) |
| 40 | $0.66(40)^2 = 1,056$ | (40, 1,056) |



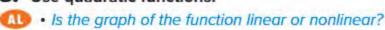
At a height of 1,000 feet, you could see approximately 39 miles.

## **Examples**

**Functions** 

### Use quadratic functions.

nonlinear



- · How do you know the graph is nonlinear? The equation is a quadratic equation and the graph is nonlinear.
- How far has the race car traveled after 1 second? 4 m 2 seconds? 16 m 4 seconds? 64 m
  - How long has the car been accelerating when it has traveled 200 meters? about 7 seconds
- Why don't you use negative numbers for this graph? time cannot be negative

#### Need Another Example?

The function  $d = 16t^2$  represents the distance d in meters that a skydiver falls in t seconds. Graph the equation. Then use the graph to find how much time it will take for the skydiver to fall 400 meters. 5 s; See Answer Appendix for graph.

### 4. Use quadratic functions.

- What method could you use to graph the function? Make a function table. Then plot the ordered pairs on a coordinate plane.
  - Is the graph of the function linear or nonlinear?
- Do negative input values make sense in this situation? Explain. No; you cannot have a negative distance.
  - What is the corresponding x value on the graph for a y value of 1,000? about 40
- Blb Suppose you were 1,200 feet in the air. About how far could you see? Sample answer: about 42 m

### Need Another Example?

The equation  $d = 16.065t^2$  describes the distance d in meters that a stone falls off a cliff over t seconds. Graph this function. Then use the graph to estimate how long it would take a stone to fall 200 meters. about 3.5 s; See Answer Appendix for graph.

# **Guided Practice**

**Formative Assessment** Use these exercises to assess students' understanding of the concepts in this lesson.



If some of your students are not ready for assignments, use the differentiated activities below.

Numbered Heads Together Assign students to 3or 4-person learning teams. Each member is assigned a number from 1 to 4. Each team completes Exercises 1–3, making sure that every member understands the steps to take in the solution. Call on a specific number from one team to present the team's solution to Exercise 1. Repeat for the remaining two Exercises.

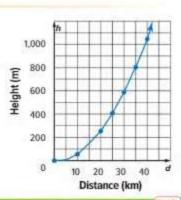
Pairs Discussion Have students work in pairs to create a graphic organizer about quadratic functions. They should include information about the graphs, equations, and anything else they can find in their research. Display the graphic organizers around the room, have students circulate, and discuss any differences in the graphic organizers. 

1, 2, 6

b. about 45 km

#### Got it? Do this problem to find out.

b. The outdoor observation deck of the Space Needle in Seattle, Washington, is 160 meters above ground level. Use the graph to estimate how far you could see from the observation deck.



## **Guided Practice**

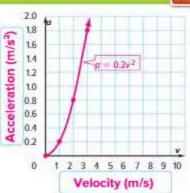


 The function a = 0.2v<sup>2</sup> models the acceleration of a carnival ride, where a is the acceleration toward the center of the ride in meters per second every second and v is the velocity in meters per second. Graph this function. Then use your graph to estimate the velocity of the ride at an acceleration of 1 meter per second



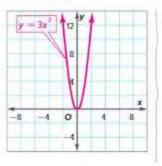
every second. (Examples 3 and 4)

Sample answer: about 2.2 mps

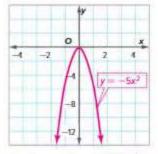


Graph each function. (Examples 1 and 2)

2. 
$$y = 3x^2$$



3. 
$$y = -5x^2$$



4. Building on the Essential Question When does the graph of a quadratic function open upward or downward? Sample answer: The graph opens upward if the coefficient of the variable that is squared is positive, downward if it is negative.

### Rate Yourself!

How confident are you about quadratic functions? Check the box that applies.





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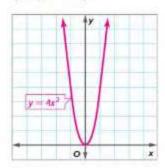
My Hamework

# **Independent Practice**

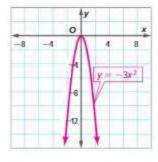
Graph each function. (Examples 1 and 2)





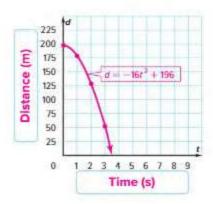


**2.** 
$$y = -3x^2$$



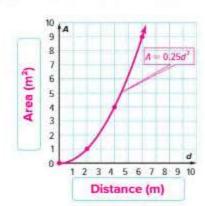
A coin is dropped from a height of 196 meters off a bridge. The function  $d = -16t^2 + 196$  models the distance d in meters the coin is from the surface of the water at time t seconds. Graph this function. Then use your graph to estimate the time it will take for the coin to reach

the water. (Examples 3 and 4) about 3.5 s



4. The area A in square meters of a projected movie on a movie screen can be represented by the equation  $A = 0.25d^2$ , where d represents the distance from a projector to the movie screen. Graph the function. Then use your graph to estimate the distance from the projector to a screen if the area of the movie is 7 square meters.

(Examples 3 and 4) about 5.2 m



- 5. Hessa has trim to make a rectangular border for a scrapbook page. The section inside the border is x centimeters long and (12 - x) centimeters wide.
  - a. Write a function to represent the area A of the section inside the border.  $A = 12x - x^2$
  - b. What should the dimensions of the section be to enclose the maximum area inside the border? (Hint: Graph the function and find the

x-coordinate of the point at the peak of the graph.) 6 cm by 6 cm

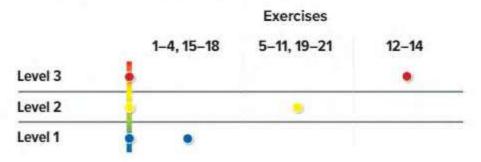
# **Practice and Apply**

### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                                |  |  |
|---------------------------------|-------------------|--------------------------------|--|--|
| (II)                            | Approaching Level | 1-4, 5-11, odd, 13, 14, 20, 21 |  |  |
| 01                              | On Level          | 1, 3, 5–11, 13, 14, 20, 21     |  |  |
| BL                              | Beyond Level      | 5–14, 20, 21                   |  |  |

## Watch Out!

Common Error If students graph a parabola in the wrong direction, remind them that it opens upward if the coefficient of  $x^2$  is positive and downward if it is negative.

| MATHEMATICAL PRACTICES  |             |
|---|-------------|
| Emphasis On   | Exercise(s) |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 12          |
| 3 Construct viable arguments and critique the reasoning of others.            | 14          |
| 4 Model with mathematics.   | 13, 19      |
| 7 Look for and make use of structure.   | 6-11        |
|   |             |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.



Have students explain the steps they would use to graph a quadratic function. See students' work.

Identify Structure Without graphing, determine whether each equation represents a linear or nonlinear function. Explain.

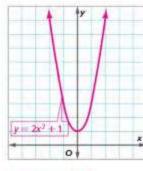
- y = 3x linear; Sample answer: The equation is written in slopeintercept form so it is a straight line.
- y = 2x<sup>2</sup> nonlinear; Sample answer:
   The function is quadratic.
- y = -3x<sup>2</sup> nonlinear; Sample answer:
   The function is quadratic.

- y = -6x linear; Sample answer: The equation is written in slope-intercept form so it is a straight line.
- 10. 5x + y = 7 linear; Sample answer: The equation can be written in slope-intercept form so it is a straight line.
  - 7x² + y = 24 nonlinear; Sample answer:
     The function is quadratic.

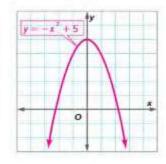


### H.O.T. Problems Higher Order Thinking

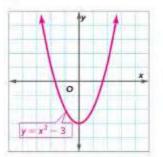
- 12. Persevere with Problems The graphs of quadratic functions may have exactly one highest point, called a maximum, or exactly one lowest point, called a minimum. Graph each quadratic equation. Determine whether each graph has a maximum or a minimum. If so, give the coordinates of each point.
- a.  $y = 2x^2 + 1$



**b.**  $y = -x^2 + 5$ 



c.  $y = x^2 - 3$ 



minimum; (0, 1) maximum; (0, 5)

- mum: (0, 5) mini
  - minimum; (0, -3)
- Model with Mathematics Write the equation of a quadratic function that opens upward and has its minimum at (0, −3.5). Sample answer: y = x² − 3.5
- 14. Reason Inductively The equation y = ax² + bx + c represents a quadratic function. What does the constant c represent? Explain. the y-intercept; Sample answer:

When a graph crosses the y-axis, x = 0. Substitute 0 for x in the equation, and y = c, so c represents the y-intercept.

My Homework

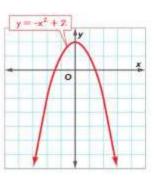
# **Extra Practice**

Graph each function.

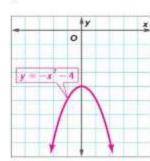
**15.** 
$$y = -x^2 + 2$$

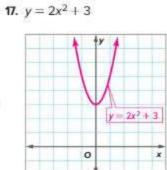
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|-----|-------------|-------|-----|
| 1   | сŧ          | -     |     |

| ×  | -x2 + 2     | y  | (x, y)   |
|----|-------------|----|----------|
| -2 | $-(-2)^2+2$ | -2 | (-2, -2) |
| -1 | $-(-1)^2+2$ | 1  | (-1, 1)  |
| 0  | $-(0)^2+2$  | 2  | (0, 2)   |
| 1  | -(1) + 2    | 1  | (1, 1)   |
| 2  | $-(2)^2+2$  | -2 | (2, -2)  |



**16.** 
$$y = -x^2 - 4$$





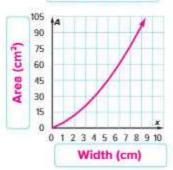
**18.** The function  $d = 0.006s^2$  represents the braking distance d in meters of a car traveling at a speed s in kilometers per second. Graph this function. Then use your graph to estimate the speed of the car if its braking distance is

12 meters. about 45 km/s



- 19. Model with Mathematics Halima is making a fabric memo board. The width of the board is x centimeters, and the length of the board is (x + 4) centimeters.
  - a. Write a function that represents the area A of the memo board.  $A = x^2 + 4x$
  - b. Graph the function.
  - c. If the width of the memo board is 8 centimeters, what is

its area? 96 cm<sup>2</sup>



# Power Up! Test Practice

Exercises 20 and 21 prepare students for more rigorous thinking needed for assessment.

20. This test item requires students to analyze and solve complex real-

| world problems through | the use of mathematical tools and models.  |
|------------------------|--|
| Depth of Knowledge     | DOK3   |
| Mathematical Practice  | MP1, MP4   |
| Scoring Rubric         |  |
| 2 points               | Students correctly complete the table, graph the function, and find the time.  |
| 1 point                | Students correctly complete the table and graph the function, but fail to find the time OR students correctly completely the table and find the time but fail to graph the function correctly OR students incorrectly complete the table, but the graph and time are correct according to the values in the table. |

21. This test item requires students to reason abstractly and

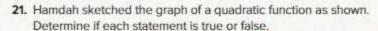
| quantitatively when pro | blem solving.                        |
|-------------------------|--------------------------------------|
| Depth of Knowledge      | DOK1                                 |
| Mathematical Practice   | MP1                                  |
| Scoring Rubric          |                                      |
| 1 point                 | Students correctly answer each part. |

# Power Up! Test Practice

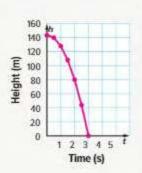
20. The height of a stone that is dropped from a 144-meter tall bridge can be modeled by the function  $h = -16t^2 + 144$ , where t is the time in seconds and h is the height of the stone above the river. Complete the table of values below and graph the function on the coordinate plane.

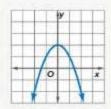
| Time, t   | 0   | 0.5 | 1   | 1.5 | 2  | 2.5 | 3 |
|-----------|-----|-----|-----|-----|----|-----|---|
| Height, h | 144 | 140 | 128 | 108 | 80 | 44  | 0 |

How long does it take for the stone to reach the river?



- a. The y-intercept is (0, 2).
- True False True False
- b. The graph opens downward, so the coefficient of  $x^2$  is positive. c. The graph represents the function
- False





### **Spiral Review**

 $y = -x^2 + 2$ .

Evaluate each expression.

**24.** 
$$8^3 = 512$$

**26.** 
$$3^3 = 27$$

27. 
$$4^2 = 16$$

**29.** 
$$6^2 = 36$$

Evaluate each expression if a = 2, b = -3, and c = 6.

30. 
$$a+b-c$$
 \_7 | 31.  $c-a+b$  4

31. 
$$c - a + b$$

# **Inquiry Lab**

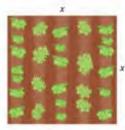
### **Graphing Technology: Families of Nonlinear Functions**



HOW are families of nonlinear functions the same as the parent function? How are families of nonlinear functions different from the parent function?

Mathematical Practices 1.3.7

Houriyya is digging a vegetable garden and wants to know how much fertilizer to buy. The equation  $y = x^2$  represents the area in square meters of the garden shown.



# Hands-On Activity 1

Families of nonlinear functions share a common characteristic based on a parent function. The parent function, or simplest function, of a family of quadratic functions is  $y = x^2$ . You can use a graphing calculator to investigate families of quadratic functions.

Graph  $y = x^2$ ,  $y = x^2 + 5$ , and  $y = x^2 - 3$  on the same screen.

Clear any existing equations from the Y= list by pressing Y= CLEAR

Step 2

Enter each equation. Press  $X,T,\theta,n$   $x^2$  ENTER,  $X,T,\theta,n$   $x^2$  + 5 ENTER, and X,T, $\theta$ ,n  $x^2$  - 3 ENTER

Step 3

Press ZOOM 6.

Copy your calculator screen on the blank screen shown.

How are the three equations related?

Sample answer: All three equations contain an

x2. However, the number added to or subtracted from  $x^2$  is different.

Describe how the graphs of the three equations are related.

Sample answer: All three are in the shape of a "U". However, the

graphs have their vertices at different points.

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### Focus narrowing the scope

Objective Use a graphing calculator to graph families of nonlinear functions.

### Coherence connecting within and across grades

### Now

Students use graphing calculators to graph families of nonlinear functions.

### Next

Students will model relationships by sketching and describing qualitative graphs.

### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 345.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance than Activity 2.

Materials: graphing calculators

# Hands-On Activity 1

Circle the Sage Poll the class to see which students have some knowledge of graphing functions. Those students (the sages) spread out around the room. Assign the rest of the students to teams. Have the teams split up with each team member going to a different sage, if possible. Have the sages lead work for Activity 1. After completing the activity, students go back to their teams and compare solutions. Students discuss how the sage may have explained the steps differently. 21, 3, 4, 5

# Hands-On Activity 2

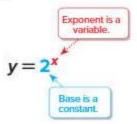
Pairs Check Have students work in pairs to complete Steps 1–4 of Activity 2. One partner completes the odd steps, while the other completes the even steps. After each pair has recorded their results and stored their graphing results, have pairs share, discuss, and check their solutions with another pair. 1,5

Pairs Check Have students work in pairs to complete Steps 1–4 of Activity 2. One partner completes the first step while the other coaches. Students switch roles for the next step. Students continue to switch roles for the remaining steps. After every two steps, the pairs check their solutions with another pair and make any necessary adjustments. 

1, 5

# Hands-On Activity 2

An exponential function is a nonlinear function in which the base is a constant and the exponent is an independent variable, x. The parent function for an exponential function is shown.



A certain type of bacteria doubles every hour. The function  $y = 2^x$  represents the total number of bacteria y at the end of every hour x. Graph the function. Then find the number of bacteria at the end of 5 hours.

Step 1 Clear any existing equations from the Y= list by pressing Y= CLEAR.

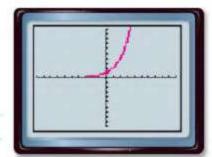
Step 2 Enter the equation, Press  $Y=2 \land X,T,\theta,n$ 

Step 3 Graph the equation in the standard viewing window. Press ZOOM 6.

Copy your calculator screen on the blank screen shown.

Describe the function by analyzing the graph. Sample answer: The function is a nonlinear

increasing function.



Step 4 Use the TABLE feature. Press 2nd GRAPH . What is the y value that corresponds to the x value of 5? 32

So, there are 32 bacteria at the end of 5 hours.

Such a workship a such

**Functions** 

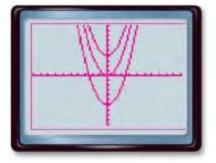


# Investigate

#### Work with a partner.

**1.** Use a graphing calculator to graph  $y = x^2$ ,  $y = x^2 - 6$ , and  $y = x^2 + 4$ . Copy your calculator screen on the blank screen shown.



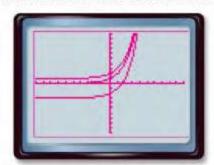


How does changing the value of c in the equation  $y = x^2 + c$  affect the graph?

Changing the value of c changes the

vertical position of the graph.

3. Use a graphing calculator to graph  $y = 2^x$ ,  $y=2^{x}+1$ , and  $y=2^{x}-3$ . Copy your calculator screen on the blank screen shown.

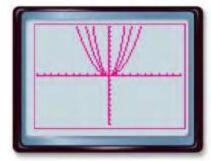


How does changing the value of c in the equation  $y = 2^{x} + c$  affect the graph?

Changing the value of c changes the

vertical position of the graph.

2. Use a graphing calculator to graph  $y = 0.5x^2$ ,  $y = x^2$ , and  $y = 2x^2$ . Copy your calculator screen on the blank screen shown.

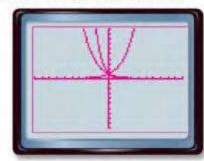


How does changing the value of a in the equation  $y = \alpha x^2$  affect the graph?

Changing the value of a changes the width

of the graph.

**4.** Use a graphing calculator to graph  $y = 0.5^x$ ,  $y = 0.25^x$  and  $y = 2^x$ . Copy your calculator screen on the blank screen shown.



How does changing the value of a to a fraction in the equation  $y = a^x$  affect the graph?

Changing the value of a to a fraction

causes the graph to slope downward.

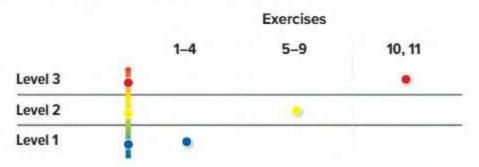
## ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# Collaborate

The Investigate and Analyze and Reflect sections are intended to be used as small-group investigations. The Create section is intended to be used as independent exercises.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.





 Group-Solo Work together as a class to complete Exercise 1 while you, or a student volunteer, demonstrates how to use a graphing calculator to graph the solution to Exercise 1. Then have students complete Exercises 2-4 individually.

31,5

Think-Pair-Write Have students work with a partner to complete Exercises 1-4. Give students time to discuss the question in each exercise. After the students finish their discussions within the pairs, have each student write their own responses to the questions. If time permits, ask for volunteers to give their answers and how their partner influenced their answer. 21,5



# **Analyze and Reflect**

- Popcorn Share Have students individually complete Exercises 5–9. Call out "Popcorn 5," so that students can quickly pop out of their chairs one at a time to share their answer. Seated students write their responses on their piece of paper after a classroom consensus. Continue to call out Popcorn until all exercises are complete. 1,5
- Pairs Discussion Have students work in pairs to complete Exercises 5–9. Have them trade their solutions with another pair of students and discuss any differences. 1,5



- BD (A) Trade-a-Problem Have students work in pairs. Each partner writes a real-world word problem that uses graphing calculator technology. Have students trade with their partners and solve each other's word problem. (2)1,4,5
- Students should be able to answer "HOW are families of nonlinear functions the same as the parent function? How are families of nonlinear functions different from the parent function?" Check for student understanding and provide guidance, if needed.

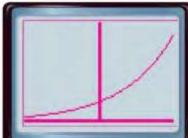


# **Analyze and Reflect**

Identify Structure Work with a partner to complete the table. Without graphing, determine which graph is wider.

|    | Equation 1           | Equation 2       | Which graph is wider? |
|----|----------------------|------------------|-----------------------|
| 5. | $y = 5x^2$           | $y = x^2$        | $y = x^2$             |
| 6. | $y = \frac{1}{3}x^2$ | $y = 3x^2$       | $y = \frac{1}{3}x^2$  |
| 7. | y = 2*               | $y = 4^{\times}$ | y = 2*                |
| 8. | y = 0.25*            | y = 0.75*        | $y = 0.75^{x}$        |

- 9. Khadija's parents started a savings account when she was born by depositing AED 100 into the account. The account makes an annual profit of 3%. The balance y in the account can be represented by the function y = 100(1.03)<sup>x</sup>, where x is the number of years.
  - a. Graph the function on your graphing calculator. Copy your calculator screen on the blank screen shown.
     (Hint: Use the x-scale -50 to 50 and the y-scale 0 to 500.)
  - b. Use the TABLE feature. How much money is in the account after 13 years? AED 146.85
  - c. Describe the function by analyzing the graph.
     Sample answer: The function is a nonlinear increasing function.





### Create

10. Model with Mathematics Write the equation of a quadratic function that is wider than  $y = \frac{2}{3}x_1^2$ . Explain how you know it is wider.

Sample answer:  $y = \frac{2}{3}x^2$ ; The value of the absolute value of the

coefficient is a smaller number so the graph is wider.

11. HOW are families of nonlinear functions the same as the parent function? How are families of nonlinear functions different from the parent function?

Sample answer: The shape of the graph is the same in a family of nonlinear functions. The location on the axes may change from one to another. The width and/or orientation may also change.

# Lesson 9 **Qualitative Graphs**

Percent

Downloaded

0

15

30

30

64

64

82

100

Time (s)

0

2

4

6

8

10

12

14

### Real-World Link

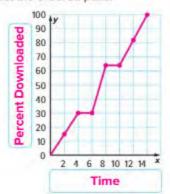
Downloads Khawla is downloading photos from her digital camera to her computer. The table shows the percent of photos downloaded for several seconds.

1. During which period(s) of time did the percent downloaded not change? between 4 and 6 seconds and between 8 and 10 seconds

2. During which period of time did the percent downloaded change the most?

between 6 and 8 seconds

Graph and connect the ordered pairs.



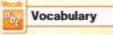
Which Mathematical Practices did you use? Shade the circle(s) that applies.

- 1) Persevere with Problems
- 2 Reason Abstractly
- 3 Construct an Argument
- Model with Mathematics www.jnob-jo.com
- (5) Use Math Tools
- (6) Attend to Precision
- (7) Make Use of Structure
- 8 Use Repeated Reasoning

# **Essential Question**

**Functions** 

HOW can we model relationships between quantities?



qualitative graphs

Mathematical Practices 1.2.3.4

### Focus narrowing the scope

Objective Sketch and describe qualitative graphs.

### Coherence connecting within and across grades

### Previous

Students graphed and analyzed linear and quadratic graphs.

### Now

Students analyze qualitative graphs.

### Next

Students will analyze graphical displays of data and make predictions from the displays.

### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 351.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **Launch the Lesson**

### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BD (IA) Think-Pair-Share Give students 1 minute to think about the ordered pairs in the table and what a graph of the ordered pairs might look like. Have them share their responses with a partner. Then call on one student to share their response within a small group or large group discussion. 21

### Alternate Strategy

Present a measuring cup and a bucket to the class. Tell the class that each is going to be used to fill a small pool. Have a class discussion about how the water level of the pool will change over time in each case. Then sketch a graph on the board to represent each situation.

# Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

### Example

### Analyze a qualitative graph.



- Describe the graph. The graph increases, then stays the same, then decreases.
  - Describe the graph in the context of the problem. The water level rises, stays the same, then falls.
  - Finally, create a scenario where this might happen. You close the drain in a bathtub, turn the water on in a bathtub, turn the water off and let the water stand. then open the drain and let the bathtub drain.



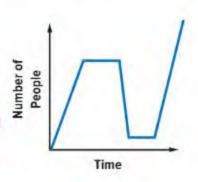
- Describe what is happening to the water in the first part of the graph. As time increases, the water level increases at a constant rate.
  - Describe what is happening to the water in the second part of the graph. As time increases, the water level stays the same.
  - Describe what is happening to the water in the third part of the graph. As time increases, the water level decreases at a constant rate.



BD • Suppose the vertical axis is labeled Temperature. Create a scenario that would describe the graph. Sample answer: Food is heated in a microwave, placed in an oven to keep it warm, then left out on a table to cool.

### Need Another Example?

The graph displays the number of people at a restaurant during the morning and afternoon. Describe the change in the number of people over time. Sample answer: The number of people increases during breakfast and then stays the same. After breakfast, the number of people decreases and then stays the same.



Finally, the number of people increases during lunch.

Work Zone

Sample answer:

Over time, the

temperature

it reaches a

increases at a

varied rate until

maximum. Then

the temperature

There are two periods

of time where the

sales decrease or

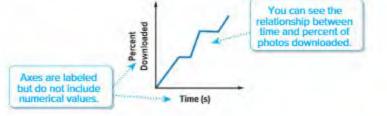
remain constant.

decreases at a

varied rate.

### **Analyze Qualitative Graphs**

The graph shown is a qualitative graph. Qualitative graphs are graphs used to represent situations that may not have numerical values or graphs in which numerical values are not included.





### Example

 The graph at the right displays the water level in a bathtub. Describe the change in the water level over time.

> At time zero, the water level in the bathtub is zero. The water level in the bathtub increases at a constant rate. Then the



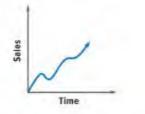
water is turned off and the water level does not change. Finally, the drain plug is pulled and the water level decreases at a constant rate until the water level is zero.

### Got it? Do these problems to find out.

a. The graph at the right displays the temperature throughout the day. Describe the change in the temperature over time.



b. The graph represents revenue from a Sample answer: local clothing store. Describe the sales Overall, the sales over time. increase steadily.



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Sketch Qualitative Graphs

Qualitative graphs represent the essential elements of a situation in a graphical form. You can sketch qualitative graphs to represent many real-world functions that are described verbally.

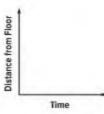


### Examples

2. A tennis ball is dropped onto the floor. On each successive bounce, it rebounds to a height less than its previous bounce height until it comes to rest on the floor. Sketch a qualitative graph to represent the situation.

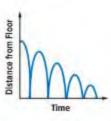
Step 1

Draw the axes. Label the vertical axis "Distance from Floor." Label the horizontal axis "Time."



Step 2

Sketch the shape of the graph. The distance from the floor starts out at a high value. The ball falls to the floor, bounces, and rebounds to a height less than its drop height. This pattern is repeated several times until the ball comes to rest on the floor.



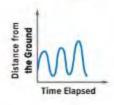
3. A child swings on a swing. Sketch a qualitative graph to represent the situation.

Step 1

Draw the axes. Label the vertical axis "Distance from the ground" and the horizontal axes "Time Elapsed."



Sketch the shape of the graph. The distance from the swing to the ground starts at a low value. The child continues to swing and creates momentum each time the swing goes back until the child on the swing stops.



**Examples** 

### Draw a qualitative graph.



**Functions** 

 Picture a tennis ball after it has dropped. How does the height of the ball after it bounces once compare to when you dropped it? The height is lower.



 Picture a tennis ball after it is dropped. What happens to it with each bounce? With each bounce, the ball bounces lower and lower.

- In this situation, what are the dependent and independent variables? Time is the independent variable, and the height of the ball is the dependent variable.
- What happens to the height of the ball as the time increases? The height of the ball rises and falls, but the maximum height is lower with each bounce.

#### Need Another Example?

A train slowly increases speed, then maintains a constant speed, then quickly speeds up. Sketch a qualitative graph to represent the sitration. See Answer Appendix.

### Draw a qualitative graph.



- In this situation, what are the independent and dependent variables? The elapsed time is the independent variable, and the distance from the ground is the dependent variable.
  - Is the graph of the function linear or nonlinear? nonlinear



 When a child is swinging, what happens to the child's distance from the ground? The distance from the ground increases and decreases continually.



• On the graph, why does each curve get closer together? The speed of the swing increases with momentum so the time that passes is shortened with each swing.

### Need Another Example?

Nasser is playing fetch with his cat. As he throws the ball away from him, the cat runs and gets the ball and brings it back. Sketch a qualitative graph to represent the situation. See Answer Appendix.

## **Guided Practice**

**Formative Assessment** Use these exercises to assess students' understanding of the concepts in this lesson.

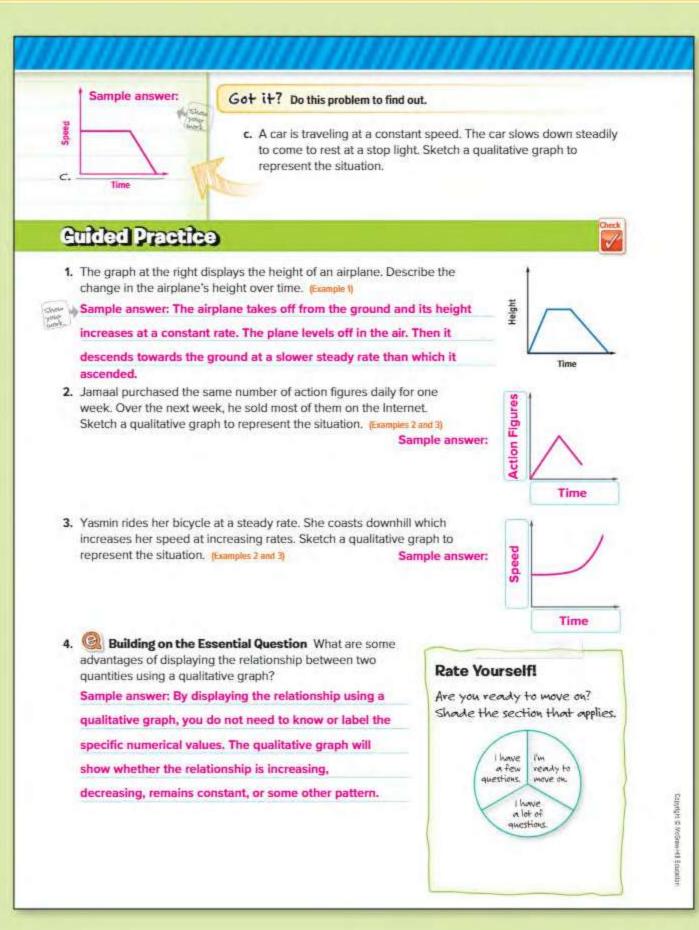


If some of your students are not ready for assignments, use the differentiated activities below.

Pairs Discussion Have students work in pairs to complete Exercises 1–3. Encourage discussion of how different interpretations of the situation can happen. Students trade their solutions with another pair of students and discuss the differences. 1, 4

Trade-a-Problem Have students create their own problem, similar to Exercises 2 and 3. Challenge them to have at least four changes in their problem. Students trade their problems, create a graph for each other's problem, and compare graphs. If the graphs are not similar, students discuss the differences. Remind students that different interpretations of the situation are possible, especially if they are not specific in their descriptions. 

1, 3, 4



Name

My Hamework

# Independent Practice

The graph below displays the distance from Hasan's home as he walks in his neighborhood. Describe the change in the distance from his home over time. (Example 1)

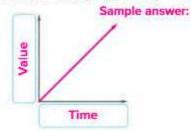




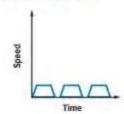
Sample answer: Hasan starts out from his home. He walks away from his home, stops, and walks further away from home.

Then he walks towards home.

A grand piano that is over 100 years old has increased in value rapidly from when it was first purchased. Sketch a qualitative graph to represent this situation. (Examples 2 and 3)



The graph below displays the speed of a city bus as it stops frequently to pick up passengers. Describe the change in the speed over time. (Example 1)



Sample answer: The speed of the bus increases at a constant rate, then remains constant, and **then slows** down. As it is picking up passengers, the speed is zero. This pattern continues.

 An athlete alternates between running and walking during a workout. Sketch a qualitative graph to represent this situation.

(Examples 2 and 3)





- Reason Abstractly Use the graph at the right which displays the rate at which Hamad hiked along a path.
  - a. What situation could the horizontal line segment represent?
    Sample answer: Hamad hikes at a steady rate.
  - b. What situation could the vertical line segment represent?
    Sample answer: Hamad suddenly stops hiking.
  - c. Did Hamad's rate increase or decrease during the first portion of his hike? Explain your reasoning.

increase; Sample answer: The graph rises from left to right at the beginning.



ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

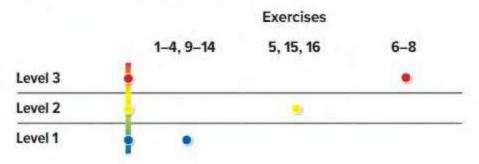
# 3 Practice and Apply

### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                    |  |
|---------------------------------|-------------------|--------------------|--|
| AL                              | Approaching Level | 1–5, 7, 8, 15, 16  |  |
| <b>O</b>                        | On Level          | 1–7 odd, 8, 15, 16 |  |
| BL                              | Beyond Level      | 5–8, 15, 16        |  |

| MATHEMATICAL PRACTICES  |             |  |
|---|-------------|--|
| Emphasis On   | Exercise(s) |  |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 6           |  |
| 2 Reason abstractly and quantitatively.                                       | 5           |  |
| 3 Construct viable arguments and critique the reasoning of others.            | 7, 8        |  |
| 4 Model with mathematics.   | 13          |  |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

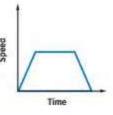
## TICKET Out the Door

Have students draw a quantitative graph on a small piece of paper. Be sure they label the axes. Then instruct the students to choose one part of their graph and write a brief description of what it represents. See students' work.

# OL.

### H.O.T. Problems Higher Order Thinking

- Persevere with Problems The graph at the right displays the speed of a car as time increases.
  - a. Draw a qualitative graph that represets the distance the car travels as time increases.



- b. Describe how the distance changes as time passes.
  Sample answer: As the time increases, the distance increases at a varied rate and then levels off when the car stops.
- Reason Inductively A tree grows steadily. When it reaches a specific height, it stops growing. Which graph displays this relationship? Explain your reasoning to a classmate.







Graph A; Sample answer: Graph A increases from left to right at a constant rate then levels off. This represents a tree growing steadily before it stops growing.

 Reason Inductively The graph below represents the height of a rocket after being launched.



Describe the change in the height of the rocket over time.

Sample answer: The height of the rocket increases as the time after launch increases.

Name

My Homework

### **Extra Practice**

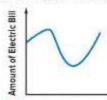
9. The graph below displays the distance Rana rode on her bike. Describe the change in the distance over time.





Rana rode her bike at a constant rate in the beginning. She then stopped riding for a period of time. Then she continued riding at a constant rate.

11. A graph of Mrs. Reham's electric bill throughout the year, starting in June, is shown below. Describe the change in the bill over time.



Sample answer:

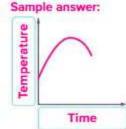
Mrs. Reham's electric bill starts out high in June,

throughout the autumn and winter. In the spring,

increases until about August, and then decreases

the electric bill increases again.

13. Model with Mathematics The outside temperature rises throughout the day at varied rates, then drops at night. Sketch a qualitative graph to represent the situation.



10. The graph below displays the temperature of a cup of hot chocolate. Describe the change in the temperature over time.



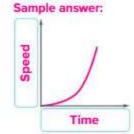
Sample answer: The temperature cools down rapidly in the beginning. Then it cools down at a slower rate and levels off.

12. The graph below displays the distance covered on a long road trip. Describe the change in the distance over time.



Sample answer: The graph shows the car traveling at a constant speed then stopping, and then moving at a faster speed. The car stops a second time, then continues traveling.

14. A lion cub is resting in the grass. He sees another lion cub nearby and races after it, picking up speed as it runs. Sketch a qualitative graph to represent the situation.



# Power Up! Test Practice

Exercises 15 and 16 prepare students for more rigorous thinking needed for the assessment.

| 15. | This test item requires students to reason abstractly and quantitatively when problem solving. |  |  |
|-----|--|--|--|
|     | Depth of Knowledge   | DOK1   |  |
|     | Mathematical Practice  | MP1  |  |
|     | Scoring Rubric   |  |  |
|     | 1 point  | Students correctly answer each part of the question. |  |

| 16. | This test item requires students to reason abstractly and quantitatively when problem solving. |  |  |
|-----|--|--|--|
|     | Depth of Knowledge   | DOK2   |  |
|     | Mathematical Practice  | MP1  |  |
|     | Scoring Rubric   |  |  |
|     | 2 points   | Students correctly answer the each part of the question. |  |
|     | 1 point  | Students correctly select 3 or 4 of the segments.        |  |

# Power Up! Test Practice

15. The graph represents the amount of pies sold by a bakery over the course of one day. Determine if each of the following statements is true or false.

True False

True False

True False

- The red segment represents sales increasing at a constant rate.
- b. The blue segment represents sales, decreasing at a constant rate.
- c. The green segment represents sales increasing, but not at a constant rate.
- 16. The graph represents Sally's activities on her way home from school on a given day. Match each statement with its corresponding segment on the graph.

Sally rides her bike from the park to Sumayya's house, Segment C

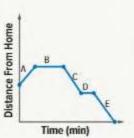
Sally rides her bike home from Sumayya's house. Segment E

Sally plays at the park. Segment B

Sally visits Sumayya at her house. Segment D

Sally rides her bike from school to the park. Segment A





### Spiral Review

Simplify each expression.

17. 
$$2(p+8)+4=\frac{2p+20}{2}$$

**18.** 
$$(18+t)(-3)+9=-3t-45$$

19. 
$$30q(2) = 60q$$

**20.** 
$$-5(n+16)-7=-5n-87$$

# 2 ST CENTURY CAREER

# in Physical Therapy

### **Physical Therapist**

Are you a compassionate person? Do you have a strong desire to help others? If so, a career as a physical therapist might be a good choice for you. Physical therapists help restore function, improve mobility, and relieve pain of patients suffering from injuries or disease. One of their jobs is to teach exercises or recommend activities to help patients regain balance, flexibility, endurance, and strength.



# Is This the Career for You?

Functions

Are you interested in a career as a physical therapist? Take some of the following courses in high school.

- Algebra
- Biology
- Chemistry
- Introduction To Physical Therapy

Turn the page to find out how math relates to a career in Physical Therapy.

### Focus narrowing the scope

**Objective** Apply mathematics to problems arising in the workplace.

This lesson emphasizes Mathematical Practice 4 Model with Mathematics.

## Coherence connecting within and across grades

### **Previous**

Students used words, tables, equations, and graphs to represent functions.

### Now

Students apply the content standard to solve problems in the workplace.

# **Rigor** pursuing concepts, fluency, and applications See the Career Project on page 356.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **Launch the Lesson**

Ask students to read the information on the student page about physical therapists and answer the following questions.

#### Ask:

- What do physical therapists do? Sample answer: They help people regain balance, flexibility, endurance, and strength by teaching patients to do specific exercises.
- What kinds of classes should you take if you want to become a physical therapist? Sample answer: Algebra, Biology, Chemistry, and Introduction to Physical Therapy

# **2** Collaborate

(I) Pairs Discussion Have students work in pairs to complete the following questions relating to Exercise 3. Have them trade their solutions with another pair of students and discuss any differences. (2) 1, 2, 4

#### Ask:

- What information do you need to solve this problem? Sample answer: I need to find the rate of change using information in the table.
- How far will the runner travel after 1 hour 15 minutes?
   11.25 miles
- RL LA Trade-a-Problem Have students create their own real-world problem similar to Exercise 1. Students trade their problems and solve. If the solutions do not agree, have students work together to find the errors. 1,4

## Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

# Career Facts

According to the U.S. Bureau of Labor Statistics, employment of physical therapists is expected to grow significantly over the next few years as the population ages and people live longer.

### Focusing on Recovery

Use the information in the table below to solve each problem.

- The function t(r) = 12r, where r is the number of repetitions, represents the total time t(r) in seconds to complete a flexibility exercise. Find t(8). Then interpret the solution. 96; It takes 96 seconds to do
  - 8 repetitions of the exercise.
- Refer to the information in Exercise 1. Make a function table to find the time it will take to complete 1, 2, 5, and 10 repetitions.

| r  | 121    | t(r) |  |
|----|--------|------|--|
| 1  | 12(1)  | 12   |  |
| 2  | 12(2)  | 24   |  |
| 5  | 12(5)  | 60   |  |
| 10 | 12(10) | 120  |  |

- Write a function to represent the distance d in kilometers a runner will travel in t minutes.
  - d(t) = 0.15t
- Refer to the function that you wrote in Exercise 3. How far will a runner travel after 80 minutes?
- Graph the function from Exercise 3. Then use the graph to estimate the distance a runner will travel after 90 minutes.

Sample answer: about 13.5 km

| Endurance Exercise:<br>Cross-Country Running |      |  |
|--|------|--|
| Time (min) Distance (km)                     |      |  |
| 15   | 2.25 |  |
| 30   | 4.5  |  |
| 45   | 6.75 |  |
| 60   | 9.0  |  |

### Career Project

It's time to update your career portfolio! Make a list of questions that you would like answered about a career in physical therapy. Then interview a physical therapist in your area. Include all the interview questions and answers in your portfolio.

List other careers that someone with an interest in physical therapy could pursue.

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**Functions** 

# Chapter Review 🗑



## **Vocabulary Check**



Complete each sentence using the vocabulary list at the beginning of the chapter.

1. A relation

is any set of ordered pairs.

- 2. The variable for the range is called the dependent variable because it depends on the domain.
- 3. Graphs used to represent situations that may not have numerical values or graphs in which numerical values are not included are called qualitative graphs
- The variable for the domain is called the independent variable because it can be any number.
- 5. Continuous data can take on any value, so there is no space between data values for a given domain.
- 6. A function in which the greatest power of the variable is 2 is called a quadratic function
- 7 A function is a relation in which every member of the domain (input values) is paired with exactly one member of the range (output value).

# **Vocabulary Check**

Roundrobin Have students work in small groups to complete the Vocabulary Check. Have Student 1 respond to Exercise 1, reading the word and sentence aloud. Have the rest of the group listen, ask any clarifying questions, and state whether they agree or disagree. If there are any disagreements, have students work together to resolve them. Repeat for each successive exercise, having students take turns. 21

## **Alternate Strategy**

To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- continuous data (Lesson 4)
- dependent variable (Lesson 3)
- function (Lesson 3)
- independent variable (Lesson 3)
- quadratic function (Lesson 8)
- qualitative graphs (Lesson 9)
- relation (Lesson 2)

# **Key Concept Check**

FOLDABLES (IA) A completed Foldable for this chapter should include a review of relations and functions.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

### **Ideas for Use**

Solo-Pair-Solo Have students complete their Foldable for the chapter, if they have not already. Then have them meet with a partner to discuss how they have completed their Foldable and how it can be used to help them review the material they learned in the chapter. Have them discuss any similarities and differences in how each of them completed their Foldable. Then have students work individually to make any desired modifications to their own Foldable. 1, 6

### Got it?

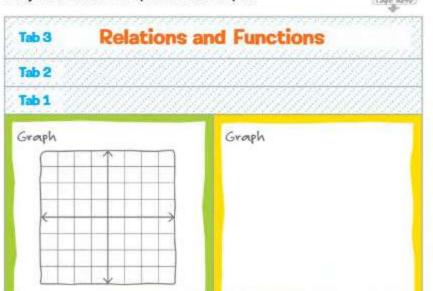
If students have trouble with Exercises 1–4, they may need help with the following concepts.

| Concept                                  | Exercise(s) |  |
|--|-------------|--|
| graphs of linear functions (Lesson 4)    | 1, 2        |  |
| graphs of quadratic functions (Lesson 8) | 3, 4        |  |

### **Key Concept Check**

### Use Your FOLDABLES

Use your Foldable to help review the chapter.



### Got it?

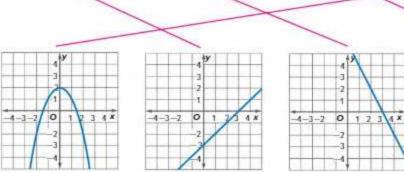
Match each equation with the correct graph.

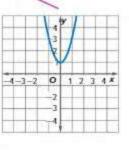
1. 
$$-x + y = -3$$

**2.** 
$$2x + y = 6$$

3. 
$$y = 2x^2 + 1$$

4. 
$$y = -x^2 + 2$$





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**Functions** 

# Power Up! Performance Task

#### **Tournament Time**

The Al-Jamal Academy School is hosting a basketball tournament. The number of teams involved, x, is still unknown. However, the total number of games, g(x), is related to the number of teams in the tournament. The relationship between the possible number of teams and the total number of games played is shown in the table.

| Teams,x     | 2 | 3 | 4 | 5 | 6 |
|-------------|---|---|---|---|---|
| Games, g(x) | 1 | 2 | 3 | 4 | 5 |

Write your answers on another piece of paper. Show all of your work to receive full credit.

#### Part A

Write a function that relates the number of teams to the total number of games

#### Part B

The function r(x) = 50x - 50 represents the cost to have two referees for each game played. What is the cost for referees if there are 2, 3, 4, 5, and 6 teams in the tournament?

#### Part C

Graph the ordered pairs for the function in Part B on a coordinate plane. Is r(x) a linear function? Explain.

#### Part D

The school wants to spend no more than AED 750 on referee fees. How many teams can enter the tournament?

# Power Up! Performance Task

This Performance-Based Assessment requires students to solve multi-step problems through abstract reasoning, precision, and perseverance. This practice scenario can be used to help students prepare for the thinking skills that will be used during assessment.

A complete scoring rubric with answers to the Exercises can be found on page PT4.



### **Answering the Essential Question**

Before answering the Essential Question, have students review their answers to the Building on the Essential Question exercises found in each lesson of the chapter.

- How can you use a graph to write an equation? (p. 272)
- How do tables and graphs represent relations? (p. 280)
- How does the domain affect the range in a function? (p. 290)
- How can functions be used to solve real-world situations? (p.300)
- What are the advantages and disadvantages to representing a function as an equation instead of a graph? (p. 314)
- How is the initial value of a function represented in a table and in a graph? (p. 322)
- How can you use a table or a graph to determine if a function is linear or nonlinear? (p. 330)
- When does the graph of a quadratic function open upward or downward? (p. 338)
- What are some advantages of displaying the relationship between two quantities using a qualitative graph? (p. 350)

### Ideas for Use

Jigsaw Assign students to a 4-person learning team. Provide students with sample topic ideas for each box in the graphic organizer, such as ordered pairs, table, graph, and domain and range. Each group member is assigned to one of these topic ideas. Then have all of the students who are assigned to each topic (from varying groups) meet together to discuss how that representation can be used to model a relationship between two quantities. Have students return to their original groups and explain what they learned in their topic groups. 2 1, 2, 4, 5

# **Track Your Progress**

Return to the beginning of the chapter to review the objectives that it addressed. Students should see that their knowledge of the key ideas has increased now that they have completed this chapter.

# Reflect



### Answering the Essential Question

Use what you learned about functions to complete the graphic organizer. List four ways functions are expressed in this chapter. Sample answers are given.

**Ordered Pairs** 

(2, 3), (-2, 4), (-3, -4), (0, 5)

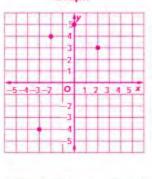
| Tal | ole |
|-----|-----|
| X   | у   |
| 2   | 3   |
| -2  | 4   |
| -3  | -4  |
| 0   | 5   |



### **Essential Question**

HOW can we model relationships between quantities?

Graph



**Domain and Range** 

D: [-3, -2, 0, 2] R: (-4, 3, 4, 5)



Answer the Essential Question. HOW can we model relationships

between quantities?

See students' work.

Green Thumb If you have a knack for gardening, volunteering in a community garden is a great way to get involved with your community and also earn a little money. In this project you will:

- Collaborate with your classmates as you research the costs involved with growing vegetables and predict possible profits.
- Share the results of your research in a creative way.
- Reflect on how you find and use patterns to model real-world situations.

By the end of this Project, you just might be a young entrepreneur!



# Collaborate

Online Work with your group to research and complete each activity. You will use your results in the Share section on the following page.

- 1. Choose a vegetable that is sold individually, and find its cost at a grocery store. Write an equation to represent the total cost as a function of the number of vegetables. Make a function table to find the cost of 1, 2, 3, 4, 5, and 6 vegetables. Then graph the ordered pairs.
- 2. Research a vegetable you would like to grow in a community garden. Find the costs involved such as buying seeds and gardening tools. Then determine how much you will charge per vegetable (or per pound) based on grocery store or farm market prices.
- 3. Based on the information you found in Exercise 2, write a linear function to represent your profit. Describe what the variables represent. Then graph and describe the function.
- 4. Research the following terms: gross profit, total revenue, and gross profit margin. Make a diagram explaining these terms. Then find your gross profit margin based on estimated gross profit and total revenue. What does your gross profit tell you?
- 5. Research the average temperatures in your area for the growing season of the vegetable you chose. Then sketch a qualitative graph that shows the change in temperature over the growing season. Include a brief explanation of your graph.

# **Launch the Project**

Objective Research the costs involved with growing vegetables and predict possible profits from selling vegetables.

#### Green Thumb

This project is designed to be completed by a group of 4 or 5 students over several days or several weeks. It utilizes concepts from the Functions domain. You may choose to complete this project after completing the chapters within this domain.



Have students work in teams to research information about growing and selling vegetables. Together, they should be able to gather the necessary information to complete Exercises 1–5. Students should show their work on a separate piece of paper.



After each group gives their presentation, have students compare the costs and profits from growing and selling different types of vegetables.

### 21st Century Skills

You may want your students to connect their projects to a 21st century skill. Check out the suggestion below and on the student page.





Students should work on their own to reflect on how the chapter from this unit and the objective of the project relate to the Essential Question.



With your group, decide on a way to share what you have learned about growing and selling vegetables. Some suggestions are listed below, but you can also think of other creative ways to present your information. Remember to show how you used mathematics to complete each of the activities in this project!

- Imagine you sell your vegetables at a farmer's market.
   Describe your experience in a blog.
- Use a budget spreadsheet to show how your vegetable can generate a profit. Include tables, equations, and graphs.



Environmental Literacy Research information about Earth's soil and the qualities needed to grow plants. Some questions to consider are:

- What type of soil allows fruits and vegetables to grow well?
- What type of soil is typically found in your area?
- What could you add to the soil to make it better for growing your plants?

Check out the note on the right to connect this project with other subjects.

# On Your Oun

# Reflect

- 6. Answer the Essential Question How can you find and use patterns to model real-world situations? See students' work.
  - a. How did you use what you learned about constructing functions in this chapter to find and use patterns to model real-world situations in this project?

b. How did you use what you learned about different representations of functions in this chapter to find and use patterns to model real-world situations in this project?

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# Essential Question

HOW can you use different measurements to solve real-life problems?



In this chapter, you will use the Pythagorean Theorem to find side lengths of right triangles and distances on the coordinate plane.



# Chapter 6

In this chapter, you will describe the effect of translations, reflections, rotations, and dilations on geometric figures.



In this chapter, you will describe transformations that produce congruent and similar figures.



In this chapter, you will find the volume and surface area of cones, cylinders, and spheres.



### **Essential Question**

At the end of this unit, students should be able to answer "How can you use different measurements to solve real-life problems?"

Each chapter explores a different essential question that assists students in answering the unit question. The lessons in each chapter include exercises that lead students to various aspects of the essential question.

#### Understand congruence and similarity using physical models, transparencies, or geometry software.

- Verify experimentally the properties of rotations, reflections, and translations:
  - a. Lines are taken to lines, and line segments to line segments of the same length.
  - b. Angles are taken to angles of the same measure.
  - c. Parallel lines are taken to parallel lines.
- 2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

continued on page 364

- Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- 4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- 5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

#### Understand and apply the Pythagorean Theorem.

- Explain a proof of the Pythagorean Theorem and its converse.
- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in realworld and mathematical problems in two and three dimensions.
- Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

# Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.



# **Unit Project Preview**

Have students brainstorm different types of rides before they make their sketch.

The Unit Project can be found at the end of Chapter 8.



# **Unit Project Preview**

**Design That Ride** Do you remember how you felt the first time you were on a roller coaster? Scared, excited, terrified? Amusement park rides come in all shapes and sizes and are thrilling to ride.

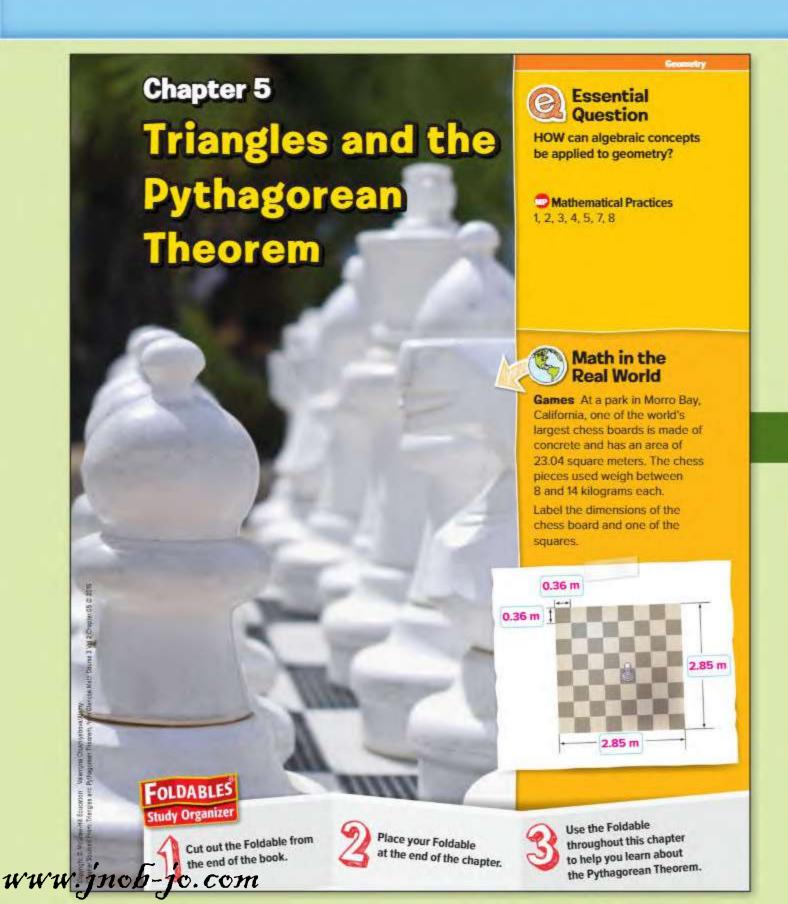
Although amusement park rides are fun, they are also designed to be very safe. Engineers apply geometric concepts and use precise measurements when designing these rides.

Later in this course, you'll complete a project to find how mathematical concepts are used to design an amusement park ride. But for now, it's time to do an activity in your book. Sketch an amusement park ride. Label some geometric shapes that can be found within your ride.



Amusement Park Ride





### Focus narrowing the scope

This chapter focuses on content from the Geometry (G) domain.

# Coherence connecting within and across grades

### Previous

Students solved multistep equations.

#### Now

Students use algebraic concepts to find relationships between lines, angles, and triangles.

### Next

Students will perform transformations on geometric figures.

### Rigor pursuing concepts, fluency, and applications

At the end of the chapter, students should be able to answer "HOW can algebraic concepts be applied to geometry?"

# **Launch the Chapter**



#### Math in the Real World

Games The chessboard is in the shape of a square. Remind students that to find the length of one side of the square, they need to find the square root of the area of the square.

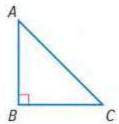
# What Tools Do You Need?

# **Vocabulary Activity**

(A) As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

**Define:** The hypotenuse is the side opposite the right angle in a right triangle.

#### Example:



#### Ask:

Which side of the above triangle is the hypotenuse? AC

### The Structure of Math

Have students read The Structure of Math section.

#### Ask:

- What kinds of questions are asked in the diamonds in a flowchart? Sample answer: In a flowchart, the questions only have yes or no answers.
- What do the answers to the questions determine? Sample answer: They determine the path to follow.
- · Where should all of the paths lead? the end oval
- What other uses can you think of for flowcharts? Sample
  answer: A flowchart can be used to clarify any process such as a
  computer program, the way a company produces goods, how to
  cook, how to write a paper in English class, and so on.

# What Tools Do You Need?

# V

exterior angles

formal proof

### Vocabulary

alternate exterior angles in alternate interior angles in converse ir corresponding angles in deductive reasoning le Distance Formula pequiangular p

inductive reasoning informal proof interior angles legs paragraph proof parallel lines

legs
paragraph proof
parallel lines
perpendicular lines
polygon

proof

theorem

triangle

transversal

two-column proof

Pythagorean Theorem

remote interior angles

regular polygon

# Study Skill: The Structure of Math

Flowchart Symbols

**Use a Flowchart** A *flowchart* is like a map that tells you how to get from the beginning of a problem to the end.

| $\Diamond$     | A diamond contains a question. You ne make a decision. | ed to stop and |
|----------------|--|----------------|
|                | A rectangle tells you what to do.                      |                |
|                | An oval indicates the beginning or end.                |                |
| Complete the   | lowchart to classify triangles b                       | y their sides. |
| The triangle i | a(n) YES Are the 3                                     | NO Are two     |

The triangle is a(n)
equilateral triangle.

YES

Are the 3

Sides the same length?

YES

The triangle is a(n)
isosceles triangle.

The triangle is a(n)
scalene triangle.

one The window C state

WW386 JANGE 5 Jong Conflithe Pythagorean Theorem

# What Do You Already Know

List three things you already know about lines, angles, and triangles in the first section. Then list three things you would like to learn about lines, angles, and triangles in the second section. See students' work.

Triangles and the Pythagorean Theorem

What I know

What I want to find out

# When Will You Use T

Here is an example of how angles and lines are used in the real world.

Activity Have you ever built a ramp for something? In the space below, draw a bicycle ramp that you would like to build. Be sure to include measurements for the lengths of the side of the ramp.

See students' work.

# **What Do You Already Know**

In this activity students assess their prior knowledge by listing three things they already know and three things they would like to learn about concepts in the chapter.

- · You may want to add a third option of "I don't know" for those students who do not have any prior knowledge of the topic.
- · After completing the chapter, have students return to this page and have them add three new facts that they learned about the topic.

# When Will You Use This

# **Activity 1**

Students use a real-world situation to understand polygons and angle relationships.

# **Activity 2**

Use the Graphic Novel to help students learn about using polygons and parallel lines to solve problems.

 What should be the first steps for Mahmoud and Majed in finding the missing angle? Sample answer: Identify the shape of the quadrilateral; identify the types of angles and lines in the ramp; set up a formula.

Use this page to determine if students have skills that are needed for the chapter.



#### **Quick Review**

Students with strong math backgrounds may opt to go directly to the Quick Check.

| REVIEW                          |                 |  |  |  |  |
|---------------------------------|-----------------|--|--|--|--|
| Example                         | Skill           |  |  |  |  |
| 1                               | Solve Equations |  |  |  |  |
| 2 Graph on the Coordinate Plane |                 |  |  |  |  |



#### **Quick Check**

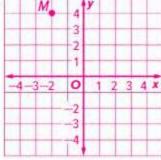
If students have difficulty with the exercises, present another example to clarify any misconceptions.

#### Exercises 1-3

Solve 90 + 62 + a = 180. 28

#### Exercises 4-9

Graph and label M(-2, 4) on a coordinate plane.



# Track Your Progress

Prior to beginning this chapter, have your students rate their knowledge of the objectives it addresses. At the end of the chapter, have your students return to these pages to rate their knowledge again. They should see that their knowledge of the key ideas has increased.

# Are You Ready





#### Quick Review

Review

### Example 1

Solve 82 + g + 41 = 180

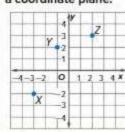
q = 57

$$82 + g + 41 = 180$$
  
 $123 + g = 180$   
**-123** = **-123**

Write the equation: Add 82 and 41. Subtraction Property of Equality

# Example 2

Graph X(-3, -2), Y(-1, 2), and Z(2, 3) on a coordinate plane.



Start at the origin. The first

#### Quick Check

#### Equations Solve each equation.

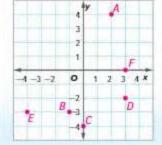
2. 
$$t + 98 + 55 = 180$$
 27

3. 
$$15 + 67 + k = 180$$
 98



Coordinate Plane Graph and label each point on the coordinate plane.

- 4. A(2, 4)
- 5. B(-1, -3)
- 6. C(0, -4)
- 7. D(3, -2)
- 9. F(3, 0) 8. E(-4, -3)



**How Did** You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.















# **Inquiry Lab**

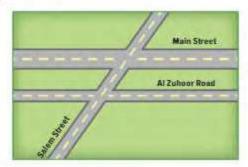
Parallel Lines



WHAT are the angle relationships formed when a third line intersects two parallel lines?

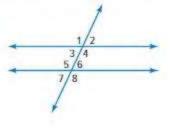


A newspaper route has two parallel streets. The streets are cut by another street as shown in the figure below.



# Hands-On Activity

Parallel lines have special angle relationships. You will examine those relationships in this activity.



Step 1

Use a protractor and angle relationships you have previously learned to find the measure of each numbered angle and record it in the table.

| Angle   | 1    | 2   | 3   | 4    | 5    | 6   | 7   | 8    |
|---------|------|-----|-----|------|------|-----|-----|------|
| Measure | 115° | 65° | 65° | 115° | 115° | 65° | 65° | 115° |

Step 2

Color the angles that have the same measure.

See students' work.

Step 3

Describe the position of the angles with the same measure.

Sample answer: Pairs of angles that have the same

measurement are vertical angles and vertical angles are

www.jnob-jo.com

### Focus narrowing the scope

Objective Examine angle relationships formed when parallel lines are cut by a transversal.

# Coherence connecting within and across grades

Students will examine angle relationships formed when parallel lines are cut by a transversal.

#### Next

Students will use angle relationships to find missing angles when parallel lines are cut by a transversal.

## Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 370.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# Launch the Lab

The activity is intended to be used as a whole-group activity.

Materials: protractor

# Hands-On Activity

of 3-4 to find the measure of an angle either by use of a protractor or by angle relationships. After each student gives an angle measure, the rest of the group gives a thumbs-up or thumbs-down to denote agreement or disagreement. If there is disagreement, students work together to resolve it. 1, 3, 5, 6

#### Ask:

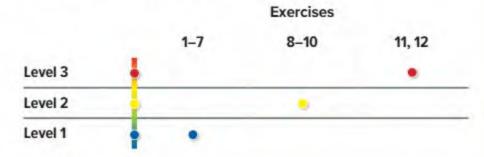
 What do you notice about the position of the angles that have the same measure? Sample answer: They are in the same location in relation to the top line and the bottom line. They are also opposite each other at the intersection of lines.

# 2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

#### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.





BL Pairs Present Have students explain how they can determine all of the angle measures if they are given only one angle measure. Have them prepare a brief oral presentation to share with the class, using illustrations. 21, 3, 5, 6, 7

#### Ask:

- What do the obtuse angles have in common? the acute angles? The obtuse angles have the same angle measure.
   The acute angles have the same measure.
- What rule(s) can you generate to determine all of the missing angle measures when you are only given one angle measure? Sample answer: Angles are either congruent or supplementary.

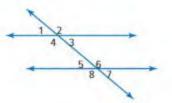


Students should be able to answer "WHAT are the angle relationships formed when a third line intersects two parallel lines?"



# Investigate

Use Math Tools Work with a partner. If the measure of ∠1 in the figure at the right is 40°, determine the measure of each given angle without using a protractor. Then check your answers by measuring with a protractor.



1. Z2 140°

2. Z3 40°

3. 24 140°

4. /5 40°

5. ∠6 140°

6. Z7 40°

7. ∠8 140°



# **Analyze and Reflect**

#### Refer to the figure above.

- What is the relationship between the two horizontal lines?Sample answer: The lines appear to be parallel.
- What is true about the measures of angles that are side by side?Sample answer: Angles that are side by side are supplementary.
- Peason Inductively Congruent angles are angles that have the same measure. Describe the position of the congruent angles.

Sample answer: Pairs of angles that appear to be congruent are in the same positions with respect to both horizontal lines.



## Create

- 11. Make a Conjecture Draw a set of parallel lines cut by another line. Estimate the measures of the eight angles formed. Check your estimates by measuring each angle with a protractor. See students' work.
- WHAT are the angle relationships formed when a third line intersects two parallel lines?

Sample answer: Eight angles are formed. Some of them add up to 180° and some of them are congruent.

Annual of the second or the State of Street

# Lesson 1 Lines

# **Vocabulary Start-Up**



When two lines intersect in a plane and form right angles they are called perpendicular lines. Two lines are called parallel lines when they are in the same plane and do not intersect.

Complete the graphic organizer. Sample answers are given.

|   | Parallel Lines                | Perpendicular Lines                            |
|---|-------------------------------|--|
| Symbols                                   | H                             | 1  |
| Define it in your<br>own words            | two lines that<br>never cross | two lines that<br>cross and form<br>90° angles |
| Draw it                                   | $\Longrightarrow$             | +  |
| Describe a<br>real-world<br>example of it | railroad<br>tracks            | an intersection where two streets cross        |

# **Essential Question**

HOW can algebraic concepts be applied to geometry?

#### Vocabulary

perpendicular lines parallel lines transversal interior angles exterior angles alternate interior angles alternate exterior angles corresponding angles

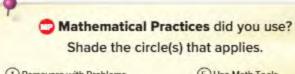
#### Math Symbols

|| is parallel to 1 is perpendicular to  $m \angle 1$  the measure of  $\angle 1$ 

Mathematical Practices

## Real-World Link

A gymnastic event in the Summer Olympics involves the parallel bars. The women compete on uneven parallel bars and the men compete on the parallel bars like the one shown. Circle the parallel lines shown in the photo at the right. See students' work.



- 1) Persevere with Problems
- 2 Reason Abstractly
- 3 Construct an Argument
- (4) Model with Mathematics
- (5) Use Math Tools (6) Attend to Precision
- (7) Make Use of Structure

### 8 Use Repeated Reasoning

### Focus narrowing the scope

Objective Identify relationships of angles formed by two parallel lines cut by a transversal.

### Coherence connecting within and across grades

#### Previous

#### Students examined angle relationships formed when parallel lines are cut by a transversal.

#### Now

Students will classify the angles formed and find missing angles when parallel lines are cut by a transversal.

#### Next

Students will explore the relationship among angles of a triangle.

## Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 375.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **Launch the Lesson**

### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

(IA) Pairs Discussion Have pairs complete the graphic organizer. Then have them share and revise their responses, if necessary, with another pair of students. Call on one pair to share their responses with the class. 31, 2, 3, 4, 5, 6

## **Alternate Strategy**

M Have students write parallel lines and perpendicular lines on the front of two index cards. On the back, have them draw an example and write the definition. Have them use these cards as a reference throughout the lesson. 21, 2, 4, 5, 6

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# **Teach the Concept**

Ask the scaffolded questions for each example to differentiate instruction.

# **Examples**

### 1. Classify angle pairs.



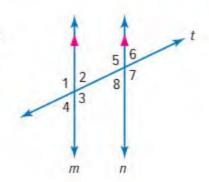
- What does the term exterior mean? outside
- Are ∠1 and ∠7 on the same side of the transversal or opposite sides? opposite sides
  - · Are the angles on the inside of the parallel lines or the outside? outside
  - What type of angles are ∠1 and ∠7? alternate exterior angles
- What is true about alternate exterior angles? They are congruent if the lines are parallel.

# 2. Classify angle pairs.

- M Look at the positions of ∠2 and ∠6 related to each of the lines. What do you notice? Sample answer: The angles are in the same position.
- Are the angles on the same side of the transversal or opposite sides? same side
  - What type of angles are ∠2 and ∠6? corresponding
- What is true about corresponding angles? They are congruent if the lines are parallel.

### Need Another Example?

Classify each pair of angles as alternate interior, alternate exterior, or corresponding. ∠3 and ∠7 corresponding ∠2 and ∠8 alternate interior



Key Concept

# **Transversals and Angles**

Work Zone

Angles

of angle 1.

A line that intersects two or more lines is called a transversal, and eight angles are formed.

Interior angles lie inside the lines. Examples: ∠3, ∠4, ∠5, ∠6

Exterior angles lie outside the lines. Examples: ∠1, ∠2, ∠7, ∠8

Alternate interior angles are interior angles that lie on opposite sides of the transversal. When the lines are parallel, their measures are equal. Examples:  $m\angle 4 = m\angle 6$ ,  $m\angle 3 = m\angle 5$ 

Alternate exterior angles are exterior angles that lie on opposite sides of the transversal. When the lines are parallel, their measures are equal.

Examples:  $m\angle 1 = m\angle 7, m\angle 2 = m\angle 8$ 

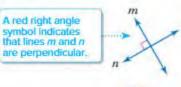
Corresponding angles are those angles that are in the same position on the two lines in relation to the transversal. When the lines are parallel, their measures are equal. Examples:  $m\angle 1 = m\angle 5$ ,  $m\angle 2 = m\angle 6$ ,  $m\angle 4 = m\angle 8, m\angle 3 = m\angle 7$ 

Parallel and **Perpendicular Lines** 

Read m/ as the measure

Read m I has line mis perpendicular to line 4 Read pll q as line pis parallel to line q.

Special notation is used to indicate perpendicular and parallel lines.



 $m \perp n$ 

Examples

Classify each pair of angles in the figure as alternate interior, alternate exterior, or corresponding.

1. ∠1 and ∠7

∠1 and ∠7 are exterior angles that lie on opposite sides of the transversal. They are alternate exterior angles.

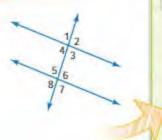
2. ∠2 and ∠6

∠2 and ∠6 are in the same position on the two lines. They are corresponding angles.

WW3W2 10Hapter 5 1 Congres and the Pythagorean Theorem

#### Got It? Do this problem to find out.

a. Classify the relationship between ∠4 and ∠6. Explain.



∠4 and ∠6 are alternate interior angles because they lie inside the two lines but on opposite sides of the transversal.

# **Find Missing Angle Measures**

When two parallel lines are cut by a transversal, special angle relationships exist. If you know the measure of one of the angles, you can find the measures of all of the angles. Suppose you know that  $m\angle 1 = 50^\circ$ . You can use that to find the measures of angles 2, 3, and 4,



#### and Reflect

In the figure, how do you know that w/5 = 509 Explain below.

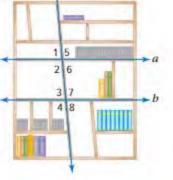
Sample answer: The lines are parallel and ∠1 and ∠5 are corresponding angles, so they have equal measures.

### Example

3. A furniture designer built the bookcase shown. Line a is parallel to line b. If  $m\angle 2 = 105^{\circ}$ , find  $m\angle 6$  and  $m\angle 3$ . Justify your answer.

Since ∠2 and ∠6 are supplementary, the sum of their measures is 180°.  $m\angle 6 = 180^{\circ} - 105^{\circ} \text{ or } 75^{\circ}$ 

Since ∠6 and ∠3 are interior angles that lie on opposite sides of the transversal, they are alternate interior angles. The measures of alternate interior angles are equal, m∠3 = 75°



#### Got It? Do this problem to find out.

b. Refer to the situation above. Find m∠4. Justify your answer.

105°: Sample answer: ∠2 and ∠4 are corresponding b, angles, so their measures are equal.

# Example

## Find missing angle measures.



- M How does ∠6 relate to ∠2? ∠6 and ∠2 are supplementary.
  - · If two angles are supplementary, what is the sum of their angle measures? 180°
  - If I know one of the angle measures is 105°, how can I find the measure of the other angle? Subtract 105° from 180°.



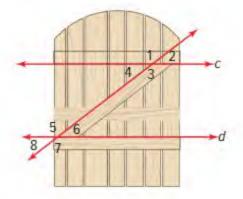
- What is the measure of ∠6? 75°
  - How does ∠3 relate to ∠6? They are alternate interior
  - What is true about the measures of alternate interior angles? The angle measures are the same if the lines are parallel.
  - What is the measure of ∠3? 75°



• Explain how to find all of the missing angle measures in the diagram. Sample answer: If  $m\angle 2 = 105^{\circ}$ , then  $m\angle 4 = 105^{\circ}$ ,  $m\angle 5 = 105^{\circ}$ , and  $m\angle 7 = 105^{\circ}$ . The other four angles, ∠1, ∠6, ∠3, and ∠8 all have a measure of 75° using corresponding angles, vertical angles, alternate interior angles, and supplementary angles.

### Need Another Example?

Mr. Mohammad installed the gate shown. Line c is parallel to line d. If  $m\angle 4 = 40^\circ$ , find  $m\angle 6$  and  $m\angle 7$ . Justify your answer.  $m\angle 6 = 40^{\circ}$  and  $m\angle 7 = 140^{\circ}$ ; Sample answer:  $\angle 4$  and  $\angle 6$  are alternate interior angles, so they are congruent. ∠6 and ∠7 are supplementary. Since  $m\angle 6 = 40^{\circ}$ ,  $m\angle 7 = 140^{\circ}$ .

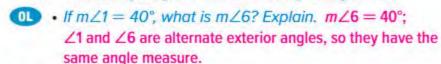


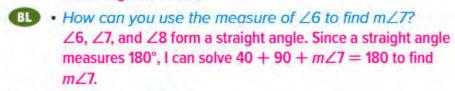
# Example

## 4. Find missing angle measures.



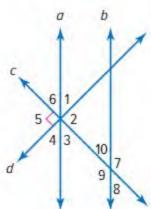
- If line a is perpendicular to line p, at what angle measure do they intersect? 90°
  - What kind of angle do ∠8, ∠7, and ∠6 form? a straight angle
  - How many degrees are in a straight angle? 180°





#### Need Another Example?

In the figure, line a is parallel to line b, and line c is perpendicular to line d. The measure of  $\angle 7$  is 125°. What is the measure of \( 47 \) 35°



# **Guided Practice**

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.



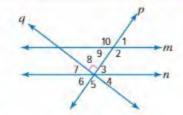
If some of your students are not ready for assignments, use the differentiated activities below.

Mhole Group Discussion Work as a large group to complete Exercises 1-4. Have students highlight in different colors which angles are congruent, which are complementary, and which are supplementary. 21,5

Round Table Students take turns completing the steps for Exercises 2 and 3, checking the previous person's work as they progress through each problem. 21,3

# Example

4. In the figure, line m is parallel to line n, and line q is perpendicular to line p. The measure of ∠1 is 40°. What is the measure of ∠7?



Since ∠1 and ∠6 are alternate exterior angles,  $m\angle 6 = 40^\circ$ .

Since ∠6, ∠7, and ∠8 form a straight line, the sum of their measures is 180°.

 $40 + 90 + m\angle 7 = 180$ 

So, m 27 is 50°.

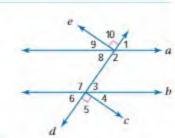
## **Guided Practice**



1. Refer to the porch stairs shown. Line m is parallel to line n and  $m \angle 7$  is 35°. Find the measure of  $\angle 1$ . Justify your answer. (Example 3) 145°; Sample answer: ∠7 and ∠5 are supplementary. So,  $m \angle 5 = 180^{\circ} - 35^{\circ}$  or 145°.  $\angle 5$  and  $\angle 1$  are corresponding angles. Since corresponding angles are have the same

measure,  $m \angle 1 = 145^{\circ}$ .

Refer to the figure at the right. Line a is parallel to line b and m∠2 is 135°. Find each given angle measure. Justify your answer. (Examples 1, 2, and 4)





2. m∠9 45°; Sample answer: ∠2 and

angles 9 and 10 are vertical angles. So, m∠9 +  $m\angle 10 = 135^{\circ}$ . So,  $m\angle 9 = 135^{\circ} - 90^{\circ}$  or  $45^{\circ}$ .

3. m∠7 135°; Sample answer: ∠2 and ∠7 are alternate interior angles. So,  $m \angle 7 = 135^{\circ}$ .

Building on the Essential Question How are the measures of the angles related when parallel lines are cut by a transversal?

Sample answer: The angles are either equal or supplementary.



How confident are you about lines and angles? Check the box that applies.





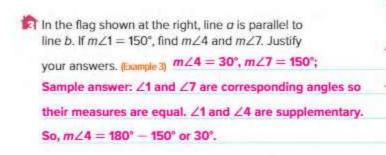


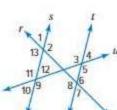
My Homework

# Independent Practice

Classify each pair of angles as alternate interior, alternate exterior, or corresponding. (Examples 1 and 2)

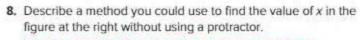
- 1. ∠2 and ∠4 corresponding
- 2. ∠4 and ∠5 alternate exterior





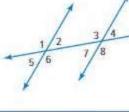
Refer to the figure at the right. Line s is parallel to line t,  $m \angle 2$  is 110° and  $m \angle 11$  is 137°. Find each given angle measure. Justify your answer. (Example 4)

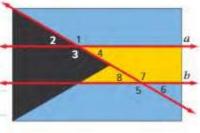
- m∠7 70°; Sample answer: ∠2 and ∠6 are corresponding angles, so they have the same measure. ∠6 and ∠7 are supplementary. So,  $m\angle 7 = 180 - 110$  or  $70^{\circ}$ .
- 5. m∠8 110°; Sample answer: ∠2 and ∠8 are alternate interior angles, so they have the same measure.
- m∠3 137°; Sample answer: ∠11 and ∠3 are corresponding angles, so they have the same measure.
- 7. The parallel lines at the right are cut by a transversal. Find the value of x.
  - a. Angles 1 and 2 are corresponding angles, m∠1 = 45°, and  $m \angle 2 = (x + 25)^{\circ}$ . 20
  - **b.** Angles 3 and 4 are alternate interior angles,  $m\angle 3 = 2x^{\circ}$ , and  $m \angle 4 = 80^{\circ}$ . 40

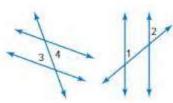


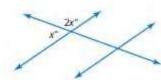
Sample answer: The two angles are supplementary.

So, 
$$x + 2x = 180^\circ$$
;  $x = 60$ .









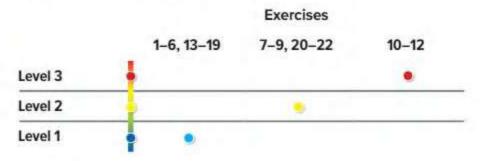
# **Practice and Apply**

### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



#### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                          |  |  |  |
|---------------------------------|-------------------|--------------------------|--|--|--|
| AD                              | Approaching Level | 1–7, 9, 11, 12, 21, 22   |  |  |  |
| OL OL                           | On Level          | 1–5 odd, 7–9, 12, 21, 22 |  |  |  |
| BL                              | Beyond Level      | 7–12, 21, 22             |  |  |  |

| • | MATHEMATICAL PRACTICES   |             |  |  |  |
|---|--|-------------|--|--|--|
|   | Emphasis On  | Exercise(s) |  |  |  |
|   | Make sense of problems and persevere in solving them.            | 10          |  |  |  |
|   | Construct viable arguments and critique the reasoning of others. | 11, 12      |  |  |  |
| 4 | Model with mathematics.  | 9, 20       |  |  |  |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to realworld situations.

#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

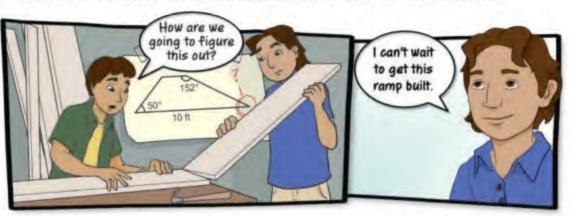
# TICKET Out the Door

Ask students to describe the pairs of congruent angles in a set of parallel lines cut by a transversal. See students' work.

# Watch Out!

Common Error Students may make errors finding missing angle measures when there is more than one transversal. Encourage students to begin by locating the given angle measure and finding the angle measures that are adjacent and vertical to that angle. They can then identify the parallel lines and use the relationships between corresponding, alternate interior, and alternate exterior angles to find the remaining angle measures.

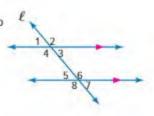
nodel with Mathematics Refer to the graphic novel frame below for Exercises a-b.



- a. Describe a method you could use to find the missing angle. The top and bottom of the ramp are parallel. The slanted part of the ramp can be considered a transversal. You can use angle relationships of parallel lines to find the measure of the missing angle.
- b. Use your method from part a to find the measure of the missing angle.

H.O.T. Problems Higher Order Thinking

- Persevere with Problems Quadrilateral ABCD is a parallelogram. Make a conjecture about the relationship of ∠1 and ∠2. Justify your reasoning. ∠1 and ∠2 are supplementary. Sample answer: Since AB and DC are parallel,  $m\angle 1 = m\angle ADE$  (alternate interior angles have the same measure). Since ∠ADE and ∠2 lie on the same line, they are supplementary, and  $m\angle ADE + m\angle 2 = 180^{\circ}$ . Substitute  $\angle 1$  for  $\angle ADE$ . Therefore,  $m\angle 1 + m\angle 2 = 180^{\circ}$ .
- Peason Inductively If two parallel lines are cut by a transversal, what relationship exists between interior angles that are on the same side of the transversal? They are supplementary.
- 12. Reason Inductively Suppose  $m \angle 1 = x^{\circ}$ . Use an informal argument to write an expression for the measure of ∠6 in the diagram at the right. Sample answer:  $\angle 1$  and  $\angle 2$  are supplementary, so  $m\angle 2 = 180^{\circ} - x^{\circ}$ .  $\angle 2$  and  $\angle 6$  are corresponding angles. So,  $m\angle 6=180^{\circ}-x^{\circ}$ .



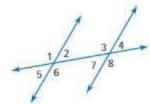
WW316 JAHOLD 5 Journal of Contract of the Pythagorean Theorem

My Homework

## **Extra Practice**

Classify each pair of angles as alternate interior, alternate exterior, or corresponding.

13. 23 and 26 alternate interior 23 and 26 are interior angles that lie on opposite sides of the transversal. They are alternate interior angles.

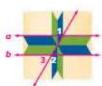


14. ∠1 and ∠3 corresponding

15. ∠2 and ∠7 alternate interior

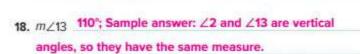
16. In the quilt design at the right, line α is parallel to line b. If  $m\angle 1 = 120^\circ$ , find  $m\angle 2$  and  $m\angle 3$ .

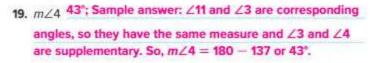
Justify your answers.  $m\angle 2 = 120^{\circ}$ ,  $m\angle 3 = 60^{\circ}$ ; Sample answer: ∠1 and ∠2 are alternate exterior angles, so they have the same measure. ∠2 and ∠3 are supplementary. So,  $m\angle 3 = 180^{\circ} - 120^{\circ}$  or 60°.

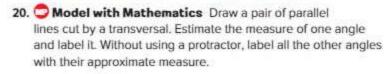


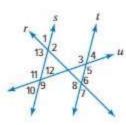
Refer to the figure at the right. Line s is parallel to line t,  $m\angle 2$  is 110° and  $m\angle 11$  is 137°. Find each given angle measure. Justify your answer.

17. m∠6 110°; Sample answer: ∠2 and ∠6 are corresponding angles, so they have the same measure.









Sample answer:

# Power Up! Test Practice

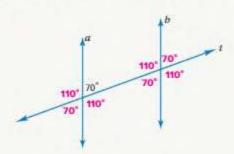
Exercises 21 and 22 prepare students for more rigorous thinking needed for assessment.

| 21. | This test item requires students to reason abstractly and quantitatively when problem solving. |   |  |  |
|-----|--|---|--|--|
|     | Depth of Knowledge   | DOK1                                    |  |  |
|     | Mathematical Practice  | MP1                                     |  |  |
|     | Scoring Rubric   |   |  |  |
|     | 1 point  | Students correctly answer the question. |  |  |

| 22. | This test item requires students to reason abstractly and<br>quantitatively when problem solving. |   |  |  |
|-----|---|---|--|--|
|     | Depth of Knowledge  | DOK1  |  |  |
| Ì   | Mathematical Practice   | MP1   |  |  |
|     | Scoring Rubric  |   |  |  |
|     | 2 points  | Students correctly label all 7 of the angles. |  |  |
|     | 1 point   | Students correctly label 5–6 of the 7 angles. |  |  |

# Power Up! Test Practice

- 21. Lines m and n are parallel and cut by the transversal p. Which of the following pairs of angles represent corresponding angles? Select all that apply.
  - ∠2 and ∠6
  - ∠4 and ∠6
  - ∠3 and ∠4
  - ∠1 and ∠5
- **22.** Lines *α* and *b* are parallel and cut by the transversal *t*. Label each of the 7 unknown angles with the correct angle measure.

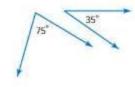


# **Spiral Review**

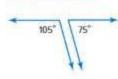
23. A poster has a triangular image with a base that measure 10 centimeters, and a height that measures 20 centimeters. What is the area of the poster?
100 cm<sup>2</sup>

Classify each pair of angles as complementary, supplementary, or neither.

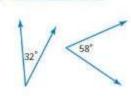
24. neither



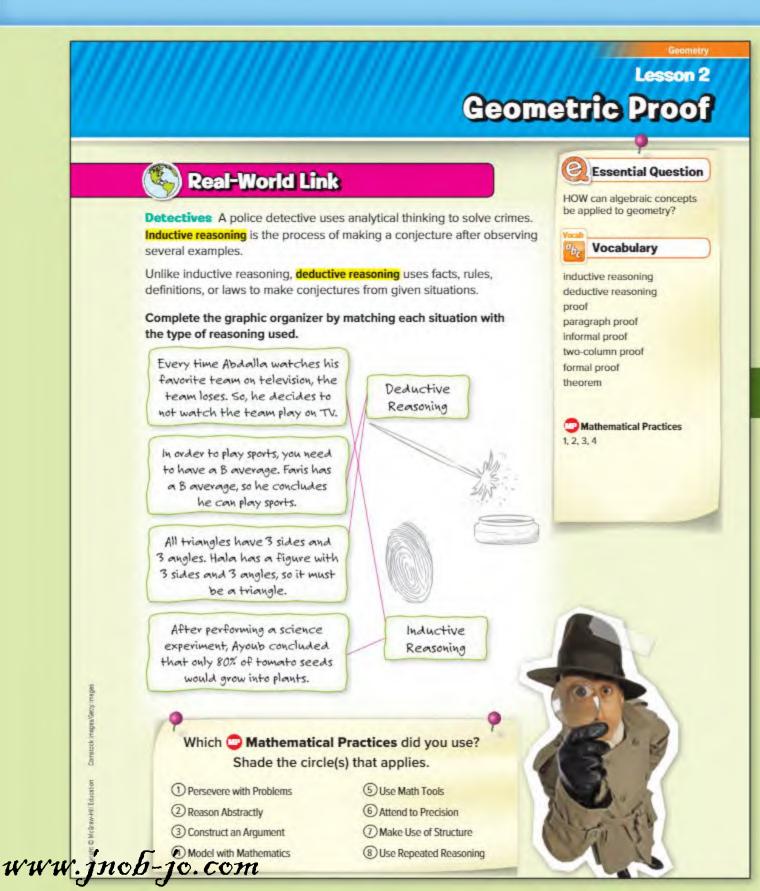
25. supplementary



26. complementary



MIN O MUSICIAN HILL EQU



### Focus narrowing the scope

Objective Write geometric proofs.

### Coherence connecting within and across grades

### Previous

Students used the properties of mathematics to justify the steps to solve an equation.

#### Now

Students use definitions. properties, and theorems to prove a hypothesis.

#### Next

Students will prove the Pythagorean Theorem and its converse.

### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 383.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# **Launch the Lesson**

### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

(A) Pairs Consult Have pairs write inductive reasoning and deductive reasoning on two index cards. Have them write the attributes of each kind of reasoning on the back of each card. Have them use their cards to determine which type of reasoning was used for each situation in the graphic organizer. 21, 5, 6

# Alternate Strategy

Bl Have students provide their own examples of when they have used inductive and deductive reasoning to solve a problem in everyday life. 56

# **Teach the Concept**

Ask the scaffolded questions for each example to differentiate instruction.

# Example

## Complete a paragraph proof.



- What is a paragraph proof? a proof that is written in paragraph form
  - What information do you know that is stated in the problem?  $m \angle 1 = m \angle 4$
  - What do you know about the relationship between ∠1 and  $\angle 2$  from the diagram? They are vertical angles.
  - What do you know about the relationship between ∠3 and  $\angle 4$  from the diagram? They are vertical angles.
  - What is true about the measures of vertical angles? They are equal.



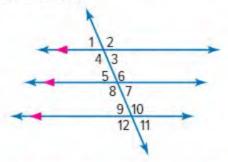
- $\bigcirc$  If  $m \angle 1 = m \angle 4$  and  $m \angle 1 = m \angle 2$ , what can you say about  $m \angle 2$  and  $m \angle 4$ ?  $m \angle 2 = m \angle 4$ 
  - If  $m\angle 2 = m\angle 4$  and  $m\angle 4 = m\angle 3$ , what can you say about m/2 and m/3? m/2 = m/3



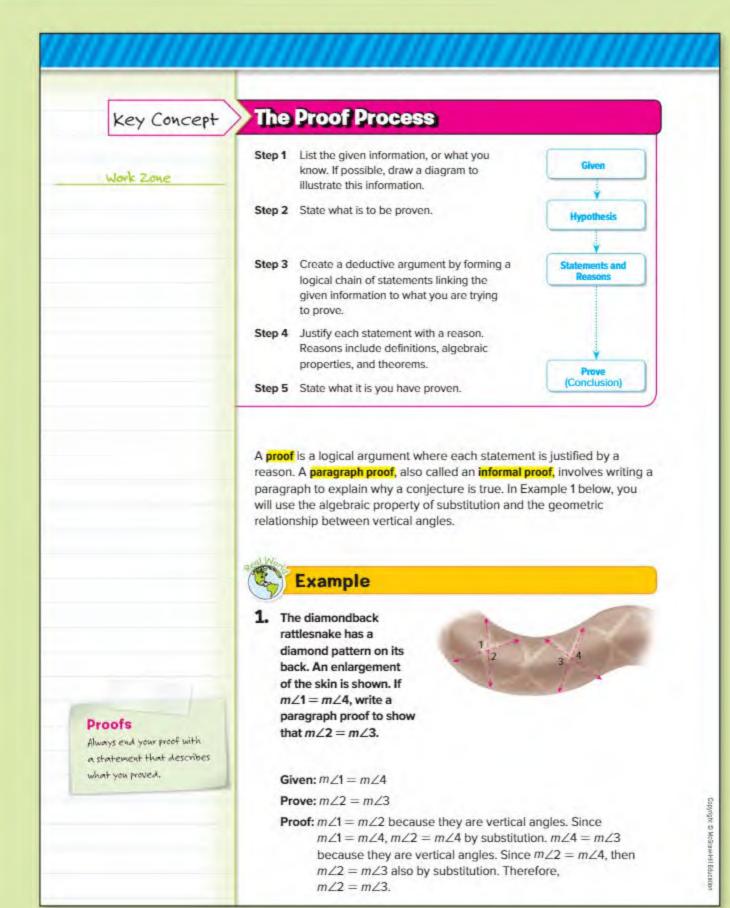
 What definitions, properties, or relationships were used in this proof? substitution; vertical angles have equal measures

### Need Another Example?

Refer to the diagram. If  $m\angle 1 = m\angle 5$ , write a paragraph proof to show that  $m \angle 1 = m \angle 11$ .



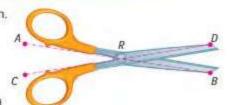
 $m\angle 1 = m\angle 9$  because they are corresponding angles.  $m\angle 9 = m\angle 11$ because they are vertical angles. Since  $m \angle 9 = m \angle 11$ , then  $m \angle 1 = m \angle 11$  by substitution.



WW386 1316 15 1 Cing & Call the Pythagorean Theorem

#### Got 1+? Do this problem to find out.

a. Refer to the diagram shown, AR = CR and DR = BR. Write a paragraph proof to show that AR + DR = CR + BR.



Given: AR = CR

DR = BR

Prove: AR + DR = CR + BR

**Proof:** You know that AR = CR and DR = BR.

AR + DR = CR + DR by the Addition Property of

Equality. So, AR + DR = CR + BR by substitution

## **Two-Column Proofs**

A two-column proof or formal proof contains statements and reasons organized in two columns. Once a statement or conjecture has been proven, it is called a theorem, and it can be used as a reason to justify statements in other proofs.

### Example

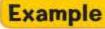
2. Write a two-column proof to show that if two angles are vertical angles, then they have the same measure.



Given: lines m and n intersect;  $\angle 1$  and  $\angle 3$  are vertical angles

Prove:  $m\angle 1 = m\angle 3$ 

|    | Statements   | Reasons                               |
|----|--|---------------------------------------|
| a. | lines $m$ and $n$ intersect;<br>$\angle 1$ and $\angle 3$ are vertical angles.                     | Given                                 |
| b. | $\angle 1$ and $\angle 2$ are a linear pair<br>and $\angle 3$ and $\angle 2$ are a linear<br>pair. | Definition of linear pair             |
| c. | $m\angle 1 + m\angle 2 = 180^{\circ}$<br>$m\angle 3 + m\angle 2 = 180^{\circ}$                     | Definition of supplementary<br>angles |
| d. | $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$  | Substitution                          |
| e. | $m\angle 1 = m\angle 3$  | Subtraction Property of<br>Equality   |



## Complete a two-column proof.



Segment Notation

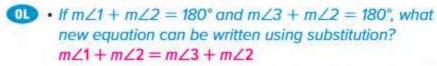
AR is read as the measure

of line segment AR

**Linear Pair** 

A linear pair of angles is a pair of adjacent angles formed by Intersecting lines.

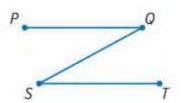
- What information are you given? lines m and n intersect; ∠1 and ∠3 are vertical angles
  - What is the relationship between ∠1 and ∠2? They are supplementary angles.
  - What is the relationship between ∠2 and ∠3? They are supplementary angles.
  - If two angles are supplementary, what is the sum of their angle measures? 180°



- What property allows you to subtract m∠2 from each side? Subtraction Property of Equality
- Do you prefer using a paragraph proof or a twocolumn proof? Explain. See students' preferences.

#### Need Another Example?

Complete the two-column proof to show that if PQ = QS and QS = ST, then PQ = ST.



Given: PQ = QS; QS = ST

Prove: PQ = ST

| Statements                 | Reasons      |
|----------------------------|--------------|
| a. $PQ = QS$ and $QS = ST$ | Given        |
| b. $PQ = ST$               | Substitution |

# **Guided Practice**

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.



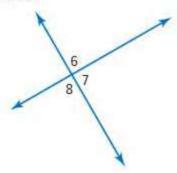
If some of your students are not ready for assignments, use the differentiated activities below.

Roundrobin Have students work in teams of three to complete Exercises 1 and 2. The first student should fill in the first blank and explain their answer, then move on to the second student, and so on. Have them trade their solutions with another team of students and discuss any differences.

1, 3, 5, 6, 7

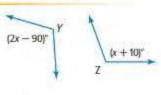
BL LA Pairs Consult Have students work in pairs. Give them the following information and ask them to create a paragraph proof or a two-column proof. 1, 3, 5, 6, 7

In the figure, two lines intersect to form four angles. If  $\angle 6$  and  $\angle 8$  are supplementary angles, prove that  $\angle 7$  is a right angle. Have them trade their proofs with another pair of students and discuss any differences.



#### Got It? Do this problem to find out.

b. The statements for a two-column proof to show that if m∠Y = m∠Z, then x = 100 are given below. Complete the proof by providing the reasons.



|    | Statements  | Reasons                          |
|----|---|----------------------------------|
| a. | $m \angle Y = m \angle Z$ ,<br>$m \angle Y = 2x - 90$ , | Given                            |
| b. | $m\angle Z = x + 10$ $2x - 90 = x + 10$                 | Substitution                     |
| c. | x - 90 = 10   | Subtraction Property of Equality |
| d. | x = 100   | Addition Property of Equality    |

### **Guided Practice**



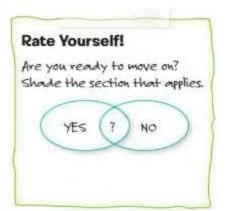
Use the figure to complete the paragraph proof. (Example 1)
 Given: m∠1 = m∠2, ∠1 and ∠2 are supplementary.

Prove: ∠1 and ∠2 are right angles.

- **Proof:**  $m\angle 1 + m\angle 2 = \frac{180^{\circ}}{}$  since they are supplementary angles. Since  $m\angle 1 = m\angle 2$ , then  $m\angle 1 + m\angle 1 = 180^{\circ}$  by **substitution**. Solving the equation gives  $m\angle 1 = \frac{90^{\circ}}{}$ . Since  $m\angle 1 = m\angle 2$ , then  $m\angle 2$  is also  $\frac{90^{\circ}}{}$ . Therefore,  $\angle 1$  and  $\angle 2$  are right angles.
- **2.** Refer to the figure above. Complete the two-column proof to show that if EG = 3x 1, ED = 2x + 4, and EG = ED, then x = 5. (Example 2)

| Stater                           | nents    | Reasons                          |  |
|----------------------------------|----------|----------------------------------|--|
| a. EG = 3x<br>ED = 2x<br>EG = EL | c + 4,   | Given                            |  |
| <b>b.</b> 3x − 1 =               | = 2x + 4 | Substitution                     |  |
| c. $x - 1 =$                     | 4        | Subtraction Property of Equality |  |
| <b>d.</b> $x = 5$                |          | Addition Property of Equality    |  |

 Building on the Essential Question How is deductive reasoning used in algebra and geometry proofs?
 Sample answer: You use facts, definitions, and properties in proofs.



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My Homework

# Independent Practice

👣 In the figure at the right, two lines intersect to form four angles. If  $m\angle 7 = 9x$  and  $m\angle 8 = 11x$ , complete the paragraph proof to show that x = 9. (Example 1)

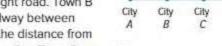


Given: Two intersecting lines with  $m\angle 7 = 9x$  and  $m\angle 8 = 11x$ 

Prove: x = 9

Proof: ∠7 and ∠8 form a straight angle so they are supplementary angles. So,  $m\angle 7 + m\angle 8 = 180^{\circ}$ definition of supplementary angles. By substitution, + 11x = 180, So, x = 9by the Division Property

Construct an Argument Four towns lie on a straight road. Town B is midway between Town A and Town C. Town C is midway between Town B and Town D. Write a paragraph proof to show the distance from Town A to Town B is the same as the distance from Town C to Town D. (Example 1)



Given: B is the midpoint of  $\overline{AC}$  and C is the midpoint of  $\overline{BD}$ .

Prove: AB = CD.

of Equality.

**Proof:** By the definition of midpoint,  $\frac{AB}{} = BC$  and  $\frac{BC}{} = CD$ . Therefore, AB = CD by substitution

3. Construct an Argument Complete the two-column proof to show that if  $\angle 1$  and  $\angle 2$  are supplementary and  $m\angle 1 = m\angle 2$ , then  $\angle 1$  and  $\angle 2$  are right angles. (Example 2)

Given:  $\angle 1$  and  $\angle 2$  are supplementary;  $m\angle 1 = m\angle 2$ 

Prove: ∠1 and ∠2 are right angles

|    | Statements                                   | Reasons                            |
|----|--|------------------------------------|
| a. | ∠1 and ∠2 are supplementary; $m$ ∠1 = $m$ ∠2 | Given                              |
| b. | $m\angle 1 + m\angle 2 = 180^{\circ}$        | Definition of supplementary angles |
| c. | $m\angle 1 + m\angle 1 = 180^{\circ}$        | Substitution                       |
| d. | 2(m∠1) = 180°                                | Simplify                           |
| e. | <i>m</i> ∠1 = 90°                            | Division Property of Equality      |
| f. | <i>m</i> ∠2 = 90°                            | $m\angle 1 = m\angle 2$ (Given)    |
| g. | ∠1 and ∠2 are right angles.                  | Definition of right angles         |

# ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

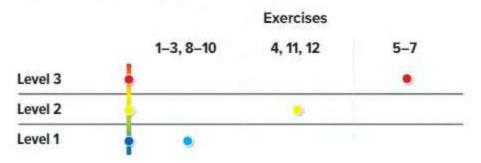
# **Practice and Apply**

### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                   |
|---------------------------------|-------------------|-------------------|
| AL                              | Approaching Level | 1–3, 5, 7, 11, 12 |
| 01                              | On Level          | 1, 3–5, 7, 11, 12 |
| B                               | Beyond Level      | 4–7, 11, 12       |

| MATHEMATICAL PRACTICES  |             |
|---|-------------|
| Emphasis On   | Exercise(s) |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 6           |
| 2 Reason abstractly and quantitatively.                                       | 5           |
| 3 Construct viable arguments and critique the reasoning of others.            | 2-4, 7-10   |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

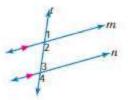
Use this activity as a closing formative assessment before dismissing students from your class.



Have students describe the differences between a paragraph proof and a two-column proof. See students' work.  Construct an Argument Complete the two-column proof to show that when two parallel lines are cut by a transversal, consecutive interior angles are supplementary.

Given: parallel lines m and n cut by transversal t

Prove: ∠2 and ∠3 are supplementary.



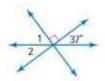
|    | Statements                         | Reasons                              |  |
|----|------------------------------------|--------------------------------------|--|
| a. | Lines m and n are parallel         | Given                                |  |
|    | and cut by transversal t           |                                      |  |
| b. | ∠1 and ∠2 form a straight angle.   | Definition of straight angle         |  |
| c. | $m \angle 1 + m \angle 2 = 180$    | Definition of supplementary angles   |  |
| d. | $m\angle 1 = m\angle 3$            | Corresponding ∠s have equal measures |  |
| e. | $m\angle 3 + m\angle 2 = 180$      | Substitution                         |  |
| f. | ∠2 and ∠3 are supplementary angles | Definition of supplementary angles   |  |

# de

### H.O.T. Problems Higher Order Thinking

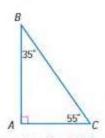
 Describe the theorem or definition you could use to find the measure of ∠2.

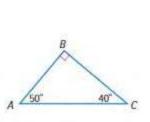
Sample answer: Vertical angles have the same measure.



 Persevere with Problems Given the right triangles shown, use inductive reasoning to make a conjecture about the sum of the measures of the two acute angles of any right triangle.



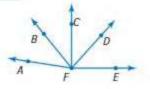




Sample answer: The sum of the measures of the acute angles of a right

triangle is 90°. So, the acute angles are complementary.

 Reason Inductively In the diagram, m∠CFE = 90° and m∠AFB = m∠CFD. Which of the following conclusions does not



have to be true?

- $m\angle AFB + m\angle DFE = 90^{\circ}$   $m\angle M\angle CFD = m\angle AFB$
- BF divides ∠AFD in half
  - w ∠CFE is a right angle.

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Name

My Homework -

# **Extra Practice**

 Construct an Argument In the figure at the right, AE = DB and C is the midpoint of AE and DB. Complete the proof to show that AC = CB.

Given: AE = DB and C is the midpoint of  $\overline{AE}$  and  $\overline{DB}$ .

Prove: AC = CB



$$AC = CE = \frac{1}{2} \frac{AE}{}$$
 and  $DC = CB = \frac{1}{2} \frac{DB}{}$  by the

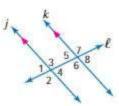
definition of midpoint. We are given AE = DB. By the Multiplication

Property of Equality,  $\frac{1}{2}AE = \frac{1}{2}DB$ . So, by substitution , AC = CB.

9. Construct an Argument Refer to the figure at the right. Complete the two-column proof to show if  $m \angle 3 = 2x - 15$  and  $m \angle 6 = x + 55$ , then x = 70.

Given:  $j \mid k$ , transversal  $\ell$ ;  $m \angle 3 = 2x - 15$ ,  $m \angle 6 = x + 55$ 

Prove: x = 70



|    | Statements   | Reasons  |
|----|--|--|
| a. | $j \mid\mid k$ , transversal $\ell$ ; $m \angle 3 = 2x - 15$ , $m \angle 6 = x + 55$ | Given  |
| b. | $m\angle 3 = m\angle 6$  | Alternate interior angles have the same measure. |
| c. | 2x - 15 = x + 55   | Substitution                                     |
| d. | x - 15 = 55  | Subtraction Property of Equality                 |
| e. | x = 70   | Addition Property of Equality                    |

Construct an Argument Refer to the figure at the right. Complete
the two-column proof to show if ∠ABE and ∠DBC are right angles, then
m∠ABD = m∠EBC.

Given: ∠ABE and ∠DBC are right angles.

Prove:  $m\angle ABD = m\angle EBC$ 



|    | Statements   | Reasons                          |  |
|----|--|----------------------------------|--|
| a. | ∠ABE and ∠DBC are right angles.                                      | Given                            |  |
| b. | $m\angle ABE = 90$ and $m\angle DBC = 90$                            | Definition of right angles       |  |
| c. | $m\angle ABD + m\angle DBE = 90$<br>$m\angle DBE + m\angle EBC = 90$ | Angle addition                   |  |
| d. | $m\angle ABD + m\angle DBE = m\angle DBE + m\angle EBC$              | Substitution                     |  |
| e. | $m\angle ABD = m\angle EBC$  | Subtraction Property of Equality |  |

# Power Up! Test Practice

Exercises 11 and 12 prepare students for more rigorous thinking needed for assessment.

- 11. This test item requires students to reason abstractly and quantitatively when problem solving.

  Depth of Knowledge DOK1

  Mathematical Practice MP1

  Scoring Rubric

  1 point Students correctly answer the question.
- 12. This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.

  Depth of Knowledge DOK2

  Mathematical Practice MP1

  Scoring Rubric

  2 points Students correctly identify all 4 steps in the proof.

  1 point Students correctly identify 3 of the 4 steps in the proof.

# Power Up! Test Practice

11. In the diagram shown, AE intersects DB at C.

Determine if each of the following conclusions will always be true. Select yes or no.

- a. m∠ACD = m∠BCE
- b. ∠ACD and ∠ECD form a linear pair.
- c. ∠DCE and ∠ACB are vertical angles.
- d. ∠ACB and ∠BCE are complementary angles.
- Select the appropriate reason for each statement of the geometric proof below.

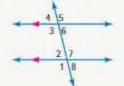
| Substitution | Division Property of Equality                     | Vertical angles<br>have equal measures.      |
|--------------|---|--|
| Given        | Alternate interior angles<br>have equal measures. | Corresponding angles<br>have equal measures. |

Given: two parallel lines cut by a transversal,

$$m\angle 1 = 2x, m\angle 3 = 94$$

Prove: x = 47

Proof:



No

No

No

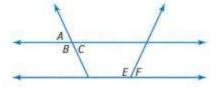
| Statements                                   | Reasons                                   |  |
|--|---|--|
| <b>a.</b> $m \angle 1 = 2x, m \angle 3 = 94$ | Given                                     |  |
| <b>b.</b> <i>m</i> ∠1 = <i>m</i> ∠3          | Corresponding angles have equal measures. |  |
| c. 2x = 94                                   | Substitution                              |  |
| d. x = 47                                    | Division Property of Equality             |  |

### **Spiral Review**

Refer to the diagram. Identify each pair of angles as adjacent, vertical, or neither.

- 13. ∠A and ∠B adjacent
- 14. ∠A and ∠C vertical
- 15. ZC and ZE neither

16. ∠E and ∠F adjacent



# **Inquiry Lab**



WHAT is the relationship among the measures of the angles of a triangle?

Mathematical Practices

Fahd has a metal bracket that is in the shape of an angle that attaches a bag to the frame of a bike. The angle of the bracket measures 35°. Fahd wonders if it will fit into the frame of the bike by the handlebars.

# Hands-On Activity

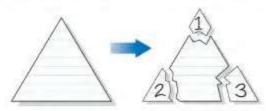
Triangle means three angles. In this Activity you will explore how the three angles of a triangle are related.

Step 1

On a separate piece of paper, draw a triangle like the one shown below.

Step 2

Label the corners 1, 2, and 3. Then tear off each corner.



Step 3

Rearrange the torn pieces so that the corners all meet at one point. Label the tom pieces with 1, 2, and 3,



What does each torn corner represent?

an angle of the triangle

The point where these corners meet is the vertex of another angle. Classify this angle as acute, right, obtuse, or straight.

Explain. Straight; the angle forms a straight line.

## Focus narrowing the scope

Objective Explore the relationship among the angles of a triangle.

# Coherence connecting within and across grades

Students explore the relationship among the angles of a triangle.

#### Next

Students will find missing angle measures in triangles.

# Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 388.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# Launch the Lab

The activity is intended to be used as a whole-group activity.

# Hands-On Activity

Think-Pair-Share Have students work in pairs to complete the Activity. Give students a few minutes to individually read through each step in the activity and think about how they would complete each step. Students then gather in pairs to discuss their responses and progress through completing each step in the activity. Students can then present their responses to the questions in Step 3 to the class. 31, 3, 4, 6.7

(B) Pairs Consult Have students work in pairs to alter the activity so that an obtuse or right triangle is used instead of an equilateral triangle. Have them note whether the type of triangle affects the angle that the three corners form. 31, 3, 4, 6, 7, 8

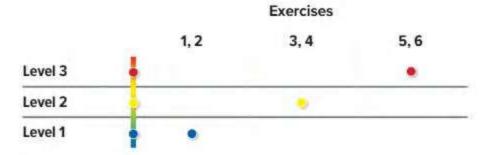
www.jnob-jo.com

# 2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.





Rally Coach Have students work in pairs to complete Exercises 1 and 2. The pair has one piece of paper and pencil. Student 1 completes the first exercise while Student 2 watches and listens, coaches and praises. Students switch roles for the second exercise. 1, 3, 4, 6, 8



BL LA Pairs Check Have students work in pairs to complete Exercises 3 and 4 and trade their answers with another pair of students to check their work. 1, 3, 5, 6, 7, 8



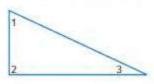
Students should be able to answer "WHAT is the relationship among the measures of the angles of a triangle?" Check for student understanding and provide guidance, if needed.



# Investigate

Work with a partner. Repeat Steps 1–3 of the Activity on the previous page for each of the following triangles. Draw or tape your results in the space provided.

See students' work.







# Collaborate

# **Analyze and Reflect**

 Reason Inductively What is the sum of the measures of the angles for each of your triangles? 180°

Verify your conjecture below by measuring each angle using a protractor.

Exercise 1: 
$$m\angle 1 + m\angle 2 + m\angle 3 = 65^{\circ} + 90^{\circ} + 25^{\circ} = 180^{\circ}$$
  
Exercise 2:  $m\angle 1 + m\angle 2 + m\angle 3 = 140^{\circ} + 15^{\circ} + 25^{\circ} = 180^{\circ}$ 

Justify Conclusions Refer to the bicycle problem on the previous page. Will the bracket fit exactly into Fahd's bike? Explain. No; 87° + 55° + 35° = 177°, so the bracket will be too small.



## Create

- 5. Ose Math Tools Find a real-world example of a triangle. Measure the angles of the triangle. What is the sum of the measures of the angles? Does your answer support your findings in this Inquiry Lab? Explain. See students' work. Answers should be close to 180, but likely not exact. Reasons may include the triangle was too large to measure accurately or the lines weren't exactly straight.
- 6. Inquiry WHAT is the relationship among the measures of the angles of a triangle? The sum of the measures of a triangle is 180°.

And the second second second second

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# Lesson 3

# **Angles of Triangles**



# **Real-World Link**

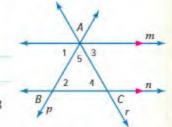
Eiman and Asma are building a bridge out of toothpicks for a science competition. Asma thinks the sides should be constructed using triangles. Use the activity to find the sum of the measures of the angles in a triangle.



Lines m and n are parallel. Lines p and r are transversals that intersect at point A.

1. What is true about the measures of ∠1 and ∠2? Explain.

They are equal because they are alternate interior angles.



What is true about the measures of ∠3 and Z4? Explain.

They are equal because they are alternate interior angles.

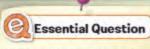
- What kind of angle is formed by ∠1, ∠5, and ∠3? Write an equation representing the relationship between the 3 angles. straight angle;  $m\angle 1 + m\angle 5 + m\angle 3 = 180$
- 4. Use the information from Exercises 1, 2, and 3 to draw a conclusion about the sum of the measures of the angles of △ABC. Explain your reasoning. The sum of the measures of the angles in  $\triangle ABC$  is 180°. Since  $m \angle 1 = m \angle 2$ ,

 $m \angle 3 = m \angle 4$ , and  $m \angle 1 + m \angle 5 + m \angle 3 = 180^{\circ}$ ,

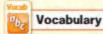
by substitution,  $m\angle 2 + m\angle 5 + m\angle 4 = 180^\circ$ .

Which Mathematical Practices did you use? Shade the circle(s) that applies.

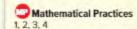
- 1) Persevere with Problems
- 2 Reason Abstractly
- 3 Construct an Argument
- Model with Mathematics www.jnob-jo.com
- (5) Use Math Tools
- 6 Attend to Precision
- Make Use of Structure
- (8) Use Repeated Reasoning



HOW can algebraic concepts be applied to geometry?



triangle interior angle exterior angle remote interior angles



### Focus narrowing the scope

Objective Find missing angle measures in triangles.

# Coherence connecting within and across grades

### Previous

Students explored the relationship of angle measures in triangles.

#### Now

Students will find missing angle measures in triangles.

#### Next

Students will examine the relationship among angles of regular polygons.

# Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 393.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

# Launch the Lesson

### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

(IA) Circle the Sage Poll students to see who has a solid understanding of angles formed when parallel lines are cut by a transversal. Have those students (the sages) spread out around the room. Place remaining students in teams. Have students report to different sages, with no two team members going to the same sage, if possible. Have the sages lead the work for the Real-World Link. Then have students report back to their teams and discuss any differences in solutions. 21, 2, 3, 4, 5, 6, 7

# **2** Teach the Concept

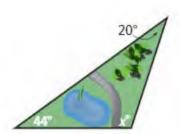
Ask the scaffolded questions for each example to differentiate instruction.

# **Examples**

- 1. Find missing angle measures in triangles.
- What is the sum of the measures of the interior angles of a triangle? 180°
  - What are the known angle measures? 55° and 90°
- What equation could be used to determine the missing angle measure?  $x^{\circ} + 55^{\circ} + 90^{\circ} = 180^{\circ}$ 
  - What is the value of x? 35
- If the measure of one known angle in a triangle is 90°, what is the sum of the other two angle measures? 90°
  - What other equation could you use to solve for x in this example? 55° + x° = 90°

### **Need Another Example?**

The city park is in the shape of a triangle. Find the value of x. 116



### 2. Use ratios to find angle measures.

- What expression could be used to represent the measure of the first angle? x the second angle? 4x the third angle? 5x
- What equation could be used to find the value of x? x + 4x + 5x = 180
  - Why can we write x + 4x + 5x as 10x? They are like terms.
- How can we check our work? Sample answer: Check that the measures 18°, 72°, and 90° are in the ratio 1:4:5.

### **Need Another Example?**

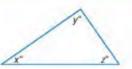
The measures of the angles of triangle *DEF* are in the ratio 1:2:3. What are the measures of the angles? 30°, 60°, and 90°

### Key Concept

# Angle Sum of a Triangle

Work Zone

The sum of the measures Mode of the interior angles of a triangle is 180°.



Symbols  $x + y + z = 180^{\circ}$ 

A **triangle** is formed by three line segments that intersect only at their endpoints. A point where the segments intersect is a vertex. The angle formed by the segments that lies inside the triangle is an **interior angle**.



Words

### Example

 Find the value of x in the Antigua and Barbuda flag.

$$x + 55 + 90 =$$
 180 Write the equation.  
 $x + 145 =$  180 Simplify.





The value of x is 35.



### Got It? Do this problem to find out.

a. In  $\triangle XYZ$ , if  $m\angle X = 72^{\circ}$  and  $m\angle Y = 74^{\circ}$ , what is  $m\angle Z$ ?



Segments

AB is read as segment AB.

below are AB, AC, and BC.

so the sides of the triangle

## Example

2. The measures of the angles of △ABC are in the ratio 1:4:5.
What are the measures of the angles?

Let x represent the measure of angle A.

Then 4x and 5x represent angle B and angle C.

$$x + 4x + 5x = 180$$
 Write the equation.  
 $10x = 180$  Collect like terms.  
 $x = 18$  Division Property of Equality

Since x = 18, 4x = 4(18) or 72, and 5x = 5(18) or 90. The measures of the angles are  $18^{\circ}$ , 72°, and 90°.

Сарупун 🗈 Мойгимни Едис

#### Got 14? Do this problem to find out.

b. The measures of the angles of △LMN are in the ratio 2:4:6. What are the measures of the angles?

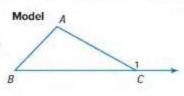


h 30°, 60°, 90°

Key Concept

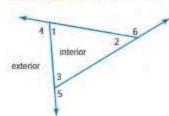
# **Exterior Angles of a Triangle**

Words The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.



 $m\angle A + m\angle B = m\angle 1$ Symbols

In addition to its three interior angles, a triangle can have an exterior angle formed by one side of the triangle and the extension of the adjacent side. Each exterior angle of the triangle has two remote interior angles that are not adjacent to the exterior angle.

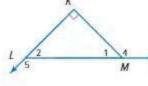


4 is an exterior angle of the triangle. Its two remote interior angles are \( \alpha \) and \( \alpha \).

 $m \angle 4 = m \angle 2 + m \angle 3$ 

### Example

3. Suppose  $m \angle 4 = 135^\circ$ . Find the measure of /2.



Angle 4 is an exterior angle. Its two remote interior angles are Z2 and ZLKM.

are 
$$\angle 2$$
 and  $\angle LKM$ .  
 $m\angle 2 + m\angle LKM = m\angle 4$  Write the equal

$$m\angle 2 = x^*, m\angle LKM = 90^*, m\angle 4 = 135^*$$

 $x + 90^{\circ} = 135^{\circ}$ 

Subtraction Property of Equality

So, the  $m\angle 2 = 45^\circ$ .

#### and Reflect

Measure 12, 13, and 14 to verify that m/2 + m23 = m24. Repeat the process for exterior angles 5 and 6. What is true about m/s and m/67

 $m \angle 5 = m \angle 1 + m \angle 2$  $m \angle 6 = m \angle 1 + m \angle 3$ 

# Example

Use exterior angles to find a missing angle.



- What is the sum of the interior angle measures of a triangle? 180°
  - · What kind of angle is angle 4? exterior
  - What are the two remote interior angles for angle 4? ∠2 and ∠LKM
  - What is the measure of \( \alpha \) KM? 90°



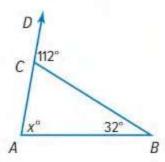
- What is true about an exterior angle and its two remote interior angles? The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.
  - What equation could be used to find the measure of ∠2? Sample answer:  $m \angle 2 + 90 = 135$



BD • Explain another way to find the measure of \( \alpha \). Sample answer: ∠4 and ∠1 form a straight line, which means that the sum of their angle measures is 180°. 135° +  $m\angle 1 = 180^\circ$ , so  $m\angle 1 = 45^\circ$ . The total degrees in a triangle are  $180^{\circ}$ ,  $45^{\circ} + 90^{\circ} + m \angle 2 = 180^{\circ}$ , so  $m \angle 2 = 45^{\circ}$ .

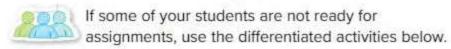
### Need Another Example?

Find the value of x in the triangle. 80



# **Guided Practice**

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.



(I) LA **Team-Pair-Solo** Have students complete Exercise 1 in a team of four students, then complete Exercises 2 and 3 in pairs. Have them complete Exercises 4 and 5 on their own and then compare answers with their original team. (2), 3, 5, 6, 7

Pairs Research Have students research or look for a real-world example of a triangle formed by transversals crossing parallel lines (a map, for example). Have them explain how known angle measures can help to find unknown angle measures in the triangle. 1, 2, 3, 4, 5, 6, 7

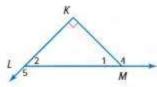
# Watch Out!

**Common Error** When working with angle ratios, students may solve for *x* and leave out calculating the measures of the three angles. Remind students to multiply *x* by each of the coefficients in the original equation to find the measures of each of the three angles. Then have them find the sum of the angles to check their work.

Mary Joseph

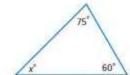
#### Got 14? Do this problem to find out.

c. Refer to the figure at the right. Suppose m∠5 = 147°. Find m∠1.



## **Guided Practice**

Find the value of x in the triangle. (Example 1)



 What is the value of x in the sail of the sailboat? (Example 1)



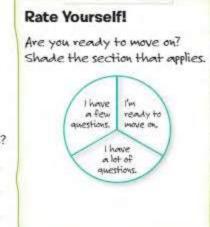
 The measures of the angles of \(\triangle LMN\) are in the ratio 1:2:5. What are the measures of the angles? (Example 2)

22.5°, 45°, 112.5°

measure.

4. Find the value of x in the triangle. (Example 3) 31





a Machala Warry Capyright © VicGray-48 Sci

WW392. JANG 5 JOING COM the Pythagorean Theorem

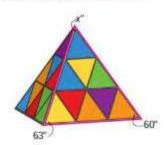
# Independent Practice

1. The top of a kite is shown below. What is the value of x? (Example 1) 55



2. A popular toy puzzle is shown below. What is the value of x? (Example 1) 57

My Homework



The measures of the angles of △RST are in the ratio 2:4:9. What are the measures of

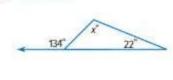
the angles? (Example 2) 24°, 48°, 108°

 The measures of the angles of △XYZ are in the ratio 3:3:6. What are the measures of the

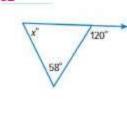
angles? (Example 2) 45°, 45°, 90°

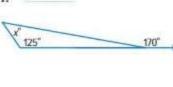
Find the value of x in each triangle. (Example 3)

5. 112



6. 62





8. In  $\triangle ABC$  the measure of angle A is 2x + 3, the measure of angle B is 4x + 2, and the measure of angle C is 2x - 1. What are the measures of

the angles?  $m\angle A = 47^{\circ}$ ,  $m\angle B = 90^{\circ}$ ,  $m\angle C = 43^{\circ}$ 

Programme Reason Abstractly What is the measure of the third angle of a triangle if one angle measures 25° and the second angle measures 50°?

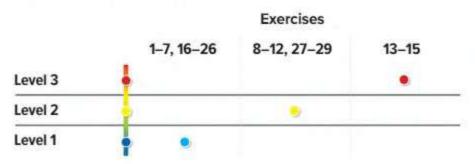
# ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE **Practice and Apply**

### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                               |
|---------------------------------|-------------------|-------------------------------|
| AL                              | Approaching Level | 1–7, 9, 11, 14, 15, 28, 29    |
| OL                              | On Level          | 1–7 odd, 8–12, 14, 15, 28, 29 |
| B                               | Beyond Level      | 8–15, 28, 29                  |

| MATHEMATICAL PRACTICES  |             |  |
|---|-------------|--|
| Emphasis On   | Exercise(s) |  |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 13          |  |
| 2 Reason abstractly and quantitatively.                                       | 9           |  |
| 3 Construct viable arguments and critique the reasoning of others.            | 14, 15, 27  |  |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

# TICKET Out the Door

Draw a triangle on the board and label the measures of two of the three angles. Have students describe the steps they would take to find the missing angle measure.

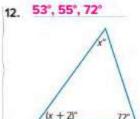
See students' work.

Find the measures of the angles in each triangle.

10. 120°, 30°, 30°



2x"



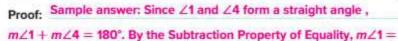


#### H.O.T. Problems Higher Order Thinking

13. Persevere with Problems Use the figure at the right to informally prove that an exterior angle of a triangle is equal to the sum of its two remote interior angles.

Given: △ABC; ∠1 is an exterior angle.

Prove:  $m\angle 1 = m\angle 2 + m\angle 3$ 



180 -  $m \angle 4$ . Since ABC is a triangle,  $m \angle 2 + m \angle 3 + m \angle 4 = 180$ . By the

Subtraction Property of Equality,  $m\angle 2 + m\angle 3 = 180 - m\angle 4$ . So by

substitution,  $m\angle 2 + m\angle 3 = m\angle 1$ .

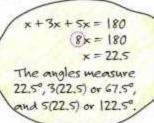
A 1 1 C

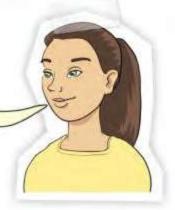
 Tind the Error Nisreen is finding the measures of the angles in a triangle that have the ratio 1:3:5.
 Circle her mistake and correct it.

$$x = 20$$

The angles measure 20°, 60°,

and 100°.





 Justify Conclusions Make a conjecture about the sum of the interior angles of a quadrilateral. Justify your reasoning.

Sample answer: The sum is 360°. Drawing the diagonal of a quadrilateral

forms two triangles. So, the sum of the interior angles is 2(180°), or 360°.

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My Homework

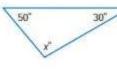
# **Extra Practice**

Find the value of x in each triangle with the given angle measures.

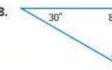
16.







100



65

19. 70°, 60°, x° 50

20. x°, 60°, 25° 95

21. x°, 35°, 25° 120

22. The measures of the angles of \( \triangle DEF \) are in the ratio 2:4:4. What are the measures of the angles?

36°, 72°, 72°

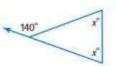
23. The measures of the angles of  $\triangle XYZ$  are in the ratio 4:5:6. What are the measures of the angles?

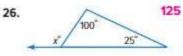
48°, 60°, 72°

Copy and Solve Find the value of x in each triangle. Show your work on a separate sheet of paper.

24.







27. Reason Inductively Apply what you know about angles and lines to find the values of x and y in the figure at the right.



$$y = 50$$



# Power Up! Test Practice

Exercises 28 and 29 prepare students for more rigorous thinking needed when taking assessment.

| 28. | This test item requires students to analyze and solve complex real- |  |
|-----|---|--|
|     | world problems through the use of mathematical tools and models.    |  |

| world problems through | the ase of mathematical tools and models.   |
|------------------------|---|
| Depth of Knowledge     | DOK2  |
| Mathematical Practices | MP1, MP4  |
| Scoring Rubric         |   |
| 2 points               | Students correctly place all values to complete the model and find the value of $x$ . |
| 1 point                | Students correctly place all values to complete the model OR find the value of x.     |

|  | This test item requires students to reason abstractly and quantitatively |  |
|--|--|--|
|  | when problem solving.  |  |

| when problem solving. |   |  |  |
|-----------------------|---|--|--|
| Depth of Knowledge    | DOK1                                    |  |  |
| Mathematical Practice | MP1                                     |  |  |
| Scoring Rubric        |   |  |  |
| 1 point               | Students correctly answer the question. |  |  |

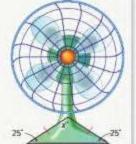
# Power Up! Test Practice

28. When viewed from the front, the base of an upright fan has a triangular face with the angle measures shown. Select the correct values to complete the model that could be used to find the value of x.

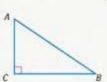


| What is the | value of x? | 130 |  |
|-------------|-------------|-----|--|

| x  | 65  |
|----|-----|
| 2  | 90  |
| 25 | 180 |



- 29. Which of the following statements are always true about the relationship between the measures of angles A and B of the right triangle shown? Select all that apply.
  - They are equivalent.
  - They are supplementary.
  - They are acute.
  - They are complementary.



## **Spiral Review**

30. The street maintenance vehicles for the city of Badr cannot safely make turns less than 70°. Should the proposed site of the new maintenance garage at the northeast corner of Park and Main be approved? Explain.



Yes; the two corners at the intersection have measures of 108° and

72°. Therefore it is within the safety limit.

31.  $\angle A$  and  $\angle B$  are complementary, and the measure of  $\angle A$  is 39°. What is the measure of  $\angle B$ ?

Solve each equation.

33. 
$$90 + 90 + (2x + 4) + (3x - 29) = 360$$
 41

Lesson 4

## **Polygons and Angles**

### **Vocabulary Start-Up**



A polygon is a simple closed figure formed by three or more line segments. The segments intersect only at their endpoints.



A map of the United States is shown. List the states that are in the shape of a polygon. Then list the number of segments that form the polygon. Sample answers are given. Some students may interpret some state borders as straight segments.

| State             | Number of Segments |
|-------------------|--------------------|
| <b>New Mexico</b> | 8                  |
| Utah              | 6                  |
| Colorado          | 4                  |
| North Dakota      | 4                  |
| Wyoming           | 4                  |

Which Mathematical Practices did you use? Shade the circle(s) that applies. (5) Use Math Tools

- 1 Persevere with Problems
- (2) Reason Abstractiv
- 3 Construct an Argument
- (4) Model with Mathematics www.jnob-jo.com
- 6 Attend to Precision
- (7) Make Use of Structure
- (8) Use Repeated Reasoning

# **Essential Question**

HOW can algebraic concepts be applied to geometry?

## Vocabulary

polygon equiangular regular polygon

Mathematical Practices

## Focus narrowing the scope

Objective Find the sum of the angle measures of a polygon and the measure of one interior angle of a regular polygon.

#### Coherence connecting within and across grades

#### Previous

#### Students used properties of triangles to find missing angle measures.

#### Now

Students find the measures of angles in polygons.

#### Next

Students use angle measures to show congruence and similarity

#### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 401.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

## **Launch the Lesson**

#### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Team Consult Organize students into three teams. Assign each team a two-dimensional figure to find in a picture: 5-sided, 6-sided, or 8-sided. You can provide the picture or they can research a picture using the Internet. Tell them that the figures should not have any curved sides. Have them present their pictures with the figures outlined to the class. 7

### Alternate Strategy

Ask students to give a few examples of states that are not polygons. Then have them justify their response. 23, 6

# **Teach the Concept**

Ask the scaffolded questions for each example to differentiate instruction.

## Example

- 1. Find the sum of the interior angle measures of a polygon.
- How many years are in a decade? 10
  - Decagon contains the same root word as decade. How many sides are in a decagon? 10
  - Draw a decagon, then draw all of the diagonals from one vertex. How many triangles did you create? 8
  - What is the sum of the angle measures for each triangle? 180°
  - · What is the sum of the interior angle measures of eight triangles? Explain how you found the sum. 1,440°; Multiply 180° by 8.
  - What is the sum of the angle measures of a decagon? 1,440°
- What equation can be used to find the sum of the interior angles measures of a polygon with n sides? S = (n-2)180
  - · What equation can be used to find the sum of the interior angle measures of a decagon? S = (10 - 2)180
- In the equation to find the sum of the measures of the interior angles of a polygon, what does n-2represent? the number of triangles you get when you draw all of the diagonals from one vertex
  - Why does the equation use n − 2 instead of just n? The number of triangles that a polygon can be separated into is not equal to the number of sides, n. It is equal to 2 less than the number of sides.

#### Need Another Example?

Find the sum of the measures of the interior angles of a 13-gon. 1,980°

#### Key Concept

Work Zone

### Interior Angle Sum of a Polygon

Words

The sum of the measures of the interior angles of a polygon is (n-2)180, where n represents the number of sides.

S = (n - 2)180Symbols

You can use the sum of the angle measures of a triangle to find the sum of the interior angle measures of various polygons. A polygon that is equilateral (all sides are the same length) and equiangular (all angles have the same measure) is called a regular polygon.

| Number<br>of Sides | Sketch of<br>Figure | Number of<br>Triangles | Sum of Angle<br>Measures |
|--------------------|---------------------|------------------------|--------------------------|
| 3                  | $\triangle$         | 1                      | 1(180") = 180"           |
| 4                  |                     | 2                      | 2(180°) = 360°           |
| 5                  |                     | 3                      | 3(180°) = 540°           |
| 6                  |                     | 4                      | 4(180") = 720"           |

#### **Everyday Use**

Deca- a prefix meaning ten, as in decade

#### Math Use

Decagon a polygon with ten sides

#### Example

1. Find the sum of the measures of the interior angles of a decagon.

S = (n - 2) 180

Write an equation

S = (10 - 2)180

A decagon has 10 sides. Replace n with 10.

S = (8)180 or 1,440

The sum of the measures of the interior angles of a decagon is 1,440°.

Got It? Do these problems to find out.

Find the sum of the interior angle measures of each polygon.

- a. hexagon
- b. octagon
- c. 15-gon

b. 1,080°

a. 720°

c. 2,340°

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#### Example

Each chamber of a bee honeycomb is a regular hexagon. Find the measure of an interior angle of a regular hexagon.

Step 1

Find the sum of the measures of the angles.

S = (n - 2)180

Write an equation.

S = (6 - 2)180

Replace n with 6.

S = (4)180 or 720

The sum of the measures of the interior angles is 720°.

Step 2

Divide 720 by 6, the number of interior angles, to find the measure of one interior angle. So, the measure of one interior angle of a regular hexagon is 720° ÷ 6 or 120°.

Got It? Do these problems to find out.

Find the measure of one interior angle in each regular polygon. Round to the nearest tenth if necessary.

d. octagon

- e. heptagon
- f. 20-gon

Model

a. 135°

e. 128.6°

£ 162"

## Exterior Angles of a Polygon

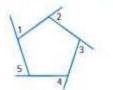
Words

In a polygon, the sum of the measures of the exterior angles, one at each vertex,

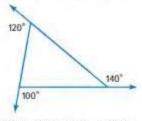
Symbols

 $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 +$ 

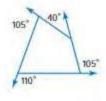
 $m \angle 5 = 360^{\circ}$ 



Regardless of the number of sides in a polygon, the sum of the exterior angle measures is equal to 360°.



 $120 + 100 + 140 = 360^{\circ}$ 



 $105 + 110 + 105 + 40 = 360^{\circ}$ 

## Key Concept



### and Reflect

Draw another quadrilateral and a pentagon. Extend the sides to show the exterior angles. Then find the sum of each figure's exterior angle measures.

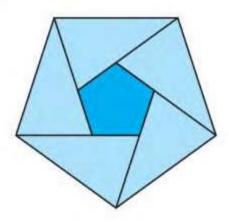
See students' work for drawings. 360°; 360°

## Example

- 2. Find the measure of one interior angle of a regular polygon.
- What is a regular polygon? a polygon with equal side lengths and equal angle measures
  - How many sides are in a hexagon?
- What equation can be used to find the sum of the interior angle measures of a hexagon? S = (6 - 2)180
  - What is the sum of the interior angle measures of a hexagon? 720°
  - · How can you find the measure of one of the interior angles of a regular hexagon? Divide 720 by 6.
- Bl Write an equation that can be used to find the measure of one angle M of a regular polygon with n  $M = \frac{(n-2) \cdot 180}{n}$

#### Need Another Example?

A designer is creating a new logo for a bank. The logo consists of a regular pentagon surrounded by isosceles triangles. Find the measure of an interior angle of a regular pentagon. 108°



## Example

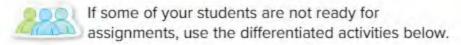
- Find the measure of one exterior angle in a regular polygon.
- What is the sum of the measures of the exterior angles of any polygon? 360°
  - · How many exterior angles are in a hexagon? 6
  - How would you find the measure of one exterior angle of a regular hexagon? Find 360 ÷ 6.
- What is the measure of each exterior angle? 60°
  - Write an equation to find the measure of one exterior angle m in a regular polygon with n sides.  $m = \frac{360}{n}$
- Can you use this method to find the measure of an exterior angle of a polygon that is not regular?
   Explain. No, the exterior angles of a polygon that is not regular are not all the same.

#### Need Another Example?

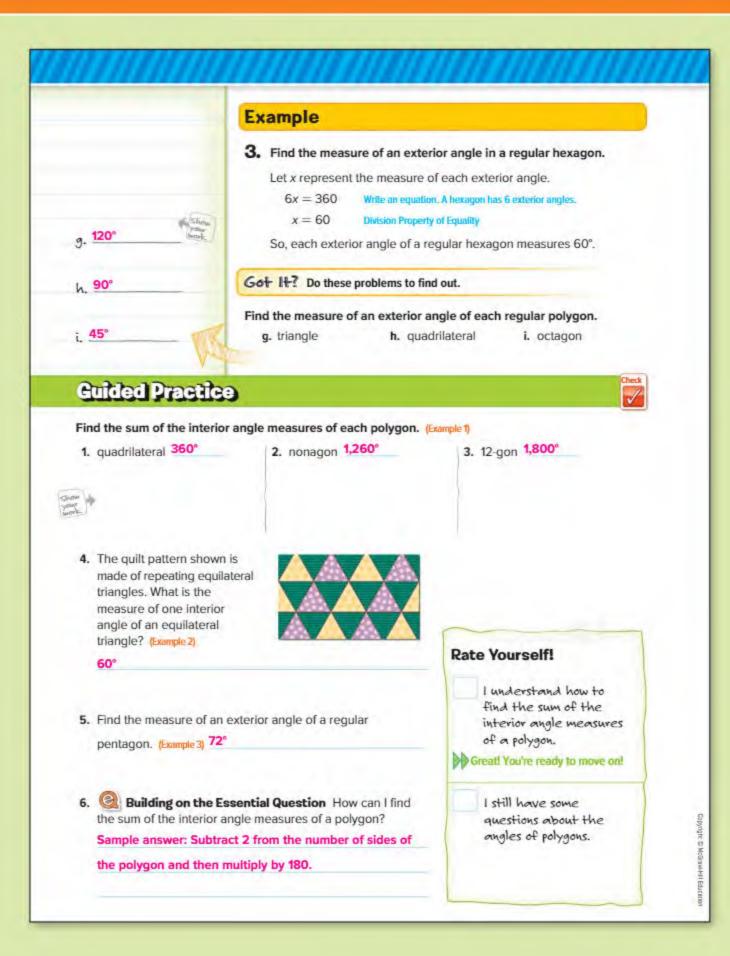
Find the measure of an exterior angle in a regular 30-gon. 12°

### **Guided Practice**

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.



- Pairs Check Have students work in pairs. Give each pair an index card. On the card, they should list the names of polygons and corresponding number of sides from triangle to decagon. Have students refer to their cards when completing Exercises 1–5. Then have them compare their answers with another pair of students.
- about tessellations in Exercises 8 and 9. Have students work in pairs to research M.C. Escher and tessellations. They should write a paragraph about the attributes of polygon tessellations. Give them sheets of colored paper or allow them to work on a computer to create their own tessellations. Display the tessellations throughout the room.



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## Independent Practice

Find the sum of the interior angle measures of each polygon. (Example 1)

1. pentagon 540°

2. 11-gon 1,620°

13-gon 1,980°

My Homework



4. The soccer ball at the right consists of repeating regular pentagons and hexagons. Find the measure of one interior angle of a pentagon.

(Example 2) 108°



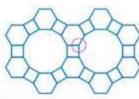
Find the measure of an exterior angle of each regular polygon. (Example 3)

decagon 36°

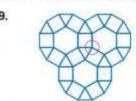
6. 20-gon 18°

7. 15-gon 24°

A tessellation is a repetitive pattern of polygons that fit together without overlapping and without gaps between them. For each tessellation, find the measure of each angle at the circled vertex. Then find the sum of the angles.



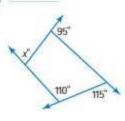
90°, 120°, 150°; 360°



60°, 90°, 90°, 120°; 360°

Find the value of x in each figure.

10. 80



ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

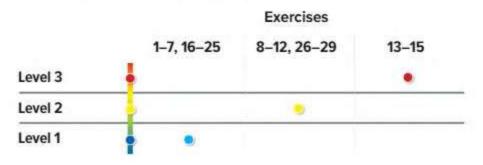
# **Practice and Apply**

#### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

#### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



#### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

|    | Differenti        | ated Homework Options         |
|----|-------------------|-------------------------------|
| AD | Approaching Level | 1–7, 9, 11, 14, 15, 28, 29    |
| 01 | On Level          | 1–7 odd, 8–12, 14, 15, 28, 29 |
| BL | Beyond Level      | 8–15, 28, 29                  |

| MATHEMATICAL PRACTICES  |             |
|---|-------------|
| Emphasis On   | Exercise(s) |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 13          |
| 3 Construct viable arguments and critique the reasoning of others.            | 14, 15, 27  |
| 4 Model with mathematics.   | 12          |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

## TICKET Out the Door

Have students explain how to find the sum of the interior angles of a polygon when they know the number of sides of the polygon. See students' work.

 Model with Mathematics Refer to the graphic novel frame below. Find the measures of the two missing angles using properties of quadrilaterals and parallel lines; 130° and 28°





#### H.O.T. Problems Higher Order Thinking

13. Persevere with Problems How many sides does a regular polygon have if the measure of an interior angle is 160°? Justify your answer.

18; 
$$\frac{(n-2)180}{n} = 160$$

$$(n-2)180 = 160n \text{ Multiplication}$$
Property of Equality
$$180n - 360 = 160n \text{ Distributive Property}$$

$$20n = 360 \text{ Properties of Equality}$$

$$n = 18 \text{ Division Property}$$
of Equality

- 14. Peason Inductively If the number of sides of a polygon increases by 1, what happens to the sum of the measures of the interior angles? It increases by 180°.
- 15. Reason Inductively Jamal drew a regular polygon and measured one of its interior angles. Explain why it is impossible for his angle measure to be 145°. Regular decagons have equal angles measuring 144° and regular 11-sided polygons have angles measuring 147.27°. 145° is between these two values so it cannot be the interior angle measure of a regular polygon.

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My Homework

## **Extra Practice**

Find the sum of the interior angle measures of each polygon.

**16.** heptagon **900°** 

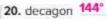
$$5 = (4 - 2)180$$

$$\leq = (7 - 2)180$$

$$s = 5 \cdot 180$$
  
 $s = 900$ 

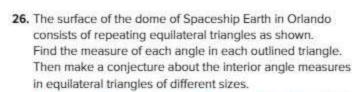
Find the measure of one interior angle in each regular polygon. Round to the nearest tenth if necessary.

19. nonagon 140°



Find the measure of an exterior angle of each regular polygon.

23. nonagon 40°



The measure of each angle in each outlined triangle is

60°. If a triangle is equilateral, the measure of each angle

will be 60" regardless of the size of the triangle.



27. Justify Conclusions What is the sum of the interior angles of nonregular hexagons? Explain your reasoning to a classmate.

Sample answer: The sum of the interior angles will still be 720° because

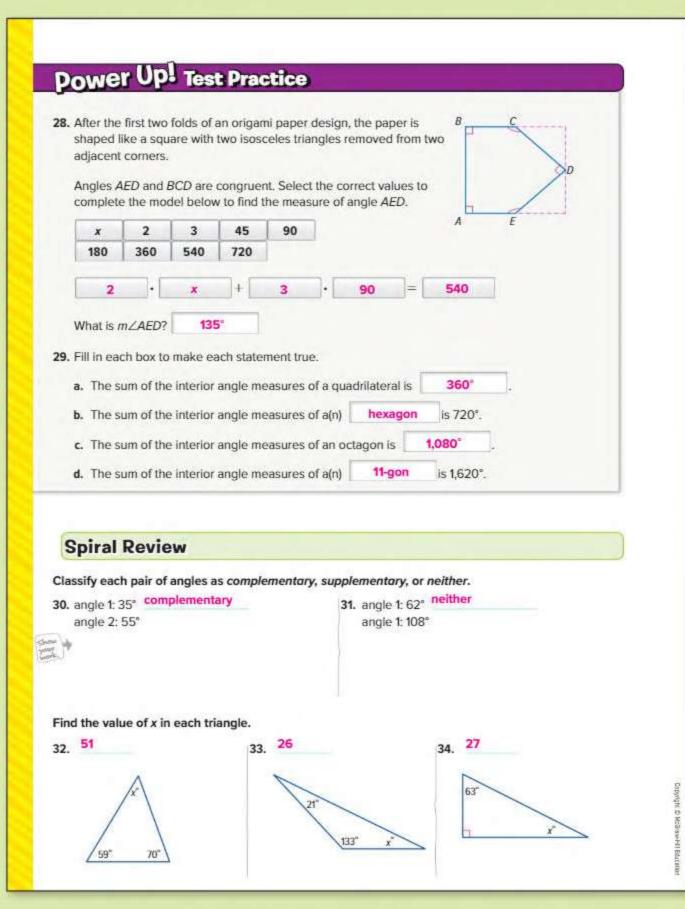
even though the figures are not regular, they are still hexagons.

# Power Up! Test Practice

Exercises 28 and 29 prepare students for more rigorous thinking needed when taking assessment.

| 28. |                        | udents to analyze and solve complex real-<br>the use of mathematical tools and models. |
|-----|------------------------|--|
|     | Depth of Knowledge     | DOK2   |
|     | Mathematical Practices | MP1, MP4   |
|     | Scoring Rubric         |  |
|     | 2 points               | Students correctly complete the model and find the measure of angle AED.               |
|     | 1 point                | Students correctly complete the model OR find the measure of angle AED.                |

| 29. | This test item requires s<br>quantitatively when pro | tudents to reason abstractly and blem solving.                     |
|-----|--|--|
|     | Depth of Knowledge                                   | DOK1   |
|     | Mathematical Practice                                | MP1  |
|     | Scoring Rubric                                       |  |
|     | 2 points   | Students correctly answer all four parts of the question.          |
|     | 1 point  | Students correctly answer three of the four parts of the question. |



## Problem-Solving Investigation Look for a Pattern

Mathematical Practices

#### Case #1 Spider Web

An activity in a low ropes course creates the inside of a spider web using string. The group members form a polygon. The strings stretch from each person to every nonadjacent member of the figure. Saeed's group has 20 members.

How many strings will saeed hold in the web?



#### Understand What are the facts?

- There are 20 group members that form a polygon.
- A string stretches from each person to every nonadjacent group member.



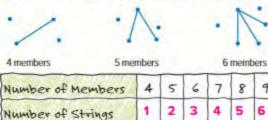
#### Plan What is your strategy to solve this problem?

Drawing a 20-gon would be difficult. Begin with a group of four members and look for a pattern. Then make a table to find the pattern.



#### **Solve** How can you apply the strategy?

Draw figures using four, five, and six members. Draw the diagonals from one member to show number of strings. Some figures are drawn for you.



How many strings will Saeed hold? 17 strings



#### Check Does the answer make sense?

Draw a 20-gon and count the number of diagonals from one vertex.

### Analyze the Strategy

Identify Repeated Reasoning How would the pattern change if Saeed was looking for the total number of strings from every person in the web?

Sample answer: If n represented the number of people, they would need

www.jnob-jo.co $m_2^{n(n-3)}$  strings.

#### Focus narrowing the scope

Objective Solve problems by using the look for a pattern strategy. This lesson emphasizes Mathematical Practice 8 Identify Repeated Reasoning.

Look for a Pattern Looking for a pattern is a good strategy for solving a variety of problems. When working with patterns, it is sometimes helpful to organize information in a table.

### Coherence connecting within and across grades

#### Now

Students solve non-routine problems.

#### Next

Students will apply the look for a pattern strategy to analyze the relationship between the side lengths of a right.

#### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 407.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

## **Launch the Lesson**

The problems on pages 405 and 406 are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance. The problem on page 405 walks students through the solution, while the problem on page 406 asks students to come up with their own solutions.

### Case #1 Spider Web

BD Have students extend the problem by answering the following question.

#### Ask:

 Write an expression that can be used to find the number of strings Chen will hold for n members. n-3

### Case #2 Follow the Bouncing Ball

Rally Coach Have students work in pairs to solve the problem. Have Student A complete the first step, speaking out loud, while Student B listens carefully, coaches, and praises. Next, have Student B complete the second step while Student A listens carefully, coaches, and praises. Partners take turns until they have solved the problem. 1, 2, 3, 4, 5, 6, 7, 8

BL (A) Pairs Discussion Have students work in pairs to answer the following extension question. (2) 1, 2, 3, 4, 5, 6, 7, 8

#### Ask:

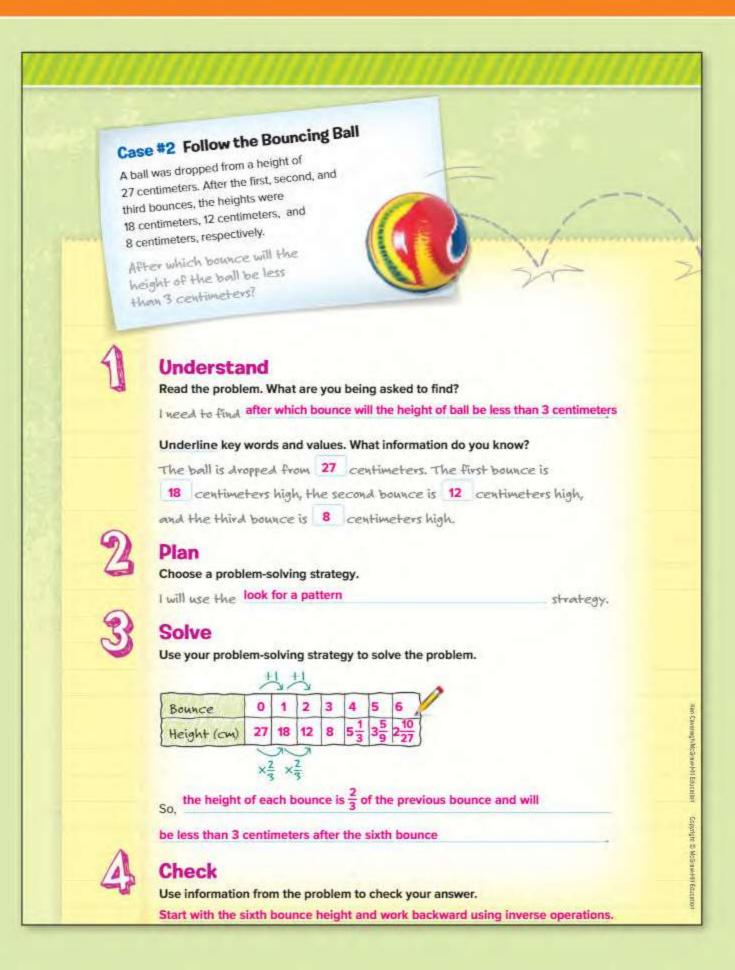
How could you solve this problem a different way? Sample answer: The height of each bounce is <sup>2</sup>/<sub>3</sub> the height of the previous bounce. I could draw a diagram to show the height of each bounce after the third bounce.

#### **Need Another Example?**

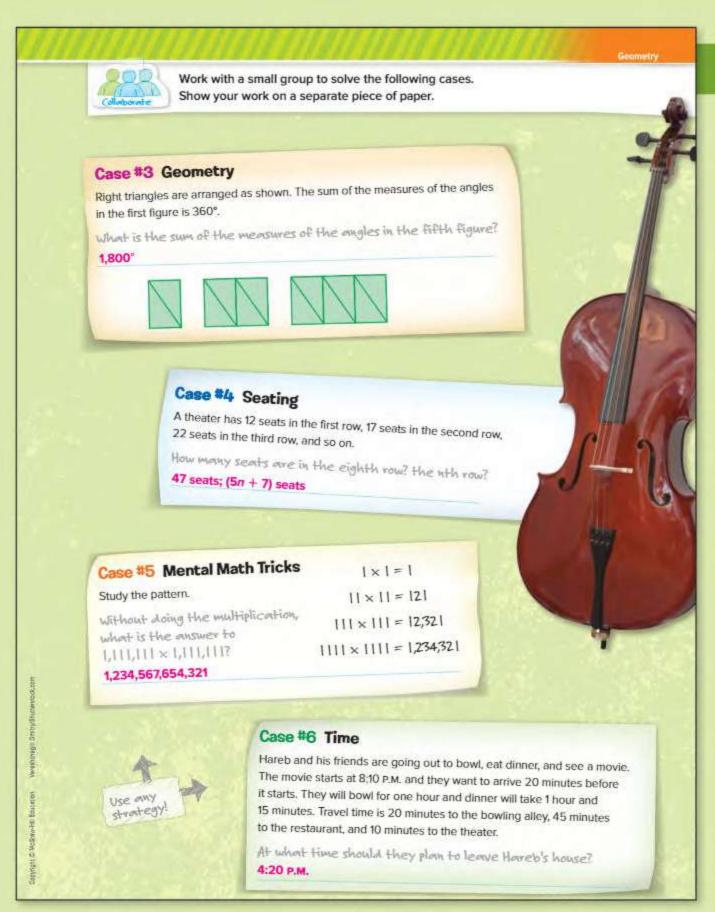
All new showerheads are required to restrict their water flow. Determine how long it will take Hidaya to use 18 liters of water.

| Number of Minutes | 1    | 2 | 3    | 4  |
|-------------------|------|---|------|----|
| Number of Liters  | 21/2 | 5 | 71/2 | 10 |

Each minute, she uses  $2\frac{1}{2}$  liters of water. She will use 18 liters of water between the  $7^{th}$  and  $8^{th}$  minute.



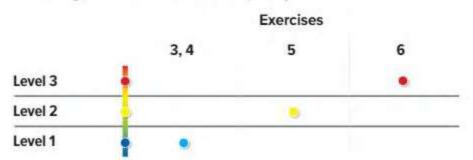
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## ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE Collaborate

#### **Levels of Complexity**

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Roundrobin Have students work in pairs to extend the pattern in Case 4 to find the number of seats in the fourth row, fifth row, sixth row, and so on, until they reach the eighth row. If they are struggling with the number of seats in the nth row, provide them with the beginning of the expression,

 $\underline{\hspace{1cm}}$  n + 7. Have them find the coefficient of n.  $\mathfrak{D}_1$ , 2, 3, 4, 5, 6, 7, 8

BD Trade-a-Problem Have students create their own real-world problem with a pattern. Then have them trade their problems with each other, solve, and compare solutions. If the solutions do not agree, students should work together to find the errors. 31, 2, 3, 4, 5, 6, 7, 8

## Mid-Chapter Check

If students have trouble with Exercises 1–10, they may need help with the following concepts.

| Concept                                    | Exercise(s) |
|--|-------------|
| parallel lines and transversals (Lesson 1) | 1, 3–8, 10  |
| polygons and angles (Lesson 4)             | 2, 9        |

## **Vocabulary Activity**

Numbered Heads Together Have students work in groups of 4 to complete Exercise 1. Each student is assigned a number from 1 to 4. Each student is also assigned one of the angle relationships, such as corresponding angles. Students are responsible to ensure that each group member understands the meaning of their angle relationship. Students should ask each other for clarification and assistance, as needed. Call on one numbered student to define their angle relationship for the class. Then have the groups complete Exercise 2. 1, 4, 6, 7

#### **Alternate Strategies**

May be students use highlighters or colored pencils to identify examples of angle relationships shown on the diagram in Exercise 1.

Have students write equations that represent the angle relationships shown on the diagram in Exercise 1.

## Mid-Chapter Check

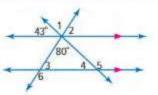
### **Vocabulary Check**



- 1. Name a pair of angles for each of the following. (Lesson 1) Sample answers: 1a-1d.
  - a. corresponding angles 2 and 6
  - b. alternate interior angles 3 and 5
  - c. vertical angles 4 and 2
  - d. alternate exterior angles 1 and 7
- List two attributes of regular polygons. (Lesson 4)
  all of the sides are the same length
  all of the angles have the same measure

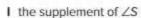
#### Skills Check and Problem Solving

Refer to the figure at the right. Find the missing measure of each angle. (Lessons 1 and 3)

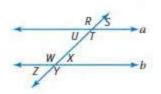


- 3. m∠1 = 80°
- 4. m∠2 = 57°
- 5. m23 = 57°

- 6. m/4 = 43°
- 7. m∠5 = 137°
- 8. m/6 = 123°
- A building is in the shape of a regular polygon with five sides. What is the measure of one of the interior angles of the building? (Lesson 4) 108°
- Ose Math Tools In the figure, line a is parallel to line b. Which of the following are equal to the measure of ∠T? (Lesson 1) I, III, IV



- II the complement of  $\angle X$
- III the angle adjacent to ZZ
- IV angle corresponding to ZR



## **Inquiry Lab**

**Right Triangle Relationships** 



#### WHAT is the relationship among the sides of a right triangle?

Mathematical Practices 1, 3, 4

Three square tents at the funfair are situated as shown below. The backs of the orange tent and the green tent form a right angle. The back of the blue tent finishes the triangle.

## Hands-On Activity

Using grid paper can help to investigate the relationship between the sides of a right triangle.

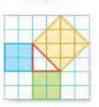


In each figure below, the sides of three squares form a right triangle.









Triangle 3

Step 2

Find the area of each square that is attached to the triangle. Record your results in the table below. The first one is already done for you. Use the figures at the right to help find the area of partial grids.





| Triangle | Area of Green<br>Square | Area of Blue<br>Square | Area of Yellow<br>Square |
|----------|-------------------------|------------------------|--------------------------|
| 1        | 1                       | 1                      | 2                        |
| 2        | 1                       | 4                      | 5                        |
| 3        | 4                       | 4                      | 8                        |

What relationship exists among the areas of the three squares

bordering each triangle? The sum of the areas of the two smaller

squares is equal to the area of the larger square.

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#### Focus narrowing the scope

Objective Model the relationship among the sides of a right triangle.

#### Coherence connecting within and across grades

Students will model the relationship among the sides of a right triangle.

#### Next

Students will use the Pythagorean Theorem and its converse to solve problems.

#### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 410.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

## **Launch the Lab**

The activity is intended to be used as a whole-group activity.

## Hands-On Activity

Three-Step Interview Have pairs complete the activity. Upon completion, have Student 1 interview Student 2, using the table and and question in Step 2 as interview questions. Then Student 1 asks Student 2 any clarification questions about the relationship and how Student 2 can verify the relationship using the values they generated in the table. 1, 3, 4, 6, 7, 8

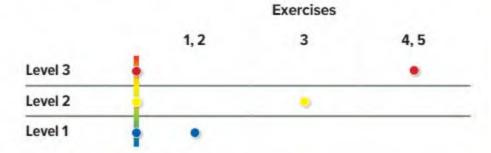
Pairs Consult Have students work with a partner to translate their verbal response to the question in Step 2 into an equation. Have them use the letters a, b, and c to represent the lengths of the sides of the triangle, letting c represent the length of the longest side. 31, 2, 4, 6, 7

# 2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

#### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.





Have each student complete Exercise 1 on their own. Then have them trade their triangles and complete their partner's triangle for Exercise 2. Then have them compare solutions and discuss any differences. 1, 3, 4, 6

## **Create**

Have students verify their conjecture in Exercise 4 by using grid paper to draw the triangle. Then have them make a conjecture about the length of the longest side of a triangle if the lengths of the two shorter sides are double that of the sides in Exercise 4. 21, 3, 4, 5, 6, 7

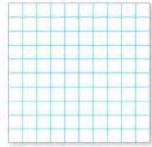
Students should be able to answer "WHAT is the relationship among the sides of a right triangle?" Check for student understanding and provide guidance, if needed.



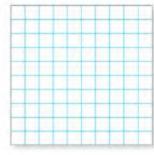
## Investigate

Work with a partner. Draw a right triangle different than those of the Activity on grid paper. Find the area of each square that is attached to the triangle. See students' work.

1.



2.



Area of square 1 =

Area of square 1 =

Area of square 2 =

Area of square 2 =

Area of square 3 =

Area of square 3 =



## **Analyze and Reflect**

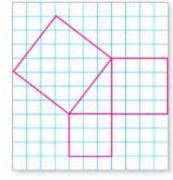
 Model with Mathematics On the grid paper shown, draw a right triangle so the two shorter sides are 3 units and 4 units long. Draw squares attached to each side of the triangle.

What is the area of each square?

9, 16, and 25 square units

What is the length of each side?

3, 4, and 5 units





#### Create

 Reason Inductively Make a conjecture about the length of the longest side of a right triangle if the lengths of the two shorter sides are 6 centimeters and 8 centimeters.

The length of the longest side would be 10 cm.

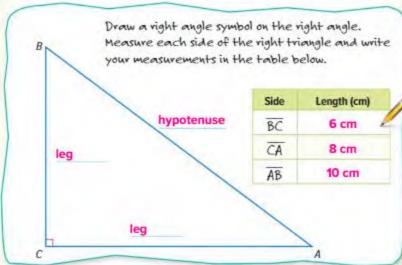
5. (Inquiry) WHAT is the relationship among the sides of a right triangle?
The sum of the squares of the two smallest sides is equal to the

square of the largest side.

Confer on the second section of the Col

Lesson 5 The Pythagorean Theorem **Essential Question Vocabulary Start-Up** HOW can algebraic concepts be applied to geometry? A right triangle is a triangle with one right angle. The legs are the sides that form the right angle. The hypotenuse is the side opposite Vocabulary the right angle. It is the longest side of the triangle.

Complete the graphic organizer. Label the legs and the hypotenuse.



# Mathematical Practices 1, 3, 4, 5

legs hypotenuse

converse

Pythagorean Theorem

## Real-World Link

When viewed from the side, the shape of some wooden waterskiing ramps is a right triangle. Suppose the height of a ramp is 90 centimeters and the length of the base of the ramp is 120 centimeters. How long do you think the ramp will be? Explain your reasoning.

Accept all reasonable answers; the length of the ramp is 150 cm.

Which Mathematical Practices did you use? Shade the circle(s) that applies. (5) Use Math Tools

- 1 Persevere with Problems
- (2) Reason Abstractiv
- 3 Construct an Argument

Model with Mathematics www.jnob-jo.com

- - 6 Attend to Precision
  - (7) Make Use of Structure
  - (8) Use Repeated Reasoning

#### Focus narrowing the scope

Objective Use the Pythagorean Theorem.

#### Coherence connecting within and across grades

#### Previous

Students used a model to explore the relationship among the sides of a right triangle.

#### Now

Students will use the Pythagorean Theorem and its converse to solve problems.

#### Next

Students will explore proofs of the Pythagorean Theorem.

#### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 415.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

## **Launch the Lesson**

#### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Pairs Discussion Ask students to draw a rectangle on a piece of paper. Then have them draw the diagonal. Ask the guestions below, then have students work in pairs to complete the Vocabulary Start-Up on the student page. 21, 2, 3, 4, 5, 6

#### Ask:

- When a diagonal is drawn, what is formed? two triangles
- How would you classify the triangles? right triangles
- Is the diagonal longer or shorter than the sides of the rectangle? longer

# **Teach the Concept**

Ask the scaffolded questions for each example to differentiate instruction.

## Example

Find a missing length in a right triangle.



- Do you need to find the length of one of the legs or the hypotenuse? hypotenuse
  - What are the lengths of the legs? 9 cm and 12 cm



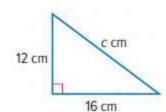
- What equation do we use to model the Pythagorean Theorem?  $a^2 + b^2 = c^2$ 
  - What value would you substitute for a in the equation? 12
  - What value would you substitute for b in the equation?



- Why can the value of c not be negative? Length must be positive, so you need to use the positive square root.
  - How do we know that our answer is reasonable? Sample answer: The hypotenuse is the longest side of a right triangle. Since 15 > 12 > 9, the answer is reasonable.

#### Need Another Example?

Write an equation you could use to find the length of the missing side of the right triangle shown. Then find the missing length. Round to the nearest tenth if necessary.  $12^2 + 16^2 = c^2$ ; 20 cm



#### Key Concept

### **Pythagorean Theorem**

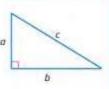
Work Zone

Right Angle

The symbol Tindicates an

angle with a measure of

In a right triangle, the sum of the Model squares of the lengths of the legs is equal to the square of the length of the hypotenuse.



Symbols

Words

$$a^2 + b^2 = c^2$$

The Pythagorean Theorem describes the relationship between the lengths of the legs and the hypotenuse for any right triangle.

You can use the Pythagorean Theorem to find the length of a side of a right triangle when you know the other two sides.

#### Examples

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

1.



 $a^2 + b^2 = c^2$ 

Pythagorean Theorem

 $12^2 + 9^2 = c^2$ 

Replace a with 12 and b with 9.

 $144 + 81 = c^2$ 

Evaluate 122 and 92

 $225 = c^2$ 

Add 81 and 144

 $\pm \sqrt{225} = c$ 

Definition of square root

Simplify.

The equation has two solutions, 15 and -15. However, the length of a side must be positive. So, the hypotenuse is 15 centimeters long.

Check

 $a^2 + b^2 = c^2$ 

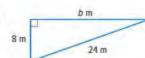
c = 15 or -15

 $12^2 + 9^2 \stackrel{?}{=} 15^2$ 

 $144 + 81 \stackrel{?}{=} 225$ 

225 = 225 V

2.



$$a^2 + b^2 = c^2$$

$$8^2 + b^2 = 24^2$$

Replace a with 8 and c with 24.

$$64 + b^2 = 576$$

Evaluate 82 and 242

Pythagorean Theorem

$$64 - 64 + b^2 = 576 - 64$$

Subtract 64 from each side.

$$b^2 = 512$$

 $b = \pm \sqrt{512}$ 

$$b \approx 22.6 \text{ or } -22.6$$

Use a calculator.

$$b \approx 22.6$$
 or  $-22.6$  Use a calc.  
The length of side  $b$  is about 22.6 meters.

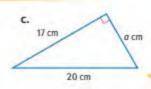
Check for

The hypotenuse is always the longest side in a right triangle. Since 22.6 is less than 24, the answer is reasonable.

#### Got It? Do these problems to find out.







$$3^2 + b^2 = 8^2$$
:

$$a^2 + 17^2 = 20^2$$
;  
c. 10.5 cm

## **Converse of Pythagorean Theorem**

Key Concept

If the sides of a triangle have lengths a, b, and c units such that  $a^2 + b^2 = c^2$ , then the triangle is a right triangle.

If you reverse the parts of the Pythagorean Theorem, you have formed its converse.

Statement: If a triangle is a right triangle, then  $a^2 + b^2 = c^2$ . Converse: If  $a^2 + b^2 = c^2$ , then the triangle is a right triangle.

The converse of the Pythagorean Theorem is also true.

## Example

#### 2. Find a missing length in a right triangle.



- Do you need to determine the length of one of the legs. or the hypotenuse? one of the legs
  - What is the length of the known leg? 8 m
  - What is the length of the hypotenuse? 24 m



- What equation do we use to model the Pythagorean Theorem?  $a^2 + b^2 = c^2$ 
  - What value would you substitute for a in the equation?
  - What value would you substitute for c in the equation?

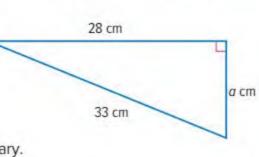


- Why do we not consider the negative square root, -22.6? The side length of a triangle cannot be negative.
  - How do we know if our answer is reasonable? Sample answer: The hypotenuse is the longest side of a right triangle. Since 22.6 is less than 24, the answer is reasonable.

#### Need Another Example?

 $a^2 + 28^2 = 33^2$ ; 17.5 cm

Write an equation you could use to find the length of the missing side of the right triangle shown. Then find the missing length. Round to the nearest tenth if necessary.



## Watch Out!

Common Error Students may think that they always substitute the known side lengths for a and b and solve for c in the Pythagorean Theorem. Remind them that c is always the hypotenuse of the triangle, so if the hypotenuse is one of the known sides, they will need to substitute the known lengths for a and c, and then solve for b.

## Example

#### 3. Use the converse of the Pythagorean Theorem.



- What are the lengths of the sides of the triangle? 5 cm, 12 cm, and 13 cm
  - Which side length is the longest? 13 cm



- If this is a right triangle, which side would be the hypotenuse? 13 cm
  - What equation do we use to determine if the triangle is a right triangle?  $a^2 + b^2 = c^2$
  - What value would you substitute for a in the equation? 5 b? 12 c? 13



 Sets of numbers that work in the Pythagorean Theorem are called Pythagorean Triples. Can you think of other values for a, b, and c that are Pythagorean Triples? Sample answer: 3, 4, and 5

#### Need Another Example?

The measures of three sides of a triangle are 24 centimeters, 7 centimeters, and 25 centimeters. Determine whether the triangle is a right triangle. yes;  $7^2 + 24^2 = 25^2$ 

## **Guided Practice**

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.



If some of your students are not ready for assignments, use the differentiated activity below.

(II) (II) Team-Pair-Solo Have students work in a team of 4 (making sure there are some approaching level students grouped with beyond level students) to complete Exercises 1 and 3. Then have them work in pairs to complete Exercises 2 and 4. Finally, have them work individually to complete Exercise 5. Upon completion, have them regroup with their original team to compare answers and resolve any differences. 1, 2, 3, 4, 6

State three measures that could be the side measures of a right triangle. Justify your answer below.

and Reflect

STOP

Sample answer: 3, 4,  $5; 3^2 + 4^2 = 5^2$ 

yes; 362 +

e. no; 42 + 52 + 7

 $48^2 = 60^2$ 

#### Example

The measures of three sides of a triangle are 5 centimeters, 12 centimeters, and 13 centimeters. Determine whether the triangle is a right triangle.

$$a^2 + b^2 = c^2$$
 Pythagorean Theorem  
 $5^2 + 12^2 \stackrel{?}{=} 13^2$   $a = 5, b = 12, c = 13$   
 $25 + 144 \stackrel{?}{=} 169$  Evaluate  $5^2$ ,  $12^2$ , and  $13^2$ .  
 $169 = 169$  ✓ Simplify.

The triangle is a right triangle.

Got It? Do these problems to find out.

Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer.

d. 36 km, 48 km, 60 km

e. 4 m 7 m, 5 m

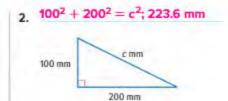
#### **Guided Practice**



Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary. (Examples 1 and 2)

1. 
$$12^2 + 16^2 = c^2$$
; 20 m





Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer. (Example 3)

- 3. 5 cm, 10 cm, 12 cm no; 52 + 102 + 122
- 4. 9 m, 40 m, 41 m yes; 92 + 402 = 412
- Building on the Essential Question What is the relationship among the legs and the hypotenuse of a right triangle?

The sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

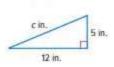


My Homework

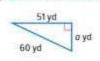
## Independent Practice

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary. (Examples 1 and 2)

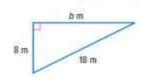
$$5^2 + 12^2 = c^2$$
; 13 cm.



2. 
$$a^2 + 51^2 = 60^2$$
; 31.6 m 3.  $a^2 + b^2 = 18^2$ ; 16.1 m



$$3.8^2 + b^2 = 18^2$$
; 16.1 m





Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer. (Example 3)

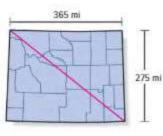
yes; 
$$28^2 + 195^2 = 197^2$$

130 cm, 122 cm, 125 cm

no; 
$$30^2 + 122^2 \neq 125^2$$

6. Calculate the length of the diagonal of the rectangle.

about 735 km



Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

$$a = 48 \text{ m}; b = 55 \text{ m}$$

$$48^2 + 55^2 = c^2$$
; 73 m

8. 
$$a = 23$$
 cm;  $b = 18$  cm

 $23^2 + 18^2 = c^2$ ; 29.2 cm

9. 
$$b = 5.1 \text{ m}; c = 12.3 \text{ m}$$

$$a^2 + 5.1^2 = 12.3^2$$
; 11.2 m

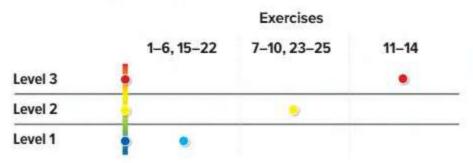
# **Practice and Apply**

#### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

#### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



#### Suggested Assignments

You can use the table below to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                              |
|---------------------------------|-------------------|------------------------------|
| AL                              | Approaching Level | 1–5, 7, 9, 12–14, 24, 25     |
| OL)                             | On Level          | 1–5 odd, 6–10, 12–14, 24, 25 |
| <b>B</b>                        | Beyond Level      | 6–14, 24, 25                 |

| MATHEMATICAL PRACTICES  |             |
|---|-------------|
| Emphasis On   | Exercise(s) |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 11          |
| 3 Construct viable arguments and critique the reasoning of others.            | 12–14, 23   |
| 5 Use appropriate tools strategically.  | 10          |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

### TICKET Out the Door

Have students determine whether a triangle with side lengths of 1 centimeter, 3 centimeters, and 3 centimeters is a right triangle. Have them justify their response. no; Sample answer:  $1^2 + 3^2 \neq 3^2$ .

### Watch Out!

**Find the Error** In Exercise 12, Amani identified the hypotenuse as one of the triangle's legs. Ask students to label the two legs of the right triangle *a* and *b* and the hypotenuse *c*. Point out that it is often helpful to write the formula before replacing variables with numbers.

10. Use Math Tools The whole numbers 3, 4, and 5 are called Pythagorean triples because they satisfy the Pythagorean Theorem. Complete the graphic organizer shown to list 4 additional sets of Pythagorean triples.

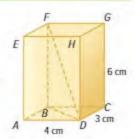
| 3 | 4  | 5  |
|---|----|----|
| 6 | 8  | 10 |
| 9 | 12 | 15 |
| 5 | 12 | 13 |
| 8 | 15 | 17 |



#### H.O.T. Problems Higher Order Thinking

 Persevere with Problems In the figure, BD is the diagonal of the base and FD is the diagonal of the figure. Find FD to the nearest tenth.

7.8 cm



 Find the Error Amani is writing an equation to find the length of the third side of the right triangle. Find her mistake and correct it.



She used the sides given as legs, when

one is a hypotenuse;  $a^2 + 3^2 = 4^2$ .

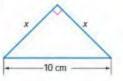


13. Justify Conclusions What does the value of x have to be for the figure to be classified as a right isosceles triangle? Justify your reasoning.

about 7.1 cm; Sample answer: The Pythagorean Theorem states that

$$c^2 = a^2 + b^2$$
. Since both legs are x inches,  $c^2 = 2x^2$ . When you replace

c with 10 and simplify,  $x \approx 7.1$ .



14. Justify Conclusions The hypotenuse of a right triangle is 23 centimeters long. Find possible measures for the legs of the triangle. Round to the nearest hundredth. Justify your answer.

Sample answer: 15 cm and 17.44 cm;  $23^2 = 529$  and

 $15^2 + 17.44^2 = 529.1536$ , So,  $23^2 \approx 15^2 + 17.44^2$ .

spyright © McGraw-H II Educado

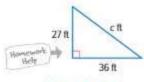
WWW. J. J. J. J. J. J. J. G. G. Wine Pythagorean Theorem

Namo

My Homework ...

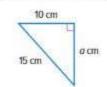
### **Extra Practice**

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

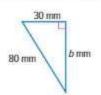


Since length cannot be negative, the length of side c is 45 meters.

16. 
$$\sigma^2 + 10^2 = 15^2$$
; 11.2 cm



17. 
$$30^2 + b^2 = 80^2$$
; 74.2 mm



Copy and Solve Determine whether each triangle is a right triangle. Justify your answer. Show your work on a separate piece of paper.

yes; 
$$24^2 + 143^2 = 145^2$$

no; 
$$135^2 + 140^2 \neq 175^2$$

21. 44 cm, 70 cm, 55 cm  
no; 
$$44^2 + 55^2 \neq 70^2$$

22. A triangle is formed by three towns, as shown on the map. Is this triangle a right triangle? Explain.

no; 
$$12^2 + 24^2 \neq 29^2$$



23. Construct an Argument Explain to a classmate why you can use any two sides of a right triangle to find the third side. Sample answer: If you know the lengths of two sides of a right triangle, you can substitute the values in the Pythagorean Theorem and find the missing side.

## Power Up! Test Practice

Exercises 24 and 25 prepare students for more rigorous thinking needed when taking assessment.

24. This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and

| constructing arguments. | <u> </u>  |
|-------------------------|---|
| Depth of Knowledge      | DOK3  |
| Mathematical Practices  | MP1, MP3, MP4, MP5  |
| Scoring Rubric          |   |
| 2 points                | Students correctly model the situation, find<br>the height of the ladder and explain their<br>response. |
| 1 point                 | Students correctly model the situation OR find the height of the ladder and explain their response.     |

25. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of

| structure.             |   |
|------------------------|---|
| Depth of Knowledge     | DOK1                                    |
| Mathematical Practices | MP1, MP2, MP6                           |
| Scoring Rubric         |   |
| 1 point                | Students correctly answer the question. |

## Power Up! Test Practice

24. The base of a 3.90-meter ladder stands 1.50 meters from a house. Sketch a drawing to model this situation.

How many meters up the side of the house does the ladder reach? Explain how drawing the picture helped you solve the problem.



3.60 m; Sample answer: Drawing and labeling the picture helps you to see how to apply the Pythagorean Theorem to solve the problem.

- 25. Which of the following lengths represent the sides of a right triangle? Select all that apply.
  - 9 cm, 12 cm, 16 cm
  - 8 cm, 15 cm, 17 cm
  - 10 cm, 24 cm, 28 cm
  - 6 cm, 8 cm, 10 cm

#### **Spiral Review**

Simplify each expression.

**26.** 
$$10^2 + 14^2 = 296$$

**27.** 
$$16^2 + 2^2 =$$
 **260**

**28.** 
$$20^2 - 17^2 = 111$$

29. The area of each square is 16 square units. Find the perimeter of the figure shown.



Find each square root. Round to the nearest tenth.

30. 
$$\sqrt{200} \approx 14.1$$

64 units

31. 
$$\sqrt{45} \approx 6.7$$

32. 
$$\sqrt{126} \approx 11.2$$

## **Inquiry Lab**

**Proofs About the Pythagorean Theorem** 



#### HOW can you prove the Pythagorean Theorem and its converse?

Mathematical Practices 1.3.7

The Pythagorean Theorem is named after a famous Greek mathematician Pythagoras who lived around 500 B.C. The properties of the theorem, however, were known by the ancient Egyptians, Babylonians, and Chinese. The following geometric proof is similar to a visual proof shown in a Chinese document written between 500 B.C. and 200 B.C.

## Hands-On Activity 1

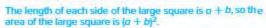
Step 1

Draw and cut out 8 copies of a right triangle. Label each pair of legs a and b, and each hypotenuse c.



Step 2

On a separate piece of paper arrange four of the triangles in a square as shown. Trace the figure formed by the hypotenuses.



Is the figure formed by the hypotenuses a square?

yes; Sample answer: The sides of the figure have the same measure c and all of the angles measure 90°.

Write an expression for the area of the inside square. c2



On the same paper, arrange the remaining triangles as shown. Draw the two figures shown by the dashed lines.

The length of each side of the large square is a + b, so the area of the large square is  $(a + b)^2$ .

Are the two figures represented by dashed line squares? Explain.

yes; Sample answer: The sides of each figure have the same measure and all of the angles measure 90°.

Write an expression for the area of the small square. 02







#### Focus narrowing the scope

Objective Prove the Pythagorean Theorem and its converse.

#### Coherence connecting within and across grades

#### Now

Students will use models and diagrams to prove the Pythagorean Thoerem.

Students will use the Pythagorean Theorem and its converse to solve

#### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 420.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

## **Launch the Lab**

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

## Hands-On Activity 1

(II) (II) Teammates Consult Have students work in small groups to complete the activity. Group one or more Approaching Level students and one or more Beyond Level students in the same groups, if possible. Have one student read each set of directions aloud, checking to see if the team understands what procedures to follow. Have students who understand the procedures check the work of those who have trouble or need clarification. For each step, have the student who is reading the directions pause and ask if students have questions. If no one on the team is able to answer a specific question, the team may ask you for help and assistance.

**1**, 4, 5



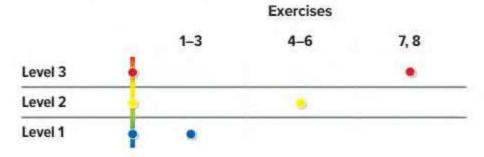


# 2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

#### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



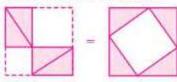


RID (IA) Teammates Consult Continue the groupings from Activity 1. After researching the Egyptian technique in Exercise 1, have students work in their small groups to demonstrate the procedure. Give groups ropes of different lengths with knots already tied. Have students trace their triangle on paper or on the board. (2) 1, 4, 5

above, give the groups ropes of different lengths that do not have any knots. Have students tie equally spaced knots and trace their triangle on paper or on the board. 21, 4, 5

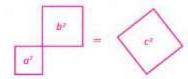
#### Step 4

Since the area of each of the two composite figures you created is  $(a + b)^2$ , the areas are equal. Use the space provided to draw each figure from Step 2 and Step 3. Place an equal sign between the two figures to show the two areas are equal.



#### Step 5

Remove the triangles from each side. Use the space provided to draw the remaining figures.



What property justifies removing the triangles from each side of

the equation? Subtraction Property of Equality

Write an algebraic equation that represents the relationship

between the figures shown in Step 5.  $a^2 + b^2 = c^2$ 

Summarize the relationship among the sides of a right triangle

measuring a units, b units, and c units.

Sample answer: The sum of the squares of the two smaller sides

is equal to the square of the largest side.



### Investigate

#### Work with a partner.

 A legend states that the ancient Egyptians could create a right triangle by using a knotted rope.
 Research this on the Internet. Describe their technique in the space provided and draw a diagram to illustrate the technique.





Sample answer: The Egyptians tied 12 knots in a rope that were equally spaced. They then laid it out so one side had 3 units, another side had

4 units and the third side had 5 units. This made a right triangle.

the state of the second section is not been second

Geometry

## Hands-On Activity 2

The converse of the Pythagorean Theorem states if a triangle has side lengths, a, b, and c units such that  $a^2 + b^2 = c^2$ , then the triangle is a right triangle. In this Activity, you will prove the converse of the Pythagorean Theorem by using a two-column proof.

Given:  $\triangle ABC$  such that  $a^2 + b^2 = c^2$ .

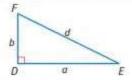
Prove:  $\triangle ABC$  is a right triangle.

Complete the proof with the correct reasons justifying each statement.

| Statements |  |
|------------|--|
|            |  |

- a. Draw a right triangle DEF so that DE is a units long and DF is b units long. Label FE as d.
- b. Write an equation that describes the relationship between the side lengths of △DEF. State the theorem that allows you to make that statement.
- c.  $a^2 + b^2 = c^2$
- **d.** If  $a^2 + b^2 = c^2$  and  $a^2 + b^2 = a^2$ , then  $a^2 = c^2$ .
- e. If  $d^2 = c^2$ , then d = c
- f. If d = c, then FE = AB.
- g. If AC = FD, CB = DE, and AB = FE, the two triangles are the same shape and size.
- h.  $m\angle C = m\angle D$
- ∠C is a right angle.
- j. \(\triangle\) ABC is a right triangle

#### Reasons



 $a^2 + b^2 = d^2$ ; Pythagorean

Theorem

Given

Substitute  $c^2$  for  $a^2 + b^2$  in the second equation.

Definition of square root

Definition of equal segments

If three sides of a triangle are the same length as the corresponding sides of another triangle, the triangles are the same shape and size.

Corresponding parts of the triangles with the same size and shape have the same measures.

Definition of right angle

Definition of right triangle

So, if a triangle has side lengths, a, b, and c units such that  $a^2 + b^2 = c^2$ , then the triangle is a right triangle.

## Hands-On Activity 2

Think-Pair-Share Have students work with a partner to complete the activity. For each step, give students about 10–20 seconds to think through how they would provide a reason for each step. Then have them share their responses with their partner, asking for support and clarification if needed from their partner, or you. You may wish to provide students with numerical measurements for the side lengths of the triangle instead of variable measurements, if students are struggling with the algebraic manipulation.

1, 2, 3, 4, 5, 6, 7

Popcorn Share Have students work in small groups to complete Activity 2. Have each student be responsible for providing the reason for each step. Have students take turns providing the reasons. When it is their turn to provide a reason, have that student stand up, verbally give the reason, and then explain the reason in their own words. 21, 2, 3, 4, 5, 6, 7



Coup-Solo Have students work as a small group to complete Exercise 2, ensuring that each group member understands how to determine whether the triangle is a right triangle. Then have students complete Exercise 3 individually. Upon completion, have them share their responses with their group to check their work. 2 1, 2, 3, 4, 5, 6, 7

#### Ask:

- What equation can you use to determine if the triangle is a right triangle?  $a^2 + b^2 = c^2$
- In Exercise 1, what length on the triangle must be substituted for c? Why? 6; Sample answer: because it is the longest side of the triangle



## **Analyze and Reflect**

(B) Value Line Students place themselves on a pretend line, using the number 10 to represent that they understand the Pythagorean Theorem and its converse completely and the number 1 to represent that they do not understand or have a lot of questions. Have students pair up with someone from the other side of the line to complete Exercises 4-6. 1, 2, 4, 5, 6, 7



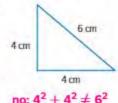
Think-Pair-Share Have students work in pairs. Give students several minutes to think through their responses to Exercises 7 and 8. Have them share their responses with their partner. Then call on one student to share their response within a small group or large group discussion. 2 1, 2, 4, 5, 6

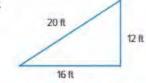
Students should be able to answer "HOW can you prove the Pythagorean Theorem and its converse?" Check for student understanding and provide guidance, if needed.



## Investigate

Work with a partner. Determine whether the following figures are right triangles. Justify your answer.





yes;  $12^2 + 16^2 = 20^2$ 



## Analyze and Reflect

Justify Conclusions Work with a partner. The whole numbers 3, 4, and 5 are called Pythagorean Triples because they satisfy the Pythagorean Theorem. Determine if each of the following is a Pythagorean Triple. Explain your reasoning.

4. 7, 24, 25

yes; Sample answer

$$7^2 + 24^2 = 25^2$$

5, 15, 20, 25

yes; Sample answer:

$$15^2 + 20^2 = 25^2$$

6. 9. 12. 16

no; Sample answer:

$$9^2 + 12^2 = 15^2$$
, not  $16^2$ 



### Create

 Identify Structure In the Activity in the "Right Triangle Relationships" Inquiry Lab, you examined the relationship between the sides of a right triangle. Compare the process used in that Activity with the one you completed when you did the "Proofs About the Pythagorean Theorum" Inquiry Lab. What kind of reasoning was used in each Activity?

Sample answer: In the first Activity I used measurement to demonstrate

the Pythagorean Theorem so that was inductive reasoning. In this

Activity, I used properties of mathematics to prove the Pythagorean

Theorem so that was deductive reasoning.

HOW can you prove the Pythagorean Theorem and its converse? Sample answer: You can use a physical model and the properties of mathematics to construct proofs of the Pythagorean Theorem and its converse.

Lesson 6

**Essential Question** 

HOW can algebraic concepts

be applied to geometry?

Mathematical Practices

## Use the Pythagorean Theorem



#### Real-World Link

Parasailing In parasailing, a towrope is used to attach a parasailer to a boat. Refer to the diagram below for Exercises 1-4.

- 1. What type of triangle is formed by the horizontal distance, the vertical height, and the length of the towrope? Explain. right triangle; Since the sum of the squares of two sides is equal to the square of the third side, the triangle is a right triangle;  $21^2 + 72^2 = 75^2$ .
- 2. Suppose the wind picks up and the parasailer rises to 17 m and remains 24 m behind the boat. Write an equation that will help you find how much towrope c the parasailer will need.
  - $50^2 + 72^2 = c^2$
- 3. Solve the equation to find the amount of rope the parasailer will need. Round to the nearest meter. 29 m
- 4. Suppose the towrope is 100 meters long and the parasailer is 70 meters above the water surface. Write an equation to find the horizontal distance b behind the boat.







Which Mathematical Practices did you use? Shade the circle(s) that applies.

- 1 Persevere with Problems
- (5) Use Math Tools
- 2 Reason Abstractly
- 6 Attend to Precision
- (3) Construct an Argument
- (7) Make Use of Structure
- Model with Mathematics
- (8) Use Repeated Reasoning

### Focus narrowing the scope

Objective Solve problems using the Pythagorean Theorem.

#### Coherence connecting within and across grades

#### Previous

Students used models and diagrams to prove the Pythagorean Theorem.

#### Now

Students will use the Pythagorean Theorem to solve problems.

Students will find the distance between two points on a coordinate plane.

### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 427.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

## Launch the Lesson

#### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Think-Pair-Solo Have students work in pairs. Give them about 20 seconds to think through their response to Exercise 1 individually, then have them share their response with a partner, taking care to make sure they justify their response. Then have them work together to complete Exercises 2 and 3. Have them work individually to complete Exercise 4. 21, 2, 3, 4, 6

### Alternate Strategy

Provide students with copies of a blank Pythagorean Theorem equation, 2 + 2 = 2 = 2 to use as they complete the exercises.

# **2** Teach the Concept

**Ask** the scaffolded questions for each example to differentiate instruction.

## **Examples**

#### 1. Solve a right triangle.

- How can you tell that the triangle is a right triangle?
   There is a right angle symbol.
- What equation can we use to model the Pythagorean Theorem?  $a^2 + b^2 = c^2$
- Why do we not use the negative square root? Sample answer: The length of a ladder cannot be negative.

#### **Need Another Example?**

Write an equation that can be used to find the length of the boat ramp. 4.2 m Then solve. Round to the nearest tenth.



$$4.2^2 + 25^2 = c^2$$
; 25.4 m

### 2. Solve a right triangle.

- Do you need to find the length of one of the legs or the hypotenuse? one of the legs
- What equation can we use to model the Pythagorean Theorem?  $a^2 + b^2 = c^2$
- Derive a different equation that you can use to solve for the leg of a right triangle, when the other two sides are known. Sample answer: When b is the unknown, you must subtract  $a^2$  from both sides to get  $b^2$  by itself;  $b^2 = c^2 a^2$ .

#### **Need Another Example?**

Write an equation that can be used to find the length of the backboard. Then solve. Round to the nearest tenth.

 $105^2 + x^2 = 208.5^2$ ; 180 cm





### Solve a Right Triangle

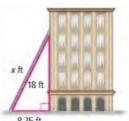
The Pythagorean Theorem can be used to solve a variety of problems. It is helpful to use a diagram to determine what part of the right triangle is unknown.



#### **Examples**

 Write an equation that can be used to find the length of the ladder. Then solve. Round to the nearest tenth.

> Notice that the distance from the building, the building itself, and the ladder form a right triangle. Use the Pythagorean Theorem.



$$a^2 + b^2 = c^2$$
 Pythagorean Theorem 8.75<sup>2</sup> + 18<sup>2</sup> =  $c^2$  Replace  $a$  with 8.75 and  $b$  with 18. 76.5625 + 324 =  $c^2$  Evaluate 8.75<sup>2</sup> and 18<sup>2</sup>.

$$400.5625 = c^2$$
 Add  $76.5625$  and  $324$ .  
 $\pm \sqrt{400.5625} = c$  Definition of square root  
 $\pm 20.0 \approx c$  Use a calculator.

Since length cannot be negative, the ladder is about 20 meters long.

Write an equation that can be used to find the height of the plane. Then solve. Round to the nearest tenth.



The distance between the planes is the hypotenuse of a right triangle. Use the Pythagorean Theorem.

$$\mathbf{a}^2 + b^2 = \mathbf{c}^2$$
 Pythagorean Theorem

 $\mathbf{10}^2 + b^2 = \mathbf{12}^2$  Replace  $a$  with 10 and  $c$  with 12.

 $\mathbf{100} + b^2 = \mathbf{144}$  Evaluate  $\mathbf{10}^2$  and  $\mathbf{12}^2$ .

 $\mathbf{b}^2 = \mathbf{44}$  Subtraction Property of Equality

 $\mathbf{b} = \pm \sqrt{44}$  Definition of square root.

 $\mathbf{b} \approx \pm 6.6$  Use a calculator.

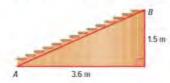
Since length cannot be negative, the height of the plane is about 6.6 kilometers.

Geometry

a. 3.9 m

#### Got It? Do this problem to find out.

a. Mr. Khalid wants to build a new banister for the staircase shown.
If the rise of the stairs of a building is 1.5 meters and the run is 3.6 meters, what will be the length of the new banister?

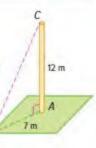


#### The Pythagorean Theorem in Three-Dimensions

You can use the Pythagorean Theorem to find missing measures in three-dimensional figures.

#### Example

3. A 12 meter flagpole is placed in the center of a square area. To stabilize the pole, a wire will stretch from the top of the pole to each corner of the square. The flagpole is 7 meters from each corner of the square. What is the length of each wire? Round to the nearest tenth.



Draw right triangle ABC. You want to find the length of each wire or BC. This is the hypotenuse of a right triangle, so use the Pythagorean Theorem.

$$AB^2 + AC^2 = BC^2$$
 Pythagorean Theorem

 $7^2 + 12^2 = BC^2$  Replace  $AB$  with 7 and  $AC$  with 12.

 $49 + 144 = BC^2$  Evaluate  $7^2$  and  $12^2$ .

 $193 = BC^2$  Simplify.

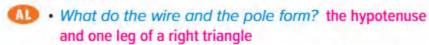
 $\pm \sqrt{193} = BC$  Definition of square root.

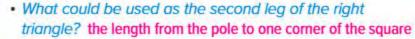
 $\pm 13.9 \approx BC$  Use a calculator.

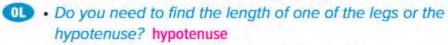
Since length cannot be riegative, the length of the wire is about 13.9 m.

## Example

## Use the Pythagorean Theorem in three dimensions.





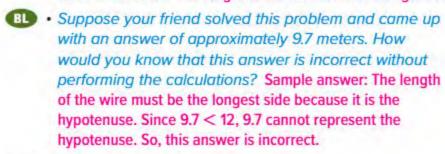


What are the lengths of the legs? 7 m and 12 m

 Use AB and AC to denote the legs of the triangle. Use BC to denote the hypotenuse. What equation can be used to find the length of the wire? AB<sup>2</sup> + AC<sup>2</sup> = BC<sup>2</sup>

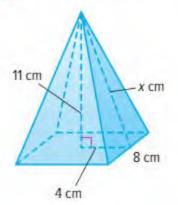
What value would you substitute for AB? 7 AC? 12

 Explain why we don't consider the negative square root to be a solution. The length of the wire cannot be negative.



#### Need Another Example?

The slant height of a pyramid is the height of each lateral face. What is the slant height of the pyramid shown? Round to the nearest tenth. 11.7 cm



## **Guided Practice**

**Formative Assessment** Use these exercises to assess students' understanding of the concepts in this lesson.



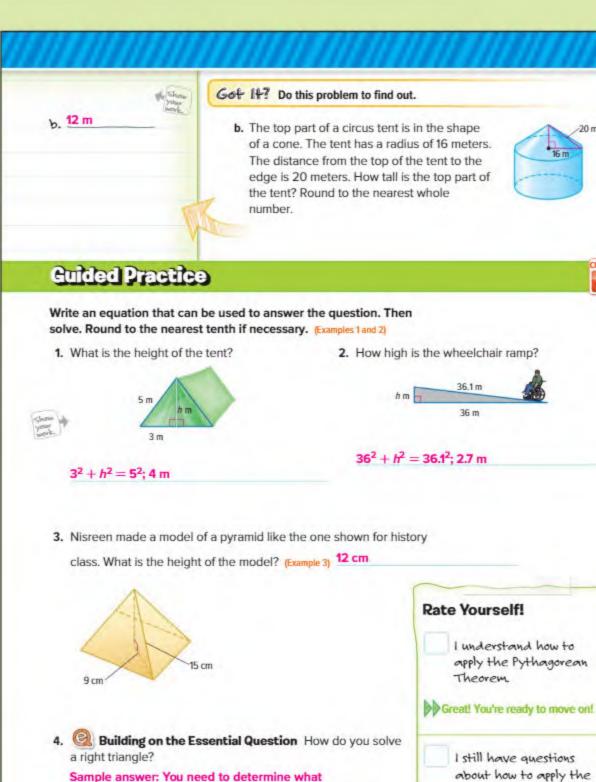
If some of your students are not ready for assignments, use the differentiated activities below.

All LA Numbered Heads Together Assign students to 3- or 4-person learning teams. Each member is assigned a number from 1 to 4. Each team completes Exercises 1–4, making sure that every member understands. Call on a specific number from one team to present the team's solution to the class. Ask each student clarifying questions to ensure understanding. 1, 2, 4, 5, 6

#### Ask:

- How did you know how to set up the equation?
   See students' work.
- What steps did you take to solve the equation?
   See students' work.
- How can you determine if your answer is reasonable?
   See students' work.

a real-world problem in which the Pythagorean Theorem must be used to solve the problem. In their problem, they should create a drawing. Then have them post the problems around the room. Pairs of students should walk around the room and select a problem, not their own, and return to their seats to solve it. After every pair has solved their problem, have pairs of students re-post the solutions around the room. The original pair locates their problem and determines if the solution is correct. 1, 2, 3, 4, 5, 6



measurements represent the legs and the hypotenuse,

and appropriately use the Pythagorean Theorem.

Pythagorean Theorem

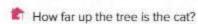
FOLDABLES Time to update your Foldable!

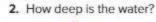
WW486 JANGE 5 Jange Carmine Pythagorean Theorem

My Homework

### Independent Practice

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary. (Examples 1 and 2)







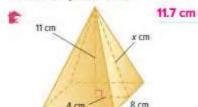


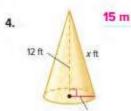


 $1.5^2 + h^2 = 3.7^2$ ; 3.4 m

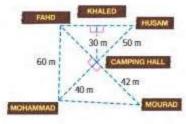
 $6^2 + x^2 = 15^2$ : 13.7 m

Find the missing measure in each figure below. Round to the nearest tenth if necessary. (Example 3)





- 5. Refer to the map of the Scout's Camp at the right. Round your answers to the nearest tenth.
  - a. How far is it from Khaled's cabin to Husam's cabin? 40 m



b. A camper in Fahd's cabin wants to visit a friend in Mohammad's cabin. How much farther is it if he walks to the

Camping Hall first? 24.7 m

6. Dustify Conclusions Ibrahim is buying a 165-centimeter-long fishing rod for his father. He wants to put it in a box so that his dad will not be able to guess what is in the box. The box he wants to use is 120 centimeters long and 120 centimeters wide. Will the pole fit in the box? Justify

yes; Sample answer: The corner of the box is a right angle. Find the length of

the diagonal using the Pythagorean Theorem.  $120^2 + 120^2 = 28,800, \sqrt{28} \approx 170.$ 

Since the fishing rod is 165 centimeters long, it will fit diagonally in the box.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

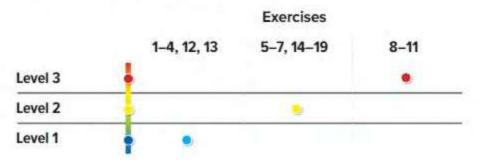
# **Practice and Apply**

#### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

#### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



#### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                          |
|---------------------------------|-------------------|--------------------------|
| AL                              | Approaching Level | 1–5, 7–9, 11, 18, 19     |
| OL)                             | On Level          | 1–5 odd, 6–9, 11, 18, 19 |
| BL                              | Beyond Level      | 5–11, 18, 19             |

## Watch Out!

Common Error Students may substitute the side lengths of the triangle for any variable and then solve. Remind them that the variables a and b must be legs of the triangle and c must be the hypotenuse.

| MATHEMATICAL PRACTICES  |             |  |
|---|-------------|--|
| Emphasis On   | Exercise(s) |  |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 10, 16, 17  |  |
| 3 Construct viable arguments and critique the reasoning of others.            | 6, 9        |  |
| 4 Model with mathematics.   | 8, 11       |  |
| 7 Look for and make use of structure.   | 7           |  |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

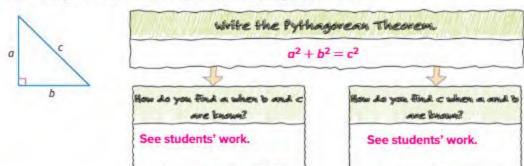
#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

### TICKET Out the Door

Ask students to write how they think using the Pythagorean Theorem will connect with the next lesson about finding the distance between two points on the coordinate plane. Use the writing prompt below. See students' work.

 The Pythagorean Theorem will help me find the distance between two points on the coordinate plane because ... 7. Didentify Structure How do you use the Pythagorean Theorem?



## H.O.T. Problems Higher Order Thinking

8. Model with Mathematics Write a real-world problem that can be solved by using the Pythagorean Theorem. Then explain how to solve the problem.
Sample answer: Nasser leaves his house. He walks 2 kilometers north, and then turns and walks 3 kilometers west. How far is Nasser from his house? Using the Pythagorean Theorem, c² = 2² + 3². Solving for c, Nasser is about 3.6 kilometers from his house.

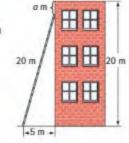
 Which One Doesn't Belong? Each set of numbers represents the side measures of a triangle. Identify the set that does not belong with the other three. Explain your reasoning.



$$3-5-7$$
;  $3^2+5^2\neq 7^2$ 

10. Persevere with Problems Suppose a ladder 20 meters long is placed against a vertical wall 20 meters high. How far would the top of the ladder move down the wall by pulling out the bottom of the ladder 5 meters? Explain your reasoning.

about 0.6 m; By solving  $20^2 = x^2 + 5^2$ , you find that the ladder reaches approximately 19.4 m up the wall. Therefore, the top of the ladder would move down 20 m - 19.4 m or 0.6 m. by pulling out the bottom of the ladder 5 meters.



 Model with Mathematics Write and solve a real-world problem that involves using the Pythagorean Theorem or its converse.

See students' work.

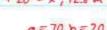
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My Homework

### **Extra Practice**

12. Write an equation to find how far the bird is from the boy. Then solve the equation. Round to the nearest tenth.

$$70^2 + 20^2 = x^2$$
; 72.8 m





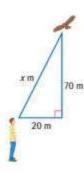
$$a = 70, b = 20, and c = x$$

$$70^{2} + 20^{2} = x^{2}$$

$$4,900 + 400 = x^{2}$$

$$5,300 = x^{2}$$

$$\sqrt{5,300} = x^{2}$$



72.8 ≈ ×

13. A hat is in the shape of a cone with dimensions shown.

Find the height of the hat. Round to the nearest tenth. 22.5 centimeters

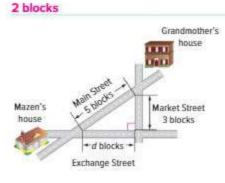


14. Mazen wants to go from his house to his grandmother's house. How much distance is

saved if he takes Main Street instead of

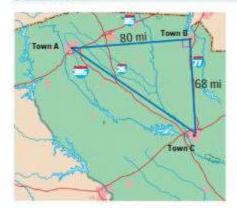
Market and Exchange?



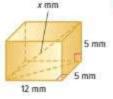


15. Suppose three towns form a right triangle. What is the distance between the two towns that form the hypotenuse?

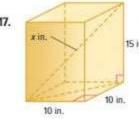
about 105 mi



Persevere with Problems Find the missing measure in each figure below. Round to the nearest tenth if necessary.



13.9 mm



20.6 cm

## Power Up! Test Practice

Exercises 18 and 19 prepare students for more rigorous thinking needed when taking assessment.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

| Depth of Knowledge     | DOK2   |
|------------------------|--|
| Mathematical Practices | MP1, MP7   |
| Scoring Rubric         |  |
| 2 points               | Students correctly label the diagram and find the perimeter. |
| 1 point                | Students correctly label the diagram OR find the perimeter.  |

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of

| structure.            |   |
|-----------------------|---|
| Depth of Knowledge    | DOK1                                    |
| Mathematical Practice | MP1                                     |
| Scoring Rubric        |   |
| 1 point               | Students correctly answer the question. |

## Power Up! Test Practice

18. Suhaila designed a decorative glass window in the shape of a kite. Select the correct measures to label the dimensions of the window.



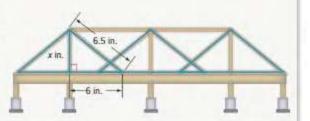
12 cm 42 cm 31 cm 45 cm 36 cm 60 cm 39 cm

What is the perimeter of the window?

168 cm

19. Ayoub is building the model bridge shown. How long must he cut the piece of wood for one of the vertical support beams, represented by x?

2.5 cm



#### Spiral Review

20. Determine whether a triangle with sides 20 centimeters, 48 centimeters, and 52 centimeters long is a right triangle. Justify your answer.

yes; 
$$20^2 + 48^2 = 52^2$$

Estimate each of the following to the nearest whole number. Justify your reasoning.

$$\sqrt{39} \approx 6$$
 22.  $-\sqrt{146} \approx -12$  6<sup>2</sup> = 36 and 7<sup>2</sup> = 49,  $-(12^2) = -144$  and

than 49, 
$$\sqrt{39} \approx 6$$
.

**22.** 
$$-\sqrt{146} \approx -12$$

$$-(12^2) = -144$$
 and

$$-(13^2) = -169$$
. Since

than 
$$-169$$
,  $-\sqrt{146} \approx -12$ .

$$3^3 = 27$$
 and  $4^3 = 64$ . Since



## Distance on the Coordinate Plane

C(7, 4)

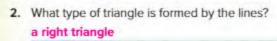


## Real-World Link

Mountain Biking Saeed was biking on a trail. A map of the trail is shown. His brother timed his ride from point A to point B.

 What do the blue and red lines on the graph represent?

The blue lines represent the horizontal and vertical distances between the two points. The red line represents the actual distance between the two points.



3. How can you find the length of AC and BC without counting the number of units?

Sample answer: subtract the x-coordinates and subtract the y-coordinates

4. What are the lengths of the two blue lines?

AC = 4 units

BC = 5.5 units

5. Write an equation using the Pythagorean Theorem that you can use to find the length of AB.

 $4^2 + 5.5^2 = c^2$ 

Which (2) Mathematical Practices did you use? Shade the circle(s) that applies.

- 1) Persevere with Problems
- 2 Reason Abstractly
- 3 Construct an Argument
- Model with Mathematics www.jnob-jo.com
- (5) Use Math Tools
- (6) Attend to Precision
- Make Use of Structure
- (8) Use Repeated Reasoning

## **Essential Question** HOW can algebraic concepts

Vocabulary

be applied to geometry?

Distance Formula

Mathematical Practices





#### Focus narrowing the scope

Objective Find the distance between two points on the coordinate plane.

## Coherence connecting within and across grades

#### Previous

Students used the Pythagorean Theorem to solve problems.

#### Now

Students will find the distance between two points on a coordinate

#### Next

Students will apply distance on the coordinate plane to transformations.

### RIGOT pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 435.

ENGAGE EXPLORE EXPLAIN FLABORATE EVALUATE

## **Launch the Lesson**

#### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Numbered Heads Together Have students work in groups of 3–4 to complete Exercises 1–5. Assign each student a number. Each student is responsible to ask for help or support and for ensuring that their teammates understand each exercise. Call on one numbered student to explain the group's responses to the class. 21, 4, 5, 6

### Alternate Strategy

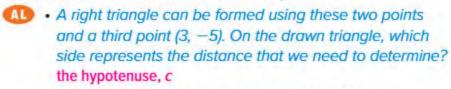
Students may benefit from a quick review of how the Pythagorean Theorem can be used to find the length of a missing side of a right triangle.

# **2** Teach the Concept

**Ask** the scaffolded questions for each example to differentiate instruction.

## Example

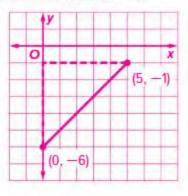
1. Find distance on the coordinate plane.



- · What is the length of the horizontal leg? 4 units
- · What is the length of the vertical leg? 5 units
- What equation can we use to model the Pythagorean Theorem?  $a^2 + b^2 = c^2$ 
  - · What value will we use for a? b? 4 units; 5 units
  - Could we have used 5 units for a and 4 units for b? yes;
     The order in which you add does not matter.
  - Why do we not use the negative square root? Distance cannot be negative.
- Could you draw a different triangle and still determine the same distance? Explain. yes; You could draw a horizontal line to the right of (3, 0) and a vertical line up from (7, —5).

#### **Need Another Example?**

Graph the ordered pairs (0, -6) and (5, -1). Then find the distance c between the points. Round to the nearest tenth. 7.1 units



Work Zone

### Find Distance on the Coordinate Plane

You can use the Pythagorean Theorem to find the distance between two points on the coordinate plane.

#### Example

 Graph the ordered pairs (3, 0) and (7, -5). Then find the distance c between the two points. Round to the nearest tenth.

$$a^2 + b^2 = c^2$$

Pythagorean Theorem

 $4^2 + 5^2 = c^2$ 

Replace a with 4 and b with 5.

 $41 = c^2$ 

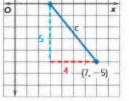
 $4^2 + 5^2 = 16 + 25$  or 41

 $\pm\sqrt{41} = \sqrt{c^2}$ 

Definition of square root

±6.4 ≈ c

Use a calculator.



The points are about 6.4 units apart.

Got It? Do this problem to find out.

a. 3.2 units

Distance

hypotenuse.

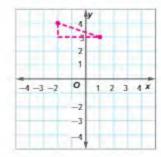
To find the distance

between two points on the

points then draw a right triangle with c as the

coordinate plane, graph the

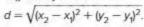
a. (1, 3), (-2, 4)



Key Concept

## **Distance Formula**

Symbols The distance d between two points with coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$ is given by the formula



Model

(x<sub>2</sub>, y<sub>2</sub>)

(x<sub>1</sub>, y<sub>1</sub>)

(x<sub>2</sub>, y<sub>2</sub>)

pyright: S McGraw-Hill Educati

You can also use the Distance Formula to find the distance between two points on the coordinate plane. You can use the model from the Key Concept box to see how the Distance Formula is based on the Pythagorean Theorem as shown below.

$$c^2 = a^2 + b^2$$

Pythagorean Theorem

$$c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Substitute. The length of side a is  $(x_2 - x_1)$ , and the length of side b is  $(y_2 - y_3)$ .

$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Definition of square root



### Example

2. On the map, each unit represents 72 kilometers. City A is located at (1.5, 2) and City B is located at (-1.5, -1.5). What is the approximate distance between City A and City B?

#### Method 1

### Use the Pythagorean

Let c represent the distance between City A and City B. Then a = 3and b = 3.5.

$$a^2 + b^2 = c^2$$

Theorem

$$3^2 + 3.5^2 = c^2$$

$$21.25 = c^2$$

$$\pm \sqrt{21.25} = \sqrt{c^2}$$

±4.6 ≈ c

#### Method 2

#### Use the Distance Formula

Let 
$$(x_1, y_1) = (1.5, 2)$$
 and  $(x_2, y_2) = (-1.5, -1.5)$ .  $c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$   $c = \sqrt{(-1.5 - 1.5)^2 + (-1.5 - 2)^2}$   $c = \sqrt{(-3)^2 + (-3.5)^2}$   $c = \sqrt{9 + 12.25}$   $c = \sqrt{21.25}$   $c \approx \pm 4.6$ 

Since each map unit equals 72 kilometers, the distance between the cities is 4.6 • 72 or about 331 kilometers.

#### Got It? Do this problem to find out.

b. K Field is located at (2.5, 3.5) and L Field at (1.5, 4.5) on a map. If each map unit is 0.16 kilometers, about how far apart are the fields?





b 0.224 kilometers

### Example

### 2. Use the Distance Formula to find actual distances.



- What ordered pair represents City A? (1.5, 2)
  - What ordered pair represents City B? (-1.5, -1.5)



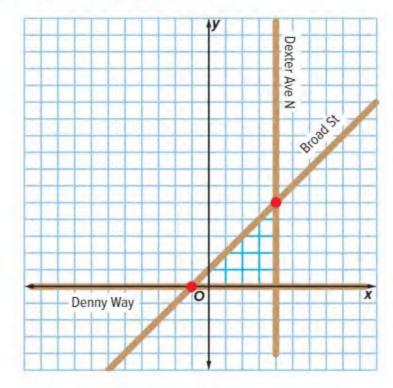
- If you make a right triangle to find the distance, what would be the length of each side? 3 units and 3.5 units
  - If you were to use the Distance Formula, what point would you use for  $(x_1, y_1)$ ? (1.5, 2)
  - What point would you use for (x2, y2)? (-1.5, -1.5)
  - · Once you find the map distance, what do you need to do to find the actual distance? Multiply 4.6 by 45.



BD • Does it matter whether we use the point (1.5, 2) for  $(x_1, y_1)$  or for  $(x_2, y_2)$ ? Explain. no; Sample answer: The distance between the two points will be the same, regardless of which point you consider, the first or second point.

### Need Another Example?

Fahd lives in Seattle, Washington. One unit on this map is 0.08 mile. Find the approximate distance he drives between Broad Street at Denny Way (-1, 0) and Broad Street at Dexter Avenue North (4, 5). 0.57 mi



### Example

### Use the Distance Formula.



• What point would you use for  $(x_2, y_2)$ ? (-3, -2)

 When you substitute the values in the formula, what expression is inside the radical sign?  $(-3-5)^2$  +  $[-2-(-4)]^2$ 

How can you simplify this expression? (-8)<sup>2</sup> + 2<sup>2</sup>, or 68

• Why do we not consider the negative square root? Distance is never negative.

 Suppose you forgot the Distance Formula. How could you determine the distance between these points? Sample answer: Use the Pythagorean Theorem.

### Need Another Example?

Use the Distance Formula to find the distance between G(-3, -2) and H(-6, 5). Round to the nearest tenth, if necessary. 7.6 units

### **Guided Practice**

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

Team-Pair-Solo Have students work in a group of 3-4 students to complete Exercise 1, ensuring that every team member understands. Then have them work in pairs to complete Exercise 2. Individually, they should complete Exercise 3 and then share their responses with their partner. 1, 4, 5, 6

Pairs Present Have students work with a partner to prepare a brief oral presentation about how the Distance Formula is derived from the Pythagorean Theorem. Their presentation should include illustrations. Have them present to the class. (2) 1, 4, 5, 6



### STOP and Reflect

Explain below how to find the length of a nonvertical and a nonhorizontal segment whose endpoints are (x1, y1) and (x2, 42)-

Sample answer: substitute the values for  $x_2$  and  $x_4$  and  $y_2$ and y, into the Distance Formula. Then simplify.

### Example

3. Use the Distance Formula to find the distance between X(5, -4) and Y(-3, -2). Round to the nearest tenth if necessary.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$XY = \sqrt{(-3-5)^2 + [-2-(-4)]^2}$$

 $(x_1, y_1) = (5, -4),$  $(x_2, y_2) = (-3, -2)$ 

$$XY = \sqrt{(-8)^2 + 2^2}$$

$$XY = \sqrt{64 + 4}$$

Evaluate (-8)2 and 22.

$$XY = \sqrt{68}$$

Add 64 and 4.

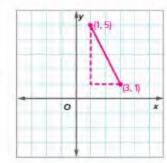
So, the distance between points X and Y is about 8.2 units.

### **Guided Practice**



1. Graph the ordered pairs (1, 5) and (3, 1). Then find the distance between the points. Round to the nearest tenth if necessary. (Example 1)

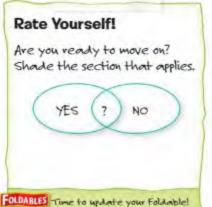
4.5 units



2. On a park map, the ranger station is located at (2.5, 3.5) and the nature center is located at (0.5, 4). Each unit in the map is equal to 0.8 kilometers. What is the approximate distance between the ranger station and the

nature center? (Examples 2 and 3) 1.6 kilometers

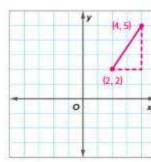
3. Building on the Essential Question How can you use the Pythagorean Theorem to find the distance between two points on the coordinate plane? Sample answer: After you plot the points, draw a right triangle. Use the Pythagorean Theorem to find the length of the hypotenuse which is the distance between the two points.

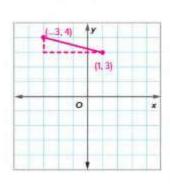


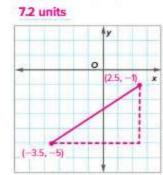
My Homework

### **Independent Practice**

Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary. (Example 1)







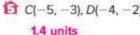


4. A ferry sets sail from an island located at (4, 12) on a map. Its destination is Ferry Landing B at (6, 2). How far will the ferry travel if each

unit on the grid is 0.5 kilometer? (Example 2) about 5.1 km

Use the Distance Formula to find the distance between each pair of points. Round to the nearest tenth if necessary. (Example 3)

7. 
$$K(8\frac{1}{2}, 12), L(-6\frac{3}{4}, 7\frac{1}{2})$$



8. Chicago, Illinois, has a longitude of 88°W and a latitude of 42°N. Indianapolis, Indiana, is located at 86°W and 40°N. At this longitude/latitude, each degree is about 85 kilometers. Find the distance between Chicago and

Indianapolis, about 240 km



# **Practice and Apply**

### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

|         |                  | Exercises    |       |
|---------|------------------|--------------|-------|
|         | 1-7, 13-19       | 8, 9, 20, 21 | 10-12 |
| Level 3 | +                |              | •     |
| Level 2 | - <mark>-</mark> |              |       |
| Level 1 |                  |              |       |

### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                           |
|---------------------------------|-------------------|---------------------------|
| AL                              | Approaching Level | 1–7, 9, 10, 12, 20, 21    |
| 01                              | On Level          | 1–7 odd, 8–10, 12, 20, 21 |
| B                               | Beyond Level      | 8–12, 20, 21              |

| MATHEMATICAL PRACTICES  |             |
|---|-------------|
| Emphasis On   | Exercise(s) |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 11          |
| 2 Reason abstractly and quantitatively.                                       | 12          |
| 3 Construct viable arguments and critique to<br>reasoning of others.          | he 9        |
| 5 Use appropriate tools strategically.  | 10, 16      |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

### TICKET Out the Door

Have students explain how their knowledge of the Pythagorean Theorem helped them to find the distance between two points on a coordinate plane.

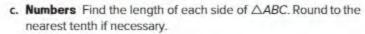
See students' work.

### Watch Out!

**Common Error** When the Distance Formula calculations are done mentally, it is easy to incorrectly subtract a negative number from another number or replace a variable with an incorrect value. Encourage students to designate which values are  $x_1, x_2, y_1$ , and  $y_2$ , write the formula and all the steps, and then evaluate the formula.

- Multiple Representations Points A(-2, 1), B(-2, 6), and C(1, 3) are the vertices of a triangle.
  - a. Graphs Graph the points A(-2, 1), B(-2, 6) and C(1, 3).
  - Words Explain how to find the length of segment BC.
     Sample answer: Use the Distance Formula and the points

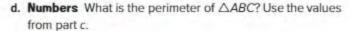
(-2, 6) and (1, 3).



 $AC \approx 3.6$  units

AB = 5 units

BC ≈ 4.2 units



perimeter = 12.8 units



### H.O.T. Problems Higher Order Thinking

10. Use Math Tools Layla needs to find the distance between the points A(-2.4, 3.7) and B(4.5, -1.4). Suggest a tool she could use to find the length. Then find the length. Explain your reasoning.

Sample answer: Calculator; it will be most helpful when squaring and

finding the square root involving decimals; about 8.6 units.

11. Persevere with Problems Apply what you have learned about distance on the coordinate plane to write the coordinates of two possible endpoints of a line segment that is neither horizontal nor vertical and has a length of 5 units.

Sample answer: (1, 2) and (4, 6)

 Reason Inductively Compare the steps to find the distance between two points on the coordinate plane by first using the Pythagorean Theorem and then using the Distance Formula.

Sample answer: To use the Pythagorean Theorem, connect the points to form

a right triangle. Then use the Pythagorean Theorem to find the length of the

hypotenuse. To use the Distance Formula, replace  $(x_1, y_1)$  and  $(x_2, y_2)$  in the

formula with the coordinates of the two endpoints and simplify.

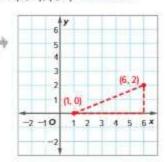
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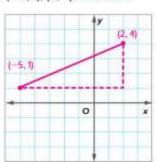
WW486- JAGE J. Grage Carmine Pythagorean Theorem

My Homework

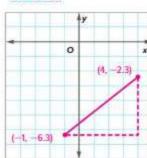
### **Extra Practice**

Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.







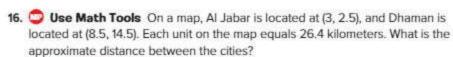


$$a^2 + b^2 = c^2$$

$$2^2 + 5^2 = c^2$$

$$29 = c^2$$

± √29 = √c2 15.4 ≈ c



about 348.5 km

Use the Distance Formula to find the distance between each pair of points. Round to the nearest tenth if necessary.

3.9 units

**19.** 
$$P(-9\frac{1}{4}, -7\frac{1}{2}), Q(-4, 5)$$

# Power Up! Test Practice

Exercises 20 and 21 prepare students for more rigorous thinking needed when taking assessment.

| 20. | This test item requires students to analyze and solve complex real-<br>world problems through the use of mathematical tools and models. |   |  |
|-----|---|---|--|
|     | Depth of Knowledge  | DOK3  |  |
|     | Mathematical Practices  | MP1, MP4, MP5   |  |
|     | Scoring Rubric  |   |  |
|     | 2 points  | Students correctly plot and connect the points and also find the shortest combined distance.  |  |
|     | 1 point   | Students correctly plot and connect the points, but fail to find the shortest combined distance OR students incorrectly plot points but draw lines and find a |  |

| 21. | This test item requires students to reason abstractly and quantitatively when problem solving. |  |  |  |
|-----|--|--|--|--|
|     | Depth of Knowledge   | DOK1   |  |  |
|     | Mathematical Practices   | ctices MP1, MP2, MP5                                 |  |  |
|     | Scoring Rubric   |  |  |  |
|     | 1 point  | Students correctly answer each part of the question. |  |  |

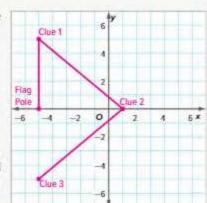
distance based on the incorrect points.

### Power Up! Test Practice

20. Mr. Mansour is using a coordinate plane to design a treasure hunt for his students. The hunt begins at the flagpole. The first clue is hidden 5 units north of the flagpole. The second clue is located 6 units east of the flagpole. Clue 2 says that Clue 3 is located 5 units south of the flagpole.

Plot the locations of the flagpole and the 3 clues on the coordinate grid and show the path students will follow with straight lines.

Each unit represents 15 meters. What is the shortest combined distance along the path from the flagpole to Clue 1 to Clue 2 to Clue 3? Round to the nearest foot if necessary.

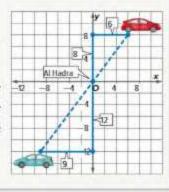


310 meters

21. Two cars leave a house in Al Hadra. The first car travels 8 kilometers north and then 6 kilometers east. The second car travels 12 kilometers south and then 9 kilometers west. Determine if each statement is true or false.

a. The first car is 10 kilometers from Al Hadra.

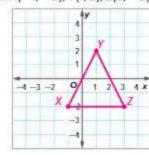
b. The second car is 15 kilometers from Al Hadra.
True
False
True
False



### **Spiral Review**

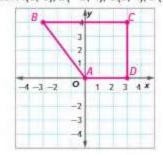
Graph each set of points on the coordinate plane. Then connect the points and identify the figure drawn. 6.6.3

22. X(-1, -2), Y(1, 2), Z(3, -2



isosceles triangle

23. A(0, 0), B(-3, 4), C(3, 4), D(3, 0)



right trapezoid

# 2 ST CENTURY CAREER

in Travel and Tourism

### **Travel Agent**

www.jnob-jo.com

Do you love to travel? Would you be interested in helping others plan their ideal vacation getaways? You should consider a career in travel and tourism. Travel agents offer advice on destination locations and make arrangements for transportation, lodging, car rentals, and tours for their clients. In addition to having personal travel experience and being knowledgeable about popular vacation destinations, travel agents also need to be detail-oriented and have excellent communication, math, and computer skills.



### Is This the Career for You?

Are you interested in a career as a travel agent? Take some of the following courses in high school.

- Algebra
- Business Software **Applications**
- Computer Technology
- Geometry

Turn the page to find out how math relates to a career in Travel and Tourism.

### Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace.

This lesson emphasizes Mathematical Practice 4 Model with Mathematics.

### Coherence connecting within and across grades

### **Previous**

Students used the Distance Formula to find lengths of sides of a triangle.

### Now

Students apply the content standard to solve problems in the workplace.

### Rigor pursuing concepts, fluency, and applications See the Career Project on page 440.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

### **Launch the Lesson**

Ask students to read the information on the student page about travel agents and answer the following questions.

#### Ask:

- What kind of classes should you take to be a travel agent? Algebra, Business Software Applications, Computer Technology, Geometry
- What is one aspect of travel that a travel agent needs to be knowledgeable about? Sample answer: A travel agent needs to be knowledgeable about popular vacation destinations.

# **2** Collaborate

circle the Sage Poll the class to see which students have knowledge about the Pythagorean Theorem. Those students (the sages) spread out around the room. Assign the rest of the students to teams. Have the teams split up with each team member going to a different sage, if possible. Have the sages lead work for Exercises 1–6. When the exercises are complete, students go back to their teams and compare solutions. Students discuss how the sages may have explained the steps differently. 1, 2, 3, 4, 5, 6, 7

### Ask:

 How do right triangles help in reading a map and finding distances? Sample answer: Right triangles help find distances that are not easily distinguishable.

BL (A) Pairs Discussion Have students work in pairs to extend the activity by answering the following question. (2)1,4,5,6,7

#### Ask:

 Describe the locations of Marathon, Cudjoe Key, and Islamorada using ordered pairs. Why is Cudjoe Key closer to Marathon than Islamorada? Sample answer: There are about 17.68 miles between Cudjoe Key and Marathon and about 20.16 miles between Islamorada and Marathon.

### Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

### Career Facts

Agents with backgrounds in computer science, geography, communication, foreign languages, or world history often have a greater chance of being hired by travel agencies. These backgrounds show that the agents have an interest in travel and culture, which clients find appealing.

### Time to Get Away!

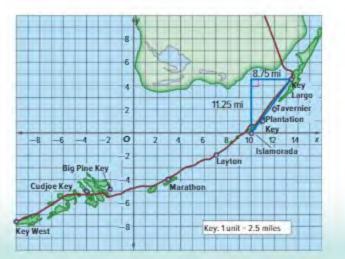
Use the map to solve each problem. Round to the nearest tenth if necessary.

- What is the approximate distance between Key Largo and Islamorada? 23 kilometers
- Draw and label a right triangle to find the distance between Plantation Key and Islamorada. Then find the approximate distance.

   5.6 kilometers
- Describe the ordered pairs that represent Layton and Plantation Key. Then find the approximate distance between Layton and Plantation Key. Layton: (7, -2); Plantation Key: (11, 1); 20 kilometers
- To the nearest 0.5 unit, name the ordered pairs that represent Key West and Cudjoe Key. Then use the ordered pairs to estimate the distance between the keys.

Sample answer: Key West (-10, -7.5); Cudjoe Key (-4, -5); 26 kilometers

- What is the approximate distance betweenKey West and Layton? Sample answer: 71.5 kilometers
- 6. What is the approximate distance between Tavernier and Big Pine Key? Sample answer: 62.5 kilometers



### Career Project

It's time to update your career portfolio! Go online and research a career as a travel agent.

Describe three things that you learned about being a travel agent that you did not know.

List several jobs that are created by the travel and tourism industry.

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### Chapter Review 🗑



### **Vocabulary Check**



Fill in the blank with the correct vocabulary term.

- 1. A line that intersects two or more lines is called a transversal
- 2 Corresponding angles are those angles that are in the same position on the two lines in relation to the transversal.
- 3 Deductive reasoning rules, definitions, or laws to make conjectures from given situations.
- 4. A statement or conjecture that has been proven and can be used as a reason to justify statements in other proofs is called
  - a theorem
- 5. The Pythagorean Theorem describes the relationship between the lengths of the legs and the hypotenuse of any right triangle.

- 6. Interior angles that lie on opposite sides of the transversal are called
  - alternate interior angles
- 7. Two lines that are in the same plane and do not intersect are called parallel lines
- 8 Inductive reasoning is the process making a conjecture after observing several examples.
- 9. The hypotenuse is the side opposite the right angle in a right triangle.
- 10. A polygon that is equilateral (all sides the same length) and equiangular (all angles have the same measure) is called a regular polygon

### **Vocabulary Check**

Rally Coach Have students work in pairs to complete the Vocabulary Check. Have Student A complete the first exercise, speaking out loud, while Student B listens carefully, coaches, and praises. Next have Student B complete the second exercise while Student A listens carefully, coaches, and praises. Partners take turns until they have completed the Vocabulary Check. 21,6

### **Alternate Strategy**

To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- alternate interior angles (Lesson 1)
- corresponding angles (Lesson 1)
- deductive reasoning (Lesson 2)
- hypotenuse (Lesson 3)
- inductive reasoning (Lesson 2)
- parallel lines (Lesson 1)
- Pythagorean Theorem (Lesson 3)
- regular polygon (Lesson 4)
- theorem (Lesson 3)
- transversal (Lesson1)

### **Key Concept Check**

FOLDABLES (IA) A completed Foldable for this chapter should include a review of the Pythagorean Theorem and the Distance Formula.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

### Ideas for Use

Have students work in pairs to discuss their Foldables. Have them practice speaking out loud in a collaborative setting by sharing how they have completed their Foldable thus far and how they could finish it. Have each student complete their Foldable and trade with their partner to discuss any similarities and differences. 6 1, 4

### Got It?

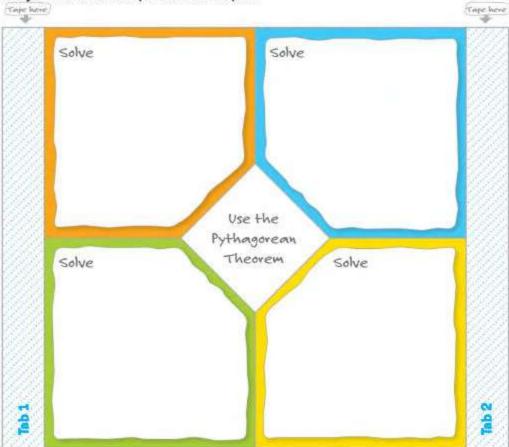
If students have trouble with Exercises 1-4, they may need help with the following concept(s).

| Concept                                    | Exercise(s) |
|--|-------------|
| identifying angle relationships (Lesson 1) | 1–4         |

### **Key Concept Check**

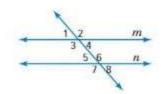
Use Your FOLDABLES

Use your Foldable to help review the chapter.



### Got it?

Use the figure at the right. Circle the correct word to complete each sentence.



- 1. Angles 2 and 6 are examples of (vertical corresponding) angles.
- 2. Angles 1 and 8 are examples of alternate (interior exterior) angles.
- 3. The angle that is corresponding to angle 8 is angle (3,4).)
- 4. If lines m and n are parallel, then angles 3 and (6,8) have equal measures.

WW442. JANGE 5 J Cong Conflict Pythagorean Theorem

### Power Up! Performance Task

#### Under Construction

The Transport Authority plans to build a highway through the city. The proposed highway will intersect Road A and Road B. Road A runs east to west, and Road B runs north to south so the two streets are perpendicular. The city created a proposal for a construction company as shown below.

#### Proposal for Brook Highway

- . The distance on Road B from Road A to the highway will be five kilometers.
- The proposed highway will make a 32-degree angle with Road A.
- . The distance on Road A from Road B to the highway will be 7.7 kilometers.

Write your answers on another piece of paper. Show all your work to receive full credit.

#### Part A

Draw a map that represents the proposed plan. Label the given information on the map. What is the measure of the angle that is between Road B and the proposed highway? Explain your answer.

#### Part B

What will be the length of the section of the highway from Road A to Road B? Round your answer to the nearest tenth of a kilometer.

#### Part C

Mr. Ali currently drives the Road B and Road A route five days a week, to and from work. If he gets 40 kilometers to 4 liters of gasoline, how many liters will he save every week when the highway is completed? Round to the nearest tenth.

#### Part D

Road C runs parallel to Road A and also intersects the highway. The city requires the construction company to put a traffic signal if the intersection forms an angle of more 150". Does the construction company have to put a signal? Explain your reasoning.

# Power Up! Performance Task

This Performance-Based Assessment requires students to solve multi-step problems through abstract reasoning, precision, and perseverance. This practice scenario can be used to help students prepare for the thinking skills that will be used during assessment.

A complete scoring rubric with answers to the Exercises can be found on page PT1.



### **Answering the Essential Question**

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- How are the measures of angles related when parallel lines are cut by a transversal? (p. 374)
- How is deductive reasoning used in algebra and geometry proofs? (p. 382)
- How can you find the missing measure of an angle in a triangle if you know the measure of two of the interior angles? (p. 392)
- How can I find the sum of the interior angle measurse of a polygon? (p. 400)
- What is the relationship among the legs and the hypotenuse of a right triangle? (p. 414)
- How do you solve a right triangle? (p. 426)
- How can you use the Pythagorean Theorem to find the distance between two points on the coordinate plane?
   (p. 434)

### **Ideas for Use**

Think-Pair-Share Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their classmate before they complete the graphic organizer.



### **Track Your Progress**

Return to the beginning of the chapter to review the objectives that it addressed. Students should see that their knowledge of the key ideas has increased now that they have completed this chapter.

### Reflect



### **Answering the Essential Question**

Use what you learned about triangles and the Pythagorean Theorem to complete the graphic organizer. List three ways you used algebra in this chapter. Draw a model to represent each way.

Sample answers are given. See students' work for models.



### **Essential Question**

HOW can algebraic concepts be applied to geometry?

| Find the missing value in a | Find the measure of an      | Use the Pythagorean           |
|-----------------------------|-----------------------------|-------------------------------|
| triangle.                   | exterior angle in a regular | Theorem to find the length of |
|                             | hexagon.                    | the missing side.             |
|                             |                             |                               |
|                             |                             |                               |
|                             |                             |                               |
|                             |                             |                               |
|                             |                             |                               |
|                             |                             |                               |
| A SHAKE SHEET IN EATHER AND |                             |                               |

|       | Answer the Essential Question. | HOW can algebraic concepts be applied |
|-------|--------------------------------|---------------------------------------|
| to ge | eometry?                       |                                       |

See students' work.

Afternoon and a second section of the second

# Chapter 6



### **Essential** Question

HOW can we best show or describe the change in position of a figure?

Mathematical Practices 1, 2, 3, 4, 5, 7, 8



### Math in the Real World

Nature Line symmetry occurs frequently in nature. A figure has line symmetry when a line can be drawn so that one half of the figure is a mirror image of the other other half.

On the figure below, draw the line of symmetry.



Study Organizer Cut out the Foldable from the end of the book.

FOLDABLES

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Place your Foldable at the end of the chapter.

Use the Foldable throughout this chapter to help you learn about transformations.

### Focus narrowing the scope

This chapter focuses on content from the Geometry (G) domain.

### Coherence connecting within and across grades

### Previous

Students used the Pythagorean Theorem.

### Now

Students study the effects of various types

### Next

Students will explore congruence and similarity

### Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

### **Launch the Chapter**

### Math in the Real World

Nature Remind students that a line of symmetry will always go through the center of an object.

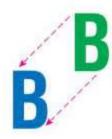
### What Tools Do You Need?

### **Vocabulary Activity**

As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

**Define:** A transformation is an operation that places an original figure, the preimage, onto a new figure, the image.

### Example:



#### Ask:

· What type of transformation is shown above? translation

### **Review Vocabulary**

Have students read the Review Vocabulary section.

Students should be able to graph ordered pairs in all four quadrants of the coordinate plane.

Remind students that the signs of the coordinates in an ordered pair indicate in which quadrant the point is located.

(+, +) Quadrant I

(-, +) Quadrant II

(-, -) Quadrant III

(+, -) Quadrant IV

### What Tools Do You Need?



### Vocabulary

angle of rotation dilation reflection center of dilation image rotation

center of cliation image rotation

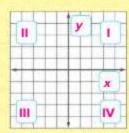
center of rotation line of reflection rotational symmetry

congruent preimage transformation

translation

### **Review Vocabulary**

**The Coordinate Plane** The x- and y-axes divide the coordinate plane into four regions called quadrants. Label the axes and the quadrants on the coordinate plane shown.



Quadrilateral ABCD has vertices A(1, 1), B(3, 5), C(4, 7), and D(2, 6).

- 1. In what quadrant is ABCD located?
- 2. Suppose you multiplied the coordinates of ABCD by  $\frac{3}{4}$ . In what quadrant would the new figure be located?
- Suppose the x-coordinates in ABCD are multiplied by -1. In what quadrant would the new figure be located?
- 4. Suppose you switched the x- and y-coordinates from Exercise 3. In what quadrant would the new figure be located?

rebladur = subalden

### **What Do You Already Know**

Read each statement. Decide whether you agree (A) or disagree (D). Place a checkmark in the appropriate column and then justify your reasoning. See students' work.

| Transformations  |   |   |      |
|--|---|---|------|
| Statement  | A | D | Why? |
| When translating a figure, every point is moved the same distance.   |   |   |      |
| All figures have at least one line of symmetry.  |   |   |      |
| A reflection is a flip of a figure over a line of reflection.  |   |   |      |
| Rotations change a figure's orientation.   |   |   |      |
| Figures that have undergone a dilation are not congruent.  |   |   |      |
| When a figure is dilated by a scale factor less<br>than 1, the dilated figure is larger than the<br>original figure. |   |   |      |

Here is an example of how transformations are used in the real world.

Activity How do you learn a series of moves in an exercise regime? Describe an exercise that you like to do. How did you learn it? Was it easy to learn?

See students' work.



### **What Do You Already**

In this activity students assess their prior knowledge by determining whether they agree or disagree with each statement about concepts in this chapter.

- · You may want to add a third option of "I don't know" for those students who do not have any prior knowledge of the content of the statement.
- · After completing the chapter, have students return to this page and see if any of their responses would change now that they have finished the chapter.

# When Will You Use Thi

### Activity

Students may not realize how much they already know about transformations. This activity shows the real-world connection with transformations.

Use this page to determine if students have skills that are needed for the chapter.



### **Quick Review**

Students with strong math backgrounds may opt to go directly to the Quick Check.

| REVIEW  | REVIEW                         |  |
|---------|--------------------------------|--|
| Example | Skill                          |  |
| 1       | Graph on the coordinate plane. |  |
| 2       | Add integers.                  |  |



#### Quick Check

If students have difficulty with the exercises, present an additional example to clarify any misconceptions.

#### Exercises 1-3

Two vertices of a rectangle are A(-2, 4) and B(3, 4). The height of the rectangle is 3 units. Graph the rectangle and label the other two vertices. See Answer Appendix.

#### Exercises 4-11

Find 12 + (-4). 8

### **Track Your Progress**

Prior to beginning this chapter, have your students rate their knowledge of the objectives it addresses. At the end of the chapter, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge of the key ideas has increased.



### Try the Quick Check below

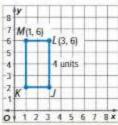


### Quick Review

Review

### Example 1

Two vertices of a rectangle are J(3, 2) and K(1, 2). The length of the rectangle is 4 units. Graph the rectangle and label the other two vertices.



### Example 2

Find 2 + (-6).

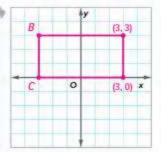
2 + (-6) = -4

|2| - |-6| = -4The sum is negative because |-6| > |2|

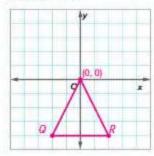
### Quick Check

### Coordinate Plane Graph each figure and label the missing vertices. Sample answers: 1 and 2

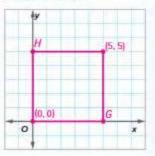
1. rectangle with vertices: B(-3, 3), C(-3, 0); side length; 6 units



2. triangle with vertices: Q(-2, -4), R(2, -4);height: 4 units



3. square with vertices: G(5, 0), H(0, 5); side lengths: 5 units



### Integers Add.

**6.** 
$$-4 + (-9) = -13$$
 **7.**  $-2 + 8 = 6$ 

9. 
$$0+(-6)=-6$$

**10.** 
$$-8+2=-6$$

11. 
$$3+(-1)=2$$

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.



















### **Inquiry Lab**

Transformations



Step 3

#### WHAT are some rigid motions of the plane?

Mathematical **Practices** 1.3

Animated movies are created using frames. Each frame changes slightly from the previous one to create the impression of movement.

### Hands-On Activity 1

In this Activity, you will make animation frames using index cards.

Arrange ten index cards in a pile. On the top card, draw a circle at the top right hand corner.



Step 2 On the next card, draw the same circle slightly down and to the left.



Repeat this for three or four more cards until your circle is at the bottom of the card. Use the remainder of the cards to draw the circle up and to the left.



Step 4 Place a rubber band around the stack, hold the stack at the rubber band, and flip the cards from front to back.



Describe what you see when you flip the cards from front to back.

Sample answer: The circle moving is like a ball

bouncing on the ground.

Look at the circles on the first and second cards and then the second and third cards. How would you describe the change in the position of the circle from one card to the next?

Sample answer: The circle moved  $\frac{1}{4}$  inch down and  $\frac{1}{4}$  inch to the left.

Did the shape or size of the circle change when you moved it? If yes, describe the change. no



### Focus narrowing the scope

Objective Identify and apply flips, slides, and turns.

### Coherence connecting within and across grades

Students will identify properties of reflections, translations, and rotations.

Students will graph transformations on a coordinate plane.

### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 451.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

### Launch the Lab

Activities 1, 2, and 3 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activities 2 and 3.

Materials: index cards, circular chips, tracing paper

### Hands-On Activity 1

Pairs Discussion Provide students with index cards and a circular chip or disk for them to trace. Have them work with a partner, following the directions in Steps 1-4 and answering the questions. Then have students compare drawings and responses to the questions with a partner. 1, 5

BD Pairs Discussion Students may wish to draw something more detailed than a circle. Have them work with a partner to draw a more complicated shape. Make sure that they can draw the shape in the same way on each index card, changing only the position of the shape. 21,5

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### Hands-On Activity 2

Paired Heads Together Have students work with a partner to complete the activity. Each student is assigned a number. Provide students with tracing paper, a protractor, and a ruler. Have them follow the steps using the given tools. Then have them complete the questions. Upon completion of the activity, call on one numbered student to share their responses with the class. 01, 5, 6, 7

BD (IA) Pairs Discussion Provide students with tracing paper, a protractor, and a ruler. Have them draw an angle that is not a right angle and label the points as in the directions. Have them follow the steps using the given tools. Then have them work in pairs to check each other's work and answer the questions. 01, 5, 6, 7

### Hands-On Activity 3

Solo to Groups Provide students with tracing paper. Have them work individually to follow the steps, making sure to rotate in the correct direction. Then have students work in small groups to discuss the questions. 21, 5, 6, 7

(B) (IA) Pairs Consult Have students work in pairs to complete the activity and the questions. Then have them answer the following extension question. 21, 5, 6, 7

### Ask:

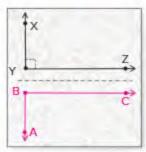
 The orientation of a figure is determined by the order in which the vertices are named. If you walk around the vertices W, Y, Z, and X in the original figure, you walk clockwise. What happens if you walk around the vertices W, Y, Z, and X after the turn? What does this tell you about the figure's orientation after the turn? Sample answer: You still walk clockwise. So, orientation is preserved after the turn.

### Hands-On Activity 2

Step 1

Draw right angle XYZ on a piece of tracing paper. Place a dashed line on the paper as shown.

Step 2 Fold the paper along the dashed line. Trace the angle onto the folded portion of the paper. Unfold and label the angle ABC so that A matches up with X, B matches up with Y, and C matches up with Z. Tape the paper to your book.



Use a protractor to find the measure of ZXYZ and ZABC. Did the

measure of the angle change after the flip? 90°; 90°; no

Use a centimeter ruler to measure the shortest distance from X and A to the dashed line. Repeat for Y and B and for Z and C. What do you notice?

See students' work; Sample answer: The distance from the

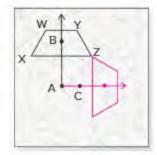
original image to the dotted line is the same as the distance

from the image to the dotted line.

### Hands-On Activity 3

Step 1

Place a piece of tracing paper over the trapezoid shown. Copy the trapezoid. Draw points A, B, and C. Draw AB



Step 2

Place the eraser end of your pencil on A. Turn the tracing paper until AB passes through C. Tape the paper to your book.

Did the shape of the trapezoid change when you moved it? If yes, describe

the change. no

Did the size of the trapezoid change when you moved

it? If yes, describe the change. no

Geometr



### Investigate

Work with a partner. Use a ruler to draw the image when each figure is moved as directed.

1. 1 centimeter down and 2 centimeters to the left.

2. 2 centimeters up and 2 centimeters to the right.







Draw the image when each figure is flipped over line  $\ell$ .

3

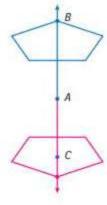


4.

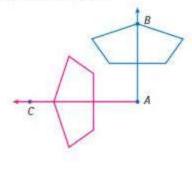


Draw the image when each pentagon is turned until  $\overrightarrow{AB}$  passes through C.

5.



6.



- 7. Refer to Exercises 1-6.
  - a. For which exercises, if any, did the size of the original figure change?

попе

- **b.** For which exercises, if any, did the shape of the original figure change?
- c. For which exercises, if any, did the orientation of the original figure change?

Exercises 3, 4, and 5

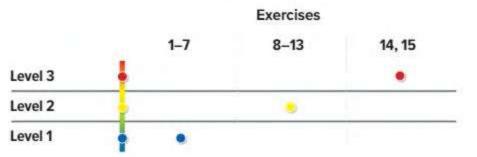
ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

### 2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.





Exercise 1 in front of the class while students watch and listen carefully. Then have students complete Exercise 2 on their own. Continue this process for Exercises 3–6. Then have students discuss Exercise 7 with a partner. \$\infty\$1, 5, 6, 7

#### Ask:

- For Exercises 3 and 4, what tool could we use in place of the dotted line to help draw the new image? a mirror
- BL LA Roundrobin Have students work in pairs. Have Student 1 complete Exercises 1, 3, and 5 while Student 2 completes Exercises 2, 4, and 6. Then have students exchange solutions and check each other's work. Have them discuss and respond to Exercise 7. 1, 5, 6, 7



### **Analyze and Reflect**

- Pairs Discussion Have students use tracing paper to complete Exercises 8 and 9 as they did in the activities. Then have them work with a partner to complete Exercises 10–12. 1, 5, 6, 7
- In which either a slide, flip, or turn has taken place. Then have students place all of their drawings in the center of the room. Have students pop up out of their seats to randomly select one drawing and classify it as a slide, flip, or turn. Have them place the drawing into a pile labeled either Slide, Flip, or Turn.

### **Create**

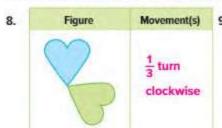
Pairs Present Have pairs research online or look indoors and outdoors for real-world examples of slides, flips, and turns. Have them explain to the class what they discovered and how they determined the type of transformation. 1, 4, 5, 6, 7

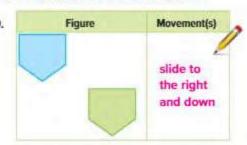
Students should be able to answer "WHAT are some rigid motions of the plane?" Check for student understanding and provide guidance, if needed.



### **Analyze and Reflect**

For each pair of figures, describe a movement or movements that will place the blue figure on top of the green figure. 8-9. Sample answers are given





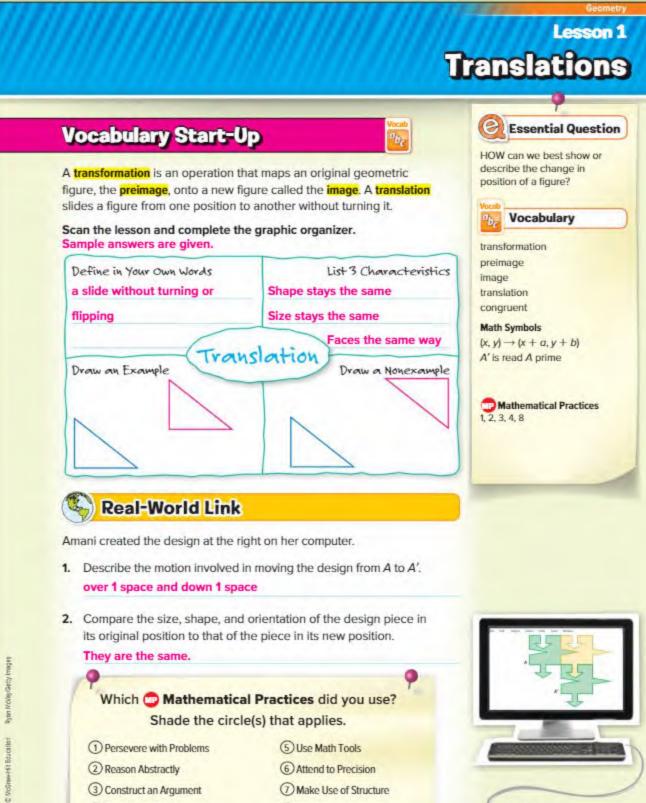
- Refer to Activity 1 and Exercises 1 and 2. Circle the word that best describes the movement of the figures: flip slide turn
- Refer to Activity 2 and Exercises 3 and 4. Circle the word that best describes the movement of the figures: flip slide turn
- 12. Refer to Activity 3 and Exercises 5 and 6. Measure one side of the original figures. Then measure that same side after the turn. Did the length of the side change after you turned it? If yes, describe the change.
- Justify Conclusions In Activity 3, WY and XZ are parallel. Were the segments still parallel after the turn? Would they still be parallel after a slide? flip? Explain.

yes; yes; Sample answer: Since the size of the figure does not change in any of the movements, the distance between the two lines is the same, so the lines remain parallel.



### Create

- 14. Reason Inductively Slides, flips, and turns are called rigid motions of the plane. Based on the Activities, describe two characteristics of a rigid motion of the plane. Sample answer: The shape and the size of a figure do not change in a rigid motion of the plane.
- 15. WHAT are some rigid motions of the plane? Slides, flips, and turns are some rigid motions of the plane.



8 Use Repeated Reasoning

### Focus narrowing the scope

Objective Graph translations on the coordinate plane.

### Coherence connecting within and across grades

### Previous

Students identified the properties of translations.

### Now

Students will graph translations on the coordinate plane.

### Next

Students will graph reflections on the coordinate plane.

### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 457.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

### **Launch the Lesson**

### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Think-Pair-Share Give students about one minute to think through their responses to the graphic organizer on the student page. Then have them share their responses with a partner. 21, 5, 6

### Alternate Strategy

 Have students describe what a translation is in everyday life, such as the translation of a word from Spanish to English. Then have them explain the meaning of the term translation in mathematics. 21, 5, 6

# **2** Teach the Concept

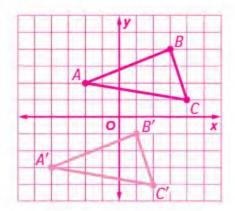
**Ask** the scaffolded questions for each example to differentiate instruction.

### Example

- 1. Translate a figure in the coordinate plane.
- How do we graph coordinate J? Start at the origin and move 3 units left and then 4 units up. coordinate K? Start at the origin and move 1 unit right and 3 units up. coordinate L? Start at the origin, move 4 units left, then 1 unit up.
- If point J(-3, 4) is moved two units right and 5 units down, what are the coordinates of point J? (-1, -1)
  - If point K(1, 3) is moved two units right and 5 units down, what are the coordinates of point K'? (3, -2)
  - If point L(-4, 1) is moved two units right and 5 units down, what are the coordinates of point L'? (-2, -4)
- Was the orientation of the image changed? no
  - Are the two images congruent? yes

### Need Another Example?

Graph  $\triangle ABC$  with vertices A(-2, 2), B(3, 4), and C(4, 1). Then graph the image of  $\triangle ABC$  after a translation 2 units left and 5 units down. Write the coordinates of its vertices. A(-4, -3), B'(1, -1), C'(2, -4)



### Key Concept

### **Translations in the Coordinate Plane**

Work Zone

Prime Symbols
Use prime symbols for
vertices in a transformed

image. A→A'

 $B \rightarrow B'$  $B \rightarrow B'$ 

A' is read A prime.

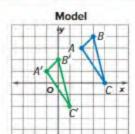
A'(0, 0), B'(-4, 5)

and C'(1, 4)

Words When a figure is translated, the x-coordinate of the preimage changes by the value of the horizontal translation a.

> The y-coordinate of the preimage changes by the vertical translation b.

Symbols  $(x, y) \longrightarrow (x + a, y + b)$ 

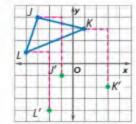


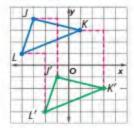
When translating a figure, every point of the preimage is moved the same distance and in the same direction. The image and the preimage are congruent. Congruent figures have the same shape and same size. So, line segments in the preimage have the same length as line segments in the image. Angles in the preimage have the same measure as angles in the image.

### Example

Graph △JKL with vertices J(-3, 4), K(1, 3), and L(-4, 1). Then graph the image of △JKL after a translation 2 units right and 5 units down. Write the coordinates of its vertices.

Move each vertex of the triangle 2 units right and 5 units down. Use prime symbols for the vertices of the image.





From the graph, the coordinates of the vertices of the image are J'(-1, -1), K'(3, -2), and L'(-2, -4).

Got it? Do this problem to find out.

a. Graph △ABC with vertices A(4, -3), B(0, 2) and C(5, 1). Then graph its image after a translation of 4 units left and 3 units up. Write the coordinates of the image.

### Example

2. Triangle XYZ has vertices X(-1, -2), Y(6, -3) and Z(2, -5). Find the vertices of  $\triangle X'Y'Z'$  after a translation of 2 units left and 1 unit up.

Use a table. Add -2 to the x-coordinates and 1 to the y-coordinates.

| Vertices of<br>△XYZ | (x+(-2),y+1)       | Vertices of<br>△X'Y'Z' |
|---------------------|--------------------|------------------------|
| X(-1, -2)           | (-1+(-2), -2+1)    | X'(-3, -1)             |
| Y(6, -3)            | (6+(-2), -3+1)     | Y'(4, -2)              |
| Z(2, -5)            | (2 + (-2), -5 + 1) | Z'(0, -4)              |

So, the vertices of  $\triangle X'Y'Z'$  are X'(-3, -1), Y'(4, -2), and Z'(0, -4).

#### Got it? Do this problem to find out.

b. Quadrilateral ABCD has vertices A(0, 0), B(2, 0), C(3, 4), and D(0, 4). Find the vertices of quadrilateral A'B'C'D' after a translation of 4 units right and 2 units down.

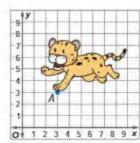


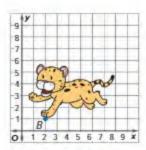
B'(6, -2), C'(7, 2), b. and D'(4, 2)



### Example

A computer image is being translated to create the illusion of movement. Use translation notation to describe the translation from point A to point B.





b = -2

Point A is located at (3, 3). Point B is located at (2, 1).

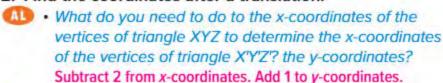
$$(x, y) \longrightarrow (x + a, y + b)$$
  
 $(3, 3) \longrightarrow (3 + a, 3 + b) \longrightarrow (2, 1)$   
 $3 + a = 2$   $3 + b = 1$ 

So, the translation is (x - 1, y - 2), 1 unit to the left and 2 units down.

a = -1

### **Examples**

Find the coordinates after a translation.



- Why do you subtract 2 from the x-coordinates? because translation is 2 units to the left
  - Why do you add 1 to the y-coordinates? because the translation is 1 unit up
- If the translation was 1 unit down instead of 1 unit up, how would it affect the operation you perform on the y-coordinates? We would subtract 1 instead of adding 1.

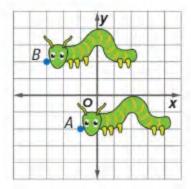
### Need Another Example?

Rectangle ABCD has vertices A(-3, 2), B(2, 2), C(2, -3), and D(-3, -3). Find the vertices of rectangle A'B'C'D' after a translation of 4 units right and 2 units down. A'(1, 0), B'(6, 0), C'(6, -5), D'(1, -5)

- Describe translations.
- What are the coordinates of point A? (3, 3) point B? (2, 1)
- What do you need to do to the x-coordinate of point A to get the x-coordinate of point B? Subtract 1. the y-coordinate? Subtract 2.
- How can you check to make sure your solution is correct? Sample answer: Check several other points on the picture to see if the movement is the same for each.

### Need Another Example?

The character below was translated from point A to point B. Use translation notation to describe the translation. (x-2, y+4)



### **Guided Practice**

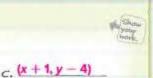
**Formative Assessment** Use these exercises to assess students' understanding of the concepts in this lesson.



If some of your students are not ready for assignments, use the differentiated activities below.

Paired Heads Together Have a student volunteer to lead a class discussion on how to translate the figure in Exercise 1. Then have students complete Exercise 2 with a partner, ensuring that each partner understands how to translate a figure. Have a student-volunteer draw the image on the board. If students are ready, have them complete Exercises 3–5 on their own. If not, have them work with a partner, while you walk around the room and monitor their progress. 1, 5, 6, 7

Trade-a-Problem Have students draw an image on a coordinate grid. Then have them write instructions in coordinate form, such as translation notation, for how to translate the image. Have students trade graph paper and translate each other's images using the translation notations only. 1, 5, 6, 7



#### Got it? Do this problem to find out.

c. Refer to the figure in Example 3. If point A was at (1, 5), use translation notation to describe the translation from point A to point B.

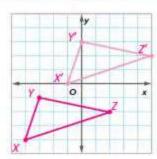
### **Guided Practice**



Graph  $\triangle XYZ$  with vertices X(-4, -4), Y(-3, -1), and Z(2, -2). Then graph the image of  $\triangle XYZ$  after each translation, and write the coordinates of its vertices. (Example 1)

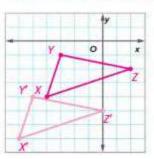
1. 3 units right and 4 units up

X'(-1, 0), Y'(0, 3), Z'(5, 2)

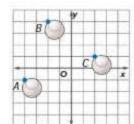


2. 2 units left and 3 units down

$$X'(-6, -7), Y'(-5, -4), Z'(0, -5)$$

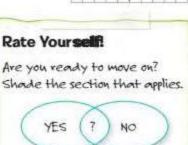


 The baseball at the right was filmed using stop-motion animation so it appears to be thrown in the air. Use translation notation to describe the translation from point A to point B. (Example 3) (x + 2, y + 5)



- Quadrilateral DEFG has vertices at D(1, 0), E(-2, -2), F(2, 4), and G(6, -3). Find the vertices of D'E'F'G' after a translation of 4 units right and 5 units down. (Example 2) Sample answer: D'(5, -5), E'(2, -7), F'(6, -1), and G'(10, -8)
- 5. Building on the Essential Question How are figures translated on the coordinate plane?
  Sample answer: They are slid up or down and right

or left.



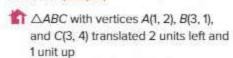
FOLDABLES Time to update your Foldable!

Buch & Accessing Street

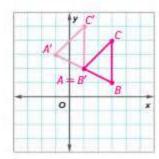
My Hamework

### **Independent Practice**

Graph each figure with the given vertices. Then graph the image of the figure after the indicated translation, and write the coordinates of its vertices. (Example 1)

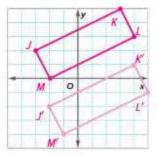


A'(-1, 3), B'(1, 2), C'(1, 5)



2. rectangle JKLM with vertices J(-3, 2), K(3, 5), L(4, 3), and M(-2, 0) translated 1 unit right and 4 units down

J'(-2, -2), K'(4, 1), L'(5, -1), M'(-1, -4)

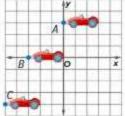


Triangle PQR has vertices P(0, 0), Q(5, -2), and R(-3, 6). Find the vertices of P'Q'R' after each translation. (Example 2)

- 3. 6 units right and 5 units up P'(6, 5), O'(11, 3), R'(3, 11)
- 4. 8 units left and 1 unit down P'(-8, -1), Q'(-3, -3), R'(-11, 5)

Use the image of the race car at the right. (Example 3)

- 5. Use translation notation to describe the translation from point A to point B. (x-3, y-3)
- 6. Use translation notation to describe the translation from point B to point C. (x-2, y-4)



 $\Omega$  Quadrilateral KLMN has vertices K(-2, -2), L(1, 1), M(0, 4), and N(-3, 5). It is first translated by (x + 2, y - 1) and then translated by (x - 3, y + 4). When a figure is translated twice, a double prime symbol is used. Find the coordinates of quadrilateral K"L"M"N" after both translations.

K''(-3, 1), L''(0, 4), M''(-1, 7), N''(-4, 8)



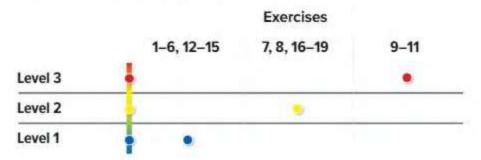
# **Practice and Apply**

### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                          |
|---------------------------------|-------------------|--------------------------|
| AL                              | Approaching Level | 1–7, 9, 11, 18, 19       |
| OL)                             | On Level          | 1–5 odd, 7–9, 11, 18, 19 |
| BD                              | Beyond Level      | 7–11, 18, 19             |

### Watch Out!

Common Error Suggest to students that they graph the original figures in one color and the translated images in another color to avoid confusion.

| MATHEMATICAL PRACTICES  |             |  |
|---|-------------|--|
| Emphasis On   | Exercise(s) |  |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 10          |  |
| 3 Construct viable arguments and critique the reasoning of others.            | 9, 11       |  |
| 4 Model with mathematics.   | 8           |  |
| 8 Look for and express regularity in repeated reasoning.                      | 17          |  |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

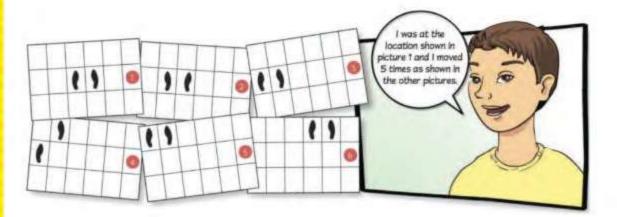
### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

### TICKET Out the Door

Have students answer the following question: If point P(-3, 2) is translated 3 units right and 2 units down, what are the coordinates of P'? (0, 0)

8. Model with Mathematics Refer to the graphic novel frame below. List the five steps the girl took and identify any transformations used in the movements. Sample answer: right crosses over left; left crosses behind right; right forward one step; left forward one step; both hop three to the right; Steps and hops are translations.



### el,

### H.O.T. Problems Higher Order Thinking

- Reason Inductively A figure is translated by (x − 5, y + 7), then by (x + 5, y − 7). Without graphing, what is the final position of the figure? Explain your reasoning to a classmate. the same as the original position of the figure; Sample answer: Since −5 and 5 are opposites, and −7 and 7 are opposites, the translations cancel each other out.
- 10. Persevere with Problems What are the coordinates of the point (x, y) after being translated m units left and n units up? (x m, y + n)
- Reason Inductively Determine whether each of the following statements is always, sometimes, or never true. Justify your reasoning.
  - a. A translation preserves orientation. always; Sample answer: Each point moves the same distance and in the same direction.
  - b. A preimage and its translated image are the same size, but not the same shape. never; Sample answer: A preimage and image in a translation will always have the same size and shape.

Name

My Homework

### **Extra Practice**

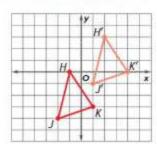
Graph each figure with the given vertices. Then graph the image of the figure after the indicated translation, and write the coordinates of its vertices.

**12.**  $\triangle HJK$  with vertices H(-1, 0), J(-2, -4)and K(1, -3) translated 3 units right and 3 units up

H'(2,3), J'(1,-1), K'(4,0)

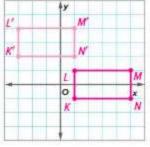
Graph each point and connect them to form a triangle. Then move each point 3 units to the right and then 3 units up. Connect them to form AHJ'K'.





13. Rectangle KLMN with vertices K(1, -1), L(1, 1), M(5, 1), and N(5, -1) translated 4 units left and 3 units up

K'(-3, 2), L'(-3, 4), M'(1, 4), N'(1, 2)



Quadrilateral ABCD has vertices A(-5, -1), B(-3, 0), C(2, -2), and D(0, -6). Find the vertices of A'B'C'D' after each translation.

- 14. 4 units up A'(-5, 3), B'(-3, 4), C'(2, 2), D'(0, -2)
- 15. 2 units right and 2 units down A'(-3, -3), B'(-1, -2), C'(4, -4), D'(2, -8)
- 16. Ismail is in Colorado exploring part of the Denver Zoo as shown. He starts at the Felines exhibit and travels 3 units to the right and 5 units up. At which exhibit is Ismail located? If the Felines exhibit is located at (3, 1), what are the coordinates of Ismail's new location? Hoofed Animals; (6, 6)



17. Cleantify Repeated Reasoning A diagram of a DNA double helix is shown below. Look for a pattern. On the diagram indicate where this pattern repeats or is translated. Find how many

translations of the original pattern are shown in the diagram. 1



## Power Up! Test Practice

Exercises 18 and 19 prepare students for more rigorous thinking needed for assessment.

18. This test item requires students to analyze and solve complex realworld problems through the use of mathematical tools and models.

| Depth of Knowledge    | DOK3   |
|-----------------------|--|
| Mathematical Practice | MP1, MP4   |
| Scoring Rubric        |  |
| 2 points              | Students correctly graph the figures and list the vertices.  |
| 1 point               | Students correctly graph the figures but fail to list the vertices OR students correctly graph one figure and list the vertices. |

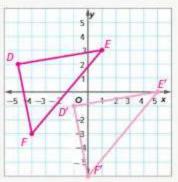
19. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

| structure.            |   |
|-----------------------|---|
| Depth of Knowledge    | DOK1                                    |
| Mathematical Practice | MP1                                     |
| Scoring Rubric        |   |
| 1 point               | Students correctly answer the question. |

### Power Up! Test Practice

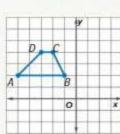
 Graph triangle DEF with vertices D(-5, 2), E(1, 3), and F(-4, -3). Then graph the image of the triangle after it is translated 4 units right and 3 units down.

What are the vertices of triangle D'E'F'?



19. Trapezoid ABCD is shown on the coordinate plan. Suppose the trapezoid is translated 3 units right and 2 units up. Which of the following are vertices of the translated figure? Select all that apply.

■ A'(-2, 4) ■ C'(0, 7) B'(1, −1)
D'(0, 6)



### **Spiral Review**

Find each sum.

**20.** 
$$-5 + 12 = 7$$

Lesson 2

### Reflections

### Real-World Link

Art Pysanky is the ancient Ukrainian art of egg decorating. Many artists use flips and line symmetry to create their designs. Use the activity to create your own pysanky design. See students' work.



The template shown represents the front view of an egg. The template has been divided into four sections.

Step 1

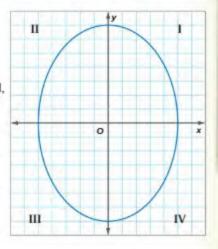
To create your egg, draw a design in Quadrant II.

Step 2

To complete Quadrant I, draw the mirror image over the y-axis.

Step 3

Repeat Steps 2 and 3 to fill in Quadrants III and N. You can create a new design or you can draw the mirror image over the x-axis.



Add color to your design by using colored pencils or markers to complete the design.

1. Line symmetry is when a figure can be folded so one side is the mirror image of the other side. Does your pysanky have line symmetry? Explain. yes; Sample answer: The design was drawn as the mirror image over the y-axis.



Which Mathematical Practices did you use? Shade the circle(s) that applies.

- 1) Persevere with Problems
- 2 Reason Abstractly
- 3 Construct an Argument
- Model with Mathematics www.jnob-jo.com
- (5) Use Math Tools
- (6) Attend to Precision (7) Make Use of Structure
- 8 Use Repeated Reasoning



### **Essential Question**

HOW can you best show or describe the change in position of a figure?



### Vocabulary

reflection line of reflection

### Math Symbols

 $(x, y) \rightarrow (x, -y)$  $(x, y) \rightarrow (-x, y)$ 

Mathematical Practices 1, 3, 4, 7



### Focus narrowing the scope

Objective Graph reflections on the coordinate plane.

### Coherence connecting within and across grades

### Previous

Students identified the properties of reflections.

### Now

Students will graph reflections on the coordinate plane.

### Next

Students will graph rotations on the coordinate plane.

### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 465.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

### **Launch the Lesson**

### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Think-Pair-Share Have students work with a partner to complete the activity. Give students about one minute to think through how their design would look in Quadrants I, III, and IV. Then have them discuss their responses with a partner. Call on several pairs of students to share their drawings with the class. 21, 5, 6, 7

### Alternate Strategy

Mave students fold their paper in half to verify that their design has line symmetry.

# **2** Teach the Concept

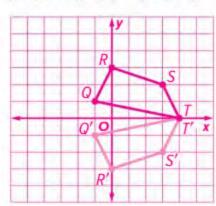
**Ask** the scaffolded questions for each example to differentiate instruction.

### **Examples**

- 1. Reflect a figure over the x-axis.
- What line in the coordinate plane are we using as the mirror? x-axis
  - When reflecting a point over the x-axis, which coordinate remains the same? x-coordinate Which coordinate changes? y-coordinate
- If point A is 2 units above the x-axis, where will it be after the reflection? 2 units below the x-axis
  - What algebraic notation explains the effect of this reflection? (x, y) → (x, -y)
  - What operation does the algebraic notation tell you to perform on the y-coordinates? Multiply by —1.
- Is the orientation of the triangle preserved?
   Explain. no; Sample answer: After the triangle is reflected, the orientation of the points are reversed from counter clockwise to clockwise.
  - · Are the figures congruent? yes

### **Need Another Example?**

Quadrilateral *QRST* has vertices Q(-1, 1), R(0, 3), S(3, 2), and T(4, 0). Graph the figure and its reflected image over the x-axis. Then find the coordinates of the vertices of the reflected image. Q'(-1, -1), R'(0, -3), S'(3, -2), T(4, 0)



Key Concept

Work Zone

Check

Check the coordinates of

y-coordinates by - 1.

 $(x, y) \rightarrow (x, -y)$ 

 $(5,2) \rightarrow (5,-2)$ 

 $(1,3) \rightarrow (1,-3)$ 

(-1,1) -+ (-1,-1) V

the image by multiplying the

### Reflections in the Coordinate Plane

Over the x-axis

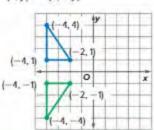
To reflect a figure over the x-axis, multiply the

y-coordinates by -1.

Symbols  $(x, y) \rightarrow (x, -y)$ 

Models

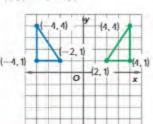
Words



Over the y-axis

To reflect a figure over the y-axis, multiply the x-coordinates by -1.

 $(x, y) \rightarrow (-x, y)$ 

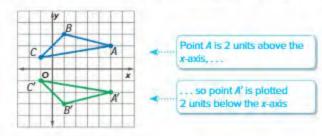


A reflection is a mirror image of the original figure. It is the result of a transformation of a figure over a line called a line of reflection. In a reflection, each point of the preimage and its image are the same distance from the line of reflection. So, in a reflection, the image is congruent to the preimage.

**Examples** 

Triangle ABC has vertices A(5, 2), B(1, 3), and C(-1, 1). Graph
the figure and its reflected image over the x-axis. Then find the
coordinates of the vertices of the reflected image.

The x-axis is the line of reflection. So, plot each vertex of A'B'C' the same distance from the x-axis as its corresponding vertex on ABC.



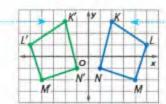
The coordinates are A'(5, -2), B'(1, -3), and C'(-1, -1).

Geometr

Quadrilateral KLMN has vertices K(2, 3), L(5, 1), M(4, -2), and N(1, -1). Graph the figure and its reflection over the y-axis. Then find the coordinates of the vertices of the reflected image.

The y-axis is the line of reflection. So, plot each vertex of K'L'M'N' the same distance from the y-axis as its corresponding vertex on KLMN.

Point K' is 2 units to the left of the

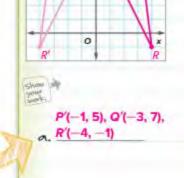


Point K is 2 units to the right of the y-axis.

The coordinates are K'(-2, 3), L'(-5, 1), M'(-4, -2), and N'(-1, -1).

Got it? Do this problem to find out.

a. Triangle PQR has vertices P(1, 5), Q(3, 7), and R(4, -1). Graph the figure and its reflection over the y-axis. Then find the coordinates of the reflected image.

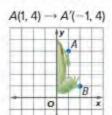


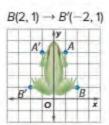


### Example

 The figure below is reflected over the y-axis. Find the coordinates of point A' and point B'. Then sketch the figure and its image on the coordinate plane.

Point A is located at (1, 4). Point B is located at (2, 1). Since the figure is being reflected over the y-axis, multiply the x-coordinates by -1.





### and Reflect

Explain below how the x and y coordinates of an image relate to the x and y coordinates of the preimage after a reflection over the y axis.

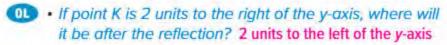
The x-coordinates are opposites and the y-coordinates are the same.

### **Examples**

### 2. Reflect a figure over the y-axis.



 When reflecting a point over the y-axis, which coordinate remains the same? y-coordinate Which coordinate changes? x-coordinate



 What algebraic notation explains the effect of this reflection? (x, y) → (-x, y)

• What is another way to determine the coordinates of the reflected points? Sample answer: Determine each point's location in respect to the origin. Point K is 2 units to the right of the origin and 3 units up. After the reflection, it will be 2 units to the left of the origin and 3 units up. Repeat this process for each point.

### Need Another Example?

Triangle XYZ has vertices X(1, 2), Y(2, 1), and Z(1, -2). Graph the figure and its reflected image over the y-axis. Then find the coordinates of the vertices of the reflected image. See Answer Appendix.

### 3. Use a reflection to sketch the figure.

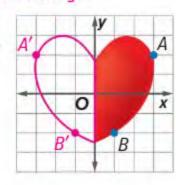
 Is the image of the frog symmetrical? Explain. Yes, the left side is a reflection of the right side.

 How do you determine the new coordinates of a point that is reflected across the y-axis? Multiply the x-coordinates by -1.

• What is another method you could use to plot A? A is 1 unit to the right and 4 units up from the origin. So, A' is 1 unit to the left and 4 units up from the origin.

### **Need Another Example?**

The figure is reflected over the y-axis. Find the coordinates of point A' and point B'. Then sketch the figure and its image on the coordinate plane. A'(-3, 2), B'(-1, -2)



### **Guided Practice**

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.



If some of your students are not ready for assignments, use the differentiated activities below.

Team-Pair-Solo Have students work in a fourperson team to complete Exercise 1, then with a partner to complete Exercise 2. Teams and partners should ensure that each person understands how to reflect a figure. Have students individually think through their response to Exercise 3, then discuss their response with their partner or group. 1, 5, 6, 7

Trade-a-Problem Have students draw their own. figure in one quadrant of the coordinate grid and plot at least 4 points. Have students trade the grid and reflect each other's figure over the x- or y-axis. Then have them work together to write the algebraic notations that explain the effect of the reflections. 0 1, 5, 6, 7

### Watch Out!

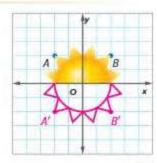
Common Error In Exercise 1, students may reflect the image over the y-axis. Remind students that when reflecting over the x-axis, the y-coordinates are multiplied by -1, not the x-coordinates.

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A'(-2, -2). B'(2, -2)

### Got it? Do this problem to find out.

b. The figure at the right is reflected over the x-axis. Find the coordinates of point A' and point B'. Then sketch the image on the coordinate plane.

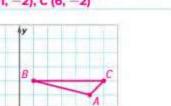


### **Guided Practice**



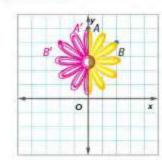
 Graph △ABC with vertices A(5, 1), B(1, 2), and C(6, 2) and its reflection over the x-axis. Then find the coordinates of the image.

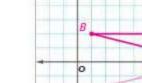
(Examples 1 and 2)



2. The figure is reflected over the y-axis. Find the coordinates of point A' and point B'. Then sketch the image on the coordinate plane. (Example 3)

A'(0, 5), B'(-2, 4)





3. Building on the Essential Question How can you determine the coordinates of a figure after a reflection over either axis?

Sample answer: If you reflect over the x-axis, keep the

x-coordinate and take the opposite of the y-coordinate.

If you reflect over the y-axis, take the opposite of the

x-coordinate and keep the y-coordinate.

### Rate Yourself!

How well do you understand reflections? Circle the image that applies.





FOLDARIES Time to update your Foldable!

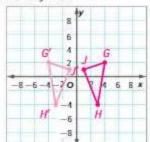
Name

My Hamework

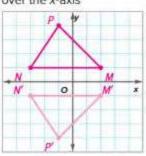
### **Independent Practice**

Graph each figure and its reflection over the indicated axis. Then find the coordinates of the reflected image. (Examples 1 and 2)

 $\bigcap \triangle GHJ$  with vertices G(4, 2), H(3, -4), and J(1, 1) over the y-axis



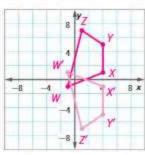
△MNP with vertices M(2, 1), N(-3, 1), and P(-1, 4) over the x-axis



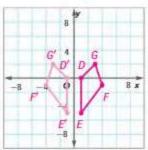
G'(-4, 2), H'(-3, -4), J'(-1, 1)



 quadrilateral WXYZ with vertices W(-1, -1). X(4, 1), Y(4, 5), and Z(1, 7) over the x-axis



 quadrilateral DEFG with vertices D(1, 0). E(1,-5), F(4,-1), and G(3,2) over the y-axis

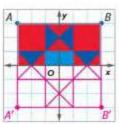


W'(-1, 1), X'(4, -1), Y'(4, -5), Z'(1, -7)

D'(-1, 0), E'(-1, -5), F'(-4, -1), G'(-3, 2)

5. The figure at the right is reflected over the x-axis. Find the coordinates of point A' and point B'. Then sketch the image on the coordinate plane. (Example 3)





Identify Structure The coordinates of a point and its image after a reflection are given. Describe the reflection as over the x-axis or y-axis.

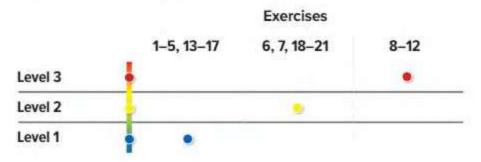
# **Practice and Apply**

### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

| Differentiated Homework Options |                   |                             |
|---------------------------------|-------------------|-----------------------------|
| AL                              | Approaching Level | 1–5, 7, 8, 10–12, 20, 21    |
| 01                              | On Level          | 1–5 odd, 6–8, 10–12, 20, 21 |
| BL                              | Beyond Level      | 6–12, 20, 21                |

| MATHEMATICAL PRACTICES  |                      |  |
|---|----------------------|--|
| Emphasis On   | Exercise(s)          |  |
| <ol> <li>Make sense of problems and persever<br/>solving them.</li> </ol> | re in 9              |  |
| 3 Construct viable arguments and critique reasoning of others.            | se the 8, 10, 11, 12 |  |
| 7 Look for and make use of structure.                                     | 6, 7, 18, 19         |  |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

### TICKET Out the Door

Ask students to compare translating a figure and reflecting a figure, and to write how previous lessons helped prepare them for concepts introduced in this lesson. Have them use the writing prompts below. See students' work.

- Reflecting a figure is similar to translating a figure because...
- Reflecting a figure is different from translating a figure because...
- What I learned about translations helped me to understand reflections in that...



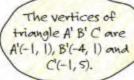
### H.O.T. Problems Higher Order Thinking

 Find the Error Mazen is finding the coordinates of the image of a triangle with vertices A(1, 1), B(4, 1) and C(1, 5) after a reflection over the x-axis. Describe his mistake and correct it.

Mazen reflected the triangle over

the y-axis. The coordinates should be

A'(1, -1), B'(4, -1) and C'(1, -5).





- Persevere with Problems Triangle JKL has vertices J(-7, 4), K(7, 1), and L(2, -2). Without graphing, find the new coordinates of the vertices of the triangle after a reflection first over the x-axis and then over the y-axis. J"(7, -4), K"(-7, -1), L"(-2, 2)
- 10. Peason Inductively Suppose you reflect a triangle in Quadrant I over the y-axis, then translate the image 2 units left and 3 units down. Is there a single transformation that maps the preimage onto the image? Explain your reasoning. no; Sample answer: If the vertices of △ABC are A(1, 2), B(3, 4), and C(1, 4), then the vertices of the final image are A"(-3, -1), B"(-5, 1), and C"(-3, 1).
- 11. Peason Inductively Suppose you reflect a nonregular figure over the x-axis and then reflect it over the y-axis. Is there a single transformation using reflections or translations that maps the preimage onto the image? Explain your reasoning.

no; Sample answer: If the vertices of  $\triangle ABC$  are A(0, 0), B(2, 2), and C(0, 4), then the vertices of the final image are A''(0, 0), B''(-2, -2), and C''(0, -4).

 Which One Doesn't Belong? Triangle ABC has vertices A(1, 2), B(1, 5), and C(4, 2) and undergoes a transformation. Circle the set of vertices that does not belong. Explain your reasoning.

A'(1,-1), B'(1,2), C'(4,-1)

A'(5, 2), B'(5, 5), C'(8, 2)

A'(1, -2), B'(1, -5), C'(4, -2)

A'(3,3), B'(3,6), C'(6,3)

Sample answer: This set is a reflection over the x-axis of  $\triangle ABC$ . The other sets are translations.

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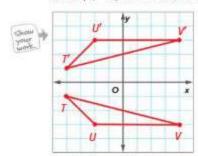
Name

My Homework

### **Extra Practice**

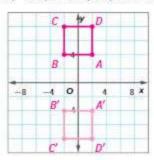
Graph each figure and its reflection over the indicated axis. Then find the coordinates of the reflected image.

13.  $\triangle TUV$  with vertices T(-4, -1), U(-2, -3), and V(4, -3) over the x-axis



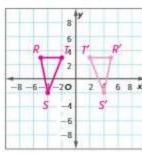
T'(-4, 1), U'(-2,3), V'(4,3)

14. square ABCD with vertices A(2, 4), B(-2, 4), C(-2, 8), and D(2, 8) over the x-axis



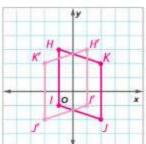
A'(2, -4), B'(-2, -4), C'(-2, -8), D'(2, -8)

**15.**  $\triangle RST$  with vertices R(-5, 3), S(-4, -2), and T(-2, 3) over the y-axis



R'(5, 3), S'(4, -2), T'(2, 3)

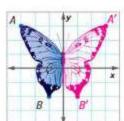
16. parallelogram HIJK with vertices H(-1, 3), I(-1, -1), J(2, -2), and K(2, 2) over the y-axis



H'(1, 3), I'(1, -1), J'(-2, -2), K'(-2, 2)

17. The figure at the right is reflected over the y-axis. Find the coordinates of point A' and point B'. Then sketch the image on the coordinate plane.

A'(3, 3), B'(1, -2)



Identify Structure The coordinates of a point and its image after a reflection are given. Describe the reflection as over the x-axis or y-axis.

**18.**  $X(-1, -4) \rightarrow X'(-1, 4)$  **x-axis** 

**19.**  $W(-4, 0) \rightarrow W'(4, 0)$  **y-axis** 

### Power Up! Test Practice

Exercises 20 and 21 prepare students for more rigorous thinking needed for assessment.

20. This test item requires students to analyze and solve complex realworld problems through the use of mathematical tools and models. Depth of Knowledge DOK2 Mathematical Practice MP1, MP4 **Scoring Rubric** Students correctly graph the figure and list 2 points the vertices. Students correctly graph the figure OR list 1 point

the vertices.

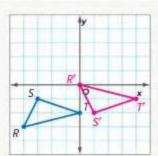
21. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure. DOK1 Depth of Knowledge Mathematical Practice MP1, MP5 **Scoring Rubric** Students correctly answer both parts of 1 point the question.

### Power Up! Test Practice

20. Graph the image of triangle RST after it is reflected over the x-axis then translated 4 units to the right and 3 units down.

What are the vertices of triangle R'S'T'?

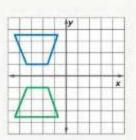
R'(0,0), S'(1,-2), T'(4,-1)



21. The figure shown at the right was transformed from Quadrant II to Quadrant III.

Fill in each box to make a true statement to describe the transformation.

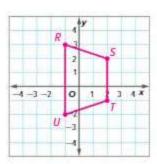
reflected x-axis The figure was



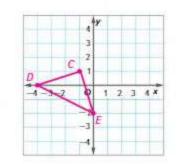
### **Spiral Review**

Graph and label each figure on the coordinate plane.

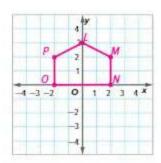
R(-1, 3), S(2, 2), T(2, -1), and U(-1, -2)



22. trapezoid RSTU with vertices 23. △CDE with vertices C(-1, 1), D(-4, 0), and E(0, -2)



24. pentagon LMNOP with vertices L(0, 3), M(2, 2), N(2, 0), O(-2, 0), and P(-2, 2)



## Problem-Solving Investigation

Act it Out

Mathematical Practices

### Case #1 Black Belt Champion

Salem's school is 3 blocks east and 4 blocks south from his house. He is taking a martial arts class that is 2 blocks east and 6 blocks north from school.

What are two different ways that Salem can travel to go from martial arts class to his house?



#### Understand What are the facts?

You know the translations involved.

- School is 3 blocks east and 4 blocks south from his house.
- Martial arts class is 2 blocks east and 6 blocks north from school.



### Plan What is your strategy to solve this problem?

Act out the situation on a coordinate plane. Plot Salem's house at (0, 0) and map out the route to his school and martial arts class. Then determine two translations that will take Salem from martial arts class to his house.

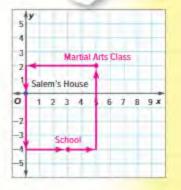


### Solve How can you apply the strategy?

What are two different ways that Salem can travel to go from martial arts class to his house?

5 blocks west and then 2 blocks south or 2 blocks

south and then 5 blocks west.





#### Check Does the answer make sense?

Begin with the point (0, 0) to represent Salem house. Use translation notation to determine the route to school, martial arts class and then back home.

## Analyze the Strategy

Make a Conjecture Suppose Salem's needed to drive 32 blocks east and 15 blocks north from school. Would it be more efficient using translation notation or acting the problem out on graph paper? Explain.

Sample answer: It would be more efficient using translation notation.

WWW. JNOb-JO. Com (x + 32, y + 15) would take up too much space on paper.

### Focus narrowing the scope

Objective Solve problems by using the act it out strategy. This lesson emphasizes Mathematical Practice 4 Model with Mathematics.

Act It Out The strategy is especially helpful for students who are kinesthetic learners. For example, they model problems by arranging students or using counters.

### Coherence connecting within and across grades

#### Now

Students apply the content standards to solve non-routine problems.

#### Next

Students will apply the act it out strategy to model transformation problems.

## RIGOT pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 471.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

## **Launch the Lesson**

The problems on pages 469 and 470 are for whole-group discussion on how to solve non-routine problems and provide scaffolded guidance. The problem on page 469 walks students through the solution, while the problem on page 470 asks students to come up with their own solutions.

## Case #1 Black Belt Champion

Extend the problem using the question below.

#### Ask:

 There is a grocery store 3 blocks east and 1 block south of Suhaila's house. If Suhaila needs to buy bread before going home, would it be better for her to stop at the store on her way to Martial Arts class, or after Martial Arts class on her way home? Explain. Sample answer: It would better for her to go to the store on her way to Martial Arts class. She still has to travel 8 blocks to get to Martial Arts class. If she waits until after Martial Arts, she will add an additional 2 blocks to her trip.

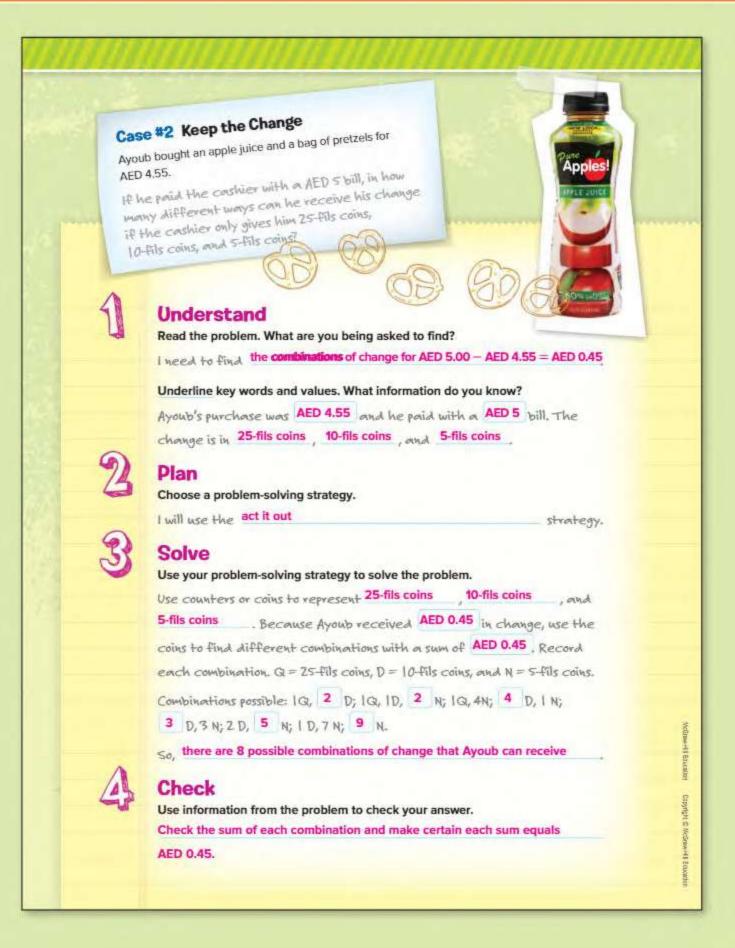
## Case #2 Keep the Change

Team Project Have students work in 3- to 4-person learning teams. Give each team counters and have them act out the problem presented in the text. Then have each team share their answers with another team, listening carefully as each solution is explained and asking for clarification if necessary. 1, 2, 3, 4, 5, 6, 7, 8

Trade-a-Problem Have students create their own problem using the act it out strategy. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. 1, 2, 3, 4, 5, 6, 7, 8

#### Need Another Example?

How many ways are there to arrange five French club members for a yearbook photo if the president and vice president must be seated in front with the other three members behind them? 12



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Work with a small group to solve the following cases. Show your work on a separate piece of paper.

#### Case #3 Picture Exchanges

The French Club took a field trip to an exhibit of French art at the museum. Five of the club members held a picture exchange to share their pictures. Saeed brought more pictures than Yousif. Nasser brought more pictures than Mansour, but fewer than Yousif. Mahmoud brought more pictures than Nasser, but not as many as Yousif.

List the picture exchange participants in order from the most pictures to the fewest.

Saeed, Yousif, Mahmoud, Nasser, Mansour

#### Case #4 Fitness

The length of a basketball court is 25.2 meters. Kareem starts at one end of the court and runs 6 meters forward and then 2.40 meters back.

How many more times will he have to do this until he reaches the end of the basketball court? What equation represents this

6 times; Sample answer: (6 - 2.4)x = 25.2, where x = the number of times



### Case #5 Dinner Parties

Wafa sent a text message to three friends inviting each of them to a dinner party. Each of those friends sent the message to three more friends. Then each of these friends sent the message to three more friends.

If two thirds of the friends receiving the text message attended the dinner, how many friends attended the dinner? 26 friends

Collaborate

#### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE



(II) Circle the Sage Poll the class to see which students have some knowledge of translations. Those students (the sages) spread out around the room. Assign the rest of the students to teams. Have the teams split up with each team member going to a different sage, if possible. Have the sages lead work for Case 4. When the case is complete, students go back to their teams and compare solutions. Students discuss how the sages may have explained the steps differently.

BD (IA) Pairs Discussion Have students work in pairs to answer the following extension question relating to Case 6. 1, 2, 3, 4, 5, 6, 7, 8

#### Ask:

1, 2, 4, 5, 6, 7, 8

 Wafa sent a text message to 2 friends inviting them to a party and they each sent a message to 2 more friends. Then each of these friends sent the message to 2 more friends. About two-thirds of the friends receiving the text message attended the party. How is the number of friends who attended this party different from the party where Wafa started by sending a message to 3 friends? Sample answer: When Wafa started with 2 friends, the total who attended the party was 9. This is a lot fewer people than the 26 who attended when she started with 3 friends.

## Mid-Chapter Check

If students have trouble with Exercises 1–5, they may need help with the following concepts.

| Concept                 | Exercise(s) |  |  |
|-------------------------|-------------|--|--|
| translations (Lesson 1) | 1, 3, 5     |  |  |
| reflections (Lesson 2)  | 2, 4, 5     |  |  |

## **Vocabulary Activity**

Think-Pair-Share Have students work in pairs to complete Exercise 1. Give them about one minute to individually think through their response. Then have them share their responses with a partner. Call on one set of pairs to share their responses with the class. 1,6

#### **Alternate Strategies**

Provide students with the coordinates of the vertices of a triangle. Have students work with a partner to perform a reflection of the triangle over the x- or y-axis. Then have students discuss the role of the line of reflection in their transformation.

BL Have students create a graphic organizer of their own choosing to show the similarities and differences of translations and reflections.

## Mid-Chapter Check

### **Vocabulary Check**



 Define transformation using the words preimage and image. (Lesson 1)

Sample answer: A transformation maps a figure, the preimage, onto a new figure called the image.

Describe the role of the line of reflection in a transformation. (Lesson 2)
 Sample answer: The line of reflection is the fixed line over which a figure is reflected.

### Skills Check and Problem Solving

Graph each triangle with the given vertices. Then graph the image after the given transformation and write the coordinates of the image's vertices.

- △ABC with vertices A(3, 5), B(4,1), and C(1, 2); translation of 3 units left and 4 units down
  - A'(0, 1), B'(1, -3), C'(-2, -2)
- △WXY with vertices W(-1, -2), X(0, -4), and Y(-3, -5); reflection over the x-axis followed by a reflection over the y-axis

W'(1, 2), X'(0, 4), Y'(3, 5)



 Persevere with Problems Point D is translated 5 units right and 2 units down, then reflected over the y-axis. Write an algebraic representation to represent the final location of point D. (Lessons 1 and 2)

 $(x, y) \rightarrow (-x - 5, y - 2)$ 

The state of the s

## **Inquiry Lab**

**Rotational Symmetry** 



#### HOW can you identify rotational symmetry?



Many products have logos so people can easily identify them. If you turn the first aid logo below 180°, will the logo look the same as the original figure?

## Hands-On Activity

A figure has rotational symmetry if it can be rotated or turned less than 360° about its center so that the figure looks exactly as it does in its original position.

Step 1

Copy the outline of the equilateral triangle onto a piece of tracing paper. Label one vertex A.



Place the tracing paper over the outline in Step 1. Put your pencil point at the center of the figure to hold the tracing paper in place. Turn the tracing paper clockwise from its original position until the two figures match. Draw and label the new figure in the space provided.

Step 3

Continue turning the tracing paper until the logo is back to its original position. Does the figure have rotational symmetry? Explain.

Yes; Sample answer; the figure was turned less than 360°

about its center and still looked like the original.

### Focus narrowing the scope

Objective Identify rotational symmetry.

## Coherence connecting within and across grades

#### Now

Students will identify properties of rotational symmetry.

Students will graph rotations on the coordinate plane.

## Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 474.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

## Launch the Lab

The activity is intended to be used as a whole-group activity.

## Hands-On Activity

(11) (LA) Teammates Consult Have students work in small teams. Give each team tracing paper. Ask a volunteer to lead the activity, showing how to rotate the paper without letting it slide. Then have them discuss the question in Step 3. Have one team share their responses with the class to initiate a class discussion about what it means for a figure to have rotational symmetry. 31, 3, 5, 6, 7

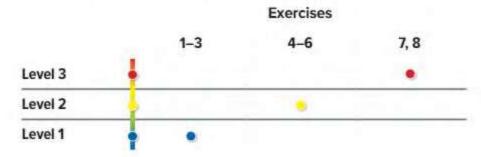
(II) (IA) Pairs Consult After completing the activity, have students work with a partner to draw and color a pattern inside the logo that will change its rotational symmetry to be 180°. 1, 5, 6, 7

## 2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

#### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



## [ Investigate

Team-Pair-Solo Give students tracing paper and have them trace the outline of each figure. As they did in the activity, have them use a pencil to hold the paper in place and rotate the paper until it matches with the original picture. Have students work in small teams to complete Exercise 1. Then have them work with a partner to complete Exercise 2, and have them work on their own to complete Exercise 3. Have them rejoin their original teams to discuss and compare responses. 1, 5, 6, 7

## **Create**

Have students design and color their own personal logo with a degree of rotational symmetry. Pass their designs around the room and have students determine the degree of rotational symmetry for each other's work. 

1, 5, 6, 7

Students should be able to answer "HOW can you identify rotational symmetry?" Check for student understanding and provide guidance, if needed.

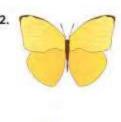


## Investigate

Work with a partner. Determine whether the figure has rotational symmetry. Write yes or no.

1.







yes



## **Analyze and Reflect**

- Reason Inductively The degree measure of an angle through which
  the figure is rotated is called the angle of rotation. Find the first angle of
  rotation of the equilateral triangle by dividing 360° by the total number of
  times the figures matched. 90°
- List the other angles of rotation of the equilateral triangle by adding the measure of the first angle of rotation to the previous angle measure.

Stop when you reach 360°, 180°, 270°

What is the angle of rotation of each figure in Exercises 1–3? Write no if there is no rotational symmetry.

Exercise 1 180°

Exercise 2 no

Exercise 3 60°



## Create

Model with Mathematics Draw two figures, one that has rotational symmetry and one that does not. See students' work.



8. HOW can you identify rotational symmetry?

Sample answer: You can identify rotational symmetry by turning the figure less than 360° and determining if the figure looks the same as the original.

# Lesson 3 Rotations

## Real-World Link

Prizes Majed is spinning the prize wheel shown below.

 A spin can be clockwise or counterclockwise. Define these two words in your own words.

clockwise rotating to the right

counterclockwise rotating to the left

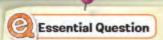
- If the section labeled 8 on the left part of the wheel spins 90° clockwise, where will it land? at the top
- If one of the sections labeled 4 makes three complete turns counterclockwise, how many degrees will it have traveled?
   1,080
- 4. Are there any points on the wheel that stay fixed, or do not move, when the wheel spins? If so, what are the points? yes; the center
- 5. Does the center of the wheel change if the wheel is spun counterclockwise as opposed to clockwise? no
- Does the distance from the center to the edge change as it spins? Explain.

no; Sample answer: The distance from the center to the edge is the radius of the circle. The size of the circle does not change as it spins so the radius does not change.

Which Mathematical Practices did you use?

Shade the circle(s) that applies.

- 1) Persevere with Problems
  - ems
- 2 Reason Abstractly
- 2) Reason Abstractly
- 3 Construct an Argument
- www.jnob-jo.com
- Use Math Tools
   Attend to Precision
- Make Use of Structure
- 8 Use Repeated Reasoning



HOW can we best show or describe the change in position of a figure?

## Vocabulary

rotation center of rotation

#### Math Symbols

 $(x, y) \rightarrow (y, -x)$   $(x, y) \rightarrow (-x, -y)$  $(x, y) \rightarrow (-y, x)$ 

Mathematical Practices



### Focus narrowing the scope

Objective Graph rotations on the coordinate plane.

### Coherence connecting within and across grades

### Previous

Students identified the properties of rotational symmetry.

#### Now

Students will graph rotations on the coordinate plane.

#### Next

Students will graph dilations on the coordinate plane.

### Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 479.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

## 1 Launch the Lesson

#### Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

front of the class or use an online spinner. Give students time to play with the spinner so that they can visualize the movements made in the activity. Then have them complete the activity with a partner. 21, 4, 5, 7

## **Alternate Strategy**

AD Have students use a clock face to visualize the meanings of clockwise and counterclockwise. 21,5

## **Teach the Concept**

Ask the scaffolded questions for each example to differentiate instruction.

## Example

#### Rotate a figure about a point.



About what point are we rotating? about vertex L



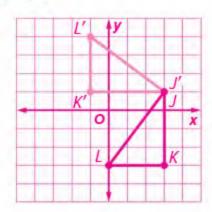
- Describe vertex M in comparison to vertex L. Vertex M is 3 units up from vertex L.
- · After the rotation, how far away from vertex L will vertex M' be? Vertex M will be 3 units down from vertex L.
- Describe vertex N in comparison to vertex L. Vertex N is 3 units up and 3 units to the right from vertex L.
- After the rotation, how far away from vertex L will vertex N' be? Vertex N' will be 3 units down and 3 units to the left from vertex L.



- After the rotation, where will vertex L' be in comparison to vertex L? Explain. Vertex L' will be in the same location as vertex L. Vertex L is the center of rotation.
  - Are the two figures congruent? yes

#### Need Another Example?

Triangle JKL has vertices J(3, 1), K(3, -3), and L(0, -3). Graph the figure and its image after a clockwise rotation of 90° about vertex J. Then give the coordinates of the vertices for  $\triangle JKL'$ . J(3, 1), K'(-1, 1), L'(-1, 4)



Work Zone

Rotations

Rotations can be described

in degrees and direction. For

example, 90° clockwise or

270' counterclockwise.

## **Rotate a Figure About a Point**

A rotation is a transformation in which a figure is rotated, or turned, about a fixed point. The center of rotation is the fixed point. A rotation does not change the size or shape of the figure. So, the preimage and the image are congruent.



### Example

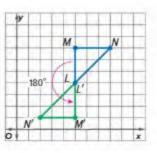
1. Triangle LMN with vertices L(5, 4), M(5, 7), and N(8, 7) represents a desk in Ibrahim's bedroom. He wants to rotate the desk counterclockwise 180° about vertex L. Graph the figure and its image. Then give the coordinates of the vertices for  $\triangle L'M'N'$ .

Step 1

Graph the original triangle.

Step 2

Graph the rotated image. Use a protractor to measure an angle of 180° with M as one point on the ray and L as the vertex. Mark off a point the same length as ML. Label this point M' as shown.



Step 3

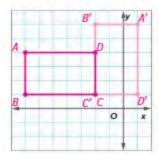
Repeat Step 2 for point N. Since L is the point at which  $\triangle LMN$  is rotated, L' will be in the same position as L.

So, the coordinates of the vertices of  $\triangle L'M'N'$  are L'(5, 4), M'(5, 1), and N'(2, 1).

A'(1, 6), B'(-2, 6), a. C'(-2, 1), D'(1, 1)

Got it? Do this problem to find out.

a. Rectangle ABCD with vertices A(-7, 4), B(-7, 1), C(-2, 1),and D(-2, 4) represents the bed in Ibrahim's room. Graph the figure and its image after a clockwise rotation of 90° about vertex C. Then give the coordinates of the vertices for rectangle A'B'C'D'.



## **Rotations About the Origin**

Key Concept

Check

the image.  $(x,y) \rightarrow (y,-x)$ 

 $(-4, 4) \rightarrow (4, 4)$ (-1,2) - (2, 1)

(-3, 1) - (1,3) V

Check the coordinates of

Words A rotation is a transformation around a fixed point. Each point of the original figure and its image are the same distance from the

center of rotation.

Models The rotations shown are clockwise rotations about the origin.

90° Rotation 180° Rotation 270° Rotation Symbols

 $(x, y) \rightarrow (-x, -y)$ 

Figures can also be rotated about the origin.

 $(x, y) \rightarrow (y, -x)$ 

### Example

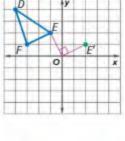
2. Triangle DEF has vertices D(-4, 4), E(-1, 2), and F(-3, 1). Graph the figure and its image after a clockwise rotation of 90° about the origin. Then give the coordinates of the vertices for  $\triangle D'E'F'$ .

Step 1

Graph  $\triangle DEF$  on a coordinate plane.

Step 2

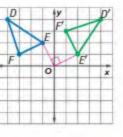
Sketch segment EO connecting point E to the origin. Sketch another segment, E'O, so that the angle between point E, O, and E' measures 90° and the segment is the same length as EO.



 $(x, y) \rightarrow (-y, x)$ 

Step 3

Repeat Step 2 for points D and F. Then connect the vertices to form  $\triangle D'E'F'$ .



So, the coordinates of the vertices of  $\triangle D'E'F'$ are D'(4, 4), E'(2, 1), and F'(1, 3).

## Example

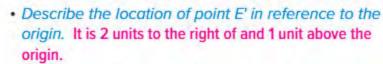
#### 2 Rotate a figure about the origin.



About what point are we rotating? about the origin



 Describe the location of point E in reference to the origin. It is 1 unit to the left of and 2 units above the origin.



 Using this as a guide, what will be the location of point F' in reference to the origin? point D'? Point F will be 1 unit to the right and 3 units above the origin. Point D' will be 4 units to the right and 4 units above the origin.



BD • Are the two figures congruent? yes

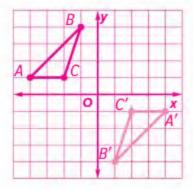


 After a 270° rotation clockwise about the origin, what does the point (x, y) become?  $(x, y) \rightarrow (-y, x)$ 

 After a 360° rotation clockwise about the origin, what does the point (x, y) become?  $(x, y) \rightarrow (x, y)$ 

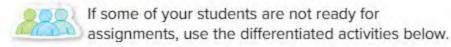
#### Need Another Example?

Triangle ABC has vertices A(-4, 1), B(-1, 4), and C(-2, 1). Graph the figure and its image after a counterclockwise rotation of 180° about the origin. Then give the coordinates of the vertices for  $\triangle A'B'C'$ . A'(4, -1), B'(1, -4), C'(2, -1)



## **Guided Practice**

**Formative Assessment** Use these exercises to assess students' understanding of the concepts in this lesson.



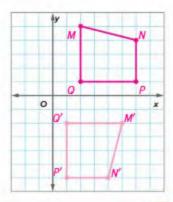
understanding how to rotate images, you may wish to complete Exercise 1 together as a whole group. You may also wish to give the students tracing paper to help them see how the image is rotated. Have them trace the original triangle and then press their pencil down on the point of rotation (vertex X for Exercise 1, the origin for Exercise 2) to rotate the paper. Then have students work with a partner to complete Exercise 2. Have them work individually to complete Exercise 3. Then have them rejoin the whole group to discuss answers and compare solutions. 1, 5, 7

BL LA Pairs Discussion For Exercises 1 and 2, have students predict the coordinates of the image after the rotation without graphing. Then have them compare the coordinates after graphing to see if their prediction was correct. 1, 3, 5, 6, 7

b. M'(5, -2), N'(4, -6),

#### Got it? Do this problem to find out.

b. Quadrilateral MNPO has vertices M(2, 5), N(6, 4), P(6, 1), and O(2, 1). Graph the figure and its image after a counterclockwise rotation of 270° about the origin. Then give the coordinates of the vertices for quadrilateral M'N'P'O'.



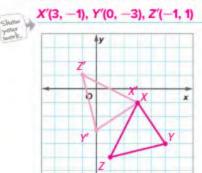
## **Guided Practice**

P'(1, -6), Q'(1, -2)

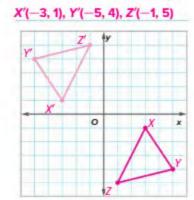


Triangle XYZ has vertices X(3, -1), Y(5, -4), and Z(1, -5). Graph  $\triangle XYZ$  and its image after each rotation. Then give the coordinates of the vertices for  $\triangle X'Y'Z'$ . (Examples 1 and 2)

1. 270° counterclockwise about vertex X



2. 180° clockwise about the origin



Sample answer: If you rotate the figure about one of the vertices, that point stays the same. If you rotate the same figure about the origin, all of the points are different unless one of the vertices is the origin.

#### Rate Yourself!

How confident are you about rotations? Check the box that applies.







FOLDABLES Time to update your Foldable!

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Name

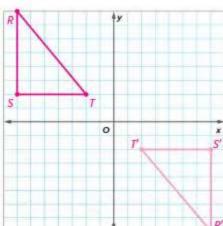
My Homework

## **Independent Practice**

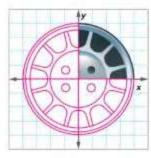
Triangle RST represents the placement of Fawzia's tricycle in the driveway and has vertices R(-7, 8), S(-7, 2), and T(-2, 2). Graph the figure and its rotated image after a clockwise rotation of 180° about the origin. Then give the coordinates of the vertices for triangle R'S'T'. (Example 2)



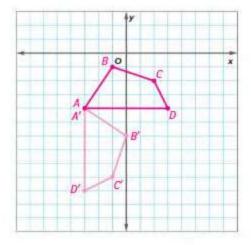
R'(7, -8), S'(7, -2), T'(2, -2)



 Model with Mathematics A partial hubcap is shown. Copy and complete the figure so that the completed hubcap has rotational symmetry of 90°, 180°, and 270°.

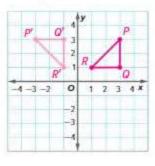


Quadrilateral ABCD has vertices at A(-3, -4), B(-1, -1), C(2, -2), and D(3, -4). Graph quadrilateral ABCD and its image after a 90° clockwise rotation about vertex A. Then give the coordinates of the vertices of the image. (Example 1)



4. The right isosceles triangle PQR has vertices P(3, 3), Q(3, 1), and R(x, y) and is rotated 90° counterclockwise about the origin. Find the missing vertex of the triangle. Then graph the triangle and its image. Sample answer:

$$R(x,y)=R(^{1}, \frac{1}{2})$$



## 3 Practice and Apply

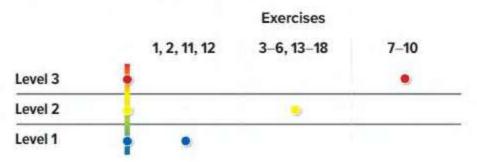
#### Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

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#### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



#### Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

|       | Differentiated Homework Options |                       |  |  |  |  |  |  |
|-------|---------------------------------|-----------------------|--|--|--|--|--|--|
| AL    | Approaching Level               | 1–3, 5, 9, 10, 17, 18 |  |  |  |  |  |  |
| OL OL | On Level                        | 1, 3–6, 9, 10, 17, 18 |  |  |  |  |  |  |
| BL    | Beyond Level                    | 3–10, 17, 18          |  |  |  |  |  |  |

## Watch Out!

Common Error Watch for students who rotate figures about a vertex rather than about the origin. Remind students to first determine the center of rotation.

| MATHEMATICAL PRACTICES  |             |  |  |  |  |  |
|---|-------------|--|--|--|--|--|
| Emphasis On   | Exercise(s) |  |  |  |  |  |
| <ol> <li>Make sense of problems and persevere in<br/>solving them.</li> </ol> | 7, 8        |  |  |  |  |  |
| 2 Reason abstractly and quantitatively.                                       | 10          |  |  |  |  |  |
| 4 Model with mathematics.   | 3, 6, 9     |  |  |  |  |  |
| 7 Look for and make use of structure.   | 13          |  |  |  |  |  |

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

#### Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

## TICKET Out the Door

Have students respond to the following question: If point T(4, -3) is rotated 90° counterclockwise about the origin, what are the coordinates of T? (3, 4)

Which capital letters in ISOSCELES produce the same letter after being rotated 180° in the plane of the page? I and N



#### H.O.T. Problems Higher Order Thinking

6. □ Persevere with Problems Triangle ABC has vertices A(0, 4), B(0, -2), and C(2, 0). The triangle is reflected over the x-axis. Then the image is rotated 180° counterclockwise about the origin. What are the coordinates of the final image?

- Persevere with Problems Triangle QRS is translated 7 units right, then rotated 90° clockwise about the origin. The vertices of triangle Q"R"S" are Q"(6, −1), R"(0, −1), and S"(0, −7). Find the coordinates of △QRS.
   Q(−6, 6), R(−6, 0), S(0, 0)
- Model with Mathematics A triangle is rotated 90° clockwise about the origin. Then the image is rotated 270° clockwise about the origin.
  - Complete the algebraic representation to explain the effect of the series of transformations performed.

$$(x,y) \rightarrow (\begin{array}{c} y \end{array}, \begin{array}{c} -x \end{array}) \rightarrow (\begin{array}{c} x \end{array}, \begin{array}{c} y \end{array})$$

- b. Based on your answer to part a, what can you conclude about a rotation of 90° followed by a rotation of 270°. They are the same as a rotation of 360°.
- 9. Reason Inductively Will a geometric figure and its rotated image always, sometimes, or never have the same perimeter? Explain your reasoning. always; Sample answer: The figure and its image have the same size and shape. Since the corresponding lengths are equal, the perimeters are the same.

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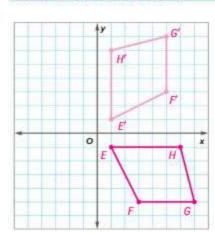
My Homework

## **Extra Practice**

10. Quadrilateral EFGH has vertices E(1, -1), F(3, -5), G(7, -5), and H(6, -1). Graph the figure and its rotated image after a counterclockwise rotation of 90° about the origin. Then give the coordinates of the vertices for quadrilateral E'F'G'H'.

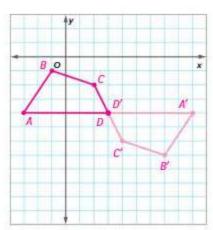
E'(1, 1), F'(5, 3), G'(5, 7), H'(1, 6)





11. Quadrilateral ABCD has vertices at A(-3, -4), B(-1, -1), C(2, -2), and D(3, -4). Graph quadrilateral ABCD and its image after a 180° counterclockwise rotation about vertex D. Then give the coordinates of the vertices of the image.

A'(9, -4), B'(7, -7), C'(4, -6), D'(3, -4)



12. Clentify Structure Identify each transformation as a translation, reflection, or rotation.









reflection

translation

rotation

Copy and Solve Triangle MNP has vertices M(1, 4), N(3, 1), and P(5, 3). Find the vertices of M'N'P' after each rotation about the origin. Show your work on a separate piece of paper.

$$P'(3, -5)$$

$$N'(-3, -4),$$

$$N'(-3, -1),$$
  
 $P'(-5, -3)$ 

## Power Up! Test Practice

Exercises 17 and 18 prepare students for more rigorous thinking needed for assessment.

- 17. This test item requires students to reason abstractly and quantitatively when problem solving.

  Depth of Knowledge DOK1

  Mathematical Practice MP1

  Scoring Rubric

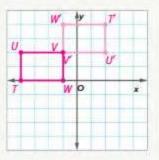
  1 point Students correctly answer each part of the question.
- 18. This test item requires students to analyze and solve complex realworld problems through the use of mathematical tools and models. Depth of Knowledge DOK3 Mathematical Practice MP1, MP4 Scoring Rubric Students correctly draw the figure and its 2 points rotation and list the coordinates. Students correctly draw the figure and its 1 point rotation but fail to list the coordinates OR students correctly draw one figure and list the coordinates OR students correctly list the coordinates but fail to draw the figures.

## Power Up! Test Practice

16. On a floor plan, TUVW with vertices T(-4, 0), U(-4, 2), V(-1, 2), and W(-1, 0) represents the location of Hiyam's bed in her bedroom. Hiyam would like to rotate her bed 180° clockwise about point V to see if she likes the new placement. Draw the bed and the rotated image on the coordinate plane.

What are the coordinates of the corners of the rotated bed?

T'(2, 4), U'(2, 2), V'(-1, 2), W'(-1, 4)



Triangle ABC is rotated 90° counterclockwise about the origin.
 Determine if each statement is true or false.

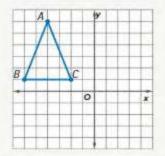
a. The image of point A is A'(-6, 4).

c. The image of point C is C'(-1, -2).

True False

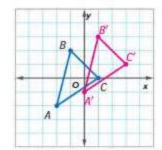
**b.** The image of point B is B'(-1, -6).

True False



## **Spiral Review**

- 18. Use the graph of △ABC shown at the right.
  - a. What are the coordinates of △A'B'C' when △ABC is reflected over the x-axis? A'(-2, 2), B'(-1, -2), C'(1, 0)
  - b. Graph the image of △ABC after it is translated 2 units right and 1 unit up.



19. Triangle FGH has vertices F(-3, 7), G(-1, 5), and H(-2, 2). Find the vertices of its image after a translation of 4 units right and 2 units down followed by a reflection over the y-axis.

F'(-1, 5), G'(-3, 3), H'(-2, 0)

พพงษะ ว่าเออร์ ว่าเรื่องแร

## **Inquiry Lab**



#### WHAT are the results of a dilation of a triangle?

Mathematical Practices 1.3.5

One way to create murals on a wall is to use a drawing grid method. Artists draw a grid on the artwork to be copied and draw a similar grid on the wall. By transferring sections of the artwork, the mural is the same shape as the artwork, but a different size.

## Hands-On Activity 1

In this Activity, you will enlarge \( \triangle ABC \) by a scale factor of 2 using grid paper. Point A will be the center point for the enlargement.

Step 1

On the grid shown below,  $\overrightarrow{AB}$  is drawn to the edge of the grid. Draw AC in the same way.



Step 2

Draw point B' on  $\overrightarrow{AB}$  so that  $\overrightarrow{AB'} = 2(\overrightarrow{AB})$ . Draw point C' on  $\overrightarrow{AC}$  so that AC' = 2(AC).

Draw B'C' to complete  $\triangle AB'C'$ .

What is the ratio of the length of  $\overline{AB}$  to the length of  $\overline{AB}$ ?

What is the ratio of the length of AC to the length of AC? 12 or 2

What is the ratio of the length of BC to the length of BC? 2

What do you notice about the ratios of corresponding sides?

www.jnob-jo.comsimilar to AABC? They are equal; yes.

### Focus narrowing the scope

Objective Identify dilations.

## Coherence connecting within and across grades

#### Now

Students will measure angles and sides to generalize the properties of a dilation.

### Next

Students will graph dilations on the coordinate plane.

## RIGOT pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 485.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

## Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

## Hands-On Activity 1

(II) (II) Circle the Sage Poll the class to see which students have some knowledge of dilations. Those students (the sages) spread out around the room. Assign the rest of the students to teams. Have the teams split up with each team member going to a different sage, if possible. Have the sages lead work for Activity 1. When the activity is complete, students go back to their teams and compare solutions. Students discuss how the sages may have explained the steps differently. 21, 5, 6, 7

#### Ask:

 How does triangle ABC compare to triangle AB'C'? Sample answer: They are the same shape but different sizes.

## Hands-On Activity 2

(I) (IA) Circle the Sage Select new sages to lead the activity based on their understanding of Activity 1. Repeat the same process, assigning new teams. (2) 1, 3, 5, 6, 7

#### Ask:

- Compare the scale factor in Activity 2 to the scale factor in Activity 1. How does the change in scale factor affect the dilation? Sample answer: The scale factor in Activity 1 was 2. The scale factor in Activity 2 is 1/2. In Activity 1, the dilation was larger than the original triangle. In Activity 2, the dilation is smaller than the original triangle.
- Rally Table Assign students to pairs. Students take turns completing the tasks in Activity 2. Have them discuss their responses to the questions in Step 4, with each student listening carefully to the other's reasoning. Have them ask for clarification or assistance, if needed. 1, 5, 6, 7

## Hands-On Activity 2

In Activity 1, you used a dilation to transform  $\triangle ABC$  by a scale factor of 2. A **dilation** is a transformation that enlarges or reduces a figure by a scale factor relative to a center point. That point is called the **center of dilation**.

In this Activity, you will draw the image of  $\triangle XYZ$  after a dilation with a scale factor of  $\frac{1}{2}$ . Point C will be the center of dilation.

Step 1

Triangle XYZ is shown below. Point C is the center of dilation. Using a ruler, draw line segments connecting C to each of the vertices of the triangle.  $\overline{CY}$  is done for you.

Step 2

Measure  $\overline{CY}$ . Draw point Y' on  $\overline{CY}$  so that  $CY' = \frac{1}{2}(CY)$ .

Step :

Repeat Step 2 for the two remaining sides. Draw point X' on  $\overline{CX}$  so that  $CX' = \frac{1}{2}(CX)$  and point Z' on  $\overline{CZ}$  so that  $CZ' = \frac{1}{2}(CZ)$ .

Step 4

Draw  $\triangle X'Y'Z'$ .

Is  $\triangle X'Y'Z'$  the same shape as  $\triangle XYZ$ ? **yes** 

Measure and compare the corresponding lengths on the original and new triangles. Describe the relationship between these measurements. Sample answer: the measures of the original triangle's side lengths are 2 times the new triangle's side lengths.

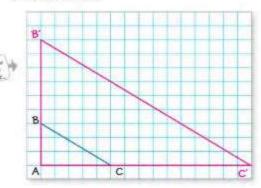
Measure and compare the corresponding angles on the original and new triangles. Describe the relationship between these measurements. The measures of the corresponding angles on the original and new triangles are the same.



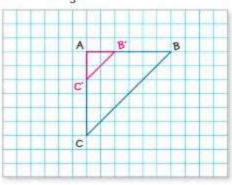
## Investigate

Work with a partner. Draw the image after a dilation with the given scale factor. Point A is the center of dilation.

1. scale factor 3.

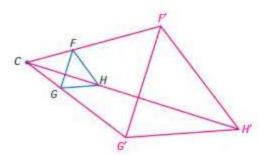


2. scale factor: 1

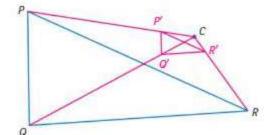


Work with a partner. Use a ruler to draw the image after a dilation with the given scale factor. Point C is the center of dilation.

3. scale factor: 3



4. scale factor: 1



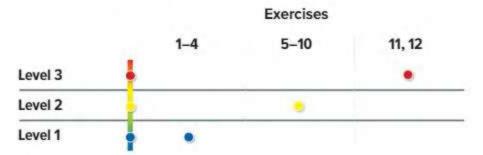
ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

## Collaborate

The Investigate and Analyze and Reflect sections are intended to be used as small-group investigations. The Create section is intended to be used as independent exercises.

#### Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.





(III) (III) Rally Coach Have students work in pairs. While Student A works through Exercise 1, Student B watches, listens, coaches, and praises. Then partners trade roles for Exercise 2. Continue for Exercises 3 and 4. 21, 5, 6, 7

#### Ask:

- What happens to the image when it is dilated by a scale factor that is a whole number greater than 1? Sample answer: The new image is bigger.
- What happens to the image when it is dilated by a scale factor that is a fraction between 0 and 1? Sample answer: The new image is smaller.
- (B) Trade-a-Problem Students draw an image with 4 or 5 sides on a grid paper and choose a scale factor for the dilation. Students trade drawings and perform the dilation on their partner's drawing. Have students compare drawings and discuss any differences in solutions. 31, 5, 6, 7



## **Analyze and Reflect**

(IA) Think-Pair-Share Give students time to complete Exercises 5–10 on their own. Then have them share answers with a partner and resolve any differences. 21, 5, 6, 7

#### Ask:

- How are ratios written? in fraction form
- Report Pairs Discussion Give students time to complete Exercises 5-10 in pairs. Then have them trade answers with another pair of students and resolve any differences. 21, 5, 6.7

#### Ask:

 For Exercise 6, explain how you can complete the table without measuring. Sample answer: Because the dilation had a scale factor of 3, I can multiply my answers in Exercise 5 by 3 to find the new side lengths.

Discuss in a small group how to answer Exercises 11 and 12. Assign one group member as the leader. The leader facilitates the discussion and makes sure that every group member understands. (2) 1, 3, 5, 6, 7

Students should be able to answer "WHAT are the results of a dilation of a triangle?" Check for student understanding and provide guidance, if needed.



## Analyze and Reflect Sample answers: 5, 6, 10

Use Math Tools For each figure from Exercise 3, measure the given side lengths in millimeters. Complete the table.

| 5. | Figure        | Side |    |    |   |
|----|---------------|------|----|----|---|
|    | Transaction 1 | FG   | GH | HF | 1 |
|    | ΔFGH          | 13   | 12 | 14 |   |

| 6. | Figure      | Side Lengths (mm) |      |      |  |  |
|----|-------------|-------------------|------|------|--|--|
|    | a sel e hal | F'G'              | G'H' | H'F' |  |  |
|    | ΔF'G'H'     | 39                | 36   | 42   |  |  |

- 7. What is the ratio of side FG to side F'G'?
- 8. What is the ratio of side GH to side G'H'?
- 9. What is the ratio of side HF to side H'F'? 3
- Measure the angles of △FGH and △F'G'H' in Exercise 3 using a protractor.

Describe the relationship between the corresponding angles.

The corresponding angles have the same measure.

| Angle Measure (*) |     |     |  |  |  |  |  |  |
|-------------------|-----|-----|--|--|--|--|--|--|
| ΖF                | ZG  | ΖH  |  |  |  |  |  |  |
| 54                | 66  | 60  |  |  |  |  |  |  |
| ZF'               | ZG' | ∠H′ |  |  |  |  |  |  |
| 54                | 66  | 60  |  |  |  |  |  |  |



### Create

- Reason Inductively Based on the Activities and Exercises, write a conjecture about the effects of a dilation on the sides and angles of a triangle. Sample answer: After a dilation, the new triangle's angles have the same measure to the original triangle's angles. The ratios of the corresponding sides are equal to the scale factor.
- 12. Inquity WHAT are the results of a dilation of a triangle? When a triangle is dilated, the resulting triangle has the same shape, but is a different size.

## 2 ST CENTURY CAREER

in Computer Animation

## **Computer Animator**

Have you ever wondered how they make animated movies look so realistic? Computer animators use computer technology and apply their artistic skills to make inanimate objects come alive. If you are interested in computer animation, you should practice drawing, study human and animal movement, and take math classes every year in high school. Tony DeRose, a computer scientist at an animation studio said, "Trigonometry helps rotate and move characters, algebra creates the special effects that make images shine and

sparkle, and calculus helps light up a scene."

## Is This the Career for You?

Are you interested in a career as a computer animator? Take some of the following courses in high school.

- 2-D Animation
- Algebra
- Calculus
- Trigonometry

Turn the page to find out how math relates to a career in Computer Animation.

### Focus narrowing the scope

**Objective** Apply mathematics to problems arising in the workplace.

This lesson emphasizes Mathematical Practice 4 Model with Mathematics.

## Coherence connecting within and across grades

### **Previous**

Students described tranformations of

#### Now

Students apply the content standard to solve problems in the workplace.

## Rigor pursuing concepts, fluency, and applications

See the Career Project on page 496.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

## 1 Launch the Lesson

Ask students to read the information on the student page about computer animation and answer the following questions.

#### Ask:

- What kinds of classes should you take to be a computer animator?
   2D Animation, Algebra, Calculus, Trigonometry
- What does a computer animator do? A computer animator uses computer technology and artistic skills to make inanimate objects come alive onscreen.
- What should a person who wants to be a computer animator do to prepare? practice drawing, study human and animal movement, and study the math needed to create the effects they are trying to achieve in their animation

Help students connect what they do today to their futures.

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## **2** Collaborate

Rally Coach Have students work in pairs to complete Exercises 1–3. Student 1 completes the first exercise while Student 2 listens, coaches, and praises. Then, Student 2 completes the next exercise while Student 1 listens, coaches, and praises. Partners take turns until the exercises are completed. 21, 5, 6, 7

**BL LA Pairs Discussion** Have students work in pairs to extend the activity by answering the following question.  $\bigcirc$  1, 3, 5, 6, 7

#### Ask:

 Describe how more than one transformation was used to create the characters below. The character on the right is a reflection and dilation of the character on the left.



## **Career Portfolio**

When students complete this page, have them add it to their Career Portfolio.

## Career Facts

As mathematics and computer techniques become more advanced, computer animation becomes more lifelike. Computer animators are able to use advanced mathematics and computer techniques to create realistic water, waves, and splash effects.

#### An Animation Sensation

Use Figures 1-3 to solve each problem.

- In Figure 1, the car is translated 8 units left and 5 units down so that it appears to be moving. What are the coordinates of A' and B' after the translation? A'(-10, -2), B'(-4, 2)
- In Figure 1, the car is translated so that A' has coordinates (-7, 2). Describe the translation as an ordered pair. Then find the coordinates of point B'. (x 5, y 1), B'(-1, 6)
- In Figure 1, the car is reflected over the x-axis in order to make its reflection appear in a pond. What are the coordinates of A' and B' after the reflection? A'(-2, -3),
   B'(4, -7)

- In Figure 2, the artist uses rotation to show the girl's golf swing. Describe the coordinates of G' if the golf club is rotated 90° clockwise about point H. (1, 6.5)
- The character in Figure 3 is enlarged by a scale factor of <sup>5</sup>/<sub>2</sub>. What are the coordinates of O' and R' after the dilation? O'(10, 15);
   R'(45, 45)
- 6. The character in Figure 3 is reduced in size by a scale factor of <sup>2</sup>/<sub>3</sub>. What is the number of units between S' and T', the width of the character's face, after the dilation?
  8<sup>2</sup>/<sub>3</sub> units

## Career Project

It's time to update your career portfolio! Choose a movie that was completely or partially computer animated. Use the Internet to research how technology was used to create the scenes in the movie. Describe any challenges that the computer animators faced.

What are some short term goals you need to achieve to become a computer animator?

Dynghi C Wodrawell

## Chapter Review 🗑



## **Vocabulary Check**



Unscramble each of the clue words to identify a term related to transformations.

GONUENRTC

| C | 0 | N | G | R | U | E | N | T |
|---|---|---|---|---|---|---|---|---|
|   |   |   |   | _ |   |   | _ | _ |

**TLODNIIA** 

| 0 | 1 | L | A | T | 1 | 0 | N |
|---|---|---|---|---|---|---|---|
|   |   |   |   |   |   |   |   |

**GEIMA** 

| Li. |   | 1 |  |
|-----|---|---|--|
| M   | A | G |  |
| 878 | _ | - |  |

PEERIGAM

| • | R | E | M | A | G | Ε |
|---|---|---|---|---|---|---|
|   |   | _ |   |   | _ | _ |

TIFROLCENE

| 12 | 120 | 100 | 121 |   | 200 | 100 |     | 1000 |   |
|----|-----|-----|-----|---|-----|-----|-----|------|---|
| R  | E   | F   | L   | E | C   | T   |     | 0    | N |
|    |     |     | 1   |   |     |     | 100 |      |   |

NOTTIRAO

| R | 0 | T | A | T | 1 | 0 | N |
|---|---|---|---|---|---|---|---|
|   |   |   |   |   |   |   |   |

LAOTANSTRIN

| - | D  | ۸ | N  | • | 1 | Λ | T. | 1 | 0 | N    |
|---|----|---|----|---|---|---|----|---|---|------|
|   | I. | A | IN | 3 | - | M |    |   | v | I.A. |

Complete each sentence using one or more of the unscrambled words above.

- 1 Translation
- is another name for a slide.
- 2. The image produced by enlarging or reducing a figure is called

a dilation

- 3. A transformation is an operation that maps an original
  - geometric figure, the preimage , onto a new figure called
  - the image
- 4. A reflection
- is the mirror image of the original figure.

## **Vocabulary Check**

 Find the Fib Have students work in groups and write down two facts and one fib using the words in the Vocabulary Check. For example, one fact could be that a translation is another name for a slide. One fib could be that the original geometric figure in a transformation is called the image. Each team member shares their facts and fib by speaking aloud to the group. The job of the group is to listen carefully, discuss, and come to a consensus to identify the fib. 31, 3, 6

## **Alternate Strategy**

(I) In To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- congruent (Lesson 1)
- dilation (Lesson 4)
- image (Lesson 1)
- preimage (Lesson 1)
- reflection (Lesson 2)
- rotation (Lesson 3)
- translation (Lesson 1)

## **Key Concept Check**

FOLDABLES LA A completed Foldable for this chapter should include translations, reflections, rotations, and dilations.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

## **Ideas for Use**

Gallery Walk Have students work with a partner to share their completed Foldables. Then have each student add or adjust anything in their Foldable based on the discussion with their partner. Display all of the Foldables around the room and have students walk around the room studying each Foldable. Have them determine if they should add anything to their Foldable based upon what they saw in others' Foldables.

1, 5, 6, 7

#### Got it?

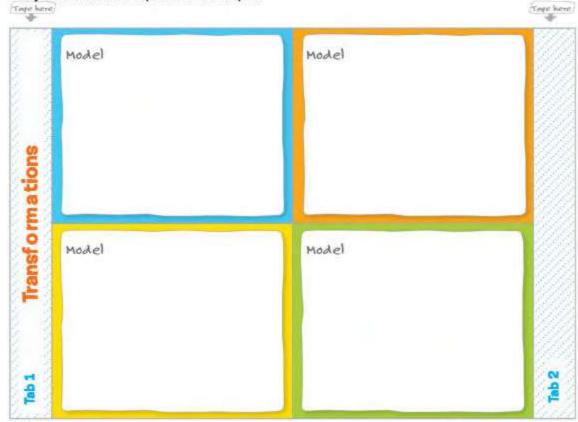
If students have trouble with Exercises 1–4, they may need help with the following concept(s).

| Concept                 | Exercise(s) |
|-------------------------|-------------|
| dilations (Lesson 4)    | 1           |
| reflections (Lesson 2)  | 2           |
| rotations (Lesson 3)    | 3           |
| translations (Lesson 1) | 4           |

## **Key Concept Check**

#### Use Your

Use your Foldable to help review the chapter.



#### Got it?

The problems below may or may not contain an error. If the problem is correct, write a "

"" by the answer. If the problem is not correct, write an "X" over the answer and correct the problem.

#### Use the figure at the right.

- The coordinates of point A after a dilation with a scale factor of 2 are (2, 6).
- The coordinates of point A after a reflection over the y-axis are (1, 3). (-1, 3)
- The coordinates of point A after a clockwise rotation 90° about the origin are (3, -1).
- The coordinates of point A after a translation of 3 units left and 2 units up are (2)5. (-2, 5)

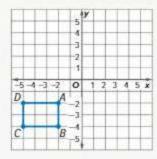
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## Power Up! Performance Task

#### Yearbook Layout

Students in Mrs. Nisreen's fifth period class are experimenting with different page layouts on a computer screen. The coordinate grid at the right represents one page of a two-page spread. One photo is already placed on the page.



Write your answers on another piece of paper. Show all of your work to receive full credit.

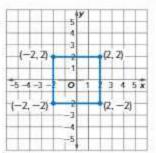
#### Part A

A second photo will be added by reflecting the original photo across the x-axis. Use a separate coordinate plane to draw and label the second photo. List the coordinates of the second photo.

#### Part B

Mrs. Nisreen wants the students to rotate the second photo 90° clockwise about the origin, then translate it two units down and one unit to the right to place a third photo. Draw and label the third photo on your coordinate plane. List the coordinates of the third photo.

Eissa placed a square photo in the center of the screen as shown. Dilate the photo so that it results in the picture taking up the whole screen. What is the scale factor? Label each point with the new coordinates.



## Power Up! Performance Task

This Performance-Based Assessment requires students to solve multi-step problems through abstract reasoning, precision, and perseverance. This practice scenario can be used to help students prepare for the thinking skills that will be used on the CCSS Assessment.

A complete scoring rubric with answers to the Exercises can be found on page PT2.



### **Answering the Essential Question**

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- How are figures translated on the coordinate plane? (p. 456)
- How can you determine the coordinates of a figure after a reflection over either axis? (p. 464)
- What is the difference between rotating a figure about a given point that is a vertex and rotating the same figure about the origin if the rotation is less than 360°? (p. 478)
- How are dilations similar to scale drawings? (p. 490)

### Ideas for Use

Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their partner before they complete the graphic organizer. 1, 5, 6, 7

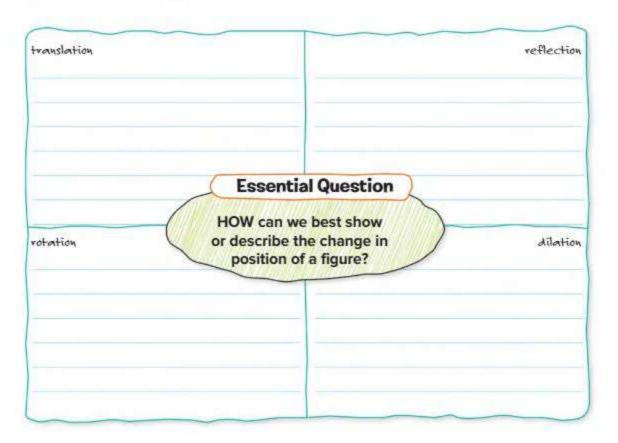
## **Track Your Progress**

Return to the beginning of the chapter to review the objectives that it addressed. Students should see that their knowledge of the key ideas has increased now that they have completed this chapter.

## Reflect

#### **Answering the Essential Question**

Use what you learned about transformations to complete the graphic organizer. Determine if you would demonstrate each transformation by using words, symbols, or models. Then give an example using your method for each transformation. See students' work.



**Answer the Essential Question.** HOW can we best show or describe the change in position of a figure?

See students' work.

## English

**accuracy** The degree of closeness of a measurement to the true value.

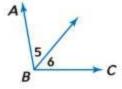
acute angle An angle whose measure is less than 90°.



acute triangle A triangle with all acute angles.

Addition Property of Equality If you add the same number to each side of an equation, the two sides remain equal.

adjacent angles Angles that share a common vertex, a common side, and do not overlap. In the figure, the adjacent angles are  $\angle 5$  and  $\angle 6$ .



algebra A branch of mathematics that involves expressions with variables.

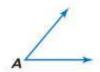
algebraic expression A combination of variables, numbers, and at least one operation.

## العربية

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دقة القياس درجة قرب الفياس إلى الفيمة الحقيقية.

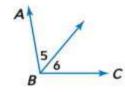
الزاوية الحادة (اوية فباسها أصغر من 90 درجة.



المثلث حاد الزوايا مثلث كل زواياه حادة.

الجمع في المعادلة إذا أضفت العدد نفسه إلى كل طرف في المعادلة. يبقى الطرفان متساويين.

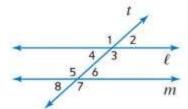
الزوایا المتجاورة الزوایا التي لها رأس مشترك، وضلع مشترك، ولا نتداخل. في الشكل التالي، الزوایا المتجاورة هي 25 و 65.



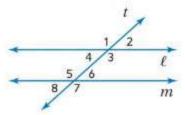
الجبر فرع من الرياضيات يحتوي على عبارات تتضمن متغيرات.

التعبير الجبري مجموعة من المتغيرات والأعداد وعملية واحدة على الأفل.

alternate exterior angles Exterior angles that lie on opposite sides of the transversal. In the figure, transversal t intersects lines I and m.  $\angle 1$  and  $\angle 7$ , and  $\angle 2$  and  $\angle 8$  are alternate exterior angles. If line I and m are parallel, then these pairs of angles are congruent.



alternate interior angles Interior angles that lie on opposite sides of the transversal. In the figure below, transversal t intersects lines I and m.  $\angle 3$  and  $\angle 5$ , and  $\angle 4$  and  $\angle 6$  are alternate interior angles. If lines I and m are parallel, then these pairs of angles are congruent.

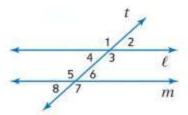


angle of rotation The degree measure of the angle through which a figure is rotated.

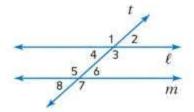
arc One of two parts of a circle separated by a central angle.

Associative Property The way in which three numbers are grouped when they are added or multiplied does not change their sum or product.

الزوایا الخارجیة المتبادلة زوایا خارجیة توجد علی جانبین مثقابلین من الفاطع. في الشکل التالي، یتداخل الفاطع t مع المستقیمین t و t الفاطع t مع المستقیمین t و t الفاطع t مع خارجیة متبادلة. إذا کان المستقیمان t و t متوازیین، فإن أزواج هذه الزوایا نکون متطابقة.



الزوایا الداخلیة البدیلة زوایا داخلیة توجد علی ضلعین متفابلین من الفاطع. فی الشکل أدناه، یتداخل الفاطع 1 مع المستفیمین 1 و 1 س 1 س 1 س 1 و 1 س 1 س 1 س 1 و 1 س 1 و 1 س 1 و 1 س متوازبین، فإن أزواج الزوایا هذه منطابقة.



زاوية الدوران المحوري مقياس بالدرجات للزاوية التي بسندير من خلالها الشكل.

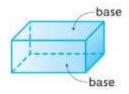
القوس أحد جزأى دائرة نفصله زاوية مركزية.

خاصية التجميع الطريقة التي يتم فيها تجميع ثلاثة أعداد عند إضافتها أو ضربها بحيث لا يتغير المجموع أو ناتج الضرب.

В

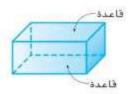
base In a power, the number that is the common factor. In  $10^3$ , the base is 10. That is,  $10^3 = 10 \times 10 \times 10$ .

base One of the two parallel congruent faces of a prism.



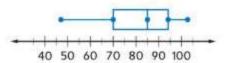
1لاً ساس عملية القوى، هو العدد الذي يتكرّر في عملية الضرب. في البثال  $^{0}$ 1, يكون الأساس هو العدد  $^{0}$ 1. بمعنى أن،  $^{0}$ 1 -  $^{0}$ 1 ×  $^{0}$ 1.

قاعدة المنشور أحد الوجهين المتطابقين المتوازيين في المنشور.



bivariate data Data with two variables, or pairs of numerical observations.

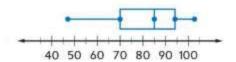
box plot A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.



العيئة المُتحازة عينة تُجمع بطريقة بُفضلها جزء أو أكثر من المجتبع الإحصائي مقارنة بالمجتبعات الأخرى.

البيانات ذات المتغيرين بيانات تحتوي على متغيرين (أو زوجين من المشاهدات العددية).

مخطط الصندوق ذي العارضين طريقة لعرض توزيع قيم البيانات بصريًا باستخدام الوسيط والزبعيّات وأطراف مجموعة البيانات، ويظهر الصندوق الوسيط بنسبة %50 من البيانات.



С

center The given point from which all points on a circle are the same distance.

**center of dilation** The center point from which dilations are performed.

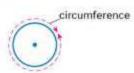
center of rotation A fixed point around which shapes move in a circular motion to a new position.

central angle An angle that intersects a circle in two points and has its vertex at the center of the circle.

circle The set of all points in a plane that are the same distance from a given point called the center.



circumference The distance around a circle.



chord A segment with endpoints that are on a circle.

coefficient The numerical factor of a term that contains a variable.

common difference The difference between any two consecutive terms in an arithmetic sequence.

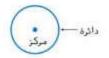
مركز الدائرة النقطة المعلومة التي تبعد عنها كل النقاط على الدائرة المسافة نفسها.

مركز تغيير الأبعاد نقطة البركز الذي يثم تغيير الأبعاد منها.

مركز الدوران المحوري نقطة ثابتة نتحرك حولها الأشكال بحركة دائرية إلى مؤقع جديد.

الزاوية المركزية زاوبة رأسها مركز وضلعاها أنصاف أقطار في الدائرة.

الدائرة مجموعة النقاط في المستوى التي لها البُعد نفسه عن نقطة معلومة ثابتة تُسمى البركز.



محيط الدائرة المسافة حول الدائرة.



الوتر قطعة طرفاها على محيط دائرة.

المعامل عامل عددي للحد الذي يحتوي على متغير.

الضرق المشترك الغرق بين حدين منتاليين في المتتالية الحسابية. وبُستِي أساس المنتالية الحسابية أ).

مقوق العلمي والنشيرات محموطة فصادح المرادعة التاريخ

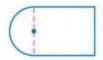
Commutative Property The order in which two numbers are added or multiplied does not change their sum or product.

complementary angles Two angles are complementary if the sum of their measures is 90°.



∠1 and ∠2 are complementary angles.

**composite figure** A figure that is made up of two or more shapes.



**composite solid** An object made up of more than one type of solid.



composition of transformations The resulting transformation when a transformation is applied to a figure and then another transformation is applied to its image.

compound event An event that consists of two or more simple events.

compound interest Interest paid on the initial principal and on interest earned in the past.

cone A three-dimensional figure with one circlular base connected by a curved surface to a single vertex.



congruent Having the same measure; if one image can be obtained by another by a sequence of rotations, reflections, or translations.

constant A term without a variable.

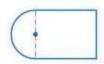
خاصية التبديل الترتيب الذي يتم به إضافة أو ضرب عددين بحيث لا يتغير مجموعهما أو ناتج ضربهما.

الزاويتان المتتامتان تكون الزاويتان متنامتين إذا كان مجموع فياسهما يساوي 90°.



1 الحو 2 كا زاويتان متنامتان.

الشكل المركب الشكل المكون من شكلين أو أكثر.



الهجسم الهركب جسم يتكون من أكثر من نوع واحد من المجسمات.



تركيب التحويلات التحويل النائج عند تطبيق تحويل على شكل ما ثم تطبيق تحويل آخر على صورة هذا الشكل.

الحدث المركب حدث مكون من حدثين بسيطين أو أكثر.

الفائدة المركبة فائدة تدفع على رأس المال الأولي وعلى الفائدة المكتسبة سابقًا.

المحروط شكل ثلاثي الأبعاد قاعدته دائرية الشكل ومتصلة بسطح منحن ورأس واحدة فقط.



المتطابق زاوية لها الغياس نفسه؛ أو شكل مماثل تبامًا لشكل أخر وينطبق عليه عن طريق سلسلة من الدورانات المحورية أو الانعكاسات أو التفسيرات.

الثابت حد لا يحتوى على متغير،

متوق العلمج والنشي © محتوظة المائح Praw-Hill Education

constant of variation A constant ratio in a direct variation.

constant rate of change

The rate of change
between any two points in a linear relationship is the
same or constant.

continuous data Data that can take on any value. There is no space between data values for a given domain. Graphs are represented by solid lines.

convenience sample A sample which includes members of the population that are easily accessed.

**converse** The converse of a theorem is formed when the parts of the theorem are reversed. The converse of the Pythagorean Theorem can be used to test whether a triangle is a right triangle. If the sides of the triangle have lengths a, b, and c, such that  $c^2 = a^2 + b^2$ , then the triangle is a right triangle.

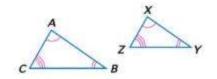
coordinate plane A coordinate system in which a horizontal number line and a vertical number line intersect at their zero points.

| x-axis | 3 | y-axis |
|--------|---|--------|
|        | 0 |        |
| -3-2-1 | 1 | origin |
|        | 3 |        |

coplanar Lines that lie in the same plane.

corresponding angles Angles that are in the same position on two parallel lines in relation to a transversal.

corresponding parts Parts of congruent or similar figures that match.



ثابت التناسب نسبة ثابتة في علاقة التناسب الخطية.

ثابت التغير نسبة ثابتة في حالة التغير الطردي.

معدل التغير الثابت تماثل أو ثبات نسبة التغير بين أي نقطتين في علاقة خطية.

البيانات المتصلة بيانات قد تأخذ أي قيبة. ولا توجد مساحة بين قيم البيانات الخاصة بمجال معين. وضئل بيانيًا عن طريق خطوط المجسم.

العينة الملائمة عينة تتضمن أفرادًا من المجتمع الإحصائي يسهل الوصول إليهم.

العكس يتشكل عكس نظرية عندما يتم عكس أجزاء من النظرية، ببكن استخدام عكس نظرية فيثاغورس لمعرفة إذا كان المثلث قائم الزاوية.إذا كانت أطوال أضلاع المثلث هي a. وb. و c. يديث إن C2 = a2 + b2.

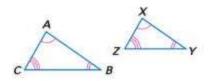
المستوى الإحداثي نظام إحداثي يكون فيه خط الأعداد الأفقي وخط الأعداد الرأسي متقاطعين في النقاط الصفرية.

| 2         | У        |
|-----------|----------|
| 2 الحور X | احور 🗸 😶 |
| 1         | 0        |
| -3-2-1    | \1 2 3 x |
| 2         | الاصل    |
| -3        |          |

مستقيهات مستوية المستقيمات التي تقع في المستوى نفسه.

الزوايا المتناظرة (وايا نقع في الجهة والموقع أنفسهما من قاطع يقطع مستقيبين متوازيين.

الأجزاء المتناظرة أجزاء من أشكال منطابقة أو منشابهة تتوافق مع بعضها.



شوق الطبع والنشر © محموطة لصالح raw-Hill Edication

counterexample A statement or example that shows a conjecture is false.

cross section The intersection of a solid and a plane.

cube root One of three equal factors of a number. If  $a^3 = b$ , then a is the cube root of b. The cube root of 64 is 4 since  $4^3 = 64$ .

cylinder A three-dimensional figure with two parallel congruent circular bases connected by a curved surface.



الهثال المضاد عبارة أو مثال بدل على فرضية غير صحيحة،

المقطع تقاطع مجسم ومستوى

الجدّر التكفييي أحد العوامل الثلاثة المتساوية في العدد. إذا كان  $a^3 = b$ . الجدر التكفيبي لــ b. الجدر التكفيبي للعدد 125 مو 5. لأن b – b.

الأسطوانة شكل ثلاثي الأبعاد يحتوي على فاعدتين دائريتين متطابقتين ومتوازيتين وتتصلان ببعضهما عن طريق سطح منحن.



D

**deductive reasoning** A system of reasoning that uses facts, rules, definitions, or properties to reach logical conclusions.

defining a variable Choosing a variable and a quantity for the variable to represent in an expression or equation.

degree A unit used to measure angles.

degree A unit used to measure temperature.

dependent events Two or more events in which the outcome of one event does affect the outcome of the other event or events.

dependent variable The variable in a relation with a value that depends on the value of the independent variable.

derived unit A unit that is derived from a measurement system base unit, such as length, mass, or time.

diagonal A line segment whose endpoints are vertices that are neither adjacent nor on the same face.

الاستدلال الاستنتاجي أسلوب النفكبر الذي يستخدم الحفائق أو القواعد أو التعريفات أو الخصائص للتوصل إلى استنتاجات صحيحة.

تعيين متفير اختبار متغير وكمية لهذا المتغير لتمثيلها في عبارة أو معادلة.

الدرجة وحدة فياس الزوايا.

الدرجة وحدة فياس درجة الحرارة.

الأحداث غير المستقلة حدثان أو أكثر نؤثر نتيجة إحداهما في نتيجة الحدث الآخر أو الأحداث الأخرى.

المتغير التابع المنغير الذي تعنمد فيمنه في علاقة على فيمة المتغير المستفل.

الوحدة المشتقة وحدة مشتقة من وحدة أساسية لأحد أنظهة القياس. مثل الطول أو الكتلة أو الزمن.

قطر ثلاثي الأبعاد قطعة مستثيمة ثلاثية الأبعاد نقطتا نهايتها عبارة عن رأسين ليسا متجاورين ولا على الوجه نقسه.



dilation A transformation that enlarges or reduces a figure by a scale factor.

dimensional analysis The process of including units of measurement when you compute.

**direct variation** A relationship between two variable quantities with a constant ratio.

**discount** The amount by which a regular price is reduced.

discrete data Data with space between possible data values. Graphs are represented by dots.

disjoint events Events that cannot happen at the same time.

Distance Formula The distance d between two points with coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

distribution A way to show the arrangement of data values.

Distributive Property To multiply a sum by a number, multiply each addend by the number outside the parentheses.

$$5(x+3) = 5x + 15$$

Division Property of Equality If you divide each side of an equation by the same nonzero number, the two sides remain equal.

domain The set of x-coordinates in a relation.

double box plot Two box plots graphed on the same number line.

قطى الدائرة المسافة المارة بالمركز داخل الدائرة، وهو وتر مار بمركز الدائرة.



تغيير الأبعاد بمقياس تحويل هندسي لتصغير أو لتكبير شكل عن طريق معامل المعياس.

التحليل البُعدي عملية تضمين وحدات قباس أثناء إجراء العمليات الحسابية.

التغيّر الطردي العلاقة بين كميني منعير ذات نسبة ثابثة.

الخصم مقدار تخفيض السعر المعتاد،

البيانات المتقطعة بيانات بها مساحة بين قيم البيانات المحتبلة، وتمثل بيانيًا عن طريق نقاط.

الأحداث الهنفصلة الأحداث التي لا يبكن أن تحدث في الوقت نفسه.

قانون المسافة بين نقطتين المسافة d بين نقطتين في المسافة d المستوى الإحداثي  $(x_1, y_1)$  و  $(x_2, y_2)$  بُمكن الحصول عليها من خلال القانون،

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

التوزيع طريقة توضح ترتيب قيم البيانات.

خاصية التوزيع ضرب الإجمالي في أي عدد. وضرب كل طرف جمع في العدد الموجود خارج الأفواس.

$$5(x+3) = 5x + 15$$

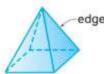
القسمة في المعادلة في حالة قسبة طرقي معادلة على العدد نفسه غير الصفرى، يبقى الطرفان متساويين.

المجال مجموعة من إحداثيات X.

مخطط الصندوق ذو العارضين المزدوج مخطط لصندوفين يتم تبئيلهما بالرسم البياني على خط الأعداد نفسه.



edge The line segment where two faces of a polyhedron intersect.



equation A mathematical sentence stating that two quantities are equal.

equiangular A polygon in which all angles are congruent.



equilateral triangle A triangle with three congruent sides.

equivalent expressions Expressions that have the same value regardless of the value(s) of the variable(s).

event An outcome is a possible result.

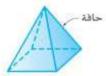
experimental probability An estimated probability based on the relative frequency of positive outcomes occurring during an experiment.

exponent In a power, the number of times the base is used as a factor. In 10<sup>3</sup>, the exponent is 3.

**exponential function** A nonlinear function in which the base is a constant and the exponent is an independent variable.

**exterior angles** The four outer angles formed by two lines cut by a transversal.

الحافة النطعة المستفيمة التي يتفايل فيها وجهان من شكل متعدد الأوجه.



المعادلة عبارة رياضية توضح وجود كميتين متساويتين

متطابق الزوايا مضلع كل زواياه منطابقة.



الهثلث متساوي الأضلاع مثلث يحنوي على ثلاثة أضلاع منطابعة.

التعابير المتكافئة تعابير لها القيمة نفسها بغض النظر عن قيمة المنغير (قيم المنغيرات).

الحدث نتيجة ممكنة.

الاحتمال التجريبي احتمال مقدر قائم على النكرار النسبي للنتائج الإيجابية التي تقع أثناء إجراء التجرية العشوائية.

الأس في عملية القوى. هو عدد البرات التي يتم فيها استخدام الأساس كعامل. وفي 103، يكون الأس 3.

الدالة الأسية دالة غير خطبة أساسها عبارة عن ثابت وأسها عبارة عن متغير مستقل.

الزوايا الخارجية أربع زوايا خارجية تتكون من خطين بقطعهما فاطع.



face A flat surface of a polyhedron.



الوجه سطح مستو لشكل متعدد الأوجه.



fair game A game where each player has an equally likely chance of winning.

five-number summary A way of characterizing a set of data that includes the minimum, first quartile, median, third quartile, and the maximum.

formal proof A two-column proof containing statements and reasons.

function A relation in which each member of the domain (input value) is paired with exactly one member of the range (output value).

function table A table organizing the domain, rule, and range of a function.

Fundamental Counting Principle Uses multiplication of the number of ways each event in an experiment can occur to find the number of possible outcomes in a sample space.

اللعبة العادلة لعبة بكون لدى كل لاعب فيها فرصة متساوية لاحتمالية القوز.

ملخص الخمسة مقاييس طربقة تصنيف مجموعة من البيانات نتضمن الحد الأدنى، والربيع الأول، والوسيط، والربيع الثالث، والحد الأقصى.

البرهان الشكلي برهان مكون من عبودين يحتويان على العبارات والاستدلالات.

الدالة علاقة يفترن فيها كل عضو في المجال (فيمة المدخل) بعضو واحد آخر في الهدى (قيمة المخرج).

جدول الدالة جدول ينظم البجال والقاعدة والمدى للدالة.

مبدأ العد الأساسى استعمال عملية الضرب لإيجاد عدد الطرق الممكنة لوقوع كل حدث في التجربة من أجل التوصل إلى عدد من النتائج المحتملة في فضاء عيني ما.

G

geometric sequence A sequence in which each term after the first is found by multiplying the previous term by a constant.

المتتالية الهندسية متالية بنم فيها إيجاد كل حد بعد الحد الأول عن طريق ضرب الحد السابق في ثابت.



half-plane The part of the coordinate plane on one side of the boundary.

hemisphere One of two congruent halves of a sphere.

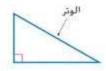
نصف المستوى الإحداثي جزء المستوى الإحداثي الواقع على جانب واحد من الحد القاصل بين الجزأين.

نصف الكرة أحد النصفين المتطابقين للكرة.

hypotenuse The side opposite the right angle in a right triangle.

hypotenuse

وتر المثلث القائم الزاوية الصلع المقابل للزاوية النائية في المثلث الغائم الزاوية.



identity An equation that is true for every value for the variable.

متطابقة عبارة رياضية صحيحة لكل قبهة من فيم المتغير.

image The resulting figure after a transformation.

independent events Two or more events in which the outcome of one event does not affect the outcome of the other event(s).

**independent variable** The variable in a function with a value that is subject to choice.

indirect measurement A technique using properties of similar polygons to find distances or lengths that are difficult to measure directly.

inductive reasoning Reasoning that uses a number of specific examples to arrive at a plausible generalization or prediction. Conclusions arrived at by inductive reasoning lack the logical certainty of those arrived at by deductive reasoning.

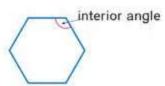
inequality A mathematical sentence that contains  $\langle , \rangle, \neq , \leq ,$  or  $\geq ,$ 

**inscribed angle** An angle that has its vertex on the circle. Its sides contain chords of the circle.

informal proof A paragraph proof.

interest The amount of money paid or earned for the use of money.

interior angle An angle inside a polygon.



**Interior angles** The four inside angles formed by two lines cut by a transversal.

**Interquartile range** A measure of variation in a set of numerical data. It is the difference between the first quartile and the third quartile.

inverse operations Pairs of operations that undo each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.

irrational number A number that cannot be expressed as the quotient  $\frac{a}{b}$ , where a and b are integers and  $b \neq 0$ .

الصورة الشكل النائج بعد التحويل.

الأحداث الهستقلة حدثان أو أكثر لا نؤثر نتيجة إحداهبا في نتيجة الحدث الآخر أو الأحداث الأخرى.

المتغير المستقل منغير في الدالة نخضع فيمنه للاختبار،

القياس غير المباشر نفنية تستخدم خصائص المضلعات المتماثلة للعثور على مسافات أو أطوال يصعب فباسها مباشرة.

الاستدلال الاستقرائي الاستدلال الذي يستخدم عددًا من الأمثلة المحددة للوصول إلى تعبيم منطقي أو توقع. تغتقر النتائج التي نتوصل إليها عن طريق الاستدلال الاستقرائي إلى اليقين المنطقي المتوفر في النتائج التي نتوصل إليها من خلال الاستدلال الاستناجي.

المتباینة عبارة ریاضیة نحتوی علی < أو > أو  $\neq$  أو  $\leq$  أو  $\geq$ .

الزاوية المحيطية زاوية رأسها على محيط الدائرة، وضلعاها وتران في الدائرة.

البرهان غير الشكلي برهان النفرة.

الشائدة مقدار المال المدفوع أو المكتسب نظير استخدام المال.

الزاوية الداخلية زاوية داخل مضلع.



الزوايا الداخلية أربع زوابا داخلية تتكون من مستفيمين يقطعهما فاطع.

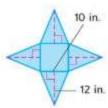
المدى الرّبيعي مغياس تشتت مجموعة من البيانات الرقمية. ويقصد به الغرق بين الرّبيّع الأول والرّبيّع الثالث.

العمليات العكسية أزواج من العمليات التي تلغي عمل بعضها البعض. يُعد الجمع والطرح عمليتين متعاكستين، كما يُعد الضرب والغسمة أيضًا عمليتين متعاكستين.

العدد غير النسبي العدد الذي لا يمكن التعبير عنه كنانج النسمة  $\frac{a}{b}$ , حبث إن a و  $b \neq 0$ .

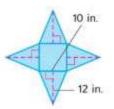


lateral area The sum of the areas of the lateral faces of a solid.



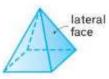
lateral area =  $4(\frac{1}{3} \times 10 \times 12) = 240$  square inch

مساحة جانبية مجموع مساحات الأوجه الجانبية لأحد المجسمات.

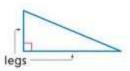


يوصة مربعة 240 = ( $\frac{1}{2} \times 10 \times 12$ ) = مساحة جانبية

lateral face Any flat surface that is not a base.



**legs** The two sides of a right triangle that form the right angle.



**like fractions** Fractions that have the same denominators.

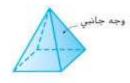
like terms Terms that contain the same variable(s) to the same powers.

linear To fall in a straight line.

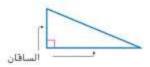
**linear equation** An equation with a graph that is a straight line.

linear function A function in which the graph of the solutions forms a line.

الوجه الجائبي أي وجه غير قاعدة المُجسّم.



ضلعا القائمة ضلعا المثلث القائم الزاوية اللذان يشكلان الزاوية القائمة.



الكسور المتشابهة الكسور التي لها المقامات نفسها.

الحدود المتشابهة حدود تتكون من نفس (المتغير) المتغيرات ومرفوعة إلى الأسس نفسها.

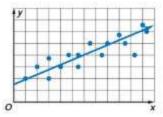
خطّى أن يكون في خط مستقيم.

المعادلة الخطية معادلة بتم تمثيلها بيانيًا بخط مستفيم

الدالة الخطية دالة بمثل الرسم البياني للحلول فيها خطأا مستفيفًا. قاموس المصطلحات/Glossary

**linear relationship** A relationship that has a straight-line graph.

line of best fit A line that is very close to most of the data points in a scatter plot.



**line of reflection** The line over which a figure is reflected.

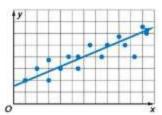
line of symmetry Each half of a figure is a mirror image of the other half when a line of symmetry is drawn.

**line symmetry** A figure has line symmetry if a line can be drawn so that one half of the figure is a mirror image of the other half.

**literal equation** An equation or formula that has more than one variable.

العلاقة الخطية علاقة يكون النمثيل البيائي فيها عبارة عن خط مستقيم.

المستقيم الأفضل تمثيلاً (خط الانتشار) خط قربب للقابة من معظم نفاط البيانات بالرسم البياني المنفرق.



خط الانعكاس الخط الذي تتكون صورة الشكل بالانعكاس عليه.

خط التناظر يعتبر كل نصف من أي شكل ببثابة صورة مطابقة للنصف الآخر عند رسم خط التناظر.

التناظر المحوري يكون للشكل تناظر محوري إذا كان يُمكن رسم خط مستقيم على الشكل بحيث يكون نصف الشكل صورة معكوسة للنصف الآخر.

المعادلة متعددة المتغيرات معادلة أو صبغة بها أكثر من منفير.

М

markup The amount the price of an item is increased above the price the store paid for the item.

mean The sum of the data divided by the number of items in the set.

mean absolute deviation The average of the absolute values of differences between the mean and each value in a data set.

measures of center Numbers that are used to describe the center of a set of data. These measures include the mean, median, and mode.

measures of variation Numbers used to describe the distribution or spread of a set of data.

هامش الربح مقدار زيادة سعر عنصر عن السعر الذي دفعه البتجر مقابل هذا العنصر.

المتوسط الحسابي مجموع البيانات منسومًا على عدد البيانات.

الانحراف المتوسط متوسط النيم المطلقة للفروق بين المنوسط الحسابي وكل قيمة في مجموعة البيانات.

مقاييس النزعة المركزية أرقام تُستخدم لوصف تمركز مجموعة من البيانات. وتنضمن هذه الفياسات المتوسط الحسابي والوسيط. والمنوال.

مقاييس التباين أرفام تستخدم لوصف توزيع أو نشر مجموعة من البيانات. مقوق العليج والنشي © محقوظة الصالح Graw-Hill Education

median A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values.

mode The number(s) or item(s) that appear most often in a set of data.

monomial A number, a variable, or a product of a number and one or more variables.

Multiplication Property of Equality If you multiply each side of an equation by the same number, the two sides remain equal.

multiplicative inverses Two numbers with a product of 1. The multiplicative inverse of  $\frac{2}{3}$  is  $\frac{3}{2}$ .

الوسيط مقياس من مقاييس النزعة المركزية في مجموعة من البيانات العددية، وسيط مجموعة من القيم هو القيمة التي تظهر في مركز القيم العددي بها أو المتوسط الحسابي للقيمتين المركزيتين، إذا كانت القائمة تحتوي على عدد زوجي من القيم.

المنوال العدد (الأعداد) أو العنصر (العناصر) الأكثر تكرارًا في مجموعة من البيانات.

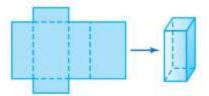
أحادي الحد عدد أو متغير أو ناتج ضرب عدد ومتغير واحد أو أكثر.

الضرب في المعادلة في حالة ضرب طرفي أية معادلة في نفس العدد، ببقى الطرفان متساوبين.

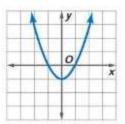
النظير الضربي عدد ناتج ضربه في العدد يساوي 1. النظير الضربي للعدد  $\frac{2}{5}$  هو  $\frac{3}{6}$  أو هو مطلوب العدد.

N

net A two-dimensional pattern of a threedimensional figure.

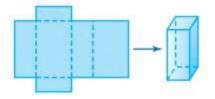


nonlinear function A function whose rate of change is not constant. The graph of a nonlinear function is not a straight line.

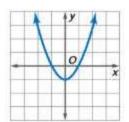


null set The empty set.

الشبكة مخطط مستوي لشكل ثلاثي الأبعاد



الدالة غير الخطية دالة بكون فيها معدل النغير لبس ثابتًا. وبكون الرسم البيائي لمعادلة غير خطية عبارة عن خط غير مستفيم.



الهجموعة الخالية المجموعة العارغة

0

obtuse angle An angle whose measure is between 90° and 180°.

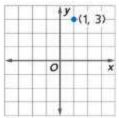
obtuse triangle A triangle with one obtuse angle.

الزاوية المنظرجة زاوية فياسها بين 90° و 180°.

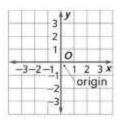
المثلث منفرج الزاوية مثلث إحدى زواياه منفرجة.

مقوق المشيخ والنشير ٥ محقو فلية لمسالح الارادية الارادية الارادية الارادية الارادية الارادية

ordered pair A pair of numbers used to locate a point in the coordinate plane. The ordered pair is written in this form: (x-coordinate, y-coordinate).



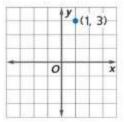
origin The point of intersection of the x-axis and y-axis in a coordinate plane.



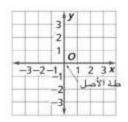
**outcome** One possible result of a probability event. For example, 4 is an outcome when a number cube is rolled.

outlier Data that are more than 1.5 times the interquartile range from the first or third quartiles.

الزوج الهرتب زوج من الأعداد يُستخدم لتحديد نقطة في المستوى الإحداثي، ويكتب الزوج المرتب على هذا الشكل: (x, y).



نقطة الأصل نقطة تقاطع البحور X مع البحور Y في مستوى إحداثي.



نتیجة غیر عشوائیة نائج مبكن لحدث محتمل.
مثال: یكون العدد 4 نتیجة فی حالة دحرجة مكعب أعداد.

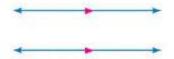
القيمة المتطرفة بيانات تكون أكبر بمقدار مرة ونصف من المدى من الزبيعي الأول أو الثالث.

P

paragraph proof A paragraph that explains why a statement or conjecture is true.

parallel Lines that never intersect no matter how far they extend.

parallel lines Lines in the same plane that never intersect or cross. The symbol | means parallel.

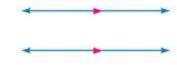


parallelogram A quadrilateral with both pairs of opposite sides parallel and congruent.

برهان الفقرة تنسبر صحة عبارة أو فرضية.

الهتوازي مستقيمات لا تتفاطع أبدًا مهما طالت البسافة بها.

المستقيمات المتوازية مستقيمات في نفس المستوى ولا تتفاطع أو تتلاقى أبدًا. يعني الرمز | نوازيًا.



متوازي الأضلاع شكل رباعي الأضلاع له زوجان من الأضلاع المتفابلة المتوازية والمتطابقة.

part = percent - whole

percent of change A ratio that compares the change in quantity to the original amount.

percent of decrease When the percent of change is negative.

percent of increase When the percent of change is positive.

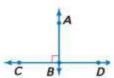
percent proportion Compares part of a quantity to the whole quantity using a percent.

perfect cube A rational number whose cube root is a whole number. 27 is a perfect cube because its cube root is 3.

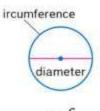
perfect square A rational number whose square root is a whole number. 25 is a perfect square because its square root is 5.

permutation An arrangement or listing in which order is important.

perpendicular lines Two lines that intersect to form right angles.



pi The ratio of the circumference of a circle to its diameter. The Greek letter  $\pi$  represents this number. The value of pi is always 3.1415926....



point-slope form An equation of the form  $y - y_1 =$  $m(x - x_1)$ , where m is the slope and  $(x_1, y_1)$  is a given point on a nonvertical line.

المعادلة المتوية صيغة مكافئة للتناسب المئوى. يتم التعبير فيها عن النسبة المثوية على صورة كسر عشري.

النسبة المئوية للتغير نسبة نفارن مقدار نغير كمية بالنسبة إلى الكم الأصلي.

النسبة الهئوية للتناقص عندما تكون النسبة المئوية للنغير سالية

النسبة المئوية للتزايد عندما تكون النسبة المئوية للتغير

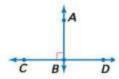
مقدار النسبة المئوية هو المفارنة بين جزء من كمية وإجمالي الكمية باستخدام نسبة مثوية.

المكعب الكامل عدد نسبي جذره التكعيبي عبارة عن عدد صحيح. ويعد العدد 27 مكعبًا كاملاً: لأن جذره التكعيبي

المربع الكامل عدد تسبى جذره التربيعي عبارة عن عدد صحيح. ويُعد العدد 25 مربعًا كاملاً لأن الجذر التربيعي للعدد 25 .5 aa

التبديل تنسيق أو وضع قائمة بلعب الترتيب فيها دورًا مهماً.

المستقيمان المتعامدان مستقيمان يتفاطعان لتكوين زوايا



(باي) نسبة محيط الدائرة إلى قطرها. ويمثل الحرف اليوناني π هذا العدد. ودائمًا ما نكون قيمة (باي) تساوي . . .3.1415926. ـُ



$$\tau = \frac{C}{d}$$

معادلة المستقيم بدلالة ميله ونقطة عليه معادلة  $(x_1, y_1)$  الشكل  $(x_1, y_1)$  ميث إن  $(x_1, y_1)$  ميث الميل و $(x_1, y_1)$  ميث الميل و هي نقطة معينة على ضلع غير متعامد. polygon A simple, closed figure formed by three or more line segments.



polyhedron A three-dimensional figure with faces that are polygons.



**power** A product of repeated factors using an exponent and a base. The power 7<sup>3</sup> is read seven to the third power, or seven cubed.

precision The ability of a measurement to be consistently reproduced.

preimage The original figure before a transformation.

principal The amount of money invested or borrowed.

prism A polyhedron with two parallel congruent faces called bases.



probability The chance that some event will happen. It is the ratio of the number of ways a certain event can occur to the number of possible outcomes

proof A logical argument in which each statement that is made is supported by a statement that is accepted as true.

property A statement that is true for any numbers.

pyramid A polyhedron with one base that is a polygon and three or more triangular faces that meet at a common vertex.



المضلع شكل مستو معلق مكون من ثلاث قطع مستغيمة أو أكثر.



متعدد الوجوه هو شكل ثلاثي الأبعاد له وجود عبارة عن مضلعات.



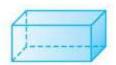
القوى ناتج ضرب عوامل متكررة باستخدام الأس والأساس. فعملية الفوى73 تُفرأ الفوة الثالثة للعدد 7 أو سبعة تكعيب.

الدقة إمكانية الحصول على نفس القياس على الدوام.

الصورة الأصلية الشكل الأصلي قبل التحويل.

رأس المال مقدار المال المستثمر أو المفترض.

المنشور مُجسّم متعدد الوجود له وجهان متوازيان ومتطابقان يطلق عليهما القاعدتان.



الاحتمال فرصة حدوث بعض الأحداث. وهو نسبة عدد الطرق الممكنة لوقوع الحدث إلى عدد النتائج الكليّة.

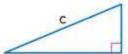
البرهان البرهان المنطقي الذي يكون فيه كل عبارة تقدمها مدعومة بعبارة مسلم بأنها صحيحة.

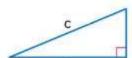
الخاصية عبارة حقيقية بالنسبة لأي أعداد.

الهرم شكل متعدد الوجود له قاعدة واحدة على شكل مضلع، وثلاثة أوجه أو أكثر على شكل مثلث تلتقي في قية مشتركة.



**Pythagorean Theorem** In a right triangle, the square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the legs a and b.  $a^2 + b^2 = c^2$ 

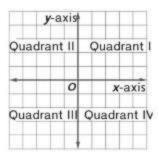


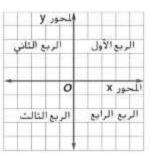


quadrants The four sections of the coordinate plane.



رُبِعيًّا ت الأفسام الأربعة التي نشكل المستوى الإحدائي.





quadratic function A function in which the greatest power of the variable is 2.

quadrilateral A closed figure with four sides and four angles.

qualitative graph A graph used to represent situations that do not necessarily have numerical values.

quantitative data Data that can be given a numerical value.

quartiles Values that divide a set of data into four equal parts.

الدالة التربيعية الدالة التي تكون قوة المتغير قبها تساوي 2.

رباعي الأضلاع شكل مستو مغلق مكون من أربعة أضلاع وأربع زوايا.

تمثيل نوعي رسم بياني يُستخدم في تمثيل حالات لا تحتوي بالضرورة على قيم عددية.

بيانات كمية بيانات يبكن إعطاؤها قبية عددية.

رباع قيم تفسم مجموعة البيانات إلى أربعة أجزاء متساوية.



radical sign The symbol used to indicate a positive square root,  $\sqrt{\phantom{a}}$ .

radius The distance from the center of a circle to any point on the circle.



رمز الجدد الرمز المستخدم للدلالة على جذر تربيعي موجب √.

نصف القطر المسافة بين مركز الدائرة وأي نقطة على الدائرة.



شوق الطبع والنشر ٥ محموطة لعمالج ٢٥٥٥ ١٥٥١ قالم ٢٥٥٠

random Outcomes occur at random if each outcome is equally likely to occur.

range The set of y-coordinates in a relation.

range The difference between the greatest number (maximum) and the least number (minimum) in a set of data.

rational number Numbers that can be written as the ratio of two integers in which the denominator is not zero. All integers, fractions, mixed numbers, and percents are rational numbers.

**real numbers** The set of rational numbers together with the set of irrational numbers.

reciprocals The multiplicative inverse of a number. The product of reciprocals is 1.

**reflection** A transformation where a figure is flipped over a line. Also called a flip.

regular polygon A polygon that is equilateral and equiangular.



regular pyramid A pyramid whose base is a regular polygon.

relation Any set of ordered pairs.

relative frequency The ratio of the number of experimental successes to the total number of experimental attempts.

remote interior angles The angles of a triangle that are not adjacent to a given exterior angle.

repeating decimal Decimal form of a rational number.

rhombus A parallelogram with four congruent sides.

عشوائي تحدث النثائج بشكل عشوائي إذا تساوت في احتمال حدوثها.

مدى العلاقة مجموعة إحداثيات y في علاقة ما.

مدى الفرق بين أكبر عدد (الأقصى) وأقل عدد (الأدنى) في مجبوعة من البيانات.

عدد نسبي العدد الذي يمكن كتابته في صورة كسر بسطه ومقامه عددان صحيحان، ولا يكون مقامه صفرًا، وتشمل الأعداد النسبية جميع الأعداد الصحيحة، والكسور، والكسور البعتلة. والنسب المؤية،

أعداد حقيقية أعداد مكونة من مجموعة الأعداد النسبية ومجموعة الأعداد غير النسبية.

نظير ضربي هو المتلوب الضربي للعدد. وناتج ضرب العدد في نظيره الضربي بساوي 1.

انعكاس تحويل هندسي يُقلب فيه الشكل على خط مستقيم. ويُطلق عليه أيضًا «القلب»..

مضلع منتظم مضلع يكون متساوي الأضلاع ومتساوي الزوايا



هرم منتظم هرم تكون فاعدته مضلعًا منتظمًا.

علاقة أي مجموعة من الأزواج المرتبة.

تكرار نسبي نسبة وقوع الحدث إلى إجمالي عدد مرات إجراء التحرية.

زوايا داخلية غير مجاورة زوايا المثلث التي تكون غير مجاورة للزاوية الخارجية المعطاة.

كسر عشري دوري الصبغة العشرية من بعض الأعداد النسبية.

مغين متوازي أضلاع له أربعة أضلاع متطابقة.

Corsw-Hill Education what the same of the same



right angle An angle whose measure is exactly

90°.

right triangle A triangle with one right angle.

rise The vertical change between any two points on a line.

rotation A transformation in which a figure is turned about a fixed point.

rotational symmetry A type of symmetry a figure has if it can be rotated less than 360° about its center and still look like the original.

run The horizontal change between any two points on a line.

مثلث قائم الزاوية مثلث إحدى زواياه فائية.

ارتشاع التغير الرأسي بين أي نقطتين على الخط.

دوران محوري تحويل هندسي ينم فيه تدوير الشكل حول نقطة ثابتة.

تشاطر دوراني نوع من أنواع التناظر الذي يتصف به الشكل إذا أمكن تدويره بزاوية أقل من 360° حول مركزه مع الاحتفاظ بشكله الأصلي.

امتداد أفقى النغير الأفنى بين أي نقطنين على البحور X.

S

sales tax An additional amount of money charged on certain goods and services.

sample A randomly-selected group chosen for the purpose of collecting data.

sample space The set of all possible outcomes of a probability experiment.

scale factor The ratio of the lengths of two corresponding sides of two similar polygons.

ضريبة الهبيعات مبلغ إضافي من المال يُفرض على سلع وخدمات معينة.

عينة مجموعة مختارة بشكل عشوائي بهدف جمع البيانات.

الغضاء العيني  $(\Omega)$  مجموعة النتائج المحتملة للتجربة الاحتمالية. ويُرمز له بالرمز أوميجا  $(\Omega)$ .

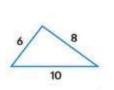
معامل المقياس نسبة طولي ضلعين متناظرين لمضلعين متشابهين.



9 12

scale factor =  $\frac{3}{2}$ 

scalene triangle A triangle with no congruent sides.



9 12

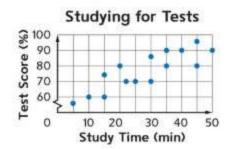
 $\frac{3}{2}$  = معامل المقباس

مثلث مختلف الأضلاع مثلث أضلاعه غير منطابقة.

مقوق العلبي والنشر ٥ محتوطة لعمالج CGraw-Hill Ethication

قاموس المصطلحات/Glossary

scatter plot A graph that shows the relationship between a data set with two variables graphed as ordered pairs on a coordinate plane.



scientific notation A compact way of writing numbers with absolute values that are very large or very small. In scientific notation,  $5,500 \text{ is } 5.5 \times 10^3$ .

selling price The amount the customer pays for an item.

semicircle An arc measuring 180°.

sequence An ordered list of numbers, such as 0, 1, 2, 3 or 2, 4, 6, 8.

similar If one image can be obtained from another by a sequence of transformations and dilations.

similar polygons Polygons that have the same shape.

similar solids Solids that have exactly the same shape, but not necessarily the same size.

simple interest Interest paid only on the initial principal of a savings account or loan.

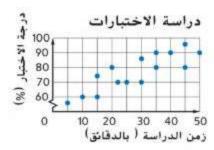
simple random sample A sample where each item or person in the population is as likely to be chosen as any other.

simplest form An algebraic expression that has no like terms and no parentheses.

simplify To perform all possible operations in an expression.

**simulation** An experiment that is designed to model the action in a given situation.

مخطط الانتشار رسم بياني يعرض العلاقة بين مجموعة بيانات ذات متغيرين، ويمثلها في صورة أزواج مرتبة على المستوى الإحداثي.



ثُمِنُ البيع مبلغ بدفعه الزبون للحصول على منتج ما،

نصف دائرة فوس فباسه 180°.

نبط قائمة مرتبة من الأعداد، مثل 0, 1, 2, 3 أو 2, 4, 6, 8.

متماثل إمكانية الحصول على صورة من صورة أخرى عن طريق سلسلة من التحويلات وتغييرات الأبعاد.

مضلعات متهاثلة مضلعات لها الشكل نفسه.

مجسمات متشابهة مجسمات لها نفس الشكل بالضبط ولكنها ليست بالضرورة بالحجم ذاته.

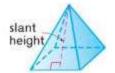
فائدة بسيطة قائدة تُدفع فقط على رأس المال الأولي لحساب الادخار أو القرض.

عينة عشوائية بسيطة عينة يكون فيها احتمال اختيار كل عنصر أو شخص في التعداد مماثلاً بالنسبة إلى جميع العناصر أو الأشخاص.

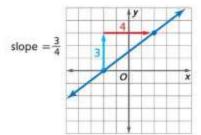
أبسط صورة تعبير جبري ليس فيه حدود متشابهة وأقواس.

تبسيط إجراء جميع العمليات المحتملة في تعبير ما.

محاكاة تجربة مصممة لصياغة نموذج من إجراء ما في حالة معينة.



slope The rate of change between any two points on a line. The ratio of the rise, or vertical change, to the run, or horizontal change.



slope-intercept form An equation written in the form y = mx + b, where m is the slope and b is the y-intercept.

solid A three-dimensional figure formed by intersecting planes.



sphere The set of all points in space that are a given distance from a given point called the center.

square root One of the two equal factors of a number. If  $a^2 = b$ , then a is the square root of b. A square root of 144 is 12 since  $12^2 = 144$ .

standard deviation A measure of variation that describes how the data deviates from the mean of the data.

standard form An equation written in the form Ax + By = C.

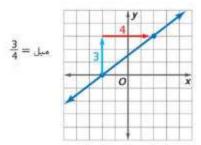
straight angle An angle whose measure is exactly 180°.



substitution An algebraic model that can be used to find the exact solution of a system of equations.



ميل معدل التغير بين أي نقطتين على الخط. نسبة الارتفاع . (أو التغير الرأسي) إلى الامتداد (أو التغير الأفقى).



معادلة المستقيم بدلالة الميل والجزء المقطوع من y = mx + b هو y = mx + b هو معادلة مكتوبة بالصيغة الميل و b هو الجزء المقطوع من محور y.

مجسم شكل ثلاثى الأبعاد يتشكل بتفاطع المستويات.



كرة مجموعة النقاط في الفراغ التي لها بعد محدد عن نقطة معلومة تُسمى المركز،

 $a^2$ = b جذر تربيعي أحد العاملين المتساويين للعدد. إذا كان  $a^2$ = b. إذن a هو الجذر التربيعي لــb. الجذر التربيعي للعدد 144 هو 12 لأن 144 a=12.

انحراف معياري مقباس الانحراف الذي بصف مدى انحراف البيانات عن المتوسّط الحسابي للبيانات.

الصورة القياسية لمعادلة المستقيم معادلة ثكنب Ax + By = C بالشكل

زاوية مستقيمة زاوية فياسها 180° بالضبط.



تعويض نموذج جبري يمكن استخدامه لإيجاد حل دفيق لنظام

Subtraction Property of Equality If you subtract the same number from each side of an equation, the two sides remain equal.

supplementary angles Two angles are supplementary if the sum of their measures is 180°.



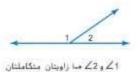
∠1 and ∠2 are supplementary angles.

**symmetric** A description of the shape of a distribution in which the left side of the distribution looks like the right side.

**system of equations** A set of two or more equations with the same variables.

الطرح في المعادلة إذا طرحت العدد نفسه من كلا طرفى المعادلة، بيقى الطرفان منساويين.

زاویتان متکاملتان نکون الزاوینان منکاملتین إذا کان مجموع قیاسهما بساوی °180.



متماثل وصف لأحد أشكال التوزيع الذي يكون فيه الجانب الأيسر من التوزيع مشابهًا للجانب الأيمن.

نظام المعادلات مجبوعة البعادلات المكونة من معادلتين 
 فأكثر والتي تحتوى على نفس المتغيرات.



term A number, a variable, or a product of numbers and variables.

term Each part of an algebraic expression separated by an addition or subtraction sign.

terminating decimal A repeating decimal where the repeating digit is zero.

theorem A statement or conjecture that can be proven.

theoretical probability Probability based on known characteristics or facts.

third quartile For a data set with median M, the third quartile is the median of the data values greater than M.

three-dimensional figure A figure with length, width, and height.

**total surface area** The sum of the areas of the surfaces of a solid.

transformation An operation that maps a geometric figure, preimage, onto a new figure, image.

حد العدد أو البنفير أو ناتج ضرب أو قسمة أعداد ومتغيرات.

حد كل جزء من التعبير الجبري تفصله علامة الجمع أو الطرح.

كسر عشري مُتناه كسر عشري ذو منازل عشرية محددة.

نظرية عبارة أو فرضية بمكن إثباتها.

احتمال نظري احتمال مبني على خصائص أو حفائق معلومة.

الرُبِيِّع الثَّالِثُ بالنسبة لمجموعة البيانات ذات الوسيط M. يكون الربع الثالث هو وسيط قيم البيانات التي تكون أكبر من قيمة M.

شكل ثلاثي الأبعاد شكل له طول وعرض وارتفاع، أو شك ل يشغل حيرًا من الفراغ.

مساحة السطح الكُليّة مجموع مساحات أسطح المجسم.

تحويل هندسي عملية صياغة الشكل الهندسي (أو الصورة الأصلية) إلى شكل جديد. صورة الشكل. transversal A line that intersects two or more other lines.

trapezoid A quadrilateral with exactly one pair of parallel sides.

tree diagram A diagram used to show the total number of possible outcomes in a probability experiment.

**triangle** A figure formed by three line segments that intersect only at their endpoints.

two-column proof A formal proof that contains statements and reasons organized in two columns. Each step is called a statement, and the properties that justify each step are called reasons.

two-step equation An equation that contains two operations.

two-step inequality An inequality that contains two operations.

two-way table A table that shows data that pertain to two different categories.

إزاحة تحويل هندسي يتم فيه تحريك الشكل من أحد مواضعه إلى آخر دون تدويره.

قاطع مستعرض مسنفيم يقطع مستقيمين فأكثر.

شبه المنحرف شكل رباعي الأضلاع مكون من زوج واحد من الأضلاع المتوازية.

مخطط الشجرة مخطط مستخدم في عرض الفضاء العيني في التجربة الاحتمالية.

مثلث مضلع مكون من ثلاث قطع مستقيمة تتقاطع فقط عند نقاط تهايتها.

برهان دو عمودين برهان شكلي يحتوي على عبارات ومبررات منظمة في عمودين. ويُطلق على كل خطوة عبارة، والخصائص التي تبرر كل خطوة يُطلق عليها مبررات.

معادلة ذات خطوتين معادلة تحتوى على عملبتين.

متبايئة ذات خطوتين منبابنة نحنوي على عملينين.

جدول ثنائي الاتجاه جدول بعرض بيانات خاصة بفتتين مختلفتين.

U

unbiased sample A sample that is selected so that it is representative of the entire population.

unit rate/ratio A rate or ratio with a denominator of 1.

univariate data Data with one variable.

unlike fractions Fractions whose denominators are different.

عينة محايدة عينة مختارة لتمثيل التعداد بالكامل.

كسر/نسبة الوحدة نسبة أوكسر منامه 1.

بيانات أحادية المتفير بيانات ذات منغبر واحد

كسور غير متشابهة كسور تختلف في المنام.

V

variable A symbol, usually a letter, used to represent a number in mathematical expressions or sentences.

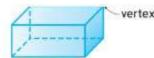
متغير رمز عادةً ما يكون حرفًا، يُستخدم في تمثيل عدد في التعاير أو العبارات الرياضية.

STAW-HIII Ethication grant and on one of the case

vertex The point where the sides of an angle meet.



vertex The point where three or more faces of a polyhedron intersect.

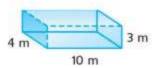


vertex The point at the tip of a cone.

vertical angles Opposite angles formed by the intersection of two lines. Vertical angles are congruent. In the figure, the vertical angles are  $\angle 1$  and  $\angle 3$ , and  $\angle 2$  and  $\angle 4$ .



**volume** The measure of the space occupied by a solid. Standard measures are cubic units such as in<sup>3</sup> or ft<sup>3</sup>.



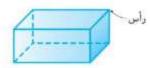
 $V = 10 \times 4 \times 3 = 120$  cubic meters

**voluntary response sample** A sample which involves only those who want to participate in the sampling.

رأس الزاوية نقطة النقاء ضلعى الزاوية.



رأس نقطة تناطع ثلاثة وجوه فقط للشكل متعدد الوجوه.

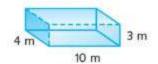


نقطة الرأس النقطة عند نهاية الشكل المخروطي.

زوايا متقابلة بالرأس زوايا منفابلة تتشكل بتفاطع خطين مستفيمين. والزوايا المتفابلة بالرأس تعد زوايا متطابقة. وتتبئل الزاويتان المتفابلتان بالرأس بهذا الشكل في  $1 \ge 6 \ge 2$  و  $2 \ge 6$ .



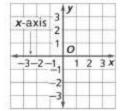
الحجم مقدار الفراغ الذي يشغله المجسم. ويكون القياس المعياري بالوحدات المكعبة مثل  $\sin^3$  أو  $\sin^3$ 



 $V = 10 \times 4 \times 3 = 120$  مثر مکعب

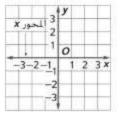
عينة الاستجابة الطوعية عينة نضم فنط الأشخاص الراغبين في المشاركة في العينة المأخوذة.

x-axis The horizontal number line that helps to form the coordinate plane.



المحور الأفقي × خط الأعداد الأففي في تشكيل المستوى الإحداثي.

X



x-intercept = 3

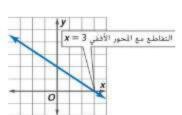
x-intercept The x-coordinate of the point where

the line crosses the x-axis.

لتقاطع مع المحور x الإحداثي x للنقطة حيث يقطع ليستقبع المحور x.







Z

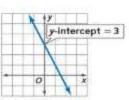
**y-axis** The vertical number line that helps to form the coordinate plane.

المحور الرأسي y خط الأعداد الرأسي في المستوى الإحداثي.



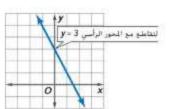
**y-coordinate** The second number of an ordered pair.

**y-intercept** The y-coordinate of the point where the line crosses the y-axis.



الإحداثي y العدد الثاني في الزوج المرتب.

التقاطع مع الهحور y الإحداثي y للنفطة حيث يقطع المستقيم البحور y.



مدوق العلمي والنشر ٥ محدوقة لصابح Original المائح WeGraw-Hill Editember ومدوق

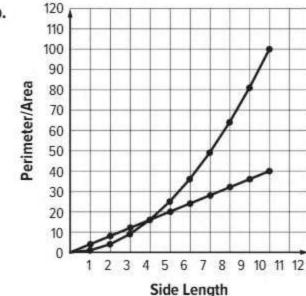
## Chapter 1 Real Numbers

#### Lesson 1-2 Extra Practice

27a.

| Side Length<br>(in.) |    |     |
|----------------------|----|-----|
| 1                    | 4  | 1   |
| 2                    | 8  | 4   |
| 3                    | 12 | 9   |
| 4                    | 16 | 16  |
| 5                    | 20 | 25  |
| 6                    | 24 | 36  |
| 7                    | 28 | 49  |
| 8                    | 32 | 64  |
| 9                    | 36 | 81  |
| 10                   | 40 | 100 |



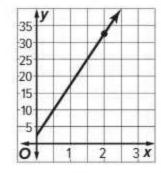


27c. Sample answer: The graph representing perimeter of a square is linear because each side length is multiplied by 4. The graph representing area of a square is nonlinear because each side length is squared and does not increase at a constant rate.

## **Chapter 3** Equations in Two Variables

### Lesson 3-4 Need Another Example?

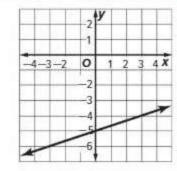
4-5.



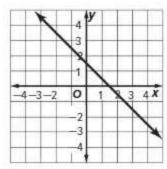
The slope 15 represents the rate of change or cost per hour. The *y*-intercept 2.5 is the charge for instruction.

### Lesson 3-4 Independent Practice

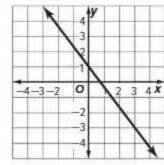
9.



10.

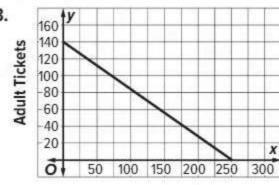


11.



### Lesson 3-5 Need Another Example?

2-3.

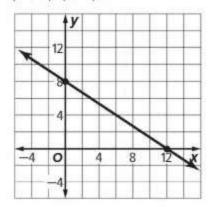


Student Tickets

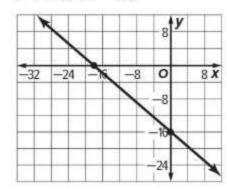
The x-intercept of 252 means that if 252 student tickets and 0 adult tickets were sold, the total sales would be \$1,260. The y-intercept of 140 means that if 0 student tickets and 140 adult tickets were sold, the total sales would be \$1,260.

#### Lesson 3-5 Extra Practice

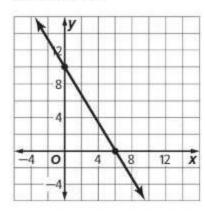
**11.** (12, 0), (0, 8);



**12.** (-18, 0), (0, -16);

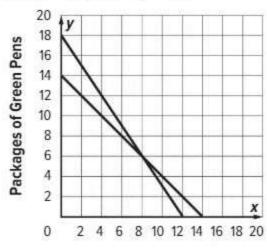


**13.** (6, 0), (0, 10);



## Lesson 3-7 Need Another Example?

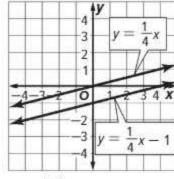
**2–3.** 
$$x + y = 14$$
;  $6x + 4y = 72$ 



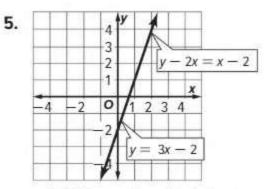
Packages of Red Pens

(8, 6); Ms. Baker bought 8 packages of red pens and 6 packages of green pens.

4.



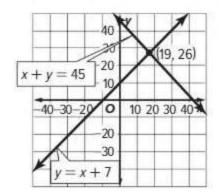
no solution



an infinite number of solutions

### Lesson 3-7 Independent Practice

Sample answer: Let x = the number of birds and y = the number of cats; x + y = 45, y = x + 7; There are 19 birds and 26 cats.



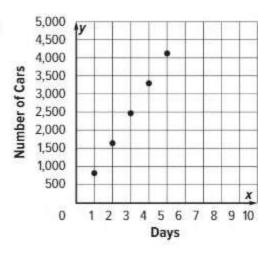
## Chapter 4 Functions

#### Lesson 4-2 Independent Practice

3.

| x | 825x   | y     |
|---|--------|-------|
| 1 | 825(1) | 825   |
| 2 | 825(2) | 1,650 |
| 3 | 825(3) | 2,475 |
| 4 | 825(4) | 3,300 |
| 5 | 825(5) | 4,125 |

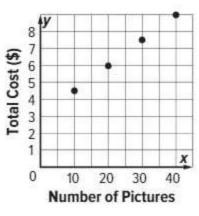
4.



6a.

| X  | 0.15x + 2.99    | y    |
|----|-----------------|------|
| 10 | 0.15(10) + 2.99 | 4.49 |
| 20 | 0.15(20) + 2.99 | 5.99 |
| 30 | 0.15(30) + 2.99 | 7.49 |
| 40 | 0.15(40) + 2.99 | 8.99 |

6b.



**6c.** 75 pictures: 0.15(75) + 2.99 = \$14.24100 pictures: 0.15(100) + 2.99 = \$17.99

## Lesson 4-3 Need Another Example?

Sample answer:

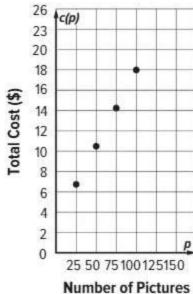
| Domain | Rule      | Range |
|--------|-----------|-------|
| x      | f(x)=4x-1 | f(x)  |
| -2     | 4(-2) - 1 | -9    |
| -1     | 4(-1) - 1 | -5    |
| 0      | 4(0) — 1  | -1    |
| 1      | 4(1) — 1  | 3     |

### Lesson 4-3 Independent Practice

8b.

| р   | c(p) = 0.15p + 2.99     | c(p)  |
|-----|-------------------------|-------|
| 25  | c(p) = 0.15(25) + 2.99  | 6.74  |
| 50  | c(p) = 0.15(50) + 2.99  | 10.49 |
| 75  | c(p) = 0.15(75) + 2.99  | 14.24 |
| 100 | c(p) = 0.15(100) + 2.99 | 17.99 |

8c.



about 145 pictures

## Lesson 4-4 Independent Practice

5b.

| N | Mountain Bike Ren | tal   |
|---|-------------------|-------|
| h | 15 + 4.25h        | c     |
| 2 | 15 + 4.25(2)      | 23.50 |
| 3 | 15 + 4.25(3)      | 27.75 |
| 4 | 15 + 4.25(4)      | 32.00 |
| 5 | 15 + 4.25(5)      | 36.25 |

|   | Scooter Rental |       |  |
|---|----------------|-------|--|
| h | 25 + 2.5h      | с     |  |
| 2 | 25 + 2.5(2)    | 30.00 |  |
| 3 | 25 + 2.5(3)    | 32.50 |  |
| 4 | 25 + 2.5(4)    | 35.00 |  |
| 5 | 25 + 2.5(5)    | 37.50 |  |

5c.

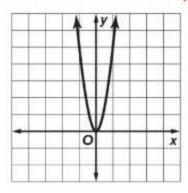
50 45 40 35 Total Cost (\$) 30 25 20 15 10 5 h 0 2 3 Time (h)

### Lesson 4-5 Need Another Example?

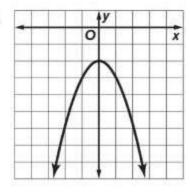
- The water garden has a flow rate of 52 gallons per minute and the koi pond has flow rate of 60 gallons per minute. The water in the koi pond has a greater rate of change.
- 2a. The function for Package A has a y-intercept of 5 and the function for Package B has a y-intercept of 0. Package A costs \$7 per person and Package B costs \$9 per person. The rate of change for Package B is greater.
- 3a. The function for the first company has a y-intercept of 5 and a rate of change of 1.5. The function for the second company has a y-intercept of 4 and a rate of change of 2. The y-intercept for the first company is greater than the y-intercept for the second company but the rate of change for the first company is less than the rate of change for the second company.

## Lesson 4-8 Need Another Example?

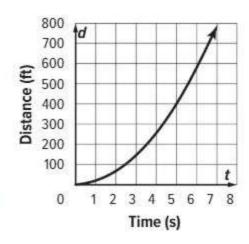
1.



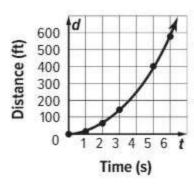
2.



3.

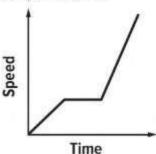


4.

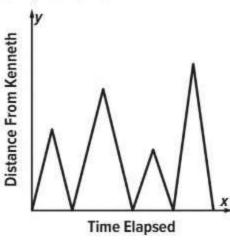


### Lesson 4-9 Need Another Example?

Sample answer:



3. Sample answer:



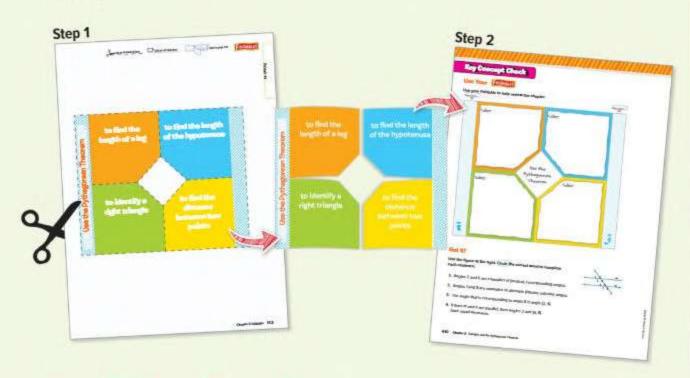
# FOLDABLES Study Organizers

### What Are Foldables and How Do I Create Them?

Foldables are three-dimensional graphic organizers that help you create study guides for each chapter in your book.

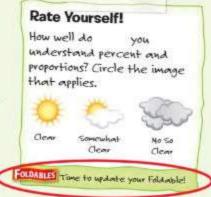
**Step 1** Go to the back of your book to find the Foldable for the chapter you are currently studying. Follow the cutting and assembly instructions at the top of the page.

**Step 2** Go to the Key Concept Check at the end of the chapter you are currently studying. Match up the tabs and attach your Foldable to this page. Dotted tabs show where to place your Foldable. Striped tabs indicate where to tape the Foldable.



# How Will I Know When to Use My Foldable?

When it's time to work on your Foldable, you will see a Foldables logo at the bottom of the **Rate Yourself!** box on the Guided Practice pages. This lets you know that it is time to update it with concepts from that lesson. Once you've completed your Foldable, use it to study for the chapter test.



## **How Do I Complete My Foldable?**

No two Foldables in your book will look alike. However, some will ask you to fill in similar information. Below are some of the instructions you'll see as you complete your Foldable. **HAVE FUN** learning math using Foldables!

#### Instructions and what they mean

Best Used to... Complete the sentence explaining

when the concept should be used.

Definition Write a definition in your own words.

Description Describe the concept using words.

Equation Write an equation that uses the concept.

You may use one already in the text or you

can make up your own.

Example Write an example about the concept. You may use

one already in the text or you can make up your own.

Formulas Write a formula that uses the concept. You may use

one already in the text.

How do I ...? Explain the steps involved in the concept.

Models Draw a model to illustrate the concept.

Picture Draw a picture to illustrate the concept.

Solve

Algebraically Write and solve an equation that uses the concept.

Symbols Write or use the symbols that pertain to the concept.

Write About It Write a definition or description in your own words.

Works Write the words that pertain to the concept.

### Meet Foldables Author Dinah Zike

Dinah Zike is known for designing hands-on manipulatives that are used nationally and internationally by teachers and parents. She is an explosion of energy and ideas. Her excitement and joy for learning inspires everyone she touches.





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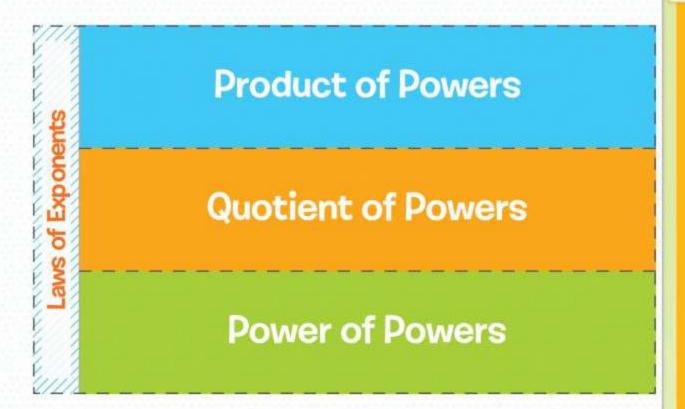
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Use this Foldable with Chapter 1.



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| Examples |          |
|----------|----------|
| Examples |          |
| Examples | page 100 |

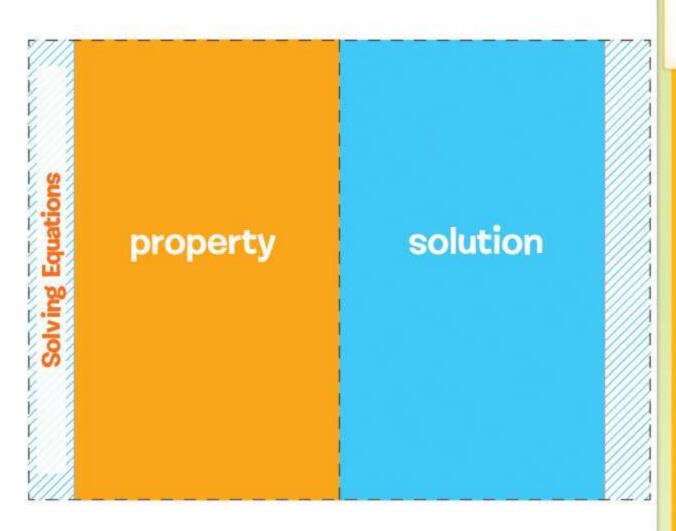
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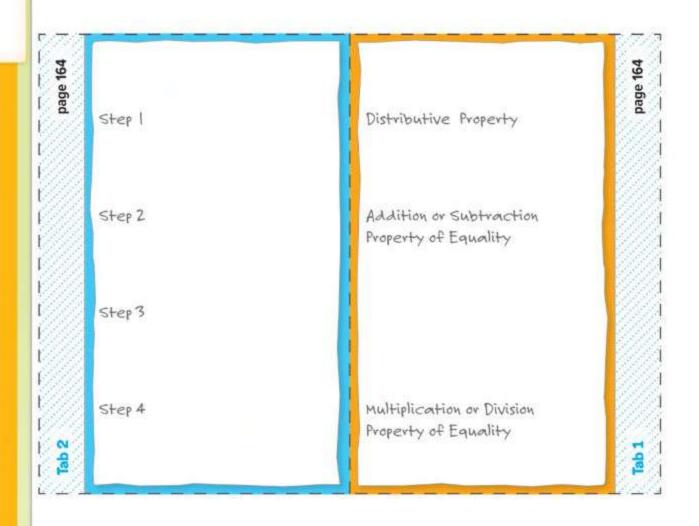
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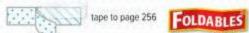
Use this Foldable with Chapter 2.





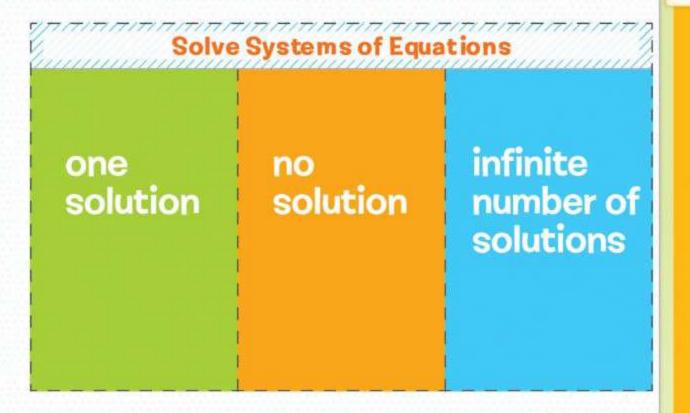
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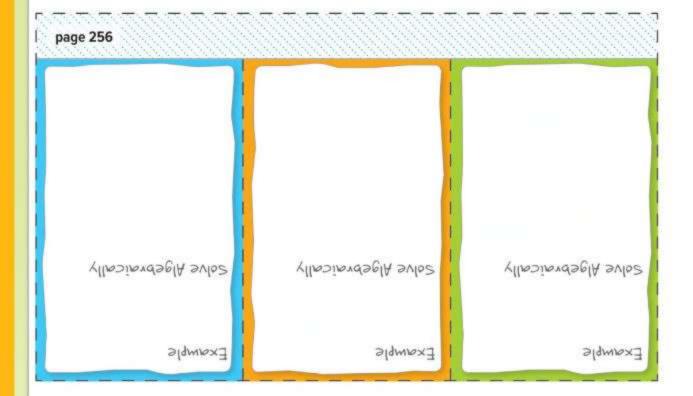




Use this Foldable with Chapter 3.







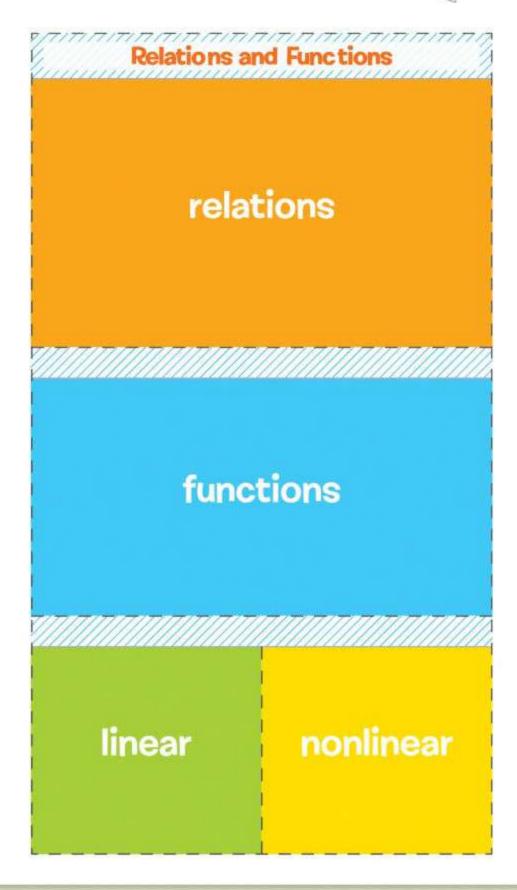
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Use this Foldable with Chapter 4.

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## **Chapter 1 Real Numbers**

## Planetary Play

| Mathe | matical P     | ractices  | MP1, MP2, MP3, MP4, MP6 |  |
|-------|---------------|-----------|-------------------------|--|
| Depth | of Knowl      | edge      | DOK3                    |  |
| Part  | Max<br>Points |           | Scoring Rubric          |  |
| Α     | 2             | Full Cred | it:                     |  |

| 200  | Max    | DANAGE MANAGEMENT  |
|------|--------|--|
| Part | Points | Scoring Rubric   |
| Α    | 2      | Full Credit:   |
|      |        | $1.5 \times 10^8 - 1.1 \times 10^8 = 4 \times 10^7$  |
|      |        | $4.5 \times 10^9 \div (5.8 \times 10^7) \approx 77.6$ times farther  |
|      |        | Earth is $4\times10^7$ kilometers farther from the Sun than it is from Venus. Neptune is 77.6 times farther from the Sun than Mercury is from the Sun. |
|      |        | Partial Credit will be given for one correct answer.   |
|      |        | No credit will be given for an incorrect answer.   |
| B 1  | 1      | Full Credit:   |
|      |        | Standard form: 0.24  |
|      |        | Fraction form: $\frac{24}{100}$ or $\frac{6}{25}$  |
|      |        | No credit will be given for an incorrect answer.   |
| С    | 1      | Full Credit:   |
|      |        | $8.27 \times 10^{14} \div 766 \approx 1.08 \times 10^{12}$   |
|      |        | The approximate volume of Earth is 1.08 $\times$ 10 <sup>12</sup> .  |
|      |        | No credit will be given for an incorrect answer.   |

| D     | 3 | Full Credit:  Mercury: $\frac{1}{1 \times 10^8} = \frac{x}{5.8 \times 10^7}$ : $x = 5.8 \times 10^{-1}$ or 0.58 cm  Venus: $\frac{1}{1 \times 10^8} = \frac{x}{1.1 \times 10^8}$ : $x = 1.1$ or 1.1 cm  Earth: $\frac{1}{1 \times 10^8} = \frac{x}{1.5 \times 10^8}$ : $x = 1.5$ or 1.5 cm  Mars: $\frac{1}{1 \times 10^8} = \frac{x}{2.3 \times 10^8}$ : $x = 2.3$ or 2.3 cm  Jupiter: $\frac{1}{1 \times 10^8} = \frac{x}{7.8 \times 10^8}$ : $x = 7.8$ or 7.8 cm  Saturn: $\frac{1}{1 \times 10^8} = \frac{x}{1.4 \times 10^8}$ : $x = 14$ or 14 cm  Neptune: $\frac{1}{1 \times 10^8} = \frac{x}{4.5 \times 10^8}$ : $x = 45$ or 45 cm  Refer to the distances shown and see students' work for the model. Students will need to use two pieces of paper joined together or a large (11 × 17) piece of paper.  Partial Credit: 2 points will be given if the student finds all of the proportions but has an error in modeling OR if the student has an error in finding between one and three of the distances and the error(s) is reflected in the model.  1 point will be given if the student finds all of the proportions but fails to model the distances.  No credit will be given for an incorrect answer. |
|-------|---|--|
| TOTAL | 7 |  |

# **Chapter 2** Equations in One Variable

## **Bowling Alley Fun**

| Mathematical Practices |               |  | MP1, MP2, MP3, MP4, MP6 |
|------------------------|---------------|--|-------------------------|
| Depth of Knowledge     |               |  | DOK3                    |
| Part                   | Max<br>Points |  | Scoring Rubric          |
| A                      | 2             | Full Credit:  The situation can be represented by the equation $5(5g+3)+25=115$ . Each family member can bowl three games. The fees for each person will be modeled by the expression $5g+3$ . There are 5 family members so the expression for the entire family is $5(5g+3)$ . Add the gas and food expenses and the result is $5(5g+3)+25$ . Set this expression equal to the total budget of \$115. $5(5g+3)+25=115$ $25g+15+25=115$ $25g+40=115$ $25g+75$ $g=3$ Partial Credit will be given for the correct equation.  No credit will be given for an incorrect answer.  |                         |
| В                      | 2             | Full Cred The situation of the situation |                         |

| С    | 1 | Full Credit:   |
|------|---|--|
|      |   | Dan's Alley: $5g + 3$  |
|      |   | Bowling Palace: $2g + 9$   |
|      |   | 5g + 3 = 2g + 9  |
|      |   | 3g + 3 = 9   |
|      |   | 3g = 6   |
|      |   | g = 2  |
|      |   | Mr. Bradley would need to bowl 2 games to pay an equal amount at each bowling alley.   |
| _    |   | No credit will be given for an incorrect answer.   |
| D    | 2 | Full Credit:   |
|      |   | A student will receive full credit for writing a reasonable scenario, solving the equation correctly, and telling what each number represents.   |
|      |   | Possible scenario: The Bradley family, plus one friend, goes bowling a different bowling alley. The cost per game is \$8, shoe rental is \$2, and they set aside \$45 for gas and food. The total budget is \$153. |
|      |   | 6 = the number of people that bowl   |
|      |   | 8g = \$8  per game   |
|      |   | 2 = \$2 shoe rental  |
|      |   | 45 = money for gas and food  |
|      |   | 153 = total budget   |
|      |   | 6(8g+2)+45=153   |
|      |   | 48g + 12 + 45 = 153  |
|      |   | 48g + 57 = 153   |
|      |   | 48g = 96<br>q = 2  |
|      |   | So they each bowled two games.   |
|      |   | Partial Credit will be given for a correct scenario.   |
|      |   | No credit will be given for an incorrect answer.   |
| OTAL | 7 |  |

# **Chapter 3 Equations in Two Variables**

## Play for a Prize

|                        | Max<br>Points | - | Scoring Rubric          |
|------------------------|---------------|---|-------------------------|
| Depth of Knowledge     |               |   | DOK3                    |
| Mathematical Practices |               |   | MP1, MP2, MP4, MP5, MP6 |

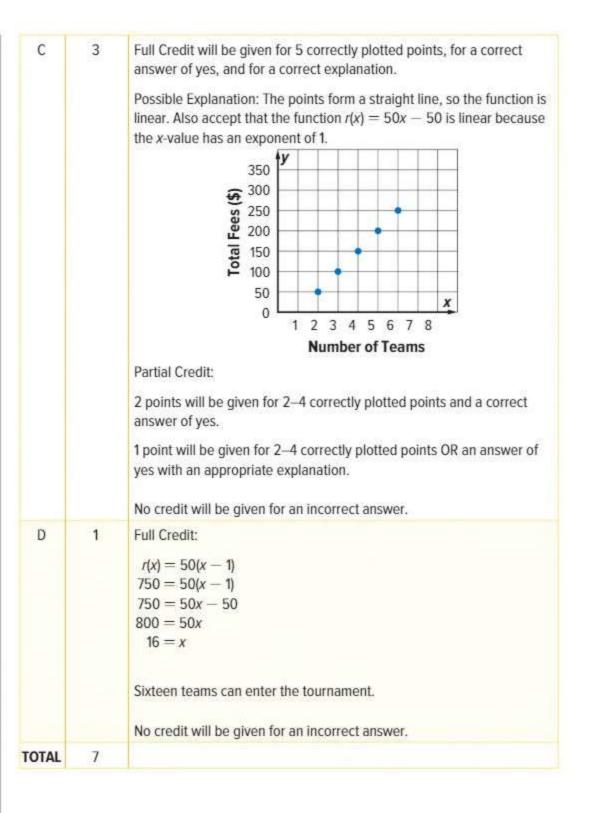
|      | of Knowle | (1.0 m) (1.0 m) (1.0 m) (1.0 m) (1.0 m) (1.0 m)  |
|------|-----------|--|
| Part | Points    | Scoring Rubric   |
| Α    | 3         | Full Credit: Game #1: $y = \frac{1}{60}x + 2$  |
|      |           |  |
|      |           | Game #2: $y = \frac{1}{40} x$  |
|      |           | The slope $\frac{1}{60}$ and y-intercept of 2 for Game 1 indicate that 2 tickets are received when the game begins and additional ticket is earned after each 60 points. |
|      |           | The slope $\frac{1}{40}$ and y-intercept of 0 for Game 2 indicate that no tickets  |
|      |           | are received when the game begins and 1 ticket is received after each  |
|      |           | 40 points.   |
|      |           | Partial Credit:  |
|      |           | 2 points will be given for a correct equation and the correct interpretation of slope and <i>y</i> -intercept for that equation.   |
|      |           | 1 point will be given for a correct equation with no other correct information.  |
|      |           | No credit will be given for an incorrect answer.   |
| В    | 1         | Full Credit:   |
|      |           | Yes; the relationship for Game 2 is proportional. Possible explanation:  |
|      |           | The function $y = \frac{1}{40}x$ has an x-intercept of zero and the graph of the   |
|      |           | function passes through the origin.  |
|      |           | No credit will be given for incorrect answer.  |

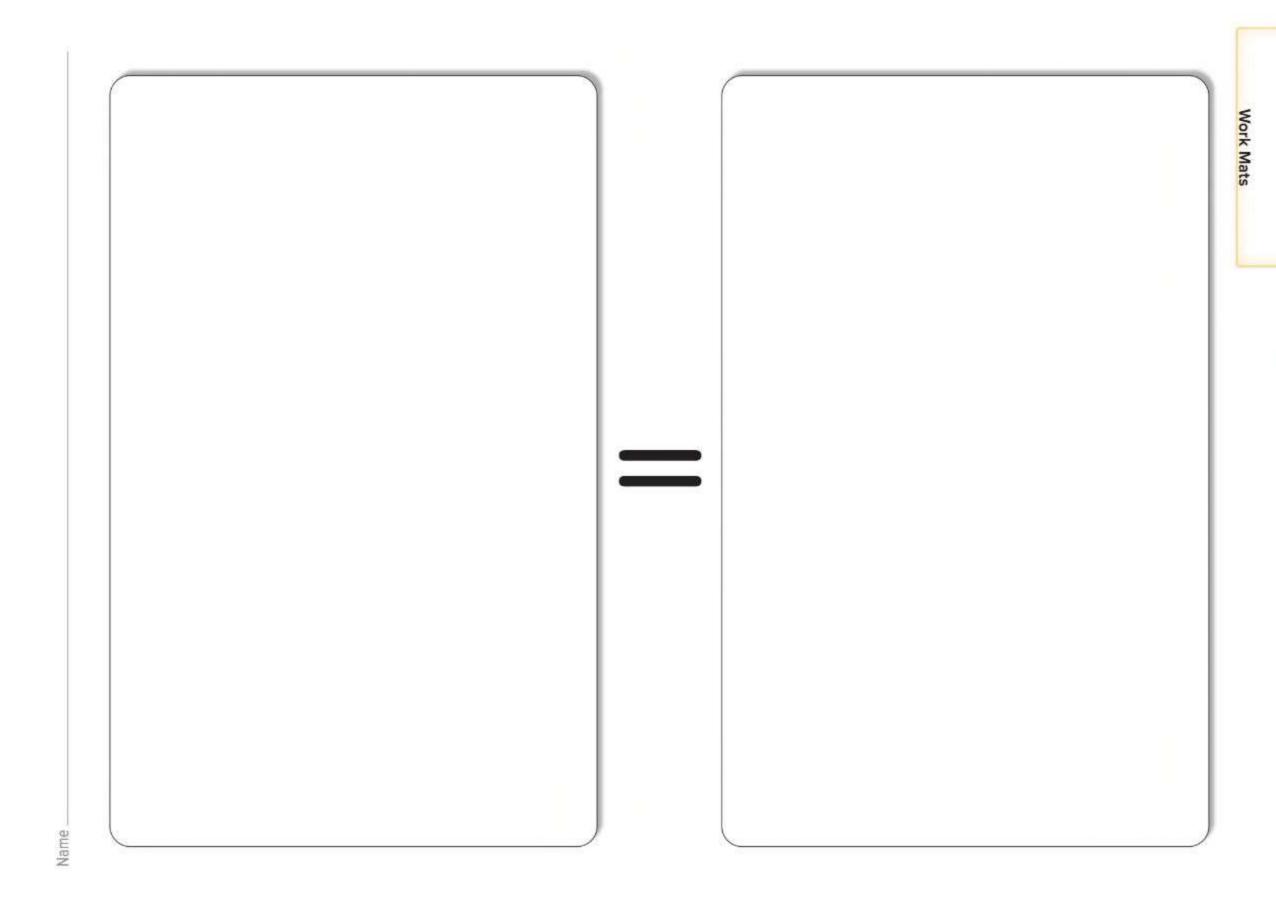
| C     | 3 | Full Credit will be given for a correct equation with work shown and solution of (240, 6), with the interpretation of the solution. $\frac{1}{60}x + 2 = \frac{1}{40}x$ $\frac{2}{120}x + 2 = \frac{3}{120}x$ $2 = \frac{3}{120}x - \frac{2}{120}x$ $2 = \frac{1}{120}x$ $(120)(2) = (120)\left(\frac{1}{120}x\right)$ $240 = x$ $y = \frac{1}{40}(240)$ $y = 6$ Possible interpretation: The solution tells what point level, 240 points, both games give an equal number of tickets, 6.  Partial Credit:  2 points will be given for a correct algebraic solution with work shown and no interpretation of the solution OR a correct algebraic solution and interpretation lacking shown work.  1 point will be given for a correct solution.  No credit will be given for incorrect answer. |
|-------|---|--|
| D     | 1 | Full Credit:  Antoine should play game #2. Possible explanation: With game #2, Antoine will receive 10 tickets after 400 points. In order to get 10 tickets with game #1, he would need 480 points.  |
| TOTAL | 8 | No credit will be given for an incorrect answer.   |
| IUIAL | Ö |  |

## Chapter 4 Functions

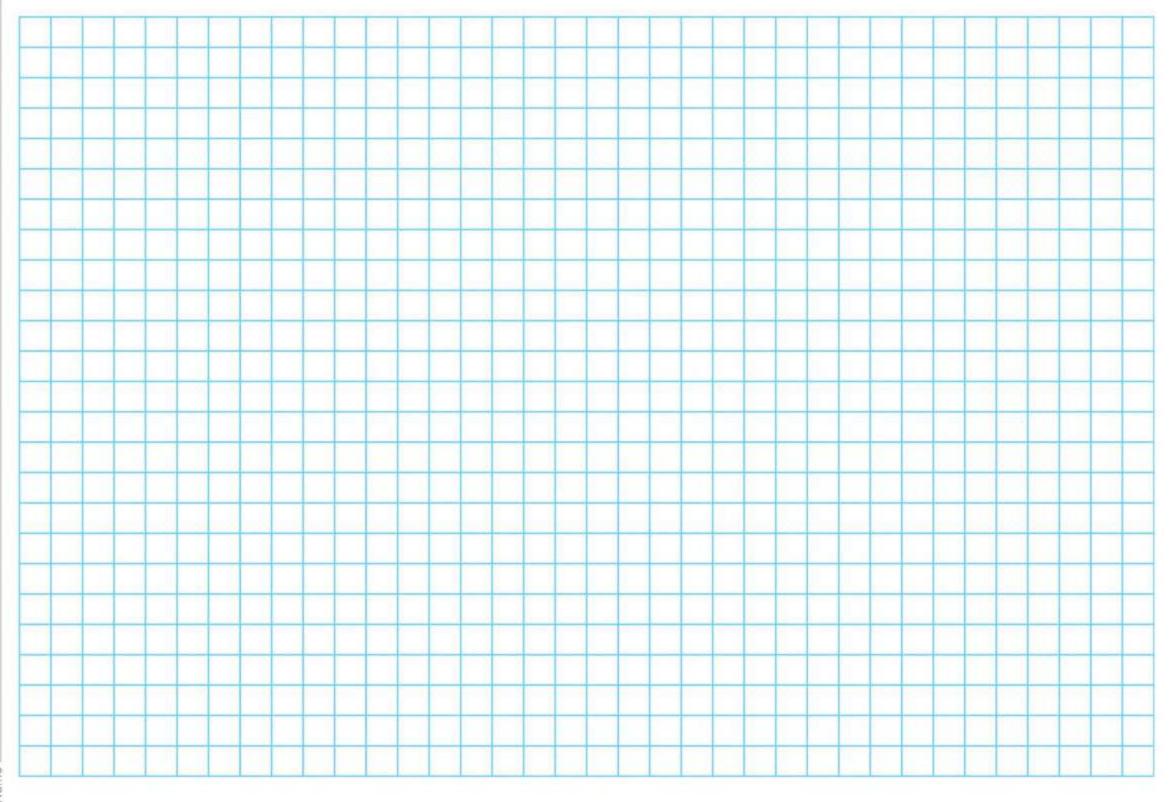
#### **Tournament Time**

| Mathematical Practices |               |  | MP1, MP2, MP3, MP4, MP5, MP6   |  |
|------------------------|---------------|--|--|--|
| Depth of Knowledge     |               |  | DOK3   |  |
| Part                   | Max<br>Points | Scoring Rubric   |  |  |
| А                      | 1             | of games   | tion $g(x) = x - 1$ relates the number of teams to the number splayed. |  |
| В                      | 2             | No credit will be given for an incorrect answer.  Full Credit: $r(x) = 50(2) - 50$ ; \$50 $r(x) = 50(3) - 50$ ; \$100 $r(x) = 50(4) - 50$ ; \$150 $r(x) = 50(5) - 50$ ; \$200 $r(x) = 50(6) - 50$ ; \$250  Partial Credit will be given for three or four correct answers.  No credit will be given for an incorrect answer. |  |  |

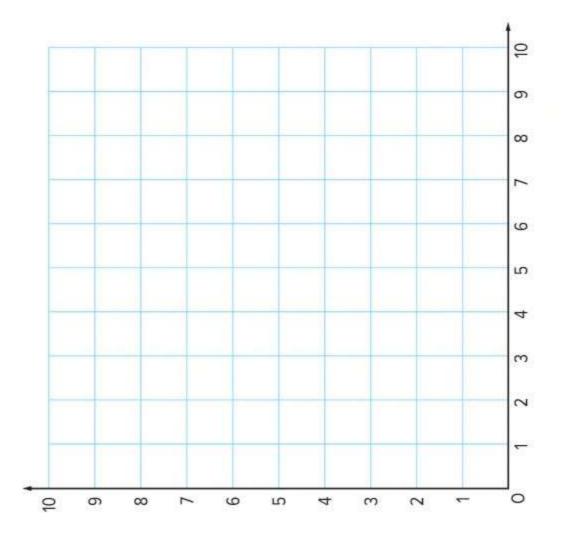


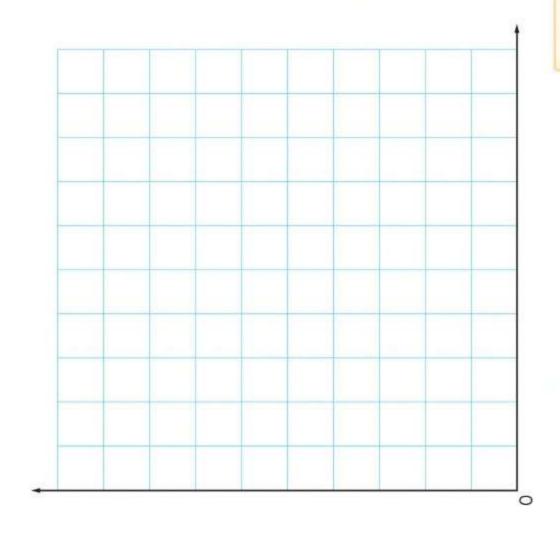


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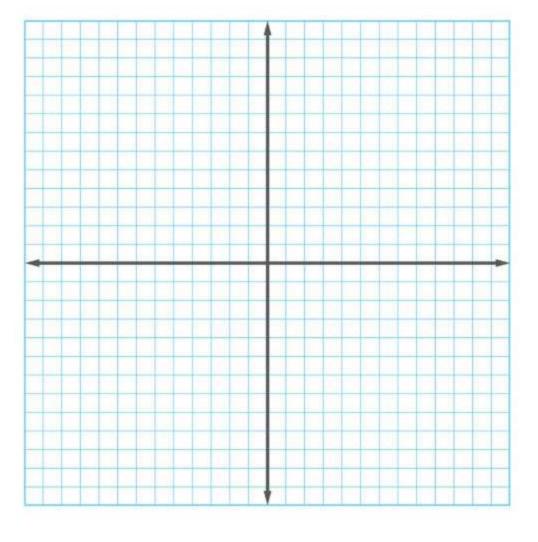


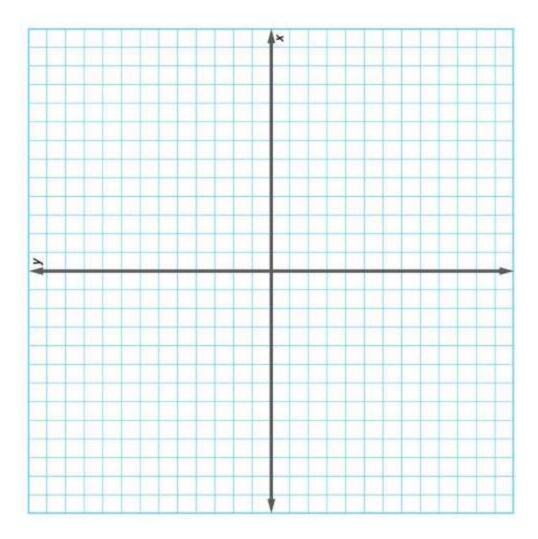
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