TOPIC 7 OPENER

FACTORS AND MULTIPLES

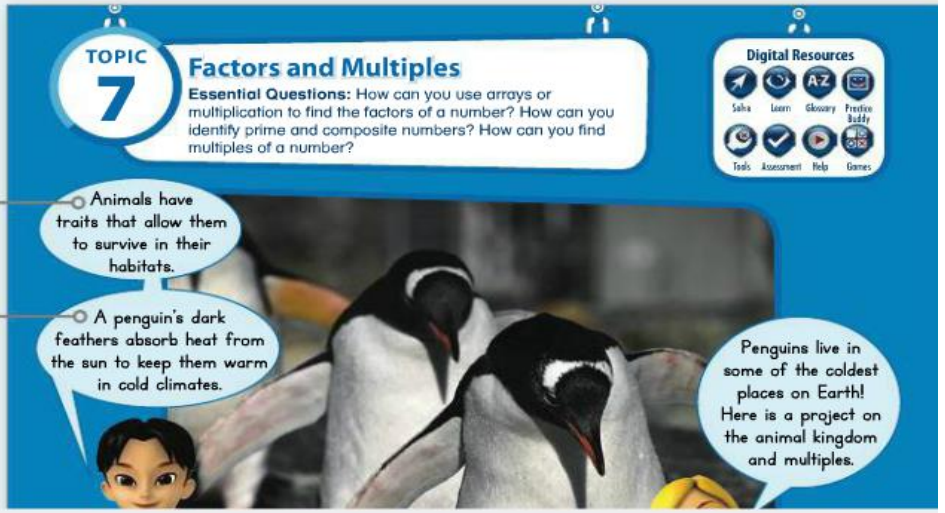
TOPIC ESSENTIAL QUESTIONS

How can you use arrays or multiplication to find the factors of a number? How can you identify prime and composite numbers? How can you find multiples of a number?

Revisit the Topic Essential Questions throughout the topic, and see a note about answering the questions in the Teacher's Edition for the Topic Assessment.

MATH AND SCIENCE PROJECT STEM

Science Theme The science theme for this project is **Plant and Animal Structures**. This theme will be revisited in the Math and Science Activities in Lessons 7-1 and 7-3 and in some



Project-Based Learning Have students work on the **Math and Science Project** over the course of several days.

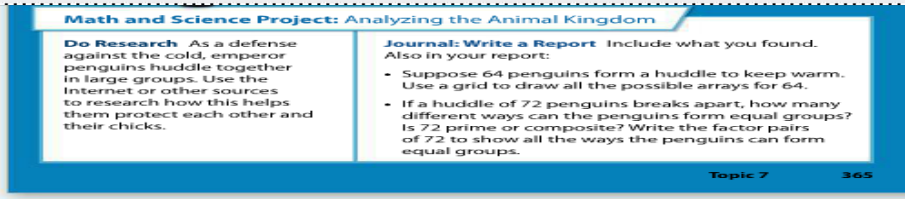
EXTENSION

Have students gather information about how some penguins trade responsibilities when working together to keep their eggs from freezing.

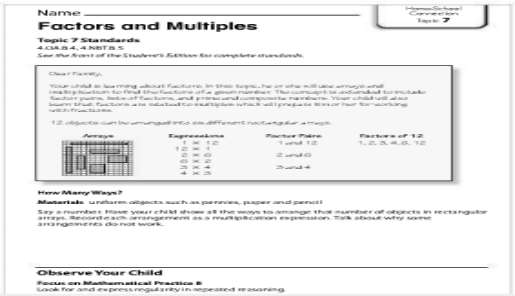
Sample Student Work for Math and Science Project

1 group of 72
2 groups of 36
3 groups of 24
4 groups of 18
6 groups of 12
8 groups of 9

72 groups of 1
36 groups of 2
24 groups of 3
18 groups of 4
12 groups of 6
9 groups of 8



Home-School Connection



Send this page home at the start of Topic 7 to give families an overview of the content in the topic.

Name _____

Review What You Know

Vocabulary

Choose the best term from the box. Write it on the blank.

- dividend
- divisor
- product
- quotient

1. The quotient is the answer to a division problem.
2. The number being divided is the dividend.
3. The divisor is the number that tells into how many groups something is being divided.

Multiplication

Find each product.

4. 8 × 4 32
5. 17 × 6 102
6. 304 × 9 2,736
7. 555 × 5 2,775
8. 22 × 26 572
9. 33 × 11 363
10. 56 × 70 3,920
11. 36 × 91 3,276
12. 27 × 48 1,296
13. 56 × 13 728
14. 12 × 19 228
15. 36 × 16 576

Division

Find each quotient.

16. 27 ÷ 3 9
17. 56 ÷ 8 7
18. 36 ÷ 4 9
19. 72 ÷ 9 8
20. 39 ÷ 3 13
21. 64 ÷ 4 16
22. 105 ÷ 5 21
23. 824 ÷ 4 206
24. 942 ÷ 3 314

My Word Cards

Use the examples for each word on the front of the card to help complete the definitions on the back.

factor

7 × 3 = 21

factors

multiple

0, 4, 8, 12, and 16 are some of the multiples of 4.

factor pairs

The factor pairs for 12 are:
1 and 12
2 and 6
3 and 4

generalize

All even numbers end in 0, 2, 4, 6, or 8.

prime number

13

factors: 1, 13

composite number

14

factors: 1, 2, 7, 14

25. 9,156 ÷ 3 **3,052** 26. 2,156 ÷ 4 **539** 27. 4,136 ÷ 8 **517**

Problem Solving

28. **MP.4 Model with Math** Cecilia bought 2 sandwiches last week and 4 sandwiches this week. She spent a total of $42. If each sandwich costs the same amount, how much did Cecilia spend on each sandwich? Write and solve equations.
$7; Sample answer: 2 + 4 = *t*; *t* = 6; 42 ÷ 6 = *m*; *m* = $7

My Word Cards

Complete each definition. Extend learning by writing your own definitions.

The product of a given number and any other whole number is called a **multiple**.

The numbers that are multiplied together to give a product are called **factors**.

To **generalize** means to make a general statement.

Factor pairs are two numbers that when multiplied together give a certain product.

A **composite number** is a whole number greater than 1 with more than two factors.

A **prime number** is a whole number greater than 1 that has exactly two factors, itself and 1.

RtI Item Analysis for Diagnosis and Intervention

Item	Standard	MDIS
1–3	4.NBT.B.6	G33
4–15	4.NBT.B.5	G47, G68
16–27	4.NBT.B.6	G53, G54, G56
28	4.OA.A.3	G37

BEFORE

1. Pose the Solve-and-Share Problem

You may wish to provide centimeter grid paper (Teaching Tool 9) and 2-color square counters (or Teaching Tool 16).

MP.5 Use Appropriate Tools Strategically Look for students who find all the possible arrays for 24 carpet squares, show the arrays on the grid, and list all the factors of 24.

2. Build Understanding

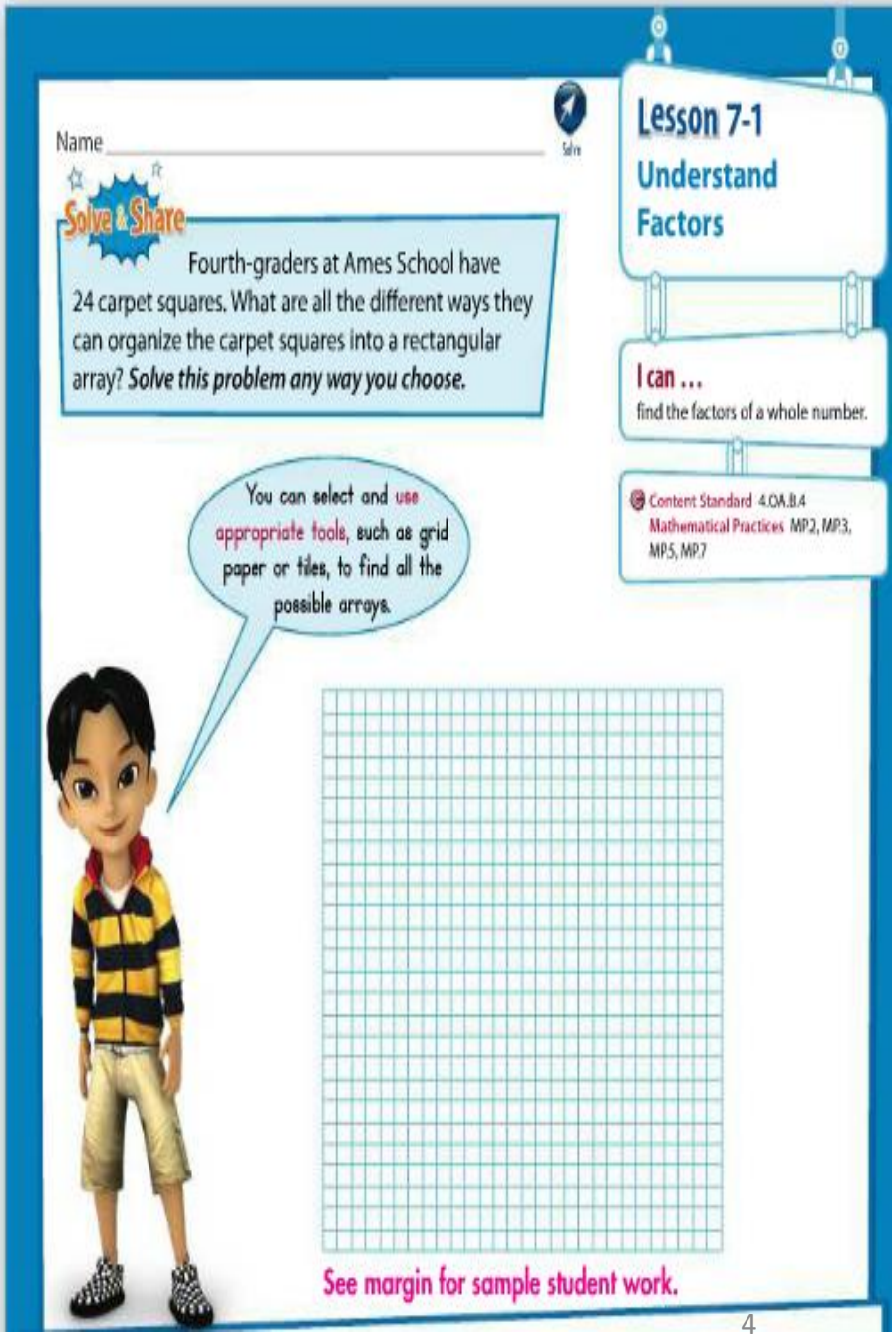
How many carpet squares will be used to create each rectangular array? [24] *How can you use the grid to help find all the possible arrays?* [Begin by drawing one row of 24 squares, then try 2 rows, 3 rows, and so on until you find all the possible arrays.]

DURING

3. Ask Guiding Questions As Needed

Is it possible to arrange the squares into 4 equal rows? [Yes] *5 equal rows?* [No] *After you find the array with 3 rows and 8 squares in each row, what other array do you immediately know you can make?* [An array with 8 rows and 3 squares in each row]

AFTER



4. Share and Discuss Solutions

Solve Start with students' solutions. If needed, project and discuss Jack's correct work.

5. Transition to the Visual Learning Bridge

You can use arrays to find the factors of a number. The dimensions of each array are factors of the number.

6. Extension for Early Finishers

Use grid paper and counters to find all the possible arrays for 20 carpet squares. [1 × 20 = 20, 2 × 10 = 20, 4 × 5 = 20, 5 × 4 = 20, 10 × 2 = 20, 20 × 1 = 20]

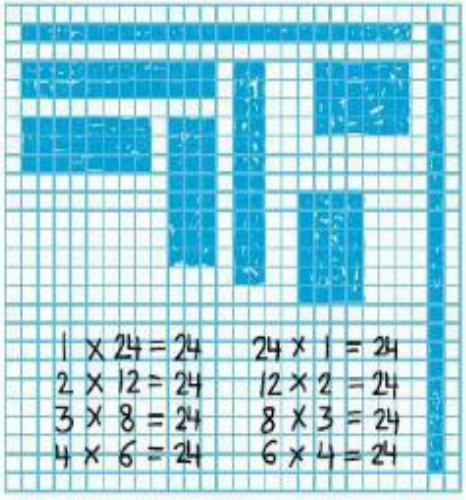
See margin for sample student work.

Look Back! MP.7 Look for Relationships What patterns do you see in the arrays?

Sample answer: The arrays can be written in reverse order. One possible array is 1 row of 24 carpet squares. Another possible array is 24 rows of 1 carpet square.

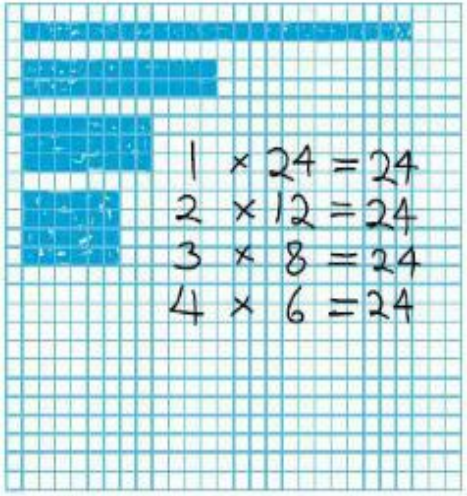
Analyze Student Work

Jack's Work



Jack uses the grid and looks for patterns to find all the possible arrays for 24.

Lydia's Work



Lydia finds half of the arrays. She does not recognize that 1 row of 24 squares is a different array than 24 rows of 1 square, etc.

MP.1 Make Sense and Persevere
Explain what "The chairs must be arranged in a rectangular array" means. [There must be the same number of chairs in each row.] *What is a factor?* [Numbers multiplied together to get a product are factors.] *How is a rectangular array of 12 chairs related to factors of 12?* [The number of rows and the number of chairs in each row are factors of 12.]

MP.8 Generalize
Is it always possible to make arrays with 1 row? [Yes] *Is it always possible to make arrays with 1 item in each row?* [Yes]

MP.8 Generalize
Is it always possible to make arrays with 2 rows? [No] *Is it always possible*

Learn Vocabulary

Essential Question

How Can You Use Arrays to Find the Factors of a Number?

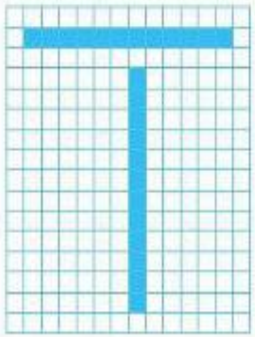
A

The music director is trying to find the best way to arrange the chairs for a performance. The chairs must be arranged in a rectangular array. Use grids to show all the ways the chairs can be arranged.

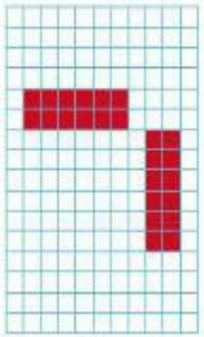
Pairs of numbers multiplied together to find a product are called factors.

12 chairs

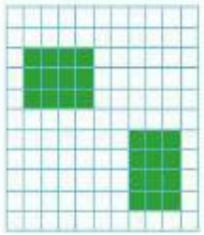
B 1 row of 12 chairs
12 rows of 1 chair



C 2 rows of 6 chairs
6 rows of 2 chairs



D 3 rows of 4 chairs
4 rows of 3 chairs



There are 6 possible ways the 12 chairs can be arranged.

The factors of 12 are 1, 2, 3, 4, 6, and 12.

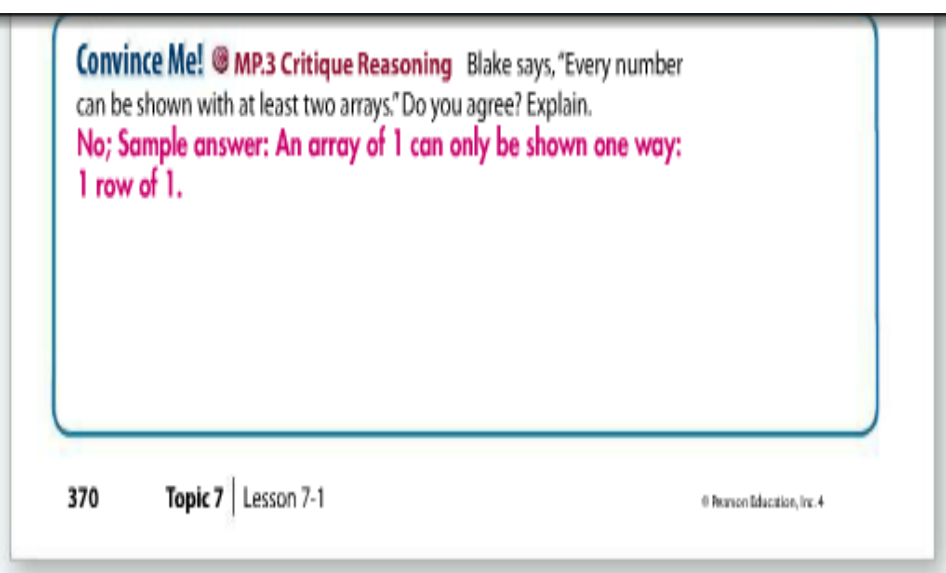
Convince Me! MP.3 Critique Reasoning Blake says, "Every number can be shown with at least two arrays." Do you agree? Explain.
No; Sample answer: An array of 1 can only be shown one way: 1 row of 1.

Visual Learning Bridge

MP.2 Reason Quantitatively
It is possible to make arrays with 1 row, 2 rows, 3 rows, and 4 rows for 12 chairs. Is it possible to make an array with 5 rows of chairs? Explain. [No; 5 rows of 2 chairs leaves 2 chairs outside of the array and 5 rows of 3 is too many chairs.] *How many possible arrays are there for 12 chairs?* [6] *How many factors does 12 have?* [6] *How do you know when you have found all the factors of a number?* [Start with an array with 1 row, then try to make arrays with 2 rows, 3 rows, and so on until the factor pairs start to repeat.]

in each row? [Yes]

MP.8 Generalize
Is it always possible to make arrays with 2 rows? [No] *Is it always possible to make arrays with 2 items in each row?* [No]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know when you have found all the factors of a number? [Start with an array with 1 row, then try to make arrays with 2 rows, 3 rows, and so on until the factor pairs start to repeat.]

Convince Me! MP.3 Critique Reasoning Students evaluate a statement made about factors and begin to explore properties of factors.

Coherence Students learn how to use arrays to find all the factors of a whole number less than 100. This connects to work in previous lessons where students used arrays to understand multiplication and division, learned basic facts for these two operations, and broke apart numbers into factors to help multiply mentally.

Revisit the Essential Question. Factors of a number can be shown by arranging counters into rows with the same number of counters in each row. The number of rows and the number of counters in each row are factors of the total number of counters.

Name _____

Guided Practice

Do You Understand?

1. MP.2 Reasoning How are the lengths of the sides of the arrays shown on the grids on the previous page related to the factors of 12?
   Sample answer: The lengths of the sides of each array are factors of 12. For example, the 3 × 4 array shows 3 and 4 are factors of 12.
2. What are the lengths of the sides of the arrays that show how 5 chairs can be arranged?
   1 and 5

Do You Know How?

For **3–4**, use grids to find all the possible arrays for each number.

3. 6
   1 × 6, 2 × 3, 3 × 2, 6 × 1
4. 16
   1 × 16, 2 × 8, 4 × 4, 8 × 2, 16 × 1

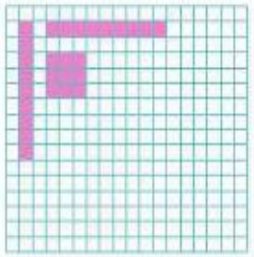
For **5–6**, use grids to find the factors of each number.

5. 45
   1, 3, 5, 9, 15, 45
6. 30
   1, 2, 3, 5, 6, 10, 15, 30

Independent Practice

For **7–8**, use the grids to find all the possible the arrays for each number. Use the arrays to help write the factors.

7. 9



1, 3, 9

8. 14

1, 2, 7, 14

For **9–14**, use grids to find the factors for each number.

9. 5 1, 5
10. 25 1, 5, 25
11. 8 1, 2, 4, 8
12. 36 1, 2, 3, 4, 6, 9, 12, 18, 36
13. 23 1, 23
14. 27 1, 3, 9, 27

*For another example, see Set A on page 401.

Math Practices and Problem Solving

15. MP.2 Reasoning Use the grid to find two numbers that have 2 and 3 as factors.

Sample answers: 6, 12, 18, 24, etc.

16. The dwarf planet Pluto takes about 90,403 days to orbit the sun. Write this number in expanded form and using number names.
    90,000 + 400 + 3; ninety thousand, four hundred three
17. David makes 17 dollars in an hour and works 25 hours each week. Linda makes 25 dollars in an hour and works 17 hours each week. How much do David and Linda make together each week? What property of multiplication does this represent?
    They make $850 each week; Commutative Property of Multiplication
18. What do you notice about the number of possible arrays and the number of factors of 22?
    Sample answer: They are equivalent. There are 4 possible arrays and 4 factors for 22.
19. Higher Order Thinking Jane says 5 is a factor of every whole number that has a 5 in the ones place. Fred says 5 is a factor of every whole number that has a 0 in the ones place. Who is correct? Explain.
    Both Jane and Fred are correct. Sample answer: Numbers with a factor of 5 have a 0 or a 5 in the ones place.

Common Core Assessment

20. Which of the following are factors of 18? Select all that apply.
    - [x] 1
    - [x] 2
    - [ ] 4
    - [x] 6
    - [ ] 23
21. Which of the following are factors of 31? Select all that apply.
    - [x] 1
    - [ ] 3
    - [ ] 7
    - [x] 31
    - [ ] 62

Error Intervention: Item 1

If students struggle to see how the two are related,

then explain that the number of rows of squares in the array is equal to the length of the rectangle and the number of squares in each row is equal to the width. Thus, the dimensions of the rectangle are factors of the area of the rectangle.

Item 4 *Why does 16 have an odd number of arrays when most numbers have an even number of arrays?* [16 has an array with 4 rows and 4 squares in each row. This array cannot be turned to form a different array.]

Reteaching Assign Reteaching Set A on p. 401.

Item 15 **MP.2 Reason Quantitatively** *What do the numbers in the problem mean?* [2 and 3 must be factors of each number found.] *How can you represent this relationship?* [Show arrays that have an appropriate number of rows and number of squares in each row.]

Item 17 *What are the hidden questions that must be answered to solve this problem?* [How much does David make in a week? How much does Linda make in a week?] *How can you solve the hidden questions?* [For each, multiply the hours they worked by the amount they make each hour.]

Item 21 **Coherence** Students can use what they have learned about quotients with remainders to solve. *Does 31 ÷ 1 have a remainder?* [No] *So, 1 is a factor of 31. Does 31 ÷ 3 have a remainder?* [Yes] *So, 3 is not a factor of 31.* Later in this topic, students will learn that 31 is a prime number because its only factors are 1 and itself.

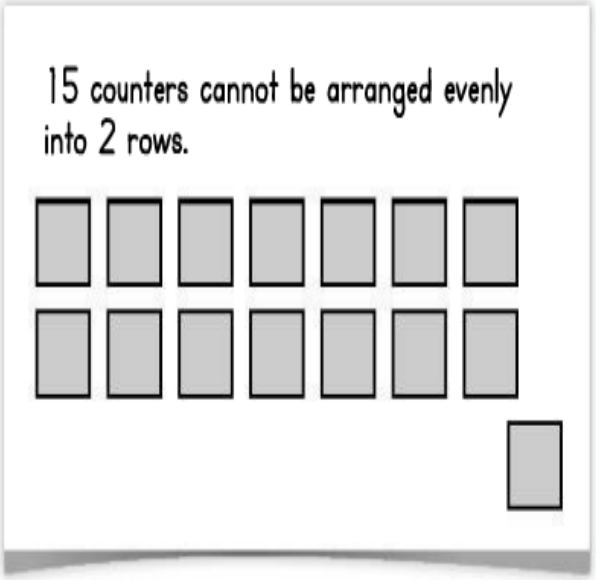
Intervention Activity

Finding All The Factors of a Number

Materials

Centimeter grid paper (Teaching Tool 9), 2-color square counters (or Teaching Tool 16)

- Display "15, 25, 30" on the board.
- *Count out 15 counters. Can you make 1 row with the same number of counters in each row?* [Yes] *What other array can you make that is similar to this?* [15 rows with 1 counter in each row] *Can you make 2 rows with the same number of counters in each row?* [No] *3 rows?* [Yes] *What other array can you make that is similar to this?* [5 rows with 3 counters in each row] *Can you make 4 rows with the same number of counters in each row?* [No] *Can you make any other arrays? Explain.* [No; I already made an array with 5 rows.] *What are all the factors of 15?* [1, 3, 5, and 15]
- Have students work in pairs to find all the factors of 25 [1, 5, and 25] and of 30 [1, 2, 3, 5, 6, 10, 15, and 30]



Reteach

Name _____

Reteach to Build Understanding 7-1

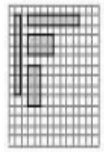
Vocabulary

1. Numbers multiplied together to find a product are called **factors**.

$4 \times 6 = 24$ (Factors) The factors are 4 and 6. The product is 24.

$2 \times 3 \times 7 = 42$ (Factors) The factors are 2, 3, and 7. The product is 42.

2. What are the factors of 8?



Use the arrays to help write the missing factors.

1 row of 8 — 1 × 8 = 8

8 rows of 1 — 8 × 1 = 8

2 rows of 4 — 2 × 4 = 8

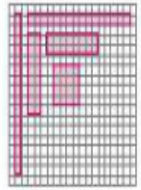
4 rows of 2 — 4 × 2 = 8

There are 4 possible ways to arrange 8.

List the factors of 8.

1, 2, 4, 8

3. Draw all the possible arrays for 16 on the grid at the right.



There are 5 possible arrays for 16.

Use the arrays to help write the factors.

1 row of 16 — 1 × 16 = 16

16 rows of 1 — 16 × 1 = 16

2 rows of 8 — 2 × 8 = 16

8 rows of 2 — 8 × 2 = 16

4 rows of 4 — 4 × 4 = 16

List the factors of 16.

1, 2, 4, 8, 16

On the Back!

4. Find all the possible factors of 18. Use a grid to help. 1, 2, 3, 6, 9, 18

RT-1

On-Level and Advanced Activity Centers O A

Name _____

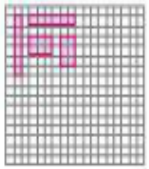
Math and Science Activity 7-1

Feeding the Animals

Did You Know? Animals have specific nutritional requirements. Zookeepers determine how much protein, fat, fiber, carbohydrates, and vitamins each animal needs. Zookeepers have to keep track of what types of foods and how much food animals eat each day.

Animal	Number of Animals
Elephant	6
Dolphin	8
Giraffe	10
Penguin	30
Snake	15

The table above shows the number of each type of animal the zookeeper feeds each day. Suppose animals could be arranged in the feeding areas in different ways.



1. Use the grid to show all the possible ways the zookeeper could arrange the elephants during feeding time.
2. What are the factors of 6?

   1, 2, 3, 6

3. How many different arrays could the zookeeper make to arrange the penguins into feeding areas during feeding time?

   8 different arrays

4. What are the factors of 30?

   1, 2, 3, 5, 6, 10, 15, 30

5. **Extension** The zookeeper would be able to arrange all but one of the animals into feeding-area arrays with two rows. For which animal would an array with two rows **NOT** be possible? Explain.

   Snake; Sample answer: The factors of 15 are 1, 3, 5, and 15. So an array with two rows is not possible.

Math and Science Activity 7-1

Math and Science Activity STEM

This activity revisits the science theme Plant and Animal Structures, introduced on page 365 in the Student's Edition.

Sample Student Work

3. 1 group of 30 penguins
   30 groups of 1 penguin
   2 groups of 15 penguins
   15 groups of 2 penguins

   3 groups of 10 penguins
   10 groups of 3 penguins
   5 groups of 6 penguins
   6 groups of 5 penguins

Whole Class BEFORE

1. Pose the Solve-and-Share Problem

You may wish to provide centimeter grid paper (Teaching Tool 9) or 2-color counters (or Teaching Tool 15).

MP.5 Use Appropriate Tools Strategically Listen and look for students who talk about multiplication facts as they use the counters and grids to find factors.

2. Build Understanding

What are you asked to find? [Find all the ways Jared can plant 20 flowers in equal rows in his garden.] *How are the different ways to arrange the flowers related to factors of 20?* [The number of flowers in each row and the number of rows are factors of 20.]

Small Group DURING

3. Ask Guiding Questions As Needed

Is 1 a factor of 20? How do you know? [Yes; $1 \times 20 = 20$] *1 and 20 are called a factor pair because their product is 20. Is there a factor pair with 2? If so, what is it?* [Yes; 2 and 10] *Is there a factor pair with 3? If so, what is it?* [No]

Whole Class AFTER

Name _____ Solve

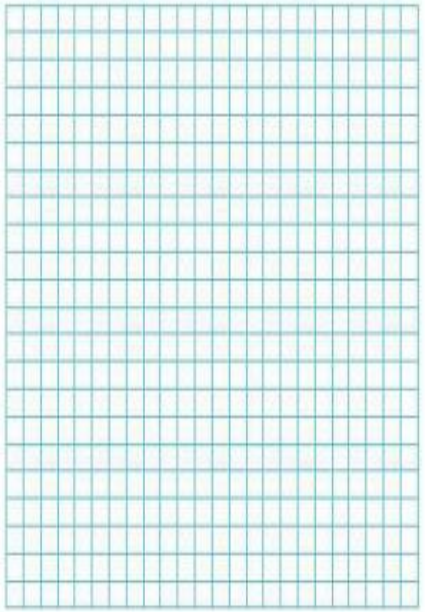
Solve & Share

Jared has 20 flowers. He wants to plant all of the flowers in equal rows in his garden. What are the different ways Jared can arrange the flowers in equal rows? ***Solve this problem any way you choose.***

Lesson 7-2
Factors

I can ...
use multiplication to find the factor pairs for a whole number.

Content Standard 4.OA.B.4
Mathematical Practices MP.1, MP.2, MP.3, MP.4, MP.5



See margin for sample student work.

Whole Class **AFTER**

4. Share and Discuss Solutions

Solve Start with students' solutions. If needed, project and discuss Tiffany's correct work.

5. Transition to the Visual Learning Bridge

Multiplication can be used to find the factors of a number.

Two numbers that when multiplied give a certain product are called a factor pair.

6. Extension for Early Finishers

In how many ways can Tami arrange 9 cards, in equal rows, in an album? What are the ways? [3 ways; 1 row of 9 cards, 9 rows of 1 card, or 3 rows of 3 cards]

See margin for sample student work.

Look Back! MP.1 **Make Sense and Persevere** How can you check you have found all the different ways Jared can plant his flowers?

Sample answer: Start with 1 row of flowers (1 × 20 = 20), then try 2 rows of flowers (2 × 10 = 20), 3 rows of flowers, 4 rows of flowers (4 × 5 = 20), and so on until you get to 20 rows of flowers (20 × 1 = 20).

Analyze Student Work

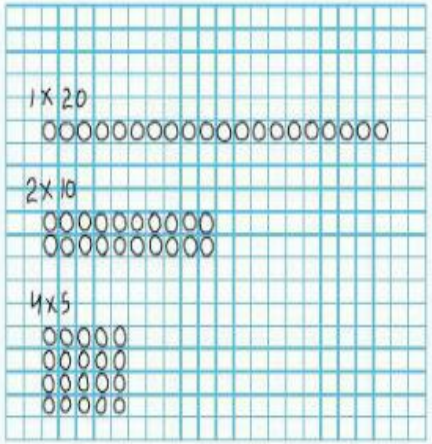
Tiffany's Work

1 x 20 is 1 row with 20 flowers.
20 x 1 is 20 rows with 1 flower.

2 x 10 is 2 rows with 10 flowers.
10 x 2 is 10 rows with 2 flowers.

4 x 5 is 4 rows with 5 flowers.
5 x 4 is 5 rows with 4 flowers.

Robin's Work



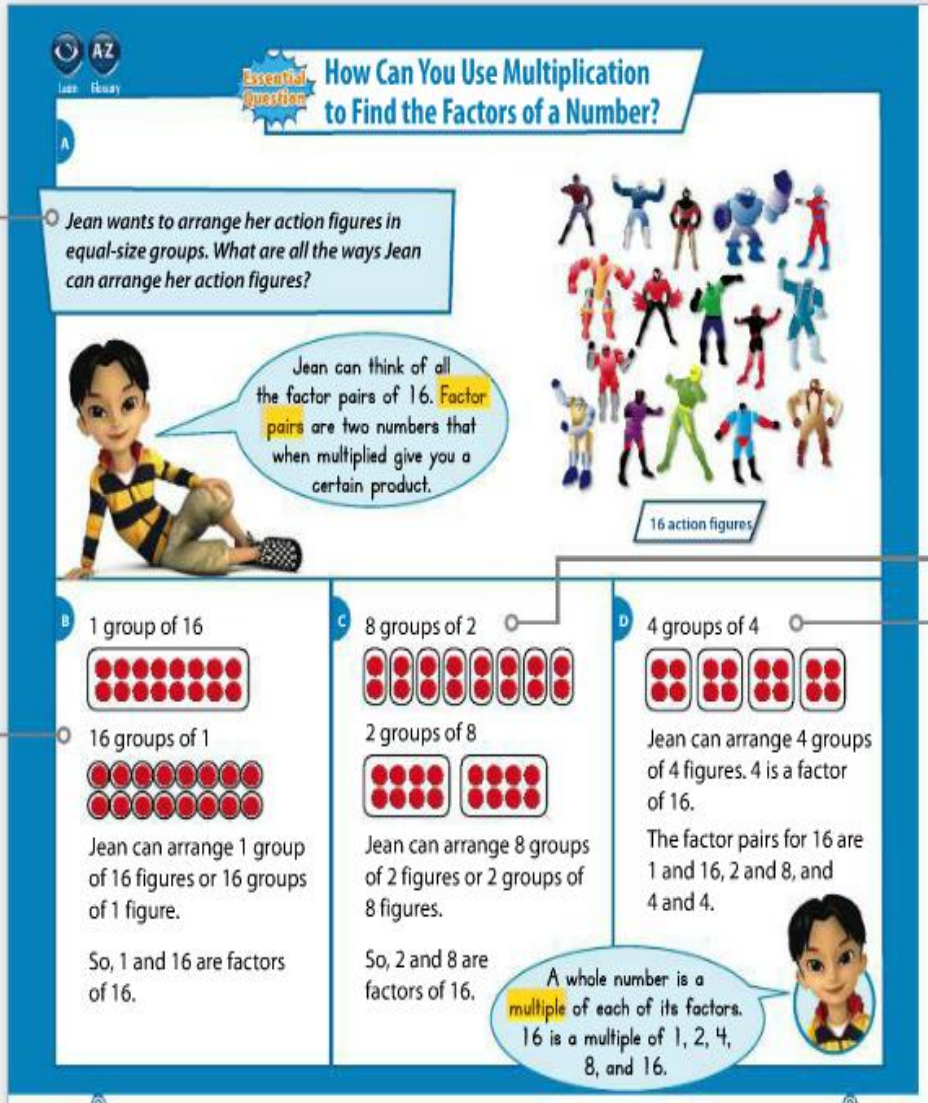
Jared can make equal rows of 1, 2, 4, 5, 10, or 20 flowers.

MP.1 Make Sense and Persevere

How many action figures does Jean have? [16] *What does Jean want to do?* [She wants to arrange the action figures in equal-size groups.] *What is a factor pair of 16? Explain why it is a factor pair.* [Sample answer: 2 and 8 are a factor pair of 16 because $2 \times 8 = 16$.]

MP.3 Construct Arguments

Why are 1 and 16 factors of 16? [$1 \times 16 = 16$ or $16 \times 1 = 16$]



Convince Me! MP.3 Construct Arguments How do you know there are no other factors for 16 other than 1, 2, 4, 8, and 16? Explain.
Sample answer: There are no other factors of 16 because there are no other factor pairs other than 1 and 16, 2 and 8, and 4 and 4 that have a product of 16.

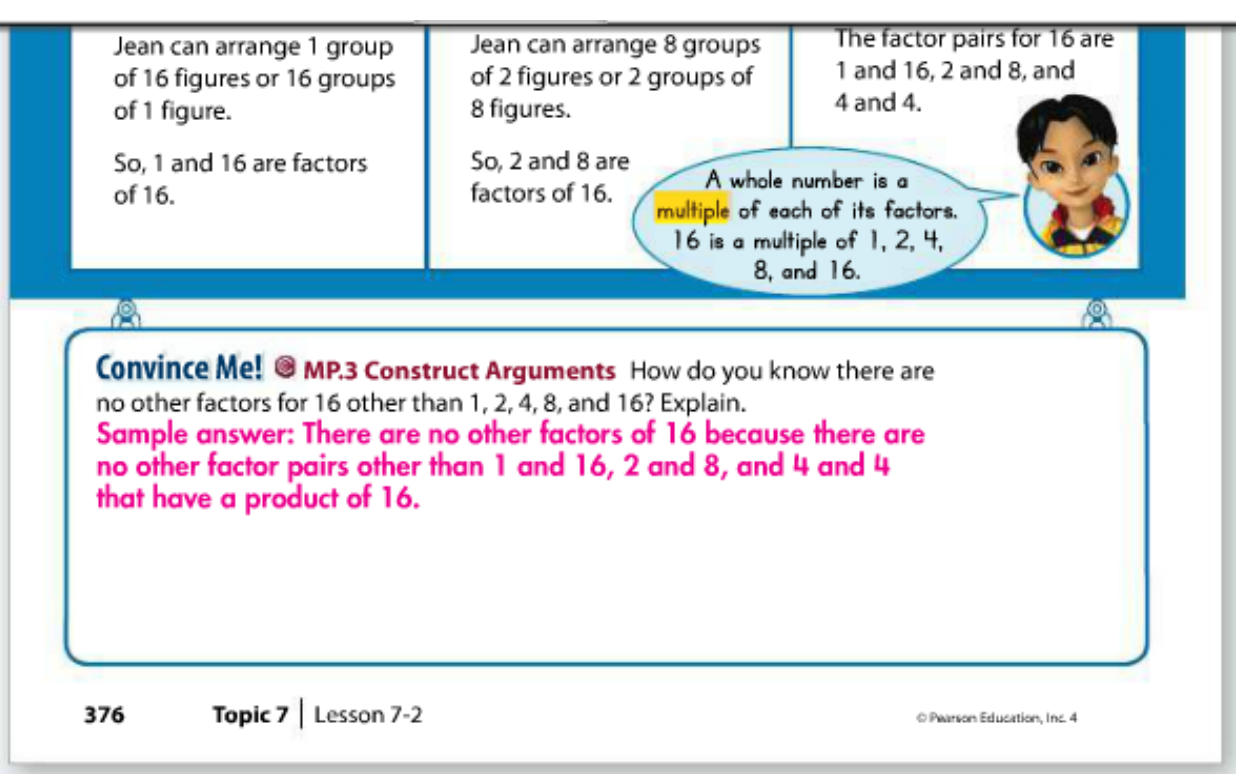
Visual Learning Bridge

Prevent Misconceptions RTI

Some students may think that 2 groups of 8 and 8 groups of 2 give 4 factors, since there are two factor pairs. Point out that this is really only one factor pair and two factors.

MP.2 Reason Quantitatively

How can you be sure you found all the factors for 16? [Try 1, 2, 3, and so on until you get to a factor you already found. For example, 1 and 2 are factors of 16. 3 is not. 4 is a factor. 5, 6, and 7 are not. 8 is a factor, but it is in a pair with 2, so it was


Jean can arrange 1 group of 16 figures or 16 groups of 1 figure.

So, 1 and 16 are factors of 16.

Jean can arrange 8 groups of 2 figures or 2 groups of 8 figures.

So, 2 and 8 are factors of 16.

The factor pairs for 16 are 1 and 16, 2 and 8, and 4 and 4.

A whole number is a multiple of each of its factors. 16 is a multiple of 1, 2, 4, 8, and 16.

Convince Me! **MP.3 Construct Arguments** How do you know there are no other factors for 16 other than 1, 2, 4, 8, and 16? Explain.

Sample answer: There are no other factors of 16 because there are no other factor pairs other than 1 and 16, 2 and 8, and 4 and 4 that have a product of 16.

376 **Topic 7** | Lesson 7-2 © Pearson Education, Inc. 4

MP.2 Reason Quantitatively

How can you be sure you found all the factors for 16? [Try 1, 2, 3, and so on until you get to a factor you already found. For example, 1 and 2 are factors of 16. 3 is not. 4 is a factor. 5, 6, and 7 are not. 8 is a factor, but it is in a pair with 2, so it was already found. Thus, 1, 16, 2, 8, and 4 are all the factors of 16.]

Convince Me! **MP.3 Construct Arguments** Students explain how an organized approach to finding factor pairs helps them determine they have found all of the possible factors for a given number.

Coherence Students learn to find all the factor pairs for a whole number, using groups of counters. This connects to work in previous lessons where students learned the meanings of the operations of multiplication and division and where they memorized basic facts. They also use what they have learned about the relationship between multiplication and division.

Essential Question

Revisit the Essential Question. Multiplication can be used to find factor pairs and thus also find factors of a given whole number.

Name _____

Guided Practice

Do You Understand?

1. Jean bought 7 more action figures. What are the different equal-size groups she can make now?
   1 group of 23 and 23 groups of 1.

2. MP.2 Reasoning What factor besides 1 does every even number have?
   2

Do You Know How?

For **3–6**, write the factors of each number. Use counters to help.

3. 2
   1, 2

4. 20
   1, 2, 4, 5, 10, 20

5. 28
   1, 2, 4, 7, 14, 28

6. 54
   1, 2, 3, 6, 9, 18, 27, 54

Independent Practice

Leveled Practice For **7–12**, write the factor pairs for each number.

7. 34
   1 and 34
   2 and 17

8. 39
   1 and 39
   3 and 13

9. 61
   1 and 61

10. 14
    1 and 14
    2 and 7

11. 22
    1 and 22
    2 and 11

12. 51
    1 and 51
    3 and 17

For **13–21**, write the factors of each number. Use counters to help as needed.

13. 6
    1, 2, 3, 6

14. 32
    1, 2, 4, 8, 16, 32

15. 83
    1, 83

16. 11
    1, 11

17. 49
    1, 7, 49

18. 25
    1, 5, 25

19. 30
    1, 2, 3, 5, 6, 10, 15, 30

20. 63
    1, 3, 7, 9, 21, 63

21. 19
    1, 19

Math Practices and Problem Solving

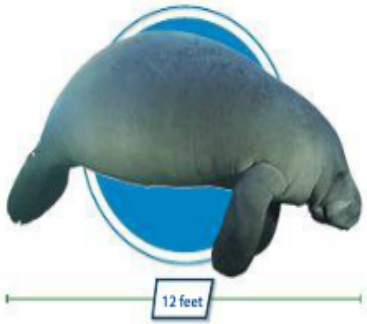
22. Irene wants to list the factors for 88. She writes 2, 4, 8, 11, 22, 44, and 88. Is Irene correct? Explain.
    No; Sample answer: 1 is also a factor, because 1 is a factor of every number.

23. **Math and Science** The roots of a plant are often the largest part of the plant. Winter rye can grow combined root tissue well over 984,000 feet in length. Write this number in expanded form.
    900,000 + 80,000 + 4,000

24. A restaurant receives a shipment of 5,000 ketchup packets. In one week, they use 1,824 packets. The next week, they use 2,352 packets. How many ketchup packets does the restaurant have left?
    824 ketchup packets

25. Any number that has 9 as a factor also has 3 as a factor. Why is this?
    Sample answer: If 9 is a factor, then so is 3, because 3 is a factor of 9.

26. **Higher Order Thinking** A mother manatee, pictured to the right, is three times as long as her baby manatee.



   a. How long is her baby manatee? Write and solve an equation.
      4 feet; Sample answer: $3 \times f = 12$; $f = 4$

   b. If a blue whale is 9 times as long as the manatee shown, how much longer is a blue whale than the manatee? Write and solve equations.
      96 feet; Sample answer: $12 \times 9 = b$; $b = 108$; $108 - 12 = m$; $m = 96$

Common Core Assessment

27. Which lists all the factors of 38?
    - Ⓐ 1, 38
    - Ⓑ 1, 2, 14, 38
    - Ⓒ 1, 2, 38
    - Ⓓ 1, 2, 19, 38

28. A store manager wants to display 45 cans of soup in an array. Which of the following shows 3 ways the cans could be displayed?
    - Ⓐ $1 \times 9, 9 \times 5, 3 \times 15$
    - Ⓑ $15 \times 3, 9 \times 1, 5 \times 9$
    - Ⓒ $5 \times 9, 3 \times 15, 9 \times 5$
    - Ⓓ $45 \times 1, 15 \times 1, 9 \times 1$

13. 6
1, 2, 3, 6

14. 32
1, 2, 4, 8, 16, 32

15. 83
1, 83

16. 11
1, 11

17. 49
1, 7, 49

18. 25
1, 5, 25

19. 30
1, 2, 3, 5, 6, 10, 15, 30

20. 63
1, 3, 7, 9, 21, 63

21. 19
1, 19

*For another example, see Set B on page 401.

Common Core Assessment

27. Which lists all the factors of 38?

Ⓐ 1, 38
Ⓑ 1, 2, 14, 38
Ⓒ 1, 2, 38
Ⓓ 1, 2, 19, 38

28. A store manager wants to display 45 cans of soup in an array. Which of the following shows 3 ways the cans could be displayed?

Ⓐ 1 × 9, 9 × 5, 3 × 15
Ⓑ 15 × 3, 9 × 1, 5 × 9
Ⓒ 5 × 9, 3 × 15, 9 × 5
Ⓓ 45 × 1, 15 × 1, 9 × 1

Error Intervention: Item 2

If students have trouble understanding the question,

then have them list 5 or 6 even numbers and their factors. *All these numbers have a factor of 1. What other number is a factor of all these numbers?* [2]

Items 3–6 Encourage students to list the factor pairs in an organized way to make sure they find all of the factors.

Reteaching Assign Reteaching Set B on p. 401.

Item 24 *What do you need to find?* [How many ketchup packets the restaurant has left] *What do you know?* [The restaurant has 5,000 packets. They used 1,824 packets one week and 2,352 packets the next week.] *What is the hidden question that must be answered to solve this problem?* [Sample answer: How many packets did the restaurant use in two weeks?] *How can you solve the hidden question?* [Add 1,824 + 2,352]

Item 25 *How are the numbers in the problem related?* [3 is a factor of 9] *How can you use numbers, objects, drawings, or actions to justify your argument?* [I can use 3 and 9 to explain.] *Is your explanation clear and complete? Explain.* [Yes; Sample answer: The reason any number that has 9 as a factor also has 3 as a factor is because 3 is a factor of 9.]

Item 27 Coherence Encourage students to use what they know about dividing a 2-digit number by a 1-digit number to complete the factor pair with 2. *You know 38 is an even number and so it has a factor of 2. How can you find the factor that pairs with 2?* [Divide 38 by 2.]

Intervention Activity I

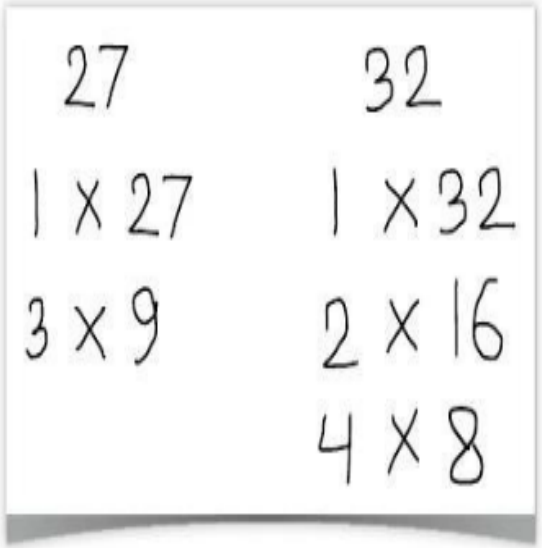
Finding Factors

Materials

2-color counters (or Teaching Tool 15)

- *Is 1 a factor of 18?* [Yes] *What is the other number in the factor pair?* [18]
- *Is 2 a factor of 18?* [Yes] *What is the other number in the factor pair?* [9]
- *Is 3 a factor of 18?* [Yes] *What is the other number in the factor pair?* [6]
- *Is 4 a factor of 18?* [No]
- *Is 5 a factor of 18?* [No]
- *Is 6 a factor of 18?* [Yes] *What is the other number in the factor pair?* [3] *Did you already find 3 and 6?* [Yes]
- *What are all the factors of 18?* [1, 2, 3, 6, 9, and 18]

- Have students work in pairs to repeat this process to find all the factors of 27 [1, 3, 9, and 27] and of 32 [1, 2, 4, 8, 16, and 32]. Provide counters as needed.



Reteach I

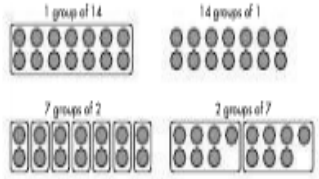
Name _____

Reteach to Build Understanding
7-2

Vocabulary

1. **Factor pairs** are two numbers that when multiplied give a certain product.
   The factors of 6 are 1, 2, 3, and 6. Find which pairs of factors have a product of 6.
   $1 \times 2 =$ 2 Are 1 and 2 a factor pair for 6? No
   $2 \times 3 =$ 6 Are 2 and 3 a factor pair for 6? Yes
2. A **multiple** is the product of a given whole number and any non-zero whole number.
   6 is a multiple of each of its factors: 1, 2, 3, and 6.
   Show how each factor can be multiplied to get a product of 6.
   Use 1 as a factor: $1 \times$ 6 $= 6$ Use 2 as a factor: $2 \times$ 3 $= 6$

3. Find the factor pairs for 14.
   First, find the factors of 14.
   1, 2, 7, 14



   Determine which pairs of factors have a product of 14.
   $1 \times$ 14 $= 14$ and 14 $\times 1 = 14$
   $2 \times$ 7 $= 14$ and 7 $\times 2 = 14$
   The factor pairs for 14 are:
   1 and 14
   2 and 7

4. Find the factor pairs for 24.
   1 group of 24 or 24 groups of 1 3 groups of 8 or 8 groups of 3
   2 groups of 12 or 12 groups of 2 4 groups of 6 or 6 groups of 4
   The factor pairs are 1 and 24, 2 and 12, 3 and 8, 4 and 6

On the Back!

5. Write the factor pairs for 40. 1 and 40, 2 and 20, 4 and 10, 5 and 8

Problem-Solving Reading Mat

Have students read the Problem-Solving Reading Mat for Topic 7 and then complete Problem-Solving Reading Activity 7-2.

See the Problem-Solving Reading Activity Guide for other suggestions on how to use this mat.

Whole Class BEFORE

1. Pose the Solve-and-Share Problem

MP.8 Generalize Look for students who use multiplication or division to find all of the factors and factor pairs for 24. They should look for a general method of knowing when to stop trying to find factors.

2. Build Understanding

How many cubicles did Jane buy? [24] *Do you need to use all of Jane's cubicles?* [Yes] *How can the cubicles be arranged?* [In rows with the same number of cubicles in each row]

Small Group DURING

3. Ask Guiding Questions As Needed

What number is a factor of every number? [1] *Can you divide 24 evenly by 2?* [Yes; $2 \times 12 = 24$] *Is 2 rows with 12 cubicles in each row the same as a 12 rows with 2 cubicles in each row? Explain.* [No; The number of cubicles is the same, but the arrangement is different.]

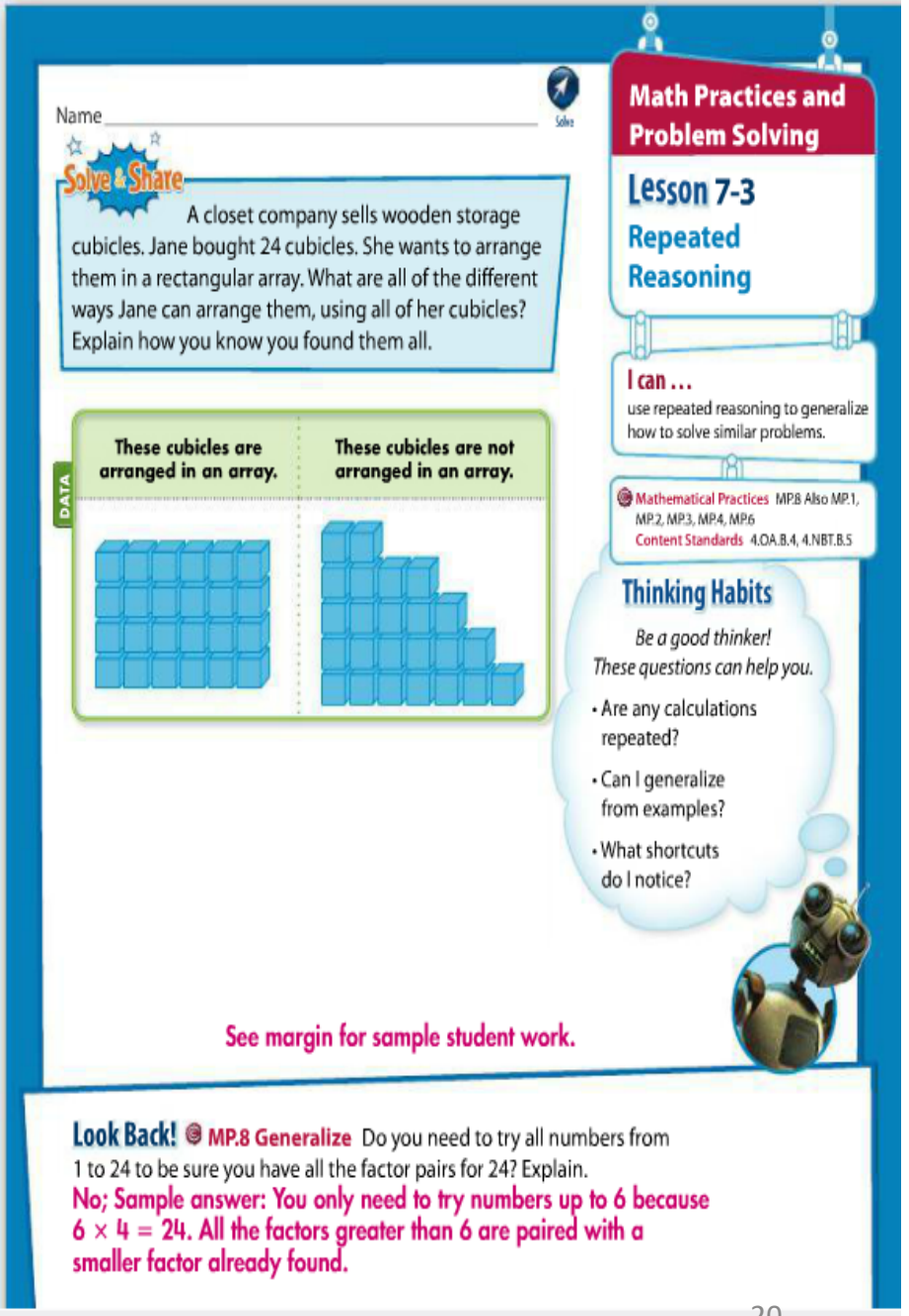
Whole Class AFTER

4. Share and Discuss Solutions

Solve Start with students' solutions. If needed, project and discuss Taylor's correct work.

5. Transition to the Visual Learning Bridge

Repeated reasoning, multiplication, and division can be used to find all the factor pairs for any whole number.



Name _____ Solve

Solve & Share

A closet company sells wooden storage cubicles. Jane bought 24 cubicles. She wants to arrange them in a rectangular array. What are all of the different ways Jane can arrange them, using all of her cubicles? Explain how you know you found them all.

DATA

These cubicles are arranged in an array.	These cubicles are not arranged in an array.

Math Practices and Problem Solving

Lesson 7-3
Repeated Reasoning

I can ...
use repeated reasoning to generalize how to solve similar problems.

Mathematical Practices MP.8 Also MP.1, MP.2, MP.3, MP.4, MP.6
Content Standards 4.OA.B.4, 4.NBT.B.5

Thinking Habits

Be a good thinker!
These questions can help you.

- Are any calculations repeated?
- Can I generalize from examples?
- What shortcuts do I notice?

See margin for sample student work.

Look Back! MP.8 Generalize Do you need to try all numbers from 1 to 24 to be sure you have all the factor pairs for 24? Explain.

No; Sample answer: You only need to try numbers up to 6 because $6 \times 4 = 24$. All the factors greater than 6 are paired with a smaller factor already found.

Repeated reasoning, multiplication, and division can be used to find all the factor pairs for any whole number.

6. Extension for Early Finishers

Find all the whole numbers greater than 2 and less than 10 that have exactly two factor pairs. Tell how you know you found them all. [4, 6, 8, and 9; Sample answer: I checked all the numbers between 2 and 10. 3, 5, and 7 only have 1 factor pair; 4 has 2 factor pairs: 1×4 and 2×2; 6 has 1×6 and 2×3; 8 has 1×8 and 2×4; 9 has 1×9 and 3×3.]

smaller factor already found.

Analyze Student Work

Taylor's Work

$24 \div 1 = 24$	1 row of 24 or 24 rows of 1
$24 \div 2 = 12$	2 rows of 12 or 12 rows of 2
$24 \div 3 = 8$	3 rows of 8 or 8 rows of 3
$24 \div 4 = 6$	4 rows of 6 or 6 rows of 4
$24 \div 5 = 4$ R4	no factor pair
$24 \div 6 = 4$	Already found

I divided over and over. I stopped at 6 because I had already found 6 and 4 when I divided by 4.

Taylor divided by 1, 2, 3, and so on. He stopped when the factor pairs started to repeat.

Tom's Work

$3 \times 8 = 24$

$4 \times 6 = 24$

$1 \times 24 = 24$

I used multiplication to find pairs for 24. Jane can put the cubicles in 3, 8, 4, 6, 1, or 24 rows.

Tom did not find all the possibilities, and did not explain how he found the different ways to arrange the cubicles.

How many trees does the gardener have to plant? [15 trees] *What does it mean to plant trees in a rectangular array?* [The trees will be planted in rows with the same number of trees in each row.] *What do you need to do?* [Find the different ways the gardener can plant the trees] *Could the gardener plant 4 rows with 5 trees in each row? Explain.* [No; That would be 20 trees and the gardener only has 15 trees to plant.]

MP.8 Generalize

What steps repeat in the problem? [Keep dividing 15 by 1, 2, and so on to find all the factors of 15.] *How might generalizing help you to solve this problem and other similar problems?* [I can figure out when to stop dividing

Visual Learning Bridge

How Can You Use Repeated Reasoning to Find All the Factors for a Number?

A

A new city park is opening. The gardener needs to select 15 trees from a nursery and plant the trees in a rectangular array. What are all the different ways the gardener can plant the trees?

Can you look for a general method to solve this problem?

I can find all the possible factors of 15 that can be arranged in a rectangular array.

B **How can I make a generalization from repeated reasoning?**

I can

- look for things that repeat in a problem.
- look for shortcuts.
- generalize from the example.

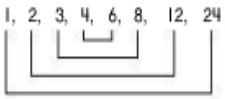
C To find all of the factors of 15, I divide 15 by divisors starting with 1. Then I use the Commutative Property to write two multiplication equations.

$1 \times 15 = 15$ and $15 \times 1 = 15$
2 is not a factor
$3 \times 5 = 15$ and $5 \times 3 = 15$
4 is not a factor
$5 \times 3 = 15$ and $3 \times 5 = 15$

I already found the factor paired with 5, 3×5 and 5×3.

The gardener has 4 different ways to plant the trees: 1×15, 15×1, 5×3, and 3×5 arrays.

Convince Me! MP.3 Construct Arguments The diagram shows all the factor pairs of 24. Use the diagram to justify the conclusion that when factor pairs start repeating, you know you have found all the factors of a number.

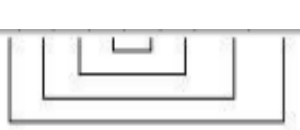


Sample answer: The factor pairs start repeating when you reach the middle factor pairs in the diagram. In this example, 4 and 6 are the middle factor pairs for 24. The factor pairs

Why should you stop dividing once you find a repeated factor pair? [When the factor pairs start to repeat, you have found all the factors for a number. Any number greater than 5 would pair with a number less than 5 to equal 15. Thus, it would have been found as a factor with the number less than 5.]

How might generalizing help you to solve this problem and other similar problems? [I can figure out when to stop dividing by possible factors without having to divide by all the numbers from 1 to 15.]

shows all the factor pairs of 24. Use the diagram to justify the conclusion that when factor pairs start repeating, you know you have found all the factors of a number.



Sample answer: The factor pairs start repeating when you reach the middle factor pairs in the diagram. In this example, 4 and 6 are the middle factor pairs for 24. The factor pairs repeat starting with 4 × 6, 6 × 4, then 8 × 3, and so on until you reach the number itself and 1 again.

Convince Me! MP.3 Construct Arguments When you generalize, you are stating something you think is true. This is not the same as justifying that the statement is true.

Coherence The behaviors listed in Box B are the same as those listed in lessons on repeated reasoning in other grades. The problem here involves finding all the factors of a number, as students have been doing earlier in this topic. Students generalize to explain how they know they have found all the possible factor pairs.

Revisit the Essential Question. Repeated reasoning leads to the generalization that all the factors of a number have been found as soon as the factor pairs start to repeat.

Name _____

Guided Practice

MP.8 Generalize

Ms. Maribel wants to arrange the 20 desks in her classroom into rows with the same number of desks in each row. She wants at least 2, but not more than 8 rows.

1. What are the factor pairs for 20? Explain how you know you have found them all.
   1 and 20, 2 and 10, 4 and 5; Sample answer: Start with 1 to find 1 and 20, try 2 to find 2 and 10, 3 is not a factor, try 4 to find 4 and 5, try 5 to find 5 and 4. Since 4 and 5 and 5 and 4 are the same factor pair, there are no more pairs to find.
2. Find the ways Ms. Maribel can arrange the desks.
   Sample answer: She can make 2 rows with 10 desks in each row, 4 rows with 5 desks in each row, and 5 rows with 4 desks in each row. Any other arrangement would not have at least 2, but not more than 8 rows.

Independent Practice

MP.8 Generalize

Kevin invited 15 friends to his birthday party. They played a game where everyone had to separate into groups. Each group had the same number of children. The game could not be played with all 16 children in one group and each group had to have more than 1 child.

3. List the factor pairs of 16 and then find the different ways Kevin and his friends could divide into groups.
   1 and 16, 2 and 8, 4 and 4; Sample answer: They can form 2 groups of 8, 4 groups of 4, and 8 groups of 2.
4. Why does 16 have an odd number of factors?
   Sample answer: One factor pair has the same number, $4 \times 4 = 16$, and so 4 pairs with itself.
5. Can you stop checking for factor pairs when you find a pair that repeats? Explain.
   Yes; Sample answer: When you find the factor pair that repeats, that means all the factors that are greater than and less than this factor pair have

Math Practices and Problem Solving

Common Core Performance Assessment

Store Displays

A pet store needs 3 displays with the products shown. The boxes of kitty litter need to be stacked with the same number of boxes in each row. There needs to be at least 3 rows with at least 3 boxes in each row. What are all the ways the boxes of kitty litter could be stacked?

50 fish bowls

48 boxes of kitty litter

88 bags of dog food

6. **MP.1 Make Sense and Persevere** What quantities are given in the problem and what do the numbers mean?
   Sample answer: 50 fish bowls, 48 boxes of kitty litter, and 88 bags of dog food. 3 is the number of displays, the minimum number of rows, and the minimum number of boxes of kitty litter in each row.
7. **MP.2 Reasoning** What are the factor pairs for 48?
   1 and 48, 2 and 24, 3 and 16, 4 and 12, 6 and 8
8. **MP.6 Be Precise** What are all the ways the boxes of kitty litter can be stacked with at least 3 rows with at least 3 boxes in each row?
   Sample answer: 3 rows with 16 boxes in each row, 4 rows with 12 boxes in each row, 6 rows with 8 boxes in each row; 8 rows with 6 boxes in each row; 12 rows with 4 boxes in each row; 16 rows with 3 boxes in each row

Kevin invited 15 friends to his birthday party. They played a game where everyone had to separate into groups. Each group had the same number of children. The game could not be played with all 16 children in one group and each group had to have more than 1 child.

3. List the factor pairs of 16 and then find the different ways Kevin and his friends could divide into groups.
1 and 16, 2 and 8, 4 and 4; Sample answer: They can form 2 groups of 8, 4 groups of 4, and 8 groups of 2.

4. Why does 16 have an odd number of factors?
Sample answer: One factor pair has the same number, $4 \times 4 = 16$, and so 4 pairs with itself.

5. Can you stop checking for factor pairs when you find a pair that repeats? Explain.
Yes; Sample answer: When you find the factor pair that repeats, that means all the factors that are greater than and less than this factor pair have already been found.

*For another example, see Set C on page 401.

litter, and 88 bags of dog food. 3 is the number of displays, the minimum number of rows, and the minimum number of boxes of kitty litter in each row.

7. MP.2 Reasoning What are the factor pairs for 48?

1 and 48, 2 and 24, 3 and 16, 4 and 12, 6 and 8

8. MP.6 Be Precise What are all the ways the boxes of kitty litter can be stacked with at least 3 rows with at least 3 boxes in each row?

Sample answer: 3 rows with 16 boxes in each row, 4 rows with 12 boxes in each row, 6 rows with 8 boxes in each row; 8 rows with 6 boxes in each row; 12 rows with 4 boxes in each row; 16 rows with 3 boxes in each row

MP.8 Repeated Reasoning Listen and look for these behaviors as evidence that students are exhibiting proficiency with MP.8.

- Notices and describes when certain calculations or steps in a procedure are repeated
- Generalizes from examples or repeated observations
- Recognizes and understands appropriate shortcuts
- Evaluates the reasonableness of intermediate results

Item 4 MP.8 Generalize If students are not sure why 16 has an odd number of factors, ask them if the factor pair 4×4 has 1 or 2 different factors.

Reteaching Assign Reteaching Set C on p. 401.

Item 6 MP.1 Make Sense and Persevere *What do you need to find?* [Find all the ways the boxes of kitty litter can be stacked with at least 3 rows and at least 3 boxes in each row.] *What is your plan for solving the problem?* [Find all the factor pairs for 48 and then see which ones satisfy the conditions in the problem.]

Item 7 MP.2 Reason Quantitatively *Why isn't there a factor pair with 5?* [When you divide 48 by 5, there is a remainder.] *How do you know you have all the factor pairs of 48?* [I checked 1, 2, 3, and so on up to 7 and 8. Since 8 was found in a factor pair with 6, there can be no other factor pairs.]

Item 8 MP.6 Be Precise *Why can't the kitty litter be stacked in 2 rows with 24 boxes in each row?* [The problem states that there need to be at least 3 rows.]

Intervention Activity I

Math Practices and Problem Solving: Repeated Reasoning

Materials

2-color counters (or Teaching Tool 15)

- Display the following problem on the board.
  Lindsey has 18 tomato plants to plant in her garden in an array. She wants at least 2 plants in each row and at least 2 rows. What are all the ways Lindsey can arrange the tomato plants in her garden?
- *What do you need to do to solve?* [Find all the factors of 18 and see which ones fit the conditions of the problem.]
- Have students work in pairs to solve. Have counters available.
- *What are all the factors of 18? How do you know?* [1, 2, 3, 6, 9, and 18; Start with 1, 2, 3, and so on to find factor pairs. When the pairs start to repeat, you have found them all.]
- Discuss the solution. *Which factor pairs are not a possible array for Lindsey's garden?* [1 × 18 or 18 × 1 because Lindsey wants at least 2 rows with at least 2 plants in each row.]

2 rows with 9 in each row,
3 rows with 6 in each row,
6 rows with 3 in each row, or
9 rows with 2 in each row.

Reteach I

Name _____

Reteach to Build Understanding
7-3

Vocabulary

1. When factor pairs start repeating, you can make a general statement, or **generalize**, that all of the factors of a number are listed.

To find all the factors for 20, begin dividing by 1. When the quotient has no remainder, you have found a factor. Use the division fact to write a related multiplication problem, and then use the Commutative Property to write another multiplication equation.

1 2 4 5 10 20

$20 \div 1 = 20$ $1 \times 20 = 20$ and $20 \times 1 = 20$
$20 \div 2 = 10$ $2 \times 10 = 20$ and $10 \times 2 = 20$
$20 \div 3 = 6$ R2 3 is not a factor.
$20 \div 4 = 5$ $4 \times 5 = 20$ and $5 \times 4 = 20$

Since you found the factor paired with 5, stop checking for factors. The factor pairs start repeating. You can generalize that all the factors for 20 have been found.

The factor pairs for 20 are
1 and 20, 2 and 10, 4 and 5

2. Find the factors for 32.

$32 \div 1 =$ 32 $1 \times 32 = 32$ and 32 $\times 1 = 32$
$32 \div 2 =$ 16 2 × 16 = 32 and 16 × 2 = 32
$32 \div 3 =$ 10 R2 3 is not a factor.
$32 \div 4 =$ 8 4 × 8 = 32 and 8 × 4 = 32

5, 6, and 7 are not factors.

$32 \div 8 =$ 4 8 × 4 = 32 and 4 × 8 = 32

What is the first factor pair that repeats? 8 and 4

The factor pairs for 32 are 1 and 32, 2 and 16, 4 and 8.

On the Back!

3. Find all the factors for 72.

1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

Name _____

Math and Science Activity 7-3

Schools of Fish

Did You Know? A group of fish is called a school. Fish travel in schools as a form of protection from predators. The idea of "safety in numbers" is especially beneficial for young and small fish. Small fish form dense schools and move in unison. This discourages predators.

1. One school of fish contains a total of 12 fish. How many different arrays can be formed with 12 fish? Draw each array.

6 different arrays; Check students' drawings.

2. List the factor pairs for a school of fish containing 48 fish. Explain how you know you have listed all of the factor pairs.

1 and 48, 2 and 24, 3 and 16, 4 and 12, 6 and 8; Sample answer: Start at 1 to find 1 and 48, try 2 to find 2 and 24, and continue until the factor pairs repeat. 6 groups of 8 is the same as 8 groups of 6, so 6 and 8 is the last factor pair.

3. **Extension** Scientists believe that a school of fish that can form an array with an equal (or close to equal) number of fish in the rows and columns will have a greater chance of survival. List all possible arrays for a school of fish containing 81 fish. List all possible arrays for a school of fish containing 92 fish. Which array provides the best chance of survival? Explain.

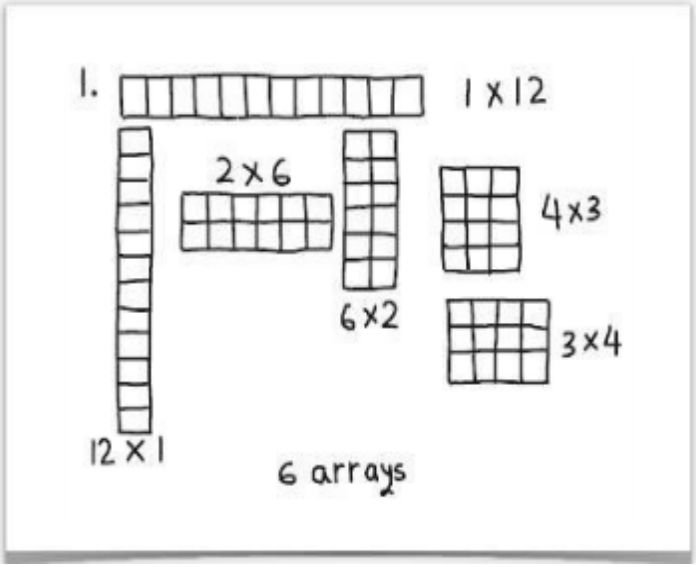
81: 1 × 81, 3 × 27, 9 × 9, 27 × 3, 81 × 1; 92: 1 × 92, 2 × 46, 4 × 23, 23 × 4, 46 × 2, 92 × 1; Sample answer: The 9 × 9 array; the rows and columns are equal.

Math and Science Activity 7-3

Math and Science Activity STEM

This activity revisits the science theme Plant and Animal Structures, introduced on page 365 in the Student's Edition.

Sample Student Work



Whole Class BEFORE

1. Pose the Solve-and-Share Problem

You may wish to provide 2-color square counters (or Teaching Tool 16).

MP.2 Reason Abstractly Listen and look for students who discuss how they found all the arrays for each number.

2. Build Understanding

When you make an array can you use only one color or more than one color? [Only 1 color] *What is the relationship between the number of factors of a number and the number of arrays that can be made with that number of tiles?* [They are equal.]

Small Group DURING

3. Ask Guiding Questions As Needed

How many arrays can Max make with his 3 blue tiles? [2 arrays] *How do you know there are only 2 arrays with 3 tiles?* [You can make an array with 1 row or with 3 rows. You cannot make an array with 2 rows. That is all the whole numbers less than 3 besides zero, which we do not use.]

Whole Class AFTER

4. Share and Discuss Solutions

Solve Start with students' solutions. If needed, project and discuss Frank's correct work.

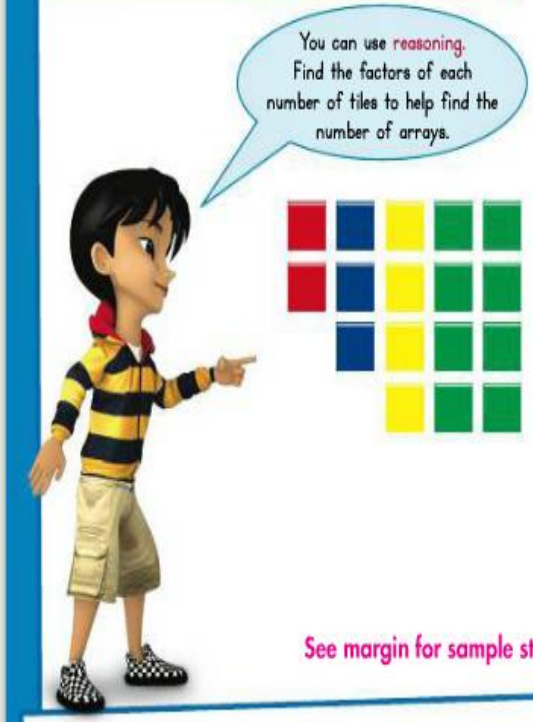
5. Transition to the Visual Learning Bridge

The number of factors is equal to the number of possible arrays.

Name _____ Solve

Solve & Share

Max has 2 red tiles, 3 blue tiles, 4 yellow tiles, and 8 green tiles. How many different arrays can Max make with each color of tile? Explain how you know you found all the arrays. ***Solve this problem any way you choose.***



See margin for sample student work.

Look Back! MP.8 Generalize What do you notice about the factors of each number of color tiles and the number of arrays?

Sample answer: The number of factors is equal to the number of arrays. 2 and 3 each have 2 factors and 2 possible arrays. 4 has 3 factors and 3 possible arrays. 8 has 4 factors and 4 possible arrays.

Lesson 7-4
Prime and Composite Numbers

I can ...
use factors to determine if a whole number is prime or composite.

Content Standard 4.OA.B.4
Mathematical Practices MP.2, MP.3, MP.7, MP.8

Solve

Frank's correct work.

5. Transition to the Visual Learning Bridge

The number of factors is equal to the number of possible arrays.

Finding all possible arrays for a number helps determine whether the number is prime or composite.

6. Extension for Early Finishers

How many arrays do 6 and 7 have? [6 has 4 arrays and 7 has 2 arrays.]

of each number of color tiles and the number of arrays?

Sample answer: The number of factors is equal to the number of arrays. 2 and 3 each have 2 factors and 2 possible arrays. 4 has 3 factors and 3 possible arrays. 8 has 4 factors and 4 possible arrays.

Analyze Student Work

Frank's Work

2 has 2 arrays 2 x 1 and 1 x 2.

3 has 2 arrays 3 x 1 and 1 x 3.

4 has 3 arrays 4 x 1, 1 x 4, and 2 x 2.

8 has 4 arrays 1 x 8, 8 x 1, 2 x 4, and 4 x 2.

I checked all the numbers until there was a repeat.

Frank used reasoning to find all the arrays for each number and to explain how he knew he found them all.

Aidan's Work

Red has 2 rows with 1 in each row.

Blue has 3 rows with 1 in each row.

yellow has 4 rows with 1 in each row.

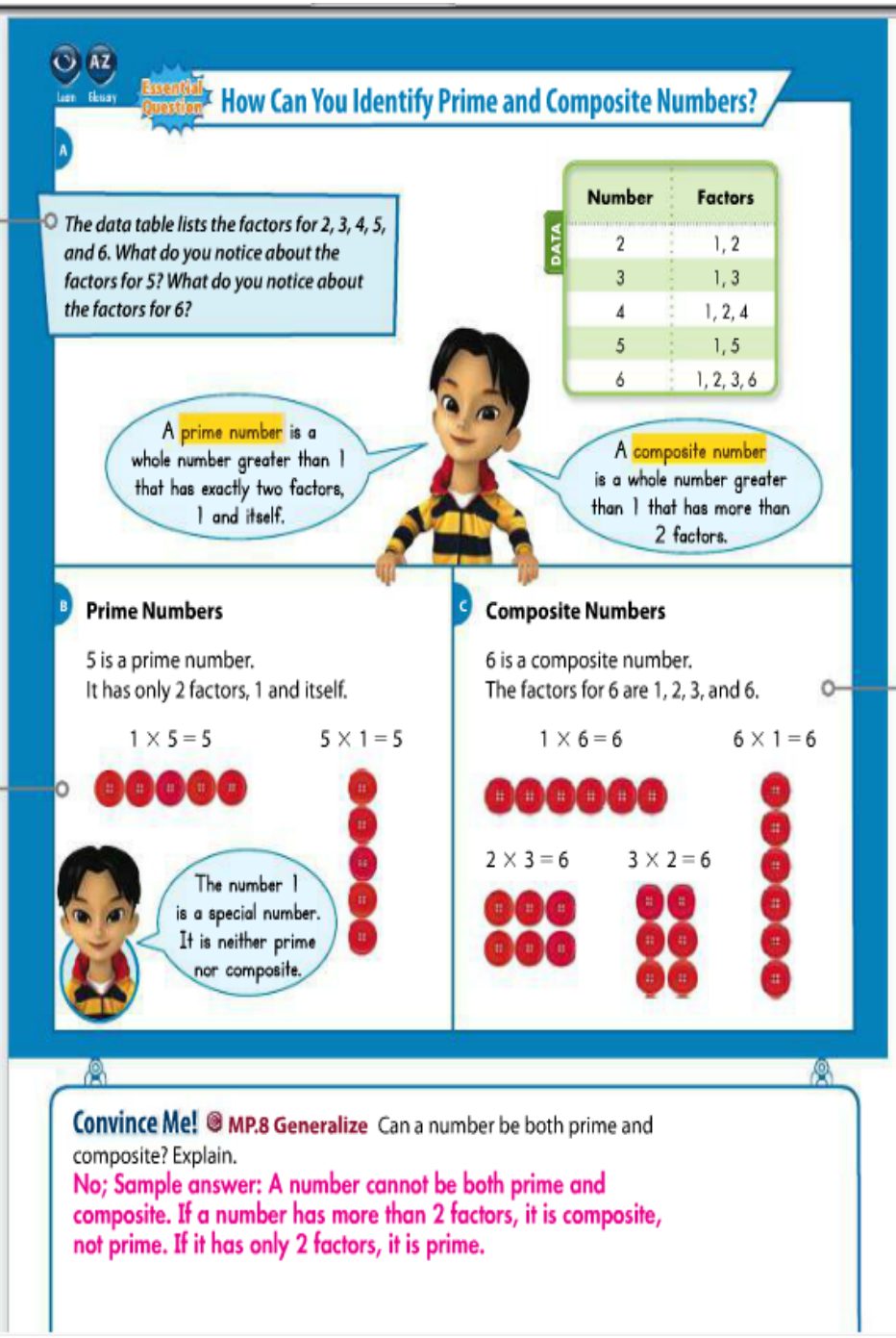
Green has 4 rows with 2 in each row.

Aidan just described the arrays shown on the Solve & Share page. He did not find all the possible arrays.

MP.1 Make Sense and Persevere

What is a prime number? [A prime number is a whole number greater than 1 that has exactly 2 factors.] *Can 1 be a prime number? Explain.* [No; It is a whole number but it is not greater than 1 and it does not have two factors.] *What is a composite number?* [A composite number is a whole number greater than 1 that has more than 2 factors.]

How many arrays are possible with 5 counters? [2 arrays] *How is the number of arrays related to the number of factors?* [The number of arrays equals the number of factors.] *Why is 5 a prime number?* [Sample answer: It is a whole number greater than 1, and it has exactly



Visual Learning Bridge

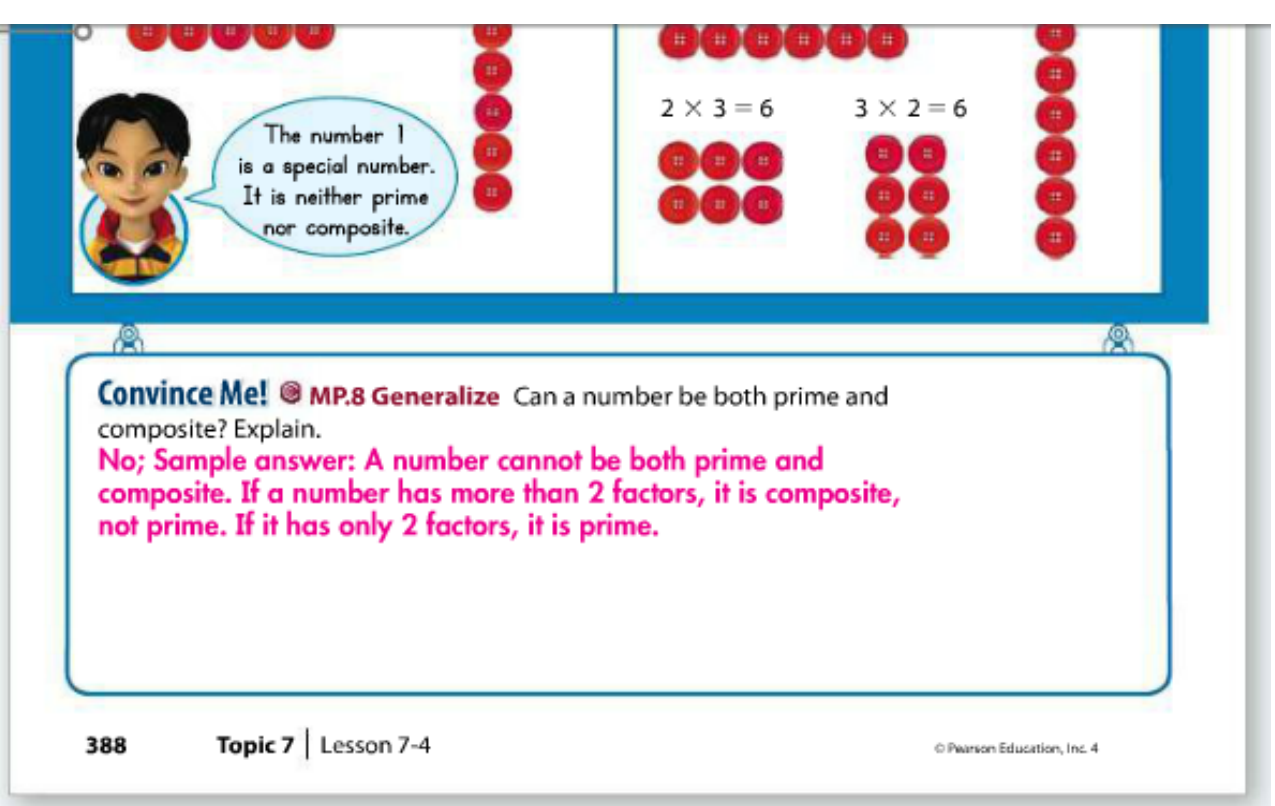
MP.6 Be Precise

How many arrays are possible with 6 counters? [4 arrays] *Why is 6 a composite number?* [Sample answer: It is a whole number greater than 1, and it has more than 2 factors.]

How many arrays are possible with 5 counters? [2 arrays] *How is the number of arrays related to the number of factors?* [The number of arrays equals the number of factors.] *Why is 5 a prime number?* [Sample answer: It is a whole number greater than 1, and it has exactly 2 factors.]

Prevent Misconceptions

Some students may want to try to classify 0 and 1 as prime or composite. Emphasize that the definitions for both terms specify "whole numbers greater than one."



Convince Me! MP.8 Generalize Students use the definitions of prime and composite numbers to generalize that all whole numbers greater than 1 are classified as one or the other.

Coherence Students identify prime and composite numbers. This connects to work in Lessons 4-1 through 4-3 where students found all the factor pairs of a number. It also involves understanding multiplication, division, and the relationship between these two operations.

Revisit the Essential Question. Arrays can show whether a whole number greater than 1 has exactly 2 factors and thus is prime or more than 2 factors and thus is composite.

Name _____

Guided Practice

Do You Understand?

1. What is the only even prime number?
   2
2. **MP.3 Construct Arguments** Write an odd number that is not prime. What makes it a composite number?
   Sample answer: 15 is an odd number because it is not divisible by 2 and a composite number because it has more than two factors.
3. Roger has 47 cars. Can he group the cars in more than 2 ways?
   No; Sample answer: 47 is a prime number. It has only 2 factors, 1 and 47.

Do You Know How?

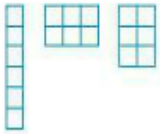
For **4–9**, tell whether each number is prime or composite.

4. 32 — **Composite**
5. 51 — **Composite**
6. 17 — **Prime**
7. 21 — **Composite**
8. 95 — **Composite**
9. 29 — **Prime**

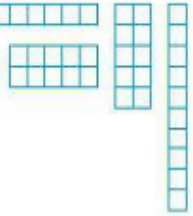
Independent Practice

Leveled Practice For **10–19**, tell whether each number is prime or composite.

10. 6 — **Composite**



11. 10 — **Composite**



12. 12 — **Composite**
13. 97 — **Prime**
14. 90 — **Composite**
15. 31 — **Prime**
16. 11 — **Prime**
17. 44 — **Composite**
18. 3 — **Prime**
19. 59 — **Prime**

Math Practices and Problem Solving

For **20–21**, use the graph at the right.

20. Which type of flower received a prime number of votes?
    Daffodils
21. How many votes are represented by the picture graph?
    21 votes

22. **MP.3 Critique Reasoning** Maria says every number in the nineties is composite. Jackie says one number in the nineties is prime. Who is correct? Explain your answer.
    Jackie; Sample answer: The only number that is prime in the nineties is 97.
23. **MP.3 Critique Reasoning** Greta says the product of two prime numbers must also be prime. Joan disagreed. Who is correct?
    Joan; Sample answer: Every number other than 1 has at least 2 factors. If a number is also the product of 2 different prime numbers, then it has at least 4 factors, so it cannot be prime.

24. Janelle has 342 pennies, 62 nickels, and 12 dimes. If Janelle exchanges her coins for dollars, how many dollars will she have? How many cents will remain?
    7 dollars; 72 cents
25. **Higher Order Thinking** Why is 1 neither a prime number nor a composite number?
    Sample answer: 1 is neither a prime number nor a composite number because it has only 1 factor, itself, 1.

Common Core Assessment

26. Select all the numbers that are prime.
    - [x] 17
    - [x] 37
    - [ ] 52
    - [ ] 63
    - [x] 89
27. Select all the numbers that are composite.
    - [x] 39
    - [x] 45
    - [x] 54
    - [ ] 61
    - [x] 92

12. 12 Composite
13. 97 Prime
14. 90 Composite
15. 31 Prime

16. 11 Prime
17. 44 Composite
18. 3 Prime
19. 59 Prime

*For another example, see Set D on page 402.

17
37
52
63
89

39
45
54
61
92

Error Intervention: Item 3

If students have difficulty deciding how many factors 47 has,

then have them try to divide 47 by 2, 3, 4, 5, 6, and 7. Point out that $7 \times 7 = 49$ and is greater than 47. Thus, any number greater than 7 would have to pair with a number less than 7 which has already been checked.

Item 13 If necessary, help students decide which numbers they need to check as possible factors for 97. They need to check numbers through 10, since $9 \times 9 < 97$ but $10 \times 10 > 97$.

Reteaching Assign Reteaching Set D on p. 402.

Items 20–21 Coherence Remind students to read the key to help interpret the graph, as they learned in an earlier grade. Since each symbol equals 2 votes, a half symbol equals 1 vote.

Item 23 MP.3 Critique Reasoning Ask questions associated with this mathematical practice. *Are there any mistakes in Greta's thinking? Explain.* [Yes; If a number is the product of two prime numbers, then the number has 4 factors: 1, the number itself, and the two prime numbers. So, the number cannot be prime.]

Item 24 *How many dollars are in 342 pennies?* [$3] *How many cents are left?* [42 cents] *How many dollars are in 62 nickels?* [$3] *How many cents are left? Explain.* [10 cents; 2 nickels are left which is equal to 10 cents] *How many dollars are in 12 dimes?* [$1] *How many cents are left? Explain.* [20 cents; 2 dimes are left which is equal to 20 cents] *Do the leftover cents make another dollar? Explain.* [No; 42 + 10 + 20 = 72 cents]

Intervention Activity

Prime and Composite Numbers

- Display "55" and "59" on the board.
- *How do you determine if a number is prime or composite?* [Find the factors of the number.]
- *What is the greatest number you need to check to see if it is a factor for each of these numbers? Explain.* [8; Sample answer: $8 \times 8 = 64$ and 64 is greater than both 55 and 59.] *Is there a number less than 8 you could use? Explain.* [No; The whole number before 8 is 7, $7 \times 7 = 49$, and 49 is less than both 55 and 59.]
- Have students work in pairs to find all the factors of each number.
- *Which number is prime?* [59] *Which number is composite?* [55]

55 has 1, 5, 11, and 55 as factors, so it is composite.
I tried 2, 3, 4, 5, 6, 7, and 8 so I know 59 does not have any other factors. 59 is prime. You get a remainder when you divide 59 by 2, 3, 4, 5, 6, 7, or 8. So its only factors are 1 and 59.

Reteach

Name _____

Reteach to Build Understanding
7-4

Vocabulary

1. A **prime number** is a whole number greater than 1 that has exactly 2 factors, 1 and itself.

Circle the prime numbers:

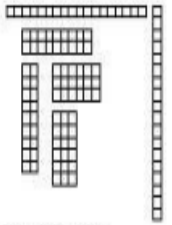
2	3	4	5	6	7	8
1 × 2	1 × 3	1 × 4 2 × 2	1 × 5	1 × 6 2 × 3	1 × 7	1 × 8 2 × 4

2. A **composite number** is a whole number greater than 1 that has more than 2 factors.

Circle the composite numbers.

2	3	4	5	6	7	8
1 × 2	1 × 3	1 × 4 2 × 2	1 × 5	1 × 6 2 × 3	1 × 7	1 × 8 2 × 4

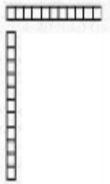
3. Is 18 a prime or a composite number? Use the arrays at the right to name the factors of 18.



Factors of 18: 1, 2, 3, 6, 9, 18

18 is a composite number because it has more than two factors.

4. Is 11 a prime or composite number? Use the arrays at the right to name the factors of 11.



Factors of 11: 1, 11

11 is a prime number because it only has two factors, 1 and the number itself.

On the Back!

5. Determine if each number is prime or composite. Draw arrays for each.

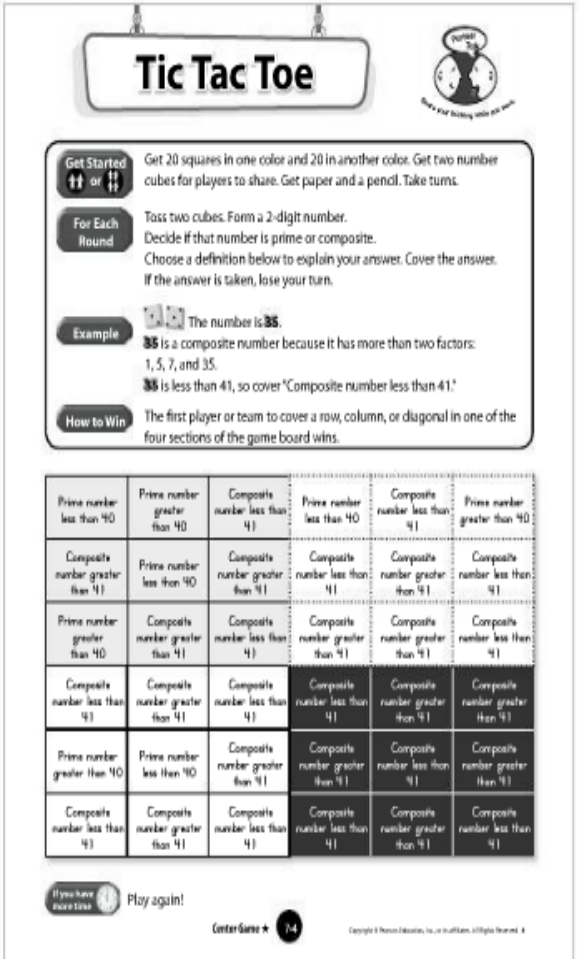
15 Composite 19 Prime Check students' arrays.

7-4

Center Games

Students work in pairs or small groups to find prime and composite numbers. Have students record the numbers, how they classified them, and their explanations as they play the game.

★ On-Level



Tic Tac Toe

Get Started Get 20 squares in one color and 20 in another color. Get two number cubes for players to share. Get paper and a pencil. Take turns.

For Each Round
Toss two cubes. Form a 2-digit number.
Decide if that number is prime or composite.
Choose a definition below to explain your answer. Cover the answer.
If the answer is taken, lose your turn.

Example The number is **35**.
35 is a composite number because it has more than two factors: 1, 5, 7, and 35.
35 is less than 41, so cover "Composite number less than 41."

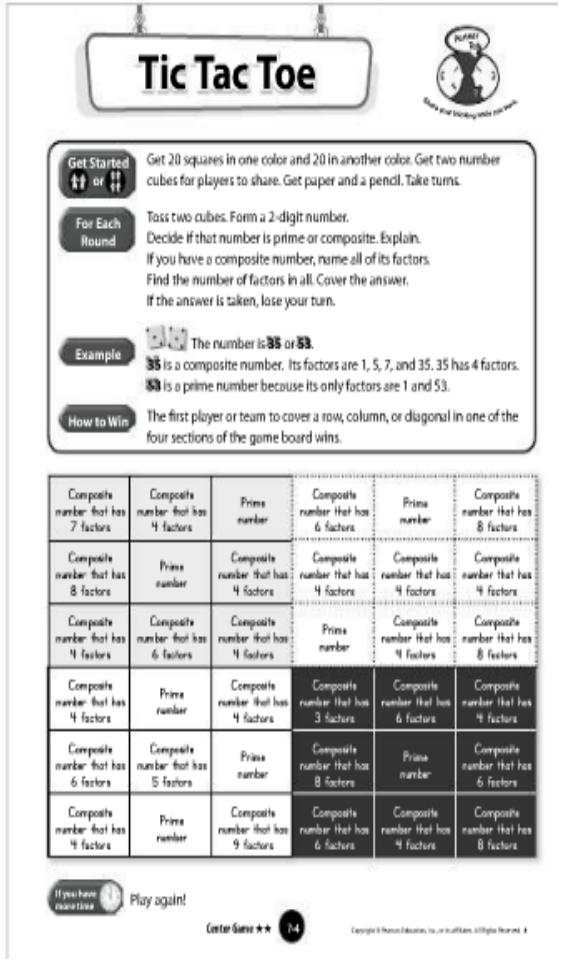
How to Win The first player or team to cover a row, column, or diagonal in one of the four sections of the game board wins.

Prime number less than 40	Prime number greater than 40	Composite number less than 41	Prime number less than 40	Composite number less than 41	Prime number greater than 40
Composite number greater than 41	Prime number less than 40	Composite number greater than 41	Composite number less than 41	Composite number greater than 41	Composite number less than 41
Prime number greater than 40	Composite number greater than 41	Composite number less than 41	Composite number greater than 41	Composite number greater than 41	Composite number less than 41
Composite number less than 41	Composite number greater than 41	Composite number less than 41	Composite number less than 41	Composite number greater than 41	Composite number greater than 41
Prime number greater than 40	Prime number less than 40	Composite number greater than 41	Composite number greater than 41	Composite number less than 41	Composite number greater than 41
Composite number less than 41	Composite number greater than 41	Composite number less than 41	Composite number less than 41	Composite number greater than 41	Composite number less than 41

If you have more time Play again!

Center Game ★ 74

★★ Advanced



Tic Tac Toe

Get Started Get 20 squares in one color and 20 in another color. Get two number cubes for players to share. Get paper and a pencil. Take turns.

For Each Round
Toss two cubes. Form a 2-digit number.
Decide if that number is prime or composite. Explain.
If you have a composite number, name all of its factors.
Find the number of factors in all. Cover the answer.
If the answer is taken, lose your turn.

Example The number is **35** or **53**.
35 is a composite number. Its factors are 1, 5, 7, and 35. 35 has 4 factors.
53 is a prime number because its only factors are 1 and 53.

How to Win The first player or team to cover a row, column, or diagonal in one of the four sections of the game board wins.

Composite number that has 7 factors	Composite number that has 4 factors	Prime number	Composite number that has 6 factors	Prime number	Composite number that has 8 factors
Composite number that has 8 factors	Prime number	Composite number that has 4 factors	Composite number that has 4 factors	Composite number that has 4 factors	Composite number that has 4 factors
Composite number that has 4 factors	Composite number that has 6 factors	Composite number that has 4 factors	Prime number	Composite number that has 4 factors	Composite number that has 8 factors
Composite number that has 4 factors	Prime number	Composite number that has 4 factors	Composite number that has 3 factors	Composite number that has 6 factors	Composite number that has 4 factors
Composite number that has 6 factors	Composite number that has 5 factors	Prime number	Composite number that has 8 factors	Prime number	Composite number that has 6 factors
Composite number that has 4 factors	Prime number	Composite number that has 9 factors	Composite number that has 6 factors	Composite number that has 4 factors	Composite number that has 8 factors

If you have more time Play again!

Center Game ★★ 74

Whole Class BEFORE

1. Pose the Solve-and-Share Problem

MP.2 Reason Quantitatively Look for students who realize that 9 is a factor of all the balls that are in play.

2. Build Understanding

How many players are there? [9] *How can you find the number of balls?* [Multiply each ball by the number of players]

Small Group DURING

3. Ask Guiding Questions As Needed

Suppose each player has 1 ball. What multiplication sentence could you write to show how many balls are in play? [1 × 9 = 9] *Suppose each player has 2 balls. What multiplication sentence could you write to show how many balls are in play?* [2 × 9 = 18] *What are the next three multiples of 9 that follow 18?* [27, 36, 45]

Whole Class AFTER

4. Share and Discuss Solutions

Solve Start with students' solutions. If needed, project and discuss Mary's correct work.

5. Transition to the Visual Learning Bridge

A multiple is a product of any two whole numbers. A whole number is a multiple of each of its factors.

Name _____ Solve

Lesson 7-5
Multiples

Solve & Share

There are 9 players on the golf practice range. If each player practices with the same number of golf balls, how many balls might be in play at the same time? ***Solve this problem any way you choose.***

I can ...
use multiplication to find multiples of a number.

Content Standard 4.OA.B.4
Mathematical Practices MP.2, MP.3, MP.4, MP.7, MP.8

Golf Balls in Play	
Balls per Player	Balls in Play
1	1 × 9 = 9 balls in play
2	2 × 9 = 18 balls in play
3	3 × 9 = 27 balls in play
4	
5	

See margin for sample student work.

Look Back! MP.8 Generalize Can you show all of the answers for the problem? Explain.
No; Sample answer: The total number of balls increases as you multiply by the number of balls with which each player practices. 1 × 9, 2 × 9, 3 × 9, and so on.

5. Transition to the Visual Learning Bridge

A multiple is a product of any two whole numbers. A whole number is a multiple of each of its factors.

6. Extension for Early Finishers

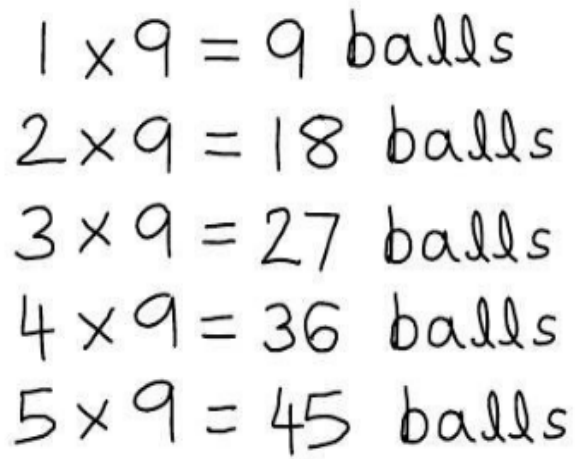
Write $1 \times 7 = 7$. Make an organized list of multiplication equations to help find the next five multiples of 7 that follow 7.
[$2 \times 7 = 14$, $3 \times 7 = 21$, $4 \times 7 = 28$, $5 \times 7 = 35$, $6 \times 7 = 42$]

Look Back! MP.8 Generalize Can you show all of the answers for the problem? Explain.

No; Sample answer: The total number of balls increases as you multiply by the number of balls with which each player practices. 1×9, 2×9, 3×9, and so on.

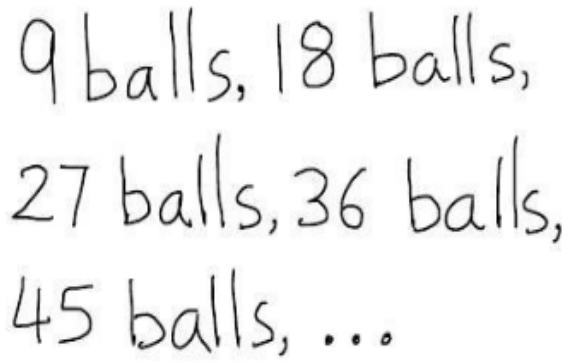
Analyze Student Work

Mary's Work



Mary uses reasoning and organizes her work in a list to find multiples of 9.

Sarah's Work



Sarah skip counts to find multiples of 9 and the number of possible balls in play.

MP.1 Make Sense and Persevere

How many minutes does it take for Car A to make one full turn? [One full turn takes 8 minutes.] *Describe how Car A travels during one full turn.* [Completely around the Ferris wheel from the bottom around to the top, and then around to the bottom again.]

In the number sentence $1 \times 8 = 8$, what does the 1 represent? [The number of full turns] *What does the product represent?* [That 8 is a multiple of 8]

What is the next multiple of 8? [16] *How many minutes does it take Car A to make two full turns?* [16 minutes]

Visual Learning Bridge

How Can You Find Multiples of a Number?

A

It takes 8 minutes for Car A to make one full turn on the Ferris wheel. If the Ferris wheel continues to turn at the same speed for the next hour, at what times during the hour will Car A return to the starting point?

A multiple is the product of a given factor and a whole number.

B Step 1

One full turn takes 8 minutes.

$1 \times 8 = 8$

8 is a multiple of 1 and 8 because $1 \times 8 = 8$.

Car A is back at the starting point after 8 minutes.

C Step 2

Two full turns take 16 minutes.

$2 \times 8 = 16$

Car A is back at the starting point after another 8 minutes.

2 and 8 are *factors* of 16. 16 is a *multiple* of 2 and 8.

D Step 3

Car A is at the starting point every 8 minutes after that:

$3 \times 8 = 24$
$4 \times 8 = 32$
$5 \times 8 = 40$
$6 \times 8 = 48$
$7 \times 8 = 56$

During the hour, Car A returns to the starting point after 8, 16, 24, 32, 40, 48, and 56 minutes.

Convince Me! MP.2 Reasoning What is the next multiple after 56? Explain why it is **NOT** used.

64; Sample answer: There are 60 minutes in 1 hour. The next multiple of 8 is greater than 60 minutes, which is greater than 1 hour.

MP.2 Reason Quantitatively

What can you do to find the remaining times that Car A will return to the starting point during the hour? [There are 60 minutes in one hour. Find the remaining multiples of 8 that are less than or equal to 60.]

Prevent Misconceptions

Some students may find multiples of 8 beyond 60. Remind them that the problem asks how many times the car will return to the starting point in one hour.

Convince Me! MP.2 Reasoning What is the next multiple after 56? Explain why it is **NOT** used.

64; Sample answer: There are 60 minutes in 1 hour. The next multiple of 8 is greater than 60 minutes, which is greater than 1 hour.

multiples of 8 beyond 60. Remind them that the problem asks how many times the car will return to the starting point in one hour.

Convince Me! MP.2 Reason Quantitatively Unlike the problem in the Solve & Share, there are a set number of multiples that are needed to solve the problem. The Ferris wheel will only return to its starting point a set number of times in an hour. Car A returns to the starting point 7 times during the hour.

Coherence Students list multiples of 8 by multiplying 8 by 1, 2, 3, and so on. This connects to work in previous lessons where students learned basic multiplication facts. Since 8 is a factor of its multiples, it also connects to the lessons about factors earlier in this topic.

Revisit the Essential Question. Multiples are the product of factors. For example, $2 \times 6 = 12$. 12 is a multiple of 2 and 6.

Name _____

Guided Practice

Do You Understand?

1. **MP.7 Look for Relationships** If the Ferris wheel in the example on the previous page turns at the same speed, will Car A return to the starting point at 75 minutes? Explain.
   No; Sample answer: 75 is not a multiple of 8.

2. Suppose the Ferris wheel speeds up so it makes one full turn every 6 minutes. When will Car A return to the starting point if the Ferris wheel continues to turn for one half hour?
   6, 12, 18, 24, and 30 minutes

Do You Know How?

For **3–6**, write five multiples of each number.

Sample answers given.

3. 2
   2, 4, 6, 8, 10
4. 9
   9, 18, 27, 36, 45
5. 3
   0, 3, 6, 9, 12
6. 10
   10, 20, 30, 40, 50

For **7–10**, tell whether the first number is a multiple of the second number.

7. 14, 2
   Yes
8. 3, 18
   No
9. 56, 9
   No
10. 42, 7
    Yes

Independent Practice

For **11–18**, write five multiples of each number.

Sample answers given.

11. 7
    7, 14, 21, 28, 35
12. 4
    4, 8, 12, 16, 20
13. 6
    6, 12, 18, 24, 30
14. 5
    5, 10, 15, 20, 25
15. 11
    11, 22, 33, 44, 55
16. 1
    1, 2, 3, 4, 5
17. 20
    20, 40, 60, 80, 100
18. 15
    15, 30, 45, 60, 75

For **19–26**, tell whether the first number is a multiple of the second number.

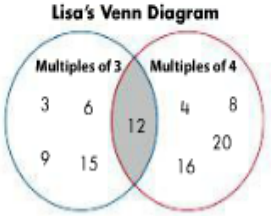
19. 44, 6
    No
20. 25, 5
    Yes
21. 30, 6
    Yes
22. 54, 9
    Yes
23. 28, 3
    No
24. 45, 5
    Yes
25. 64, 7
    No
26. 48, 8
    Yes

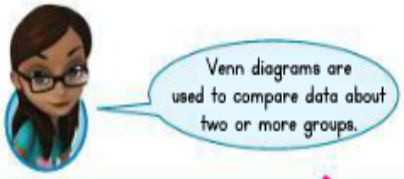
Math Practices and Problem Solving

27. Name three numbers that are a multiple of 2 and a multiple of 5.
    Sample answer: 10, 20, 30

28. **MP.3 Critique Reasoning** Lindsay says all numbers that are multiples of 4 have 2 as a factor. Is Lindsay correct? Explain.
    Yes; Sample answer: All multiples of 4 are even and all even numbers have 2 as a factor.

29. **MP.4 Model with Math** Lisa made a Venn diagram showing five multiples of 3 and five multiples of 4. What does the shaded section in her diagram show?
    Numbers that are multiples of both 3 and 4





30. Describe how 20,000 and 2,000 are related.
    Sample answer: 20,000 is 10 times as great as 2,000 because it is ten groups of 2,000; 2,000 × 10 = 20,000.

31. **Higher Order Thinking** Isabel wrote this mystery problem: The quotient is a multiple of 6. The dividend is a multiple of 9. The divisor is a factor of 12. Find one possible solution to Isabel's mystery problem.
    Sample answer: 54 ÷ 3 = 18

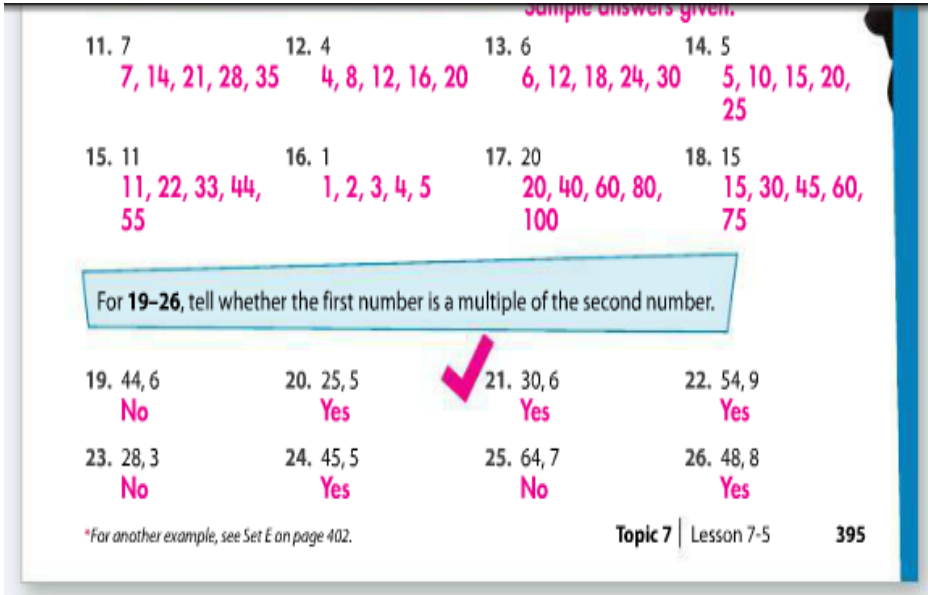
Common Core Assessment

32. Latifa and John played a game of multiples. Each player picks a number card and says a multiple of that number. Latifa picked a 9. Write all the multiples of 9 from the box.

Multiples of 9					
9	36	45			
9	17	29	36	45	51

33. A roller-coaster ride completes a full loop every 3 minutes. Seth listed multiples of 3 to determine when the ride would be back at its starting point. Write all the multiples of 3 from the box.

Multiples of 3					
9	12	33			
9	11	12	13	19	33



11. 7
7, 14, 21, 28, 35

12. 4
4, 8, 12, 16, 20

13. 6
6, 12, 18, 24, 30

14. 5
5, 10, 15, 20, 25

15. 11
11, 22, 33, 44, 55

16. 1
1, 2, 3, 4, 5

17. 20
20, 40, 60, 80, 100

18. 15
15, 30, 45, 60, 75

For **19–26**, tell whether the first number is a multiple of the second number.

19. 44, 6
No

20. 25, 5
Yes

21. 30, 6
Yes

22. 54, 9
Yes

23. 28, 3
No

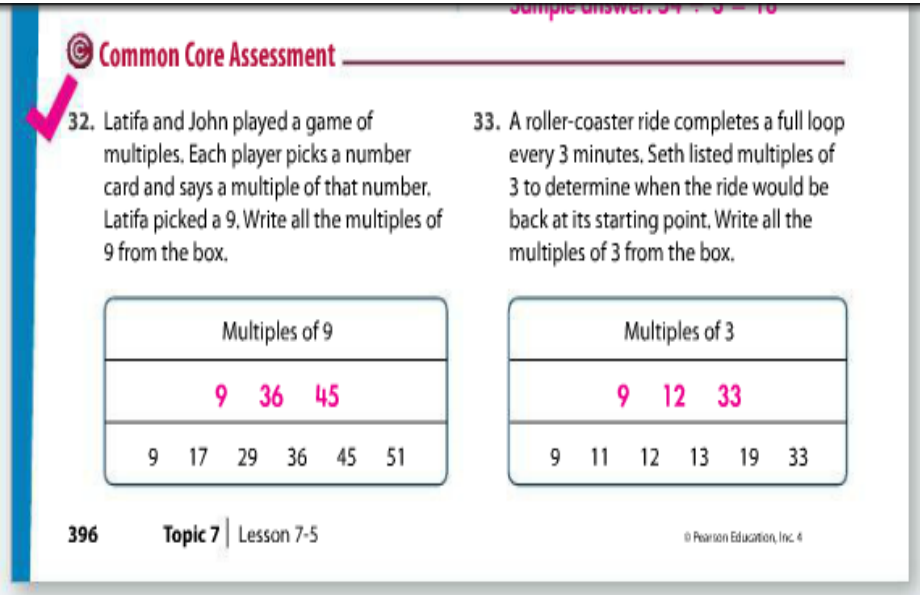
24. 45, 5
Yes

25. 64, 7
No

26. 48, 8
Yes

*For another example, see Set E on page 402.

Topic 7 | Lesson 7-5 395



Common Core Assessment

32. Latifa and John played a game of multiples. Each player picks a number card and says a multiple of that number. Latifa picked a 9. Write all the multiples of 9 from the box.

Multiples of 9
9 36 45
9 17 29 36 45 51

33. A roller-coaster ride completes a full loop every 3 minutes. Seth listed multiples of 3 to determine when the ride would be back at its starting point. Write all the multiples of 3 from the box.

Multiples of 3
9 12 33
9 11 12 13 19 33

396 Topic 7 | Lesson 7-5

Error Intervention: Item 1

If students have trouble determining if 75 is a multiple of 8,

then remind them that 8 must be a factor of 75 for 75 to be a multiple of 8. Encourage students to see if 75 can be divided by 8 without a remainder.

Items 7–10 Coherence Students can use what they know about quotients with remainders. *Is there a remainder when 14 is divided by 2?* [No] *So, 14 is a multiple of 2. Is there is a remainder when 56 is divided by 9?* [Yes] So, 56 is not a multiple of 9.

Reteaching Assign Reteaching Set E on p. 402.

Item 29 **MP.4 Model with Math** *How can I use math I know to help solve this problem?* [To find multiples that 3 and 4 have in common, make a sequential list of multiples of 3 and a sequential list of multiples of 4, then find numbers in common.]

Item 30 *Is 2,000 a multiple or a factor of 20,000? Explain.* [A factor; Sample answer: Because 20,000 is 10 times as great as 2,000. 2,000 can be multiplied by 10 to get 20,000. 2,000 therefore is a factor of 20,000.]

Item 31 **Higher Order Thinking** If students have difficulty knowing where to start, encourage them to make sequential lists of multiples of 6 and 9. Students should then list the factors of 12 and use their lists to find possible solutions.

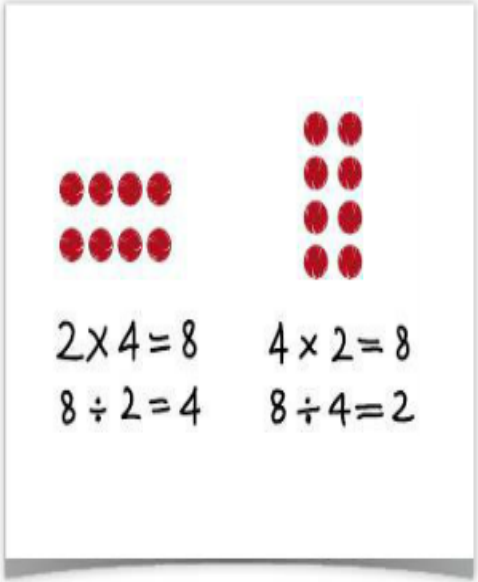
Intervention Activity I

Relating Factors and Multiples

Materials

2-color counters (or Teaching Tool 15)

- Have students use counters to create an array to represent 2 × 4 and write the related multiplication and division sentences.
- Have students turn their arrays to represent 4 × 2 and write the related multiplication and division sentences.
- Discuss how the multiplication sentences show that a product is a multiple of its factors.
- Discuss how the division sentences show that a divisor and a quotient are factors of the dividend.
- Help students to create other arrays and identify factors and multiples.



Reteach I

Name _____

Reteach to Build Understanding 7-5

Vocabulary

1. A **multiple** is the product of a given factor and a whole number. You can use a multiplication chart to help find some multiples for numbers.

   List some multiples of 5.

   Step 1: Find the column (or row) for 5.

   Step 2: All the numbers in that column (or row) are multiples of 5.

   Multiples of 5, in the chart, are:

   5, 10, 15, 20, 25, 30, 35, 40, 45

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

2. What are some multiples of 8? Use the multiplication chart above.

   Step 1: Find the column (or row) for 8.

   Step 2: All the numbers in that column (or row) are multiples of 8.

   In the chart, the multiples of 8 are:

   8, 16, 24, 32, 40, 48, 56, 64, 72

3. Is 48 a multiple of 6?

   Think: What number times 6 equals 48? 8

   48 is a multiple of 6 because 6 times 8 equals 48.

4. Is 39 a multiple of 9?

   Think: What number times 9 equals 39?

   39 is not a multiple of 9 because 39 is not the product of any number times 9

On the Back!

5. Write 5 multiples of each number. Sample answers given.

   6: 6, 12, 18, 24, 30

   12: 12, 24, 36, 48, 60

   15: 15, 30, 45, 60, 75

Problem-Solving Reading Mat

Have students read the Problem-Solving Reading Mat for Topic 7 and then complete Problem-Solving Reading Activity 7-5.

See the Problem-Solving Reading Activity Guide for other suggestions on how to use this mat.

FLUENCY PRACTICE ACTIVITY

Students practice fluently adding and subtracting whole numbers during an activity that reinforces mathematical practices.

Common Core Standards

Content Standard **4.NBT.B.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Mathematical Practices **MP.2, MP.6, MP.7**

Getting Started Ask students to work with a partner. Tell them to record their answers and shade the path on their own page. Go over the directions.

Both students should solve each problem and record their work. Tell students to take turns choosing which square to try next.

As Students Do the Activity Remind students that the path may go up, down, left, or right. There may be several options they must try before they find the square with the problem that follows the rule. Remind students to compare and discuss answers.

Encourage students to use mental-math strategies to help choose the squares that are on the path. Some students may find all of the answers first and then shade the path. Allow this strategy as it provides the same fluency practice.

Another Activity Have students work together to revise problems and write a new rule for shading a different path from Start to Finish.

Name _____

TOPIC 7 **Fluency Practice Activity**

Follow the Path

Shade a path from **START** to **FINISH**. Follow the sums and differences that are correct. You can only move up, down, right, or left.

I can ...
add and subtract multi-digit whole numbers.

Content Standard 4.NBT.B.4

Start				
573 + 417 990	685 − 559 137 126	808 + 123 921 931	609 − 541 48 68	501 + 469 170 970
491 − 188 303	347 + 607 954	948 − 558 410 390	505 + 125 620 630	987 − 696 311 291
764 + 346 1,000 1,110	994 − 405 589	874 + 721 1,595	894 − 455 449 439	369 + 290 669 659
668 − 485 253 183	762 + 901 2,663 1,663	941 − 725 216	640 + 89 729	537 − 271 806 266
119 + 679	977 − 239	987 + 111	812 − 99	335 + 25

As Students Do the Activity Remind students that the path may go up, down, left, or right. There may be several options they must try before they find the square with the problem that follows the rule. Remind students to compare and discuss answers.

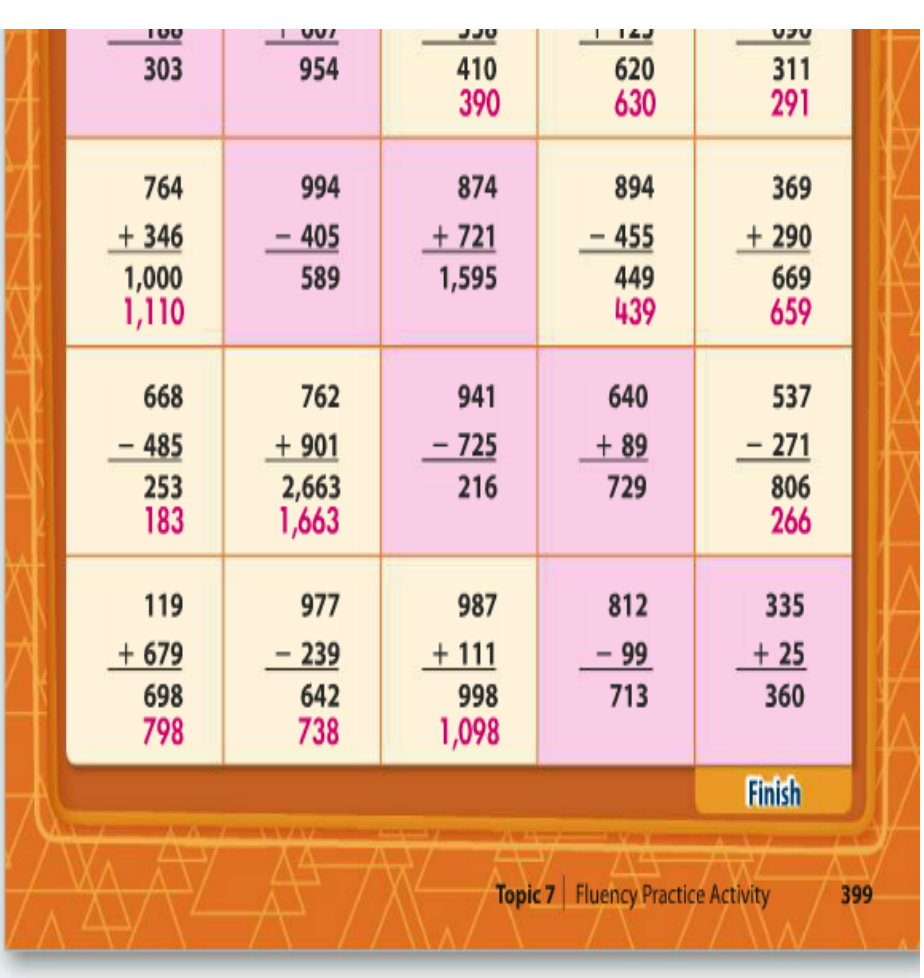
Encourage students to use mental-math strategies to help choose the squares that are on the path. Some students may find all of the answers first and then shade the path. Allow this strategy as it provides the same fluency practice.

Another Activity Have students work together to revise problems and write a new rule for shading a different path from Start to Finish.

Extra Challenge *Create your own Follow the Path activity. Start with a rule. Write problems to fit the rule so you can shade a path from Start to Finish. Then trade your activity with your partner and complete your partner's Follow the Path activity.*

Games **Online Game** The Game Center at PearsonRealize.com provides opportunities for fluency practice.

Practice Buddy **Steps to Fluency Success** To ensure all students achieve fluency, see pages 43K–43N for additional resources including practice/assessment masters and online practice/assessment on fluency subskills. You can also use the ExamView® CD-ROM to generate worksheets with multiple-choice or free-response items on fluency subskills.



303	954	410 390	620 630	311 291
764 + 346 1,000 1,110	994 − 405 589	874 + 721 1,595	894 − 455 449 439	369 + 290 669 659
668 − 485 253 183	762 + 901 2,663 1,663	941 − 725 216	640 + 89 729	537 − 271 806 266
119 + 679 698 798	977 − 239 642 738	987 + 111 998 1,098	812 − 99 713	335 + 25 360
				Finish

TOPIC 7

Vocabulary Review

Glossary

Word List

- array
- composite number
- factor
- factor pairs
- generalization
- multiple
- prime number
- whole number

Understand Vocabulary

1. Cross out the numbers that are **NOT** factors of 16.

1 2 ~~3~~ 4 8

2. Cross out the numbers that are **NOT** multiples of 3.

3 6 9 ~~18~~ ~~23~~

3. Cross out the numbers that are **NOT** whole numbers.

~~$\frac{1}{4}$~~ ~~$\frac{1}{2}$~~ 7 ~~$17\frac{1}{5}$~~ 6,219

4. Cross out the numbers that are **NOT** factor pairs for 24.

1 and 24 2 and 12 3 and ~~6~~ 4 and ~~8~~ 4 and 6

Label each example with a term from the Word List.

5. 13 — prime or whole number

6. 12 — composite or whole number

7. ●●● — array

8. When factor pairs begin repeating, I have found all the pairs for a number. — generalization

Use Vocabulary in Writing

9. Marisol says 23 is both a prime and a composite number because 2 and 3 are both prime. Use at least 3 terms from the Word List to explain the error in Marisol's reasoning.

Sample answer:
When determining if a number is *prime* or *composite*, you look at the value of the number, not the value of each place in the number. 23 is prime because it has only 2 *factors*.

Students review vocabulary words used in the topic.

Oral Language Before students complete the page, you might reinforce oral language through a class discussion involving one or more of the following activities.

- Have students define the terms in their own words.
- Have some students ask math questions that use the words, and have some students answer the questions using the math words.
- Play a "What's My Word?" guessing game in which you or a student thinks about one of the terms from the Word List and says a clue that others listen to before they guess the word.
- Play a "Right or Wrong?" game in which you or a student says a sentence that uses one of the words correctly or incorrectly. Then others say "right" or "wrong."

Writing in Math After students complete the page, you might further reinforce writing in math by doing one or more of the following activities.

- Give students a non-example of terms from the Word List, and have students correct the non-example making it a correct example.
- Have student pairs write each term on a note card and each term's definition on another note card. After all of the terms have been written and defined, allow students time to play a matching game where the cards are face down, and students take turns turning two cards face up trying to match each term with its definition.

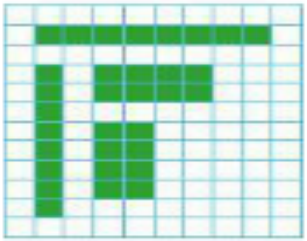
Name _____

TOPIC 7

Reteaching

Set A pages 369–374

Draw arrays to find all the factors of 8.



1 row of 8
8 rows of 1

2 rows of 4
4 rows of 2

The factors of 8 are 1, 2, 4, and 8.

Remember that 1 is a factor of every number.

Use grid paper to find the factors of each number.

1. 26 1, 2, 13, 26
2. 9 1, 3, 9
3. 37 1, 37
4. 24 1, 2, 3, 4, 6, 8, 12, 24
5. 19 1, 19

Set B pages 375–380

Find the factor pairs for 12.

1 and 12
2 and 6
3 and 4

The factors of 12 are 1, 2, 3, 4, 6, and 12.

Remember you can use counters or grids to make arrays and find the factors of a number.

Find the factors of each number.

1. 45 1, 3, 5, 9, 15, 45
2. 40 1, 2, 4, 5, 8, 10, 20, 40
3. 56 1, 2, 4, 7, 8, 14, 28, 56
4. 63 1, 3, 7, 9, 21, 63

Set C pages 381–386

Think about these questions to help you use **repeated reasoning**.

Thinking Habits

- Are any calculations repeated?
- Can I generalize from examples?
- What shortcuts do I notice?

Remember to look for repeating factors when dividing to find the factor pairs of a number.

A valet has 34 cars to park in a rectangular array.

1. What are the different ways the valet could park the cars?
   1 row of 34 cars; 34 rows of 1 car; 2 rows of 17 cars; 17 rows of 2 cars
2. How do you know when you can stop looking for factors of a number?
   When the factor pairs begin to repeat.

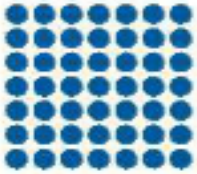
Set D pages 387–392

Is 49 prime or composite?

To determine if 49 is prime or composite, find whether 49 has factors other than 1 and 49.

49 is composite because it is divisible by 7.

$49 = 7 \times 7$



Remember you can use an array to determine if a number is prime or composite.

Tell whether each number is prime or composite.

1. 13 **Prime**
2. 25 **Composite**
3. 55 **Composite**
4. 2 **Prime**
5. 29 **Prime**
6. 23 **Prime**
7. 64 **Composite**
8. 99 **Composite**
9. 5 **Prime**
10. 21 **Composite**

Set E pages 393–398

Find five multiples of 7.

Use multiplication.

$7 \times 1 = 7$

$7 \times 2 = 14$

$7 \times 3 = 21$

$7 \times 4 = 28$

$7 \times 5 = 35$

Remember that to find multiples of a number, multiply the number by any whole number.

Find five multiples of each number.

Sample answers given.

1. 3 **3, 6, 9, 12, 15**
2. 6 **6, 12, 18, 24, 30**
3. 4 **4, 8, 12, 16, 20**
4. 9 **9, 18, 27, 36, 45**

Tell whether the first number is a multiple of the second number.

5. 22, 2 **Yes**
6. 29, 3 **No**
7. 25, 5 **Yes**
8. 40, 8 **Yes**

Name _____

TOPIC 7

Assessment

1. Courtney has 36 photos to arrange on a gallery wall.

Part A

How many arrays can Courtney make with the 36 photos? List all the possible arrays. **2 points**

9 arrays; 1 × 36, 2 × 18, 3 × 12, 4 × 9, 6 × 6, 9 × 4, 12 × 3, 18 × 2, 36 × 1

Part B

How many factors are there for 36? Write them. What do you notice about the number of factors of 36 and the number of arrays Courtney can make with the photos? **3 points**

9 factors; 1, 2, 3, 4, 6, 9, 12, 18, 36; Sample answer: The number of factors is the same as the number of arrays.

Part C

Write all the factor pairs for 36. Is 36 prime or composite? Explain. **3 points**

1 and 36, 2 and 18, 3 and 12, 4 and 9, 6 and 6; Composite; Sample answer: 36 is composite because it has more than one factor pair.

2. Peter wrote 4 sets of numbers. Which sets show only multiples of 6? **1 point**

2a. 6, 12, 18, 24	● Yes	○ No
2b. 6, 16, 26, 36	○ Yes	● No
2c. 1, 2, 3, 6	○ Yes	● No
2d. 6, 60, 66, 600	● Yes	○ No

3. Which statement is true? **1 point**

- (A) The only factors of 3 are 3 and 1.
- (B) The only factors of 4 are 4 and 1.
- (C) The only factors of 6 are 6 and 1.
- (D) The only factors of 8 are 8 and 1.

4. Choose the correct word from the box to complete each statement. **1 point**

Prime Composite

19 is a prime number.

12 is a composite number.

33 is a composite number.

17 is a prime number.

5. The dividend is a multiple of 4. The divisor is a factor of 12. The quotient is a factor of 18. Choose numbers from the box to find one possible solution. **1 point**

Sample answer given.

36 ÷ 4 = 9

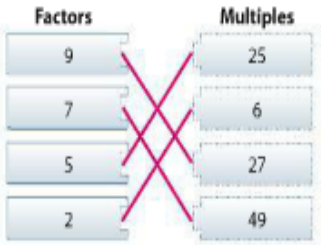
Dividend Divisor Quotient

2 3 4 6 8 9 12 36

6. Write 3 multiples and 3 factors for 24. **2 points**

Sample answer: Multiples: 24, 48, 72; Factors: 1, 2, 3

7. Corky wrote a list of factors and a list of multiples. Draw lines to match the factors with the multiples. **1 point**



8. Select all the true statements. **1 point**

- [x] A composite number has at least 3 factors.
- [ ] All prime numbers are odd.
- [ ] 99 is a prime number.
- [x] 2 is the smallest prime number.
- [x] All even numbers greater than 2 are composite.
- [x] All prime numbers have 2 factors.

9. Martika says factors and multiples are related. Use the equation 6 × 7 = 42 to describe the relationship between factors and multiples. **1 point**

Sample answer: 6 and 7 are factors of 42. 42 is a multiple of 6 and 7.

10. Which lists all the factors of 25? **1 point**

- (A) 1, 25
- (B) 1, 5, 25
- (C) 1, 10, 25
- (D) 1, 25, 50

11. Carter lives on a street where all the house numbers are multiples of 6. Name two possible house numbers between 601 and 650. Explain. **2 points**

606 and 612; Sample answer: Multiples of 6 can be divided by 6 without a remainder. 606 ÷ 6 = 101 and 612 ÷ 6 = 102

12. Write each number in the correct answer space to show factors of 27 and 35. **1 point**

Factors of 27	Factors of 35
3 9 27	5 7 35

3 5 7 9 27 35

13. Javier says all odd numbers greater than 2 and less than 20 are prime. Find an odd number greater than 2 and less than 20 that is **NOT** prime. Explain why the number is not prime. **2 points**

9; Sample answer: 9 is not prime because it has more than 2 factors: 1, 3, and 9. 9 is composite.

Topic Assessment Masters

Name _____ Topic 7 Assessment

1. Margie has 64 rectangular stepping stones to arrange in an array in her backyard.

Part A

How many arrays can Margie make with the 64 stepping stones? List all the possible arrays. **2 points**

7 arrays; 1 × 64, 2 × 32, 4 × 16, 8 × 8, 16 × 4, 32 × 2, 64 × 1

Part B

How many factors are there for 64? Write them. What do you notice about the number of factors of 64 and the number of arrays Margie can make with the stepping stones? **3 points**

7 factors; 1, 2, 4, 8, 16, 32, 64; Sample answer: The number of factors is the same as the number of arrays.

Part C

Write all the factor pairs for 64. Is 64 prime or composite? Explain. **3 points**

1 and 64, 2 and 32, 4 and 16, 8 and 8; Composite; Sample answer: 64 is composite because it has more than one factor pair.

2. Paige wrote 4 sets of numbers. Which sets show only multiples of 8? **1 point**

2a. 8, 16, 24, 32	● Yes	○ No
2b. 16, 32, 40, 64	● Yes	○ No
2c. 1, 2, 4, 8	○ Yes	● No
2d. 8, 88, 888, 8888	● Yes	○ No

3. Which statement is true? **1 point**

- Ⓐ The only factors of 4 are 4 and 1.
- ● The only factors of 7 are 7 and 1.
- Ⓒ The only factors of 16 are 16 and 1.
- Ⓓ The only factors of 32 are 32 and 1.

4. Choose the correct word from the box to complete each statement. **1 point**

Prime Composite

39 is a **prime** number.

51 is a **composite** number.

62 is a **composite** number.

91 is a **composite** number.

5. The dividend is a multiple of 2. The divisor is a factor of 16. The quotient is a factor of 24. Choose numbers from the box to find one possible solution. **1 point**

Sample answer given.

12 (Dividend) ÷ **4** (Divisor) = **3** (Quotient)

2 3 4 6 8 9 12 32

Assessment 1 of 2

6. Write 3 multiples and 3 factors for 12. **2 points**

Sample answer: Multiples: 12, 24, 36 Factors: 1, 2, 3

7. Krista wrote a list of factors and a list of multiples. Draw lines to match the factors with the multiples. **1 point**

Factors	Multiples
3	14
5	22
7	12
11	50

8. Select all the true statements. **1 point**

- ■ All prime numbers have 2 factors.
- ■ A composite number has at least 3 factors.
- ☐ All composite numbers are odd.
- ■ 99 is a composite number.
- ☐ 3 is the least prime number.
- ■ All even numbers greater than 2 are composite.

9. Kurt says factors and multiples are related. Use the equation $8 \times 5 = 40$ to describe the relationship between factors and multiples. **1 point**

Sample answer: 8 and 5 are factors of 40. 40 is a multiple of 8 and 5.

10. Which lists all the factors of 36? **1 point**

- Ⓐ 1, 36
- Ⓑ 1, 2, 4, 6, 36
- ● 1, 2, 3, 4, 6, 9, 12, 18, 36
- Ⓓ 36, 72, 108, 144, 180, 216

11. Pete's Pastries sells cupcakes in packs of 4. A caterer needs between 401 and 415 cupcakes for dessert. Name two possible numbers between 401 and 415 that are multiples of 4. Explain. **2 points**

404 and 408; Sample answer: Multiples of 4 can be divided by 4 without a remainder. 404 ÷ 4 = 101 and 408 ÷ 4 = 102

12. Write each number in the correct answer space to show factors of 18 and 49. **1 point**

Factors of 18	Factors of 49
2 3 6 18	7 49

2 3 6 7 18 49

13. Jared says all even numbers less than 20 are composite. Find an even number less than 20 that is **NOT** composite. Explain why the number is not composite. **2 points**

2; Sample answer: 2 is not composite because it only has 2 factors: 1 and 2.

Assessment 2 of 2

Name _____

TOPIC 7

Performance Assessment

Arranging Cars to Sell

Ms. Ortiz owns a car dealership. The dealership has the inventory of cars listed in the **Ortiz Car Dealership** table.

1. Ms. Ortiz wants to arrange all of the trucks in the front lot. She would like to have same number of trucks in each row.

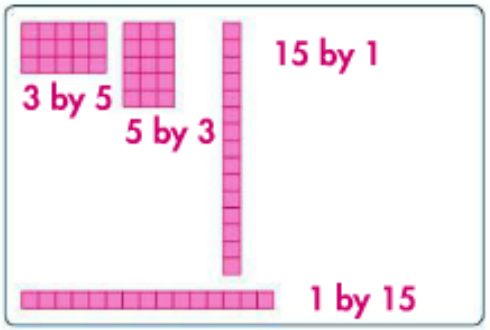
Part A

How many different ways can the trucks be arranged in the front lot if the same number of trucks are parked in each row? 1 point

4 ways

Part B

What are all the ways the trucks can be arranged? Draw and label the different arrays. 2 points



Part C

Ms. Ortiz would like the arrangement to have more than 2 rows of trucks but less than 6 rows. What are the ways the trucks can be arranged? Explain. 2 points

5 rows of 3 trucks and 3 rows of 5 trucks; Sample answer: 1 row of 15 trucks and 15 rows of 1 truck do not meet Mrs. Ortiz's requirements.

DATA

Ortiz Car Dealership

Type of Vehicle	Number Dealership Has
Compact	40
Sedan	36
SUV	23
Truck	15

2. As Ms. Ortiz sells sedans, those sedans remaining are parked in different arrangements.

Complete the **Arranging Sedans** table to find the number of ways Ms. Ortiz can arrange each number of sedans in the front lot so there are at least 2 rows and more than one sedan in each row. 1 point

Arranging Sedans

Sedans Sold	Number Left	Number of Arrangements	Arrangements
1	35	2	5 rows of 7; 7 rows of 5
2	34	2	2 rows of 17; 17 rows of 2
3	33	2	3 rows of 11; 11 rows of 3
4	32	4	2 rows of 16; 16 rows of 2; 4 rows of 8; 8 rows of 4
5	31	0	None
6	30	6	2 rows of 15; 15 rows of 2; 3 rows of 10; 10 rows of 3; 5 rows of 6; 6 rows of 5
7	29	0	None

Topic Performance Assessment Masters

Name _____

Topic 7
Performance Assessment

Marching Band

The sections of the marching band are shown in the **Marching Band** table.

Marching Band

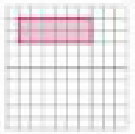
Section	Number of Members
Brass	24
Percussion	12
Woodwind	19

1. The band director needs to decide how to arrange the members of the marching band into rows with the same number of members in each row. Each row should have between 5 and 8 members.

Part A

What are all the ways the band director could arrange the members of the percussion section? Show the different ways on the grid. **2 points**

2 rows with 6 members in each row



Part B

What are all the ways the band director could arrange the members of the brass section? Explain how you know you have all possible arrangements. **2 points**

3 rows with 8 members and 4 rows with 6 members; Sample answer: First I found all the factor pairs for 24: 1 and 24, 2 and 12, 3 and 8, 4 and 6; When factor pairs start repeating, I found all the factors for a number. 1 row of 24 members and 2 rows of 12 members both have too many members in each row. 24 rows of 1 member, 12 rows of 2 members, 8 rows of 3 members, and 6 rows of 4 members do not have enough members in each row. That leaves 3 rows of 8 members and 4 rows of 6 members.

Part C

What are all the ways the band director could arrange the members of the woodwind section? Explain. **2 points**

The director cannot arrange the woodwind section into equal rows that have between 5 and 8 members in each row; 19 is a prime number, its only factors are 1 and 19. 1 row of 19 members has too many members in each row, and 19 rows of 1 member has too few members per row.

2. One of the members of the woodwind section cannot march this Friday night.

Part A

Complete the **Number of Members by Section** table to show that one member is missing from the woodwind section. **1 point**

Number of Members by Section

Section	Members
Brass	24
Percussion	12
Woodwind	18
Total	54

Part B

For this Friday, what are all the ways the band director could arrange the members of the woodwind section? Remember that each row can only have between 5 and 8 members. **1 point**

3 rows of 6 members

Part C

The band director wants to try a new formation. The new formation must have at least 3 members but not more than 9 members in each row. List all the possible ways the band director could arrange the total members. **1 point**

6 rows of 9 members, 9 rows of 6 members, 18 rows of 3 members