UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

YEAR OF TOLERANCE

TEACHER EDITION

2018 - 2019

McGraw-Hill Education

Mathematics

General Stream

United Arab Emirates Edition

6

2019
عام التسامح

2018

فخر الوطن

Mc Graw Hill Education

مجموعات فخر الوطن وعام زايد

Teacher Edition

McGraw-Hill Education

Mathematics

General Stream

United Arab Emirates Edition

مجموعات فخر الوطن وعام زايد

Mc Graw Hill Education

Geometry

Essential Question

HOW can you use different measurements to solve real-life problems?

Chapter 8

Area

A composite figure can be decomposed into triangles and other shapes. In this chapter, you will find the area of triangles, quadrilaterals, and composite figures.

Chapter 11

Volume and Surface Area

Prisms and pyramids are examples of three-dimensional figures. In this chapter, you will find the volume and surface area of three-dimensional figures in the context of solving real-world and mathematical problems.

Chapter 9
Area

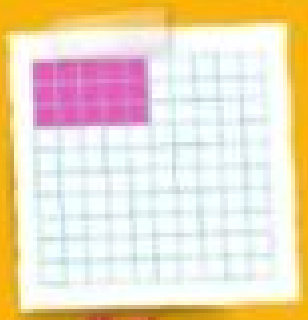
Essential Question

HOW does measurement help you solve problems in everyday life?

Mathematical Practices
1, 2, 3, 4, 5, 6, 7, 8

Math in the Real World

Gardens A garden designer plants dahlias in a 5 meter by 3 meter plot. What area of the garden do the dahlias cover? In the diagram below, shade the area covered by dahlias.



15 m²

Area = _____

FOLDABLES
Study Organizer

1. Cut out the Foldable from the end of the book.
2. Place your Foldable at the end of the chapter.
3. Use the Foldable throughout this chapter to help you learn about area.

Are You Ready?

Try the Quick Check below.

Quick Review

Review

Example 1

Find the area of the rectangle.

$A = \ell w$ Area of a rectangle

$A = 9 \cdot 6$ Replace ℓ with 9 and w with 6.

$A = 54$ Multiply.

The area of the rectangle is 54 square meters.

Example 2

Find $\frac{1}{2} \times 16$.

$\frac{1}{2} \times 16 = \frac{1}{2} \times \frac{16}{1}$ Write 16 as $\frac{16}{1}$.

$= \frac{1 \times 8}{2 \times 1}$ Divide the numerator and the denominator by 2.

$= \frac{8}{1}$ or 8 Simplify.

Quick Check

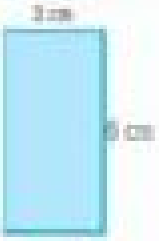
Area **Find the area of each rectangle.**

1. 

32 cm²

2. 

90 cm²

3. 

18 cm²

4. The playing area of a board game is a rectangle with a length of 35 centimeters and a width of 50 centimeters. What is the area of the board game? 1750 cm²

Fractions **Multiply. Write in simplest form.**

5. $\frac{1}{2} \times 28 =$ 14

6. $\frac{1}{3} \times 27 =$ 9

7. $\frac{1}{7} \times 84 =$ 12

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7

Inquiry Lab
Area of Parallelograms

HOW does finding the area of a parallelogram relate to finding the area of a rectangle?

Mathematical Practices 1, 2, 3, 5

Hala wants to make a banner in the shape of a parallelogram. Her parallelogram has a base of 2 meters and a height of 3 meters. What is the area of her parallelogram?

Hands-On Activity 1

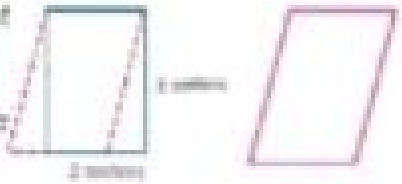
Another type of quadrilateral is a *parallelogram*. A parallelogram has opposite sides parallel and congruent.

Parallelograms	Not Parallelograms
	

Make a parallelogram to represent Hala's banner.

Step 1 Start with a rectangle. Trace the rectangle shown at the right.

Step 2 Cut a triangle from one side of the rectangle you traced and move it to the other side to form a parallelogram. Tape the parallelogram to the right.



The rectangle was rearranged to form the parallelogram. Nothing was removed or added, so the parallelogram has the same area as the rectangle.

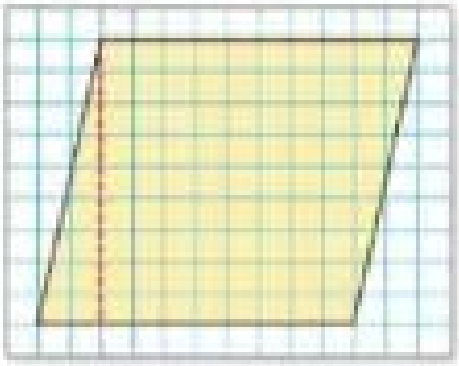
Step 3 Multiply the base and height of the parallelogram to find the area. The base of the parallelogram is 2 meters and the height is 3 meters.

2 meters × 3 meters = 6 square meters

Hands-On Activity 2

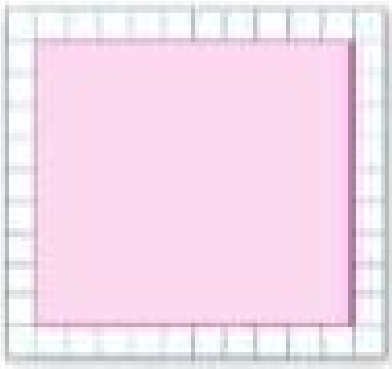
Find the area of the parallelogram below.

Step 1 Trace the parallelogram on grid paper and cut it out.



Step 2 Fold and cut along the dotted line.

Step 3 Move the triangle to the right to make a rectangle. Tape the rectangle in the space provided.



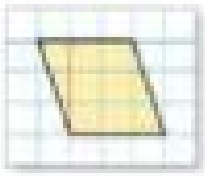
Step 4 Count the number of square units in the rectangle.

The area is 90 square units.

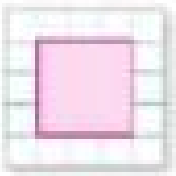
Hands-On Activity 3

Find the area of the parallelogram below.

Step 1 Trace the parallelogram and cut it out.



Step 2 Fold and cut along the dotted line. Then move the triangle to the right to make a rectangle. Tape it in the space provided.



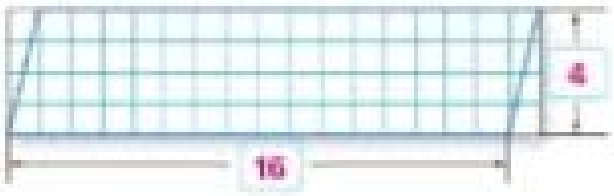
Step 3 Count the number of square units in the rectangle.

The area is 9 square units.

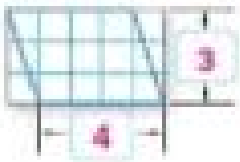
Investigate

Use Math Tools Work with a partner. Find the area of each parallelogram.

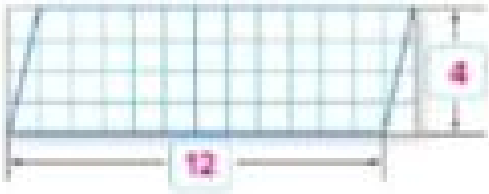
1. $A =$ 64 square units



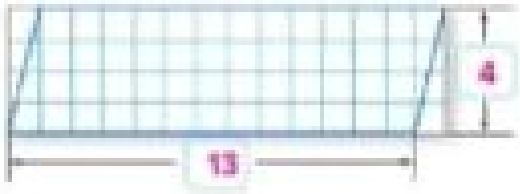
2. $A =$ 12 square units



3. $A =$ 48 square units

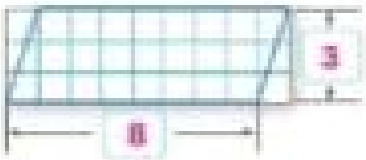


4. $A =$ 52 square units

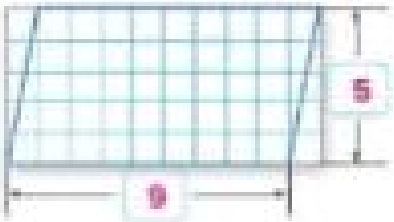


5. $A =$ 8 square units

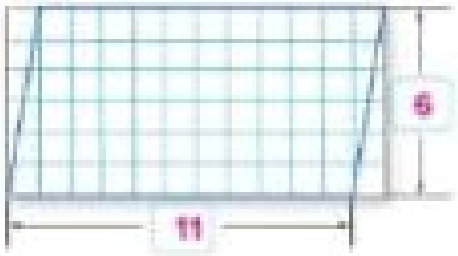
6. $A =$ 24 square units



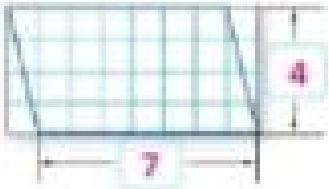
7. $A =$ 45 square units



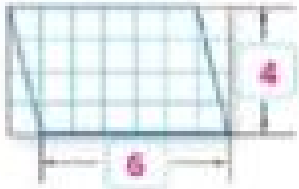
8. $A =$ 66 square units



9. $A =$ 28 square units



10. $A =$ 24 square units



Analyze and Reflect

The table shows the dimensions of several rectangles and the corresponding dimensions of several parallelograms if each rectangle was rearranged to form a parallelogram. Work with a partner to complete the table. The first one is done for you.

	Rectangle	Length (ℓ)	Width (w)	Parallelogram	Base (b)	Height (h)	Area (units^2)
	Rectangle 1	6	2	Parallelogram 1	6	2	12
11.	Rectangle 2	12	4	Parallelogram 2	12	4	48
12.	Rectangle 3	7	3	Parallelogram 3	7	3	21
13.	Rectangle 4	5	4	Parallelogram 4	5	4	20
14.	Rectangle 5	10	6	Parallelogram 5	10	6	60
15.	Rectangle 6	6	4	Parallelogram 6	6	4	24
16.	Rectangle 7	15	9	Parallelogram 7	15	9	135
17.	Rectangle 8	9	3	Parallelogram 8	9	3	27

18. A rectangle was rearranged to form a parallelogram. How is the height of the parallelogram similar to and different from the width of the rectangle?
Sample answer: They have the same measure. The parallelogram's height may or may not be one of its side lengths. The rectangle's width is one of its side lengths.

19. **Reason Abstractly** If you were to draw three different parallelograms, each with a base of 6 units and a height of 4 units, how would the areas compare? The areas would be equal.

Create

20. **Reason Inductively** Write a rule that gives the area of a parallelogram.
Area = base × height

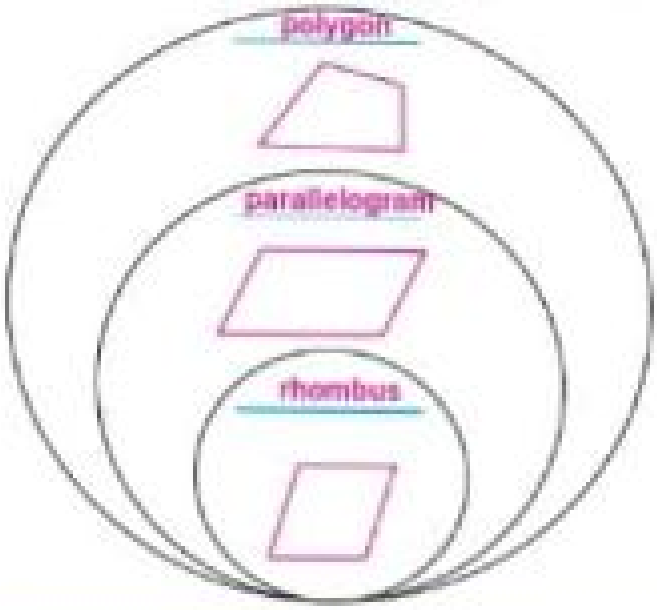
21. **HOW** does finding the area of a parallelogram relate to finding the area of a rectangle?
Sample answer: Finding the area of a parallelogram is similar to finding the area of a rectangle. Instead of multiplying the length and the width, a parallelogram's area is found by multiplying the base and the height.

Lesson 1

Area of Parallelograms

Vocabulary Start-Up

A **polygon** is a closed figure formed by 3 or more straight lines. A **parallelogram** is a quadrilateral with opposite sides parallel and opposite sides the same length. A **rhombus** is a parallelogram with four equal sides. Fill in the lines in the diagram with polygon, parallelogram, or rhombus and draw an example of each.



Essential Question

HOW does measurement help you solve problems in everyday life?

Vocabulary

polygon
parallelogram
rhombus
base
height
formula

Mathematical Practices

1, 2, 4, 7

Real-World Link

Stairs Expert skateboarders can slide down the railings of stairs safely. A parallelogram is used to build a staircase. How many sets of parallel lines are shown in the parallelogram to the right?

2 sets

Which Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Key Concept

Area of a Parallelogram

Words The area A of a parallelogram is the product of its base b and its height h.

Model

Symbols $A = bh$

The area of a parallelogram is related to the area of a rectangle as you discovered in the previous Inquiry Lab.

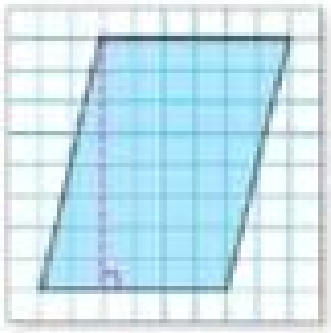
The **base** of a parallelogram can be any one of its sides.

The **height** is the perpendicular distance from the base to the opposite side.

Parallelograms include special quadrilaterals, such as rectangles, squares, and rhombi.

Examples

1. **Find the area of the parallelogram.**



The base is 6 units, and the height is 8 units.

$A = bh$ Area of parallelogram

$A = 6 \cdot 8$ Replace b with 6 and h with 8.

$A = 48$ Multiply.

The area is 48 square units or 48 units^2.

Area Measurement

An area might be measured in square inches, square centimeters, square miles, and so on.

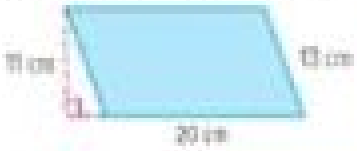
For example,

square inch = inch^2

square centimeter = cm^2

square mile = mi^2

2. Find the area of the parallelogram.



Estimate $A \approx 20 \cdot 10 = 200$ cm²

$A = bh$ Area of parallelogram

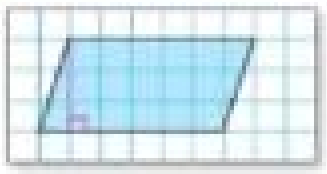
$A = 20 \cdot 11$ Replace b with 20 and h with 11

$A = 220$ Check for Reasonableness: $220 \approx 200$ ✓

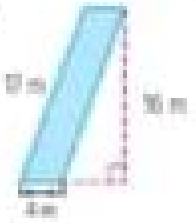
The area is 220 square centimeters or 220 cm².

Get It? Do these problems to find out.

a.



b.



a. 18 units² ____

b. 64 m² ____

Find Missing Dimensions

A **formula** is an equation that shows a relationship among certain quantities. To find missing dimensions, use the formula for the area of a parallelogram. Replace the variables with the known measurements. Then solve the equation for the remaining variable.

Example

3. Find the missing dimension of the parallelogram.

$A = bh$ Area of a parallelogram

$45 = 9 \cdot h$ Replace A with 45 and b with 9.

$\frac{45}{9} = \frac{9 \cdot h}{9}$ Divide each side by 9.

$5 = h$ Simplify

So, the height is 5 centimeters.

Checking Your Work

To check your work, replace b and h in the formula with 9 and 5.

$A = bh$

$A = 9 \cdot 5$

$A = 45$ ✓

Get It? Do these problems to find out.

c.

d.

c. 5 cm ____

d. 12 m ____

Height of Parallelograms

Example

4. **Samiha is painting a replica of the national flag of Trinidad and Tobago for a research project. Find the area of the black stripe.**

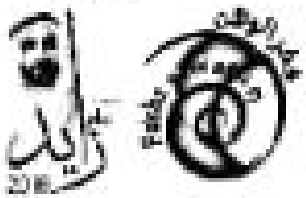
The black stripe is shaped like a parallelogram. So, use the formula $A = bh$.

$A = bh$ Area of parallelogram

$A = 6\frac{3}{4} \cdot 12$ Replace b with $6\frac{3}{4}$ and h with 12.

$A = 81$ $6\frac{3}{4} \cdot 12 = \frac{27}{4} \cdot 12 = 81$

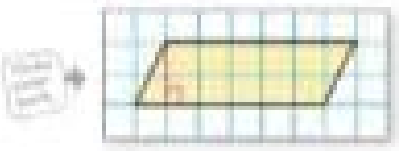
The area of the flag that is black is 81 square centimeters.



Practice

Find the area of each parallelogram. (Examples 1 and 2)

1. 12 units²



2. 50 m²

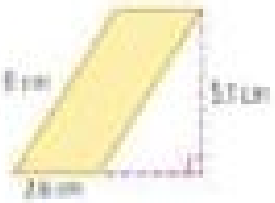
3. 77 m²

4. Find the height of a parallelogram if its base is 35 centimeters and its area is 700 square centimeters. (Example 1)

20 cm

5. The size of the parallelogram piece in a set of tangrams is shown at the right. Find the area of the piece. (Example 4)

13.26 cm²



6. **Building on the Essential Question** How are parallelograms related to triangles and rectangles?

Sample answer: Parallelograms can be decomposed into triangles, or composed into rectangles. You can find the area of parallelograms using the relationship to triangles and rectangles.

Rate Yourself!

How confident are you about the area of parallelograms? Shade the ring on the target.

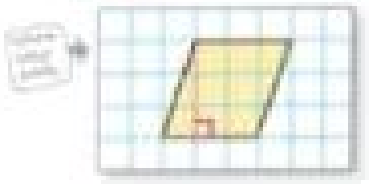
Check: Time to square your knowledge!

Name _____ My Homework _____

Independent Practice

Find the area of each parallelogram. (Examples 1 and 2)

1. 9 units²



2. base, 6 millimeters; height, 4 millimeters
24 mm²

3. 72 cm²

4. Find the base of a parallelogram with an area of 24 square meters and height 3 meters. (Example 3) 8 m

5. Find the area of the parking space shown to the right. (Example 4) 15.125 m²

6. STEM An architect designed three different parallelogram-shaped brick patios. Write the missing dimensions in the table.

Patio	Base (m)	Height (m)	Area (m²)
1	4.75	2.8	13.3
2	3.75	3.4	12.8
3	3.1	4.5	14

7. The base of a building is shaped like a parallelogram. The first floor has an area of 1,575 m². If the base of this parallelogram is 75 m, can its height be 21 m? Explain.
yes; In order for the area of the first floor to be 1,575 m² and the base 75 m, the height must be 1,575 m² ÷ 75 or 21 m.

8. **Identify Structure** Draw and label a parallelogram with a base twice as long as the height and an area less than 60 square centimeters. Find the area. Sample answer:

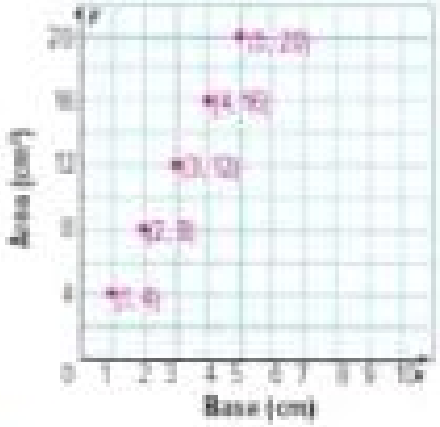
50 cm²

9. **Multiple Representations** Draw five parallelograms that each have a height of 4 centimeters and different base measurements on centimeter grid paper. **Sample answers are given.**

a. **Table** Make a table with a column for base, height, and area.

Base (cm)	Height (cm)	Area (cm²)
1	4	4
2	4	8
3	4	12
4	4	16
5	4	20

b. **Graph** Graph the ordered pairs (base, area).



c. **Words** Describe the graph. **It appears to form a line.**

H.O.T. Problems Higher Order Thinking

10. **Persevere with Problems** If $x = 5$ and $y < x$, which figure has the greater area? Explain your reasoning.

the rectangle; The area of the rectangle is 5y and the area of the parallelogram cannot be greater than 5y since y is less than 5 and the height of the parallelogram is not greater than y.

11. **Reason Inductively** Explain how the formula for the area of a parallelogram is related to the formula for the area of a rectangle.

Sample answer: Both parallelograms and rectangles have bases and heights. So, the formula $A = bh$ can be used for both figures. The height of a rectangle is the length of one of its sides while the height of a parallelogram is the length of the altitude.

12. **Reason Inductively** Give an example of a triangle and a parallelogram that have the same area. Describe the bases and heights of each figure. Then state the area.

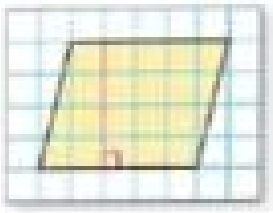
Sample answer: A triangle with a base of 8 units and a height of 3 units has the same area as a parallelogram with a base of 4 units and a height of 3 units. 12 units²

Name _____ My Homework _____

Extra Practice

Find the area of each parallelogram.

13. 20 units²



$A = bh$
$A = 5 \cdot 4$
$A = 20$

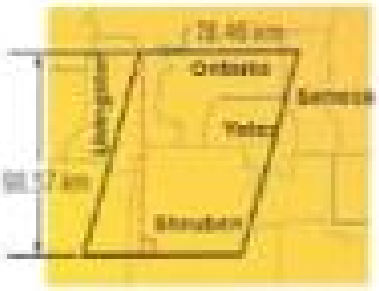
14. 48 m²

15. base, 12 centimeters; height, 15 centimeters
180 cm²

16. Find the height of a parallelogram with base 6.75 meters and an area of 218.7 square meters.
32.4 m

17. Find the area of a parallelogram with base 15 meters and height $21\frac{3}{5}$ meters.
325 m²

18. What is the area of the region shown on the map? 7,733.8022 km²



19. What is the height of the parallelogram-shaped pattern block shown below?
25 mm

Draw and label each figure. Then find the area. Sample answers: 20–21

20. a parallelogram with an equal base and height and an area greater than 64 square meters
81 m²

21. a parallelogram with a base four times the height and an area less than 200 square meters
196 m²

Identify Structure **Find the area of the shaded region in each figure.**

22. 227 m²

23. 84 cm²

Power Up! Test Practice

24. The table shows the dimensions of 4 parallelograms. Sort the parallelograms from least to greatest area.

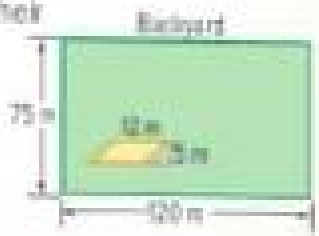
	Parallelogram	Area (in²)
Least	D	67.5
	B	84.5
	A	104.5
Greatest	C	116

Parallelogram	Base (cm)	Height (cm)
A	4.75	22
B	13	6.5
C	7.25	16
D	5	13.5

Which parallelogram has the greatest area? parallelogram C

25. A family has a flower garden in the shape of a parallelogram in their backyard. They planted grass in the rest of the yard.

Fill in the boxes to complete each statement.



a. The total area of the backyard is 9,000 square meters.

b. The area of the flower garden is 60 square meters.

c. The area of the backyard that is planted with grass is 8,940 square meters.

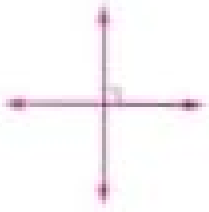
Spiral Review

Draw each pair of lines. 6.G.1 Sample answers: 26–28

26. intersecting

27. parallel

28. perpendicular

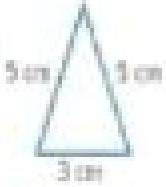


29. Rana has 22 songs in her music library. Noura has half as many. How many songs does Noura have in her music library?

11 songs

30. Name and describe the figure based on the lengths of its sides.

Isosceles triangle; Sample answer: It is a closed three-sided figure with two congruent sides.



Inquiry Lab
Area of Triangles

inquiry **HOW can you use the area of a parallelogram to find the area of a triangle?**

Mathematical Practices 1, 3, 7, 8

Omar is making a mosaic and is cutting rectangular tiles to make triangular tiles. He wants to find the area of the triangular tiles he is cutting.

What do you know? Omar is cutting rectangular tiles.

What do you need to know? How to use the area of a related rectangle to find the area of a triangle.

Hands-On Activity 1

Omar starts with a rectangular piece that is 10 centimeters by 15 centimeters, similar to the size of an index card.

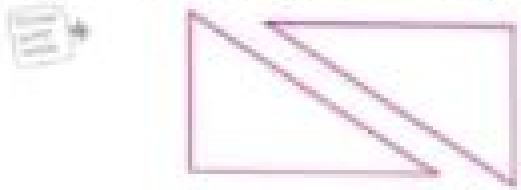
Step 1 Find the area of an index card.

A = length × width

A = 15 centimeters × 10 centimeters

A = 150 square centimeters

Step 2 Use an index card. Draw a diagonal line across your index card from one corner to another. Then cut across the line. Draw the resulting figures in the space below.



Step 3 Find the area of one of the remaining triangles. The triangle is exactly half the size of the related rectangle.

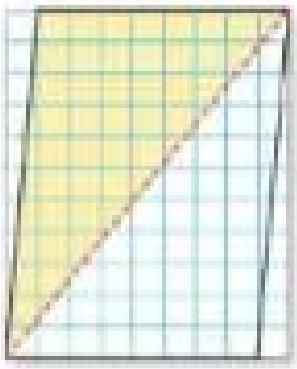
So, the area of the rectangle can be divided by 2 to find the area of one triangle.

The area is 150 ÷ 2, or 75 square centimeters.

Hands-On Activity 2

You can also find the area of a triangle from the area of a related parallelogram.

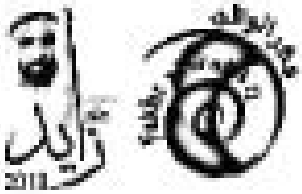
Step 1 Copy the parallelogram shown on grid paper.



Step 2 Draw a diagonal as shown by the dashed line. Cut out the parallelogram. The area of the parallelogram is 88 square units.

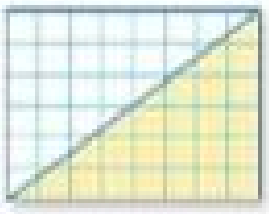
Step 3 Cut along the diagonal to form two triangles. Then find the area of one triangle. The triangle is half the size of the parallelogram. So, the area of the parallelogram can be divided by 2 to find the area of one triangle.

The area of one triangle is 88 ÷ 2 or 44 square units.



Investigate

Work with a partner to find the area of each shaded triangle.

1. 

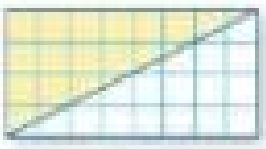
length: 8
width: 6

area: 8 × 6 = 48
area of triangle = 24 square units

2. 

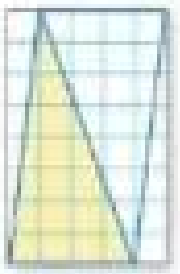
base: 5
height: 4

area: 5 × 4 = 20
area of triangle = 10 square units

3. 

length: 8
width: 4

area: 8 × 4 = 32
area of triangle = 16 square units

4. 

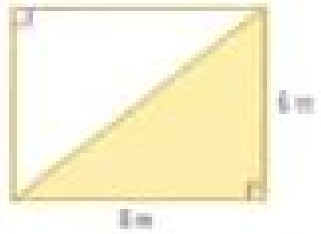
base: 4
height: 8

area: 4 × 8 = 32
area of triangle = 16 square units

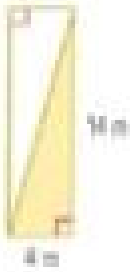
Investigate

Work with a partner to find the area of each shaded triangle.

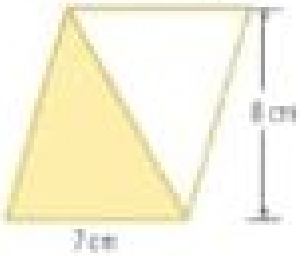
5. $A =$ 24 square meters



6. $A =$ 28 square meters



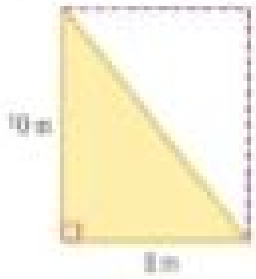
7. $A =$ 28 square centimeters



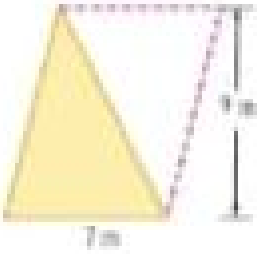
8. $A =$ 6 square meters

Identify Structure Draw dotted lines to show the parallelogram or rectangle that can be used to find the area of each triangle. Then find the area of each triangle.

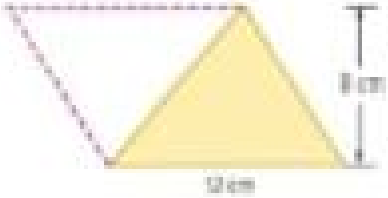
9. $A =$ 40 square meters



10. $A =$ 31.5 square meters



11. $A =$ 48 square centimeters



12. $A =$ 24 square meters

Analyze and Reflect

The table shows the dimensions of several parallelograms. Use the area of each parallelogram to find the missing information for each triangle. Work with a partner to complete the table. The first one is already done for you.

	Parallelogram	Base, b	Height, h	Area of Parallelogram (units squared)	Triangle created with diagonal	Base, b	Height, h	Area of Each Triangle (units squared)
	A	4	5	20	A	4	5	10
13.	B	4	6	24	B	4	6	12
14.	C	2	5	10	C	2	5	5
15.	D	3	4	12	D	3	4	6
16.	E	6	3	18	E	6	3	9
17.	F	8	5	40	F	8	5	20
18.	G	5	7	35	G	5	7	17.5
19.	H	9	7	63	H	9	7	31.5
20.	I	11	5	55	I	11	5	27.5

21. **Reason Inductively** How is the area of the parallelogram related to the area of a triangle with the same base and height?

Sample answer: The area of the parallelogram is the product of the base and the height. The area of each triangle formed is the product of the base and the height divided by 2.

Create

22. **Identify Repeated Reasoning** Write a formula that relates the area A of a triangle to the lengths of its base b and height h.

$A = \frac{bh}{2}$ or $A = \frac{1}{2}bh$

23. **EQUITY** HOW can you use the area of a parallelogram to find the area of a triangle?

The area of the triangle is one half the area of a parallelogram with the same base and height.

Lesson 2

Area of Triangles

Real-World Link

Biosphere The Biosphere 2 complex in Tucson, Arizona, researches Earth and its living systems. Sections of the building are interlocking triangles of the same size.

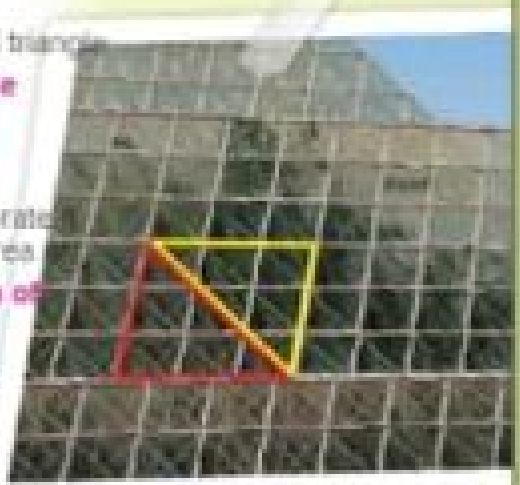
1. There are two triangles that are outlined in the photo. They have the same size and the same shape.

2. Draw the figure formed by the two triangles.

3. How many small triangles make up the outlined parallelogram? How many small triangles make up each outlined triangle? 18; 9

4. Describe the relationship between the area of one outlined triangle and the area of the outlined parallelogram. Sample answer: The area of the triangle is half the area of the parallelogram.

5. Draw another parallelogram like the one in the photo. Separate it into two triangles. Describe the relationship between the area of one triangle and the parallelogram. Sample answer: The area of the triangle is half the area of the parallelogram.



Essential Question

HOW does measurement help you solve problems in everyday life?

Vocabulary

congruent

Mathematical Practices

1, 2, 4, 6

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1 Persevere with Problems	5 Use Math Tools
2 Reason Abstractly	6 Attend to Precision
3 Construct an Argument	7 Make Use of Structure
4 Model with Mathematics	8 Use Repeated Reasoning

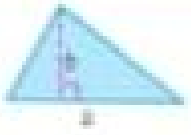
Key Concept

Area of a Triangle

Words The area A of a triangle is one half the product of the base b and its height h.

Symbols $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

Model



Work Zone

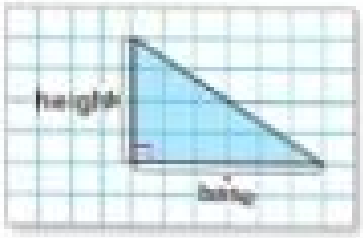
Congruent figures are figures that are the same shape and size.

A parallelogram can be formed by two congruent triangles. Since congruent triangles have the same area, the area of a triangle is one half the area of the parallelogram.

The base of a triangle can be any one of its sides. The height is the perpendicular distance from that base to the opposite vertex.

Examples

1. **Find the area of the triangle.**



By counting, you find that the measure of the base is 6 units and the height is 4 units.

$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(6)(4)$ Replace b with 6 and h with 4.

$A = \frac{1}{2}(24)$ Multiply.

$A = 12$ Multiply.

The area of the triangle is 12 square units.

Mental Math

You can use mental math to multiply $\frac{1}{2}(24)$. Think: Half of 24 is 12, and 12×1 is 12.

2. Find the area of the triangle.

$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(12.1)(6.4)$ Replace b with 12 and h with 6.4

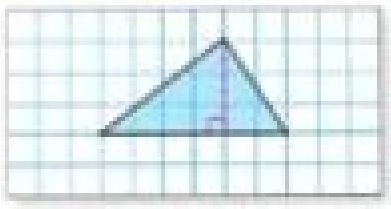
$A = \frac{1}{2}(77.44)$ Multiply

$A = 38.72$ Divide $\frac{1}{2}(77.44)$ or $77.44 \div 2$, or 38.72

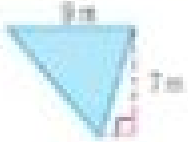
The area of the triangle is 38.72 square meters.

Got it? Do these problems to find out.

a.



b.



a. 9 units²

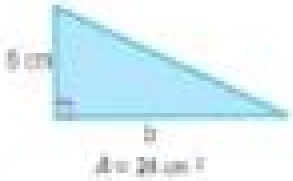
b. 31.5 m²

Find Missing Dimensions

Use the formula for the area of a triangle to find missing dimensions.

Example

3. Find the missing dimension of the triangle.



$A = \frac{bh}{2}$ Area of a triangle

$24 = \frac{b \cdot 6}{2}$ Replace A with 24 and h with 6.

$2 \cdot 24 = \frac{b \cdot 6}{2} \cdot 2$ Multiply each side by 2.

$48 = b \cdot 6$ Simplify.

$\frac{48}{6} = \frac{b \cdot 6}{6}$ Divide each side by 6.

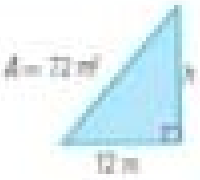
$8 = b$ Simplify.

So, the base is 8 centimeters.

Got it? Do these problems to find out.

c.

d.



c. 10 m

d. 12 m

Check for Reasonableness

To check your work, see if it is safe that multiplying and what to find that work.

Example

4. The front of a camping tent has the dimensions shown. How much material was used to make the front of the tent?

$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(5)(3)$ Replace b with 5 and h with 3.

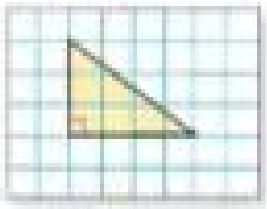
$A = \frac{1}{2}(15)$ or 7.5 Multiply

The front of the tent has an area of 7.5 square meters.

Guided Practice

Find the area of each triangle. (Examples 1 and 3)

1. 6 units²



2. 48 m²

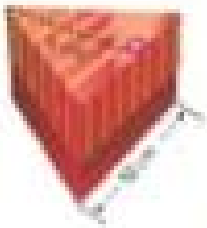
3. 97.75 m²

4. Faris designs uniquely-shaped ceramic floor tiles. What is the base of the tile shown? (Example 3)

20 cm.

5. Nabila made a triangular paper box as shown. What is the area of the top of the box? (Example 4)

45 cm²



6. **Building on the Essential Question** How is the formula for the area of a triangle related to the formula for the area of a parallelogram?

Sample answer: A parallelogram can be decomposed into two congruent triangles. So, the formula for the area of a triangle, $A = \frac{1}{2}bh$, is one-half the area of a parallelogram, $A = bh$.

Rate Yourself!

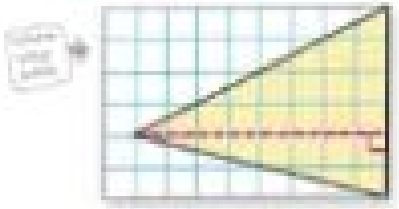
- ☐ I understand how to find the area of a triangle.
  - Great! You're ready to move on!
- ☐ I still have some questions about the area of a triangle.
  - Talk to your classmates or teacher.

Name _____ My Homework _____

Independent Practice

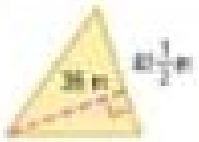
Find the area of each triangle. Examples 1 and 2

1. 24 units²



2. 198.4 cm²

3. 747 m²



Find the missing dimension of each triangle described. Example 3

4. height: 14 cm
area: 245 cm²
35 cm

5. base: 27 cm
area: 256.5 cm²
19 cm

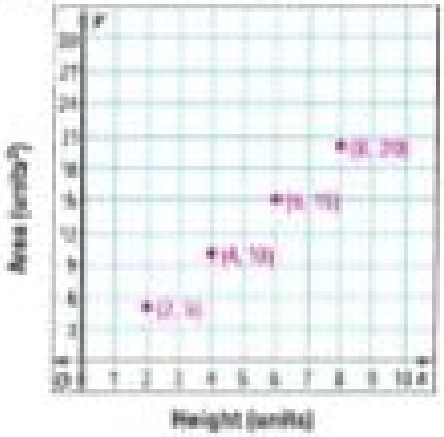
6. Amer is going to help his father shingle the roof of their house. What is the area of the triangular portion of one end of the roof? Example 4
14 m²

Multiple Representations The table shows the areas of a triangle where the base of the triangle stays the same but the height changes.

Area of Triangles		
Base (units)	Height (units), x	Area (units²), y
5	2	5
5	4	10
5	6	15
5	8	20
5	x	y

a. **Algebra** Write an algebraic expression that can be used to find the area of a triangle that has a base of 5 units and a height of x units. $\frac{5x}{2}$

b. **Graph** Graph the ordered pairs (height, area).



c. **Words** Describe the graph.
The points appear to form a line.

8. What is the area of the triangle on the flag of the Philippines in centimeters? Explain your reasoning. 3,375 cm²

Sample answer: The base of the triangle is 90 centimeters. The height is 75 centimeters. So, the area is $\frac{1}{2}(90)(75)$ or 3,375 square centimeters.

H.O.T. Problems Higher Order Thinking

9. **Find the Error** Faleh is finding the base of the triangle shown. Its area is 100 square meters. Find his mistake and correct it.

The formula is $\frac{1}{2}bh$, not bh.

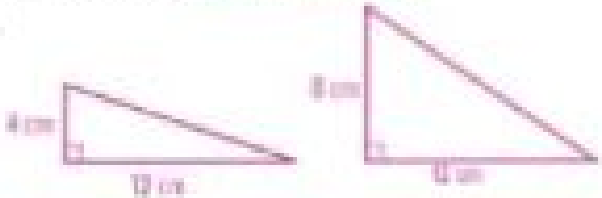
$100 = \frac{b \cdot 20}{2}$

$b = 10$ m

10. **Persevere with Problems** How can you use triangles to find the area of the hexagon shown? Draw a diagram to support your answer.

Sample answer: The hexagon is formed by six congruent triangles. If the hexagon is divided into six triangles, find the area of one triangle and multiply by 6.

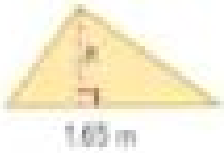
11. **Identify Repeated Reasoning** Draw a triangle and label its base and height. Draw another triangle that has the same base, but a height twice that of the first triangle. Find the area of each triangle. Then write a ratio that expresses the area of the first triangle to the area of the second triangle. Sample answer:



Area of first triangle is 24 cm². Area of second triangle is 48 cm².

1:2 or $\frac{1}{2}$

12. **Reason Inductively** The triangle shown has an area of 0.825 square meters. What is the height, in centimeters? 100 cm

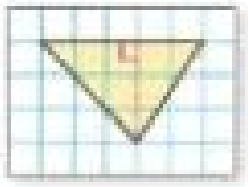


Name _____ My Homework _____

Extra Practice

Find the area of each triangle.

13. $\frac{7}{2}$ units²



$A = \frac{bh}{2}$

$A = \frac{7 \times 3}{2}$

$A = \frac{21}{2}$ or $10\frac{1}{2}$

14. 45 cm²

15. 87.5 m²

Find the missing dimension of each triangle described.

16. height: 7 cm, area: 21 cm²

6 cm

17. base: 11 m, area: 115.5 m²

21 m

18. base: 14.2 m, area: 63.9 m²

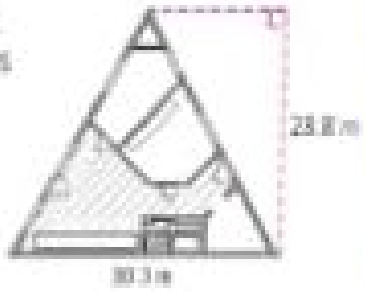
9 m

19. height: 11 cm, area: 260.15 cm²

47.3 cm

20. STEAM An architect is designing a building on a triangular plot of land. If the base of the triangle is 30.3 meters and the height is 28.8 meters, find the available floor area of the building.

436.32 m²



21. A flower bed in a parking lot is shaped like a triangle as shown.

a. Find the area of the flower bed in square centimeters.

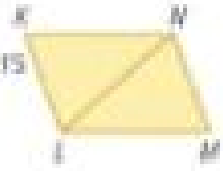
30,000 cm²

b. If one bag of topsoil covers 10,000 square centimeters how many bags are needed to cover this flower bed?

3 bags

22. **Identify Repeated Reasoning** Refer to parallelogram *KLMN* at the right. If the area of parallelogram *KLMN* is 35 square centimeters what is the area of triangle *KLN*?

17.5 cm²



Power Up! Test Practice

23. The table shows the areas of a triangle where the height of the triangle stays the same but the base changes. What expression can be used to find the area of a triangle that has a height of 7 units and a base of x units? Explain your reasoning.

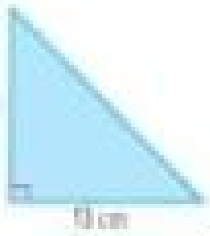
$\frac{7x}{2}$; Sample answer: The area is the product of the height (7), the base (x), and $\frac{1}{2}$ or $\frac{7x}{2}$.

Areas of Triangles

Height (units)	Base (units)	Area (square units)
7	3	$\frac{21}{2}$
7	3	$10\frac{1}{2}$
7	4	14
7	5	$17\frac{1}{2}$
7	x	$\frac{7x}{2}$

24. Nareen cut a triangle out of construction paper for an art project. The area of the triangle is 84.5 square centimeters.

Select the correct values to complete the formula below to find the height of the triangle.



$\frac{1}{2}$	2	13	26	84.5	h

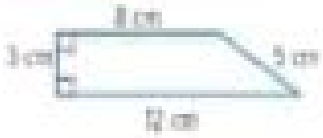
84.5 = $\frac{1}{2}$ · 13 · h

What is the height of the triangle? 13 cm

Spiral Review

Identify each figure below as a *rectangle*, *rhombus*, or *trapezoid*.

25. trapezoid



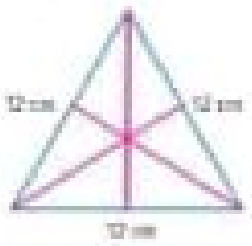
26. rectangle

27. rhombus

28. Sultan's floor rug has four 90° angles. All four sides are 46 centimeters long. The rug has two sets of parallel sides. What shape is Sultan's floor rug?

square

29. How many lines of symmetry can be drawn for the figure shown? Draw them on the figure. 3 lines



Inquiry Lab
Area of Trapezoids

Inquiry **HOW can you use the area of a parallelogram to find the area of a corresponding trapezoid?**

Mathematical Practices 1, 4, 5, 7

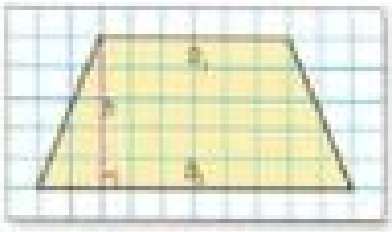
Maysa is building a garden in the shape of a trapezoid. The garden is 180 centimeters wide in the back, 300 centimeters wide in the front, and 150 centimeters from back to front. She wants to find the area of the garden.

Hands-On Activity 1

Find the area of a trapezoid by drawing the related parallelogram.

Step 1 Trace the trapezoid below on grid paper. Label the height h and label the bases b_1 and b_2.

A trapezoid has two bases, b_1 and b_2. The height h of a trapezoid is the perpendicular distance between the bases.



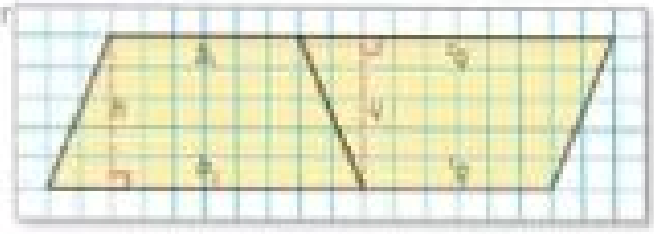
The shorter base, b_1, represents the garden width of 180 centimeters.

The longer base, b_2, represents the garden width of 300 centimeters.

The height h represents the garden dimension of 150 centimeters.

Step 2 Cut out another trapezoid that is identical to the one in Step 1.

Step 3 Tape the trapezoids together as shown.



Step 4 Find the area of the parallelogram. Then divide by 2 to find the area of each trapezoid.

480 × 150 = 72,0000 72,000 ÷ 2 = 36,000

So, the area of the garden is 36,000 square centimeters.

Hands-On Activity 2

Discover the formula for the area of a trapezoid.

Step 1 What figure is formed by the two trapezoids in Activity 1? parallelogram

Write an addition expression to represent the length of the base of the entire figure. $b_1 + b_2$

Step 2 Write a formula for the area A of the parallelogram using b and h. $A = (b_1 + b_2)h$

Step 3 How does the area of each trapezoid compare to the area of the parallelogram? Sample answer: The area of each trapezoid is one-half the area of the parallelogram.

Step 4 Write a formula for the area A of each trapezoid using b and h. $A = \frac{(b_1 + b_2)h}{2}$ or $A = \frac{1}{2}(b_1 + b_2)h$

Hands-On Activity 3

Another way to find the area of a trapezoid is to deconstruct it to determine which figures form the trapezoid. Find the area of the trapezoid shown below.

Step 1 The trapezoid is made up of one rectangle and two congruent triangles. Find the area of the shapes that make up the trapezoid.

The area of the rectangle is $6 \times 5 = 30$ square centimeters.

The area of each triangle is $\frac{3 \times 5}{2} = 7.5$ square centimeters.

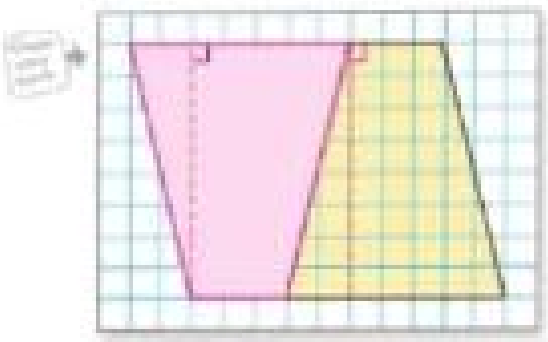
Step 2 Add the areas.

$30 + 7.5 + 7.5 = 45$ square centimeters

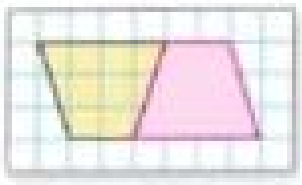
Investigate

Use Math Tools Work with a partner. Find the area of each trapezoid by drawing the related parallelogram.

1. $A =$ 40 square units



2. $A =$ 9 square units



Work with a partner. Find the area of each trapezoid by using the formula.

3. $A = \frac{(6 + 12)7}{2}$

$A =$ 63 square units

4. $A = \frac{(11 + 14)8}{2}$

$A =$ 100 square units

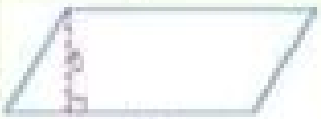
Work with a partner. Decompose each trapezoid to find the area.

5. $A =$ 40 square units

6. $A =$ 72 square units

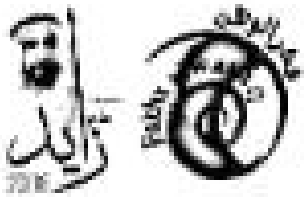
Analyze and Reflect

The table shows the dimensions of several parallelograms and corresponding trapezoids. Work with a partner to complete the table. The first one is done for you.

	Dimensions of Parallelogram	Area of Parallelogram	Length of Trapezoid b_1	Length of Trapezoid b_2	Trapezoid Height	Area of Trapezoid
		28	2	5	4	14
7.		66	5	6	6	33
8.		60	8	4	5	30
9.	$b_1 = 11$ $b_2 = 5$	33	7	4	3	16.5

10. **Reason Inductively** Compare the dimensions of the parallelogram to the dimensions of the corresponding trapezoid. What pattern do you see in the table? **Sample answer: To find the length of the base of a parallelogram, add the length of the bases of the related trapezoid. The height of a parallelogram and its related trapezoid are the same.**

11. **Reason Inductively** Compare the area of the parallelogram to the area of the corresponding trapezoid. What pattern do you see in the table? **Sample answer: Divide the area of the parallelogram by 2 to find the area of the related trapezoid.**



Create

12. **Identify Structure** Write the formula for the area A of a trapezoid with bases b_1 and b_2 and height h.

$A = \frac{(b_1 + b_2)h}{2}$ or $A = \frac{1}{2}(b_1 + b_2)h$

13. **Explain** HOW can you use the area of a parallelogram to find the area of a corresponding trapezoid?

Sample answer: The trapezoid is one half the area of the related parallelogram.

Lesson 3

Area of Trapezoids

Real-World Link

Window Seat Naja has a bay window in her room. The window seat is in the shape of a trapezoid. She needs to measure the seat in order to sew a cushion for the seat. The blue trapezoid in the diagram below represents the dimensions of the window seat.

Use the diagram below to describe the relationship between trapezoids and rectangles.

Essential Question

HOW does measurement help you solve problems in everyday life?

Mathematical Practices

1, 2, 3, 4, 5, 6

1. Find the dimensions of each figure.

Trapezoid		Rectangle	
base 1:	3 units	length:	8 units
base 2:	5 units	height:	3 units
height:	3 units		

2. What is the relationship between the measures of the rectangle and the measures of the trapezoid?

Sample answer: The sum of the two bases is equal to the length of the rectangle. The height of the trapezoid is equal to the height of the rectangle.

3. **Make a Conjecture** How is the area of a trapezoid related to the area of a rectangle?

Sample answer: Since two equal trapezoids make one rectangle, the area of a trapezoid is half the area of the related rectangle.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

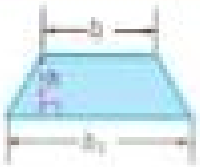
- ① Persevere with Problems
- ② Reason Abstractly
- ③ Construct an Argument
- ④ Model with Mathematics
- ⑤ Use Math Tools
- ⑥ Attend to Precision
- ⑦ Make Use of Structure
- ⑧ Use Repeated Reasoning

Key Concept

Area of a Trapezoid

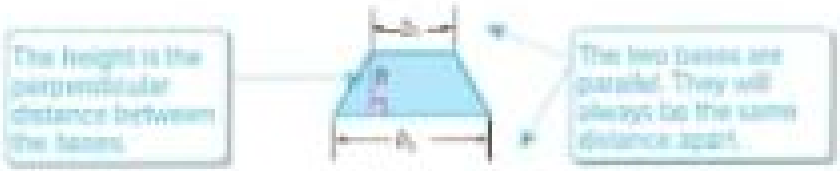
Words The area A of a trapezoid is one half the product of the height h and the sum of the bases b_1 and b_2.

Model



Symbols $A = \frac{1}{2}h(b_1 + b_2)$

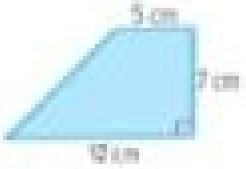
A trapezoid has two bases, b_1 and b_2. The height of a trapezoid is the distance between the bases.



When finding the area of a trapezoid, it is important to follow the order of operations. In the formula, the bases are to be added before multiplying by $\frac{1}{2}$ of the height h.

Examples

1. Find the area of the trapezoid.



The bases are 5 centimeters and 12 centimeters. The height is 7 centimeters.

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

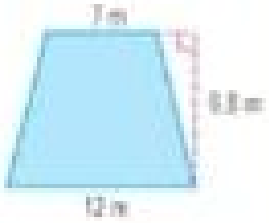
$A = \frac{1}{2}(7)(5 + 12)$ Replace A with 7, b_1 with 5, and b_2 with 12.

$A = \frac{1}{2}(7)(17)$ Add 5 and 12.

$A = 59.5$ Multiply.

The area of the trapezoid is 59.5 square centimeters.

2. Find the area of the trapezoid.



$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

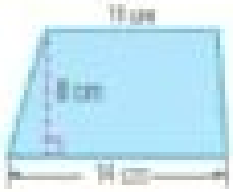
$A = \frac{1}{2}(9.8)(7 + 12)$ Replace h with 9.8, b_1 with 7, and b_2 with 12

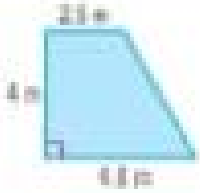
$A = \frac{1}{2}(9.8)(19)$ Add 7 and 12.

$A = 93.1$ Multiply.

So, the area of the trapezoid is 93.1 square meters.

Get it? Do these problems to find out.

a. 

b. 

c. 

a. 100 cm²

b. 14.6 m²

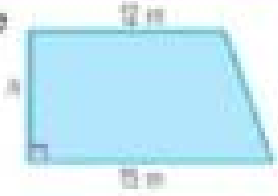
c. 0.225 m²

Find the Missing Height

Use the related formula, $h = \frac{2A}{b_1 + b_2}$, to find the height of a trapezoid.

Example

3. The trapezoid has an area of 108 square meters. Find the height.



$h = \frac{2A}{b_1 + b_2}$ Height of a trapezoid

$h = \frac{2(108)}{12 + 15}$ Replace A with 108, b_1 with 12, and b_2 with 15.

$h = \frac{216}{27}$ Multiply 2 and 108. Add 12 and 15.

$h = 8$ Divide.

So, the height of the trapezoid is 8 meters.

Be Precise

Check your answer by using the formula for the area of a trapezoid.

Get it? Do these problems to find out.

d. $A = 24$ cm²
$b_1 = 4$ cm
$b_2 = 12$ cm
$h = ?$

e. $A = 21$ m²
$b_1 = 2$ m
$b_2 = 5$ m
$h = ?$

d. 3 cm

e. 6 m

Mental Math

To multiply $\frac{1}{2}(77)(26)$, it is easier to use the Commutative Property to regroup the factors as $\frac{1}{2}(26)(77)$ and take half of 26 instead of half of 77.

Example

4. The shape of Osceola County, Florida, resembles a trapezoid. Find the approximate area of this county.

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$A = \frac{1}{2}(82)(77 + 26)$ Replace h with 82, b_1 with 77, and b_2 with 26.

$A = \frac{1}{2}(82)(103)$ Add 77 and 26.

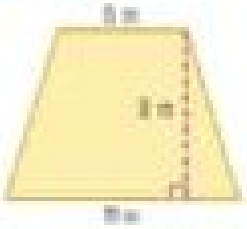
$A = 4{,}223$ Multiply.

So, the approximate area of the county is 4,223 square kilometers.

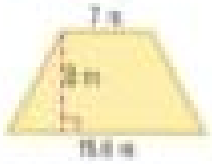
Guided Practice

Find the area of each trapezoid. Round to the nearest tenth if necessary. (Ex 1 and 2)

1. 68 m²



2. 90.4 mi²



3. A trapezoid has an area of 15 square meters. If the bases are 4 meters and 6 meters, what is the height of the trapezoid? (Example 3) 3 meters

4. In the National Hockey League, goaltenders can play the puck behind the goal line only in a trapezoid-shaped area, as shown at the right. Find the area of the trapezoid. (Example 4)

22.77 m²

5. **Building on the Essential Question** How is the formula for the area of a trapezoid related to the formula for the area of a parallelogram?

Sample answer: A parallelogram can be decomposed into two congruent trapezoids. So the area of each trapezoid is one half the area of the parallelogram.

Rate Yourself!

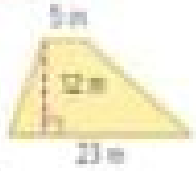
Are you ready to move on? Shade the section that applies.

Need to update your flowchart?

Independent Practice

Find the area of each trapezoid. Round to the nearest tenth if necessary. (Examples 1 and 2)

1. 168 m²



2. 4.5 cm²

3. 112 m²

4. A trapezoid has an area of 150 square meters. If the bases are 14 meters and 16 meters, what is the height of the trapezoid? (Example 3)

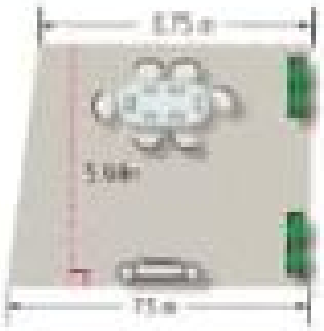
10 m

5. A trapezoid has an area of 400 square millimeters. The bases are 14 millimeters and 36 millimeters. What is the height of the trapezoid? (Example 3)

16 mm

6. Find the area of the patio shown. (Example 4)

42.3 m²

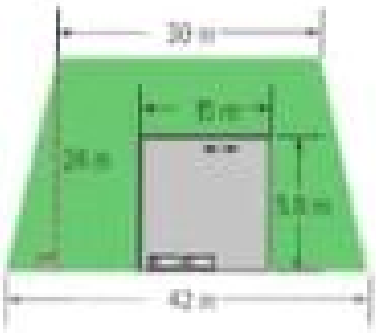


7. Use the diagram that shows the lawn that surrounds an office building.

a. What is the area of the lawn? 630 m²

b. If one bag of grass seed covers 180 square meters, how many bags are needed to seed the lawn?

4 bags



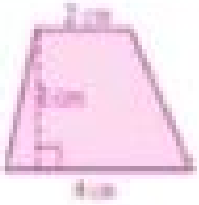
8. **Reason Abstractly** Tiles are being placed in front of a fireplace to create a trapezoidal hearth. The hearth will have a height of 60 centimeters and bases that are 120 centimeters and 150 centimeters. If the tiles cover 100 square centimeters, how many tiles will be needed?

81 tiles

Draw and label each figure. Then find the area. Sample answers: 9–13

9. a trapezoid with no right angles and an area less than 12 square centimeters

$A = 9$ cm²



10. a trapezoid with a right angle and an area greater than 40 square centimeters

$A = 50$ cm²

H.O.T. Problems Higher Order Thinking

11. **Persevere with Problems** Apply what you know about rounding to explain how to estimate the height h of the trapezoid shown if the area is 235.5 m².

The lengths of the bases can be rounded to 20 m and 30 m, respectively. The area can be rounded to 250. Divide 250 by (20 + 30), or 50, and then multiply by 2. The height h is about 10 m.

12. **Identify Repeated Reasoning** Find two possible lengths of the bases of a trapezoid with a height of 1 meter and an area of 9 square meters. Explain how you found your answer. Since $9 = \frac{1}{2}(1)(b_1 + b_2)$, the possible lengths have a sum of 18; Sample answers: 2 m and 16 m, or 8 m and 10 m

13. **Reason Abstractly** How can you use the formula for area of a parallelogram to determine the area of a trapezoid if you forgot the formula for area of a trapezoid? By knowing the formula for the area of a parallelogram is $A = bh$, I can draw two congruent trapezoids and rotate one so they create a parallelogram. After multiplying the base and height, I can divide by 2 to find the area of the trapezoid.

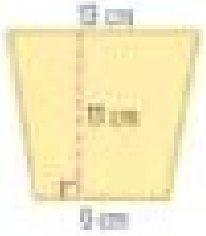
14. **Reason Inductively** The area of a trapezoid is 36 square centimeters. The height is 4 centimeters and one base is twice the length of the other base. What are the lengths of the bases? 6 cm and 12 cm.

Name _____ My Homework _____

Extra Practice

Find the area of each figure. Round to the nearest tenth if necessary.

15. 121 cm²



$A = \frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}(11)(13 + 9)$

$A = \frac{1}{2}(11)(22)$

$A = 121$

16. 161.5 m²

17. 187.6 m²

18. A trapezoid has an area of 50 square centimeters. The bases are 3 centimeters and 7 centimeters. What is the height of the trapezoid?

10 centimeters

19. A trapezoid has an area of 18 square kilometers. The bases are 5 kilometers and 7 kilometers. What is the height of the trapezoid?

3 kilometers

20. A county is shaped like a trapezoid. Its northern border is about 9.6 kilometers across, and the southern border is approximately 25 kilometers across. The distance from the southern border to the northern border is about 90 kilometers. Find the approximate area of the county.

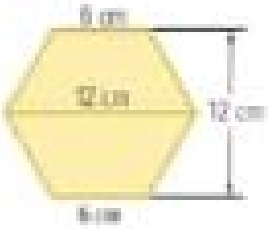
1,557 km²

21. A play tent is shown. How much fabric was used to make the front and back of the play tent?

11,840 cm²

C Identify Structure Each figure below is made up of congruent trapezoids. Find the area of each figure.

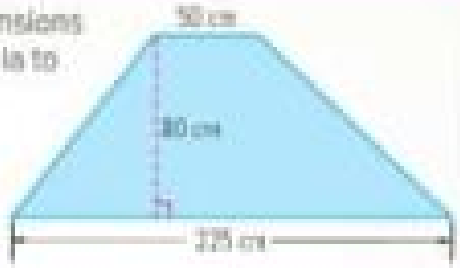
22. 108 cm²



23. 100 cm²

Power Up! Test Practice

24. A piece of sod is shaped like a trapezoid with the dimensions shown. Select the correct values to complete the formula to find the area of the piece of sod.



$\frac{1}{2}$	2	50	80	225

$A = \frac{1}{2} \cdot 80 \cdot (50 + 225)$

What is the area of the piece of sod? 11,000 cm²

25. Lamis designed the bags shown for her salon products.

Determine if each statement is true or false.

a. It takes 800 cm² of fabric to make the front of the small bag. ■ True ☐ False

b. It takes 1700 cm² of fabric to make the front of the large bag. ☐ True ■ False

Spiral Review

Add or multiply.

26. 5 + 6.2 + 8.8 = 20

27. 8 × 8 × 4 = 256

28. 725 + 315 + 4 = 1,044

29. Lamya is building a rectangular frame for her favorite photograph. The frame is 17.5 centimeters wide and 12.5 centimeters long.

What is the perimeter of the frame? 60 cm

Problem-Solving Investigation
Draw a Diagram

Mathematical Practices 1, 4, 7

Case #1 Amazing Array

A designer wants to arrange 12 mosaic tiles into a rectangular shape with the least perimeter possible.

What are the dimensions of the rectangle?

1 Understand *What are the facts?*

Twelve tiles will be arranged with the least perimeter possible.

2 Plan *What is your strategy to solve this problem?*

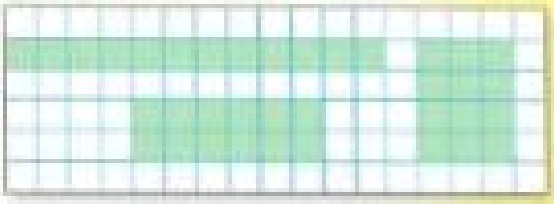
Use graph paper. Make diagrams of 12 squares to represent 12 tiles.

3 Solve *How can you apply the strategy?*

A rectangle with dimensions of 12 and 1 has a perimeter of 26.

A rectangle with dimensions of 3 and 4 has a perimeter of 14.

A rectangle with dimensions of 2 and 6 has a perimeter of 16.



So, the least perimeter has dimensions of 3 and 4.

4 Check *Does the answer make sense?*

Use addition to check your answer.

$3 + 4 + 3 + 4 = 14$ $2 + 6 + 2 + 6 = 16$ $12 + 1 + 12 + 1 = 26$

Analyze the Strategy

Identify Structure Describe a design with a perimeter and an area of 16.

Sample answer: The designer would use 4 tiles in each of 4 rows to arrange the tiles, so the arrangement would be a square.

Case #2 Dynamic Dimensions

For a school assignment, Tarek has to give three different possibilities for the dimensions of a rectangle that has a perimeter of 28 meters and an area greater than 30 square meters. One of the diagrams he drew is shown at the right.

What are two other possibilities for the dimensions of the rectangle?

1 Understand

Read the problem. What are you being asked to find?

I need to find two possibilities for the dimensions of the rectangle.

Underline key words and values in the problem. What information do you know?

The perimeter of the rectangle is 28 meters, and the area is greater than 30 m².

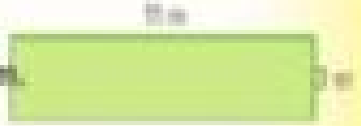
2 Plan

Choose a problem-solving strategy.

I will use the draw a diagram strategy.

3 Solve

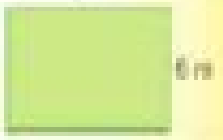
Use your problem-solving strategy to solve the problem.



Draw rectangles with perimeters of 28 meters. Then, multiply length times width to find the area.

The product must be greater than 30.

3 + 3 + 11 + 11 = 28; 11 × 3 = 33; 33 > 30; 12 + 12 + 2 + 2 = 28; 12 × 2 = 24; 24 < 30; 6 + 6 + 8 + 8 = 28; 6 × 8 = 48; 48 > 30



So, the dimensions of two possible rectangles are 3 m × 11 m and 6 m × 8 m.

4 Check

Use information from the problem to check your answer.

Reread the problem. Check that both conditions have been met.

Perimeter: 28 = 28 Area: 33 > 30 and 48 > 30

Mid-Chapter Check

Vocabulary Check

1. **Be Precise** Define *polygon*. Give an example of a figure that is a polygon and an example of a figure that is not a polygon. (Lesson 1)

   A polygon is a closed figure formed with 3 or more straight lines. Sample answer: A triangle is a polygon, but a circle is not a polygon.

2. Fill in the blank in the sentence below with the correct term. (Lesson 2)

   Congruent figures have the **same** size and the **same** shape.

Skills Check and Problem Solving

Find the area of each figure. (Lessons 1 and 2)

3. **50 cm²**

4. **48 m²**

5. **66 m²**

Find the missing dimension of each figure. (Lesson 1 and 3)

6. parallelogram: $b = 5\frac{1}{4}$ m; $A = 12$ m²

   $b = 2\frac{2}{7}$ m

7. trapezoid: $b_1 = 3$ m; $b_2 = 4$ m; $A = 7$ m²

   $h = 2$ m

8. **Model with Mathematics** corner table is in the shape of a trapezoid. Find the area of the tabletop. (Lesson 3)

   1,050 cm²

9. **Reason Inductively** The area of a triangle is 56 square centimeters. Give all possible sets of whole number dimensions for the base and height of the triangle. (Lesson 3)

   1 cm and 112 cm, 2 cm and 56 cm, 4 cm and 28 cm, 7 cm and 16 cm, 8 cm and 14 cm

Lesson 4

Changes in Dimension

Real-World Link

Construction Mr. Mohammad is building a rectangular cat house. The floor of the cat house is 120 centimeters long and 60 centimeters wide.

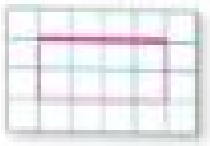
Essential Question

HOW does measurement help you solve problems in everyday life?

Mathematical Practices

1, 2, 3, 4, 7

1. Draw the floor of the cat house on the graph paper below.



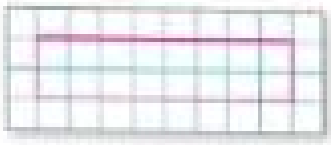
2. Add the lengths of the sides to find the perimeter.

360 centimeters

3. Multiply the length and width to find the area.

7,200 square centimeters

4. Mr. Mohammad doubles the width of the cat house. Draw the new floor below.



5. How did the perimeter and area of the floors change from the first to the second cat house?

The perimeter of the second house is 240 centimeters greater. The area of the second house is twice the area of the first.

Which Mathematical Practices did you use? Shade the circle(s) that applies.

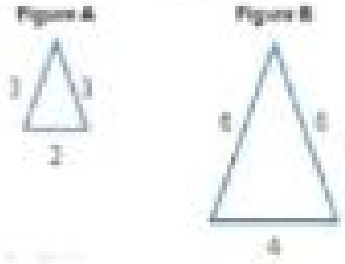
- 1 Persevere with Problems
- 2 Reason Abstractly
- 3 Construct an Argument
- 4 Model with Mathematics
- 5 Use Math Tools
- 6 Attend to Precision
- 7 Make Use of Structure
- 8 Use Repeated Reasoning

Key Concept

Changing Dimensions: Effect on Perime

Words If the dimensions of a polygon are multiplied by x, then the perimeter of the polygon changes by a factor of x.

Model



Example The dimensions of Figure A are multiplied by 2 to produce the dimensions of Figure B.

$$\underbrace{\text{perimeter of Figure A}}_{8} \cdot \underbrace{2}_{\cdot\,2} = \underbrace{\text{perimeter of Figure B}}_{16}$$

Notice that all the dimensions of the figure must change using the same factor, x.

Example

1. Suppose the side lengths of the parallelogram at the right are tripled. What effect would this have on the perimeter? Justify your answer.

The dimensions are 3 times greater.

original perimeter: $2(4) + 2(3) = 14$ cm.

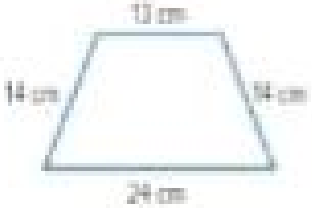
new perimeter: $2(12) + 2(9) = 42$ cm.

compare perimeters: $42 \div 14 = 3$

So, the perimeter is 3 times the perimeter of the original figure.

Got it? Do this problem to find out.

a. Suppose the side lengths of the trapezoid at the right are multiplied by $\frac{1}{2}$. What effect would this have on the perimeter? Justify your answer.



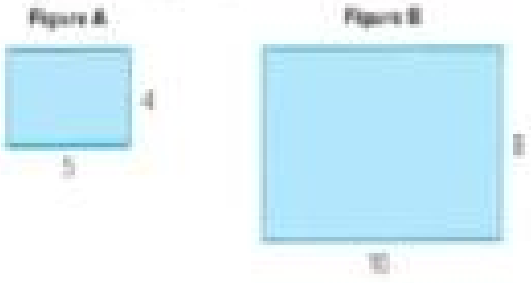
The perimeter is $\frac{1}{2}$ the original perimeter. The perimeter of the original figure is 65 cm and the perimeter of the new figure is 32.5 cm; $65 \text{ cm} \div 32.5 \text{ cm} = \frac{1}{2}$

Changing Dimensions: Effect on Area

Key Concept

Words If the dimensions of a polygon are multiplied by x, then the area of the polygon changes by x^2.

Model



Example The dimensions of Figure A are multiplied by 2 to produce the dimensions of Figure B.

$$\underbrace{\text{area of Figure A}}_{20} \cdot 2^2 = \underbrace{\text{area of Figure B}}_{80}$$
$$\cdot 4 =$$

Notice that all the dimensions of the figure must change using the same factor, x.

Example

2. The side lengths of the triangle at the right are multiplied by 5. What effect would this have on the area? Justify your answer.

The dimensions are 5 times greater.

original area $\frac{1}{2} \cdot 2 \cdot 1 = 1 \text{ cm}^2$

new area $\frac{1}{2} \cdot 10 \cdot 5 = 25 \text{ cm}^2$

compare areas:
$25 \text{ cm}^2 \div 1 \text{ cm}^2 = 25$ or 5^2

So, the area is 5^2 or 25 times the area of the original figure.

Get it? Do this problem to find out.

b. A rectangle measures 2 meters by 4 meters. Suppose the side lengths are multiplied by 2.5. What effect would this have on the area? Justify your answer.

The area is 2.5^2 or 6.25 times greater. The area of the original figure is 8 m^2 and the area of the new figure is 50 m^2.

b. $50 \text{ m}^2 \div 8 \text{ m}^2 = 6.25$

Example

3. **A stop sign is in the shape of a regular octagon. Sign A shown at the right has an area of 2000 square centimeters. What is the area of sign B?**

Since $20 \times 1.5 = 30$, the area of sign B is 1.5 times the area of sign A.

$2000 \times 1.5^2 = 2000 \times 2.25$ or 4,500

So, the area of sign B is 4,500 square centimeters.

Guided Practice

Refer to the figure at the right for Exercises 1 and 2. Justify your answers. (Examples 1-2)

1. Each side length is doubled. Describe the change in the perimeter.

The perimeter is doubled. The perimeter of the original figure is 15.4 cm and the perimeter of the new figure is 30.8 cm; 30.8 cm ÷ 15.4 cm = 2.

2. Each side length is tripled. Describe the change in the area.

The area is 3 or 9 times greater. The area of the original figure is 10 cm^2 and the area of the new figure is 90 cm^2; 90 cm^2 ÷ 10 cm^2 = 9.

3. Different sizes of regular hexagons are used in a quilt. Each small hexagon has side lengths of 4 centimeters and an area of 41.6 square centimeters. Each large hexagon has side lengths of 8 centimeters. What is the area of each large hexagon? (Example 3) 166.4 cm^2

Rate Yourself!

How confident are you about changes in dimensions? Check the box that applies.

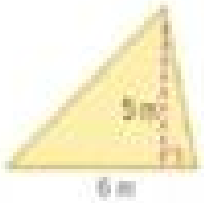
4. **Building on the Essential Question** How can exponents help you find the area of a rectangle if each side length is multiplied by x? Sample answer: The original area is multiplied by x^2 to find the new area.

Independent Practice

1. Each side length of the parallelogram at the right is multiplied by 4. Describe the change in the perimeter. Justify your answer. (Example 1)

The perimeter is 4 times greater. The perimeter of the original figure is 36 cm and the perimeter of the new figure is 144 cm; 144 cm ÷ 36 cm = 4.

2. The base and height of the triangle at the right are multiplied by 4. Describe the change in the area. Justify your answer. (Example 2)



The area is 4 or 16 times greater. The area of the original figure is 15 m² and the area of the new figure is 240 m² ÷ 15 m² = 16.

3. Each side length of the rectangle is multiplied by $\frac{1}{3}$. Describe the change in the area. Justify your answer. (Example 2)

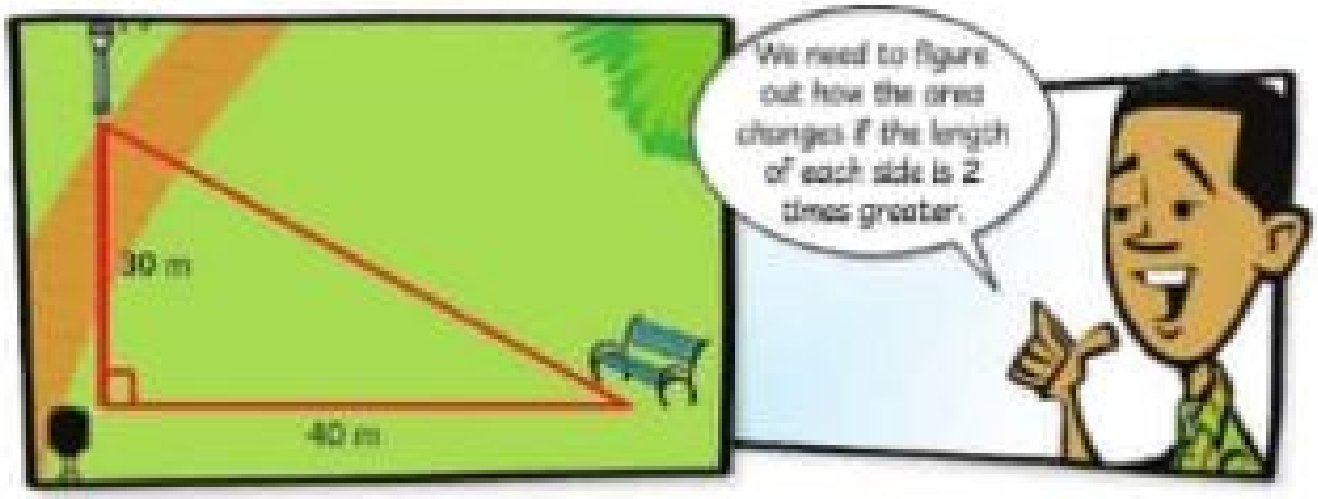
The area is multiplied by $\frac{1}{3} \times \frac{1}{3}$ or $\frac{1}{9}$ the original area. The area of the original figure is 315 m² and the area of the new figure is 35 m² 35 m² ÷ 315 m² = $\frac{1}{9}$.

4. Different sizes of regular pentagons are used in a stained glass window. Each small pentagon has side lengths of 4 centimeters and an area of 27.5 square centimeters. Each large pentagon has side lengths of 8 centimeters. What is the area of each large pentagon? (Example 3)

110 cm²

5. **Justify Conclusions** A dollhouse has a bed with dimensions the size of a queen-size bed. A queen-size bed has an area of 30,000 square centimeters, and a length of 200 centimeters. What are the side lengths of the dollhouse bed? Justify your answer. Use the area and the length to find the width of the queen-size bed. The width of the bed is 4,800 ÷ 80, or 60 centimeters. So, the width of the dollhouse bed is $60 \times \frac{1}{12}$, or 5 centimeters. The length of the dollhouse bed is $80 \times \frac{1}{12}$, or $6\frac{2}{3}$ centimeters.

6. **Reason Abstractly** Refer to the graphic novel frame below for Exercises a–b.



a. What is the original area of the triangle? 600 m²

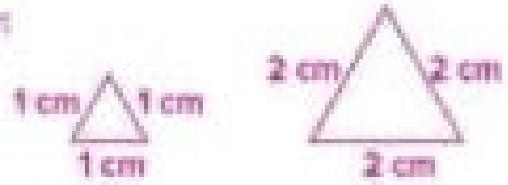
b. What is the new area if the sides are all two times longer?

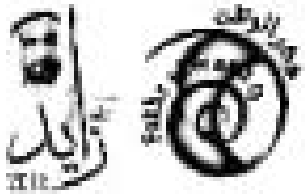
2,400 m²

H.O.T. Problems Higher Order Thinking

7. **Identify Structure** Sketch a triangle with the side lengths labeled. Sketch and label another triangle that has a perimeter two times greater than the perimeter of the first triangle.

Sample answer:

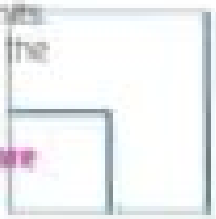




8. **Persevere with Problems** The corresponding side lengths of two figures have a ratio $\frac{a}{b}$. What is the ratio of the perimeters? the ratio of the areas?

$\frac{a}{b}; \frac{a^2}{b^2}$

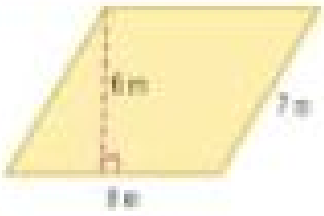
9. **Reason Inductively** The larger square shown has a perimeter of 48 units. The smaller square inside has a perimeter that is 2 times smaller. What are the side lengths of the larger and smaller square? Explain.



larger square: 12 units; smaller square: 6 units; Sample answer: The lengths of a squares' sides are equal. Divide 48 by 4 to get a side length of 12. The side length of the smaller square is half as big, so 6 units.

Extra Practice

Refer to the parallelogram at the right for Exercises 10–12. Justify your answers.



10. Suppose the base and height are each multiplied by $\frac{1}{2}$. What effect would this have on the area?

original area: 8 • 6 or 48 square meters

new dimensions: base = $\frac{1}{2}$ of 8 m, height = $\frac{1}{2}$ of 6 m

new area: 4 • 3 or 12 square meters; $\frac{1}{4}$ of 48 m² = 12 m²

So, the area is $\frac{1}{2} \cdot \frac{1}{2}$ or $\frac{1}{4}$ times the area of the original figure.

11. Suppose the side lengths are multiplied by 6. Describe the change in the perimeter. **The perimeter is 6 times greater. The perimeter of the original figure is 30 m and the perimeter of the new figure is 180 m; 180 m ÷ 30 m = 6.**

12. Suppose the base and height are each multiplied by 3.5. Describe the change in the area. **The area is 3.5 or 12.25 times greater. The area of the original figure is 48 m² and the area of the new figure is 588 m² 588 m² ÷ 48 m² = 12.25.**

13. Refer to the triangle at the right. Suppose the side lengths and height of the triangle were divided by 4. What effect would this have on the perimeter? the area? Justify your answer.

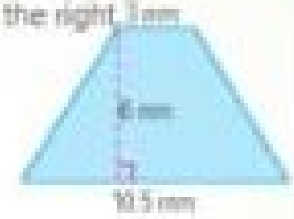
The perimeter is $\frac{1}{4}$ the original perimeter. The perimeter of the original figure is 80 m and the perimeter of the new figure is 20 m; $\frac{1}{4}$ • 80 m = 20 m. The area is $\frac{1}{4} \cdot \frac{1}{4}$ or $\frac{1}{16}$ the original area. The area of the original figure is 240 m² and the area of the new figure is 15 m² 15 m² ÷ 240 m² = $\frac{1}{16}$.

14. Justify Conclusions A model car has a windshield with dimensions $\frac{1}{15}$ the size of a real car windshield. The rectangular windshield of the real car has an area of about 14,955 square centimeters, with a width of 152.5 centimeters. What are the side lengths of the model car's windshield? Round to the nearest hundredth. Justify your answer.

Use the area and the length to find the width of the model car's windshield. The width of the windshield is 14,955 ÷ 152.5, or 98 centimeters. So, the width of the model car's windshield is 98 ÷ 15 or 5.44 centimeters. The length of the model car's windshield is 152.5 ÷ 15 or 8.47 centimeters .

Power Up! Test Practice

15. Fill in the boxes to complete each statement about the trapezoid at the right.



a. When the dimensions of the trapezoid are multiplied by 2, the area is [4] times greater.

b. When the dimensions of the trapezoid are multiplied by [4], the area is 16 times greater.

c. When the dimensions of the trapezoid are multiplied by 5, the area is [25] times greater.

16. The side lengths of triangle *A* are equal. The side lengths of triangle *B* are also equal. Triangle *A* has a perimeter of 9 meters. Triangle *B* has a perimeter of 27 meters. Select the correct values to make each statement true.

3	23.4
6	27
9	35.1
11.7	

a. The length of each side in triangle *A* is [3] meters.

b. The length of each side in triangle *B* is [9] meters.

c. The area of triangle *A* is about 3.9 square meters. The area of triangle *B* is about [35.1] square meters.

Spiral Review

Graph the opposite of each number on a number line.

17. 0

18. −7

19. 5

20. Graph 2 and 9. Then use the number line to find the distance between 9 and 2. **7 units**

21. Majed and his dad are playing catch on the football field. Majed is standing on the 10 meter line. His dad is standing on the 25 meter line. How far is Majed from his dad? If his dad moves to the 20 meter line, what is the distance between them now?

15 m; 10 m

Lesson 5

Polygons on the Coordinate Plane

Real-World Link

Maps Graph points on a coordinate plane to draw a map of an outdoor stadium. Complete the table to identify each shape.

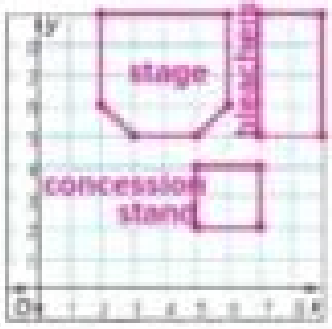
Location	Vertices	Shape
Stage	(2, 6), (2, 9), (5, 9), (6, 6), (5, 5), (3, 5)	hexagon
Bleachers	(2, 5), (2, 9), (9, 9), (9, 5)	rectangle
Concession Stand	(5, 2), (5, 4), (7, 4), (7, 2)	square

Essential Question

How does measurement help you solve problems in everyday life?

Mathematical Practices

1, 2, 3, 4, 5, 7



1. Find the dimensions of the bleachers.

   Length 4 units Height 2 units

2. The length of the line from point (2, 6) to point (2, 9) is 3 units long. How can you use the y-coordinates to find the length of the line?

   Sample answer: Subtract 6 from 9. The difference is 3.

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

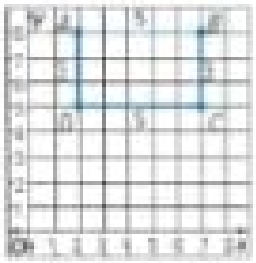
Work Zone

Find Perimeter

You can use the coordinates of a figure to find its dimensions by finding the distance between two points. To find the distance between two points with the same *x*-coordinates, subtract their *y*-coordinates. To find the distance between two points with the same *y*-coordinates, subtract their *x*-coordinates.

Examples

1. **A rectangle has vertices *A*(2, 8), *B*(7, 8), *C*(7, 5), and *D*(2, 5). Use the coordinates to find the length of each side. Then find the perimeter of the rectangle.**



Width: Find the length of the horizontal lines.

$\overline{AB}$ is 5 units long. $\overline{CD}$ is 5 units long.

Length: Find the length of the vertical lines.

$\overline{BC}$ is 3 units long. $\overline{DA}$ is 3 units long.

Add the lengths of each side to find the perimeter.

$5 + 5 + 3 + 3 = 16$ units

So, rectangle *ABCD* has a perimeter of 16 units.

2. **Rectangle *ABCD* has vertices *A*(2, 1), *B*(2, 5), *C*(4, 5), and *D*(4, 1). Use the coordinates to find the length of each side. Then find the perimeter of the rectangle.**

Width: Subtract *y*-coordinates.

AB: $5 - 1 = 4$ units *CD*: $5 - 1 = 4$ units

Length: Subtract *x*-coordinates.

AD: $4 - 2 = 2$ units *BC*: $4 - 2 = 2$ units

Add the lengths of each side to find the perimeter.

$4 + 2 + 4 + 2 = 12$ units

Got It? **Do these problems to find out.**

Use the coordinates to find the length of each side. Then find the perimeter of the rectangle.

a. *E*(3, 6), *F*(3, 8), *G*(7, 8), *H*(7, 6)

b. *I*(1, 4), *J*(1, 9), *K*(8, 9), *L*(8, 4)

Perimeter and Area

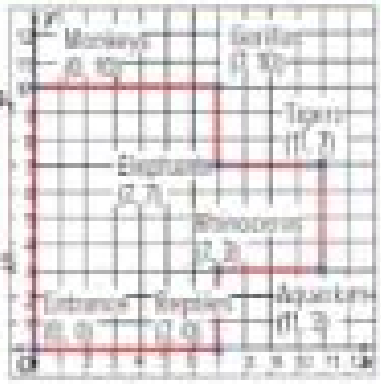
Explain how to find the perimeter of a rectangle. How is the number of square units needed to cover the surface enclosed by a rectangle related to its perimeter?

EF = 2 units, *FG* = 4 units, *GH* = 2 units, *HE* = 4 units; 12 units

b. *IJ* = 5 units, *JK* = 7 units, *KL* = 5 units, *LI* = 7 units; 24 units

Example

3. **Each grid square on the zoo map has a length of 60 meters. Find the total distance, in meters, around the zoo.**



When x-coordinates are the same, subtract the y-coordinates. When y-coordinates are the same, subtract the x-coordinates.

$10 + 7 + 3 + 4 + 4 + 4 + 3 + 7 = 42$ units

Multiply by 60 meters to find the total distance.

$42 \times 60 = 2{,}520$ meters. The total distance is 2,520 meters.

Get it? **Do this problem to find out.**

c. The coordinates of the vertices of a garden are (0, 1), (8, 4), and (8, 1). If each unit represents 30 centimeters, find the perimeter in centimeters of the garden.

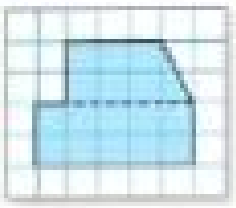
c. 660 centimeters

Find Area

You can find the area of a figure that has been drawn grid paper or graphed on the coordinate plane.

Example

4. **Find the area of the figure in square units.**



The figure can be separated into a rectangle and a trapezoid.

Area of rectangle

$A = \ell \times w$

$A = 5 \times 2$ or 10

Area of trapezoid

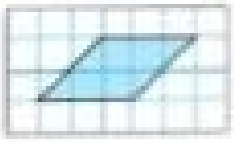
$A = \frac{1}{2}h(b + b)$

$A = \frac{1}{2}(2)(3+4)$ or 7

So, the area of the figure is 10 + 7 or 17 square units.

Get it? **Do this problem to find out.**

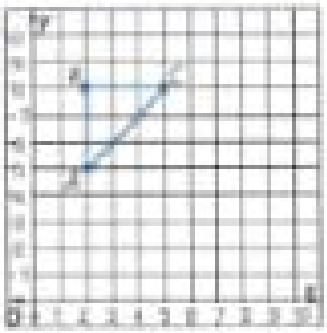
d. Find the area, in square units, of the figure at the right.



d. 6 square units

Example

5. **A figure has vertices $A(2, 5)$, $B(2, 8)$, and $C(5, 8)$. Graph the figure and classify it. Then find the area.**



Plot the points. Connect the vertices. The figure is a right triangle.

The height from point A to point B is 3 units. The base from point B to point C is 3 units.

$A = \frac{1}{2}bh$ Area formula of a triangle

$A = \frac{1}{2}(3)(3)$ Replace b with 3 and h with 3

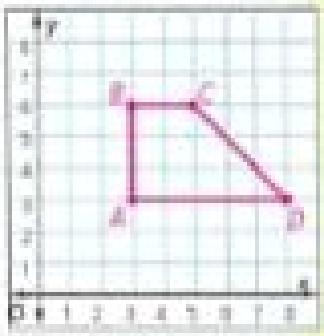
$A = 4.5$ Multiply

Triangle ABC has an area of 4.5 square units.

Got It? **Do this problem to find out.**

Graph the figure and classify it. Then find the area.

a. $A(3, 3)$, $B(3, 6)$, $C(5, 6)$, $D(8, 3)$



a. trapezoid; 10.5 units²

Guided Practice

Use the coordinates to find the length of each side of the rectangle. Then find the perimeter. Examples 1 and 2

1. $L(3, 3)$, $M(3, 5)$, $N(7, 5)$, $P(7, 3)$

$LM = 2$ units, $MN = 4$ units, $NP = 2$ units, $PL = 4$ units; 12 units

2. $P(3, 0)$, $Q(6, 0)$, $R(6, 7)$, $S(3, 7)$

$PQ = 3$ units, $QR = 7$ units, $RS = 3$ units, $SP = 7$ units; 20 units

3. Hideya is building a fence around the perimeter of her yard for her horse. The coordinates of the vertices of the yard are (0, 0), (0, 10), (5, 10), and (5, 0). If each grid square has a length of 30 meters, find the amount of wire, in meters, needed for the fence. What is the shape of her yard? Example 3

900 meters; rectangle

4. **Building on the Essential Question** How can coordinates help you to find the area of figures on the coordinate plane?

Sample answer: Coordinates can be used to identify a figure and find the lengths of the sides. The lengths of the sides can be used in the area formulas for various figures.

Rate Yourself!

How well do you understand polygons on the coordinate plane? Circle the image that applies.

Clear | Somewhat Clear | Not So Clear

Name _____ My Homework _____

Independent Practice

Use the coordinates to find the length of each side of the rectangle. Then find the perimeter. (Examples 1 and 2)

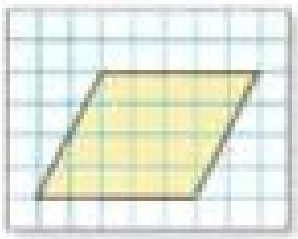
1. D(1, 2), E(1, 7), F(4, 7), G(4, 2)
   DE = 5 units, EF = 3 units, FG = 5 units, GD = 3 units; 16 units

2. Q(0, 0), R(4, 0), S(4, 4), T(0, 4)
   QR = 4 units, RS = 4 units, ST = 4 units, TQ = 4 units; 16 units

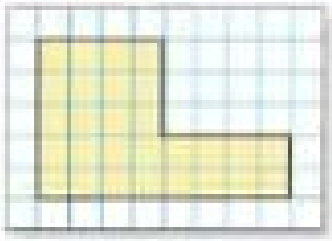
3. Fawzia is building a rectangular picture frame for her favorite photo. The coordinates of the vertices of the frame are (0, 0), (0, 8), (12, 8), and (12, 0). Each grid square has a length of 3 centimeters. Find the amount of wood, in centimeters, needed for the perimeter. (Example 3)
   120 cm

Find the area of each figure in square units. (Example 4)

4. 20 square units

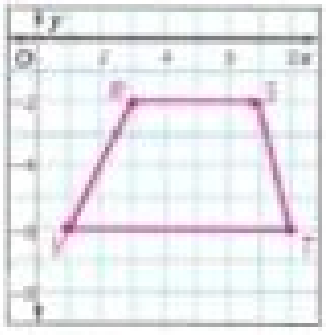


5. 28 square units

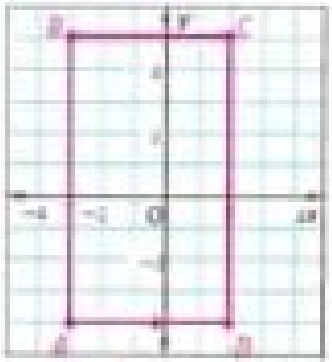


Graph each figure and classify it. Then find the area. (Example 5)

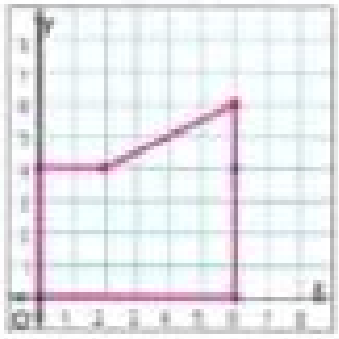
6. R(3, −2), S(7, −2), T(8, −6), V(1, −6)
   trapezoid; 22 units



A(−3, −4), B(−3, 5), C(2, 5), D(2, −4)
rectangle; 45 units



8. **Use Math Tools** A rectangle has a perimeter of 20 units. The coordinates of three of the vertices are (0, 0), (6, 0), and (6, 4) as shown on the graph.



a. What is the coordinate of the missing vertex?
(0, 4)

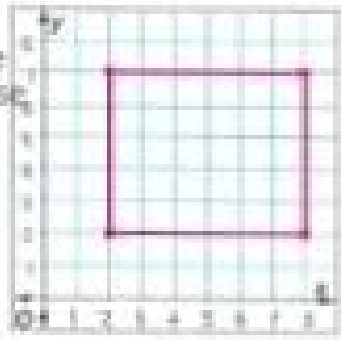
b. Plot points (6, 6) and (2, 4). Connect these points to create a composite figure.

c. What is the area of the composite figure?
28 square units

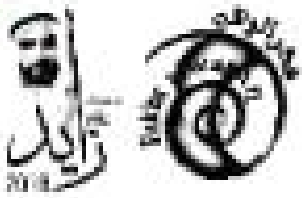
H.O.T. Problems Higher Order Thinking

9. **Use Math Tools** Draw a rectangle on a coordinate plane that has a perimeter of 16 units. Label all of the vertices with the coordinates. Then find the area of the rectangle.
See students' work.

10. **Persevere with Problems** A certain rectangle has a perimeter of 22 units and an area of 30 square units. Two of the vertices have coordinates at (2, 2) and (2, 7). Find the two missing coordinates. Use the coordinate plane to support your answer.
Sample answer: (8, 2) and (8, 7)



11. **Identify Structure** Explain the steps you would use to find the perimeter of a rectangle using the coordinates of the vertices.
Sample answer: Subtract the *x*-coordinates of the points with the same *y*-coordinates to find the length of 2 of the sides and then subtract the *y*-coordinates of the points with the same *x*-coordinates to find the length of the other 2 sides. Then find the sum of all 4 sides to find the perimeter.



12. **Persevere with Problems** Rectangle *QRST* has vertices *Q*(3, 2) and *S*(7, 8).

a. Give two possible coordinates for vertices *R* and *T*.
Sample answer: *R*(3, 8) and *T*(7, 2)

b. Find the perimeter and area of the rectangle.
20 units; 24 units

Extra Practice

Use the coordinates to find the length of each side of the rectangle. Then find the perimeter.

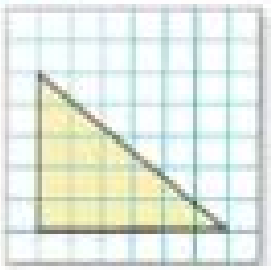
13. $A(5, 2)$, $B(5, 4)$, $C(2, 4)$, $D(2, 2)$
$AB = 2$ units, $BC = 3$ units, $CD = 2$ units, $DA = 3$ units; 10 units

14. $M(1, 1)$, $N(1, 9)$, $P(7, 9)$, $Q(7, 1)$
$MN = 8$ units, $NP = 6$ units, $PQ = 8$ units, $QM = 6$ units; 28 units

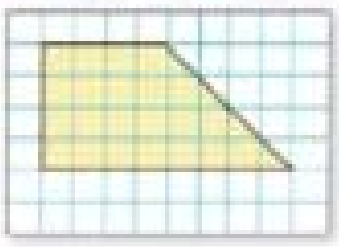
15. **Reason Abstractly** Hamdan is creating a border around his rectangular patio with paver bricks. The coordinates of the vertices of the patio are (1, 5), (6, 5), (6, 1), and (1, 1). Each grid square has a length of 90 centimeters. Find the amount of brick, in centimeters, needed for the perimeter. **54 meters**

Find the area of each figure in square units.

16. **15 square units**

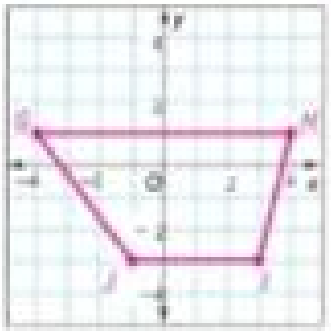


17. **24 square units**

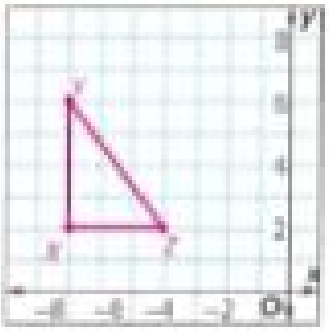


Graph each figure and classify it. Then find the area.

18. $G(-4, 1)$, $H(4, 1)$, $I(3, -3)$, $J(-1, -3)$
trapezoid; 24 units



19. $X(-7, 2)$, $Y(-7, 6)$, $Z(-4, 2)$
right triangle; 6 units



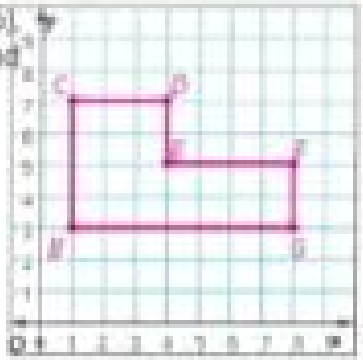
Copy and Solve **Graph each figure and classify it. Then find the area.**

20. $K(-2, 2)$, $L(3, 2)$, $M(2, -2)$, $N(-3, -2)$ **20–21. See margin.**

21. $O(-2, 4)$, $R(0, -2)$, $S(-4, -2)$

Power Up! Test Practice

22. Figure *BCDEFG* has vertices located at *B*(1, 3), *C*(1, 7), *D*(4, 7), *E*(4, 5), *F*(8, 5), and *G*(8, 3). Sketch the figure on the coordinate plane and connect the vertices.



What is the area of the figure?

20 square units

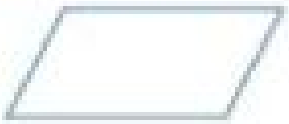
23. A quadrilateral has vertices with coordinates of *A*(8, 5), *B*(7, 2), *C*(4, 2), and *D*(2, 5). Which of the following are characteristics of the quadrilateral? Select all that apply.

- [x] one set of parallel sides
- [ ] two sets of parallel sides
- [x] four vertices
- [x] two acute angles

Spiral Review

Describe the sides of each figure using the terms *parallel*, *perpendicular*, and *congruent*.

24. parallelogram Opposite sides are congruent and parallel.



25. trapezoid No sides are congruent. One pair of opposite sides is parallel.

26. Mr. Khalaf's garden is surrounded by a fence. The fence makes four right angles at each corner. All four sides of the fence are 14 meters long. What shape best describes Mr. Khalaf's garden?

square

27. Rashid drew the logo to the right. The blue figure has two pairs of parallel sides, two pairs of congruent sides, and four right angles. What is the shape of the blue figure?

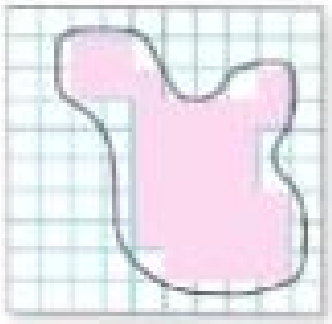
rectangle

Inquiry Lab
Area of Irregular Figures

inquiry **HOW can you estimate the area of an irregular figure?**

Mathematical Practices 1, 3, 4, 5

Jamal's family is putting a koi pond in their backyard. They need to estimate the area of the pond to know how many fish they can put in the pond. A scale drawing of the pond is shown below. In the drawing, each square represents 900 square centimeters.



What do you know? 900 square centimeters.

What do you need to know? the area of the pond

Hands-On Activity 1

Step 1 Shade and count the number of whole squares the pond covers.

33

Step 2 Estimate the number of whole squares covered by the partial squares altogether. 13

Step 3 Add your answers from Steps 1 and 2.

33 + 13 = 46

So, the area of the pond is about 41,400 square centimeters.

Hands-On Activity 2

Another way to estimate the area of an irregular figure is to separate the figure into simpler shapes. Then find the sum of these areas.

Step 1 First, separate the figure into a triangle and a rectangle.

Step 2 Find the area of each figure.

Area of a triangle

$A = \frac{1}{2}bh$

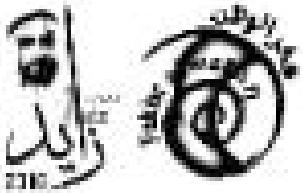
$= \frac{1}{2} \cdot 200 \cdot 311$ $b = 300 - 100$ or 200; $h = 481 - 170$ or 311

$= 31{,}100$ Simplify.

Area of rectangle

$A = lw$

$= 300 \cdot 170$ or $51{,}000$ $l = 300$ and $w = 170$



Step 3 Add to find the total area.

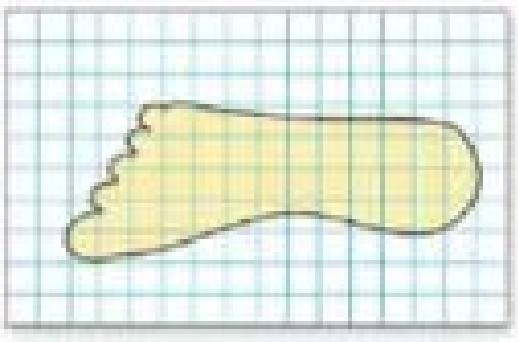
31,100 + 51,000 = 82,100

The area of Idaho is about 82,100 square kilometers.

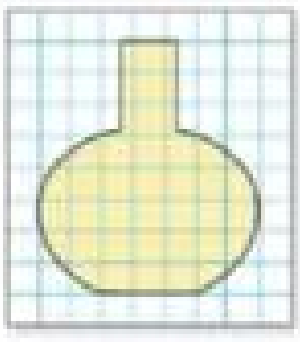
Investigate

Use Math Tools Work with a partner to estimate the area of each irregular figure. Sample answers given.

1. $A \approx$ 38 square units



2. $A \approx$ 36 square units



Investigate

Work with a partner to estimate the area of each irregular figure. Sample answers given.

3. $A \approx$ 160,000 square kilometers

4. $A \approx$ 9 square centimeters

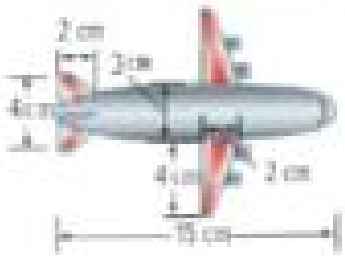
5. $A \approx$ 225 square centimeters

6. $A \approx$ 14 square centimeters

7. $A \approx$ 57 square centimeters

8. $A \approx$ 32 square centimeters

9. $A \approx$ 55 square centimeters

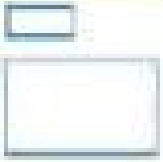
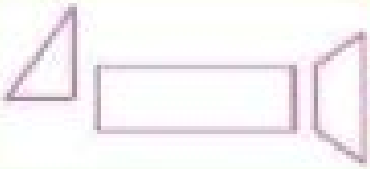


10. $A \approx$ 112.5 square centimeters

Analyze and Reflect

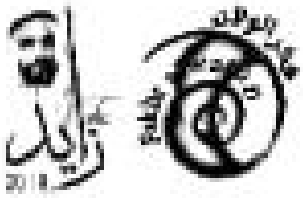
Sample answers: 11–14

Work with a partner to complete the table. The first one is done for you.

	Irregular Figure	Draw the simpler shapes you can make.	Area of Each Simpler Shape	Estimated Area of Irregular Figure
			$8 \times 5 = 24$ $12 \times 4 = 48$	72 square centimeters
11.			$\frac{5 \times 6}{2} = 15$ $15 \times 6 = 90$	105 square centimeters
12.			$\frac{4(7+5)}{2} = 24$ $\frac{4(7+5)}{2} = 24$ $9 \times 5 = 45$	93 square centimeters
13.			$\frac{1 \times 1}{2} = 0.5$ $\frac{2(3+2)}{2} = 5$ $6 \times 2 = 12$	104 square centimeters

14. **Reason Inductively** Suha solves Exercise 11 by subtracting the area of two triangles from the area of a large rectangle and finds the answer 105 square centimeters. How does Suha's answer compare to your answer for Exercise 11?

The answers are the same.



Create

15. **Model with Mathematics** Draw an irregular figure. Write a problem about your figure. Then have a classmate solve the problem.

See students' work.

16. **Inquiry** HOW can you estimate the area of an irregular figure?

Sample answer: Estimate the area of an irregular figure by dividing it into simpler shapes. Find the area of each of the simpler shapes and add each area to estimate the total area of the figure.

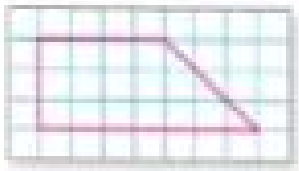
Lesson 6

Area of Composite Figures

Vocabulary Start-Up

A **composite figure** is a figure made of two or more two-dimensional figures. The composite figure shown to the right is made of two rectangles.

Draw a composite figure made of a rectangle and a right triangle on the graph paper below. Sample answer is given.



Essential Question

HOW does measurement help you solve problems in everyday life?

Vocabulary

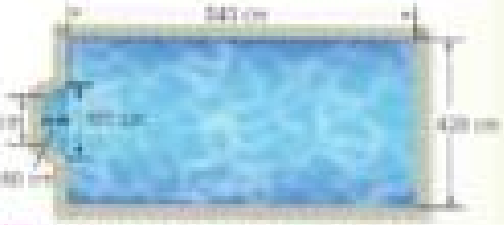
composite figure

Mathematical Practices

1, 2, 3, 4, 6, 7

Real-World Link

Pools The dimensions of the city pool are shown.



1. What two-dimensional figures are used to make the shape of the pool?
   The pool is a rectangle with a trapezoid at the entrance.

2. How could you determine the area of the pool floor?
   Separate the shape into a rectangle and a trapezoid, find the area of each, and add to find the total area.

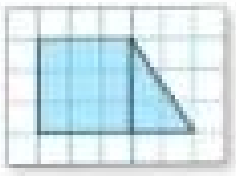
Which Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Work Zone

Find the Area of a Composite Figure

You can decompose some trapezoids into a square and a triangle to find the area.



Area of Square

$A = \ell \cdot w$

$A = 3 \cdot 3$, or 9

Area of Triangle

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(2)(3)$, or 3

Then add the area of the square and the area of the triangle to find the area of the trapezoid. The area of the trapezoid is 9 + 3 or 12 square units.

You can find the area of a composite figure using the same strategy. To find the area of a composite figure, separate it into figures with areas you know how to find. Then add those areas.

Example

1. Find the area of the figure at the right.

The figure can be separated into a rectangle and a triangle. Find the area of each.

Area of Rectangle

$A = \ell w$

$A = 10 \cdot 6$ or 60

Area of Triangle

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(4)(4)$ or 8

The base of the triangle is 10 − 6 or 4 centimeters.

The area is 60 + 8 or 68 square centimeters.

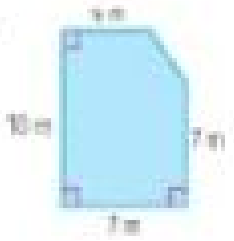
Get it? Do these problems to find out.

Find the area of each figure.

a. 60 cm²

b. 67 m²

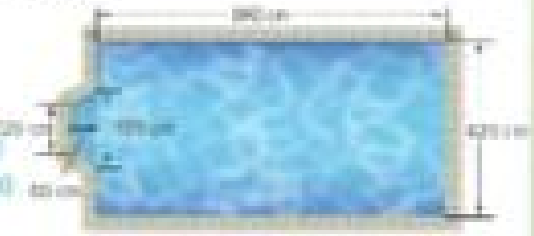
b.



Example

2. **Find the area of the pool's floor.**

Separate the figure into a rectangle and a trapezoid.



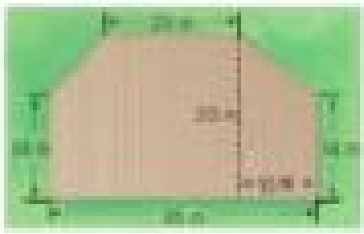
Rectangle: 840 × 420 or 352,800

Trapezoid: $\frac{1}{2}(600)(300 + 180)$ or 9,000

So, the area of the pool's floor is 352,800 + 9,000 or 361,800 square centimeters.

Get it? **Do this problem to find out.**

c.



c. 672 m²

Find the Area of Overlapping Figures

To find the area of overlapping figures, decompose the figures.

Example

3. **Find the area of the figure at the right.**

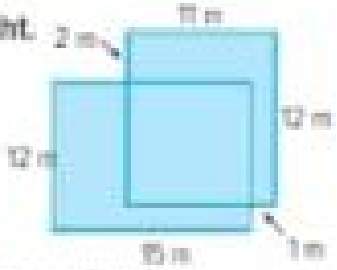
Square: 11 × 12 or 132

Rectangle: 15 × 12 or 180

The sum of the areas: 132 + 180 or 312

Overlapping area: 6 × 7 or 42

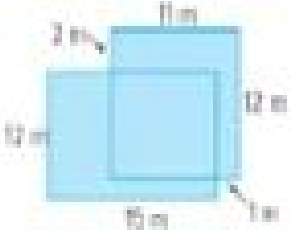
Subtract the overlapping area: 324 − 42 = 282



So, the area of the figure is 282 square centimeters.

Get it? **Do this problem to find out.**

d.



d. 65 m²

Be Precise

It is important not to count the area of the overlapping section twice when finding the area of overlapping figures.

Example

4. **Husam and his brother Hasan are neighbors in an apartment complex where they share a patio. What is the area of both apartments and the patio?**

Each apartment:
17 × 14 or 238

The sum of the areas:
238 + 238 or 476

Patio: 7 × 7 or 49

Subtract the overlapping area. 476 − 49 = 427

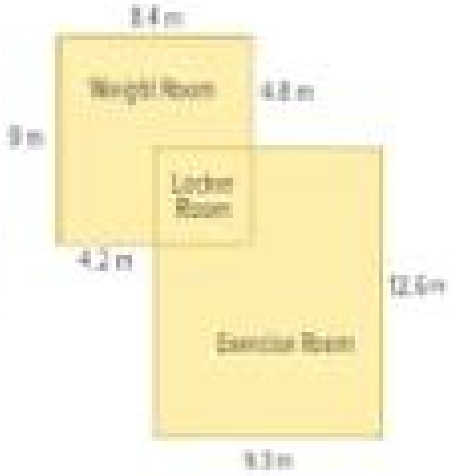
So, the total area is 427 square meters.

Guided Practice

1. The manager of an apartment complex will install new carpeting in a studio apartment. The floor plan is shown at the right. What is the total area that needs to be carpeted? (Examples 1 and 2)

30.15 m²

2. Finn Fitness has an entrance to the locker room from both the exercise room and the weight room. What is the total area of Finn Fitness? (Examples 3 and 4)



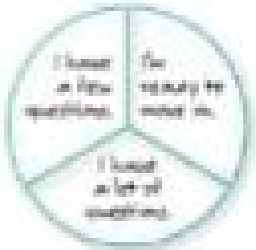
181.9 m²

3. **Building on the Essential Question** How can you decompose figures to find area?

Sample answer: Decompose figures into areas that you know how to find. Then add to find areas of composite figures, or subtract areas of overlapping figures.

Rate Yourself!

Are you ready to move on? Choose the section that applies.

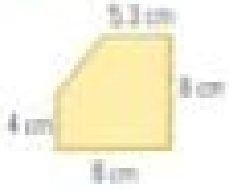


Name _____ My Homework _____

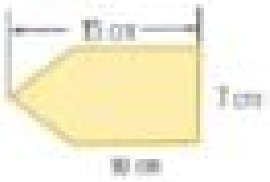
Independent Practice

Find the area of each figure. Round to the nearest tenth if necessary. (Example 1)

1. 58.6 cm²

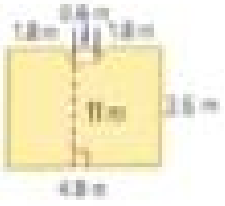


2. 87.5 cm²



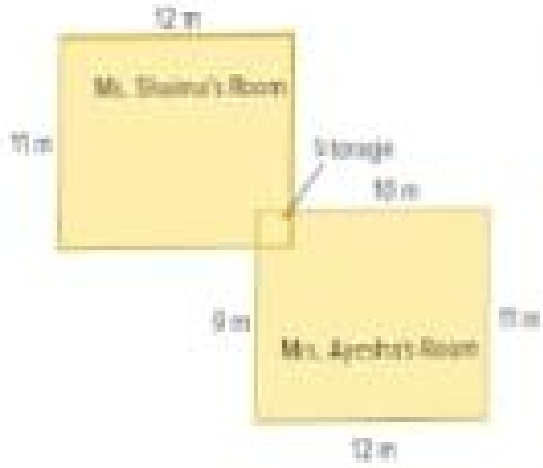
3. The floor plan of a kitchen is shown at the right. If the entire kitchen floor is to be tiled, how many square feet of tile are needed? (Example 2)

17 m²



4. Ms. Shaima and Mrs. Ayesha both teach sixth grade math. They share a storage closet. What is the total area of both rooms and the storage closet? (Examples 3 and 4)

360 m²



5. The diagram shows one side of a storage barn.

a. This side needs to be painted. Find the total area to be painted. 43.75 m²

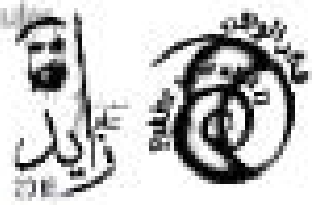
b. Each can of paint costs AED 20 and covers 32 square meters. Find the total cost to paint this side once. Justify your answer.

43.75 ÷ 32 ≈ 1.4; Since only whole cans of paint can be purchased, you will need 2 cans of paint. At AED 20 each, the cost will be 2× AED 20 or AED 40.

6. **Reason Abstractly** Refer to the graphic novel frame below for Exercises a–b.

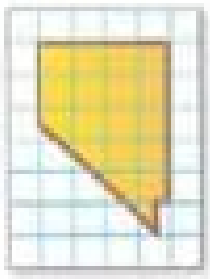
a. The first clue was hidden in a triangular section of the park with an area of 54 square meters. The second clue was hidden in a rectangular section with a height of 9 m and a width of 7 m. What was the area of the rectangular section? 63 m²

b. What is the total search area? 117 m²



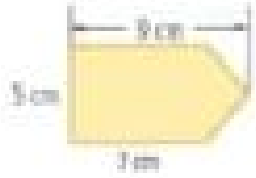
H.O.T. Problems Higher Order Thinking

7. **Persevere with Problems** Describe how to separate the figure into simpler figures. Then estimate the area. One square unit equals 6,144 square kilometers. Justify your answer.



Sample answer: Add the areas of a rectangle and a triangle. Area of rectangle: 3 × 4 = 12; Area of triangle: $\frac{1}{2} \times 3 \times 3 = 4.5$; 12 + 4.5 = 16.5. So, an approximate area is 16.5 • 6,144 or 101,376 km²

8. **Identify Structure** Describe how you would find the area of the figure shown at the right. Sample answer: Separate it into a rectangle and a triangle, find the area of each, and add.



9. **Make a Conjecture** Refer to the composite figure at the right. Make a conjecture about how the area of the composite figure changes if each dimension given is doubled. Then test your conjecture by doubling the dimensions and finding the area.

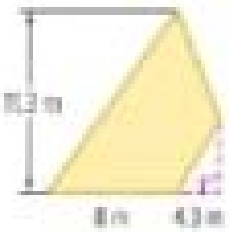
The area is multiplied by 4. Original area: 159.9 cm²; new area: 639.6 cm²

Name _____ My Homework _____

Extra Practice

Find the area of each figure. Round to the nearest tenth if necessary.

10. 69.5 m²



$A = \frac{1}{2}(8)(11.3) = 45.2$

$A = \frac{1}{2}(4.2)(11.3) \approx 24.3$

$45.2 + 24.3 = 69.5$

11. 66.2 m²

12. The diagram gives the dimensions of a swimming pool. If a cover is needed for the pool, what will be the approximate area of the cover? 705 m²

13. At the local zoo, the aquarium can be seen from the Reptile Room and the Amphibian Arena. What is the total area of both rooms and the aquarium?
1,010 m²

14. **Persevere with Problems**
The diagram shows one wall of Abeer's living room.

a. This wall needs to be painted. Find the total area to be painted.
12.9 m²

b. Each quart of paint costs AED 8 and covers 8 square meters. Find the total cost to paint this wall once. Justify your answer.
12.9 ÷ 8 ≈ 1.6; Since only whole quarts of paint can be purchased, you will need 2 quarts of paint. At AED 8 each, the cost will be 2 × AED 8 or AED 16.

Power Up! Test Practice

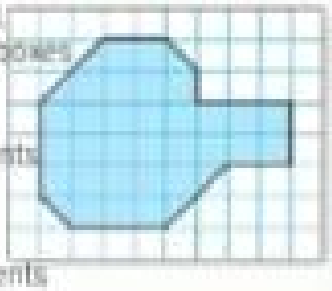
15. A window has the dimensions shown. Determine if each statement is true or false.

a. The area of the trapezoidal section of the window is 1,620 square centimeters. ☐ True ■ False

b. The area of the rectangular section of the window is 10,800 square centimeters. ■ True ☐ False

c. The area of the entire window is 5,940 square centimeters. ☐ True ■ False

16. The shaded part of the grid represents the plans for a fish pond. Each square on the grid represents 5 square meters. Fill in the boxes to complete each statement.



a. There are 30 full squares in the pond. This represents an area of 150 square meters.

b. There are 6 half squares in the pond. This represents an area of 15 square meters.

c. What is the total area of the fish pond? 165 m²

Review

Multiply.

17. $36 \times 12 = 432$

18. $15 \times 71 = 1{,}065$

19. $72 \times 200 = 14{,}400$

20. Find the volume of the rectangular prism.

105 cm³

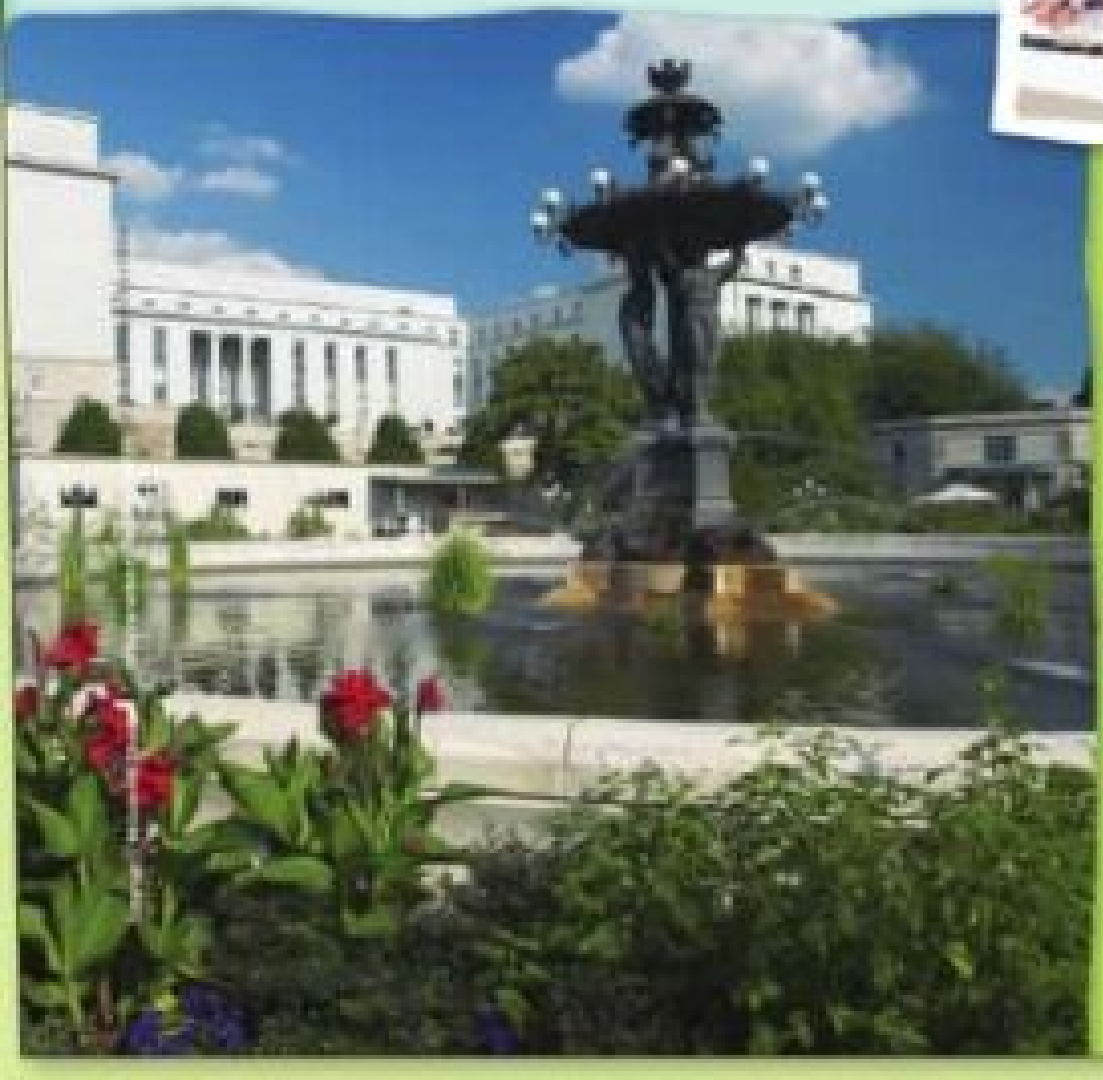
21. Hiking burns about 144 Calories each half hour. About how many Calories can be burned if someone hikes 3 days a week for an hour?

864 Calories

21ST CENTURY CAREER in Community Planning

Parks and Recreation Planner

Do you enjoy thinking about how your community might look 10 years in the future? If so, a career in parks and recreation planning might be a perfect fit for you. Most planners are employed by local governments. They assess the best use for the land and create short and long term plans for various parks and recreation areas. They make recommendations based on the location of roads, schools, and residential areas. A parks and recreation planner uses mathematics, science, and computer software to complete their work.



College & Career READINESS

Is This the Career for You?

Are you interested in a career as a parks and recreation planner? Take some of the following courses in high school.

- Economics
- Environmental Design
- Geometry

Turn the page to find out how math relates to a career in Community Planning.

You be the Parks and Recreation Planner!

For each problem, use the information in the designs.

1. What is the area of the playground in Design 2? **168 m²**

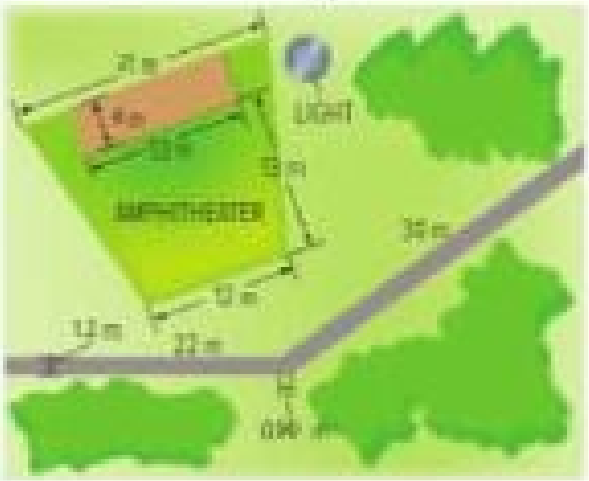
2. In Design 2, how much larger is the area of the soccer field than the area of the playground? **5,832 m²**

3. In Design 1, the amphitheater has a stage. What is the area of the amphitheater without the stage? **228.5 m²**

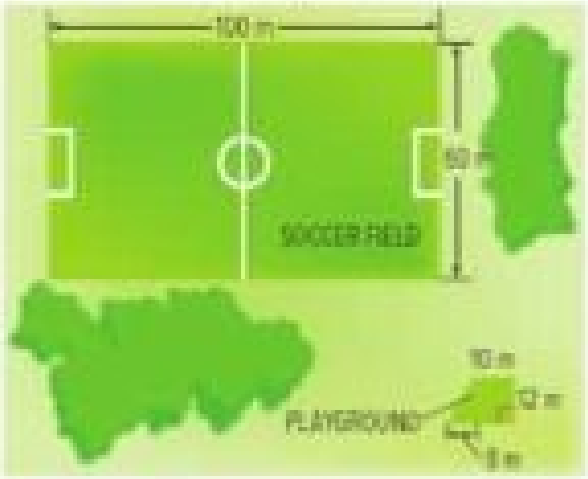
4. The cost of building the amphitheater including the stage is AED 225 a square meters. The budget provided to build the amphitheater is AED 65,000. Are they within budget? Explain.

Yes; Sample answer: The cost of building the amphitheater is 280.6 AED 225, or AED 63,112.50 Since this is less than AED 65,000, they are within budget.

Design 1

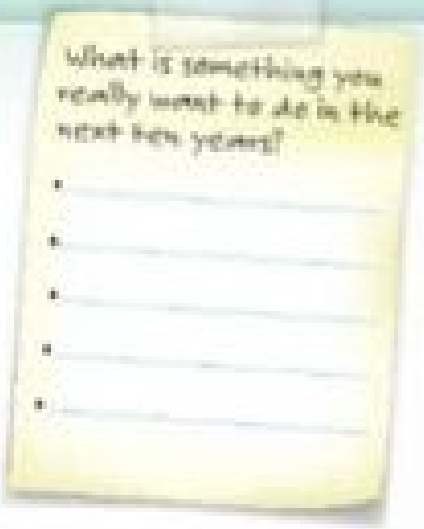


Design 2



Career Project

It's time to update your career portfolio! Create your own park with trees, sports fields, and paths. Search online for Park Planning to get ideas of what you can do and how much it might cost.



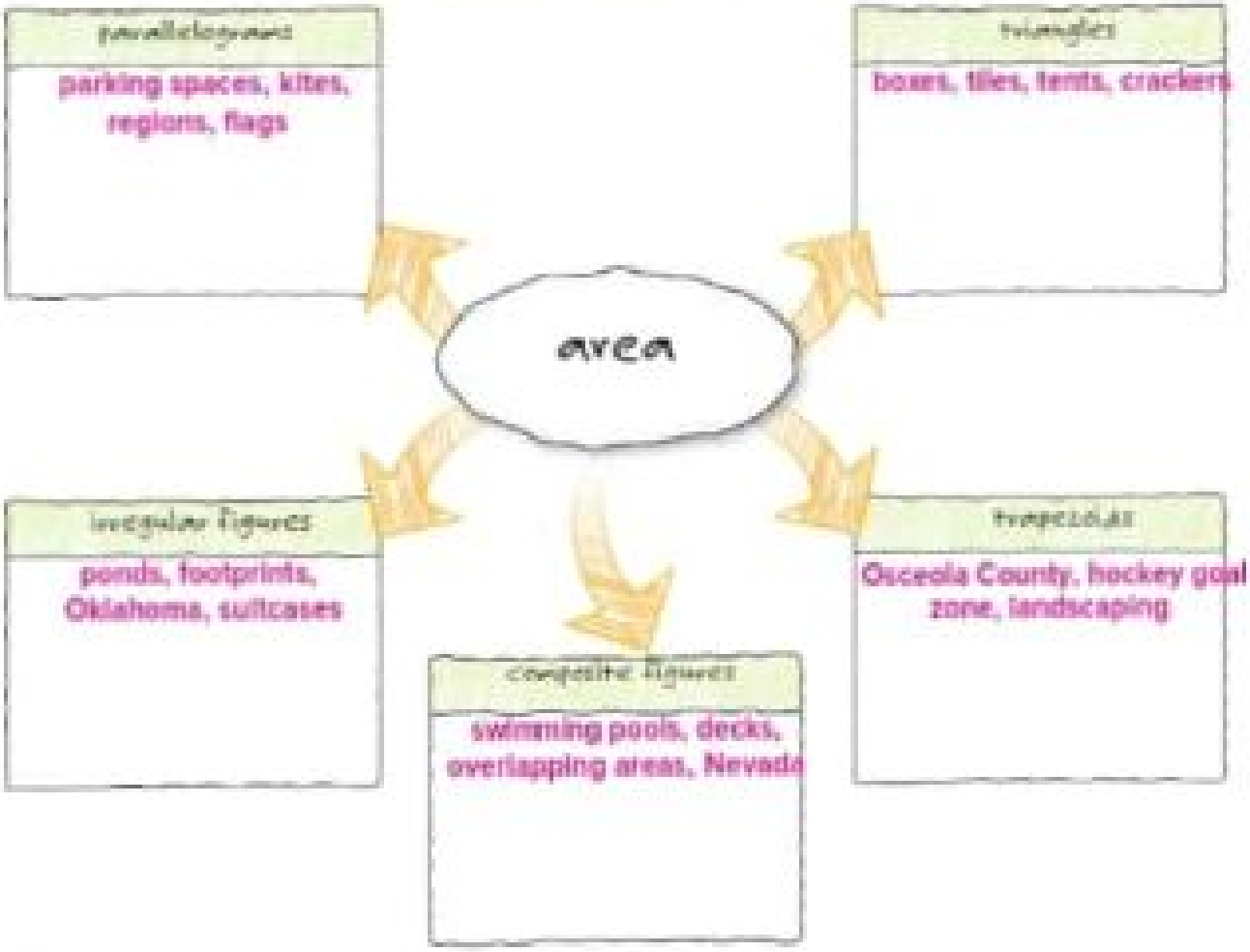
Reflect

Answering the Essential Question

Use what you learned about area to complete the graphic organizer. List several real-world examples for each figure. Sample answers are given.

Essential Question

HOW does measurement help you solve problems in everyday life?



Answer the Essential Question. HOW does measurement help you solve problems in everyday life?

See students' work.

Chapter 10

Volume and Surface Area

Essential Question

HOW is shape important when measuring a figure?

Mathematical Practices
1, 2, 3, 4, 5, 6, 7, 8

Math in the Real World

Aquariums Two-dimensional figures have area, while three-dimensional figures have volume and surface area.

A 75 liter aquarium can measure 75 centimeters wide, 25 centimeters deep, and 40 centimeters high. What is the area of the bottom of the aquarium?

FOLDABLES
Study Organizer

1. Cut out the Foldable on page FL11 of this book.
2. Place your Foldable on page 734.
3. Use the Foldable throughout this chapter to help you learn about volume and surface area.

Are You Ready?

Try the Quick Check below.

Quick Review

Example 1

Find $16 \times 2.5 \times 8$.

$16 \times 2.5 = 40$ Multiply 16 and 2.5.
$40 \times 8 = 320$ Multiply the product by 8.

Example 2

Evaluate $(6 \times 4) + (3 \times 5)$.

$(6 \times 4) + (3 \times 5) = 24 + 15$ Multiply.
$= 39$ Add.

Quick Check

Decimals **Multiply.**

1. $3 \times 5.5 \times 13 =$ 214.5

2. $9.8 \times 4 \times 15 =$ 588

3. $18 \times 1.6 \times 6 =$ 172.8

4. Khamis earned AED 7.25 for each hour he worked. If he worked 8 hours a week for 4 weeks, how much did he earn?

AED 232

Numerical Expressions **Evaluate each expression.**

5. $(3 \times 12) + (4 \times 2) =$ 44

6. $(9 \times 7) + (6 \times 4) =$ 87

7. $(15 \times 3) + (8 \times 7) =$ 101

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7

Inquiry Lab
Volume of Rectangular Prisms

Inquiry **HOW can you use models to find volume?**

Mathematical Practices 1, 3, 4

Mohammad is designing a section of a building. The section is 2 meters wide, 3 meters long, and 6 meters tall. What is the volume of the building's section?

Hands-On Activity 1

You can use centimeter cubes to find the **volume** of the cabinet. Volume is the amount of space inside a three-dimensional figure. Volume is measured in **cubic units**. Each cube of your model represents 1 cubic meter.

Step 1 Build a model that is 2 cubes wide, 3 cubes long, and 6 cubes tall.

Step 2 Count the number of cubes used to build the model. The model uses 36 cubes.

So, the volume of the cabinet is 36 cubic meters.

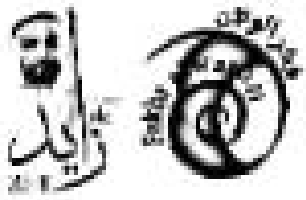
Find the product of the dimensions of the cabinet.

$2 \times 3 \times 6 = 36$

The product is the same as the volume.

Work with a partner. Use 36 cubes. Build all the possible prisms with a volume of 36 cubic units. List the dimensions below. Use each set of factors only once.
Answer order may vary. Factor order may vary.

$6 \times 2 \times 3 = 36$
$2 \times 2 \times 9 = 36$
$3 \times 4 \times 3 = 36$
$1 \times 4 \times 9 = 36$
$12 \times 3 \times 1 = 36$
$36 \times 1 \times 1 = 36$
$18 \times 2 \times 1 = 36$
$1 \times 6 \times 6 = 36$



Hands-On Activity 2

You can find the volume of rectangular prisms with fractional side lengths.

Step 1 The model to the right is $2\frac{1}{2}$ cubes long, 1 cube wide, and 1 cube tall.

Step 2 Count the number of cubes used to build the model.

The model uses $2\frac{1}{2}$ cubes.

So, the volume of the model is $2\frac{1}{2}$ cubic meters.

Compare the product of the dimensions of the prism with its volume.

$2\frac{1}{2} \times 1 \times 1 = 2\frac{1}{2}$

They are the same.

Hands-On Activity 3

You can use cubes of candy to find the volume of rectangular prisms with fractional sides.

Step 1 Cut one piece of candy into two halves.

Step 2 Make a model that is $\frac{1}{2}$ cubes long, 2 cubes wide, and 1 cube tall. Draw a picture of your model.

Step 3 Count the number of cubes used to build the model. The model uses 4 whole cubes and 2 half-cubes.

Two halves equal one whole. So, a total of 5 cubes were used.

So, the volume of the prism is 5 cubic units.

Compare the product of the dimensions of the prism with its volume.

$2\frac{1}{2} \times 2 \times 1 = 5$

They are the same.

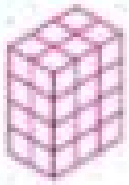
Investigate

Model with Mathematics Work with a partner. Use models to determine the volume of each prism. Draw a diagram of each model in the space provided.

1. length: 1
height: 1
width: 1
volume: 1 cubic unit

2. length: 2
height: 4
width: 1
volume: 8 cubic units

3. length: 3
height: 4
width: 2
volume: 24 cubic units



4. length: $\frac{1}{2}$
height: 1
width: 1
volume: $\frac{1}{2}$ cubic unit

5. length: $2\frac{1}{2}$
height: 4
width: 1
volume: 10 cubic units

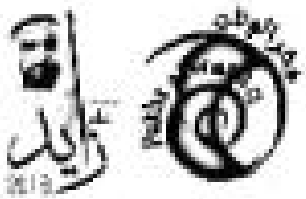
6. length: $3\frac{1}{2}$
height: 2
width: 2
volume: 14 cubic units

Analyze and Reflect

Sample answers: 11–16

Work with a partner to complete the table. Use models, if needed. The first one is done for you.

	Prism	Height (units)	Length (units)	Width (units)	Volume (cubic units)
	A	6	3	2	36
7.	B	$2\frac{1}{2}$	$1\frac{1}{2}$	2	$7\frac{1}{2}$
8.	C	5	$1\frac{1}{2}$	2	15
9.	D	2	5	$1\frac{1}{2}$	15
10.	E	5	3	4	60



11. Compare the dimensions for prism **C** to the dimensions of prism **D**. Compare the volume of the two prisms. What do you notice?

The dimensions used the same values and the volumes of both prisms were the same.

12. The length and width of prisms **B** and **C** are equal. Compare the height of the two prisms. How does the change in height affect the change in volume?

Prism B is half the height of prism C. The volume of prism B is half that of prism C.

13. Compare the dimensions for prism **B** to the dimensions of prism **E**. Compare the volume of the two prisms. What do you notice?

The dimensions of prism E are double those of prism B. The volume of prism E is 8 times greater than the volume of prism B.

14. **Reason Inductively** Describe the relationship between the number of cubes needed and the dimensions of the prism.

The number of cubes needed to make a prism is equal to the product of the length, width, and height of the prism.

Create

15. **Model with Mathematics** Write a real-world problem that involves volume of rectangular prisms. Include the dimensions and the volume of the rectangular prism in your response.

Reham has a pencil case that is 7 centimeters high, 30 centimeters long, and 15 centimeters wide. What is the volume of the pencil case? The volume is 3,150 cubic centimeters.

16. **Essential Question** HOW can you use models to find volume?

You can use cubes to model the size of the dimensions of a rectangular prism.

The dimensions tell the height, width, and length of the prism. The volume is determined by the number of cubes that were used to build the model.

Lesson 1

Volume of Rectangular Prisms

Vocabulary Start-Up

Sample answers are given.

Define: volume	When would you use volume?
the amount of space inside a three-dimensional figure	to find how much something can hold

volume

Example	Nonexample
how much space is inside an aquarium	how much space on the floor an aquarium covers

Essential Question

HOW is shape important when measuring a figure?

Vocabulary

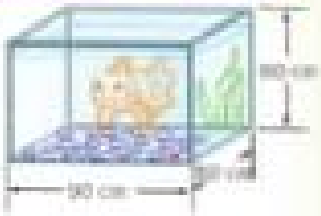
three-dimensional figure
prism
rectangular prism
volume
cubic units

Mathematical Practices

1, 3, 4, 5, 6, 7

Real-World Link

Aquarium The dimensions of an aquarium are shown.



1. What is the area of the base of the aquarium? 5,400 square centimeters

2. What is the height of the aquarium? 60 centimeters

3. Fill in the blanks to find the volume.

90 (height) × 60 (width) × 60 (height) = 324,000 cm³

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

- ① Persevere with Problems
- ② Reason Abstractly
- ③ Construct an Argument
- ④ Model with Mathematics
- ⑤ Use Math Tools
- ⑥ Attend to Precision
- ⑦ Make Use of Structure
- ⑧ Use Repeated Reasoning

Key Concept

Volume of a Rectangular Prism

Words The volume V of a rectangular prism is the product of its length ℓ, width w, and height h.

Symbols $V = \ell wh$ or $V = Bh$

Model

A **three-dimensional figure** has length, width, and height. A **prism** is a three-dimensional figure with two parallel bases that are congruent polygons. In a **rectangular prism**, the bases are congruent rectangles.

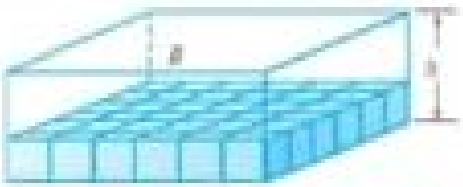
Volume is the amount of space inside a three-dimensional figure. It is measured in **cubic units**, which can be written using abbreviations and an exponent of 3, such as units^3 or cm^3.

Cubes

Cubes are special rectangular prisms. All three of their lengths are equal. So, the volume of a cube can be written using the formula $V = s^3$.

Decomposing the prism tells you the number of cubes of a given size it will take to fill the prism. The volume of a rectangular prism is related to its dimensions, length, width, and height.

Another method to decompose a rectangular prism is to find the area of the base (B) and multiply it by the height (h).

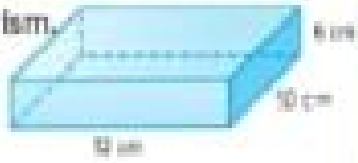


$V = Bh$ = number of rows of cubes needed to fill the prism

area of the base, or the number of cubes needed to cover the base

Example

1. **Find the volume of the rectangular prism.**



B, or the area of the base, is $10 \cdot 12$ or 120 square centimeters. The height of the prism is 6 centimeters.

$V = Bh$ Volume of rectangular prism

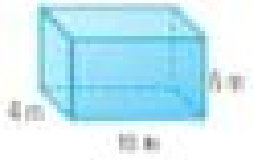
$V = 120 \times 6$ Replace B with 120 and h with 6.

$V = 720$ Multiply

The volume is 720 cubic centimeters.

Get it? **Do these problems to find out.**

a. 

b. 

Decomposing Figures

You can think of the volume of a figure as consisting of the composite units. Each unit contains the area of the base, 12 cm^2, multiplied by a height of 1 cm.

a. 125 cm^3

b. 240 m^3

Example

2. **A cereal box has the dimensions shown. What is the volume of the cereal box?**

Estimate $10 \times 3 \times 10 = 300$

$V = \ell wh$ Volume of a rectangular prism

$V = 8 \times 3\frac{1}{4} \times 12\frac{1}{2}$ Replace ℓ with 8, w with $3\frac{1}{4}$, and h with $12\frac{1}{2}$.

$V = \frac{8}{1} \times \frac{13}{4} \times \frac{25}{2}$ Write as improper fractions. Then divide out common factors.

$V = \frac{325}{1}$ or 325 Multiply

The volume of the cereal box is 325 cubic centimeters.

Check for Reasonableness: $300 \approx 300$ ✓

Get it? **Do this problem to find out.**

c. Find the volume of a container that measures 4 centimeters long, 5 centimeters high, and $\frac{1}{2}$ centimeters wide.

c. 170 cm^3

Find Missing Dimensions

To find missing dimensions of a rectangular prism, replace the variables with known measurements. Then solve for the unknown measurement.

Example

3. Find the missing dimension of the prism.

$V = 84 \text{ m}^3$

$V = \ell wh$ Volume of rectangular prism

$84 = 6 \times 4 \times h$ Replace V with 84, ℓ with 6, and w with 4.

$84 = 24h$ Multiply.

$\frac{84}{24} = \frac{24h}{24}$ Divide each side by 24.

$3.5 = h$ Simplify.

The height of the prism is 3.5 meters.

Check $6 \times 4 \times 3.5 = 84 \text{ m}^3$ ✓

Get It? Do this problem to find out.

a. $V = 94.5 \text{ km}^3$, $\ell = 7$ km, $h = 3$ km, $w = ?$

a. 4.5 km

Guided Practice

1. A rectangular kitchen sink is 64 centimeters long, 50 centimeters wide, and 25 centimeters deep. Find the amount of water that can be contained in the sink. (Examples 1 and 2) 80,000 cm³ or 80 liters

2. Find the missing dimension of a rectangular prism with a volume of 126 cubic centimeters, a width of 7 centimeters, and a height of 2 centimeters. (Example 3) 9 cm

3. **Building on the Essential Question** Why can you use either the formula $V = \ell wh$ or $V = Bh$ to find the volume of a rectangular prism?

Sample answer: The area of the base can be represented as $\ell \times w$ or as B. To find the volume of the prism, multiply the area of the base by the height of the prism.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

Name _____ My Homework _____

Independent Practice

Find the volume of each prism. (Example 1)

1. 132 mi

2. 618.75 mi

3. 171 cm

4. A fishing tackle box is 13 centimeters long, 6 centimeters wide, and $2\frac{1}{2}$ centimeters high. What is the volume of the tackle box? (Example 2)

195 cm

5. Find the length of a rectangular prism having a volume of 2,830.5 cubic meters, width of 18.5 meters, and height of 9 meters. (Example 3)

17 m

Find the missing dimension of each prism. (Example 3)

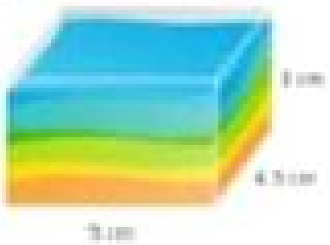
6. 6 cm

7. 3 mm

8. **Be Precise** In Japan, farmers have created watermelons in the shape of rectangular prisms. Find the volume of a prism-shaped watermelon in cubic centimeters if its length is 25 centimeters, its width is 20 centimeters, and its height is 22 centimeters.

11,000 cm

The glass container shown is filled to a height of 2.25 centimeters.



a. How much sand is currently in the container?

50.625 cm

b. How much more sand could the container hold before it overflows? 16.875 cm

c. What percent of the container is filled with sand? 25%

10. **Identify Structure** Refer to the graphic novel frame below for Exercises a–c.

a. Muna chose the box on the left. If it is 20 centimeters long, 20 centimeters wide, and 20 centimeters tall, what is the volume of Muna's box?

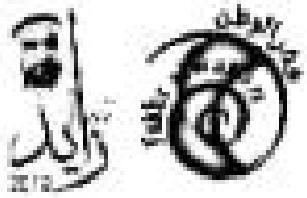
8000 cm³

b. Huda chose the box on the right. If it is 20 centimeters long, 15 centimeters wide, and 25 centimeters tall, what is the volume of Huda's box?

7500 cm³

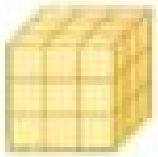
c. Who received more popcorn, Muna or Huda? How much more?

Muna; 500 cm³



H.O.T. Problems Higher Order Thinking

11. **Persevere with Problems** Refer to the prism at the right. If all the dimensions of the prism doubled, would the volume double? Explain your reasoning.



No; the volume of the figure is 27 cubic units. If the dimensions doubled, the volume would be 216 cubic units, eight times greater.

12. **Justify Conclusions** Which has the greater volume: a prism with a length of 5 centimeters, a width of 4 centimeters, and a height of 10 centimeters; or a prism with a length of 10 centimeters, a width of 5 centimeters, and a height of 4 centimeters?

Justify your selection. They both have the same volume. volume of the first prism: $5 \times 4 \times 10$ or 200 cm³; volume of the second prism: $10 \times 5 \times 4$ or 200 cm³

13. **Model with Mathematics** Write a real-world problem in which you need to find the volume of a right rectangular prism. Solve your problem.

Sample answer: A gift box is 7 centimeters long, 9 centimeters wide, and 4 centimeters tall. What is the volume of the gift box?; 252 cm

Extra Practice

Find the volume of each prism.

14. 101.84 cm³

$V = lwh$

$V = 7 \times 4\frac{1}{2} \times 3\frac{3}{5}$

$V = \frac{7}{1} \times \frac{21}{5} \times \frac{18}{5}$

$V = \frac{2,646}{25}$

$V = 101.84$

15. 1,430 m³

16. 6,664.77 m³

17. Find the volume of the pet carrier shown at the right.

37,833.75 cm³

18. What is the width of a rectangular prism with a length of 13 meters, volume of 11,232 cubic meters, and height of 36 meters?

24 m

19. A canyon is 120 kilometers long, as much as 20 kilometers wide, and has a maximum depth of more than 0.15 kilometer. What is the approximate volume of this canyon?

360 km³

20. **Use Math Tools** Use the table at the right.

a. What is the approximate volume of the small truck?

Sample answer: $3 \times 2.75 \times 2.5 = 21$ km³

b. Ayman's family is moving, and they estimate that they will need a truck with about 36 cubic meters. Which truck would be best for them to rent?

3-bedroom moving truck

c. About how many cubic meters greater is the volume of the Mega Moving Truck than the 2-bedroom moving truck?

$7.5 \times 2.75 \times 2.8 = 58$ m³

$4.5 \times 2.75 \times 2.2 = 27$ m³

58 m³ $- 27$ m³ $= 31$ m³

Inside Dimensions of Moving Trucks			
Truck	Length (m)	Width (m)	Height (m)
Van	3	2.5	2
Small Truck	3.3	2.75	2.5
2-Bedroom Moving Truck	4.5	2.75	2.2
3-Bedroom Moving Truck	7	2.5	2.9
Mega Moving Truck	7.5	2.75	2.8

Power Up! Test Practice

21. The volume of the rectangular prism shown is 2,520 cubic centimeters. Fill in the boxes to complete each statement.

a. To find the width of the prism, divide 2,520 by the product of 14 and 9.

b. The width of the prism is 20 centimeters.

22. A pet carrier company is creating a new size carrier. It has a length of 27 centimeters, a width of 7 centimeters, and a volume of 6,426 cubic centimeters. Select values to complete the formula below to find the height h of the carrier.

7
27
6,426
h

$6{,}426 = 27 \times 7 \times h$

What is the height of the pet carrier? 34 cm

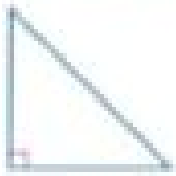
Spiral Review

Classify each triangle by the measure of the angles.

23. acute triangle

24. obtuse triangle

25. right triangle



26. Draw the next figure in the pattern below.

27. Triangles are often used in designing bridges. Classify the triangle shown by the measure of its sides. Explain.

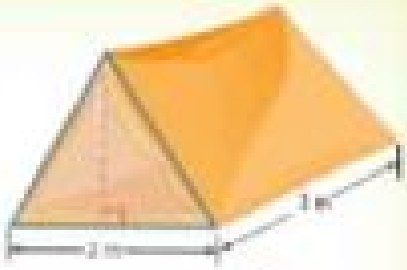
isosceles; Sample answer: The triangle has two congruent sides.

Lesson 2

Volume of Triangular Prisms

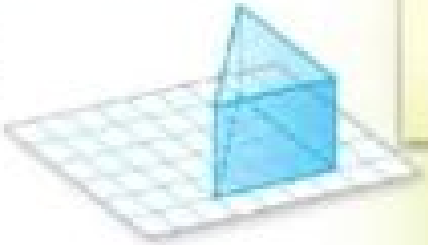
Real-World Link

Camping Ali has a pup tent like the one shown. The opening of the tent has a base and a height of 2 meters. The length of the tent is 3 meters.



What is the area of the front triangular face? 2 square meters

On a piece of grid paper, draw a right triangle with a base and height of 4 units as shown.



1. What is the area of the triangle?
   8 units²

2. Suppose you cover the triangle with cubes the size of one square on the grid paper. How many cubes would you use? (*Hint*: You can cut and reassemble the cubes.) 8 cubes

3. How many cubes would you use if you had 4 layers? 32 cubes

4. **Make a Conjecture** Write a formula to find the volume of a triangular prism. $V = Bh$

Essential Question

How is shape important when measuring a figure?

Vocabulary

triangular prism

Mathematical Practices

1, 3, 4, 5, 8

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

- ① Persevere with Problems
- ② Reason Abstractly
- ③ Construct an Argument
- ④ Model with Mathematics
- ⑤ Use Math Tools
- ⑥ Attend to Precision
- ⑦ Make Use of Structure
- ⑧ Use Repeated Reasoning

Key Concept

Volume of a Triangular Prism

Words The volume V of a triangular prism is the area of the base B times the height h.

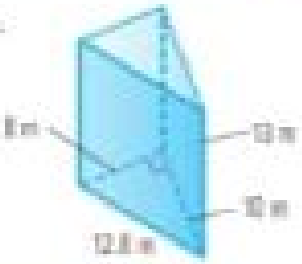
Symbols $V = Bh$, where B is the area of the base

Model

In a **triangular prism** the bases are congruent triangles. The diagram below shows that the volume of a triangular prism is also the product of the area of the base B and the height h of the prism.

Example

1. **Find the volume of the triangular prism.**



The area of the triangle $\frac{1}{2} \cdot 8 \cdot 10$,
so B is $\frac{1}{2} \cdot 8 \cdot 10$.

$V = Bh$ Volume of a prism

$V = \frac{1}{2} \cdot 8 \cdot 10 h$ Replace B with $\frac{1}{2} \cdot 8 \cdot 10$.

$V = \frac{1}{2} \cdot 8 \cdot 10 \cdot 13$ Replace h with 13, the height of the prism.

$V = 520$ Multiply

The volume is 520 cubic meters or 520 m

Got it? Do these problems to find out.

a.

b.

Base

Before finding the volume of a triangular prism, identify the base. In Exercise 9, the base is the one that "sits on" the ground. The base is one of the parallel faces.

a. 49 m²

b. 280 m²

Example

2. **A large skateboard ramp is shown. Find the volume of the triangular prism.**

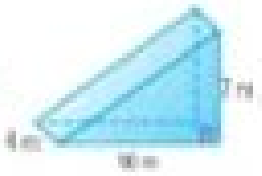
The base is a triangle with a base length of 10 meters and a height of 7 meters. The height of the prism is 4 meters.

$V = Bh$ — Volume of a prism

$V = \left(\frac{1}{2} \cdot 10 \cdot 7\right)h$ — Replace B with $\frac{1}{2} \cdot 10 \cdot 7$

$V = \left(\frac{1}{2} \cdot 10 \cdot 7\right)4$ — Replace h with 4, the height of the prism

$V = 140$ — Multiply



The volume is 140 cubic meters or 140 m^3.

Get it? Do this problem to find out.

c. Find the volume of a triangular prism-shaped model whose base has an area of 32 square centimeters and a height of 6 centimeters.

c. 192 cm^3

Find Missing Dimensions

To find missing dimensions of a triangular prism, replace the variables with known measurements. Then solve for the unknown measurement.

Example

3. **Find the height of the triangular prism.**

$V = Bh$ — Volume of a triangular prism

$V = \left(\frac{1}{2} \cdot 1 \cdot 0.3\right)h$ — Replace B with $\frac{1}{2} \cdot 1 \cdot 0.3$

$12 = \left(\frac{1}{2} \cdot 1 \cdot 0.3\right)h$ — Replace V with 12.

$12 = 0.15h$ — Multiply

$\frac{12}{0.15} = \frac{0.15h}{0.15}$ — Divide each side by 0.15.

$80 = h$ — Simplify

So, the height of the prism is 80 cm.

Get it? Do this problem to find out.

Find the missing dimension of the triangular prism.

d. $V = 55\ km^3$, base length = 2 km, base height = 5 km, h = ?

d. 11 km

Example

4. Ayoub bought a cheese wedge for his Dinner party. The cheese wedge has the dimensions shown. The volume of the cheese wedge is 54 cubic centimeters. What is the height of the cheese wedge?

$V = Bh$ Volume of a triangular prism

$54 = \left(\frac{1}{2} \cdot 3 \cdot 4\right)h$ Replace V with 54, and B with $\frac{1}{2} \cdot 3 \cdot 4$

$54 = 6h$ Multiply.

$\frac{54}{6} = \frac{6h}{6}$ Divide each side by 6.

$9 = h$ Simplify.

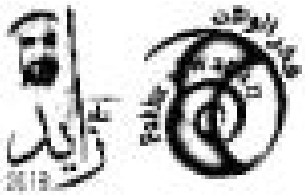
So, the height of the cheese wedge is 9 centimeters.

Guided Practice

Find the volume of each prism. Round to the nearest tenth if necessary. (Example 1)

1. 45 cm³

2. 12 m³



3. Mazen has a triangular-shaped piece of cheesecake in his lunch. Find the volume of the piece of cheesecake. (Example 2)

36 cm³

4. Find the base length of a shipping box in the shape of a triangular prism. The shipping box has a volume of 7.56 cubic meters, a base height of 2.1 meters, and a height of 3 meters. (Examples 3 and 4)

2.4 m

5. **Building on the Essential Question** How is the area of a triangle related to the volume of a triangular prism?

Sample answer: To find the volume of a triangular prism, you multiply the area of the triangular base B times the height h of the prism.

Rate Yourself!

How well do you understand volume of triangular prisms? Circle the image that applies.

Follow-up: Time to adjust your flexibility

Name _____ My Homework _____

Independent Practice

Find the volume of each prism. Round to the nearest tenth if necessary.

1. 336 m³

2. 140.4 m³

3. 104.0 cm³

4. A wheelchair ramp is in the shape of a triangular prism. It has a base area of 37.4 square meters and a height of 5 meters. Find the volume of the ramp. (Example 2)

187 m³

5. A triangular prism has a height of 9 centimeters. The triangular base has a base of 3 centimeters and a height of 8 centimeters. Find the volume of the prism. (Example 2)

108 cm³

Find the missing dimension of each triangular prism. (Example 3)

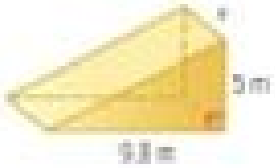
6. x = 2 m

V = 30 m³

7. x = 8 cm

V = 390 cm³

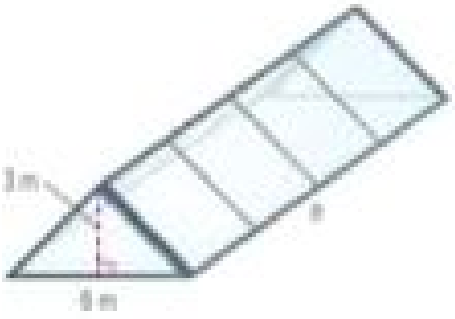
8. x = 4 m



V = 98 m³

9. Eissa's greenhouse has the dimensions shown. The volume of the greenhouse is 90 cubic meters. Find the missing dimension of the greenhouse. (Example 4)

10 m



10. **Be Precise** Noreen built the model house shown.

a. What is the volume of the first floor?

9,000 cm³

b. What is the volume of the attic space?

3,600 cm³

H.O.T. Problems Higher Order Thinking

11. **Find the Error** Huda is finding the volume of the triangular prism. Find her mistake and correct it.

To find the base area, Huda should have multiplied by $\frac{1}{2}$. The base area of the prism is 6 cm^2, not 12 cm^2. So, the volume of the prism is 42 cm^3.

12. **Identify Repeated Reasoning** A rectangular prism and a triangular prism each have a volume of 210 cubic meters. Find possible sets of dimensions for each prism.

Sample answer: Rectangular prism: length, 7; width, 5; height, 6; triangular prism: area of base, 35 sq. meters; height, 6 meters

13. **Persevere with Problems** A candy company sells mints in two different containers. Which container shown below holds more mints? Justify your answer.

The rectangular prism will hold more mints than the triangular prism. The rectangular prism has a volume of 144 cm^3 while the triangular prism has a volume of 72 cm^3.

14. **Persevere with Problems** Explain a method you could use to find the volume of the prism below. Then find the volume of the prism.

Sample answer: The formula for the volume of a prism is $V = Bh$, where B is the area of the base. Since the base is a trapezoid, replace B with $\frac{1}{2}h(b_1 + b_2)$, substitute, and simplify; 364 cm^3

Extra Practice

Find the volume of each prism. Round to the nearest tenth if necessary.

15. 146.5 m³

$V = Bh$

$V = \left(\frac{1}{2} \cdot 7 \cdot 8\right)(11)$

$V = 146.5$

16. 595 m³

17. 380 cm³

18. A candle is in the shape of a triangular prism. The base has an area of 30 square centimeters. The candle has a height of 6 centimeters. Find the volume of the candle.

180 cm³

19. A box is in the shape of a triangular prism. The triangular base has a base length of 14 centimeters and a base height of 22 centimeters. The box is 675 centimeters tall. What is the volume of the box?

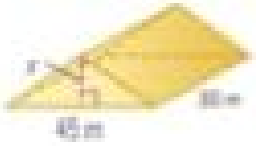
10,395 cm³

Find the missing dimension of each triangular prism.

20. x = 12 cm

$V = 6{,}300$ cm³

21. x = 15 m



$V = 10{,}125$ m³

22. x = 2 cm

$V = 3.5$ m³

23. What is the volume of the A-frame tent shown?

48 m³

24. **Be Precise** A covered bridge has the dimensions shown.

a. What is the volume of the bottom section rounded to the nearest tenth? 15,505.2 m³

b. What is the volume of the top section, rounded to the nearest tenth? 2,235.5 m³

Power Up! Test Practice

25. A triangular prism has a volume of 240 cubic meters. Which of the following are possible dimensions for the area of the base and the height of the prism? Select all that apply.

- [x] $B = 48$ m², $h = 5$ m
- [x] $B = 24$ m², $h = 10$ m
- [x] $B = 12$ m², $h = 20$ m
- [ ] $B = 50$ m², $h = 4$ m

26. A kitchen cabinet manufacturer offers three different size corner cabinets with the dimension shown below. Sort the volume of the cabinets from least to greatest.

	Cabinet	Volume (in³)
Least		
Greatest		

Cabinet A

Cabinet B

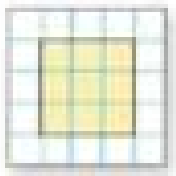
Cabinet C

Which cabinet has the greatest volume? Cabinet B

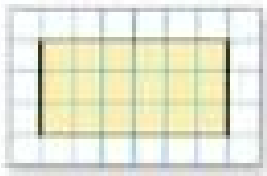
Spiral Review

Find the area of each figure.

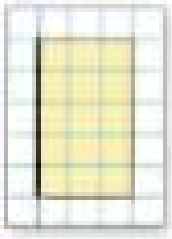
27. 9 units²



28. 18 units²



29. 15 units²



30. Najat is building a birdhouse. The nails she uses are 2.5 centimeters long. The wood board is 30 centimeters long. How many times smaller are the nails compared to the wood? 12 times

Case #2 Contain Your Fun

A storage container is made from plastic that measures 45 centimeters long, 60 centimeters wide, and 75 centimeters high.

Find the surface area of the plastic container including the lid.

1 Understand

Read the problem. What are you being asked to find?

I need to find the surface area of the container, including the lid.

Underline key words and values in the problem. What information do you know?

The storage container measures 45 cm long, 60 cm wide, and 75 cm high.

2 Plan

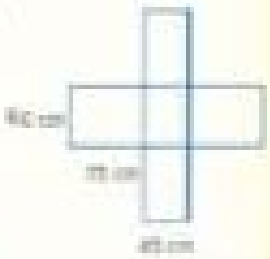
Choose a problem-solving strategy.

I will use the make a model strategy.

3 Solve

Use your problem-solving strategy to solve the problem.

Make a model of the container using a net. Then find the area of each rectangle to find the total surface area.



front and back: $2(60 \times 75) = 9{,}000$

left and right: $2(60 \times 45) = 5{,}400$

top and bottom: $45 \times 75 + 6{,}750$

Sum of 6 sides: $9{,}000 + 5{,}400 + 6{,}750 = 21{,}150$

So, the surface area of the container is 21,150 square centimeters.

4 Check

Use information from the problem to check your answer.

Substitute known values into the surface area formula to check your answer.

$S.A. = (2 \times 60 \times 75) + (2 \times 45 \times 75) + (2 \times 60 \times 45) = 21{,}150\ cm$

Work with a small group to solve the following cases. Show your work on a separate piece of paper.

Case #3 Assembly

Khalid is helping set up 7 rows of chairs for a school assembly. There are eight chairs in the first row. Each row after that has two more chairs than the previous row.

If he has 100 chairs, can he set up enough rows? Explain.

yes; Sample answer: 8 + 10 + 12 + 14 + 16 + 18 + 20 = 98;

Since 98 < 100, there are enough chairs.

Case #4 Paper

Nasser took a piece of notebook paper and cut it in half. Then he placed the 2 pieces on top of each other and cut them in half again to have 4 pieces of paper.

If he could keep cutting the paper in this manner, how many pieces of paper would he have after 6 cuts?

64 pieces

Case #5 Sports

Rashid is packing a crate with boxes of miniature golf putters. Each box has a height of 0.5 meters, a width of 0.5 meters, and a length of 1 meter.

How many boxes can Rashid fit in a crate that is 2 meters high, 2 meters wide, and 1 meter long?

16 boxes

Case #6 Patterns

Draw the next geometric figure in the pattern.

Use any shapes!

Mid-Chapter Check

Vocabulary Check

1. **Be Precise** Define *three-dimensional figure*. Give an example of a figure that is a three-dimensional figure and an example of a figure that is not a three-dimensional figure.

A three-dimensional figure is a solid figure that has length, width, and height. Sample answer: rectangular prism; rectangle

Fill in the blanks in the sentences below with the correct terms.

2. Volume is the amount of space inside a three-dimensional figure.

3. Volume is measured in cubic units.

Skills Check and Problem Solving

Find the volume of each prism. Round to the nearest tenth if necessary.

4. 4.2 cm³

5. 191.4 cm³

6. 6.4 cm³

Find the missing dimension of each figure.

7. rectangular prism: $V = 80$ m³; length = 5 m; width = 4 m

$h =$ 4 m

8. triangular prism: $V = 42$ cm³; base length = 2 cm; base height = 6 cm

$h =$ 7 cm

9. **Persevere with Problems** Khawla is mailing a candle that is in the shape of a triangular prism as shown. She put the candle in a rectangular box that measures 3 centimeters by 5 centimeters by 7 centimeters and places foam pieces around the candle. Find the volume of the foam pieces needed to fill the space between the candle and the box. 85 cm³

Inquiry Lab
Surface Area of Rectangular Prisms

inquiry **HOW can you use nets to find surface area?**

Mathematical Practices 1, 3, 4

If you want to know the amount of cereal that can fit in the box, you would find the volume. But if you want to know how much cardboard is needed to make the box, you would find the **surface area**.

Hands-On Activity 1

One way to find the surface area is to use a **net**. Nets are two-dimensional patterns of three-dimensional figures. When you construct a net, you are decomposing the three-dimensional figure into separate shapes.

Sample answers are provided for Activity 1.

Step 1 Use a box in the shape of a rectangular prism. Measure and record the length, width, and height of the box on the lines below.

Length: 6 centimeters

Width: 2 centimeters

Height: 12 centimeters

Step 2 Using a marker, label the top, bottom, front, back, and side faces of the box.

Step 3 Using scissors, carefully cut along three edges of the top face and then cut down each vertical edge.

Step 4 Measure and record the area of each face, using the dimensions of the box shown in the table.

Face	Length	Width	Area of Face
Front	12 cm	6 cm	72 cm^2
Back	12 cm	6 cm	72 cm^2
Side 1	2 cm	12 cm	24 cm^2
Side 2	2 cm	12 cm	24 cm^2
Top	6 cm	2 cm	12 cm^2
Bottom	6 cm	2 cm	12 cm^2

Step 5 Add the areas of each face to find the surface area of the box.

72 + 72 + 24 + 24 + 12 + 12 = 216

So, the surface area of the box is 216 square centimeters.

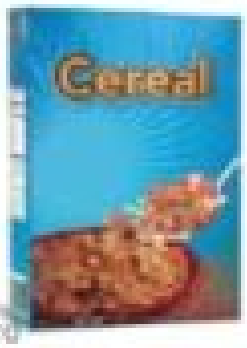
Inquiry Lab

Surface Area of Rectangular Prisms

inquiry **HOW can you use nets to find surface area?**

Mathematical Practices 1, 2, 4

If you want to know the amount of cereal that can fit in the box, you would find the volume. But if you want to know how much cardboard is needed to make the box, you would find the **surface area**.



Hands-On Activity 1

One way to find the surface area is to use a **net**. Nets are two-dimensional patterns of three-dimensional figures. When you construct a net, you are decomposing the three-dimensional figure into separate shapes.

Sample answers are provided for Activity 1.

Step 1 Use a box in the shape of a rectangular prism. Measure and record the length, width, and height of the box on the lines below.

Length: 6 centimeters

Width: 2 centimeters

Height: 12 centimeters

Step 2 Using a marker, label the top, bottom, front, back, and side faces of the box.

Step 3 Using scissors, carefully cut along three edges of the top face and then cut down each vertical edge.

Step 4 Measure and record the area of each face, using the dimensions of the box shown in the table.

Face	Length	Width	Area of Face
Front	12 cm	6 cm	72 cm^2
Back	12 cm	6 cm	72 cm^2
Side 1	2 cm	12 cm	24 cm^2
Side 2	2 cm	12 cm	24 cm^2
Top	6 cm	2 cm	12 cm^2
Bottom	6 cm	2 cm	12 cm^2

Step 5 Add the areas of each face to find the surface area of the box.

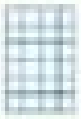
72 + 72 + 24 + 24 + 12 + 12 = 216

So, the surface area of the box is 216 square centimeters.

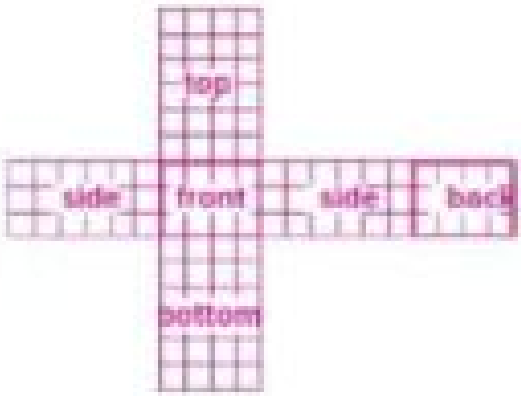
Hands-On Activity 2

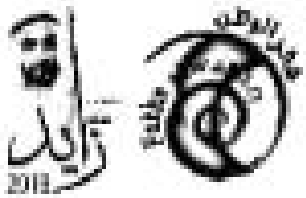
Orthogonal drawings consist of separate views of an object taken from different angles. You can make a net from orthogonal drawings.

Step 1 Find the dimensions of each side of a rectangular prism from the orthogonal drawing.

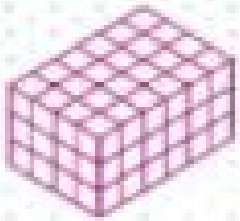
Orthogonal Drawing		
View	Drawing	Dimensions
Front and Back		3 × 4
Sides		3 × 6
Top and Bottom		6 × 4

Step 2 Use grid paper to draw a net from the orthogonal drawing. Trace and cut out your drawing and tape it in the space below. Check the dimensions of each face using the information in the table.





Step 3 Fold the net into a three-dimensional figure. Draw the resulting figure in the space provided.



So, the figure is rectangular prism.

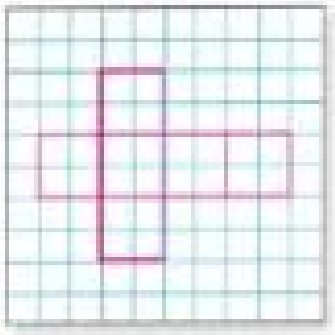
It has a surface area of 108 square units.

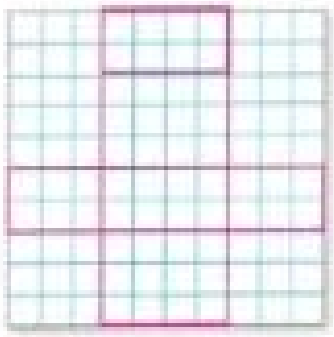
Investigate

Model with Mathematics Work with a partner. Use a net to determine the surface area of each prism. Draw a net of each prism on the provided grid.

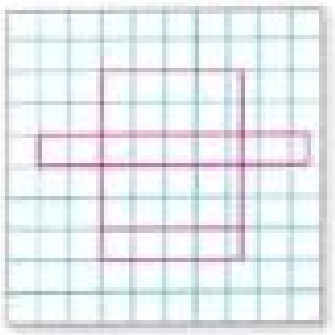
1. 24 mm^2

2. 52 cm^2

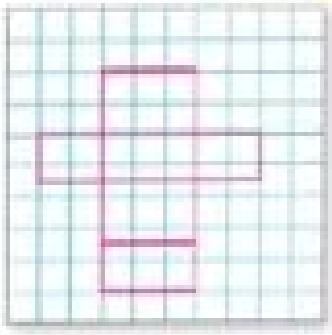




3. 31 m^2



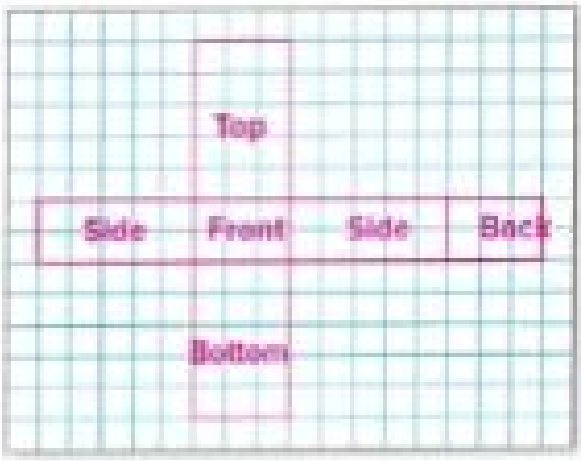
4. 27 m^2



Draw a net on the grid from the orthogonal drawing. Then find the surface area of the prism.

5. 62 square units

Orthogonal Drawing	
View	Drawing
Front and Back	
Sides	
Top and Bottom	



Analyze and Reflect

Sample answers: 10, 12, 14

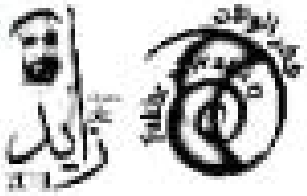
Work with a partner to complete the table. The first one is done for you.

	Dimensions of Rectangular Prism	Area of Top (units²)	Area of Bottom (units²)	Area of Side 1 (units²)	Area of Side 2 (units²)	Area of Front (units²)	Area of Back (units²)	Surface Area (units²)
	1 × 2 × 5	2	2	4	4	5	5	22
6.	2 × 2 × 5	4	4	6	6	6	6	32
7.	3 × 3 × 3	9	9	9	9	9	9	54
8.	3 × 2 × 8	6	6	16	16	24	24	92
9.	6 × 6 × 6	36	36	36	36	36	36	216

10. Compare the surface area for Exercise 7 to the surface area for Exercise 9. How does doubling each dimension affect the surface area?

The prism in Exercise 9 has a surface area that is four times that of the prism in Exercise 7. Doubling each dimension multiplies the area of each face by four, which multiplies the surface area by four.

11. **Reason Inductively** Write a formula to find the surface area of a rectangular prism. Use your formula to find the surface area of the prism in Activity 2. $SA = 2lw + 2lh + 2wh$; 108 square units



Create

12. **Model with Mathematics** Write a real-world problem that involves the surface area of rectangular prisms. Provide the dimensions and the surface area.

Ghaya is wrapping a box that is 3 centimeters high, 12 centimeters long, and 6 centimeters wide. What is the surface area of the box?; 252 square centimeters

13. Will the surface area of a cube ever have the same numerical value as the volume of the cube?

yes; Sample answer: When the side length of a cube is 6 units, the surface area and volume have the same numerical value, 216.

14. **Inquiry** HOW can you use nets to find surface area?

Nets help you to determine the area of each face of a rectangular prism. Then add the areas of the faces to find the surface area.

Lesson 3

Surface Area of Rectangular Prisms

Vocabulary Start-Up

Sample answers are given.

Define surface	Define area
the outer face, outside or exterior boundary of a thing	the number of square units needed to cover the surface of a closed figure
What is surface area?	Example:
the sum of the areas of all the surfaces (faces) of a three-dimensional figure	the amount of wrapping paper to cover a box

Essential Question

HOW is shape important when measuring a figure?

Vocabulary

surface area

Mathematical Practices

1, 3, 4, 6

Real-World Link

Gifts Abeer is wrapping a gift for her sister's graduation. She places it in a box with the measurements shown.

1. What is the area of one face of the box?
   100 square centimeters

2. How many faces does the box have? 6

3. What operations would you use to find the surface area of the box?
   Sample answer: multiplication and addition

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

- ① Persevere with Problems
- ② Reason Abstractly
- ③ Construct an Argument
- ④ Model with Mathematics
- ⑤ Use Math Tools
- ⑥ Attend to Precision
- ⑦ Make Use of Structure
- ⑧ Use Repeated Reasoning

Key Concept

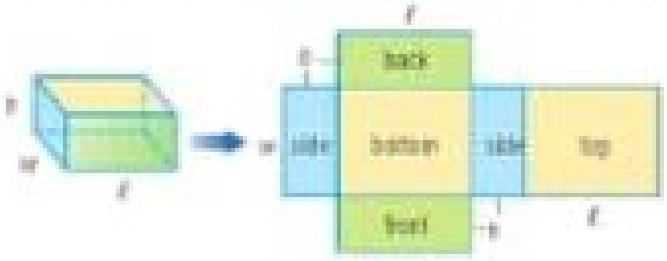
Surface Area of a Rectangular Prism

Words The surface area S.A. of a rectangular prism with length ℓ, width w, and height h is the sum of the areas of the faces.

Model

Symbols $S.A. = 2\ell h + 2\ell w + 2hw$

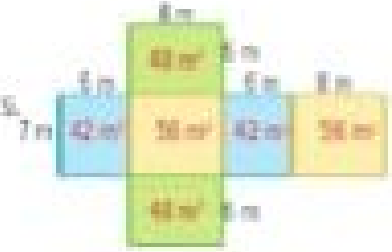
The surface area of a prism is the sum of the areas of its faces.



front and back: $\ell h + \ell h = 2\ell h$
top and bottom: $\ell w + \ell w = 2\ell w$
two sides: $hw + hw = 2hw$
$\Big\}\ 2\ell h + 2\ell w + 2hw$

Example

1. Find the surface area of the rectangular prism.



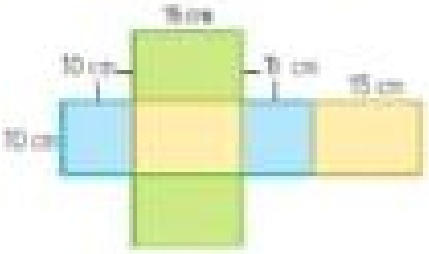
Find the area of each pair of faces.
front and back: $2(\blacksquare 6) = 2(48)$
top and bottom: $2(\blacksquare 8) = 2(56)$
sides: $2(7 \cdot 6) = 2(42)$

$48 + 48 + 56 + 56 + 42 + 42 = 292$ Add the areas of each face.

So, the surface area is 292 square meters.

Got it? Do this problem to find out.

a. Find the surface area of the rectangular prism.



Nets

The net shows that a rectangular prism has six faces. The faces can be grouped as three pairs of congruent faces. The colors indicate which faces are congruent.

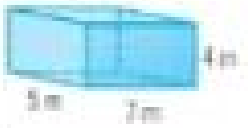
a. 800 cm²

Find Surface Area Using a Formula

You can use nets or models to find the surface area of a rectangular prism. You can also use the surface area formula, $S.A. = 2\ell h + 2\ell w + 2hw$.

Examples

2. **Find the surface area of the rectangular prism.**



Find the area of each face.

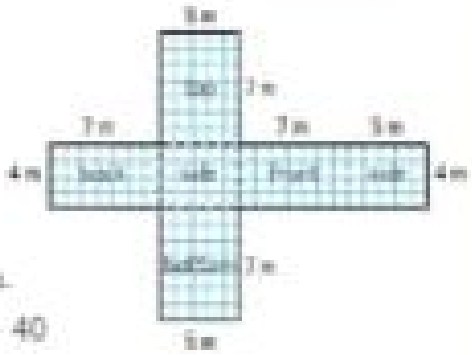
front and back:
$2\ell h = 2(7)(4)$ or 56

top and bottom:
$2\ell w = 2(7)(5)$ or 70

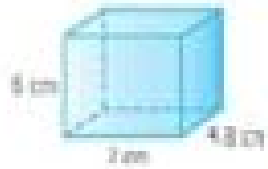
left and right sides:
$2hw = 2(4)(5)$ or 40

Add to find the surface area.

The surface area is 56 + 70 + 40 or 166 square meters.



3. **Find the surface area of the rectangular prism.**



To find the area of each face, determine the dimensions.

$\ell = 7$, $w = 4.8$, $h = 6$

front and back: $2\ell h = 2($ 7 $)($ 6 $)$ or 84

top and bottom: $2\ell w = 2($ 7 $)($ 4.8 $)$ or 67.2

two sides: $2hw = 2($ 6 $)($ 4.8 $)$ or 57.6

Add to find the surface area.

84 + 67.2 + 57.6 or 208.8 square centimeters

Got it? Do this problem to find out.

b. Find the surface area of the rectangular prism.

b. 52 m²

Example

4. **STEM** A geode is being sent as a gift. It is packed in a box that measures 7 centimeters long, 3 centimeters wide, and 16 centimeters tall. What is the surface area of the box?

$S.A. = 2lh + 2lw + 2hw$ Surface area of a prism

$S.A. = 2(7)(16) + 2(7)(3) + 2(16)(3)$ $l = 7, w = 3, h = 16$

$S.A. = 14(16) + 14(3) + 32(3)$ Multiply

$S.A. = 224 + 42 + 96$ Multiply

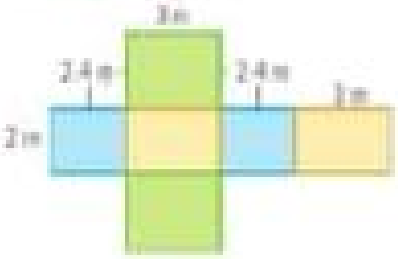
$S.A. = 362$ Add

The surface area of the box is 362 square centimeters.

Guided Practice

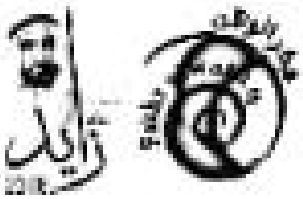
Find the surface area of each rectangular prism. Example 1–3

1. 36 m²



2. 300.75 m²

3. 298 cm²



4. Salem keeps his diecast car in a glass display case as shown. What is the surface area of the glass, including the bottom? Example 4

390 cm²

5. **Building on the Essential Question** What is the relationship between area and surface area?

Sample answer: Surface area is calculated for a three-dimensional figure. It is the sum of the areas of the surfaces that make up the three-dimensional figure.

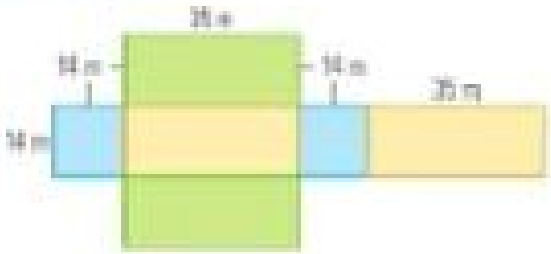
Rate Yourself!

Are you ready to move on? Should the section Hunt app?

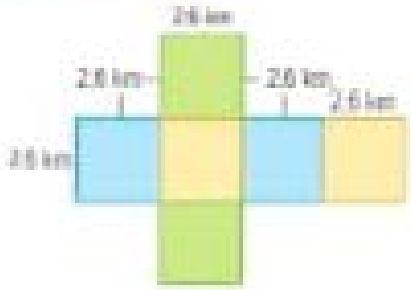
Independent Practice

Find the surface area of each rectangular prism.

1. 2,352 m²



2. 40.56 km²



3. 3,668.94 m²

4. 256 cm²

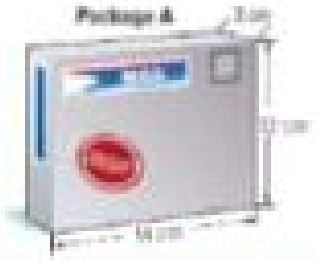
5. A game box for video games is shaped like a rectangular prism. What is the surface area of the game box?

1,162 cm²

6. **Justify Conclusions** Shaime estimates that the surface area of a rectangular prism with a length of 13.2 meters, a width of 6 meters, and a height of 8 meters is about 460 square meters. Is her estimate reasonable? Explain your reasoning.

Yes; the approximate surface area of the rectangular prism is (2 × 13 × 6) + (2 × 13 × 8) + (2 × 6 × 8) or 460 m².

7. **Justify Conclusions** Find the surface area of each shipping package. Which package has the greater surface area? Does the same package have a greater volume? Explain your reasoning to a classmate.



Package A: 492 cm²; Package B: 404 cm²; Package A has a greater surface area. No, the volume of Package B is greater.

8. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–c.

a. The box on the left is 20 centimeters long, 20 centimeters wide, and 20 centimeters tall. What is the surface area of the box? 2000 cm²

b. The box on the right is 20 centimeters long, 15 centimeters wide, and 25 centimeters tall. What is the surface area of the box? 2050 cm²

c. How much more surface area does the larger container have?
50 cm²

H.O.T. Problems Higher Order Thinking

Persevere with Problems All of the triangular faces of the figure are congruent.

9. What is the area of one of the triangular faces? the square face?
48 cm²; 144 cm²

10. Use what you know about finding the surface area of a rectangular prism to find the surface area of the square pyramid.
336 cm²

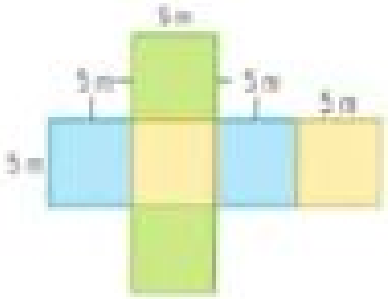
11. **Model with Mathematics** Sketch two prisms such that one has a greater volume and the other has a greater surface area. Include real-world units. See students' work.

Name _____ My Homework _____

Extra Practice

Find the surface area of each rectangular prism.

12. 150 m^2

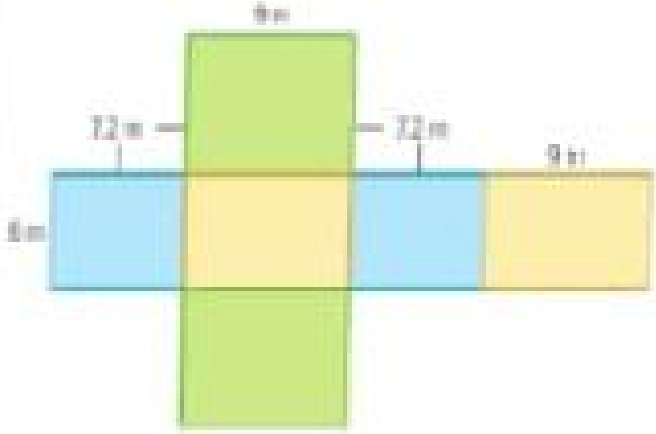


$2(5)(5) + 2(5)(5) + 2(5)(5)$

$= 50 + 50 + 50$

$= 150$

13. 324 m^2



14. 142 m^2

15. 384.62 cm^2

16. Suha is going to paint her younger sister's toy chest, including the bottom. What is the approximate surface area that she will paint? 20,544 cm^2

17. **Identify Repeated Reasoning** Keikha is making a bird nesting box for her backyard.

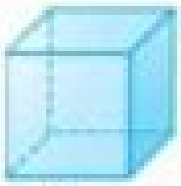
a. What is the surface area of the nesting box, including the hole? 1,822.5 cm^2

b. What is the surface area if the width of 18 centimeters is doubled? 3082.5 cm^2

c. What is the surface area if the width of 18 centimeters is half as great? 1192.5 cm^2

Power Up! Test Practice

18. A company is experimenting with two new boxes for packaged merchandise. Each box is a cube with the side lengths shown.



Select the correct values to complete each statement.

a. The surface area of the smaller box is 864 square centimeters.

b. The surface area of the larger box is 1,944 square centimeters.

c. The ratio of the side lengths of the smaller box to the side lengths of the larger box, in lowest terms, is 2 to 3.

d. The ratio of the surface area of the smaller box to the surface area of the larger box, in lowest terms, is 4 to 9.

2	9
3	864
4	1,728
6	1,944
8	5,832

Are the ratios in parts c and d the same? Did you expect them to be the same? Explain your reasoning.

no; Sample answer: No, since to find the surface area, the side lengths are squared for each face. It makes sense that the ratios are different.

19. Which measure(s) can be classified as surface area? Select all that apply.

- ☐ the amount of water in a lake
- ■ the amount of wrapping paper needed to cover a box
- ■ the amount of paint needed to cover a statue
- ☐ the amount of space needed to build a playset

Spiral Review

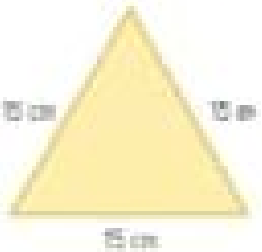
Add or multiply.

20. $14 \times 16 =$ 224

21. $72 + 62 + 84 =$ 218

22. $27 \times 63 =$ 1,701

23. Classify the triangle by the measure of its sides. Explain.



equilateral; Sample answer: All three sides measure 15 centimeters.

Inquiry Lab
Nets of Triangular Prisms

Inquiry **HOW is the area of a triangle related to the surface area of a triangular prism?**

Mathematical Practices 1, 3, 4, 7

A computer hardware company packages batteries and cords in boxes shaped like triangular prisms. You can use nets and drawings to determine the surface area of the box.

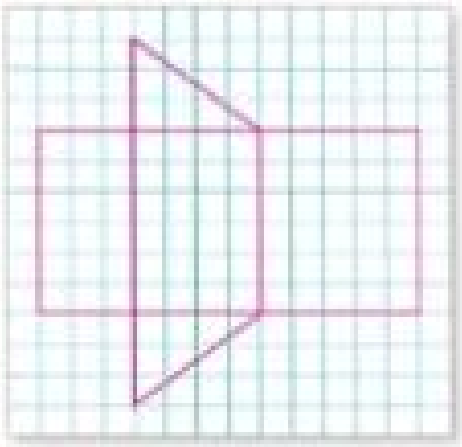
Hands-On Activity

Use orthogonal drawings to find the surface area of a triangular prism. A *triangular prism* is a prism that has triangular bases.

Step 1 Find the dimensions of each side of the triangular prism from the orthogonal drawing.

Orthogonal Drawing							
View	Drawing	Dimensions (cm)	Area of Face (cm²)	View	Drawing	Dimensions (cm)	Area of Face (cm²)
Back		base = 3 height = 4	$\frac{1}{2}(3 \times 4) = 6$	Bottom		length = 6 width = 5	$6 \times 5 = 30$
Left		length = 6 width = 3	$6 \times 3 = 18$	Right		length = 6 width = 4	$6 \times 4 = 24$

Step 2 Use grid paper to draw a net. Check the dimensions of each face using the information in the table.



Step 3 Add the area of each face to find the surface area of the figure. Remember, there are two bases.

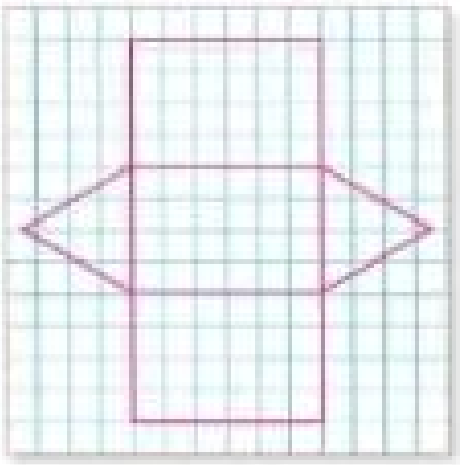
6 + 6 + 18 + 24 + 30 = 84

So, the surface area is 84 square units.

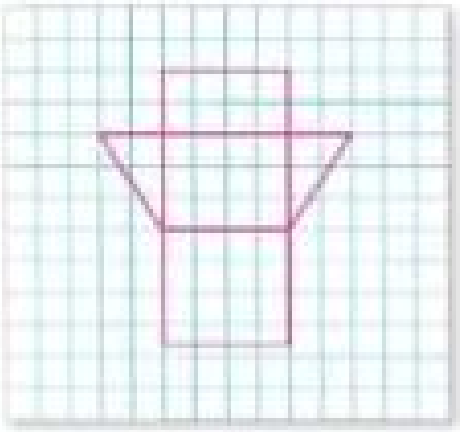
Investigate

Model with Mathematics Work with a partner. Use nets to determine the surface area of each prism. Draw a net of each prism on the provided grid paper.

1. 86 cm²



2. 40.4 cm²



Create

3. **Identify Structure** Explain how to find the surface area of a triangular prism, using only the dimensions of the figure. Use the dimensions in Exercise 2 to explain your answer.

Sample answer: Multiply to find the area of each face. Add the area of each face to find the surface area of the prism. 3 + 14.4 + 8 + 12 = 40.4

4. **Explain** HOW is the area of a triangle related to the surface area of a triangular prism?

Sample answer: If you know the area of a triangle, you can find the area of the bases of a triangular prism. Add the area of the bases of a triangular prism to the combined area of the other faces to find the surface area of the figure.

Lesson 4

Surface Area of Triangular Prisms

Real-World Link

Ramp Husam and his dad are building a ramp to move his dirt bike onto a trailer.

Essential Question

HOW is shape important when measuring a figure?

Mathematical Practices

1, 2, 3, 4, 6

Fill in the table by drawing the sides of the ramp and naming the shape of each face.

	Face	Draw the Face	Shape of the Face
1.	Front		triangle
2.	Back		triangle
3.	Top		rectangle
4.	Bottom		rectangle
5.	Side		rectangle

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1 Persevere with Problems	5 Use Math Tools
2 Reason Abstractly	6 Attend to Precision
3 Construct an Argument	7 Make Use of Structure
4 Model with Mathematics	8 Use Repeated Reasoning

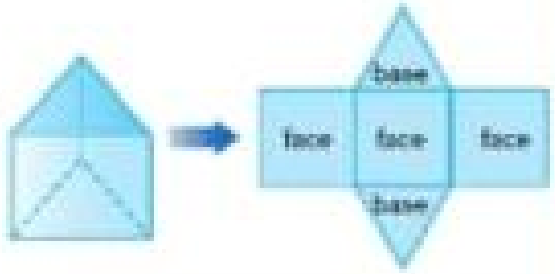
Key Concept

Surface Area of a Triangular Prism

Words

The surface area of a triangular prism is the sum of the areas of the two triangular bases and the three rectangular faces.

Model

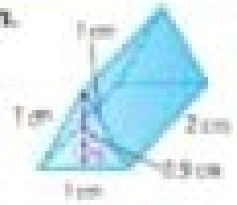


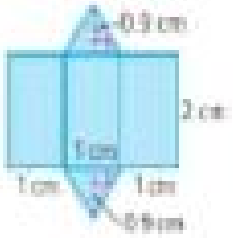
A triangular prism is a prism that has triangular bases. When the bases are equilateral triangles, the areas of the three rectangular faces are equal. You can use a net to find the surface area of a triangular prism.

Example

1. Find the surface area of the triangular prism.

To find the surface area of the triangular prism, find the area of each face and add.





area of each triangular base: $\frac{1}{2}(1)(0.9) = 0.45$

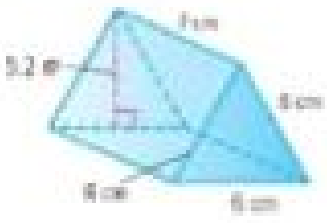
area of each rectangular face: $1(2) = 2$

Add to find the surface area.

$0.45 + 0.45 + 2 + 2 + 2 = 6.9$ square centimeters

Got it? Do this problem to find out.

a. Find the surface area of the triangular prism.



a. 157.2 cm²

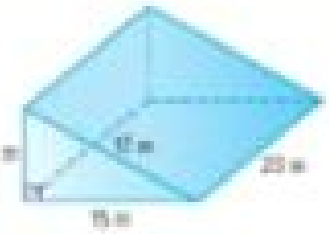
Surface Area of Other Triangular Prisms

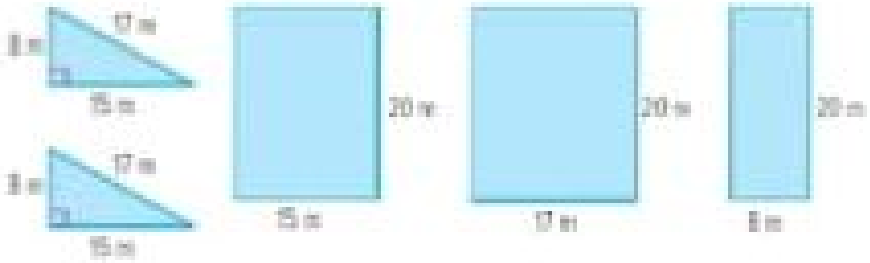
You can also find the surface area of any triangular prism by adding the areas of all the sides of the prism using an orthogonal drawing.

Example

2. **Find the surface area of the triangular prism.**

Find the area of each face and add. For this prism, each rectangular face has a different area.





area of each triangular base: $\frac{1}{2}(15)(8) = 60$

area of the rectangular faces: $15(20) = 300$

$17(20) = 340$

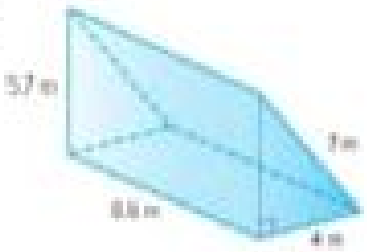
$8(20) = 160$

Add to find the surface area.

$60 + 60 + 300 + 340 + 160 = 920$ square meters

Got it? Do this problem to find out.

Find the surface area of each triangular prism.

b. 

c. 

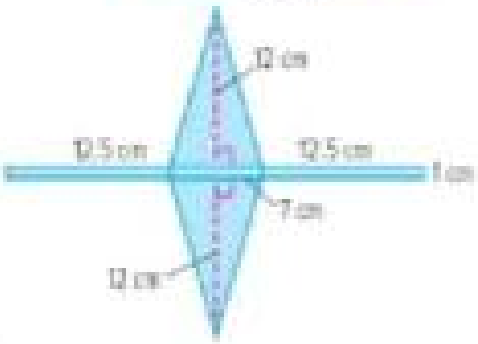
b. 166.42 m²

c. 59.6 m²

Example

3. **A bakery boxes pastries in a triangular prism box, as shown. Find the amount of cardboard used to make a pastry box.**

Sketch and label the bases and faces of the triangular prism. Then add the areas of the polygons.



Surface area $= 2\left(\frac{1}{2} \cdot 7 \cdot 12\right) + 2(1 \cdot 12.5) + (1 \cdot 7)$

$= 84 + 25 + 7$ or 116

So, 116 square centimeters of cardboard is needed to make a

Guided Practice

1. Find the surface area of the triangular prism. (Example 1) 231.5 m²

2. A skateboarding ramp is in the shape of a triangular prism. If the entire ramp is to be painted, what is the surface area to be painted? (Example 2) 4,704 cm²

3. **Building on the Essential Question** How is the area of a rectangle related to the surface area of a triangular prism? Sample answer: A triangular prism has three rectangular faces. You can use the area of a rectangle to find the area of the three rectangular faces of a triangular prism.

Rate Yourself!

How confident are you about surface area of triangular prisms? Check the box that applies.

Independent Practice

Find the surface area of each triangular prism. (Examples 1–2)

1. **1,152 m²**

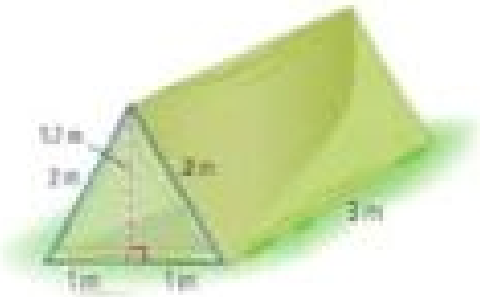
2. **4.8 cm²**

3. **13.6 m²**

4. **533.6 cm²**

5. A tent is in the shape of a triangular prism. About how much canvas, including the floor, is used to make the tent? (Example 3)

about 21.4 m²



6. A decorative gift box is in the shape of a triangular prism as shown. What is the surface area of the box? (Example 3)

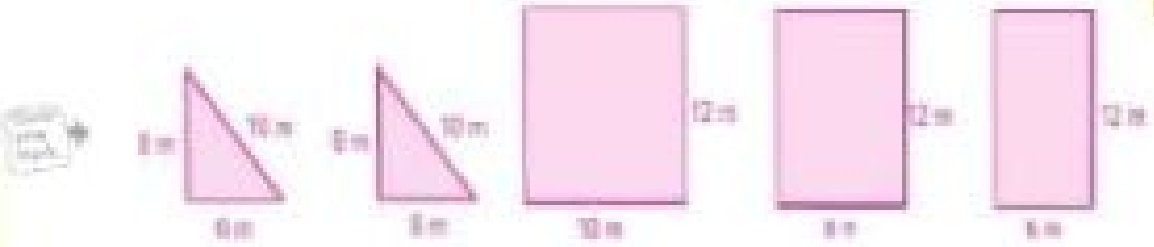
225 cm²

7. A mailer for posters is a triangular prism as shown. Find the surface area of the mailer. (Example 3)

279.2 cm²

8. **Multiple Representations** The figure shows the dimensions of a triangular prism.

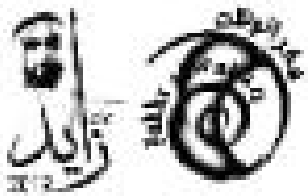
a. **Models** Draw a model of the faces and bases of the triangular prism.



b. **Words** Describe the triangular prism. **Sample answer: The prism has right triangle bases each with an area of 24 m^2. The rectangular faces have an area of 72 m^2, 120 m^2, and 96 m^2. The total surface area is 336 m^2.**

c. **Numbers** Find the surface area of the triangular prism using addition. **24 + 24 + 72 + 120 + 96 = 336; 336 m^2**

9. The surface area of a right triangular prism is 228 square centimeters. The base is a right triangle with a base height of 6 centimeters and a base length of 8 centimeters. The length of the third side of the base is 10 centimeters. Find the height of the prism. **7.5 cm**



H.O.T. Problems Higher Order Thinking

10. **Reason Abstractly** Describe the dimensions of a triangular prism that has a surface area between 550 square centimeters and 700 square centimeters.
Sample answer: triangular base that measures 9 centimeters, 12 centimeters, and 15 centimeters, and a height of 14 centimeters

11. **Persevere with Problems** Sketch and label two triangular prisms such that one has a greater volume and the other has a greater surface area.
See students' work. Sample answer: Prism A with bases that are right triangles that measure 3 by 4 by 5 and with a height of 1. Prism B with bases that are right triangles that measure 1 by 1 by 1.4 and with a height of 10. Prism A has a greater volume while Prism B has a greater surface area.

12. **Justify Conclusions** Hamad is painting a decorative box with the dimensions shown at the right. A can of paint covers about 1,000 square centimeters. Does he have enough to paint the rectangular faces of his box with three coats of paint? Justify your answer.
yes; Sample answer: The area of the rectangular faces is about 216 cm^2. Three coats would need to cover 648 cm^2, which is much less than the 1,000 cm^2.

Extra Practice

Be Precise **Find the surface area of each triangular prism. Round to the nearest tenth if necessary.**

13. 557 m²

area of each base: $\frac{1}{2} \cdot 10 \cdot 8.7 = 45.5\ m^2$

area of each face: $15 \cdot 10 = 150\ m^2$

surface area $= 2(45.5) + 3(150)$
$= 557\ m^2$

14. 351.1 m²

area of each base: $\frac{1}{2} \cdot 11.5 \cdot 6 = 33.1\ m^2$

area of faces: $11.5 \cdot 9.5 = 107.25\ m^2$
$8.5 \cdot 9.5 = 80.75\ m^2$
$8 \cdot 9.5 = 76\ m^2$

surface area $= 33.1 + 33.1 + 107.25 +$
$80.75 + 76 = 351.1\ m^2$

15. 70.8 cm²

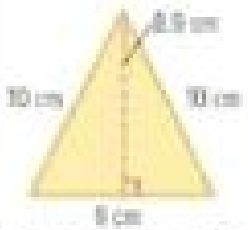
16. 540 m²

17. 282.7 cm²

18. 409.2 m²

Copy and Solve **Find the surface area of each triangular prism using the base triangles shown. Show your work on a separate piece of paper.**

19.



height of prism: 12 cm **428.8 cm²**

20.

height of prism: 15 m **510 m²**

Power Up! Test Practice

21. A triangular prism has the dimensions shown. Determine if each statement is true or false.

a. The combined areas of the bases is 54 m. ☐ True ■ False

b. The areas of the rectangular faces are 90 square meters, 120 square meters and 180 square meters. ☐ True ■ False

c. The surface area of the prism is 360 square meters. ■ True ☐ False

22. The attic of the house shown below has a solid wood floor.

Select values to complete the model below to find how much wood is needed to make the roof of the house and the floor of the attic.

2	8	50
3.5	10	80
5	14	100

Attic Floor: 8 × 10 = 80 m²

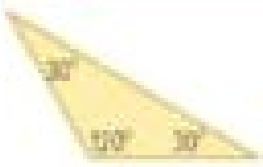
Roof: 2 × 5 × 10 = 100 m²

How many square meters of wood are needed to build the roof and the floor of the attic? 180 m²

Spiral Review

Identify each triangle as *acute*, *right*, or *obtuse*.

23. obtuse



24. acute

25. right

26. A certain two-dimensional figure has two pairs of parallel lines, four right angles, and four congruent sides. What is the figure? square

Inquiry Lab
Nets of Pyramids

Inquiry **HOW is the area of a triangle related to the surface area of a square pyramid?**

Mathematical Practices 1, 3, 4

Omar is designing a paper weight that is shaped like a square pyramid.

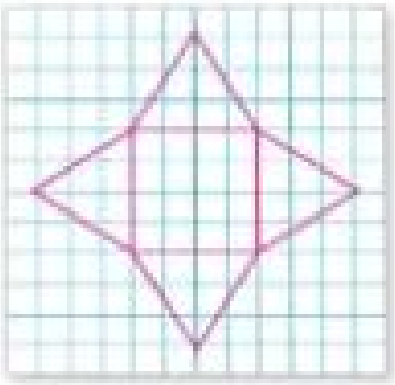
Hands-On Activity

Use orthogonal drawings to find the surface area of a square pyramid.
A **square pyramid** is a three-dimensional figure with a square base and four triangular faces.

Step 1 Find the dimensions of each side of the square pyramid from the orthogonal drawing.

Orthogonal Drawing			
View	Drawing	Dimensions (m)	Area of Face (m²)
Base	4 m, 4 m	length = 4 width = 4	$4 \times 4 = 16$
Triangular Face	3 m, h, 4 m	height = 3 base = 4	$\frac{1}{2}(3 \times 4) = 6$

Step 2 Use grid paper to draw a net. Let 1 unit on the grid represent 1 meter. Check the dimensions of each face using the information in the table.



Step 3 Add the area of each face to find the surface area of the figure. Remember, there are four triangular faces.

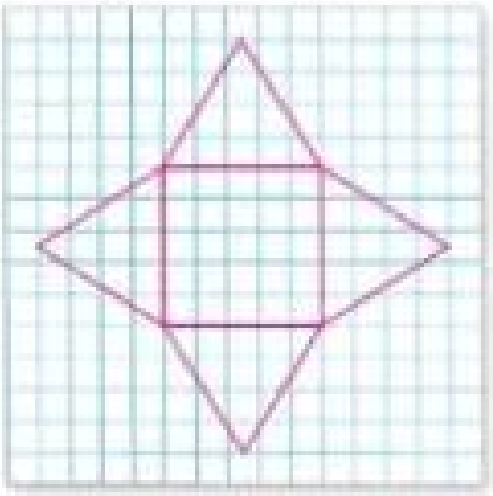
16 + 4 × 6 = 40

So, the surface area is 40 square meters.

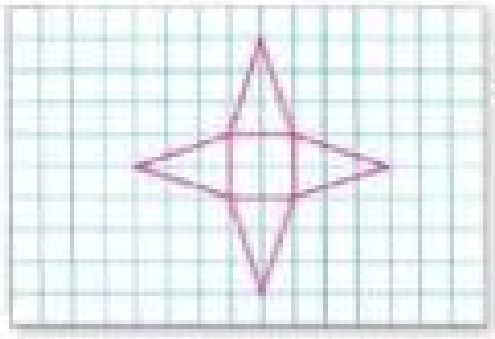
Investigate

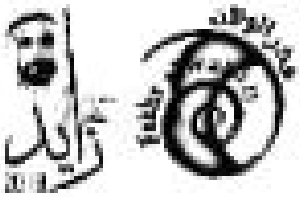
Model with Mathematics Work with a partner. Use nets to determine the surface area of each pyramid. Draw a net of each pyramid on the provided grid paper.

1. 65 cm²



2. 16 m²





Create

3. **Construct an Argument** Explain how to find the surface area of a square pyramid, without creating a net. Use the dimensions in Exercise 1 to explain your answer.

Sample answer: Multiply to find the area of each face. The area of each triangular face is $\frac{1}{2}(5 \times 4)$, or 10 square cm. Since all four triangular faces are congruent, you can find the area of one face, multiply by 4, and then add the product to the area of the base to find the surface area of the pyramid. $(4 \cdot 10) + (5 \times 5) = 40 + 25 = 65$ square cm

4. **Inquiry** HOW is the area of a triangle related to the surface area of a square pyramid? Sample answer: If you know the area of a triangle, you can find the area of the triangular faces of the pyramid. Add the area of the square base to the area of the triangular faces to find the surface area of the square pyramid.

Lesson 5

Surface Area of Pyramids

Vocabulary Start-Up

Pyramids Ancient Egyptians built pyramids, such as the one shown in the photo below. A right square pyramid has a square base and four isosceles triangles that make up the lateral faces. **lateral surface area** is the sum of the areas of all its lateral faces. The height of each lateral face is called **slant height**.

Essential Question

HOW do measurements help you describe real-world objects?

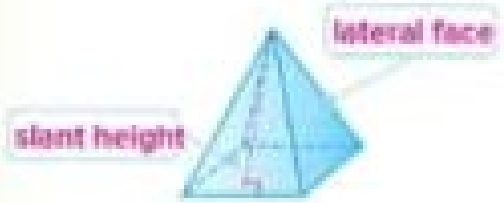
Vocabulary

lateral surface area
slant height
regular pyramid

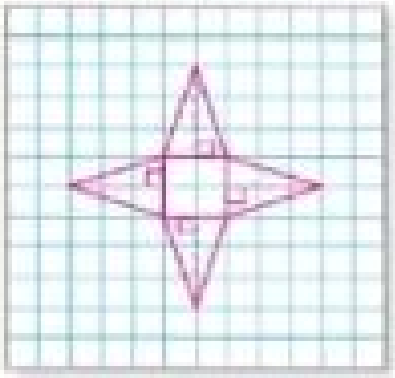
Mathematical Practices

1, 2, 4, 5

1. Fill in the blanks on the diagram below with the terms *slant height* and *lateral face*.



2. Draw a net of a square pyramid.
Sample answer:



Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

- ○ Persevere with Problems
- ○ Reason Abstractly
- ○ Construct an Argument
- ○ Model with Mathematics
- ○ Use Math Tools
- ○ Attend to Precision
- ○ Make Use of Structure
- ○ Use Repeated Reasoning

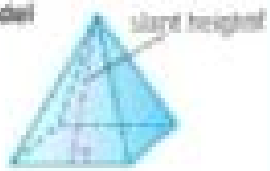
Key Concept

Surface Area of a Pyramid

Lateral Area

Words The lateral surface area $L.A.$ of a regular pyramid is half the perimeter P of the base times the slant height ℓ.

Model



Symbols $L.A. = \frac{1}{2}P\ell$

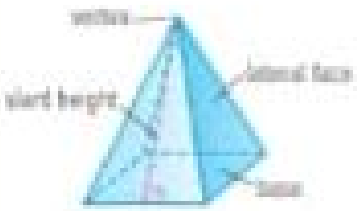
Total Surface Area

Words The total surface area $S.A.$ of a regular pyramid is the lateral area $L.A.$ plus the area of the base B.

Symbols $S.A. = B + L.A.$ or $S.A. = B + \frac{1}{2}P\ell$

A **regular pyramid** is a pyramid with a base that is a regular polygon.

Model of Regular Square Pyramid



Net of Regular Square Pyramid

To find the lateral area $L.A.$ of a regular pyramid, refer to the net. The lateral area is the sum of the areas of the triangles.

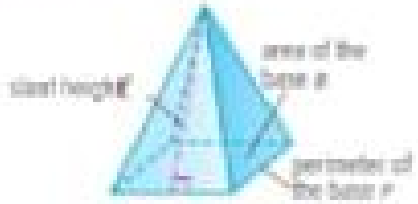
$L.A. = 4\left(\frac{1}{2}s\ell\right)$ Area of the lateral faces

$L.A. = \frac{1}{2}(4s)\ell$ Commutative Property of Multiplication

$L.A. = \frac{1}{2}P\ell$ The perimeter of the base P is $4s$.

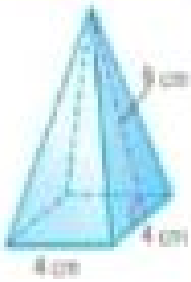
The total surface area of a regular pyramid is the lateral surface area $L.A.$ plus the area of the base B.

$S.A. = B + \frac{1}{2}P\ell$



Examples

1. **Find the total surface area of the pyramid. Round to the nearest tenth.**



$S.A. = B + \frac{1}{2}P\ell$ Surface area of a pyramid

$S.A. = 16 + \frac{1}{2}(16 \cdot 9)$ $B = 4 \cdot 4$, $P = 4 \cdot 4$ or 16, $\ell = 9$

$S.A. = 88$ Simplify

The surface area is 88 square centimeters.

2. **Find the total surface area of the pyramid with a base area of 111 square meters.**

$S.A. = B + \frac{1}{2}P\ell$ Surface area of a pyramid

$S.A. = 111 + \frac{1}{2}(48 \cdot 20)$ $B = 111$, $P = 10 + 10 + 10$ or 48, $\ell = 20$

$S.A. = 591$ Simplify

The surface area of the pyramid is 591 square meters.

3. **Find the total surface area of the pyramid.**

$S.A. = B + \frac{1}{2}P\ell$ Surface area of a pyramid

$S.A. = 43.5 + \frac{1}{2}P\ell$ $B = \frac{1}{2} \cdot 10 \cdot 8.7$ or 43.5

$S.A. = 43.5 + \frac{1}{2}(30 \cdot 12)$ $P = 10 + 10 + 10$ or 30, $\ell = 12$

$S.A. = 223.5$ Simplify

The surface area is 223.5 square meters.

Get it? **Do these problems to find out.**

a. Find the surface area of a square pyramid that has a slant height of 8 centimeters and a base length of 5 centimeters.

b. Find the total surface area of the pyramid shown.

Perimeter of a Square

The formula for the perimeter of a square is $P = 4s$.

a. 105 cm²

b. 332.4 m²

Example

4. Ali is wrapping gift boxes that are square pyramids for party favors. They have a slant height of 3 centimeters and base edges 2.5 centimeters long. How many square centimeters of card stock are used to make one gift box?

$S.A. = B + \frac{1}{2}P\ell$ Surface area of a pyramid

$S.A. = 6.25 + \frac{1}{2}(10)(3)$ $B = 2.5^2$ or 6.25, $P = 4(2.5)$ or 10, $\ell = 3$

$S.A. = 21.25$ Simplify

So, 21.25 square centimeters of card stock are used to make one gift box.

Get it? Do this problem to find out.

c. Adrian purchased a bottle of perfume that is in the shape of a square pyramid. The slant height of the bottle is 11.25 centimeters and the base is 5 centimeters. Find the surface area.

c. 137.5 cm²

Guided Practice

Find the total surface area of each pyramid. Round to the nearest tenth.

Examples 1–3

1. 6.1 cm, 6.4 cm, 6.4 cm — 119.0 cm²

2. 9 mm, 7.8 mm, 12 mm, 9 mm, 6 mm — 140.4 mm²

3. The Washington Monument is an obelisk with a square pyramid top. The slant height of the pyramid is 16.65 meters, and the square base has sides of 10.35 meters. Find the lateral area of the pyramid. Example 4 — 344.7 m²

4. **Building on the Essential Question** Justify the formula for the surface area of a pyramid.

The formula is based on finding the area of each face and the base and then adding them.

Rate Yourself!

How confident are you about finding the surface area of pyramids? Check the box that applies.

Need to review your notes!

Name _____ My Homework _____

Independent Practice

Find the total surface area of each pyramid. Round to the nearest tenth. (Examples 1–3)

1. 

96 cm²

2. 

507.5 mm²

3. 

328 cm²

4. 

90.3 m²

5. A triangular pyramid has a slant height of 0.75 meter. The equilateral triangular base has a perimeter of 1.2 meters and an area of about 0.07 square meter. Find the approximate surface area. (Example 4)

0.52 m²

6. The gemstone shown is a square pyramid that has a base with sides 3.4 centimeters long. The slant height of the pyramid is 3.8 centimeters. Find the surface area of the gemstone. (Example 4)

37.4 cm²

7. Mazen is building a birdhouse for a class project. The birdhouse is a regular hexagonal pyramid. The base has side lengths of 7.5 centimeters and an area of about 150 square centimeters. The slant height is 15 centimeters. Find the approximate surface area of the birdhouse. (Example 4)

488 cm²

8. **Persevere with Problems** square pyramid has a surface area of 175 square centimeters. The square base has side lengths of 5 centimeters. Find the slant height of the pyramid.

15 cm

9. A square pyramid has a lateral area of 107.25 square centimeters and a slant height of 8.25 centimeters. Find the length of each side of its base.

6.5 cm

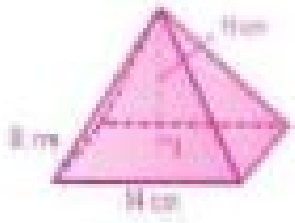
H.O.T. Problems Higher Order Thinking

10. **Justify Conclusions** Suppose you could climb to the top of the Great Pyramid of Giza in Egypt. Which path would be shorter, climbing a lateral edge or the slant height? Justify your response.

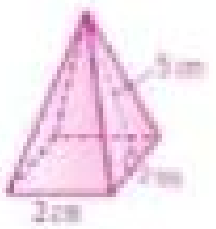
It would be shorter to climb up the slant height. The bottom of the slant height is closer to the center of the base of the pyramid than the bottom of the lateral edge.

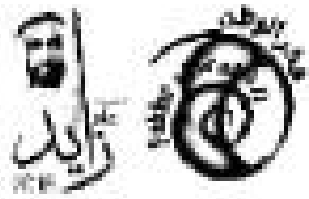
11. **Model with Mathematics** Draw a rectangular pyramid and a square pyramid. Explain the differences between the two.

Rectangular Pyramid



Square Pyramid

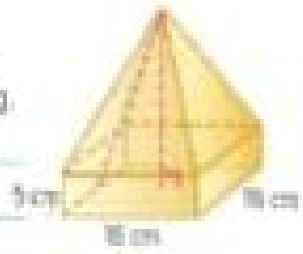




Sample answer: Both a square pyramid and a rectangular pyramid have isosceles triangles as their lateral faces. All the lateral faces are congruent on a square pyramid but, on a rectangular pyramid, the opposite pairs of lateral faces are congruent.

12. **Persevere with Problems** The total height of the figure shown is 20 centimeters and the slant height is 17 centimeters. Which has a greater surface area, the prism or the pyramid? Explain your reasoning.

The prism; the surface area of the prism is 576 and the lateral surface area of the pyramid is 544 cm



Name _____ My Homework _____

Extra Practice

Find the total surface area of each pyramid. Round to the nearest tenth.

13. 

197.1 m²

14. 

18 cm²

15. 

765 cm²

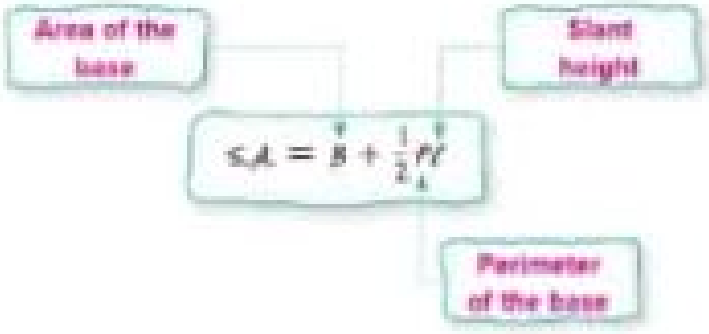
16. 

13.7 m²

17. A square pyramid has a slant height of $\frac{2}{3}$ meters. The base has side lengths of $\frac{1}{4}$ meters. Find the surface area. 26.1 m²

18. A building in San Francisco is shaped like a square pyramid. It has a slant height of 256.8 meters and each side of its base is 43.5 meters. Find the lateral area of the building. 22,344 m²

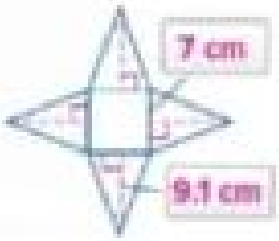
19. **Use Math Tools** Complete the organizer below to help you remember what each part of the formula for the surface area of a pyramid represents.



Power Up! Test Practice

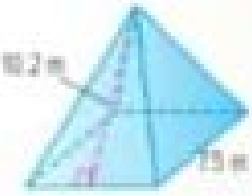
20. The base of a square pyramid has a perimeter of 28 centimeters. The height of the pyramid is 2.1 centimeters longer than the side length of the base. Label the net of the pyramid with the correct dimensions.

2.1 cm	7 cm
4 cm	9.1 cm
4.9 cm	28 cm



What is the total surface area of the pyramid? 126.4 cm²

21. An entertainment company is constructing a tent in the shape of a square pyramid, without a floor, to be used at a dish party. Determine if each statement is true or false.



a. The area of the ground covered by the tent is 56.25 square meters. ■ True □ False

b. The area of each triangular face of the tent is 38.25 square meters. ■ True □ False

c. The amount of material needed to make the tent is 209.25 square meters. □ True ■ False

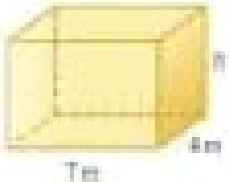
Spiral Review

Find the surface area of each prism. 6.G.4

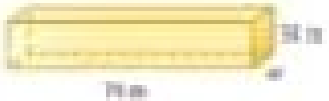
22. 13,890 cm²

23. 456 m²

24. The volume of the prism shown below is 140 cubic meters. Find the height of the prism. 7.G.6 5 m



25. The volume of the prism shown below is 10,360 cubic meters. Find the width of the prism. 7.G.6 10 m



Inquiry Lab 5
Volume of Pyramids

inquiry **WHAT is the relationship between the volume of a prism and the volume of a pyramid with the same base area and height?**

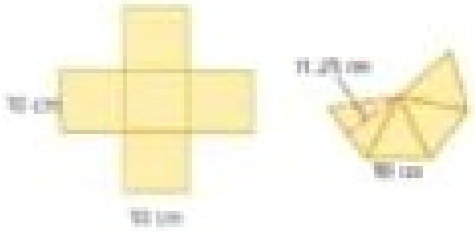
Mathematical Practices 1, 2, 5

A movie theater offers two different containers of popcorn: a square prism and a square pyramid. Both containers are 10 centimeters tall and have a base area of 100 square centimeters. Determine the container that holds more popcorn.

Hands-On Activity

Nets are two-dimensional patterns of three-dimensional figures.

Step 1 Draw the nets of the popcorn containers shown below onto card stock. Cut out and tape each net to form its shape. The prism and pyramid will be open. The pyramid is composed of 4 congruent isosceles triangles with bases of 10 centimeters and heights of 11.25 centimeters.



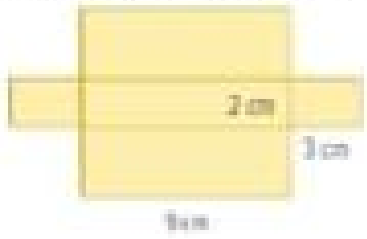
Step 2 Fill the pyramid with rice. Pour the rice from the pyramid into the prism and repeat until the prism is full. Slide a ruler across the top to level the amount.

It took 3 pyramids of rice to fill the prism.

So, the square prism container holds more popcorn than the square pyramid container.

Investigate

Use Math Tools Work with a partner to repeat the Activity with the rectangular prism and the rectangular pyramid shown.



1. How many pyramids of rice did it take to fill the prism?
   Sample answer: It took three pyramids of rice to fill the prism.

2. What is true about the bases of your rectangular prism and rectangular pyramid? the heights?
   Sample answer: The bases and the heights appear to be the same.

3. Refer to the Activity. What is true about the bases of the square prism and square pyramid? the heights?
   Sample answer: The bases and the heights are the same.

Analyze and Reflect

4. What fraction of the volume of the rectangular prism is the volume of the rectangular pyramid? $\frac{1}{3}$

5. Refer to the Activity. What fraction of the volume of the square prism is the volume of the square pyramid? $\frac{1}{3}$

Create

6. **Reason Inductively** How can you find volume of a pyramid given a prism with the same base area and height? Write a formula for the volume of a pyramid based on the formula for the volume of a prism.
   Sample answer: Multiply the volume of the prism by $\frac{1}{3}$. $V = \frac{1}{3}Bh$

7. WHAT is the relationship between the volume of a prism and the volume of a pyramid with the same base area and height?
   Sample answer: The volume of the pyramid is $\frac{1}{3}$ the volume of the prism.

Volume of Pyramids

Real-World Link

Sand Sculpture Mohammad is helping his mother build a sand sculpture at the beach in the shape of a pyramid. The square pyramid has a base with a length and width of 30 centimeters each and a height of 35 centimeters.

Essential Question

HOW do measurements help you describe real-world objects?

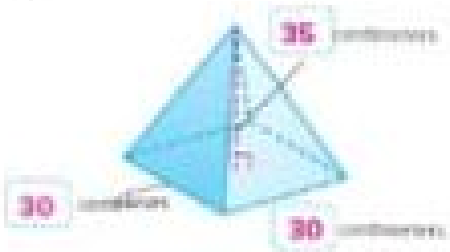
Vocabulary

lateral face

Mathematical Practices

1, 3, 4, 6

1. Label the dimensions of the sand sculpture on the square pyramid below.



2. What is the area of the base of the pyramid?

   900 cm²

3. What is the volume of a square prism with the same dimensions as the pyramid?

   31,500 cm³

Which Mathematical Practices did you use? Shade the circle(s) that applies.

- ① Persevere with Problems
- ② Reason Abstractly
- ③ Construct an Argument
- ④ Model with Mathematics
- ⑤ Use Math Tools
- ⑥ Attend to Precision
- ⑦ Make Use of Structure
- ⑧ Use Repeated Reasoning

Key Concept

Volume of a Pyramid

Words The volume V of a pyramid is one third the area of the base B times the height of the pyramid h.

Symbols $V = \frac{1}{3}Bh$

Model

In a polyhedron, any face that is not a base is called a **lateral face**. The lateral faces of a pyramid meet at a common vertex. The height of a pyramid is the distance from the vertex perpendicular to the base.

Examples

1. **Find the volume of the pyramid. Round to the nearest tenth.**

$V = \frac{1}{3}Bh$ Volume of a pyramid

$V = \frac{1}{3}(3.2 \cdot 1.4)2.8$ $B = 3.2 \cdot 1.4$, $h = 2.8$

$V = 4.2$ Simplify

The volume is about 4.2 cubic centimeters.

2. **Find the volume of the pyramid. Round to the nearest tenth.**

$V = \frac{1}{3}Bh$ Volume of a pyramid

$V = \frac{1}{3}(\frac{1}{2} \cdot 8.1 \cdot 6.4)11$ $B = \frac{1}{2} \cdot 8.1 \cdot 6.4$, $h = 11$

$V = 95.04$ Simplify

The volume is about 95.0 cubic meters.

Got it? **Do this problem to find out.**

a. Find the volume of a pyramid that has a height of 9 centimeters and a rectangular base with a length of 7 centimeters and a width of 3 centimeters.

a. 63 cm³

Key Concept

Volume of a Pyramid

Words The volume V of a pyramid is one third the area of the base B times the height of the pyramid h.

Symbols $V = \frac{1}{3}Bh$

Model

In a polyhedron, any face that is not a base is called a **lateral face**. The lateral faces of a pyramid meet at a common vertex. The height of a pyramid is the distance from the vertex perpendicular to the base.

Examples

1. **Find the volume of the pyramid. Round to the nearest tenth.**

$V = \frac{1}{3}Bh$ Volume of a pyramid

$V = \frac{1}{3}(3.2 \cdot 1.4)2.8$ $B = 3.2 \cdot 1.4$, $h = 2.8$

$V = 4.2$ Simplify

The volume is about 4.2 cubic centimeters.

2. **Find the volume of the pyramid. Round to the nearest tenth.**

$V = \frac{1}{3}Bh$ Volume of a pyramid

$V = \frac{1}{3}(\frac{1}{2} \cdot 8.1 \cdot 6.4)11$ $B = \frac{1}{2} \cdot 8.1 \cdot 6.4$, $h = 11$

$V = 95.04$ Simplify

The volume is about 95.0 cubic meters.

Got it? **Do this problem to find out.**

a. Find the volume of a pyramid that has a height of 9 centimeters and a rectangular base with a length of 7 centimeters and a width of 3 centimeters.

a. 63 cm³

Find the Height of a Pyramid

You can also use the formula for the volume of a pyramid to find a missing height.

Examples

3. **The rectangular pyramid shown has a volume of 90 cubic centimeters. Find the height of the pyramid.**

$V = \frac{1}{3}Bh$ Volume of a pyramid

$90 = \frac{1}{3}(9 \cdot 5)h$ $V = 90, B = 9 \cdot 5$

$90 = 15h$ Multiply

$\frac{90}{15} = \frac{15h}{15}$ Divide by 15.

$6 = h$ Simplify

The height of the pyramid is 6 centimeters.

4. **A triangular pyramid has a volume of 44 cubic meters. It has an 8-meter base and a 3-meter height. Find the height of the pyramid.**

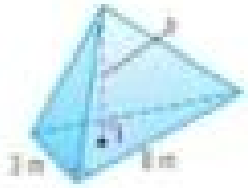
$V = \frac{1}{3}Bh$ Volume of a pyramid

$44 = \frac{1}{3}(\frac{1}{2} \cdot 8 \cdot 3)h$ $V = 44, B = \frac{1}{2} \cdot 8 \cdot 3$

$44 = 4h$ Multiply

$\frac{44}{4} = \frac{4h}{4}$ Divide by 4.

$11 = h$ Simplify



The height of the pyramid is 11 meters.

Multiplying Fractions

To find $\frac{1}{3}$ of $\frac{1}{2}$, find $\frac{1}{3} \cdot \frac{1}{2}$.

To find $\frac{2}{3}$ of $\frac{1}{2}$, find $\frac{2}{3} \cdot \frac{1}{2}$.

To find $\frac{1}{4}$ of 24, find $\frac{1}{4} \cdot 24$.

Get it? **Do these problems to find out.**

b. A triangular pyramid has a volume of 840 cubic centimeters. The triangular base has a base length of 20 centimeters and a height of 21 centimeters. Find the height of the pyramid.

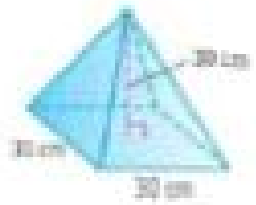
b. 12 cm

c. A rectangular pyramid has a volume of 525 cubic meters. It has a base of 25 meters by 18 meters. Find the height of the pyramid.

c. 3.5 m

Example

5. Kamilah is making a model of the Food Guide Pyramid for a class project. Find the volume of the square pyramid.



$V = \frac{1}{3}Bh$ Volume of a pyramid

$V = \frac{1}{3}(30 \cdot 30)30$ $B = 30 \cdot 30, h = 30$

$V = 9{,}000$ Multiply

The volume is 9,000 cubic centimeters.

Guided Practice

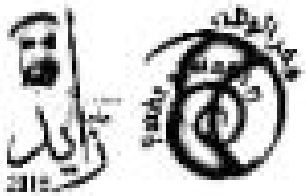
Find the volume of each pyramid. Round to the nearest tenth if necessary. (Examples 1 and 2)

1.

2,683.3 m³

2.

3.8 cm³



Find the height of each pyramid. (Examples 3 and 4)

3. square pyramid: volume 1,024 ft³, base edge 16 cm 12 cm

4. triangular pyramid: volume 48 ft³, base edge 9 cm, base height 4 cm 8 cm

5. The Transamerica Pyramid is a skyscraper in San Francisco. The rectangular base has a length of 52.5 meters and a width of 36 meters. The height is 256 meters. Find the volume of the building. (Example 5) 161,217 m³

6. **Building on the Essential Question** When you are finding the volume of a pyramid, why is it important to know the shape of the base of the pyramid?

Sample answer: In the formula $V = \frac{1}{3}Bh$, B will be different depending on the shape of the base.

Rate Yourself!

How well do you understand volume of pyramids? Circle the image that applies.

Then: To adjust your fluency

Name _____ My Homework _____

Independent Practice

Find the volume of each pyramid. Round to the nearest tenth if necessary. (Examples 1 and 2)

1. 

80 m³

2. 

109.3 m³

3. 

42 m³

4. 

410.7 cm³

Find the height of each pyramid. (Examples 3 and 4)

5. rectangular pyramid: volume 448 in^3; base edge 12 cm; base length 8 cm
14 cm

6. triangular pyramid: volume 270 in^3; base edge 15 cm; height of base 4 cm
27 cm

7. A glass pyramid has a height of 10 centimeters. Its rectangular base has a length of 7.5 centimeters and a width of 6.25 centimeters. Find the volume of glass used to create the pyramid. (Example 5)
156.25 cm³

8. The Pyramid Arena in Memphis, Tennessee, is a square pyramid that is 96.3 meters tall. The base has 180-meter sides. Find the volume of the pyramid. (Example 5)
1,040,040 m³

9. **Reason Inductively** A rectangular pyramid has a length of 14 centimeters, a width of 9 centimeters, and a height of 10 centimeters. Explain the effect on the volume if each dimension were doubled.

The volume is eight times greater; Sample answer: Since each dimension is two times greater, the volume is 2 × 2 × 2 or eight times greater.

10. Find the height of a square pyramid that has a volume of $\frac{3}{5}$ cubic meters and a base with 4 meter sides.

$\frac{9}{80}$ meters

H.O.T. Problems Higher Order Thinking

11. **Be Precise** A rectangular pyramid has a volume of 160 cubic meters. Find two possible sets of measurements for the base area and height of the pyramid.

Sample answer: first set: area of the base, 40 m; height of the pyramid, 12 m; second set: area of the base, 30 m; height of the pyramid, 16 m

12. **Persevere with Problems** A square pyramid and a cube have the same bases and volumes. How are their heights related? Explain.

The height of the pyramid is 3 times the height of the cube.

13. **Reason Inductively** The two figures shown have congruent bases. How does the volume of the two square pyramids in Figure B compare to the volume of the square pyramid in Figure A?

Figure A

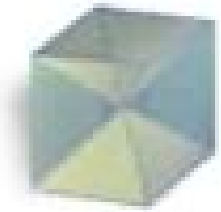


Figure B

The volumes are the same.

14. **Reason Inductively** Determine whether the following statement is true or false. Explain your reasoning.

The volumes of a rectangular-based pyramid and a triangular-based pyramid with congruent heights and equal base areas are equal.

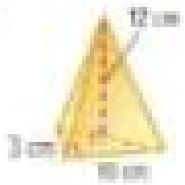
true; Sample answer: Both volumes are found using the formula $\frac{1}{3}Bh$. If the base area and the height are congruent, then the volumes are also congruent.

Name _____ My Homework _____

Extra Practice

Find the volume of each pyramid. Round to the nearest tenth if necessary.

15. 60 cm³



$V = \frac{1}{3}Bh$

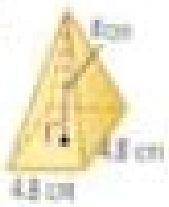
$V = \frac{1}{3}\left(\frac{1}{2}\right)(10)(12)$

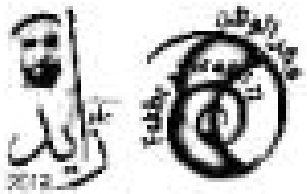
$V = 60$

16. 43.3 cm³

17. 195 m³

18. 61.4 cm³





Find the height of each pyramid.

19. square pyramid: volume 297 m³; area of the base 81 m²

11 m

20. hexagonal pyramid: volume 1,320 m³; area of the base 120 m²

33 m

21. square pyramid: volume 550 cm³; area of the base 75 cm²

22 cm

22. rectangular pyramid: volume 3,800 m³; area of the base 300 m²

38 m

23. An ancient stone pyramid has a height of 13.6 meters. The edges of the square base are 16.5 meters. Find the volume of the stone pyramid.

1,234.2 m³

Power Up! Test Practice

24. The table shows the base dimensions and heights of 4 rectangular pyramids. Sort the pyramids from least to greatest volume.

Pyramid	ℓ (cm)	w (cm)	h (cm)
A	4	9	5
B	6	6	7
C	5	5	9
D	3	8	12

	Pyramid	Volume (cm³)
Least	A	60
	D	72
	C	75
Greatest	B	84

Which pyramid has the greatest volume? Pyramid B

25. The rectangular pyramid shown has a volume of 1,560 cubic centimeters. What is the height of the pyramid? Explain how you found your answer.

24 cm; Sample answer: Replace V with 1,560 and B with 13×15 in the formula $V = \frac{1}{3}Bh$. Then solve for h.

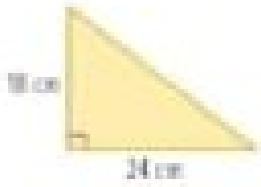
Spiral Review

Find the area of each figure.

26.

15 m²

27.

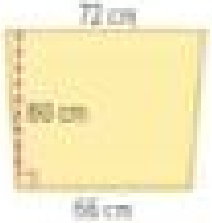


216 cm²

28.

162.5 m²

29.



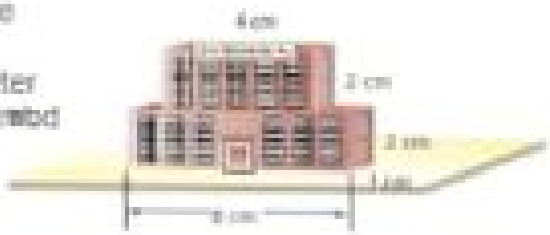
4140 cm²

Inquiry Lab 7
Composite Figures

inquiry **HOW can you find the volume and surface area of a composite figure?**

Mathematical Practices 1, 2, 4

A company made a model of a new office building. The building is composed of rectangular prisms. You can use centimeter cubes to find the volume of the building model.



Hands-On Activity 1

The model is a composite figure because it is made from two rectangular prisms.

Step 1 Model the top and bottom rectangular prisms using cubes.

Step 2 Count the cubes to find the dimensions. Write the dimensions in the table below. Then use the cube models to find the volume of both prisms. Write these measures in the table below step 3.

Step 3 Use the table to find the volume of the entire building model. Write these measures in the composite row of the table.

Model	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
Bottom	6	1	2	12
Top	4	1	2	8
Composite				20

Mr. Obaid's class made a model of a house.
The model was composed of a rectangular prism and a triangular prism. Determine the volume and surface area of the model house.

Hands-On Activity 2

Step 1 Use a rectangular prism to model the bottom of the house. Use a triangular prism to model the top of the house.

Step 2 Complete the tables below using the models from Step 1.

Prism	Length (cm)	Width (cm)	Height (cm)
Rectangular	4	3	3

Prism	Length (cm)	Base (cm)	Height (cm)
Triangular	4	3	2

Step 3 Use the information from the tables and the models to find the total volume of the model house.

36 cm³ + 12 cm³ = 48 cm³

36 cm³: Volume of Rectangular Prism
12 cm³: Volume of Triangular Prism
48 cm³: Total Volume

Step 4 Use the information from the tables and the models to find the total surface area of the model house.

66 cm² + 38 cm² − 24 cm² = 80 cm²

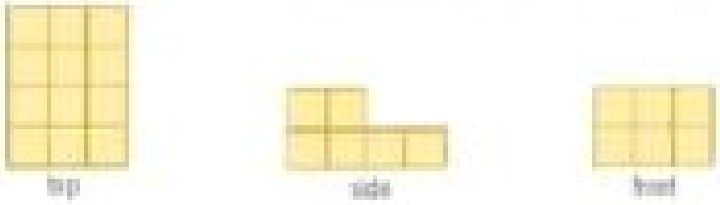
66 cm²: Surface Area of Rectangular Prism
38 cm²: Surface Area of Triangular Prism
24 cm²: Areas where Prisms Connect
80 cm²: Total Surface Area

The total volume of the model house is 48 cubic centimeters. The total surface area is 80 square centimeters.

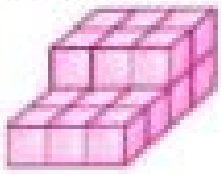
Investigate

Work with a partner.

1. **Model with Mathematics** Use the top, side, and front views to build a figure using centimeter cubes. See students' work for model.



a. Make a sketch of the figure you built.



b. Find the volume and surface area of the figure.

Volume: 18 cm³ Surface Area: 48 cm², including the bottom

Refer to the figure at the right for Exercises 2–4.

2. The figure is comprised of a cube and a square pyramid.

3. Complete the following to find the volume of the figure.

a. The volume of the cube is 1,000 cubic centimeters.

b. The volume of the square pyramid is 250 cubic centimeters.

c. So, the volume of the composite figure is 1,250 cubic centimeters.

4. Complete the following to find the surface area of the figure.

a. The surface area of the cube is 600 square centimeters.

b. The surface area of the square pyramid is 280 square centimeters.

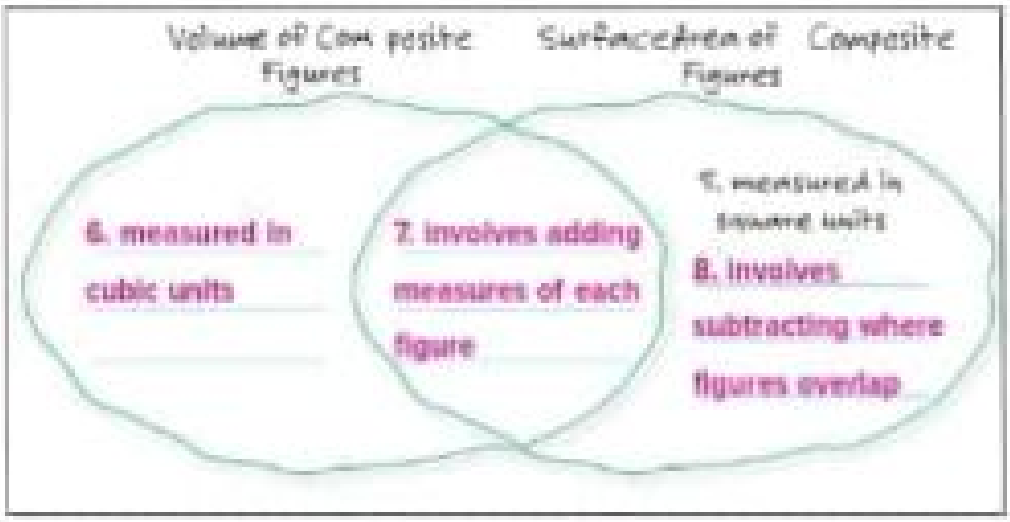
c. The area where the figures overlap is 200 square centimeters.

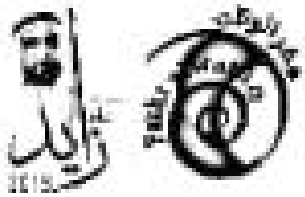
d. The surface area of the composite figure is 680 square centimeters.

Analyze and Reflect

Reason Inductively Work with a partner. Write each of the following statements in the correct location. One statement is done for you.

5. *measured in square units*
6. *measured in cubic units*
7. *involves adding measures of each figure*
8. *involves subtracting where figures overlap*





Create

9. **Model with Mathematics** Describe a real-world situation where it might be necessary to use a model or a drawing to find the volume or surface area.

Sample answer: Architects use models and drawings to determine volume and surface area of large buildings.

10. **Explain** HOW can you find the volume and surface area of a composite figure?

Sample answer: Add the volumes and surface areas of the figures that make up the composite figure. When finding surface area, be sure to subtract the overlapping areas.

Lesson 7

Volume and Surface Area of Composite Figure

Real-World Link

Abdalla and his sister Maysoun are making a bat house for their backyard like the one shown. They need to determine the surface area to find how much wood they will need.

Essential Question

HOW do measurements help you describe real-world objects?

Mathematical Practices

1, 5, 6

1. Look at the largest bat house. What three-dimensional figures make up the bat house?

   rectangular prism; triangular prism

2. What method could you use to find the surface area of the bat house?

   Find the area of each face and add. Then subtract the faces that overlap.

3. Suppose you wanted to find the volume of the bat house. What method could you use?

   Find the volume of each shape separately and then add.

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1 Persevere with Problems	5 Use Math Tools
2 Reason Abstractly	6 Attend to Precision
3 Construct an Argument	7 Make Use of Structure
4 Model with Mathematics	8 Use Repeated Reasoning

Work Zone

Volume of a Composite Figure

The volume of a composite figure can be found by decomposing the figure into solids whose volumes you know how to find.

Examples

1. **Find the volume of the composite figure.**

Find the volume of each prism.

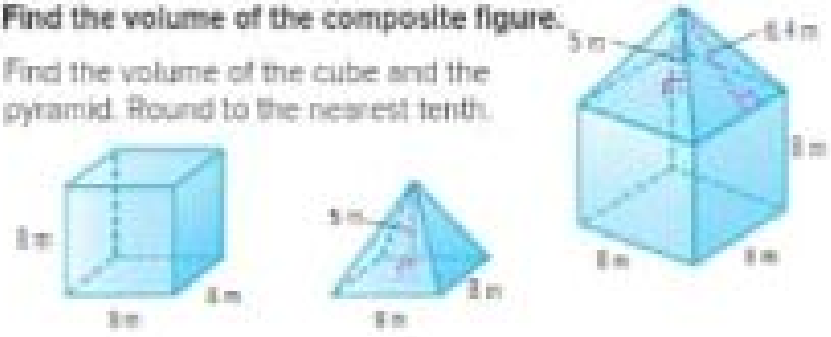
$V = \ell wh$ $V = \ell wh$

$V = 8 \cdot 6 \cdot 16$ or 768 $V = 8 \cdot 6 \cdot 8$ or 384

The volume is 768 + 384 or 1,152 cubic centimeters.

2. **Find the volume of the composite figure.**

Find the volume of the cube and the pyramid. Round to the nearest tenth.



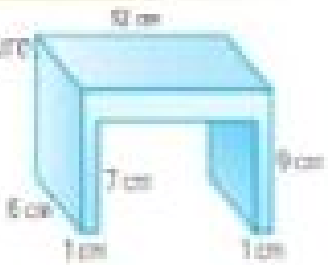
$V = \ell wh$ $V = \frac{1}{3}\ell wh$

$V = 8 \cdot 8 \cdot 8$ or 512 $V = \frac{1}{3}(8 \cdot 8)5$ or 106.7

The volume is 512 + 106.7 or 618.7 cubic meters.

Got it? **Do this problem to find out.**

a. Find the volume of the composite figure.



228 cm³

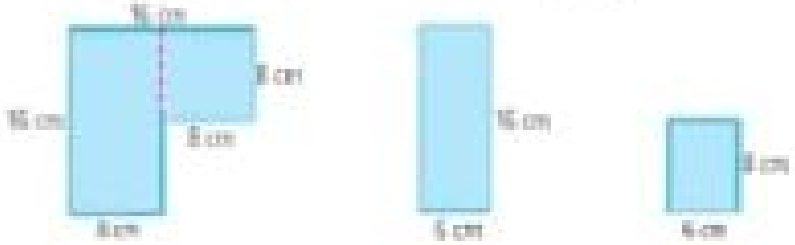
Surface Area of a Composite Figure

You can also find the surface area of composite figures by finding the areas of the faces that make up the composite figure.

Examples

3. **Find the surface area of the figure in Example 1.**

The surface is made up of three different polygons.



$A = \ell w + \ell w$ | $A = \ell w$ | $A = \ell w$

$A = (8 \cdot 16) + (8 \cdot 8)$ | $A = 6 \cdot 16$ | $A = 6 \cdot 8$

$A = 128 + 64$ or 192 | $A = 96$ | $A = 48$

The total surface area is $2(192) + 2(96) + 4(48)$ or 768 square centimeters.

4. **Find the surface area of the composite figure in Example 2.**

The figure is made up of two different polygons.

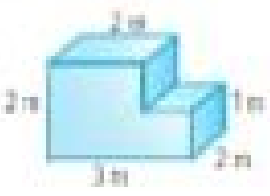
$A = \ell w$ | $A = \frac{1}{2}bh$

$A = 8 \cdot 8$ or 64 | $A = \frac{1}{2} \cdot 8 \cdot 6.4$ or 25.6

The total surface area is $5(64) + 4(25.6)$ or 422.4 square meters.

Get It? **Do this problem to find out.**

b. Find the surface area of the steps that are represented by the composite figure shown.



Surface Area

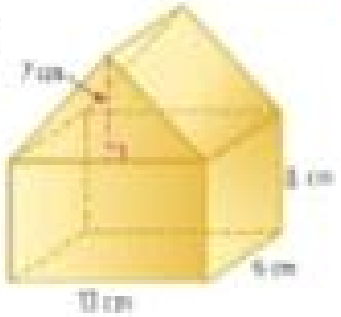
To make it easier to see each face, sketch the faces and label the dimensions of each.

b. 30 m²

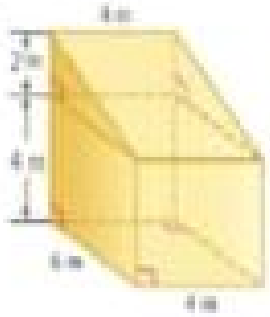
Guided Practice

Find the volume of each composite figure. Round to the nearest tenth if necessary. (Examples 1 and 2)

1. 897 cm³

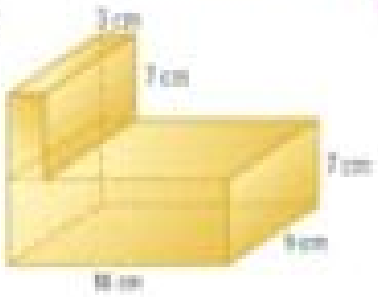


2. 120 m³

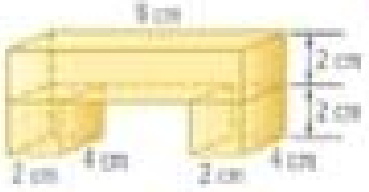


Find the surface area of each composite figure. Round to the nearest tenth if necessary. (Examples 3 and 4)

3. 870 cm²



4. 172 cm²

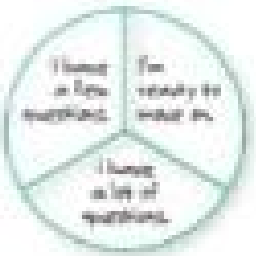


5. **Building on the Essential Question** How do the previous lessons in this chapter help you find the surface area and volume of a composite figure?

Sample answer: We can decompose the composite figure into three-dimensional shapes for which we already know how to find the surface area and volume.

Rate Yourself!

Are you ready to move on? Shade the section that applies.



Name _____ My Homework _____

Independent Practice

Find the volume of each composite figure. Round to the nearest tenth if necessary. (Examples 1 and 2)

1. 

2.3 m³

2. 

13 m³

Find the surface area of each composite figure. Round to the nearest tenth if necessary. (Examples 3 and 4)

3. 

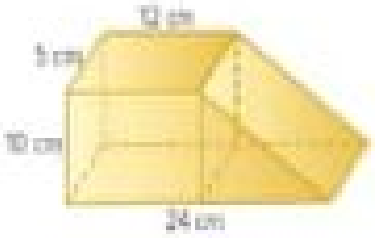
2,600 m²

4. 

720 m²

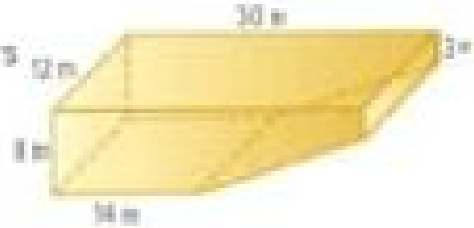
Find the volume of the figure at the right in cubic meters. Round to the nearest tenth. (Examples 1 and 2)

0.0009 m³



6. **Reason Inductively** The swimming pool at the right is being filled with water. Find the number of cubic meters that it will take to fill the swimming pool. (Hint: The area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$.) (Examples 1 and 2)

2,400 m³

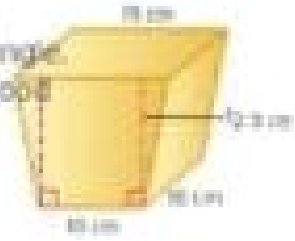


Copy and Solve **For Exercises 7–8, show your work on a separate piece of paper. Round to the nearest tenth.** (Examples 1–4)

7. Find the surface area of the figure in Exercise 1. 10.4 m²

8. Find the volume of the figure in Exercise 4. 972 m³

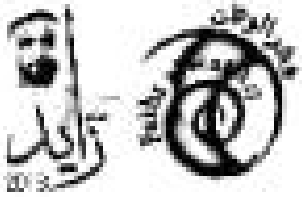
9. A carryout container is shown. The bottom base is a 10-centimeters square and the top base is a 10-centimeters by 15-centimeters rectangle. The height of the container is 12.5 centimeters. Find the volume of food that it holds. 1,562.5 cm³



10. Refer to the house shown. Find the surface area and volume of the house. Do not include the bottom of the house when calculating the surface area. volume: 1,380 ft³; surface area: 616.4 ft²

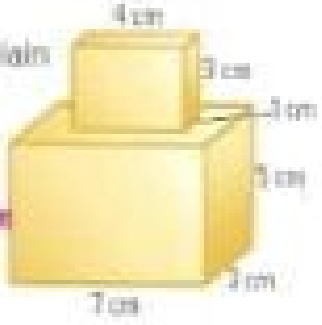
H.O.T. Problems Higher Order Thinking

11. **Model with Mathematics** Draw a composite figure that is made up of a cube and a square pyramid. Label its dimensions and find the volume of the figure. See students' work.



12. **Persevere with Problems** Draw an example of a composite figure that has a volume between 250 and 300 cubic u... See students' work.

13. **Construct an Argument** Will the surface area of the figure at the right be greater than or less than 180 square centimeters? Explain your reasoning. less than; Sample answer: The combined surface area of the two separate prisms is 180 cm², since they share a surface, the area of that surface is not included in the total surface area. The total surface area will be less than 180 cm.



Extra Practice

Find the volume of each composite figure. Round to the nearest tenth if necessary.

14. 450 cm³

Rectangular Prism

$V = lwh$

$V = 5 \cdot 10 \cdot 7$

$V = 350$

Triangular Prism

$V = Bh$

$V = \frac{1}{2} \cdot 10 \cdot 4 \cdot 5$

$V = 100$

Total volume = 350 + 100 or 450 cm³

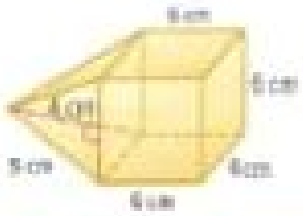
15. 100 cm²

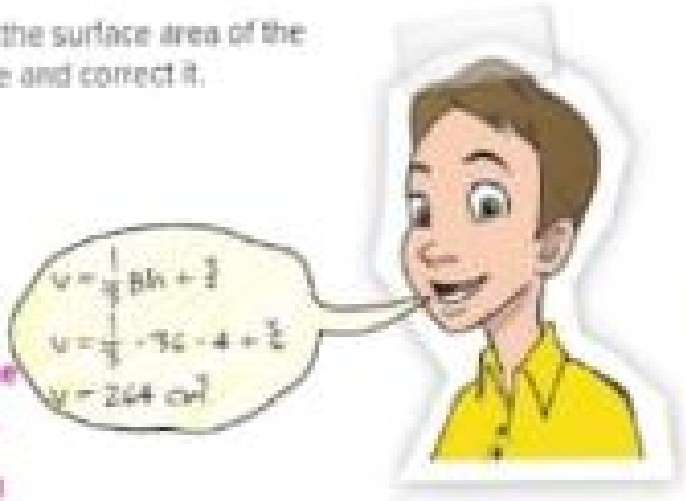
Find the surface area of each composite figure. Round to the nearest tenth if necessary.

16. 292 m²

17. 280.2 cm²

18. Find the Error Abdulkarim is finding the surface area of the composite figure shown. Find his mistake and correct it.





Seth incorrectly found the volume of the composite figure instead of the surface area. The actual surface area is 228 cm²

Power Up! Test Practice

19. Refer to the composite figure with the dimensions shown. Fill in the boxes to complete each statement.

a. The volume of the composite figure is 1.68 m³.

b. The total surface area of the composite figure is 19.28 m².

20. Refer to the composite figure with the dimensions shown.

Select the correct values to complete the expression to find the volume of the figure.

$V = \boxed{2} \cdot \frac{1}{3} \cdot \boxed{6} \cdot \boxed{6} \cdot \boxed{8}$

2
3
4
6
8

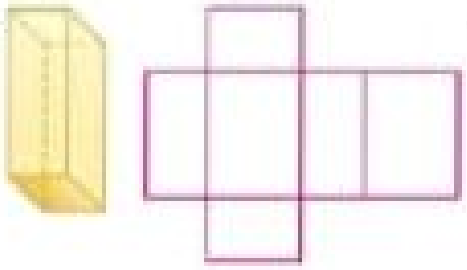
What is the volume of the composite figure?

192 mm³

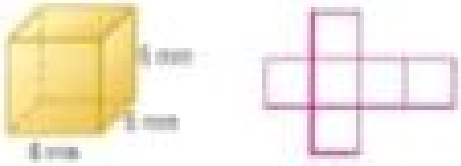
Spiral Review

Draw a net for each figure. 6.G.4

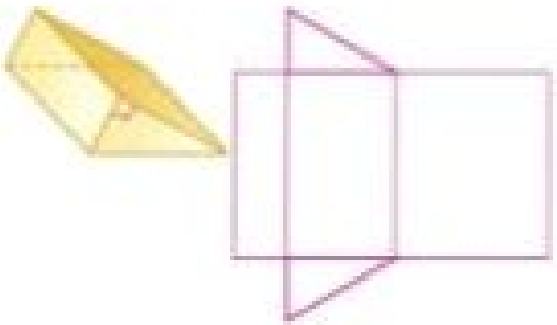
21.



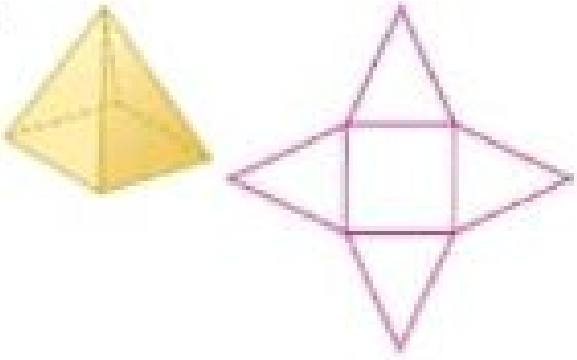
22.



23.



24.



21ST CENTURY CAREER in Design

Interior Designer

Do you like coming up with new ways to decorate your room, or are you always rearranging the furniture? You could have a career doing just that by becoming an interior designer. Interior designers plan the interior space and furnishings of homes, offices, and other places. Their designs are based on the client's specifications, tastes, and budget. Interior designers are responsible for recommending color schemes, furniture, lighting, and remodeling options. Many interior designers also develop their own product lines such as furniture, bedding, and accessories.

Is This the Career for You?

Are you interested in a career as an interior designer? Take some of the following courses in high school.

- Algebra
- Geometry
- Interior Design
- Intro to CAD

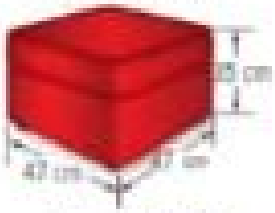
Turn the page to find out how math relates to a career in Design.

You be the Designer!

Use the labeled figures to solve each problem. Round to the nearest tenth if necessary.

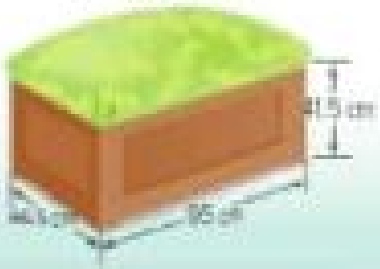
1. A client wants to buy the rectangular ottoman with the most storage area inside. Which one should she choose? Explain your reasoning. Purple; its volume is 84,000 dm compared to the red ottoman, which has a volume of 77,315 cm
2. Find the volume of the blanket chest. 183,326.25 cm
3. What is the volume of the toy chest? How does it compare to the volume of the blanket chest? 83,895 cm; it is about half the size.
4. A designer is having the red ottoman reupholstered. If the bottom is not covered, estimate the amount of fabric needed. 8,789 cm
5. How much fabric is needed to cover the purple ottoman? 41,800 cm
6. How much greater is the surface area of the paisley blanket chest than the surface area of the toy chest? 7,470.5 cm

Red Ottoman



Purple Ottoman

Blanket Chest



Toy Chest

Career Project

It's time to update your career portfolio! Use grid paper to make a scale drawing of a room in your home. Model the furniture using squares, rectangles, and triangles drawn to scale. Cut out each shape and use them to create different room arrangements. Then, tape the pieces onto the grid paper. Describe the room's color scheme and style.

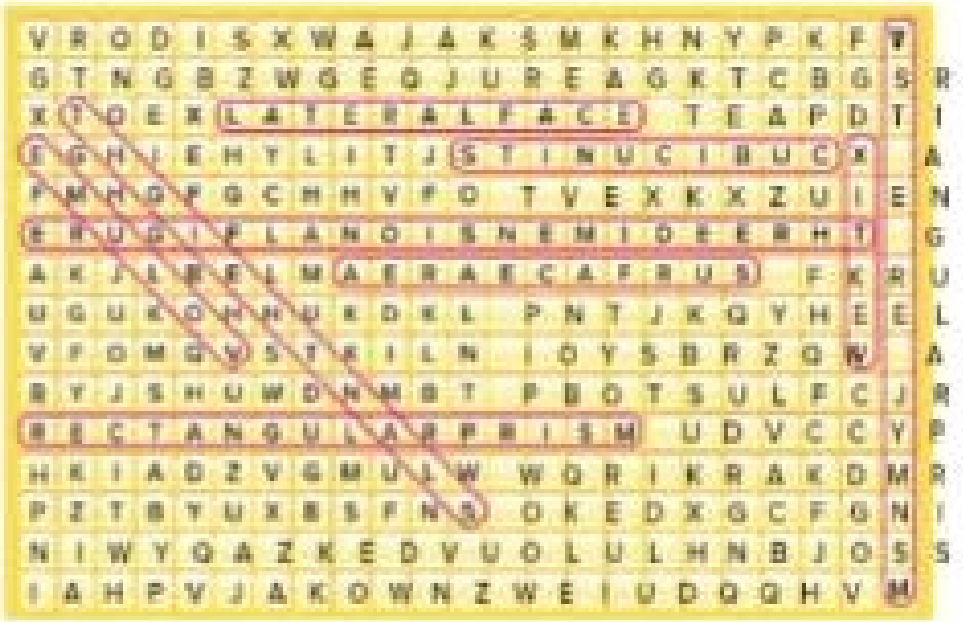
Do you think you would enjoy a career as an interior designer? Why or why not?

Chapter Review

Vocabulary Check

Complete each sentence using the vocabulary list at the beginning of the chapter. Then circle the word that completes the sentence in the word search.

1. A figure with length, width, and height is a three-dimensional figure.
2. Surface area is the sum of the area of all the faces of a three-dimensional figure.
3. The amount of space inside a three-dimensional figure is a volume.
4. A prism that has triangular bases is a triangular prism.
5. A rectangular prism is a prism that has rectangular bases.
6. Volume is measured in cubic units.
7. The point where three or more faces intersect is the vertex.
8. The slant height is the height of each lateral face.
9. Any face that is not a base is a lateral face.



Reflect

Answering the Essential Question

Use what you learned about volume and surface area to complete the graphic organizer.

Essential Question

HOW is shape important when measuring a figure?

	Draw it.	How do you find the volume?	How do you find the surface area?
rectangular prism		Sample answer: Use the formula $V = \ell wh$ or the formula $V = Bh$.	Sample answer: Use the formula $S = 2\ell w + 2\ell h + 2wh$.
triangular prism		Sample answer: Use the formula $V = Bh$.	Sample answer: Find the area of each base and the area of the 3 rectangular faces and add.

Answer the Essential Question HOW is shape important when measuring a figure?

See students' work.

Statistics and Probability

Essential Question

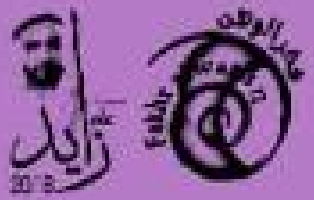
WHY is learning mathematics important?

Chapter 11
Statistical Measures

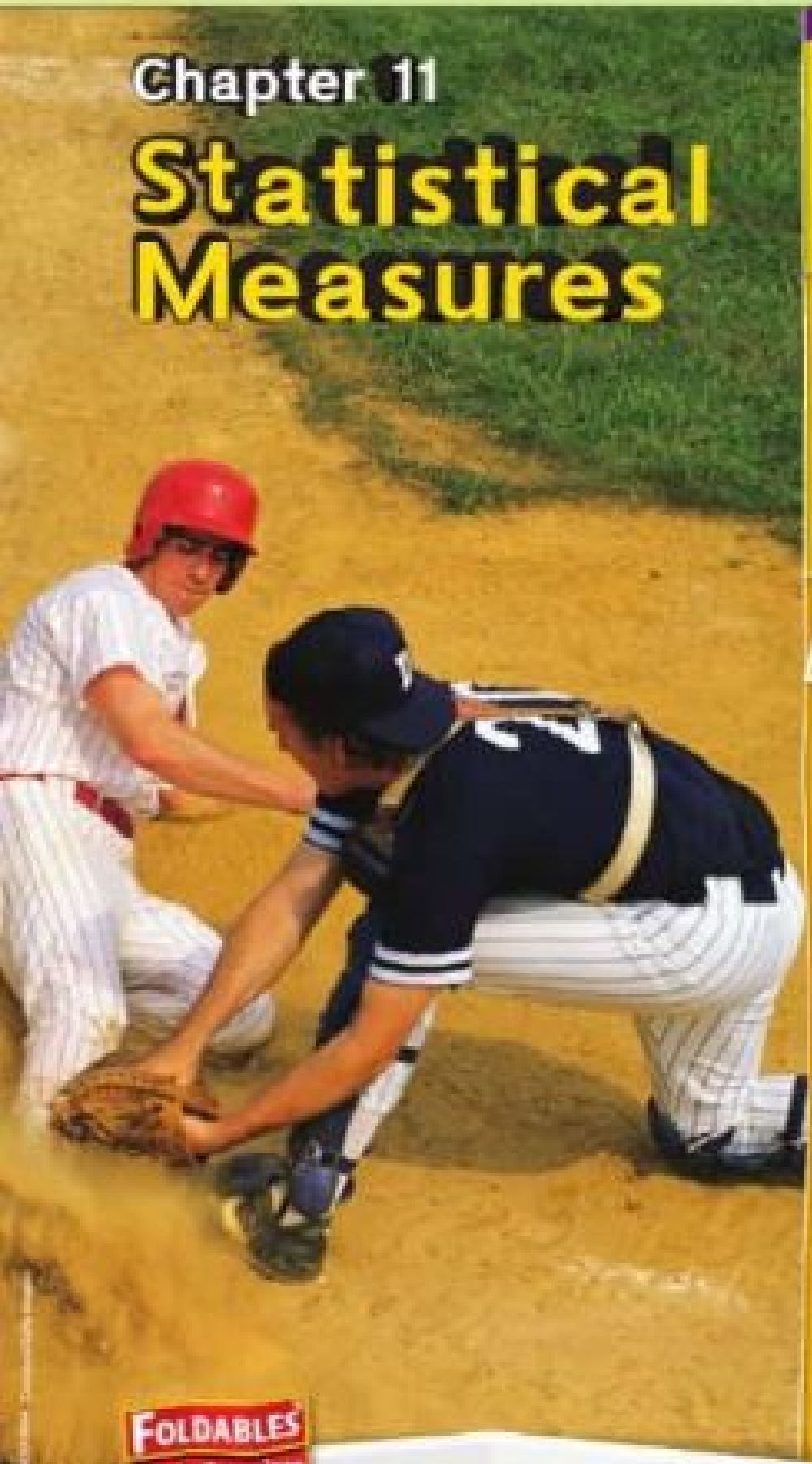
Statistical data has a distribution that can be described by its center or by its spread. In this chapter, you will find and use measures of center and measures of variation to describe sets of data.

Chapter 12
Statistical Displays

Statistical data can be represented in a variety of ways. In this chapter, you will represent and analyze data using line plots, histograms, and box plots.



Chapter 11
Statistical Measures



Essential Question

HOW are the mean, median, and mode helpful in describing data?

Mathematical Practices
1, 2, 3, 4, 5, 6

Math in the Real World

Sports A baseball team scored 9, 6, 8, 16, and 5 points in 5 games. Plot the scores on the number line.

FOLDABLES
Study Organizer

1. Cut out the Foldable on page FL13 of this book.
2. Place your Foldable on page 856.
3. Use the Foldable throughout this chapter to help you learn about statistical measures.

What Tools Do You Need?

Vocabulary

average
first quartile
interquartile range
mean
mean absolute deviation
measure of center
measures of variation
median
mode
outliers
quartiles
range
statistical question
third quartile

Review Vocabulary

Graphic Organizer One way to remember vocabulary terms is to connect them to an opposite term or example. Use this information to complete the graphic organizer.

Sample answers are given.

quotient

Definition
The result of a division problem.

Opposite
product—The result of a multiplication problem.

Example
In $6 \div 2 = 3$, 3 is the quotient.

Are You Ready

Try the Skills Check below.

Quick Review

Review

Example 1

Find 12.53 + 9.87 + 16.24 + 22.12.

12.53
9.87
16.24
+ 22.12
60.76

Add.

Example 2

Eiman read 56.5 pages of her book on Monday and Tuesday. If she read the same amount of pages each day, how many pages did she average each day?

56.5 ÷ 2 = 28.25 Divide the total number of pages by the number of days.

Eiman averaged 28.25 pages per day.

Quick Check

Add Decimals **Find each sum.**

1. 6.20 + 31.59 + 11.11 + 19.85 =

68.75

2. 22.69 + 15.45 + 9.87 + 26.79 =

74.8

3. Asma went to the baseball game. She paid AED 10.50 for admission. She bought a drink for AED 2.75, a bag of popcorn for AED 4.60, and a sandwich for AED 3.75. How much did she spend in total?

AED 21.60

Divide Decimals **Find each quotient.**

4. 79.2 ÷ 6 =

13.2

5. 72.60 ÷ 3 =

24.20

6. 240.5 ÷ 13 =

18.5

7. Ibrahim's family drove 345.6 kilometers on their vacation. They drove the same amount each of the 3 days. How many kilometers did they drive each day?

115.2 kilometers

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7

Inquiry Lab
Statistical Questions

inquiry **HOW are surveys created to collect and analyze data?**

Mathematical Practices 1, 3, 4

Anderson Advertising is collecting information for a pizza shop. They want to know the number of toppings most customers prefer on their pizza. They will use this information to determine the weekly special.

Hands-On Activity 1

Statistics deals with collecting, organizing, and interpreting pieces of information, or ***data***. One way to collect data is by asking statistical questions. A **statistical question** a question that anticipates and accounts for a variety of answers.

The table below gives some examples of statistical questions and questions that are *not* statistical questions.

Statistical Questions	Not Statistical Questions
How many text messages do you send each day?	What is the height in meters of the tallest mountain in Colorado?
What is the minimum driving age for each state in the United States?	How many people attended last night's jazz concert?

Create a survey similar to the one Anderson Advertising would use to survey your classmates. Consider a cheese pizza with no additional toppings as a pizza with one topping.

Step 1 Write a statistical question. *How many toppings do you like on your pizza?*

Step 2 Survey your classmates.

Step 3 Record the results in the table to the right. Add additional numbers of toppings to the table as necessary.
See students' work.

How Many Toppings Do You Like on Your Pizza?	
Number of Toppings	Number of Responses

Why is the following a statistical question? *How many toppings do you like on your pizza?*

It anticipates and accounts for a variety of answers, such as 1, 2, 3, or more toppings.

Hands-On Activity 2

Sometimes a set of data can be organized into intervals to more easily organize it. This often happens when the set of data has a wide range of values.

Suppose you want to determine the number of video games each of your math classmates has at home.

Step 1 Write the statistical question. ***How many different video games do you own?***

Step 2 Survey your classmates.

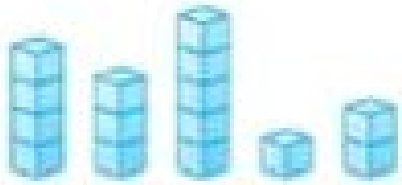
Step 3 Record the results in the table to the right.
See students' work.

How Many Different Video Games Do You Own?	
Number of Video Games	**Number of Responses**
less than 5	
5–9	
10–14	
15 or more	

Hands-On Activity 3

You can use surveys to provide information about patterns in the responses. Suppose you surveyed five students using the statistical question, ***How many Web sites did you visit before school this morning?*** The students said 4, 3, 5, 1, and 2 Web sites. If the total amount was equally distributed among all five students, how many Web sites did each student visit?

Step 1 Make a stack of centimeter cubes to represent the number of Web sites visited by each student as shown.



Step 2 Move the cubes so that each stack has the same number of cubes. Draw your models in the space below.

There are five stacks with 3 cubes in each stack. So, if the responses were equally distributed, each student visited 3 Web sites before school.

Investigate

Work with a partner. State whether each question is a statistical question. Explain your reasoning.

1. Who was the first president of the United States?

   no; Sample answer: The question does not anticipate a variety of answers since there was only one first president of the United States.

2. How much time do the students in my school spend on the Internet each night?

   yes; Sample answer: The question anticipates a variety of answers, such as 45 minutes or 2 hours.

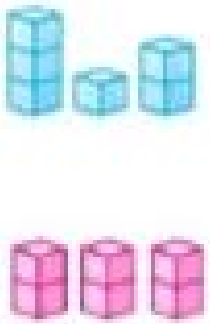
3. What is the height of the tallest waterslide at Wild Rides Water Park?

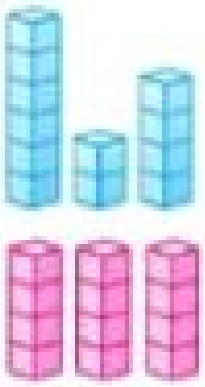
   no; Sample answer: The question does not anticipate a variety of answers since there is only one height for the tallest waterslide.

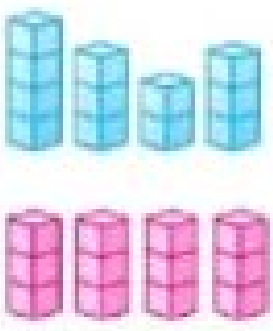
4. What are the cabin rental prices for each of the state parks in Kentucky?

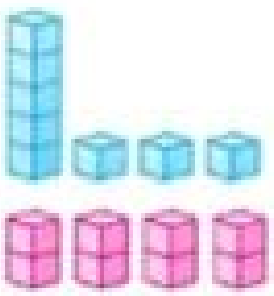
   yes; Sample answer: The question anticipates a variety of answers, such as AED 75 or AED 100 per night.

Work with a partner. Determine the equal share if the total number of centimeter cubes were equally distributed among the groups. Draw your models in the space provided.

5. 

6. 

7. 

8. 

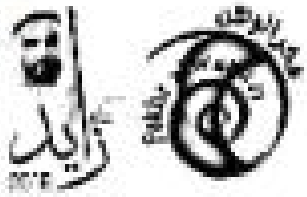
Analyze and Reflect

Work with a partner to determine the equal share for each exercise. Use centimeter cubes or counters if needed. The first one is done for you.

	Scenario	Responses	Response Total	Number of Responses	Equal Share
	Rainfall (centimeters)	7, 5, 2, 6	$7 + 5 + 2 + 6 = 20$	4	5
9.	Books Read	8, 7, 3	$8 + 7 + 3 = 18$	3	6
10.	Eggs Hatched	5, 2, 3, 6	$5 + 2 + 3 + 6 = 16$	4	4
11.	States Visited	1, 4, 2, 5, 3	$1 + 4 + 2 + 5 + 3 = 15$	5	3
12.	Photos Taken	5, 3, 7, 2, 4, 3	$5 + 3 + 7 + 2 + 4 + 3 = 24$	6	4
13.	Kilometers Hiked	11, 12, 8, 9	$11 + 12 + 8 + 9 = 40$	4	10

14. **Reason Inductively** Compare the answers you provided in the table above. How does the response total and the number of responses relate to the equal share? Write a rule you can use to evenly distribute a data set without using centimeter cubes.

Sample answer: You can find the equal share by dividing the response total by the number of responses.



Create

15. **Model with Mathematics** Write a survey question that yields data without variability. Rewrite the question so that it yields data with variability.

Sample answer: How many seasons are there during the year?

What is your favorite season of the year?

16. **Model with Mathematics** Write a real-world problem that involves equal shares. Find the equal share of your data set.

Sample answer: Fatema surveyed several of her volleyball teammates. She asked the question, *How many hours do you exercise each week?* The responses were 6, 4, 5, 7, 3, 6, and 4. The equal share of the data set is 5 hours each week.

17. **Equity** HOW are surveys created to collect and analyze data?

Sample answer: Surveys ask statistical questions. Statistical questions anticipate a variety of responses. You can compare the responses individually or by using intervals. Finding the equal share of the

provide information about response patterns

Lesson 1

Mean

Real-World Link

Music Amani and her friends downloaded songs for 6 weeks, as shown in the table.

Number of Songs Downloaded Each Week					
12	6	10	9	4	1

1. How many total songs were downloaded? 42 songs

2. On average, how many songs did they download each week?

42	÷	6	=	7
total		number of weeks		average per week

3. On the number line below, draw an arrow that points to the average. Plot the number of songs downloaded on the number line.

4. How far below the average is 1? 4? 6? How far above the average is 9? 10? 12? 6, 3, 1, 2, 3, 5

5. What is the sum of the distances between the average and the points below the average? above the average? 10; 10

6. Explain why the average is the balance point of the data.
Sample answer: the sum of the distances from 7 of all the data points above 7 is 10 and the sum of all the data points below 7 is 10. So, the data above and below 7 are balanced.

Essential Question

HOW are the mean, median, and mode helpful in describing data?

Vocabulary

mean

average

Mathematical Practices

1, 2, 3, 4, 6

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

- 1 Persevere with Problems
- 2 Reason Abstractly
- 3 Construct an Argument
- 4 Model with Mathematics
- 5 Use Math Tools
- 6 Attend to Precision
- 7 Make Use of Structure
- 8 Use Repeated Reasoning

Key Concept

Mean

The **mean** of a data set is the sum of the data divided by the number of pieces of data. It is the balance point for the data set.

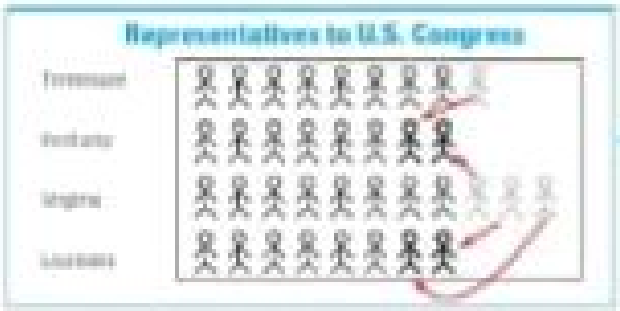
Work Zone

On the previous page, you found a single number to describe the number of songs downloaded each week. **Average** or mean, summarizes the data using a single number.

You can find the mean of a set of data shown in different displays such as pictographs and dot plots.

Example

1. Find the mean number of representatives for the four states shown in the pictograph.



Move the figures to equally distribute the total number of representatives among the four states.

Each state has a mean or average of 8 representatives.

Including Data

Even if a data value is 0, it should be counted in the total number of pieces of data.

Got it? Do this problem to find out.

a. The table shows the number of CDs a group of friends bought. Find the mean number of CDs the group bought.

Number of CDs Purchased		
3	4	6
0	2	

a. 3

Examples

2. **The dot plot shows the recorded high temperatures for six days in Little Rock, Arkansas. Find the mean temperature.**

$$\text{mean} = \frac{45 + 45 + 47 + 49 + 50 + 52}{6} \quad \leftarrow \text{sum of the data} \quad \leftarrow \text{number of data items}$$

$$= \frac{288}{6} \text{ or } 48 \quad \text{Simplify}$$

The mean is 48 degrees. So, all of the data values can be summarized with a single number, 48.

3. **The dot plot shows the number of runs a baseball team had for each game of a 4-game series. Find the mean number of runs for the series.**

$$\text{mean} = \frac{1 + 3 + 4 + 4}{4} \quad \leftarrow \text{sum of the data} \quad \leftarrow \text{number of data items}$$

$$= \frac{12}{4} \text{ or } 3 \quad \text{Simplify}$$

The mean number of runs for the series is 3.

Got it? **Do this problem to find out.**

b. The dot plot shows the number of books Amal read each week of a reading challenge. Find the mean number of books she read.

b. 3 books

Dot Plots

In a dot plot, individual data values are represented as dots above a number line.

STOP and Reflect

The mean is sometimes described as the balance point. Explain below what this means using the data set {2, 2, 3, 4, 10}.

Sample answer: This means that the sum of the distances from 5 of all the points above 5 is equal to the sum of the distances of all the data points below 5.

Example

4. The number of minutes Amna spent talking on her cell phone each month for the past five months were 494, 502, 486, 690, and 478. Suppose the mean for six months was 532 minutes. How many minutes did she talk on her cell phone during the sixth month?

If the mean is 532, the sum of the six pieces of data must be 532×6 or 3,192. You can create a bar diagram.

$3{,}192 - (494 + 502 + 486 + 690 + 478) = 3{,}192 - 2{,}650$

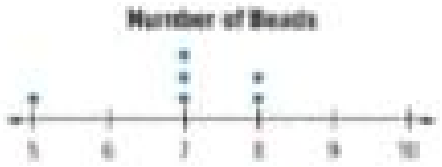
$= 542$

Amna talked 542 minutes during the sixth month.

Guided Practice

1. The dot plot shows the number of beads sold. Find the mean number of beads.

7



2. The table shows the greatest depths of four of the five oceans in the world. If the average greatest depth is 8.094 kilometers, what is the greatest depth of the Southern Ocean? 7.24 km

Ocean	Greatest Depth (km)
Pacific	10.92
Atlantic	9.22
Indian	7.46
Arctic	5.63
Southern	■

3. **Building on the Essential Question** Why is it helpful to find the mean of a data set?

Sample answer: The mean gives the average of the data set, which is a summary of all the data using a single number.

Rate Yourself!

How confident are you about finding the mean of a data set? Check the one that applies.

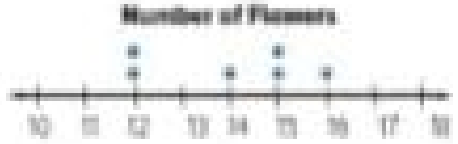
Reason: Then adjust your response!

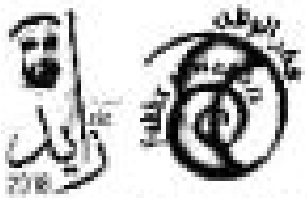
Independent Practice

Find the mean for each data set.

1. 88%

2. 14

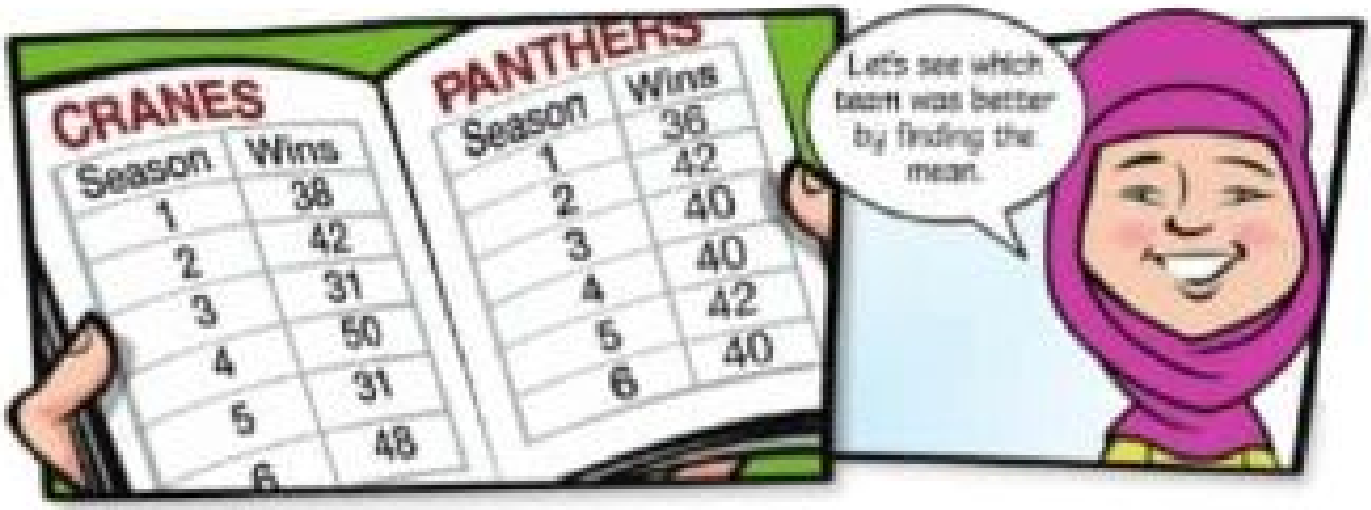




Financial Literacy Buthaina babysat nine times. She earned AED 15, AED 20, AED 10, AED 12, AED 20, AED 16, AED 80, and AED 18 for eight babysitting jobs. How much did she earn the ninth time if the mean of the data set is AED 24?

AED 25

4. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.



a. What is the mean number of wins for the Cranes? and for the Panthers?

40; 40

b. Based on your answer for part a, is the mean a good measure for determining which team has the better record? Explain.

no; Both means are equal.

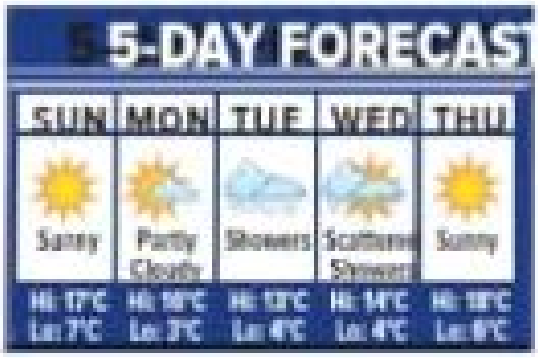
5. A *stem-and-leaf plot* is a display that organizes data from least to greatest. The digits of the least place value form the leaves, and the next place-value digits form the stems. The stem-and-leaf plot shows Bedria's scores on several tests. Find the mean test score.

Stem	Leaf
7	8
8	5 8 9
9	2 6

7|8 = 78

88

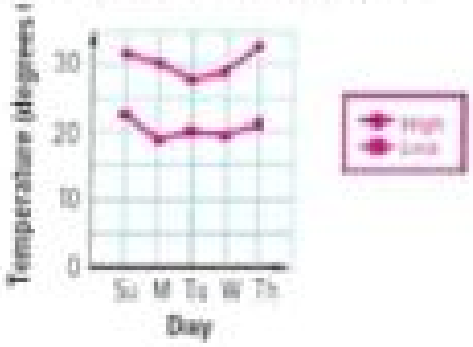
6. **Multiple Representations** The graphic shows the 5-day forecast.



a. **Numbers** What is the difference between the mean high and mean low temperature for this 5-day period? Justify your answer.

10.8°C; The mean high temperature was 15.6°C and the mean low temperature was 4.8°C; therefore, the difference is 10.8°C.

b. **Graph** Make a double-line graph of the high and low temperatures for the 5-day period.



H.O.T. Problems Higher Order Thinking

7. **Reason Abstractly** Create a data set that has five values. The mean of the data set should be 34.

Sample answer: pages read: 27, 38, 26, 39, 40

8. **Persevere with Problems** The mean of a set of data is 45 years. Find the missing numbers in the data set {40, 45, 48, ?, 54, ?, 45}. Explain the method or strategy you used.

Sample answer: 41 and 42; I used the *work backward* strategy.

9. **Reason Inductively** If 99 students had a mean quiz score of 82, how much is the mean score increased by the addition of a single score of 99? Explain.

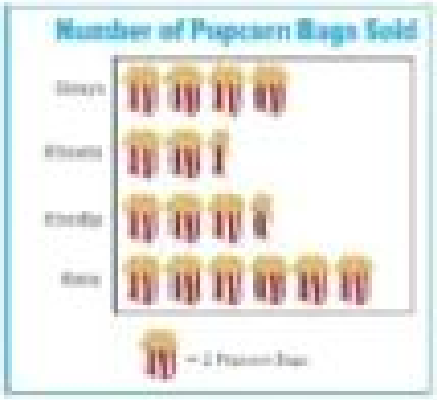
0.17; Sample answer: The sum of the scores for the 99 students must be 82 × 99 or 8,118. Adding the score of 99, the sum of the 100 students is 8,217. The new mean is then 82.17. The mean increased by 82.17 − 82 or 0.17.

Name _____ My Homework _____

Extra Practice

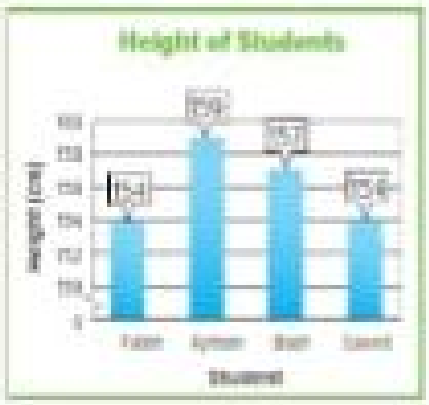
Find the mean for each data set.

10. 8 bags

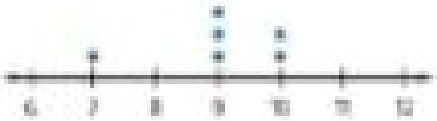


$\frac{8 + 6 + 7 + 12}{4} = 8$

11. 156 cm.



12. 9 cards



13. 26 tickets

14. **Be Precise** The table shows the approximate heights of some of the tallest U.S. trees.

Tallest Trees in U.S.	
Tree	Height (m)
Western Red Cedar	49
Coast Redwood	97
Monterey Cypress	30
California Laurel	34
Sitka Spruce	61
Port Orford Cedar	67

a. Find the mean of the data. 56.3 m

b. Find the mean if the Coast Redwood is not included in the data set. 48.2 m

c. How does the height of the Coast Redwood affect the mean of the data? Sample answer: An extreme value that is higher than the other values causes the mean of the data to be higher than most of the values in the table. So, the mean is less representative of the data.

d. Suppose Blue Spruce was included in the list and the mean decreased to 50.3 m. What is the height of the Blue Spruce?
14.1 m

Power Up! Test Practice

15. The table shows the money raised by each type of booth at a craft sale. The mean amount raised per booth was AED 59. How much money was raised by the T-shirt booth? Explain how you found your answer.

AED 80; Sample answer: Multiply 59 by 6 and subtract the other amounts given in the table.

Booth	Money Raised (AED)
Artwork	58
Candles	47
Holiday decorations	54
Jewelry	70
Picture frames	45
T-shirts	?

16. The table shows the number of points scored by a football team during their first 4 games.

Game	1	2	3	4
Points Scored	24	30	22	28

Select values to complete the model below to find the mean number of points scored per game.

$$\frac{\boxed{24} + \boxed{30} + \boxed{22} + \boxed{28}}{\boxed{4}} = \boxed{26}$$

1	24
2	26
3	28
4	30
22	32

An average of **26** points were scored per game.

Common Core Spiral Review

Compare the numbers using < or >.

17. 18 > 16

18. 65 > 63

19. 22 < 28

20. 34 > 31

21. 75 < 79

22. 67 > 57

23. The table shows the distances from Louisville to several cities.

City	Distance (kilometers)
Charlotte	763
Cincinnati	161
Indianapolis	183
Lexington	121
St. Louis	426

a. How much farther is it from Louisville to Charlotte than from Louisville to Lexington? **642 kilometers**

b. Which city is the greatest distance from Louisville? **Charlotte**

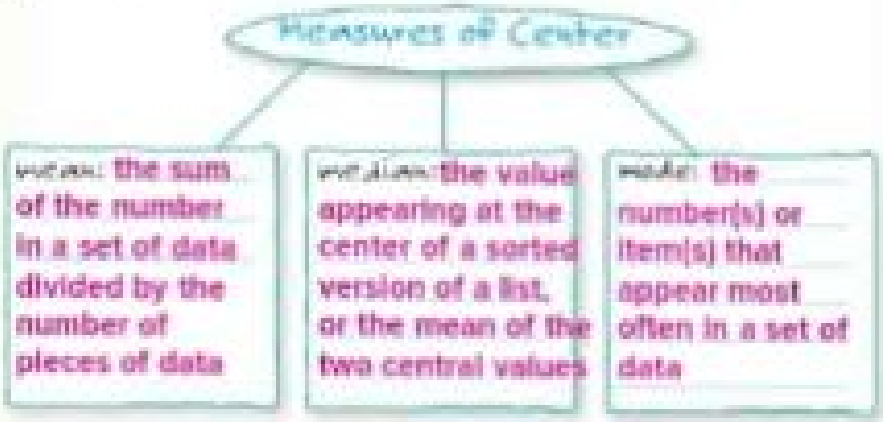
Lesson 2

Median and Mode

Vocabulary Start-Up

A data set can also be described by its median or its mode. The mean, median, and mode are called **measures of center** because they describe the center of a set of data.

Find the definition of each term in the glossary. Then complete the graphic organizer.



Essential Question

HOW are the mean, median, and mode helpful in describing data?

Vocabulary

measure of center
median
mode

Mathematical Practices
1, 3, 4, 5, 6

Real-World Link

Hurricanes The table shows the number of Atlantic hurricanes in different years.

Atlantic Hurricanes						
5	15	9	7	4	9	8

1. Order the data from least to greatest. Circle the number in the middle of your list. 4, 5, 7, 8, 9, 9, 15

2. Find the mean. Compare the middle number to the mean of the data. Round to the nearest hundredth if necessary.
8.14; Sample answer: 8 is less than the mean of the data.

Which Mathematical Practices did you use? Shade the circle(s) that applies.

- Persevere with Problems
- Use Math Tools
- Reason Abstractly
- Attend to Precision
- Construct an Argument
- Make Use of Structure
- Model with Mathematics
- Use Repeated Reasoning

Key Concept

Median and Mode

The **median** of a list of values is the value appearing at the center of a sorted version of the list, or the mean of the two central values, if the list contains an even number of values.

The **mode** is the number or numbers that occur most often.

Just as mean is one value used to summarize a data set, the median and mode also summarize a data set with a single number. If there is more than one number that occurs with the same frequency, a data set may have more than one mode.

Examples

1. **The table shows the number of monkeys at eleven different zoos. Find the median and mode of the data.**

Number of Monkeys					
28	36	18	25	12	44
18	42	34	16	30	

Order the data from least to greatest.

Median 12, 16, 18, 18, 25, 28, 30, 34, 36, 42, 44 — 28 is in the center

Mode 12, 16, 18, 18, 25, 28, 30, 34, 36, 42, 44 — 18 occurs most often

The median is 28 monkeys. The mode is 18 monkeys.

2. **Hamdah recorded her scores on 7 tests in the table. Find the median and mode of the data.**

Test Scores			
93	88	94	93
85	97	90	

Order the data from least to greatest.

85 88 90 93 93 94 97

Circle the number in the center. This is the median.

Circle the most frequently occurring numbers. This value is the mode.

The median is a score of 93. The mode is a score of 93.

Get It? **Do this problem to find out.**

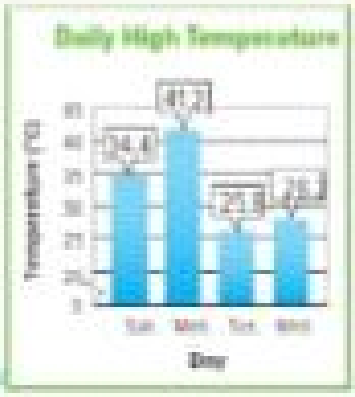
a. The list shows the number of stories in the 11 tallest buildings in Springfield. Find the median and mode of the data.

40, 38, 40, 37, 33, 30, 20, 24, 21, 17, 19

a. median: 30 stories; mode: 40 stories

Examples

3. **Find the median and mode of the temperatures displayed in the graph.**



Median 25.8, 28.2, 34.4, 41.2

$\frac{28.2 + 34.4}{2} = \frac{62.6}{2} = 31.3°$

There are an even number of data values. So, to find the median, find the mean of the two central values.

Mode There is no mode.

4. **Hareb researched the average precipitation in several states. Find and compare the median and mode of the average precipitation.**

State	Precipitation (cm)	State	Precipitation (cm)
Alabama	58.3	Louisiana	60.1
Florida	54.5	Maine	42.2
Georgia	50.7	Michigan	32.8
Kentucky	48.9	Missouri	42.2

Median 32.8, 42.2, 42.2, 48.9, 50.7, 54.5, 58.3, 60.1

$\frac{48.9 + 50.7}{2} = \frac{99.6}{2} = 49.8$

Mode 32.8, 42.2, 42.2, 48.9, 50.7, 54.5, 58.3, 60.1

The median is 49.8 centimeters and the mode is 42.2 centimeters. The median is 7.6 centimeters greater than the mode.

Got it? **Do these problems to find out.**

b. Find the median and mode of the costs in the table.

Cost of Backpacks (AED)			
16.78	48.75	31.42	18.38
22.89	31.25	28.54	26.79

c. Find and compare the median and mode of the costs in the table.

Cost of Juice (AED)			
1.65	1.97	2.45	2.87
2.35	3.75	2.49	2.87

b. median: AED 27.67; mode: none

c. The median is AED 2.47 and the mode is AED 2.87. The median is AED 0.40 less than the mode.

Example

5. Describe the daily high temperatures using the measures of center.

Daily High Temperature (°C)			
42	43	37	35
41	34	41	

Mean $\frac{42 + 43 + 37 + 35 + 41 + 34 + 41}{7} = \frac{273}{7}$ or 39°

Median 34, 35, 37, (41), 41, 42, 43

Mode 34, 35, 37, (41, 41), 42, 43

The median and mode both equal 41 degrees. They are both 2 degrees greater than the mean. The data follows the measures of center in that the temperatures are close to the measures of center.

Got It? Do this problem to find out.

d. Describe the cost of CDs using the measures of center.

Cost of CDs (AED)		
11.95	12.89	19.99
19.99	12.59	18.49

d. The mean, AED 15.98, and median, AED 15.69, are close and lower than the mode, AED 19.99, which is about AED 4 higher. The median and mean are in the middle of the data set, while the data are grouped near the extremes.

Guided Practice

1. Find and compare the median and mode for the following set of data. monthly spending: AED 46, AED 62, AED 62, AED 57, AED 50, AED 42, AED 56, AED 40 (Examples 1–4)

AED 53; AED 62; The median is AED 9 less than the mode.

2. Describe the daily high temperatures using the measures of center. (Example 5)

Daily High Temperature (°C)			
34	35	31	36
31	34	33	

The measures vary by one degree each. The median is the highest at 33°, the mode is the lowest at 31°, and the mean is 32°. The data follows the measures of center in that they are close to the measures of center.

3. **Building on the Essential Question** How are mean and median similar? Sample answer: Both are one number used to summarize a data set.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

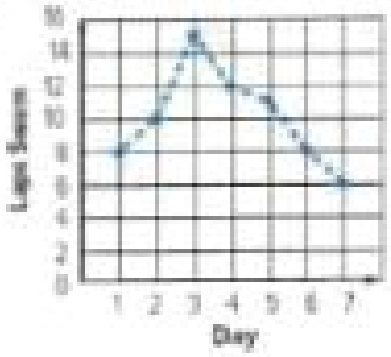
Time to update your Foldable!

Name _____ My Homework _____

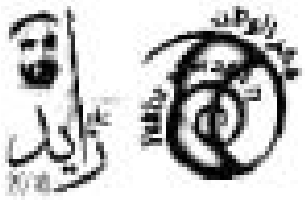
Independent Practice

Find and compare median and mode for each set of data. (ex 1–4)

1. math test scores: 97, 85, 92, 89; none; There is no mode to compare.

2. 

median: 10; mode: 8; The median is 2 laps more than the mode.



3. Describe the average speeds using the measures of center. (ex 5)

The values are close. The median and mode are equal, 44 km/h, and the mean of about 45.6 km/h, is slightly more. The data follows the measures of center in that they are close to the measures of center.

Average Speeds (km/h)			
40	52	44	46
52	40	44	50
41	44	44	50

4. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.

a. Find the median and mode for each team's wins.

Cranes: median: 40, mode: 31; Panthers: median: 40, mode: 40

b. Which team had the better record? Justify your response.

Sample answer: The Panthers had the better record. Even though the mean and median are the same, the Panthers' mode is higher.

A newspaper claims that during seven days, the high temperature in Lexington was typically 6° warmer than the high temperature in Louisville. What measure was used to make this claim? Justify your answer. **Mode; The mode of the temperatures in Louisville is 20° and the mode for Lexington's temperatures is 26°. Since 26° − 20° = 6°, the mode was used to make this claim.**

Daily High Temperatures (°C)							
Louisville				Lexington			
25	0	30	22	30	23	25	24
20	34	20		21	26	26	

6. **Use Math Tools** Use the Internet to find the high temperatures for each of the last seven days in a city near you. Then find the median high temperature. **See students' work.**

H.O.T. Problems Higher Order Thinking

7. **Persevere with Problems** The ticket prices for a concert series were AED 12, AED 37, AED 45, AED 18, AED 8, AED 25, and AED 18. What was the ticket price of the eighth and final concert in this series if the set of 8 prices had a mean of AED 23, a mode of AED 18, a median of AED 19.5? **AED 21**

8. **Construct an Argument** One evening at a local pizzeria, the following number of toppings were ordered on each large pizza.

3, 0, 1, 1, 2, 5, 4, 3, 1, 0, 0, 1, 1, 2, 2, 3, 6, 4, 3, 2, 0, 2, 1, 3

Determine whether each statement is **true** or **false**. Explain your reasoning.

a. The greatest number of people ordered a pizza with 1 topping.
True; the mode of the data set is 1.

b. Half the customers ordered pizzas with 3 or more toppings, and half the customers ordered pizzas with less than 3 toppings.
False; the median of the data is 2.

9. **Justify Conclusions** In the data set {3, 7, 4, 2, 31, 5, 4}, which measure best describes the set of data: mean, median, or mode? Explain your reasoning. **Sample answer: The median or mode best represents the data. The mean, 8, is greater than all but one of the data values.**

10. **Model with Mathematics** Create a list of six values where the mean, median, and mode are 45, and only two of the values are the same.
Sample answer: 42, 43, 45, 45, 47, 48

Name _____ My Homework

Extra Practice

Find and compare median and mode for each set of data.

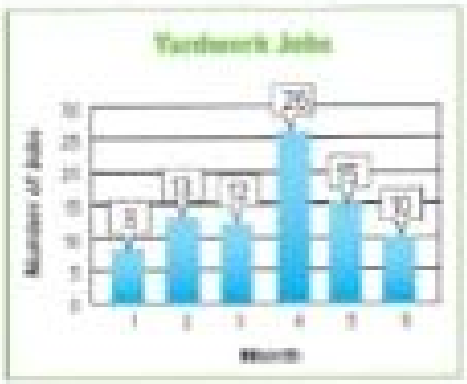
11. age of employees: 23, 22, 15, 44, 44; median: 23; mode: 44; The mode is 21 years more than the median.

median is: 22, 23, 44, 44
mode is: 22, 23, 44, 44

12. minutes spent on homework: 18, 20, 22, 11, 19, 18, 18

18; 18; The median and mode are equal.

13.



median: 12.5; mode: none; There is no mode to compare.

14. Describe the test grades using the measures of center.

Test Grades			
100	77	80	65
87	85	85	82
100	97	85	78

The mean, 85.67, and median, 85, are very close. One of the modes, 85, is equal to the median, but the other is the highest score. The data follows the measures of center in that they are close to the measures of center.

15. **Be Precise** Fill in the graphic organizer with the description. The first one is done for you. Sample answers are given.

measures of center	numbers used to describe the center of data
mean	the average of a data set
median	the value appearing at the center of a sorted version of a list, or the mean of the two central values, if the list contains an even number of values
mode	the most frequently occurring number or numbers of a data set

Power Up! Test Practice

16. The list of data shows the number of schools in 12 different counties.

Number of Schools in Different Counties			
4	3	6	10
3	14	8	5
7	11	7	8

Arrange the data values in order from least to greatest.

3, 3, 4, 5, 6, 7, 7, 8, 8, 10, 11, 14

What are the two middle numbers in the data set? 7, 7

What is the median number of schools in the 12 counties? 7

17. The table shows the number of concerts performed each year by a band. Determine if each statement is true or false.

Year	Number of Concerts	Year	Number of Concerts
1	142	5	124
2	142	6	138
3	136	7	136
4	136	8	150

a. The median is 135 concert ☐ True ■ False e

b. The mode is 136 concerts. ■ True ☐ False e

c. The mean is 138 concerts. ■ True ☐ False

Common Core Spiral Review

Find the greatest number in the data set.

18. {23, 35, 31, 28, 26, 34}
35

19. {56, 58, 49, 50, 56, 57}
58

20. {78, 81, 79, 84, 82, 83}
84

Find the least number in the data set.

21. {62, 58, 56, 61, 59, 57}
56

22. {24, 29, 22, 26, 23, 24}
22

23. {56, 58, 52, 54, 53, 57}
52

24. The table shows the distances Houriyya biked each day. What is the greatest distance she biked during the week?

Day	Distance (kilometers)
Monday	5.2
Tuesday	3.5
Wednesday	4.9
Thursday	3.8
Friday	3.2

5.2 kilometers

25. It is 230 kilometers from Columbus to Cleveland and 172 kilometers from Columbus to Cincinnati. How much further is it from Columbus to Cleveland than Columbus to Cincinnati?

58 kilometers

Problem-Solving Investigation

Use Logical Reasoning

Mathematical Practices 1, 3, 4

Case #1 Speak to Me

Rana surveyed 15 students with the statistical question, "Do you speak Spanish, French, both languages, or neither language?" Four students speak French, seven students speak Spanish, and two students speak both languages.

Use a Venn diagram to find how many students speak neither Spanish nor French.

1 Understand *What are the facts?*

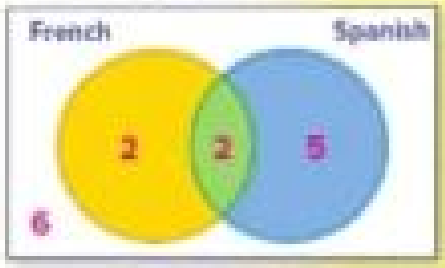
- You know 7 classmates speak Spanish and 4 classmates speak French.
- You know that 2 students speak both languages.

2 Plan *What is your strategy to solve this problem?*

Make a Venn diagram to organize the information. Use logical reasoning to find the answer.

3 Solve *How can you apply the strategy?*

Draw and label two overlapping circles to represent the two languages. Since 2 students speak both languages, place a 2 in the section that is part of both circles. Use subtraction to determine the number for each of the other sections.



only French: 4 − 2 = 2

only Spanish? 7 − 2 = 5

neither: 15 − 2 − 2 − 5 = 6

So, 6 students speak neither French nor Spanish.

4 Check *Does the answer make sense?*

Check each circle to see if the appropriate number of students is represented.

Analyze the Strategy

Reason Inductively Explain why Rana's question, "Do you speak Spanish, French, both languages, or neither language?" is a statistical question.

Sample answer: The question anticipates a variety of answers.

Case #2 Battle of the Mascots

Hamad conducted a survey of 85 students about a new school mascot. The results showed that 40 students liked Tigers, and 31 students liked Bears. Of those students, 12 liked both Tigers and Bears.

How many students liked neither Tigers nor Bears?

1 Understand

Read the problem. What are you being asked to find?

I need to find the number of students who liked neither Tigers nor Bears as a school mascot.

Underline key words and values in the problem. What information do you know?

85 students were surveyed. In the survey, 40 students said they liked Tigers, 31 said they liked Bears, and 12 said they liked both.

2 Plan

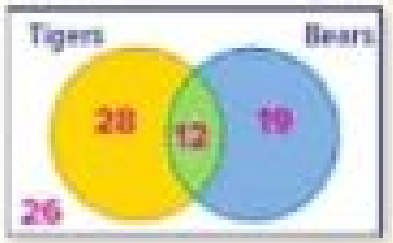
Choose a problem-solving strategy.

I will use the logical reasoning strategy.

3 Solve

Use your problem-solving strategy and a Venn diagram to solve the problem.

Draw and label two overlapping circles to represent the two mascots. Since 12 students said they liked both mascots, place a 12 in the section that is part of both circles. Subtract to find the numbers for the other sections.



only tigers: $40 - 12 = 28$ only bears: $31 - 12 = 19$

neither tigers nor bears: $85 - 28 - 12 - 19 = 26$

So, 26 students liked neither tigers nor bears as the school mascot.

4 Check

Use information from the problem to check your answer.

Check each circle to see if the appropriate number of students is represented. Both: 12; Tigers: $28 + 12 = 40$; Bears: $19 + 12 = 31$; Neither: $85 - 28 - 19 - 12 = 26$

Work with a small group to solve the following cases.
Show your work on a separate piece of paper.

Case #3 Marketing

A survey showed that 70 customers bought white bread, 63 bought wheat bread, and 35 bought rye bread. Of those who bought exactly two types of bread, 12 bought wheat and white, 5 bought white and rye, and 7 bought wheat and rye. Two customers bought all three.

How many customers bought only wheat bread?

42 customers

Case #4 Pets

Dr. Sally is a veterinarian. One week she treated 20 rabbits, 16 cats, and 11 birds. Some owners had more than one pet, as shown in the table.

How many owners had only a rabbit as a pet?

6 owners

Pet	Number of Owners
rabbit and cat	7
rabbit and bird	5
cat and bird	3
rabbit, cat, and bird	2

Case #5 Sports

The Student Council surveyed a group of 24 students by asking the statistical question, "Do you like softball, basketball, both, or neither?" The results showed that 14 students liked softball, and 18 liked basketball. Of these, 8 liked both.

How many students liked just softball and how many liked just basketball?

6 students; 10 students

Use any strategy!

Case #6 Money

Khalid has AED 138.22 in his savings account. He deposits AED 10.75 every week and withdraws AED 31.68 every four weeks.

What will his balance be in 8 weeks?

AED 160.86

Mid-Chapter Check

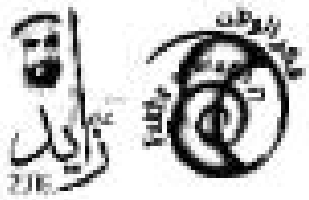
Vocabulary Check

1. Define **mean**. Then determine the mean of the following data set. (22, 18, 38, 6, 24, 18) (Lesson 1)

   The mean, or average, is the sum of the data divided by the number of pieces of data; 21

2. Fill in the blank in the sentence below with the correct term. (Lesson 2)

   The mode is the number or numbers that occur most often in a set.



Skills Check and Problem Solving

Find the mean of each data set. (Lesson 1)

3. number of home runs by baseball players in season: 43, 21, 35, 15, 35

   29.8

4. number of different birds spotted: 7, 10, 13, 9, 12, 3

   9

Find the median and mode for each set of data. (Lesson 2)

5. hours spent studying: 4, 2, 5, 7, 1

   median: 4; mode: none

6. heights of buildings in meters: 35, 42, 40, 25, 42, 54, 50 median: 42; mode: 42

7. **Use Math Tools** Use the table that shows the lengths of different lizards. Find and compare the median and mode of the data. (Lesson 2)

   median: 14.5; mode: 14; The median is 0.5 cm larger than the mode.

Lizard Length (cm)			
14	12	14	14
10	18	11	16
30	12	19	15

8. **Persevere with Problems** The table shows the number of minutes spent doing different exercises. The mean time spent exercising was 18.2 minutes. How many minutes were spent doing sit-ups? (Lesson 2)

   15 min

Daily Exercises	
Exercise	Time (min)
Pull-ups	8
Push-ups	10
Running	38
Sit-ups	[illegible]
Weight lifting	20

Lesson 3

Measures of Variation

Vocabulary Start-Up

Measures of variation are used to describe the distribution, or spread, of the data. They describe how the values of a data set vary with a single number. A quartile is one measure of variation.

Look in a dictionary and find words that begin with quar-. Write two of the words and their definitions. Sample answers are given.

Word beginning with quar-	Definition
quarter	one of the four equal parts into which anything is or may be divided
quarterly	occurring, done, paid, etc., at the end of every quarter of a year

Based on the definitions you found, fill in the blank below.

Quartiles are values that divide a set of data into four equal parts.

Essential Question

HOW are the mean, median, and mode helpful in describing data?

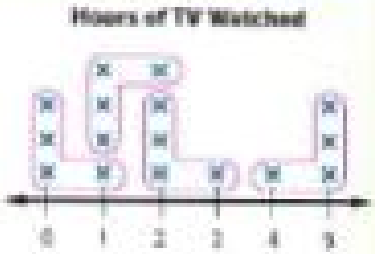
Vocabulary

measures of variation
quartiles
first quartile
third quartile
interquartile range
range
outliers

Mathematical Practices
1 2 3 4 5

Real-World Link

Surveys Khalaf asked his classmates how many hours of TV they watch on a typical day.



1. Divide the data into 4 equal parts. Draw a circle around each part. Sample answer is given.
2. How many data values are in each group? 4

Which Mathematical Practices did you use? Shade the circle(s) that applies.

- 1 Persevere with Problems
- 2 Reason Abstractly
- 3 Construct an Argument
- 4 Model with Mathematics
- 5 Use Math Tools
- 6 Attend to Precision
- 7 Make Use of Structure
- 8 Use Repeated Reasoning

Key Concept

Measures of Variation

Quartiles are values that divide the data set into four equal parts.

First and Third Quartiles

The first and third quartiles are the medians of the data values less than the median and the data values greater than the median, respectively.

Interquartile Range (IQR)

The distance between the first and third quartiles of the data set.

Range

The difference between the greatest and least data values.

Work Zone

Measures of variation of a data set are shown below.

One fourth of the data lie below the first quartile and one fourth of the data lie above the third quartile. So, one half of the data lie between the first quartile and third quartile.

Example

1. Find the measures of variation for the data.

Animal	Speed (km/h)
cheetah	113
lion	80
cat	48
elephant	40
mouse	13
spider	2

Range 113 − 2 or 111 km/h

Quartiles Order the numbers.

Q_1 median = 44 Q_3

2 13 40 48 80 113

Interquartile Range 80 13 or 67 $Q_3 - Q_1$

The range is 111, the median is 44, the first quartile is 13, the third quartile is 80, and the IQR is 67.

Interquartile Range

If the interquartile range is low, the middle data are grouped closely together.

Got it? Do this problem to find out.

a. Determine the measures of variation for the data 64, 61, 67, 59, 60, 58, 57, 71, 56, and 62.

range: 15; median: 60.5; Q_1: 58; Q_3: 64; IQR: 6

Find Outliers and Analyze Data

An **outlier** is a data value that is either much greater or much less than the other values in a data set. If a data value is more than 1.5 times the value of the interquartile range beyond the quartiles, it is an outlier.

Example

2. **The ages of candidates in an election are 23, 48, 49, 55, 57, 63, and 72. Name any outliers in the data.**

Find the interquartile range: 63 − 48 = 15

Multiply the interquartile range by 1.5: 1.5 × 15 = 22.5

Subtract 22.5 from the first quartile and add 22.5 to the third quartile to find the limits for the outliers.

48 − 22.5 = 25.5 63 + 22.5 = 85.5

The only age beyond the limits is 23. So, it is the only outlier.

Get it? **Do this problem to find out.**

b. The lengths, in meters, of various bridges are 88, 251, 376, and 1,121. Name any outliers in the data set.

b. none

Example

3. **The table shows a set of scores on a science test in two different classrooms. Compare and contrast their measures of variation.**

Room A	Room B
72	63
100	80
67	79
84	83
65	98
78	87
92	73
87	81
80	65

Find the measures of variation for both rooms.

	Room A	Room B
Range	100 − 65 = 35	98 − 63 = 35
Median	80	81
Q_3	$\frac{87 + 92}{2} = 89.5$	$\frac{87 + 93}{2} = 90$
Q_1	$\frac{67 + 72}{2} = 69.5$	$\frac{65 + 73}{2} = 69$
IQR	89.5 − 69.5 = 20	90 − 69 = 21

Both classrooms have a range of 35 points, but Room B has an interquartile range of 21 points while Room A's interquartile range is 20 points. There are slight differences in the medians as well as the third and first quartiles.

Stop and Reflect

Which measure of center would most likely be affected by an outlier? Explain below.

Sample answer: mean; The median is the middl number, so it is usually not affected by an extreme value, the mod is the most frequently occurring number, so an extreme value will not change it, but an extrem value could change the mean more significantly

c. range: 24 and 21; median 10 and 9.5; Q_3: 21 and 19; Q_1: 3 and 4; IQR: 18 and 15; The medians are close, but the temperatures are more spread out in the Antelope data.

Got it? Do this problem to find out.

c. Temperatures for the first half of the year are given for Antelope, Montana, and Augusta, Maine. Compare and contrast the measures of variation of the two cities.

Month	Antelope, MT	Augusta, ME
January	3	3
February	3	4
March	6	5
April	14	14
May	21	19
June	26	24

Guided Practice

1. The average wind speeds for several cities in Pennsylvania are given in the table.

Wind Speed	
Pennsylvania City	Speed (km/h)
Allentown	8.9
Erie	11.0
Harrisburg	7.5
Middletown	7.7
Philadelphia	9.5
Pittsburgh	9.0
Williamsport	7.6

   a. Find the range of the data. 3.5

   b. Find the median and the first and third quartiles. 8.9; 7.6; 9.5

   c. Find the interquartile range. 1.9

   d. Identify any outliers in the data. none

2. The heights of several types of deciduous trees, in meters, are 40, 25, 15, 22, 50, and 30. The heights of several types of evergreen trees, in meters, are 60, 75, 45, 80, 75, and 70. Compare and contrast the measures of variation of both kinds of trees. ranges: 35 and 35; medians: 27.5 and 72.5; 40 and 75; Q_1 22 and 60; IQR: 18 and 15. Both sets of data have the same ranges of height and close interquartile ranges even though most of the palm trees are shorter than most of the pine trees.

3. **Building on the Essential Question** Describe the difference between measure of center and measure of variation. Sample answer: A measure of center summarizes a set of data with a single number, but a measure of variation uses a single number to describe how the values vary.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

Need to ... [illegible]

Name _____ My Homework _____

Independent Practice

1. The table shows the number of golf courses in various states.

Number of Golf Courses			
California	1,117	New York	954
Florida	1,465	North Carolina	650
Georgia	513	Ohio	893
Iowa	427	South Carolina	456
Michigan	1,038	Texas	1,038

a. Find the range of the data. 1,038

b. Find the median and the first and third quartiles.
923.5; 513; 1,038

c. Find the interquartile range. 525

d. Name any outliers in the data. none

For each data set, find the median, the first and third quartiles, and the interquartile range.

2. texts per day: 24, 53, 38, 12, 31, 19, 26
median: 26; Q_1: 19; Q_3: 38; IQR: 19

3. daily attendance at the water park: 346, 250, 433, 369, 422, 298
median: 357.5; Q_1: 298; Q_3: 422; IQR: 124

4. The table shows the number of minutes of exercise for each person. Compare and contrast the measures of variation for both weeks.

Minutes of Exercise		
	Week 1	Week 2
Sumayya	45	30
Sindyya	40	55
Abeer	45	35
Suha	55	60
Shaikha	60	45
Alia	60	75

ranges: 50 and 45; medians: both 50; Q_1: 45 and 35; Q_3: both 60; IQR: 15 and 25; Sample answer: Both sets of data have the same median number of minutes while the middle data in the first set are grouped more closely than the middle data in the second.

5. STEM The table shows the number of known moons for each planet in our solar system. Use the measures of variation to describe the data.

Known Moons of Planets			
Mercury	0	Jupiter	63
Venus	0	Saturn	34
Earth	1	Uranus	27
Mars	2	Neptune	13

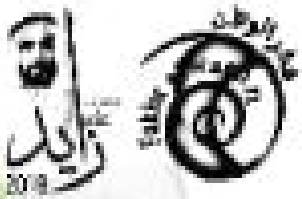
range: 63; median: 7.5; Q_1: 30.5; Q_3: 0.5; IQR: 30; Sample answer: The number of moons for each planet varies greatly. The IQR and range are both large.

6. **Use Math Tools** The double stem-and-leaf plot, where the stem is in the middle and the leaves are on either side, shows the high temperatures for two cities in the same week. Use the measures of variation to describe the data in the stem-and-leaf plot.

Minneapolis		Columbus
5 3 1 0	2	5 7 9 9
6 4	3	7
3	4	8
	5	
	6	2

3|3 = 33° 2|5 = 25°

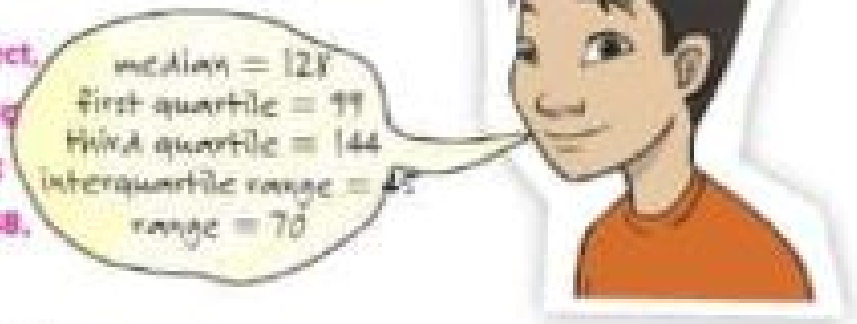
Minneapolis—range: 23 degrees; IQR: 16 degrees; median: 25; Q_1: 21 degrees; Columbus—range: 37 degrees; Q_3: 48 degrees; median: 29; IQR: 27 degrees; Sample answer: The Minneapolis temperatures are closer together than the Columbus temperatures.



H.O.T. Problems Higher Order Thinking

7. **Find the Error** Rasheed was finding the measures of variation of the following set of data: 89, 93, 99, 110, 128, 135, 144, 152, and 159. Find his mistake and correct it.

Sample answer: The median is correct, but Rasheed included it when finding the third and first quartiles. The first quartile is 96, the third quartile is 148, and the interquartile range is 52.



8. **Reason Abstractly** Create a list of data with at least six numbers that has an interquartile range of 15 and two outliers.

Sample answer: 6, 30, 33, 41, 45, 71

9. **Persevere with Problems** How is finding the first and third quartiles similar to finding the median? **Sample answer: The third quartile is the median of the upper half of the data and the first quartile is the median of the lower half of the data.**

10. **Reason Inductively** Explain why the median is not affected by very high or very low values in the data. **Sample answer: The median is the middle number of a data set. Very high values and very low values typically fall at the beginning or end of a data set. Therefore, the median is not significantly affected by these values.**

11. **Reason Inductively** Determine the range and IQR of each data set. Which measure of variation tells you more about the distribution of the data values? Explain.

Data Set A	Data Set B
1, 2, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 12, 19, 21	1, 2, 8, 11, 12, 12, 12, 13, 18, 18, 18, 19, 20, 21

Set A—range: 20; IQR: 4; Set B—range: 20; IQR: 1; Sample answer: The IQR tells more information, specifically that the middle half of the data in Set B are closer together than the middle half of the data in Set A.

Name _____ My Homework _____

Extra Practice

12. The table shows the countries with the most internet users.

Millions of Internet Users	
China	99.8
Germany	41.80
India	36.97
Japan	78.05
South Korea	31.67
United Kingdom	33.11
United States	185.55

a. Find the range of the data.

185,000,000 | 185,000,000 − 31,670,000 = 185,000,000

b. Find the median and the first and third quartiles.

41,000,000; 99,110,000; 99,800,000

31.67 99.11 36.97 41.80 78.05 99.8 185.55

Q_1 Median Q_3

c. Find the interquartile range.

44,670,000 99,800,000 − 99,110,000 = 44,670,000

d. Name any outliers in the data. none

13. **Use Math Tools** The table shows teams in the National Football Conference (NFC) and the American Football Conference (AFC).

Penalties By NFL Teams			
NFC		AFC	
Dallas Cowboys	104	New England Patriots	78
Arizona Cardinals	127	Indianapolis Colts	67
Green Bay Packers	113	Jacksonville Jaguars	76
New Orleans Saints	48	San Diego Chargers	94
New York Giants	77	Cleveland Browns	114
Seattle Seahawks	59	Pittsburgh Steelers	80
Minnesota Vikings	86	Houston Texans	82

a. Which conference had a greater range of penalties? NFC

b. Find the measures of variation for each conference. NFC—median: 86, @ 113, Q_3 68, IQR: 45; AFC—median: 80, Q_3 94, Q_1 76, IQR: 18

c. Compare and contrast the measures of variation for each conference.

Sample answer: The AFC had a median of 80 penalties and the NFC had a median of 86 penalties. The AFC had an IQR of 18 penalties while the NFC had an IQR of 45 penalties. The ranges were 47 penalties for the AFC and 78 penalties for the NFC.

14. Find the median, the first and third quartiles, and the interquartile range for the cost of admission: AED 13.95, AED 24.59, AED 19.99, AED 29.98, AED 23.95, AED 28.99.

median: AED 24.27; @AED 19.99; Q_3 AED 28.99; IQR: AED 9.00

Power Up! Test Practice

15. The number of games won by 10 chess players is shown below.

13, 15, 2, 7, 5, 9, 11, 10, 12, 1

Which of the following statements are true? Select all that apply.

- [x] Half of the players won more than 10.5 games and half won less than 10.5 games.
- [x] The range of the data is 13 games.
- [x] There are no outliers.
- [ ] Only one fourth of the players won more than 7 games.

16. The data at the right shows the number of students engaged in different sports.

Number of Students in Different Sports				
8	12	20	10	6
15	12	9	10	22

a. Order the values from least to greatest.

6, 8, 9, 10, 10, 12, 12, 15, 20, 22

b. Find the range of the data. 16

c. Find the median and the first and third quartiles. 11, 9, 15

d. Find is the interquartile range. 6

Spiral Review

Divide.

17. 160 ÷ 5 = 32

18. 188 ÷ 8 = 23.5

19. 133 ÷ 7 = 19

20. 87.5 ÷ 5 = 17.5

21. 136.5 ÷ 7 = 19.5

22. 74.4 ÷ 6 = 12.4

23. Refer to the table. How much farther did the Zayed's family drive on Friday than on Saturday?

Day	Distance (kilometers)
Thursday	68
Friday	193
Saturday	26
Sunday	95

167 kilometers

24. Refer to the table. How many more hours did Ayesha work in week 2 than in week 3?

Week	Hours Worked
1	12
2	16
3	9

7 hours

Lesson 4

Mean Absolute Deviation

Real-World Link

Basketball The tables show the number of points two teams scored.

Abeer's Team			
52	48	60	50
56	54	58	62

Alia's Team			
51	48	60	49
59	50	62	61

Essential Question

HOW are the mean, median, and mode helpful in describing data?

Vocabulary

mean absolute deviation

Mathematical Practices

1, 2, 3, 4, 5, 6

1. Plot each set of data on a number line.

Abeer's Team

Alia's Team

2. Find the mean of each set of data. Plot the means on the number lines with a star.

3. Find the range of each set of data. Abeer's Team: 14; Alia's Team: 14

4. Refer to the number lines. Compare and contrast each set of data.
Sample answer: the mean, 55 points, and the range, 14 points, are the same for each set. The data for Alia's team are more clustered while the data for Abeer's team are more spread out.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Find Mean Absolute Deviation

You have used the interquartile range to describe the spread of a set of data. You can also use the mean absolute deviation. **Mean absolute deviation** of a set of data is the average distance between each data value and the mean.

Example

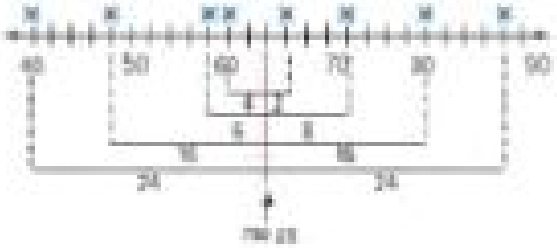
1. **The table shows the maximum speeds of eight roller coasters. Find the mean absolute deviation of the set of data. Describe what the mean absolute deviation represents.**

Maximum Speeds of Roller Coasters (km/h)			
58	88	40	60
72	66	80	48

Step 1 Find the mean.

$$\frac{58 + 88 + 40 + 60 + 72 + 66 + 80 + 48}{8} = 64$$

Step 2 Find the absolute value of the differences between each value in the data set and the mean. Each data value is represented by an "x".



Step 3 Find the average of the absolute values of the differences between each value in the data set and the mean.

$$\frac{24 + 16 + 6 + 4 + 2 + 8 + 16 + 24}{8} = 12.5$$

The mean absolute deviation is 12.5. This means that the average distance each data value is from the mean is 12.5 kilometers per hour.

Got it? **Do this problem to find out.**

a. The table shows speeds of ten birds. Find the mean absolute deviation of the data. Round to the nearest hundredth. Describe what the mean absolute deviation represents.

Speeds of Ten Birds (km/h)				
88	77	65	70	65
72	95	80	106	68

a. 10.92 kilometers per hour; Sample answer: The average distance each data value is from the mean is 10.92 kilometers per hour.

Compare Variation

You can compare the mean absolute deviations for two data sets. A data set with a smaller mean absolute deviation has data values that are closer to the mean than a data set with a greater mean absolute deviation.

Example

2. **The top five salaries and the bottom five salaries for a group of professional racing car drivers are shown in the table below. Salaries are in millions of dirhams and are rounded to the nearest hundredth.**

Professional Racing Driver Salaries (millions of AED)									
Top Five Salaries					Bottom Five Salaries				
33.00	24.29	22.60	20.63	16.50	0.45	0.44	0.43	0.41	0.41

a. Find the mean absolute deviation for each set of data. Round to the nearest hundredth.

Find the mean of the top five salaries.

$$\frac{33.00 + 24.29 + 22.60 + 20.63 + 16.50}{5} \approx 23.40$$

The mean is about AED 23.40 million.

Find the mean absolute deviation of the top five salaries.

$$\frac{9.60 + 0.89 + 0.80 + 2.77 + 5.90}{5} \approx 4.19$$

The mean absolute deviation is about AED 4.19 million.

Find the mean of the bottom five salaries.

$$\frac{0.45 + 0.44 + 0.43 + 0.41 + 0.41}{5} \approx 0.43$$

The mean is about AED 0.43 million.

Find the mean absolute deviation of the bottom five salaries.

$$\frac{0.02 + 0.01 + 0 + 0.02 + 0.02}{5} \approx 0.01$$

The mean absolute deviation is about AED 0.01 million.

b. Write a few sentences comparing their variation.

The mean absolute deviation for the bottom five salaries is much less than that for the top five salaries. The data for the bottom five salaries are closer together than the data for the top five salaries.

Mean Absolute Deviation

The absolute value of the differences between each data value and the mean for the six five salaries are calculated below.

|33.00 − 23.40| = 9.60

|24.29 − 23.40| = 0.89

|22.60 − 23.40| = 0.80

|20.63 − 23.40| = 2.77

|16.50 − 23.40| = 6.90

comedy: 4.16 minutes; drama: 12.24 minutes; b. Sample answer: The mean absolute deviation of the running times of comedies is less than the mean absolute deviation of the running times of dramas. The running times for comedies are closer together.

Got it? Do this problem to find out.

b. The table shows the running time in minutes for two kinds of movies. Find the mean absolute deviation for each set of data. Round to the nearest hundredth. Then write a few sentences comparing their variation.

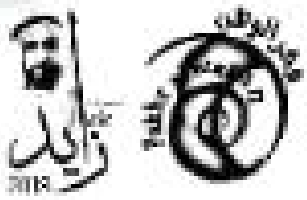
Running Time for Movies (min)										
Comedy						Drama				
90	95	88	100	98		115	120	150	135	144

Guided Practice

1. Find the mean absolute deviation for the set of data. Round to the nearest hundredth if necessary. Then describe what the mean absolute deviation represents.

Number of Daily Visitors to a Web Site				
172	149	138	160	122

18.48 visitors; Sample answer: The average distance each data value is from the mean is 18.48 visitors.



2. The table shows the height of rollercoasters at two different parks. Find the mean absolute deviation for each set of data. Round to the nearest hundredth. Then write a few sentences comparing their variation. (Example 2)

Height of Rollercoasters (meters)										
Park A						Park B				
75	95	80	110	88		120	108	94	135	126

Park A: 10.32 meters; Park B: 12.48 meters; Sample answer: The mean absolute deviation of the heights at Park A is less than the mean absolute deviation of the heights at Park B. The heights at Park A are closer together.

3. **Building on the Essential Question** What does the mean absolute deviation tell you about a set of data?

Sample answer: It tells the average distance of each data value from the mean, which lets you know if the data values are close together and close to the mean, or close to the extremes and farther from the mean.

Rate Yourself!

- ☐ I understand how to find the mean absolute deviation.
- Great! You're ready to move on.
- ☐ I still have questions about finding the mean absolute deviation.

Name _____ My Homework _____

Independent Practice

Find the mean absolute deviation for each set of data. Round to the nearest hundredth if necessary. Then describe what the mean absolute deviation represents.

1.

Known Moons of Planets			
0	0	1	2
63	34	27	13

17.88 moons;

Sample answer: The average distance each data value is from the mean is 17.88 moons.

2.

Hard Drive (gigabytes)			
640	250	500	640
720	640	250	720

158.75 gigabytes;

Sample answer: The average distance each da value is from the mean is 158.75 gigabytes.

3. The table shows the lengths of the longest bridges in the United States and in Europe. Find the mean absolute deviation for each set of data. Round to the nearest hundredth if necessary. Then write a few sentences comparing their variation.

Longest Bridges (kilometers)										
United States						Europe				
38.4	36.7	29.3	24.1	17.7		12.2	10.7	7.8	6.8	6.6
12.9	11.3	10.9	8.9	8.9		6.1	5.1	5.0	4.3	3.9

United States: 9.77 km; Europe: 2.87 km; Sample answer: The mean absolute deviation in bridge lengths in the U.S. is greater than the mean absolute deviation of the bridge lengths in Europe. The lengths of the bridges in Europe are closer to the mean.

For Exercises 4–7, refer to the table that shows the recent population, in millions, of the ten largest U.S. cities.

Population of Largest U.S. Cities (millions)					
1.5	3.8	1.3	1.6	2.8	
1.4	0.9	2.3	8.4	1.3	

4. Find the mean absolute deviation. Round to the nearest hundredth.

1.50 million

5. How many data values are closer than one mean absolute deviation away from the mean? eight

6. Which population is farthest from the mean? How far away from the mean is that population? Round to the nearest hundredth.

8.40 million; 5.86 million

7. Are there any populations that are more than twice the mean absolute deviation from the mean? Explain. yes; Sample answer: Twice the mean absolute deviation is 2× 1.50 million, or 3.00 million. Since 5.86 million > 3.00 million, the population of 8.4 million is greater than 3.00 million away from the mean.

Be Precise For Exercises 8 and 9, look up the word *deviate* in a dictionary or online.

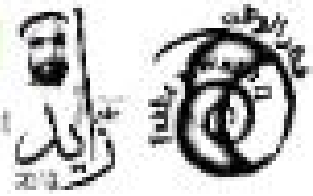
8. What does the word *deviate* mean? How can it help you remember what the mean absolute deviation refers to?

to differ from; Sample answer: The mean absolute deviation describes how the data values differ from the mean.

9. How does the word *absolute* help you to remember how to calculate the mean absolute deviation?

Sample answer: It helps me to remember to take the absolute value of the difference between each data value and the mean.

H.O.T. Problems Higher Order Thinking



10. **Reason Abstractly** Create two sets of data, each with five values, that satisfy the following conditions.

The mean absolute deviation of Set A is less than the mean absolute deviation of Set B.

The mean of Set A is greater than the mean of Set B.

Sample answer: Set A: 12, 13, 15, 16, 19; Set B: 4, 8, 10, 15, 23

Persevere with Problems For Exercises 11 and 12, refer to the table that shows the recorded speeds of several cars on a busy street.

Recorded Speeds (km/h)					
35	38	41	35	36	55

11. Calculate the mean absolute deviation both with and without the data value of 55. Round to the nearest hundredth if necessary.

with the data value of 55: 5.33 kilometers per hour; without the data value of 55: 2 kilometers per hour

12. Explain how including the value of 55 affects the mean absolute deviation.

Sample answer: When the value of 55 was included, the mean absolute deviation increased. Because the value of 55 is so much greater then the other data values, the larger mean absolute deviation shows that the data are more spread out when the value of 55 is included as opposed to when it is not included.

13. **Construct an Argument** Explain why the mean absolute deviation is calculated using absolute value.

Sample answer: The mean absolute deviation is the average distance that each data value is from the mean. Since distance cannot be negative, the absolute values of the differences are used.

14. **Persevere with Problems** The table shows the high temperatures for 6 days. If the high temperature for day 7 is 21°C, how does the mean absolute deviation change?

High Temperature (°C)					
15	18	32	28	29	25

The mean absolute deviation increases from 4 to about 4.6.

Name _____ My Homework _____

Extra Practice

Use Math Tools **Find the mean absolute deviation for each set of data. Round to the nearest hundredth if necessary. Then describe what the mean absolute deviation represents.**

15.

Digital Camera Prices (AED)				
140	125	190	148	156
212	178	188	196	224

AED 26.76; The average distance each data value is from the mean is AED 26.76.

mean: $\frac{140 + 125 + 190 + 148 + 156 + 212 + 178 + 188 + 196 + 224}{10} = \text{AED } 179.70$

mean absolute deviation: $\frac{39.7 + 54.7 + 10.3 + 31.7 + 23.7 + 18.3 + 8.3 + 9.3 + 16.3 + 44.3}{10} = 26.76$

16.

Grand Slam Singles Titles Won				
14	8	7	6	5
10	11	8	8	6

2.02 Grand Slam titles; Sample answer: The average distance each data value is from the mean is 2.02 Grand Slam titles.

Copy and Solve **Find the mean absolute deviation for each set of data. Round to the nearest hundredth. Then write a few sentences comparing their variation.** **See margin.**

17. The table shows the amount of money raised by the homerooms for two grade levels at a middle school.

Money Raised (AED)											
Sixth Grade						Seventh Grade					
88	116	94	108	112	124	144	91	97	122	128	132

18. The table shows the number of points scored each game for two different basketball teams.

Number of Points Scored											
Lakeside Panthers						Jefferson Eagles					
44	38	54	48	26	36	58	42	64	62	70	40

Power Up! Test Practice

19. Which of the following statements are true concerning the mean absolute deviation of a set of data? Select all that apply.

- ☐ It describes the variation of the data around the median.
- ☐ It describes the absolute value of the mean.
- ■ It describes the variation of the data around the mean.
- ■ It describes the average distance between each data value and the mean.

20. The table shows the prices for parking at three different beaches along the same coastline. Select the correct values to complete the model below to find the mean absolute deviation of the data.

Beach Parking (AED)		
2.50	3.75	3.50

0.25	2.75	1
0.50	3.00	2
0.75	3.25	3
1.00	3.50	4
2.50	3.75	5

Find the mean:

$$\frac{2.50 + 3.75 + 3.50}{3} = 3.25$$

Find the absolute values of the differences between each data value and the mean:

$$2.50 - 3.25 = 0.75$$
$$3.75 - 3.25 = 0.50$$
$$3.50 - 3.25 = 0.25$$

Find the mean of the absolute values of the differences:

$$\frac{0.75 + 0.50 + 0.25}{3} = 0.50$$

What is the mean absolute deviation of the data? AED0.50

Common Core Spiral Review

21. The table shows the number of different cones Delightful Dips ice cream shop sold in one afternoon. What is the total number of cones sold? 235 cones

Flavor	Number of Cones
Chocolate	57
Cookie Crunch	49
Fudge Swirl	48
Strawberry	37
Vanilla	51

22. The hiking club wanted to cover a different trail each day for a week. On Monday they hiked 2.3 kilometers, on Tuesday they hiked 1.8 kilometers, on Wednesday they hiked 3.2 kilometers, on Thursday they hiked 1.4 kilometers, and on Friday they hiked 2.8 kilometers. What is the total distance they hiked?

11.5 kilometers

Lesson 5

Appropriate Measures

Real-World Link

Recycling The green committee had a recycling drive where they collected aluminum cans, plastic bottles, newspapers, and batteries. The weights collected on the first day are shown.

Essential Question

HOW are the mean, median, and mode helpful in describing data?

Mathematical Practices

1, 3, 4

1. Find the mean weight collected. **13.925 kg**

2. If the newspapers are not included, find the mean weight rounded to the nearest hundredth. **12.07 kg**

3. How does the weight of the newspapers affect the mean?
Sample answer: It causes the mean to be about 2 kilograms higher if it is included in the data set.

4. What is the median for the data set? How does the median differ if the newspapers are not included?
12.6; Sample answer: The median is 0.4 kilograms less if the newspapers are not included.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

- ① Persevere with Problems
- ② Reason Abstractly
- ③ Construct an Argument
- ④ Model with Mathematics
- ⑤ Use Math Tools
- ⑥ Attend to Precision
- ⑦ Make Use of Structure
- ⑧ Use Repeated Reasoning

key Concept

Using Mean, Median, and Mode

Measure	Most appropriate when...
mean	• the data have no extreme values.
median	• the data have extreme values. • there are no big gaps in the middle of the data.
mode	• data have many repeated numbers.

Work Zone

Sometimes, one measure is more appropriate than others to use to summarize a data set.

Examples

1. The table shows the number of medals won by the U.S. Which measure of center best represents the data? Then find the measure of center.

Year	1992	1996	2000	2004	2008
Number of Medals	112	101	97	103	110

Since the set of data has no extreme values or numbers that are repeated, the mean would best represent the data.

Mean $\frac{112 + 101 + 97 + 103 + 110}{5} = \frac{523}{5}$ or $104\frac{3}{5}$

The mean number of medals won is $104\frac{3}{5}$ medals.

2. The table shows a cooking pot's water temperature over seven minutes. Which measure of center best represents the data? Then find the measure of center.

Water Temperature (°C)				
82	85	82	81	
82	82	78		

In the set of data, there are no extreme values. There is a temperature repeated four times, so the mode 82° is the measure of center that best represents the data.

Got it? Do this problem to find out.

a. The prices of several DVDs are AED 22.50, AED 21.95, AED 25.00, AED 21.95, AED 19.95, AED 21.95, and AED 21.50. Which measure of center best represents the data? Justify your selection. Then find the measure of center.

Since there are many repeated prices, the mode AED 21.95 is the measure of center that best represents the data.

Outliers and Appropriate Measure

Sometimes data sets contain outliers. Outliers are deviations from the majority of the data set. The outlier may affect the measures of center.

Examples

The table shows average life spans of some animals.

Average Life Span	
Animal	**Life Span (years)**
African elephant	35
Bottlenose dolphin	30
Chimpanzee	50
Galapagos tortoise	200
Gorilla	30
Gray whale	70
Horse	20

3. Identify the outlier in the data set.

Compared to the other values, 200 years is extremely high. So, it is an outlier.

4. Determine how the outlier affects the mean, median, and mode of the data.

Find the mean, median, and mode with and without the outlier.

With the outlier

Mean $\frac{35 + 30 + 50 + 200 + 30 + 70 + 20}{7} \approx 62$

Median 35

Mode 30

Without the outlier

Mean $\frac{35 + 30 + 50 + 30 + 70 + 20}{6} = 39$

Median 32.5

Mode 30

The mean life span decreased by 6239 or 23 years. The median life span decreased by 35 32.5 or 2.5 years. The mode did not change.

5. Which measure of center best describes the data with and without the outlier? Justify your selection.

The mean was affected the most with the outlier. The median life span changed very little with and without the outlier, so it best describes the data in both cases. The mode does not describe the data very well since there were only two repeated numbers.

Outliers

Stop and Reflect

Sample answer: If the outlier is removed from the data set, the mean may change significantly, but the median may not change at all or only change slightly, so it would be more representative of the data.

Get it? Do these problems to find out.

The prices of some new athletic shoes are shown in the table.

Price of Athletic Shoes		
AED 51.95	AED 47.50	AED 46.50
AED 48.50	AED 52.95	AED 78.95
	AED 39.95	

b. AED 78.95

b. Identify the outlier in the data set.

c. Determine how the outlier affects the mean, median, and mode of the data. without the outlier: mean: AED 47.89, median: AED 48.00, mode: none; with the outlier: mean: AED 52.33, median: AED 48.50, mode: none

d. Tell which measure of center best describes the data with and without the outlier. The mean price was affected the most with the outlier. The median price changed very little with and without the outlier, so it best describes the data in both cases.

Guided Practice

1. The table shows the temperatures at which eight different solid substances melt.

Melting Temperature (°C)			
175	325	325	350
350	350	400	450

a. Identify the outlier in the data set. 175°

b. Determine how the outlier affects the mean, median, and mode of the data. without the outlier: mean: about 364.3°, median: 350°, mode: 350°; with the outlier: mean: about 340.6°, median: 350°, mode: 350°

c. Tell which measure of center best describes the data with and without the outlier. Justify your selection. The median temperature and mode best describe the data with and without the outlier. The mean temperature is misleading because of the outlier.

Rate Yourself!

How well do you understand choosing the appropriate measure of center for a data set? Circle the image that applies.

Clear | Somewhat Clear | Not So Clear

2. **Building on the Essential Question** How does an outlier affect the mean, median, and mode of a data set? Sample answer: An outlier affects the mean the greatest. It can affect the median some. The outlier does not affect the mode if it was not the mode.

Name _____ My Homework _____

Independent Practice

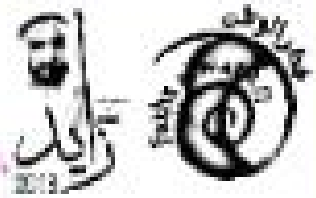
1. The number of minutes spent studying are: 60, 70, 45, 60, 80, 35, and 45. Find the measure of center that best represents the data. Justify your selection and then find the measure of center.

The mean best represents the data. There are no extreme values.

mean: 56.4 minutes

2. The table shows monthly rainfall in centimeters for five months. Identify the outlier in the data set. Determine how the outlier affects the mean, median, and mode of the data. Then tell which measure of center best describes the data with and without the outlier. Round to the nearest hundredth. Justify your selection.

Month	June	July	Aug.	Sept.	Oct.	Nov.
Rainfall (cm)	6.14	7.19	6.83	8.38	6.42	2.43



outlier: 2.43 cm; without the outlier: mean: 7.36 cm, median: 7.19 cm, mode: none; with the outlier: mean: 6.54 cm, median: 6.83 cm, mode: none; The mean rainfall best describes the data without the outlier. The median rainfall best describes the data with the outlier.

The table shows the average depth of several lakes.

Lake	Depth (m)
Lake A	148
Lake B	19
Lake C	43
Lake D	62
Lake E	16
Lake F	34

a. Identify the outlier in the data set. **148**

b. Determine how the outlier affects the mean, median, mode, and range of the data. **With the outlier, the mean is 216.83 m, the median is 33.5 m, there is no mode, and the range is 1,138. Without the outlier, the mean is 30.6 m, the median is 24 m, there is no mode, and the range is 52.**

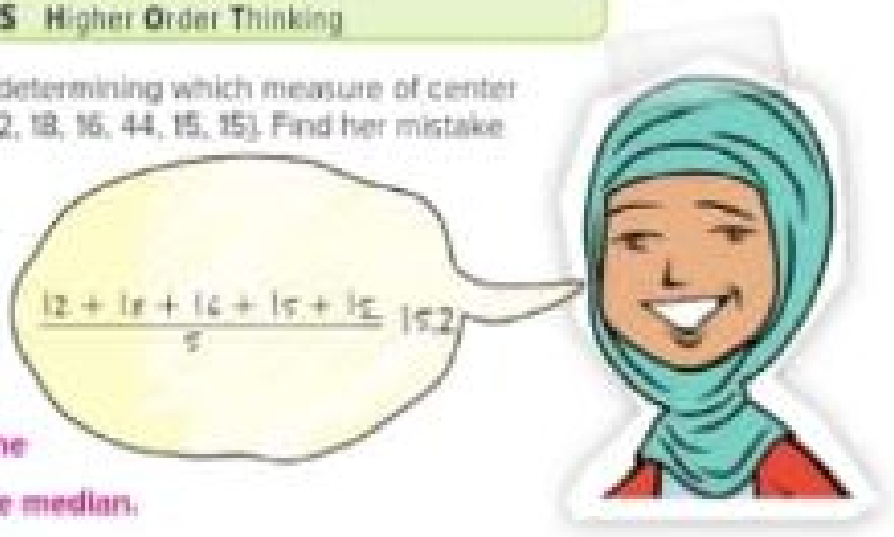
c. Tell which measure of center best describes the data with and without the outlier. **With the outlier, the best measure is the median; without the outlier, the best measure is the mean.**

4. **Construct an Argument** Fill in the graphic organizer below.

Measure of Center	How an outlier affects it
mean	The outlier can make the mean significantly greater or less than the mean would be without the outlier.
median	An outlier can change the median slightly or not at all, depending upon the spread of the data.
mode	An outlier that is a single value does not affect the mode.

H.O.T. Problems Higher Order Thinking

5. **Find the Error** Ghaya is determining which measure of center best describes the data set {12, 18, 16, 44, 15, 15}. Find her mistake and correct it.



Ghaya did not include the outlier. The mean is 20. The median, which is 15.5, best describes the data because the outlier affects the mean more than it affects the median.

6. **Justify Conclusions** Determine whether the following statement is true or false. If true, explain your reasoning. If false, give a counterexample.

Of mean, median, and mode, the median will always be most affected by outliers.

false; Sample answer: Usually the mean is most affected by an outlier. In the data set 1, 4, 5, 5, 6, and 24, the mean is 7.5 with the outlier and 4.2 without the outlier.

7. **Persevere with Problems** Add three data values to the following data set so the mean increases by 10 and the median does not change.

42, 37, 32, 29, 20

Sample answer: 125, 32, and 19

8. **Model with Mathematics** Use the Internet to find some real-world data. Record your data in the space below. See students' work.

a. Find the mean, median, and mode of your data set.

b. Are there any outliers? If so, how do they affect the measures of center?

c. Which measure of center best describes the data with and without the outlier?

Name _____ My Homework _____

Extra Practice

9. The number of songs downloaded per month by a group of friends were 8, 12, 6, 4, 2, 0, and 10. Find the measure of center that best represents the data. Justify your selection then find the measure of center. Since the set of data has no extreme values or numbers that are identical, the mean or median, 6 songs, would best represent the data.

Example

There are no extreme values and no repeated numbers.

mean: $\frac{8 + 2 + 4 + 6 + 8 + 10 + 12}{7} = 6$

median: 0, 2, 4, (6), 8, 10, 12

10. The ages of participants in a relay race are 12, 15, 14, 13, 15, 12, 22, 16, and 11. Identify the outlier in the data set. Determine how the outlier affects the mean, median, and mode of the data. Then tell which measure of center best describes the data with and without the outlier. **outlier: 22 years; without the outlier: mean: 13.5 years, median: 13.5 years, mode: 12 years and 15 years; with the outlier: mean: 14.4 years, median: 14 years, mode: 12 years and 15 years; The mode best describes the data with and without the outlier, because it doesn't change.**

11. **Justify Conclusions** The table shows the high temperatures during one week. Round to the nearest hundredth if necessary.

High Temperatures
29° 27° 29° 29° 28° 29° 62°

a. Identify the outlier in the data set. **62°**

b. Determine how the outlier affects the mean, median, mode, and range of the data. **With the outlier, the mean is 32.71°, the median is 29°, the mode is 29° and the range is 37°. Without the outlier, the mean is 27.83°, the median is 28.5°, the mode is 29°, and the range is 4°.**

c. Tell which measure of center best describes the data with and without the outlier. Explain your reasoning to a classmate. **Sample answer: With the outlier, the best measure is the mode; without the outlier, the best measure is the mode; the outlier does not affect the mode, but affects the mean and median.**

Power Up! Test Practice

12. The table shows the number of points scored by a basketball team during their first 6 games. Determine if each statement is true or false.

Points Scored		
79	83	79
85	41	77

a. The median or mode is the best measure of center to represent the data. ■ True ☐ False

b. The range is affected by the outlier. ■ True ☐ False

c. The mean is the measure of center least affected by the outlier. ☐ True ■ False

13. For each data set, select the most appropriate measure of center.

mean
median
mode

a. mp3 player prices: AED 45, AED 249, AED 77, AED 55, AED 24, AED 36, AED 0 — median

b. years of teaching experience: 19, 5, 7, 24, 20, 3, 28, 2, 1 — mean

c. forecast high temperatures: 72°, 74°, 73°, 74°, 74°, 75° — mode

Spiral Review

Find the total of each set of numbers.

14. {19, 16, 24, 22, 18} 99

15. {54, 48, 52, 57, 49} 260

16. {9, 5, 6, 7, 4, 11, 7} 49

17. {31, 36, 28, 34, 25} 154

18. Graph the numbers 15, 18, 22, 19, and 16 on the number line.

19. The table shows the number of tickets sold to the school musical on three days. How many total tickets were sold?

203 tickets

Day	Number of Tickets Sold
Wednesday	56
Thursday	79
Friday	68

21ST CENTURY CAREER in Marine Biology

Marine Biologist

Do all the unusual and amazing creatures in the ocean fascinate you? Do you think you would be good at coming up with your own experiments to test theories about them? If so, a career in marine biology might be something to think about! A marine biologist studies plants and animals that live in the ocean. These include everything from microscopic plankton to multi-ton whales. Marine biologists study organisms that live in the tiny layers of the surface and those that live thousands of meters below the surface.

Is This the Career for You?

If you would like to be a marine biologist, you may want to take some of the following courses in high school.

- Biology
- Calculus
- Chemistry
- Marine Science
- Statistics

Turn the page to find out how math relates to a career in Marine Biology.

Ready to Make Waves?

Use the information in the line plot and the table to solve each problem. Round to the nearest tenth if necessary.

1. Find the mean of the pipefish data. **8.3 cm**

2. Find the median and mode of the pipefish data. **8.4 cm; 8.4 cm**

3. What is the range of the pipefish data? Would you describe the data as spread out or close in value? Explain. **1.2 cm; Sample answer: Compared to the values in the data set, the range is large. So, the data are spread out. This can also be seen in the line plot.**

4. Identify the outlier in the artificial reef data. Find the mean with and without the outlier. **Sample answer: 351; with outlier: 68.7; without outlier: 59.9**

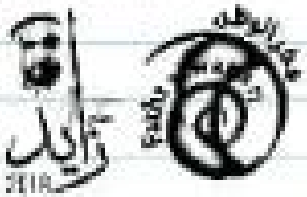
5. Describe how the outlier affects the mean in Exercise 4. **With the outlier, the mean is about 8.8 greater than without the outlier. The mean without the outlier is better representative of the data.**

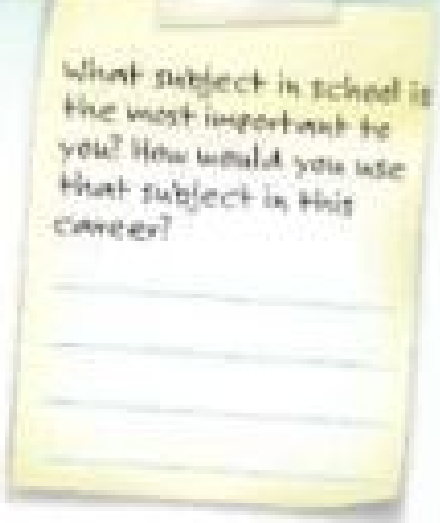
6. Find the median and mode of the artificial reef data. Which better represents the data? Explain. **median: 61; mode: 34; The median better represents the data because half the data are less than the median and half the data are greater than the median.**

Number of Artificial Reefs in Florida Counties						
198	62	108	34	29	73	173
96	97	9	46	21	22	60
8	83	31	79	67	61	15
105	63	34	351	13	126	36
25	12	82	35	4		

Career Project

It's time to update your career portfolio! Use the Internet or another source to research several careers in marine biology. Write a brief summary comparing and contrasting the careers.

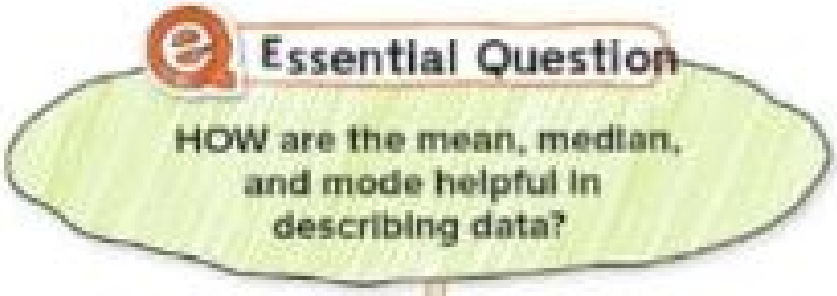




Reflect

Answering the Essential Question

Use what you learned about mean, median, and mode to complete the graphic organizer. Sample answers are given.



	mean	median	mode
definition	The sum of the numbers in a set of data divided by the number of pieces of data.	The value appearing at the center of a sorted version of a list, or the mean of the two central values, if the list contains an even number of values.	The number(s) or item(s) that appear most often in a data set.
When is it appropriate to use?	when there are no extreme values	in a large data set with extreme values	when there are several repeated values
How does an outlier affect it?	can alter the mean significantly	usually minimally, if at all	unless the mode is the outlier, it doesn't affect it

Answer the Essential Question. HOW are the mean, median, and mode helpful in describing data?

See students' work.

Statistics and Probability

Chapter 12
Statistical Displays

Essential Question

WHY is it important to carefully evaluate graphs?

Mathematical Practices
1, 2, 3, 4, 5, 6, 7

Math in the Real World

Roller Coaster The table shows the drop of several different roller coasters.

Roller Coaster	Drop (m)
Anaconda	44
Mind Eraser	29
Scorpion	18
Thunderbolt	21

Draw bars to represent the drop of each roller coaster.

FOLDABLES
Study Organizer

1. Cut out the Foldable on page FL15 of this book.
2. Place your Foldable on page 922.
3. Use the Foldable throughout this chapter to help you learn about statistical displays.

What Tools Do You Need?

Vocabulary

box plot
cluster
distribution
dot plot
frequency distribution
gap
histogram
line graph
line plot
peak
symmetric

Review Vocabulary

Using a graphic organizer can help you remember important vocabulary terms. Fill in the graphic organizer for the word *graph*.

graph

Definition

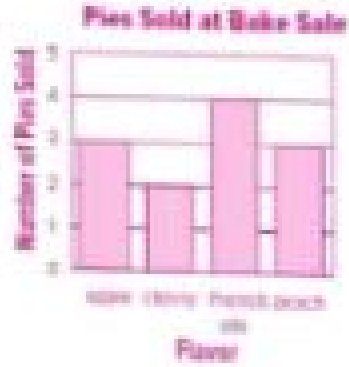
To place a point named by an ordered pair; a statistical display.

Example

Sample answer: you can graph sales from a bake sale or points on a coordinate plane to form a line.

Picture

Sample answer:



Are You Ready?

Try Her Quick Check below.

Quick Review

Review

Example 1

Find the mean of the data set.

{15, 30, 20, 25, 30}

$15 + 30 + 20 + 25 + 30 = 120$ Add.

$\frac{120}{5} = 24$ Divide.

The mean is 24.

Example 2

Find the median of the data set.

{65, 57, 33, 41, 49}

33 41 (49) 57 65 Order the numbers from least to greatest.

The number in the middle is 49, so 49 is the median.

Quick Check

Mean **Find the mean of each data set.**

1. {8, 13, 21, 12, 29, 13}
   16
2. {52, 76, 61, 58, 68}
   63
3. {35, 18, 22, 20, 36, 31}
   27

4. Ahmed's social studies grades during one quarter are shown in the table. What is his mean score for the quarter?
   94%

Social Studies Grades (%)					
94	89	96	93	90	99
87	97	95	93	96	97

Median **Find the median of each data set.**

5. {56, 61, 54, 54, 58, 59}
   57
6. {124, 131, 114, 148, 126}
   126
7. {85, 79, 82, 90, 84, 87}
   84.5

8. The table shows the high temperatures in a certain city for a week. What is the median temperature?
   22°C

High Temperature (°C)						
31	24	16	12	22	22	25

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7 8

Lesson 1

Line Plots

Real-World Link

Activities Students in Mr. Badr's class were asked how many after-school activities they have. Their responses are shown in the table.

Step 1 Use the data to complete the frequency table.

Number of Activities			
0	2	1	3
1	1	3	4
2	1	0	1
2	3	2	1

→

Number of Activities	
Number	Tally
0	||
1	𝍸 ||
2	𝍸
3	𝍸
4	|

Step 2 Turn the table so the number of activities is along the bottom on a number line. Instead of tally marks, place Xs above the number line. The Xs for 0 activities have been placed for you.

The data is now represented in a *line plot*.

Essential Question

WHY is it important to carefully evaluate graphs?

Vocabulary

line plot
dot plot

Mathematical Practices
1, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Make a Line Plot

One way to give a picture of data is to make a line plot. A **line plot** is a visual display of a distribution of data values where each data value is shown as a dot or other mark, usually an X, above a number line. A line plot is also known as **dot plot**.

Example

1. Yasmin asked her class how many pets they had. The results are shown in the table. Make a line plot of the data. Then describe the data presented in the graph.

Number of Pets					
3	2	2	1	3	1
0	1	0	2	3	4
0	1	1	4	2	2
1	2	2	3	0	2

Step 1 Draw and label a number line.

Step 2 Place as many Xs above each number as there are responses for that number. Include a title.

Step 3 Describe the data. 24 students responded to the question. No one has more than 4 pets. Four students have no pets. The response given most is 2 pets. This represents the mode.

Get It? Do this problem to find out.

a. Fahed asked the members of his 4-H club how many projects they were taking. The results are shown in the table. Make a line plot of the data. Then describe the data in the graph.

Number of Projects				
2	4	3	3	1
0	5	4	2	2
1	3	2	1	2

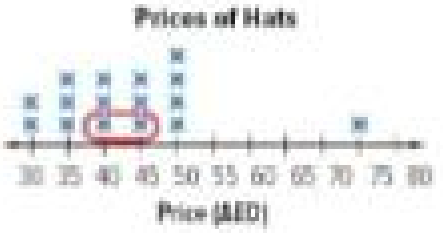
a. Sample answer: 15 members responded. No one is taking more than 5 projects. 1 member is not taking any projects. The response given most is 2 projects. This represents the mode.

Analyze Line Plots

You can describe a set of data using measures of center as well as measures of variability. The range of the data and any outliers are also useful in describing the data.

Examples

The line plot shows the prices of hats.



2. Find the median and mode of the data. Then describe the data using them.

There are 16 hat prices represented in the line plot. The median is between the 8th and 9th pieces of data.

The two middle numbers, shown on the line plot, are 40 and 45. So, the median is AED 42.50. This means that half of the hats cost more than AED 42.50 and half cost less than AED 42.50.

The number that appears most often is 50. So, the mode of the data is 50. This means that more hats cost AED 50 than any other price.

3. Find the range and any outliers of the data. Then describe the data using them.

The range of the prices is AED 75−AED 30 or AED 45. The limits for the outlier are AED 12.50 and AED 72.50. So, AED 75 is an outlier.

Get it? Do this problem to find out.

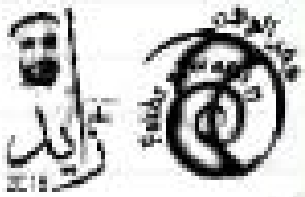
b. The line plot shows the number of magazines each member of the student council sold. Find the median, mode, range, and any outliers of the data. Then describe the data using them.

Stop and Reflect

Suppose two sets of data have the same median but different ranges. What can you conclude about the sets? Explain below.

Sample answer: The values greater than the median and the values less than the median are different for the two sets.

There are 14 numbers represented in the line plot. The median is 20, so half of the student council members sold more than 20 magazines and half sold less than 20. The mode is 20, which means more members sold 20 magazines than any other number of magazines. The range is 8, and there are no outliers.



Sample answer: There are 10 prices represented. mean: AED 35; median: AED 30; mode: AED 25; Q_1: AED 25; Q_3: AED 40; The IQR is AED 40 − AED 25 or AED 15. The majority of the data is from AED 25- AED 45. The median is most representative of the data since there is an outlier at AED 65.

Example

4. The line plot shows the amount Yousif deposited in his savings account each month. Describe the data. Include measures of center and variability.

The mean is AED 46.67. The median is AED 47.50, and the mode is AED 50. So, the majority of the data are close to the measures of center.

The range of the data is AED 75−AED 35 or AED 40. The interquartile range is $Q_3 - Q_1$ or AED 50- AED 37.50= AED 12.50. So, half of the amounts are between AED 37.50 and AED 50. There is one outlier at AED 75.

Get it? Do this problem to find out.

c. The line plot shows the prices of sweaters in a store. Describe the data. Include measures of center and variability.

Guided Practice

Sample answers: 1 and 2

1. Make a line plot for the set of data. Describe the data. Include measures of center and variability.

Calories in Serving of Peanut Butter			
190	160	210	220
200	185	190	190
185	200	190	210
190	185	200	200

There are 16 peanut butters represented. median: 190; mode: 190; Most of the data is close to the medium and mode, range: 50; @187.5; Q_3: 200; IQR is 200-187.5 or 12.5 Calories; Since there is an outlier at 160, the median or mode is most representative of the data.

2. **Building on the Essential Question** How is using a line plot useful to analyze data? The median, mode, first and third quartiles are easily seen on a line plot, as are the range of the data and the presence of any outliers.

Rate Yourself!

How confident were you about line plots? Check the box that says it.

Independent Practice

Make a line plot for each set of data. Find the median, mode, range, and any outliers of the data shown in the line plot. Then describe the data using them.

1. Length of summer camps in days:
7, 7, 12, 10, 5, 10, 5, 7, 10, 9, 7, 9, 6, 10, 5, 8, 7, and 8

median: 7.5; mode: 7; range: 7; no outlier; There are a total of 18 summer camps represented. The median means that one-half of the summer camps are longer than 7.5 days and one-half are less. More camps are 7 days than any other number of days.

2.

Students' Estimates of Room Length (m)				
10	11	12	12	13
13	13	14	14	14
15	15	15	15	15
16	16	16	17	17
17	17	18	18	25

median: 15; mode: 15; range: 15; outlier: 25; There are 25 room lengths, in meters, represented. The median means that one-half of the rooms are greater than 15 meters and one-half are less. More rooms are 15 meters than any other length.

3. The line plot shows the number of songs in play lists. Describe the data. Include measures of center and variability.

Sample answer: There are 15 play lists represented. mean: 40; median: 40; modes: 40 and 42; So, the majority of the data is close to the measures of center. 38; Q_3: 42; IQR: 4, which means half the playlists have between 38 and 42 songs; there is an outlier at 25.

Inductive Reasoning The number of runs a softball team scored in their last five games is shown in the line plot. How many runs would the team need to score in the next game so that each statement is true?

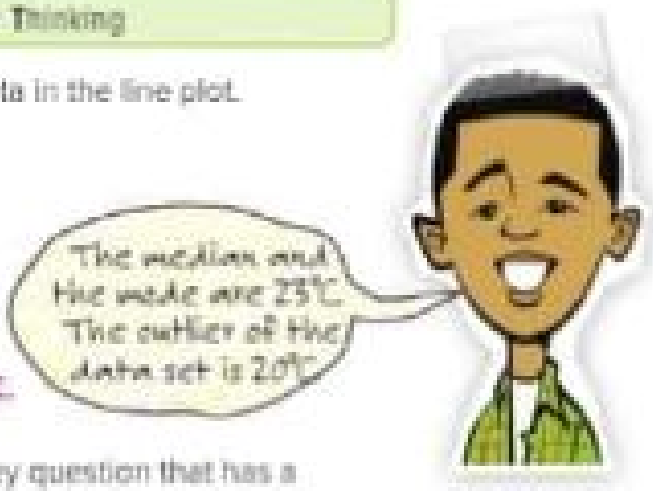
4. The range is 10 4 or 16

5. Another mode is 11 11

6. The median is 9.5 10

H.O.T. Problems Higher Order Thinking

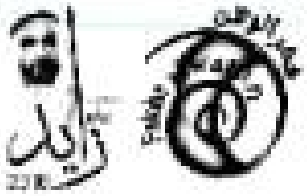
7. **Find the Error** Omar is analyzing the data in the line plot. Find his mistake and correct it.



The outlier of the data set is 29°C, not 20°C.

8. **Model with Mathematics** Write a survey question that has a numerical answer. Some examples are "How many CDs do you have?" or "How many meters long is your bedroom?" Ask your friends and family the question. Record the results and organize the data in a line plot. Use the line plot to make conclusions about your data. For example, describe the data using the measures of center and variability.

See students' work.



9. **Persevere with Problems** There are several sizes of flying disks in a collection. The range is 8 centimeters. The median is 22 centimeters. The smallest size is 16 centimeters. What is the largest disk in the collection?

24 cm

10. **Construct an Argument** Determine whether the statement is true or false. Explain.

Line plots display individual data.

true; Sample answer: Each piece of data is represented as a dot or an X on the line plot.

11. **Reason Inductively** The line plot shows the number of student visitors to the National Wildlife Refuge each day for two weeks. If the four Xs at 56 were not included in the data set, which measure of center would be most affected? Justify your response.

mode; Sample answer: With the four values, the mean is 61.36, the median is 62, and the mode is 56. Without the four values, the mean is 63.5, the median is 63.5, and the modes are 62, 65, and 68. Not including the four values changes the mode more drastically.

Extra Practice

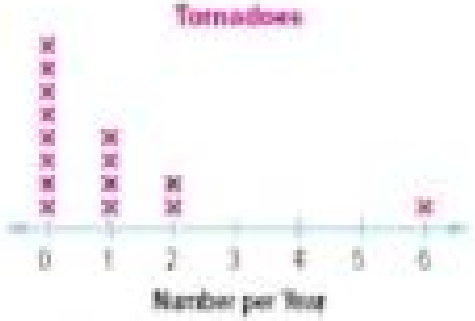
Make a line plot for each set of data. Find the median, mode, range, and any outliers of the data shown in the line plot. Then describe the data using them.

12. Daily high temperatures in degrees Celsius:
31, 32, 34, 32, 32, 28, 31, 27, 28, 31, 28, 32, 36, 35, 32, 33, 28, 29, 29, 33, 34, 36, 32, and 34

median: 32°C; mode: 32°C; range: 9°C; no outliers; The number of temperatures, in °C, represented is 24. The median means half the daily high temperatures are greater than 32°C and half are less. Most days had a high of 32°C than any other temperature.

13.

Number of Tornadoes				
0	1	1	1	6
0	0	0	0	0
2	1	2	0	0

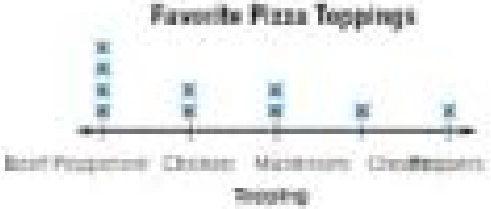


median: 0; mode: 0; range: 6; outlier: 6 There were 15 tornadoes represented. The median means that half the number of tornadoes was greater than zero and half the number of tornadoes was zero.

Copy and Solve Describe the data in the line plots. Show your work on a separate piece of paper. See margin.

14. The line plot shows the number of hours students spend watching TV each night. Describe the data. Include measures of center and variability. Round to the nearest tenth if necessary.

15. Justify Conclusions The line plot shows students' favorite pizza toppings. Which can you find using the line plot: the median, mode, range, or outlier(s)? Explain. Then write a sentence or two to describe the data set. Explain your reasoning to a classmate.



Power Up! Test Practice

16. The table shows the number of stories in 15 skyscrapers. Construct a line plot of the data.

Number of Stories				
54	88	80	88	70
78	101	69	88	85
103	73	88	110	80

What are the median, first quartile, third quartile, and interquartile range of the data? 85, 73, 88, 15

17. The line plot shows the number of weekly chores that some students have. Determine if each statement is true or false.

a. The median number of chores is 2. ■ True □ False

b. The range of the data is 4. ■ True □ False

c. The interquartile range of the data is 2. □ True ■ False

Spiral Review

Fill in each ○ with >, <, or = to make a true statement.

18. 26 > 19

19. 89 < 92

20. 5.6 < 6.5

21. 11.5 < 105

22. 47 > 44

23. 1.52 < 14.8

24. The table shows the number of days several students attended an exercise class during a month. How many students attended a class less than 15 days? 6 students

Number of Days			
16	21	18	6
19	15	8	11
16	4	20	22
12	19	21	9

25. Seven friends compared their test scores. The scores they received were 89, 97, 93, 95, 90, 88, 91. How many people had scores greater than 90? 4

Lesson 2

Histograms

Real-World Link

Concerts Nabila researched the average price of concert tickets. The table shows the results.

Average Ticket Prices of Top 10 Money-Earning Concerts				
AED 83.87	AED 68.54	AED 51.53	AED 62.10	AED 59.58
AED 47.22	AED 66.58	AED 88.49	AED 50.63	AED 98.98

1. Fill in the tally column and frequency column on the frequency table.

Average Ticket Prices of Top 10 Money-Earning Concerts		
Price	Tally	Frequency
AED 25.00–AED 41.99	|	1
AED 42.00–AED 74.99	𝍸 ||	7
AED 75.00–AED 99.99	||	2

2. What does each tally mark represent? the concert

3. What is one advantage of using the frequency table?
Sample answer: You can easily tell how many concerts had an average ticket price less than AED 50.

4. What is one advantage of using the first table?
Sample answer: You can identify specific average ticket prices.

Essential Question

Why is it important to carefully evaluate graphs?

Vocabulary

histogram
frequency distribution

Mathematical Practices
1, 3, 4, 5, 6

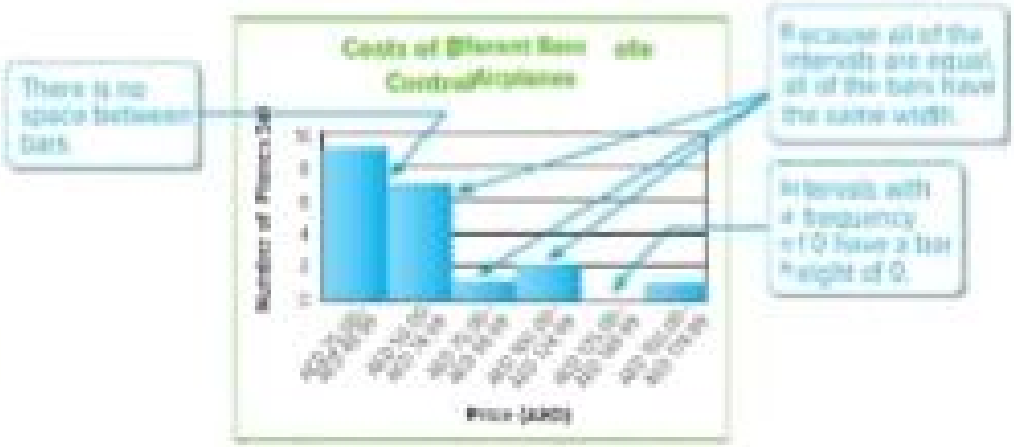
Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Interpret Data

Data from a frequency table can be displayed as a histogram. A **histogram** is a type of bar graph used to display numerical data that have been organized into equal intervals. These intervals allow you to see the **frequency distribution** of the data, or how many pieces of data are in each interval.



Example

1. Refer to the histogram above. Describe the histogram. How many remote control airplanes cost at least AED 100?

There are 9+7 + 1+ 2 + 1 or 20 prices recorded.
More remote control airplanes had prices between AED 25.00 and AED 49.99 than any other range. There were no airplanes recorded with a price between AED 125.00 and AED 149.99.

Two remote control airplanes had prices between AED 100.00–AED 124.99 and one remote control airplane had a price between AED 150.00–AED 174.99. So, 2 1, or 3, remote control airplanes had prices that were at least AED 100.

Get it? Do this problem to find out.

A. 16 airplanes

a. Refer to the histogram above. How many remote control airplanes cost less than AED 75?

Construct a Histogram

You can use data from a table to construct a histogram.

Example

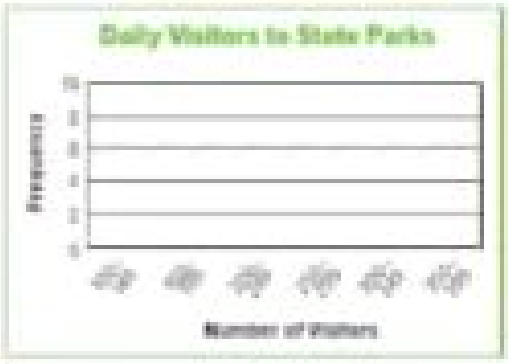
2. **The table shows the number of daily visitors to selected state parks. Draw a histogram to represent the data.**

Daily Visitors to Selected State Parks				
108	309	171	152	236
165	244	263	212	161
327	185	192	226	137
193	235	207	382	241

Step 1 Make a frequency table to organize the data. Use a scale from 100 through 399 with an interval of 50.

Daily Visitors to Selected State Parks		
Visitors	Tally	Frequency
100–149	|	2
150–199	𝍸 ||	7
200–249	𝍸 ||	8
250–299	|	1
300–349	|	1
350–399	|	1

Step 2 Draw and label a horizontal and vertical axis. Include a title. Show the intervals from the frequency table on the horizontal axis. Label the vertical axis to show the frequencies.



Step 3 For each interval, draw a bar whose height is given by the frequencies.

Scales and Intervals

It is important to choose a scale that includes all of the numbers in the data set. The interval should organize the data to make it easy to compare.

TOC and Reflect

When is a histogram more useful than a table with individual data? Explain below.

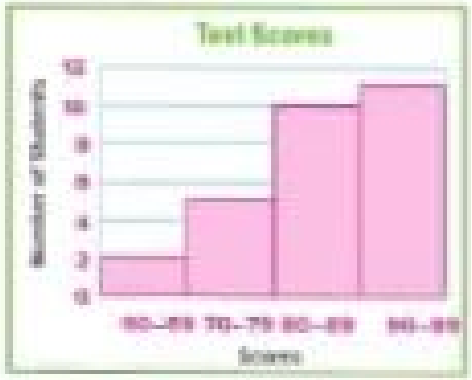
Sample answer: Because it is more visual, a histogram is more useful than a table when you are trying to show a general trend.

Get it? Do this problem to find out.

b. The table at the right shows a set of test scores. Choose intervals, make a frequency table, and construct a histogram to represent the data.

Test Scores						
72	97	80	86	92	98	88
76	79	82	91	83	90	76
81	94	96	92	72	83	85
65	91	92	68	86	89	97

Test Scores		
Score	Tally	Frequency
60–69	||	2
70–79	𝍸	5
80–89	𝍸 𝍸	10
90–99	𝍸 𝍸 |	11



Guided Practice

1. The frequency table below shows the number of books read on vacation by the students in Mrs. Maha's class.

a. Draw a histogram to represent the data.

b. Describe the histogram. Sample answer: The number of books read by 30 students were recorded. More students read between 3 and 5 books than any other range.

c. How many students read six or m one book? 30 students

Number of Books Read		
Books	Tally	Frequency
0–2	𝍸 |	6
3–5	𝍸 𝍸	10
6–8	𝍸 |	7
9–11	||	3
12–14	|||	4

2. **Building on the Essential Question** Why would you create a frequency table before creating a histogram?

Sample answer: A frequency table shows the frequency for each range. This information is used to create a histogram.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

Name _____ My Homework _____

Independent Practice

For Exercises 1–4, use the histogram at the right.

1. Describe the histogram. Sample answer: 24 cyclists participated. No one finished with a time lower than 60 minutes.

2. Which interval has 7 cyclists? 65–69 minutes

3. Which interval represents the greatest number of cyclists? 60–64 minutes

4. How many cyclists had a time less than 70 minutes? 16 cyclists

Draw a histogram to represent the set of data.

5.

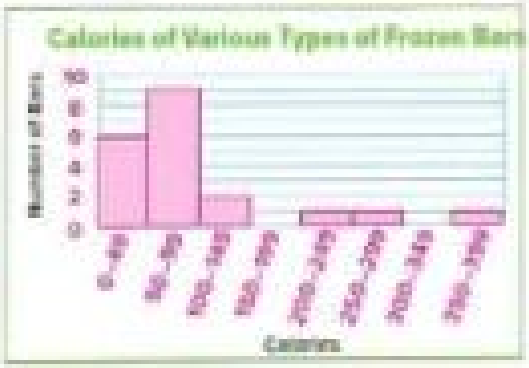
Number of States Visited by Students in Ali's Class		
Number of States	Tally	Frequency
0–4	𝍸 III	9
5–9	III	3
10–14	𝍸	5
15–19	III	3
20–24	𝍸 I	6
25–29	I	1

Use Math Tools For Exercises 6 and 7, refer to the histograms below.

6. About how many students from both grades earned AED 600 or more? 24 students

7. Which grade had more students earn between AED 400 and AED 599? 6th grade

8. **Be Precise** The following data provides the number of Calories of various types of frozen bars. (25, 35, 200, 280, 80, 80, 90, 40, 45, 50, 50, 60, 90, 100, 120, 40, 45, 60, 70, 350)



a. Draw a histogram to represent the data.

b. Find the measures of center.

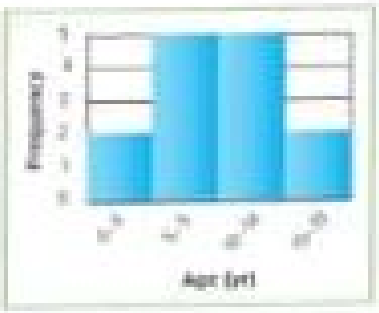
mean: 95.5; median: 65; modes: 40, 45, 50, 60, 80, and 90

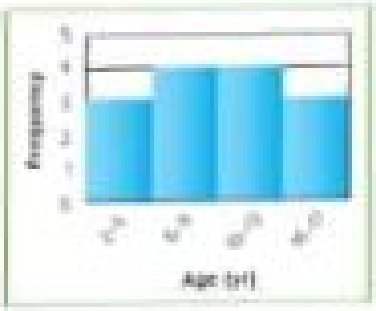
c. Can you find the measures of center only from the histogram? Explain.

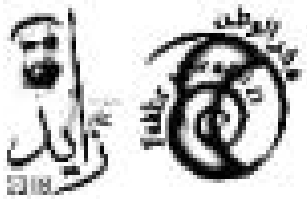
no; Sample answer: Individual data values are not shown in the histogram, so the measures of center cannot be found.

H.O.T. Problems Higher Order Thinking

9. **Persevere with Problems** Give a set of data that could be represented by both histograms below.







Sample answer: ages of students at summer camp: 3, 4, 5, 7, 7, 8, 8, 10, 10, 11, 13, 14, 15, 15

10. **Justify Conclusions** Identify the interval that is not equal to the other three. Explain your reasoning.

15–19	30–34	40–45	45–49

40–45; All the other intervals represent 5 whole numbers.

11. **Reason Inductively** The table shows a set of plant heights. Describe two different sets of intervals that can be used in representing the set in a histogram. Compare and contrast the two sets of intervals.

Plant Heights (cm)		
12	7	15
8	24	41
16	18	27
43	33	11
24	10	22

Sample answer: One set of intervals would be from 0 to 45, with intervals of 5. Another set would be from 0 to 50 with intervals of 10. If smaller intervals are used, less data values will be in each interval, therefore the bars of the histogram will be shorter.

Name _____ My Homework _____

Extra Practice

For Exercises 12–16, use the histogram.

12. Describe the histogram. The ages of 32 players were collected. One player is older than 35. Five were 36 or younger.

Add each of the frequencies to find the total players.
6 + 11 + 4 + 9 + 1 = 32

13. Which interval represents the greatest number of players?
24–27 years

14. Which interval has 4 players? 28–31 years

15. How many players are younger than 28? 17

16. How many players have ages in the interval 32–35? 9

Model with Mathematics Draw a histogram to represent the set of data.

17.

Number of Home runs in a Season		
Homeruns	Tally	Frequency
0–9	𝍸 𝍸 ||	12
10–19	𝍸 𝍸	10
20–29	𝍸 |||	9
30–39	𝍸 |||	9
40–49	𝍸 |	6

18. Find the Error Ilia is analyzing the frequency table below. Find her mistake and correct it.

Distances from Home to School (km)	Tally	Frequency
0.5–0.5	𝍸 ||	7
0.6–1.0	||	3
1.1–1.5	𝍸	5
1.6–2.0	||	3

15 people live less than 1.6 kilometers from school. Since 1.5 is within an interval, we do not know how many of the 5 people live less than 1.5 kilometers from school in that range.

Power Up! Test Practice

19. The histogram shows the number of goals scored by the top players on a soccer team. Explain why there is not a bar for the interval of 30–44 goals.

Sample answer: There were no players who scored 30–44 goals in their career.

20. The table shows the number of sit-ups each member of a gym class completed in one minute. Choose an appropriate scale and intervals and construct a histogram of the data.**Sample answer:**

Number of Sit-Ups in One Minute				
30	15	34	22	28
20	25	26	31	29
27	30	19	22	28
32	31	27	23	26

Spiral Review

Divide.

21. $126 \div 3 =$ **42**

22. $477 \div 9 =$ **53**

23. $162 \div 6 =$ **27**

24. $327 \div 5 =$ **65.4**

25. $195 \div 2 =$ **97.5**

26. $842 \div 4 =$ **210.5**

27. Jamel, Hereb, and Rashid bought a bag of apples. Jamal kept 0.25 of the apples, and Rashid kept 0.5 of the apples. Who kept more of the apples?

Rashid

Lesson 3

Box Plots

Real-World Link

Football The table shows the number of touchdowns scored by each of the 16 teams in the National Football Conference in a recent year.

Number of Touchdowns							
47	45	35	38	28	54	48	34
49	44	27	34	37	44	26	38

1. Plot the data on a line plot.

2. Find the median, lower extreme, upper extreme, first quartile and third quartile of the data. Place a star on the number line for each value.

median: 37.5
lower extreme: 24
upper extreme: 54
first quartile: 31
third quartile: 46.5

3. What percent of the teams scored less than 31 touchdowns?

25%

4. What percent of the teams scored more than 37.5 touchdowns?

50%

Essential Question

WHY is it important to carefully evaluate graphs?

Vocabulary

box plot

Mathematical Practices

1, 2, 3, 4, 7

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Construct a Box Plot

A **box plot** or box-and-whisker plot, uses a number line to show the distribution of a set of data by using the median, quartiles, and extreme values. A *box* is drawn around the quartile values, and the *whiskers* extend from each quartile to the extreme data points that are not outliers. The median is marked with a vertical line. The figure below is a box plot.

Box plots separate data into four parts. Even though the parts may differ in length, each contains 25% of the data. The box shows the middle 50% of the data.

Example

Common Misconception

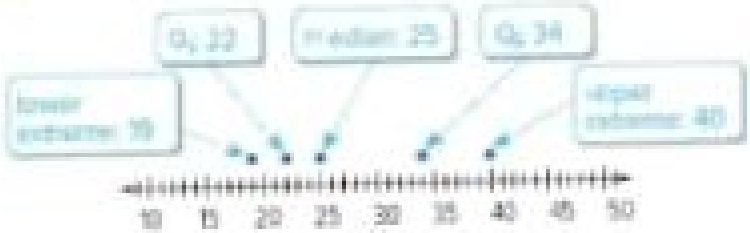
You may think that the median always divides the box in half. However, the median may not divide the box in half because the data may be clustered around the quartile.

1. Draw a box plot of the car speed data.

25 35 27 22 34 40 20 19 23 25 30

Step 1 Order the numbers from least to greatest. Then draw a number line that covers the range of the data.

Step 2 Find the median, the extremes, and the first and third quartiles. Mark these points above the number line.



Step 3 Draw the box so that it includes the quartile values. Draw a vertical line through the box at the median value. Extend the whiskers from each quartile to the extreme data points. Include a title.

Get it? Do this problem to find out.

a. Draw a box plot of the data set below.

(AED 20, AED 25, AED 22, AED 30, AED 15, AED 18, AED AED 17, AED 30, AED 27, AED 15)

a.

Interpret Data

Though a box plot does not show individual data, you can use it to interpret data.

Examples

Refer to the box plot in Example 1.

2. Half of the drivers were driving faster than what speed?

Half of the 11 drivers were driving faster than 25 kilometers per hour.

3. What does the box plot's length tell about the data?

The length of the left half of the box plot is short. This means that the speeds of the slowest half of the cars are concentrated. The speeds of the fastest half of the cars are spread out.

Box Plots

- If the length of a whisker in the box is short, the values of the data in that part are concentrated.
- If the length of a whisker in the box is long, the values of the data in that part are spread out.

Get it? Do this problem to find out.

b. What percent were driving faster than 34 kilometers per?

b. 25%

Example

4. The box plot below shows the daily attendance at a fitness club. Find the median and the measures of variability. Then describe the data.

The median is 72.5. The first quartile is 65 and the third quartile is 80. The range is 54 and the interquartile range is 15. There is an outlier at 110. Both whiskers are approximately the same size so the data, without the outlier, is spread evenly below and above the quartiles.

Outliers

If the data set includes outliers, then the whiskers will not extend to the outliers, just to the nearest data point. Outliers are represented with an asterisk (*) or like this.

c. median: 8.5; Q_1 5.5; Q_3 11; range: 12; IQR: 5.5; no outlier; Sample answer: The left side of the data is more spread out and the right side is more concentrated.

Get it? Do this problem to find out.

c. The number of games won in the American Football Conference in a recent year is displayed below. Find the median and the measures of variability. Then describe the data.

American Football Conference Wins

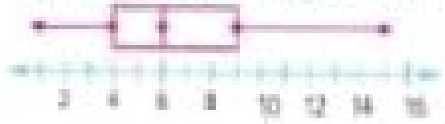
Guided Practice

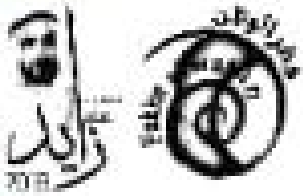
1. Use the table.

Depth of Recent Earthquakes (km)						
5	15	1	11	2	7	3
9	5	4	8	10	5	7

a. Make a box plot of the data.

Depths of Earthquakes (km)





b. What percent of the earthquakes were between 4 and 9 kilometers deep? 50%

c. Write a sentence explaining what the length of the box plot means.

Sample answer: The length of the box plot shows that the depths of the earthquakes are not concentrated around a certain depth.

2. Find the median and the measures of variability for the box plot shown. Then describe the data.

Average Daily Temperatures for One Month

median: 27; Q_1 25; Q_3 33; range: 18; IQR: 8; Sample answer: The right half of the data is more spread out and the left half is more concentrated. The median is close to the first quartile. There are no outliers.

3. **Building on the Essential Question** How is the information you can learn from a box plot different from what you can learn from the same set of data shown in a line plot?

Sample answer: A box plot summarizes the data and shows how the data are spread out. A line plot lists all the data.

Rate Yourself!

How confident are you about making and interpreting box plots? Check the box that applies.

Name _____ My Homework _____

Independent Practice

Draw a box plot for each set of data.

1. {65, 92, 74, 61, 55, 35, 88, 99, 97, 100, 96}

2.

Cost of MP3 Players (AED)	
95	55
105	100
85	158
122	174
165	162

3. The table shows the length of coastline for a group of 13 countries.

Length of Coastline (km)	
28	110
580	127
100	301
228	40
31	187
192	112
13	

a. Make a box plot of the data.

b. Half of the countries have a coastline less than how many kilometers?

127 km

c. Write a sentence describing what the length of the box plot tells about the number of kilometers of coastline for the group of countries.

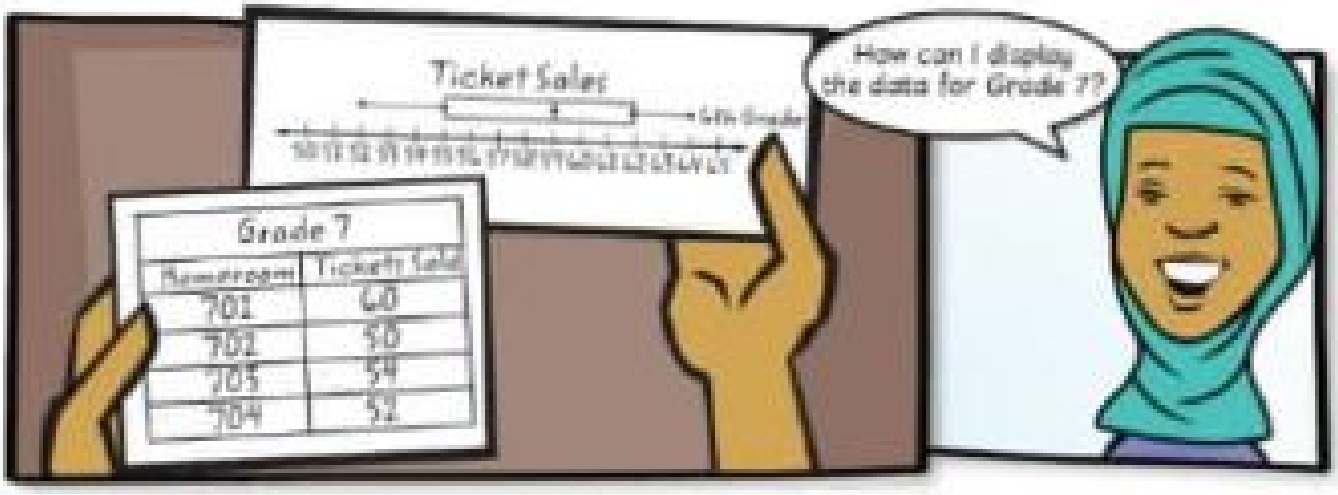
Sample answer: The length of the box plot shows that the number of kilometers of coastline for the top 25% of countries varies greatly. The number of kilometers of coastline for the bottom 25% of countries is concentrated.

4. The amount of Calories for a serving of certain fruits is displayed. Find the median and the measures of variability. Then describe the data.

median: 75; Q_1: 50; Q_3: 140; range: 400; IQR: 90; There is an outlier at 425.

Sample answer: The right half of the data is more spread out and the left half of the data is more concentrated. The median is closer to the first quartile.

5. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.



a. Draw a box plot using the data for Grade 7.

b. Compare the box plots. Which grade sold more tickets? Explain.

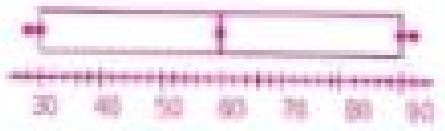
Grade 6; Sample answer: The median, upper extreme, and first and third quartiles are higher for the grade 6 data.

H.O.T. Problems Higher Order Thinking

6. **Persevere with Problems** Write a set of data that contains 12 values for which the box plot has no whiskers. State the median, first and third quartiles, and lower and upper extremes.

Sample answer: {60, 60, 60, 60, 60, 70, 75, 80, 85, 85, 85, 85}; median = 72.5; Q_1 = lower extreme = 60; Q_3 = upper extreme = 85

7. **Reason Abstractly** Write a set of data that, when displayed in a box plot, will result in a long box and short whiskers. Draw the box plot.



Sample answer: {28, 30, 52, 68, 90, 92}

8. **Reason Inductively** What can you conclude from a box plot where the length of the left box and whisker is the same as the length of the right box and whisker?

Sample answer: Both halves of the data have an equal spread.

Name _____ My Homework _____

Extra Practice

Draw a box plot for each set of data.

9. {26, 22, 31, 36, 22, 27, 15, 36, 32, 29, 30}

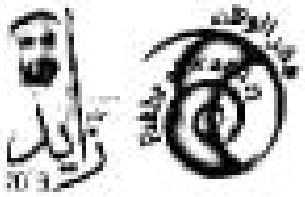
15; 22; 24; 26; 30; 32; 36; 36

median: 29; Q1: 22; Q3: 32

Mark the median, Q1, Q3, and extremes above the number line. Draw a box around the quartiles and a line through the center of the median. Connect the extremes to the box with a line.

10.

Height of Waves (cm)		
50	51	77
72	55	65
43	78	67
40	81	68
63	73	59



11. The box plot below summarizes math test scores.

a. What was the greatest test score? 96

b. Explain why the median is not in the middle of the box.

Sample answer: The scores were closer together between 82 and 86.

c. What percent of the scores were between 71 and 96? 75%

d. Half of the scores were higher than what score? 82

12. **Identify Structure** Find the median, first and third quartiles, and the interquartile range for the set of data in the table. Create a box plot of the data.

Words Typed Per Minute		
80	42	65
72	63	81
67	73	40
51	68	59
77	55	78

median: 67; Q1: 55; Q3: 77; IQR: 22

Words Typed Per Minute

Power Up! Test Practice

13. Which of the following statements are true about the box plot? Select all that apply.

- [x] Half of the data are greater than 62.
- [x] Half of the data are in the interval 62–74.
- [ ] There are more data values in the interval 52–62 than there are in the interval 62–74.
- [x] The value 74 is the maximum value.

14. The table shows the heights, in centimeters, of tomato plants in a garden.

Height (cm)				
62	70	60	68	64
64	53	65	51	67
60	59	57	65	61

a. Construct a box plot of the data.

b. What are the minimum, first quartile, median, third quartile, and maximum of the data?

51, 59, 62, 65, 70

Spiral Review

Find the total of each set of numbers.

15. (6, 8, 7, 9, 2, 4)
36

16. (15, 20, 35, 24, 31)
125

17. (16, 25, 35, 28, 31, 27)
162

18. (56, 58, 63, 51, 52)
280

19. (84, 106, 98, 88)
376

20. (34, 68, 23, 18, 57)
200

21. The table shows the number of tickets each member of the drama club sold. How many members sold more than 50 tickets?

Tickets Sold				
26	32	18	53	28
35	42	29	38	50
49	51	21	34	46
42	52	50	36	20

3 members

22. Suha jumped rope for 6 minutes on Monday, 12 minutes on Tuesday, 7 minutes on Wednesday, 10 minutes on Thursday, and 8 minutes on Friday. Graph her times on a number line.

Problem-Solving Investigation
Use a Graph

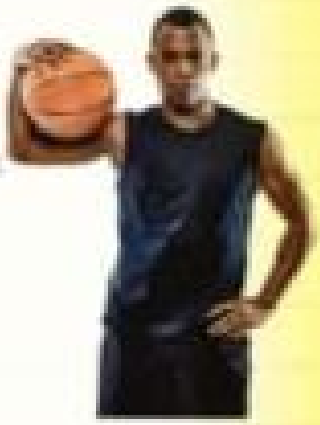
Mathematical Practices 1, 3, 4

Case #1 Basketball

Jessie's brother is on the basketball team and he is making a display of the number of points the team scored in each game last year. He uses the information in the table to make a line plot.

What score occurred most frequently?

Number of Points Scored			
35	35	42	21
49	35	21	24
34	35	21	



1 Understand *What are the facts?*

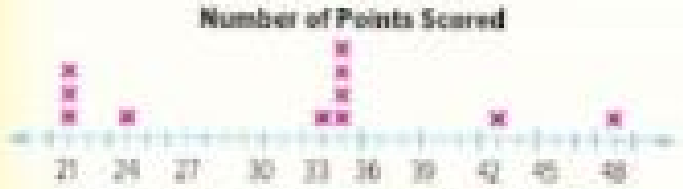
The range of the points is 49 − 21, or 28.

2 Plan *What is your strategy to solve this problem?*

Make a line plot to see which score occurs most frequently. Use the range to label the line plot from 20 to 50.

3 Solve *How can you apply the strategy?*

Plot each score on the line plot.



The score occurring most frequently is 35.

4 Check *Does the answer make sense?*

The team scored 35 points four times. No other score occurred four or more times. So, the answer is reasonable.

Analyze the Strategy

Reason Inductively How would the results change if the team played a twelfth game and scored 21 points?

Sample answer: There would be two modes, 21 and 35.

Case #2 Life Span

Different animals have different average life spans. The average life spans of several animals are shown in the table.

How many more animals have an average life span between 11 and 15 years than those that have an average life span between 1 and 5 years?

Average Life Span (years)	
Camel	12
Deer	10
Chicken	12
Fox	9
Gorilla	20
Horse	20
Kangaroo	7
Lion	15
Lobster	15
Mouse	2
Salamander	10
Polar Bear	20
Rabbit	5

1 Understand

Read the problem. What are you being asked to find?

I need to find how many more animals have a life span of 11–15 years than those that have an average life span between 1 and 5 years.

What information do you know?

Animals with 11–15 year life span: camel, chicken, lion, lobster

Animals with 1–5 year life span: mouse and rabbit

2 Plan

Choose a problem-solving strategy.

I will use the use a graph strategy.

3 Solve

Use your problem-solving strategy to solve the problem.

Make a histogram. Use intervals of 1–5 years, 6–10 years, 11–15 years, and 16–20 years.

So, there are 2 more animals with an average life span between 11–15 years than with an average life span between 1–5 years.

4 Check

Use information from the problem to check your answer.

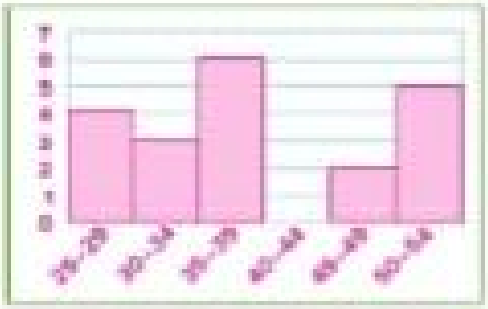
There are four animals with an average life span between 11 and 15 years and two animals, mice and rabbits, with an average life span between 1 and 5 years.

Mid-Chapter Check

Vocabulary Check

1. **Be Precise** Define *histogram*. Use the data set {26, 37, 35, 49, 54, 53, 30, 36, 31, 28, 29, 33, 38, 47, 54, 50, 37, 26, 35, 51} to make a histogram. (Lesson 2)

A type of bar graph used to display numerical data that have been organized into equal intervals.



Skills Check and Problem Solving

Make a line plot for each set of data. Then describe the data. (Lesson 1)

2. {36, 43, 39, 47, 34, 43, 47, 39, 34, 43}

Sample answer: There are 10 pieces of data; mean: 40.5; median: 41; mode: 43; The median means that one half of the data is greater than 41 and one half is less.

3. {63, 54, 57, 63, 52, 59, 52, 63, 61, 54}

There are 10 pieces of data; mean: 57.8; median: 58; mode: 63; The median means that one half of the data is greater than 58 and one half is less.

4. The histogram shows a movie theater's attendance each time a movie is shown. Describe the data in the histogram. (Lesson 2) Sample answer: More movies had an attendance between 1 and 20 people than any other range. Less movies had an attendance between 41 and 60 people than any other range.

5. **Persevere with Problems** In a box plot, the first quartile, median, and third quartile are x, y, and 70, respectively. Give possible values for x and y according to each of the following conditions. (Lesson 3)

a. The median separates the box into two quartiles, each with the same range. Sample answer: $x = 60$, $y = 65$

b. The box between the median and the third quartile is twice as long as the box between the median and the lower quartile. Sample answer: $x = 64$, $y = 66$

Lesson 4

Shape of Data Distributions

Vocabulary Start-Up

The **distribution** of a set of data shows the arrangemen t of data values. The words below show some of the ways the distribut on of data can be described. Match the ords below to their definitions.

cluster	The left side of the distribution looks like the right side.
gap	The numbers that have no data value.
peak	The most frequently occurring values, or mode.
symmetry	Data that are grouped closely together.

Essential Question

WHY is it important to carefully evaluate graphs?

Vocabulary

distribution
symmetric distribution
cluster
gap
peak

Mathematical Practices

1, 3, 4, 5, 7

Real-World Link

Parasailing The line plot shows the costs for parasailing for different companies on a certain beach. **Sample answers are given.**

1. Draw a vertical line through the middle of the data. What do you notice?
   The left side of the data looks like the right side of the data.
2. Use one of the words shown above to write a sentence about the data.
   There are no gaps in the data.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

- 1 Persevere with Problems
- 2 Reason Abstractly
- 3 Construct an Argument
- 4 Model with Mathematics
- 5 Use Math Tools
- 6 Attend to Precision
- 7 Make Use of Structure
- 8 Use Repeated Reasoning

Describe the Shape of a Distributio

Data that are evenly distributed between the left side and the right side have a **symmetric distribution**. The distribution shown has a **cluster** of several data values within the interval 10–12. The **gaps** 9 and 13 have no data values. The value 10 is a **peak** because it is the most frequently-occurring value.

Examples

Describe the shape of each distribution.

1. **The line plot shows the temperature in degrees Celsius in a city over several days.**

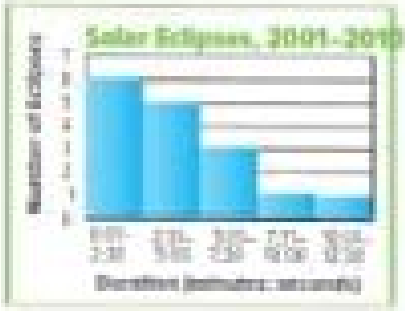
You can use clusters, gaps, peaks, outliers and symmetry to describe the shape. The shape of the distribution is not symmetric because the left side of the data does not look like the right side of the data. There is a gap from 19–21. There are clusters from 16–18 and 22–25. The distribution has a peak at 22. There are no outliers.

2. **The box plot shows the number of visitors to a gift shop in one month.**

You cannot identify gaps, peaks, and clusters.
Each box and whisker has the same length. So, the data is evenly distributed. The distribution is symmetric since the left side of the data looks like the right side. There are no outliers.

Got it? **Do this problem to find out.**

a. Use clusters, gaps, peaks, outliers, and symmetry to describe the shape of the distribution at the right.



The shape of the distribution is not symmetric. There is a cluster from 0:01–7:30 minutes. The distribution has a peak on the left side of the data at 0:01–2:30 minutes. There are no gaps and no outliers.

Measures of Center and Spread

Key Concept

Use the following flow chart to decide which measures of center and spread are most appropriate to describe a data distribution.

Is the data distribution symmetric?

Yes → Use the **mean** to describe the center. Use the **mean absolute deviation** to describe the spread.

No → Use the **median** to describe the center. Use the **interquartile range** to describe the spread.

If there is an outlier, the distribution is not usually symmetric.

Example

3. **The line plot shows the number of states visited by students in a class.**

a. Choose the appropriate measures to describe the center and spread of the distribution. Justify your response based on the shape of the distribution.

The data are not symmetric and there is an outlier, 19. The median and interquartile range are appropriate measures to use.

b. Write a few sentences describing the center and spread of the distribution using the appropriate measures.

The median is 12 states. The first quartile is 11. The third quartile is 13. The interquartile range is 13–11, or 2 states.

The data are centered around 12 states. The spread of the data around the center is about 2 states.

Got it? **Do this problem to find out.**

b. Choose the appropriate measures to describe the center and spread of the distribution. Justify your response based on the shape of the distribution. Then describe the center and spread.

Stop and Reflect

Explain below which measures are most appropriate to describe the center and spread of a symmetric distribution.

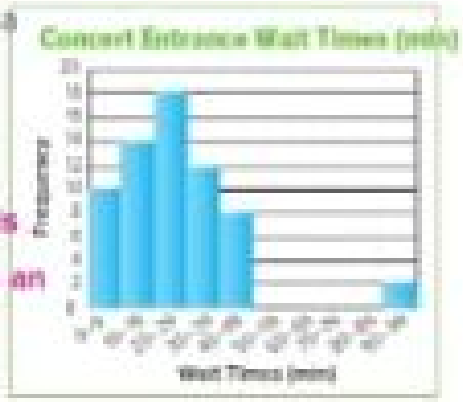
Sample answer: The mean and the absolute deviation are appropriate to use to describe the center and spread of the distribution.

mean and mean absolute deviation; Sample answer: The distribution is symmetric and there are no outliers. The data are centered around the mean of 29 years. The spread of the data around the center is about 1.7 years, which is the mean absolute deviation.

b.

Guided Practice

1. The histogram shows the wait times in minutes for entering a concert. Describe the shape of the distribution.



Sample answer: The shape of the distribution is not symmetric. There is a cluster from 0–49 minutes. The distribution has a gap from 50–89. The peak of the data is on the left side of the data in the interval 20–29. There is an outlier in the interval 90–99.

2. The box plot shows the weights of the gorillas in a troop. Describe the shape of the distribution.

Sample answer: The shape of the distribution is not symmetric since the lengths of each box and whisker are not the same. There is an outlier at 20.

3. The line plot shows the number of hours several students spent on the Internet during the week.

   a. Choose the appropriate measures to describe the center and spread of the distribution. Justify your response based on the shape of the distribution. **mean and mean absolute deviation; Sample answer: The distribution is symmetric and there are no outliers.**

   b. Write a few sentences describing the center and spread of the distribution using the appropriate measures. Round to the nearest tenth if necessary.

Sample answer: The data are centered around the mean of 4 hours. The spread of the data around the center is about 1.2 hours, which is the mean absolute deviation.

4. **Building on the Essential Question** Why does the choice of measure of center and spread vary based on the type of data display? **Sample answer: Different displays show different information. Median can be found in line plots, histograms, and box plots, but mean can only be found when looking at individual data. Not all displays show gaps, clusters, and peaks.**

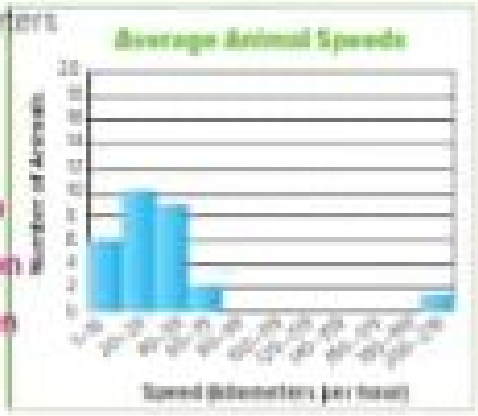
Rate Yourself!

How well do you understand how to describe the shape of a distribution? Circle the image that applies.

Name _____ My Homework _____

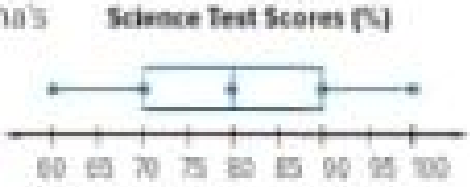
Independent Practice

1. The histogram shows the average animal speeds in kilometers per hour of several animals. Describe the shape of the distribution.



Sample answer: The shape of the distribution is not symmetric. There is a cluster from 1–79. The distribution has a gap from 80–199. The peak of the distribution is on the left side of the data in the interval 20–39. There is an outlier in the interval 200–219.

2. The box plot shows the science test scores for Mrs. Ayesha's students. Describe the shape of the distribution.



Sample answer: The shape of the distribution is symmetric. The left side of the data looks like the right side. There are no outliers.

3. The line plot shows the number of text messages sent by different students in one day.

a. Choose the appropriate measures to describe the center and spread of the distribution. Justify your response based on the shape of the distribution.

median and interquartile range; Sample answer: The distribution is not symmetric.

b. Write a few sentences describing the center and spread of the distribution using the appropriate measures.

Sample answer: The data are centered around 23.5 text messages. The spread of the data around the center is about 3 text messages.

4. **Identify Structure** Fill in the graphic organizer to show when to use each measure regarding the shape of the distribution.

Measure:	Symmetric or not symmetric
mean	symmetric
median	not symmetric
interquartile range	not symmetric
mean absolute deviation	symmetric

5. A distribution that is not symmetric is called *skewed*. A distribution that is *skewed left* shows data that is more spread out on the left side than on the right side. A distribution that is *skewed right* shows data that is more spread out on the right side than on the left side. The box plot shows the heights in meters of several trees.

a. Explain how you know the distribution is not symmetric.

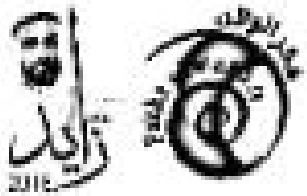
Sample answer: The lengths of the whiskers are not equal.

b. Is the distribution skewed left or skewed right? Explain.

skewed left; Sample answer: The data are more spread out on the left side due to the long left whisker.

c. Use appropriate measures to describe the center and spread of the distribution. Justify your choice of measure based on the shape of the distribution.

Sample answer: Use the median and interquartile range to describe the center and spread since the distribution is not symmetric. The data are centered around 40 meters. The spread of the data around the center is 10 meters.



H.O.T. Problems Higher Order Thinking

6. **Model with Mathematics** Draw a line plot for which the median is the most appropriate measure to describe the center of the distribution.

See students' work; Distributions should not be symmetric.

7. **Persevere with Problems** Explain why you cannot describe the specific location of the center and spread of the box plot shown using the most appropriate measures.

Sample answer: The distribution is symmetric. The appropriate measures to describe the center and spread are the mean and mean absolute deviation. A box plot shows the location of the median and interquartile range but it does not show the location of the mean or the mean absolute deviation.

8. **Justify Conclusions** Ida created the dot plot shown to represent the ages of the staff of the community pool. She concludes that since there is a peak at 19, the median is 19. She also concludes the two data values that are 25 to be outliers, so there are no gaps. Evaluate her conclusions.

Sample answer: The median of the data set is 19.5. Peaks in a data set do not necessarily correspond to the median. The data values of 25 are outliers, but there is a gap between 21 and 25.

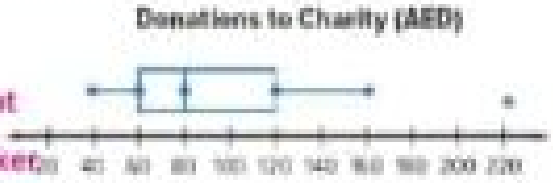
Name _____ My Homework _____

Extra Practice

9. The line plot shows the prices for several DVDs. Describe the shape of the distribution. Sample answer: The shape of the distribution is symmetric. The left side of the data looks like the right side. There is a cluster from AED 13–AED 15. There are gaps in the data. The peak of the distribution is AED 14. There are no outliers.

10. The box plot shows donations to charity. Describe the shape of the distribution.

Sample answer: The shape of the distribution is not symmetric since the lengths of each box and whisker are not the same. There is an outlier at 220.



11. The line plot shows the number of kilometers Rana ran each week.

a. Choose the appropriate measures to describe the center and spread of the distribution. Justify your response based on the shape of the distribution. mean and mean absolute deviation; Sample answer: The distribution is symmetric and there are no outliers.

b. Write a few sentences describing the center and spread of the distribution using the appropriate measures. Round to the nearest tenth if necessary. Sample answer: The data are centered around 31 kilometers. The spread of the data around the center is about 1.3 kilometers.

12. **Use Math Tools** The line plot shows the number of siblings for 18 students.

a. Explain how you know the distribution is not symmetric. Sample answer: The left side of the distribution does not look like the right side.

b. Is the distribution skewed left or skewed right? Explain. skewed right; Sample answer: The right side shows data that is more spread out than the left side.

c. Use appropriate measures to describe the center and spread of the distribution. Justify your choice of measure based on the shape of the distribution. Sample answer: Use the median and interquartile range to describe the center and spread since the distribution is not symmetric. The data are centered around 1.5 siblings. The spread of the data around the center is 2 siblings.

Power Up! Test Practice

13. Which of the following statements are true about the box plot? Select all that apply.

- [x] The distribution has an outlier.
- [x] The distribution has a gap of data.
- [ ] The distribution is symmetric.

14. The line plot shows the speeds of several cyclists.

interquartile range
mean
mean absolute deviation
median
not symmetric
symmetric

Select the correct term to complete each statement.

a. The distribution is **not symmetric**.

b. The **median** should be used to describe the center of the data distribution.

c. The **interquartile range** should be used to describe the spread of the data.

Spiral Review

Graph the points on the coordinate plane.

15. $P(2, 4)$

16. $K(4, 9)$

17. $G(1, 8)$

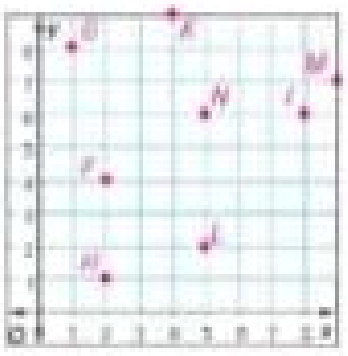
18. $L(5, 2)$

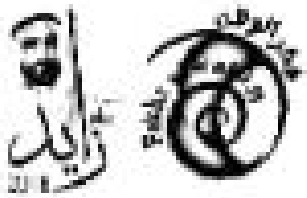
19. $H(2, 1)$

20. $M(9, 7)$

21. $I(8, 6)$

22. $N(5, 6)$





23. Reham is working on a small scrapbook. She completes 3 scrapbook pages each hour. How many pages will she complete in 12 hours?

36 pages

Inquiry Lab
Collect Data

Inquiry **HOW do you answer a statistical question?**

Mathematical Practices 1, 3, 4

Ghaliya surveyed students in the cafeteria lunch line. She asked the statistical question, *How many photos are currently stored in your cell phone?* She wants to organize the data and choose an appropriate way to display the results of her survey.

Hands-On Activity

You can collect, organize, display, and interpret data in order to answer a statistical question.

Step 1 Make a data collection plan. Ghaliya chose to survey students in the cafeteria.

Step 2 Collect the data. The results of the survey are provided below.

55, 47, 58, 50, 66, 47, 54, 64, 47, 65,
43, 44, 51, 81, 54, 45, 57, 52, 58, 60

Step 3 Organize the data. Place the values in order from least to greatest.
43, 44, 45, 47, 47, 47, 50, 51, 52, 54, 54, 55, 57, 58, 58, 60, 64, 65, 66, 81

Step 4 Describe the data. There were a total of **20** responses. The responses measure the number **photos**. The data was collected using **survey**. One attribute of the data is the median, which is **54** photos. Another attribute is the interquartile range, which is **12** photos. There is an outlier at **81** photos.

Step 5 Create a display of the data. Explain why a box plot would be an appropriate display of Ghaliya's data. **Sample answer: The range of the data is reasonable for a box plot. This graph will indicate the median, quartiles, and extreme values.**

Investigate

Work with a partner. Collect data in order to answer a statistical question. Sample answers: 1–6

1. Write a statistical question.

   How many movies have you seen in the last three months?

2. Collect the data and record the results in a table.

How many movies have you seen in the last three months?	
Number of Movies	Number of Responses
0–2	4
3–5	6
6–8	9
9–11	1

3. Create a display of the data.

Analyze and Reflect

4. **Model with Mathematics** Write a few sentences describing your results. Include the number of responses you recorded, how the responses were measured and/or gathered, and their overall pattern.

   There were 20 responses. They were gathered using a survey and the responses were measured in the number of movies seen. Overall, more people saw 6–8 movies over any other interval.

5. **Reason Inductively** Write a few sentences describing the center and spread of the distribution.

   There are no gaps in the data. The data has a peak in the 6–8 range. The median is 3–5 movies and the mode is 6–8 movies. The data is not symmetrical.

Create

6. **Apply** HOW do you answer a statistical question?

   Create a data collection plan. Next, collect data from as many people as possible. Finally, create and share an organized display of the data.

Lesson 5

Interpret Line Graphs

Real-World Link

Golf The table shows the prize money for winners of a golf Tournament.

Money Won by Golf Tournament Winners	
Year	Amount (AED)
2005	1,170,000
2006	1,225,000
2007	1,305,000
2008	1,305,000
2009	1,350,000
2010	1,350,000

AED 55,000
AED 80,000
AED 0
AED 45,000
AED 0

1. Fill in the difference in prize money between each consecutive year on the lines above.

2. If the data were plotted, would the points (year, amount) form a straight line? Explain.

   No; Some years there isn't an increase and when there is an increase, it is not constant.

3. The Masters Tournament is held once a year. If a line graph is made of these data, will there be any realistic data between tournament dates? Explain.

   No; Sample answer: no money is awarded in between tournaments.

Essential Question

WHY is it important to carefully evaluate graphs?

Vocabulary

line graph

Mathematical Practices

1, 3, 4

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

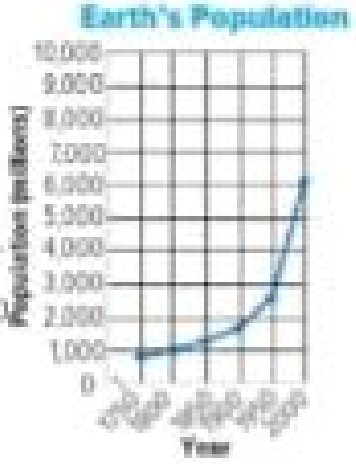
Make a Line Graph

A **line graph** is used to show how a set of data changes over a period of time. To make a line graph, decide on a scale and interval. Then graph pairs of data and draw a line to connect each point.

Example

1. **Make a line graph of the data of Earth's Population. Describe the change in Earth's population from 1750 to 2000.**

Earth's Population						
Year	1750	1800	1850	1930	1990	2000
Population (millions)	790	980	1,260	1,650	2,555	6,080



Step 1 The data include numbers from 790 million to 6,080 million. So, a scale from 0 to 10,000 million and an interval of 1,000 million are reasonable.

Step 2 Let the horizontal axis represent the year. Let the vertical axis represent the population. Label the horizontal and vertical axes.

Step 3 Plot and connect the points for each year.

Step 4 Label the graph with a title.

Earth's population has increased drastically from 1750 to 2000.

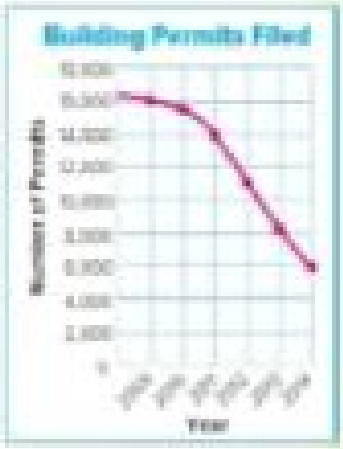
Got it? **Do this problem to find out.**

a. Make a line graph of the data. Describe the change in the number of building permits filed from 2005 to 2010.

Number of Building Permits Filed in a Major City						
Year	2009	2010	2011	2012	2013	2014
Building Permits Filed	16,000	15,500	13,900	11,000	8,200	5,900

Line Graphs

The bars in a bar graph are used to show the differences between data values and may list data over (a) values between data points.



a. Sample answer: The number of building permits filed is decreasing. A sharper decrease in the number of permits occurred in 2012 than in any other year listed.

Interpret Line Graphs

By observing the upward or downward slant of the lines connecting the points, you can describe trends in the data and predict future events.

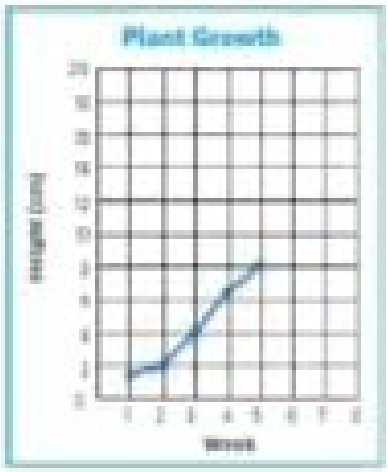
Example

2. **The line graph below shows the cost of tuition at a college during several years. Describe the trend. Then predict how much tuition will cost in 2020.**

Notice that the increase from 2002 through 2012 is fairly steady. By extending the graph, you can predict that tuition in 2020 will cost a student about AED 11,500.

Got it? **Do this problem to find out.**

b. The line graph shows the growth of a plant over several weeks. Describe the trend. Then predict how tall the plant will be at 7 weeks.



b. Sample answer: The plant grows slowly at first then increases steadily; 11 cm

Example

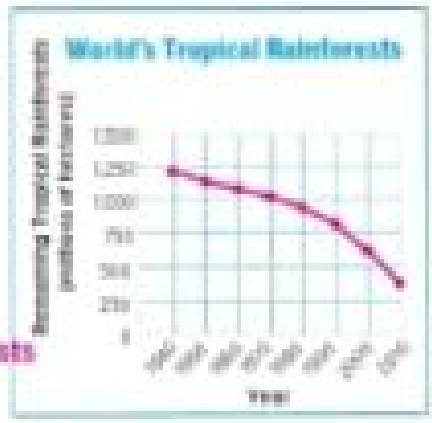
3. What does the graph tell you about the popularity of skateboarding?

The graph shows that skateboard sales have been increasing each year. You can assume that the popularity of the sport is increasing.

Guided Practice

1. Make a line graph of the data.

World's Tropical Rainforests								
Year	1940	1950	1960	1970	1980	1990	2000	2010
Remaining Tropical Rainforests (millions of hectares)	1363	1109	1052	961	896	728	587	338



2. Describe the change in the world's remaining rainforests from 1940 to 2010.
Sample answer: The size of the rainforests has rapidly decreased from 1940 to 2010.

3. Describe the trend in the remaining tropical rainforests.
Sample answer: The size of the rainforests is decreasing.

4. Predict how many millions of hectares there will be left in 2020.
about 100 million hectares

5. What does the graph tell you about future changes in the remaining rainforests?
Sample answer: The rainforests are decreasing in size. You can expect this pattern to continue.

6. **Building on the Essential Question** How can you use line graphs to predict data?
Sample answer: By analyzing the trend and extending the line, you can predict future data.

Rate Yourself!

☐ I understand how to interpret line graphs.

Great! You're ready to move on!

☐ I still have some questions about interpreting line graphs.

Name _____ My Homework _____

Independent Practice

1. Make a line graph of the data. Then describe the change in the total amount Salama saved from Week 1 to Week 5.

Sample answer: Sally's total savings increased slowly for Weeks 1 and 2, then increased more dramatically for Weeks 3 and 4 with a slower increase for Week 5.

Sally's Savings	
Week	Total Amount (AED)
1	50
2	54
3	75
4	98
5	100

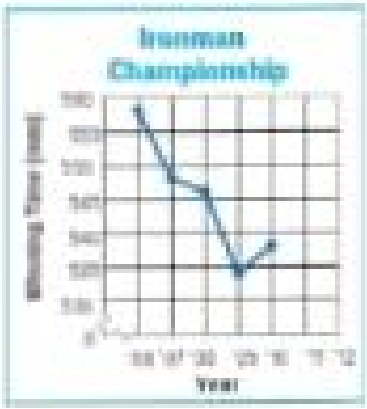
2. Use the graph at the right.

a. Describe the change in the winning times from 2006 to 2010.

Sample answer: Most years, the winning times decreased. From 2009 to 2010, the winning time increased.

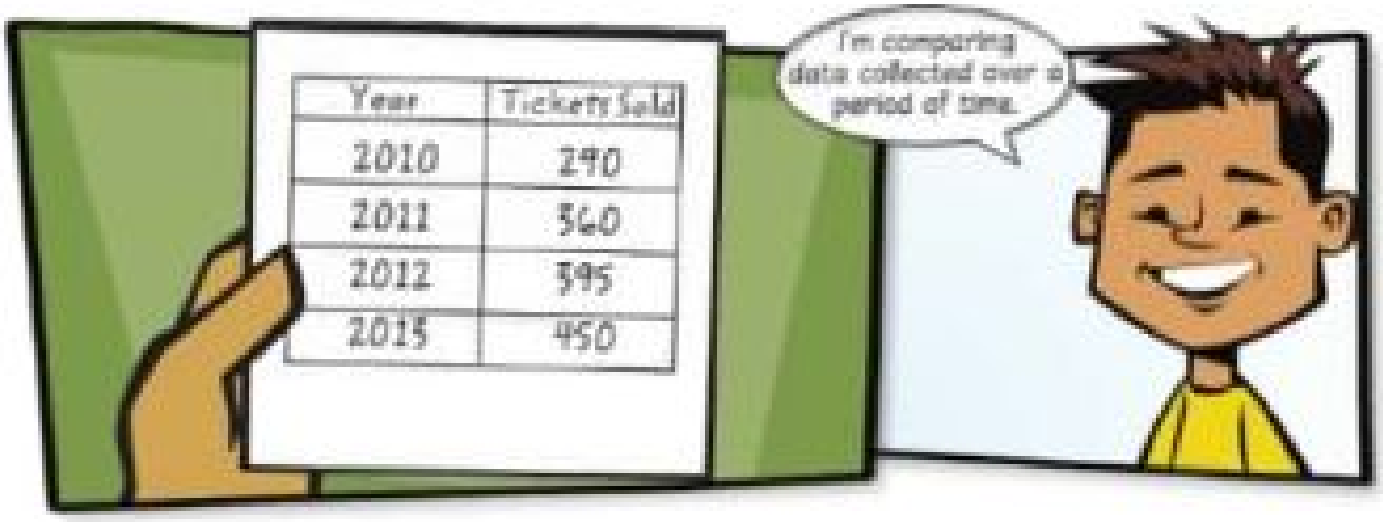
b. Predict the winning time in 2015. about 525 min

c. Predict when the winning time will be less than 500 minutes. about 2019



Copy and Solve **For Exercise 3, show your work on a separate piece of paper.**

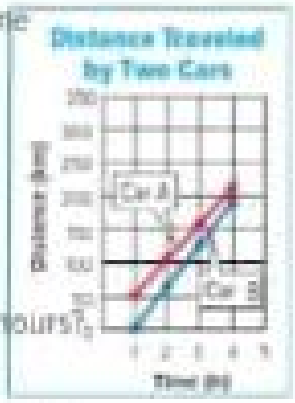
3. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b. **See Answer Appendix.**



a. Use the information in the table and draw a line graph to show the changes in ticket sales over the past four years.

b. Predict what the ticket sales will be in 2015.

4. Use the graph that shows the distance traveled by two cars on the same freeway headed in the same direction.



a. Predict the distance traveled by Car A after 5 hours.
about 260 km

b. Predict the distance traveled by Car B after 5 hours.
about 250 km

c. How many kilometers do you think Car A will have traveled after 8 hours?
about 410 km

d. Based on the graph, after how many hours will Car B have traveled about 360 kilometers?
about 7 h

e. Based on the graph, which car will reach a distance of 500 kilometers first? Explain your reasoning.
Car B; Car B will reach a distance of 500 kilometers shortly after 9 hours. It will take Car A shortly under 10 hours to travel 500 kilometers

H.O.T. Problems Higher Order Thinking

5. **Justify Conclusions** Can changes to the vertical scale or interval affect the appearance of a line graph? Justify your reasoning with examples.
Sample answer: If the vertical scale is much higher than the highest value, it makes the graph flatter. Changing the interval does not affect the graph.

6. **Persevere with Problems** Refer to the graph for Exercise 4. What can you conclude about the point at which the red and blue lines cross?
Sample answer: The point at which the lines cross represents the time when both cars will have traveled the same distance.

7. **Construct an Argument** Explain why line graphs are often used to make predictions.
Sample answer: Line graphs are often used to make a prediction because they show changes over time and they allow the viewer to see data trends and, thus, make predictions.

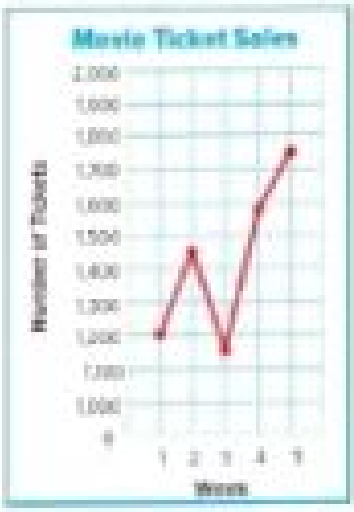
8. **Model with Mathematics** Give an example of a set of data that is best represented in a line graph. Then make a line graph that could represent that data.
See students' work.

Name _____ My Homework _____

Extra Practice

9. **Model with Mathematics** Make a line graph of the data. Describe the change in the online sales of movie tickets for Weeks 1 to 5.

Online Sales of Movie Tickets	
Week	Number of Tickets
1	1,200
2	1,450
3	1,150
4	1,575
5	1,750



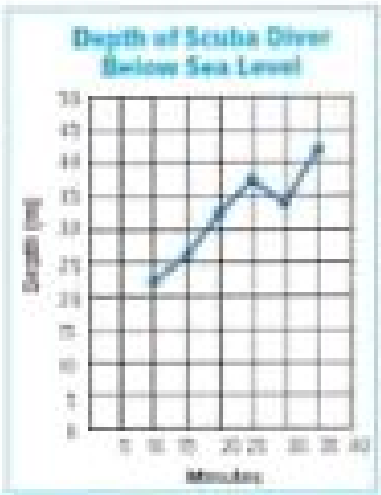
The online sales of movie tickets increased from week 1 to week 2, decreased in week 3 and then increased again for weeks 4 and 5.

10. Use the graph at the right.

a. Describe the change in depth from 10 minutes to 35 minutes.
Sample answer: For most of the time, the depth increased. Between 25 and 30 minutes, the depth decreased.

b. Predict the depth at 45 minutes. about 50 m

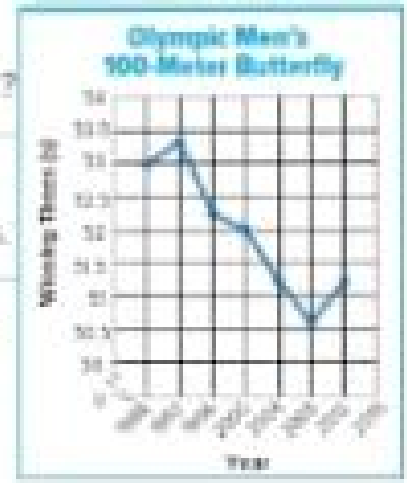
c. Predict when the depth will be more than 65 meters.
about 65 min



11. Use the line graph at the right.

a. Between which years did the winning time change the most? Explain your reasoning. 1992 and 1996; The winning time decreased by about 1 second.

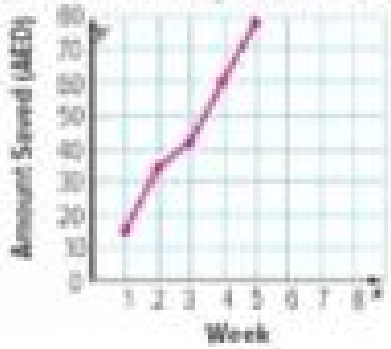
b. Make a prediction of the winning time in the 2020 Olympics. Explain your reasoning. Sample answer: 48.50 seconds; Based on the trend from 1992 to 2008, the winning time decreased.



Power Up! Test Practice

12. The table shows the amount of money Shaima has saved after 5 weeks. Construct a line graph of the data.

Week	Amount Saved (AED)
1	15
2	34
3	42
4	60
5	78



Predict how much Shaima will have saved after Week 8.

Sample answer: AED 125

13. The graph shows the amount of time Wisa spent studying last week. Determine if each statement is true or false.

a. The amount of time increased the most from Thursday to Friday. ■ True ☐ False

b. Khawla spent the same amount of time studying on Monday and Wednesday. ☐ True ■ False

c. The amount of time decreased from Monday to Tuesday. ☐ True ■ False

Spiral Review

Find the greatest number in the set.

14. {23, 34, 41, 25, 36}
41

15. {65, 58, 64, 56, 62}
65

16. {18, 16, 22, 19, 24}
24

Find the total of the set of numbers.

17. {95, 88, 97, 89, 91}
460

18. {56, 71, 68, 62, 74}
331

19. {33, 36, 38, 29, 27}
163

20. The table shows the kilometers the Khalifa family traveled each day. What is the total number of kilometers they traveled?
321 kilometers

Day	Kilometers
Saturday	125
Sunday	84
Monday	112

21. Khadija can make 24 cookies in 30 minutes. At this rate, how many cookies can she make in 90 minutes?
72 cookies

Lesson 6

Select an Appropriate Display

Real-World Link

Animals The displays show the measured speed of six animals.

Animal Speeds	
Speeds	Number of Animals
1–5	0
6–10	0
11–15	2
16–20	0
21–25	0
26–30	1
31–35	2

Essential Question

WHY is it important to carefully evaluate graphs?

Mathematical Practices

1, 3, 4, 5, 6

1. Use the bar graph to fill in the "Number of Animals" column in the table.

2. Which display allows you to find a rabbit's speed?

   the bar graph

3. In which display is it easier to find the number of animals with a measured speed of 15 kilometers per hour or less? Explain.

   the table; In order to find the number of animals with a measured speed of 15 or less, you can add three entries on the table.

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Key Concept

Statistical Displays

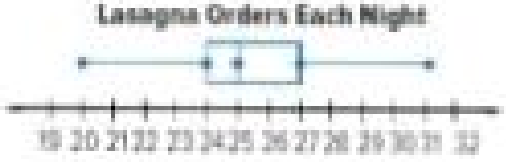
Type of Display	Best used to
Bar Graph	show the number of items in specific categories
Box Plot	show measures of variation for a set of data, also useful for very large sets of data
Histogram	show frequency of data divided into equal intervals
Line Graph	show change over a period of time
Line Plot	show how many times each number occurs

Work Zone

Data can often be displayed in several different ways. The display you choose depends on your data and what you want to show.

Example

1. Which display allows you to tell the mode of the data?



The line plot shows each night's data. The number of orders that occurs most frequently is 27. The box plot shows the spread of the data, but does not show individual data so it does not show the mode.

Got it? Do this problem to find out.

a. Which of the above displays allows you to easily find the median of the data?

box plot

Examples

2. **A survey compared different brands of hair shampoo. The table shows the number of first-choice responses for each brand. Select an appropriate type of display to compare the number of responses. Justify your choice.**

Favorite Shampoo Survey			
Brand	Responses	Brand	Responses
A	35	D	24
B	12	E	8
C	42	F	11

These data show the number of responses for each brand. A bar graph would be the best display to compare the responses.

3. **Make the appropriate display of the data.**

Step 1 Draw and label horizontal and vertical axes. Add a title.

Step 2 Draw a bar to represent the number of responses for each brand.

Got it? **Do these problems to find out.**

The table shows the quiz scores of Mr. Tarek's math class.

Math Quiz Scores											
70	70	75	80	100	85	85	65	75	85	95	90
90	100	85	90	90	95	80	85	90	85	90	75

b. Select an appropriate type of display to allow you to find the number of students with a score of 85. Explain your choice.

c. Make the appropriate display of the data.

STOP and Reflect

What type of data are best represented in a bar graph? Explain below.

Sample answer: data that have numbers in specific categories

b. line plot; It will show the number of times the value of 85 occurs.

Guided Practice

1. Which display makes it easier to determine the greatest number of calendars sold? Justify your reasoning.

line plot; The line plot shows the greatest value as a numerical value rather than a bar.

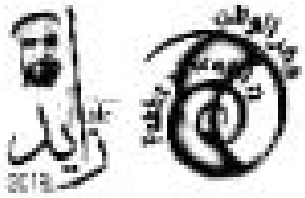
Select an appropriate type of display for data gathered about each situation. Justify your reasoning.

2. the favorite cafeteria lunch item of the sixth-grade students bar graph; It is easier to look for the tallest bar than for the greatest value.

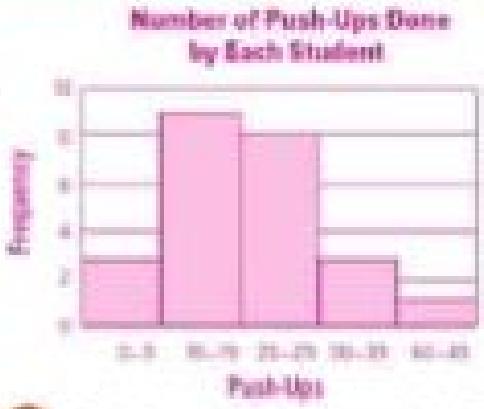
3. the temperature from 6 A.M. to 12:00 P.M. line graph; A line graph shows the trend over time.

4. Select and make an appropriate display for the following data.

Number of Push-Ups Done by Each Student											
15	20	8	11	6	25	22	12	14	16	21	25
18	35	40	20	25	15	10	5	18	20	31	28



Sample answer: histogram



5. **Building on the Essential Question** Why is it important to choose the appropriate display for a set of data?
Sample answer: The type of data influences which displays are appropriate, as does what you want the display to show.

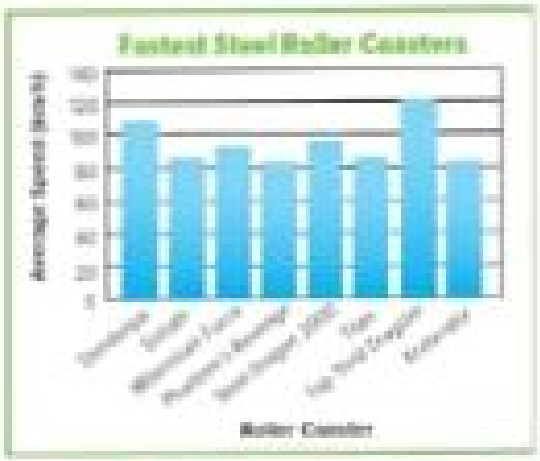
Rate Yourself!

How confident were you about selecting an appropriate display? Shade in the ring on the target.

Name _____ My Homework _____

Independent Practice

1. Which display makes it easier to compare the average speeds of Top Thrill Dragster and Millennium Force? Justify your reasoning.



bar graph; The bar graph shows the maximum speeds, not just the interval in which the data occurs.

Select an appropriate type of display for data gathered about each situation. Justify your reasoning.

2. the test scores each student had on a language arts test

line plot; The display can show individual scores.

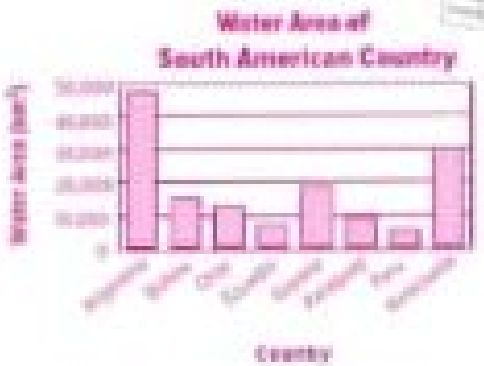
3. the median age of people who voted in an election

box plot; A box plot easily displays the median.

Use Math Tools Select and make an appropriate type of display for the situation.

4.

South American Country	Water Area (km^2)	South American Country	Water Area (km^2)
Argentina	47,710	Guyana	18,120
Bolivia	15,280	Paraguay	9,490
Chile	12,200	Peru	5,220
Ecuador	6,720	Venezuela	30,000



Sample answer: bar graph

5. **Use Math Tools** Use the Internet or another source to find a set of data that is displayed in a bar graph, line graph, frequency table, or circle graph. Was the most appropriate type of display used? What other ways might these same data be displayed?

See students' work.

6. **Be Precise** Fill in the graphic organizer below.

Display	Best for showing
line plot	individual data
histogram	a large amount of data displayed in intervals
box plot	the median and spread of a data set
bar graph	amount within a category

7. Display the data in the bar graph using another type of display. Compare the advantages of each display.

Sample answer: The line plot allows you to easily see how many countries have a given number of neighbors. The bar graph, however, allows you to see the number of neighbors for each given country.

H.O.T. Problems Higher Order Thinking

8. **Construct an Argument** Determine whether the following statement is *true* or *false*. If true, explain your reasoning. If false, give a counterexample.

Any set of data can be displayed using a line graph.

false; Sample answer: To compare the price of five different cell phones, a line graph would not be appropriate because these data do not show change over a period of time.

9. **Persevere with Problems** Which type of display allows you to easily find the mode of the data? Explain your reasoning.

Sample answer: line plot; You can easily locate the values with the most Xs to find the mode.

10. **Reason Inductively** The table shows the number of each type of plant at a botanical garden. The director of the garden would like to add cacti so that the relative frequency of the plant is 50%. How many cactus plants should the director add?

Type of Plant	Frequency
Rose	13
Cactus	18
Palm	4
Ferns	15

14 cactus plants

Extra Practice

11. Which display makes it easier to see the median distance? Justify your reasoning.

box plot; The median is easily seen on the box plot as the line in the box.

Select an appropriate type of display for data gathered about each situation. Justify your reasoning.

12. the amount of sales a company has over 6 months

line graph; A line graph shows the trend over time.

13. the prices of five different brands of tennis shoes at an athletic store

bar graph; A bar graph allows for the prices to be compared.

14. the amount in a savings account over a year

line graph; A line graph shows the trend over time.

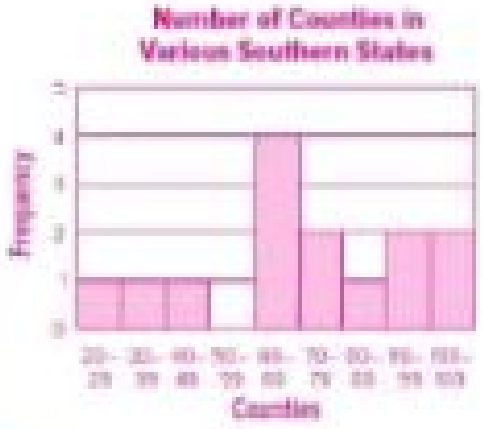
15. the shape of the distribution of a team's soccer goals for one season

Sample answer: box plot; A box plot easily shows the spread of data.

Model with Mathematics Select and make an appropriate type of display for the situation.

16.

Number of Counties in Various Southern States	
67	67
95	82
33	64
63	29
46	100
75	77
95	105



Sample answer: histogram

Power Up! Test Practice

17. The table shows the heights of 15 different sheep. Complete each statement with the most appropriate type of data display.

Height of Sheep (cm)				
24	26	22	22	21
24	25	24	23	23
18	26	25	22	24

a. A histogram would be most appropriate to show the data divided into equal intervals.

b. A line plot would be most appropriate to show how many times each height occurs.

c. A box plot would be most appropriate to show the distribution and spread of the data.

18. Match each situation to the type of display that would represent it.

bar graph
histogram
line graph
line plot

the favorite subject of the students in Mrs. Muna's home room — bar graph

the weight a lamb gains in one year — line graph

the number of hits Obaid got in each game this baseball season — line plot

the number of each type of sandwich a deli sells during lunch — bar graph

Spiral Review

Divide.

19. $36 \div 12 =$ 3

20. $108 \div 12 =$ 9

21. $138 \div 23 =$ 6

22. $204 \div 17 =$ 12

23. $192 \div 12 =$ 16

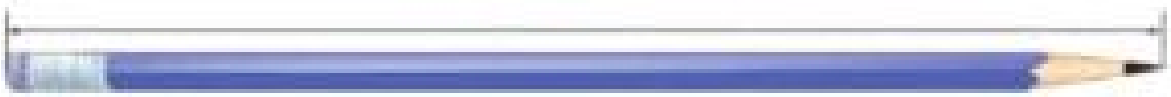
24. $390 \div 15 =$ 26

25. $324 \div 36 =$ 9

26. $540 \div 36 =$ 15

27. $792 \div 12 =$ 66

28. Measure the pencil below to the nearest centimeter. Then represent your measurement in meters. 15 cm; 0.15 m



Inquiry Lab
Use Appropriate Units and Tools

inquiry **HOW do you determine a measureable attribute?**

Mathematical Practices 1, 3, 4

Each item in a backpack has different attributes, such as color, size, and weight. Some of the attributes of the objects can be measured.

Hands-On Activity

You can choose the appropriate unit and tool to measure an object.

Step 1 Select an object in your classroom such as a desk, book, backpack, or trash can.

Step 2 List all of the measureable attributes of your object in the Step 3 table, for example length, weight or mass, time, or capacity.

Step 3 Select an appropriate tool and measure each attribute. Record each measure using appropriate units in the table below.

Sample answers given.

Object	Attribute	Tool	Measurement
textbook	weight	electronic scale	9 kg
textbook	length	ruler	29.5 cm
textbook	thickness	ruler	3.7 cm

Step 4 Choose a different object with at least one attribute that requires the use of a different tool to measure. Then repeat steps 1–3.

Object	Attribute	Tool	Measurement
pencil sharpener	length	ruler	15 cm
pencil sharpener	width	ruler	6.3 cm
pencil sharpener	time it takes to sharpen a new pencil	stopwatch	8 seconds

Step 5 Write and solve a real-world problem in which one of your measurements is needed to solve the problem.

Sample answer: Estimate how long it would take each student in the class to sharpen a new pencil before a test. 200 – 300 seconds or 5 minutes

Investigate

Sample answers: 1–5

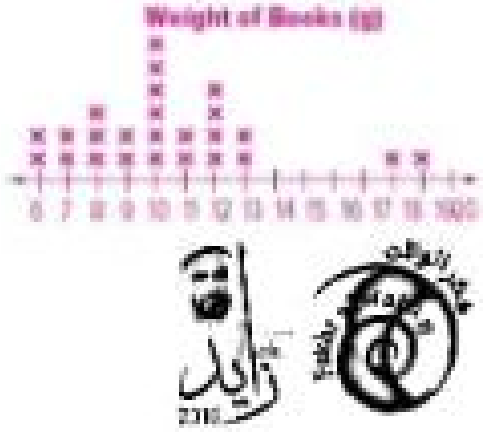
Work with a partner. Choose an attribute common to several similar objects and use the appropriate unit and tool to measure.

1. Choose a set of objects and a measurable attribute.

   the weight of each classmates' pen

2. Measure the attribute and record the results in a table. Then create a display of the data.

Weight of Different Books (g)				
6	13	8	10	11
18	7	10	12	13
9	10	8	8	12
19	12	7	10	6
10	9	11	12	10



Analyze and Reflect

3. **Model with Mathematics** Write a few sentences describing your data. Include the number of observations, how the data was measured, and the overall pattern of the data.

   25 books were weighed to the nearest gram on an electronic scale. The data is clustered from 6–13 g. There is a gap from 14–17 g. There is a peak at 10 g. The median is 10 gram. Most of the data is clustered near the median. Two data points, 18 and 19 g, deviate from the overall pattern.

4. **Make a Conjecture** Explain how the way you measured the objects influenced the shape of the display.

   If the data were measured to the nearest tenth of an gram, the data might be more spread out and there might be a different median and mode.

Create

5. **How?** How do you determine a measurable attribute?

   First, examine the attributes of an object and select an attribute that can be measured. Use an appropriate tool to measure and select the scale you will use.

21ST CENTURY CAREER in Environmental Science

Environmental Engineer

Are you concerned about protecting the environment? If so, you should think about a career in environmental science. Environmental engineers apply engineering principles along with biology and chemistry to develop solutions for improving the air, water, and land. They are involved in pollution control, recycling, and waste disposal. Environmental engineers also determine methods for conserving resources and for reducing environmental damage caused by construction and industry.

Is This the Career for You?

Are you interested in a career as an environmental engineer? Take some of the following courses in high school.

- Algebra
- Biology
- Environmental Science
- Environmental History

Turn the page to find out how math relates to a career in Environmental Science.

Thinking Green!

Use the information in the table to solve each problem. Round to the nearest tenth if necessary.

1. Find the mean, median, and mode of the percent of recycled glass dat**mean: 19.9; median: 17; mode: 17**

2. If Lee County is removed from the recycled aluminum cans data, which changes the most: the mean, median, or mode? Does this make sense? Explain your reasoning.

   The mean changes the most, since it goes from 18.6 to about 13.7. The median changes only slightly, decreasing from 14 to 13. This makes sense because 48 is an extreme value, and so it affects the mean.

3. Find the range, quartiles, and interquartile range of the percent of recycled newspapers data. **range: 59; Q_2 66; Q_1 28; IQR: 38**

4. Find any outliers in the percent of recycled plastic bottles data.**53**

5. Make a box plot of the percent of recycled glass data.

6. Refer to the box plot you made in Exercise 5. Compare the parts of the box and the lengths of the whiskers. What does this tell you about the data? **The data between the first quartile and the median are less spread out than the data between the median and the third quartile. The whiskers are the same length, so the spread of data above the third quartile and below the first quartile is the same.**

Percent of Materials That Are Recycled				
County	Aluminum Cans	Glass	Newspapers	Plastic Bottles
Broward	15	13	41	7
Dade	4	17	28	15
Duval	31	17	81	7
Hillsborough	14	21	38	23
Lee	48	16	66	53
Orange	12	28	33	16
Polk	6	26	22	8

Career Project

It's time to update your career portfolio! Describe an environmental issue that concerns you. Explain how you, as an environmental engineer, would work to resolve this issue. Then research how the issue is being addressed by environmental scientists today.