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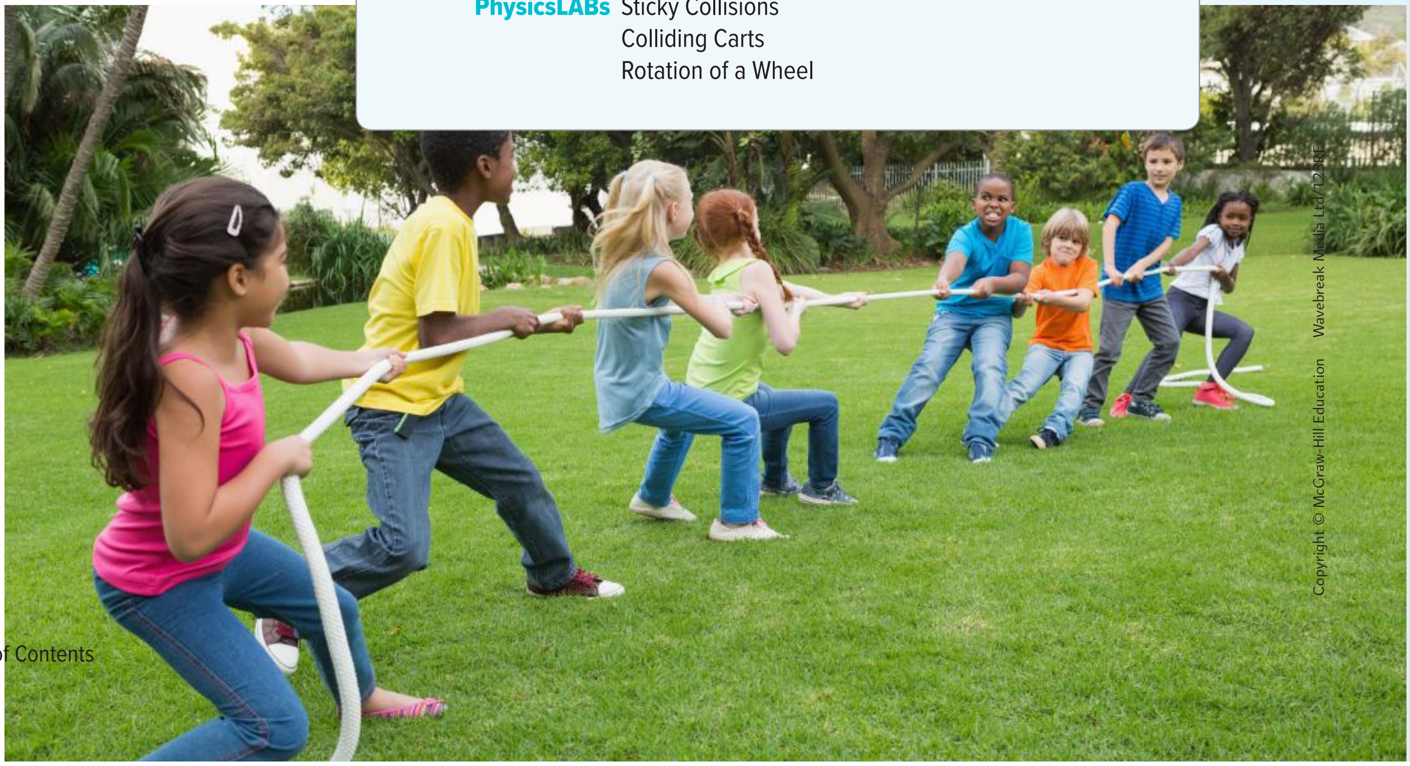

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CHAPTER 1

Motion in Two Dimensions

BIG IDEA You can use vectors and Newton's laws to describe projectile motion and circular motion.

SECTIONS

1 Projectile Motion

2 Circular Motion

3 Relative Velocity

LaunchLAB

PROJECTILE MOTION

What does the path of a projectile, such as a ball that is thrown, look like?

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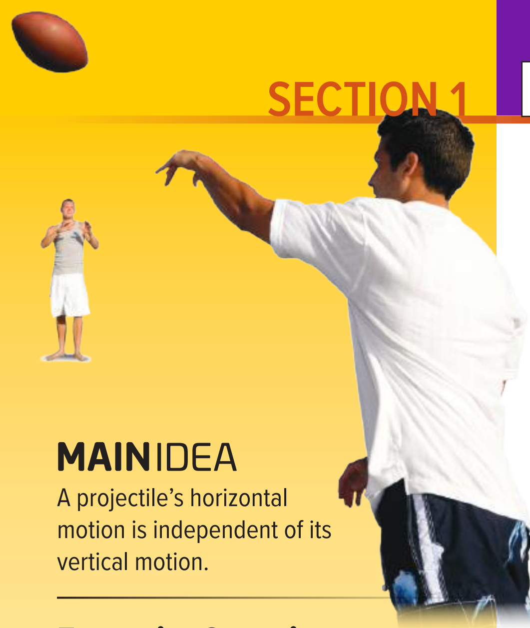


SECTION 1

Projectile Motion

PHYSICS 4 YOU **When you throw a softball or a football, it travels in an arc. These tossed balls are projectiles. The word *projectile* comes from the Latin prefix *pro–*, meaning "forward", and the Latin root *ject* meaning "to throw."**

MAIN IDEA

A projectile's horizontal motion is independent of its vertical motion.

Essential Questions

- How are the vertical and horizontal motions of a projectile related?
- What are the relationships between a projectile's height, time in the air, initial velocity, and horizontal distance traveled?

Review Vocabulary

motion diagram a series of images showing the positions of a moving object taken at regular time intervals

New Vocabulary

projectile
trajectory

Path of a Projectile

A hopping frog, a tossed snowball, and an arrow shot from a bow all move along similar paths. Each path rises and then falls, always curving downward along a parabolic path. An object shot through the air is called a **projectile.** You can draw a free-body diagram of a launched projectile and identify the forces acting on it. If you ignore air resistance, after an initial force launches a projectile, the only force on it as it moves through the air is gravity. Gravity causes the object to curve downward. Its path through space is called its **trajectory.** You can determine a projectile's trajectory if you know its initial velocity. In this chapter, you will study two types of projectile motion. The top of **Figure 1** shows water that is launched as a projectile horizontally. The bottom of the figure shows water launched as a projectile at an angle. In both cases, gravity curves the path downward along a parabolic path.

Figure 1 A projectile launched horizontally immediately curves downward, but if it is launched upward at an angle, it rises and then falls, always curving downward.

Independence of Motion in Two Dimensions

Think about two softball players warming up for a game, tossing high fly balls back and forth. What does the path of the ball through the air look like? Because the ball is a projectile, it has a parabolic path. Imagine you are standing directly behind one of the players and you are watching the softball as it is being tossed. What would the motion of the ball look like? You would see it go up and back down, just like any object that is tossed straight up in the air. If you were watching the softball from a hot-air balloon high above the field, what motion would you see then? You would see the ball move from one player to the other at a constant speed, just like any object that is given an initial horizontal velocity, such as a hockey puck sliding across ice. The motion of projectiles is a combination of these two motions.

Why do projectiles behave in this way? After a softball leaves a player's hand, what forces are exerted on the ball? If you ignore air resistance, there are no contact forces on the ball. There is only the field force of gravity in the downward direction. How does this affect the ball's motion? Gravity causes the ball to have a downward acceleration.

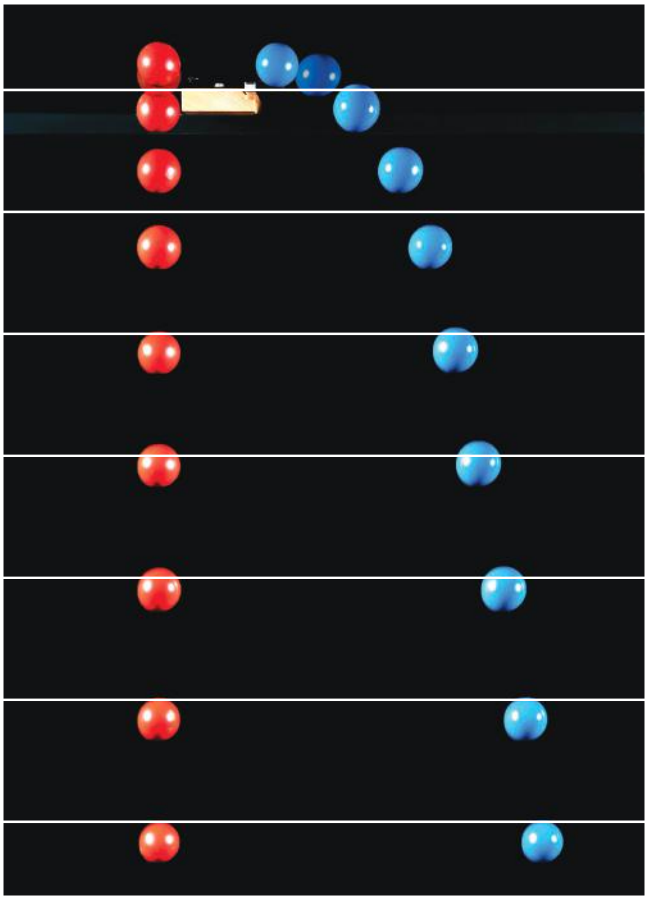


Figure 2 The ball on the left was dropped with no initial velocity. The ball on the right was given an initial horizontal velocity. The balls have the same vertical motion as they fall.

Identify What is the vertical velocity of the balls after falling for 1 s?

Comparing motion diagrams The trajectories of two balls are shown in **Figure 2.** The red ball was dropped, and the blue ball was given an initial horizontal velocity. What is similar about the two paths? Look at their vertical positions. The horizontal lines indicate the equal vertical distances. At each moment that a picture was taken, the heights of the two balls were the same. Because the change in vertical position was the same for both, their average vertical velocities during each interval were also the same. The increasingly large distance traveled vertically by the balls, from one time interval to the next, shows that they were accelerating downward due to the force of gravity.

Notice that the horizontal motion of the launched ball does not affect its vertical motion. A projectile launched horizontally has initial horizontal velocity, but it has no initial vertical velocity. Therefore, its vertical motion is like that of an object dropped from rest. Just like the red ball, the blue ball has a downward velocity that increases regularly because of the acceleration due to gravity.

READING CHECK **Explain** why a dropped object has the same vertical velocity as an object launched horizontally.

MiniLABs

OVER THE EDGE
Does mass affect the motion of a projectile?

PROJECTILE PATH
Are the horizontal and vertical motions of a projectile related?

Vectors In Two Dimensions

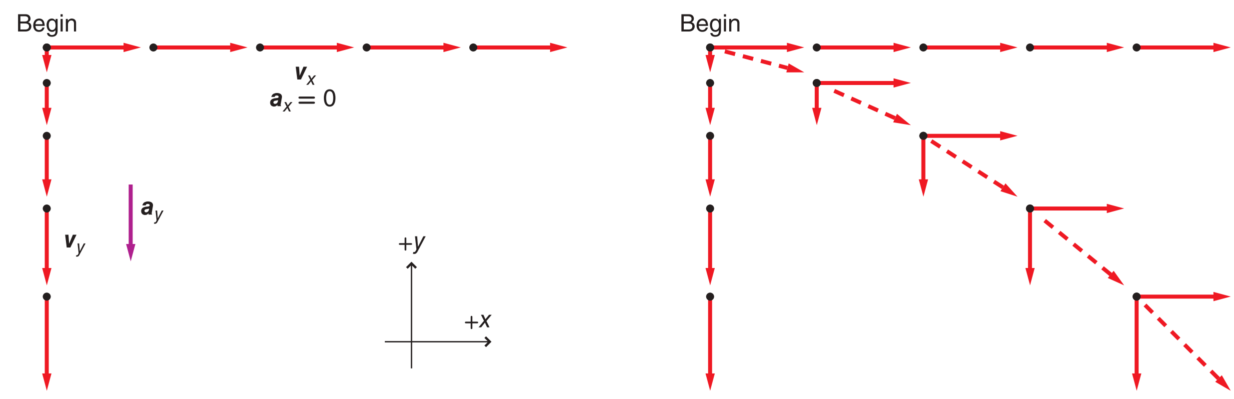


Figure 3 To describe the motion of a horizontally launched projectile, the *x*- and *y*-components can be treated independently. The resultant vectors of the projectile are tangent to a parabola.

Decide What is the value of a_y?

PhysicsLABs

LAUNCH AN INVESTIGATION

FORENSICS LAB How can physics reconstruct a projectile's launch?

ON TARGET

DESIGN YOUR OWN LAB What factors affect projectile motion?

Horizontally Launched Projectiles

Imagine a person standing near the edge of a cliff and kicking a pebble horizontally. Like all horizontally launched projectiles, the pebble will have an initial horizontal velocity, but it will not have an initial vertical velocity. What will happen to the pebble as it falls from the cliff?

Separate motion diagrams Recall that the horizontal motion of a projectile does not affect its vertical motion. It is therefore easier to analyze the horizontal motion and the vertical motion separately. Separate motion diagrams for the *x*-components and *y*-components of a horizontally launched projectile, such as a pebble kicked off a cliff, are shown on the left in **Figure 3.**

Horizontal motion Notice the horizontal vectors in the diagram on the left. Each of the velocity vectors is the same length, which indicates that the object's velocity is not changing. The pebble is not accelerating horizontally. This constant velocity in the horizontal direction is exactly what should be expected, because after the initial kick, there is no horizontal force acting on the pebble. (In reality, the pebble's speed would decrease slightly because of air resistance, but remember that we are ignoring air resistance in this chapter.)

READING CHECK **Explain** why the horizontal motion of a projectile is constant.

Vertical motion Now look at the vertical velocity vectors in the diagram on the left. Each velocity vector has a slightly longer length than the one above it. The changing length shows that the object's velocity is increasing and accelerating downward. Again, this is what should be expected, because in this case the force of gravity is acting on the pebble.

Parabolic path When the *x*- and *y*-components of the object's motion are treated independently, each path is a straight line. The diagram on the right in **Figure 3** shows the actual parabolic path. The horizontal and vertical components at each moment are added to form the total velocity vector at that moment. You can see how the combination of constant horizontal velocity and uniform vertical acceleration produces a trajectory that has a parabolic shape.

PROBLEM-SOLVING STRATEGIES

MOTION IN TWO DIMENSIONS

When solving projectile problems, use the following strategies.

1. Draw a motion diagram with vectors for the projectile at its initial position and its final position. If the projectile is launched at an angle, also show its maximum height and the initial angle.
2. Consider vertical and horizontal motion independently. List known and unknown variables.
3. For horizontal motion, the acceleration is $a_x = 0.0\ \text{m/s}^2$. If the projectile is launched at an angle, its initial vertical velocity and its vertical velocity when it falls back to that same height have the same magnitude but different direction: $v_{yi} = -v_{yf}$.
4. For vertical motion, $a_y = -9.8\ \text{m/s}^2$ (if you choose up as positive). If the projectile is launched at an angle, its vertical velocity at its highest point is zero: $v_{y,\max} = 0$.
5. Choose the motion equations that will enable you to find the unknown variables. Apply them to vertical and horizontal motion separately. Remember that time is the same for horizontal and vertical motion. Solving for time in one of the dimensions identifies the time for the other dimension.
6. Sometimes it is useful to apply the motion equations to part of the projectile's path. You can choose any initial and final points to use in the equations.

Motion Equations

Horizontal (constant speed)

$$\mathbf{x}_f = \mathbf{v}t + \mathbf{x}_i$$

Vertical (constant acceleration)

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t$$

$$\mathbf{x}_f = \mathbf{x}_i + \mathbf{v}_i t_f + \frac{1}{2}\mathbf{a}t^2$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

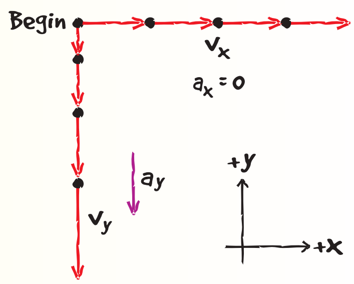
EXAMPLE 1

A SLIDING PLATE You are preparing breakfast and slide a plate on the countertop. Unfortunately, you slide it too fast, and it flies off the end of the countertop. If the countertop is 1.05 m above the floor and it leaves the top at 0.74 m/s, how long does it take to fall, and how far from the end of the counter does it land?

1 ANALYZE AND SKETCH THE PROBLEM

Draw horizontal and vertical motion diagrams. Choose the coordinate system so that the origin is at the top of the countertop. Choose the positive x direction in the direction of horizontal velocity and the positive y direction up.

KNOWN		UNKNOWN
$x_i = y_i = 0$ m	$a_x = 0\ \text{m/s}^2$	$t = ?$
$v_{xi} = 0.75$ m/s	$a_y = -9.8\ \text{m/s}^2$	$x_f = ?$
$v_{yi} = 0$ m/s	$y_f = -1.05$ m	



2 SOLVE FOR THE UNKNOWN

Use the equation of motion in the y direction to find the time of fall.

$$y_f = y_i + \frac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2(y_f - y_i)}{a_y}}$$ ◀ *Rearrange the equation to solve for time.*

$$= \sqrt{\frac{2(-1.05\ \text{m} - 0\ \text{m})}{-9.8\ \text{m/s}^2}} = 0.46\ \text{s}$$ ◀ *Substitute $y_f = -1.05$ m, $y_i = 0$ m, $a_y = -9.8\ \text{m/s}^2$.*

Use the equation of motion in the x direction to find where the plate hits the floor.

$$\mathbf{x}_f = \mathbf{v}_x t = (0.74\ \text{m/s to the right})(0.46\ \text{s}) = 0.34\ \text{m to the right of the counter}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** Time is measured in seconds. Position in measured in meters.
- **Do the signs make sense?** Both are positive. The position sign agrees with the coordinate choice.
- **Are the magnitudes realistic?** A fall of about a meter takes about 0.5 s. During this time, the horizontal displacement of the plate would be about 0.5 s × 0.74 m/s.

APPLICATIONS

1. You throw a stone horizontally at a speed of 5.0 m/s from the top of a cliff that is 78.4 m high.

a. How long does it take the stone to reach the bottom of the cliff?

b. How far from the base of the cliff does the stone hit the ground?

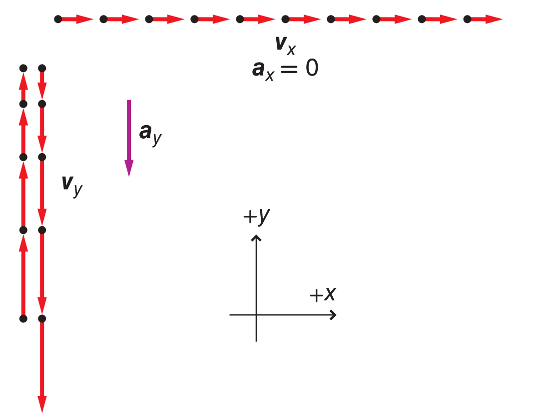
c. What are the horizontal and vertical components of the stone's velocity just before it hits the ground?

2. Samia and her friend are working at an assembly plant making wooden toy giraffes. At the end of the line, the giraffes go horizontally off the edge of a conveyor belt and fall into a box below. If the box is 0.60 m below the level of the conveyor belt and 0.40 m away from it, what must be the horizontal velocity of giraffes as they leave the conveyor belt?

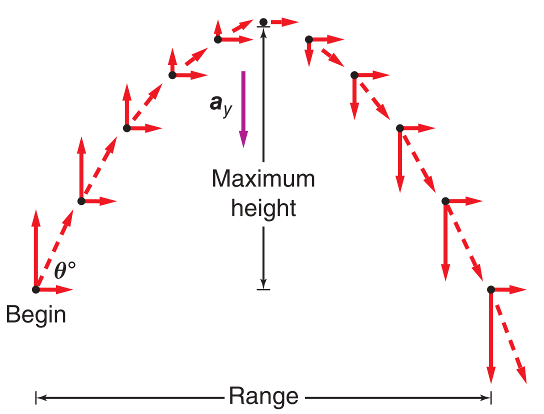
3. CHALLENGE You are visiting a friend from elementary school who now lives in a small town. One local amusement is the ice-cream parlor, where Mourad, the short-order cook, slides his completed ice-cream sundaes down the counter at a constant speed of 2.0 m/s to the servers. (The counter is kept very well polished for this purpose.) If the servers catch the sundaes 7.0 cm from the edge of the counter, how far do they fall from the edge of the counter to the point at which the servers catch them?

Figure 4 When a projectile is launched at an upward angle, its parabolic path is upward and then downward. The up-and-down motion is clearly represented in the vertical component of the vector diagram.

■ Vertical and Horizontal Components



■ Parabolic Trajectory



Angled Launches

When a projectile is launched at an angle, the initial velocity has a vertical component as well as a horizontal component. If the object is launched upward, like a ball tossed straight up in the air, it rises with slowing speed, reaches the top of its path where its speed is momentarily zero, and descends with increasing speed.

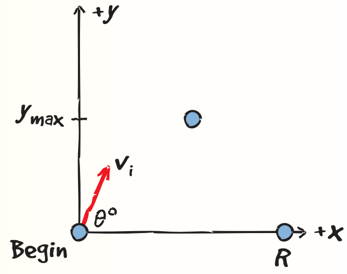
Separate motion diagrams The upper diagram of **Figure 4** shows the separate vertical- and horizontal-motion diagrams for the trajectory. In the coordinate system, the *x*-axis is horizontal and the *y*-axis is vertical. Note the symmetry. At each point in the vertical direction, the velocity of the object as it is moving upward has the same magnitude as when it is moving downward. The only difference is that the directions of the two velocities are opposite. When solving problems, it is sometimes useful to consider symmetry to determine unknown quantities.

Parabolic path The lower diagram of **Figure 4** defines two quantities associated with the trajectory. One is the maximum height, which is the height of the projectile when the vertical velocity is zero and the projectile has only its horizontal-velocity component. The other quantity depicted is the range (*R*), which is the horizontal distance the projectile travels when the initial and final heights are the same. Not shown is the flight time, which is how much time the projectile is in the air. For football punts, flight time often is called hang time.

☑ **READING CHECK** At what point of a projectile's trajectory is its vertical velocity zero?

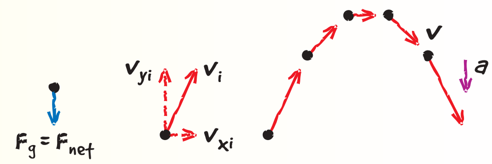
EXAMPLE 2

THE FLIGHT OF A BALL A ball is launched at 4.5 m/s at 66° above the horizontal. It starts and lands at the same distance from the ground. What are the maximum height above its launch level and the flight time of the ball?



1 ANALYZE AND SKETCH THE PROBLEM

- Establish a coordinate system with the initial position of the ball at the origin.
- Show the positions of the ball at the beginning, at the maximum height, and at the end of the flight. Show the direction of $\boldsymbol{F}_{net}$.
- Draw a motion diagram showing $\boldsymbol{v}$ and $\boldsymbol{a}$.



KNOWN			UNKNOWN
$y_i = 0.0$ m	$\theta_i = 66°$	$v_{y,\,max} = 0.0$ m	$y_{max} = ?$
$v_i = 4.5$ m/s	$a_y = -9.8\ \text{m/s}^2$		$t = ?$

2 SOLVE FOR THE UNKNOWN

Find the y-component of v_i.

$$v_{yi} = v_i(\sin\theta_i)$$
$$= (4.5\ \text{m/s})(\sin 66°) = 4.1\ \text{m/s}$$

◀ Substitute $v_i = 4.5$ m/s, $\theta_i = 66°$.

Use symmetry to find the y-component of v_f.

$$v_{yf} = -v_{yi} = -4.1\ \text{m/s}$$

Solve for the maximum height.

$$v_{y,\,max}{}^2 = v_{yi}{}^2 + 2a_y(y_{max} - y_i)$$
$$(0.0\ \text{m/s})^2 = v_{yi}{}^2 + 2a_y(y_{max} - 0.0\ \text{m})$$
$$y_{max} = -\frac{v_{yi}{}^2}{2a_y}$$
$$= -\frac{(4.1\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} = 0.86\ \text{m}$$

◀ Substitute $v_{yi} = 4.1$ m/s; $a_y = -9.8\ \text{m/s}^2$.

Solve for the time to return to the launching height.

$$v_{yf} = v_{yi} + a_y t$$
$$t = \frac{v_{yf} - v_{yi}}{a_y}$$
$$= \frac{-4.1\ \text{m/s} - 4.1\ \text{m/s}}{-9.8\ \text{m/s}^2} = 0.84\ \text{s}$$

◀ Substitute $v_{yf} = -4.1$ m/s; $v_{yi} = 4.1$ m/s; $a_y = -9.8\ \text{m/s}^2$.

3 EVALUATE THE ANSWER

Are the magnitudes realistic? For an object that rises less than 1 m, a time of less than 1 s is reasonable.

APPLICATIONS

4. A player kicks a football from ground level with an initial velocity of 27.0 m/s, 30.0° above the horizontal, as shown in **Figure 5.** Find each of the following. Assume that forces from the air on the ball are negligible.

a. the ball's hang time

b. the ball's maximum height

c. the horizontal distance the ball travels before hitting the ground

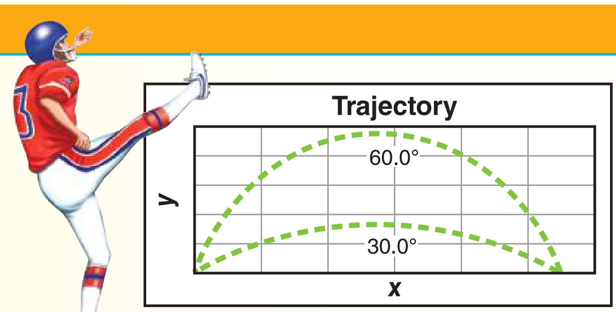
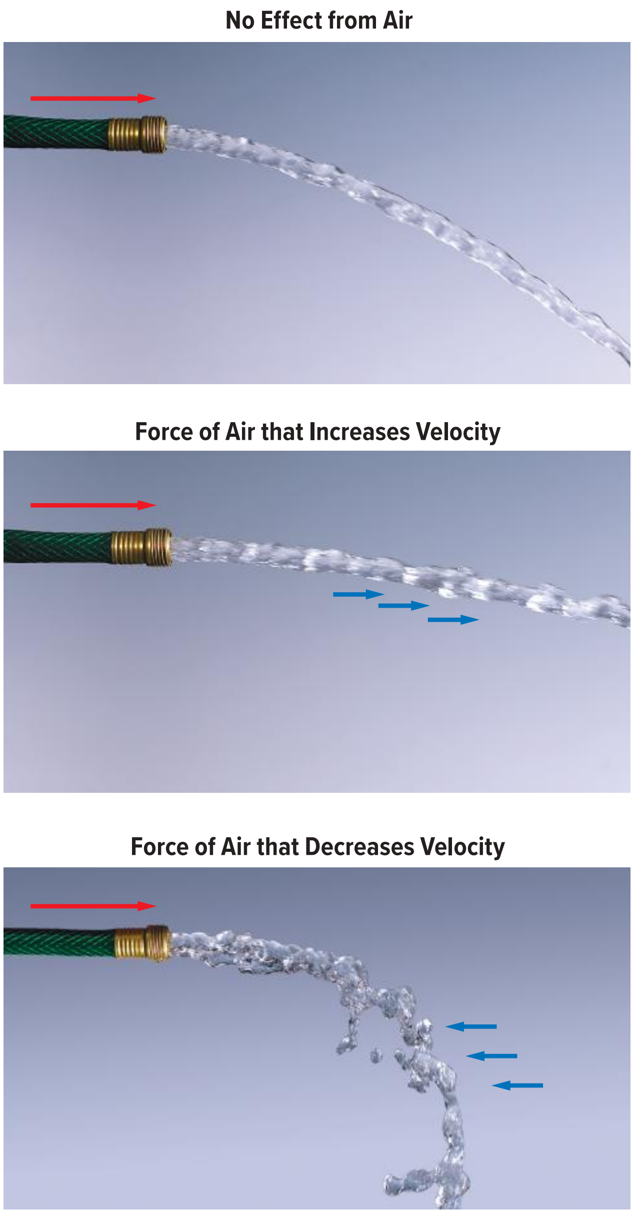


Figure 5

5. The player in the previous problem then kicks the ball with the same speed but at 60.0° from the horizontal. What is the ball's hang time, horizontal distance traveled, and maximum height?

6. CHALLENGE A rock is thrown from a 50.0 m high cliff with an initial velocity of 7.0 m/s at an angle of 53.0° above the horizontal. Find its velocity when it hits the ground below.

Figure 6 Forces from air can increase or decrease the velocity of a moving object.



Forces from Air

The effect of forces due to air has been ignored so far in this chapter, but think about why a kite stays in the air or why a parachute helps a skydiver fall safely to the ground. Forces from the air can significantly change the motion of an object.

What happens if there is wind? Moving air can change the motion of a projectile. Consider the three cases shown in **Figure 6.** In the top photo, water is flowing from the hose pipe with almost no effect from air. In the middle photo, wind is blowing in the same direction as the water's initial movement. The path of the water changes because the air exerts a force on the water in the same direction as its motion. The horizontal distance the water travels increases because the force increases the water's horizontal speed. The direction of the wind changes in the bottom photo. The horizontal distance the water travels decreases because the air exerts a force in the direction opposite the water's motion.

What if the direction of wind is at an angle relative to a moving object? The horizontal component of the wind affects only the horizontal motion of an object. The vertical component of the wind affects only the vertical motion of the object. In the case of the water, for example, a strong updraft could decrease the downward speed of the water.

The effects shown in **Figure 6** occur because the air is moving enough to significantly change the motion of the water. Even air that is not moving, however, can have a significant effect on some moving objects. A piece of paper held horizontally and dropped, for example, falls slowly because of air resistance. The air resistance increases as the surface area of the object that faces the moving air increases.

SECTION 1 REVIEW

7. **MAIN**IDEA Two baseballs are pitched horizontally from the same height but at different speeds. The faster ball crosses home plate within the strike zone, but the slower ball is below the batter's knees. Why do the balls pass the batter at different heights?

8. **Free-Body Diagram** An ice cube slides without friction across a table at a constant velocity. It slides off the table and lands on the floor. Draw free-body and motion diagrams of the ice cube at two points on the table and at two points in the air.

9. **Projectile Motion** A tennis ball is thrown out a window 28 m above the ground at an initial velocity of 15.0 m/s and 20.0° below the horizontal. How far does the ball move horizontally before it hits the ground?

10. **Projectile Motion** A softball player tosses a ball into the air with an initial velocity of 11.0 m/s, as shown in **Figure 7.** What will be the ball's maximum height?

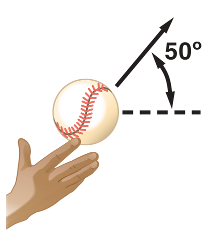


Figure 7

11. **Critical Thinking** Suppose an object is thrown with the same initial velocity and direction on Earth and on the Moon, where the acceleration due to gravity is one-sixth its value on Earth. How will vertical velocity, time of flight, maximum height, and horizontal distance change?

SECTION 2 Circular Motion

PHYSICS 4 YOU Many amusement park and carnival rides spin. When the ride is spinning, forces from the walls or sides of the ride keep the riders moving in a circular path.

MAINIDEA

An object in circular motion has an acceleration toward the circle's center due to an unbalanced force toward the circle's center.

Essential Questions

- Why is an object moving in a circle at a constant speed accelerating?
- How does centripetal acceleration depend upon the object's speed and the radius of the circle?
- What causes centripetal acceleration?

Review Vocabulary

average velocity the change in position divided by the time during which the change occurred; the slope of an object's position-time graph

New Vocabulary

uniform circular motion
centripetal acceleration
centripetal force

Describing Circular Motion

Consider an object moving in a circle at a constant speed, such as a stone being whirled on the end of a string or a fixed horse on a carousel. Are these objects accelerating? At first, you might think they are not because their speeds do not change. But remember that acceleration is related to the change in velocity, not just the change in speed. Because their directions are changing, the objects must be accelerating.

Uniform circular motion is the movement of an object at a constant speed around a circle with a fixed radius. The position of an object in uniform circular motion, relative to the center of the circle, is given by the position vector $\boldsymbol{r}$. Remember that a position vector is a displacement vector with its tail at the origin. Two position vectors, $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, at the beginning and end of a time interval are shown on the left in **Figure 8.** As the object moves around the circle, the length of the position vector does not change, but its direction does. The diagram also shows two instantaneous velocity vectors. Notice that each velocity vector is tangent to the circular path, at a right angle to the corresponding position vector.

To determine the object's velocity, you first need to find its displacement vector over a time interval. You know that a moving object's average velocity is defined as $\frac{\Delta \boldsymbol{x}}{\Delta t}$, so for an object in circular motion, $\bar{\boldsymbol{v}} = \frac{\Delta \boldsymbol{r}}{\Delta t}$. The right side of **Figure 8** shows $\Delta \boldsymbol{r}$ drawn as the displacement from $\boldsymbol{r}_1$ to $\boldsymbol{r}_2$ during a time interval. The velocity for this time interval has the same direction as the displacement, but its length would be different because it is divided by Δt.

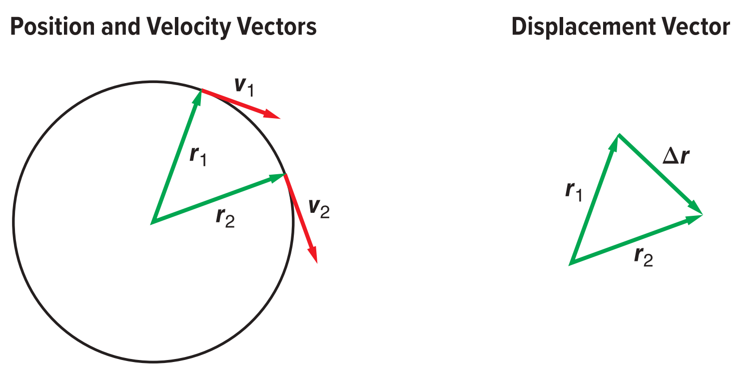


Figure 8 For an object in uniform circular motion, the velocity is tangent to the circle. It is in the same direction as the displacement.

Analyze How can you tell from the diagram that the motion is uniform?

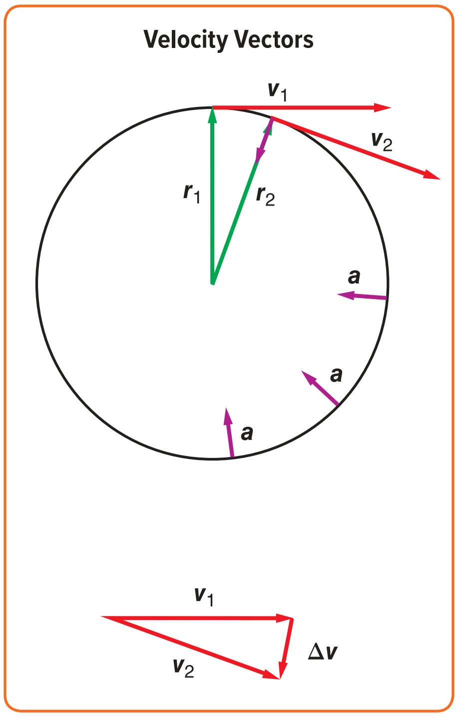


Figure 9 The acceleration of an object in uniform circular motion is the change in velocity divided by the time interval. The direction of centripetal acceleration is always toward the center of the circle.

Centripetal Acceleration

You have read that a velocity vector of an object in uniform circular motion is tangent to the circle. What is the direction of the acceleration? **Figure 9** shows the velocity vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ at the beginning and end of a time interval. The difference in the two vectors ($\Delta\boldsymbol{v}$) is found by subtracting the vectors, as shown at the bottom of the figure. The average acceleration $\left(\bar{\boldsymbol{a}} = \frac{\Delta\boldsymbol{v}}{\Delta t}\right)$ for this time interval is in the same direction as $\Delta\boldsymbol{v}$. For a very small time interval, $\Delta\boldsymbol{v}$ is so small that $\boldsymbol{a}$ points toward the center of the circle.

Repeat this process for several other time intervals when the object is in different locations on the circle. As the object moves around the circle, the direction of the acceleration vector changes, but it always points toward the center of the circle. For this reason, the acceleration of an object in uniform circular motion is called center-seeking or **centripetal acceleration.**

Magnitude of acceleration What is the magnitude of an object's centripetal acceleration? Look at the starting points of the velocity vectors in the top of **Figure 9.** Notice the triangle the position vectors at those points make with the center of the circle. An identical triangle is formed by the velocity vectors in the bottom of **Figure 9.** The angle between $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ is the same as that between $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. Therefore, similar triangles are formed by subtracting the two sets of vectors, and the ratios of the lengths of two corresponding sides are equal. Thus, $\frac{\Delta r}{r} = \frac{\Delta v}{v}$. The equation is not changed if both sides are divided by Δt.

$$\frac{\Delta r}{r\Delta t} = \frac{\Delta v}{v\Delta t}$$

But, $v = \frac{\Delta r}{\Delta t}$ and $a = \frac{\Delta v}{\Delta t}$

$$\left(\frac{1}{r}\right)\left(\frac{\Delta r}{\Delta t}\right) = \left(\frac{1}{v}\right)\left(\frac{\Delta v}{\Delta t}\right)$$

Substituting $v = \frac{\Delta r}{\Delta t}$ in the left-hand side and $a = \frac{\Delta v}{\Delta t}$ in the right-hand side gives the following equation:

$$\frac{v}{r} = \frac{a}{v}$$

Solve for acceleration, and use the symbol a_c for centripetal acceleration.

CENTRIPETAL ACCELERATION

Centripetal acceleration always points to the center of the circle. Its magnitude is equal to the square of the velocity divided by the radius of motion.

$$a_c = \frac{v^2}{r}$$

Period of revolution One way to describe the magnitude of the velocity of an object moving in a circle is to measure its period (T), the time needed for the object to make one complete revolution. During this time, the object travels a distance equal to the circumference of the circle ($2\pi r$). The velocity, then, is represented by $v = \frac{2\pi r}{T}$. If you substitute for v in the equation for centripetal acceleration, you obtain the following equation:

$$a_c = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2}$$

Figure 10 As the hammer thrower swings the ball around, tension in the chain is the force that causes the ball to have an inward acceleration.

Predict Neglecting air resistance, how would the horizontal acceleration and velocity of the hammer change if the thrower released the chain?

Centripetal force Because the acceleration of an object moving in a circle is always in the direction of the net force acting on it, there must be a net force toward the center of the circle. This force can be provided by any number of agents. For Earth circling the Sun, the force is the Sun's gravitational force on Earth. When a hammer thrower swings the hammer, as in **Figure 10,** the force is the tension in the chain attached to the massive ball. When an object moves in a circle, the net force toward the center of the circle is called the **centripetal force.** To accurately analyze centripetal acceleration situations, you must identify the agent of the force that causes the acceleration. Then you can apply Newton's second law for the component in the direction of the acceleration in the following way.

NEWTON'S SECOND LAW FOR CIRCULAR MOTION
The net centripetal force on an object moving in a circle is equal to the object's mass times the centripetal acceleration.

$$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}_{\text{c}}$$

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CENTRIPETAL FORCE
What keeps an object moving when you swing it in a circle?

Direction of acceleration When solving problems, you have found it useful to choose a coordinate system with one axis in the direction of the acceleration. For circular motion, the direction of the acceleration is always toward the center of the circle. Rather than labeling this axis *x* or *y,* call it *c,* for centripetal acceleration. The other axis is in the direction of the velocity, tangent to the circle. It is labeled *tang* for tangential. You will apply Newton's second law in these directions, just as you did in the two-dimensional problems you have solved before. Remember that centripetal force is just another name for the net force in the centripetal direction. It is the sum of all the real forces, those for which you can identify agents that act along the centripetal axis.

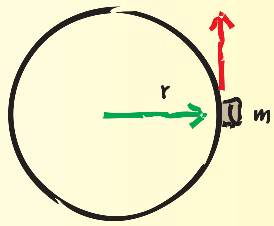
In the case of the hammer thrower in **Figure 10,** in what direction does the hammer fly when the chain is released? Once the contact force of the chain is gone, there is no force accelerating the hammer toward the center of the circle, so the hammer flies off in the direction of its velocity, which is tangent to the circle. Remember, if you cannot identify the agent of a force, then it does not exist.

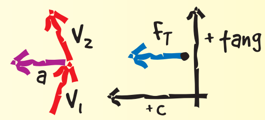
EXAMPLE 3

UNIFORM CIRCULAR MOTION A 13 g rubber stopper is attached to a 0.93 m string. The stopper is swung in a horizontal circle, making one revolution in 1.18 s. Find the magnitude of the tension force exerted by the string on the stopper.

1 ANALYZE AND SKETCH THE PROBLEM

- Draw a free-body diagram for the swinging stopper.
- Include the radius and the direction of motion.
- Establish a coordinate system labeled *tang* and *c*. The directions of a_c and F_T are parallel to *c*.





KNOWN	UNKNOWN
$m = 13$ g	$F_T = ?$
$r = 0.93$ m	
$T = 1.18$ s	

2 SOLVE FOR THE UNKNOWN

Find the magnitude of the centripetal acceleration.

$$a_c = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2(0.93\ \text{m})}{(1.18\ \text{s})^2}$$ ◀ Substitute $r = 0.93$ m, $T = 1.18$ s.

$$= 26\ \text{m/s}^2$$

Use Newton's second law to find the magnitude of the tension in the string.

$$F_T = ma_c$$

$$= (0.013\ \text{kg})(26\ \text{m/s}^2)$$ ◀ Substitute $m = 0.013$ kg, $a_c = 26$ m/s^2.

$$= 0.34\ \text{N}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** Dimensional analysis verifies that a_c is in meters per second squared and F_T is in newtons.
- **Do the signs make sense?** The signs should all be positive.
- **Are the magnitudes realistic?** The force is almost three times the weight of the stopper, and the acceleration is almost three times that of gravity, which is reasonable for such a light object.

APPLICATIONS

12. A runner moving at a speed of 8.8 m/s rounds a bend with a radius of 25 m. What is the centripetal acceleration of the runner, and what agent exerts the centripetal force on the runner?

13. An airplane traveling at 201 m/s makes a turn. What is the smallest radius of the circular path (in kilometers) the pilot can make and keep the centripetal acceleration under 5.0 m/s^2?

14. A 45 kg merry-go-round worker stands on the ride's platform 6.3 m from the center, as shown in **Figure 11.** If her speed (v_{worker}) as she goes around the circle is 4.1 m/s, what is the force of friction (F_f) necessary to keep her from falling off the platform?

15. A 16 g ball at the end of a 1.4 m string is swung in a horizontal circle. It revolves once every 1.09 s. What is the magnitude of the string's tension?

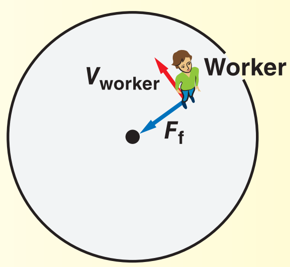


Figure 11

16. CHALLENGE A car racing on a flat track travels at 22 m/s around a curve with a 56 m radius. Find the car's centripetal acceleration. What minimum coefficient of static friction between the tires and the road is necessary for the car to round the curve without slipping?

Centrifugal "Force"

If a car makes a sharp left turn, a passenger on the right side might be thrown against the right door. Is there an outward force on the passenger? Consider a similar situation. If a car in which you are riding stops suddenly, you will be thrown forward into your safety belt. Is there a forward force on you? No, because according to Newton's first law, you will continue moving with the same velocity unless there is a net force acting on you. The safety belt applies the force that accelerates you to a stop.

Figure 12 shows a car turning left as viewed from above. A passenger in the car would continue to move straight ahead if it were not for the force of the door acting in the direction of the acceleration. As the car goes around the curve, the car and the passenger are in circular motion, and the passenger experiences centripetal acceleration. Recall that centripetal acceleration is always directed toward the center of the circle. There is no outward force on the passenger.

If, however, you think about similar situations that you have experienced, you know that it feels as if a force is pushing you outward. The so-called centrifugal, or outward, force is a fictitious, nonexistent force. You feel as if you are being pushed only because you are accelerating relative to your surroundings. There is no real force because there is no agent exerting a force.

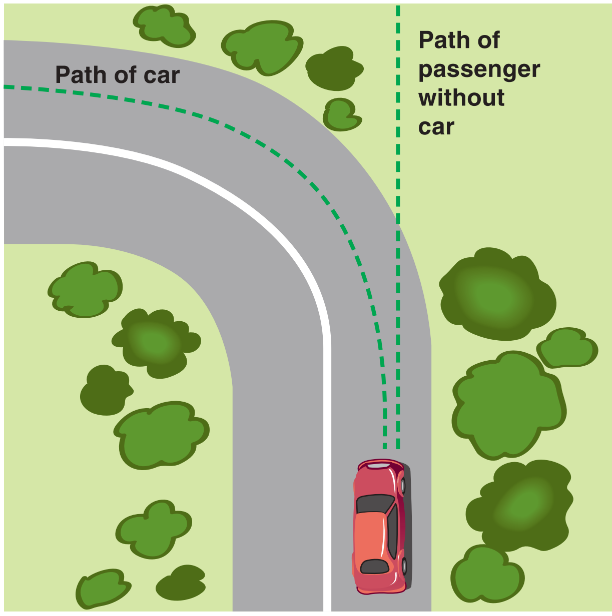


Figure 12 When a car moves around a curve, a passenger feels a fictitious centrifugal force directed outward. In fact, the force on the passenger is the centripetal force, which is directed toward the center of the circle and is exerted by the seat on which the person is sitting.

SECTION 2 REVIEW

17. **MAIN**IDEA If you attach a ball to a rope and swing it at a constant speed in a circle above your head, the ball is in uniform circular motion. In which direction does it accelerate? What force causes the acceleration?

18. **Uniform Circular Motion** What is the direction of the force that acts on the clothes in the spin cycle of a top-load washing machine? What exerts the force?

19. **Centripetal Acceleration** A newspaper article states that when turning a corner, a driver must be careful to balance the centripetal and centrifugal forces to keep from skidding. Write a letter to the editor that describes physics errors in this article.

20. **Free-Body Diagram** You are sitting in the back seat of a car going around a curve to the right. Sketch motion and free-body diagrams to answer these questions:

a. What is the direction of your acceleration?

b. What is the direction of the net force on you?

c. What exerts this force?

21. **Centripetal Acceleration** An object swings in a horizontal circle, supported by a 1.8 m string. It completes a revolution in 2.2 s. What is the object's centripetal acceleration?

22. **Centripetal Force** The 40.0 g stone in **Figure 13** is whirled horizontally at a speed of 2.2 m/s. What is the tension in the string?

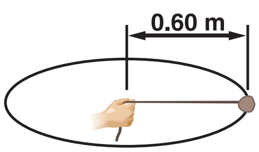


Figure 13

23. **Amusement-Park Ride** A ride at an amusement park has people stand around a 4.0 m radius circle with their backs to a wall. The ride then spins them with a 1.7 s period of revolution. What are the centripetal acceleration and velocity of the riders?

24. **Centripetal Force** A bowling ball has a mass of 7.3 kg. What force must you exert to move it at a speed of 2.5 m/s around a circle with a radius of 0.75 m?

25. **Critical Thinking** Because of Earth's daily rotation, you always move with uniform circular motion. What is the agent that supplies the force that causes your centripetal acceleration? If you are standing on a scale, how does the circular motion affect the scale's measure of your weight?

SECTION 3

Relative Velocity

MAIN IDEA

An object's velocity depends on the reference frame chosen.

Essential Questions

- What is relative velocity?
- How do you find the velocities of an object in different reference frames?

Review Vocabulary

resultant a vector that results from the sum of two other vectors

New Vocabulary

reference frame

PHYSICS 4 YOU **Have you ever noticed that people riding along with you on an escalator don't seem to be moving, while people going in the opposite direction seem to be moving very fast? These people may in fact have the same speed relative to the ground, but their velocities relative to you are very different.**

Relative Motion in One Dimension

Suppose you are in a school bus that is traveling at a velocity of 8 m/s in a positive direction. You walk with a velocity of 1 m/s toward the front of the bus. If a friend is standing on the side of the road watching the bus go by, how fast would your friend say you are moving? If the bus is traveling at 8 m/s, its speed as measured by your friend in a coordinate system fixed to the road is 8 m/s. When you are standing still on the bus, your speed relative to the road is also 8 m/s, but your speed relative to the bus is zero. How can your speed be different?

Different reference frames In this example, your motion is viewed from different coordinate systems. A coordinate system from which motion is viewed is a **reference frame.** Walking at 1 m/s toward the front of the bus means your velocity is measured in the reference frame of the bus. Your velocity in the road's reference frame is different. You can rephrase the problem as follows: given the velocity of the bus relative to the road and your velocity relative to the bus, what is your velocity relative to the road?

A vector representation of this problem is shown in **Figure 14.** If right is positive, your speed relative to the road is 9 m/s, the sum of 8 m/s and 1 m/s. Suppose that you now walk at the same speed toward the rear of the bus. What would be your velocity relative to the road? **Figure 14** shows that because the two velocities are in opposite directions, the resultant speed is 7 m/s, the difference between 8 m/s and 1 m/s. You can see that when the velocities are along the same line, simple addition or subtraction can be used to determine the relative velocity.

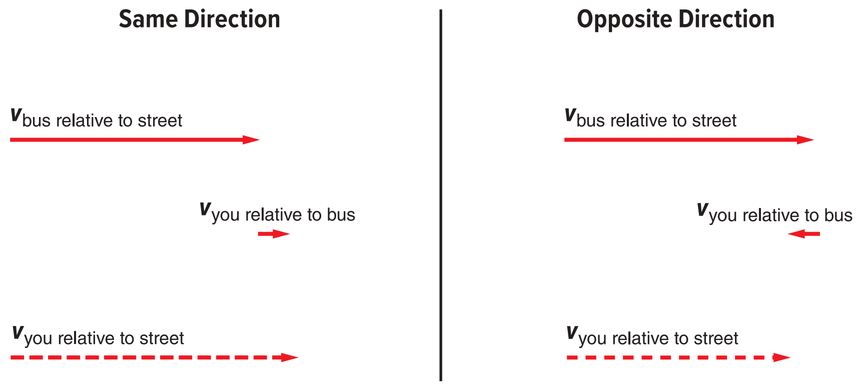


Figure 14 When an object moves in a moving reference frame, you add the velocities if they are in the same direction. You subtract one velocity from the other if they are in opposite directions.

Recall What do the lengths of the velocity vectors indicate?

PHYSICS CHALLENGE

TENSION IN A ROPE Amir whirls a stone of mass *m* on a rope in a horizontal circle above his head such that the stone is at a height *h* above the ground. The circle has a radius of *r*, and the magnitude of the tension in the rope is *T*. Suddenly the rope breaks, and the stone falls to the ground. The stone travels a horizontal distance *x* from the time the rope breaks until it impacts the ground. Find a mathematical expression for *x* in terms of *T*, *r*, *m*, and *h*. Does your expression change if Amir is walking 0.50 m/s relative to the ground?

Combining velocity vectors Take a closer look at how the relative velocities in **Figure 14** were obtained. Can you find a mathematical rule to describe how velocities are combined when the motion is in a moving reference frame? For the situation in which you are walking in a bus, you can designate the velocity of the bus relative to the road as $\boldsymbol{v}_{b/r}$. You can designate your velocity relative to the bus as $\boldsymbol{v}_{y/b}$ and the velocity of you relative to the road as $\boldsymbol{v}_{y/r}$. To find the velocity of you relative to the road in both cases, you added the velocity vectors of you relative to the bus and the bus relative to the road. Mathematically, this is represented as $\boldsymbol{v}_{y/b} + \boldsymbol{v}_{b/r} = \boldsymbol{v}_{y/r}$. The more general form of this equation is as follows.

RELATIVE VELOCITY
The relative velocity of object a to object c is the vector sum of object a's velocity relative to object b and object b's velocity relative to object c.

$$\boldsymbol{v}_{a/b} + \boldsymbol{v}_{b/c} = \boldsymbol{v}_{a/c}$$

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MOVING REFERENCE FRAME

How can you describe motion in a moving reference frame?

Relative Motion in Two Dimensions

Adding relative velocities also applies to motion in two dimensions. As with one-dimensional motion, you first draw a vector diagram to describe the motion, and then you solve the problem mathematically.

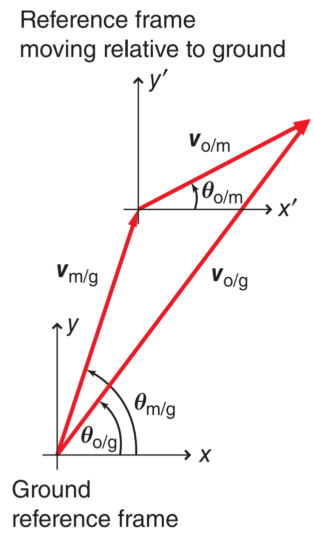
Vector diagrams The method of drawing vector diagrams for relative motion in two dimensions is shown in **Figure 15.** The velocity vectors are drawn tip-to-tail. The reference frame from which you are viewing the motion, often called the ground reference frame, is considered to be at rest. One vector describes the velocity of the second reference frame relative to ground. The second vector describes the motion in that moving reference frame. The resultant shows the relative velocity, which is the velocity relative to the ground reference frame.

READING CHECK **Decide** Can a moving car be a reference frame?

An example is the relative motion of an airplane. Airline pilots cannot expect to reach their destinations by simply aiming their planes along a compass direction. They must take into account the plane's speed relative to the air, which is given by their airspeed indicators, and their direction of flight relative to the air. They also must consider the velocity of the wind at the altitude they are flying relative to the ground. These two vectors must be combined to obtain the velocity of the airplane relative to the ground. The resultant vector tells the pilot how fast and in what direction the plane must travel relative to the ground to reach its destination. A similar situation occurs for boats that are traveling on water that is flowing.

Figure 15 Vectors are placed tip-to-tail to find the relative velocity vector for two-dimensional motion. The subscript o/g refers to an object relative to ground, o/m refers to an object relative to a moving reference frame, and m/g refers to the moving frame relative to ground.

Analyze How would the resultant vector change if the ground reference frame were considered to be the moving reference frame?



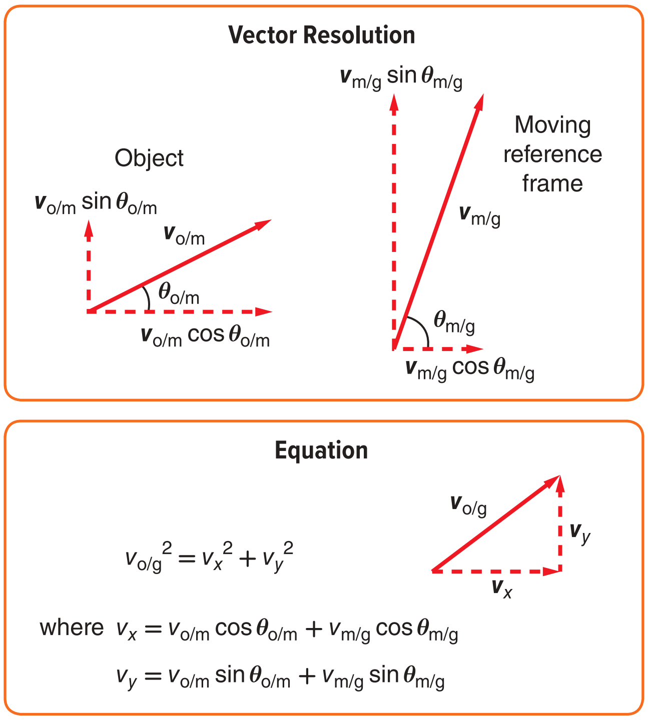


Figure 16 To find the velocity of an object in a moving reference frame, resolve the vectors into *x*-and *y*-components.

Combining velocities You can use the equations in **Figure 16** to solve problems for relative motion in two dimensions. The velocity of a reference frame moving relative to the ground is labeled $\boldsymbol{v}_{m/g}$. The velocity of an object in the moving frame is labeled $\boldsymbol{v}_{o/m}$. The relative velocity equation gives the object's velocity relative to the ground: $\boldsymbol{v}_{o/g} = \boldsymbol{v}_{o/m} + \boldsymbol{v}_{m/g}$.

To determine the magnitude of the object's velocity relative to the ground ($v_{o/g}$), first resolve the velocity vectors of the object and the moving reference frame into *x*- and *y*-components. Then apply the Pythagorean theorem. The general equation is shown in **Figure 16,** but for many problems the equation is simpler because the vectors are along an axis. As shown in the example problem below, you can find the angle of the object's velocity relative to the ground by observing the vector diagram and applying a trigonometric relationship.

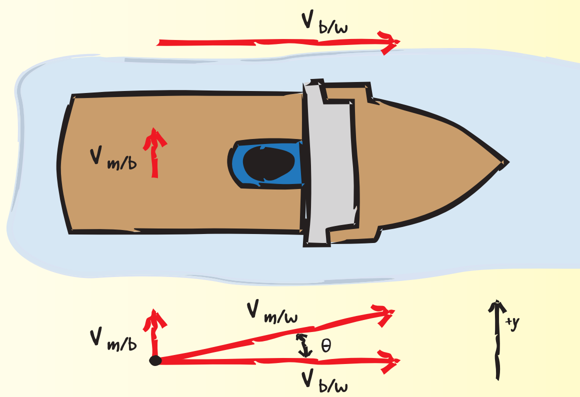
☑ **READING CHECK Explain** How are vectors used to describe relative motion in two dimensions?

EXAMPLE 4

RELATIVE VELOCITY OF A MARBLE Rodina and Dalia are riding on a ferry boat traveling east at 4.0 m/s. Dalia rolls a marble with a velocity of 0.75 m/s north, straight across the deck of the boat to Rodina. What is the velocity of the marble relative to the water?

1 ANALYZE AND SKETCH THE PROBLEM

Establish a coordinate system. Draw vectors for the velocities.



KNOWN		UNKNOWN
$v_{b/w} = 4.0$ m/s	$v_{m/b} = 0.75$ m/s	$v_{m/w} = ?$

2 SOLVE FOR THE UNKNOWN

The velocities are perpendicular, so we can use the Pythagorean theorem.

$$v_{m/w}^2 = v_{b/w}^2 + v_{m/b}^2$$

$$v_{m/w} = \sqrt{v_{b/w}^2 + v_{m/b}^2}$$

$$= \sqrt{(4.0\text{ m/s})^2 + (0.75\text{ m/s})^2} = 4.1\text{ m/s}$$

◀ Substitute $v_{b/w} = 4.0$ m/s, $v_{m/b} = 0.75$ m/s.

Find the angle of the marble's velocity.

$$\theta = \tan^{-1}\left(\frac{v_{m/b}}{v_{b/w}}\right)$$

$$= \tan^{-1}\left(\frac{0.75\text{ m/s}}{4.0\text{ m/s}}\right) = 11° \text{ north of east}$$

The marble travels 4.1 m/s at 11° north of east.

3 EVALUATE THE ANSWER

- **Are the units correct?** Dimensional analysis verifies units of meters per second for velocity.
- **Do the signs make sense?** The signs should all be positive.
- **Are the magnitudes realistic?** The resulting velocity is of the same order of magnitude as the velocities given in the problem and slightly larger than the larger of the two.

APPLICATIONS

26. You are riding in a bus moving slowly through heavy traffic at 2.0 m/s. You hurry to the front of the bus at 4.0 m/s relative to the bus. What is your speed relative to the street?

27. Ahmed is pulling a toy wagon through a neighborhood at a speed of 0.75 m/s. A caterpillar in the wagon is crawling toward the rear of the wagon at a rate of 2.0 cm/s. What is the caterpillar's velocity relative to the ground?

28. A boat is rowed directly upriver at a speed of 2.5 m/s relative to the water. Viewers on the shore see that the boat is moving at only 0.5 m/s relative to the shore. What is the speed of the river? Is it moving with or against the boat?

29. A boat is traveling east at a speed of 3.8 m/s. A person walks across the boat with a velocity of 1.3 m/s south.

a. What is the person's speed relative to the water?

b. In what direction, relative to the ground, does the person walk?

30. An airplane flies due north at 150 km/h relative to the air. There is a wind blowing at 75 km/h to the east relative to the ground. What is the plane's speed relative to the ground?

31. CHALLENGE The airplane in **Figure 17** flies at 200.0 km/h relative to the air. What is the velocity of the plane relative to the ground if it flies during the following wind conditions?

a. a 50.0 km/h tailwind

b. a 50.0 km/h headwind

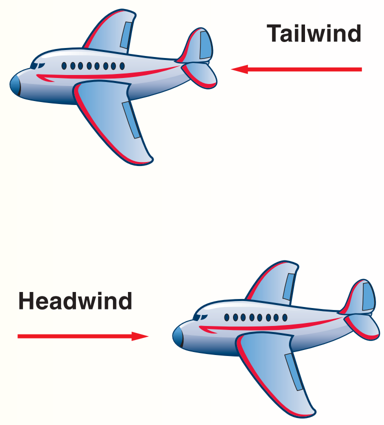


Figure 17

SECTION 3 REVIEW

32. MAIN IDEA A plane has a speed of 285 km/h west relative to the air. A wind blows 25 km/h east relative to the ground. What are the plane's speed and direction relative to the ground?

33. Relative Velocity A fishing boat with a maximum speed of 3 m/s relative to the water is in a river that is flowing at 2 m/s. What is the maximum speed the boat can obtain relative to the shore? The minimum speed? Give the direction of the boat, relative to the river's current, for the maximum speed and the minimum speed relative to the shore.

34. Relative Velocity of a Boat A motorboat heads due west at 13 m/s relative to a river that flows due north at 5.0 m/s. What is the velocity (both magnitude and direction) of the motorboat relative to the shore?

35. Boating You are boating on a river that flows toward the east. Because of your knowledge of physics, you head your boat 53° west of north and have a velocity of 6.0 m/s due north relative to the shore.

a. What is the velocity of the current?

b. What is the speed of your boat relative to the water?

36. Boating Hazem is riding on a ferry boat that is traveling east at 3.8 m/s. He walks north across the deck of the boat at 0.62 m/s. What is Hazem's velocity relative to the water?

37. Relative Velocity An airplane flies due south at 175 km/h relative to the air. There is a wind blowing at 85 km/h to the east relative to the ground. What are the plane's speed and direction relative to the ground?

38. A Plane's Relative Velocity An airplane flies due north at 235 km/h relative to the air. There is a wind blowing at 65 km/h to the northeast relative to the ground. What are the plane's speed and direction relative to the ground?

39. Critical Thinking You are piloting the boat in **Figure 18** across a fast-moving river. You want to reach a pier directly opposite your starting point. Describe how you would navigate the boat in terms of the components of your velocity relative to the water.

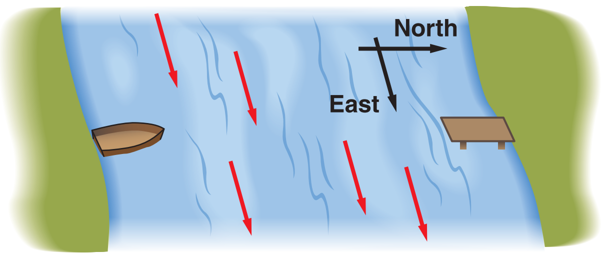


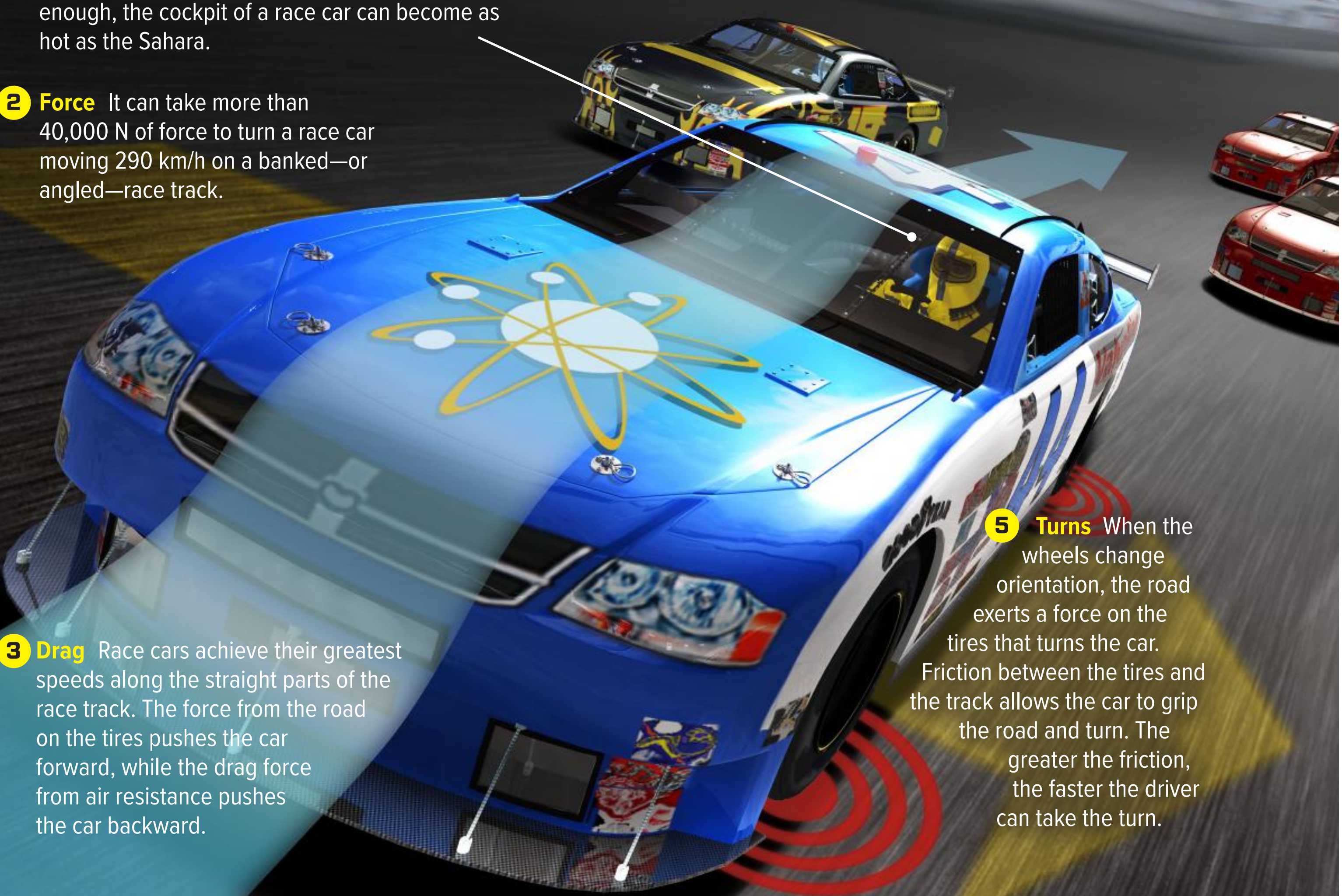
Figure 18

ON THE **JOB**

Need for SPEED

Race-Car Driver

The job of a race-car driver is more than just pushing down the gas pedal and following the curve of the track. Managing the extreme forces at work while driving a car at speeds of nearly 320 kilometers per hour takes endurance, strength, and fast reflexes—especially during the turns.



1 Heat A race-car driver wears a helmet to protect the head from impact, a full-body suit that protects against fire, and gloves to improve steering-wheel grip. As if all this gear isn't hot enough, the cockpit of a race car can become as hot as the Sahara.

2 Force It can take more than 40,000 N of force to turn a race car moving 290 km/h on a banked—or angled—race track.

3 Drag Race cars achieve their greatest speeds along the straight parts of the race track. The force from the road on the tires pushes the car forward, while the drag force from air resistance pushes the car backward.

5 Turns When the wheels change orientation, the road exerts a force on the tires that turns the car. Friction between the tires and the track allows the car to grip the road and turn. The greater the friction, the faster the driver can take the turn.

4 Grip The gravitational force pulls the car downward, producing friction between the track and the small area of the tire that touches it, called the contact patch. Air flowing around the car body also produces a downward force on the car, which results in an increased normal force and increased friction at the contact patch.

GOING**FURTHER** >>>

Research Compare at least two different kinds of auto racing events, such as drag racing and stock car, in terms of the different forces at work due to the different car styles, track structures, and racing rules.

CHAPTER 1

STUDY GUIDE

BIG IDEA **You can use vectors and Newton's laws to describe projectile motion and circular motion.**

VOCABULARY

- **projectile**
- **trajectory**

SECTION 1 Projectile Motion

MAIN IDEA A projectile's horizontal motion is independent of its vertical motion.

- The vertical and horizontal motions of a projectile are independent. When there is no air resistance, the horizontal motion component does not experience an acceleration and has constant velocity; the vertical motion component of a projectile experiences a constant acceleration under these same conditions.
- The curved flight path a projectile follows is called a trajectory and is a parabola. The height, time of flight, initial velocity, and horizontal distance of this path are related by the equations of motion. The horizontal distance a projectile travels before returning to its initial height depends on the acceleration due to gravity and on both components of the initial velocity.

VOCABULARY

- **uniform circular motion**
- **centripetal acceleration**
- **centripetal force**

SECTION 2 Circular Motion

MAIN IDEA An object in circular motion has an acceleration toward the circle's center due to an unbalanced force toward the circle's center.

- An object moving in a circle at a constant speed has an acceleration toward the center of the circle because the direction of its velocity is constantly changing.
- Acceleration toward the center of the circle is called centripetal acceleration. It depends directly on the square of the object's speed and inversely on the radius of the circle.

$$a_c = \frac{v^2}{r}$$

- A net force must be exerted by external agents toward the circle's center to cause centripetal acceleration.

$$\boldsymbol{F}_{net} = m\boldsymbol{a}_c$$

VOCABULARY

- **reference frame**

SECTION 3 Relative Velocity

MAIN IDEA An object's velocity depends on the reference frame chosen.

- A coordinate system from which you view motion is called a reference frame. Relative velocity is the velocity of an object observed in a different, moving reference frame.
- You can use vector addition to solve motion problems of an object in a moving reference frame.

SECTION 1 Projectile Motion

Mastering Concepts

40. Some students believe the force that starts the motion of a projectile, such as the kick given a soccer ball, remains with the ball. Is this a correct viewpoint? Present arguments for or against.

41. Consider the trajectory of the cannonball shown in **Figure 19.**

a. Where is the magnitude of the vertical-velocity component largest?

b. Where is the magnitude of the horizontal-velocity component largest?

c. Where is the vertical velocity smallest?

d. Where is the magnitude of the acceleration smallest?

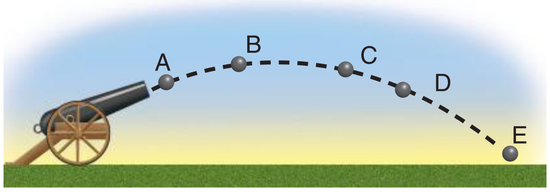


Figure 19

42. Trajectory Describe how forces cause the trajectory of an object launched horizontally to be different from the trajectory of an object launched upward at an angle.

43. Reverse Problem Write a physics problem with real-life objects for which the following equations would be part of the solution. *Hint: The two equations describe the same object.*

$$x = (1.5\ \text{m/s})t \qquad 8.0\ \text{m} = \frac{1}{2}(9.8\ \text{m/s}^2)t^2$$

44. An airplane pilot flying at constant velocity and altitude drops a heavy crate. Ignoring air resistance, where will the plane be relative to the crate when the crate hits the ground? Draw the path of the crate as seen by an observer on the ground.

Mastering Problems

45. You accidentally throw your car keys horizontally at 8.0 m/s from a cliff 64 m high. How far from the base of the cliff should you look for the keys?

46. A dart player throws a dart horizontally at 12.4 m/s. The dart hits the board 0.32 m below the height from which it was thrown. How far away is the player from the board?

47. The toy car in **Figure 20** runs off the edge of a table that is 1.225 m high. The car lands 0.400 m from the base of the table.

a. How long did it take the car to fall?

b. How fast was the car going on the table?

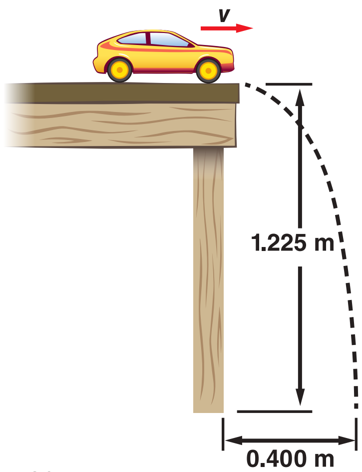


Figure 20

48. Swimming You took a running leap off a high-diving platform. You were running at 2.8 m/s and hit the water 2.6 s later. How high was the platform, and how far from the edge of the platform did you hit the water? Assume your initial velocity is horizontal. Ignore air resistance.

49. Archery An arrow is shot at 30.0° above the horizontal. Its velocity is 49 m/s, and it hits the target.

a. What is the maximum height the arrow will attain?

b. The target is at the height from which the arrow was shot. How far away is it?

50. BIGIDEA A pitched ball is hit by a batter at a 45° angle and just clears the outfield fence, 98 m away. If the top of the fence is at the same height as the pitch, find the velocity of the ball when it left the bat. Ignore air resistance.

51. At-Sea Rescue An airplane traveling 1001 m above the ocean at 125 km/h is going to drop a box of supplies to shipwrecked victims below.

a. How many seconds before the plane is directly overhead should the box be dropped?

b. What is the horizontal distance between the plane and the victims when the box is dropped?

52. Diving Divers dive from a cliff that is 61 m high. What is the minimum horizontal veloc ity a diver must have to enter the water at least 23 m from the cliff?

53. Jump Shot A basketball player is trying to make a half-court jump shot and releases the ball at the height of the basket. Assume that the ball is launched at an angle of 51.0° above the horizontal and a horizontal distance of 14.0 m from the basket. What speed must the player give the ball in order to make the shot?

54. The two baseballs in **Figure 21** were hit with the same speed, 25 m/s. Draw separate graphs of y versus t and x versus t for each ball.

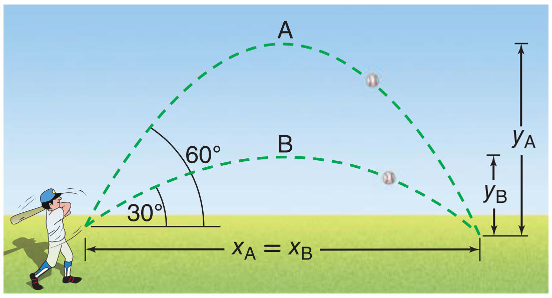


Figure 21

SECTION 2 Circular Motion

Mastering Concepts

55. Can you go around a curve with the following accelerations? Explain.

a. zero acceleration vector

b. constant acceleration vector

56. To obtain uniform circular motion, how must the net force that acts on a moving object depend on the speed of the object?

57. Suppose you whirl a yo-yo about your head in a horizontal circle.

a. In what direction must a force act on the yo-yo?

b. What exerts the force?

c. If you let go of the string on the yo-yo, in which direction would the toy travel? Use Newton's laws in your answer.

Mastering Problems

58. Car Racing A 615 kg racing car completes one lap in a time of 14.3 s around a circular track that has a radius of 50.0 m. Assume the race car moves at a constant speed.

a. What is the acceleration of the car?

b. What force must the track exert on the tires to produce this acceleration?

59. Ranking Task Rank the following objects according to their centripetal accelerations, from least to greatest. Specifically indicate any ties.

A: a 0.50 kg stone moving in a circle of radius 0.6 m at a speed of 2.0 m/s

B: a 0.50 kg stone moving in a circle of radius 1.2 m at a speed of 3.0 m/s

C: a 0.60 kg stone moving in a circle of radius 0.8 m at a speed of 2.4 m/s

D: a 0.75 kg stone moving in a circle of radius 1.2 m at a speed of 3.0 m/s

E: a 0.75 kg stone moving in a circle of radius 0.6 m at a speed of 2.4 m/s

60. Hammer Throw An athlete whirls a 7.00 kg hammer 1.8 m from the axis of rotation in a horizontal circle, as shown in **Figure 22.** If the hammer makes one revolution in 1.0 s, what is the centripetal acceleration of the hammer? What is the tension in the chain?

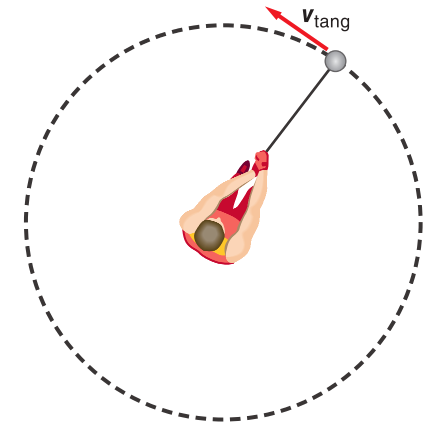


Figure 22

61. A rotating rod that is 15.3 cm long is spun with its axis through one end of the rod. The other end of the rod has a constant speed of 2010 m/s (4500 mph).

a. What is the centripetal acceleration of the end of the rod?

b. If you were to attach a 1.0 g object to the end of the rod, what force would be needed to hold it on the rod?

62. A carnival clown rides a motorcycle down a ramp and then up and around a large, vertical loop. If the loop has a radius of 18 m, what is the slowest speed the rider can have at the top of the loop so that the motorcycle stays in contact with the track and avoids falling? *Hint: At this slowest speed, the track exerts no force on the motorcycle at the top of the loop.*

63. A 75 kg pilot flies a plane in a loop as shown in **Figure 23.** At the top of the loop, when the plane is completely upside-down for an instant, the pilot hangs freely in the seat and does not push against the seat belt. The airspeed indicator reads 120 m/s. What is the radius of the plane's loop?

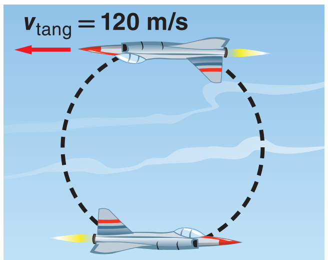


Figure 23

SECTION 3 Relative Velocity

Mastering Concepts

64. Why is it that a car traveling in the opposite direction as the car in which you are riding on the freeway often looks like it is moving faster than the speed limit?

Mastering Problems

65. Ali and Ahmed are sitting by a river and decide to have a race. Ali will run down the shore to a dock, 1.5 km away, then turn around and run back. Ahmed will also race to the dock and back, but he will row a boat in the river, which has a current of 2.0 m/s. If Ali's running speed is equal to Ahmed's rowing speed in still water, which is 4.0 m/s, what will be the outcome of the race? Assume they both turn instantaneously.

66. Crossing a River You row a boat, such as the one in **Figure 24,** perpendicular to the shore of a river that flows at 3.0 m/s. The velocity of your boat is 4.0 m/s relative to the water.

a. What is the velocity of your boat relative to the shore?

b. What is the component of your velocity parallel to the shore? Perpendicular to it?

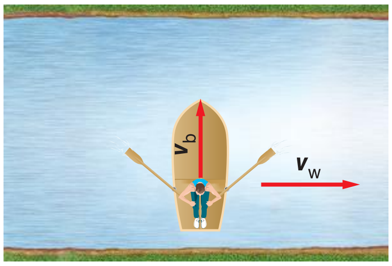


Figure 24

67. Air Travel You are piloting a small plane, and you want to reach an airport 450 km due south in 3.0 h. A wind is blowing from the west at 50.0 km/h. What heading and airspeed should you choose to reach your destination in time?

68. Problem Posing Complete this problem so that it can be solved using the concept of relative velocity: "Salem is on the west bank of a 55-m-wide river with a current of 0.7 m/s ...."

Applying Concepts

69. Projectile Motion Explain how horizontal motion can be uniform while vertical motion is accelerated. How will projectile motion be affected when drag due to air resistance is taken into consideration?

70. Baseball A batter hits a pop-up straight up over home plate at an initial speed of 20 m/s. The ball is caught by the catcher at the same height at which it was hit. At what velocity does the ball land in the catcher's mitt? Neglect air resistance.

71. Fastball In baseball, a fastball takes about 0.5 s to reach the plate. Assuming that such a pitch is thrown horizontally, compare the distance the ball falls in the first 0.25 s with the distance it falls in the second 0.25 s.

72. You throw a rock horizontally. In a second horizontal throw, you throw the rock harder and give it even more speed.

a. How will the time it takes the rock to hit the ground be affected? Ignore air resistance.

b. How will the increased speed affect the distance from where the rock left your hand to where the rock hits the ground?

73. Field Biology A zoologist standing on a cliff aims a tranquilizer gun at a monkey hanging from a tree branch that is in the gun's range. The barrel of the gun is horizontal. Just as the zoologist pulls the trigger, the monkey lets go and begins to fall. Will the dart hit the monkey? Ignore air resistance.

74. Football A player throws a ball at 24 m/s at a 45° angle. If it takes the ball 3.0 s to reach the top of its path and the ball is caught at the same height at which it is thrown, how long is it in the air? Ignore air resistance.

75. Track and Field You are working on improving your performance in the long jump and believe that the information in this chapter can help. Does the height that you reach make any difference to your jump? What influences the length of your jump?

76. **Driving on a Freeway** Explain why it is that when you pass a car going in the same direction as you on the freeway, it takes a longer time than when you pass a car going in the opposite direction.

77. Imagine you are sitting in a car tossing a ball straight up into the air.

a. If the car is moving at a constant velocity, will the ball land in front of, behind, or in your hand?

b. If the car rounds a curve at a constant speed, where will the ball land?

78. You swing one yo-yo around your head in a horizontal circle. Then you swing another yo-yo with twice the mass of the first one, but you don't change the length of the string or the period. How do the tensions in the strings differ?

79. **Car Racing** The curves on a race track are banked to make it easier for cars to go around the curves at high speeds. Draw a free-body diagram of a car on a banked curve. From the motion diagram, find the direction of the acceleration.

a. What exerts the force in the direction of the acceleration?

b. Can you have such a force without friction?

Mixed Review

80. Early skeptics of the idea of a rotating Earth said that the fast spin of Earth would throw people at the equator into space. The radius of Earth is about 6.38×10^3 km. Show why this idea is wrong by calculating the following.

a. the speed of a 97 kg person at the equator

b. the force needed to accelerate the person in the circle

c. the weight of the person

d. the normal force of Earth on the person, that is, the person's apparent weight

81. **Firing a Missile** An airplane moving at 375 m/s relative to the ground fires a missile forward at a speed of 782 m/s relative to the plane. What is the missile's speed relative to the ground?

82. **Rocketry** A rocket in outer space that is moving at a speed of 1.25 km/s relative to an observer fires its motor. Hot gases are expelled out the back at 2.75 km/s relative to the rocket. What is the speed of the gases relative to the observer?

83. A 1.13 kg ball is swung vertically from a 0.50-m cord in uniform circular motion at a speed of 2.4 m/s. What is the tension in the cord at the bottom of the ball's motion?

84. Two horses, initially separated by 500.0 m, are running toward each other, each moving with a constant speed of 2.5 m/s. A dragonfly, moving with a constant speed of 3.0 m/s, flies from the nose of one horse to the other, then turns around instantaneously and flies back to the other horse. It continues to fly back and forth until the horses run into each other. What distance does the dragonfly fly during this time?

85. **Banked Roads** Curves on roads often are banked to help prevent cars from slipping off the road. If the speed limit for a particular curve of radius 36.0 m is 15.7 m/s (35 mph), at what angle should the road be banked so that cars will stay on a circular path even if there were no friction between the road and the tires? If the speed limit was increased to 20.1 m/s (45 mph), at what angle should the road be banked?

86. The 1.45 kg ball in **Figure 25** is suspended from a 0.80-m string and swung in a horizontal circle at a constant speed.

a. What is the tension in the string?

b. What is the speed of the ball?

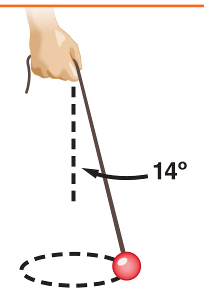


Figure 25

87. A baseball is hit directly in line with an outfielder at an angle of 35.0° above the horizontal with an initial speed of 22.0 m/s. The outfielder starts running as soon as the ball is hit at a constant speed of 2.5 m/s and barely catches the ball. Assuming that the ball is caught at the same height at which it was hit, what was the initial separation between the hitter and the outfielder? *Hint: There are two possible answers.*

88. Mohamed has a bag of marbles at the top of a 60.0 m tall building. Meanwhile, Tareq is in a hot-air balloon 20.0 m from the base of the building and untethers the balloon, so it begins to rise at a constant speed. Mohamed tosses the bag of marbles horizontally with a speed of 7.3 m/s just as the balloon begins its ascent. What must the velocity of the balloon be for Tareq to easily catch the bag?

Thinking Critically

89. Apply Concepts Consider a roller-coaster loop like the one in **Figure 26.** Are the cars traveling through the loop in uniform circular motion? Explain.

Figure 26

90. Apply Computers and Calculators A baseball player hits a belt-high (1.0 m) fastball down the left-field line. The player hits the ball with an initial velocity of 42.0 m/s at an angle 26° above the horizontal. The left-field wall is 96.0 m from home plate at the foul pole and is 14 m high. Write the equation for the height of the ball (y) as a function of its distance from home plate (x). Use a computer or graphing calculator to plot the path of the ball. Trace along the path to find how high above the ground the ball is when it is at the wall.

a. Is the hit a home run?

b. What is the minimum speed at which the ball could be hit and clear the wall?

c. If the initial velocity of the ball is 42.0 m/s, for what range of angles will the ball go over the wall?

91. Analyze Albert Einstein showed that the rule you learned for the addition of velocities does not work for objects moving near the speed of light. For example, if a rocket moving at speed v_A releases a missile that has speed v_B relative to the rocket, then the speed of the missile relative to an observer that is at rest is given by $v = \frac{v_A + v_B}{1 + \frac{v_A v_B}{c^2}}$, where c is the speed of light, 3.00×10^8 m/s. This formula gives the correct values for objects moving at slow speeds as well. Suppose a rocket moving at 11 km/s shoots a laser beam out in front of it. What speed would an unmoving observer find for the laser light? Suppose that a rocket moves at a speed $\frac{c}{2}$, half the speed of light, and shoots a missile forward at a speed of $\frac{c}{2}$ relative to the rocket. How fast would the missile be moving relative to a fixed observer?

92. Analyze and Conclude A ball on a light string moves in a vertical circle. Analyze and describe the motion of this system. Be sure to consider the effects of gravity and tension. Is this system in uniform circular motion? Explain your answer.

Writing In Physics

93. Roller Coasters The vertical loops on most roller coasters are not circular in shape. Research and explain the physics behind this design choice.

94. Many amusement-park rides utilize centripetal acceleration to create thrills for the park's customers. Choose two rides other than roller coasters that involve circular motion, and explain how the physics of circular motion creates the sensations for the riders.

Cumulative Review

95. Multiply or divide, as indicated, using significant figures correctly.

a. $(5\times10^8\ \text{m})(4.2\times10^7\ \text{m})$

b. $(1.67\times10^{-2}\ \text{km})(8.5\times10^{-6}\ \text{km})$

c. $\frac{2.6\times10^4\ \text{kg}}{9.4\times10^3\ \text{m}^3}$

d. $\frac{6.3\times10^{-1}\ \text{m}}{3.8\times10^2\ \text{s}}$

96. Plot the data in **Table 1** on a position-time graph. Find the average speed in the time interval between 0.0 s and 5.0 s.

Table 1 Position v. Time

Clock Reading *t* (s)	Position *x* (m)
0.0	30
1.0	30
2.0	35
3.0	45
4.0	60
5.0	70

97. Moustafa and his older brother Amir are at the grocery store. Moustafa, with mass 17.0 kg, likes to hang on the front of the cart while Amir pushes it, even though both boys know this is not safe. Amir pushes the 12.4 kg cart with his brother on it such that they accelerate at a rate of 0.20 m/s^2.

a. With what force is Amir pushing?

b. What is the force the cart exerts on Moustafa?

MULTIPLE CHOICE

1. A 1.60-m-tall girl throws a football at an angle of 41.0° from the horizontal and at an initial speed of 9.40 m/s. What is the horizontal distance between the girl and the spot when the ball is again at the height above the ground from which the girl threw it?

A. 4.55 m **C.** 8.90 m

B. 5.90 m **D.** 10.5 m

2. A child is sitting on a merry-go-round 2.8 m from the center. If the tangential velocity of the ride is 0.89 m/s, what is the centripetal acceleration of the dragonfly?

A. 0.11 m/s^2 **C.** 0.32 m/s^2

B. 0.28 m/s^2 **D.** 2.2 m/s^2

3. The force exerted by a 2.0 m massless string on a 0.82 kg object being swung in a horizontal circle is 4.0 N. What is the tangential velocity of the object?

A. 2.8 m/s **C.** 4.9 m/s

B. 3.1 m/s **D.** 9.8 m/s

4. A 1000 kg car enters an 80.0 m-radius curve at 20.0 m/s. What centripetal force must be supplied by friction so the car does not skid?

A. 5.0 N **C.** 5.0×10^3 N

B. 2.5×10^2 N **D.** 1.0×10^3 N

5. A jogger on a riverside path sees a rowing team coming toward him. Relative to the ground, the jogger is running at 10 km/h west and the boat is sailing at 20 km/h east. How quickly does the jogger approach the boat?

A. 10 km/h **C.** 20 km/h

B. 30 km/h **D.** 40 km/h

6. What is the maximum height obtained by a 125-g apple that is slung from a slingshot at an angle of 78° from the horizontal with an initial velocity of 18 m/s?

A. 0.70 m **C.** 32 m

B. 16 m **D.** 33 m

7. An orange is dropped at the same time and from the same height that a bullet is shot from a gun. Which of the following is true?

A. The acceleration due to gravity is greater for the orange because the orange is heavier.

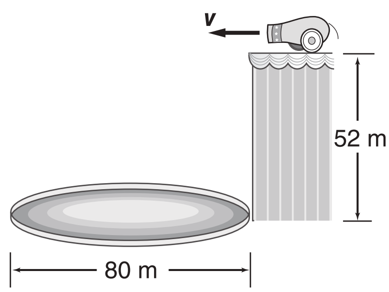
B. Gravity acts less on the bullet than on the orange because the bullet is moving so quickly.

C. The velocities will be the same.

D. The two objects will hit the ground at the same time.

FREE RESPONSE

8. A lead cannonball is shot horizontally at a speed of 25 m/s out of the circus cannon, shown in the figure, on the high-wire platform on one side of a circus ring. If the platform is 52 m above the 80-m diameter ring, will the performers need to adjust their cannon so that the ball will land inside the ring instead of past it? Explain.



9. A mythical TV character swings a 5.6 kg mace on the end of a magically massless 86-cm chain in a horizontal circle above his head. The mace makes one full revolution in 1.8 s. Find the tension in the magical chain.

CHAPTER 2

Gravitation

BIG IDEA Gravity is an attractive field force that acts between objects with mass.

SECTIONS

1 Planetary Motion and Gravitation

2 Using the Law of Universal Gravitation

LaunchLAB

MODEL MERCURY'S MOTION

How can measurements of angles and distances be used to draw a model of an orbit?

Chapter Sourced From: 7. Gravitation, Chapter 7, from Glencoe Physics: Principles & Problems, Student Edition ©2017

SECTION 1 Planetary Motion and Gravitation

PHYSICS 4 YOU Our solar system includes the Sun, Earth and seven other major planets, dwarf planets, and interplanetary dust and gas. Various moons orbit the planets. What holds all this together?

MAIN IDEA

The gravitational force between two objects is proportional to the product of their masses divided by the square of the distance between them.

Essential Questions

- What is the relationship between a planet's orbital radius and period?
- What is Newton's law of universal gravitation, and how does it relate to Kepler's laws?
- Why was Cavendish's investigation important?

Review Vocabulary

Newton's third law states all forces come in pairs and that the two forces in a pair act on different objects, are equal in strength, and are opposite in direction

New Vocabulary

Kepler's first law
Kepler's second law
Kepler's third law
gravitational force
law of universal gravitation

Early Observations

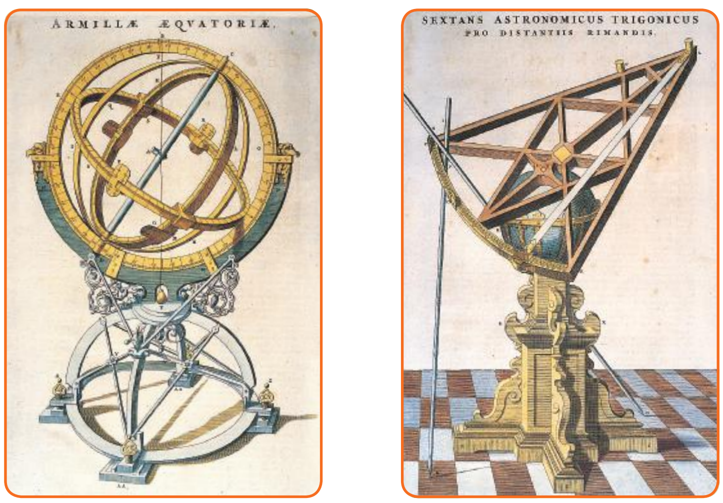
In ancient times, the Sun, the Moon, the planets, and the stars were assumed to revolve around Earth. Nicholas Copernicus, a Polish astronomer, noticed that the best available observations of the movements of planets did not fully agree with the Earth-centered model.

The results of his many years of work were published in 1543, when Copernicus was on his deathbed. His book showed that the motion of planets is much more easily understood by assuming that Earth and other planets revolve around the Sun. His model helped explain phenomena such as the inner planets Mercury and Venus always appearing near the Sun. Copernicus's view advanced our understanding of planetary motion. He incorrectly assumed, however, that planetary orbits are circular. This assumption did not fit well with observations, and modification of Copernicus's model was necessary to make it accurate.

Tycho Brahe was born a few years after Copernicus died. As a boy of 14 in Denmark, Tycho observed an eclipse of the Sun on August 21, 1560. The fact that it had been predicted inspired him toward a career in astronomy.

As Tycho studied astronomy, he realized that the charts of the time did not accurately predict astronomical events. Tycho recognized that measurements were required from one location over a long period of time. He was granted an estate on the Danish island of Hven and the funding to build an early research institute. Telescopes had not been invented, so to make measurements, Tycho used huge instruments that he designed and built in his own shop, such as those shown in **Figure 1.** Tycho is credited with the most accurate measurements of the time.

Figure 1 Instruments such as these were used by Tycho to measure the positions of planets.



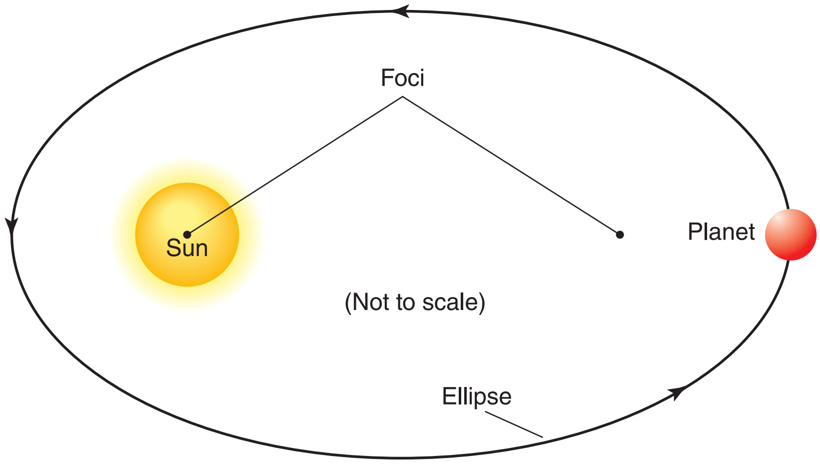


Figure 2 The orbit of each planet is an ellipse, with the Sun at one focus.

Kepler's Laws

In 1600 Tycho moved to Prague where Johannes Kepler, a 29-year-old German, became one of his assistants. Kepler analyzed Tycho's observations. After Tycho's death in 1601, Kepler continued to study Tycho's data and used geometry and mathematics to explain the motion of the planets. After seven years of careful analysis of Tycho's data on Mars, Kepler discovered the laws that describe the motion of every planet and satellite, natural or artificial. Here, the laws are presented in terms of planets.

Kepler's first law states that the paths of the planets are ellipses, with the Sun at one focus. An ellipse has two foci, as shown in **Figure 2.** Although exaggerated ellipses are used in the diagrams, Earth's actual orbit is very nearly circular. You would not be able to distinguish it from a circle visually.

Kepler found that the planets move faster when they are closer to the Sun and slower when they are farther away from the Sun. **Kepler's second law** states that an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals, as illustrated in **Figure 3.**

☑ READING CHECK **Compare** the distances traveled from point 1 to point 2 and from point 6 to point 7 in **Figure 3.** Through which distance would Earth be traveling fastest?

A period is the time it takes for one revolution of an orbiting body. Kepler also discovered a mathematical relationship between periods of planets and their mean distances away from the Sun.

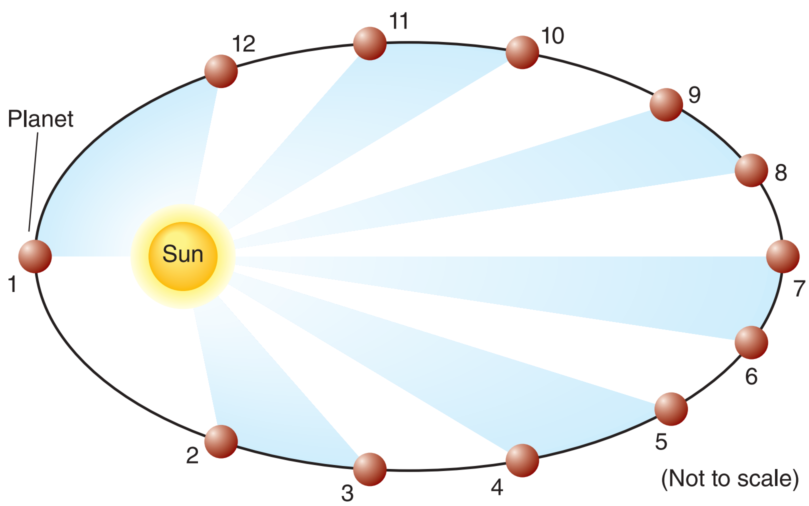


Figure 3 Kepler found that elliptical orbits sweep out equal areas in equal time periods.

Explain why the equal time areas are shaped differently.

Table 1 Solar System Data

Name	Average Radius (m)	Mass (kg)	Average Distance from the Sun (m)
Sun	6.96×10^{8}	1.99×10^{30}	—
Mercury	2.44×10^{6}	3.30×10^{23}	5.79×10^{10}
Venus	6.05×10^{6}	4.87×10^{24}	1.08×10^{11}
Earth	6.38×10^{6}	5.97×10^{24}	1.50×10^{11}
Mars	3.40×10^{6}	6.42×10^{23}	2.28×10^{11}
Jupiter	7.15×10^{7}	1.90×10^{27}	7.78×10^{11}
Saturn	6.03×10^{7}	5.69×10^{26}	1.43×10^{12}
Uranus	2.56×10^{7}	8.68×10^{25}	2.87×10^{12}
Neptune	2.48×10^{7}	1.02×10^{26}	4.50×10^{12}

Kepler's third law states that the square of the ratio of the periods of any two planets revolving about the Sun is equal to the cube of the ratio of their average distances from the Sun. Thus, if the periods of the planets are T_A and T_B and their average distances from the Sun are r_A and r_B, Kepler's third law can be expressed as follows.

KEPLER'S THIRD LAW

The square of the ratio of the period of planet A to the period of planet B is equal to the cube of the ratio of the distance between the centers of planet A and the Sun to the distance between the centers of planet B and the Sun.

$$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$$

Note that Kepler's first two laws apply to each planet, moon, and satellite individually. The third law, however, relates the motion of two objects around a single body. For example, it can be used to compare the planets' distances from the Sun, shown in **Table 1,** to their periods around the Sun. It also can be used to compare distances and periods of the Moon and artificial satellites orbiting Earth.

PhysicsLAB

MODELING ORBITS

What is the shape of the orbits of planets and satellites in the solar system?

Comet periods Comets are classified as long-period comets or short-period comets based on orbital periods. Long-period comets have orbital periods longer than 200 years and short-period comets have orbital periods shorter than 200 years. Comet Hale-Bopp, shown in **Figure 4,** with a period of approximately 2400 years, is an example of a long-period comet. Comet Halley, with a period of 76 years, is an example of a short-period comet. Comets also obey Kepler's laws. Unlike planets, however, comets have highly elliptical orbits.

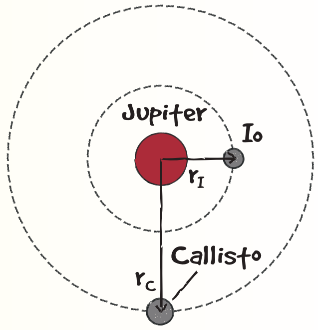
Figure 4 Hale-Bopp is a long-period comet, with a period of 2400 years. This photo was taken in 1997, when Hale-Bopp was highly visible.

EXAMPLE 1

CALLISTO'S DISTANCE FROM JUPITER Galileo measured the orbital radii of Jupiter's moons using the diameter of Jupiter as a unit of measure. He found that Io, the closest moon to Jupiter, has a period of 1.8 days and is 4.2 units from the center of Jupiter. Callisto, the fourth moon from Jupiter, has a period of 16.7 days. Using the same units that Galileo used, predict Callisto's distance from Jupiter.

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the orbits of Io and Callisto.
- Label the radii.



KNOWN

$T_C = 16.7$ days

$T_I = 1.8$ days

$r_I = 4.2$ units

UNKNOWN

$r_C = ?$

2 SOLVE FOR CALLISTO'S DISTANCE FROM JUPITER

Solve Kepler's third law for r_C.

$$\left(\frac{T_C}{T_I}\right)^2 = \left(\frac{r_C}{r_I}\right)^3$$

$$r_C{}^3 = r_I{}^3\left(\frac{T_C}{T_I}\right)^2$$

$$r_C = \sqrt[3]{r_I{}^3\left(\frac{T_C}{T_I}\right)^2}$$

◀ Substitute $r_I = 4.2$ units, $T_C = 16.7$ days, $T_I = 1.8$ days

$$= \sqrt[3]{(4.2\text{ units})^3\left(\frac{16.7\text{ days}}{1.8\text{ days}}\right)^2}$$

$$= \sqrt[3]{6.4\times10^3\text{ units}^3}$$

$$= 19\text{ units}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** r_C should be in Galileo's units, like r_I.
- **Is the magnitude realistic?** The period is larger, so the radius should be larger.

APPLICATIONS

1. If Ganymede, one of Jupiter's moons, has a period of 32 days, how many units is its orbital radius? Use the information given in Example 1.

2. An asteroid revolves around the Sun with a mean orbital radius twice that of Earth's. Predict the period of the asteroid in Earth years.

3. Venus has a period of revolution of 225 Earth days. Find the distance between the Sun and Venus as a multiple of Earth's average distance from the Sun.

4. Uranus requires 84 years to circle the Sun. Find Uranus's average distance from the Sun as a multiple of Earth's average distance from the Sun.

5. From **Table 1** you can find that, on average, Mars is 1.52 times as far from the Sun as Earth is. Predict the time required for Mars to orbit the Sun in Earth days.

6. The Moon has a period of 27.3 days and a mean distance of 3.9×10^5 km from its center to the center of Earth.

a. Use Kepler's laws to find the period of a satellite in orbit 6.70×10^3 km from the center of Earth.

b. How far above Earth's surface is this satellite?

7. CHALLENGE Using the data in the previous problem for the period and radius of revolution of the Moon, predict what the mean distance from Earth's center would be for an artificial satellite that has a period of exactly 1.00 day.

Newton's Law of Universal Gravitation

In 1666, Isaac Newton began his studies of planetary motion. It has been said that seeing an apple fall made Newton wonder if the force that caused the apple to fall might extend to the Moon, or even beyond. He found that the magnitude of the force (F_g) on a planet due to the Sun varies inversely with the square of the distance (r) between the centers of the planet and the Sun. That is, F_g is proportional to $\frac{1}{r^2}$. The force ($\boldsymbol{F}_g$) acts in the direction of the line connecting the centers of the two objects, as shown in **Figure 5.**

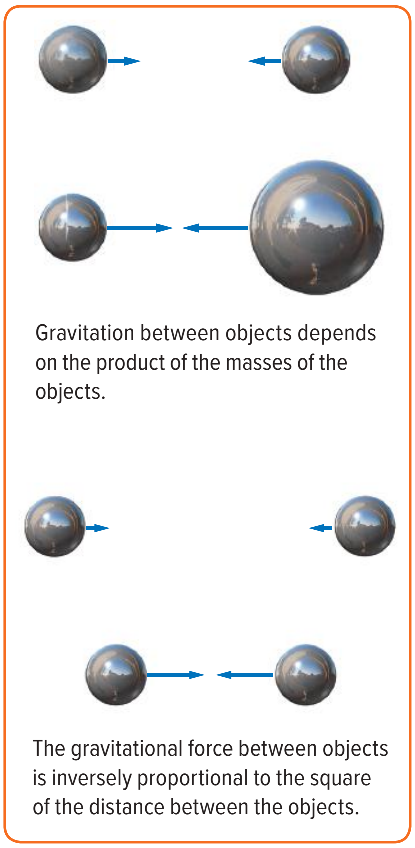


Figure 5 Mass and distance affect the magnitude of the gravitational force between objects.

The force of attraction between two objects must be proportional to the objects' masses and is known as the **gravitational force.**

Newton was confident that the same force of attraction would act between any two objects anywhere in the universe. He proposed the **law of universal gravitation,** which states that objects attract other objects with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them as shown below.

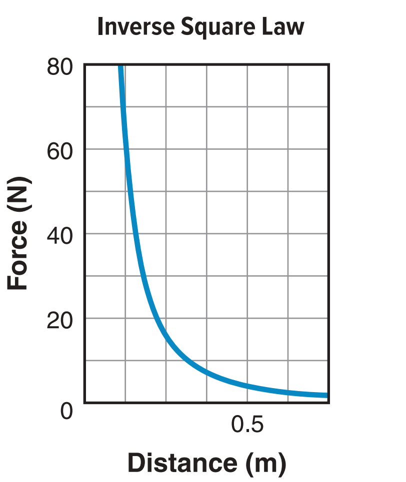
LAW OF UNIVERSAL GRAVITATION

The gravitational force is equal to the universal gravitational constant, times the mass of object 1, times the mass of object 2, divided by the distance between the centers of the objects, squared.

$$F_g = \frac{Gm_1m_2}{r^2}$$

According to Newton's equation, F is directly proportional to m_1 and m_2. If the mass of a planet near the Sun doubles, the force of attraction doubles. Use the Connecting Math to Physics feature below to examine how changing one variable affects another. **Figure 6** illustrates the inverse square relationship graphically. The term G is the universal gravitational constant and will be discussed in the next sections.

Figure 6 This is a graphical representation of the inverse square relationship.



CONNECTING MATH TO PHYSICS

Direct and Inverse Relationships Newton's law of universal gravitation has both direct and inverse relationships.

$F_g \propto m_1m_2$		$F_g \propto \frac{1}{r^2}$	
Change	**Result**	**Change**	**Result**
$(2m_1)m_2$	$2F_g$	$2r$	$\frac{1}{4}F_g$
$(3m_1)m_2$	$3F_g$	$3r$	$\frac{1}{9}F_g$
$(2m_1)(3m_2)$	$6F_g$	$\frac{1}{2}r$	$4F_g$
$\left(\frac{1}{2}\right)m_1m_2$	$\frac{1}{2}F_g$	$\frac{1}{3}r$	$9F_g$

Universal Gravitation and Kepler's Third Law

Newton stated the law of universal gravitation in terms that applied to the motion of planets about the Sun. This agreed with Kepler's third law and confirmed that Newton's law fit the best observations of the day.

Consider a planet orbiting the Sun, as shown in **Figure 7.** Newton's second law of motion, $F_{net} = ma$, can be written as $F_{net} = m_p a_c$, where F_{net} is the magnitude of the gravitational force, m_p is the mass of the planet, and a_c is the centripetal acceleration of the planet. Forsimplicity, assume circular orbits. Recall from your study of uniform circular motion that for a circular orbit $a_c = \frac{4\pi^2 r}{T^2}$. This means that $F_{net} = m_p a_c$ may now be written $F_{net} = \frac{m_p 4\pi^2 r}{T^2}$. In this equation, T is the time in seconds required for the planet to make one complete revolution about the Sun. If you set the right side of this equation equal to the right side of the law of universal gravitation, you arrive at the following result:

$$\frac{Gm_S m_p}{r^2} = \frac{m_p 4\pi^2 r}{T^2}$$

$$T^2 = \left(\frac{4\pi^2}{Gm_S}\right) r^3$$

$$T = \sqrt{\left(\frac{4\pi^2}{Gm_S}\right) r^3}$$

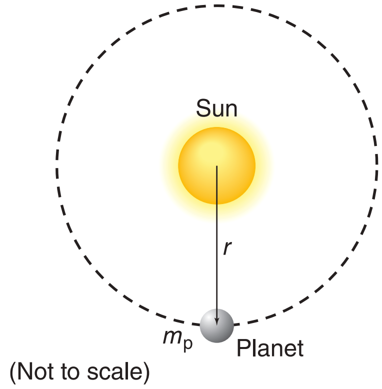


Figure 7 A planet with mass m_p and average distance from the Sun r orbits the Sun. The mass of the Sun is m_S.

The period of a planet orbiting the Sun can be expressed as follows.

PERIOD OF A PLANET ORBITING THE SUN

The period of a planet orbiting the Sun is equal to 2π times the square root of the average distance from the Sun cubed, divided by the product of the universal gravitational constant and the mass of the Sun.

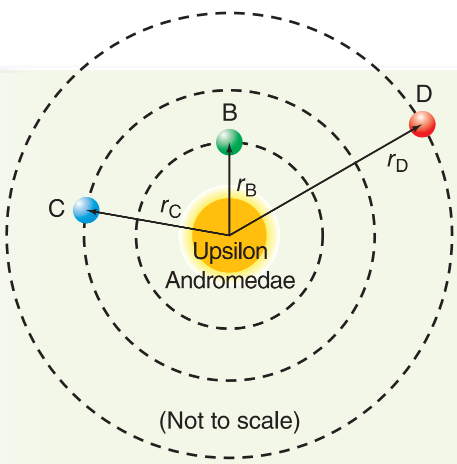
$$T = 2\pi\sqrt{\frac{r^3}{Gm_S}}$$

Squaring both sides makes it apparent that this equation is Kepler's third law of planetary motion: the square of the period is proportional to the cube of the distance that separates the masses. The factor $\frac{4\pi^2}{Gm_S}$ depends on the mass of the Sun and the universal gravitational constant. Newton found that this factor applied to elliptical orbits as well.

PHYSICS CHALLENGE

Astronomers have detected three planets that orbit the star Upsilon Andromedae. Planet B has an average orbital radius of 0.0595 AU and a period of 4.6171 days. Planet C has an average orbital radius of 0.832 AU and a period of 241.33 days. Planet D has an average orbital radius of 2.53 AU and a period of 1278.1 days. (Distances are given in astronomical units (AU)—Earth's average distance from the Sun. The distance from Earth to the Sun is 1.00 AU.)

1. Do these planets obey Kepler's third law?
2. Find the mass of the star Upsilon Andromedae in units of the Sun's mass. *Hint: compare* $\frac{r^3}{T^2}$ *for these planets with that of Earth in the same units (AU and days).*



Measuring the Universal Gravitational Constant

How large is the constant G? As you know, the force of gravitational attraction between two objects on Earth is relatively small. The slightest attraction, even between two massive bowling balls, is almost impossible to detect. In fact, it took 100 years from the time of Newton's work for scientists to develop an apparatus that was sensitive enough to measure the force of gravitational attraction.

Cavendish's apparatus In 1798 English scientist Henry Cavendish used equipment similar to the apparatus shown in **Figure 8** to measure the gravitational force between two objects. The apparatus has a horizontal rod with small lead spheres attached to each end. The rod is suspended at its midpoint so that it can rotate. Because the rod is suspended by a thin wire, the rod and spheres are very sensitive to horizontal forces.

To measure G, two large spheres are placed in a fixed position close to each of the two small spheres, as shown in **Figure 8.** The force of attraction between the large and small spheres causes the rod to rotate. When the force required to twist the wire equals the gravitational force between the spheres, the rod stops rotating. By measuring the angle through which the rod turns, the attractive force between the objects can be calculated.

READING CHECK **Explain** why the rod and sphere in Cavendish's apparatus must be sensitive to horizontal forces.

The angle through which the rod turns is measured by using a beam of light that is reflected from the mirror. The distances between the sphere's centers and the force can both be measured. The masses of the spheres are known. By substituting the values for force, mass, and distance into Newton's law of universal gravitation, an experimental value for G is found: when m_1 and m_2 are measured in kilograms, r in meters, and F in newtons, $G = 6.67\times10^{-11}\ N{\bullet}m^2/kg^2$.

Cavendish Balance

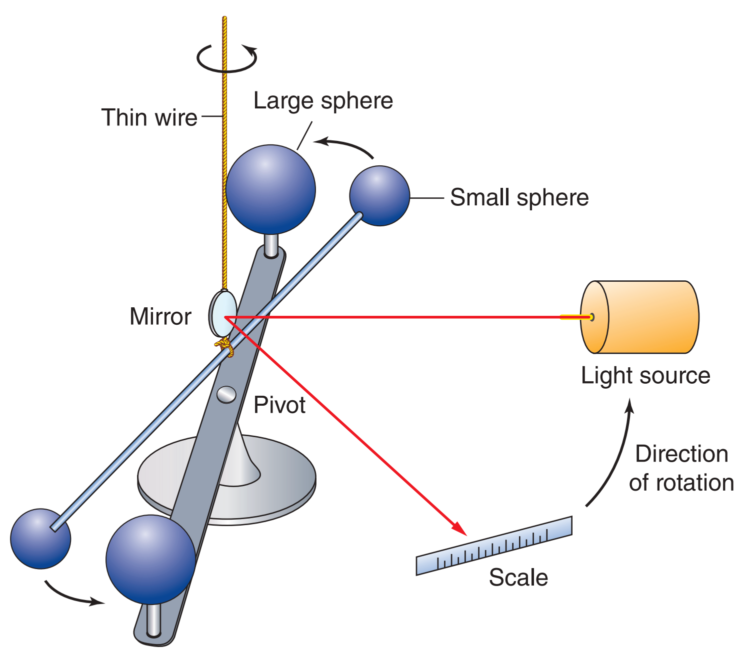


Figure 8 A Cavendish balance uses a light source and a mirror to measure the movement of the spheres.

The importance of G Cavendish's investigation often is called "weighing Earth" because it helped determine Earth's mass. Once the value of G is known, not only the mass of Earth, but also the mass of the Sun can be determined. In addition, the gravitational force between any two objects can be calculated by using Newton's law of universal gravitation. For example, the attractive gravitational force (F_g) between two bowling balls of mass 7.26 kg, with their centers separated by 0.30 m, can be calculated as follows:

$$F_g = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.26\ \text{kg})(7.26\ \text{kg})}{(0.30\ \text{m})^2} = 3.9\times10^{-8}\ \text{N}$$

You know that on Earth's surface, the weight of an object of mass m is a measure of Earth's gravitational attraction: $F_g = mg$. If Earth's mass is represented by m_E and Earth's radius is represented by r_E, the following is true:

$$F_g = \frac{Gm_Em}{{r_E}^2} = mg, \text{ and so } g = \frac{Gm_E}{{r_E}^2}$$

This equation can be rearranged to solve for m_E.

$$m_E = \frac{g{r_E}^2}{G}$$

Using $g = 9.8$ N/kg, $r_E = 6.38\times10^6$ m, and $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$, the following result is obtained for Earth's mass:

$$m_E = \frac{(9.8\ \text{N/kg})(6.38\times10^6\ \text{m})^2}{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 5.98\times10^{24}\ \text{kg}$$

When you compare the mass of Earth to that of a bowling ball, you can see why the gravitational attraction between everyday objects is not easily observed. Cavendish's investigation determined the value of G, confirmed Newton's prediction that a gravitational force exists between any two objects, and helped calculate the mass of Earth **(Figure 9).**

Figure 9 Cavendish's investigations helped calculate the mass of Earth.

SECTION 1 REVIEW

8. **MAIN IDEA** What is the gravitational force between two 15-kg balls whose centers are 35 m apart? What fraction is this of the weight of one ball?

9. **Neptune's Orbital Period** Neptune orbits the Sun at an average distance given in **Figure 10,** which allows gases, such as methane, to condense and form an atmosphere. If the mass of the Sun is 1.99×10^{30} kg, calculate the period of Neptune's orbit.

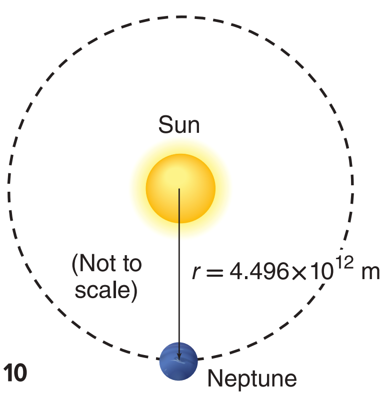


Figure 10

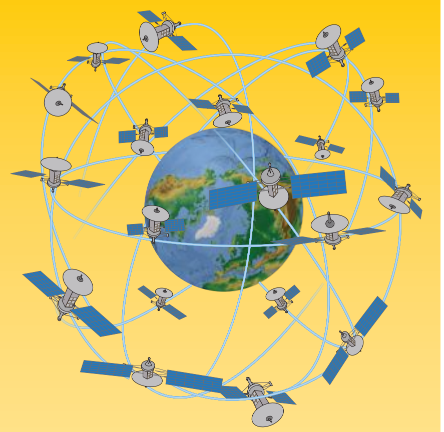
10. **Gravity** If Earth began to shrink, but its mass remained the same, what would happen to the value of g on Earth's surface?

11. **Universal Gravitational Constant** Cavendish did his investigation using lead spheres. Suppose he had replaced the lead spheres with copper spheres of equal mass. Would his value of G be the same or different? Explain.

12. Kepler's three statements and Newton's equation for gravitational attraction are called laws. Were they ever theories? Will they ever become theories?

13. **Critical Thinking** Picking up a rock requires less effort on the Moon than on Earth.
   a. How will the Moon's gravitational force affect the path of the rock if it is thrown horizontally?
   b. If the thrower accidentally drops the rock on her toe, will it hurt more or less than it would on Earth? Explain.

SECTION 2 Using the Law of Universal Gravitation



MAIN IDEA
All objects are surrounded by a gravitational field that affects the motions of other objects.

Essential Questions

- How can you describe orbital motion?
- How are gravitational mass and inertial mass alike, and how are they different?
- How is gravitational force explained, and what did Einstein propose about gravitational force?

Review Vocabulary

centripetal acceleration the center-seeking acceleration of an object moving in a circle at a constant speed

New Vocabulary

inertial mass
gravitational mass

PHYSICS 4 YOU Have you ever used a device to locate your position or to map where you want to go? Where does that device get information? The Global Positioning System (GPS) consists of many satellites circling Earth. GPS satellites give accurate position data anywhere on or near Earth.

Orbits of Planets and Satellites

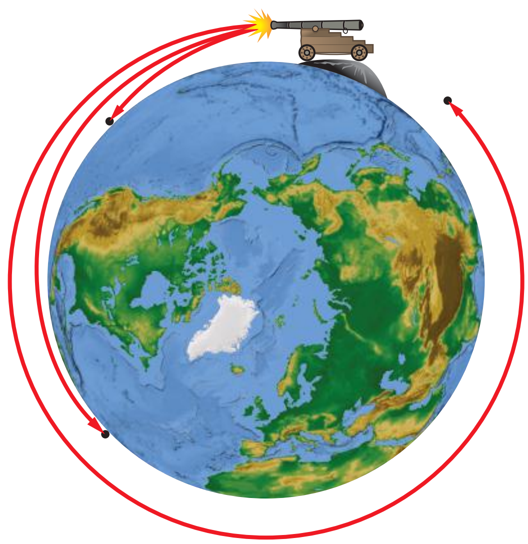
The planet Uranus was discovered in 1781. By 1830 it was clear that the law of gravitation didn't correctly predict its orbit. Two astronomers proposed that Uranus was being attracted by the Sun and by an undiscovered planet. They calculated the orbit of such a planet in 1845, and, one year later, astronomers at the Berlin Observatory found the planet now called Neptune. How is it possible for planets, such as Neptune and Uranus, to remain in orbit around the Sun?

Newton used a drawing similar to the one shown in **Figure 11** to illustrate a thought experiment on the motion of satellites. Imagine a cannon, perched high atop a mountain, firing a cannonball horizontally with a given horizontal speed. The cannonball is a projectile, and its motion has both vertical and horizontal components. Like all projectiles on Earth, it would follow a parabolic trajectory and fall back to the ground.

If the cannonball's horizontal speed were increased, it would travel farther across the surface of Earth and still fall back to the ground. If an extremely powerful cannon were used, however, the cannonball would travel all the way around Earth and keep going. It would fall toward Earth at the same rate that Earth's surface curves away. In other words, the curvature of the projectile would continue to just match the curvature of Earth so that the cannonball would never get any closer to or farther away from Earth's curved surface. The cannonball would, therefore, be in orbit.

Figure 11 Newton imagined a projectile launched parallel to Earth. If it has enough speed it will fall toward Earth with a curvature that matches the curvature of Earth's surface.

Identify the factor that is not considered in this example.



Newton's thought experiment ignored air resistance. For the cannonball to be free of air resistance, the mountain on which the cannon is perched would have to be more than 150 km above Earth's surface. By way of comparison, the mountain would have to be much taller than the peak of Mount Everest, the world's tallest mountain, which is only 8.85 km in height. A cannonball launched from a mountain that is 150 km above Earth's surface would encounter little or no air resistance at an altitude of 150 km because the mountain would be above most of the atmosphere. Thus, a cannonball or any object or satellite at or above this altitude could orbit Earth.

A satellite's speed A satellite in an orbit that is always the same height above Earth moves in uniform circular motion. Recall that its centripetal acceleration is given by $a_c = \frac{v^2}{r}$. Newton's second law, $F_{net} = ma_c$, can thus be rewritten $F_{net} = \frac{mv^2}{r}$. If Earth's mass is m_E, then this expression combined with Newton's law of universal gravitation produces the following equation:

$$\frac{Gm_Em}{r^2} = \frac{mv^2}{r}$$

Solving for the speed of a satellite in circular orbit about Earth (v) yields the following.

SPEED OF A SATELLITE ORBITING EARTH

The speed of a satellite orbiting Earth is equal to the square root of the universal gravitational constant times the mass of Earth, divided by the radius of the orbit.

$$v = \sqrt{\frac{Gm_E}{r}}$$

A satellite's orbital period A satellite's orbit around Earth is similar to a planet's orbit about the Sun. Recall that the period of a planet orbiting the Sun is expressed by the following equation:

$$T = 2\pi\sqrt{\frac{r^3}{Gm_S}}$$

Thus, the period for a satellite orbiting Earth is given by the following equation.

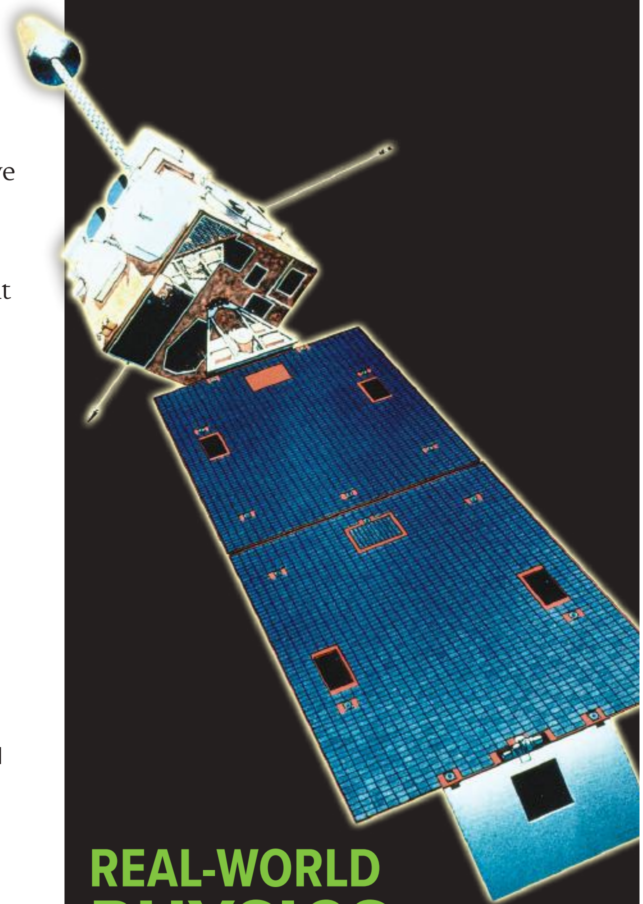
PERIOD OF A SATELLITE ORBITING EARTH

The period for a satellite orbiting Earth is equal to 2π times the square root of the radius of the orbit cubed, divided by the product of the universal gravitational constant and the mass of Earth.

$$T = 2\pi\sqrt{\frac{r^3}{Gm_E}}$$

The equations for the speed and period of a satellite can be used for any object in orbit about another. The mass of the central body will replace m_E in the equations, and r will be the distance between the centers of the orbiting body and the central body. Orbital speed (v) and period (T) are independent of the mass of the satellite.

☑ **READING CHECK** **Describe** how the mass of a satellite affects that satellite's orbital speed and period.



REAL-WORLD PHYSICS

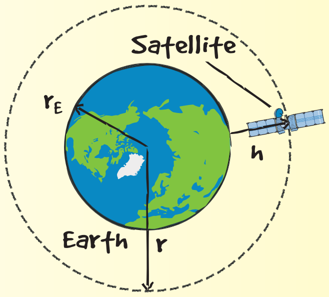
GEOSYNCHRONOUS ORBIT The GOES weather satellites orbit Earth once a day at an altitude of 35,785 km. The orbital speed of the satellite matches Earth's rate of rotation. Thus, to an observer on Earth, the satellite appears to remain above one spot. Dish antennas on Earth can be directed to one point in the sky and remain in a fixed position as the satellite orbits.

EXAMPLE 2

ORBITAL SPEED AND PERIOD Assume that a satellite orbits Earth 225 km above its surface. Given that the mass of Earth is 5.97×10^{24} kg and the radius of Earth is 6.38×10^{6} m, what are the satellite's orbital speed and period?

1 ANALYZE AND SKETCH THE PROBLEM

Sketch the situation showing the height of the satellite's orbit.



KNOWN	UNKNOWN
$h = 2.25\times10^{5}$ m	$v = ?$
$r_E = 6.38\times10^{6}$ m	$T = ?$
$m_E = 5.97\times10^{24}$ kg	
$G = 6.67\times10^{-11}$ N•m²/kg²	

2 SOLVE FOR ORBITAL SPEED AND PERIOD

Determine the orbital radius by adding the height of the satellite's orbit to Earth's radius.

$$r = h + r_E$$
$$= 2.25\times10^{5}\text{ m} + 6.38\times10^{6}\text{ m} = 6.60\times10^{6}\text{ m}$$

◀ *Substitute $h = 2.25\times10^{5}$ m and $r_E = 6.38\times10^{6}$ m.*

Solve for the speed.

$$v = \sqrt{\frac{Gm_E}{r}}$$
$$= \sqrt{\frac{(6.67\times10^{-11}\text{ N·m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}{6.60\times10^{6}\text{ m}}}$$
$$= 7.77\times10^{3}\text{ m/s}$$

◀ *Substitute $G = 6.67\times10^{-11}$ N·m²/kg², $m_E = 5.97\times10^{24}$ kg, and $r = 6.60\times10^{6}$ m.*

Solve for the period.

$$T = 2\pi\sqrt{\frac{r^3}{Gm_E}}$$
$$= 2\pi\sqrt{\frac{(6.60\times10^{6}\text{ m})^3}{(6.67\times10^{-11}\text{ N·m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}}$$
$$= 5.34\times10^{3}\text{ s}$$

◀ *Substitute $r = 6.60\times10^{6}$ m, $G = 6.67\times10^{-11}$ N·m²/kg², and $m_E = 5.97\times10^{24}$ kg.*

This is approximately 89 min, or 1.5 h.

3 EVALUATE THE ANSWER

Are the units correct? The unit for speed is meters per second, and the unit for period is seconds.

APPLICATIONS

For the following problems, assume a circular orbit for all calculations.

14. Suppose that the satellite in Example 2 is moved to an orbit that is 24 km larger in radius than its previous orbit.

a. What is its speed?

b. Is this faster or slower than its previous speed?

c. Why do you think this is so?

15. Uranus has 27 known moons. One of these moons is Miranda, which orbits at a radius of 1.29×10^{8} m. Uranus has a mass of 8.68×10^{25} kg. Find the orbital speed of Miranda. How many Earth days does it take Miranda to complete one orbit?

16. Use Newton's thought experiment on the motion of satellites to solve the following.

a. Calculate the speed that a satellite shot from a cannon must have to orbit Earth 150 km above its surface.

b. How long, in seconds and minutes, would it take for the satellite to complete one orbit and return to the cannon?

17. CHALLENGE Use the data for Mercury in **Table 1** to find the following.

a. the speed of a satellite that is in orbit 260 km above Mercury's surface

b. the period of the satellite

CONNECTION TO EARTH SCIENCE *Landsat 7,* shown in **Figure 12,** is an artificial satellite that provides images of Earth's continental surfaces. *Landsat 7* images have been used to create maps, study land use, and monitor resources and global changes. The *Landsat 7* system enables researchers to monitor small-scale processes, such as deforestation, on a global scale. Satellites, such as *Landsat 7,* are accelerated to the speeds necessary for them to achieve orbit by large rockets, such as shuttle-booster rockets. Because the acceleration of any mass must follow Newton's second law of motion, $F_{net} = ma$, more force is required to launch a more massive satellite into orbit. Thus, the mass of a satellite is limited by the capability of the rocket used to launch it.

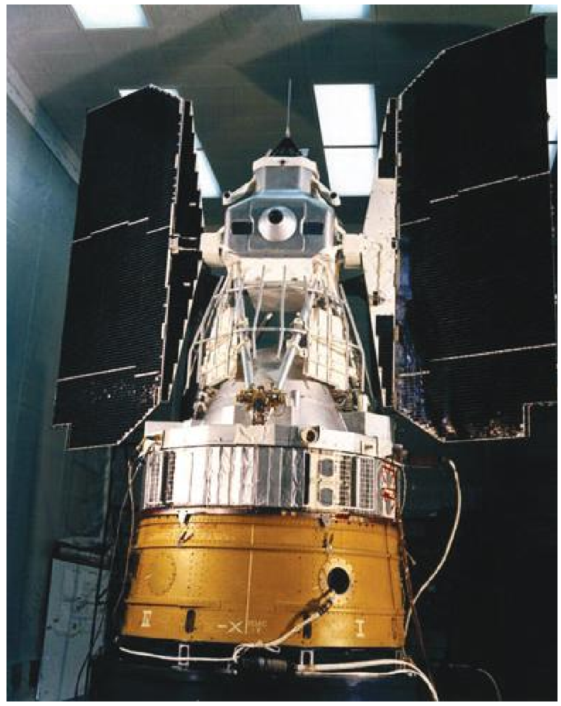


Figure 12 *Landsat 7* is capable of providing up to 532 images of Earth per day.

Free-Fall Acceleration

The acceleration of objects due to Earth's gravity can be found by using Newton's law of universal gravitation and his second law of motion. For a free-falling object of mass m, the following is true:

$$F = \frac{Gm_Em}{r^2} = ma, \text{ so } a = \frac{Gm_E}{r^2}$$

If you set $a = g_E$ and $r = r_E$ on Earth's surface, the following equation can be written:

$$g = \frac{Gm_E}{{r_E}^2}, \text{ thus, } m_E = \frac{g{r_E}^2}{G}$$

You saw above that $a = \frac{Gm_E}{r^2}$ for a free-falling object. Substitution of the above expression for m_E yields the following:

$$a = \frac{G\left(\frac{g{r_E}^2}{G}\right)}{r^2}$$

$$a = g\left(\frac{r_E}{r}\right)^2$$

On the surface of Earth, $r = r_E$ and so $a = g$. But, as you move farther from Earth's center, r becomes larger than r_E, and the free-fall acceleration is reduced according to this inverse square relationship. What happens to your mass as you move farther and farther from Earth's center?

Weight and weightlessness You may have seen photos similar to **Figure 13** in which astronauts are on a spacecraft in an environment often called zero-g or weightlessness. The spacecraft orbits about 400 km above Earth's surface. At that distance, $g = 8.7$ N/kg, only slightly less than that on Earth's surface. Earth's gravitational force is certainly not zero in the shuttle. In fact, gravity causes the shuttle to orbit Earth. Why, then, do the astronauts appear to have no weight?

Remember that you sense weight when something, such as the floor or your chair, exerts a contact force on you. But if you, your chair, and the floor all are accelerating toward Earth together, then no contact forces are exerted on you. Thus, your apparent weight is zero and you experience apparent weightlessness. Similarly, the astronauts experience apparent weightlessness as the shuttle and everything in it falls freely toward Earth.

Figure 13 Astronauts in orbit around Earth are in free fall because their spacecraft and everything in it is accelerating toward Earth along with the astronauts. That is, the floor is constantly falling from beneath their feet.

MiniLABs

WEIGHTLESS WATER
What are the effects of weightlessness in free fall?

WEIGHT IN FREE FALL
What is the effect of free fall on mass?

The Gravitational Field

Recall from studying motion that many common forces are contact forces. Friction is exerted where two objects touch; for example, the floor and your chair or desk push on you when you are in contact with them. Gravity, however, is different. It acts on an apple falling from a tree and on the Moon in orbit. In other words, gravity acts over a distance. It acts between objects that are not touching or that are not close together. Newton was puzzled by this concept. He wondered how the Sun could exert a force on planet Earth, which is hundreds of millions of kilometers away.

Field concept The answer to the puzzle arose from a study of magnetism. In the nineteenth century, Michael Faraday developed the concept of a field to explain how a magnet attracts objects. Later, the field concept was applied to gravity.

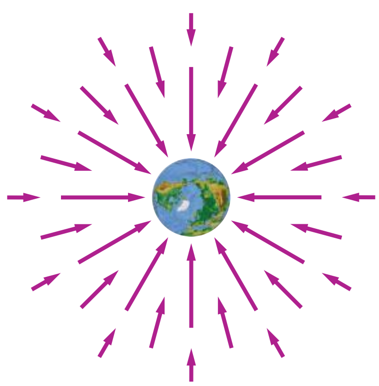
Any object with mass is surrounded by a gravitational field, which exerts a force that is directly proportional to the mass of the object and inversely proportional to the square of the distance from the object's center. Another object experiences a force due to the interaction between its mass and the gravitational field ($\boldsymbol{g}$) at its location. The direction of $\boldsymbol{g}$ and the gravitational force is toward the center of the object producing the field. Gravitational field strength is expressed by the following equation.

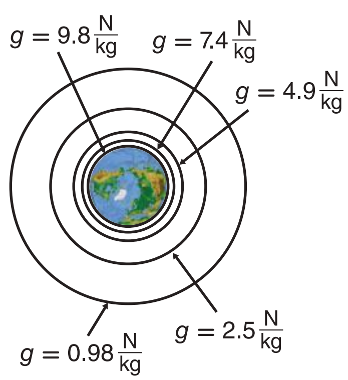
GRAVITATIONAL FIELD
The gravitational field strength produced by an object is equal to the universal gravitational constant times the object's mass, divided by the square of the distance from the object's center.

$$g = \frac{Gm}{r^2}$$

Figure 14 Earth's gravitational field can be represented by vectors pointing toward Earth's center. The decrease in $\boldsymbol{g}$'s magnitude follows an inverse-square relationship as the distance from Earth's center increases.

Explain why the value of $\boldsymbol{g}$ never reaches zero.





Suppose the gravitational field is created by the Sun. Then a planet of mass m in the Sun's gravitational field has a force exerted on it that depends on its mass and the magnitude of the gravitational field at its location. That is, $\boldsymbol{F}_g = m\boldsymbol{g}$, toward the Sun. The force is caused by the interaction of the planet's mass with the gravitational field at its location, not with the Sun millions of kilometers away. To find the gravitational field caused by more than one object, calculate all gravitational fields and add them as vectors.

The gravitational field is measured by placing an object with a small mass (m) in the gravitational field and measuring the force ($\boldsymbol{F}_g$) on it. The gravitational field is calculated using $\boldsymbol{g} = \frac{\boldsymbol{F}_g}{m}$. The gravitational field is measured in units of newtons per kilogram (N/kg).

On Earth's surface, the strength of the gravitational field is 9.8 N/kg, and its direction is toward Earth's center. The field can be represented by a vector of length g pointing toward the center of the object producing the field. You can picture the gravitational field produced by Earth as a collection of vectors surrounding Earth and pointing toward it, as shown in **Figure 14.** The strength of Earth's gravitational field varies inversely with the square of the distance from Earth's center. Earth's gravitational field depends on Earth's mass but not on the mass of the object experiencing it.

Two Kinds of Mass

You read that mass can be defined as the slope of a graph of force versus acceleration. That is, mass is equal to the net force exerted on an object divided by its acceleration. This kind of mass, related to the inertia of an object, is called inertial mass and is represented by the following equation.

INERTIAL MASS
Inertial mass is equal to the net force exerted on the object divided by the acceleration of the object.

$$m_{\text{inertial}} = \frac{F_{\text{net}}}{a}$$

Inertial mass You know that it is much easier to push an empty cardboard box across the floor than it is to push one that is full of books. The full box has greater inertial mass than the empty one. The **inertial mass** of an object is a measure of the object's resistance to any type of force. Inertial mass of an object is measured by exerting a force on the object and measuring the object's acceleration. The more inertial mass an object has, the less acceleration it undergoes as a result of a net force exerted on it.

Gravitational mass Newton's law of universal gravitation, $F_g = \frac{Gm_1m_2}{r^2}$, also involves mass–but a different kind of mass. Mass as used in the law of universal gravitation is a quantity that measures an object's response to gravitational force and is called **gravitational mass.** It can be measured by using a simple balance, such as the one shown in **Figure 15.** If you measure the magnitude of the attractive force exerted on an object by another object of mass *m,* at a distance *r,* then you can define the gravitational mass in the following way.

GRAVITATIONAL MASS
The gravitational mass of an object is equal to the distance between the centers of the objects squared, times the gravitational force, divided by the product of the universal gravitational constant, times the mass of the other object.

$$m_g = \frac{r^2F_g}{Gm}$$

How different are these two kinds of mass? Suppose you have a watermelon in the trunk of your car. If you accelerate the car forward, the watermelon will roll backward relative to the trunk. This is a result of its inertial mass–its resistance to acceleration. Now, suppose your car climbs a steep hill at a constant speed. The watermelon will again roll backward. But this time, it moves as a result of its gravitational mass. The watermelon is pulled downward toward Earth.

Newton made the claim that inertial mass and gravitational mass are equal in magnitude. This hypothesis is called the principle of equivalence. All investigations conducted so far have yielded data that support this principle. Most of the time we refer simply to the mass of an object. Albert Einstein also was intrigued by the principle of equivalence and made it a central point in his theory of gravity.

PhysicsLABs

HOW CAN YOU MEASURE MASS?
How is an inertial balance used to measure mass?

INERTIAL MASS AND GRAVITATIONAL MASS
How can you determine the relationship between inertial mass and gravitational mass?

Figure 15 A simple balance is used to determine the gravitational mass of an object.

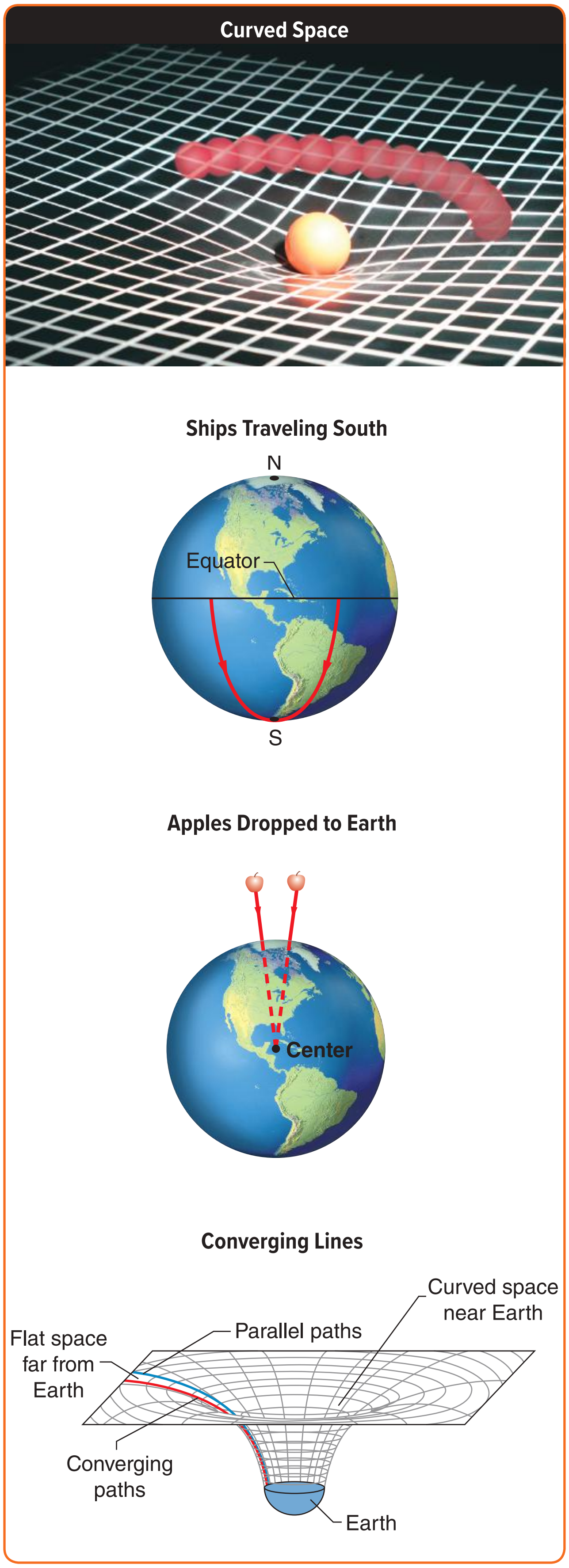


Figure 16 Visualizing how space is curved is difficult. Analogies can help you understand difficult concepts.

Einstein's Theory of Gravity

Newton's law of universal gravitation allows us to calculate the gravitational force that exists between two objects because of their masses. Newton was puzzled, however, as to how two objects could exert forces on each other if those two objects were millions of kilometers away from each other. Albert Einstein proposed that gravity is not a force but rather an effect of space itself. According to Einstein's explanation of gravity, mass changes the space around it. Mass causes space to be curved, and other bodies are accelerated because of the way they follow this curved space.

Curved space One way to picture how mass affects space is to model three-dimensional space as a large, two-dimensional sheet, as shown in the top part of **Figure 16.** The yellow ball on the sheet represents a massive object. The ball forms an indentation on the sheet. A red ball rolling across the sheet simulates the motion of an object in space. If the red ball moves near the sagging region of the sheet, it will be accelerated. In a similar way, Earth and the Sun are attracted to each other because of the way space is distorted by the two objects.

Ships traveling south The following is another analogy that might help you understand the curvature of space. Suppose you watch from space as two ships travel due south from the equator. At the equator, the ships are separated by 4000 km. As they approach the South Pole, the distance decreases to 1 km. To the sailors, their paths are straight lines, but because of Earth's curvature, they travel in a curve, as viewed far from Earth's surface, as in **Figure 16.**

Apples dropped to Earth Consider a similar motion. Two apples are dropped to Earth, initially traveling in parallel paths, as in **Figure 16.** As they approach Earth, they are pulled toward Earth's center. Their paths converge.

Converging lines This convergence can be attributed to the curvature of space near Earth. Far from any massive object, such as a planet or star, space is flat, and parallel lines remain parallel. Then they begin to converge. In flat space, the parallel lines would remain parallel. In curved space, the lines converge.

Einstein's theory or explanation, called the general theory of relativity, makes many predictions about how massive objects affect one another. In every test conducted to date, Einstein's theory has been shown to give the correct results.

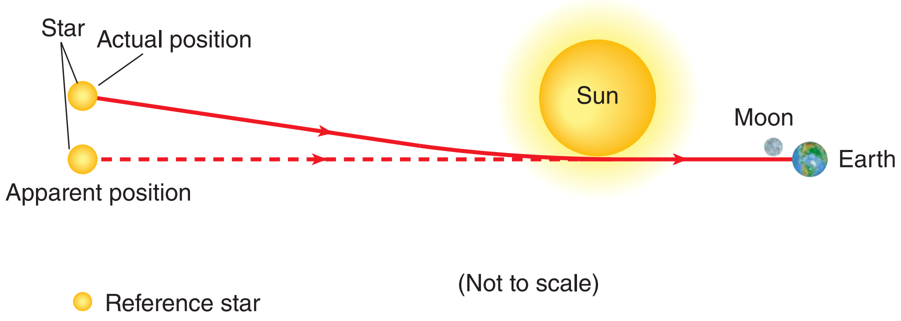


Figure 17 Light is bent around massive objects in space, altering the apparent position of the light's source.

Describe how this effect contradicts your experience of light's behavior.

Deflection of light Einstein's theory predicts that massive objects deflect and bend light. Light follows the curvature of space around the massive object and is deflected, as shown in **Figure 17.** In 1919, during an eclipse of the Sun, astronomers found that light from distant stars that passed near the Sun was deflected an amount that agreed with Einstein's predictions.

Another result of general relativity is the effect of gravity on light from extremely massive objects. If an object is massive and dense enough, the light leaving it is totally bent back to the object. No light ever escapes the object. Objects such as these, called black holes, have been identified as a result of their effects on nearby stars. Black holes have been detected through the radiation produced when matter is pulled into them.

While Einstein's theory provides very accurate predictions of gravity's effects, it is still incomplete. It does not explain the origin of mass or how mass curves space. Physicists are working to understand the deeper meaning of gravity and the origin of mass itself.

SECTION 2 REVIEW

18. MAINIDEA The Moon is 3.9×10^5 km from Earth's center and Earth is 14.96×10^7 km from the Sun's center. The masses of Earth and the Sun are 5.97×10^{24} kg and 1.99×10^{30} kg, respectively. During a full moon, the Sun, Earth, and the Moon are in line with each other, as shown in **Figure 18.**

a. Find the ratio of the gravitational fields due to Earth and the Sun at the center of the Moon.

b. What is the net gravitational field due to the Sun and Earth at the center of the Moon?

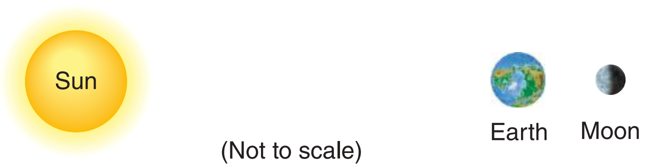


Figure 18

19. Apparent Weightlessness Chairs in an orbiting spacecraft are weightless. If you were on board such a spacecraft and you were barefoot, would you stub your toe if you kicked a chair? Explain.

20. Gravitational Field The mass of the Moon is 7.3×10^{22} kg and its radius is 1785 km. What is the strength of the gravitational field on the surface of the Moon?

21. Orbital Period and Speed Two satellites are in circular orbits about Earth. One is 150 km above the surface, the other is 160 km.

a. Which satellite has the larger orbital period?

b. Which has the greater speed?

22. Theories and Laws Why is Einstein's description of gravity called a theory, while Newton's is a law?

23. Astronaut What would be the strength of Earth's gravitational field at a point where an 80.0-kg astronaut would experience a 25.0 percent reduction in weight?

24. A Satellite's Mass When the first artificial satellite was launched into orbit by the former Soviet Union in 1957, U.S. president Dwight D. Eisenhower asked scientists to calculate the mass of the satellite. Would they have been able to make this calculation? Explain.

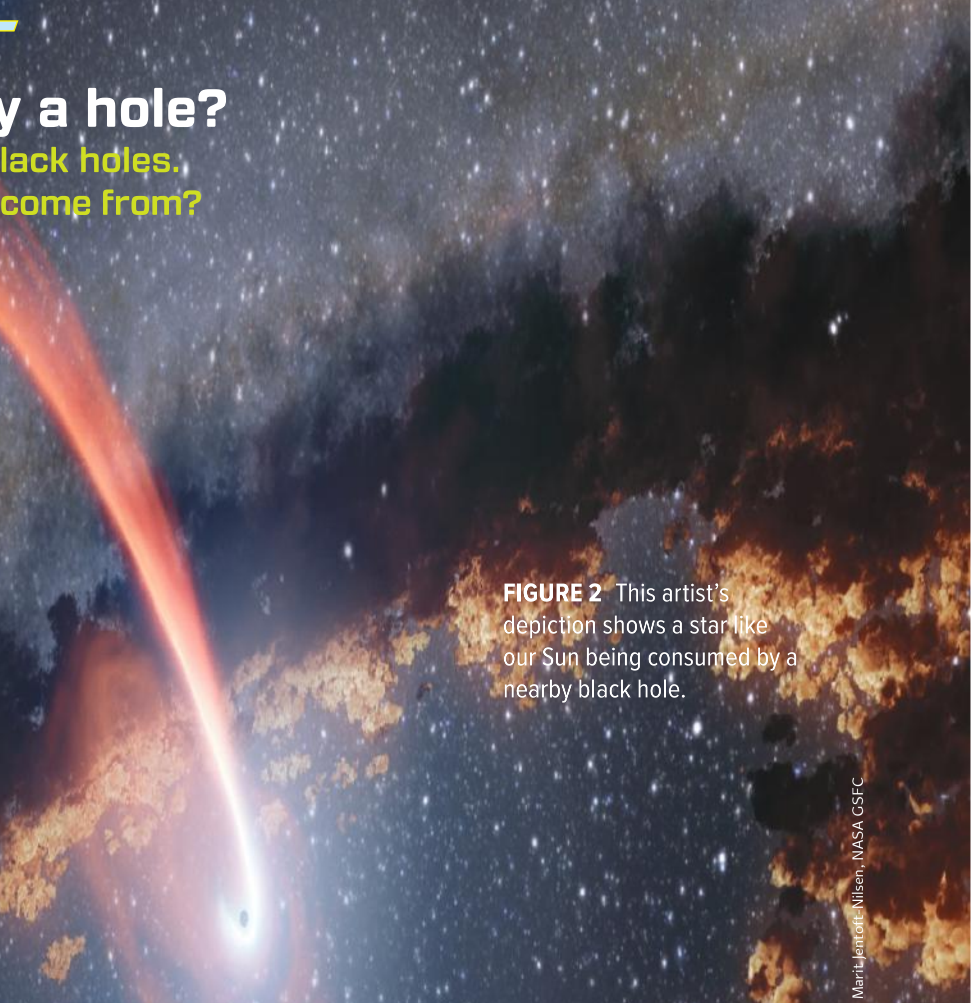
25. Critical Thinking It is easier to launch a satellite from Earth into an orbit that circles eastward than it is to launch one that circles westward. Explain.

A CLOSER **LOOK**

NO ESCAPE

Is a BLACK HOLE really a hole?

You may have wondered about black holes. What are they and where do they come from?



A star explodes What happens when a giant star runs low on fuel? The star cannot maintain its temperature, causing it to collapse under its own weight. The resulting explosion, called a supernova, is usually brighter than the entire galaxy it is in.

If what's left of the star is massive enough (more than three times the mass of the Sun), the remnant becomes one of the strangest objects in the universe: a black hole.

Escape from Earth Imagine standing on the surface of Earth and throwing a ball straight up. As the ball moves up, gravitational force robs the ball of its upward velocity. Finally, when the upward velocity reaches zero, the ball begins to fall back down. **Figure 1** illustrates this.

Of course, this happens due to the limitations of your throwing arm. If you could throw the ball fast enough (about 11,000 m/s), it would not fall back to Earth but instead would escape Earth entirely. This speed is the escape velocity from the surface of our planet.

Escape from a black hole? A black hole is a very compact, massive object with an escape velocity so high that nothing, not even light, can escape. The image in **Figure 2** shows a star being swallowed by a black hole. Anything that passes through the boundary of the influence of a black hole is truly lost forever.

Are black holes really holes? NO! They are extremely dense objects in space.

FIGURE 2 This artist's depiction shows a star like our Sun being consumed by a nearby black hole.

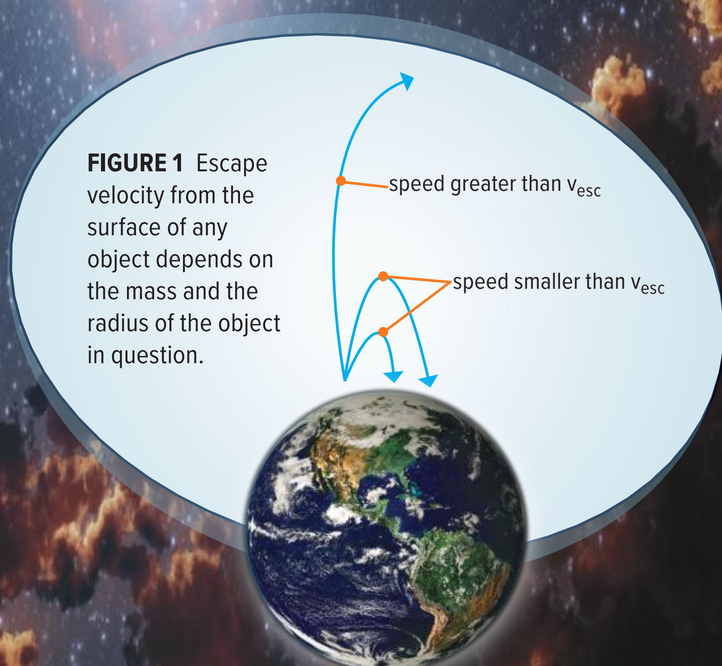


FIGURE 1 Escape velocity from the surface of any object depends on the mass and the radius of the object in question.

GOING**FURTHER** >>>

Research Light traveling through a vacuum cannot slow down. So how, then, does a black hole prevent a light beam from escaping? To find out, investigate and write about an effect known as the gravitational red shift.

CHAPTER 2

STUDY GUIDE

BIGIDEA **Gravity is an attractive field force that acts between objects with mass.**

VOCABULARY

- **Kepler's first law**
- **Kepler's second law**
- **Kepler's third law**
- **gravitational force**
- **law of universal gravitation**

SECTION 1 Planetary Motion and Gravitation

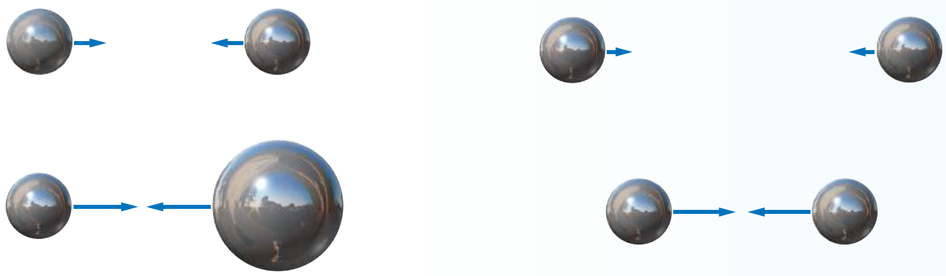
MAINIDEA The gravitational force between two objects is proportional to the product of their masses divided by the square of the distance between them.

- Kepler's first law states that planets move in elliptical orbits, with the Sun at one focus, and Kepler's second law states that an imaginary line from the Sun to a planet sweeps out equal areas in equal times. Kepler's third law states that the square of the ratio of the periods of any two planets is equal to the cube of the ratio of the distances between the centers of the planets and the center of the Sun.

$$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$$

- Newton's law of universal gravitation can be used to rewrite Kepler's third law to relate the radius and period of a planet to the mass of the Sun. Newton's law of universal gravitation states that the gravitational force between any two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. The force is attractive and along a line connecting the centers of the masses.

$$F = \frac{Gm_1m_2}{r^2}$$



- Cavendish's investigation determined the value of G, confirmed Newton's prediction that a gravitational force exists between two objects, and helped calculate the mass of Earth.

VOCABULARY

- **inertial mass**
- **gravitational mass**

SECTION 2 Using the Law of Universal Gravitation

MAINIDEA All objects are surrounded by a gravitational field that affects the motions of other objects.

- The speed and period of a satellite in circular orbit describe orbital motion. Orbital speed and period for any object in orbit around another are calculated with Newton's second law.
- Gravitational mass and inertial mass are two essentially different concepts. The gravitational and inertial masses of an object, however, are numerically equal.
- All objects have gravitational fields surrounding them. Any object within a gravitational field experiences a gravitational force exerted on it by the gravitational field. Einstein's general theory of relativity explains gravitational force as a property of space itself.

SECTION 1
Planetary Motion and Gravitation

Mastering Concepts

26. Problem Posing Complete this problem so that it can be solved using Kepler's third law: "Suppose a new planet was found orbiting the Sun in the region between Jupiter and Saturn . . ."

27. In 1609 Galileo looked through his telescope at Jupiter and saw four moons. The name of one of the moons that he saw is Io. Restate Kepler's first law for Io and Jupiter.

28. Earth moves more slowly in its orbit during summer in the northern hemisphere than it does during winter. Is it closer to the Sun in summer or in winter?

29. Is the area swept out per unit of time by Earth moving around the Sun equal to the area swept out per unit of time by Mars moving around the Sun? Explain your answer.

30. Why did Newton think that a force must act on the Moon?

31. How did Cavendish demonstrate that a gravitational force of attraction exists between two small objects?

32. What happens to the gravitational force between two masses when the distance between the masses is doubled?

33. According to Newton's version of Kepler's third law, how would the ratio $\frac{T^2}{r^3}$ change if the mass of the Sun were doubled?

Mastering Problems

34. Jupiter is 5.2 times farther from the Sun than Earth is. Find the length of Jupiter's orbital period in Earth years.

35. The dwarf planet Pluto has a mean distance from the Sun of 5.87×10^{12} m. What is its orbital period of Pluto around the Sun in years?

36. Use **Table 1** to compute the gravitational force that the Sun exerts on Jupiter.

37. The gravitational force between two electrons that are 1.00 m apart is 5.54×10^{-71} N. Find the mass of an electron.

38. Two bowling balls each have a mass of 6.8 kg. The centers of the bowling balls are located 21.8 cm apart. What gravitational force do the two bowling balls exert on each other?

39. Figure 19 shows a Cavendish apparatus like the one used to find G. It has a large lead sphere that is 5.9 kg in mass and a small one with a mass of 0.047 kg. Their centers are separated by 0.055 m. Find the force of attraction between them.

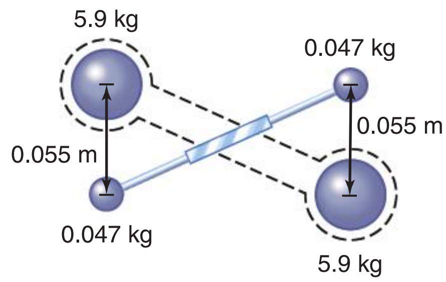


Figure 19

40. Assume that your mass is 50.0 kg. Earth's mass is 5.97×10^{24} kg, and its radius is 6.38×10^{6} m.

a. What is the force of gravitational attraction between you and Earth?

b. What is your weight?

41. A 1.0-kg mass weighs 9.8 N on Earth's surface, and the radius of Earth is roughly 6.4×10^{6} m.

a. Calculate the mass of Earth.

b. Calculate the average density of Earth.

42. Tom's mass is 70.0 kg, and Sally's mass is 50.0 kg. Tom and Sally are standing 20.0 m apart on the dance floor. Sally looks up and sees Tom. She feels an attraction. Supposing that the attraction is gravitational, find its size. Assume that both Tom and Sally can be replaced by spherical masses.

43. BIGIDEA The centers of two balls are 2.0 m apart, as shown in **Figure 20.** One ball has a mass of 8.0 kg. The other has a mass of 6.0 kg. What is the gravitational force between them?

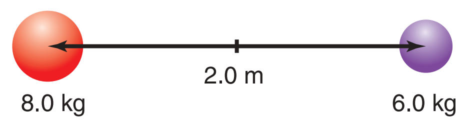


Figure 20

44. The star HD102272 has two planets. Planet A has a period of 127.5 days and a mean orbital radius of 0.615 AU. Planet B has a period of 520 days and a mean orbital radius of 1.57 AU. What is the mass of the star in units of the Sun's mass?

45. If a small planet, D, were located 8.0 times as far from the Sun as Earth is, how many years would it take the planet to orbit the Sun?

46. The Moon's center is 3.9×10^8 m from Earth's center. The Moon is 1.5×10^8 km from the Sun's center. If the mass of the Moon is 7.3×10^{22} kg, find the ratio of the gravitational forces exerted by Earth and the Sun on the Moon.

47. **Ranking Task** Using the solar system data in the reference tables at the end of the book, rank the following pairs of planets according to the gravitational force they exert on each other, from least to greatest. Specifically indicate any ties.

A. Mercury and Venus, when 5.0×10^7 km apart

B. Jupiter and Saturn, when 6.6×10^8 km apart

C. Jupiter and Earth, when 6.3×10^8 km apart

D. Mercury and Earth, when 9.2×10^7 km apart

E. Jupiter and Mercury, when 7.2×10^8 km apart

48. Two spheres are placed so that their centers are 2.6 m apart. The gravitational force between the two spheres is 2.75×10^{-12} N. What is the mass of each sphere if one of the spheres is twice the mass of the other sphere?

49. **Toy Boat** A force of 40.0 N is required to pull a 10.0 kg wooden toy boat at a constant velocity across a smooth glass surface on Earth. What is the force that would be required to pull the same wooden toy boat across the same glass surface on the planet Jupiter?

50. Mimas, one of Saturn's moons, has an orbital radius of 1.87×10^8 m and an orbital period of 23.0 h. Use Newton's version of Kepler's third law to find Saturn's mass.

51. **Halley's Comet** Every 76 years, comet Halley is visible from Earth. Find the average distance of the comet from the Sun in astronomical units. (AU is equal to the Earth's average distance from the Sun. The distance from Earth to the Sun is defined as 1.00 AU.)

52. Area is measured in m^2, so the rate at which area is swept out by a planet or satellite is measured in m^2/s.

a. How quickly is an area swept out by Earth in its orbit about the Sun?

b. How quickly is an area swept out by the Moon in its orbit about Earth? Use 3.9×10^8 m as the average distance between Earth and the Moon and 27.33 days as the period of the Moon.

53. The orbital radius of Earth's Moon is 3.9×10^8 m. Use Newton's version of Kepler's third law to calculate the period of Earth's Moon if the orbital radius were doubled.

SECTION 2

Using the Law of Universal Gravitation

Mastering Concepts

54. How do you answer the question, "What keeps a satellite up?"

55. A satellite is orbiting Earth. On which of the following does its speed depend?

a. mass of the satellite

b. distance from Earth

c. mass of Earth

56. What provides the force that causes the centripetal acceleration of a satellite in orbit?

57. During space flight, astronauts often refer to forces as multiples of the force of gravity on Earth's surface. What does a force of 5*g* mean to an astronaut?

58. Newton assumed that a gravitational force acts directly between Earth and the Moon. How does Einstein's view of the attractive force between the two bodies differ from Newton's view?

59. Show that the units of *g*, previously given as N/kg, are also m/s^2.

60. If Earth were twice as massive but remained the same size, what would happen to the value of *g*?

Mastering Problems

61. **Satellite** A geosynchronous satellite is one that appears to remain over one spot on Earth, as shown in **Figure 21.** Assume that a geosynchronous satellite has an orbital radius of 4.23×10^7 m.

a. Calculate its speed in orbit.

b. Calculate its period.

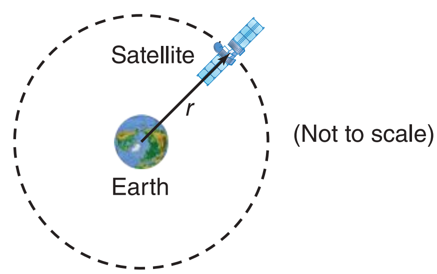


Figure 21

62. **Asteroid** The dwarf planet Ceres has a mass of 7×10^{20} kg and a radius of 500 km.

a. What is *g* on the surface of Ceres?

b. How much would a 90 kg astronaut weigh on Ceres?

63. The Moon's mass is 7.34×10^{22} kg, and it has an orbital radius of 3.9×10^{8} m from Earth. Earth's mass is 5.97×10^{24} kg.

a. Calculate the gravitational force of attraction between Earth and the Moon.

b. Find the magnitudes of Earth's gravitational field at the Moon.

64. Reverse Problem Write a physics problem with real-life objects for which the following equation would be part of the solution:

$$8.3\times10^{3}\ \text{m/s} = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{r}}$$

65. The radius of Earth is about 6.38×10^{3} km. A spacecraft with a weight of 7.20×10^{3} N travels away from Earth. What is the weight of the spacecraft at each of the following distances from the surface of Earth?

a. 6.38×10^{3} km

b. 1.28×10^{4} km

c. 2.55×10^{4} km

66. Rocket How high does a rocket have to go above Earth's surface before its weight is half of what it is on Earth?

67. Two satellites of equal mass are put into orbit 30.0 m apart. The gravitational force between them is 2.0×10^{-7} N.

a. What is the mass of each satellite?

b. What is the initial acceleration given to each satellite by gravitational force?

68. Two large spheres are suspended close to each other. Their centers are 4.0 m apart, as shown in **Figure 22.** One sphere weighs 9.8×10^{2} N. The other sphere weighs 1.96×10^{2} N. What is the gravitational force between them?

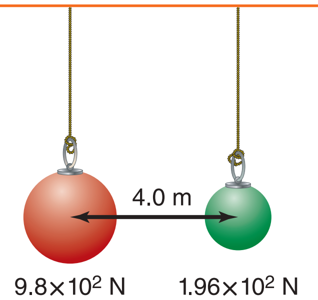


Figure 22

69. Suppose the centers of Earth and the Moon are 3.9×10^{8} m apart, and the gravitational force between them is about 1.9×10^{20} N. What is the approximate mass of the Moon?

70. On the surface of the Moon, a 91.0 kg physics teacher weighs only 145.6 N. What is the value of the Moon's gravitational field at its surface?

71. The mass of an electron is 9.1×10^{-31} kg. The mass of a proton is 1.7×10^{-27} kg. An electron and a proton are about 0.59×10^{-10} m apart in a hydrogen atom. What gravitational force exists between the proton and the electron of a hydrogen atom?

72. Consider two spherical 8.0 kg objects that are 5.0 m apart.

a. What is the gravitational force between the two objects?

b. What is the gravitational force between them when they are 5.0×10^{1} m apart?

73. If you weigh 637 N on Earth's surface, how much would you weigh on the planet Mars? Mars has a mass of 6.42×10^{23} kg and a radius of 3.40×10^{6} m.

74. Find the value of g, the gravitational field at Earth's surface, in the following situations.

a. Earth's mass is triple its actual value, but its radius remains the same.

b. Earth's radius is tripled, but its mass remains the same.

c. Both the mass and the radius of Earth are doubled.

Applying Concepts

75. Acceleration The force of gravity acting on an object near Earth's surface is proportional to the mass of the object. **Figure 23** shows a table-tennis ball and a golf ball in free fall. Why does a golf ball not fall faster than a table-tennis ball?

Figure 23

76. What information do you need to find the mass of Jupiter using Newton's version of Kepler's third law?

77. Why was the mass of the dwarf planet Pluto not known until a satellite of Pluto was discovered?

78. A satellite is one Earth radius above Earth's surface. How does the acceleration due to gravity at that location compare to acceleration at the surface of Earth?

79. What would happen to the value of G if Earth were twice as massive, but remained the same size?

80. An object in Earth's gravitational field doubles in mass. How does the force exerted by the field on the object change?

81. Decide whether each of the orbits shown in **Figure 24** is a possible orbit for a planet.

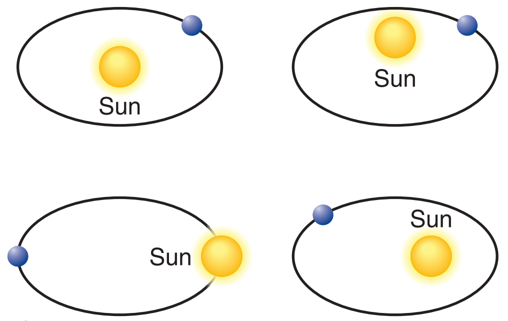


Figure 24

82. The Moon and Earth are attracted to each other by gravitational force. Does the more massive Earth attract the Moon with a greater force than the Moon attracts Earth? Explain.

83. **Figure 25** shows a satellite orbiting Earth. Examine the equation $v = \sqrt{\frac{Gm_E}{r}}$, relating the speed of an orbiting satellite and its distance from the center of Earth. Does a satellite with a large or small orbital radius have the greater velocity?

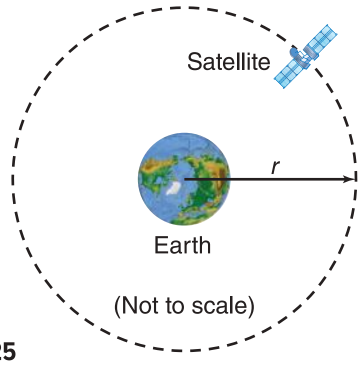


Figure 25

84. **Space Shuttle** If a space shuttle goes into a higher orbit, what happens to the shuttle's period?

85. Jupiter has about 300 times the mass of Earth and about ten times Earth's radius. Estimate the size of *g* on the surface of Jupiter.

86. Mars has about one-ninth the mass of Earth. **Figure 26** shows satellite M, which orbits Mars with the same orbital radius as satellite E, which orbits Earth. Which satellite has a smaller period?

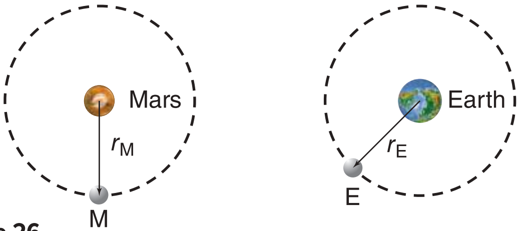


Figure 26

87. **Weight** Suppose that yesterday your body had a mass of 50.0 kg. This morning you stepped on a scale and found that you had gained weight. Assume your location has not changed.

   a. What happened, if anything, to your mass?

   b. What happened, if anything, to the ratio of your weight to your mass?

88. As an astronaut in an orbiting space shuttle, how would you go about "dropping" an object down to Earth?

89. **Weather Satellites** The weather pictures you see on television come from a spacecraft that is in a stationary position relative to Earth, 35,700 km above the equator. Explain how the satellite can stay in exactly the same position. What would happen if it were closer? Farther out? *Hint: Draw a pictorial model.*

Mixed Review

90. Use the information for Earth to calculate the mass of the Sun, using Newton's version of Kepler's third law.

91. The Moon's mass is 7.3×10^{22} kg and its radius is 1738 km. Suppose you perform Newton's thought experiment in which a cannonball is fired horizontally from a very high mountain on the Moon.

   a. How fast would the cannonball have to be fired to remain in orbit?

   b. How long would it take the cannonball to return to the cannon?

92. **Car Races** You want to construct a flat, circular race track. If a car can reach speeds of 12 m/s, what is the smallest radius of a track for which the coefficient of friction is 0.50?

93. ***Apollo 11*** On July 19, 1969, *Apollo 11* was adjusted to orbit the Moon at a height of 111 km. The Moon's radius is 1738 km, and its mass is 7.3×10^{22} kg.

a. What was the period of *Apollo 11* in minutes?

b. At what velocity did *Apollo 11* orbit the Moon?

94. The Moon's period is one month. Answer the following assuming the mass of Earth is doubled.

a. What would the Moon's period be in months?

b. Where would a satellite with an orbital period of one month be located?

c. How would the length of a year on Earth change?

95. Satellite A satellite is in orbit, as in **Figure 27,** with an orbital radius that is half that of the Moon's. Find the satellite's period in units of the Moon's period.

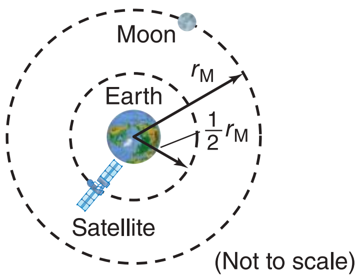


Figure 27

96. How fast would a planet of Earth's mass and size have to spin so that an object at the equator would be weightless? Give the period in minutes.

Thinking Critically

97. Make and Use Graphs Use Newton's law of universal gravitation to find an equation where x is equal to an object's distance from Earth's center and y is its acceleration due to gravity. Use a graphing calculator to graph this equation, using 6400–6600 km as the range for x and 9–10 m/s^2 as the range for y. The equation should be of the form $y = c\left(\frac{1}{x^2}\right)$. Use this graph and find y for these locations: sea level, 6400 km; the top of Mt. Everest, 6410 km; a satellite in typical orbit, 6500 km; a satellite in higher orbit, 6600 km.

98. Suppose the Sun were to disappear–its mass destroyed. If the gravitational force were action at a distance, Earth would experience the loss of the gravitational force of the Sun immediately. But, if the force were caused by a field or Einstein's curvature of space, the information that the Sun was gone would travel at the speed of light. How long would it take this information to reach Earth?

99. Analyze and Conclude The tides on Earth are caused by the pull of the Moon. Is this statement true?

a. Determine the forces (in newtons) that the Moon and the Sun exert on a mass (m) of water on Earth. Answer in terms of m.

b. Which celestial body, the Sun or the Moon, has a greater pull on the waters of Earth?

c. What is the difference in force exerted by the Moon on water at the near surface and water at the far surface (on the opposite side) of Earth, as illustrated in **Figure 28.** Answer in terms of m.

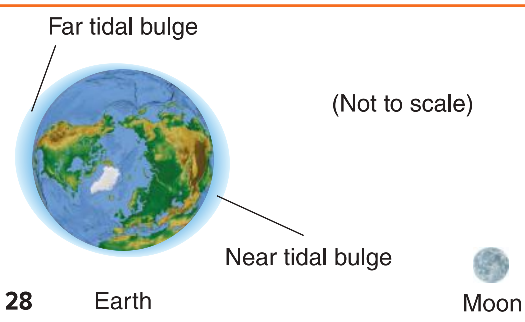


Figure 28

d. Determine the difference in force exerted by the Sun on water at the near surface and on water at the far surface (on the opposite side) of Earth.

e. Which celestial body has a greater difference in pull from one side of Earth to the other?

f. Why is it misleading to say the tides result from the pull of the Moon? Make a correct statement to explain how the Moon causes tides on Earth.

Writing in Physics

100. Research and report the history of how the distance between the Sun and Earth was determined.

101. Explore the discovery of planets around other stars. What methods did the astronomers use? What measurements did they take? How did they use Kepler's third law?

Cumulative Review

102. Airplanes A jet left Dubai at 2:20 P.M. and landed in Doha at 3:15 P.M. on the same day. If the jet's average speed was 441.0 km/h, what is the distance between the cities?

103. Wael wants to weigh his brother Amir. He agrees to stand on a scale, but only if they ride in an elevator. He steps on the scale while the elevator accelerates upward at 1.75 m/s^2. The scale reads 716 N. What is Amir's usual weight?

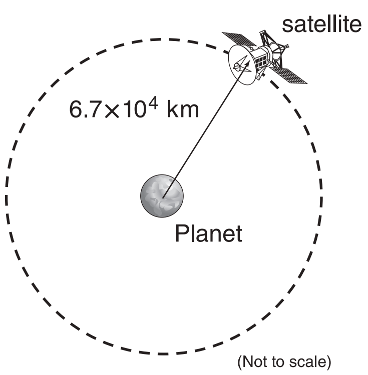
MULTIPLE CHOICE

1. Two satellites are in orbit around a planet. One satellite has an orbital radius of 8.0×10^6 m. The period of revolution for this satellite is 1.0×10^6 s. The other satellite has an orbital radius of 2.0×10^7 m. What is this satellite's period of revolution?

A. 5.0×10^5 s | **C.** 4.0×10^6 s
B. 2.5×10^6 s | **D.** 1.3×10^7 s

2. The illustration below shows a satellite in orbit around a small planet. The satellite's orbital radius is 6.7×10^4 km and its speed is 2.0×10^5 m/s. What is the mass of the planet around which the satellite orbits? ($G = 6.67\times10^{-11}$ N•m²/kg²)

A. 2.5×10^{18} kg | **C.** 2.5×10^{23} kg
B. 4.0×10^{20} kg | **D.** 4.0×10^{28} kg



3. Two satellites are in orbit around the same planet. Satellite A has a mass of 1.5×10^2 kg, and satellite B has a mass of 4.5×10^3 kg. The mass of the planet is 6.6×10^{24} kg. Both satellites have the same orbital radius of 6.8×10^6 m. What is the difference in the orbital periods of the satellites?

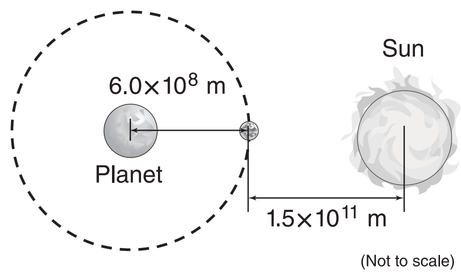
A. no difference | **C.** 2.2×10^2 s
B. 1.5×10^2 s | **D.** 3.0×10^2 s

4. A moon revolves around a planet with a speed of 9.0×10^3 m/s. The distance from the moon to the center of the planet is 5.43×10^6 m. What is the orbital period of the moon?

A. $1.2\pi\times10^2$ s | **C.** $1.2\pi\times10^3$ s
B. $6.0\pi\times10^2$ s | **D.** $1.2\pi\times10^9$ s

5. A moon in orbit around a planet experiences a gravitational force not only from the planet, but also from the Sun. The illustration below shows a moon during a solar eclipse, when the planet, the moon, and the Sun are aligned. The moon has a mass of about 3.9×10^{21} kg. The mass of the planet is 2.4×10^{26} kg, and the mass of the Sun is 1.99×10^{30} kg. The distance from the moon to the center of the planet is 6.0×10^8 m, and the distance from the moon to the Sun is 1.5×10^{11} m. What is the ratio of the gravitational force on the moon due to the planet, compared to its gravitational force due to the Sun during the solar eclipse?

A. 0.5 | **C.** 5.0
B. 2.5 | **D.** 7.5



FREE RESPONSE

6. Two satellites are in orbit around a planet. Satellite S_1 takes 20 days to orbit the planet at a distance of 2×10^5 km from the center of the planet. Satellite S_2 takes 160 days to orbit the planet. What is the distance of satellite S_2 from the center of the planet?

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CHAPTER 3

Rotational Motion

BIG IDEA **Applying a torque to an object causes a change in that object's angular velocity.**

SECTIONS

LaunchLAB

ROLLING OBJECTS
How do different objects rotate as they roll?

Chapter Sourced From: 6. Rotational Motion, Chapter 8, from Glencoe Physics: Principles & Problems, Student Edition ©2017

Describing Rotational Motion

PHYSICS 4 YOU **Many cars have a tachometer that displays the rate at which the motor's shaft rotates. This speed is measured in thousands of revolutions per minute. Why might a driver need to know this information?**

MAIN IDEA

Angular displacement, angular velocity, and angular acceleration all help describe angular motion.

Essential Questions

- What is angular displacement?
- What is average angular velocity?
- What is average angular acceleration, and how is it related to angular velocity?

Review Vocabulary

displacement change in position having both magnitude and direction; it is equal to the final position minus the initial position

New Vocabulary

radian
angular displacement
angular velocity
angular acceleration

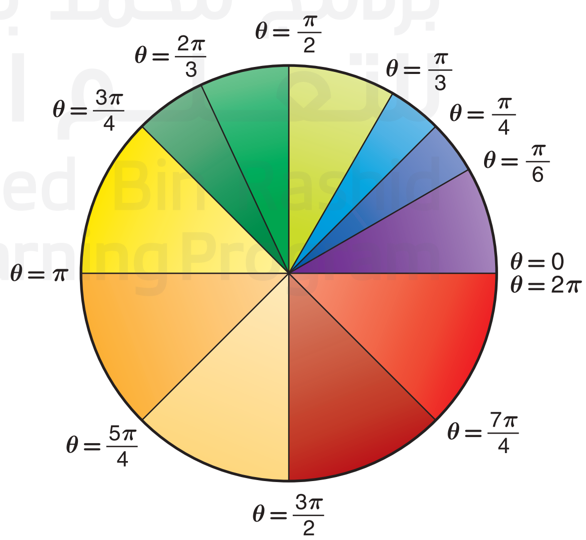
Angular Displacement

You probably have observed a spinning object many times. How would you measure such an object's rotation? Find a circular object, such as a DVD. Mark one point on the edge of the DVD so that you can keep track of its position. Rotate the DVD to the left (counterclockwise), and as you do so, watch the location of the mark. When the mark returns to its original position, the DVD has made one complete revolution.

Measuring revolution How can you measure a fraction of one revolution? It can be measured in several different ways, but the two most used are degrees and radians. A degree is $\frac{1}{360}$ of a revolution and is the usual scale marking on a protractor. In mathematics and physics, the radian is related to the ratio of the circumference of a circle to its radius. In one revolution, a point on the edge of a wheel travels a distance equal to 2π times the radius of the wheel. For this reason, the **radian** is defined as $\frac{1}{(2\pi)}$ of a revolution. One complete revolution is an angle of 2π radians. The radian is abbreviated *rad.*

The Greek letter theta (θ) is used to represent the angle of revolution. **Figure 1** shows the angles in radians for several common fractions of a revolution. Note that counterclockwise rotation is designated as positive, while clockwise is negative. As an object rotates, the change in the angle is called the object's **angular displacement.**

Figure 1 A fraction of a revolution can be measured in degrees or radians. Some common angles are shown below measured in radians. Each angle is measured in the counterclockwise direction from $\theta = 0$.



■ Measuring Distance

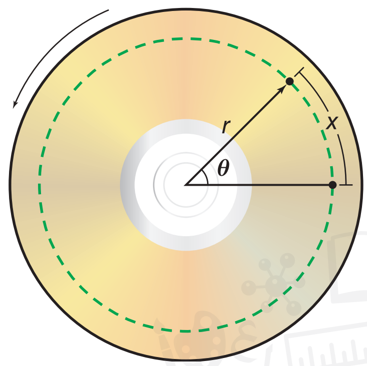


Figure 2 The dashed line shows the path of a point on a DVD as the DVD rotates counterclockwise about its center. The point is located a distance *r* from the center of the DVD and moves a distance *x* as it rotates.

Explain what the variables *r, x,* and θ represent.

Earth's revolution As you know, Earth turns one complete revolution, or 2π rad, in 24 h. In 12 h, it rotates through π rad. Through what angle does Earth rotate in 6 h? Because 6 h is one-fourth of a day, Earth rotates through an angle of $\frac{\pi}{2}$ rad during that period. Earth's rotation as seen from the North Pole is positive. Is it positive or negative when viewed from the South Pole?

READING CHECK **Identify** the angle that Earth rotates in 48 h.

Measuring distance How far does a point on a rotating object move? You already found that a point on the edge of an object moves 2π times the radius in one revolution. In general, for rotation through an angle (θ), a point at a distance r from the center, as shown in **Figure 2,** moves a distance given by $x = r\theta$. If r is measured in meters, you might think that multiplying it by θ rad would result in x being measured in m·rad. However, this is not the case. Radians indicate the dimensionless ratio between x and r. Thus, x is measured in meters.

Angular Velocity

How fast does a DVD spin? How can you determine its speed of rotation? Recall that velocity is displacement divided by the time taken to make the displacement. Likewise, the **angular velocity** of an object is angular displacement divided by the time taken to make the angular displacement. The angular velocity of an object is given by the following ratio, and is represented by the Greek letter omega (ω).

AVERAGE ANGULAR VELOCITY OF AN OBJECT

The angular velocity equals the angular displacement divided by the time required to make the rotation.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Recall that if the velocity changes over a time interval, the average velocity is not equal to the instantaneous velocity at any given instant. Similarly, the angular velocity calculated in this way is actually the average angular velocity over a time interval (Δt). Instantaneous angular velocity equals the slope of a graph of angular position versus time.

READING CHECK **Define** angular velocity in your own words.

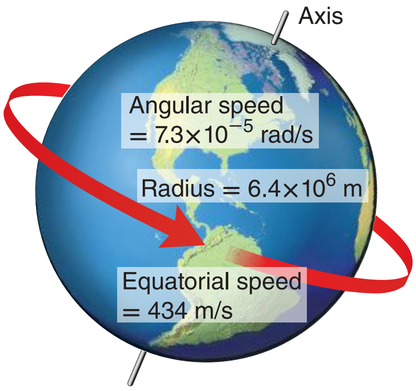


Figure 3 Earth is a rotating, rigid body and all parts rotate at the same rate.

Earth's angular velocity Angular velocity is measured in rad/s. You can calculate Earth's angular velocity as follows:

$$\omega_E = \frac{(2\pi \text{ rad})}{(24.0 \text{ h})(3600 \text{ s/h})} = 7.27\times10^{-5} \text{ rad/s}$$

In the same way that counterclockwise rotation produces positive angular displacement, it also results in positive angular velocity.

If an object's angular velocity is ω, then the linear velocity of a point a distance r from the axis of rotation is given by $v = r\omega$. The speed at which an object on Earth's equator moves as a result of Earth's rotation is given by $v = r\omega = (6.38\times10^{6} \text{ m/rad})(7.27\times10^{-5} \text{ rad/s}) = 464$ m/s. Earth is an example of a rotating, rigid body, as shown in **Figure 3.** Even though different points at different latitudes on Earth do not move the same distance in each revolution, all points rotate through the same angle. All parts of a rigid body rotate at the same rate. The Sun is not a rigid body. Different parts of the Sun rotate at different angular velocities. Most objects that we will consider in this chapter are rigid bodies.

Angular Acceleration

What if angular velocity is changing? For example, a car could accelerate from 0.0 m/s to 25 m/s in 15 s. In the same 15 s, the angular velocity of the car's 0.64 m diameter wheels would change from 0.0 rad/s to 78 rad/s. The wheels would undergo **angular acceleration,** which is the change in angular velocity divided by the time required to make the change. Angular acceleration (α) is represented by the following equation and is measured in rad/s^2.

AVERAGE ANGULAR ACCELERATION OF AN OBJECT
Angular acceleration is equal to the change in angular velocity divided by the time required to make that change.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

If the change in angular velocity is positive, then the angular acceleration also is positive. Angular acceleration defined in this way is also the average angular velocity over the time interval Δt. One way to find the instantaneous angular acceleration is to find the slope of a graph of angular velocity as a function of time. The linear acceleration of a point at a distance (r) from the axis of an object with angular acceleration (α) is given by $a = r\alpha$. **Table 1** is a summary of linear and angular relationships discussed previously in this section.

☑ **READING CHECK** **Compare** the angular velocity and angular acceleration of a rotating body.

Table 1 Linear and Angular Measures

Quantity	Linear	Angular	Relationship
Displacement	x (m)	θ (rad)	$x = r\theta$
Velocity	v (m/s)	ω (rad/s)	$v = r\omega$
Acceleration	a (m/s^2)	α (rad/s^2)	$a = r\alpha$

APPLICATIONS

1. What is the angular displacement of each of the following hands of a clock in 1.00 h? State your answer in three significant digits.

a. the second hand

b. the minute hand

c. the hour hand

2. A rotating toy above a baby's crib makes one complete counterclockwise rotation in 1 min.

a. What is its angular displacement in 3 min?

b. What is the toy's angular velocity in rad/min?

c. If the toy is turned off, does it have positive or negative angular acceleration? Explain.

3. If a truck has a linear acceleration of 1.85 m/s^2 and the wheels have an angular acceleration of 5.23 rad/s^2, what is the diameter of the truck's wheels?

4. The truck in the previous problem is towing a trailer with wheels that have a diameter of 48 cm.

a. How does the linear acceleration of the trailer compare with that of the truck?

b. How do the angular accelerations of the wheels of the trailer and the wheels of the truck compare?

5. CHALLENGE You replace the tires on your car with tires of larger diameter. After you change the tires, how will the angular velocity and number of revolutions be different, for trips at the same speed and over the same distance?

Angular frequency An object can revolve many times in a given amount of time. For instance, a spinning wheel may complete several revolutions in 1 min. The number of complete revolutions made by an object in 1 s is called angular frequency. Angular frequency is defined as $f \equiv \frac{\omega}{2\pi}$.

One example of such a rotating object is a computer hard drive. Listen carefully when you start a computer. You often will hear the hard drive spinning. Hard drive frequencies are measured in revolutions per minute (RPM). Inexpensive hard drives rotate at 4800, 5400, and 7200 RPM. More advanced hard drives operate at 10,000 or 15,000 RPM. The faster the hard drive rotates, the quicker the hard drive can access or store information.

SECTION 1 REVIEW

6. MAINIDEA The Moon rotates once on its axis in 27.3 days. Its radius is 1.74×10^6 m.

a. What is the period of the Moon's rotation in seconds?

b. What is the frequency of the Moon's rotation in rad/s?

c. A rock sits on the surface at the Moon's equator. What is its linear speed due to the Moon's rotation?

d. Compare this speed with the speed of a person at Earth's equator due to Earth's rotation.

7. Angular Displacement A news program lasts 2 h. During that time, what is the angular displacement of each of the following?

a. the hour hand

b. the minute hand

c. the second hand

8. Angular Acceleration In the spin cycle of a clothes washer, the drum turns at 635 rev/min. If the lid of the washer is opened, the motor is turned off. If the drum requires 8.0 s to slow to a stop, what is the angular acceleration of the drum?

9. Angular Displacement Do all parts of the minute hand on a watch, shown in **Figure 4,** have the same angular displacement? Do they move the same linear distance? Explain.

Figure 4

10. Critical Thinking A CD-ROM has a spiral track that starts 2.7 cm from the center of the disk and ends 5.5 cm from the center. The disk drive must turn the disk so that the linear velocity of the track is a constant 1.4 m/s. Find the following.

a. the angular velocity of the disk (in rad/s and rev/min) for the start of the track

b. the disk's angular velocity at the end of the track

c. the disk's angular acceleration if the disk is played for 76 min

SECTION 2
Rotational Dynamics

PHYSICS 4 YOU **Have you ever watched a washing machine spin? During the rinse cycle, it starts spinning slowly, but its angular velocity soon increases until the clothes are a blur. The rotational motion causes water in the clothes to move to the drum. The holes in the drum allow the water to drain from the washer.**

MAIN IDEA

Torques cause changes in angular velocity.

Essential Questions

- What is torque?
- How is the moment of inertia related to rotational motion?
- How are torque, the moment of inertia, and Newton's second law for rotational motion related?

Review Vocabulary

magnitude a measure of size

New Vocabulary

lever arm
torque
moment of inertia
Newton's second law for rotational motion

Force and Angular Velocity

How do you start the rotation of an object? That is, how do you increase its angular velocity? A toy top is a handy round object that is easy to spin. If you wrap a string around it and pull hard, you could make the top spin rapidly. The force of the string is exerted at the outer edge of the top and at right angles to the line from the center of the top to the point where the string leaves the top's surface.

You have learned that a force changes the velocity of a point object. In the case of a toy top, a force that is exerted in a very specific way changes the angular velocity of an extended object. An extended object is an object that has a definite shape and size.

Consider how you open a door: you exert a force. How can you exert the force to open the door most easily? To get the most effect from the least force, you exert the force as far from the axis of rotation as possible, as shown in **Figure 5.** In this case, the axis of rotation is an imaginary vertical line through the hinges. The doorknob is near the outer edge of the door. You exert the force on the doorknob at right angles to the door. Thus, the magnitude of the force, the distance from the axis to the point where the force is exerted, and the direction of the force determine the change in angular velocity.

Figure 5 When opening a door that is free to rotate about its hinges, apply the force farthest from the hinges, at an angle perpendicular to the door.

■ Lever Arm Length

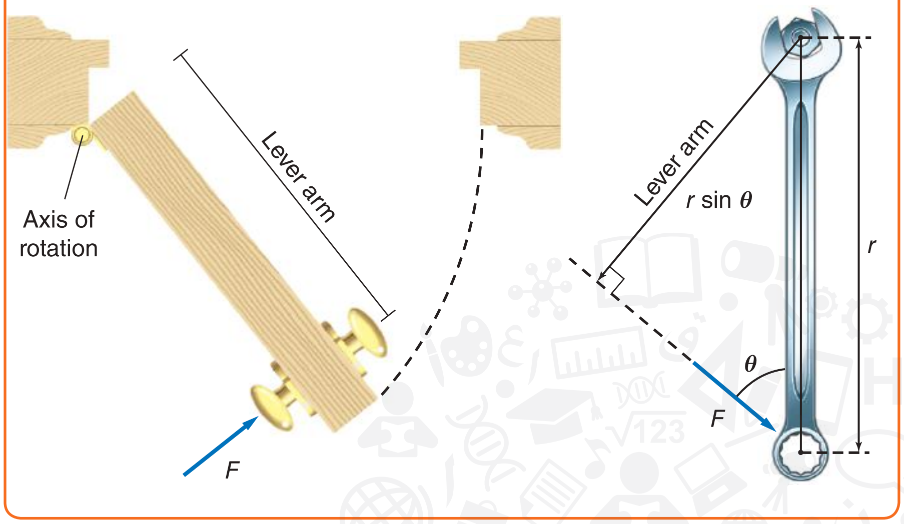


Figure 6 The lever arm is the perpendicular distance from the axis of rotation to the point where the force is exerted. For the door, the lever arm is along the width of the door, from the hinge to the point where the force is exerted. For the wrench, the lever arm is equal to $r \sin \theta$, when the angle (θ) between the force and the radius of rotation is not equal to 90°.

Explain why the formula $r \sin \theta$ is used to find the length of the lever arm.

Lever arm For a given applied force, the change in angular velocity depends on the **lever arm,** which is the perpendicular distance from the axis of rotation to the point where the force is exerted. If the force is perpendicular to the radius of rotation, as it was with the toy top, then the lever arm is the distance from the axis (r). For the door example, the lever arm is the distance from the hinges to the point where you exert the force, as illustrated on the left in **Figure 6.** If a force is not exerted perpendicular to the radius, the length of the lever arm is reduced. You must use mathematics to find the length of the lever arm.

To find the lever arm, you must extend the line of the force until it forms a right angle with a line from the center of rotation, as shown in **Figure 6.** The distance between this intersection and the axis is the lever arm. Using trigonometry, the lever arm (L) can be calculated by the equation $L = r \sin \theta$. In this equation, r is the distance from the axis of rotation to the point where the force is exerted, and θ is the angle between the force and the radius from the axis of rotation to the point where the force is applied.

☑ READING CHECK **Explain** what each of the variables represent in the equation $L = r \sin \theta$.

Torque The term **torque** describes the combination of force and lever arm that can cause an object to rotate. The magnitude of a torque is the product of the force and the perpendicular lever arm. Because force is measured in newtons and distance is measured in meters, torque is measured in newton-meters (N•m). Torque is represented by the Greek letter tau (τ) and is represented by the equation shown below.

TORQUE

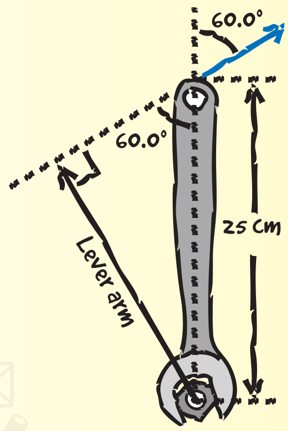
Torque is equal to the force F times the lever arm ($r \sin \theta$).

$$\tau = Fr \sin \theta$$

☑ READING CHECK **Identify** what each of the variables in the torque equation—τ, F, r, and θ—represents.

EXAMPLE 1

LEVER ARM A bolt on a car engine must be tightened with a torque of 35 N•m. You use a 25 cm-long wrench and pull the end of the wrench at an angle of 60.0° to the handle of the wrench. How long is the lever arm, and how much force must you exert?



1 ANALYZE AND SKETCH THE PROBLEM

Sketch the situation. Find the lever arm by extending the force vector backward until a line that is perpendicular to it intersects the axis of rotation.

KNOWN	UNKNOWN
$r = 0.25$ m $\tau = 35$ N•m	$L = ?$
$\theta = 60.0°$	$F = ?$

2 SOLVE FOR THE UNKNOWN

Solve for the length of the lever arm.

$L = r \sin \theta$

$= (0.25 \text{ m})(\sin 60.0°)$ ◀ *Substitute $r = 0.25$ m and $\theta = 60.0°$ into the equation. Then, solve the equation.*

$= 0.22$ m

Solve for the force.

$\tau = Fr \sin \theta$

$F = \frac{\tau}{(r \sin \theta)}$

$= \frac{(35 \text{ N•m})}{(0.25 \text{ m})(\sin 60.0°)}$ ◀ *Substitute $\tau = 35$ N•m, $r = 0.25$ m, and $\theta = 60.0°$ into the equation.*

$= 1.6 \times 10^2$ N ◀ *Then, solve the equation. Remember to use significant digits.*

3 EVALUATE THE ANSWER

- **Are the units correct?** Force is measured in newtons.
- **Does the sign make sense?** Only the magnitude of the force needed to rotate the wrench clockwise is calculated.

APPLICATIONS

11. Consider the wrench in Example 1. What force is needed if it is applied to the wrench pointing perpendicular to the wrench?

12. If a torque of 55.0 N•m is required to turn a bolt and the largest force you can exert is 135 N, how long a lever arm must you use to turn the bolt?

13. You have a 0.234 m long wrench. A job requires a torque of 32.4 N•m, and you can exert a force of 232 N.

a. What is the smallest angle, with respect to the handle of the wrench, at which you can pull on the wrench and get the job done?

b. A friend can exert 275 N. What is the smallest angle she can use to accomplish the job?

14. You stand on a bicycle pedal, as shown in **Figure 7.** Your mass is 65 kg. If the pedal makes an angle of 35° above the horizontal and the pedal is 18 cm from the center of the chain ring, how much torque would you exert?

15. CHALLENGE If the pedal in the previous problem is horizontal, how much torque would you exert? How much torque would you exert when the pedal is vertical?

Figure 7

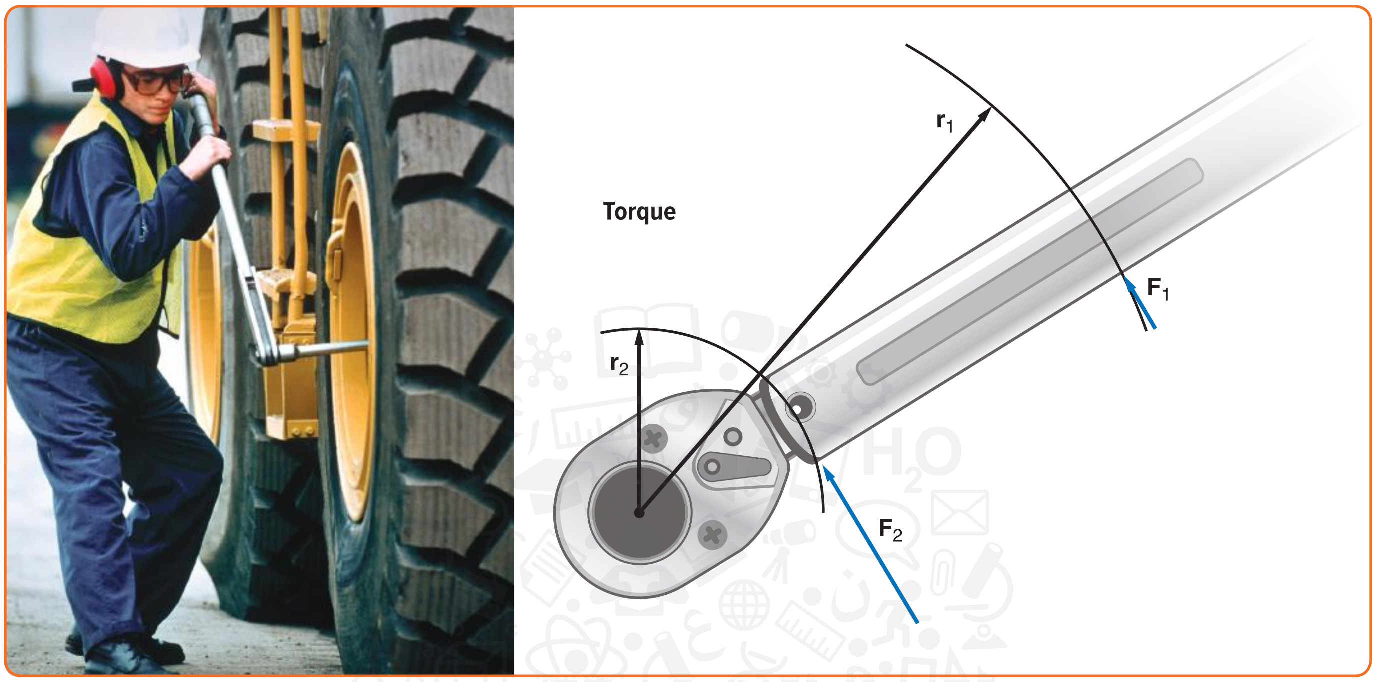


Figure 8 This worker uses a long wrench because it requires him to exert less force to tighten and loosen the nut. The wrench has a long lever arm, and less force is required if the force is applied farther from the axis of rotation (the center of the nut).

Finding Net Torque

Figure 8 shows a practical application of increasing torque to make a task easier. For another example of torque, try the following investigation. Collect two pencils, some coins, and some transparent tape. Tape two identical coins to the ends of the pencil and balance it on the second pencil, as shown in **Figure 9.** Each coin exerts a torque that is equal to the distance from the balance point to the center of the coin (r) times its weight (F_g), as follows:

$$\tau = rF_g$$

But the torques are equal and opposite in direction. Thus, the net torque is zero:

$$\tau_1 - \tau_2 = 0$$
or
$$r_1F_{g1} - r_2F_{g2} = 0$$

How can you make the pencil rotate? You could add a second coin on top of one of the two coins, thereby making the two forces different. You also could slide the balance point toward one end or the other of the pencil, thereby making the two lever arms of different length.

PhysicsLABs

LEVERAGE

FORENSICS LAB Can a person use a simple lever to open a heavy locked door?

TORQUES

Can you measure forces that produce torque?

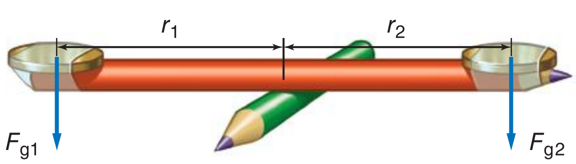


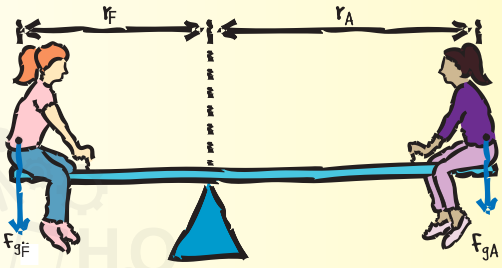
Figure 9 The torque exerted by the first coin ($F_{g1}r_1$) is equal and opposite in direction to the torque exerted by the second coin ($F_{g2}r_2$) when the pencil is balanced.

EXAMPLE 2

BALANCING TORQUES Fatema (56 kg) and Ayesha (43 kg) want to balance on a 1.75m long seesaw. Where should they place the pivot point of the seesaw? Assume that the seesaw is massless.

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Draw and label the vectors.



KNOWN	**UNKNOWN**
$m_F = 56$ kg	$r_F = ?$
$m_A = 43$ kg	$r_A = ?$
$r_F + r_A = 1.75$ m	

2 SOLVE FOR THE UNKNOWN

Find the two forces.

Fatema:

$$F_{gF} = m_K g$$
$$= (56\text{ kg})(9.8\text{ N/kg})$$
$$= 5.5\times10^2\text{ N}$$

◀ Substitute the known values into the equation: $m_F = 56$ kg, $g = 9.8$ N/kg.

Ayesha:

$$F_{gA} = m_A g$$
$$= (43\text{ kg})(9.8\text{ N/kg})$$
$$= 4.2\times10^2\text{ N}$$

◀ Substitute the known values into the equation: $m_A = 43$ kg, $g = 9.8$ N/kg.

Define Fatema's distance in terms of the length of the seesaw and Ayesha's distance.

$$r_F = 1.75\text{ m} - r_A$$

When there is no rotation, the sum of the torques is zero.

$$F_{gF}r_F = F_{gA}r_A$$
$$F_{gF}r_F - F_{gA}r_A = 0.0\text{ N•m}$$
$$F_{gF}(1.75\text{ m} - r_A) - F_{gA}r_A = 0.0\text{ N•m}$$

◀ Substitute the relationship between Fatema's distance in terms of Ayesha's distance in to the equation: $r_F = 1.75$ m - r_A.

Solve for r_A.

$$F_{gF}(1.75\text{ m}) - F_{gF}(r_A) - F_{gA}r_A = 0.0\text{ N•m}$$
$$F_{gF}r_A + F_{gA}r_A = F_{gF}(1.75\text{ m})$$
$$(F_{gF} + F_{gA})r_A = F_{gF}(1.75\text{ m})$$
$$r_A = \frac{F_{gF}(1.75\text{ m})}{(F_{gF} + F_{gA})}$$
$$= \frac{(5.5\times10^2\text{ N})(1.75\text{ m})}{(5.5\times10^2\text{ N} + 4.2\times10^2\text{ N})}$$
$$= 0.99\text{ m}$$

◀ Substitute $F_{gF} = 5.5\times10^2$ N and $F_{gA} = 4.2\times10^2$ N.

3 EVALUATE THE ANSWER

- **Are the units correct?** Distance is measured in meters.
- **Do the signs make sense?** Distances are positive.
- **Is the magnitude realistic?** Ayesha is about 1 m from the center, so Fatema is about 0.75 m away from it. Because Ayesha's weight is greater than Fatema's weight, the lever arm on Fatema's side should be shorter. Ayesha is farther from the pivot, as expected.

APPLICATIONS

16. Fares, whose mass is 43 kg, sits 1.8 m from a pivot at the center of a seesaw. Fahd, whose mass is 52 kg, wants to seesaw with Fares. How far from the center of the seesaw should Fahd sit?

17. A bicycle-chain wheel has a radius of 7.70 cm. If the chain exerts a 35.0-N force on the wheel in the clockwise direction, what torque is needed to keep the wheel from turning?

18. Two stationary baskets of fruit hang from strings going around pulleys of different diameters, as shown in **Figure 10.** What is the mass of basket A?

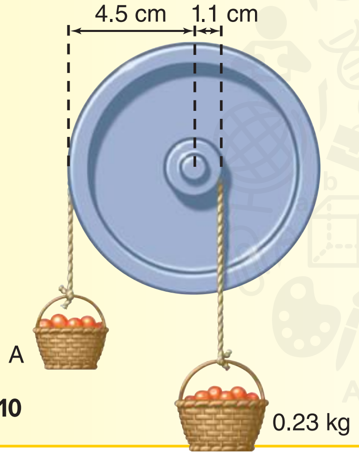


Figure 10

19. Suppose the radius of the larger pulley in problem 18 was increased to 6.0 cm. What is the mass of basket A now?

20. CHALLENGE A bicyclist, of mass 65.0 kg, stands on the pedal of a bicycle. The crank, which is 0.170 m long, makes a 45.0° angle with the vertical, as shown in **Figure 11.** The crank is attached to the chain wheel, which has a radius of 9.70 cm. What force must the chain exert to keep the wheel from turning?

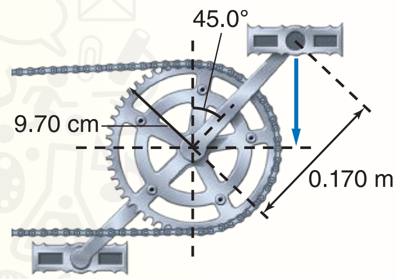


Figure 11

The Moment of Inertia

If you exert a force on a point mass, its acceleration will be inversely proportional to its mass. How does an extended object rotate when a torque is exerted on it? To observe firsthand, recover the pencil, the coins, and the transparent tape that you used earlier in this chapter. First, tape the coins at the ends of the pencil. Hold the pencil between your thumb and forefinger, and wiggle it back and forth. Take note of the forces that your thumb and forefinger exert. These forces create torques that change the angular velocity of the pencil and coins.

Now move the coins so that they are only 1 or 2 cm apart. Wiggle the pencil as before. Did you notice that the pencil is now easier to rotate? The torque that was required was much less this time. The mass of an object is not the only factor that determines how much torque is needed to change its angular velocity; the distribution or location of the mass also is important.

The resistance to rotation is called the **moment of inertia,** which is represented by the symbol I and has units of mass times the square of the distance. For a point object located at a distance (r) from the axis of rotation, the moment of inertia is given by the following equation.

MOMENT OF INERTIA OF A POINT MASS

The moment of inertia of a point mass is equal to the mass of the object times the square of the object's distance from the axis of rotation.

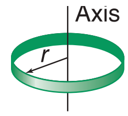
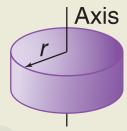
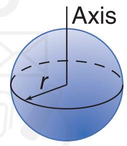
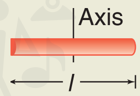
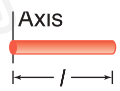
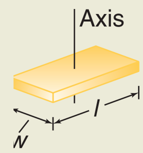
$$I = mr^2$$

MiniLAB

BALANCING TORQUES

Can you find the equilibrium point on a beam?

Table 2 Moments of Inertia for Various Objects

Object	Location of Axis	Diagram	Moment of Inertia
Thin hoop of radius r	through central diameter		mr^2
Solid, uniform cylinder of radius r	through center		$\left(\frac{1}{2}\right)mr^2$
Uniform sphere of radius r	through center		$\left(\frac{2}{5}\right)mr^2$
Long, uniform rod of length l	through center		$\left(\frac{1}{12}\right)ml^2$
Long, uniform rod of length l	through end		$\left(\frac{1}{3}\right)ml^2$
Thin, rectangular plate of length l and width w	through center		$\left(\frac{1}{12}\right)m(l^2 + w^2)$

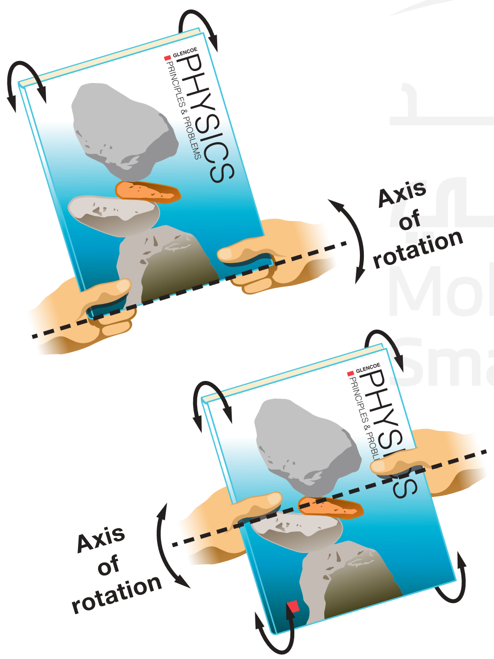


Figure 12 The moment of inertia of a book depends on the axis of rotation. The moment of inertia of the book on the top is larger than the moment of inertia of the book on the bottom because the average distance of the book's mass from the rotational axis is larger.

Identify which book requires more torque to rotate it and why.

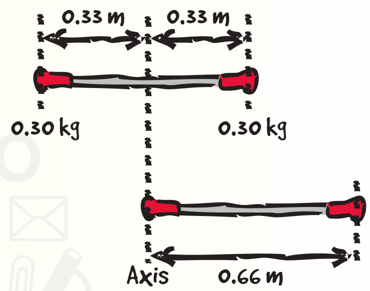
Moment of inertia and mass As you have seen, the moments of inertia for extended objects, such as the pencil and coins, depend on how far the masses are from the axis of rotation. A bicycle wheel, for example, has almost all of its mass in the rim and tire. Its moment of inertia about its axle is almost exactly equal to mr^2, where r is the radius of the wheel. For most objects, however, the mass is distributed closer to the axis so the moment of inertia is less than mr^2. For example, as shown in **Table 2,** for a solid cylinder of radius r, $I = \left(\frac{1}{2}\right)mr^2$, while for a solid sphere, $I = \left(\frac{2}{5}\right)mr^2$.

☑ **READING CHECK** **Write** the equation for the moment of inertia of a hoop.

Moment of inertia and rotational axis The moment of inertia also depends on the location and direction of the rotational axis, as illustrated in **Figure 12.** To observe this firsthand, hold a book in the upright position by placing your hands at the bottom of the book. Feel the torque needed to rock the book toward you and then away from you. Now put your hands in the middle of the book and feel the torque needed to rock the book toward you and then away from you. Note that much less torque is needed when your hands are placed in the middle of the book because the average distance of the book's mass from the rotational axis is much less in this case.

EXAMPLE 3

MOMENT OF INERTIA A simplified model of a twirling baton is a thin rod with two round objects at each end. The length of the baton is 0.66 m, and the mass of each object is 0.30 kg. Find the moment of inertia of the baton as it is rotated about an axis at the midpoint between the round objects and perpendicular to the rod. What is the moment of inertia of the baton if the axis is moved to one end of the rod? Which is greater? The mass of the rod is negligible compared to the masses of the objects at the ends.



1 ANALYZE AND SKETCH THE PROBLEM

Sketch the situation. Show the baton with the two different axes of rotation and the distances from the axes of rotation to the masses.

KNOWN	UNKNOWN
$m = 0.30$ kg	$I = ?$
$l = 0.66$ m	

2 SOLVE FOR THE UNKNOWN

Calculate the moment of inertia of each mass separately.

Rotating about the center of the rod:

$$r = \left(\frac{1}{2}\right)l$$

$$= \left(\frac{1}{2}\right)(0.66\text{ m})$$ ◄ *Substitute the known value, $l = 0.66$ m, into the equation.*

$$= 0.33\text{ m}$$

$$I_{\text{single mass}} = mr^2$$

$$= (0.30\text{ kg})(0.33\text{ m})^2$$ ◄ *Substitute $m = 0.30$ kg and $r = 0.33$ m into the equation.*

$$= 0.033\text{ kg}\cdot\text{m}^2$$

Find the moment of inertia of the baton.

$$I = 2I_{\text{single mass}}$$

$$= 2(0.033\text{ kg}\cdot\text{m}^2)$$ ◄ *Substitute $I_{\text{single mass}} = 0.033$ kg•m^2 into the equation.*

$$= 0.066\text{ kg}\cdot\text{m}^2$$

Rotating about one end of the rod:

$$I_{\text{single mass}} = mr^2$$

$$= (0.30\text{ kg})(0.66\text{ m})^2$$ ◄ *Substitute $m = 0.30$ kg and $r = 0.66$ m into the equation.*

$$= 0.13\text{ kg}\cdot\text{m}^2$$

The $I_{\text{single mass}} = mr^2$

$$= (0.30\text{ kg})(0.0\text{ m})^2$$

$$= 0 \text{ for the other mass.}$$

Find the moment of inertia of the baton.

$$I = I_{\text{single mass}} + 0$$

$$= 0.13\text{ kg}\cdot\text{m}^2$$

The moment of inertia is greater when the baton is swung around one end.

3 EVALUATE THE ANSWER

- **Are the units correct?** Moment of inertia is measured in kg•m^2.
- **Is the magnitude realistic?** Masses and distances are small, and so are the moments of inertia. Doubling the distance increases the moment of inertia by a factor of 4. Thus, doubling the distance increases the moment of inertia more than having only one mass decreases the moment of inertia.

APPLICATIONS

21. Two children of equal masses sit 0.3 m from the center of a seesaw. Assuming that their masses are much greater than that of the seesaw, by how much is the moment of inertia increased when they sit 0.6 m from the center? Ignore the moment of inertia for the seesaw.

22. Suppose there are two balls with equal diameters and masses. One is solid, and the other is hollow, with all its mass distributed at its surface. Are the moments of inertia of the balls equal? If not, which is greater?

23. Calculate the moments of inertia for each object below using the formulas in **Table 2.** Each object has a radius of 2.0 m and a mass of 1.0 kg.

a. a thin hoop

b. a solid, uniform cylinder

c. a solid, uniform sphere

24. CHALLENGE **Figure 13** shows three equal-mass spheres on a rod of very small mass. Consider the moment of inertia of the system, first when it is rotated about sphere A and then when it is rotated about sphere C.

a. Are the moments of inertia the same or different? Explain. If the moments of inertia are different, in which case is the moment of inertia greater?

b. Each sphere has a mass of 0.10 kg. The distance between spheres A and C is 0.20 m. Find the moment of inertia in the following instances: rotation about sphere A, rotation about sphere C.

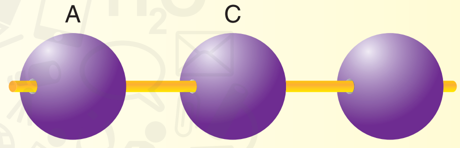


Figure 13

Newton's Second Law for Rotational Motion

Newton's second law for linear motion is expressed as $a = \frac{F_{net}}{m}$. If you rewrite this equation to represent rotational motion, acceleration is replaced by angular acceleration (α) force is replaced by net torque (τ_{net}) and mass is replaced by moment of inertia (I). Angular acceleration is directly proportional to the net torque and inversely proportional to the moment of inertia as stated in **Newton's second law for rotational motion.** This law is expressed by the following equation.

NEWTON'S SECOND LAW FOR ROTATIONAL MOTION

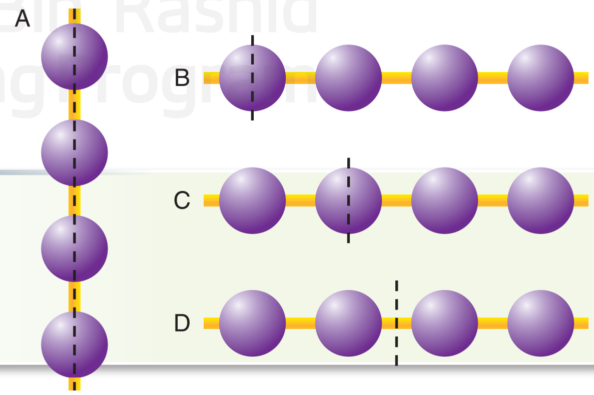
The angular acceleration of an object about a particular axis equals the net torque on the object divided by the moment of inertia.

$$\alpha = \frac{\tau_{net}}{I}$$

If the torque on an object and the angular velocity of that object are in the same direction, then the angular velocity of the object increases. If the torque and angular velocity are in different directions, then the angular velocity decreases.

PHYSICS CHALLENGE

Moments of Inertia Rank the objects shown in the diagram from least to greatest according to their moments of inertia about the indicated axes. All spheres have equal masses and all separations are the same. Assume that the rod's mass is negligible.



EXAMPLE 4

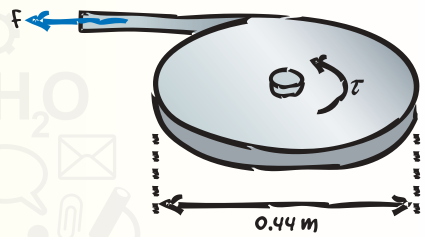
TORQUE A solid steel wheel is free to rotate about a motionless central axis. It has a mass of 15 kg and a diameter of 0.44 m and starts at rest. You want to increase this wheel's rotation about its central axis to 8.0 rev/s in 15 s.

a. What torque must be applied to the wheel?

b. If you apply the torque by wrapping a strap around the outside of the wheel, how much force should you exert on the strap?

1 ANALYZE AND SKETCH THE PROBLEM

Sketch the situation. The torque must be applied in a counterclockwise direction; force must be exerted as shown.



KNOWN	UNKNOWN
$m = 15$ kg	$\alpha = ?$
$r = \left(\frac{1}{2}\right)(0.44\text{ m}) = 0.22\text{ m}$	$I = ?$
$\omega_i = 0.0$ rad/s	$\tau = ?$
$\omega_f = 2\pi(8.0\text{ rev/s})$	$F = ?$
$t = 15$ s	

2 SOLVE FOR THE UNKNOWN

a. Solve for angular acceleration.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$= \frac{(16\pi\text{ rad/s} - (0.0\text{ rad/s}))}{15\text{ s}}$$

◀ *Substitute $\omega_f = 16\pi$ rad/s and $\omega_i = 0.0$ rad/s into the equation.*

$$= 3.4\text{ rad/s}^2$$

Solve for the moment of inertia.

$$I = \left(\frac{1}{2}\right)mr^2$$

$$= \left(\frac{1}{2}\right)(15\text{ kg})(0.22\text{ m})^2$$

◀ *Substitute $m = 15$ kg and $r = 0.22$ m into the equation.*

$$= 0.36\text{ kg}\cdot\text{m}^2$$

Solve for torque.

$$\tau = I\alpha$$

$$= (0.36\text{ kg}\cdot\text{m}^2)(3.4\text{ rad/s}^2)$$

◀ *Substitute $I = 0.36$ kg•m² and $\alpha = 3.4$ rad/s² into the equation.*

$$= 1.2\text{ kg}\cdot\text{m}^2\text{/s}^2$$

$$= 1.2\text{ N}\cdot\text{m}$$

b. Solve for force.

$$\tau = Fr$$

$$F = \frac{\tau}{r}$$

$$= \frac{(1.2\text{ N}\cdot\text{m})}{(0.22\text{ m})}$$

◀ *Substitute $\tau = 1.2$ N•m and $r = 0.22$ m into the equation.*

$$= 5.5\text{ N}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** Torque is measured in N•m and force is measured in N.
- **Is the magnitude realistic?** Despite its large mass, the small size of the wheel makes it relatively easy to spin.

APPLICATIONS

25. Consider the wheel in Example 4. If the force on the strap were twice as great, what would be the angular frequency of the wheel after 15 s?

26. A solid wheel accelerates at 3.25 rad/s^2 when a force of 4.5 N exerts a torque on it. If the wheel is replaced by a wheel which has all of its mass on the rim, the moment of inertia is given by $I = mr^2$. If the same angular velocity were desired, what force should be exerted on the strap?

27. A bicycle wheel on a repair bench can be accelerated either by pulling on the chain that is on the gear or by pulling on a string wrapped around the tire. The tire's radius is 0.38 m, while the radius of the gear is 0.14 m. What force would you need to pull on the string to produce the same acceleration you obtained with a force of 15 N on the chain?

28. The bicycle wheel in the previous problem is used with a smaller gear whose radius is 0.11 m. The wheel can be accelerated either by pulling on the chain that is on the gear or by pulling a string that is wrapped around the tire. If you obtained the needed acceleration with a force of 15 N on the chain, what force would you need to exert on the string?

29. A chain is wrapped around a pulley and pulled with a force of 16.0 N. The pulley has a radius of 0.20 m. The pulley's rotational speed increases from 0.0 to 17.0 rev/min in 5.00 s. What is the moment of inertia of the pulley?

30. CHALLENGE A disk with a moment of inertia of 0.26 kg•m^2 is attached to a smaller disk mounted on the same axle. The smaller disk has a diameter of 0.180 m and a mass of 2.5 kg. A strap is wrapped around the smaller disk, as shown in **Figure 14.** Find the force needed to give this system an angular acceleration of 2.57 rad/s^2.

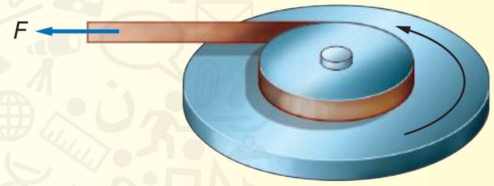


Figure 14

SECTION 2 REVIEW

31. MAIN IDEA Omar enters a revolving door that is not moving. Explain where and how Omar should push to produce a torque with the least amount of force.

32. Lever Arm You open a door by pushing at a right angle to the door. Your friend pushes at the same place, but at an angle of 55° from the perpendicular. If both you and your friend exert the same torque on the door, how do the forces you and your friend applied compare?

33. The solid wheel, shown in **Figure 15,** has a mass of 5.2 kg and a diameter of 0.55 m. It is at rest, and you need it to rotate at 12 rev/s in 35 s.

a. What torque do you need to apply to the wheel?

b. If a nylon strap is wrapped around the outside of the wheel, how much force do you need to exert on the strap?

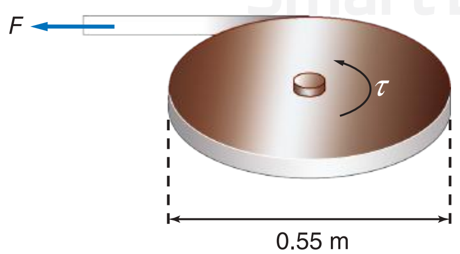


Figure 15

34. Net Torque Two people are pulling on ropes wrapped around the edge of a large wheel. The wheel has a mass of 12 kg and a diameter of 2.4 m. One person pulls in a clockwise direction with a 43 N force, while the other pulls in a counterclockwise direction with a 67 N force. What is the net torque on the wheel?

35. Moment of Inertia Refer to **Table 2,** and rank the moments of inertia from least to greatest of the following objects: a sphere, a wheel with almost all of its mass at the rim, and a solid disk. All have equal masses and diameters. Explain the advantage of using the object with the least moment of inertia.

36. Newton's Second Law for Rotational Motion A rope is wrapped around a pulley and pulled with a force of 13.0 N. The pulley's radius is 0.150 m. The pulley's rotational speed increases from 0.0 to 14.0 rev/min in 4.50 s. What is the moment of inertia of the pulley?

37. Critical Thinking A ball on an extremely low-friction, tilted surface will slide downhill without rotating. If the surface is rough, however, the ball will roll. Explain why, using a free-body diagram.

SECTION 3 Equilibrium

PHYSICS 4 YOU This gymnast moves and shifts his body during his routine to control his movements. Here, he has increased his moment of inertia to help establish balance. At other times, he decreases his moment of inertia to facilitate rapid changes in position.

MAIN IDEA

An object in static equilibrium experiences a net force of zero and a net torque of zero.

Essential Questions

- What is center of mass?
- How does the location of the center of mass affect the stability of an object?
- What are the conditions for equilibrium?
- How do rotating frames of reference give rise to apparent forces?

Review Vocabulary

torque a measure of how effectively a force causes rotation; the magnitude is equal to the force times the lever arm

New Vocabulary

center of mass
centrifugal "force"
Coriolis "force"

The Center of Mass

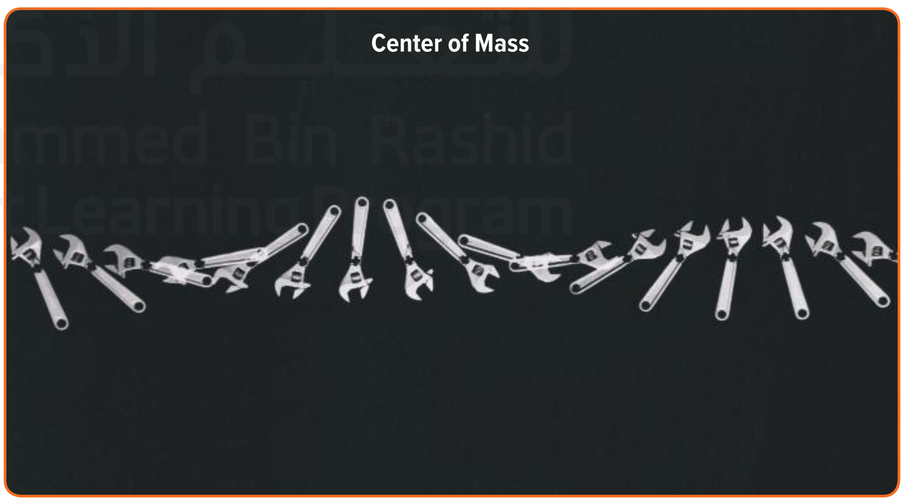
Why are some vehicles more likely than others to roll over when involved in an accident? What causes a vehicle to roll over? The answers are important to the engineers who design safe vehicles. In this section, you will learn some of the factors that cause an object to tip over.

How does a freely moving object rotate around its center of mass? A wrench may spin about its handle or end-over-end. Does any single point on the wrench follow a straight path? **Figure 16** shows the path of the wrench. You can see that there is a single point whose path traces a straight line, as if the wrench could be replaced by a point particle at that location. The black *X* in the photo represents this point. The point on the object that moves in the same way that a point particle would move is the **center of mass** of an object.

Locating the center of mass How can you locate the center of mass of an object? First, suspend the object from any point. When the object stops swinging, the center of mass is somewhere along the vertical line drawn from the suspension point. Draw the line. Then, suspend the object from another point. Again, the center of mass must be directly below this point. Draw a second vertical line. The center of mass is at the point where the two lines cross. The wrench and all other objects that are freely moving through space rotate about an axis that goes through their center of mass. Where would you think the center of mass of a person is located?

☑ **READING CHECK Paraphrase** the definition of the center of mass.

Figure 16 The path of the center of mass of a wrench is a straight line.



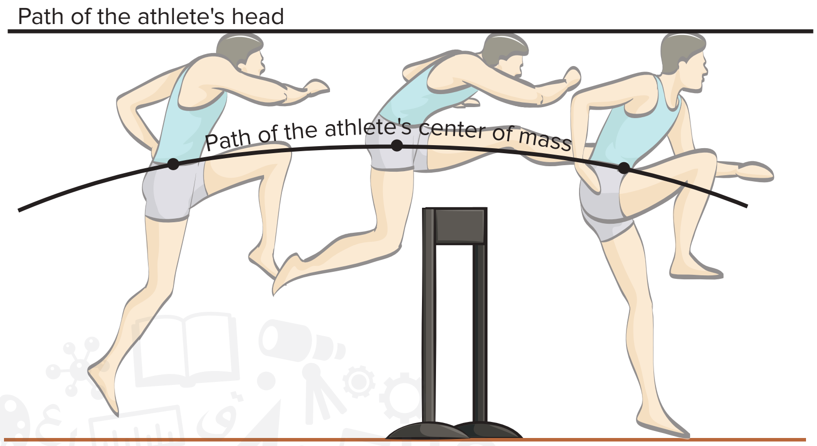


Figure 17 The upward motion of the athlete's head is less than the upward motion of the center of mass. Thus, the head and torso move in a nearly horizontal path. This creates an illusion of floating.

The human body's center of mass For a person who is standing with his arms hanging straight down, the center of mass is a few centimeters below the navel, midway between the front and back of the person's body. The center of mass is farther below the navel for women than men, which often results in better balance for women than men. Because the human body is flexible, however, its center of mass is not fixed. If you raise your hands above your head, your center of mass rises 6 to 10 cm. An athlete, for example, can appear to be floating on air by changing his center of mass in a leap. By raising his arms and legs while in the air, as shown in **Figure 17,** the athlete moves his center of mass up. The path of the center of mass is a parabola, but the athlete's head stays at almost the same height for a surprisingly long time.

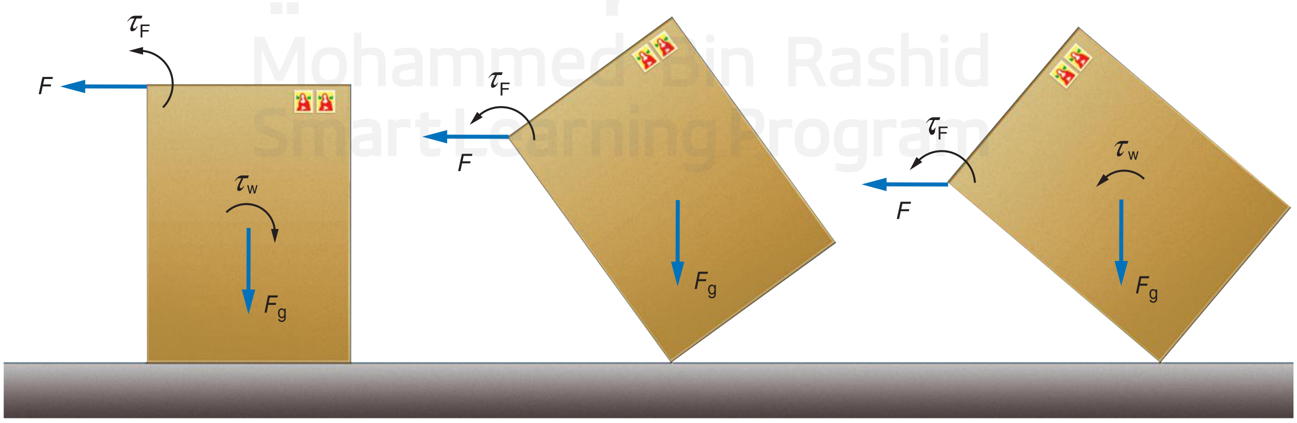
REAL-WORLD PHYSICS

THE FOSBURY FLOP In high jumping, a technique called the Fosbury flop allows a high jumper to clear the bar when it is placed at the highest position. This is possible because the athlete's center of mass passes below the bar as he or she somersaults over the bar, with his or her back toward it.

Center of Mass and Stability

What factors determine whether a vehicle is stable or prone to roll over in an accident? To understand the problem, think about tipping over a box. A tall, narrow box standing on end tips more easily than a low, broad box. Why? To tip a box, as shown in **Figure 18,** you must rotate it about a corner. You pull at the top with a force (***F***) applying a torque (τ_F). The weight of the box, acting on the center of mass ($\boldsymbol{F}_g$) applies an opposing torque (τ_w). When the center of mass is directly above the point of support, τ_w is zero. The only torque is the one applied by you. As the box rotates farther, its center of mass is no longer above its base of support, and both torques act in the same direction. At this point, the box tips over rapidly.

Figure 18 The curved arrows show the direction of the torque produced by the force exerted to tip over a box.



Stability An object is said to be stable if a large external force is required to tip it. The box in **Figure 18** is stable as long as the direction of the torque due to its weight (τ_W) tends to keep it upright. This occurs as long as the box's center of mass lies above its base. To tip the box over, you must rotate its center of mass around the axis of rotation until it is no longer above the base of the box. To rotate the box, you must lift its center of mass. The broader the base, the more stable the object is. Passengers on a city bus, for example, often stand with their feet spread apart to avoid toppling over as the bus starts and stops and weaves through traffic.

Why do vehicles roll over? **Figure 19** shows two vehicles about to roll over. Note that the one with the higher center of mass does not have to be tilted very far for its center of mass to be outside its base–its center of mass does not have to be raised as much as the other vehicle's center of mass. As demonstrated by the vehicles, the lower the location of an object's center of mass, the greater its stability.

You are stable when you stand flat on your feet. When you stand on tiptoe, however, your center of mass moves forward directly above the balls of your feet, and you have very little stability. In judo, aikido, and other martial arts, the fighter uses torque to rotate the opponent into an unstable position, where the opponent's center of mass does not lie above his or her feet. A small person can use torque, rather than force, to defend himself or herself against a stronger person.

In summary, if the center of mass is not located above the base of an object, it is unstable and will roll over without additional torque. If the center of mass is above the base of the object, it is stable. If the base of the object is very narrow and the center of mass is high, then the object might be stable, but the slightest force will cause it to tip over.

READING CHECK **Describe** when an object is the most stable.

MiniLAB

SPINNING TOPS

Where is a spinning object's center of mass?

PhysicsLAB

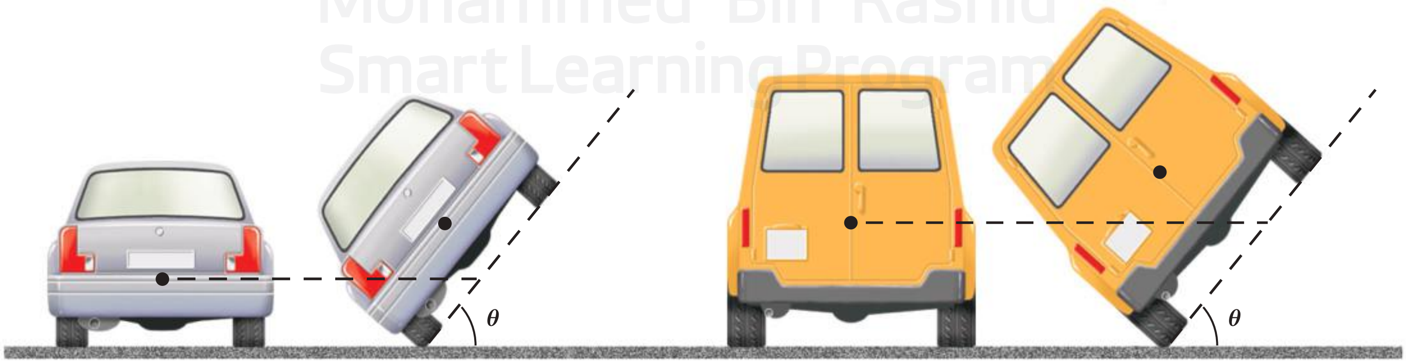
EQUILIBRIUM

Can you make the net torque on scaffolding zero so that it will not rotate?

Conditions for Equilibrium

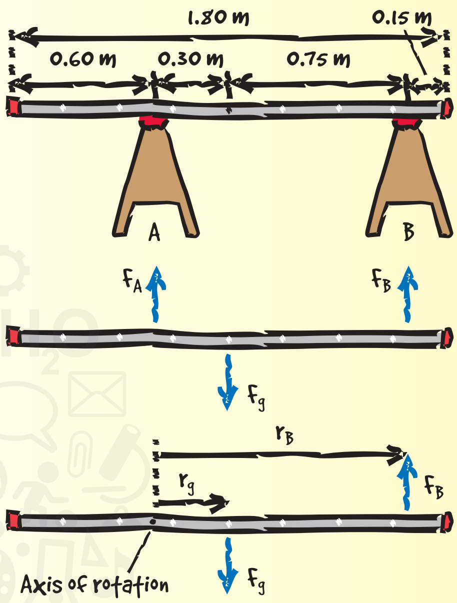
If your pen is at rest, what is needed to keep it at rest? You could either hold it up or place it on a desk or some other surface. An upward force must be exerted on the pen to balance the downward force of gravity. You must also hold the pen so that it will not rotate. An object is said to be in static equilibrium if both its velocity and angular velocity are zero or constant. Thus, for an object to be in static equilibrium, it must meet two conditions. First, it must be in translational equilibrium; that is, the net force exerted on the object must be zero. Second, it must be in rotational equilibrium; that is, the net torque exerted on the object must be zero.

Figure 19 Larger vehicles have a higher center of mass than smaller ones. The higher the center of mass, the smaller the tilt needed to cause the vehicle's center of mass to move outside its base and cause the vehicle to roll over.



EXAMPLE 5

STATIC EQUILIBRIUM A 5.8 kg ladder, 1.80 m long, rests on two sawhorses. Sawhorse A is 0.60 m from one end of the ladder, and sawhorse B is 0.15 m from the other end of the ladder. What force does each sawhorse exert on the ladder?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Choose the axis of rotation at the point where F_A acts on the ladder. Thus, the torque due to F_A is zero.

KNOWN	UNKNOWN
$m = 5.8$ kg	$F_A = ?$
$l = 1.80$ m	$F_B = ?$
$l_A = 0.60$ m	
$l_B = 0.15$ m	

2 SOLVE FOR THE UNKNOWN

For a ladder that has a constant density, the center of mass is at the center rung.

The net force is the sum of all forces on the ladder.

The ladder is in translational equilibrium, so the net force exerted on it is zero.

$$F_{net} = F_A + F_B + (-F_g)$$

$$0.0\text{ N} = F_A + F_B + (-F_g)$$ ◀ *The ladder is in translational equilibrium, so the net force exerted on it is zero.*

Solve for F_A.

$$F_A = F_g - F_B$$

Find the torques due to F_g and F_B.

$$\tau_g = -F_g r_g$$ ◀ *τ_g is in the clockwise direction.*

$$\tau_B = +F_B r_B$$ ◀ *τ_B is in the counterclockwise direction.*

The net torque is the sum of all torques on the object.

$$\tau_{net} = \tau_B + \tau_g$$

$$0.0\text{ N•m} = \tau_B + \tau_g$$ ◀ *The ladder is in rotational equilibrium, so $\tau_{net} = 0.0$ N•m.*

$$\tau_B = -\tau_g$$

$$F_B r_B = F_g r_g$$ ◀ *Substitute $\tau_B = r_B F_B$ and $\tau_g = -r_g F_g$ into the equation.*

Solve for F_B.

$$F_B = \frac{F_g r_g}{r_B}$$

$$= \frac{r_g mg}{r_B}$$ ◀ *Substitute $F_g = mg$*

Using the expression $F_A = F_g - F_B$, substitute in the expressions for F_B and F_g.

$$F_A = F_g - F_B$$

$$= F_g - \frac{r_g mg}{r_B}$$ ◀ *Substitute $F_B = \frac{r_g mg}{r_B}$ into the equation.*

$$= mg - \frac{r_g mg}{r_B}$$ ◀ *Substitute $F_g = mg$ into the equation.*

$$= mg\left(1 - \frac{r_g}{r_B}\right)$$

Solve for r_g.

$$r_g = \left(\frac{1}{2}\right)l - l_A$$
$$= 0.90 \text{ m} - 0.60 \text{ m}$$
$$= 0.30 \text{ m}$$

◀ *For a ladder, which has a constant density, the center of mass is at the center rung.*

◀ *Substitute $\frac{l}{2} = 0.90$ m and $l_A = 0.60$ m into the equation.*

Solve for r_B.

$$r_B = (0.90 \text{ m} - l_B) + (0.90 \text{ m} - l_A)$$
$$= (0.90 \text{ m} - 0.15 \text{ m}) + (0.90 \text{ m} - 0.60 \text{ m})$$
$$= 0.75 \text{ m} + 0.30 \text{ m}$$
$$= 1.05 \text{ m}$$

◀ *Substitute $l_B = 0.15$ m and $l_A = 0.60$ m into the equation.*

Calculate F_B.

$$F_B = \frac{r_g mg}{r_B}$$
$$= \frac{(0.30 \text{ m})(5.8 \text{ kg})(9.8 \text{ N/kg})}{(1.05 \text{ m})}$$
$$= 16 \text{ N}$$

◀ *Substitute $r_g = 0.30$ m, $m = 5.8$ kg, $g = 9.8$ N/kg, and $r_B = 1.05$ m into the equation.*

Calculate F_A.

$$F_A = mg\left(1 - \frac{r_g}{r_B}\right)$$
$$= (5.8 \text{ kg})(9.8 \text{ N/kg})\left(1 - \frac{(0.30 \text{ m})}{(1.05 \text{ m})}\right)$$
$$= 41 \text{ N}$$

◀ *Substitute $r_g = 0.30$ m, $m = 5.8$ kg, $g = 9.8$ N/kg, and $r_B = 1.05$ m into the equation.*

3 EVALUATE THE ANSWER

- **Are the units correct?** Forces are measured in newtons.
- **Do the signs make sense?** Both forces are upward.
- **Is the magnitude realistic?** The forces add up to the weight of the ladder, and the force exerted by the sawhorse closer to the center of mass is greater, which makes sense.

APPLICATIONS

38. What would be the forces exerted by the two sawhorses if the ladder in Example 5 had a mass of 11.4 kg?

39. A 7.3 kg ladder, 1.92 m long, rests on two sawhorses, as shown in **Figure 20.** Sawhorse A, on the left, is located 0.30 m from the end, and sawhorse B, on the right, is located 0.45 m from the other end. Choose the axis of rotation to be the center of mass of the ladder.

a. What are the torques acting on the ladder?

b. Write the equation for rotational equilibrium.

c. Solve the equation for F_A in terms of F_g.

d. How would the forces exerted by the two sawhorses change if A were moved very close to, but not directly under, the center of mass?

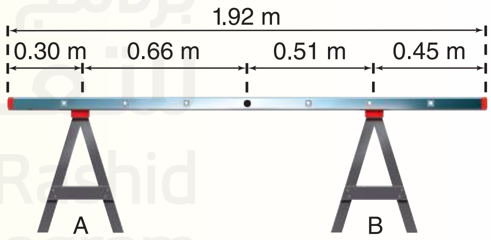


Figure 20

40. A 4.5m long wooden plank with a 24 kg mass is supported in two places. One support is directly under the center of the board, and the other is at one end. What are the forces exerted by the two supports?

41. CHALLENGE A 85 kg diver walks to the end of a diving board. The board, which is 3.5 m long with a mass of 14 kg, is supported at the center of mass of the board and at one end. What are the forces on the two supports?

Rotating Frames of Reference

When you sit on a spinning amusement-park ride, it feels as if a strong force is pushing you to the outside. A pebble on the floor of the ride would accelerate outward without a horizontal force being exerted on it in the same direction. The pebble would not move in a straight line relative to the floor of the ride. In other words, Newton's laws of motion would not seem to apply. This is because in the ride your point of view, called your frame of reference, is rotating. Newton's laws are valid only in non-rotating or nonaccelerated frames of reference.

Motion in a rotating reference frame is important to us because Earth rotates. The effects of the rotation of Earth are too small to be noticed in the classroom or lab, but they are significant influences on the motion of the atmosphere and, therefore, on climate and weather.

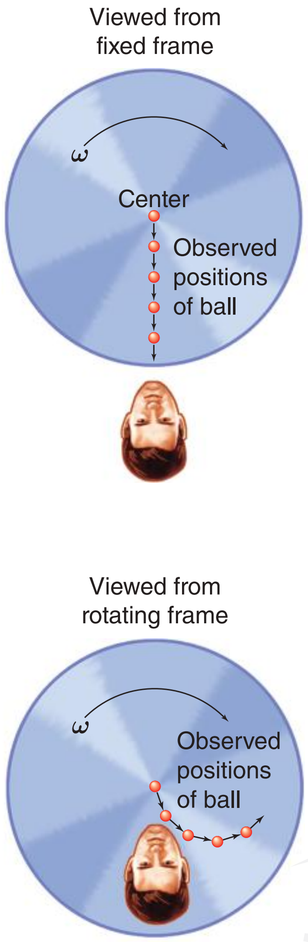


Figure 21 The Coriolis "force" is not a real force. It exists only in rotating reference frames.

Centrifugal "Force"

Suppose you fasten one end of a spring to the center of a rotating platform. An object lies on the platform and is attached to the other end of the spring. As the platform rotates, an observer on the platform sees the object stretch the spring. The observer might think that some force toward the outside of the platform is pulling on the object. This apparent force seems to pull on a moving object but does not exert a physical outward push on it. This apparent force, which seems to push an object outward, is observed only in rotating frames of reference and is called the **centrifugal "force",** which is not a real force.

As the platform rotates, an observer on the ground would see things differently. This observer sees the object moving in a circle, and it accelerates toward the center because of the force of the spring. As you know, this centripetal acceleration is given by $a_c = \frac{v^2}{r}$. It also can be written in terms of angular velocity, as $a_c = \omega^2 r$. Centripetal acceleration is proportional to the distance from the axis of rotation and depends on the square of the angular velocity. Thus, if you double the rotational frequency, the angular acceleration increases by a factor of four.

☑ **READING CHECK** **Define** centrifugal force in your own words.

The Coriolis "Force"

A second effect of rotation is shown in **Figure 21.** Suppose a person standing at the center of a rotating disk throws a ball toward the edge of the disk. Consider the horizontal motion of the ball as seen by two observers, and ignore the vertical motion of the ball as it falls. An observer standing outside the disk sees the ball travel in a straight line at a constant speed toward the edge of the disk. However, the other observer, who is stationed on the disk and rotating with it, sees the ball follow a curved path at a constant speed. The apparent force that seems to deflect a moving object from its path and is observed only in rotating frames of reference is called the **Coriolis "force."** Like the centrifugal "force," the Coriolis "force" is not a real force. It seems to exist because we observe a deflection in horizontal motion when we are in a rotating frame of reference.

Coriolis “force” due to Earth’s rotation

Suppose a cannon is fired from a point on the equator toward a target due north of it. If the projectile were fired directly northward, it would also have an eastward velocity component because of the rotation of Earth. Recall that Earth is actually rotating beneath the projectile.

This eastward speed is greater at the equator than at any other latitude. Thus, as the projectile moves northward, it also moves eastward faster than points on Earth below it do. The result is that the projectile lands east of the target as shown in **Figure 22.**

READING CHECK **Describe** how you would aim a projectile to compensate for the rotation of Earth.

While an observer in space would see Earth’s rotation, an observer on Earth could claim that the projectile missed the target because of the Coriolis “force” on the rocket. Note that for objects moving toward the equator, the direction of the apparent force is westward. A projectile will land west of the target when fired due south.

CONNECTION TO EARTH SCIENCE The direction of winds around high- and low-pressure areas results from the Coriolis “force.” Winds flow from areas of high to low pressure. Because of the Coriolis “force” in the northern hemisphere, winds from the south go to the east of low-pressure areas. Winds from the north end up west of low-pressure areas. Therefore, winds rotate counter-clockwise around low-pressure areas in the northern hemisphere. In the southern hemisphere, however, winds rotate clockwise around low-pressure areas. This is why tropical cyclones, or hurricanes as they are also called, rotate clockwise in the southern hemisphere and counterclockwise in the northern hemisphere.

Figure 22 An observer on Earth sees the Coriolis “force” cause a projectile fired due north to deflect to the right of the intended target.

SECTION 3 REVIEW

42. MAINIDEA Give an example of an object for each of the following conditions.

a. rotational equilibrium, but not translational equilibrium

b. translational equilibrium, but not rotational equilibrium

43. Center of Mass Can the center of mass of an object be located in an area where the object has no mass? Explain.

44. Stability of an Object Why is a modified vehicle with its body raised high on risers less stable than a similar vehicle with its body at normal height?

45. Center of Mass Where is the center of mass on a roll of masking tape?

46. Locating the Center of Mass Describe how you would find the center of mass of this textbook.

47. Rotating Frames of Reference A coin is placed on a rotating, old-fashioned record turntable. At the highest speed, the coin starts sliding outward. What are the forces acting on the coin?

48. Critical Thinking You have read about how the spin of Earth on its axis affects the winds. Predict the direction of the flow of surface ocean currents in the northern and southern hemispheres.

HOW IT **WORKS**

Spin-Cycle

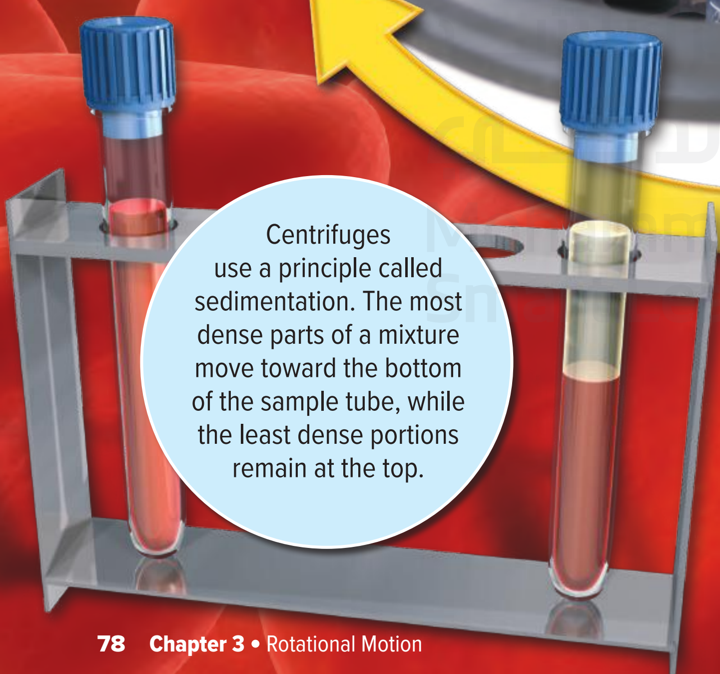
CENTRIFUGES

Powerful physics is at work in devices known as laboratory centrifuges.

One of the main uses of laboratory centrifuges is separating blood into its components for analysis. Some labs also use centrifuges to separate DNA and proteins from the samples.

Centrifuges must be balanced with a sample on one side offset by a sample or a blank on the other. Much like a washing machine, an out-of-balance centrifuge can quickly malfunction.

Laboratory centrifuges can spin very quickly, up to thousands of revolutions per minute (rpm), producing accelerations of hundreds or thousands of times the free-fall acceleration.



Centrifuges use a principle called sedimentation. The most dense parts of a mixture move toward the bottom of the sample tube, while the least dense portions remain at the top.

GOINGFURTHER >>>

Research Interview a technician at a pathology laboratory or a student at a local college or university to learn how centrifuges are used in separating samples to be analyzed.

CHAPTER 3 STUDY GUIDE

BIGIDEA Applying a torque to an object causes a change in that object's angular velocity.

VOCABULARY
- radian
- angular displacement
- angular velocity
- angular acceleration

SECTION 1 Describing Rotational Motion

MAINIDEA Angular displacement, angular velocity, and angular acceleration all help describe angular motion.

- Angular displacement is the change in the angle (θ) as an object rotates. It is usually measured in degrees or radians.
- Average angular velocity is the object's angular displacement divided by the time taken to make the angular displacement. Average angular velocity is represented by the Greek letter omega (ω) and is determined by the following equation:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- Average angular acceleration is the change in angular velocity divided by the time required to make the change.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

VOCABULARY
- lever arm
- torque
- moment of inertia
- Newton's second law for rotational motion

SECTION 2 Rotational Dynamics

MAINIDEA Torques cause changes in angular velocity.

- Torque describes the combination of a force and a lever arm that can cause an object to rotate. Torque is represented by the Greek letter tau (τ) and is determined by the following equation:

$$\tau = Fr \sin \theta$$

- The moment of inertia is a point object's resistance to changes in angular velocity. The moment of inertia is represented by the letter I and for a point mass, it is represented by the following equation:

$$I = mr^2$$

- Newton's second law for rotational motion states that angular acceleration is directly proportional to the net torque and inversely proportional to the moment of inertia.

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

VOCABULARY
- center of mass
- centrifugal "force"
- Coriolis "force"

SECTION 3 Equilibrium

MAINIDEA An object in static equilibrium experiences a net force of zero and a net torque of zero.

- The center of mass of an object is the point on the object that moves in the same way that a point particle would move.
- An object is stable against rollover if its center of mass is above its base.
- An object in equilibrium has no net force exerted on it and there is no net torque acting on it.
- Centrifugal "force" and the Coriolis "force" are two apparent, but nonexistent, forces that seem to exist when a object is analyzed from a rotating frame of reference.

SECTION 1 Describing Rotational Motion

Mastering Concepts

49. BIGIDEA A bicycle wheel rotates at a constant 25 rev/min. Is its angular velocity decreasing, increasing, or constant?

50. A toy rotates at a constant 5 rev/min. Is its angular acceleration positive, negative, or zero?

51. Do all parts of Earth move at the same rate? Explain.

52. A unicycle wheel rotates at a constant 14 rev/min. Is the total acceleration of a point on the tire inward, outward, tangential, or zero?

Mastering Problems

53. On a test stand a bicycle wheel is being rotated about its axle so that a point on the edge moves through 0.210 m. The radius of the wheel is 0.350 m, as shown in **Figure 23.** Through what angle (in radians) is the wheel rotated?

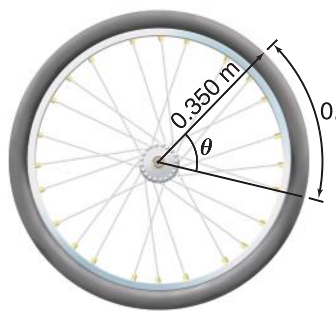


Figure 23

54. The outer edge of a truck tire that has a radius of 45 cm has a velocity of 23 m/s. What is the angular velocity of the tire in rad/s?

55. A steering wheel is rotated through 128°, as shown in **Figure 24.** Its radius is 22 cm. How far would a point on the steering wheel's edge move?

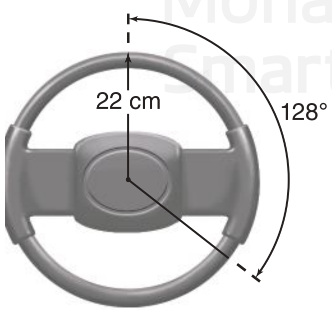


Figure 24

56. Propeller A propeller spins at 1880 rev/min.

a. What is its angular velocity in rad/s?

b. What is the angular displacement of the propeller in 2.50 s?

57. The propeller in the previous problem slows from 475 rev/min to 187 rev/min in 4.00 s. What is its angular acceleration?

58. The automobile wheel shown in **Figure 25** rotates at 2.50 rad/s. How fast does a point 7.00 cm from the center travel?

Figure 25

59. Washing Machine A washing machine's two spin cycles are 328 rev/min and 542 rev/min. The diameter of the drum is 0.43 m.

a. What is the ratio of the centripetal accelerations for the fast and slow spin cycles? Recall that $a_c = \frac{v^2}{r}$ and $v = r\omega$.

b. What is the ratio of the linear velocity of an object at the surface of the drum for the fast and slow spin cycles?

c. Find the maximum centripetal acceleration in terms of g for the washing machine.

60. A laboratory ultracentrifuge is designed to produce a centripetal acceleration of $0.35\times10^6\ g$ at a distance of 2.50 cm from the axis. What angular velocity in revolutions per minute is required?

SECTION 2 Rotational Dynamics

Mastering Concepts

61. Think about some possible rotations of your textbook. Are the moments of inertia about these three axes the same or different? Explain.

62. An auto repair manual specifies the torque needed to tighten bolts on an engine. Why does it not mention force?

63. Ranking Task Rank the torques on the five doors shown in **Figure 26** from least to greatest. Note that the magnitude of all the forces is the same.

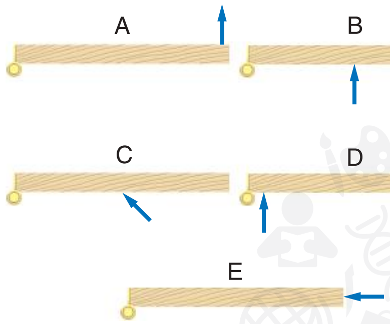


Figure 26

Mastering Problems

64. Wrench A bolt is to be tightened with a torque of 8.0 N•m. If you have a wrench that is 0.35 m long, what is the least amount of force you must exert?

65. What is the torque on a bolt produced by a 15-N force exerted perpendicular to a wrench that is 25 cm long, as shown in **Figure 27?**

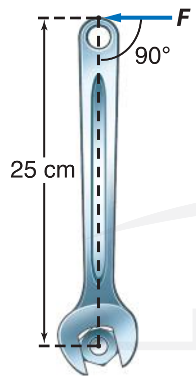


Figure 27

66. A bicycle wheel with a radius of 38 cm is given an angular acceleration of 2.67 rad/s^2 by applying a force of 0.35 N on the edge of the wheel. What is the wheel's moment of inertia?

67. Toy Top A toy top consists of a rod with a diameter of 8.0 mm and a disk of mass 0.0125 kg and a diameter of 3.5 cm. The moment of inertia of the rod can be neglected. The top is spun by wrapping a string around the rod and pulling it with a velocity that increases from zero to 3.0 m/s over 0.50 s.

a. What is the resulting angular velocity of the top?

b. What force was exerted on the string?

68. A toy consisting of two balls, each 0.45 kg, at the ends of a 0.46 m-long, thin, lightweight rod is shown in **Figure 28.** Find the moment of inertia of the toy. The moment of inertia is to be found about the center of the rod.

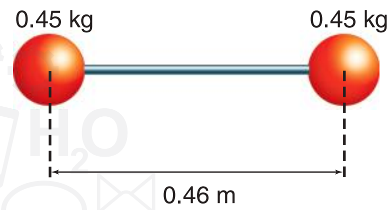


Figure 28

SECTION 3 Equilibrium

Mastering Concepts

69. To balance a car's wheel, it is placed on a vertical shaft and weights are added to make the wheel horizontal. Why is this equivalent to moving the center of mass until it is at the center of the wheel?

70. A stunt driver maneuvers a monster truck so that it is traveling on only two wheels. Where is the center of mass of the truck?

71. Suppose you stand flat-footed, then you rise and balance on tiptoe. If you stand with your toes touching a wall, you cannot balance on tiptoe. Explain.

72. Why does a gymnast appear to be floating on air when he raises his arms above his head in a leap?

73. Why is a vehicle with wheels that have a large diameter more likely to roll over than a vehicle with wheels that have a smaller diameter?

Mastering Problems

74. A 12.5 kg board, 4.00 m long, is being held up on one end by Ahmed. He calls for help in lifting the board, and Marwan responds.

a. What is the least force that Marwan could exert to lift the board to the horizontal position? What part of the board should he lift to exert this force?

b. What is the greatest force that Marwan could exert to lift the board to the horizontal position? What part of the board should he lift to exert this force?

75. A car's specifications state that its weight distribution is 53 percent on the front tires and 47 percent on the rear tires. The wheel base is 2.46 m. Where is the car's center of mass?

76. Two people are holding up the ends of a 4.25 kg wooden board that is 1.75 m long. A 6.00 kg box sits on the board, 0.50 m from one end, as shown in **Figure 29.** What forces do the two people exert?

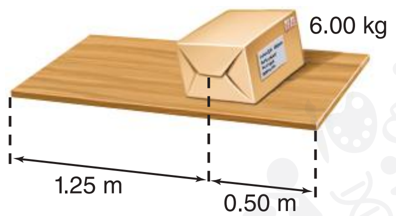


Figure 29

Applying Concepts

77. Two gears are in contact and rotating. One is larger than the other, as shown in **Figure 30.** Compare their angular velocities. Also compare the linear velocities of two teeth that are in contact.

Figure 30

78. How can you experimentally find the moment of inertia of an object?

79. Bicycle Wheels Three bicycle wheels have masses that are distributed in three different ways: mostly at the rim, uniformly, and mostly at the hub. The wheels all have the same mass. If equal torques are applied to them, which one will have the greatest angular acceleration? Which one will have the least?

80. Bowling Ball When a bowling ball leaves a bowler's hand, it does not spin. After it has gone about half the length of the lane, however, it does spin. Explain how its rotation rate increased.

81. Flat Tire Suppose your car has a flat tire. You find a lug wrench to remove the nuts from the bolt studs. You cannot turn the nuts. Your friend suggests ways you might produce enough torque to turn them. What three ways might your friend suggest?

82. Why can you ignore forces that act on the axis of rotation of an object in static equilibrium when determining the net torque?

83. Tightrope Walkers Tightrope walkers often carry long poles that sag so that the ends are lower than the center as shown in **Figure 31.** How does a pole increase the tightrope walker's stability? *Hint: Consider both center of mass and moment of inertia.*

Figure 31

84. Merry-Go-Round While riding a merry-go-round, you toss a key to a friend standing on the ground. For your friend to be able to catch the key, should you toss it a second or two before you reach the spot where your friend is standing or wait until your friend is directly beside you? Explain.

85. In solving problems about static equilibrium, why is the axis of rotation often placed at a point where one or more forces are acting on the object?

86. Ranking Task Rank the objects in **Figure 32** according to their moments of inertia, from least to greatest. Each has a radius of (R) and total mass (M), which is uniformly distributed throughout the shaded region. Specifically indicate any ties.

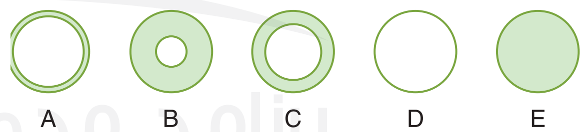


Figure 32

Mixed Review

87. A wooden door of mass m and length l is held horizontally by Haytham and Adham. Haytham suddenly drops his end.

a. What is the angular acceleration of the door just after Haytham lets go?

b. Is the acceleration constant? Explain.

88. Hard Drive A hard drive on a computer spins at 7200 rpm (revolutions per minute). If the drive is designed to start from rest and reach operating speed in 1.5 s, what is the angular acceleration of the disk?

89. Topsoil Ten bags of topsoil, each weighing 75 N, are placed on a 2.43 m-long sheet of wood. They are stacked 0.50 m from one end of the sheet of wood, as shown in **Figure 33.** Two people lift the sheet of wood, one at each end. Ignoring the weight of the wood, how much force must each person exert?

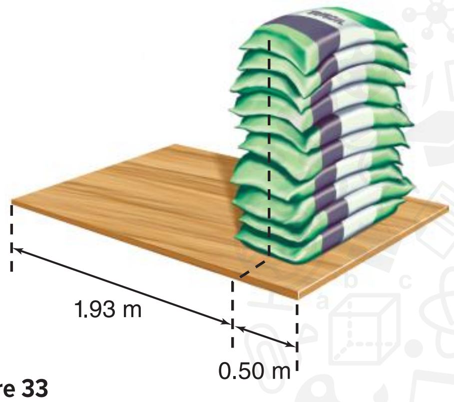


Figure 33

90. A steel beam that is 6.50 m long weighs 325 N. It rests on two supports, 3.00 m apart, with equal amounts of the beam extending from each end. Suki, who weighs 575 N, stands on the beam in the center and then walks toward one end. How close to the end can she walk before the beam begins to tip?

91. The second hand on a watch is 12 mm long. What is the velocity of its tip?

92. A cylinder with a 50 cm diameter, as shown in **Figure 34,** is at rest on a surface. A rope is wrapped around the cylinder and pulled. The cylinder rolls without slipping.

a. After the rope has been pulled a distance of 2.50 m at a constant speed, how far has the center of mass of the cylinder moved?

b. If the rope was pulled a distance of 2.50 m in 1.25 s, how fast was the center of mass of the cylinder moving?

c. What is the angular velocity of the cylinder?

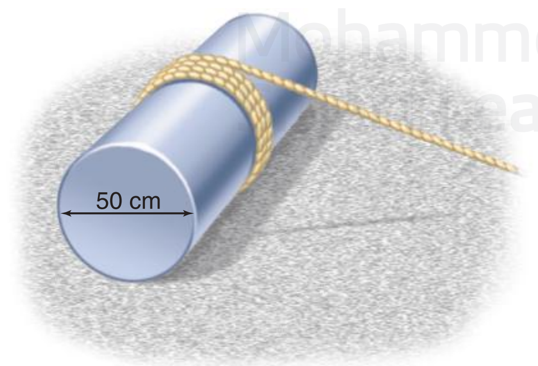


Figure 34

93. Basketball A basketball is rolled down the court. A regulation basketball has a diameter of 24.1 cm, a mass of 0.60 kg, and a moment of inertia of 5.8×10^{-3} kg•m^2. The basketball's initial velocity is 2.5 m/s.

a. What is its initial angular velocity?

b. The ball rolls a total of 12 m. How many revolutions does it make?

c. What is its total angular displacement?

94. The basketball in the previous problem stops rolling after traveling 12 m.

a. If its acceleration was constant, what was its angular acceleration?

b. What torque was acting on it as it was slowing down?

95. Speedometers Most speedometers in automobiles measure the angular velocity of the transmission and convert it to speed. How will increasing the diameter of the tires affect the reading of the speedometer?

96. A box is dragged across the floor using a rope that is a distance *h* above the floor. The coefficient of friction is 0.35. The box is 0.50 m high and 0.25 m wide. Find the force that just tips the box.

97. Lumber You buy a 2.44 m-long piece of 10 cm × 10 cm lumber. Your friend buys a piece of the same size and cuts it into two lengths, each 1.22 m long, as shown in **Figure 35.** You each carry your lumber on your shoulders.

a. Which load is easier to lift? Why?

b. Both you and your friend apply a torque with your hands to keep the lumber from rotating. Which load is easier to keep from rotating? Why?

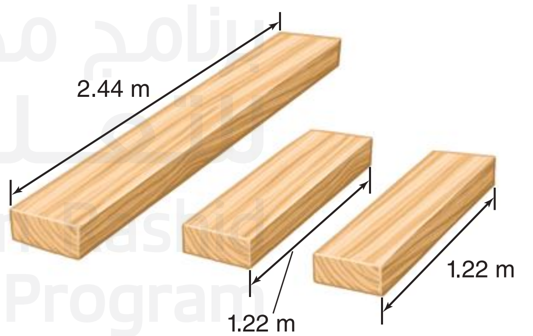


Figure 35

98. Surfboard Fares and Mourad carry a surfboard that is 2.43 m long and weighs 143 N. Mourad lifts one end with a force of 57 N.

a. What force must Fares exert?

b. What part of the board should Fares lift?

Thinking Critically

99. Analyze and Conclude A banner is suspended from a horizontal, pivoted pole, as shown in **Figure 36.** The pole is 2.10 m long and weighs 175 N. The banner, which weighs 105 N, is suspended 1.80 m from the pivot point or axis of rotation. What is the tension in the cable supporting the pole?

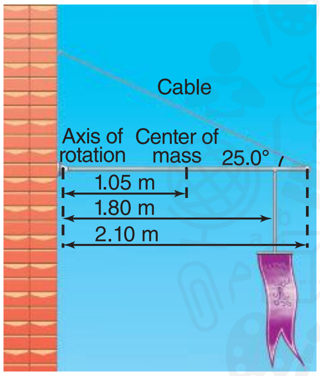


Figure 36

100. Analyze and Conclude A pivoted lamp pole is shown in **Figure 37.** The pole weighs 27 N, and the lamp weighs 64 N.

a. What is the torque caused by each force?

b. Determine the tension in the rope supporting the lamp pole.

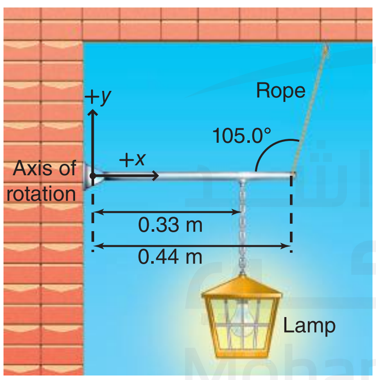


Figure 37

101. Reverse Problem Write a physics problem with real-life objects for which the following equation would be part of the solution:

$$\Delta\theta = \left(20\ \frac{\text{rad}}{\text{s}}\right)(4\ \text{s}) - \frac{1}{2}\left(3.5\ \frac{\text{rad}}{\text{s}^2}\right)(4\ \text{s})^2$$

102. Problem Posing Complete this problem so that it can be solved using the concept of torque: "A painter carries a 3.0 m, 12 kg ladder… ."

103. Apply Concepts Consider a point on the edge of a wheel rotating about its axis.

a. Under what conditions can the centripetal acceleration be zero?

b. Under what conditions can the tangential (linear) acceleration be zero?

c. Can the tangential acceleration be nonzero while the centripetal acceleration is zero? Explain.

d. Can the centripetal acceleration be nonzero while the tangential acceleration is zero? Explain.

104. Apply Concepts When you apply the brakes in a car, the front end dips. Why?

Writing in Physics

105. Astronomers know that if a natural satellite is too close to a planet, it will be torn apart by tidal forces. The difference in the gravitational force on the part of the satellite nearest the planet and the part farthest from the planet is stronger than the forces holding the satellite together. Research the Roche limit, and determine how close the Moon would have to orbit Earth to be at the Roche limit.

106. Automobile engines are rated by the torque they produce. Research and explain why torque is an important quantity to measure.

Cumulative Review

107. Two blocks, one of mass 2.0 kg and the other of mass 3.0 kg, are tied together with a massless rope. This rope is strung over a massless, resistance-free pulley. The blocks are released from rest. Find the following:

a. the tension in the rope

b. the acceleration of the blocks

108. Fares sits on a seesaw. At what angle, relative to the vertical, will the component of his weight parallel to the length of the seesaw be equal to one-third the perpendicular component of his weight?

109. The pilot of a plane wants to reach an airport 325 km due north in 2.75 h. A wind is blowing from the west at 30.0 km/h. What heading and airspeed should be chosen to reach the destination on time?

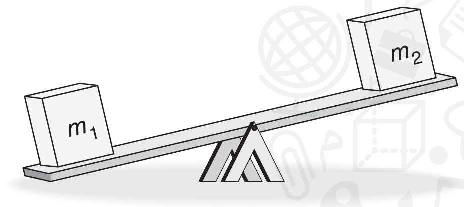
110. A 60.0 kg speed skater with a velocity of 18.0 m/s skates into a curve of 20.0 m radius. How much friction must be exerted between the skates and the ice for him to negotiate the curve?

MULTIPLE CHOICE

1. The illustration below shows two boxes on opposite ends of a massless board that is 3.0 m long. The board is supported in the middle by a fulcrum. The box on the left has a mass (m_1) of 25 kg, and the box on the right has a mass (m_2) of 15 kg. How far should the fulcrum be positioned from the left side of the board in order to balance the masses horizontally?

A. 0.38 m **C.** 1.1 m

B. 0.60 m **D.** 1.9 m



2. A force of 60 N is exerted on one end of a 1.0 m-long lever. The other end of the lever is attached to a rotating rod that is perpendicular to the lever. By pushing down on the end of the lever, you can rotate the rod. If the force on the lever is exerted at an angle of 30° to the perpendicular to the lever, what torque is exerted on the rod? (sin 30° = 0.5; cos 30° = 0.87; tan 30° = 0.58)

A. 30 N **C.** 60 N

B. 52 N **D.** 69 N

3. A child attempts to use a wrench to remove a nut on a bicycle. Removing the nut requires a torque of 10 N•m. The maximum force the child is capable of exerting at a 90° angle is 50 N. What is the length of the wrench the child must use to remove the nut?

A. 0.1 m **C.** 0.2 m

B. 0.15 m **D.** 0.25 m

4. A car moves a distance of 420 m. Each tire on the car has a diameter of 42 cm. Which shows how many revolutions each tire makes as they move that distance?

A. $\left(\frac{(5.0\times10^1)}{\pi}\right)$ rev **C.** $\left(\frac{(1.5\times10^2)}{\pi}\right)$ rev

B. $\left(\frac{(1.0\times10^2)}{\pi}\right)$ rev **D.** $\left(\frac{(1.0\times10^3)}{\pi}\right)$ rev

5. A thin hoop with a mass of 5.0 kg rotates about a perpendicular axis through its center. A force of 25 N is exerted tangentially to the hoop. If the hoop's radius is 2.0 m, what is its angular acceleration?

A. 1.3 rad/s **C.** 5.0 rad/s

B. 2.5 rad/s **D.** 6.3 rad/s

6. Two of the tires on a farmer's tractor have diameters of 1.5 m. If the farmer drives the tractor at a linear velocity of 3.0 m/s, what is the angular velocity of each tire?

A. 2.0 rad/s **C.** 4.0 rad/s

B. 2.3 rad/s **D.** 4.5 rad/s

FREE RESPONSE

7. You use a 25 cm long wrench to remove the lug nuts on a car wheel, as shown in the illustration below. If you pull up on the end of the wrench with a force of 2.0×10^2 N at an angle of 30°, what is the torque on the wrench? (sin 30°= 0.5, cos 30° = 0.87)