تجميع هيكل فيزياء

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1.	Explain Kepler's Second Law which states that an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals.	Student Book	165

Tycho Brahe realized that the charts of the time did not accurately predict astronomical events. He recognized that measurements were required from one location over a long period of time. Tycho was granted an estate on the Danish island of Hven and the funding to build an early research institute. Telescopes had not been invented, so to make measurements, Tycho used huge instruments that he designed and built in his own shop, like those shown in **Figure 1.** Tycho is credited with the most accurate measurements of the time.

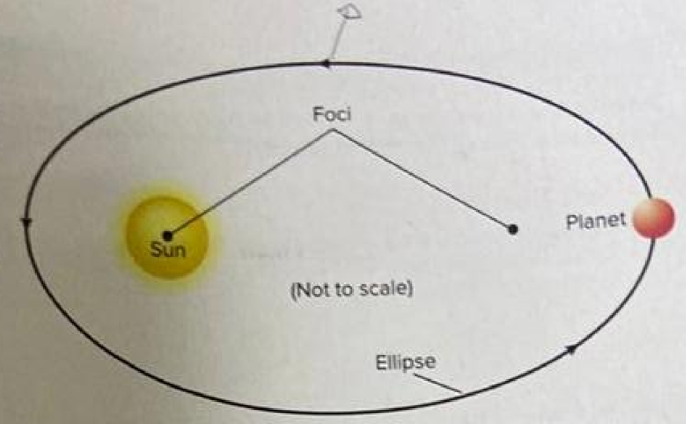


Figure 2 The orbit of each planet is an ellipse, with the Sun at one focus.

Kepler's Laws

In 1600 Tycho moved to Prague where Johannes Kepler, a 29-year-old German, became one of his assistants. Kepler analyzed Tycho's observations. After Tycho's death in 1601, Kepler continued to study Tycho's data and used geometry and mathematics to explain the motion of the planets. After seven years of careful analysis of Tycho's data on Mars, Kepler discovered the laws that describe the motion of every planet and satellite, natural or artificial. Here, the laws are presented in terms of planets.

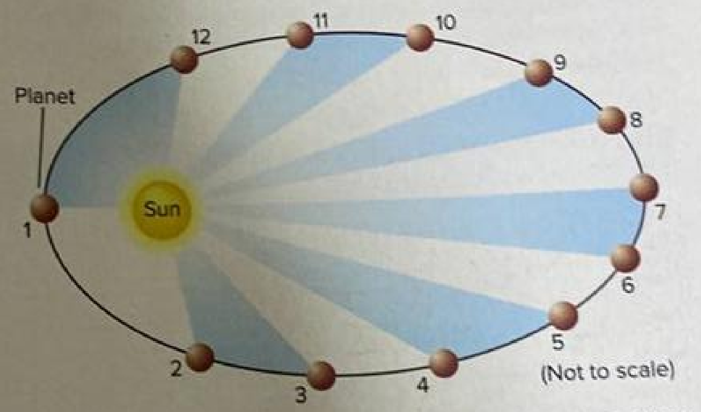


Figure 3 Kepler found that elliptical orbits sweep out equal areas in equal time periods.
Explain *why the equal time areas are shaped differently.*

Kepler's first law states that the paths of the planets are ellipses, with the Sun at one focus. An ellipse has two foci, as shown in **Figure 2.** Although exaggerated ellipses are used in the diagrams, Earth's actual orbit is very nearly circular. You would not be able to distinguish it from a circle visually.

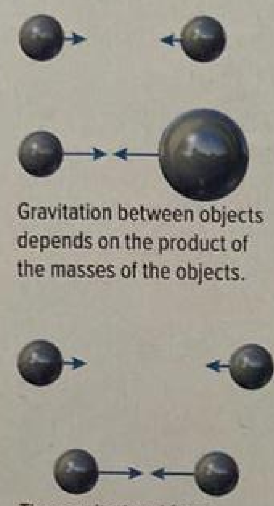
Kepler found that orbits might change due to gravitational effects from, or collisions with, other objects in the solar system. He also found that the planets move faster when they are closer to the Sun and slower when they are farther away from the Sun. **Kepler's second law** states that an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals, as illustrated in **Figure 3.**

A period is the time it takes for one revolution of an orbiting body. Kepler also discovered a mathematical relationship between periods of planets and their mean distances away from the Sun.

Get It?
Describe the common feature that Kepler's first law found concerning the paths of orbiting objects around the Sun.

2.	Explain the law of universal gravitation and write it in equation form $\left[F_g = G\frac{m_1 m_2}{r^2}\right]$	Student Book	168

Newton's Law of Universal Gravitation Fg



Gravitation between objects depends on the product of the masses of the objects.

The gravitational force between objects is inversely proportional to the square of the distance between the objects.

Figure 5 Mass and distance affect the magnitude of the gravitational force between objects.

In 1666, Isaac Newton began his studies of planetary motion. It has been said that seeing an apple fall made Newton wonder if the force that caused the apple to fall might extend to the Moon, or even beyond. He found that the magnitude of the force (F_g) on a planet due to the Sun varies inversely with the square of the distance (r) between the centers of the planet and the Sun. That is, F_g is proportional to $\frac{1}{r^2}$. The force (F_g) acts in the direction of the line connecting the centers of the two objects, as shown in **Figure 5.**

Newton found that both the apple's and the Moon's accelerations agree with the $\frac{1}{r^2}$ relationship. According to his own third law, the force Earth exerts on the apple is exactly the same as the force the apple exerts on Earth. Even though these forces are exactly the same, you can easily observe the effect of the force on the apple because it has much lower mass than Earth. The force of attraction between two objects must be proportional to the objects' masses and is known as the **gravitational force.**

Newton was confident that the same force of attraction would act between any two objects anywhere in the universe. He proposed the **law of universal gravitation,** which states that objects attract other objects with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. Newton's law of universal gravitation, shown below, provides the mathematical models to describe and predict the effects of gravitational forces between distant objects.

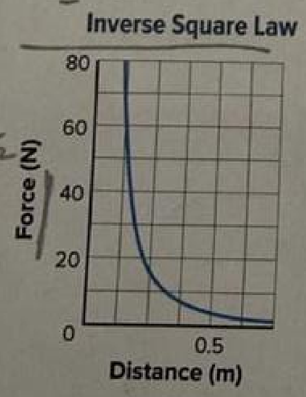


Figure 6 This is a graphical representation of the inverse square relationship.

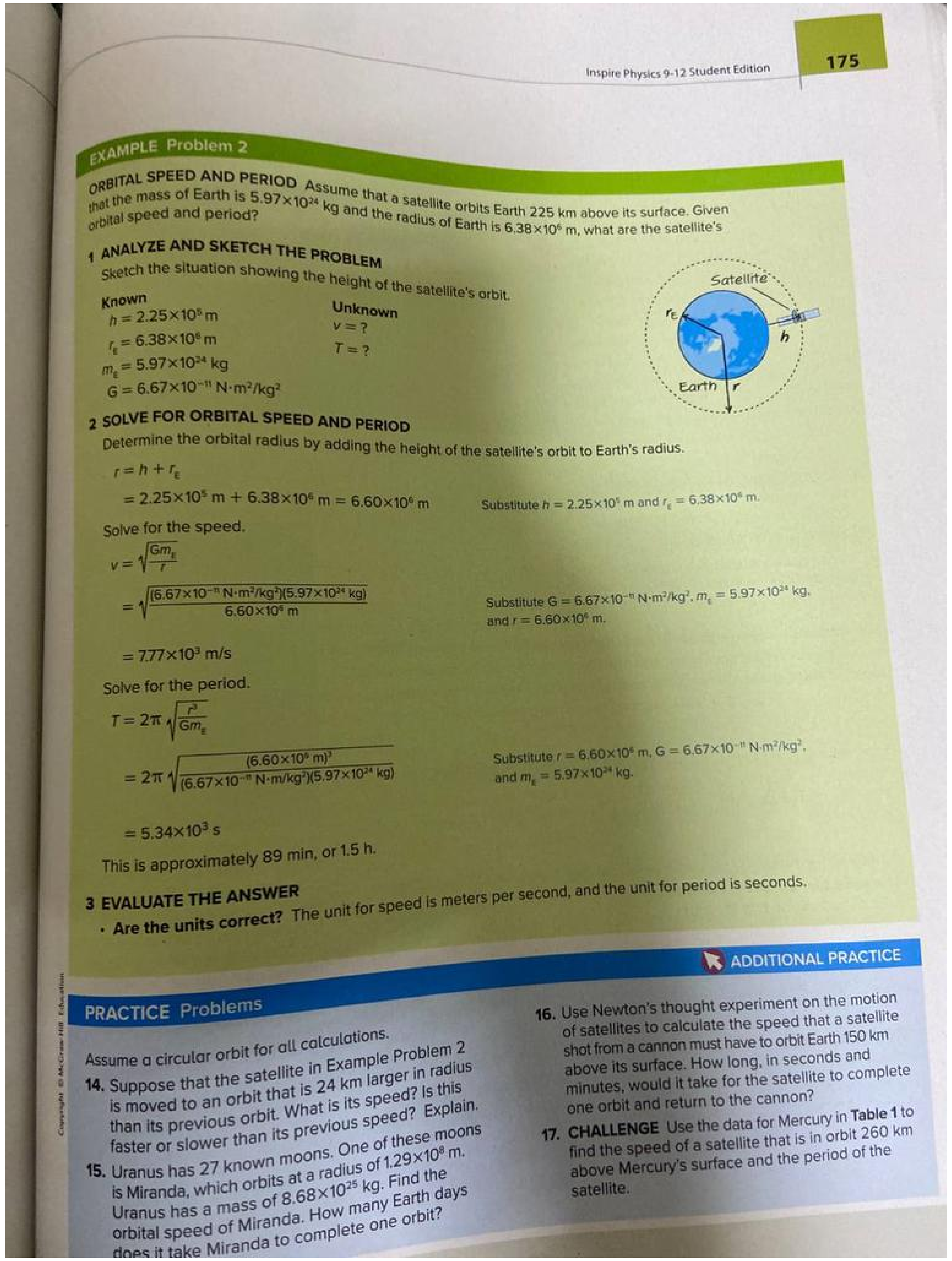
Law of Universal Gravitation

The gravitational force is equal to the universal gravitational constant, times the mass of object 1, times the mass of object 2, divided by the distance between the centers of the objects, squared.

$$F_g = \frac{Gm_1m_2}{r^2}$$

According to Newton's equation, F is directly proportional to m_1 and m_2. If the mass of a planet near the Sun doubles, the force of attraction doubles. Use the Connecting Math to Physics feature on the next page to examine how changing one variable affects another. **Figure 6** illustrates the inverse square relationship graphically. The term G is the universal gravitational constant and will be discussed on the following pages.

3.	Calculate the orbital period of a satellite.	Example Problem (2) Check your Progress Q.8	175 172



EXAMPLE Problem 2

ORBITAL SPEED AND PERIOD Assume that a satellite orbits Earth 225 km above its surface. Given that the mass of Earth is 5.97×10^{24} kg and the radius of Earth is 6.38×10^{6} m, what are the satellite's orbital speed and period?

1 ANALYZE AND SKETCH THE PROBLEM

Sketch the situation showing the height of the satellite's orbit.

Known

$h = 2.25\times10^{5}$ m

$r_E = 6.38\times10^{6}$ m

$m_E = 5.97\times10^{24}$ kg

$G = 6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2$

Unknown

$v = ?$

$T = ?$

2 SOLVE FOR ORBITAL SPEED AND PERIOD

Determine the orbital radius by adding the height of the satellite's orbit to Earth's radius.

$$r = h + r_E$$

$$= 2.25\times10^{5}\ \text{m} + 6.38\times10^{6}\ \text{m} = 6.60\times10^{6}\ \text{m}$$

Substitute $h = 2.25\times10^{5}$ m and $r_E = 6.38\times10^{6}$ m.

Solve for the speed.

$$v = \sqrt{\frac{Gm_E}{r}}$$

$$= \sqrt{\frac{(6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{6.60\times10^{6}\ \text{m}}}$$

Substitute $G = 6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2$, $m_E = 5.97\times10^{24}$ kg, and $r = 6.60\times10^{6}$ m.

$$= 7.77\times10^{3}\ \text{m/s}$$

Solve for the period.

$$T = 2\pi\sqrt{\frac{r^3}{Gm_E}}$$

$$= 2\pi\sqrt{\frac{(6.60\times10^{6}\ \text{m})^3}{(6.67\times10^{-11}\ \text{N·m/kg}^2)(5.97\times10^{24}\ \text{kg})}}$$

Substitute $r = 6.60\times10^{6}$ m, $G = 6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2$, and $m_E = 5.97\times10^{24}$ kg.

$$= 5.34\times10^{3}\ \text{s}$$

This is approximately 89 min, or 1.5 h.

3 EVALUATE THE ANSWER

- **Are the units correct?** The unit for speed is meters per second, and the unit for period is seconds.

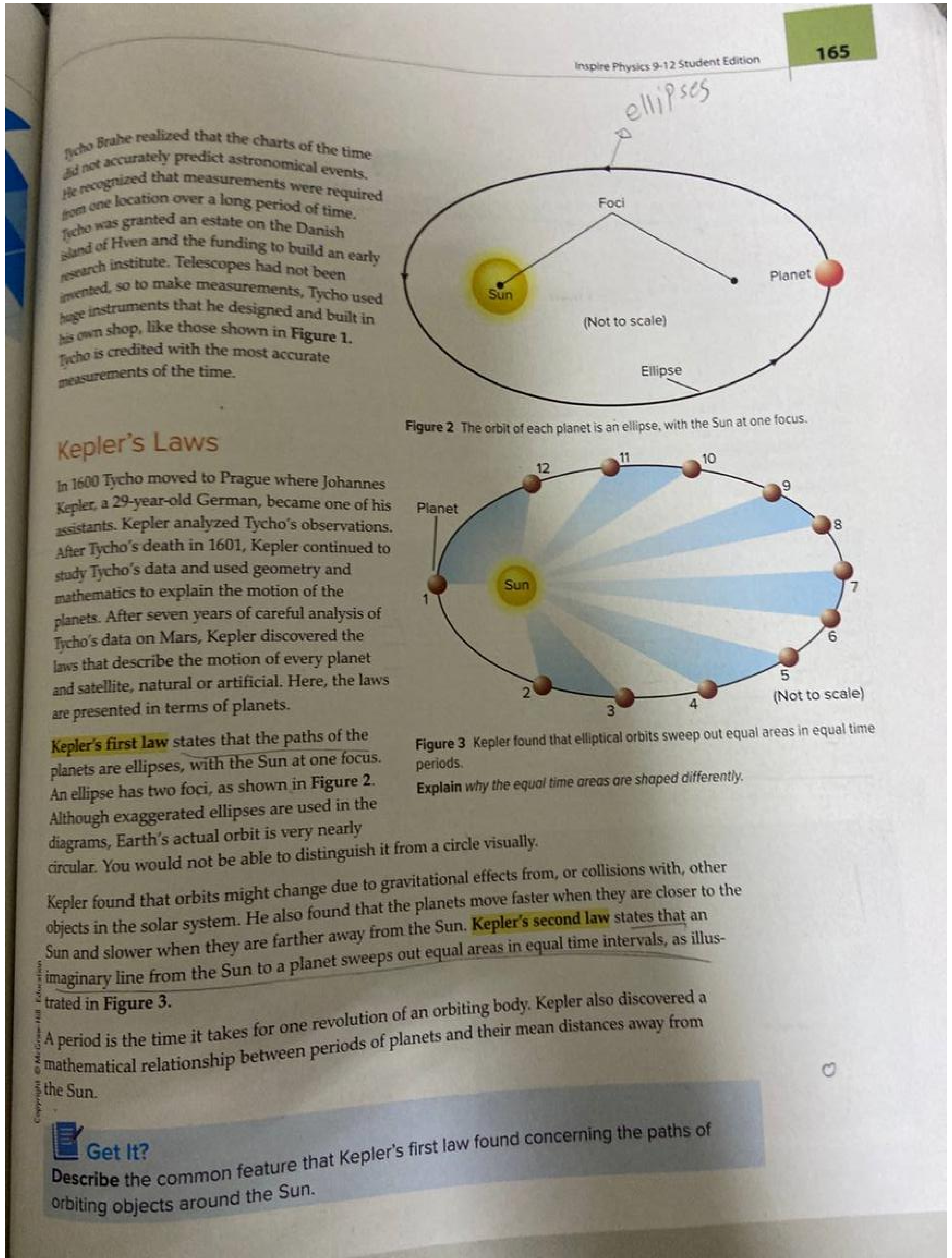
ADDITIONAL PRACTICE

PRACTICE Problems

Assume a circular orbit for all calculations.

14. Suppose that the satellite in Example Problem 2 is moved to an orbit that is 24 km larger in radius than its previous orbit. What is its speed? Is this faster or slower than its previous speed? Explain.
15. Uranus has 27 known moons. One of these moons is Miranda, which orbits at a radius of 1.29×10^{8} m. Uranus has a mass of 8.68×10^{25} kg. Find the orbital speed of Miranda. How many Earth days does it take Miranda to complete one orbit?
16. Use Newton's thought experiment on the motion of satellites to calculate the speed that a satellite shot from a cannon must have to orbit Earth 150 km above its surface. How long, in seconds and minutes, would it take for the satellite to complete one orbit and return to the cannon?
17. **CHALLENGE** Use the data for Mercury in **Table 1** to find the speed of a satellite that is in orbit 260 km above Mercury's surface and the period of the satellite.

4.	Explain Kepler's First Law which states that the planets follow elliptical paths with the sun at one focus.	Student Book	165



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Get It?
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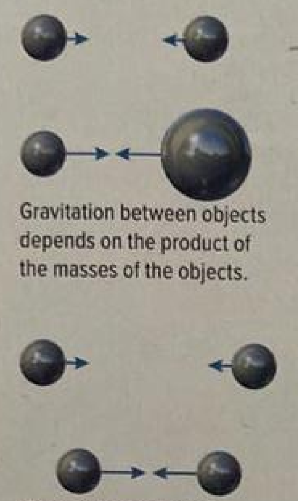
5.	Define gravitational force as the force of attraction between two objects of given mass.	Student Book	168

Newton's Law of Universal Gravitation



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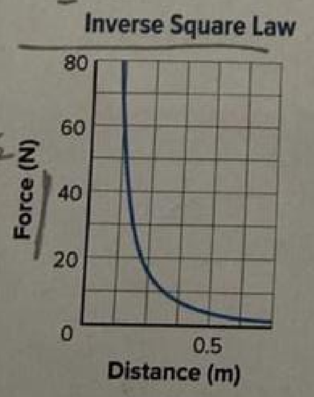

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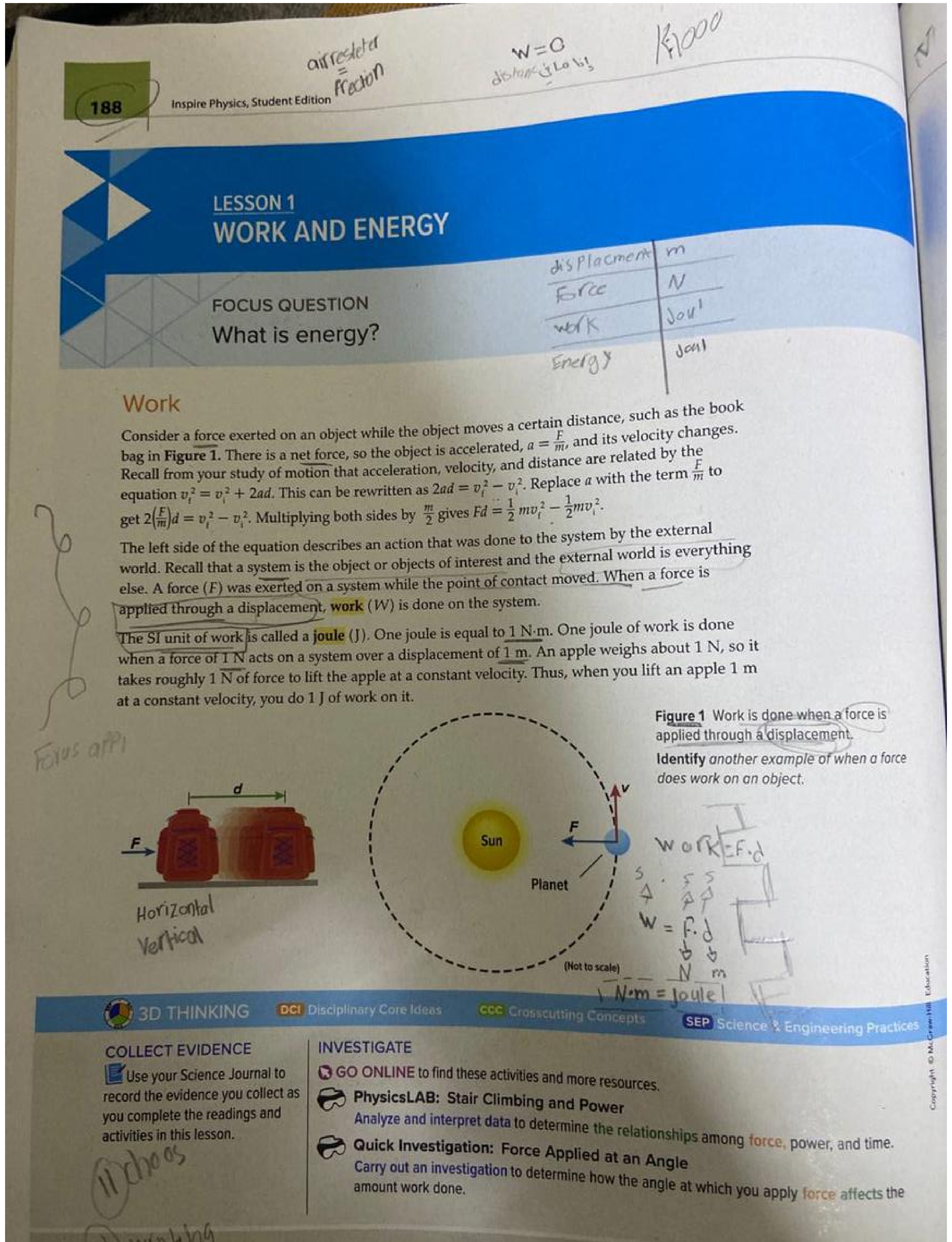
6.	Identify work as a scalar quantity measured in N.m or Joule (J).	Student Book	188



LESSON 1

WORK AND ENERGY

FOCUS QUESTION

What is energy?

Work

Consider a force exerted on an object while the object moves a certain distance, such as the book bag in **Figure 1**. There is a net force, so the object is accelerated, $a = \frac{F}{m}$, and its velocity changes. Recall from your study of motion that acceleration, velocity, and distance are related by the equation $v_f^2 = v_i^2 + 2ad$. This can be rewritten as $2ad = v_f^2 - v_i^2$. Replace a with the term $\frac{F}{m}$ to get $2\left(\frac{F}{m}\right)d = v_f^2 - v_i^2$. Multiplying both sides by $\frac{m}{2}$ gives $Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$.

The left side of the equation describes an action that was done to the system by the external world. Recall that a system is the object or objects of interest and the external world is everything else. A force (F) was exerted on a system while the point of contact moved. When a force is applied through a displacement, **work** (W) is done on the system.

The SI unit of work is called a **joule** (J). One joule is equal to 1 N·m. One joule of work is done when a force of 1 N acts on a system over a displacement of 1 m. An apple weighs about 1 N, so it takes roughly 1 N of force to lift the apple at a constant velocity. Thus, when you lift an apple 1 m at a constant velocity, you do 1 J of work on it.

Figure 1 Work is done when a force is applied through a displacement.

Identify *another example of when a force does work on an object.*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

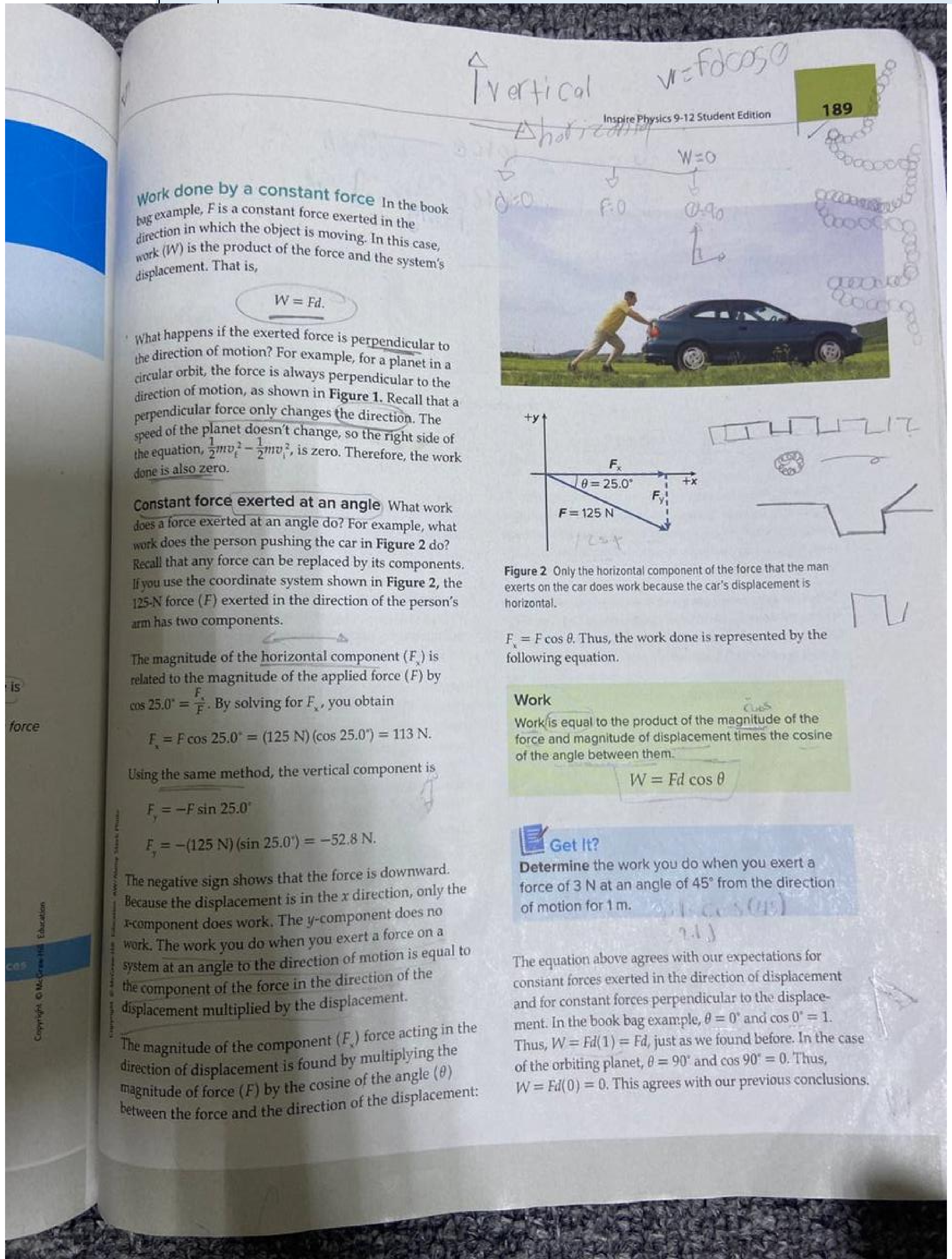
PhysicsLAB: Stair Climbing and Power

Analyze and interpret data to determine the relationships among force, power, and time.

Quick Investigation: Force Applied at an Angle

Carry out an investigation to determine how the angle at which you apply force affects the amount work done.

7.	Illustrate when work is positive, negative or zero with suitable examples.	Student Book Practice Problem (9)	189 193



Work done by a constant force In the book bag example, F is a constant force exerted in the direction in which the object is moving. In this case, work (W) is the product of the force and the system's displacement. That is,

$$W = Fd.$$

What happens if the exerted force is perpendicular to the direction of motion? For example, for a planet in a circular orbit, the force is always perpendicular to the direction of motion, as shown in **Figure 1**. Recall that a perpendicular force only changes the direction. The speed of the planet doesn't change, so the right side of the equation, $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, is zero. Therefore, the work done is also zero.

Constant force exerted at an angle What work does a force exerted at an angle do? For example, what work does the person pushing the car in **Figure 2** do? Recall that any force can be replaced by its components. If you use the coordinate system shown in **Figure 2**, the 125-N force (F) exerted in the direction of the person's arm has two components.

The magnitude of the horizontal component (F_x) is related to the magnitude of the applied force (F) by $\cos 25.0° = \frac{F_x}{F}$. By solving for F_x, you obtain

$$F_x = F\cos 25.0° = (125\ N)(\cos 25.0°) = 113\ N.$$

Using the same method, the vertical component is

$$F_y = -F\sin 25.0°$$

$$F_y = -(125\ N)(\sin 25.0°) = -52.8\ N.$$

The negative sign shows that the force is downward. Because the displacement is in the x direction, only the x-component does work. The y-component does no work. The work you do when you exert a force on a system at an angle to the direction of motion is equal to the component of the force in the direction of the displacement multiplied by the displacement.

The magnitude of the component (F_x) force acting in the direction of displacement is found by multiplying the magnitude of force (F) by the cosine of the angle (θ) between the force and the direction of the displacement:

Figure 2 Only the horizontal component of the force that the man exerts on the car does work because the car's displacement is horizontal.

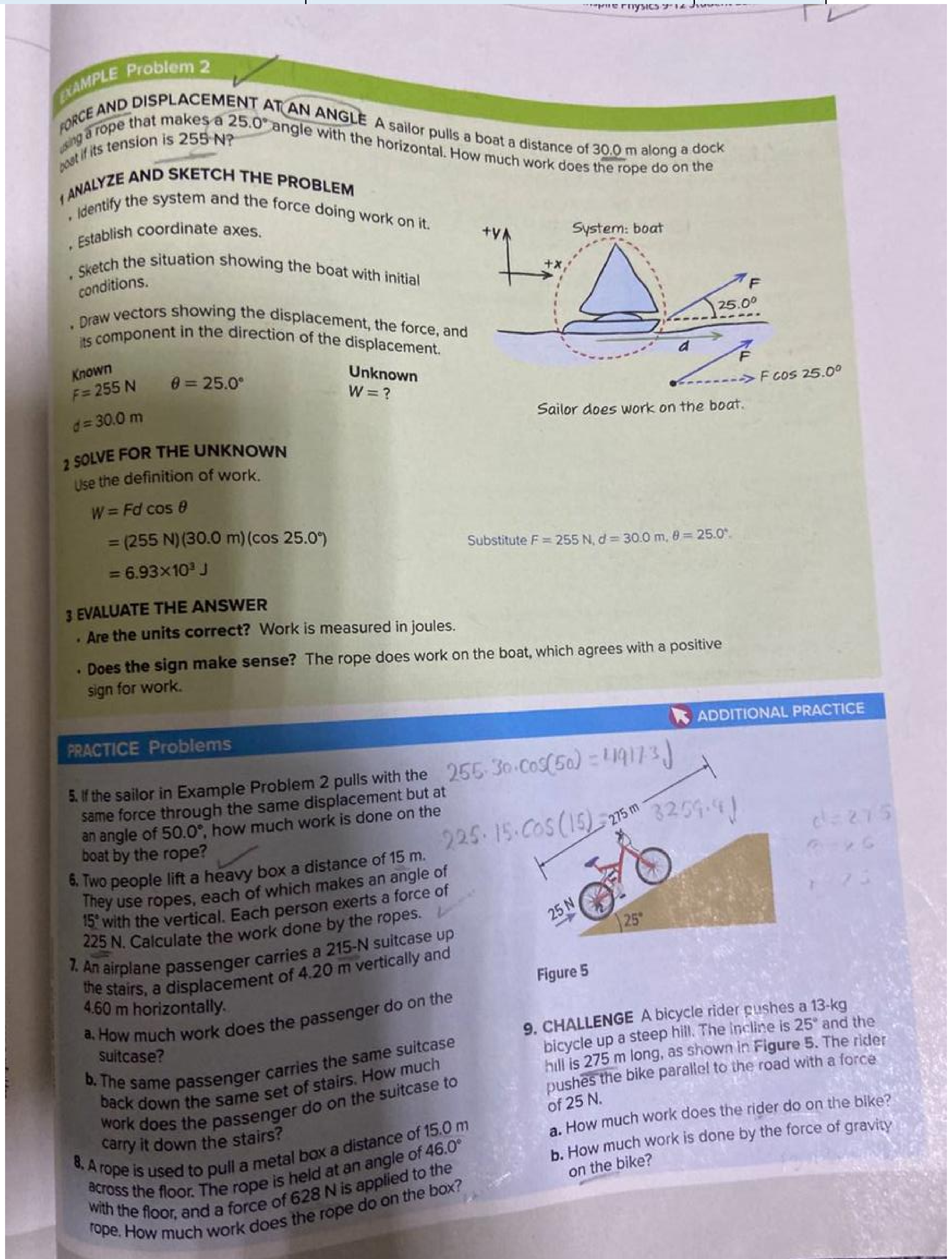
$F_x = F\cos\theta$. Thus, the work done is represented by the following equation.

Work
Work is equal to the product of the magnitude of the force and magnitude of displacement times the cosine of the angle between them.

$$W = Fd\cos\theta$$

Get It?
Determine the work you do when you exert a force of 3 N at an angle of 45° from the direction of motion for 1 m.

The equation above agrees with our expectations for constant forces exerted in the direction of displacement and for constant forces perpendicular to the displacement. In the book bag example, $\theta = 0°$ and $\cos 0° = 1$. Thus, $W = Fd(1) = Fd$, just as we found before. In the case of the orbiting planet, $\theta = 90°$ and $\cos 90° = 0$. Thus, $W = Fd(0) = 0$. This agrees with our previous conclusions.



EXAMPLE Problem 2

FORCE AND DISPLACEMENT AT AN ANGLE A sailor pulls a boat a distance of 30.0 m along a dock using a rope that makes a 25.0° angle with the horizontal. How much work does the rope do on the boat if its tension is 255 N?

1 ANALYZE AND SKETCH THE PROBLEM
- Identify the system and the force doing work on it.
- Establish coordinate axes.
- Sketch the situation showing the boat with initial conditions.
- Draw vectors showing the displacement, the force, and its component in the direction of the displacement.

Known		Unknown
$F = 255$ N	$\theta = 25.0°$	$W = ?$
$d = 30.0$ m		

2 SOLVE FOR THE UNKNOWN
Use the definition of work.

$$W = Fd\cos\theta$$
$$= (255\ N)(30.0\ m)(\cos 25.0°) \quad \text{Substitute } F = 255\ N,\ d = 30.0\ m,\ \theta = 25.0°.$$
$$= 6.93\times10^3\ J$$

3 EVALUATE THE ANSWER
- **Are the units correct?** Work is measured in joules.
- **Does the sign make sense?** The rope does work on the boat, which agrees with a positive sign for work.

PRACTICE Problems — ADDITIONAL PRACTICE

5. If the sailor in Example Problem 2 pulls with the same force through the same displacement but at an angle of 50.0°, how much work is done on the boat by the rope?
6. Two people lift a heavy box a distance of 15 m. They use ropes, each of which makes an angle of 15° with the vertical. Each person exerts a force of 225 N. Calculate the work done by the ropes.
7. An airplane passenger carries a 215-N suitcase up the stairs, a displacement of 4.20 m vertically and 4.60 m horizontally.
   a. How much work does the passenger do on the suitcase?
   b. The same passenger carries the same suitcase back down the same set of stairs. How much work does the passenger do on the suitcase to carry it down the stairs?
8. A rope is used to pull a metal box a distance of 15.0 m across the floor. The rope is held at an angle of 46.0° with the floor, and a force of 628 N is applied to the rope. How much work does the rope do on the box?

Figure 5

9. **CHALLENGE** A bicycle rider pushes a 13-kg bicycle up a steep hill. The incline is 25° and the hill is 275 m long, as shown in **Figure 5**. The rider pushes the bike parallel to the road with a force of 25 N.
   a. How much work does the rider do on the bike?
   b. How much work is done by the force of gravity on the bike?

8.	Determine graphically the work done by a force from the area of force versus displacement graph.	Student Book	191

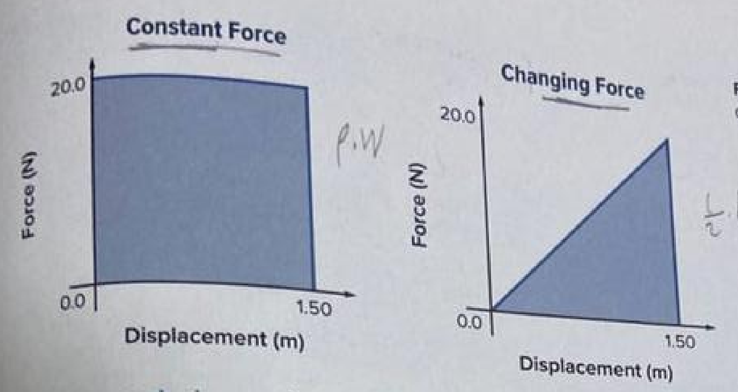


Figure 4 The area under a force-displacement graph is equal to the work.

تمثل البياني

Area

Finding work done when forces change In the last example, the force changed but we could determine the work done in each segment. But what if the force changes in a more complicated way?

A graph of force versus displacement lets you determine the work done by a force. This graphical method can be used to solve problems in which the force is changing. The left graph in **Figure 4** shows the work done by a constant force of 20.0 N that is exerted to lift an object 1.50 m. The work done by this force is represented by

$$W = Fd = (20.0\ \text{N})(1.50\ \text{m}) = 30.0\ \text{J}.$$

Note that the shaded area under the left graph is also equal to (20.0 N)(1.50 m), or 30.0 J. The area under a force-displacement graph is equal to the work done by that force.

This is true even if the force changes. The right graph in **Figure 4** shows the force exerted by a spring that varies linearly from 0.0 to 20.0 N as it is compressed 1.50 m. The work done by the force that compressed the spring is the area under the graph, which is the area of a triangle, $\left(\frac{1}{2}\right)$(base)(altitude), or

$$W = \left(\tfrac{1}{2}\right)(20.0\ \text{N})(1.50\ \text{m}) = 15.0\ \text{J}.$$

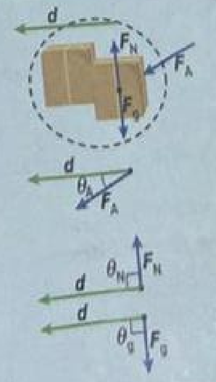
Use the problem-solving strategy below when you solve problems related to work.

PROBLEM-SOLVING STRATEGY

Work

When solving problems related to work, use the following strategy:

1. Identify and sketch the system. Show any forces doing work on the system.
2. Establish a coordinate system. Draw the displacement vectors of the system and each force vector doing work on the system.
3. Find the angle (θ) between each force and displacement.
4. Calculate the work done by each force using $W = Fd \cos \theta$.
5. Calculate the net work done.



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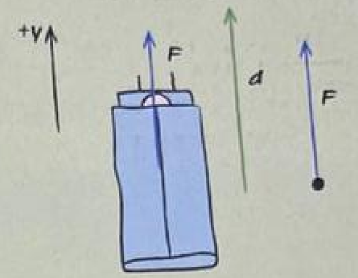
9.	Apply the relationship between power, the work done by a force, and the time interval in which that work is done ($P = \frac{W}{\Delta t}$)	Example Problem (3) Practice Problem (17)	197 197

EXAMPLE Problem 3

POWER An electric motor lifts an elevator 9.00 m in 15.0 s by exerting an upward force of 1.20×10^4 N. What power does the motor produce in kW?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation showing the system as the elevator with its initial conditions.
- Establish a coordinate system with up as positive.
- Draw a vector diagram for the force and displacement.



Known
$d = 9.00$ m
$t = 15.0$ s
$F = 1.20\times10^4$ N

Unknown
$P = ?$

2 SOLVE FOR THE UNKNOWN

Use the definition of power.

$$P = \frac{W}{t}$$

$$= \frac{Fd}{t}$$ Substitute $W = Fd \cos 0° = Fd$.

$$= \frac{(1.20\times10^4\ \text{N})(9.00\ \text{m})}{(15.0\ \text{s})}$$ Substitute $F = 1.20\times10^4$ N, $d = 9.00$ m, $t = 15.0$ s.

$$= 7.20\ \text{kW}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** Power is measured in joules per second, or watts.
- **Does the sign make sense?** The positive sign agrees with the upward direction of the force.

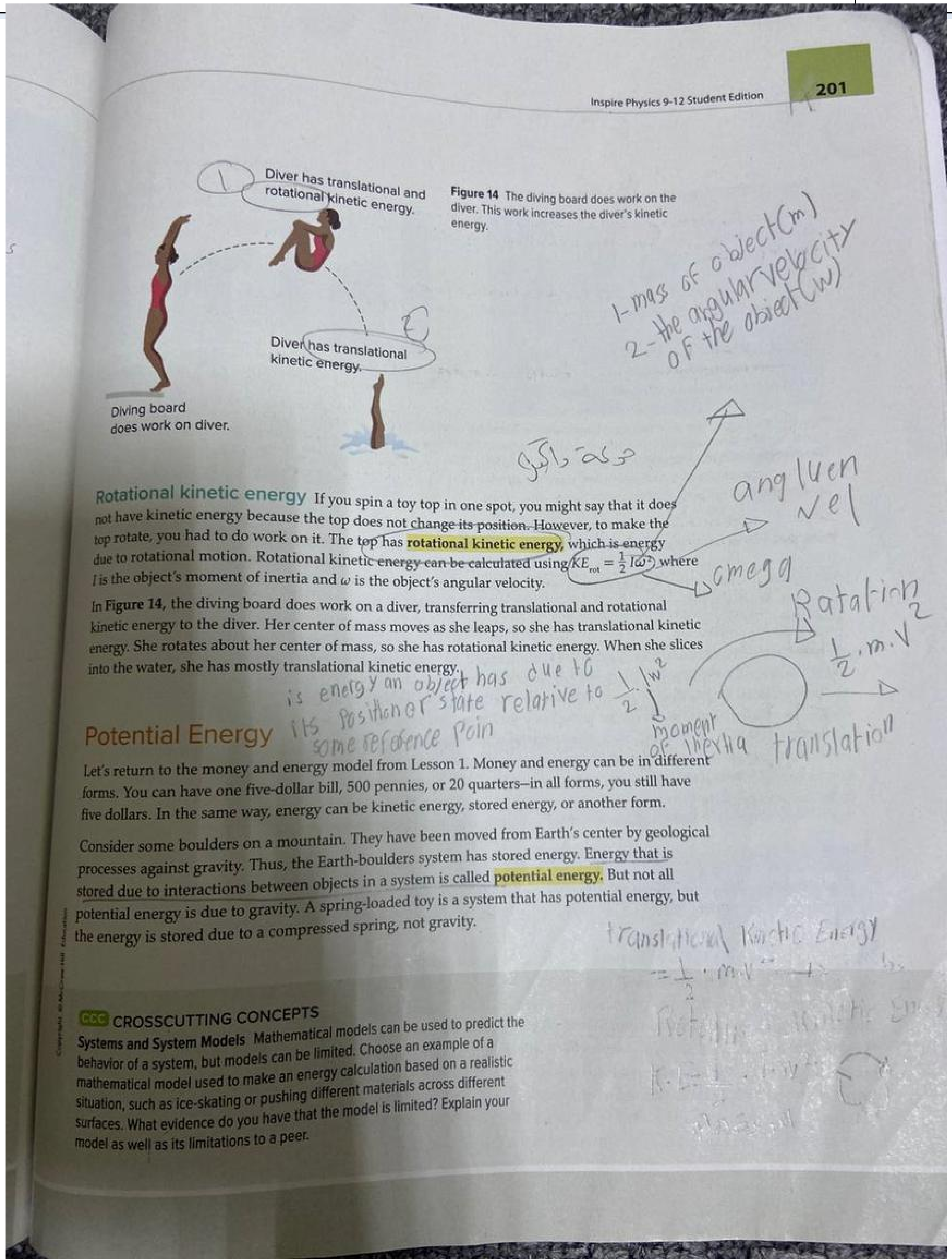
PRACTICE Problems

ADDITIONAL PRACTICE

14. A cable attached to a motor lifts a 575-N box up a distance of 20.0 m. The box moves with a constant velocity and the job is done in 10.0 s. What power is developed by the motor in W and kW?
15. You push a wheelbarrow a distance of 60.0 m at a constant speed for 25.0 s by exerting a 145-N force horizontally.
    a. What power do you develop?
    b. If you move the wheelbarrow twice as fast, how much power is developed?
16. What power does a pump develop to lift 35 L of water per minute from a depth of 110 m? (One liter of water has a mass of 1.00 kg.)
17. An electric motor develops 65 kW of power as it lifts a loaded elevator 17.5 m in 35 s. How much force does the motor exert?
18. **CHALLENGE** A winch designed to be mounted on a truck, as shown in **Figure 10**, is advertised as being able to exert a 6.8×10^3-N force and to develop a power of 0.30 kW. How long would it take the truck and the winch to pull an object 15 m?

Figure 10

10.	Relate the rotational kinetic energy to the object's moment of inertia and it's angular velocity: ($K_{rotational} = \frac{1}{2}I\omega^2$)	Student Book	201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201

Diver has translational and rotational kinetic energy.

Diver has translational kinetic energy.

Diving board does work on diver.

Figure 14 The diving board does work on the diver. This work increases the diver's kinetic energy.

Rotational kinetic energy If you spin a toy top in one spot, you might say that it does not have kinetic energy because the top does not change its position. However, to make the top rotate, you had to do work on it. The top has **rotational kinetic energy**, which is energy due to rotational motion. Rotational kinetic energy can be calculated using $KE_{rot} = \frac{1}{2}I\omega^2$, where I is the object's moment of inertia and ω is the object's angular velocity.

In **Figure 14**, the diving board does work on a diver, transferring translational and rotational kinetic energy to the diver. Her center of mass moves as she leaps, so she has translational kinetic energy. She rotates about her center of mass, so she has rotational kinetic energy. When she slices into the water, she has mostly translational kinetic energy.

Potential Energy

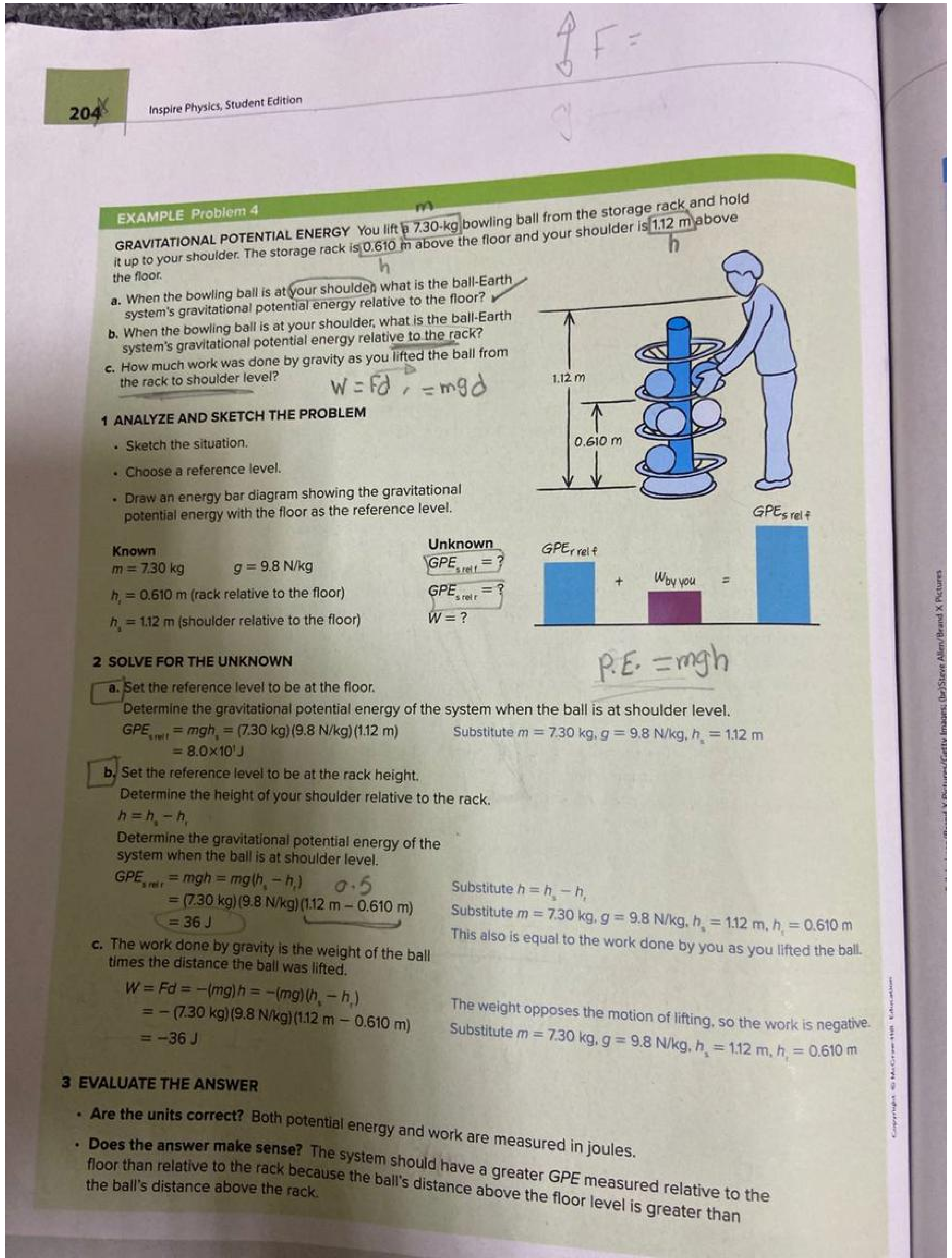
Let's return to the money and energy model from Lesson 1. Money and energy can be in different forms. You can have one five-dollar bill, 500 pennies, or 20 quarters—in all forms, you still have five dollars. In the same way, energy can be kinetic energy, stored energy, or another form.

Consider some boulders on a mountain. They have been moved from Earth's center by geological processes against gravity. Thus, the Earth-boulders system has stored energy. Energy that is stored due to interactions between objects in a system is called **potential energy**. But not all potential energy is due to gravity. A spring-loaded toy is a system that has potential energy, but the energy is stored due to a compressed spring, not gravity.

CCC CROSSCUTTING CONCEPTS

Systems and System Models Mathematical models can be used to predict the behavior of a system, but models can be limited. Choose an example of a realistic mathematical model used to make an energy calculation based on a realistic situation, such as ice-skating or pushing different materials across different surfaces. What evidence do you have that the model is limited? Explain your model as well as its limitations to a peer.

11.	Relate the gravitational potential energy to the mass of the object and its height above or below a reference level ($GPE = mgh$)	Example Problem (4)	204



EXAMPLE Problem 4

GRAVITATIONAL POTENTIAL ENERGY You lift a 7.30-kg bowling ball from the storage rack and hold it up to your shoulder. The storage rack is 0.610 m above the floor and your shoulder is 1.12 m above the floor.

a. When the bowling ball is at your shoulder, what is the ball-Earth system's gravitational potential energy relative to the floor?

b. When the bowling ball is at your shoulder, what is the ball-Earth system's gravitational potential energy relative to the rack?

c. How much work was done by gravity as you lifted the ball from the rack to shoulder level?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Choose a reference level.
- Draw an energy bar diagram showing the gravitational potential energy with the floor as the reference level.

Known		Unknown
$m = 7.30$ kg	$g = 9.8$ N/kg	$GPE_{s\,rel\,f} = ?$
$h_r = 0.610$ m (rack relative to the floor)		$GPE_{s\,rel\,r} = ?$
$h_s = 1.12$ m (shoulder relative to the floor)		$W = ?$

2 SOLVE FOR THE UNKNOWN

a. Set the reference level to be at the floor.

Determine the gravitational potential energy of the system when the ball is at shoulder level.

$GPE_{s\,rel\,f} = mgh_s = (7.30\text{ kg})(9.8\text{ N/kg})(1.12\text{ m})$ Substitute $m = 7.30$ kg, $g = 9.8$ N/kg, $h_s = 1.12$ m

$= 8.0\times10^1$ J

b. Set the reference level to be at the rack height.

Determine the height of your shoulder relative to the rack.

$h = h_s - h_r$

Determine the gravitational potential energy of the system when the ball is at shoulder level.

$GPE_{s\,rel\,r} = mgh = mg(h_s - h_r)$ Substitute $h = h_s - h_r$

$= (7.30\text{ kg})(9.8\text{ N/kg})(1.12\text{ m} - 0.610\text{ m})$ Substitute $m = 7.30$ kg, $g = 9.8$ N/kg, $h_s = 1.12$ m, $h_r = 0.610$ m

$= 36$ J This also is equal to the work done by you as you lifted the ball.

c. The work done by gravity is the weight of the ball times the distance the ball was lifted.

$W = Fd = -(mg)h = -(mg)(h_s - h_r)$ The weight opposes the motion of lifting, so the work is negative.

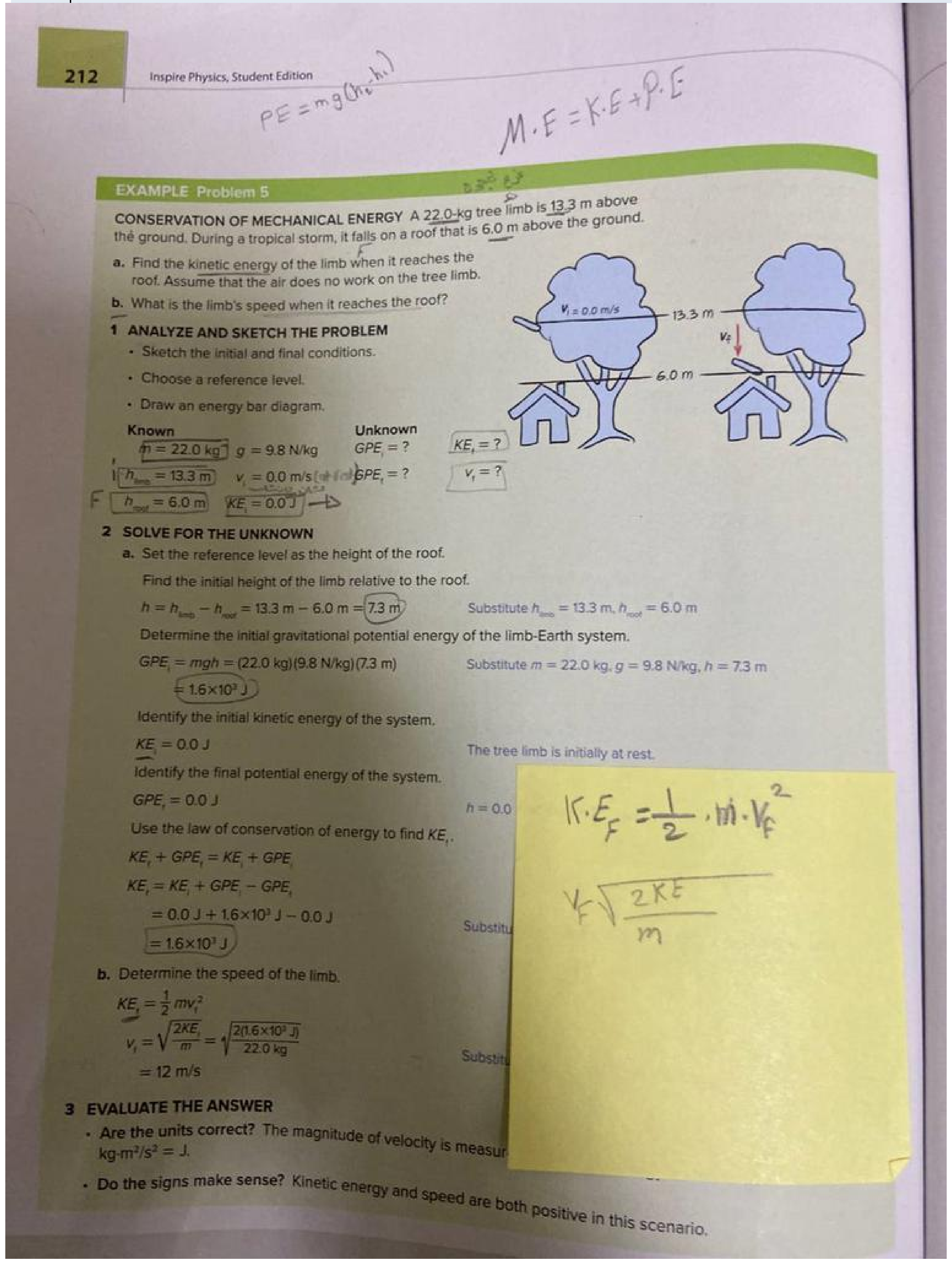
$= -(7.30\text{ kg})(9.8\text{ N/kg})(1.12\text{ m} - 0.610\text{ m})$ Substitute $m = 7.30$ kg, $g = 9.8$ N/kg, $h_s = 1.12$ m, $h_r = 0.610$ m

$= -36$ J

3 EVALUATE THE ANSWER

- **Are the units correct?** Both potential energy and work are measured in joules.
- **Does the answer make sense?** The system should have a greater GPE measured relative to the floor than relative to the rack because the ball's distance above the floor level is greater than the ball's distance above the rack.

12.	Apply the law of conservation of mechanical energy to solve problems ($KE_i + PE_i = KE_f + PE_f$).	Example Problem (5) Check your Progress Q.51	212 217



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$PE = mg(h_f - h_i)$

$M.E = K.E + P.E$

EXAMPLE Problem 5

CONSERVATION OF MECHANICAL ENERGY A 22.0-kg tree limb is 13.3 m above the ground. During a tropical storm, it falls on a roof that is 6.0 m above the ground.

a. Find the kinetic energy of the limb when it reaches the roof. Assume that the air does no work on the tree limb.

b. What is the limb's speed when it reaches the roof?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the initial and final conditions.
- Choose a reference level.
- Draw an energy bar diagram.

Known		Unknown	
$m = 22.0$ kg	$g = 9.8$ N/kg	$GPE_i = ?$	$KE_f = ?$
$h_{limb} = 13.3$ m	$v_i = 0.0$ m/s	$GPE_f = ?$	$v_f = ?$
$h_{roof} = 6.0$ m	$KE_i = 0.0$ J		

2 SOLVE FOR THE UNKNOWN

a. Set the reference level as the height of the roof.

Find the initial height of the limb relative to the roof.

$h = h_{limb} - h_{roof} = 13.3\text{ m} - 6.0\text{ m} = 7.3\text{ m}$ Substitute $h_{limb} = 13.3$ m, $h_{roof} = 6.0$ m

Determine the initial gravitational potential energy of the limb-Earth system.

$GPE_i = mgh = (22.0\text{ kg})(9.8\text{ N/kg})(7.3\text{ m})$ Substitute $m = 22.0$ kg, $g = 9.8$ N/kg, $h = 7.3$ m

$= 1.6\times10^3\text{ J}$

Identify the initial kinetic energy of the system.

$KE_i = 0.0$ J The tree limb is initially at rest.

Identify the final potential energy of the system.

$GPE_f = 0.0$ J $h = 0.0$

Use the law of conservation of energy to find KE_f.

$KE_f + GPE_f = KE_i + GPE_i$

$KE_f = KE_i + GPE_i - GPE_f$

$= 0.0\text{ J} + 1.6\times10^3\text{ J} - 0.0\text{ J}$

$= 1.6\times10^3\text{ J}$ Substitu

b. Determine the speed of the limb.

$KE_f = \frac{1}{2}mv_f^2$

$v_f = \sqrt{\frac{2KE_f}{m}} = \sqrt{\frac{2(1.6\times10^3\text{ J})}{22.0\text{ kg}}}$ Substit

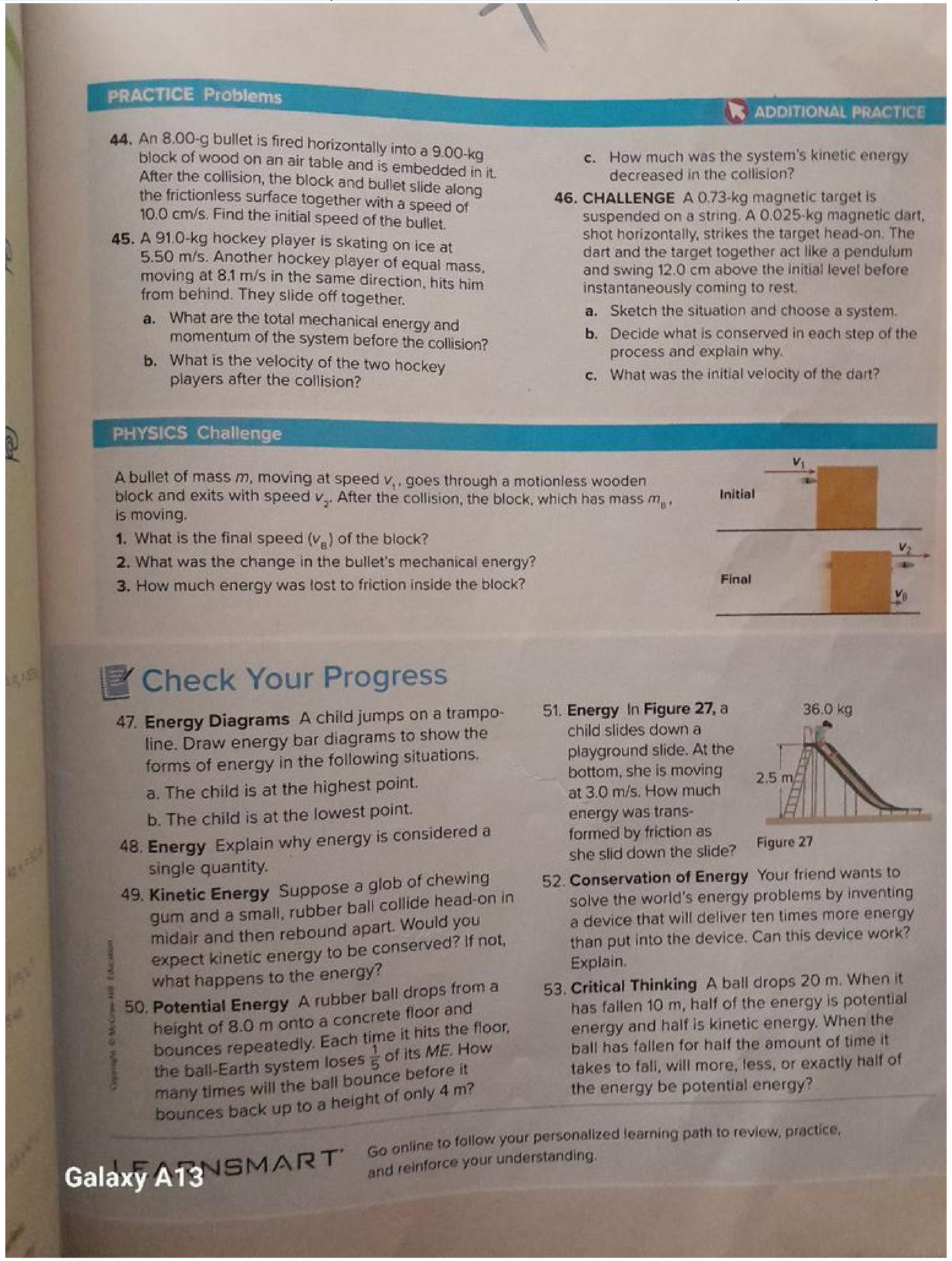
$= 12$ m/s

$K.E_f = \frac{1}{2} \cdot m \cdot v_f^2$

$v_f = \sqrt{\frac{2KE}{m}}$

3 EVALUATE THE ANSWER

- **Are the units correct?** The magnitude of velocity is measur kg·m²/s² = J.
- **Do the signs make sense?** Kinetic energy and speed are both positive in this scenario.



PRACTICE Problems ADDITIONAL PRACTICE

44. An 8.00-g bullet is fired horizontally into a 9.00-kg block of wood on an air table and is embedded in it. After the collision, the block and bullet slide along the frictionless surface together with a speed of 10.0 cm/s. Find the initial speed of the bullet.

45. A 91.0-kg hockey player is skating on ice at 5.50 m/s. Another hockey player of equal mass, moving at 8.1 m/s in the same direction, hits him from behind. They slide off together.
 a. What are the total mechanical energy and momentum of the system before the collision?
 b. What is the velocity of the two hockey players after the collision?
 c. How much was the system's kinetic energy decreased in the collision?

46. **CHALLENGE** A 0.73-kg magnetic target is suspended on a string. A 0.025-kg magnetic dart, shot horizontally, strikes the target head-on. The dart and the target together act like a pendulum and swing 12.0 cm above the initial level before instantaneously coming to rest.
 a. Sketch the situation and choose a system.
 b. Decide what is conserved in each step of the process and explain why.
 c. What was the initial velocity of the dart?

PHYSICS Challenge

A bullet of mass m, moving at speed v_1, goes through a motionless wooden block and exits with speed v_2. After the collision, the block, which has mass m_B, is moving.

1. What is the final speed (v_B) of the block?
2. What was the change in the bullet's mechanical energy?
3. How much energy was lost to friction inside the block?

Check Your Progress

47. **Energy Diagrams** A child jumps on a trampoline. Draw energy bar diagrams to show the forms of energy in the following situations.
 a. The child is at the highest point.
 b. The child is at the lowest point.

48. **Energy** Explain why energy is considered a single quantity.

49. **Kinetic Energy** Suppose a glob of chewing gum and a small, rubber ball collide head-on in midair and then rebound apart. Would you expect kinetic energy to be conserved? If not, what happens to the energy?

50. **Potential Energy** A rubber ball drops from a height of 8.0 m onto a concrete floor and bounces repeatedly. Each time it hits the floor, the ball-Earth system loses $\frac{1}{5}$ of its ME. How many times will the ball bounce before it bounces back up to a height of only 4 m?

51. **Energy** In **Figure 27**, a child slides down a playground slide. At the bottom, she is moving at 3.0 m/s. How much energy was transformed by friction as she slid down the slide?

Figure 27

52. **Conservation of Energy** Your friend wants to solve the world's energy problems by inventing a device that will deliver ten times more energy than put into the device. Can this device work? Explain.

53. **Critical Thinking** A ball drops 20 m. When it has fallen 10 m, half of the energy is potential energy and half is kinetic energy. When the ball has fallen for half the amount of time it takes to fall, will more, less, or exactly half of the energy be potential energy?

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Galaxy A13

13.	Define kinetic energy and apply the relationship between a particle's kinetic energy, mass, and speed ($KE = \frac{1}{2}mv^2$).	Student Book	200

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$KE = \frac{1}{2}mv^2$

Translational and Rotational Kinetic Energy

In the examples we have considered so far, moving objects that were changing position had kinetic energy ($\frac{1}{2}mv^2$) due to their motion. What about energy due to an object's changing position?

Translational kinetic energy Energy due to changing position is called **translational kinetic energy** and can be represented by the following equation.

Translational Kinetic Energy

A system's translational kinetic energy is equal to one-half times the system's mass multiplied by the system's speed squared.

$$KE_{trans} = \frac{1}{2}mv^2$$

$KE_{rot} = \frac{1}{2}I\omega^2$

Translational kinetic energy is proportional to the object's mass. For example, a 7.26-kg bowling ball thrown through the air has more translational kinetic energy than a 0.148-kg baseball, like the one shown in **Figure 13**, moving with the same speed.

An object's translational kinetic energy is also proportional to the square of the object's speed. A car moving at 20 m/s has four times the translational kinetic energy of the same car moving at 10 m/s.

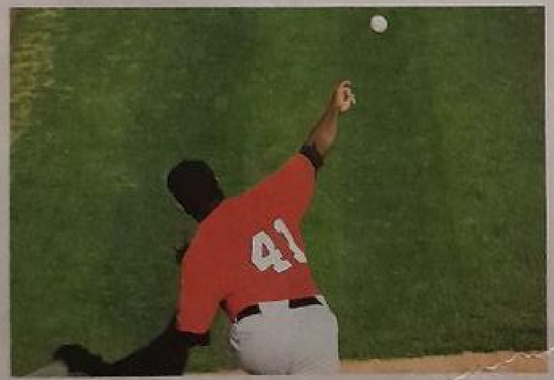


Figure 13 A baseball has less translational kinetic energy than a bowling ball moving with the same speed.

GPE = mgh

1. mass GPE ∝ m
2. hieght h
3. gravity field (9.8)

Get It?

Explain Using the equation for translational kinetic energy, show ... a car moving at 20 m/s has four times the translational kinetic energy of the same car moving at 10 m/s.

SCIENCE USAGE v. COMMON USAGE

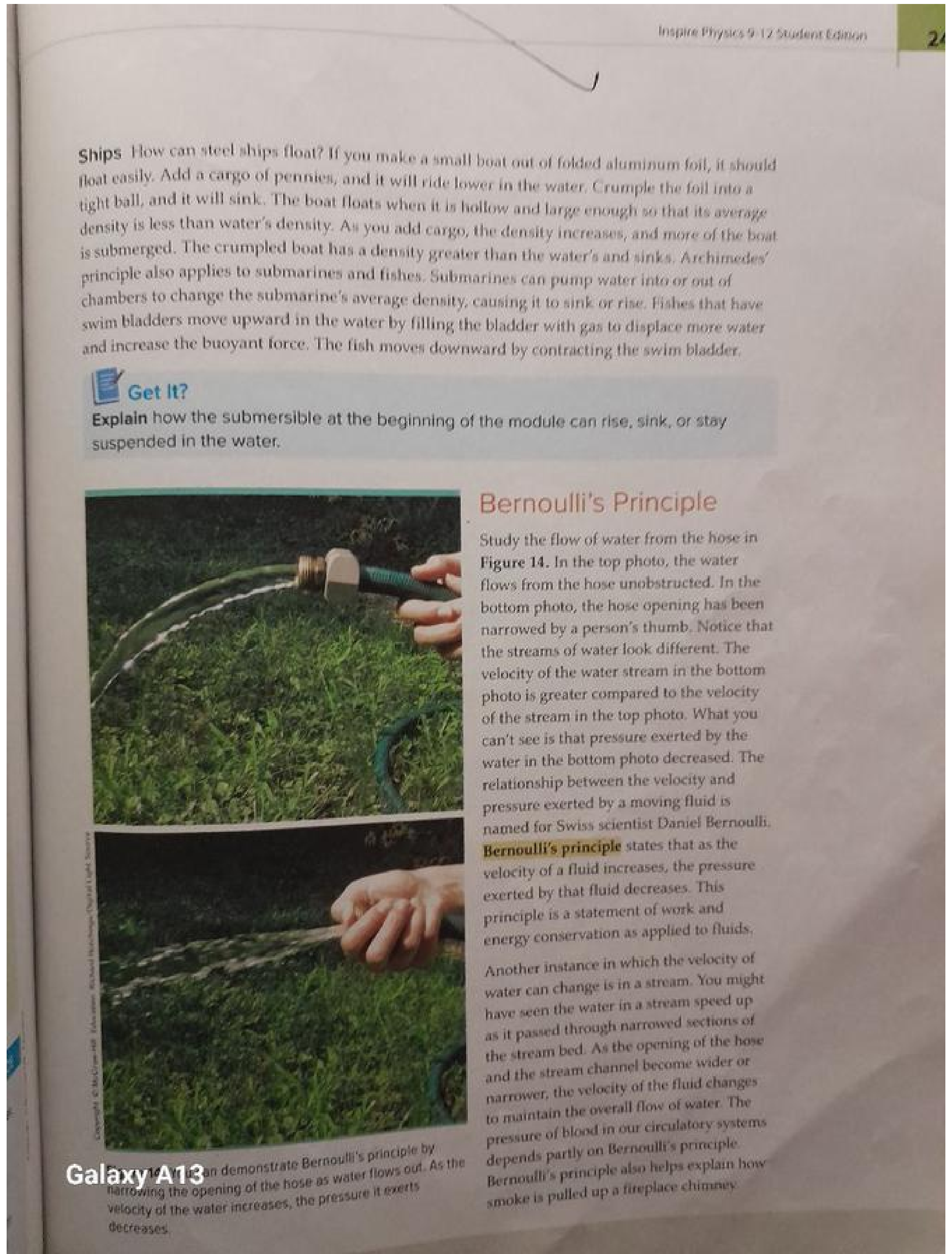
Energy

Science usage: the ability of a system to produce a change in itself or the world around it
The kinetic energy of the soccer ball decreased as it slowed down.

Common usage: the capacity of acting or being active
The young students had a lot of energy during recess.

[redacted]

14.	Describe the relationship between the velocity of a fluid and the pressure it exerts according to Bernoulli's principle	Student Book Check your Progress Q.38	249-250 251



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Ships How can steel ships float? If you make a small boat out of folded aluminum foil, it should float easily. Add a cargo of pennies, and it will ride lower in the water. Crumple the foil into a tight ball, and it will sink. The boat floats when it is hollow and large enough so that its average density is less than water's density. As you add cargo, the density increases, and more of the boat is submerged. The crumpled boat has a density greater than the water's and sinks. Archimedes' principle also applies to submarines and fishes. Submarines can pump water into or out of chambers to change the submarine's average density, causing it to sink or rise. Fishes that have swim bladders move upward in the water by filling the bladder with gas to displace more water and increase the buoyant force. The fish moves downward by contracting the swim bladder.

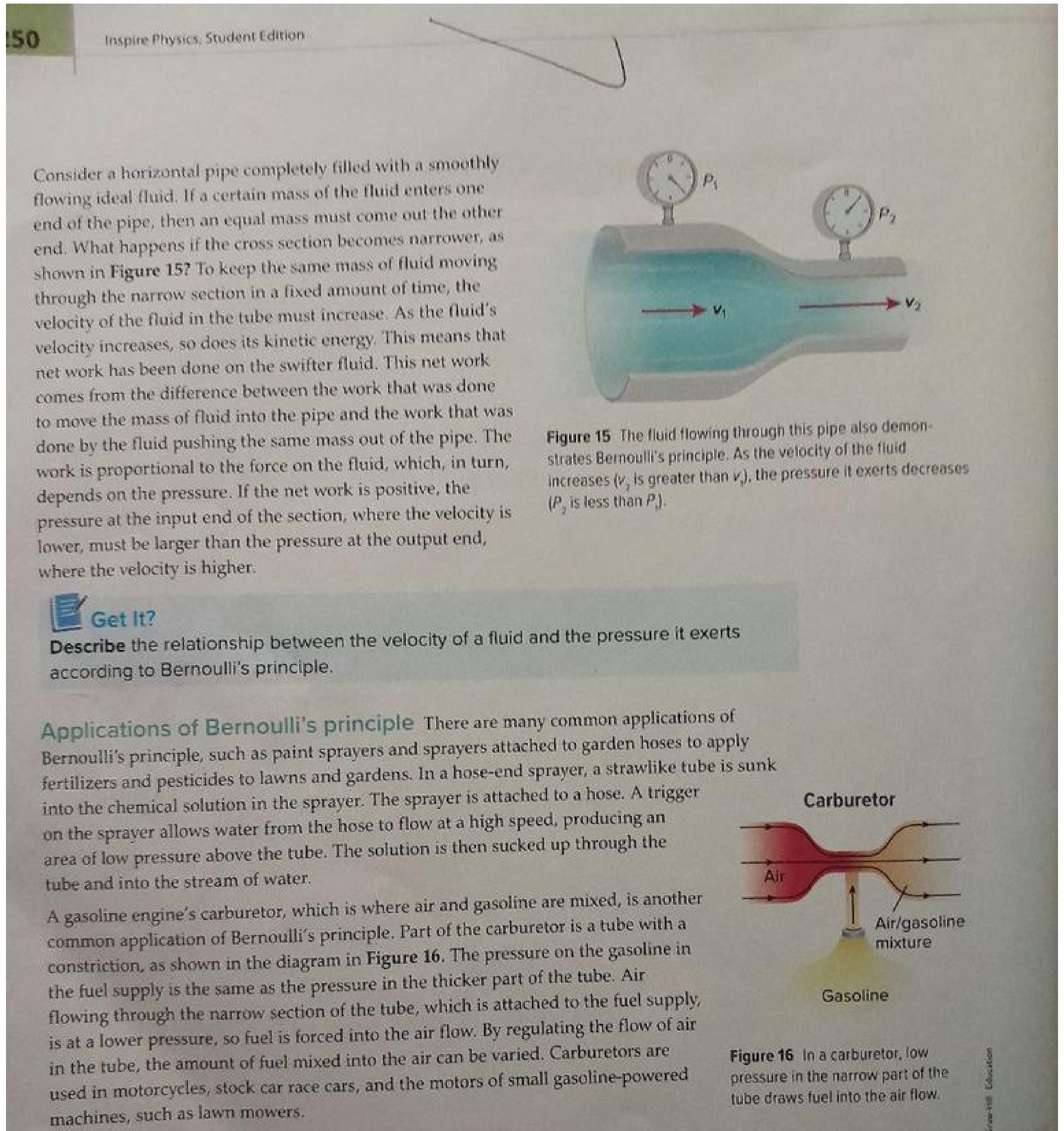
Get It?
Explain how the submersible at the beginning of the module can rise, sink, or stay suspended in the water.

Bernoulli's Principle

Study the flow of water from the hose in **Figure 14.** In the top photo, the water flows from the hose unobstructed. In the bottom photo, the hose opening has been narrowed by a person's thumb. Notice that the streams of water look different. The velocity of the water stream in the bottom photo is greater compared to the velocity of the stream in the top photo. What you can't see is that pressure exerted by the water in the bottom photo decreased. The relationship between the velocity and pressure exerted by a moving fluid is named for Swiss scientist Daniel Bernoulli. **Bernoulli's principle** states that as the velocity of a fluid increases, the pressure exerted by that fluid decreases. This principle is a statement of work and energy conservation as applied to fluids.

Another instance in which the velocity of water can change is in a stream. You might have seen the water in a stream speed up as it passed through narrowed sections of the stream bed. As the opening of the hose and the stream channel become wider or narrower, the velocity of the fluid changes to maintain the overall flow of water. The pressure of blood in our circulatory systems depends partly on Bernoulli's principle. Bernoulli's principle also helps explain how smoke is pulled up a fireplace chimney.

... demonstrate Bernoulli's principle by narrowing the opening of the hose as water flows out. As the velocity of the water increases, the pressure it exerts decreases.



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Consider a horizontal pipe completely filled with a smoothly flowing ideal fluid. If a certain mass of the fluid enters one end of the pipe, then an equal mass must come out the other end. What happens if the cross section becomes narrower, as shown in **Figure 15**? To keep the same mass of fluid moving through the narrow section in a fixed amount of time, the velocity of the fluid in the tube must increase. As the fluid's velocity increases, so does its kinetic energy. This means that net work has been done on the swifter fluid. This net work comes from the difference between the work that was done to move the mass of fluid into the pipe and the work that was done by the fluid pushing the same mass out of the pipe. The work is proportional to the force on the fluid, which, in turn, depends on the pressure. If the net work is positive, the pressure at the input end of the section, where the velocity is lower, must be larger than the pressure at the output end, where the velocity is higher.

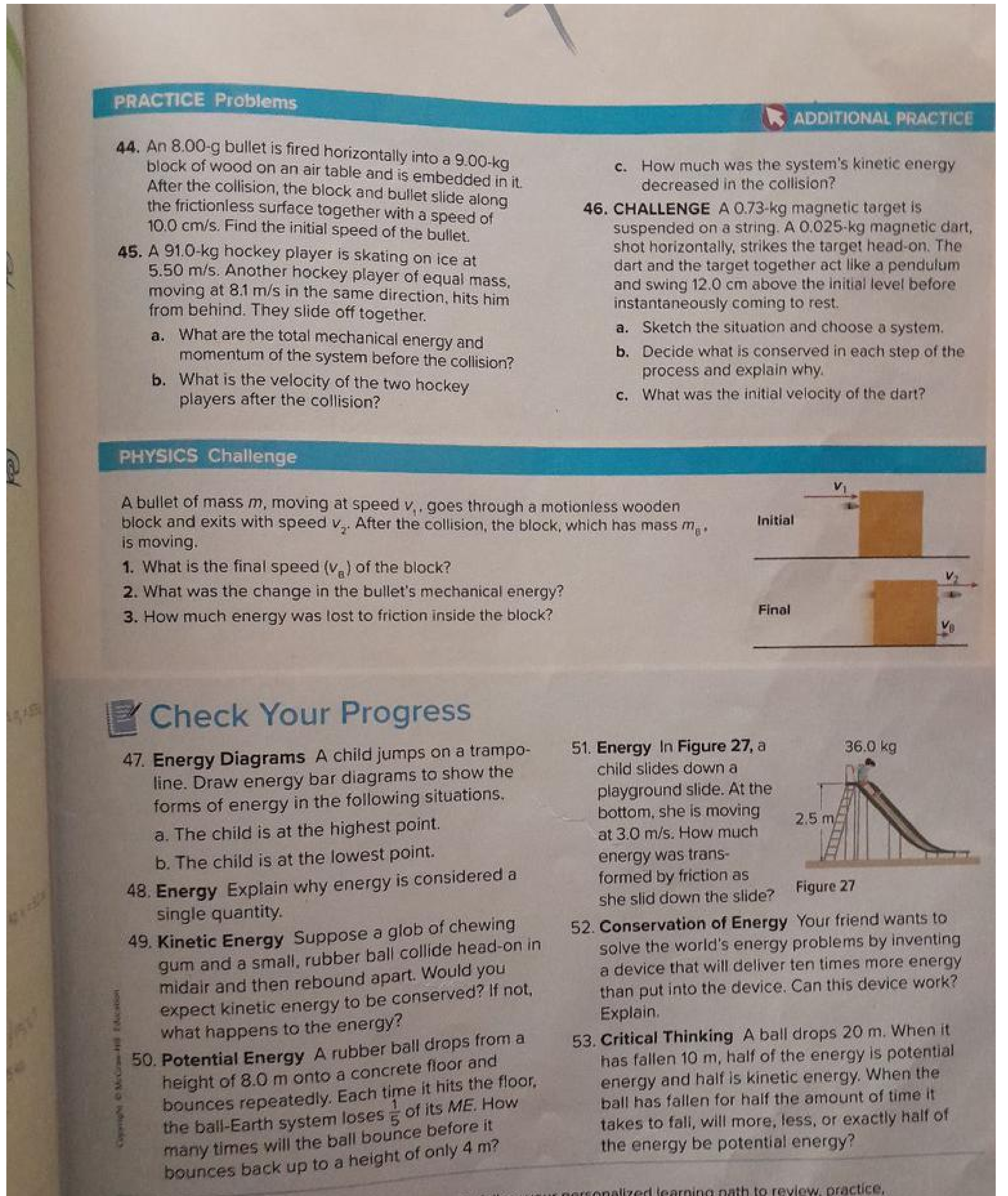
Figure 15 The fluid flowing through this pipe also demonstrates Bernoulli's principle. As the velocity of the fluid increases (v_2 is greater than v_1), the pressure it exerts decreases (P_2 is less than P_1).

Get It?
Describe the relationship between the velocity of a fluid and the pressure it exerts according to Bernoulli's principle.

Applications of Bernoulli's principle There are many common applications of Bernoulli's principle, such as paint sprayers and sprayers attached to garden hoses to apply fertilizers and pesticides to lawns and gardens. In a hose-end sprayer, a strawlike tube is sunk into the chemical solution in the sprayer. The sprayer is attached to a hose. A trigger on the sprayer allows water from the hose to flow at a high speed, producing an area of low pressure above the tube. The solution is then sucked up through the tube and into the stream of water.

A gasoline engine's carburetor, which is where air and gasoline are mixed, is another common application of Bernoulli's principle. Part of the carburetor is a tube with a constriction, as shown in the diagram in **Figure 16.** The pressure on the gasoline in the fuel supply is the same as the pressure in the thicker part of the tube. Air flowing through the narrow section of the tube, which is attached to the fuel supply, is at a lower pressure, so fuel is forced into the air flow. By regulating the flow of air in the tube, the amount of fuel mixed into the air can be varied. Carburetors are used in motorcycles, stock car race cars, and the motors of small gasoline-powered machines, such as lawn mowers.

Figure 16 In a carburetor, low pressure in the narrow part of the tube draws fuel into the air flow.



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15.	(1) Recall Pascal's principle. (2) Apply Pascal's principle to hydraulic systems to solve problems.	Student Book	244

LESSON 3

FLUIDS AT REST AND IN MOTION

FOCUS QUESTION

What principles describe the behavior and motion of fluids?

Fluids at Rest

If you have ever dived deep into a swimming pool or a lake, you likely felt pressure on your ears. Blaise Pascal, a French physician, found that the pressure at a point in a fluid depends on its depth in the fluid and is unrelated to the shape of the fluid's container. He also noted that any change in pressure applied at any point on a confined fluid is transferred undiminished throughout the fluid, a fact that is now known as **Pascal's principle.** One application of Pascal's principle is using fluids in machines to multiply forces. In the hydraulic system shown in **Figure 10,** a fluid is confined to two connecting chambers. Each chamber has a piston that is free to move, and the pistons have different surface areas. Recall that if a force (F_1) is exerted on the first piston with a surface area of A_1, the pressure (P_1) exerted on the fluid is $P_1 = \frac{F_1}{A_1}$. The pressure exerted by the fluid on the second piston, with a surface area A_2, is $P_2 = \frac{F_2}{A_2}$.

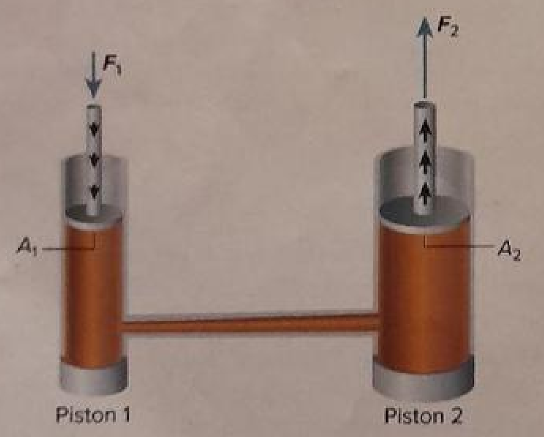


Figure 10 As F_1 exerts pressure on the smaller piston (piston 1), the pressure is transmitted throughout the fluid. As a result, a multiplied force (F_2) is exerted on the larger piston (piston 2).

Infer *How would F_2 change if F_1 increased? Explain why.*

3D THINKING | DCI Disciplinary Core Ideas | CCC Crosscutting Concepts | SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

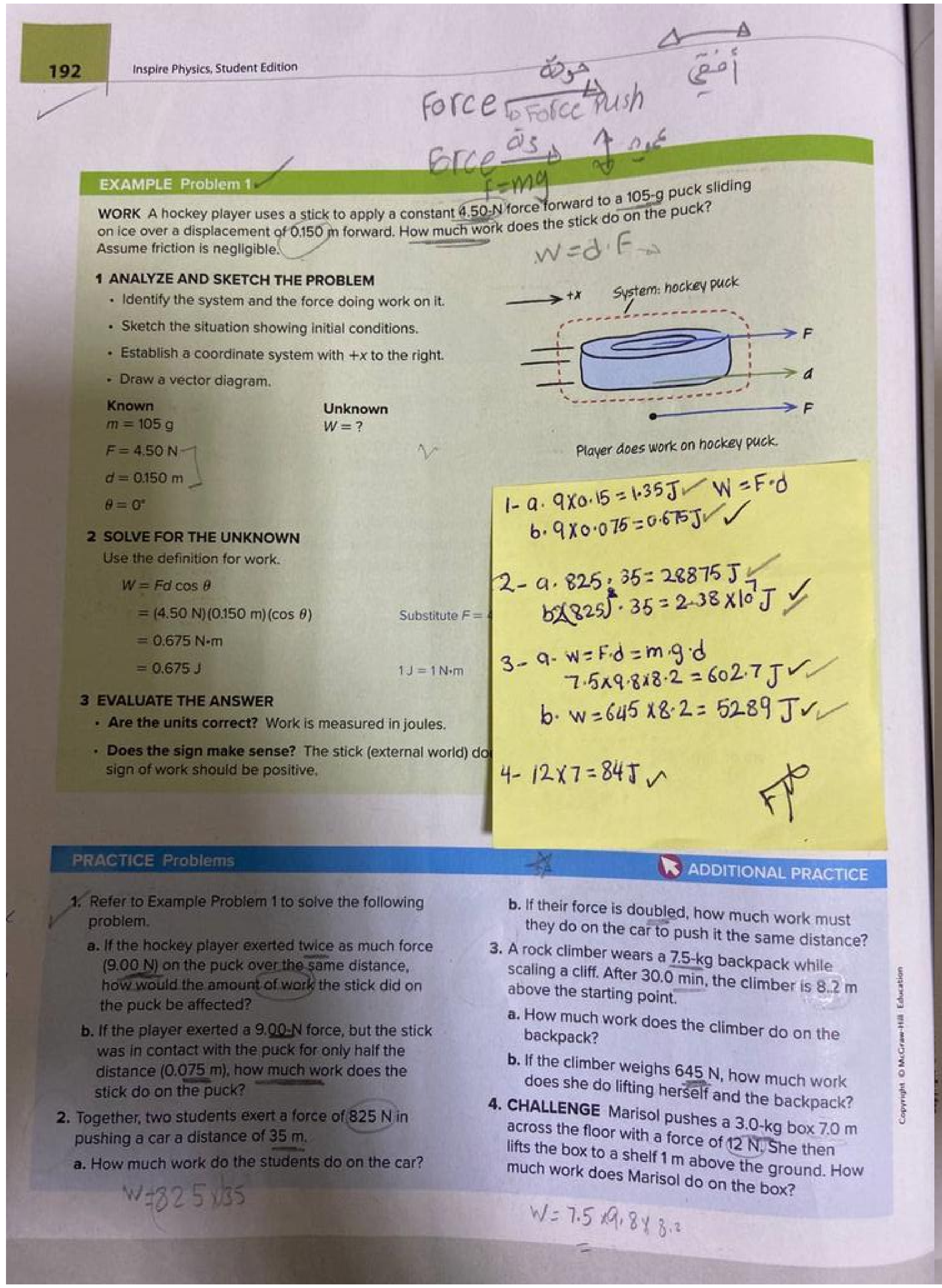
PhysicsLAB: Under Pressure

Carry out an investigation to determine the effect of force in a fluid.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

16	(1) Apply the relationship between a force $\vec{F}$ and the work done on a system by the force when the system undergoes a displacement $\vec{d}$, $W = \lvert\vec{F}\rvert \times \lvert\vec{d}\rvert \cos(\theta)$ where θ is the angle between the direction of the force and the direction of displacement. (2) Apply the work–energy theorem to relate the net work done on a system and the resulting change in kinetic energy	Example Problem (1) Example Problem (2) Practice Problem (11)	192 193 196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EXAMPLE Problem 1

WORK A hockey player uses a stick to apply a constant 4.50-N force forward to a 105-g puck sliding on ice over a displacement of 0.150 m forward. How much work does the stick do on the puck? Assume friction is negligible.

1 ANALYZE AND SKETCH THE PROBLEM
- Identify the system and the force doing work on it.
- Sketch the situation showing initial conditions.
- Establish a coordinate system with +x to the right.
- Draw a vector diagram.

Known: $m = 105$ g, $F = 4.50$ N, $d = 0.150$ m, $\theta = 0°$
Unknown: $W = ?$

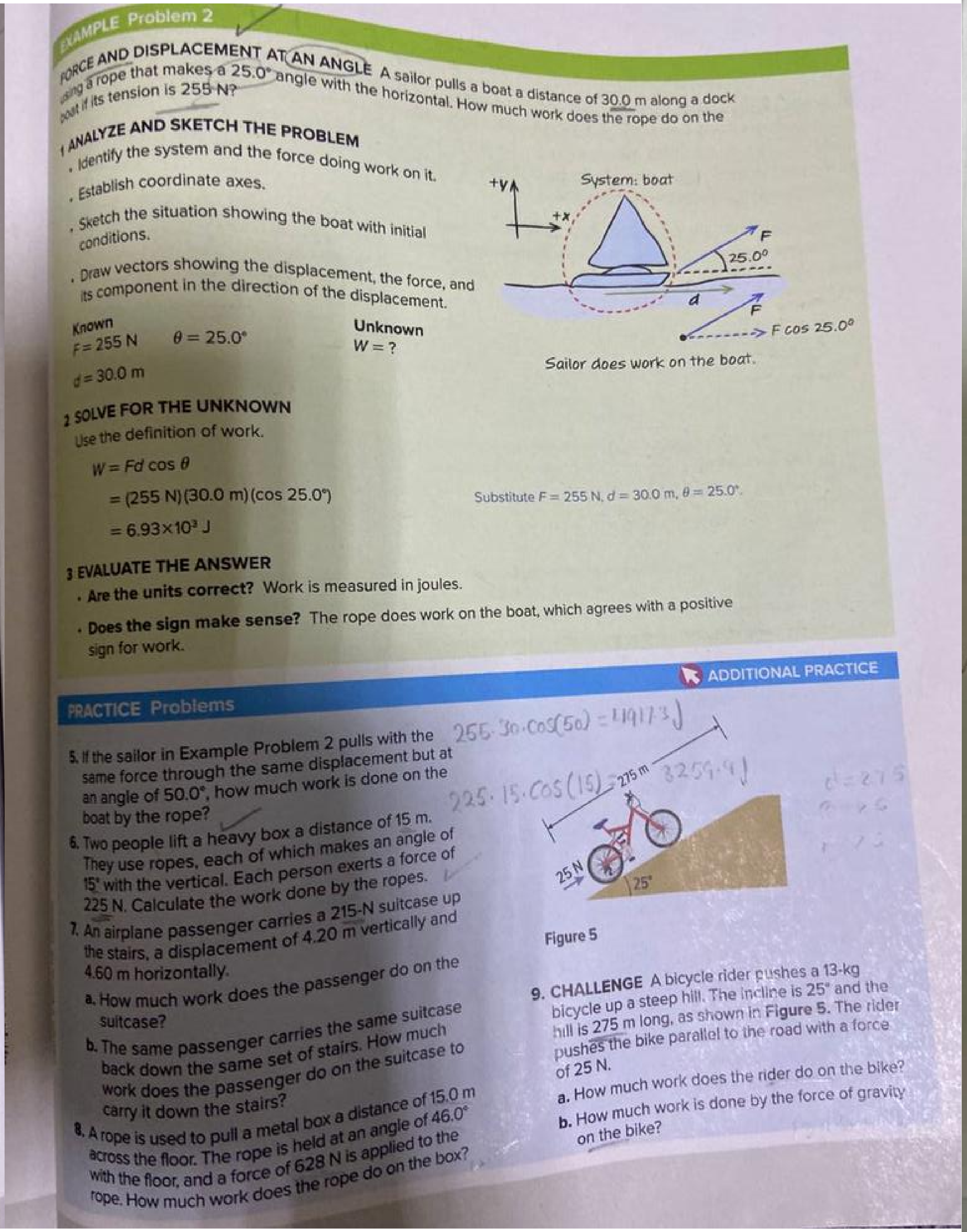
System: hockey puck. Player does work on hockey puck.

2 SOLVE FOR THE UNKNOWN
Use the definition for work.
$W = Fd\cos\theta$
$= (4.50\ \text{N})(0.150\ \text{m})(\cos\theta)$
$= 0.675$ N·m
$= 0.675$ J (1 J = 1 N·m)

3 EVALUATE THE ANSWER
- Are the units correct? Work is measured in joules.
- Does the sign make sense? The stick (external world) does work ... sign of work should be positive.

PRACTICE Problems — ADDITIONAL PRACTICE

1. Refer to Example Problem 1 to solve the following problem.
a. If the hockey player exerted twice as much force (9.00 N) on the puck over the same distance, how would the amount of work the stick did on the puck be affected?
b. If the player exerted a 9.00-N force, but the stick was in contact with the puck for only half the distance (0.075 m), how much work does the stick do on the puck?
2. Together, two students exert a force of 825 N in pushing a car a distance of 35 m.
a. How much work do the students do on the car?
b. If their force is doubled, how much work must they do on the car to push it the same distance?
3. A rock climber wears a 7.5-kg backpack while scaling a cliff. After 30.0 min, the climber is 8.2 m above the starting point.
a. How much work does the climber do on the backpack?
b. If the climber weighs 645 N, how much work does she do lifting herself and the backpack?
4. CHALLENGE Marisol pushes a 3.0-kg box 7.0 m across the floor with a force of 12 N. She then lifts the box to a shelf 1 m above the ground. How much work does Marisol do on the box?


EXAMPLE Problem 2

FORCE AND DISPLACEMENT AT AN ANGLE A sailor pulls a boat a distance of 30.0 m along a dock using a rope that makes a 25.0° angle with the horizontal. How much work does the rope do on the boat if its tension is 255 N?

1 ANALYZE AND SKETCH THE PROBLEM
- Identify the system and the force doing work on it.
- Establish coordinate axes.
- Sketch the situation showing the boat with initial conditions.
- Draw vectors showing the displacement, the force, and its component in the direction of the displacement.

Known: $F = 255$ N, $\theta = 25.0°$, $d = 30.0$ m
Unknown: $W = ?$

System: boat. Sailor does work on the boat. $F\cos 25.0°$

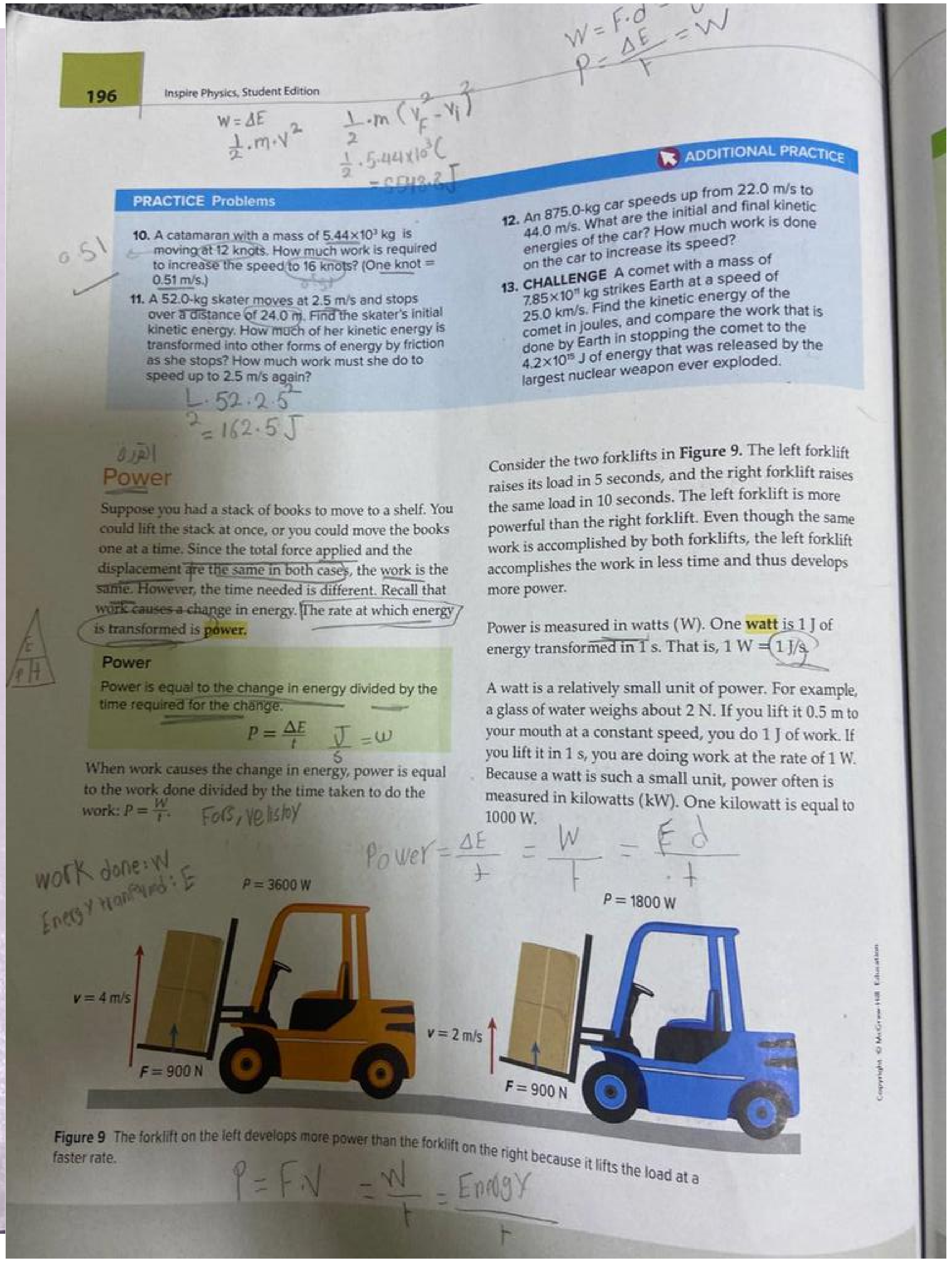
2 SOLVE FOR THE UNKNOWN
Use the definition of work.
$W = Fd\cos\theta$
$= (255\ \text{N})(30.0\ \text{m})(\cos 25.0°)$ Substitute $F = 255$ N, $d = 30.0$ m, $\theta = 25.0°$.
$= 6.93\times10^3$ J

3 EVALUATE THE ANSWER
- Are the units correct? Work is measured in joules.
- Does the sign make sense? The rope does work on the boat, which agrees with a positive sign for work.

PRACTICE Problems — ADDITIONAL PRACTICE

5. If the sailor in Example Problem 2 pulls with the same force through the same displacement but at an angle of 50.0°, how much work is done on the boat by the rope?
6. Two people lift a heavy box a distance of 15 m. They use ropes, each of which makes an angle of 15° with the vertical. Each person exerts a force of 225 N. Calculate the work done by the ropes.
7. An airplane passenger carries a 215-N suitcase up the stairs, a displacement of 4.20 m vertically and 4.60 m horizontally.
a. How much work does the passenger do on the suitcase?
b. The same passenger carries the same suitcase back down the same set of stairs. How much work does the passenger do on the suitcase to carry it down the stairs?
8. A rope is used to pull a metal box a distance of 15.0 m across the floor. The rope is held at an angle of 46.0° with the floor, and a force of 628 N is applied to the rope. How much work does the rope do on the box?
9. CHALLENGE A bicycle rider pushes a 13-kg bicycle up a steep hill. The incline is 25° and the hill is 275 m long, as shown in Figure 5. The rider pushes the bike parallel to the road with a force of 25 N.
a. How much work does the rider do on the bike?
b. How much work is done by the force of gravity on the bike?

Figure 5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PRACTICE Problems — ADDITIONAL PRACTICE

10. A catamaran with a mass of 5.44×10^3 kg is moving at 12 knots. How much work is required to increase the speed to 16 knots? (One knot = 0.51 m/s.)
11. A 52.0-kg skater moves at 2.5 m/s and stops over a distance of 24.0 m. Find the skater's initial kinetic energy. How much of her kinetic energy is transformed into other forms of energy by friction as she stops? How much work must she do to speed up to 2.5 m/s again?
12. An 875.0-kg car speeds up from 22.0 m/s to 44.0 m/s. What are the initial and final kinetic energies of the car? How much work is done on the car to increase its speed?
13. CHALLENGE A comet with a mass of 7.85×10^{11} kg strikes Earth at a speed of 25.0 km/s. Find the kinetic energy of the comet in joules, and compare the work that is done by Earth in stopping the comet to the 4.2×10^{15} J of energy that was released by the largest nuclear weapon ever exploded.

Power

Suppose you had a stack of books to move to a shelf. You could lift the stack at once, or you could move the books one at a time. Since the total force applied and the displacement are the same in both cases, the work is the same. However, the time needed is different. Recall that work causes a change in energy. The rate at which energy is transformed is power.

Power
Power is equal to the change in energy divided by the time required for the change.
$P = \frac{\Delta E}{t}$

When work causes the change in energy, power is equal to the work done divided by the time taken to do the work: $P = \frac{W}{t}$.

Consider the two forklifts in Figure 9. The left forklift raises its load in 5 seconds, and the right forklift raises the same load in 10 seconds. The left forklift is more powerful than the right forklift. Even though the same work is accomplished by both forklifts, the left forklift accomplishes the work in less time and thus develops more power.

Power is measured in watts (W). One watt is 1 J of energy transformed in 1 s. That is, 1 W = 1 J/s.

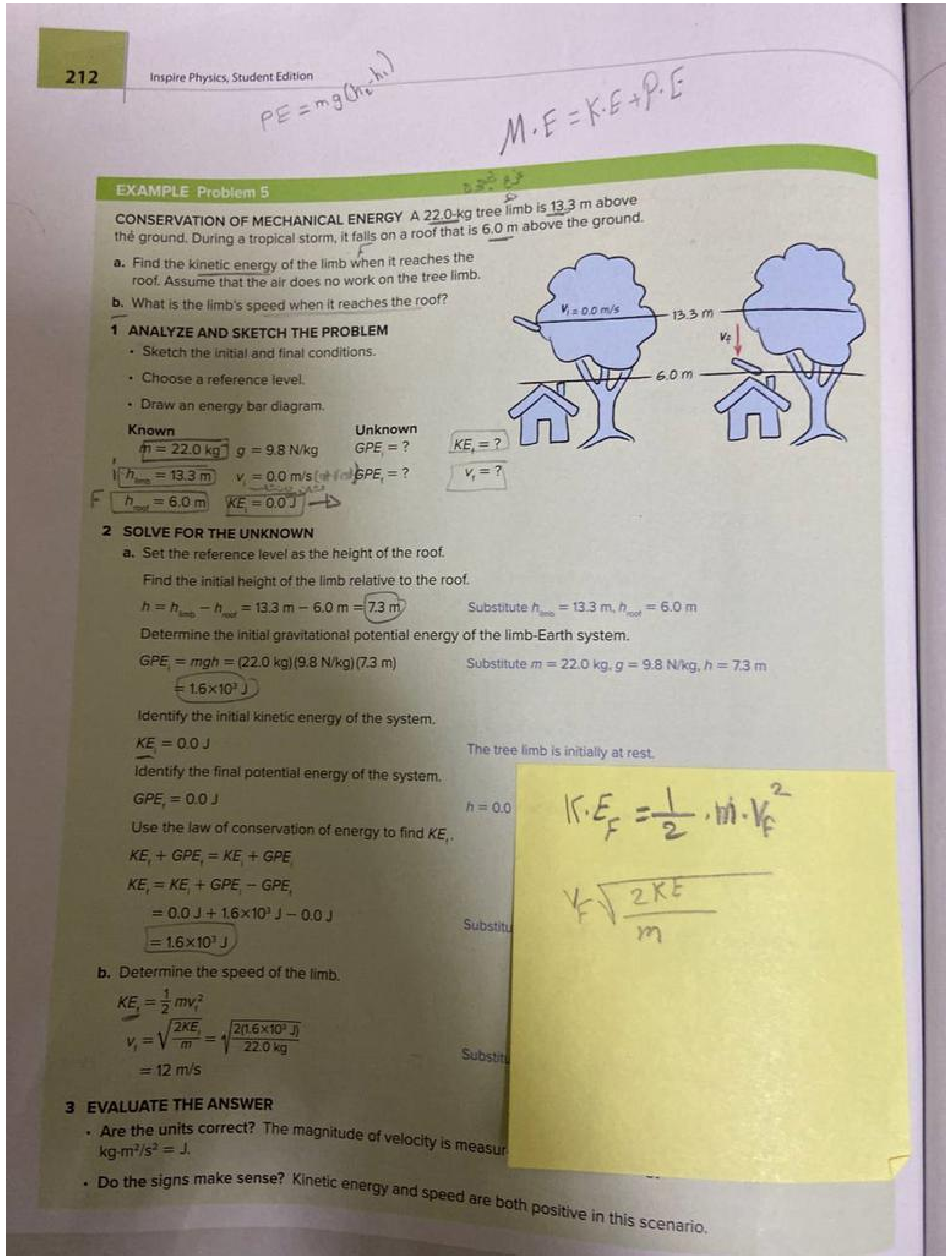
A watt is a relatively small unit of power. For example, a glass of water weighs about 2 N. If you lift it 0.5 m to your mouth at a constant speed, you do 1 J of work. If you lift it in 1 s, you are doing work at the rate of 1 W. Because a watt is such a small unit, power often is measured in kilowatts (kW). One kilowatt is equal to 1000 W.

P = 3600 W, v = 4 m/s, F = 900 N; P = 1800 W, v = 2 m/s, F = 900 N

Figure 9 The forklift on the left develops more power than the forklift on the right because it lifts the load at a faster rate.

تعليم

17	(1) Apply the law of conservation of mechanical energy to solve problems ($KE_i + PE_i = KE_f + PE_f$). (2) Apply the law of conservation of energy to examples like roller coaster rides, ski slopes, inclined planes/ hills, and pendulums	Example Problem (5) Practice Problem (41)	212 213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EXAMPLE Problem 5

CONSERVATION OF MECHANICAL ENERGY A 22.0-kg tree limb is 13.3 m above the ground. During a tropical storm, it falls on a roof that is 6.0 m above the ground.

a. Find the kinetic energy of the limb when it reaches the roof. Assume that the air does no work on the tree limb.

b. What is the limb's speed when it reaches the roof?

1 ANALYZE AND SKETCH THE PROBLEM
- Sketch the initial and final conditions.
- Choose a reference level.
- Draw an energy bar diagram.

Known: $m = 22.0$ kg, $g = 9.8$ N/kg, $h_{limb} = 13.3$ m, $v_i = 0.0$ m/s, $h_{roof} = 6.0$ m, $KE_i = 0.0$ J

Unknown: $GPE_i = ?$, $GPE_f = ?$, $KE_f = ?$, $v_f = ?$

2 SOLVE FOR THE UNKNOWN

a. Set the reference level as the height of the roof.

Find the initial height of the limb relative to the roof.

$h = h_{limb} - h_{roof} = 13.3\text{ m} - 6.0\text{ m} = 7.3\text{ m}$ Substitute $h_{limb} = 13.3$ m, $h_{roof} = 6.0$ m

Determine the initial gravitational potential energy of the limb-Earth system.

$GPE_i = mgh = (22.0\text{ kg})(9.8\text{ N/kg})(7.3\text{ m}) = 1.6\times10^3$ J Substitute $m = 22.0$ kg, $g = 9.8$ N/kg, $h = 7.3$ m

Identify the initial kinetic energy of the system.

$KE_i = 0.0$ J The tree limb is initially at rest.

Identify the final potential energy of the system.

$GPE_f = 0.0$ J

Use the law of conservation of energy to find KE_f.

$KE_i + GPE_i = KE_f + GPE_f$

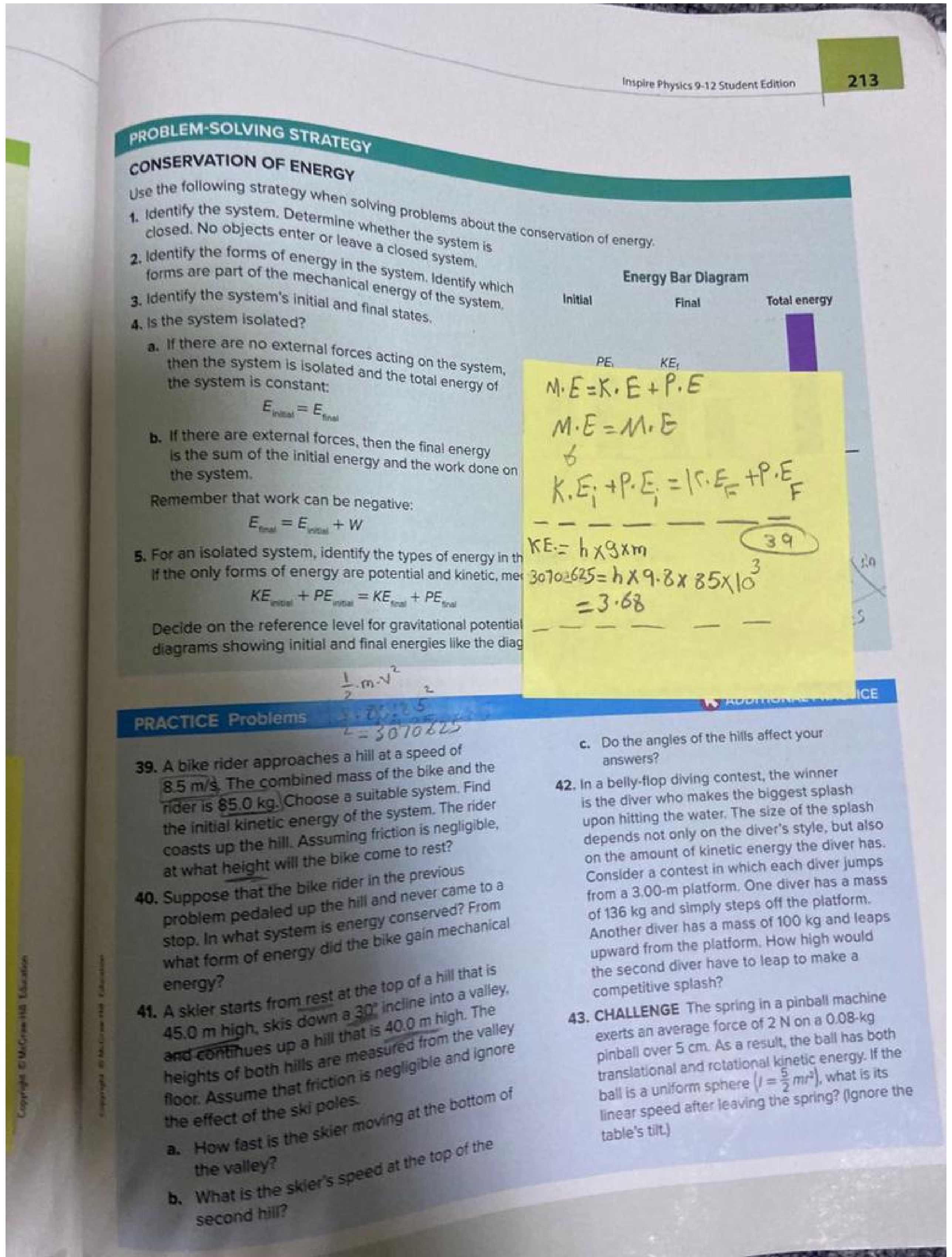
$KE_f = KE_i + GPE_i - GPE_f = 0.0\text{ J} + 1.6\times10^3\text{ J} - 0.0\text{ J} = 1.6\times10^3$ J

b. Determine the speed of the limb.

$KE_f = \frac{1}{2}mv_f^2$

$v_f = \sqrt{\frac{2KE_f}{m}} = \sqrt{\frac{2(1.6\times10^3\text{ J})}{22.0\text{ kg}}} = 12$ m/s

3 EVALUATE THE ANSWER
- Are the units correct? The magnitude of velocity is measur... kg·m²/s² = J.
- Do the signs make sense? Kinetic energy and speed are both positive in this scenario.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PROBLEM-SOLVING STRATEGY

CONSERVATION OF ENERGY

Use the following strategy when solving problems about the conservation of energy.

1. Identify the system. Determine whether the system is closed. No objects enter or leave a closed system.
2. Identify the forms of energy in the system. Identify which forms are part of the mechanical energy of the system.
3. Identify the system's initial and final states.
4. Is the system isolated?
   a. If there are no external forces acting on the system, then the system is isolated and the total energy of the system is constant: $E_{initial} = E_{final}$
   b. If there are external forces, then the final energy is the sum of the initial energy and the work done on the system. Remember that work can be negative: $E_{final} = E_{initial} + W$
5. For an isolated system, identify the types of energy in th... If the only forms of energy are potential and kinetic, me... $KE_{initial} + PE_{initial} = KE_{final} + PE_{final}$

Decide on the reference level for gravitational potential... diagrams showing initial and final energies like the diag...

Energy Bar Diagram: Initial, Final, Total energy

PRACTICE Problems

39. A bike rider approaches a hill at a speed of 8.5 m/s. The combined mass of the bike and the rider is 85.0 kg. Choose a suitable system. Find the initial kinetic energy of the system. The rider coasts up the hill. Assuming friction is negligible, at what height will the bike come to rest?

40. Suppose that the bike rider in the previous problem pedaled up the hill and never came to a stop. In what system is energy conserved? From what form of energy did the bike gain mechanical energy?

41. A skier starts from rest at the top of a hill that is 45.0 m high, skis down a 30° incline into a valley, and continues up a hill that is 40.0 m high. The heights of both hills are measured from the valley floor. Assume that friction is negligible and ignore the effect of the ski poles.
   a. How fast is the skier moving at the bottom of the valley?
   b. What is the skier's speed at the top of the second hill?
   c. Do the angles of the hills affect your answers?

42. In a belly-flop diving contest, the winner is the diver who makes the biggest splash upon hitting the water. The size of the splash depends not only on the diver's style, but also on the amount of kinetic energy the diver has. Consider a contest in which each diver jumps from a 3.00-m platform. One diver has a mass of 136 kg and simply steps off the platform. Another diver has a mass of 100 kg and leaps upward from the platform. How high would the second diver have to leap to make a competitive splash?

43. CHALLENGE The spring in a pinball machine exerts an average force of 2 N on a 0.08-kg pinball over 5 cm. As a result, the ball has both translational and rotational kinetic energy. If the ball is a uniform sphere ($I = \frac{2}{5}mr^2$), what is its linear speed after leaving the spring? (Ignore the table's tilt.)

18	(1) Explain Kepler's First Law which states that the planets follow elliptical paths with the sun at one focus. (2) Explain Kepler's Second Law which states that an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals. (3) Explain Kepler's Third Law which states that the square of the ratio of the periods of any two planets revolving about the Sun is equal to the cube of the ratio of their average distances from the Sun and write it in equation form $\left[\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3\right]$. (4) Explain the law of universal gravitation and write it in equation form $\left[F_g = G\frac{m_1 m_2}{r^2}\right]$	Student Book	165-168

Tycho Brahe realized that the charts of the time did not accurately predict astronomical events. He recognized that measurements were required from one location over a long period of time. Tycho was granted an estate on the Danish island of Hven and the funding to build an early research institute. Telescopes had not been invented, so to make measurements, Tycho used huge instruments that he designed and built in his own shop, like those shown in **Figure 1.** Tycho is credited with the most accurate measurements of the time.

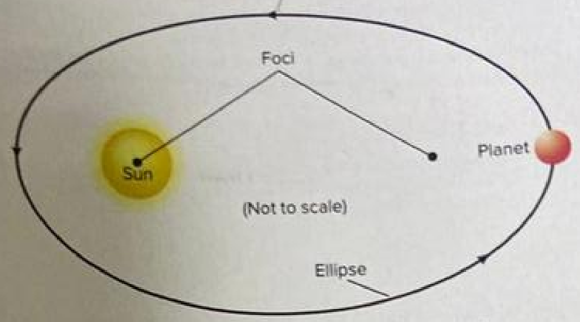


Figure 2 The orbit of each planet is an ellipse, with the Sun at one focus.

Kepler's Laws

In 1600 Tycho moved to Prague where Johannes Kepler, a 29-year-old German, became one of his assistants. Kepler analyzed Tycho's observations. After Tycho's death in 1601, Kepler continued to study Tycho's data and used geometry and mathematics to explain the motion of the planets. After seven years of careful analysis of Tycho's data on Mars, Kepler discovered the laws that describe the motion of every planet and satellite, natural or artificial. Here, the laws are presented in terms of planets.

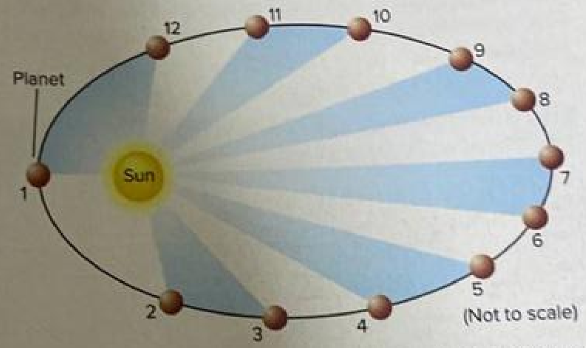


Figure 3 Kepler found that elliptical orbits sweep out equal areas in equal time periods.
Explain *why the equal time areas are shaped differently.*

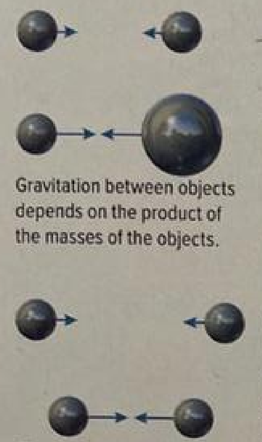
Kepler's first law states that the paths of the planets are ellipses, with the Sun at one focus. An ellipse has two foci, as shown in **Figure 2.** Although exaggerated ellipses are used in the diagrams, Earth's actual orbit is very nearly circular. You would not be able to distinguish it from a circle visually.

Kepler found that orbits might change due to gravitational effects from, or collisions with, other objects in the solar system. He also found that the planets move faster when they are closer to the Sun and slower when they are farther away from the Sun. **Kepler's second law** states that an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals, as illustrated in **Figure 3.**

A period is the time it takes for one revolution of an orbiting body. Kepler also discovered a mathematical relationship between periods of planets and their mean distances away from the Sun.

Get It?
Describe the common feature that Kepler's first law found concerning the paths of orbiting objects around the Sun.

Newton's Law of Universal Gravitation



Gravitation between objects depends on the product of the masses of the objects.

The gravitational force between objects is inversely proportional to the square of the distance between the objects.

Figure 5 Mass and distance affect the magnitude of the gravitational force between objects.

In 1666, Isaac Newton began his studies of planetary motion. It has been said that seeing an apple fall made Newton wonder if the force that caused the apple to fall might extend to the Moon, or even beyond. He found that the magnitude of the force (F_g) on a planet due to the Sun varies inversely with the square of the distance (r) between the centers of the planet and the Sun. That is, F_g is proportional to $\frac{1}{r^2}$. The force (F_g) acts in the direction of the line connecting the centers of the two objects, as shown in **Figure 5.**

Newton found that both the apple's and the Moon's accelerations agree with the $\frac{1}{r^2}$ relationship. According to his own third law, the force Earth exerts on the apple is exactly the same as the force the apple exerts on Earth. Even though these forces are exactly the same, you can easily observe the effect of the force on the apple because it has much lower mass than Earth. The force of attraction between two objects must be proportional to the objects' masses and is known as the **gravitational force.**

Newton was confident that the same force of attraction would act between any two objects anywhere in the universe. He proposed the **law of universal gravitation,** which states that objects attract other objects with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. Newton's law of universal gravitation, shown below, provides the mathematical models to describe and predict the effects of gravitational forces between distant objects.

Inverse Square Law

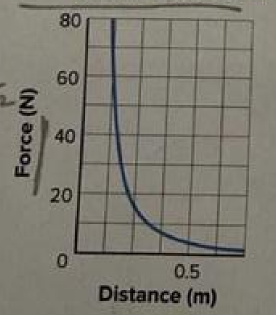


Figure 6 This is a graphical representation of the inverse square relationship.

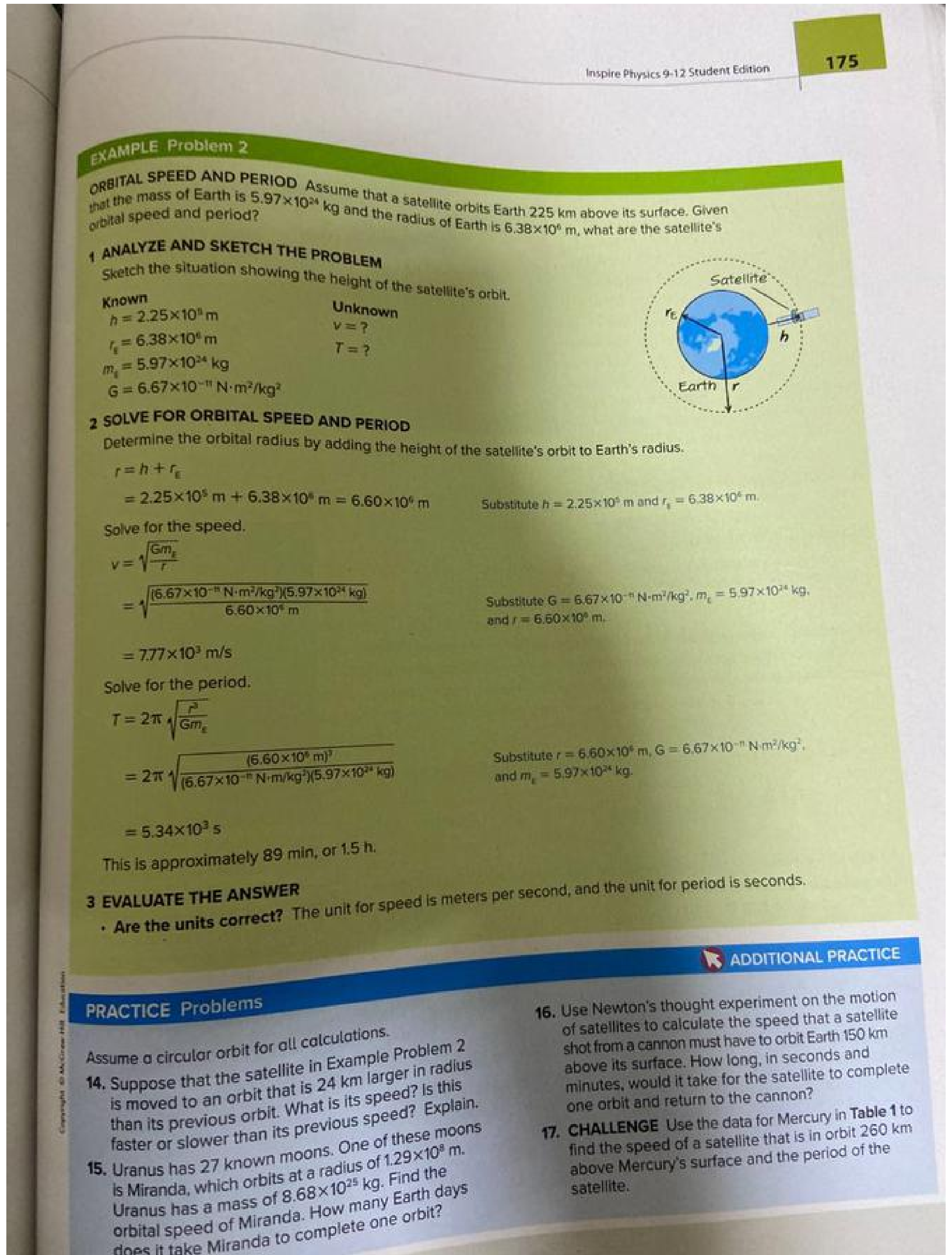
Law of Universal Gravitation

The gravitational force is equal to the universal gravitational constant, times the mass of object 1, times the mass of object 2, divided by the distance between the centers of the objects, squared.

$$F_g = \frac{Gm_1m_2}{r^2}$$

According to Newton's equation, F is directly proportional to m_1 and m_2. If the mass of a planet near the Sun doubles, the force of attraction doubles. Use the Connecting Math to Physics feature on the next page to examine how changing one variable affects another. **Figure 6** illustrates the inverse square relationship graphically. The term G is the universal gravitational constant and will be discussed on the following pages.

19	**(1)** Calculate the orbital period of a satellite. **(2)** Define pressure as the perpendicular component of a force on a surface divided by the area of the surface: $\left(P = \frac{F}{A}\right)$	Example Problem (2) Check your Progress Q.8 Example Problem (1) Practice Problem (3)	175 172 234 235



EXAMPLE Problem 2

ORBITAL SPEED AND PERIOD Assume that a satellite orbits Earth 225 km above its surface. Given that the mass of Earth is 5.97×10^{24} kg and the radius of Earth is 6.38×10^{6} m, what are the satellite's orbital speed and period?

1 ANALYZE AND SKETCH THE PROBLEM

Sketch the situation showing the height of the satellite's orbit.

Known: $h = 2.25\times10^{5}$ m; $r_E = 6.38\times10^{6}$ m; $m_E = 5.97\times10^{24}$ kg; $G = 6.67\times10^{-11}$ N·m²/kg²

Unknown: $v = ?$; $T = ?$

2 SOLVE FOR ORBITAL SPEED AND PERIOD

Determine the orbital radius by adding the height of the satellite's orbit to Earth's radius.

$r = h + r_E$

$= 2.25\times10^{5}\text{ m} + 6.38\times10^{6}\text{ m} = 6.60\times10^{6}\text{ m}$ — Substitute $h = 2.25\times10^{5}$ m and $r_E = 6.38\times10^{6}$ m.

Solve for the speed.

$v = \sqrt{\frac{Gm_E}{r}}$

$= \sqrt{\frac{(6.67\times10^{-11}\text{ N·m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}{6.60\times10^{6}\text{ m}}}$ — Substitute $G = 6.67\times10^{-11}$ N·m²/kg², $m_E = 5.97\times10^{24}$ kg, and $r = 6.60\times10^{6}$ m.

$= 7.77\times10^{3}$ m/s

Solve for the period.

$T = 2\pi\sqrt{\frac{r^3}{Gm_E}}$

$= 2\pi\sqrt{\frac{(6.60\times10^{6}\text{ m})^3}{(6.67\times10^{-11}\text{ N·m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}}$ — Substitute $r = 6.60\times10^{6}$ m, $G = 6.67\times10^{-11}$ N·m²/kg², and $m_E = 5.97\times10^{24}$ kg.

$= 5.34\times10^{3}$ s

This is approximately 89 min, or 1.5 h.

3 EVALUATE THE ANSWER

- Are the units correct? The unit for speed is meters per second, and the unit for period is seconds.

ADDITIONAL PRACTICE

PRACTICE Problems

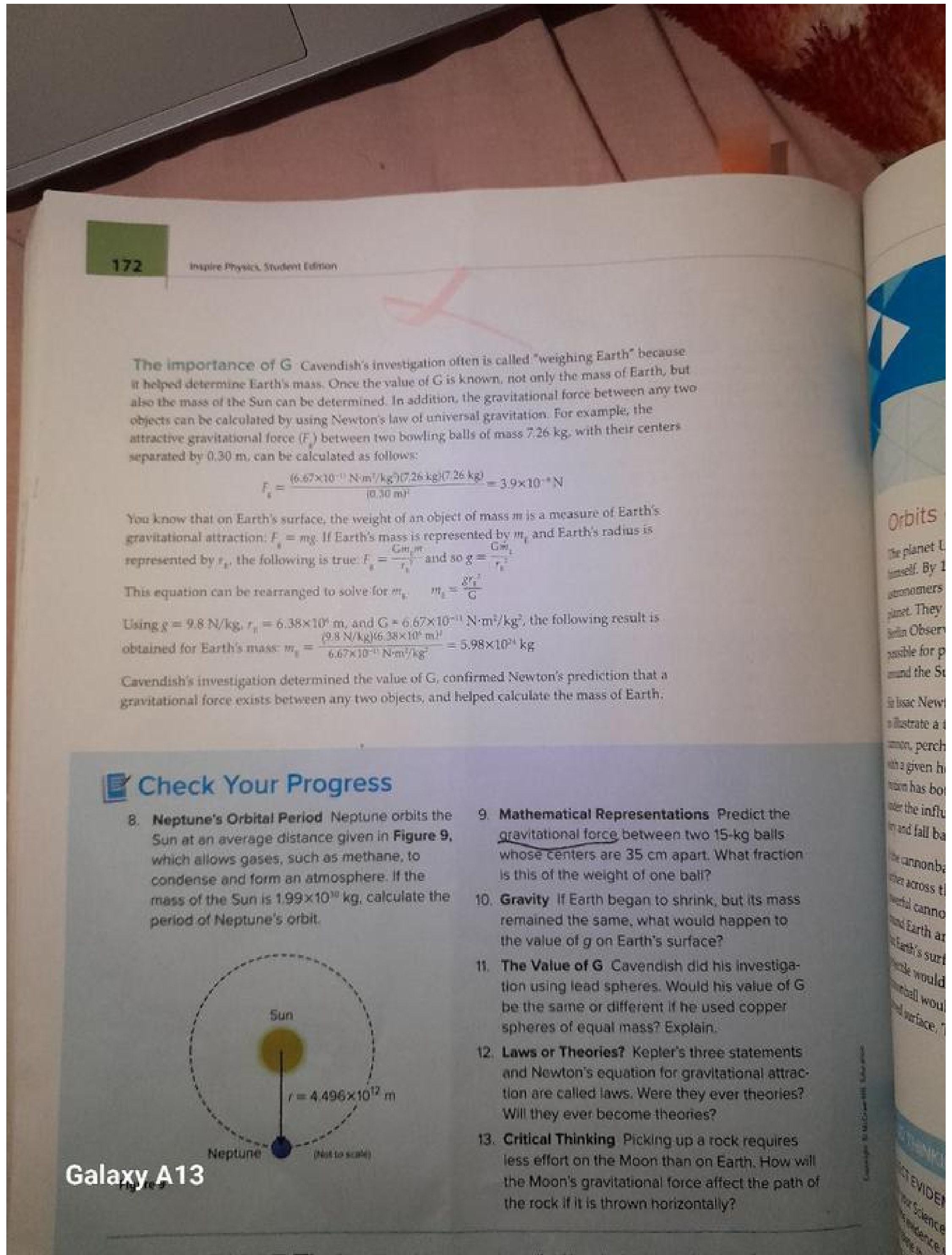
Assume a circular orbit for all calculations.

14. Suppose that the satellite in Example Problem 2 is moved to an orbit that is 24 km larger in radius than its previous orbit. What is its speed? Is this faster or slower than its previous speed? Explain.

15. Uranus has 27 known moons. One of these moons is Miranda, which orbits at a radius of 1.29×10^{8} m. Uranus has a mass of 8.68×10^{25} kg. Find the orbital speed of Miranda. How many Earth days does it take Miranda to complete one orbit?

16. Use Newton's thought experiment on the motion of satellites to calculate the speed that a satellite shot from a cannon must have to orbit Earth 150 km above its surface. How long, in seconds and minutes, would it take for the satellite to complete one orbit and return to the cannon?

17. CHALLENGE Use the data for Mercury in Table 1 to find the speed of a satellite that is in orbit 260 km above Mercury's surface and the period of the satellite.



The importance of G Cavendish's investigation often is called "weighing Earth" because it helped determine Earth's mass. Once the value of G is known, not only the mass of Earth, but also the mass of the Sun can be determined. In addition, the gravitational force between any two objects can be calculated by using Newton's law of universal gravitation. For example, the attractive gravitational force (F_g) between two bowling balls of mass 7.26 kg, with their centers separated by 0.30 m, can be calculated as follows:

$F_g = \frac{(6.67\times10^{-11}\text{ N·m}^2/\text{kg}^2)(7.26\text{ kg})(7.26\text{ kg})}{(0.30\text{ m})^2} = 3.9\times10^{-8}\text{ N}$

You know that on Earth's surface, the weight of an object of mass m is a measure of Earth's gravitational attraction: $F_g = mg$. If Earth's mass is represented by m_E and Earth's radius is represented by r_E, the following is true: $F_g = \frac{Gm_Em}{r_E^2}$ and so $g = \frac{Gm_E}{r_E^2}$

This equation can be rearranged to solve for m_E. $m_E = \frac{gr_E^2}{G}$

Using $g = 9.8$ N/kg, $r_E = 6.38\times10^{6}$ m, and $G = 6.67\times10^{-11}$ N·m²/kg², the following result is obtained for Earth's mass: $m_E = \frac{(9.8\text{ N/kg})(6.38\times10^{6}\text{ m})^2}{6.67\times10^{-11}\text{ N·m}^2/\text{kg}^2} = 5.98\times10^{24}\text{ kg}$

Cavendish's investigation determined the value of G, confirmed Newton's prediction that a gravitational force exists between any two objects, and helped calculate the mass of Earth.

Check Your Progress

8. Neptune's Orbital Period Neptune orbits the Sun at an average distance given in Figure 9, which allows gases, such as methane, to condense and form an atmosphere. If the mass of the Sun is 1.99×10^{30} kg, calculate the period of Neptune's orbit.

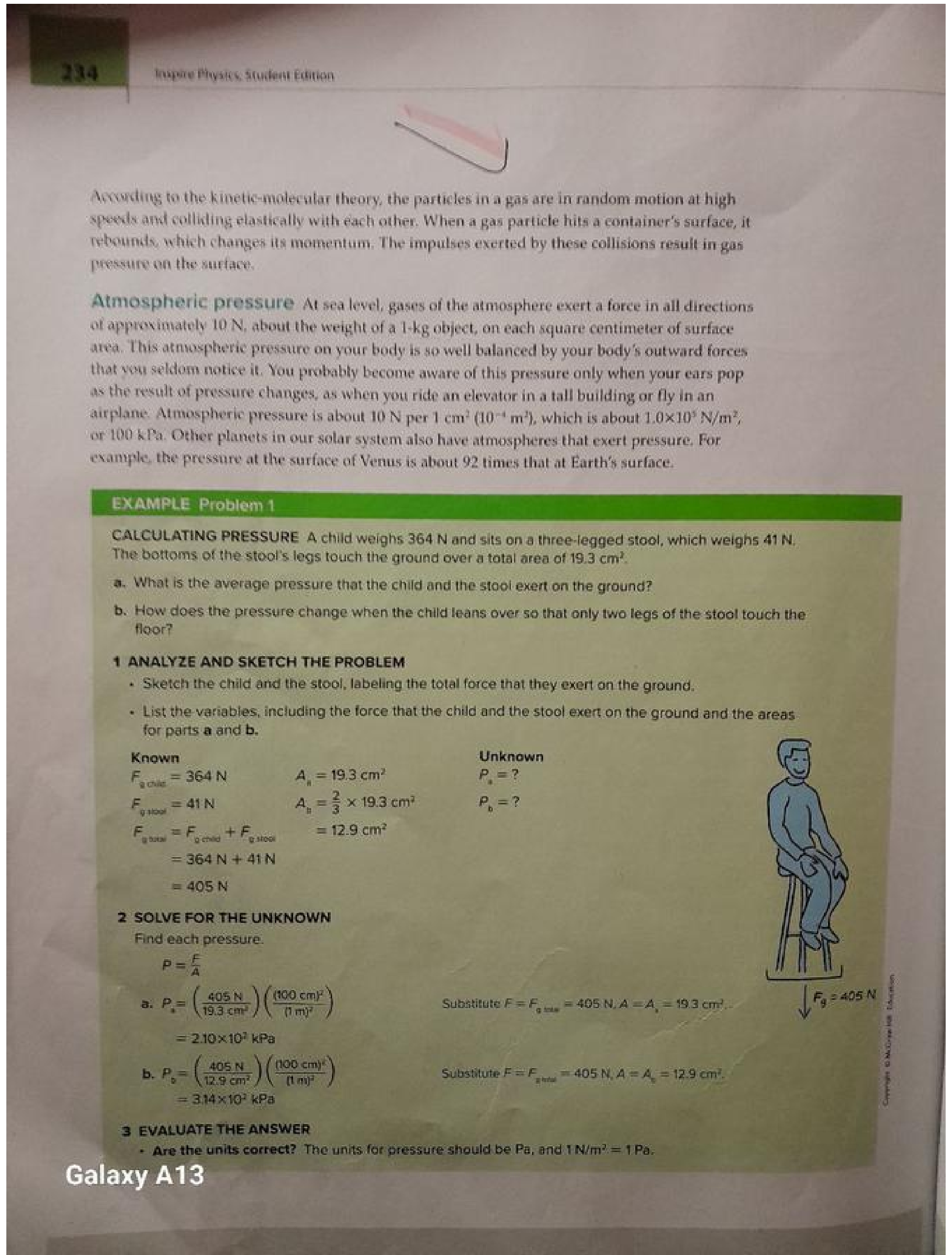
9. Mathematical Representations Predict the gravitational force between two 15-kg balls whose centers are 35 cm apart. What fraction is this of the weight of one ball?

10. Gravity If Earth began to shrink, but its mass remained the same, what would happen to the value of g on Earth's surface?

11. The Value of G Cavendish did his investigation using lead spheres. Would his value of G be the same or different if he used copper spheres of equal mass? Explain.

12. Laws or Theories? Kepler's three statements and Newton's equation for gravitational attraction are called laws. Were they ever theories? Will they ever become theories?

13. Critical Thinking Picking up a rock requires less effort on the Moon than on Earth. How will the Moon's gravitational force affect the path of the rock if it is thrown horizontally?



According to the kinetic-molecular theory, the particles in a gas are in random motion at high speeds and colliding elastically with each other. When a gas particle hits a container's surface, it rebounds, which changes its momentum. The impulses exerted by these collisions result in gas pressure on the surface.

Atmospheric pressure At sea level, gases of the atmosphere exert a force in all directions of approximately 10 N, about the weight of a 1-kg object, on each square centimeter of surface area. This atmospheric pressure on your body is so well balanced by your body's outward forces that you seldom notice it. You probably become aware of this pressure only when your ears pop as the result of pressure changes, as when you ride an elevator in a tall building or fly in an airplane. Atmospheric pressure is about 10 N per 1 cm² (10^{-4} m²), which is about 1.0×10^{5} N/m², or 100 kPa. Other planets in our solar system also have atmospheres that exert pressure. For example, the pressure at the surface of Venus is about 92 times that at Earth's surface.

EXAMPLE Problem 1

CALCULATING PRESSURE A child weighs 364 N and sits on a three-legged stool, which weighs 41 N. The bottoms of the stool's legs touch the ground over a total area of 19.3 cm².

a. What is the average pressure that the child and the stool exert on the ground?

b. How does the pressure change when the child leans over so that only two legs of the stool touch the floor?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the child and the stool, labeling the total force that they exert on the ground.
- List the variables, including the force that the child and the stool exert on the ground and the areas for parts a and b.

Known: $F_{g\,child} = 364$ N; $F_{g\,stool} = 41$ N; $F_{g\,total} = F_{g\,child} + F_{g\,stool} = 364\text{ N} + 41\text{ N} = 405\text{ N}$; $A_a = 19.3$ cm²; $A_b = \frac{2}{3}\times19.3$ cm² $= 12.9$ cm²

Unknown: $P_a = ?$; $P_b = ?$

2 SOLVE FOR THE UNKNOWN

Find each pressure.

$P = \frac{F}{A}$

a. $P_a = \left(\frac{405\text{ N}}{19.3\text{ cm}^2}\right)\left(\frac{(100\text{ cm})^2}{(1\text{ m})^2}\right)$ — Substitute $F = F_{g\,total} = 405$ N, $A = A_a = 19.3$ cm².

$= 2.10\times10^{2}$ kPa

b. $P_b = \left(\frac{405\text{ N}}{12.9\text{ cm}^2}\right)\left(\frac{(100\text{ cm})^2}{(1\text{ m})^2}\right)$ — Substitute $F = F_{g\,total} = 405$ N, $A = A_b = 12.9$ cm².

$= 3.14\times10^{2}$ kPa

3 EVALUATE THE ANSWER

- Are the units correct? The units for pressure should be Pa, and 1 N/m² = 1 Pa.

19	(1) Calculate the orbital period of a satellite. (2) Define pressure as the perpendicular component of a force on a surface divided by the area of the surface: $\left(P = \frac{F}{A}\right)$	Example Problem (2) Check your Progress Q.8 Example Problem (1) Practice Problem (3)	175 172 234 235

PRACTICE Problems

ADDITIONAL PRACTICE

1. The atmospheric pressure at sea level is about 1.0×10^5 Pa. What is the force at sea level that air exerts on the top of a desk that is 152 cm long and 76 cm wide?
2. A car tire makes contact with the ground on a rectangular area of 12 cm by 18 cm. If the car's mass is 925 kg, what pressure does the car exert on the ground as it rests on all four tires?
3. A lead brick, 5.0 cm × 10.0 cm × 20.0 cm, rests on the ground on its smallest face. Lead has a density of 11.8 g/cm³. What pressure does the brick exert on the ground?
4. Suppose that during a storm, the atmospheric pressure suddenly drops by 15 percent outside. What net force would be exerted on a front door to a house that is 195 cm high and 91 cm wide? In what direction would this force be exerted?
5. **CHALLENGE** Large pieces of industrial equipment are placed on wide steel plates that spread the weight of the equipment over larger areas. If an engineer plans to install a 454-kg device on a floor that is rated to withstand additional pressure of 5.0×10^4 Pa, how large should the steel support plate be?

The Gas Laws

Think about a container of gas that is held at a constant temperature. If you reduced the volume, what would happen to the pressure of the gas? There would be more collisions between the particles and the container's walls, and so the pressure would increase. Similarly, if you increased the volume, there would be fewer collisions, decreasing the pressure. This inverse relationship was found by seventeenth-century chemist and physicist Robert Boyle. Because the product of inversely related variables is a constant, Boyle's law can be written PV = constant, or $P_1V_1 = P_2V_2$. The subscripts that you see in the gas laws help keep track of different variables, such as pressure and volume, as they change throughout a problem. The relationship between the pressure and the volume of a gas is critical to the scuba diver in **Figure 3.**

About 100 years after Boyle's work, Jacques Charles cooled a gas and found that the volume shrank by $\frac{1}{273}$ of its original volume for every degree cooled, which is a linear relationship. At the time, Charles could not cool gases to the extremely low temperatures achieved in modern laboratories. In order to see what lower limits might be possible, he extended, or extrapolated, the graph of his data to these temperatures. This extrapolation suggested that if the temperature were reduced to −273°C, a gas would have zero volume. The temperature at which a gas would have zero volume is now called absolute zero, which is represented by the zero of the Kelvin temperature scale.

Figure 3 The gas in the tank on a diver's back is at high pressure. The regulator in the diver's mouth reduces the pressure, making the pressure of the gas the diver breathes equal to that of the water pressure. Bubbles are emitted from the regulator as the diver exhales.

These experiments indicated that under constant pressure, the volume of a sample of gas varies directly with its Kelvin temperature, a result that is now called Charles's law. Charles's law can be written $\frac{V}{T}$ = constant, or $\frac{V_1}{T_1} = \frac{V_2}{T_2}$.

Combined gas law Combining Boyle's law and Charles's law relates the pressure, temperature, and volume of a fixed amount of ideal gas, which leads to the **combined gas law.**

Combined Gas Law

For a fixed amount of an ideal gas, the pressure times the volume, divided by the Kelvin ... equals a constant.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \text{constant}$$

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