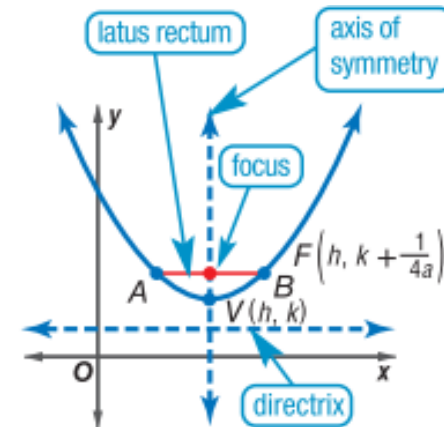
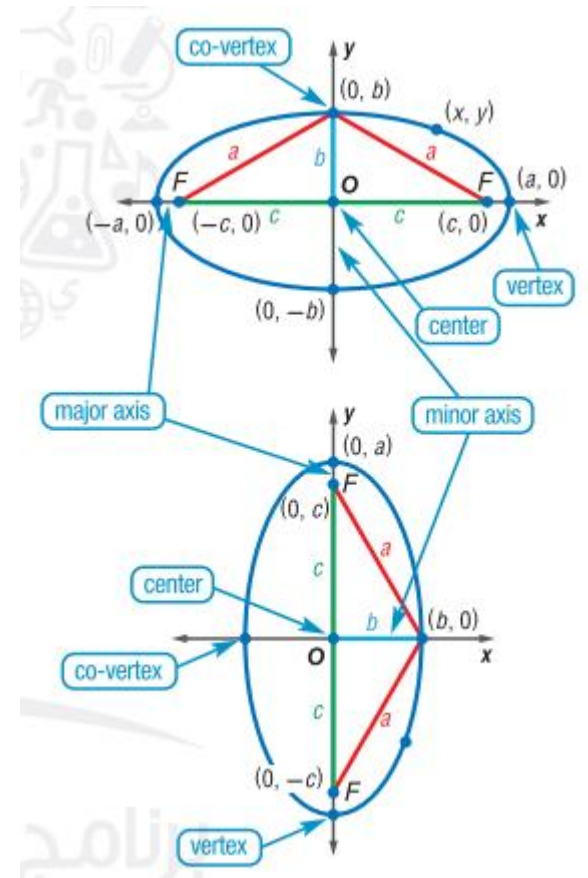
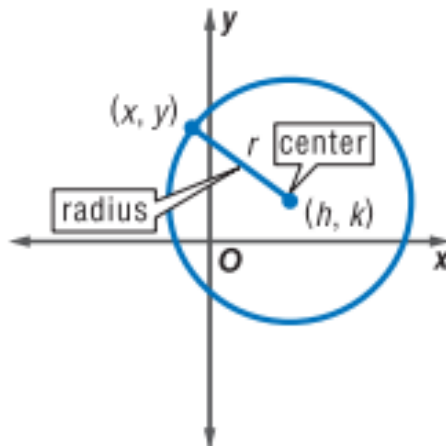
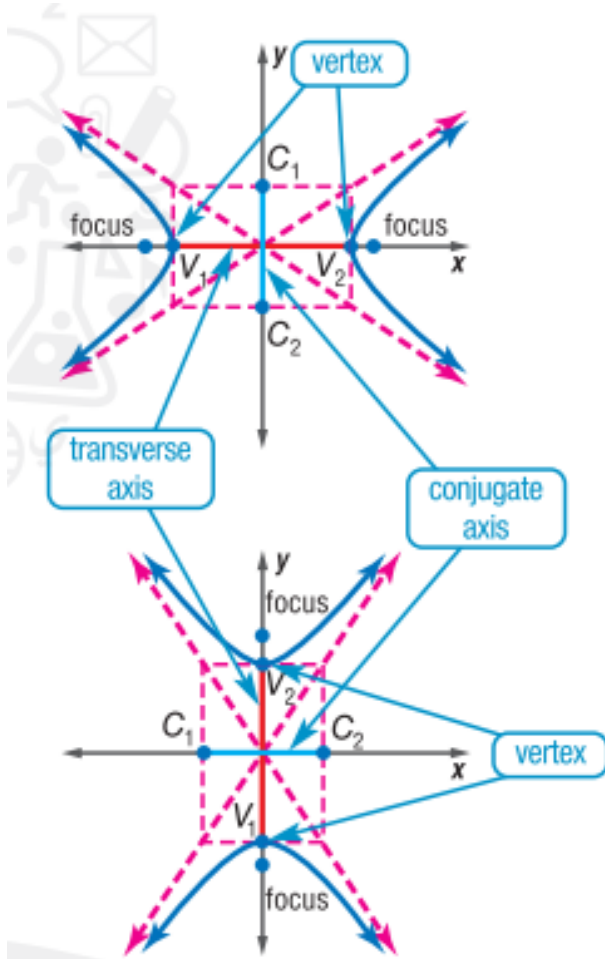


# EO Term 2 2023-2024 11ADV

Chapter 6 (8-9-10-11-19)  
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Al Samha School



**Example 1** Analyze the Equation of a Parabola

Write  $y = 2x^2 - 12x + 6$  in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

$$y = 2x^2 - 12x + 6$$

Original equation

$$= 2(x^2 - 6x) + 6$$

Factor 2 from the  $x$ - and  $x^2$ -terms.

$$= 2(x^2 - 6x + \blacksquare) + 6 - 2(\blacksquare)$$

Complete the square on the right side.

$$= 2(x^2 - 6x + 9) + 6 - 2(9)$$

The 9 added when you complete the square is multiplied by 2.

$$= 2(x - 3)^2 - 12$$

Factor.

The vertex of this parabola is located at  $(3, -12)$ , and the equation of the axis of symmetry is  $x = 3$ . The parabola opens upward.

**Guided Practice**

- Write  $y = 4x^2 + 16x + 34$  in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

**Check Your Understanding**

**Example 1** Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

- $y = 2x^2 - 24x + 40$

- $y = 3x^2 - 6x - 4$

- $x = y^2 - 8y - 11$

- $x + 3y^2 + 12y = 18$

1.  $y = 2(x - 6)^2 - 32$  الرأس

$(-32, 6)$ ؛ محور التماثل؛

$x = 6$ ؛ مفتوح إلى الأعلى

2.  $y = 3(x - 1)^2 - 7$  الرأس

$(1, -7)$ ؛ محور التماثل؛  $x = 1$ ؛

مفتوح إلى الأعلى

3.  $x = (y - 4)^2 - 27$  الرأس

$(-27, 4)$ ؛ محور التماثل؛  $y =$

4؛ مفتوح إلى الجهة اليمنى

4.  $x = -3(y + 2)^2 + 30$

الرأس  $(30, -2)$ ؛ محور

التماثل؛  $y = -2$ ؛ مفتوح إلى

الجهة اليسرى

**Example 4** Write an Equation of a Parabola

Write an equation for a parabola with vertex at  $(-2, -4)$  and directrix  $y = 1$ . Then graph the equation.

The directrix is a horizontal line, so the equation of the parabola is of the form  $y = a(x - h)^2 + k$ . Find  $a$ ,  $h$ , and  $k$ .

- The vertex is at  $(-2, -4)$ , so  $h = -2$  and  $k = -4$ .

- Use the equation of the directrix to find  $a$ .

$$y = k - \frac{1}{4a}$$

Equation of directrix

$$1 = -4 - \frac{1}{4a}$$

Replace  $y$  with 1 and  $k$  with  $-4$ .

$$5 = -\frac{1}{4a}$$

Add 4 to each side.

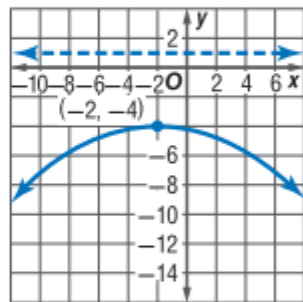
$$20a = -1$$

Multiply each side by 4a.

$$a = -\frac{1}{20}$$

Divide each side by 20.

So, the equation of the parabola is  $y = -\frac{1}{20}(x + 2)^2 - 4$ .

**Guided Practice**

Write an equation for each parabola described below. Then graph the equation.

- 4A. vertex  $(1, 3)$ , focus  $(1, 5)$

- 4B. focus  $(5, 6)$ , directrix  $x = -2$

**Example 4**

Write an equation for each parabola described below. Then graph the equation.

26. vertex  $(0, 1)$ , focus  $(0, 4)$

27. vertex  $(1, 8)$ , directrix  $y = 3$

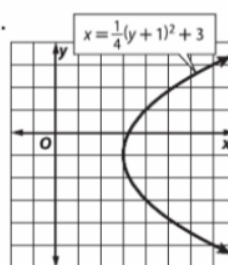
28. focus  $(-2, -4)$ , directrix  $x = -6$

29. focus  $(2, 4)$ , directrix  $x = 10$

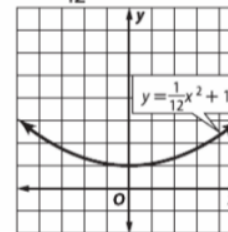
30. vertex  $(-6, 0)$ , directrix  $x = 2$

31. vertex  $(9, 6)$ , focus  $(9, 5)$

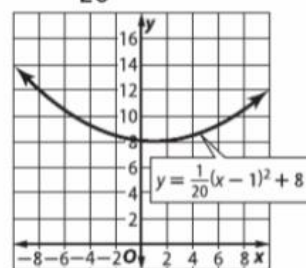
25.



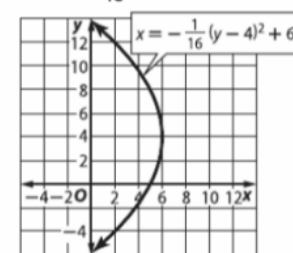
26.  $y = \frac{1}{12}x^2 + 1$



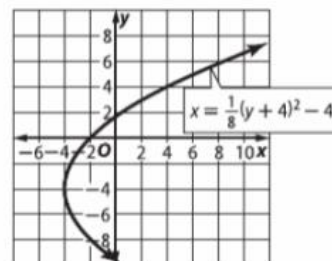
27.  $y = \frac{1}{20}(x - 1)^2 + 8$



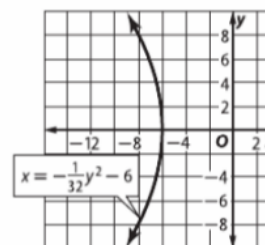
29.  $x = -\frac{1}{16}(y - 4)^2 + 6$



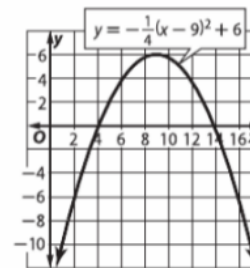
28.  $x = \frac{1}{8}(y + 4)^2 - 4$



30.  $x = -\frac{1}{32}y^2 - 6$



31.  $y = -\frac{1}{4}(x - 9)^2 + 6$



### Real-World Example 1 Write an Equation Given the Radius

**DELIVERY** Appliances + More offers free delivery within 35 kilometers of the store. The Abu Dhabi store is located 100 kilometers north and 45 kilometers east of the corporate office. Write an equation to represent the delivery boundary of the Abu Dhabi store if the origin of the coordinate system is the corporate office.

Since the corporate office is at  $(0, 0)$ , the Abu Dhabi store is at  $(45, 100)$ . The boundary of the delivery region is the circle centered at  $(45, 100)$  with radius 35 kilometers.

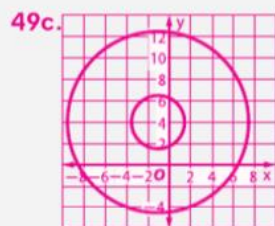
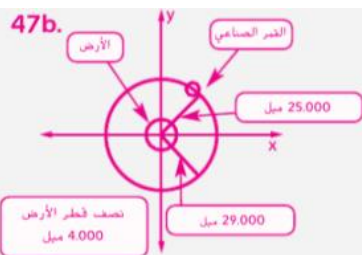
$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{Equation of a circle}$$

$$(x - 45)^2 + (y - 100)^2 = 35^2 \quad (h, k) = (45, 100) \text{ and } r = 35$$

$$(x - 45)^2 + (y - 100)^2 = 1225 \quad \text{Simplify.}$$

### Guided Practice

- Wi-Fi** A certain wi-fi phone has a range of 30 kilometers in any direction. If the phone is 4 kilometers south and 3 kilometers west of headquarters, write an equation to represent the area within which the phone can operate via the Wi-Fi system.



### التبرير المنطقي

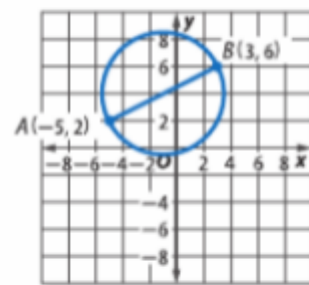
48. **التبرير المنطقي** افترض أن بث محطة راديو غير محجوب يمكنه الانتقال لمسافة 120 mi. افترض أن المحطة متمركزة عند نقطة الأصل.

$$x^2 + y^2 = 14,400$$

a. اكتب معادلة لتمثيل حدود منطقة البث باستخدام المركز كنقطة الأصل.

b. إذا علمت أن برج البث يبعد عن المحطة الحالية مسافة 40 mi شرقًا و 10 mi جنوبًا. وتعمل الإشارة المعززة على بث

$$49a. (x + 1)^2 + (y - 4)^2 = 36 + 16\sqrt{5} \quad \text{بمسافة } 40\sqrt{5} \text{ mi. اكتب معادلة التي تمثل منطقة البث الجديدة.}$$



49. **الهندسة** الدوائر متحدة المركز هي دوائر لها المركز ذاته ولكن بأنصاف أقطار مختلفة. ارجع إلى التمثيل البياني الموضح حيث  $\overline{AB}$  هو قطر الدائرة.

a. اكتب معادلة لدائرة تتحد في المركز مع الدائرة الموضحة على اليسار. ولكن نصف قطرها أكبر بمقدار 4 وحدات.

b. اكتب معادلة لدائرة تتحد في المركز مع الدائرة الموضحة على اليسار. ولكن نصف قطرها أصغر بمقدار وحدتين.

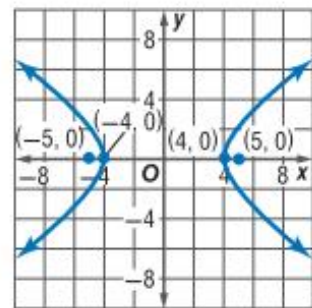
### انظر الحاشية.

$$c. \text{ مثل معادلتين الدائرتين a و b في الجوابين 49a و 49b. } (x + 1)^2 + (y - 4)^2 = 24 - 8\sqrt{5} \quad \text{50. } (x + 35)^2 + (y - 40)^2 = 3025$$

50. **الزلازل** تبعد روز بول عن وسط مدينة تقريبًا بمسافة 35 mi غربًا و 4 mi شمالًا. افترض حدوث زلزال يبعد مركزه عن استاد المدينة بمسافة 55 mi تقريبًا. افترض أن نقطة أصل المستوى الإحداثي تقع عند مركز وسط المدينة. اكتب معادلة لمجموعة النقاط التي يمكن أن تكون مركز الزلزال.

**Example 1** Write an Equation Given Vertices and Foci

Write an equation for the hyperbola shown in the graph.

**Step 1** Find the center.

The vertices are equidistant from the center.

The center is at  $(0, 0)$ .

**Step 2** Find the values of  $a$ ,  $b$ , and  $c$ .

The value of  $a$  is the distance between a vertex and the center, or 4 units.

The value of  $c$  is the distance between a focus and the center, or 5 units.

$$c^2 = a^2 + b^2 \quad \text{Equation relating } a, b, \text{ and } c \text{ for a hyperbola}$$

$$5^2 = 4^2 + b^2 \quad c = 5 \text{ and } a = 4$$

$$9 = b^2 \quad \text{Subtract } 4^2 \text{ from each side.}$$

**Step 3** Write the equation.

The transverse axis is horizontal, so the equation is  $\frac{x^2}{16} - \frac{y^2}{9} = 1$ .

**Guided Practice**

- Write an equation for a hyperbola with vertices at  $(6, 0)$  and  $(-6, 0)$  and foci at  $(8, 0)$  and  $(-8, 0)$ .

**Example 2** Write an Equation Given Asymptotes

The asymptotes for a vertical hyperbola are  $y = \frac{5}{3}x$  and  $y = -\frac{5}{3}x$  and the vertices are at  $(0, 5)$  and  $(0, -5)$ . Write the equation for the hyperbola.

**Step 1** Find the center.

The vertices are equidistant from the center.

The center of the hyperbola is at  $(0, 0)$ .

**Step 2** Find the values of  $a$  and  $b$ .

The hyperbola is vertical, so  $a = 5$ .

From the asymptotes,  $b = 3$ .

The value of  $c$  is not needed.

**Step 3** Write the equation.

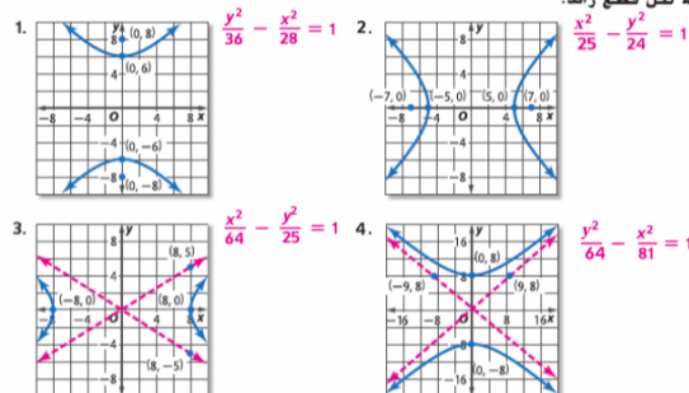
The equation for the hyperbola is  $\frac{y^2}{25} - \frac{x^2}{9} = 1$ .

**Guided Practice**

- The asymptotes for a horizontal hyperbola are  $y = \frac{7}{9}x$  and  $y = -\frac{7}{9}x$ . The vertices are  $(9, 0)$  and  $(-9, 0)$ . Write an equation for the hyperbola.

## التحقق من فهمك

المثالان 1-2 اكتب معادلة لكل قطع زائد.



## Example 4 Graph an Ellipse

Find the coordinates of the center and foci, and the lengths of the major and minor axes of an ellipse with equation  $25x^2 + 9y^2 + 250x - 36y + 436 = 0$ . Then graph the ellipse.

**Step 1** Write in standard form. Complete the square for each variable to write this equation in standard form.

$$25x^2 + 9y^2 + 250x - 36y + 436 = 0$$

$$25x^2 + 250x + 9y^2 - 36y = -436$$

$$25(x^2 + 10x) + 9(y^2 - 4y) = -436$$

$$25(x^2 + 10x + \blacksquare) + 9(y^2 - 4y + \blacksquare) = -436 + 25(\blacksquare) + 9(\blacksquare)$$

$$25(x^2 + 10x + 25) + 9(y^2 - 4y + 4) = -436 + 25(25) + 9(4)$$

$$25(x + 5)^2 + 9(y - 2)^2 = 225$$

$$\frac{(x + 5)^2}{9} + \frac{(y - 2)^2}{25} = 1$$

Original equation

Associative Property

Distributive Property

Complete the squares.

 $5^2 = 25$  and  $(-2)^2 = 4$ 

Write as perfect squares.

Divide each side by 225.

**Step 2** Find the center.

$h = -5$  and  $k = 2$ , so the center of the ellipse is at  $(-5, 2)$ .

**Step 3** Find the lengths of the axes and graph.

The ellipse is vertical.

$a^2 = 25$ , so  $a = 5$ .  $b^2 = 9$ , so  $b = 3$ .

The length of the major axis is  $2 \cdot 5$  or 10.

The length of the minor axis is  $2 \cdot 3$  or 6.

The vertices are at  $(-5, 7)$  and  $(-5, -3)$ .

The co-vertices are at  $(-2, 2)$  and  $(-8, 2)$ .

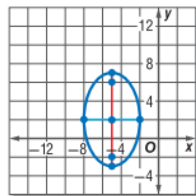
**Step 4** Find the foci.

$$c^2 = 25 - 9 \text{ or } 16, \text{ so } c = 4.$$

The foci are at  $(-5, 6)$  and  $(-5, -2)$ .

**Step 5** Graph the ellipse.

Draw the ellipse that passes through the vertices and co-vertices.



## Example 4

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

24.  $\frac{(x-3)^2}{36} + \frac{(y-2)^2}{128} = 1$

26.  $\frac{x^2}{27} + \frac{(y-5)^2}{64} = 1$

28.  $3x^2 + y^2 - 6x - 8y - 5 = 0$

30.  $7x^2 + y^2 - 56x + 6y + 93 = 0$

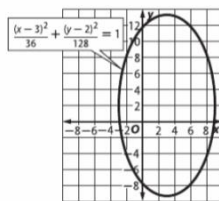
25.  $\frac{(x+6)^2}{50} + \frac{(y-3)^2}{72} = 1$

27.  $\frac{(x+4)^2}{16} + \frac{y^2}{75} = 1$

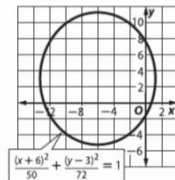
29.  $3x^2 + 4y^2 - 18x + 24y + 3 = 0$

31.  $3x^2 + 2y^2 + 12x - 20y + 14 = 0$

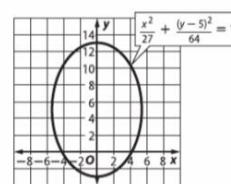
24. المركز (3, 2) البؤرتان (3, 11.59) و (3, -7.59). المحور الأكبر: ≈22.63. المحور الأصغر: 12



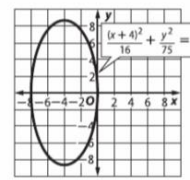
25. المركز (-6, 3) البؤرتان (-6, 7.69) و (-6, -1.69). المحور الأكبر: ≈16.97. المحور الأصغر: ≈14.14



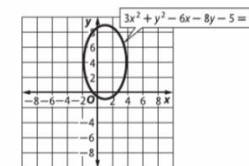
26. المركز (0, 5) البؤرتان (0, 11.08) و (0, -1.08). المحور الأكبر: 16. المحور الأصغر: ≈10.39



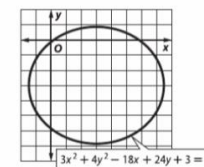
27. المركز (-4, 0) البؤرتان (-4, 7.68) و (-4, -7.68). المحور الأكبر: ≈17.32. المحور الأصغر: 8



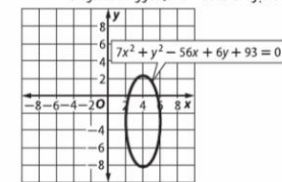
28. المركز (1, 4) البؤرتان (1, 8) و (1, 0). المحور الأكبر: ≈9.80. المحور الأصغر: ≈5.66



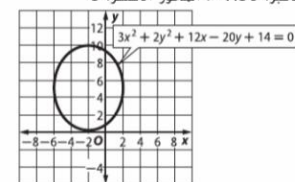
29. المركز (3, -3) البؤرتان (5.24, -3) و (0.76, -3). المحور الأكبر: ≈8.94. المحور الأصغر: ≈7.75



30. المركز (4, -3) البؤرتان (4, 1.90) و (4, -7.90). المحور الأكبر: ≈10.58. المحور الأصغر: 4



31. المركز (-2, 5) البؤرتان (-2, 7.83) و (-2, 2.17). المحور الأكبر: ≈9.80. المحور الأصغر: 8



## Guided Practice

4. Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation  $x^2 + 4y^2 - 2x + 24y + 21 = 0$ . Then graph the ellipse.