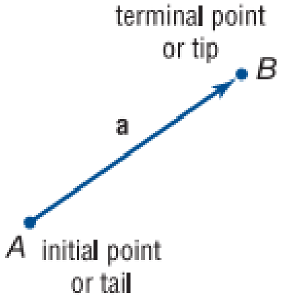
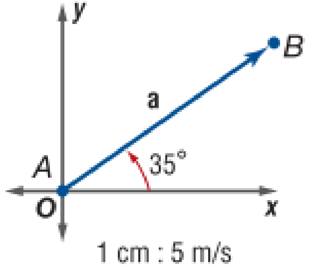
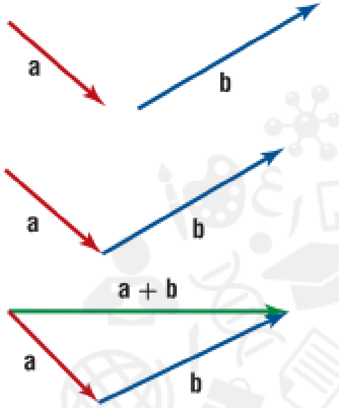
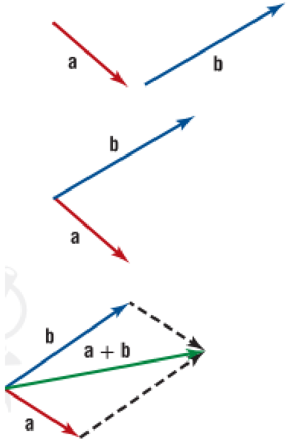
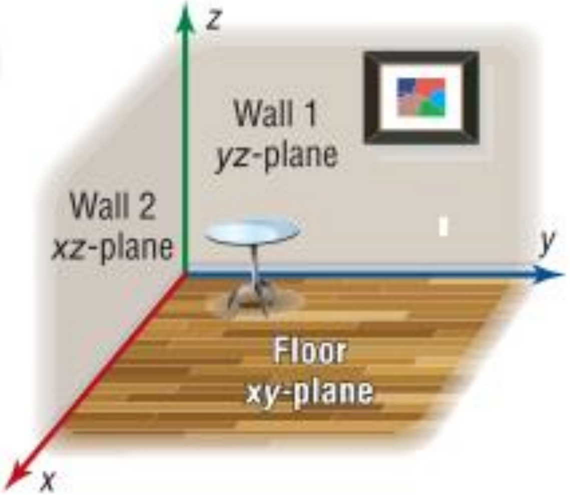
EO Term 2 2023-2024 11ADV







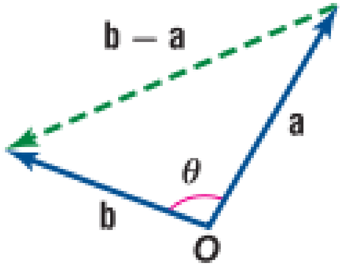


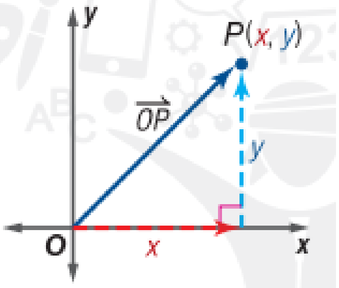


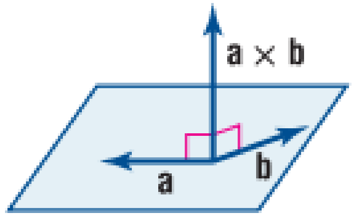
Chapter 7 Vectors(12-13-14-15-20)

Teacher :Atika Alalawneh

Al Samha School







12	Multiple Solve vector problems and resolve vectors into their rectangular components	Exercises (38-44)	P417

Draw a diagram that shows the resolution of each vector into its rectangular components. Then find the magnitudes of the vector's horizontal and vertical components. (Example 6)

38. $2\frac{1}{8}$ centimeters at 310° to the horizontal

39. 1.5 centimeters at a bearing of N49°E

40. 3.2 centimeters per hour at a bearing of S78°W

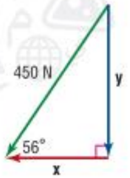
41. $\frac{3}{4}$ centimeters per minute at a bearing of 255°

Real-World Example 6 Resolve a Force into Rectangular Components

LAWN CARE Eiman is pushing the handle of a lawn mower with a force of 450 newtons at an angle of 56° with the ground.

a. Draw a diagram that shows the resolution of the force that Eiman exerts into its rectangular components.

Eiman's push can be resolved into a horizontal push **x** forward and a vertical push **y** downward as shown.



b. Find the magnitudes of the horizontal and vertical components of the force.

The horizontal and vertical components of the force form a right triangle. Use the sine or cosine ratios to find the magnitude of each force.

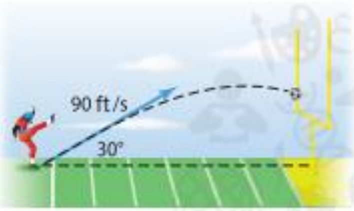
$\cos 56° = \frac{\lvert x \rvert}{450}$	Right triangle definitions of cosine and sine	$\sin 56° = \frac{\lvert y \rvert}{450}$
$\lvert x \rvert = 450 \cos 56°$	Solve for x and y.	$\lvert y \rvert = 450 \sin 56°$
$\lvert x \rvert \approx 252$	Use a calculator.	$\lvert y \rvert \approx 373$

The magnitude of the horizontal component is about 252 newtons, and the magnitude of the vertical component is about 373 newtons.

GuidedPractice

6. **SOCCER** A player kicks a football so that it leaves the ground with a velocity of 44 feet per second at an angle of 33° with the ground.

42. **SOCCER** For a field goal attempt, a ball is kicked with the velocity shown in the diagram below



a. Draw a diagram that shows the resolution of this force into its rectangular components.

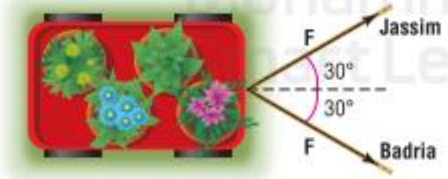
b. Find the magnitudes of the horizontal and vertical components. (Example 6)

43. **CLEANING** Buthaina is pushing the handle of a push broom with a force of 190 newtons at an angle of 33° with the ground. (Example 6)

a. Draw a diagram that shows the resolution of this force into its rectangular components.

b. Find the magnitudes of the horizontal and vertical components.

44. **GARDENING** Jassim and his sister Badria are pulling a wagon full of plants. Each person pulls on the wagon with equal force at an angle of 30° with the axis of the wagon. The resultant force is 120 newtons.



a. How much force is each person exerting?

b. If each person exerts a force of 75 newtons, what is the resultant force?

c. How will the resultant force be affected if Jassim and Badria move closer together?

13	Multiple	Represent and operate with vectors in the coordinate plane	Exercises (1-10)	P425

Example 1 Express a Vector in Component Form

Find the component form of $\overrightarrow{AB}$ with initial point $A(-4, 2)$ and terminal point $B(3, -5)$.

$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$ — Component form

$= \langle 3 - (-4), -5 - 2 \rangle$ — $(x_1, y_1) = (-4, 2)$ and $(x_2, y_2) = (3, -5)$

$= \langle 7, -7 \rangle$ — Subtract.

GuidedPractice

Find the component form of $\overrightarrow{AB}$ with the given initial and terminal points.

1A. $A(-2, -7), B(6, 1)$

1B. $A(0, 8), B(-9, -3)$

Example 2 Find the Magnitude of a Vector

Find the magnitude of $\overrightarrow{AB}$ with initial point $A(-4, 2)$ and terminal point $B(3, -5)$.

$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ — Distance Formula

$= \sqrt{[3 - (-4)]^2 + (-5 - 2)^2}$ — $(x_1, y_1) = (-4, 2)$ and $(x_2, y_2) = (3, -5)$

$= \sqrt{98}$ or about 9.9 — Simplify.

CHECK From Example 1, you know that $\overrightarrow{AB} = \langle 7, -7 \rangle$. $|\overrightarrow{AB}| = \sqrt{7^2 + (-7)^2}$ or $\sqrt{98}$. ✓

GuidedPractice

Find the magnitude of $\overrightarrow{AB}$ with the given initial and terminal points.

2A. $A(-2, -7), B(6, 1)$

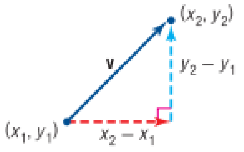
2B. $A(0, 8), B(-9, -3)$

KeyConcept Magnitude of a Vector in the Coordinate Plane

If **v** is a vector with initial point (x_1, y_1) and terminal point (x_2, y_2), then the magnitude of **v** is given by

$|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

If **v** has a component form of $\langle a, b \rangle$, then $|\mathbf{v}| = \sqrt{a^2 + b^2}$.



Find the component form and magnitude of $\overrightarrow{AB}$ with the given initial and terminal points. (Examples 1 and 2)

1. $A(-3, 1), B(4, 5)$

2. $A(2, -7), B(-6, 9)$

3. $A(10, -2), B(3, -5)$

4. $A(-2, 7), B(-9, -1)$

5. $A(-5, -4), B(8, -2)$

6. $A(-2, 6), B(1, 10)$

7. $A(2.5, -3), B(-4, 1.5)$

8. $A(-4.3, 1.8), B(9.4, -6.2)$

9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$

10. $A\left(\frac{3}{5}, -\frac{2}{5}\right), B(-1, 7)$

14	Multiple	Write a vector as a linear combination of unit vectors	Exercises (38-43)	P425

Example 6 Find Component Form

Find the component form of the vector v with magnitude 10 and direction angle 120°.

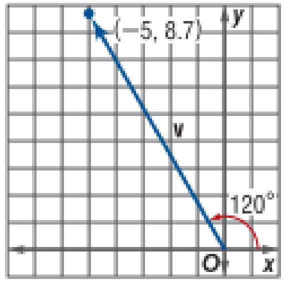
$\mathbf{v} = \langle |\mathbf{v}| \cos\theta, |\mathbf{v}| \sin\theta \rangle$ — Component form of v in terms of |v| and θ

$= \langle 10 \cos 120°, 10 \sin 120° \rangle$ — $|\mathbf{v}| = 10$ and $\theta = 120°$

$= \left\langle 10\left(-\frac{1}{2}\right), 10\left(\frac{\sqrt{3}}{2}\right)\right\rangle$ — $\cos 120° = -\frac{1}{2}$ and $\sin 120° = \frac{\sqrt{3}}{2}$

$= \langle -5, 5\sqrt{3} \rangle$ — Simplify.

CHECK Graph $\mathbf{v} = \langle -5, 5\sqrt{3} \rangle \approx \langle -5, 8.7 \rangle$. The measure of the angle v makes with the positive x-axis is about 120° as shown, and $|\mathbf{v}| = \sqrt{(-5)^2 + (5\sqrt{3})^2}$ or 10. ✔



GuidedPractice

Find the component form of v with the given magnitude and direction angle.

6A. $|\mathbf{v}| = 8, \theta = 45°$

6B. $|\mathbf{v}| = 24, \theta = 210°$

Find the component form of v with the given magnitude and direction angle. (Example 6)

38. $|\mathbf{v}| = 12, \theta = 60°$

39. $|\mathbf{v}| = 4, \theta = 135°$

40. $|\mathbf{v}| = 6, \theta = 240°$

41. $|\mathbf{v}| = 16, \theta = 330°$

42. $|\mathbf{v}| = 28, \theta = 273°$

43. $|\mathbf{v}| = 15, \theta = 125°$

15	Multiple	Express vectors algebraically and operate with vectors in space	Exercises (36-47)	P442

Example 5 Operations with Vectors in Space

Find each of the following for $\mathbf{y} = \langle 3, -6, 2 \rangle$, $\mathbf{w} = \langle -1, 4, -4 \rangle$, and $\mathbf{z} = \langle -2, 0, 5 \rangle$.

a. $4\mathbf{y} + 2\mathbf{z}$

$4\mathbf{y} + 2\mathbf{z} = 4\langle 3, -6, 2 \rangle + 2\langle -2, 0, 5 \rangle$ — Substitute.

$= \langle 12, -24, 8 \rangle + \langle -4, 0, 10 \rangle$ or $\langle 8, -24, 18 \rangle$ — Scalar multiplication and vector addition

b. $2\mathbf{w} - \mathbf{z} + 3\mathbf{y}$

$2\mathbf{w} - \mathbf{z} + 3\mathbf{y} = 2\langle -1, 4, -4 \rangle - \langle -2, 0, 5 \rangle + 3\langle 3, -6, 2 \rangle$ — Substitute.

$= \langle -2, 8, -8 \rangle + \langle 2, 0, -5 \rangle + \langle 9, -18, 6 \rangle$ — Scalar multiplication

$= \langle 9, -10, -7 \rangle$ — Vector addition

GuidedPractice

5A. $4\mathbf{w} - 8\mathbf{z}$

5B. $3\mathbf{y} + 3\mathbf{z} - 6\mathbf{w}$

KeyConcept Vector Operations in Space

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, and any scalar k, then

Vector Addition: $\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

Vector Subtraction: $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b}) = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$

Scalar Multiplication: $k\mathbf{a} = \langle ka_1, ka_2, ka_3 \rangle$

Find each of the following for $\mathbf{a} = \langle -5, -4, 3 \rangle$, $\mathbf{b} = \langle 6, -2, -7 \rangle$, and $\mathbf{c} = \langle -2, 2, 4 \rangle$. (Example 5)

36. $6\mathbf{a} - 7\mathbf{b} + 8\mathbf{c}$

37. $7\mathbf{a} - 5\mathbf{b}$

38. $2\mathbf{a} + 5\mathbf{b} - 9\mathbf{c}$

39. $6\mathbf{b} + 4\mathbf{c} - 4\mathbf{a}$

40. $8\mathbf{a} - 5\mathbf{b} - \mathbf{c}$

41. $-6\mathbf{a} + \mathbf{b} + 7\mathbf{c}$

Find each of the following for $\mathbf{x} = -9\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, $\mathbf{y} = 6\mathbf{i} - 2\mathbf{j} - 7\mathbf{k}$, and $\mathbf{z} = -2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$. (Example 5)

42. $7\mathbf{x} + 6\mathbf{y}$

43. $3\mathbf{x} - 5\mathbf{y} + 3\mathbf{z}$

44. $4\mathbf{x} + 3\mathbf{y} + 2\mathbf{z}$

45. $-8\mathbf{x} - 2\mathbf{y} + 5\mathbf{z}$

46. $-6\mathbf{y} - 9\mathbf{z}$

47. $-\mathbf{x} - 4\mathbf{y} - \mathbf{z}$

20	Multiple Find the dot product of two vectors and use the dot product to find the angle between them	Exercises (16-23)	P434

Example 3 Find the Cross Product of Two Vectors

Find the cross product of $\mathbf{u} = \langle 3, -2, 1\rangle$ and $\mathbf{v} = \langle -3, 3, 1\rangle$. Then show that $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ -3 & 3 & 1 \end{vmatrix}$$ $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

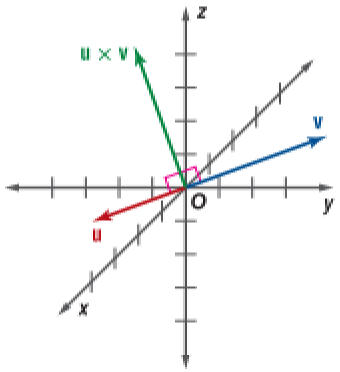
$$= \begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 3 & 1 \\ -3 & 1 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 3 & -2 \\ -3 & 3 \end{vmatrix}\mathbf{k}$$ Determinant of a 3 × 3 matrix

$$= (-2 - 3)\mathbf{i} - [3 - (-3)]\mathbf{j} + (9 - 6)\mathbf{k}$$ Determinants of 2 × 2 matrices

$$= -5\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$$ Simplify.

$$= \langle -5, -6, 3\rangle$$ Component form

In the graph of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u} \times \mathbf{v}$, $\mathbf{u} \times \mathbf{v}$ is orthogonal to $\mathbf{u}$ and $\mathbf{u} \times \mathbf{v}$ is orthogonal to $\mathbf{v}$.



To show that $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$, find the dot product of $\mathbf{u} \times \mathbf{v}$ with $\mathbf{u}$ and $\mathbf{u} \times \mathbf{v}$ with $\mathbf{v}$.

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u}$
$= \langle -5, -6, 3\rangle \cdot \langle 3, -2, 1\rangle$
$= -5(3) + (-6)(-2) + 3(1)$
$= -15 + 12 + 3$ or 0 ✓

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$
$= \langle -5, -6, 3\rangle \cdot \langle -3, 3, 1\rangle$
$= -5(-3) + (-6)(3) + 3(1)$
$= 15 + (-18) + 3$ or 0 ✓

Because both dot products are zero, the vectors are orthogonal.

GuidedPractice

Find the cross product of $\mathbf{u}$ and $\mathbf{v}$. Then show that $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

3A. $\mathbf{u} = \langle 4, 2, -1\rangle$, $\mathbf{v} = \langle 5, 1, 4\rangle$

3B. $\mathbf{u} = \langle -2, -1, -3\rangle$, $\mathbf{v} = \langle 5, 1, 4\rangle$

Find the cross product of $\mathbf{u}$ and $\mathbf{v}$. Then show that $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$. (Example 3)

16. $\mathbf{u} = \langle -1, 3, 5\rangle$, $\mathbf{v} = \langle 2, -6, -3\rangle$

17. $\mathbf{u} = \langle 4, 7, -2\rangle$, $\mathbf{v} = \langle -5, 9, 1\rangle$

18. $\mathbf{u} = \langle 3, -6, 2\rangle$, $\mathbf{v} = \langle 1, 5, -8\rangle$

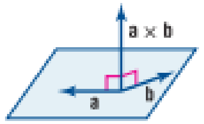
19. $\mathbf{u} = \langle 5, -8, 0\rangle$, $\mathbf{v} = \langle -4, -2, 7\rangle$

20. $\mathbf{u} = -2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$, $\mathbf{v} = 7\mathbf{i} + \mathbf{j} - 6\mathbf{k}$

21. $\mathbf{u} = -4\mathbf{i} + \mathbf{j} + 8\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$

2 Cross Products

Another important product involving vectors in space is the cross product. Unlike the dot product, the **cross product** of two vectors $\mathbf{a}$ and $\mathbf{b}$ in space, denoted $\mathbf{a} \times \mathbf{b}$ and read *a cross b*, is a vector, not a scalar. The vector $\mathbf{a} \times \mathbf{b}$ is perpendicular to the plane containing vectors $\mathbf{a}$ and $\mathbf{b}$.



KeyConcept Cross Product of Vectors in Space

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, the cross product of $\mathbf{a}$ and $\mathbf{b}$ is the vector

$$\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} - (a_1b_3 - a_3b_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}.$$

If we apply the formula for calculating the determinant of a 3 × 3 matrix to the following *determinant form* involving $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, and the components of $\mathbf{a}$ and $\mathbf{b}$, we arrive at the same formula for $\mathbf{a} \times \mathbf{b}$.

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Put the unit vectors i, j, and k in Row 1.
Put the components of a in Row 2.
Put the components of b in Row 3.

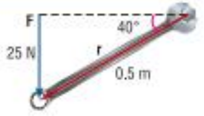
$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\mathbf{k}$$ Apply the formula for a 3 × 3 determinant.

$$= (a_2b_3 - a_3b_2)\mathbf{i} - (a_1b_3 - a_3b_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$$ Compute each 2 × 2 determinant.

20	Writing Find the dot product of two vectors and use the dot product to find the angle between them	Exercises (16-23)	P434

Real-World Example 4 **Torque Using Cross Product**

AUTO REPAIR **Tarek uses a lug wrench to tighten a lug nut. The wrench he uses is 50 centimeters or 0.5 meter long. Find the magnitude and direction of the torque about the lug nut if he applies a force of 25 newtons straight down to the end of the handle when it is 40° below the positive *x*-axis as shown.**



Step 1 Graph each vector in standard position (Figure 8.5.1).

Step 2 Determine the component form of each vector.

The component form of the vector representing the directed distance from the axis of rotation to the end of the handle can be found using the triangle in Figure 8.5.2 and trigonometry. Vector **r** is therefore $\langle 0.5 \cos 40°, 0, -0.5 \sin 40° \rangle$ or about $\langle 0.38, 0, -0.32 \rangle$. The vector representing the force applied to the end of the handle is 25 newtons straight down, so $\mathbf{F} = \langle 0, 0, -25 \rangle$.

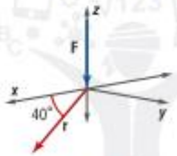


Figure 8.5.1

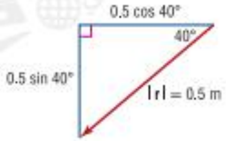


Figure 8.5.2

Step 3 Use the cross product of these vectors to find the vector representing the torque about the lug nut.

$\mathbf{T} = \mathbf{r} \times \mathbf{F}$ — Torque Cross Product Formula

$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.38 & 0 & -0.32 \\ 0 & 0 & -25 \end{vmatrix}$ — Cross product of r and F

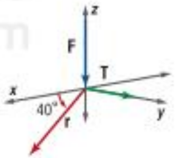
$= \begin{vmatrix} 0 & -0.32 \\ 0 & -25 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 0.38 & -0.32 \\ 0 & -25 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 0.38 & 0 \\ 0 & 0 \end{vmatrix}\mathbf{k}$ — Determinant of a 3 × 3 matrix

$= 0\mathbf{i} - (-9.5)\mathbf{j} + 0\mathbf{k}$ — Determinants of 2 × 2 matrices

$= \langle 0, 9.5, 0 \rangle$ — Component form

Step 4 Find the magnitude and direction of the torque vector.

The component form of the torque vector $\langle 0, 9.5, 0 \rangle$ tells us that the magnitude of the vector is about 9.5 newton-meters parallel to the positive *y*-axis as shown.



Guided Practice

4. **AUTO REPAIR** Find the magnitude of the torque if Tarek applied the same amount of force to the end of the handle straight down when the handle makes a 40° angle above the positive *x*-axis as shown in Figure 8.5.3.

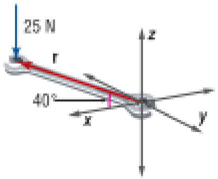
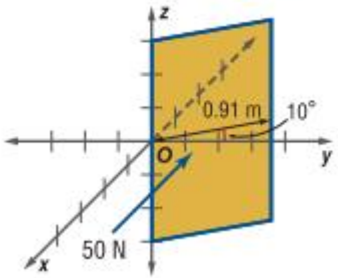
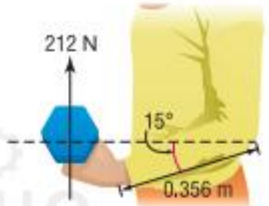


Figure 8.5.3

22. **RESTAURANTS** A restaurant server applies 50 newtons of force to open a door. Find the magnitude and direction of the torque about the door's hinge. (Example 4)



23. **WEIGHTLIFTING** A weightlifter doing bicep curls applies 212 newtons of force to lift the dumbbell. The weightlifter's forearm is 0.356 meters long and he begins the bicep curl with his elbow bent at a 15° angle below the horizontal in the direction of the positive *x*-axis. (Example 4)



a. Find the vector representing the torque about the weightlifter's elbow in component form.

b. Find the magnitude and direction of the torque.