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G12 Adv Term 2 (2023-24)
End of Term (EoT) Questions

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G12 Adv Term 2
Part 1: MCQ & Part 2 FRQ
EoT2 2023-24

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Number of MCQ عدد الأسئلة الموضوعية	15
Marks of MCQ درجة الأسئلة الموضوعية	4
Number of FRQ عدد الأسئلة المقالية	5
Marks per FRQ الدرجات للأسئلة المقالية	(5-10)
Type of All Questions نوع كافة الأسئلة	MCQ/ الأسئلة الموضوعية
	FRQ/ الأسئلة المقالية
Maximum Overall Grade الدرجة القصوى الممكنة	100
Exam Duration - مدة الامتحان	150 minutes
Mode of Implementation -طريقة التطبيق	SwiftAssess & Paper-Based
Calculator	Allowed
الآلة الحاسبة	مسموحة

Question* السؤال*	Learning Outcome/Performance Criteria** ناتج التعلم/ معايير الأداء**	Reference(s) in the Student Book (English Version) المرجع في كتاب الطالب (النسخة الانجليزية)	
		Example/Exercise مثال/تمرين	Page الصفحة
1	Find the critical points of a given function إيجاد الأعداد الحرجة لدالة معطاة	(3-6)	258
2	Find the absolute extrema of a given function إيجاد القيم القصوى المطلقة لدالة معطاة	(25-34)	258
3	Identify increasing and decreasing functions التعرف على مفهومي الدالة المتناقصة والدالة المتزايدة	(1-10)	267
4	Find the local extrema of a given function using the First Derivative test إيجاد القيم القصوى المحلية لدالة معينة باستخدام اختبار المشتقة الأولى	(11-20)	267
5	Learn the notion of an Inflection Point and find one التعريف على مفهوم نقطة الانعطاف وإيجادها	(1-8)	276
6	Determine the concavity of a function using the first and second derivatives تحديد فترات التقعر إلى أعلى وإلى أسفل لدالة معينة باستخدام المشتقتين الأولى والثانية	(45,46)	276
7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

Question* السؤال*	Learning Outcome/Performance Criteria** ناتج التعلم/ معايير الأداء**	Reference(s) in the Student Book (English Version) المرجع في كتاب الطالب (النسخة الانجليزية)	
		Example/Exercise مثال/تمرين	Page الصفحة
8	Solve mathematical and real-life problems on related rates حل مسائل اقتصادية وعلمية على القيم القصوى	Example 9.7 (33-36)	312 314
9	Find the antiderivative of a given function إيجاد عكس المشتقة لدالة معطاة	(5-28)	329
10	Understand the notion of indefinite integral as finding an antiderivative التعرف على مفهوم التكامل غير المحدود بصفته عكس المشتقة	(45-48)	330
11	Use the sigma notation to compute basic summation استخدام رمز المجموع سيجما لإيجاد المجاميع البسيطة	(5-18)	337
12	Estimate the area under a curve on a given interval using rectangles تقدير المساحة تحت المنحنى لدالة في فترة محددة باستخدام المستطيلات	(35-38)	345
13	التعرف على خصائص التكامل المحدود Learn the properties of definite integrals	(23,24)+(35-38)	356
14	Apply the Integral Mean Value Theorem تطبيق نظرية القيمة المتوسطة في التكامل	(25-28)+(33,34)	356
15	Learn the Fundamental Theorem of Calculus (Part I) and use it to compute various definite integrals التعرف على النظرية الأساسية الأولى للتفاضل والتكامل وتطبيقها على دوال متنوعة لإيجاد تكاملات محدود	(1-18)	366

Question* السؤال*		Learning Outcome/Performance Criteria** ناتج التعلم/ معايير الأداء**	Reference(s) in the Student Book (English Version) المرجع في كتاب الطالب (النسخة الانجليزية)	
			Example/Exercise مثال/تمرين	Page الصفحة
FRQ - الأسئلة المقالية				
	16	Solve mathematical and real-life optimization problems حل مسائل رياضية وحياتية على القيم القصوى لإيجاد القيم المثلى	(1-7) (8,9)	296 297
	17	Solve mathematical and real-life problems on related rates حل مسائل رياضية وحياتية على المعدلات المرتبطة	(1-13)	303
	18	Solve mathematical and real-life problems on related rates حل مسائل اقتصادية وعلمية على القيم القصوى	Example 9.8 37 38	312 314 315
	19	Compute the area under a curve using summations and limits إيجاد المساحة تحت المنحنى لدالة باستخدام المجاميع والنهايات	Example 3.2 (11-14)	341 344
	20	Learn the Fundamental Theorem of Calculus (Part II) and use it to compute derivatives of functions defined as definite integrals التعرف على النظرية الأساسية الثانية للتفاضل والتكامل وتطبيقها على دوال معرفة كتكاملات محدودة لإيجاد مشتقاتها	(25-32)	366

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Part 1: MCQ

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Question 1

Find the critical points of a given function

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Exercise 19 - 30

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In exercises 3–6, find all critical numbers by hand. Use your knowledge of the type of graph (e.g., parabola or cubic) to determine whether the critical number represents a local maximum, local minimum or neither.

3. (a) $f(x) = x^2 + 5x - 1$

(b) $f(x) = -x^2 + 4x + 2$

In exercises 3–6, find all critical numbers by hand. Use your knowledge of the type of graph (e.g., parabola or cubic) to determine whether the critical number represents a local maximum, local minimum or neither.

4. (a) $f(x) = x^3 - 3x + 1$

(b) $f(x) = -x^3 + 6x^2 + 2$

In exercises 3–6, find all critical numbers by hand. Use your knowledge of the type of graph (e.g., parabola or cubic) to determine whether the critical number represents a local maximum, local minimum or neither.

5. (a) $f(x) = x^3 - 3x^2 + 6x$

(b) $f(x) = -x^3 + 3x^2 - 3x$

In exercises 3–6, find all critical numbers by hand. Use your knowledge of the type of graph (e.g., parabola or cubic) to determine whether the critical number represents a local maximum, local minimum or neither.

6. (a) $f(x) = x^4 - 2x^2 + 1$

(b) $f(x) = x^4 - 3x^3 + 2$

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Question 2

Find the absolute extrema of a given function

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Exercise 19 - 30

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In exercises 25–34, find the absolute extrema of the given function on each indicated interval.

25. $f(x) = x^3 - 3x + 1$ on (a) $[0, 2]$ and (b) $[-3, 2]$

In exercises 25–34, find the absolute extrema of the given function on each indicated interval.

26. $f(x) = x^4 - 8x^2 + 2$ on (a) $[-3, 1]$ and (b) $[-1, 3]$

In exercises 25–34, find the absolute extrema of the given function on each indicated interval.

27. $f(x) = x^{2/3}$ on (a) $[-4, -2]$ and (b) $[-1, 3]$

In exercises 25–34, find the absolute extrema of the given function on each indicated interval.

28. $f(x) = \sin x + \cos x$ on (a) $[0, 2\pi]$ and (b) $[\pi/2, \pi]$

In exercises 25–34, find the absolute extrema of the given function on each indicated interval.

29. $f(x) = e^{-x^2}$ on (a) $[0, 2]$ and (b) $[-3, 2]$

In exercises 25–34, find the absolute extrema of the given function on each indicated interval.

30. $f(x) = x^2 e^{-4x}$ on (a) $[-2, 0]$ and (b) $[0, 4]$

In exercises 25–34, find the absolute extrema of the given function on each indicated interval.

31. $f(x) = \dfrac{3x^2}{x-3}$ on (a) $[-2, 2]$ and (b) $[2, 8]$

In exercises 25–34, find the absolute extrema of the given function on each indicated interval.

32. $f(x) = \tan^{-1}(x^2)$ on (a) $[0, 1]$ and (b) $[-3, 4]$

In exercises 25–34, find the absolute extrema of the given function on each indicated interval.

33. $f(x) = \dfrac{x}{x^2 + 1}$ on (a) $[0, 2]$ and (b) $[-3, 3]$

In exercises 25–34, find the absolute extrema of the given function on each indicated interval.

34. $f(x) = \dfrac{3x}{x^2 + 16}$ on (a) $[0, 2]$ and (b) $[0, 6]$

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Question 3

Identify increasing and decreasing functions

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Exercise 11 - 18

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In exercises 1–10, find (by hand) the intervals where the function is increasing and decreasing. Use this information to determine all local extrema and sketch a graph.

1. $y = x^3 - 3x + 2$

In exercises 1–10, find (by hand) the intervals where the function is increasing and decreasing. Use this information to determine all local extrema and sketch a graph.

2. $y = x^3 + 2x^2 + 1$

In exercises 1–10, find (by hand) the intervals where the function is increasing and decreasing. Use this information to determine all local extrema and sketch a graph.

3. $y = x^4 - 8x^2 + 1$

In exercises 1–10, find (by hand) the intervals where the function is increasing and decreasing. Use this information to determine all local extrema and sketch a graph.

4. $y = x^3 - 3x^2 - 9x + 1$

In exercises 1–10, find (by hand) the intervals where the function is increasing and decreasing. Use this information to determine all local extrema and sketch a graph.

5. $y = (x+1)^{2/3}$

In exercises 1–10, find (by hand) the intervals where the function is increasing and decreasing. Use this information to determine all local extrema and sketch a graph.

6. $y = (x-1)^{1/3}$

In exercises 1–10, find (by hand) the intervals where the function is increasing and decreasing. Use this information to determine all local extrema and sketch a graph.

7. $y = \sin x + \cos x$

In exercises 1–10, find (by hand) the intervals where the function is increasing and decreasing. Use this information to determine all local extrema and sketch a graph.

8. $y = \sin^2 x$

In exercises 1–10, find (by hand) the intervals where the function is increasing and decreasing. Use this information to determine all local extrema and sketch a graph.

9. $y = e^{x^2-1}$

In exercises 1–10, find (by hand) the intervals where the function is increasing and decreasing. Use this information to determine all local extrema and sketch a graph.

10. $y = \ln(x^2 - 1)$

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Question 4

Find the local extrema of a given function using the First Derivative test

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Exercise 18 - 35

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In exercises 11–20, find (by hand) all critical numbers and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

11. $y = x^4 + 4x^3 - 2$

In exercises 11–20, find (by hand) all critical numbers and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

12. $y = x^5 - 5x^2 + 1$

In exercises 11–20, find (by hand) all critical numbers and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

13. $y = xe^{-2x}$

In exercises 11–20, find (by hand) all critical numbers and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

14. $y = x^2 e^{-x}$

In exercises 11–20, find (by hand) all critical numbers and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

15. $y = \tan^{-1}(x^2)$

In exercises 11–20, find (by hand) all critical numbers and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

16. $y = \sin^{-1}\left(1 - \frac{1}{x^2}\right)$

In exercises 11–20, find (by hand) all critical numbers and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

17. $y = \dfrac{x}{1+x^3}$

In exercises 11–20, find (by hand) all critical numbers and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

18. $y = \dfrac{x}{1 + x^4}$

In exercises 11–20, find (by hand) all critical numbers and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

19. $y = \sqrt{x^3 + 3x^2}$

In exercises 11–20, find (by hand) all critical numbers and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

20. $y = x^{4/3} + 4x^{1/3}$

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Question 5

Learn the notion of an Inflection Point and find one

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Exercise 18 - 35

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5	Learn the notion of an Inflection Point and find one التعريف على مفهوم نقطة الانعطاف وإيجادها	(1-8)	276

In exercises 1–8, determine the intervals where the graph of the given function is concave up and concave down, and identify inflection points.

1. $f(x) = x^3 - 3x^2 + 4x - 1$

5	Learn the notion of an Inflection Point and find one التعريف على مفهوم نقطة الانعطاف وإيجادها	(1-8)	276

In exercises 1–8, determine the intervals where the graph of the given function is concave up and concave down, and identify inflection points.

2. $f(x) = x^4 - 6x^2 + 2x + 3$

5	Learn the notion of an Inflection Point and find one التعريف على مفهوم نقطة الانعطاف وإيجادها	(1-8)	276

In exercises 1–8, determine the intervals where the graph of the given function is concave up and concave down, and identify inflection points.

3. $f(x) = x + 1/x$

5	Learn the notion of an Inflection Point and find one التعريف على مفهوم نقطة الانعطاف وإيجادها	(1-8)	276

In exercises 1–8, determine the intervals where the graph of the given function is concave up and concave down, and identify inflection points.

4. $f(x) = x + 3(1 - x)^{1/3}$

5	Learn the notion of an Inflection Point and find one التعريف على مفهوم نقطة الانعطاف وإيجادها	(1-8)	276

In exercises 1–8, determine the intervals where the graph of the given function is concave up and concave down, and identify inflection points.

5. $f(x) = \sin x - \cos x$

5	Learn the notion of an Inflection Point and find one التعريف على مفهوم نقطة الانعطاف وإيجادها	(1-8)	276

In exercises 1–8, determine the intervals where the graph of the given function is concave up and concave down, and identify inflection points.

6. $f(x) = \tan^{-1}(x^2)$

5	Learn the notion of an Inflection Point and find one التعريف على مفهوم نقطة الانعطاف وإيجادها	(1-8)	276

In exercises 1–8, determine the intervals where the graph of the given function is concave up and concave down, and identify inflection points.

7. $f(x) = x^{4/3} + 4x^{1/3}$

5	Learn the notion of an Inflection Point and find one التعريف على مفهوم نقطة الانعطاف وايجادها	(1-8)	276

In exercises 1–8, determine the intervals where the graph of the given function is concave up and concave down, and identify inflection points.

8. $f(x) = xe^{-4x}$

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Question 6

Determine the concavity of a function using the first and second derivatives

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Exercise 18 - 35

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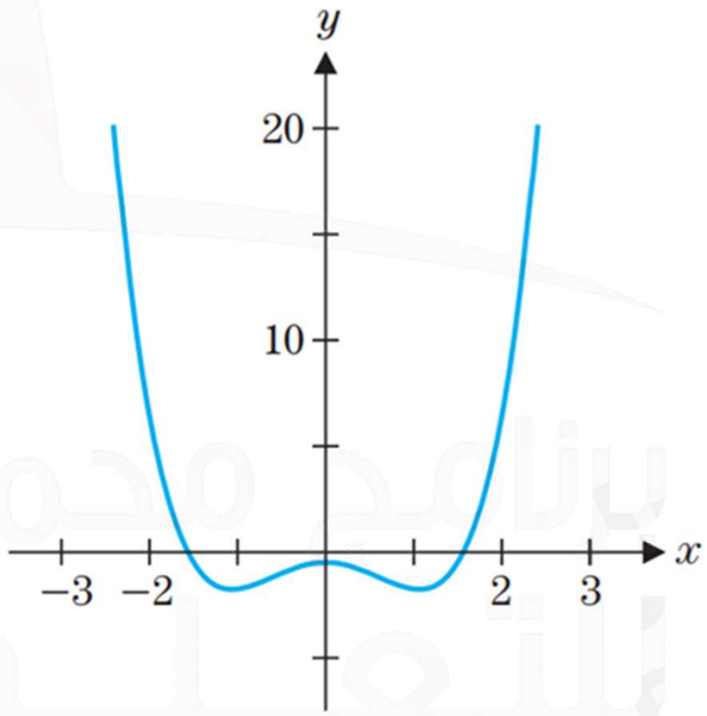
6	Determine the concavity of a function using the first and second derivatives تحديد فترات التقعر إلى أعلى وإلى أسفل لدالة معينة باستخدام المشتقتين الأولى والثانية	(45,46)	276

In exercises 45 and 46, estimate the intervals of increase and decrease, the locations of local extrema, intervals of concavity and locations of inflection points.

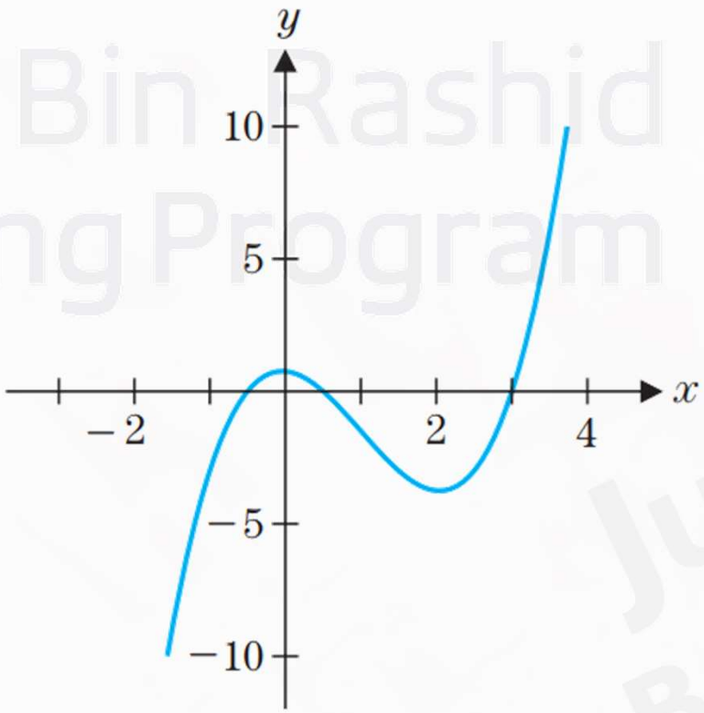
45.



6	Determine the concavity of a function using the first and second derivatives تحديد فترات التقعر إلى أعلى وإلى أسفل لدالة معينة باستخدام المشتقتين الأولى والثانية	(45,46)	276

In exercises 45 and 46, estimate the intervals of increase and decrease, the locations of local extrema, intervals of concavity and locations of inflection points.

46.



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Question 7

Sketch the graph of a given function using its properties and its first and second derivative

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Exercise 1 - 6

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7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

1. $f(x) = x^3 - 3x^2 + 3x$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

2. $f(x) = x^4 - 3x^2 + 2$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

3. $f(x) = x^5 - 2x^3 + 1$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

4. $f(x) = x^4 + 4x^3 - 1$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

5. $f(x) = x + \dfrac{4}{x}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

6. $f(x) = \dfrac{x^2 - 1}{x}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

7. $f(x) = \dfrac{x^2 + 4}{x^3}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

8. $f(x) = \dfrac{x-4}{x^3}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

9. $f(x) = \dfrac{2x}{x^2 - 1}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

10. $f(x) = \dfrac{3x^2}{x^2+1}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

11. $f(x) = x + \sin x$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

12. $f(x) = \sin x - \cos x$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

13. $f(x) = x \ln x$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

14. $f(x) = x \ln x^2$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

15. $f(x) = \sqrt{x^2 + 1}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

16. $f(x) = \sqrt{2x - 1}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

17. $f(x) = \sqrt[3]{x^3 - 3x^2 + 2x}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

18. $f(x) = \sqrt{x^3 - 3x^2 + 2x}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

19. $f(x) = x^{5/3} - 5x^{2/3}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

20. $f(x) = x^3 - \frac{3}{400}x$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

21. $f(x) = e^{-2/x}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

22. $f(x) = e^{1/x^2}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 23–36, determine all significant features (approximately if necessary) and sketch a graph.

23. $f(x) = \dfrac{1}{x^3 - 3x^2 - 9x + 1}$

In exercises 23–36, determine all significant features (approximately if necessary) and sketch a graph.

24. $f(x) = \dfrac{1}{x^3 + 3x^2 + 4x + 1}$

In exercises 23–36, determine all significant features (approximately if necessary) and sketch a graph.

25. $f(x) = (x^3 - 3x^2 + 2x)^{2/3}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 23–36, determine all significant features (approximately if necessary) and sketch a graph.

26. $f(x) = x^6 - 10x^5 - 7x^4 + 80x^3 + 12x^2 - 192x$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 23–36, determine all significant features (approximately if necessary) and sketch a graph.

27. $f(x) = \dfrac{x^2 + 1}{3x^2 - 1}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 23–36, determine all significant features (approximately if necessary) and sketch a graph.

28. $f(x) = \dfrac{5x}{x^3 - x + 1}$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 49–52, find a function whose graph has the given asymptotes.

49. $x = 1,\ x = 2$ and $y = 3$

In exercises 49–52, find a function whose graph has the given asymptotes.

50. $x = -1$, $x = 1$ and $y = 0$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 49–52, find a function whose graph has the given asymptotes.

51. $x = -1,\ x = 1,\ y = -2$ and $y = 2$

7	Sketch the graph of a given function using its properties and its first and second derivative رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها	(1-28) (49-52)	286 287

In exercises 49–52, find a function whose graph has the given asymptotes.

52. $x = 1,\ y = 2$ and $x = 3$

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Question 8

Solve mathematical and real-life problems on related rates

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Exercise 18 - 35

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EXAMPLE 9.7 Modeling Electrical Current in a Wire

The electrical circuit shown in Figure 4.103 includes a 14-ohm resistor, a 2-henry inductor, a 0.05-farad capacitor and a battery supplying 232 volts of AC current modeled by the oscillating function 232 sin $2t$, where t is measured in seconds. Find the current in the circuit at any time t.

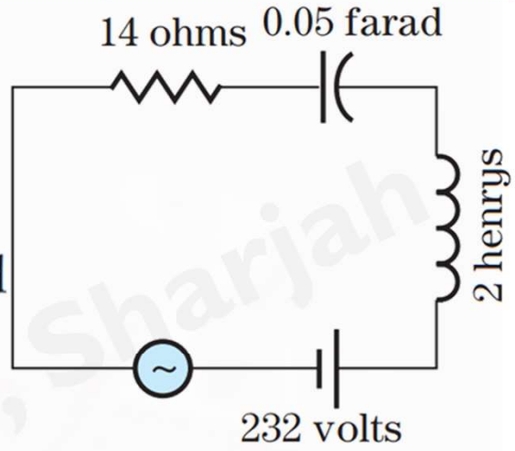

FIGURE 4.103
A simple electrical circuit

Solution It can be shown (using the elementary laws of electricity) that the charge in this circuit is given by

$$Q(t) = 10e^{-5t} + 2te^{-2t} + 3\sin 2t - 7\cos 2t \text{ coulombs.}$$

The current is then

$$Q'(t) = -50e^{-5t} + 2e^{-2t} - 4te^{-2t} + 6\cos 2t + 14\sin 2t \text{ amps (coulombs per second).}$$

33. Suppose that the charge in an electrical circuit is $Q(t) = e^{-2t}(\cos 3t - 2 \sin 3t)$ coulombs. Find the current.

34. Suppose that the charge in an electrical circuit is $Q(t) = e^t(3\cos 2t + \sin 2t)$ coulombs. Find the current.

35. Suppose that the charge at a particular location in an electrical circuit is $Q(t) = e^{-3t}\cos 2t + 4\sin 3t$ coulombs. What happens to this function as $t \to \infty$? Explain why the term $e^{-3t}\cos 2t$ is called a **transient** term and $4\sin 3t$ is known as the **steady-state** or **asymptotic** value of the charge function. Find the transient and steady-state values of the current function.

36. As in exercise 35, find the steady-state and transient values of the current function if the charge function is given by $Q(t) = e^{-2t}(\cos t - 2\sin t) + te^{-3t} + 2\cos 4t$.

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Question 9

Find the antiderivative of a given function

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Exercise 10 - 18

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In exercises 5–28, find the general antiderivative.

5. $\int (3x^4 - 3x)\,dx$

6. $\int (x^3 - 2)\,dx$

In exercises 5–28, find the general antiderivative.

7. $\int \left(3\sqrt{x} - \frac{1}{x^4}\right) dx$

8. $\int \left(2x^{-2} + \frac{1}{\sqrt{x}}\right) dx$

In exercises 5–28, find the general antiderivative.

9. $\int \frac{x^{1/3} - 3}{x^{2/3}}\, dx$

10. $\int \frac{x + 2x^{3/4}}{x^{5/4}}\, dx$

In exercises 5–28, find the general antiderivative.

11. $\int (2\sin x + \cos x)\,dx$

12. $\int (3\cos x - \sin x)\,dx$

In exercises 5–28, find the general antiderivative.

13. $\int 2 \sec x \tan x \, dx$

14. $\int \frac{4}{\sqrt{1 - x^2}} \, dx$

In exercises 5–28, find the general antiderivative.

15. $\int 5 \sec^2 x \, dx$

16. $\int 4 \dfrac{\cos x}{\sin^2 x} \, dx$

In exercises 5–28, find the general antiderivative.

17. $\int (3e^x - 2)\,dx$

18. $\int (4x - 2e^x)\,dx$

In exercises 5–28, find the general antiderivative.

19. $\int (3\cos x - 1/x)\,dx$

20. $\int (2x^{-1} + \sin x)\,dx$

In exercises 5–28, find the general antiderivative.

21. $\int \frac{4x}{x^2+4}\,dx$

22. $\int \frac{3}{4x^2+4}\,dx$

In exercises 5–28, find the general antiderivative.

23. $\int \frac{\cos x}{\sin x}\, dx$

24. $\int \left(2\cos\ x - \sqrt{e^{2x}}\right) dx$

In exercises 5–28, find the general antiderivative.

25. $\int \frac{e^x}{e^x + 3}\, dx$

26. $\int \frac{e^x + 3}{e^x}\, dx$

In exercises 5–28, find the general antiderivative.

27. $\int x^{1/4}(x^{5/4} - 4)\,dx$

28. $\int x^{2/3}(x^{-4/3} - 3)\,dx$

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Question 10

Understand the notion of indefinite integral as finding an antiderivative

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Exercise 18 - 35

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10	Understand the notion of indefinite integral as finding an antiderivative التعرف على مفهوم التكامل غير المحدود بصفته عكس المشتقة	(45-48)	330

45. Determine the position function if the velocity function is $v(t) = 3 - 12t$ and the initial position is $s(0) = 3$.

10	Understand the notion of indefinite integral as finding an antiderivative التعرف على مفهوم التكامل غير المحدود بصفته عكس المشتقة	(45-48)	330

46. Determine the position function if the velocity function is $v(t) = 3e^{-t} - 2$ and the initial position is $s(0) = 0$.

47. Determine the position function if the acceleration function is $a(t) = 3 \sin t + 1$, the initial velocity is $v(0) = 0$ and the initial position is $s(0) = 4$.

10	Understand the notion of indefinite integral as finding an antiderivative التعرف على مفهوم التكامل غير المحدود بصفته عكس المشتقة	(45-48)	330

48. Determine the position function if the acceleration function is $a(t) = t^2 + 1$, the initial velocity is $v(0) = 4$ and the initial position is $s(0) = 0$.

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Question 11

Use the sigma notation to compute basic summation

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Exercise 1 - 6

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In exercises 5–8, write out all terms and compute the sums.

5. $\sum_{i=1}^{6} 3i^2$

6. $\sum_{i=3}^{7} (i^2 + i)$

In exercises 5–8, write out all terms and compute the sums.

7. $\sum_{i=6}^{10}(4i+2)$

8. $\sum_{i=6}^{8}(i^2+2)$

In exercises 9–18, use summation rules to compute the sum.

9. $\sum_{i=1}^{70}(3i-1)$

10. $\sum_{i=1}^{45}(3i-4)$

In exercises 9–18, use summation rules to compute the sum.

11. $\sum_{i=1}^{40} (4 - i^2)$

12. $\sum_{i=1}^{50} (8 - i)$

In exercises 9–18, use summation rules to compute the sum.

13. $\sum_{n=1}^{100} (n^2 - 3n + 2)$

14. $\sum_{n=1}^{140} (n^2 + 2n - 4)$

In exercises 9–18, use summation rules to compute the sum.

15. $\sum_{i=3}^{30}[(i-3)^2+i-3]$

16. $\sum_{i=4}^{20}(i-3)(i+3)$

In exercises 9–18, use summation rules to compute the sum.

17. $\sum_{k=3}^{n}(k^2-3)$

18. $\sum_{k=0}^{n}(k^2+5)$

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Question 12

Estimate the area under a curve on a given interval using rectangles

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Exercise 18 - 35

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In exercises 35–38, use the given function values to estimate the area under the curve using left-endpoint and right-endpoint evaluation.

35.

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(x)$	2.0	2.4	2.6	2.7	2.6	2.4	2.0	1.4	0.6

In exercises 35–38, use the given function values to estimate the area under the curve using left-endpoint and right-endpoint evaluation.

36.

x	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
$f(x)$	2.0	2.2	1.6	1.4	1.6	2.0	2.2	2.4	2.0

In exercises 35–38, use the given function values to estimate the area under the curve using left-endpoint and right-endpoint evaluation.

37.

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
$f(x)$	1.8	1.4	1.1	0.7	1.2	1.4	1.8	2.4	2.6

In exercises 35–38, use the given function values to estimate the area under the curve using left-endpoint and right-endpoint evaluation.

38.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6
$f(x)$	0.0	0.4	0.6	0.8	1.2	1.4	1.2	1.4	1.0

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Question 13

Learn the properties of definite integrals

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Exercise 21 - 30

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In exercises 23 and 24, compute $\int_0^4 f(x)\,dx$.

23. $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 4 & \text{if } x \geq 1 \end{cases}$

In exercises 23 and 24, compute $\int_0^4 f(x)\,dx$.

24. $f(x) = \begin{cases} 2 & \text{if } x \leq 2 \\ 3x & \text{if } x > 2 \end{cases}$

In exercises 35 and 36, use Theorem 4.2 to write the expression as a single integral.

35. (a) $\int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx$ (b) $\int_0^3 f(x)\,dx - \int_2^3 f(x)\,dx$

In exercises 35 and 36, use Theorem 4.2 to write the expression as a single integral.

36. (a) $\int_0^2 f(x)\,dx + \int_2^1 f(x)\,dx$ (b) $\int_{-1}^2 f(x)\,dx + \int_2^3 f(x)\,dx$

In exercises 37 and 38, assume that $\int_1^3 f(x)\,dx = 3$ and $\int_1^3 g(x)\,dx = -2$ and find

37. (a) $\int_1^3 [f(x) + g(x)]\,dx$ (b) $\int_1^3 [2f(x) - g(x)]\,dx$

In exercises 37 and 38, assume that $\int_1^3 f(x)\,dx = 3$ and $\int_1^3 g(x)\,dx = -2$ and find

38. (a) $\int_1^3 [f(x) - g(x)]\,dx$ (b) $\int_1^3 [4g(x) - 3f(x)]\,dx$

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Question 14

Apply the Integral Mean Value Theorem

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Exercise 16 - 27

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In exercises 25–28, compute the average value of the function on the given interval.

25. $f(x) = 2x + 1, [0, 4]$

26. $f(x) = x^2 + 2x, [0, 1]$

In exercises 25–28, compute the average value of the function on the given interval.

27. $f(x) = x^2 - 1, [1, 3]$

28. $f(x) = 2x - 2x^2, [0, 1]$

In exercises 33 and 34, find a value of c that satisfies the conclusion of the Integral Mean Value Theorem.

33. $\int_0^2 3x^2\,dx\,(=8)$

In exercises 33 and 34, find a value of c that satisfies the conclusion of the Integral Mean Value Theorem.

34. $\int_{-1}^{1} (x^2 - 2x)\, dx \; (= \frac{2}{3})$

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Question 15

Learn the Fundamental Theorem of Calculus (Part I) and use it to compute various definite integrals

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Exercise 1 - 16

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In exercises 1–18, use Part I of the Fundamental Theorem to compute each integral exactly.

1. $\int_0^2 (2x - 3)\, dx$

2. $\int_0^3 (x^2 - 2)\, dx$

In exercises 1–18, use Part I of the Fundamental Theorem to compute each integral exactly.

3. $\int_{-1}^{1} (x^3 + 2x)\, dx$

4. $\int_{0}^{2} (x^3 + 3x - 1)\, dx$

In exercises 1–18, use Part I of the Fundamental Theorem to compute each integral exactly.

5. $\int_1^4 \left(x\sqrt{x} + \frac{3}{x}\right) dx$

6. $\int_1^2 \left(4x - \frac{2}{x^2}\right) dx$

In exercises 1–18, use Part I of the Fundamental Theorem to compute each integral exactly.

7. $\int_0^1 (6e^{-3x} + 4)\, dx$

8. $\int_0^2 \left(\frac{e^{2x} - 2e^{3x}}{e^{3x}} \right) dx$

In exercises 1–18, use Part I of the Fundamental Theorem to compute each integral exactly.

9. $\int_{\pi/2}^{\pi} (2\sin x - \cos x)\, dx$

10. $\int_{\pi/4}^{\pi/2} 3\csc x \cot x\, dx$

In exercises 1–18, use Part I of the Fundamental Theorem to compute each integral exactly.

11. $\int_0^{\pi/4} \sec t \tan t \, dt$

12. $\int_0^{\pi/4} \sec^2 t \, dt$

In exercises 1–18, use Part I of the Fundamental Theorem to compute each integral exactly.

13. $\displaystyle\int_0^{1/2} \frac{3}{\sqrt{1-x^2}}\,dx$

14. $\displaystyle\int_{-1}^{1} \frac{4}{1+x^2}\,dx$

In exercises 1–18, use Part I of the Fundamental Theorem to compute each integral exactly.

15. $\int_1^4 \frac{t-3}{t}\,dt$

16. $\int_0^4 t(t-2)\,dt$

In exercises 1–18, use Part I of the Fundamental Theorem to compute each integral exactly.

17. $\int_0^t (e^{x/2})^2 \, dx$

18. $\int_0^t (\sin^2 x + \cos^2 x) \, dx$

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G12 Adv Term 2

Part 2: Writing (FRQ)

End of Term 2023-24

Justin Dsouza

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Grade 12Adv

Question 16

Solve mathematical and real-life optimization problems

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Exercise 25 - 36

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16	Solve mathematical and real-life optimization problems حل مسائل رياضية وحياتية على القيم القصوى لإيجاد القيم المثلى	(1-7) (8,9)	296 297

1. A three-sided fence is to be built next to a straight section of river, which forms the fourth side of a rectangular region. The enclosed area is to equal 1800 ft^2. Find the minimum perimeter and the dimensions of the corresponding enclosure.

2. A three-sided fence is to be built next to a straight section of river, which forms the fourth side of a rectangular region. There is 96 feet of fencing available. Find the maximum enclosed area and the dimensions of the corresponding enclosure.

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3. A two-pen corral is to be built. The outline of the corral forms two identical adjoining rectangles. If there is 120 ft of fencing available, what dimensions of the corral will maximize the enclosed area?

4. A showroom for a department store is to be rectangular with walls on three sides, 6-ft door openings on the two facing sides and a 10-ft door opening on the remaining wall. The showroom is to have 800 ft^2 of floor space. What dimensions will minimize the length of wall used?

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5. Show that the rectangle of maximum area for a given perimeter P is always a square.

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6. Show that the rectangle of minimum perimeter for a given area A is always a square.

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7. A box with no top is to be built by taking a 6 in-by-10 in sheet of cardboard, cutting x-in squares out of each corner and folding up the sides. Find the value of x that maximizes the volume of the box.

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8. A box with no top is to be built by taking a 12 in-by-16 in sheet of cardboard, cutting x-inch squares out of each corner and folding up the sides. Find the value of x that maximizes the volume of the box.

9. (a) A box with no top is built by taking a 6″-by-6″ piece of cardboard, cutting x-in. squares out of each corner and folding up the sides. The four x-in. squares are then taped together to form a second box (with no top or bottom). Find the value of x that maximizes the sum of the volumes of the boxes. (b) Repeat the problem starting with a 4 in-by-6 in piece of cardboard.

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Question 17

Solve mathematical and real-life problems on related rates

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Exercise 1 - 8

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1. Oil spills out of a tanker at the rate of 120 gl/min per minute. The oil spreads in a circle with a thickness of $\frac{1}{4}''$. Given that 1 ft^3 equals 7.5 gallons, determine the rate at which the radius of the spill is increasing when the radius reaches (a) 100 ft and (b) 200 ft. Explain why the rate decreases as the radius increases.

2. Oil spills out of a tanker at the rate of 90 gallon per minute. The oil spreads in a circle with a thickness of $\frac{1}{8}''$. Determine the rate at which the radius of the spill is increasing when the radius reaches 100 feet.

3. Oil spills out of a tanker at the rate of g gallons per minute. The oil spreads in a circle with a thickness of $\frac{1}{4}''$. (a) Given that the radius of the spill is increasing at a rate of 0.6 ft/min when the radius equals 100 feet, determine the value of g. (b) If the thickness of the oil is doubled, how does the rate of increase of the radius change?

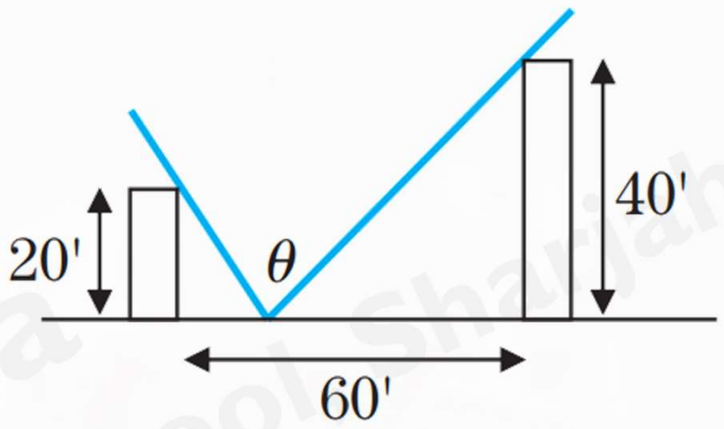
4. Assume that the infected area of an injury is circular. (a) If the radius of the infected area is 3 mm and growing at a rate of 1 mm/hr, at what rate is the infected area increasing? (b) Find the rate of increase of the infected area when the radius reaches 6 mm. Explain in commonsense terms why this rate is larger than that of part (a).

5. Suppose that a raindrop evaporates in such a way that it maintains a spherical shape. Given that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$ and its surface area is $A = 4\pi r^2$, if the radius changes in time, show that $V' = Ar'$. If the rate of evaporation (V') is proportional to the surface area, show that the radius changes at a constant rate.

6. Suppose a forest fire spreads in a circle with radius changing at a rate of 5 ft/min. When the radius reaches 200 feet, at what rate is the area of the burning region increasing?

7. A 10 ft ladder leans against the side of a building as in example 8.2. If the bottom of the ladder is pulled away from the wall at the rate of 3 ft/sec and the ladder remains in contact with the wall, (a) find the rate at which the top of the ladder is dropping when the bottom is 6 ft from the wall. (b) Find the rate at which the angle between the ladder and the horizontal is changing when the bottom of the ladder is 6 ft from the wall.

8. Two buildings of height 20 ft and 40 ft, respectively, are 60 ft apart. Suppose that the intensity of light at a point between the buildings is proportional to the angle θ in the figure. (a) If a person is moving from right to left at 4 ft/s, at what rate is θ changing when the person is exactly halfway between the two buildings? (b) Find the location at which the angle θ is maximum.



9. A plane is located $x = 40$ mile (horizontally) away from an airport at an altitude of h mile. Radar at the airport detects that the distance $s(t)$ between the plane and airport is changing at the rate of $s'(t) = -240$ mph. (a) If the plane flies toward the airport at the constant altitude $h = 4$, what is the speed $|x'(t)|$ of the airplane? (b) Repeat with a height of 6 mile. Based on your answers, how important is it to know the actual height of the airplane?

10. (a) Rework example 8.3 if the police car is not moving. Does this make the radar gun's measurement more accurate?
(b) Show that the radar gun of example 8.3 gives the correct speed if the police car is located at the origin.

11. Show that the radar gun of example 8.3 gives the correct speed if the police car is at $x = \frac{1}{2}$ moving at a speed of $(\sqrt{2} - 1)\ 50$ mph.

12. Find a position and speed for which the radar gun of example 8.3 has a slower reading than the actual speed.

13. For a small company spending AED x thousand per year in advertising, suppose that annual sales in thousands of dollars equal $s = 60 - 40e^{-0.05x}$. The three most recent yearly advertising figures are given in the table.

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Question 18

Solve mathematical and real-life problems on related rates

Page 312, 314 & 315
Exercise 37 & 38
Example 9.8

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18	Solve mathematical and real-life problems on related rates حل مسائل اقتصادية وعلمية على القيم القصوى	Example 9.8	312
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EXAMPLE 9.8 Finding the Maximum Rate of Population Growth

Suppose that a population grows according to the equation $p'(t) = 2p(t)[1 - p(t)]$ (the logistic equation with $r = 2$). Find the population for which the growth rate is a maximum. Interpret this point graphically.

Solution To clarify the problem, we write the population growth rate as

$$f(p) = 2p(1 - p).$$

Our aim is then to find the population $p \geq 0$ that maximizes $f(p)$. We have

$$\begin{aligned} f'(p) &= 2(1)(1 - p) + 2p(-1) \\ &= 2(1 - 2p) \end{aligned}$$

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EXAMPLE 9.8 Finding the Maximum Rate of Population Growth

Suppose that a population grows according to the equation $p'(t) = 2p(t)[1 - p(t)]$ (the logistic equation with $r = 2$). Find the population for which the growth rate is a maximum. Interpret this point graphically.

and so, the only critical number is $p = \frac{1}{2}$.

Notice that the graph of $y = f(p)$ is a parabola opening downward and hence, the critical number must correspond to the absolute maximum.

In Figure 4.104, observe that the height $p = \frac{1}{2}$ corresponds to the portion of the graph with maximum slope.

Also, notice that this point is an inflection point on the graph.

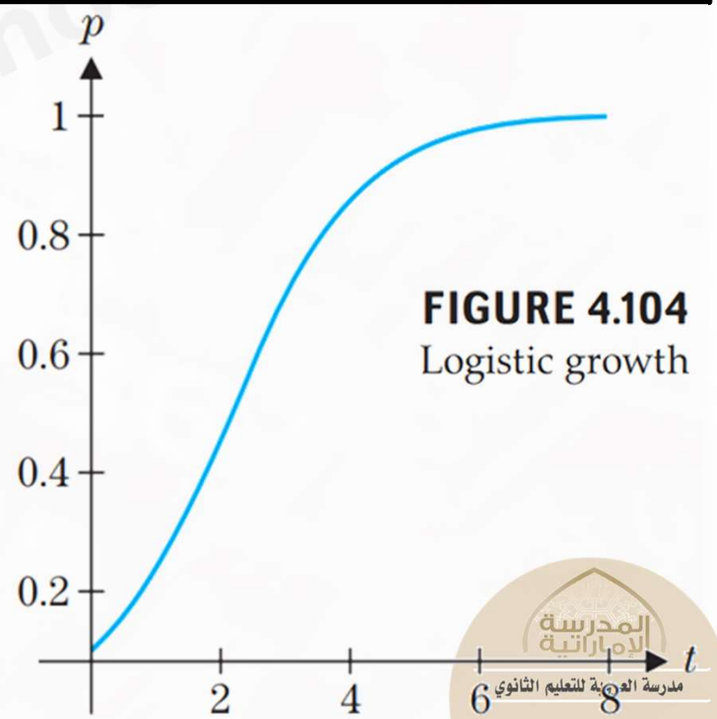

FIGURE 4.104
Logistic growth

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EXAMPLE 9.8 Finding the Maximum Rate of Population Growth

Suppose that a population grows according to the equation $p'(t) = 2p(t)[1 - p(t)]$ (the logistic equation with $r = 2$). Find the population for which the growth rate is a maximum. Interpret this point graphically.

We can verify this by noting that we solved the equation $f'(p) = 0$, where $f(p)$ equals $p'(t)$. Therefore, $p = \frac{1}{2}$ is the p-value corresponding to the solution of $p''(t) = 0$. This fact can be of value to population biologists. If they are tracking a population that reaches an inflection point, then (assuming that the logistic equation gives an accurate model) the population will eventually double in size.

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37. Suppose that a population grows according to the logistic equation $p'(t) = 4p(t)[5 - p(t)]$. Find the population at which the population growth rate is a maximum.

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38. Suppose that a population grows according to the logistic equation $p'(t) = 2p(t)[7 - 2p(t)]$. Find the population at which the population growth rate is a maximum.

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Question 19

Compute the area under a curve using summations and limits

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Exercise 1 - 8

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EXAMPLE 3.2 Computing the Area Exactly

Find the area under the curve $y = f(x) = 2x - 2x^2$ on the interval $[0, 1]$.

Solution Here, using n subintervals, we have

$$\Delta x = \frac{1-0}{n} = \frac{1}{n}$$

and so, $x_0 = 0, x_1 = \frac{1}{n}, x_2 = x_1 + \Delta x = \frac{2}{n}$ and so on. Then, $x_i = \frac{i}{n}$, for $i = 0, 1, 2, \ldots, n$. From (3.1), the area is approximately

EXAMPLE 3.2 Computing the Area Exactly

Find the area under the curve $y = f(x) = 2x - 2x^2$ on the interval $[0, 1]$.

$$A \approx A_n = \sum_{i=1}^{n} f\left(\frac{i}{n}\right)\left(\frac{1}{n}\right)$$

$$= \sum_{i=1}^{n} \left[2\frac{i}{n} - 2\left(\frac{i}{n}\right)^2\right]\left(\frac{1}{n}\right)$$

$$= \sum_{i=1}^{n} \left[2\left(\frac{i}{n}\right)\left(\frac{1}{n}\right)\right] - \sum_{i=1}^{n} \left[2\left(\frac{i^2}{n^2}\right)\left(\frac{1}{n}\right)\right]$$

$$= \frac{2}{n^2}\sum_{i=1}^{n} i - \frac{2}{n^3}\sum_{i=1}^{n} i^2$$

EXAMPLE 3.2 Computing the Area Exactly

Find the area under the curve $y = f(x) = 2x - 2x^2$ on the interval $[0, 1]$.

$$= \frac{2}{n^2}\frac{n(n+1)}{2} - \frac{2}{n^3}\frac{n(n+1)(2n+1)}{6}$$ From Theorem 2.1 (ii) and (iii).

$$= \frac{n+1}{n} - \frac{(n+1)(2n+1)}{3n^2}$$

$$= \frac{(n+1)(n-1)}{3n^2}.$$

Since we have a formula for A_n, for any n, we can compute various values with ease.

EXAMPLE 3.2 Computing the Area Exactly

Find the area under the curve $y = f(x) = 2x - 2x^2$ on the interval $[0, 1]$.

$$A_{200} = \frac{(201)(199)}{3(40{,}000)} = 0.333325,$$

$$A_{500} = \frac{(501)(499)}{3(250{,}000)} = 0.333332$$

$$\lim_{n\to\infty} A_n = \lim_{n\to\infty} \frac{n^2 - 1}{3n^2}$$

$$= \lim_{n\to\infty} \frac{1 - 1/n^2}{3} = \frac{1}{3}.$$

Therefore, the exact area in Figure 5.8 is 1/3, as we had suspected.

In exercises 11–14, use Riemann sums and a limit to compute the exact area under the curve.

11. $y = x^2 + 1$ on (a) $[0, 1]$; (b) $[0, 2]$; (c) $[1, 3]$

In exercises 11–14, use Riemann sums and a limit to compute the exact area under the curve.

12. $y = x^2 + 3x$ on (a) $[0, 1]$; (b) $[0, 2]$; (c) $[1, 3]$

19	Compute the area under a curve using summations and limits إيجاد المساحة تحت المنحنى لدالة باستخدام المجاميع والنهايات	Example 3.2 (11-14)	341 344

In exercises 11–14, use Riemann sums and a limit to compute the exact area under the curve.

13. $y = 2x^2 + 1$ on (a) $[0, 1]$; (b) $[-1, 1]$; (c) $[1, 3]$

In exercises 11–14, use Riemann sums and a limit to compute the exact area under the curve.

14. $y = 4x^2 - x$ on (a) $[0, 1]$; (b) $[-1, 1]$; (c) $[1, 3]$

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Question 20

Learn the Fundamental Theorem of Calculus (Part II) and use it to compute derivatives of functions defined as definite integrals

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Exercise 1 - 8

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In exercises 25–32, find the derivative $f'(x)$.

25. $f(x) = \int_0^x (t^2 - 3t + 2)\, dt$

In exercises 25–32, find the derivative $f'(x)$.

26. $f(x) = \int_2^x (t^2 - 3t - 4)\, dt$

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In exercises 25–32, find the derivative $f'(x)$.

27. $f(x) = \int_0^{x^2} (e^{-t^2} + 1)\, dt$

In exercises 25–32, find the derivative $f'(x)$.

28. $f(x) = \int_x^2 \sec t \, dt$

20	Learn the Fundamental Theorem of Calculus (Part II) and use it to compute derivatives of functions defined as definite integrals التعرف على النظرية الأساسية الثانية للتفاضل والتكامل وتطبيقها على دوال معرفة كتكاملات محدودة لإيجاد مشتقاتها	(25-32)	366

In exercises 25–32, find the derivative $f'(x)$.

29. $f(x) = \int_{e^x}^{2-x} \sin t^2 \, dt$

In exercises 25–32, find the derivative $f'(x)$.

30. $f(x) = \int_{2-x}^{xe^x} e^{2t}\,dt$

In exercises 25–32, find the derivative $f'(x)$.

31. $f(x) = \int_{x^2}^{x^3} \sin(3t)\, dt$

In exercises 25–32, find the derivative $f'(x)$.

32. $f(x) = \int_{3x}^{\sin x} (t^2 + 4)\, dt$

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