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Reveal MATH®

UAE Edition
Grade 7 General
Volume 2
Student Edition

Mc
Graw
Hill

Interactive Student Edition

Reveal MATH®

Course 2 • Volume 2

McGraw Hill

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Send all inquiries to:
McGraw-Hill Education
STEM Learning Solutions Center
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Interactive Student Edition, Volume 2

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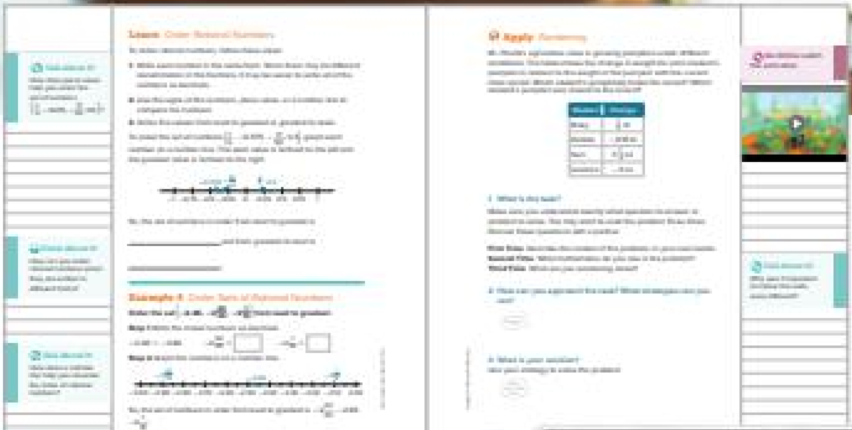
Proportional Relationships

Essential Question

What does it mean for two quantities to be in a proportional relationship?

Reveal Math® Makes Math Meaningful...



Interactive Student Edition

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Web Sketchpad® Powered by The Geometer's Sketchpad®- Dynamic, exploratory, visual activities embedded at point of use within the lesson.

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Personal Tutors – See and hear a teacher explain how to solve problems.

eTools – Math tools are available to help you solve problems and develop concepts.

Module 2

Solve Percent Problems

Essential Question

How can percent describe the change of a quantity?

Module 3

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Essential Question

How are operations with integers related to operations with whole numbers?

Module 4

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Essential Question

How are operations with rational numbers related to operations with integers?

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Essential Question

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Module 5

Simplify Algebraic Expressions

Essential Question

Why is it beneficial to rewrite expressions in different forms?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY	Before			After		
■ — I don't know. ◆ — I've heard of it. ★ — I know it!	■	◆	★	■	◆	★
simplifying algebraic expressions by combining like terms						
using the Distributive Property to expand linear expressions						
adding linear expressions						
subtracting linear expressions						
finding the greatest common factors of monomials						
factoring linear expressions						
simplifying linear expressions						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about simplifying algebraic expressions.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ coefficient
☐ constant
☐ factor
☐ factored form
☐ greatest common factor
☐ like terms
☐ linear expression
☐ monomial
☐ simplest form
☐ term

Are You Ready?

Study the Quick Review to see if you are ready to start this module.
Then complete the Quick Check.

Quick Review

Example 1

Subtract integers.

Simplify $-15 - (-3)$.

$-15 - (-3)$

$= -15 + 3$ Add the additive inverse.

$= -12$ Find the difference of the absolute values. The sign of the sum is negative because −15 has a greater absolute value than 3.

Example 2

Multiply integers.

Simplify $6(-7)$.

$6(-7) = -42$ The product is negative because the signs of the factors are different.

Quick Check

1. Simplify $24 - 81$.

2. Simplify $37 - (-16)$.

3. Simplify $-5(-8)$.

4. Simplify $-4(11)$.

How Did You Do?

Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right.

① ② ③ ④

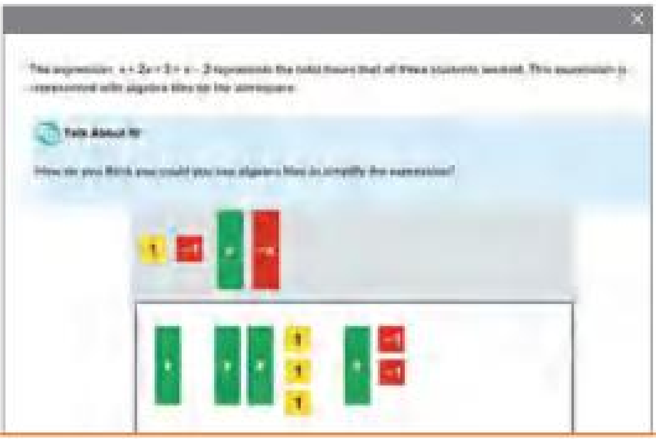
Simplify Algebraic Expressions

I Can... simplify algebraic expressions by identifying and combining like terms.

What Vocabulary Will You Learn?
coefficient
constant
like terms
simplest form
term

Explore Use Algebra Tiles to Add Integers

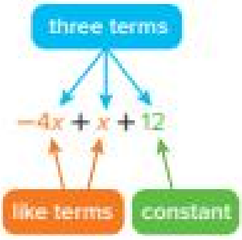
Online Activity You will use algebra tiles to explore how to simplify algebraic expressions.



Learn Like Terms

When addition or subtraction signs separate an algebraic expression into parts, each part is called a **term. Like terms** contain the same variables to the same powers. For example, $3x^2$ and $-7x^2$ are like terms because the variables and their exponents are the same. But, $5y^3$ and $9y^4$ are not like terms because the exponents are different.

The numerical factor of a term that contains a variable is called the **coefficient** of the variable. For example, in the term $3x^2$, 3 is the coefficient. A term without a variable is called a **constant**. Constant terms are also like terms.



(continued on next page)

Your Notes

Sort the terms by writing like terms in the appropriate bins.

5 $\quad 8x \quad -9x^2 \quad 6x \quad -12 \quad 30x^2 \quad -1.5x \quad \frac{1}{2}$

$4x^2$	$-2x$	3

Talk About It!
Why are the expressions $3x$ and $3x^2$ *not* like terms?

Learn Combine Like Terms

An algebraic expression is in **simplest form** if it has no like terms and no parentheses.

Go Online Watch the animation to learn how the Distributive Property can be used to combine like terms.

$6n - 1 - 8n + 9$

$= 6n + (-1) + (-8n) + 9$ Rewrite each subtraction as addition.

$= 6n + (-8n) + (-1) + 9$ Apply the Commutative Property.

$= [6 + (-8)]n + (-1) + 9$ Apply the Distributive Property.

$= -2n + 8$ Simplify.

Talk About It!
Without using the Distributive Property, what is another way you could add $6n$ and $-8n$?

Example 1 Combine Like Terms

The cost of a jacket j after a 5% markup can be represented by the expression $j + 0.05j$.

Simplify the expression.

$j + 0.05j = 1j + 0.05j$ Identity Property: $j = 1j$

$= 1.05j$ Combine like terms.

Increasing the jacket's price by 5% is the same as multiplying the price by ________ .

Suppose the original cost of the jacket is $35. What is the cost of the jacket after the 5% markup? $ ☐

Talk About It!
When might it be more advantageous to simplify the expression then evaluate versus evaluating first, then simplifying?

Check

The cost of a new pair of shoes after a 3% markup can be represented by the expression $c + 0.03c$. Simplify the expression.

Show your work here

Example 2 Combine Like Terms

Simplify $-5x + y + 6 - 5y - 3$.

$-5x + y + 6 - 5y - 3$ Write the expression.

$= -5x + y + 6 + (-5y) \boxed{} (\boxed{})$ Rewrite subtraction as addition.

$= -5x + y + (\boxed{}) + \boxed{} + (-3)$ Commutative Property

$= -5x + (\boxed{}) + \boxed{}$ Combine like terms.

$= -5x - 4y + 3$ Because parentheses are in the expression, it is not simplified. Rewrite using subtraction.

So, $-5x + y + 6 - 5y - 3 = -5x - 4y + 3$.

Talk About It!

In Steps 2 and 3, why must subtraction be written as addition in order to use the Commutative Property? Why is the Commutative Property used?

Check

Simplify $-5 - 3w + 9 - 6z + 8w - 4z$.

Ⓐ $4 - 9w + 4z$

Ⓑ $4 + 5w - 10z$

Ⓒ $4 + 5w + 10z$

Ⓓ $-4 - 5w - 10z$

Show your work here

Go Online You can complete an Extra Example online.

Think About It!
How would you begin simplifying the expression?

Talk About It!
In Step 4, why were two different common denominators found?

Example 3 Combine Like Terms

Simplify $\frac{3}{4}a - \frac{2}{3} - \frac{1}{2}a + \frac{5}{6}$.

$\frac{3}{4}a - \frac{2}{3} - \frac{1}{2}a + \frac{5}{6}$	Write the expression.
$= \frac{3}{4}a + \left(-\frac{2}{3}\right) + \left(-\frac{1}{2}a\right) + \frac{5}{6}$	Rewrite subtraction as addition.
$= \frac{3}{4}a + \left(-\frac{1}{2}a\right) + \left(-\frac{2}{3}\right) + \frac{5}{6}$	Commutative Property
$= \frac{3}{4}a + \left(-\frac{2}{4}a\right) + \left(-\frac{4}{6}\right) + \frac{5}{6}$	Rewrite fractions with common denominators.
$= \frac{1}{4}a + \frac{1}{6}$	Combine like terms.

So, $\frac{3}{4}a - \frac{2}{3} - \frac{1}{2}a + \frac{5}{6} =$ ________.

Check

Simplify $-\frac{1}{4}m + \frac{5}{6} + \frac{3}{8}m - \frac{2}{3}$.

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Reflect on the process of simplifying algebraic expressions containing fractions. What concepts did you use? How are they used to simplify an expression?

Record your observations here

Learn Expand Linear Expressions

The Distributive Property can be used to expand linear expressions.

You learned about this property in an earlier grade.

Words	
The Distributive Property states that to multiply a sum or difference by a number, multiply each term inside the parentheses by the number outside the parentheses.	
Symbols	**Examples**
$a(b + c) = ab + ac$ $a(b - c) = ab - ac$	$4(x + 2) = 4 \cdot x + 4 \cdot 2$ $= 4x + 8$ $3(x - 5) = 3 \cdot x - 3 \cdot 5$ $= 3x - 15$

Fill in the boxes to model the Distributive Property.

$2(x + 2) = \boxed{}(x) + \boxed{}(2)$ Distributive Property

$= \boxed{} + \boxed{}$ Simplify.

Example 4 Distribute Over Addition

Use the Distributive Property to expand $4(-3x + 6)$.

$4(-3x + 6) = \boxed{}(-3x) + \boxed{}(6)$ Distributive Property

$= -12x + 24$ Simplify.

So, $4(-3x + 6) =$ ________.

Think About It!
What terms will be multiplied by 4 when you expand the expression?

Talk About It!
In the second step, why is it helpful to use parentheses when expanding the expression?

Check

Use the Distributive Property to expand $9(-5a + 3b)$.

Show your work here

Go Online You can complete an Extra Example online.

Example 5 Distribute Over Subtraction

Use the Distributive Property to expand $(2x - 5y)3$.

$(2x - 5y)3 = [(2x + (-5y)]3$	Rewrite subtraction as addition.
$= 3(2x) + 3(-5y)$	Distributive Property
$= 6x + (-15y)$	Multiply.
$= 6x - 15y$	Because parentheses are in the expression, it is not simplified. Rewrite using subtraction.

Talk About It!
What is another way you can expand the expression $(2x - 5y)3$ without first rewriting the subtraction as addition?

So, $(2x - 5y)3 =$ ______________.

Check

Use the Distributive Property to expand $(-4w - 7)5$.

Show your work here

Example 6 Distribute Negative Numbers

Use the Distributive Property to expand $-5(2x - 9)$.

$-5(2x - 9) = -5[2x + (-9)]$	Rewrite subtraction as addition.
$= -5(2x) + -5(-9)$	Distributive Property
$= -10x + 45$	Simplify.

Talk About It!
What mistake might be made if you do not rewrite subtraction as addition?

So, $-5(2x - 9) =$ ______________.

Check

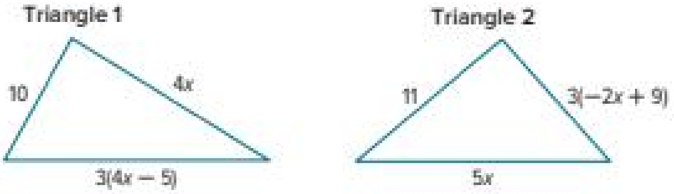
Use the Distributive Property to expand $-6(-8y + 10)$.

Show your work here

Go Online You can complete an Extra Example online.

Apply Geometry

The side lengths of two triangles are shown. Represent the perimeter of each triangle with an expression in simplest form. Which triangle has a greater perimeter if $x = 3$? Will this be true if $x = 2$? Justify your response.



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

Talk About It!

What properties did you use when solving this problem?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

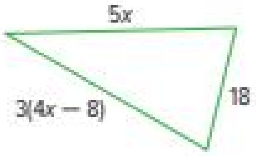
Write About It! Write an argument that can be used to defend your solution.

Check

The side lengths of two triangles are shown. Select the perimeter of each triangle with an expression in simplest form.

Triangle 1

Triangle 2



Ⓐ The perimeter of Triangle 1 is $-2x + 91$.
The perimeter of Triangle 2 is $17x - 6$.

Ⓑ The perimeter of Triangle 1 is $4x + 34$.
The perimeter of Triangle 2 is $9x + 10$.

Ⓒ The perimeter of Triangle 1 is $-2x + 19$.
The perimeter of Triangle 2 is $17x - 24$.

Ⓓ The perimeter of Triangle 1 is $14x + 91$.
The perimeter of Triangle 2 is $17x - 24$.

Which triangle has a greater perimeter if $x = 5$?

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Where in the lesson did you feel most confident? Why?

Record your observations here

Name ________________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

1. The cost of a set of DVDs after a 25% markup can be represented by the expression $c + 0.25c$. Simplify the expression. (Example 1)

2. The cost of a new robotic toy after an 8% markup can be represented by the expression $r + 0.08r$. Simplify the expression. (Example 1)

Simplify each expression. (Examples 2 and 3)

3. $-y + 9z - 16y - 25z + 4$

4. $8z + x - 5 - 9z + 2$

5. $5c - 3d - 12c + d - 6$

6. $-\frac{3}{4}x - \frac{1}{3} + \frac{7}{8}x - \frac{1}{2}$

7. $\frac{1}{4} + \frac{9}{10}y - \frac{3}{5}y + \frac{7}{8}$

8. $-\frac{1}{2}a + \frac{2}{5} + \frac{5}{6}a - \frac{1}{10}$

Use the Distributive Property to expand each expression. (Examples 4–6)

9. $2(-3x + 5)$

10. $6(-4x + 3y)$

11. $(3y - 2z)5$

12. $(-2x - 7)4$

13. $-7(x - 2)$

14. $-3(8x - 4)$

Test Practice

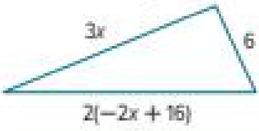
15. Table Item The table shows the side lengths of a triangle. The perimeter of the triangle is $6a + 3$. Write an expression in simplest form for the length of Side 3.

Triangle Side	Length (units)
1	$2(a + 3)$
2	$3a - 1$
3	

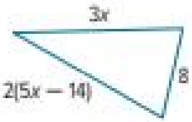
Apply

16. The side lengths of two triangles are shown. Represent the perimeter of each triangle with an expression in simplest form. Which triangle has a greater perimeter if $x = 4$? Will this be true if $x = 5$? Justify your response.

Triangle 1

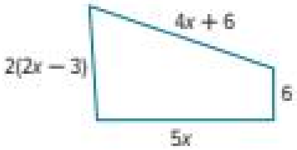


Triangle 2

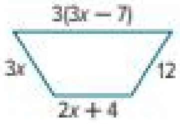


17. The side lengths of two quadrilaterals are shown. Represent the perimeter of each quadrilateral with an expression in simplest form. Which quadrilateral has a greater perimeter if $x = 3$? Will this be true if $x = 4$? Justify your response.

Quadrilateral 1



Quadrilateral 2



Higher-Order Thinking Problems

18. Create Write an expression with at least three unlike terms and then simplify the expression.

19. Find the Error A student simplified the expression $5x - 3(x + 4)$ to $2x + 12$. Find the student's error and correct it.

20. Justify Conclusions Is the following statement *true* or *false*? If false, explain.

When using the Distributive Property, if the term outside the parentheses is negative, then the sign of each term inside the parentheses will not change.

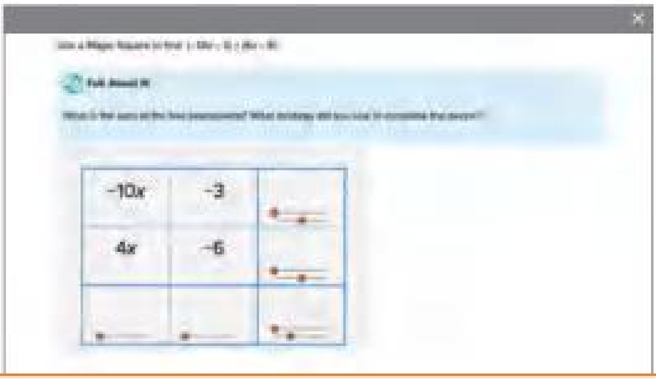
Add Linear Expressions

I Can... use different methods to add linear expressions.

What Vocabulary Will You Learn?
linear expression

Explore Add Expressions

Online Activity You will use Web Sketchpad to explore how to add linear expressions.



−10x	−3	
4x	−6	

Learn Add Linear Expressions

A **linear expression** is an algebraic expression in which each term is a constant or the product of a constant and the variable raised to the first power. When simplified, a linear expression cannot contain a variable in the denominator of a fraction.

Sort the expressions by writing each one in the appropriate bin. Examples of each type are given.

$-5x^2$ $-4x + 3$ $\frac{1}{2}x - 5$ $\frac{6}{x}$ $x^3 + 2$

Linear Expressions	Nonlinear Expressions
$5x$	$5mn$
$3x + 2$	$x^4 - 7$

Talk About It!
Why are $-5x^2$ and $\frac{6}{x}$ *not* linear expressions?

(continued on next page)

Your Notes

Go Online Watch the animation to learn how to add linear expressions.

The animation shows how to add the expressions $(4x + 2) + (5x - 7)$.

$$\begin{array}{rll} & (4x + 2) + (5x - 7) & \\ = & (4x + 2) + [5x + (-7)] & \text{Rewrite each subtraction as addition.} \\ = & 4x + 2 & \text{Arrange like terms in columns.} \\ & \underline{(+)\ 5x + (-7)} & \\ & 9x + (-5) \text{ or } 9x - 5 & \text{Add the like terms in each column.} \end{array}$$

Talk About It!

When you add the expressions $4x + 2$ and $5x - 7$, the answer is $9x + (-5)$ or $9x - 5$. Why can we rewrite $9x + (-5)$ as $9x - 5$?

Example 1 Add Linear Expressions

Find $(4x - 2) + (-7x - 3)$.

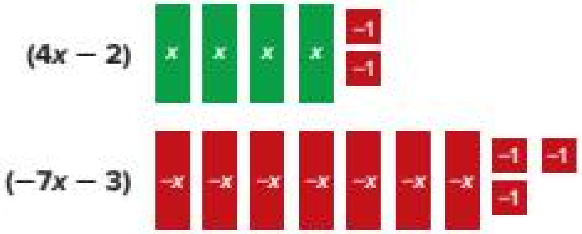
Method 1 Use algebra tiles.

Use algebra tiles to add $(4x - 2) + (-7x - 3)$.

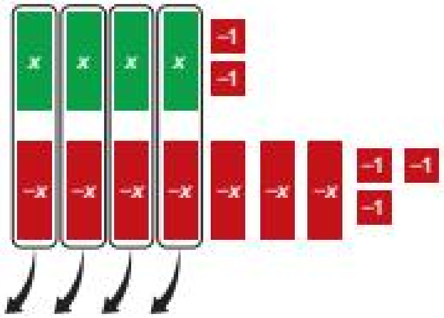
Think About It!

How would you begin finding the sum?

Step 1 Model each expression using tiles.



Step 2 Remove zero pairs.



There are three $-x$-tiles and five -1-tiles remaining.

So, $(4x - 2) + (-7x - 3) =$ ________.

(continued on next page)

Method 2 Arrange terms in columns.

$(4x - 2) + (-7x - 3)$ Write the original expression.

$= [4x + (-2)] + [-7x + (-3)]$ Rewrite subtraction as addition.

$$\begin{array}{rl} & = 4x + (-2) \\ (+) & \underline{-7x + (-3)} \\ & -3x + (-5) \end{array}$$

Arrange like terms in columns.

Add.

So, $(4x - 2) + (-7x - 3) =$ ______________________.

Talk About It!

How are the processes for Method 1 and Method 2 similar?

Could you simplify this expression another way? Explain.

Check

Find $(5x - 4) + (-2x + 2)$.

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Are you ready to move on to the next Example? If yes, what have you learned that you think will help you? If no, what questions do you still have? How can you get those questions answered?

Record your observations here

Example 2 Add Linear Expressions

Find $\left(\frac{1}{3}x + 9\right) + \left(\frac{5}{12}x - 4\right)$.

Think About It!

What are the like terms in the expression?

Rewrite subtraction as addition.

$$\left(\frac{1}{3}x + 9\right) + \left(\frac{5}{12}x - 4\right)$$

Write the original expression.

$$= \left(\frac{1}{3}x + 9\right) + \left[\frac{5}{12}x + (-4)\right]$$

Rewrite subtraction as addition.

Arrange like terms in columns.

$$\begin{array}{rcrl} \frac{1}{3}x + 9 & \rightarrow & & \boxed{}\,x + 9 \\ +\frac{5}{12}x + (-4) & \rightarrow & + & \boxed{}\,x + (-4) \\ \hline & & & \frac{9}{12}x + 5 \\ & & & \boxed{}\,x + 5 \end{array}$$

Rewrite using a common denominator.

Add.

Simplify.

Talk About It!

You can also add linear expressions by arranging like terms in columns without first rewriting subtraction as addition. Compare and contrast the two methods.

So, $\left(\frac{1}{3}x + 9\right) + \left(\frac{5}{12}x - 4\right) = \frac{3}{4}x + 5$.

Check

Find $\left(\frac{1}{4}x + 4\right) + \left(\frac{2}{3}x - 8\right)$.

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare what you learned today with something similar you learned in an earlier module or grade. How are they similar? How are they different?

Record your observations here

Apply Theater

The drama club is selling tickets for their latest production. They are also accepting additional cash donations. They plan to save 20% of the money from all ticket sales and donations for their spring trip. Ticket sales and donations from two performances are represented in the table, where t represents the cost of a ticket. If tickets cost $9, how much money will the drama club have available for the spring trip?

Performance	Ticket Sales and Donations
Friday Night	$92t + 109$
Saturday Night	$34t + 13$

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It! Compare and contrast your method for solving this problem with a classmate's method.

Check

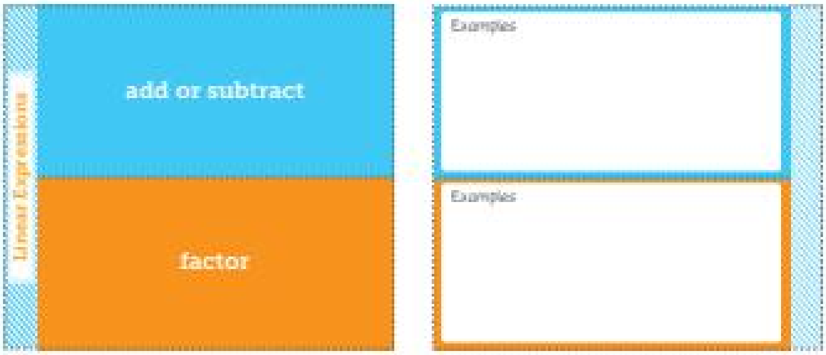
The football team is selling tickets for their next two home games. They also sell food at the concession stand during the games. They plan to save 30% of the money from all ticket sales and concession stand sales for their summer camp. Ticket and concession sales for two home games are represented in the table, where t represents the cost of a ticket. If tickets cost $5, how much money will the football team have for their summer camp?

Home Game	Ticket and Concession Sales
Week 1	$(75t + 130)$
Week 2	$(82t + 115)$

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Name ____________________ Period __________ Date __________

Practice

Go Online You can complete your homework online.

Add. (Examples 1 and 2)

1. $(8x + 9) + (-6x - 2)$

2. $(5x + 4) + (-8x - 2)$

3. $(-7x + 1) + (4x - 5)$

4. $(-3x - 9) + (4x + 8)$

5. $(-5x + 4) + (-9x - 3)$

6. $(-2x + 10) + (-8x -1)$

7. $\left(\frac{1}{4}x - 3\right) + \left(\frac{3}{16}x + 5\right)$

8. $\left(\frac{1}{2}x - 3\right) + \left(\frac{1}{6}x + 1\right)$

9. $\left(4x + \frac{3}{4}\right) + \left(-3x - \frac{5}{12}\right)$

10. $\left(-9x - \frac{4}{5}\right) + \left(2x + \frac{2}{3}\right)$

11. $\left(\frac{1}{3}x - 3\right) + \left(-\frac{3}{4}x - 5\right)$

12. $\left(-5x - \frac{2}{3}\right)+ \left(-4x - \frac{1}{9}\right)$

Test Practice

13. Open Response The table shows the length and width of a rectangle. Write a simplified expression for the perimeter of the rectangle.

Dimension	Measurement (units)
Length	$3x + 6$
Width	$2x - 4$

Apply

14. Jade and Chet get a weekly allowance plus x dollars for each time the pair walks the dog. They plan to save 40% of their combined earnings in one week to purchase a new app for their smart tablet. Their earnings in a certain week are represented in the table. If their parents pay $2.50 each time they walk the dog, how much money will they have to purchase the app?

	Earnings ($)
Jade	$8 + 2x$
Chet	$4x + 6$

15. Elsa is selling bracelets at craft shows to raise money for an animal shelter. She is also accepting additional cash donations. She plans to give 75% of the money from all bracelet sales and donations to the shelter. She will use the remaining money to buy more supplies. Bracelet sales and donations from the first craft show are represented by the expression $24n + 32$, where n represents the amount Elsa charges for each bracelet. The second craft show sales and donations is represented by the expression $40n + 56$. If Elsa charges $4 for each bracelet, how much money will she donate to the animal shelter?

16. MP **Identify Structure** Write two linear expressions that have a sum of $6x + 9$.

17. MP **Identify Structure** What linear expression would you need to add to $(-6x + 3)$ to have a sum of $-x$?

18. **Which One Doesn't Belong?** Identify the linear expression that is not equivalent to the other three. Explain your reasoning.

a. $(2x - 1) + (-3x + 7)$

b. $(-5x + 3) + (4x + 3)$

c. $(5x - 6) + (-6x + 12)$

d. $(-5x - 1) + (6x + 7)$

19. MP **Reason Inductively** When will the sum of two linear expressions with only x-terms be zero?

Lesson 5-3

Subtract Linear Expressions

I Can... use different methods to subtract linear expressions.

Learn Additive Inverses of Expressions

When two expressions have a sum of zero, the expressions are additive inverses.

To find the additive inverse of an expression, use the Distributive Property to multiply the expression by −1.

Find the additive inverse of $4x + 2$.

$-1(4x + 2) = \boxed{}(4x) + (\boxed{})(2)$ Multiply the expression by −1.

$= \boxed{} + (\boxed{})$ Simplify.

$= -4x - 2$ Definition of subtraction

You can check to see if $4x + 2$ and $-4x - 2$ are additive inverses by adding them together to see if their sum is zero.

$$\begin{array}{rr} & 4x + 2 \\ (+) & \underline{-4x - 2} \\ & 0x + 0 \text{ or } 0 \end{array}$$

Arrange like terms in columns.

Add.

Because $4x + 2$ and $-4x - 2$ have a sum of zero, they are additive inverses.

Talk About It!

How could you find the additive inverse of an expression mentally?

Pause and Reflect

Give at least one example of an expression and its additive inverse. Then show how you would check your work.

Record your observations here

Your Notes

Example 1 Find the Additive Inverse of Expressions

Find the additive inverse of $5x - 7$.

To find the additive inverse, multiply the expression by -1.

$$
\begin{aligned}
-1(5x - 7) &= -1[5x + (-7)] && \text{Rewrite subtraction as addition.}\\
&= -1(5x) + (-1)(-7) && \text{Distributive Property}\\
&= -5x + 7 && \text{Simplify.}
\end{aligned}
$$

So, the additive inverse of $5x - 7$ is $-5x + 7$.

Check

Find the additive inverse of $-7x + 3$.

Show your work here

Go Online You can complete an Extra Example online.

Talk About It!
How is the process for subtracting linear expressions the same as adding? How is it different?

Learn Subtract Linear Expressions

When subtracting integers, you add the opposite, or the additive inverse. The same process is used when subtracting linear expressions.

Go Online Watch the animation to learn how to subtract linear expressions.

The animation shows how to subtract $(8y - 5) - (3y + 4)$.

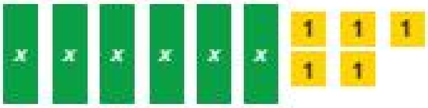
$$
\begin{array}{rl}
(8y - 5) & \text{Arrange like terms in columns.}\\
-\ \underline{(3y + 4)} & \\
 & \\
8y - 5 & \\
(+)\ \underline{-3y - 4} & \text{Add the additive inverse.}\\
5y - 9 & \text{Add.}
\end{array}
$$

Example 2 Subtract Linear Expressions

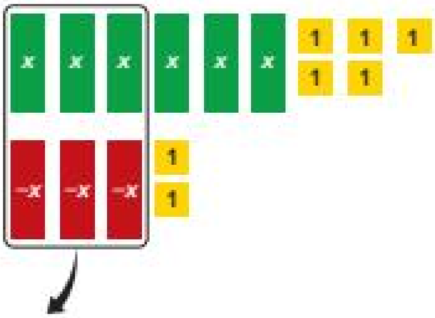
Find $(6x + 5) - (3x - 2)$.

Method 1 Use algebra tiles.

Step 1 Model $6x + 5$ using algebra tiles.



Step 2 To subtract $3x - 2$, add the additive inverse, or $-3x + 2$. Then, remove any zero pairs.



So, $(6x + 5) - (3x - 2) = 3x + 7$.

Method 2 Arrange terms in columns.

$(6x + 5) - (3x - 2)$ Write the expression.

$= (6x + 5) + (-3x + 2)$ The additive inverse of $(3x - 2)$ is $(-3x + 2)$.

$$\begin{array}{rr} & 6x + 5 \\ (+) & -3x + 2 \\ \hline & 3x + 7 \end{array}$$

Arrange like terms in columns.

Add.

So, $(6x + 5) - (3x - 2) =$ ______________.

Check

Find $(-3x + 9) - (8x - 7)$.

Show your work here

Go Online You can complete an Extra Example online.

Think About It!

How would you begin finding the difference?

Talk About It!

Compare the methods for subtracting linear expressions.

- Method 1: Use algebra tiles to subtract linear expressions.
- Method 2: Arrange terms in columns.

Example 3 Subtract Linear Expressions

Find $\left(\frac{2}{3}x + \frac{1}{2}\right) - \left(\frac{1}{6}x - \frac{3}{8}\right)$.

Rewrite using the additive inverse.

$\left(\frac{2}{3}x + \frac{1}{2}\right) - \left(\frac{1}{6}x - \frac{3}{8}\right)$	Write the expression.
$= \left(\frac{2}{3}x + \frac{1}{2}\right) \square \left(\square\right)$	The additive inverse of $\left(\frac{1}{6}x - \frac{3}{8}\right)$ is $\left(-\frac{1}{6}x + \frac{3}{8}\right)$.

Arrange like terms in columns.

$$\begin{array}{rcr} \frac{2}{3}x + \frac{1}{2} & \rightarrow & \frac{4}{6}x + \frac{4}{8} \\ (+)\ \underline{-\frac{1}{6}x + \frac{3}{8}} & \rightarrow & (+)\ \underline{-\frac{1}{6}x + \frac{3}{8}} \end{array}$$

Rewrite using the common denominator.

$\square\, x + \square$ Add.

$\square\, x + \square$ Simplify.

So, $\left(\frac{2}{3}x + \frac{1}{2}\right) - \left(\frac{1}{6}x - \frac{3}{8}\right) = \frac{1}{2}x + \frac{7}{8}$.

Check

Find $\left(-\frac{1}{6}x + \frac{3}{4}\right) - \left(\frac{1}{2}x - \frac{1}{8}\right)$.

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Show how rewriting an expression using the additive inverse is beneficial when subtracting linear expressions.

Record your observations here

Apply Sales

Bonnie owns a T-shirt shop where she sells plain T-shirts and printed T-shirts. The table shows the cost per shirt and the number of each type of shirt sold over *w* weeks. After 25 weeks, how much more did she earn in sales of printed T-shirts than in sales of plain T-shirts?

Style	Cost ($)	Number Sold
Plain	15	$5w - 4$
Printed	15	$8w + 3$

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

[illegible]

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It!
How do you know just by looking at the expressions that Bonnie sold more printed T-shirts than plain T-shirts?

Check

The table shows a bakery's sales of sugar cookies and chocolate chip cookies sold in *h* hours.

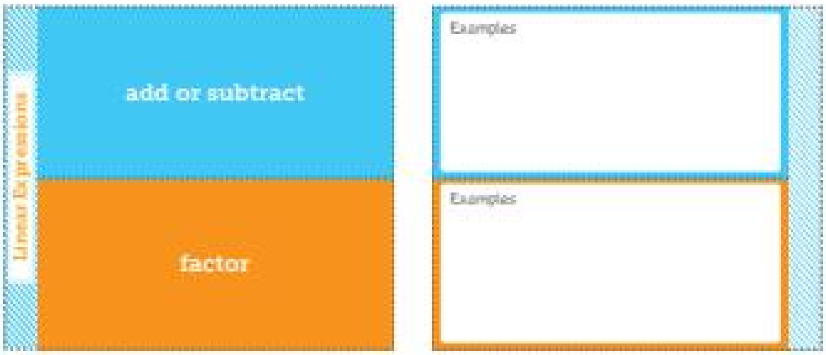
Cookie Sales		
Flavor	**Cost ($)**	**Number Sold**
Sugar	1.15	$6h - 5$
Chocolate Chip	1.15	$10h + 6$

After 15 hours, how much more did the bakery earn in sales of chocolate chip cookies than in sales of sugar cookies?

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Math History Minute

Mathematician and astronomer **Muhammad al-Khwarizmi (around 780–850)** wrote the first known text in elementary algebra. The word *algebra* is derived from the word *al-jabr*, part of the title of this text. It means *reunion of broken parts* in Arabic. His texts were influential in bringing algebraic knowledge to Europe and were the first Arabic mathematics texts translated into Latin.

Name ______________________ Period ____________ Date ____________

Practice

Go Online You can complete your homework online.

Find the additive inverse of each linear expression. (Example 1)

1. $3x - 6$

2. $-9x + 3$

3. $-4x - 8$

Subtract. (Examples 2 and 3)

4. $(8x + 9) - (6x - 2)$

5. $(3x - 4) - (x - 5)$

6. $(-5x - 9) - (-6x - 1)$

7. $(-7x - 14) - (x - 5)$

8. $(-8x + 2) - (-5x + 7)$

9. $\left(\frac{3}{5}x + \frac{3}{4}\right) - \left(\frac{1}{3}x - \frac{1}{8}\right)$

10. $\left(\frac{1}{10}x - \frac{2}{3}\right) - \left(\frac{4}{5}x - \frac{1}{6}\right)$

11. $\left(-\frac{5}{6}x - \frac{7}{12}\right) - \left(-\frac{1}{3}x - \frac{1}{4}\right)$

12. $\left(-\frac{1}{2}x + \frac{7}{10}\right) - \left(-\frac{3}{4}x - \frac{1}{5}\right)$

Test Practice

13. Open Response The table shows the scores of two teams in a trivia challenge at the end of the first half. How many more points did the Huskies score than the Bobcats?

Team	Points Scored
Bobcats	$2x - 7$
Huskies	$5x - 3$

Apply

14. The table shows the sales of plain and Asiago cheese bagels at a bakery for *h* hours. After 6 hours, how much more will the bakery have made in sales of Asiago cheese bagels than the sales of plain bagels?

Bagel Sales

Bagel	Cost ($)	Number Sold After *h* hours
Asiago Cheese	1.50	$12h + 7$
Plain	1.50	$7h - 4$

15. Derek owns a snack shop where he sells tins of buttered and caramel popcorn. The table shows the number of each type of popcorn sold over *w* weeks. After 12 weeks, how much more will he have made in sales of buttered popcorn than the sales of caramel popcorn?

Popcorn Sales

Popcorn	Cost ($)	Number Sold Over *w* Weeks
Buttered	11	$8w + 9$
Caramel	11	$6w - 1$

16. **Identify Structure** Write two linear expressions that have a difference of $x + 1$.

17. **Identify Structure** What linear expression would you need to subtract from $(5x + 3)$ to have a difference of $-x$?

18. **Which One Doesn't Belong?** Identify the linear expression that does not belong with the other three. Explain your reasoning.

a. $(-5x + 3) - (-7x - 1)$

b. $(-3x + 3) - (-5x - 2)$

c. $(x - 6) - (-x - 10)$

d. $(-7x + 2) - (-9x - 2)$

19. **Find the Error** A student simplified the expression $(6x - 2) - (-x + 5)$ to $7x + 3$. Find the student's error and correct it.

Factor Linear Expressions

I Can... use GCF to factor linear expressions.

What Vocabulary Will You Learn?
factor
factored form
greatest common factor
monomial

Learn Monomials

A **monomial** is a number, a variable, or a product of a number and one or more variables. For example, $2x$ is a monomial because it is a product of 2 and x. The expression $x + 4$ is *not* a monomial because it is the sum of two monomials.

Sort the expressions by writing each one in the appropriate bin. Examples of each type are given.

$-12 \qquad 60x \qquad \frac{3}{4}x - 7 \qquad -\frac{2}{3}x \qquad -6x + \frac{1}{2}$

Monomials	Not Monomials
25	$x + 4$
$2x$	$2x + 5$

Learn Greatest Common Factor of Monomials

To **factor** a number means to write it as a product of its factors. A monomial can be factored using the same method you would use to factor a number.

The **greatest common factor** (GCF) of two monomials is the greatest monomial that is a factor of both. The greatest common factor also includes any variables that the monomials have in common.

Number	Monomial
The GCF of 25 and 30 is 5.	The GCF of $25x$ and $30xy$ is $5x$.

Talk About It!
Based on what you know about finding the GCF of numbers, how do you think you can find the GCF of monomials?

Your Notes

Think About It!

How would you begin finding the GCF?

Example 1 Find the GCF of Monomials

Find the GCF of 12*y* and 30*y*.

Step 1 Find the GCF of the coefficients.

Circle the factors of 12 and underline the factors of 30.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

The common factors of 12 and 30 are numbers that you both circled and underlined. Find the greatest of these numbers.

The greatest common factor of the coefficients is 6.

Step 2 Find the GCF of the variables.

The common variable of 12y and 30y is y.

Step 3 Find the GCF of the monomials.

To find the GCF of the monomials, multiply the GCF of the coefficients, ________, by the common variable, ________.

So, the GCF of 12y and 30y is 6y.

Check

Find the GCF of 24a and 32a.

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Did you make any errors when completing the Check exercise? What can you do to make sure you don't repeat that error in the future?

Record your observations here

Example 2 Find the GCF of Monomials

Use prime factorization to find the GCF of 18*a* and 20*ab*.

Step 1 Identify common factors.

Complete the prime factorization for each term.

$18a = 2 \cdot 3 \cdot 3 \cdot a$

$20ab = 2 \cdot 2 \cdot 5 \cdot a \cdot b$

The common factors in each term are 2 and a.

Step 2 Multiply the common factors.

The GCF of $18a$ and $20ab$ is $2 \cdot a$ or ________.

Check

Use prime factorization to find the GCF of $42xy$ and $14y$.

Show your work here

Think About It!

What variable(s) are common in each term?

Talk About It!

The factor 2 is common in each term. Why can only one 2 of the term 20*ab* be selected as a common factor?

Go Online You can complete an Extra Example online.

Explore Factor Linear Expressions

Online Activity You will use algebra tiles to explore how to factor linear expressions.

Learn Factor Linear Expressions

You can use the Distributive Property and work backward to express a linear expression as a product of its factors. A linear expression is in **factored form** when it is expressed as the product of its factors.

Go Online Watch the animation to learn how to factor $12a + 6b$.

Step 1 Find the GCF of the terms.

$12a = 2 \cdot 2 \cdot 3 \cdot a$ Write the prime factorization of each term.

$6b = 2 \cdot 3 \cdot b$ Circle the common factors.

$2 \cdot 3$ or 6 Multiply the common factors to find the GCF.

Step 2 Write each term as a product with the GCF as a factor.

$$12a + 6b = 6(2a) + 6(b)$$

Step 3 Apply the Distributive Property.

$$6(2a) + 6(b) = 6(2a + b)$$

So, the factored form of $12a + 6b$ is $6(2a + b)$.

Think About It!
What is the GCF of $3x$ and 9?

Example 3 Factor Linear Expressions

Factor $3x + 9$.

$3x = 3 \cdot x$ Write the prime factorization of each term.

$9 = 3 \cdot 3$ Circle the common factors.

3 Multiply, if necessary, the common factors to find the GCF.

$3x + 9 = 3(\boxed{}) + 3(\boxed{})$ Write each term as a product of its factors.

$= 3(x + 3)$ Distributive Property

So, $3x + 9$ is ______________

Talk About It!
How can you check to see if the factored form is correct?

Check

Factor $6x + 14$.

Go Online You can complete an Extra Example online.

Example 4 Expressions With No Common Factors

Factor $12x + 7y$.

Write the prime factorization of each term.

$12x = 2 \cdot 2 \cdot 3 \cdot x$

$7y = 7 \cdot y$

What, if any, are the common factors?

Because there are no common factors, $12x + 7y$ cannot be factored.

Check

Which expression represents the factored form of $12x + 11$?

Ⓐ $12(x + 11)$

Ⓑ $12(x - 11)$

Ⓒ $6(2x + 11)$

Ⓓ The expression cannot be factored.

Go Online You can complete an Extra Example online.

Pause and Reflect

Describe how you could determine that an expression cannot be factored mentally.

Record your observations here

Think About It!

How do you find the GCF of two monomials?

Talk About It!

Write another expression that cannot be factored. Explain why it cannot be factored.

Example 5 Factor Linear Expressions

Factor $\frac{1}{4}x + \frac{1}{4}$.

To factor $\frac{1}{4}x + \frac{1}{4}$, write each term as a product of the GCF and its remaining factors. The GCF of the terms is $\frac{1}{4}$.

$$\frac{1}{4}x + \frac{1}{4} = \frac{1}{4}\left(\boxed{}\right) + \frac{1}{4}\left(\boxed{}\right)$$

Write each term as a product of its factors.

$$= \frac{1}{4}\left(\boxed{}\right)$$

Distributive Property

So, $\frac{1}{4}x + \frac{1}{4} = \frac{1}{4}(x + 1)$.

Think About It!
How would you begin factoring the expression?

Talk About It!
When you factor out the common factor of $\frac{1}{4}$, where does the 1 come from in the expression $\frac{1}{4}(x + 1)$?

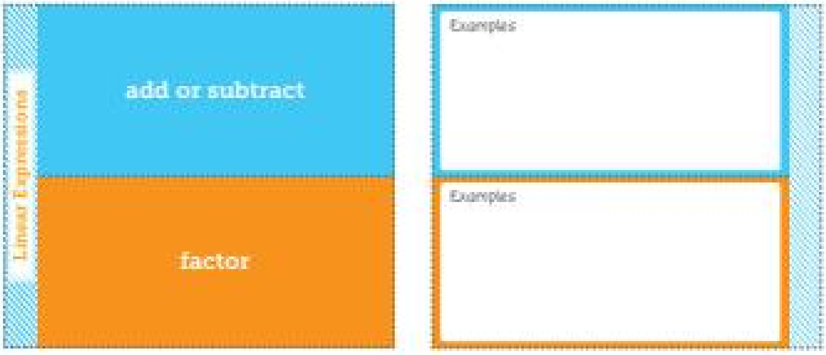
Check

Factor $\frac{2}{3}x - \frac{2}{3}$.

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Name ______________________ Period __________ Date __________

Practice

Go Online You can complete your homework online.

Find the GCF of each pair of monomials. (Example 1)

1. $4y, 12y$

2. $48x, 32x$

3. $16mn, 24m$

Use prime factorization to find the GCF of each pair of monomials. (Example 2)

4. $8xy, 12x$

5. $14ab, 28ab$

6. $27cd, 72cde$

Factor each expression. If the expression cannot be factored, write *cannot be factored*. (Examples 3–5)

7. $5x + 35$

8. $8x - 14$

9. $3x + 11y$

10. $32x - 15$

11. $72x - 18xy$

12. $45xy - 81y$

13. $25x + 14y$

14. $\frac{1}{3}x - \frac{1}{3}$

15. $\frac{1}{2}x + \frac{1}{2}$

Test Practice

16. Multiselect Select all of the expressions that cannot be factored.

☐ $7x - 14y$

☐ $9x + 31$

☐ $4x - 5y + 2z$

☐ $27x - 18y$

☐ $15x - 28y$

☐ $24x + 12x$

Apply

17. The total cost for Baydan and three of her friends to go ice skating can be represented by the expression $4x + 36$. The four friends pay an amount x to rent the ice skates and an admission fee. How much is the admission fee for one person?

18. The amount, in dollars, of Marisa's savings account can be represented by the expression $5x + 40$. Marisa saved the same amount x each month for a period of 5 months. Her mother contributed an additional amount each month to Marisa's savings account. How much did her mother contribute each month?

19. **Identify Structure** Write two monomials whose greatest common factor is $3x$.

20. **Identify Structure** What expression, in factored form, is $3x(3 + 7y)$?

21. **Which One Doesn't Belong?** Identify the expression that does not belong with the other three. Explain your reasoning.

a. $7x + 35$

b. $3x - 27$

c. $7x + 3$

d. $7x + 21$

22. **Find the Error** A student is factoring $18x + 6x$. Find the student's mistake and correct it.

$$18x + 6x = 6x(3)$$
$$= 18x$$

Lesson 5-5

Combine Operations with Linear Expressions

I Can... combine operations to simplify linear expressions.

Example 1 Combine Operations to Simplify Expressions

Simplify $-2(x + 3) + 8x$. Write your answer in factored form.

$-2(x + 3) + 8x = -2x - 6 + 8x$ Distributive Property

$= 6x - 6$ Combine like terms.

$= 6(x - 1)$ Write in factored form.

So, $-2(x + 3) + 8x =$ ____________.

Think About It!
How would you begin simplifying the expression?

Talk About It!
Why was the Distributive Property performed first?

Check

Simplify $-3(4x - 9) + 30x$. Write your answer in factored form.

Go Online You can complete an Extra Example online.

Example 2 Combine Operations to Simplify Expressions

Simplify $\frac{3}{8}x + \frac{1}{2}\left(\frac{1}{4}x - 2\right)$.

$\frac{3}{8}x + \frac{1}{2}\left(\frac{1}{4}x - 2\right)$ Write the expression.

$= \frac{3}{8}x + \left(\frac{1}{2}\right)\left(\frac{1}{4}x\right) - \left(\frac{1}{2}\right)(2)$ Distributive Property

$= \frac{3}{8}x + \frac{1}{8}x - 1$ Multiply.

$= \frac{4}{8}x - 1$ Combine like terms.

$= \frac{1}{2}x - 1$ Simplify.

So, $\frac{3}{8}x + \frac{1}{2}\left(\frac{1}{4}x - 2\right) = \frac{1}{2}x - 1$.

Talk About It!
Why is the answer $\frac{4}{8}x - 1$ not completely simplified?

Your Notes

Think About It!
How would you begin simplifying the expression?

Check

Simplify $\frac{5}{12}a - \frac{1}{4}\left(\frac{2}{3}a - 8\right)$.

Show your work here

Example 3 Combine Operations to Simplify Expressions

Simplify $\frac{2}{3}(18x - 12) - (6x + 7)$. Write your answer in factored form.

$\frac{2}{3}(18x - 12) - (6x + 7)$

$= (12x - 8) - (6x + 7)$ Distributive Property

$= (12x - 8) + (-6x - 7)$ Add the additive inverse.

$$\begin{array}{rrr} = & 12x & -\ 8 \\ (+) & -6x & -\ 7 \\ \hline & 6x & -\ 15 \end{array}$$

Arrange like terms in columns.

Add.

$= 3(2x - 5)$ Write in factored form.

So, $\frac{2}{3}(18x - 12) - (6x + 7) =$ ____________ .

Check

Simplify $\frac{2}{3}(27x - 45) - (4x - 9)$. Write your answer in factored form.

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

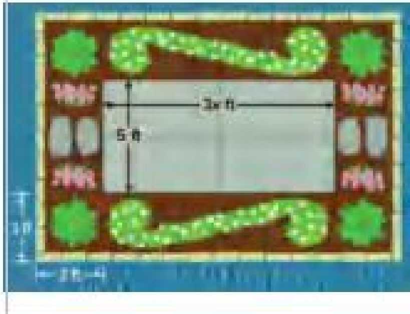
What part(s) of the lesson made you want to learn more? Why?

Record your observations here

Apply Gardening

A garden consists of a rectangular sitting region surrounded by a flower border. The sitting region has a length of $3x$ feet and a width of 5 feet. The flower border is 3 feet wide. Write an expression, in factored form, that represents the area of the flower border.

Go Online watch the animation.



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

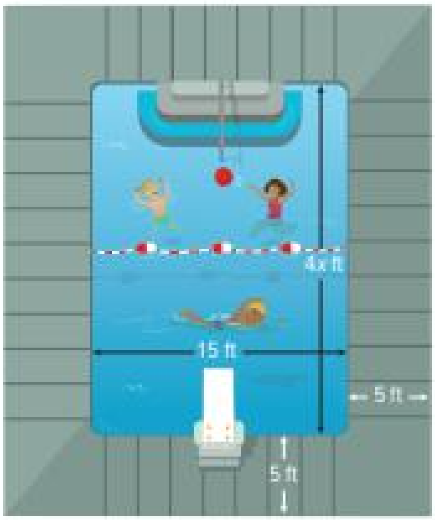
Talk About It!
Why can't you find a numerical value for the area of the flower border?

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Check

The diagram shows a walkway that is 5 feet wide surrounding a rectangular swimming pool. Write an expression, in factored form, that represents the area of the walkway.



Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Write a real-world problem that uses the concepts from today's lesson. Explain how you came up with that problem. Exchange problems with a classmate and solve each other's problem.

Record your observations here

Name _______________ Period _______ Date _______

Practice

Go Online You can complete your homework online.

Simplify each expression. For Exercises 1–4 and 9–12, write your answer in factored form. (Examples 1–3)

1. $3(x + 4) + 5x$

2. $-4(x + 1) + 6x$

3. $-5(2x - 6) + 25x$

4. $2(-8x - 3) + 18x$

5. $\frac{1}{6}x + \frac{3}{4}\left(\frac{1}{2}x - 4\right)$

6. $\frac{2}{3}\left(6x - \frac{1}{6}\right) + 3x$

7. $\frac{5}{8}x + \frac{1}{2}\left(\frac{1}{4}x + 10\right)$

8. $\frac{2}{5}\left(10x + \frac{3}{4}\right) - 2x$

9. $\frac{3}{4}(24x + 28) - (4x - 1)$

10. $-\frac{1}{2}(32x - 40) + (20x - 4)$

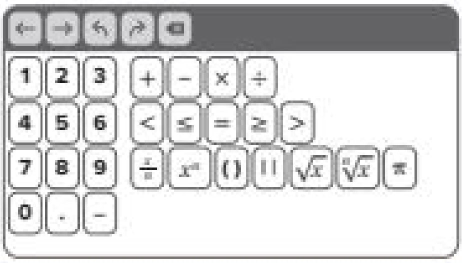
11. $\frac{2}{3}(9x - 15) - (-6x + 2)$

12. $-\frac{4}{5}(30x - 40) + (42x + 4)$

Test Practice

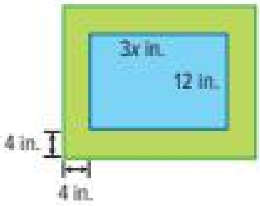
13. Equation Editor The table shows the area of two rugs a preschool teacher has in her room. She places the two rugs together to make one big rug. What is the area in square units of the new rug in factored form?

Rug	Area (square units)
Blue	$8\left(x + \frac{1}{2}\right)$
Red	$10\left(x + \frac{2}{5}\right)$

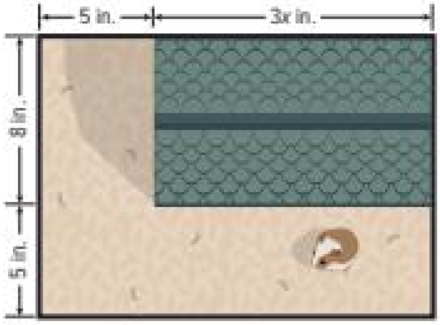


Apply

14. The diagram shows a border that is 4 inches wide surrounding Annie's painting. Write an expression, in factored form, that represents the area of the border.



15. Raul's hamster's house sits in the corner of its cage as shown in the diagram. The area around the house is 5 inches wide. Write an expression, in factored form, that represents the area around the house.



16. Create Write and simplify a linear expression with more than one operation.

17. MP **Find the Error** A student is simplifying the expression below. Find the student's error and correct it.

$$-3(x + 2) + 6x = -3x + 2 + 6x$$
$$= 3x + 2$$

18. A student said that $\frac{4}{6}x + 1$ is written in simplest form. Is the student correct? Explain why or why not.

19. MP **Persevere with Problems** Write each expression in factored form.

a. $\frac{1}{2}x + 6$

b. $\frac{3}{4}x - 18$

Review

Foldables Use your Foldable to help review the module.

Linear Expressions

Explanation

Explanation

Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.

Write about a question you still have.

Reflect on the Module

Use what you learned about expressions to complete the graphic organizer.

Essential Question

Why is it beneficial to rewrite expressions in different forms?

Combine Like Terms

Example

How is it helpful?

Distributive Property

Example

How is it helpful?

Add Linear Expressions

Example

How is it helpful?

Simplify Linear Expressions

Subtract Linear Expressions

Example

How is it helpful?

Factor Linear Expressions

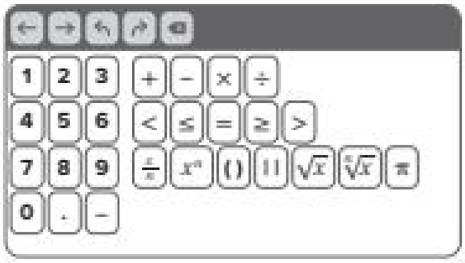
Example

How is it helpful?

Name ______________________ Period ____________ Date ____________

Test Practice

1. **Equation Editor** The cost of Noah's lunch, c, after a 15% tip can be represented by the expression $c + 0.15c$. What is this expression written in simplest form? (Lesson 1)



2. **Multiple Choice** Which of the following correctly shows the result when $-6(5a - 3b)$ is expanded using the Distributive Property? (Lesson 1)

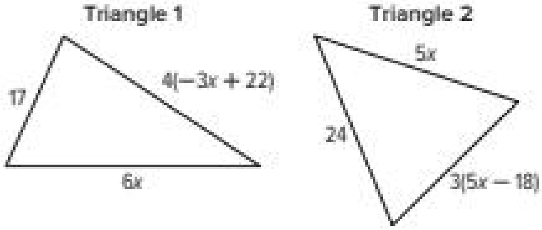
   Ⓐ $-30a - 18b$

   Ⓑ $-30a + 18b$

   Ⓒ $-6a - 30b$

   Ⓓ $-a - 9b$

3. **Open Response** Two triangles are shown. (Lesson 1)



   A. Represent the perimeter of each triangle as an algebraic expression written in simplest form.

   B. If $x = 5$, which triangle has a greater perimeter?

4. **Multiselect** Which of the following expressions are linear? Select all that apply. (Lesson 2)

   ☐ $8x - 1$

   ☐ $-3x$

   ☐ $7xy$

   ☐ $9x^2$

   ☐ $4x + 3$

   ☐ $\frac{1}{x}$

5. **Multiple Choice** What is the simplest form of $(-7x + 3) + (11x - 4)$? (Lesson 2)

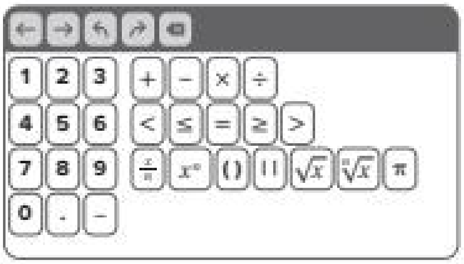
   Ⓐ $4x - 7$

   Ⓑ $4x + 7$

   Ⓒ $4x - 1$

   Ⓓ $18x + 1$

6. **Equation Editor** What is $(-3x + 5) - (-7x + 2)$? (Lesson 3)



7. **Open Response** What is $\left(\frac{1}{4}x - 7\right) - \left(\frac{3}{4}x - 5\right)$? Explain how you found your answer. (Lesson 3)

8. Open Response The table shows the sales of retractable leashes and standard leashes at a pet store for *w* weeks. (Lesson 3)

Style	Price ($)	Number Sold
Retractable	17	$8w + 5$
Standard	9	$3w + 2$

A. Write an expression that represents the difference between the number of retractable leashes sold and the number of standard leashes sold.

B. After 15 weeks, how many more retractable leashes has the store sold?

9. Multiple Choice Which of the following is the GCF of $16m$ and $40mn$? (Lesson 4)

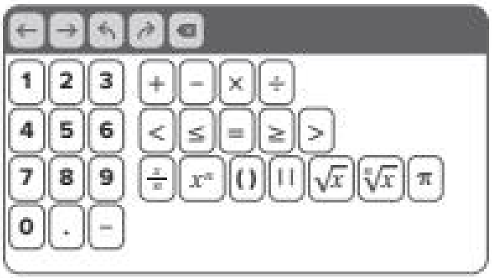
Ⓐ 8

Ⓑ $4m$

Ⓒ $8m$

Ⓓ $4mn$

10. Equation Editor What is the GCF of $20x$ and $25x$? (Lesson 4)



11. Multiple Choice What is $10xy - 15y$ written in factored form? (Lesson 4)

Ⓐ $5y(2x - 3)$

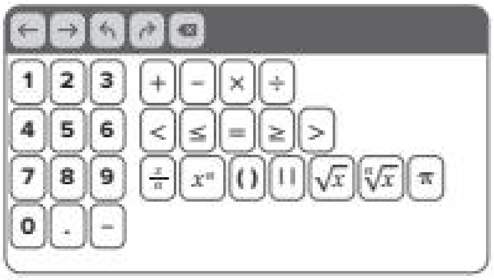
Ⓑ $5(2x - 3y)$

Ⓒ $10y(x - 3)$

Ⓓ cannot be factored

12. Open Response Factor $12m + 5n$. Explain how you found your answer. (Lesson 4)

13. Equation Editor Simplify $3(2x - 1) - 4x$. Write your answer in factored form. (Lesson 5)



14. Open Response A swimming pool has a 3-foot wide cement walkway around its perimeter as shown. (Lesson 5)

Write an expression, in factored form, that represents the area (in square feet) of the cement walkway.

Module 6

Write and Solve Equations

Essential Question

How can equations be used to solve everyday problems?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY

■ — I don't know. ◆ — I've heard of it. ★ — I know it!

	Before			After		
	■	◆	★	■	◆	★
solving one-step equations involving integers and rational numbers						
writing one-step equations, involving integers and rational numbers, to represent real-world problems						
solving two-step equations in the form of $px + q = r$						
writing two-step equations in the form of $px + q = r$ to represent real-world problems						
solving two-step equations in the form of $p(x + q) = r$						
writing two-step equations in the form of $p(x + q) = r$ to represent real-world problems						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about writing and solving equations.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ Addition Property of Equality
- ☐ defining a variable
- ☐ Division Property of Equality
- ☐ equation
- ☐ equivalent equations
- ☐ Inverse Property of Multiplication
- ☐ Multiplication Property of Equality
- ☐ solution
- ☐ Subtraction Property of Equality
- ☐ two-step equation

Are You Ready?

Study the Quick Review to see if you are ready to start this module.
Then complete the Quick Check.

Quick Review

Example 1

Write algebraic expressions.

Write the phrase *six points more than Jake's score* as an algebraic expression.

Let j represent the number of points Jake scored.

The phrase *more than* implies addition.

The expression is $j + 6$.

Example 2

Determine solutions to equations.

Is 1, 2, or 3 the solution of the equation $x - 2 = 1$?

$1 - 2 \overset{?}{=} 1$ Substitute 1 for x in the equation.

$-1 \neq 1$

$2 - 2 \overset{?}{=} 1$ Substitute 2 for x in the equation.

$0 \neq 1$

$3 - 2 \overset{?}{=} 1$ Substitute 3 for x in the equation.

$1 = 1$

The solution is 3 because replacing x with 3 results in a true sentence.

Quick Check

1. Write the phrase *a number n decreased by fifteen* as an algebraic expression.

2. Is 5, 6, or 7 the solution of the equation $x + 12 = 17$?

How Did You Do?

Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right.

① ②

Write and Solve One-Step Equations

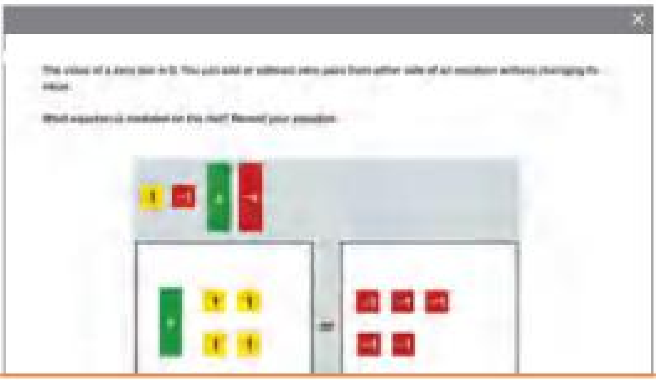
I Can... write one-step equations involving integers and rational numbers and use inverse operations to solve the equations.

What Vocabulary Will You Learn?

- Addition Property of Equality
- defining a variable
- Division Property of Equality
- equation
- equivalent equations
- Inverse Property of Multiplcation
- Multiplication Property of Equality
- solution
- Subtraction Property of Equality

Explore Solve One-Step Equations Using Algebra Tiles

Online Activity You will use algebra tiles to explore how to model and solve one-step equations.



Learn Properties of Equality

You can use the properties of equality to solve equations algebraically.

Property	Words	Symbols
Addition Property of Equality	Two sides of an equation remain equal when you add the same number to each side.	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	Two sides of an equation remain equal when you subtract the same number from each side.	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	Two sides of an equation remain equal if you multiply each side by the same number.	If $a = b$, then $ac = bc$.
Division Property of Equality	Two sides of an equation remain equal when you divide each side by the same nonzero number.	If $a = b$, and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Your Notes

Learn Equations

An **equation** is a sentence stating that two quantities are equal. The value of a variable that makes an equation true is called the **solution** of the equation.

The equations $x + 2 = 6$ and $x = 4$ are **equivalent equations** because they have the same solution, $x = 4$.

Example 1 Solve One-Step Addition Equations

Solve $-5 = b + 8$.

$-5 = b + 8$	Write the equation.
$\underline{-8 \qquad -8}$	Subtraction Property of Equality
$-13 = b$	Simplify.

Check your solution.

$-5 = b + 8$	Write the original equation.
$-5 \overset{?}{=} \boxed{} + 8$	Replace b with -13.
$-5 = \boxed{}$	The sentence is true.

So, the solution of the equation is $b = -13$.

Think About It!
What operation is paired with the variable? How do you undo that operation?

Talk About It!
Why are $-5 = b + 8$ and $-13 = b$ equivalent equations?

Check

Solve $-7 = x + 4$.

Show your work here

Go Online You can complete an Extra Example online.

Example 2 Solve One-Step Subtraction Equations

Solve $x - 4 = -2$.

$x - 4 = -2$	Write the equation.
$\underline{\;+4 \quad +4\;}$	Addition Property of Equality
$x \;\; = 2$	Simplify.

Check your solution.

$x - 4 = -2$	Write the original equation.
$2 - 4 \stackrel{?}{=} -2$	Replace x with 2.
$-2 = -2$	The sentence is true.

So, the solution of the equation is $x =$ ______________.

Talk About It!

How do you know the solution is correct?

Check

Solve $q - 8 = -9$.

Show your work here

Go Online You can complete an Extra Example online.

Example 3 Solve One-Step Multiplication Equations

Solve $-8y = 24$.

$-8y = 24$	Write the equation.
$\frac{-8y}{-8} = \frac{24}{-8}$	Division Property of Equality
$y = \square$	Simplify.

Check your solution.

$-8y = 24$	Write the original equation.
$-8(-3) \stackrel{?}{=} 24$	Replace y with -3.
$\square = 24$	The sentence is true.

So, the solution of the equation is $y = -3$.

Talk About It!

Why is the solution of the equation negative?

Check

Solve $-6a = 36$.

Show your work here

Example 4 Solve One-Step Division Equations

Solve $\frac{a}{-4} = -9$.

Think About It!
What property will you use to solve the equation? Why?

$\frac{a}{-4} = -9$	Write the equation.
$\frac{a}{-4}(-4) = -9(-4)$	Multiplication Property of Equality
$a = \boxed{}$	Simplify.

Talk About It!
Why does $\frac{a}{-4}(-4)$ equal a?

Check your solution.

$\frac{a}{-4} = -9$	Write the original equation.
$\frac{\boxed{}}{-4} \overset{?}{=} -9$	Replace a with 36.
$-9 = -9$	The sentence is true.

So, the solution of the equation is $a = 36$.

Check

Solve $\frac{y}{-3} = -8$.

Show your work here

Go Online You can complete an Extra Example online.

Learn Multiplicative Inverse

Two numbers with a product of 1 are called reciprocals, or multiplicative inverses. The **Inverse Property of Multiplication** can be used to solve equations involving rational number coefficients.

Words	Numbers	Symbols
The product of a number and its multiplicative inverse is 1.	$\frac{7}{8} \times \frac{8}{7} = 1$ $-\frac{3}{2}\left(-\frac{2}{3}\right) = 1$	$\frac{a}{b} \cdot \frac{b}{a} = 1$, where a and $b \neq 0$

Example 5 Solve Equations with Fractional Coefficients

Solve $-1\frac{7}{8}x = 4\frac{1}{2}$.

If the coefficient in a multiplication equation is a fraction, multiply each side by the reciprocal of the coefficient.

$-1\frac{7}{8}x = 4\frac{1}{2}$	Write the equation.
$-\frac{15}{8}x = \frac{9}{2}$	Write the mixed numbers as improper fractions.
$\left(-\frac{8}{15}\right) \cdot \left(-\frac{15}{8}\right)x = \left(-\frac{8}{15}\right) \cdot \frac{9}{2}$	Multiply each side by the reciprocal of $-\frac{15}{8}$, $-\frac{8}{15}$.
$x = -\frac{72}{30}$	Simplify.
$x = -2\frac{2}{5}$	Simplify.

So, the solution of the equation is $x = -2\frac{2}{5}$.

Think About It!

Before you begin to solve for x, why is it helpful to write the mixed numbers as improper fractions?

Talk About It!

How do you know if your solution is correct?

Check

Solve $-\frac{7}{8}x = -\frac{21}{64}$.

Show your work here

Go Online You can complete an Extra Example online.

Example 6 Solve Equations with Decimal Coefficients

Solve $0.25x = -16$.

As with whole numbers, if the coefficient in a multiplication equation is a decimal, divide each side by the coefficient.

$0.25x = -16$	Write the equation.
$\frac{0.25x}{0.25} = \frac{-16}{0.25}$	Division Property of Equality
$x = -64$	Simplify.

So, the solution of the equation is $x =$ ________ .

Check

Solve $-4.7k = -10.81$.

Show your work here

Go Online You can complete an Extra Example online.

Think About It!

When working with fractions, what step do you need to take before you add or subtract?

Talk About It!

In the second line of the solution, why was $-\frac{1}{5}$ written as $-\frac{2}{10}$?

Example 7 Solve Equations with Rational Numbers

Solve $-\frac{1}{5} = x - \frac{3}{10}$.

$-\frac{1}{5} = x - \frac{3}{10}$	Write the equation.
$-\frac{2}{10} = x - \frac{3}{10}$	Rename $-\frac{1}{5}$ as $-\frac{2}{10}$.
$+\frac{3}{10} \qquad +\frac{3}{10}$	Addition Property of Equality
$\frac{1}{10} = x$	Simplify.

So, the solution of the equation is $x =$ ________.

Check

Solve $-\frac{5}{6} + m = -\frac{2}{3}$.

Show your work here

Go Online You can complete an Extra Example online.

Learn Write One-Step Equations

Many real-world situations can be represented with equations. Consider the following problem. The highest recorded temperature in Warsaw, Missouri, is 118° Fahrenheit. This is 158° greater than the city's lowest recorded temperature. What is the lowest recorded temperature?

The steps show how to model the problem with an equation to represent this real-world situation.

Words
Describe the mathematics of the problem. The highest recorded temperature is 158° greater than the lowest recorded temperature.
Variable
Define the variable to represent the unknown quantity. Let t represent the lowest recorded temperature.
Equation
Translate the words into an algebraic equation. $118 = 158 + t$

Choosing a variable and a quantity for the variable to represent in an equation is called **defining a variable**.

Math History Minute

Mary Everest Boole (1832–1916) was a self-taught mathematician and the author of many texts on mathematics education. She supported cooperative learning and encouraged students to explore math through entertaining activities. Her book *Philosophy and Fun of Algebra* explained algebra through topics that were of interest to students.

Pause and Reflect

Explain to a partner which words or phrases in the problem you used to determine the numbers and symbols needed to write the equation. Then write the equation in a different way. Discuss with your partner how the equations can be written differently but have the same solution.

Record your observations here

Example 8 Write and Solve One-Step Equations

A stingray is at a certain depth in the ocean. After ascending $5\frac{1}{2}$ feet, it is now at a depth of $-14\frac{5}{6}$ feet.

Think About It!
What is the unknown in the problem?

Write and solve an equation to determine the stingray's initial depth.

Part A Write an equation.

The initial depth plus $5\frac{1}{2}$ feet equals $-14\frac{5}{6}$ feet.

$$d + 5\frac{1}{2} = -14\frac{5}{6}$$

Part B Solve the equation.

Talk About It!
How can you check your answer for reasonableness?

$$\begin{array}{rcll}
d + 5\frac{1}{2} &=& -14\frac{5}{6} & \text{Write the equation.} \\
d + 5\frac{3}{6} &=& -14\frac{5}{6} & \text{Rename the fractions with a common denominator.} \\
-5\frac{3}{6} && -5\frac{3}{6} & \text{Subtraction Property of Equality} \\
\hline
d &=& -19\frac{8}{6} & \text{Simplify.} \\
d &=& -20\frac{1}{3} & \text{Rename the fraction.}
\end{array}$$

So, the stingray's initial depth was ________ feet.

Check

At the end of the week, Madison had −$55.98 in her checking account. This is $202.64 less than the amount she had at the beginning of the week.

Part A

Which equation can be used to determine the amount of money *m* Madison had at the beginning of the week?

Ⓐ $-55.98 = m - 202.64$
Ⓑ $-55.98 = m + 202.64$
Ⓒ $202.64 = m - 55.98$
Ⓓ $-202.64 = m - 55.98$

Part B

How much money did Madison have at the beginning of the week?

Go Online You can complete an Extra Example online.

Name ____________________ Period __________ Date __________

Practice

Go Online You can complete your homework online.

Solve each equation. Check your solution. (Examples 1–7)

1. $6 + y = -8$

2. $-12 = 4 + c$

3. $p - 11 = -5$

4. $12 = z - 8$

5. $-7x = 56$

6. $-20 = -5b$

7. $\frac{d}{-9} = -6$

8. $15 = \frac{z}{-8}$

9. $2\frac{4}{5}x = -1\frac{1}{4}$

10. $-6 = \frac{3}{5}y$

11. $-6 = 0.2b$

12. $-0.8n = 2.8$

13. $c - 5.3 = -6.4$

14. $-\frac{1}{3} = -\frac{5}{6} + w$

15. $n + 7.1 = 8.6$

16. The table shows the balance of Rob's checking account at the end of the day. This is \$95.50 less than the amount he had at the beginning of the day. Write and solve an equation to determine the balance at the beginning of the day. (Example 8)

Time	Balance (\$)
Start of Day	?
End of Day	4.50

Test Practice

17. Open Response A diver is at a certain depth in the ocean. After ascending $10\frac{3}{4}$ feet, the diver is now at a depth of $-56\frac{1}{2}$ feet. Write and solve an equation to determine the diver's initial depth.

Apply

18. The table shows the lengths of hiking trails. It took Lannie 1.9 hours to hike Ranger Cove Trail. It took Ava 4 hours to hike Mohawk Trail. Write and solve an equation to find the average hiking rate for Lannie. Write and solve an equation to find the average hiking rate for Ava. What is the difference between their average hiking rates? Round to the nearest tenth.

Trail	Distance (miles)
Mohawk	6.8
Ranger Cove	4.8
Willow Oak	5.7

19. Monica earned twice as much as Samuel walking dogs. The amount Samuel earned was $7 more than Kara earned. Suppose Monica earned $48.50. Write and solve an equation to find the amount Samuel earned. Write and solve an equation to find the amount Kara earned. What is the difference between the greatest amount earned and the least amount earned walking dogs?

20. Create Write an addition equation and a subtraction equation that each have 4 as a solution.

21. **Find the Error** A student solved $\frac{1}{2}x = -40$. Find the student's mistake and correct it.

$$\frac{1}{2}x \div 2 = -40 \div 2$$
$$x = -20$$

22. **Persevere with Problems** Solve $2|x| = 16$. Explain your reasoning.

23. **Reason Abstractly** Suppose $a + b = 10$ and the value of a increases by 1. If the sum remains the same, what must happen to the value of b? Explain your reasoning.

Solve Two-Step Equations: $px + q = r$

I Can... use inverse operations to solve two-step equations of the form $px + q = r$.

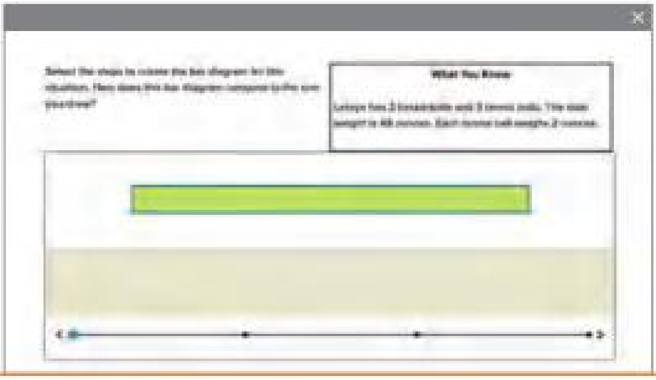
What Vocabulary Will You Learn?

order of operations

two-step equation

Explore Solve Two-Step Equations Using Bar Diagrams

Online Activity You will use bar diagrams to explore how to represent and solve two-step equations.



Explore Solve Two-Step Equations Using Algebra Tiles

Online Activity You will use algebra tiles to explore how to represent and solve two-step equations.

Your Notes

Talk About It!
Compare and contrast the equations.
$5x + 3 = 13$ and $5x = 10$.

Talk About It!
What property allows you to subtract 3 from each side of the equation?

Learn Two-Step Equations

A **two-step equation**, such as $5x + 3 = 13$, has two operations paired with the variable. In this case, the operations are multiplication and addition. To solve a two-step equation, undo the operations in reverse order of the **order of operations**.

Go Online Watch the animation to see how to solve the two-step equation $5x + 3 = 13$.

Equation	Steps
$5x + 3 = 13$ $\underline{\quad - 3 = -3}$ $5x = 10$	Undo the addition.
$\frac{5x}{5} = \frac{10}{5}$	Undo the multiplication.
$x = 2$	Simplify.
$5(2) + 3 \overset{?}{=} 13$ $13 = 13$ ✓	Check the solution.

Example 1 Solve Two-Step Equations

Solve $2x + 3 = 9$. Check your solution.

$2x + 3 = 9$	Write the equation.
$\underline{\quad - 3 \quad - 3}$	Subtract 3 from each side.
$2x = 6$	Simplify.
$\frac{2x}{2} = \frac{6}{2}$	Divide each side by 2.
$x = 3$	Simplify.

So, the solution of the equation is $x =$ ________.

Check your solution by substituting 3 for x in the equation.

$2(3) + 3 \overset{?}{=} 9$

$6 + 3 = 9$ ✓

Because $6 + 3 = 9$ is a true statement, the solution is correct.

Check

Solve $-3x + 5 = 14$. Check your solution.

Show your work here

Go Online You can complete an Extra Example online.

Example 2 Solve Two-Step Equations

Solve $-2y - 7 = 3$. Check your solution.

$-2y - 7 = 3$	Write the equation.
$\underline{\quad +7 \quad +7}$	Addition Property of Equality
$-2y = \boxed{}$	Simplify.
$\frac{-2y}{-2} = \frac{10}{-2}$	Division Property of Equality
$y = \boxed{}$	Simplify.

So, the solution of the equation is $y = -5$.

Check your solution by substituting -5 for y in the equation.

$-2(-5) - 7 \overset{?}{=} 3$

$10 - 7 = 3$ ✓

Because $10 - 7 = 3$ is a true statement, the solution is correct.

Check

Solve $5w - 8 = -3$.

Show your work here

Go Online You can complete an Extra Example online.

Think About It!
What two operations are paired with the variable?

Talk About It!
In the fourth line of the solution, why was each side of the equation divided by -2 instead of 2?

Example 3 Solve Two-Step Equations

Solve $5x - 3.8 = -6.4$. Check your solution.

$5x - 3.8 = -6.4$	Write the equation.
$\underline{\quad +3.8 \quad +3.8}$	Addition Property of Equality
$5x = -2.6$	Simplify.
$\frac{5x}{5} = \frac{-2.6}{5}$	Division Property of Equality
$x = \boxed{}$	Simplify.

So, the solution of the equation is $x = -0.52$.

Check your solution by substituting -0.52 for x in the equation.

$5(-0.52) - 3.8 \overset{?}{=} -6.4$

$-2.6 - 3.8 = -6.4$ ✓

Because $-2.6 - 3.8 = -6.4$ is a true statement, the solution is correct.

Check

Solve $-6x - 8.1 = -5.7$.

Show your work here

Example 4 Solve Two-Step Equations

Solve $4 + \frac{1}{5}r = -1$. Check your solution.

$4 + \frac{1}{5}r = -1$	Write the equation.
$\underline{-4 \qquad\quad -4}$	Subtraction Property of Equality
$\frac{1}{5}r = -5$	Simplify.
$5 \cdot \frac{1}{5}r = 5 \cdot (-5)$	Multiplication Property of Equality
$r = -25$	Simplify.

Think About It!

What do you notice about this equation?

Talk About It!

In the fourth line of the solution, why was each side of the equation multiplied by 5? Describe another strategy you can use to solve the equation.

So, the solution of the equation is $r = -25$.

Check your solution by substituting -25 for r in the equation.

$4 + \frac{1}{5}(-25) \stackrel{?}{=} -1$

$4 + (-5) \quad = -1$ ✓

Because $4 + (-5) = -1$ is a true statement, the solution is correct.

Check

Solve $-2 + \frac{2}{3}w = 10$.

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Create a two-step equation involving a fractional coefficient. Trade your equation with a partner. Solve each other's equations and explain to each other how you handled the fractional coefficient.

Record your observations here

Learn Two-Step Equations: Arithmetic Method and Algebraic Method

Using only numbers and operations to solve a problem is an arithmetic method.

Using variables to solve the problem is an algebraic method. Consider the following problem.

Natalie is participating in a jog-a-thon. Her goal is to raise $125. She has collected $80 and will receive $3 for each lap around the track she jogs. How many laps does she need to jog to reach her goal?

Arithmetic Method

The amount Natalie still needs to reach her goal can be represented by the following equation.

$\$125 - \$80 = \$45$

She will earn $3 for every lap she jogs. So, she needs to jog $\$45 \div 3$, or 15 laps to reach her goal.

Algebraic Method

Let x represent the number of laps Natalie needs to jog around the track to reach her goal. Write and solve an equation.

$$
\begin{aligned}
125 &= 80 + 3x && \text{Write the equation.} \\
\underline{-80\ } & \underline{\ -80\qquad} && \text{Subtract 80 from each side.} \\
\frac{45}{3} &= \frac{3x}{3} && \text{Divide each side by 3.} \\
15 &= x && \text{Simplify.}
\end{aligned}
$$

So, using either method, Natalie needs to jog 15 laps to reach her goal.

Pause and Reflect

How are the arithmetic and algebraic methods similar and different?

Record your observations here

(continued on next page)

Consider the following problem.

Rashan is saving money to buy a skateboard that costs \$85. He has already saved \$40. He plans to save the same amount each week for three weeks. How much should Rashan save each week?

Arithmetic Method

Solve the problem using the arithmetic method. Describe the steps you used.

Algebraic Method

Solve the problem using the algebraic method.

Let x represent the amount saved each week. The equation $40 + 3x = 85$ represents this situation. Solve the equation to find how much Rashan should save each week.

$$40 + 3x = 85$$

So, using either method, Rashan should save \$15 each week.

Talk About It!

Compare and contrast the arithmetic method and algebraic method used to solve the problem.

Pause and Reflect

Where did you encounter struggle in this lesson, and how did you deal with it? Write down any questions you still have.

Apply Chemistry

The table shows the freezing point of several solvents in degrees Fahrenheit (°F) or degrees Celsius (°C). The formula $F = 1.8C + 32$ can be used to convert between degrees Fahrenheit and degrees Celsius. What is the difference, in degrees Celsius, between acetone's freezing point and carbon disulfide's freezing point?

Solvent	Freezing Point
Acetone	−138.64°F
Carbon Disulfide	−111.5°C
Lauric Acid	44°C
Water	32°F

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

Talk About It!
Why did you need to convert acetone's freezing point to degrees Celsius?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Check

The table shows the average temperature of several planets in degrees Fahrenheit (°F) or degrees Celsius (°C). The formula $F = 1.8C + 32$ can be used to convert between degrees Fahrenheit and degrees Celsius.

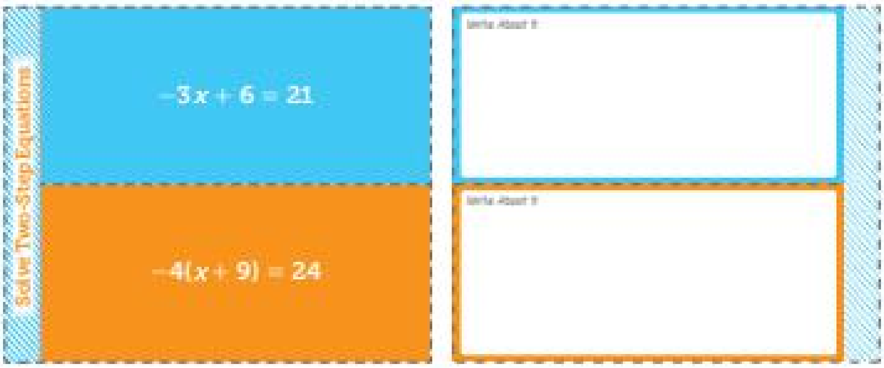
Planet	Average Temperature
Earth	57°F
Jupiter	−202°F
Mars	−63°C
Neptune	−200°C

What is the difference, in degrees Celsius, between Earth's average temperature and Mars' average temperature? Round to the nearest tenth.

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Name ______________________ Period __________ Date __________

Practice

Go Online You can complete your homework online.

Solve each equation. Check your solution. (Examples 1–4)

1. $5x + 2 = 17$

2. $19 = 4x + 3$

3. $-18 = 6 + 6x$

4. $-3x - 9 = -15$

5. $-6x - 7 = 17$

6. $-5 = 3x - 14$

7. $3.8 = 2x - 11.2$

8. $5x - 3.3 = 7.2$

9. $1.3x + 1.5 = 5.4$

10. $2 + \frac{1}{6}x = -4$

11. $-\frac{1}{2}x - 7 = 18$

12. $-9 = \frac{2}{7}x + 5$

Test Practice

13. Open Response The table shows the costs of a membership and fruit baskets at a discount warehouse club. Mrs. Williams paid a total of $105 for her annual membership fee and several fruit baskets as gifts for her coworkers. Solve the equation $15x + 30 = 105$ to find the number of fruit baskets Mrs. Williams purchased.

Item	Cost ($)
Membership Fee	30
Fruit Basket	15

Apply

14. The table shows the record high temperature, in degrees Fahrenheit (°F) or degrees Celsius (°C), of certain states. The formula $F = 1.8C + 32$ can be used to convert between degrees Celsius and degrees Fahrenheit. What is the difference, in degrees Celsius, between Nevada's record high temperature and Alaska's record high temperature? Round to the nearest tenth.

State	Record High Temperature
Alaska	38°C
Florida	109°F
Nevada	125°F
Texas	49°C

15. The table shows the boiling point of several liquids in degrees Fahrenheit (°F) or degrees Celsius (°C). The formula $F = 1.8C + 32$ can be used to convert between degrees Fahrenheit and degrees Celsius. What is the difference, in degrees Celsius, between petroleum's boiling point and iodine's boiling point?

Liquid	Boiling Point
Aniline	363.8°F
Iodine	184.3°C
Petroleum	410°F
Petrol	95°C

16. **Create** Write a real-world problem that could be represented by the equation $3x + 12 = 30$. Then solve the equation.

17. **Find the Error** A student is solving $-5 + 2x = 15$. Find the student's mistake and correct it.

$$-5 + 2x = 15$$
$$-5 + (-5) + 2x = 15 + (-5)$$
$$2x = 10$$
$$x = 5$$

18. **Identify Structure** Write a two-step equation that can be solved using the Addition Property of Equality and Division Property of Equality. Justify your reasoning.

19. Evaluate 4(3) − 1. Then solve the equation $4x - 1 = 13$. How are the problems and solutions alike? How are they different?

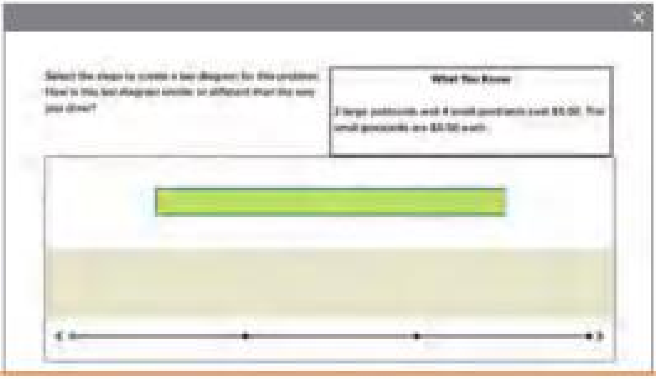
Lesson 6-3

Write and Solve Two-Step Equations: $px + q = r$

I Can... write two-step equations of the form $px + q = r$ and use inverse operations to solve the equations.

Explore Write Two-Step Equations

Online Activity You will use bar diagrams to explore how to write two-step equations to model and solve real-world problems.



Learn Write Two-Step Equations

Some real-world situations can be represented by two-step equations. Consider the following problem.

A caterer is preparing a dinner for a party. She charges an initial fee of \$16 and \$8.25 per person. How many people can attend a dinner that costs \$131.50?

The table shows how to model the problem with a two-step equation.

Words
Describe the mathematics of the problem. The initial fee of \$16 plus \$8.25 per person equals \$131.50.
Variable
Define the variable to represent the unknown quantity. Let p represent the number of people.
Equation
Translate the words into an algebraic equation. $16 + 8.25p = 131.50$

Your Notes

Talk About It!
How can you use the bar diagram to check your answer?

Talk About It!
Why was it important to subtract $27 before dividing by $8.50?

Example 1 Write and Solve Two-Step Equations

Toya had her birthday party at the movies. It cost $27 for pizza and $8.50 per friend for the movie tickets.

Write and solve an equation to determine how many friends Toya had at her party if she spent $78.

Part A Write an equation.

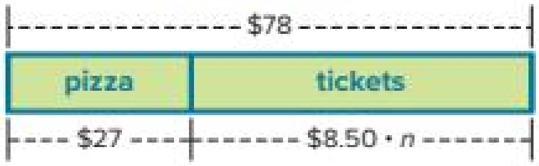
Words

The cost of pizza plus the cost per friend times the number of friends equals $78.

Variable

Let n represent the number of friends.

Bar Diagram



Equation

$$27 + 8.50n = 78$$

Part B Solve the equation.

$27 + 8.50n = \quad 78$	Write the equation.
$\underline{-27 \qquad\qquad -27}$	Subtraction Property of Equality
$8.50n = 51$	Simplify.
$\frac{8.50n}{8.50} = \frac{51}{8.50}$	Division Property of Equality
$n = 6$	Simplify.

So, Toya had ________ friends at her party.

Check the solution.
If Toya had ________ friends at her party, then the cost of the movie tickets was 6($8.50), or $51. Adding the cost of the pizza means the total cost was $51 + $27 or $78, which is what Toya spent. The solution is correct.

Check

Cassidy went to a football game with some of her friends. The tickets cost \$6.50 each, and they spent \$17.50 on snacks. The total amount paid was \$63.00. Write and solve an equation to determine the number of people p that went to the game.

Part A Write an equation that can be used to determine how many people p went to the game.

Part B How many people went to the game?

Go Online You can complete an Extra Example online.

Example 2 Write and Solve Two-Step Equations

Diego's aquarium contains $30\frac{1}{2}$ gallons of water. He drains the water at a rate of 5 gallons per minute for cleaning.

Write and solve an equation to determine in how many minutes the amount of water will reach $10\frac{1}{2}$ gallons.

Part A Write an equation.

Words
$30\frac{1}{2}$ gallons minus 5 gallons per minute equals $10\frac{1}{2}$ gallons.
Variable
Let m represent the number of minutes.
Equation
$30\frac{1}{2} - 5m = 10\frac{1}{2}$

Part B Solve the equation.

$30\frac{1}{2} - 5m = 10\frac{1}{2}$	Write the equation.
$\underline{-30\frac{1}{2} \qquad\qquad -30\frac{1}{2}}$	Subtraction Property of Equality
$-5m = -20$	Simplify.
$\frac{-5m}{-5} = \frac{-20}{-5}$	Division Property of Equality
$m = 4$	Simplify.

So, it will take ________ minutes to drain the tank to a level of $10\frac{1}{2}$ gallons.

Think About It!
What is the unknown in this problem?

Talk About It!
How can you check your answer for reasonableness?

Check

Amelia started with \$54, and spent \$6 each day at camp. She has \$18 left. Write and solve an equation to find how many days d Amelia was at camp.

Part A Which equation can be used to determine how many days d she was at camp?

Ⓐ $6 + 54d = 18$

Ⓑ $54 + 6d = 18$

Ⓒ $6 - 54d = 18$

Ⓓ $54 - 6d = 18$

Part B How many days was Amelia at camp?

Show your work here

Go Online You can complete an Extra Example online.

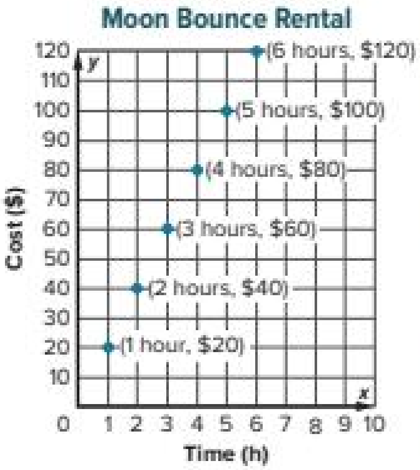
Pause and Reflect

Compare and contrast writing and solving two-step equations with writing and solving one-step equations.

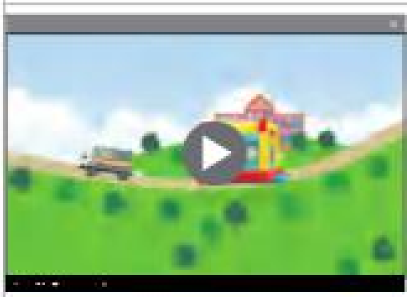
Record your observations here

Apply Budgets

The students at Worthingway Middle School would like to rent a moon bounce for an end-of-year party. The graph shows the cost to rent the moon bounce for locations within the regular delivery area. To deliver the moon bounce to locations beyond the regular delivery area, the company charges a $50 delivery fee. Worthingway Middle School is located beyond the regular delivery area. The students have $200 to spend. For how many full hours can they rent the moon bounce?



Go Online Watch the animation.



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

Talk About It!

How can you use the graph to determine the answer?

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

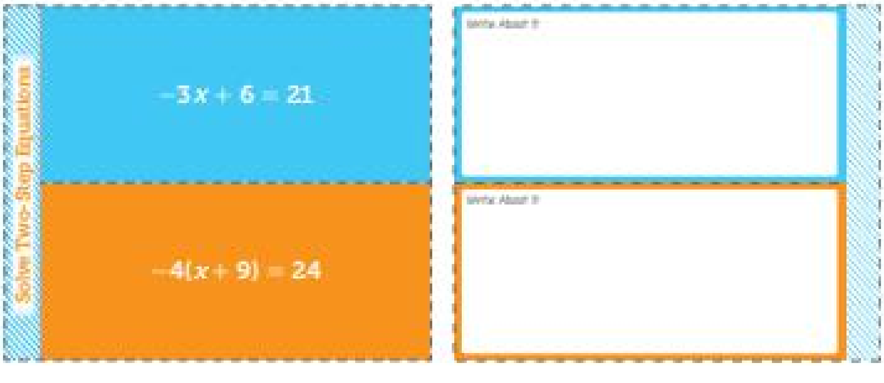
Check

Oliver earns an hourly wage, as shown in the graph. In addition to his hourly wage, he is eligible for bonuses. If he received a $100 bonus award for his performance and worked 20 hours, how much did he earn?

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Name ______________________ Period __________ Date __________

Practice

Go Online You can complete your homework online.

Write a two-step equation to represent each problem. Then solve the problem.

1. Easton went to a concert with some of his friends. The tickets cost $29.50 each, and they spent a total of $15 on parking. The total amount spent was $133. Determine how many people went to the concert. (Example 1)

2. Ishi bought a $6.95 canvas and 8 tubes of paint. She spent a total of $24.95 on the canvas and paints. Determine the cost of each tube of paint. (Example 1)

3. A taxi service charges $1.50 plus $0.60 per mile for a trip to the airport. The total charge is $13.50. Determine how many miles it is to the airport. (Example 1)

4. At the market, Meyer buys a bunch of bananas for $0.65 per pound and a frozen pizza for $4.99. The total for his purchase was $6.94, without tax. Determine how many pounds of bananas Meyer bought. (Example 1)

5. A hot air balloon is at an altitude of $100\frac{1}{5}$ yards. The balloon's altitude decreases by $10\frac{4}{5}$ yards every minute. Determine the number of minutes it will take the balloon to reach an altitude of 57 yards. (Example 2)

6. The current temperature is 48°F. It is expected to drop 1.5°F each hour. Determine in how many hours the temperature will be 36°F. (Example 2)

7. Mariko and her friend spent $24.50 on lunch. Their lunches cost the same amount, and they used a $4 off coupon. Determine the cost of each lunch. (Example 2)

Test Practice

8. **Open Response** The table shows the amount of water Joel had in his bathtub to wash his dog and his desired water level. If the water drains at a rate of 14 gallons per minute, how many minutes will it take the tub to drain to his desired level?

Starting Water Level	42 gallons
Desired Water Level	28 gallons

Apply

9. A face painting artist's fee for parties is shown in the table. Customers are also charged a $25 reservation fee. A school has $175 to spend on a face painting artist for their carnival. For how many full hours can they hire the artist?

Hours	Fee ($)
2	49.00
3	73.50
4	98.00
5	122.50

10. The table shows the cost of boarding a dog at a dog kennel. Owners are also charged a $10 registration fee. The Kittle family boarded their dog at the kennel for 7 days. What was the total, with registration fee, of boarding their dog?

Days	Cost ($)
3	97.50
4	130.00
5	162.50
6	195.00

11. Write a real-world problem that could be represented by the equation $2x + 5 = 35$. Then solve the equation.

12. MP **Persevere with Problems** Mia discovered that if she takes half her age and adds 3, it produces the same results as taking one-fourth of her age and adding 7. How old is Mia?

13. MP **Multiple Representations** At the start of a school trip, a minibus contains 28 gallons of gas. Each hour, the minibus uses 6 gallons of gas. The bus will stop when there are 4 gallons of gas left.

a. Make a table to show how many gallons of gas are remaining after 1, 2, and 3 hours.

b. Write and solve an equation to find how many hours will pass before the minibus will have to stop for gas.

14. **Create** Write and solve a real-world problem that involves decimals and can be solved using a two-step equation.

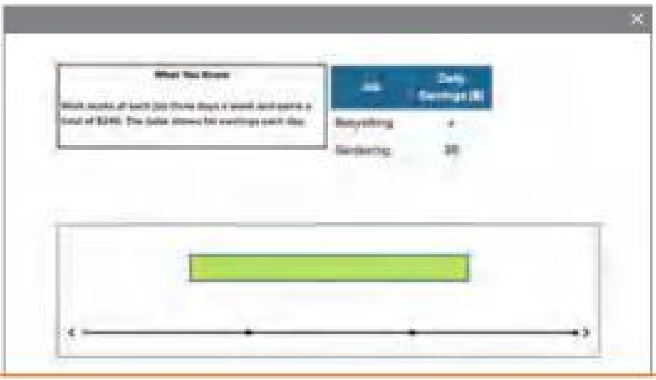
Lesson 6-4

Solve Two-Step Equations: $p(x + q) = r$

I Can... use inverse operations to solve two-step equations of the form $p(x + q) = r$.

Explore Solve Two-Step Equations Using Bar Diagrams

Online Activity You will use bar diagrams to explore how to model and solve two-step equations with parentheses.



Explore Solve Two-Step Equations Using Algebra Tiles

Online Activity You will use algebra tiles to explore how to model and solve two-step equations with parentheses.

Your Notes

Learn Two-Step Equations

An equation like $2(x + 6) = 14$ is in the form $p(x + q) = r$. It contains two factors, p and $(x + q)$, and is considered a two-step equation because two steps are needed to solve the equation.

Go Online Watch the animation to learn how to solve two-step equations with parentheses.

Follow the steps to solve the equation $3(x + 2) = -18$ using the Distributive Property.

Equation	Steps
$3(x + 2) = -18$	
$3x + 6 = -18$	Apply the Distributive Property.
$\underline{-6 \quad -6}$	Solve the resulting two-step equation.
$\frac{3x}{3} = \frac{-24}{3}$	
$x = -8$	

Follow the steps to solve the equation using the properties of equality.

Equation	Steps
$\frac{3(x + 2)}{3} = \frac{-18}{3}$	Undo the multiplication.
$x + 2 = -6$	Simplify.
$\underline{-2 \quad -2}$	Undo the addition.
$x = -8$	

Example 1 Solve Two-Step Equations

Solve $3(x + 5) = 45$. Check your solution.

Method 1 Use the Division Property of Equality first.

$3(x + 5) = 45$	Write the equation.
$\frac{3(x + 5)}{3} = \frac{45}{3}$	Division Property of Equality
$x + 5 = 15$	Simplify.
$\underline{-5 \quad -5}$	Subtraction Property of Equality
$x = 10$	Simplify.

Think About It!

How is the equation $3(x + 5) = 45$ different from the equation $3x + 5 = 45$?

(continued on next page)

Method 2 Use the Distributive Property first.

$3(x + 5) = 45$	Write the equation.
$3x + 15 = 45$	Distributive Property
$\underline{-15 \quad -15}$	Subtraction Property of Equality
$3x = 30$	Simplify.
$\frac{3x}{3} = \frac{30}{3}$	Division Property of Equality
$x = 10$	Simplify.

So, using either method, the solution of the equation is $x = 10$.

Check your solution by substituting the result back into the original equation.

$3(x + 5) = 45$	Write the original equation.
$3(10 + 5) \stackrel{?}{=} 45$	Replace x with 10.
$45 = 45$ ✓	The sentence is true.

Talk About It!
Compare and contrast the two methods used to solve the equation.

Check

Solve $2(x + 4) = -20$.

Show your work here

Go Online You can complete an Extra Example online.

Example 2 Solve Two-Step Equations

Solve $5(n - 2) = -30$. Check your solution.

Think About It!
How would you begin solving the equation?

Method 1 Use the Division Property of Equality first.

$5(n - 2) = -30$	Write the equation.
$\frac{5(n-2)}{5} = \frac{-30}{5}$	Division Property of Equality
$n - 2 = -6$	Simplify.
$\underline{+2 \quad +2}$	Addition Property of Equality
$n = -4$	Simplify.

(continued on next page)

Talk About It!
When might it be advantageous to use one method over the other?

Method 2 Use the Distributive Property.

$5(n-2)=-30$	Write the equation.
$5n-10=-30$	Distributive Property
$\underline{+10 \quad +10}$	Addition Property of Equality
$5n=-20$	Simplify.
$\frac{5n}{5}=\frac{-20}{5}$	Division Property of Equality
$n=-4$	Simplify.

So, using either method, the solution of the equation is $n=-4$.

Check your solution by substituting the result back into the original equation.

$5(n-2)=-30$	Write the original equation.
$5(-4-2)\stackrel{?}{=}-30$	Replace n with -4.
$-30=-30$ ✓	The sentence is true.

Check

Solve $3(x-6)=-12$.

Show your work here

Go Online You can complete an Extra Example online.

Example 3 Solve Two–Step Equations

Solve $0.2(c-3)=-10$. Check your solution.

$0.2(c-3)=-10$	Write the equation.
$\frac{0.2(c-3)}{0.2}=\frac{-10}{0.2}$	Division Property of Equality
$c-3=-50$	Simplify.
$\underline{+3 \quad +3}$	Addition Property of Equality
$c=-47$	Simplify.

Talk About It!
Describe an advantage of dividing each side by 0.2 as the first step to solving the equation, instead of using the Distributive Property to expand the expression.

So, the solution of the equation is $c=-47$.

Check your solution by substituting the result back into the original equation.

$0.2(c-3)=-10$	Write the original equation.
$0.2(-47-3)\stackrel{?}{=}-10$	Replace c with -47.
$-10=-10$ ✓	The sentence is true.

Check

Solve $0.8(m - 5) = 10$.

Show your work here

Example 4 Solve Two-Step Equations

Solve $\frac{2}{3}(n + 6) = 10$. Check your solution.

$\frac{2}{3}(n + 6) = 10$	Write the equation.
$\frac{3}{2} \cdot \frac{2}{3}(n + 6) = \frac{3}{2} \cdot 10$	Multiplication Property of Equality
$(n + 6) = \frac{3}{2} \cdot \frac{10}{1}$	$\frac{3}{2} \cdot \frac{2}{3} = 1$; write 10 as $\frac{10}{1}$.
$n + 6 = 15$	Simplify.
$\underline{-6 \quad -6}$	Subtraction Property of Equality
$n = 9$	Simplify.

So, the solution to the equation is $n = 9$.

Check your solution by substituting the result back into the original equation.

$\frac{2}{3}(n + 6) = 10$	Write the original equation.
$\frac{2}{3}(9 + 6) \overset{?}{=} 10$	Replace n with 9.
$10 = 10$ ✓	The sentence is true.

Think About It!

How can you use the reciprocal of $\frac{2}{3}$ to solve this equation?

Talk About It!

Why is the use of reciprocals important in solving the equation?

Check

Solve $\frac{1}{4}(d - 3) = -15$.

Show your work here

Go Online You can complete an Extra Example online.

Learn Two-Step Equations: Arithmetic Method and Algebraic Method

Using only numbers and operations to solve a problem is an arithmetic method. Using variables to solve the problem is an algebraic method. Consider the following problem.

Three friends went to a band party at a local farm. Each student spent the same amount of money and a total of \$21 altogether. Each student bought a hot dog for \$5. If they each also bought a hay ride ticket, how much did each hay ride ticket cost?

The table demonstrates how to use each of the methods to solve this problem.

Talk About It!

Compare and contrast the arithmetic method and algebraic method used to solve the problem.

Arithmetic Method	Algebraic Method
Divide the total amount spent by 3 to find the amount each friend spent. \$21 ÷ \$3 = \$7 Then subtract the cost of a hot dog from \$7 to find the cost of a hay ride ticket: \$7 − \$5 = \$2 So, each hay ride ticket cost \$2.	Let x represent the cost of a hay ride ticket. Then the equation $3(x + 5) = 21$ represents this situation. Solve the equation to find the cost of a hay ride ticket. $3(x + 5) = 21$ $\frac{3(x+5)}{3} = \frac{21}{3}$ $x + 5 = 7$ $\underline{-5 \quad -5}$ $x = 2$

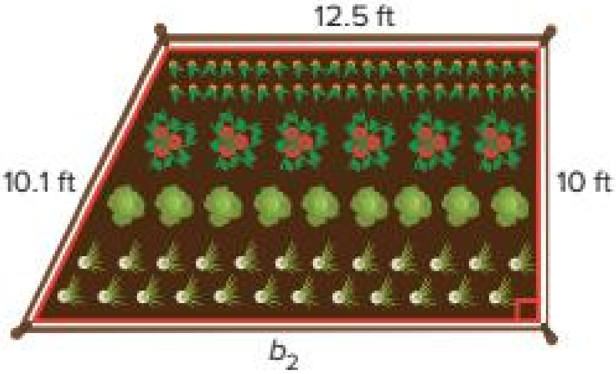
Pause and Reflect

Did you ask questions about today's lesson? Why or why not?

Record your observations here

Apply Gardening

The plans for a school vegetable garden, shaped like a trapezoid, are shown. The area of the garden is 132.5 square feet. A border is planned around the perimeter of the garden. How many feet of material are needed for the border?



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It!
How can you use the area formula for a trapezoid to help you solve this problem?

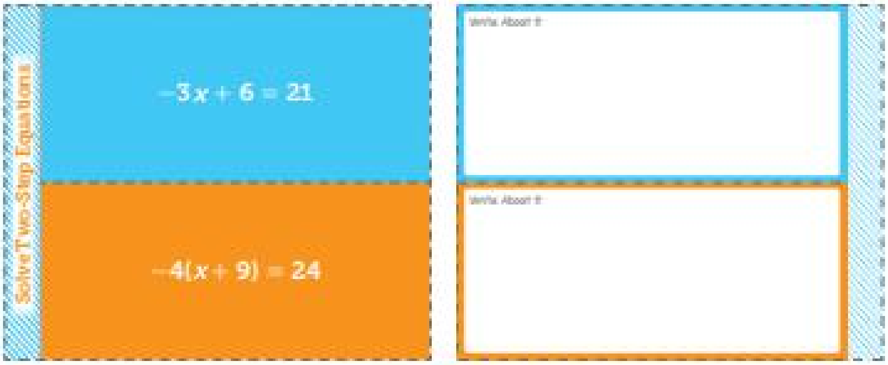
Check

The shape of Augello County resembles a trapezoid. The approximate area is 770 square miles. What is the perimeter of the county?

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Name ______________________ Period ____________ Date ____________

Practice

Go Online You can complete your homework online.

Solve each equation. Check your solution. (Examples 1–4)

1. $4(x + 8) = 44$

2. $7(x + 8) = 49$

3. $-2(x + 4) = 18$

4. $10(x - 5) = -80$

5. $-5(x - 10) = -35$

6. $-9(x - 4) = 81$

7. $0.4(x - 7) = 18$

8. $-0.25(8 + x) = 14$

9. $-0.8(10 - x) = 36$

10. $\frac{1}{2}(x - 4) = 5$

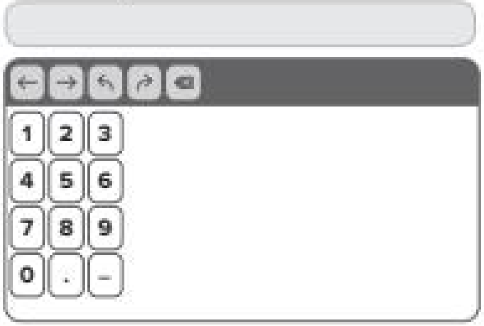
11. $\frac{4}{5}(x + 7) = 20$

12. $-\frac{7}{9}(x + 3) = 14$

Test Practice

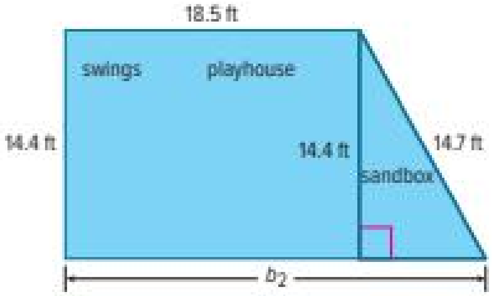
13. Equation Editor The reading rug in Mrs. Shaw's room has a perimeter of 28 meters. The table gives the length and width of the rug. If the equation $4(x + 3.3) = 28$ can be used to find the perimeter of the rug, what is the value of x?

Dimension	Measurement (m)
Length	$(x + 3.3)$
Width	$(x + 3.3)$

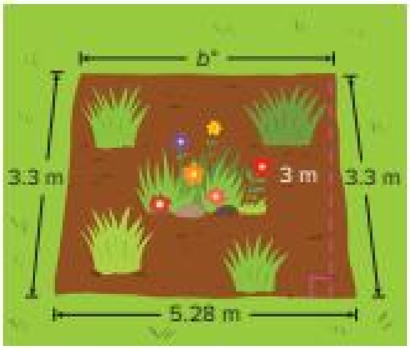


Apply

14. The plan for the Thorne's backyard play area, shaped like a trapezoid, is shown. The area of the play area is 286.56 square feet. A border is planned around the perimeter of the play area. How many feet of material are needed for the border?



15. A landscape architect designed a flower garden in the shape of a trapezoid. The area of the garden is 13.92 square meters. A fence is planned around the perimeter of the garden. How many meters of fencing are needed?



16. Write a real-world problem that could be represented by the equation $6(x + 3.5) = 57$. Then solve the equation.

17. **MP** **Find the Error** A student is solving $-2(x - 5) = 12$. Find the student's mistake and correct it.

$$-2(x - 5) = 12$$
$$-2x - 5 = 12$$
$$-2x - 5 + 5 = 12 + 5$$
$$-2x = 17$$
$$x = -8.5$$

18. **MP** **Justify Conclusions** Suppose for some value of x the solution to the equation $2.5(y - x) = 0$ is $y = 6$. What must be true about x? Justify your conclusion.

19. **MP** **Persevere with Problems** Solve each equation.

a. $2.5(x + 4) + x = 38$

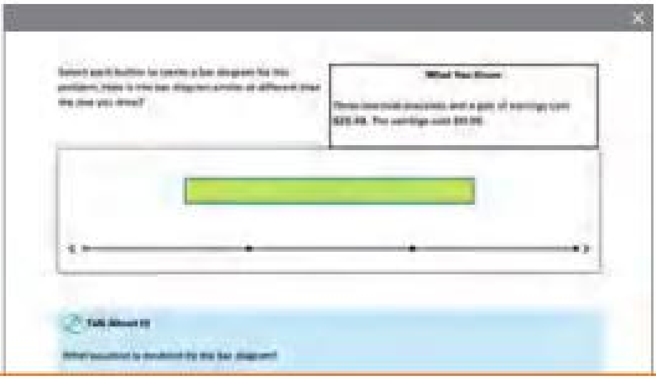
b. $6.1(x - 2) + x = 51.7$

Write and Solve Two-Step Equations: $p(x + q) = r$

I Can... write two-step equations of the form $p(x + q) = r$ and use inverse operations to solve the equations.

Explore Write Two-Step Equations

Online Activity You will use bar diagrams to write two-step equations with parentheses to represent real-world problems.



Learn Write Two-Step Equations

Some real-world situations can be modeled by two-step equations of the form $p(x + q) = r$. Consider the following problem.

Mr. Vargas takes his class of 24 students ice skating. Each student pays an entrance fee to enter the rink and a \$4.75 fee to rent skates. The total cost for the students to enter the rink and rent skates is \$234. What is the ice-skating rink's entrance fee?

Words
Describe the mathematics of the problem. 24 times the total cost for each student is \$234.
Variable
Define the variable to represent the unknown quantity. Let f represent the entrance fee. So, $f + 4.75$ is the total cost for each student.
Equation
Translate the words into an algebraic equation. $24(f + 4.75) = 234$

Your Notes

Think About It!

What will the variable represent in this problem?

Example 1 Write and Solve Two-Step Equations

Mackenzie drives the same distance to and from school each day. She also drives $1\frac{3}{4}$ miles round trip each day to go to the library. During a 5-day school week, Mackenzie drives a total of 50 miles.

Write and solve an equation to determine the total distance to and from school.

Part A Write an equation.

Words

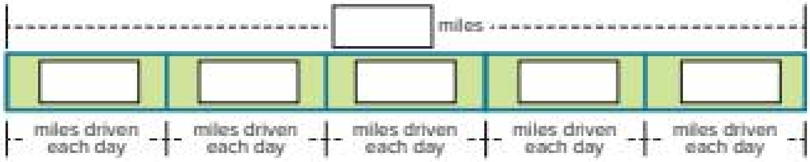
5 times the total distance each day equals 50 miles.

Variable

Let d represent the total distance to and from school.
So, $d + 1\frac{3}{4}$ is the total distance she drives each day.

Bar Diagram

Complete the bar diagram to assist you in writing the equation.



Equation

The equation is $5\left(d + 1\frac{3}{4}\right) = 50$.

Talk About It!

How does the bar diagram help you write the equation $5\left(d + 1\frac{3}{4}\right) = 50$?

Part B Solve the equation.

$5\left(d + 1\frac{3}{4}\right) = 50$	Write the equation.
$\frac{5\left(d + 1\frac{3}{4}\right)}{5} = \frac{50}{5}$	Division Property of Equality
$d + 1\frac{3}{4} = 10$	Simplify.
$-1\frac{3}{4} \quad -1\frac{3}{4}$	Subtraction Property of Equality
$d = 8\frac{1}{4}$	Simplify.

So, the distance to and from school is ________ miles.

Check

Mrs. Byers is making 5 costumes that each require $1\frac{3}{8}$ yards of blue fabric and a certain amount of red fabric. She will use $8\frac{3}{4}$ yards in all. Write and solve an equation to determine the number of yards of red fabric *r* she will need for each costume.

Part A Which equation can be used to determine how many yards of red fabric *r* are needed for each costume?

Ⓐ $5\left(1\frac{3}{8} + r\right) = 8\frac{3}{4}$

Ⓑ $5\left(1\frac{3}{8} - r\right) = 8\frac{3}{4}$

Ⓒ $1\frac{3}{8}(5 + r) = 8\frac{3}{4}$

Ⓓ $1\frac{3}{8}(5 - r) = 8\frac{3}{4}$

Part B How many yards of red fabric are needed for each costume?

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Explain how you wrote and solved the equation in the Check problem. Did you use a bar diagram? Why or why not? Can you solve the problem in a different way from what was shown in the Example? If so, explain.

Record your observations here

Example 2 Write and Solve Two-Step Equations

Think About It!
What are the operations you will use to write the equation?

Jamal and two cousins received the same amount of money to go to a movie. Each boy spent \$15. Afterward, the boys had \$30 altogether.

Write and solve an equation to find the amount of money each boy received.

Part A Write an equation.

Words
3 times the amount of money each boy has left to spend equals \$30.
Variable
Let m represent the amount of money each boy received. So, $m - 15$ is the amount of money each boy has left to spend.
Equation
The equation is $3(\boxed{}) = 30$.

Part B Solve the equation.

Talk About It!
Does the solution make sense in the context of the problem?

$3(m - 15) = 30$	Write the equation.
$\frac{3(m-15)}{3} = \frac{30}{3}$	Division Property of Equality
$m - 15 = 10$	Simplify.
$+15 \quad +15$	Addition Property of Equality
$m = 25$	Simplify.

So, each boy received \$ ________.

Check

Mr. Singh had three sheets of stickers. He gave 20 stickers from each sheet to his students and has 12 total stickers left. Write and solve an equation to find the total number of stickers s there were originally on each sheet.

Part A What equation can be used to determine how many stickers s were originally on each sheet?

Ⓐ $3(s - 20) = 12$

Ⓑ $3(s + 20) = 12$

Ⓒ $20(s - 3) = 12$

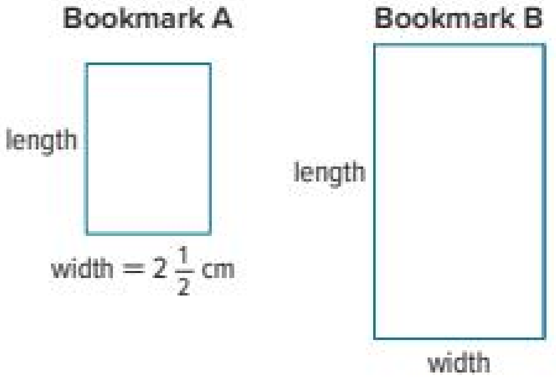
Ⓓ $20(s + 3) = 12$

Part B How many stickers were originally on each sheet?

Go Online You can complete an Extra Example online.

Apply Perimeter

Pierre uses two rectangular pieces of paper as bookmarks. The width of Bookmark B is equal to the length of Bookmark A. The length of Bookmark B is equal to half the perimeter of Bookmark A. The perimeter of Bookmark A is 14 centimeters. What is the perimeter of Bookmark B?



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

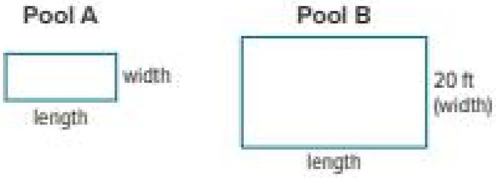
Write About It! Write an argument that can be used to defend your solution.

Talk About It!

How can you find the length of Bookmark A?

Check

Two rectangular swimming pools are shown. The length of Pool A is equal to the width of Pool B. The width of Pool A is $\frac{1}{4}$ the length of Pool B. The perimeter of Pool B is 120 feet. What is the perimeter of Pool A?



Show your work here

Go Online You can complete an Extra Example online.

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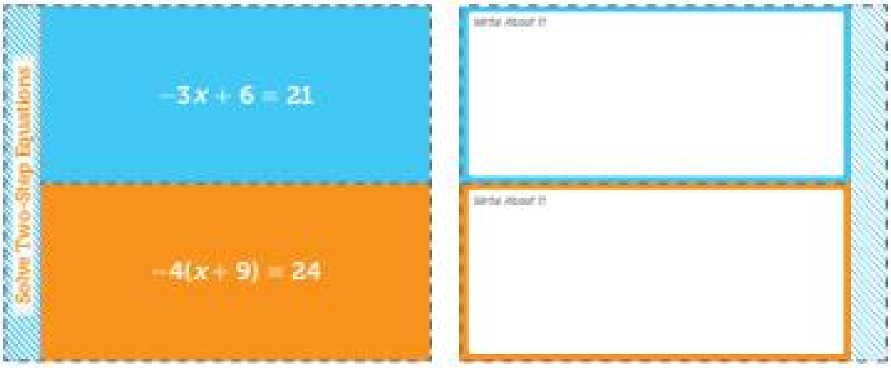


Name ________________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

Write a two-step equation to represent each problem. Then solve the problem.

1. Ayana is making 6 scarves that each require $1\frac{1}{4}$ yards of purple fabric and a certain amount of blue fabric. She will use 10 yards in all. Determine how many yards of blue fabric are needed for each scarf. (Example 1)

2. Sara is making 3 batches of chocolate chip cookies and 3 batches of oatmeal cookies. Each batch of chocolate chip cookies uses $2\frac{1}{4}$ cups of flour. She will use $12\frac{3}{4}$ cups of flour for all six batches. Determine how many cups of flour are needed for each batch of oatmeal cookies. (Example 1)

3. Pete is making 8 identical fruit baskets as gifts. Each basket contains some apples and 12 oranges. Pete uses a total of 168 pieces of fruit to make the baskets. Determine the number of apples that are in each basket. (Example 1)

4. A teacher is making 7 identical supply boxes for each table in her classroom. Each box contains some pencils and 11 pens. The teacher uses a total of 182 pencils and pens. Determine the number of pencils that are in each box. (Example 1)

5. Javier bought 3 bags of balloons for a party. He used 8 balloons from each bag. Determine how many balloons were originally in each bag if there were 21 balloons left over. (Example 2)

6. Vera and her three sisters received the same amount of money to go to the school festival. Each girl spent $12. Afterward, the girls had $24 altogether. Determine the amount of money each girl received. (Example 2)

7. Zak buys 6 gallons of fruit punch. He has coupons for $0.55 off the regular price of each gallon of fruit punch. After using the coupons, the total cost of the fruit punch is $8.70. Determine the regular price of a gallon of fruit punch. (Example 2)

Test Practice

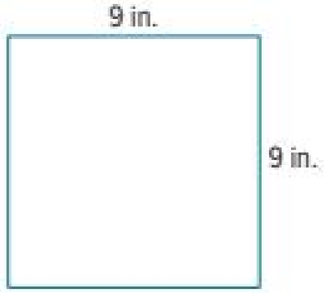
8. **Open Response** Mrs. James buys 5 hat and glove sets for charity. She has coupons for $1.50 off the regular price of each set. After using the coupons, the total cost is $48.75. Determine the regular price of a hat and glove set.

Item	Cost ($)
Hat and glove set	p
Scarf	9.99

Apply

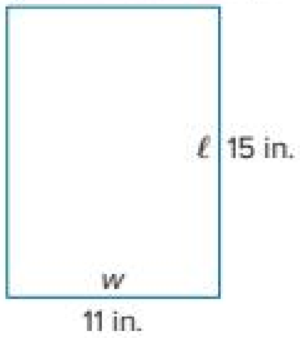
9. Olive and Ryan have picture frames with the same perimeter. Olive's picture frame has a width 1.25 times the width of Ryan's picture frame. What is the length ℓ of Olive's picture frame?

Ryan's Picture Frame



10. Becky and Lewis's hamster cages have the same perimeter. Lewis's cage has a length 0.75 times the length of Becky's cage. What is the width w of Lewis's cage?

Becky's Hamster's Cage



11. **Create** Write a real-world problem that could be represented by the equation $12(x + 2.50) = 78$. Then solve the equation.

12. **Persevere with Problems** Keith is 5 years older than Trina. Two times the sum of their ages is 62. Write and solve an equation to find Keith's age.

13. A student solved the equation $8.9(x - 4.2) = 35.99$ and found the solution to be 22.89. Explain how you can use estimation to show that the solution is incorrect.

14. **Create** Write and solve a real-world problem that involves decimals and can be solved using an equation in the form $p(x + q) = r$.

Module 6 • Write and Solve Equations

Review

Foldables Use your Foldable to help review the module.

Solve Two-Step Equations

Solve

Solve

Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.

Write about a question you still have.

Reflect on the Module

Use what you learned about writing and solving equations to complete the graphic organizer.

Essential Question

How can equations be used to solve everyday problems?

One-Step Equations

Explain how to solve $x + 6 = 12$.

Explain how to solve $n - 4 = 9$.

Explain how to solve $7m = 49$.

Explain how to solve $\frac{k}{5} = -1.5$.

Two-Step Equations

Explain how to solve $3h - 7 = 24$.

Explain how to solve $2(a + 5) = 20$.

What are the steps for writing an equation from a real-world problem?

Name ____________________ Period __________ Date __________

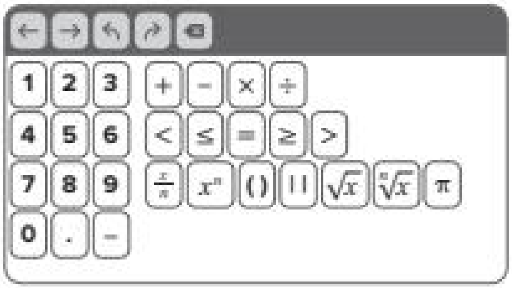
Test Practice

1. **Open Response** Consider the equation $-7 = w + 5$. (Lesson 1)

   A. Solve the equation.

   B. Explain how you can check your solution to the equation. Then check the solution.

2. **Equation Editor** Solve $-3\frac{1}{2} = -1\frac{1}{4}b$ for b. Express the solution as a fraction or mixed number in simplest form. (Lesson 1)



3. **Multiple Choice** What is the solution to the equation $-5p = 10$? (Lesson 1)

   Ⓐ $p = -50$

   Ⓑ $p = -2$

   Ⓒ $p = 2$

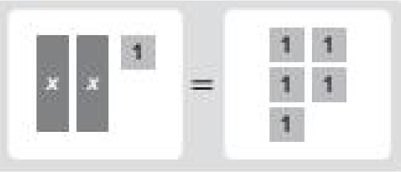
   Ⓓ $p = 15$

4. **Open Response** The table shows the record high temperatures, in degrees Fahrenheit (°F) or degrees Celsius (°C), of certain states. The formula $F = 1.8C + 32$ can be used to convert between temperature scales. (Lesson 2)

State	Record High Temperature
Alabama	112°F
Connecticut	106°F
Hawaii	37.8°C
Kansas	49.4°C

What is the difference, in degrees Celsius, between Alabama's record high temperature and Hawaii's record high temperature? Round to the nearest tenth if necessary.

5. **Multiple Choice** Consider the model shown below. (Lesson 2)



   A. Which equation is modeled on the algebra mat?

   Ⓐ $2x - 1 = 5$

   Ⓑ $2x = 5$

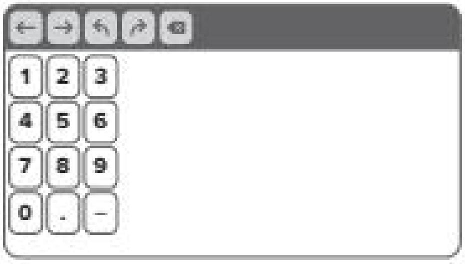
   Ⓒ $2x + 1 = 5$

   Ⓓ $2(x + 1) = 5$

   B. What is the solution to the equation?

6. **Open Response** Solve the equation $-7t - 1 = 20$. (Lesson 2)

7. **Equation Editor** Carlos wants to rent a video game console for \$17.50 and some video games for \$5.25 each. He has \$49 to spend. How many games can he rent? (Lesson 3)



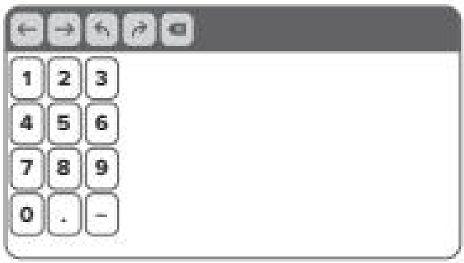
8. **Open Response** Regina had her birthday party at a family fun center. It costs \$34.50 for food plus \$9.25 per person for game and ride tickets. (Lesson 3)

A. If the total cost was \$108.50, how many people attended the party?

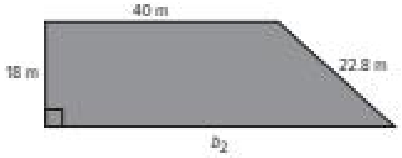
B. Explain how you can check your solution of the number of people who attended Regina's party. Then check the solution.

9. **Open Response** The current temperature is 73°F. It is expected to drop 3.5°F each hour. Define a variable and write an equation that can be used to find the number of hours it will take for the temperature to reach 59°F. Then solve the equation. (Lesson 3)

10. **Equation Editor** The solution to the equation $-2(x + 11) = -8$ is $x =$ _. (Lesson 4)



11. **Open Response** A parking lot is shaped like a trapezoid as shown. If the area of the parking lot is 846 square meters, what is the perimeter? (*Hint*: The formula for the area of a trapezoid is $A = \frac{h}{2}(b_1 + b_2)$). (Lesson 4)



12. **Multiple Choice** Terrance buys 6 bottles of sports drink. He has coupons for \$0.55 off the regular price of each bottle. After using the coupons, the total cost of the sports drinks is \$4.44. (Lesson 5)

A. Which equation can be used to find the regular price p of a bottle of sports drink?

Ⓐ $6(p + 0.55) = 4.44$

Ⓑ $6(p - 0.55) = 4.44$

Ⓒ $6(0.55 - p) = 4.44$

Ⓓ $0.55(p + 6) = 4.44$

B. What is the regular price per bottle of sports drink?

Module 7

Write and Solve Inequalities

Essential Question

How are the solutions to inequalities different from the solutions to equations?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY

■ — I don't know. ◆ — I've heard of it. ★ — I know it!

	Before			After		
	■	◆	★	■	◆	★
graphing inequalities						
solving one-step addition and subtraction inequalities						
writing one-step addition and subtraction inequalities						
solving one-step multiplication and division inequalities						
writing one-step multiplication and division inequalities						
solving two-step inequalities						
writing two-step inequalities						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about writing and solving equations.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ Addition Property of Inequality
☐ Division Property of Inequality
☐ inequality
☐ Multiplication Property of Inequality
☐ Subtraction Property of Inequality
☐ two-step inequality

Are You Ready?

Study the Quick Review to see if you are ready to start this module.
Then complete the Quick Check.

Quick Review

Example 1

Solve one-step addition equations.

Solve $d + 9 = -8\frac{1}{4}$.

$d + 9 = -8\frac{1}{4}$	Write the equation.
$d + 9 - 9 = -8\frac{1}{4} - 9$	Subtraction Property of Equality
$d = -17\frac{1}{4}$	Simplify.

Example 2

Solve one-step multiplication equations.

Solve $-2x = 6$.

$-2x = 6$	Write the equation.
$\frac{-2x}{-2} = \frac{6}{-2}$	Division Property of Equality
$x = -3$	Simplify.

Quick Check

1. After descending 8.25 feet, a bird is now at a height of 16.5 feet. What was the initial height of the bird?

2. Solve $-8x = -72$.

How Did You Do?

Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right.

① ②

Lesson 7-1

Solve One-Step Addition and Subtraction Inequalities

I Can... use inverse operations to solve one-step addition and subtraction inequalities.

What Vocabulary Will You Learn?

Addition Property of Inequality
inequality
Subtraction Property of Inequality

Explore Addition and Subtraction Properties of Inequality

Online Activity You will use Web Sketchpad to explore the effects of adding or subtracting a number from each side of an inequality.

Learn Inequalities

An **inequality** is a mathematical sentence that compares quantities. Inequalities contain the symbols <, >, ≤, or ≥.

The table shows the meaning of each inequality symbol.

Symbol	Meaning
$<$	is less than $4 < 8$
$>$	is greater than $3 > -2$
$\leq$	is less than or equal to $-6 \leq 1$ or $5 \leq 5$
$\geq$	is greater than or equal to $9 \geq 6$ or $-7 \geq -7$

Your Notes

Talk About It!

When graphing an inequality on the number line, how do you know whether to use an open dot or a closed dot?

Talk About It!

If you were asked to graph the solution $x > 35$, what range of values can you use to create the number line? Explain.

Talk About It!

Are the Addition and Subtraction Properties of Inequality true for the symbols ≥ and ≤? Explain.

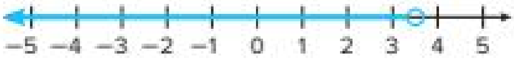
Learn Graph Inequalities

Go Online Watch the animation to learn how to graph inequalities on the number line.

Follow the steps to graph the inequality $x < 3.5$.

Step 1 Place the endpoint as an open dot. For an inequality that contains a $<$ or $>$ symbol, an open dot is used to show that the number is not a solution.

Step 2 Draw an arrow that points to the left to show that numbers less than 3.5 are part of the solution.



Follow the steps to graph the inequality $x \geq -1$.

Step 1 Place the endpoint as a closed dot. For an inequality that contains a $\leq$ or $\geq$ symbol, a closed dot is used to show that the number is a solution.

Step 2 Draw an arrow that points to the right to show that values greater than −1 are solutions.

Learn Subtraction and Addition Properties of Inequality

Solving an inequality means finding values for the variable that make the inequality true. You can solve addition inequalities by using the **Subtraction Property of Inequality**. You can solve subtraction inequalities by using the **Addition Property of Inequality**.

Words	When you add or subtract the same number from each side of an inequality, the inequality remains true.	
Symbols	For all numbers a, b, and c, if $a > b$, then $a - c > b - c$ and $a + c > b + c$, if $a < b$, then $a - c < b - c$ and $a + c < b + c$.	
Examples	$\begin{array}{rcr} 2 & < & 5 \\ \underline{-4} & & \underline{-4} \\ -2 & < & 1 \end{array}$	$\begin{array}{rcr} 4 & > & -1 \\ \underline{+3} & & \underline{+3} \\ 7 & > & 2 \end{array}$

These properties are also true for $a \geq b$ and $a \leq b$.

Example 1 Solve and Graph Addition Inequalities

Solve $x + 3 > 10$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$$\begin{array}{rll} x + 3 > 10 & & \text{Write the inequality.} \\ \underline{-3 \quad -3} & & \text{Subtraction Property of Inequality} \\ x > 7 & & \text{Simplify.} \end{array}$$

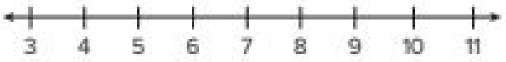
The solution of the inequality $x + 3 > 10$ is $x >$ ________.

You can check the solution $x > 7$ by substituting a number greater than 7 into the original inequality. Try using 8.

$$\begin{array}{rll} x + 3 > 10 & & \text{Write the original inequality.} \\ 8 + 3 \overset{?}{>} 10 & & \text{Substitute 8 for } x. \\ 11 > 10 & & \text{Simplify.} \end{array}$$

Part B Graph the solution set on a number line.

To graph $x > 7$, draw an open dot at 7 and an arrow pointing to the right. This shows that 7 is not part of the solution, but values greater than 7 are part of the solution.



Talk About It!

Which of the following numbers are solutions of the inequality $x > 7$? Explain why the values you chose are solutions.

7

−7

7.01

7.001

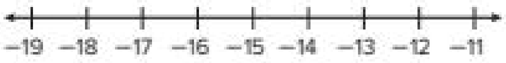
Check

Solve $6 + h \leq -9$ and graph the solution set.

Part A Solve $6 + h \leq -9$.

Show your work here

Part B Graph the solution set.



Go Online You can complete an Extra Example online.

Example 2 Solve and Graph Addition Inequalities

Solve $0.4 + y \leq -9.6$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$$\begin{array}{rll} 0.4 + y \leq & -9.6 & \text{Write the inequality.} \\ \underline{-\,0.4 \qquad\quad} & \underline{-\,0.4} & \text{Subtraction Property of Inequality} \\ y \leq & -10 & \text{Simplify.} \end{array}$$

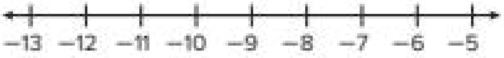
The solution of the inequality $0.4 + y \leq -9.6$ is $y \leq$ ________.

You can check the solution $y \leq -10$ by substituting a number less than or equal to -10 into the original inequality.

$$\begin{array}{rl} 0.4 + y \leq -9.6 & \text{Write the original inequality.} \\ 0.4 + (-10) \overset{?}{\leq} -9.6 & \text{Substitute } -10 \text{ for } y. \\ -9.6 \leq -9.6 & \text{Simplify.} \end{array}$$

Part B Graph the solution set on a number line.

To graph $y \leq -10$, draw a closed dot at -10 and an arrow pointing to the left. This shows that -10 is part of the solution, and values less than -10 are part of the solution.



Talk About It!
Why does the range of the number line extend from −13 to −5? Can you use a different range?

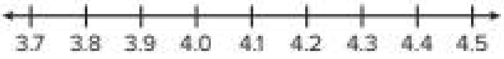
Check

Solve $x + 1.3 < 5.4$ and graph the solution set.

Part A Solve $x + 1.3 < 5.4$.

Show your work here

Part B Graph the solution set.



Go Online You can complete an Extra Example online.

Example 3 Solve and Graph Subtraction Inequalities

Solve $-6 \geq n - 5$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$-6 \geq n - 5$	Write the inequality.
$+5 \qquad +5$	Addition Property of Inequality
$\boxed{} \geq \boxed{}$	Simplify.

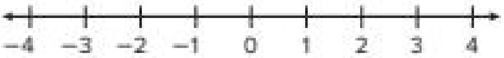
The solution of the inequality $-6 \geq n - 5$ is $-1 \geq n$ or $n \leq -1$.

You can check this solution by substituting a number less than or equal to -1 into the original inequality.

$-6 \geq n - 5$	Write the original inequality.
$-6 \overset{?}{\geq} -3 - 5$	Replace n with -3.
$-6 \geq -8$	Simplify.

Part B Graph the solution set on a number line.

To graph $n \leq -1$, draw a closed dot at -1 and an arrow pointing to the left. This shows that -1 is part of the solution, and values less than -1 are part of the solution.



Talk About It!

You can write $-1 \geq n$ as $n \leq -1$. Which way will help you visualize the solution on a number line? Explain your reasoning.

Talk About It!

Why is $-1 \geq n$ equivalent to $n \leq -1$? Substitute values for n that verify that the two inequalities are equivalent.

Check

Solve $x - 2.3 \geq -8.5$ and graph the solution set.

Part A Solve $x - 2.3 \geq -8.5$.

Show your work here

Part B Graph the solution set.

Go Online You can complete an Extra Example online.

Math History Minute

The Inca empire of 15th and 16th century South America used knotted and colored strings called *quipus* to keep complex records of everything from the empire's population to the amount of food a village had in store for lean seasons.

Pause and Reflect

Compare and contrast solving one-step addition and one-step subtraction inequalities to one-step addition and one-step subtraction equations. Create a graphic organizer.

Apply Crafting

Rafael has more than 5 feet of wire. He uses 1.5 feet for a project. Solve the inequality $x + 1.5 > 5$ to find the amount of wire x, in feet, that Rafael has left. How many inches of wire does he have left?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

Talk About It! How do you know the answer is not 3.5?

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Check

Caroline has at least 6 yards of ribbon. She uses 4.5 yards to make hair bows for the cheerleaders. Solve the inequality $4.5 + x \geq 6$ to find the amount of ribbon x, in yards, that Caroline has left. What is the minimum number of feet of ribbon Caroline has left?

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

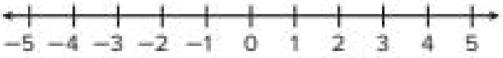
Name ______________________ Period __________ Date __________

Practice

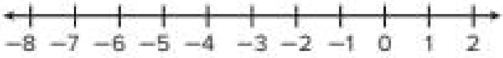
Go Online You can complete your homework online.

Solve each inequality. Graph the solution set on a number line. (Examples 1–3)

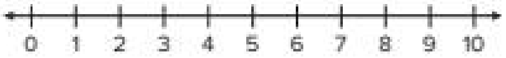
1. $x + 5 < 7$



2. $1 > x + 6$



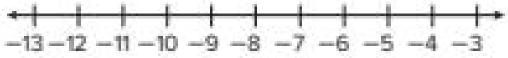
3. $x + 8 \geq 14$



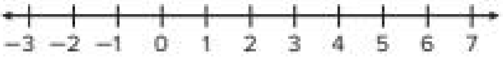
4. $5 \leq x + 12$

5. $x + 5.4 < -1.6$

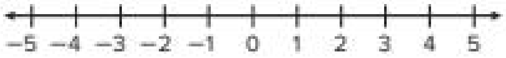
6. $x + 7.5 > -2.5$



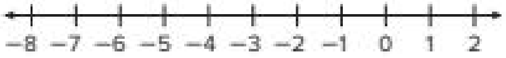
7. $3 \leq \frac{1}{3} + x$



8. $4 \geq x + \frac{3}{4}$

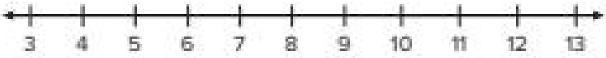


9. $x - 3 \leq -8$



10. $4 \leq x - 7$

11. $6.9 < x - 2.3$

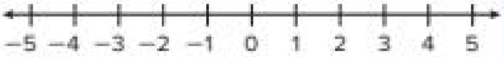


Test Practice

12. Grid Solve the inequality and graph the solution on the number line.

A. Solve the inequality $x - \frac{3}{4} < \frac{1}{2}$.

B. Graph the solution on the number line.



Apply

13. Renee started with more than 4 pounds of brown sugar. The table shows the amount of brown sugar she used when making cookies. Solve the inequality $x + 1\frac{1}{4} > 4$ to find the amount of brown sugar x, in pounds, that Renee has remaining. How many ounces of brown sugar does she have left?

	Brown Sugar (pounds)
Amount Used	$1\frac{1}{4}$
Amount Remaining	x

14. To prepare for a dance competition, a dance team needs to practice at least 12.75 hours a week. The team has already practiced 10.5 hours this week. Solve the inequality $10.5 + x \geq 12.75$ to find the amount of time x, in hours, the team has left to practice. What is the minimum number of minutes the team needs to practice?

15. Write an addition inequality and a subtraction inequality that each have the solution set graphed below.

16. MP **Reason Abstractly** Compare and contrast the solutions $x < 2$ and $x \leq 2$.

17. MP **Persevere with Problems** Solve the inequality $x - y < z$ for x.

18. MP **Make an Argument** Make an argument for why the order of an inequality is important.

Lesson 7-2

Write and Solve One-Step Addition and Subtraction Inequalities

I Can... write one-step addition and subtraction inequalities from real-world situations and use inverse operations to solve the inequalities.

Learn Write Inequalities

Inequalities can be used to represent real-world situations. The table shows common phrases that describe each inequality.

Symbols	Phrases
<	is less than is fewer than
>	is greater than is more than exceeds
≤	is less than or equal to is no more than is at most
≥	is greater than or equal to is no less than is at least

The table outlines the steps used to write an inequality from a real-world situation.

Words	Describe the mathematics of the problem.
Variable	Define a variable to represent the unknown quantity.
Inequality	Translate the words into an inequality.

For example, a certain semi-truck can hold no more than 25 tons of material. This can be represented by the inequality $x \leq 25$, where x represents the weight of the material the truck is carrying.

Talk About It!

How do you know which inequality symbol to use when representing a real-world situation?

Your Notes

Think About It!
What key word(s) tell you which inequality symbol to use?

Example 1 Write and Solve One-Step Addition Inequalities

Dylan can spend at most \$18 to ride go-karts and play games at the state fair. Suppose the go-karts cost \$5.50.

Write and solve an inequality to determine the amount Dylan can spend on games, if he rides go-karts once. Then interpret the solution.

Part A Write the inequality.

Words	Cost of go-kart ride plus cost of games is less than or equal to the total amount he can spend.
Variable	Let x represent the cost of the games.
Inequality	$5.5 + x \leq 18$

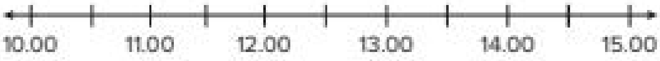
Part B Solve the inequality.

$5.5 + x \leq 18$	Write the inequality.
$-5.5 \qquad -5.5$	Subtraction Property of Inequality
$x \leq 12.5$	Simplify.

The solution of the inequality $5.5 + x \leq 18$ is __________.

Part C Interpret the solution.

Graph the solution set of $x \leq 12.50$ on the number line.



The greatest value that is part of the solution set is 12.50.

So, the most Dylan can spend on games is \$12.50.

Check

Hannah's exercise goal is to walk at least $6\frac{1}{2}$ miles this week. She has already walked $2\frac{3}{4}$ miles this week. Write and solve an inequality to determine the distance Hannah needs to walk to meet or exceed her goal. Then interpret the solution.

Part A Write an inequality that can be used to find how many more miles she needs to walk this week to meet or exceed her goal.

Part B What is the solution of the inequality in Part A?

Part C What is the correct interpretation of the solution of the inequality?

Ⓐ Hannah needs to walk at least $3\frac{3}{4}$ miles.

Ⓑ Hannah needs to walk no more than $3\frac{3}{4}$ miles.

Ⓒ Hannah needs to walk exactly $3\frac{3}{4}$ miles.

Ⓓ Hannah needs to walk less than $3\frac{3}{4}$ miles.

Go Online You can complete an Extra Example online.

Example 2 Write and Solve One-Step Subtraction Inequalities

Caleb owns two types of dogs. The difference in height of his Yorkshire Terrier and his Great Dane is at least 25 inches. Caleb's Yorkshire Terrier has a height of $6\frac{1}{4}$ inches.

Write and solve an inequality to determine the possible height of the Great Dane. Then interpret the solution.

Part A Write an inequality.

Words	The Great Dane's height minus the Yorkshire Terrier's height is at least 25 inches.
Variable	Let h represent the height of the Great Dane.
Inequality	$h - 6\frac{1}{4} \geq 25$

Think About It!
What inequality symbol will you use to write the inequality?

(continued on next page)

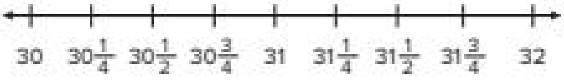
Part B Solve the inequality.

$h - 6\frac{1}{4} \ge 25$	Write the inequality.
$+\boxed{} \quad +\boxed{}$	Addition Property of Inequality
$h \ge 31\frac{1}{4}$	Simplify.

The solution of the inequality $h - 6\frac{1}{4} \ge 25$ is $h \ge 31\frac{1}{4}$.

Part C Interpret the solution.

Graph the solution set.



So, the height of the Great Dane is at least $31\frac{1}{4}$ inches.

Check

Gabriella paid $17.25 for a sweatshirt that was on sale. The difference between the original price and the sale price was at most $8.50. Write and solve an inequality to determine the possible price of the sweatshirt. Then interpret the solution.

Part A Write an inequality that can be used to find the highest original price of the sweatshirt.

Part B What is the solution of the inequality in Part A?

Show your work here

Part C The original price of the sweatshirt was at ________________.

Go Online You can complete an Extra Example online.

Apply Elevators

Passenger	Weight (lb)
1	126
2	182
3	78
4	135
5	63

The maximum weight capacity of the elevator in Maya's apartment building is 900 pounds. One morning she and five other people are on the elevator. Then two more passengers get on the elevator. If Maya weighs 108 pounds, what could be the maximum sum of the weights of the two additional passengers without exceeding the maximum weight capacity?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Go Online Watch the animation.

Talk About It!

Are the following sets of values possible solutions to the problem? Explain why or why not.

- 115 and 93
- 120 and 90
- 35 and 165

Check

Hayley's reading log is shown. She is required to read at least 2 hours each week. Which could be the amount of time she needs to read on Saturday and Sunday to meet her reading goal?

Day	Number of Minutes
Monday	20
Tuesday	25
Wednesday	15
Thursday	10
Friday	20
Saturday	x
Sunday	y

Ⓐ Saturday: 5 minutes; Sunday: 10 minutes

Ⓑ Saturday: 10 minutes; Sunday: 10 minutes

Ⓒ Saturday: 10 minutes; Sunday: 15 minutes

Ⓓ Saturday: 15 minutes; Sunday: 20 minutes

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

Name ______________________ Period __________ Date __________

Practice

Go Online You can complete your homework online.

Solve each problem by first writing an inequality.

1. Gabe went to the amusement park with $40 to spend. His ticket cost $26.50. Determine how much Gabe can spend on souvenirs and snacks. Then interpret the solution. (Example 1)

2. Drew practices piano at least 45 minutes per day. He has already practiced 18.5 minutes today. Determine how much longer he will have to practice. Then interpret the solution. (Example 1)

3. A dolphin is swimming at a depth of −50 feet and then ascends a certain number of feet to a depth above −35 feet. Determine the number of feet the dolphin ascended. Then interpret the solution. (Example 1)

4. Elena's account balance with her parents is −$5.50. She adds a certain amount of money to her balance by mowing the lawn. Elena now has an account balance less than $20. Determine a possible amount she earned mowing the lawn. Then interpret the solution. (Example 1)

5. Linda has two cats. The difference in weight of her Maine Coon and Siberian is at least 6 pounds. Linda's Siberian has a weight of $8\frac{3}{4}$ pounds. Determine the possible weight of the Maine Coon. Then interpret the solution. (Example 2)

6. The Hendersons have a sedan and a minivan. The difference in mileage of the two vehicles is greater than 4,500 miles. The minivan has 12,755.25 miles. Determine the possible number of miles on the sedan. Then interpret the solution. (Example 2)

7. The difference between the monthly high and low temperatures was less than 27° Fahrenheit. The monthly low temperature was −2° Fahrenheit. Determine the possible monthly high temperature. Then interpret the solution. (Example 2)

Test Practice

8. Open Response Teddy has two piggy banks. The difference in the amount of money between the two banks is no more than $10. One piggy bank has $7.31 in it. Determine the possible amount of money in the other piggy bank. Then interpret the solution.

Apply

9. Zeg has $9.20 left on a gift card to the candy store. He has the following items in his shopping basket: 2 giant lollipops, 3 popcorn balls, 1 candy bar, and 5 candy sticks. Zeg wants to buy two more items. Name two more possible items he can buy using his gift card.

Item	Cost ($)
Candy Bars	0.99
Candy Stick	0.45
Giant Lollipops	1.75
Popcorn Ball	0.50

10. Jen is packing a box for her brother at college. The package must weigh less than 12 pounds. The table shows the weight of items she has already placed in the box. She has the following items she wants to pack: a sweatshirt that weighs 1.25 pounds, a package of socks that weighs 1.2 pounds, and a book that weighs 0.75 pound. Jen places one more item in the box. Which of the items could she send?

Item	Weight (pounds)
Boots	5.2
Jeans	1.25
Sweater	2.05
Running Shoes	2.25

11. Write a real-world problem that could have the solution $x \leq 10$.

12. MP **Reason Inductively** There is space for 120 students to go on a field trip. Currently, 74 students have signed up. Can 46 more students sign up for the field trip? Explain your reasoning.

13. **Create** Write and solve a real-world problem that involves a one-step addition inequality.

14. MP **Persevere with Problems** William is 3 feet 1 inch tall and would like to ride a roller coaster. Riders must be at least 42 inches tall to ride the coaster. Write and solve an addition inequality to determine how much taller William must be to ride the coaster.

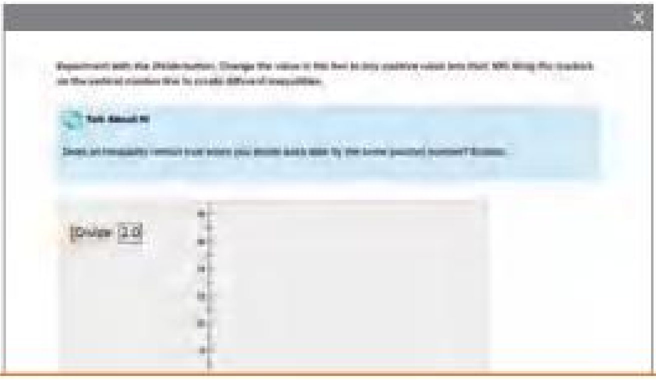
Lesson 7-3

Solve One-Step Multiplication and Division Inequalities with Positive Coefficients

I Can... use inverse operations to solve one-step multiplication and division inequalities with positive coefficients.

Explore Multiplication and Division Properties of Inequality

Online Activity You will use Web Sketchpad to determine if multiplying or dividing each side of an inequality by the same positive number keeps the inequality true.



Learn Division and Multiplication Properties of Inequality

You can solve multiplication inequalities by using the **Division Property of Inequality**. You can solve division inequalities by using the **Multiplication Property of Inequality**.

The table outlines the process of using the Division and Multiplication Properties of Inequality with *positive* coefficients.

Words	An inequality remains true when you divide or multiply each side of the inequality by the same positive number.	
Symbols	For all numbers a, b, and c, where $c > 0$, if $a > b$, then $\frac{a}{c} > \frac{b}{c}$ and $ac > bc$. if $a < b$, then $\frac{a}{c} < \frac{b}{c}$ and $ac < bc$.	
Examples	$9 < 15$ $\frac{9}{3} < \frac{15}{3}$ $3 < 5$	$10 > 7$ $2(10) > 2(7)$ $20 > 14$

These properties are also true for $a \geq b$ and $a \leq b$.

What Vocabulary Will You Learn?

Division Property of Inequality

Multiplication Property of Inequality

Talk About It!

The Division Property of Inequality states:

For all numbers a, b, and c, where $c > 0$,

1. if $a > b$, then $\frac{a}{c} > \frac{b}{c}$.
2. if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.

What does the inequality $c > 0$ mean?

Example 1 Solve and Graph Multiplication Inequalities

Solve $8x \leq 40$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$8x \leq 40$ Write the inequality.

$\frac{8x}{8} \leq \frac{40}{8}$ Division Property of Inequality

$x \leq 5$ Simplify.

The solution of the inequality $8x \leq 40$ is ________.

You can check this solution by substituting a number less than or equal to 5 into the original inequality. Try using 4.

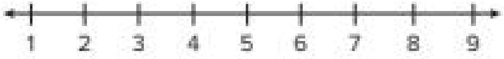
$8x \leq 40$ Write the inequality.

$8(4) \overset{?}{\leq} 40$ Replace x with 4.

$32 \leq 40$ Simplify.

Part B Graph the solution set on a number line.

To graph $x \leq 5$, place a closed dot at 5 and draw an arrow to the left. This shows that the values less than, and including, 5 are part of the solution.



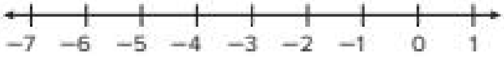
Check

Solve $12x \geq -36$ and graph the solution set.

Part A Solve $12x \geq -36$.

Show your work here

Part B Graph the solution set.



Go Online You can complete an Extra Example online.

Your Notes

Think About It!
What step(s) do you need to take in order to solve the inequality?

Talk About It!
What property allows you to divide each side of the inequality by the same number?

Example 2 Solve and Graph Division Inequalities

Solve $\frac{d}{2} > 7$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$\frac{d}{2} > 7$ Write the inequality.

$2\left(\frac{d}{2}\right) > 2(7)$ Multiplication Property of Inequality

$d > 14$ Simplify.

The solution of the inequality $\frac{d}{2} > 7$ is ________.

You can check this solution by substituting a number greater than 14 into the original inequality. Try using 15.

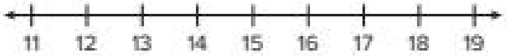
$\frac{d}{2} > 7$ Write the inequality.

$\frac{15}{2} \overset{?}{>} 7$ Replace *d* with 15.

$7.5 > 7$ Simplify.

Part B Graph the solution set on a number line.

To graph $d > 14$, place an open dot on 14 and draw an arrow pointing to the right to show that all values greater than, but not including, 14 are part of the solution.



Check

Solve $-6 \geq \frac{x}{7}$ and graph the solution set.

Part A Solve $-6 \geq \frac{x}{7}$.

Show your work here

Part B Graph the solution set.

Think About It!

What step(s) do you need to take in order to solve the inequality?

Talk About It!

Would the inequality be true if 14 was substituted for *d*? Why or why not?

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare and contrast solving one-step multiplication and division inequalities to one-step multiplication and division equations. Create a graphic organizer.

Record your observations here

Apply Fashion

Scarlett has \$60 to buy T-shirts and tie-dye kits. She needs to buy an equal number of each item. If each T-shirt costs \$2 and each tie-dye kit costs \$9, then the inequality $2x + 9x \leq 60$ can be used to find the number of each item she can buy. How much money will she have left after buying the greatest number of pairs of T-shirts and tie-dye kits?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

Talk About It!
What does the inequality tell about the relationship between the number of T-shirts and tie-dye kits?

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Check

Ryan has \$63 to buy an equal number of posters and poster frames. Each poster costs \$4 and each frame costs \$10. The inequality $4x + 10x \leq 63$ can be used to find the number of each item he can buy. How much money will he have left after buying the greatest number of pairs of posters and frames?

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

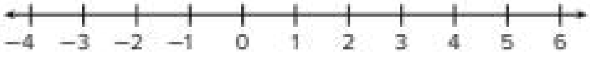
Name ______________________ Period ____________ Date ____________

Practice

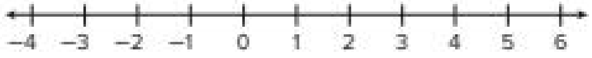
Go Online You can complete your homework online.

Solve each inequality. Graph the solution set on a number line. (Examples 1 and 2)

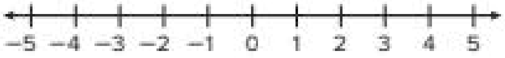
1. $3x > 12$



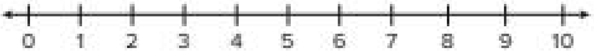
2. $60 \geq 12x$



3. $-14 \geq 7x$



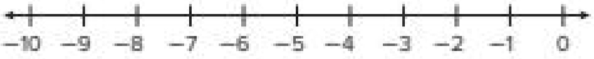
4. $2 \leq 0.25x$



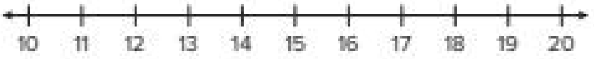
5. $1.1x < -4.4$

6. $\frac{x}{6} \geq 2$

7. $\frac{x}{2} > -4$



8. $\frac{x}{3} < 5$



9. $\frac{x}{4} \leq 1.6$

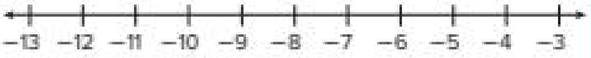
Test Practice

10. Grid Solve the inequality and graph the solution set on a number line.

$-2.4 \geq \frac{x}{5}$

A. Solve the inequality.

B. Graph the solution set on a number line.



Apply

11. Oneida is buying hats and gloves for charity. She needs to buy an equal number of each item. The table shows the cost of the items. If she has \$50 to spend on the hats and gloves, then the inequality $5x + 3x \leq 50$ can be used to find the number of each item she can buy. How much money will she have left after buying the greatest number of hats and gloves?

Item	Cost (\$)
Gloves	3.00
Hat	5.00

12. Ben needs to buy an equal number of sunglasses and beach balls to place in gift bags. Each pair of sunglasses costs \$4 and each beach ball costs \$2.50. If he has \$45 to spend on the sunglasses and beach balls, then the inequality $4x + 2.5x \leq 45$ can be used to find the number of each item he can buy. How much money will he have left after buying the greatest number of sunglasses and beach balls?

13. **Identify Structure** Write two different inequalities that have the solution $x < 10$. One inequality should be able to be solved using a multiplication property and the other should be able to be solved using a division property.

14. **Persevere with Problems** A student scored 98, 94, 96, and 88 points out of 100 on the last four science tests. What score must the student score on the fifth test to have an average of at least 94 points?

15. **Reason Abstractly** The inequalities $3x < 1$ and $6x < 2$ are equivalent inequalities. Write another inequality that is equivalent to $3x < 1$ and $6x < 2$.

16. **Find the Error** A student solved the inequality shown below. Find the student's mistake and correct it.

$$\frac{x}{2} < 4$$

$$\frac{x}{2} \div 2 < 4 \div 2$$

$$x < 2$$

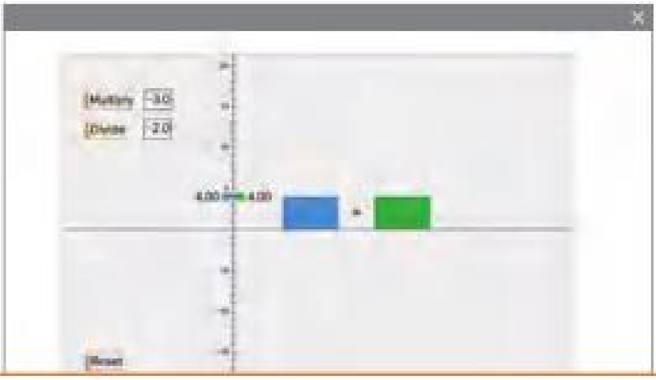
Lesson 7-4

Solve One-Step Multiplication and Division Inequalities with Negative Coefficients

I Can... use inverse operations to solve one-step multiplication and division inequalities with negative coefficients.

Explore Multiply and Divide Inequalities by Negative Numbers

Online Activity You will use Web Sketchpad to explore how multiplying and dividing each side of an inequality by the same negative number affects the inequality.



Learn Division and Multiplication Properties of Inequality

The table outlines the process of using the Division and Multiplication Properties of Inequality when dividing or multiplying each side of an inequality by a *negative* number.

Words	When you divide or multiply each side of an inequality by the same *negative* number, the inequality symbol must be reversed for the inequality to remain true.	
Symbols	For all numbers a, b, and c, where $c < 0$, if $a < b$, then $\frac{a}{c} > \frac{b}{c}$ and $ac > bc$, if $a > b$, then $\frac{a}{c} < \frac{b}{c}$ and $ac < bc$.	
Examples	$18 > -12$ $\frac{18}{-3} < \frac{-12}{-3}$ $-6 < 4$	$-4 < 5$ $-4(-3) > 5(-3)$ $12 > -15$

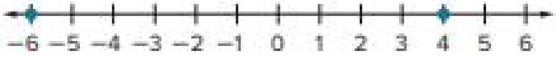
These properties are also true for $a \leq b$ and $a \geq b$.

(continued on next page)

Your Notes

Go Online Watch the animation to understand why you need to reverse the inequality symbol when you divide each side of an inequality by the same negative number.

Step 1 Start by graphing the numbers on each side of the true inequality $-6 < 4$.



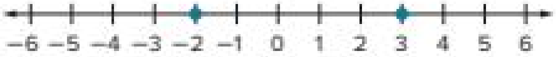
Step 2 Divide each side of the inequality by the same negative number, for example, -2.

$$\frac{-6}{-2} \overset{?}{<} \frac{4}{-2}$$

Step 3 Simplify each side of the inequality. Is the inequality still true?

$$3 \overset{?}{<} -2$$

Step 4 Graph each side of the inequality to determine that it is NOT true. The number 3 is not less than -2.



Step 5 Make the inequality true by reversing the inequality symbol. The number 3 is greater than -2.

$$3 > -2$$

(continued on next page)

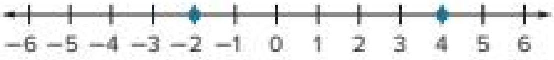
Talk About It!
Will the inequality symbol need to be reversed when solving the inequality $3x > -9$? Why or why not?

Pause and Reflect

You just observed that dividing each side of the inequality by -2 results in a false inequality, unless the inequality symbol is reversed. Do you think that this will always be true for any inequality and/or for any negative divisor? Create your own inequalities with which to experiment and verify whether or not this will always be true.

Go Online Watch the animation to understand why you need to reverse the inequality symbol when you multiply each side of an inequality by the same negative number.

Step 1 Start by graphing the numbers on each side of the true inequality $-2 < 4$.



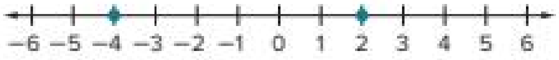
Step 2 Multiply each side of the inequality by the same negative number, for example, −1.

$$-1(-2) \overset{?}{<} -1(4)$$

Step 3 Simplify each side of the inequality. Is the inequality still true?

$$2 \overset{?}{<} -4$$

Step 4 Graph each side of the inequality to determine that it is NOT true. The number 2 is not less than −4.



Step 5 Make the inequality true by reversing the inequality symbol. The number 2 is greater than −4.

$$2 > -4$$

Talk About It!

Use your understanding of opposites to explain why the inequality symbol is reversed when multiplying each side of the inequality by −1.

Pause and Reflect

You just observed that multiplying each side of the inequality by −1 results in a false inequality, unless the inequality symbol is reversed. Do you think that this will always be true for any inequality and/or for any negative number? Create your own inequalities with which to experiment and verify whether or not this will always be true.

Think About It!

What is important to remember when multiplying or dividing each side of an inequality by a negative number?

Talk About It!

How can you use the graph of the solution to check your work?

Example 1 Multiplication Inequalities with Negative Coefficients

Solve $-2x < 10$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

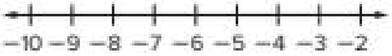
$-2x < 10$	Write the inequality.
$\frac{-2x}{-2} > \frac{10}{-2}$	Divide each side by −2 and reverse the inequality symbol.
$x > -5$	Simplify.

The solution of the inequality $-2x < 10$ is $x > -5$.

You can check the solution by substituting a number greater than −5 into the original inequality.

$-2x < 10$	Write the inequality.
$-2(-3) \overset{?}{<} 10$	Replace x with −3.
$6 < 10$	Simplify.

Part B Graph the solution set on a number line.



Check

Solve $-4.1x > 12.3$ and graph the solution set.

Part A Solve $-4.1x > 12.3$.

Show your work here

Part B Graph the solution set.

Go Online You can complete an Extra Example online.

Example 2 Division Inequalities with Negative Coefficients

Solve $\frac{x}{-4} \geq 3$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$\frac{x}{-4} \geq 3$ Write the inequality.

$\boxed{}\left(\frac{x}{-4}\right) \leq \boxed{}(3)$ Multiplication Property of Inequality

$x \leq -12$ Simplify.

The solution of the inequality $\frac{x}{-4} \geq 3$ is ________.

You can check the solution by substituting a number less than or equal to −12 into the original inequality.

$\frac{x}{-4} \geq 3$ Write the inequality.

$\frac{\boxed{}}{-4} \overset{?}{\geq} 3$ Replace x with −20.

$5 \geq 3$ Simplify.

Part B Graph the solution set on a number line.

To graph the solution $x \leq -12$, place a closed dot at −12 and draw an arrow to the left to show that all values less than, and including, −12 are part of the solution.

Check

Solve $\frac{x}{-2.25} \leq -4$ and graph the solution set.

Part A Solve $\frac{x}{-2.25} \leq -4$.

Show your work here

Part B Graph the solution set.

Go Online You can complete an Extra Example online.

Think About It!

What step(s) do you need to take in order to solve the inequality?

Pause and Reflect

Compare and contrast solving one-step multiplication and one-step division inequalities with positive coefficients to solving one-step multiplication and one-step division inequalities with negative coefficients. Create a graphic organizer.

Record your observations here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

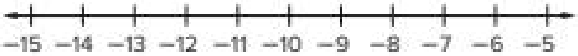
Name ______________________ Period __________ Date __________

Practice

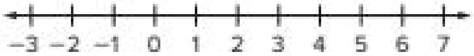
Go Online You can complete your homework online.

Solve each inequality. Graph the solution set on a number line. (Examples 1 and 2)

1. $-6x > 66$

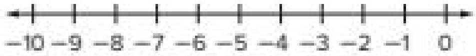


2. $-12 \leq -3x$

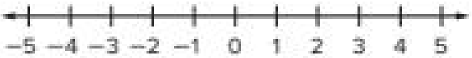


3. $-4x \geq -36$

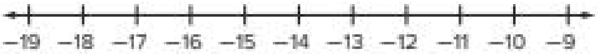
4. $3 > -0.4x$



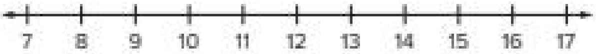
5. $-2.2x \leq -6.6$



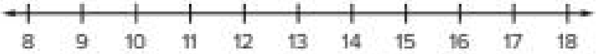
6. $\frac{x}{-8} > 2$



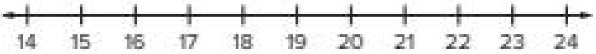
7. $\frac{x}{-5} \geq -3$



8. $\frac{x}{-2} < -6$

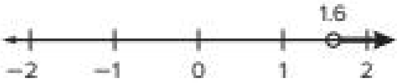


9. $\frac{x}{-8} \leq -2.4$



Test Practice

10. Multiselect Select all of the inequalities whose solution sets can be represented by the number line below.



☐ $-10x \leq 16$

☐ $-8 > \frac{x}{-0.2}$

☐ $-10x < -16$

☐ $-8 < \frac{x}{0.2}$

☐ $-8 \leq \frac{x}{-0.2}$

Apply

11. In an online game, you receive -3 points for every banana your character slips on during a race. The total points from slipping on bananas during one race were at most -27. When the character slips, it takes 2 seconds for him to resume the race. Write and solve an inequality to find the least number of bananas your character slipped on. At least how many seconds of race time were lost?

12. After a number of months, a bank balance was $-\$46.83$. Each month, the bank charges a \$5.95 fee for having a negative balance. Write and solve an inequality to determine at most how many months the fee was charged. What was the balance before the fees were charged?

13. **Identify Structure** Write two different inequalities that each has a solution of $x \le 4$. One inequality should involve multiplication with a negative coefficient and the other should involve division with a negative coefficient.

14. **Persevere with Problems** Solve each inequality.

a. $-2.7x - 3.4x > -24.583$

b. $-8\frac{1}{5}x + 3\frac{1}{3}x \le \frac{14}{15}$

15. **Which One Doesn't Belong?** Identify the inequality that does not belong with the other three. Explain your reasoning.

A. $-6x \le -36$

B. $\frac{x}{2} \ge 3$

C. $\frac{x}{3} \ge 2$

D. $\frac{x}{-3} \ge -2$

16. **Find the Error** A student solved the inequality shown below. Find the student's mistake and correct it.

$$-25 \le -5x$$

$$\frac{-25}{-5} \le \frac{-5x}{-5}$$

$$5 \le x$$

Lesson 7-5

Write and Solve One-Step Multiplication and Division Inequalities

I Can... write one-step multiplication and division inequalities from real-world situations and use inverse operations to solve the inequalities.

Example 1 Write and Solve One-Step Multiplication Inequalities

Ling earns $15 per hour working at the zoo.

Write and solve an inequality to determine the number of hours Ling must work in a week to earn at least $225. Then interpret the solution.

Part A Write an inequality.

Words	The amount earned per hour times the number of hours is at least the amount earned each week.
Variable	Let x represent the number of hours.
Inequality	$15x \leq 225$

Part B Solve the inequality.

$15x \geq 225$ Write the inequality.

$\frac{15x}{15} \geq \frac{225}{15}$ Division Property of Inequality

$x \geq \square$ Simplify.

The solution of the inequality $15x \geq 225$ is $x \geq 15$.

Part C Interpret the solution.

Graph the solution set of $x \geq 15$ on the number line.

Use the graph to interpret the solution.

So, Ling must work ________________.

Think About It!

What key word(s) tell you which inequality symbol to use?

Talk About It!

When solving the inequality, did you need to reverse the inequality symbol? Explain.

Your Notes

Check

Dominic has invited 12 friends to his birthday party. How much can he spend per person on party favors if his budget is no more than $75?

Write and solve an inequality to determine the budget for each person. Then interpret the solution.

Part A Which inequality can be used to find how much he can spend on favors?

Ⓐ $12x \leq 75$

Ⓑ $\frac{x}{12} \geq 75$

Ⓒ $\frac{x}{12} \leq 75$

Ⓓ $12x \geq 75$

Part B What is the solution of the inequality in Part A?

Ⓐ $x \geq 6.25$

Ⓑ $x \leq 900$

Ⓒ $x \geq 900$

Ⓓ $x \leq 6.25$

Show your work here

Part C Interpret the solution.

Dominic can spend ________________ per person on party favors.

Go Online You can complete an Extra Example online.

Pause and Reflect

Did you make any errors when completing the Check exercise? What can you do to make sure you don't repeat these errors in the future?

Record your observations here

Example 2 Write and Solve One-Step Division Inequalities

Mrs. Miller is buying wax to make candles. She wants to make at least 35 candles and needs 6.5 ounces of wax for each candle.

Write and solve an inequality to determine the amount of wax she needs to buy. Then interpret the solution.

Part A Write an inequality.

Words	The total amount of wax divided by the amount for each candle is at least the number of candles.
Variable	Let x represent the total amount of wax.
Inequality	$\frac{x}{6.5} \geq 35$

Part B Solve the inequality.

$\frac{x}{6.5} \geq 35$ Write the inequality.

$(\square)\frac{x}{6.5} \geq (\square)35$ Multiplication Property of Inequality

$x \geq 227.5$ Simplify.

The solution of the inequality $\frac{x}{6.5} \geq 35$ is $x \geq 227.5$.

Part C Interpret the solution.

Graph the solution set of $x \geq 227.5$ on the number line.

Use the graph to interpret the solution.

So, Mrs. Miller needs ________________ ounces of candle wax.

Think About It!

How would you begin writing the inequality?

Check

Thomas wants to make popsicles from a juice mixture. He needs 7.5 ounces of juice for each popsicle and wants to make no more than 12 popsicles. How much juice should he make?

Write and solve an inequality to determine the amount of juice he needs to make. Then interpret the solution.

Part A Which inequality can be used to determine how much juice he should make?

Ⓐ $\frac{x}{7.5} \geq 12$

Ⓑ $7.5x \leq 12$

Ⓒ $7.5x \geq 12$

Ⓓ $\frac{x}{7.5} \leq 12$

Part B What is the solution of the inequality in Part A?

Ⓐ $x \leq 90$

Ⓑ $x \geq 90$

Ⓒ $x \geq 1.6$

Ⓓ $x \leq 1.6$

Show your work here

Part C Interpret the solution.

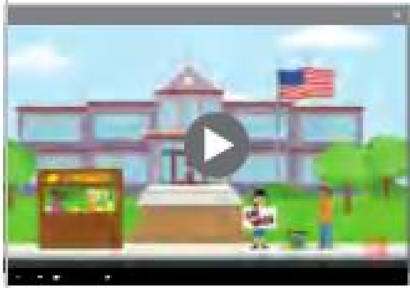
Thomas needs to make ________________ ounces of juice.

Go Online You can complete an Extra Example online.

Apply Fundraising

The students at Westlake Middle School are raising money for buses to go on a science field trip. Each bus holds 44 students and costs $150 for the day. If at least 275 students go on the trip, how much money will they need to raise for buses?

Go Online
Watch the animation.



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

Talk About It!
If the number of students participating in the trip increases by 35 students, will more money need to be raised? Explain your reasoning.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Check

Hudson needs to rent tables for a family reunion. Each table seats 8 people and costs $8.75 to rent. If at least 85 people attend the reunion, how much will it cost to rent tables?

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

Name _______________ Period _______ Date _______

Practice

Go Online You can complete your homework online.

Solve each problem by first writing an inequality.

1. Hermes earns $6 an hour for babysitting. He wants to earn at least $168 for a new video game system. Determine the number of hours he must babysit to earn enough money for the video game system. Then interpret the solution. (Example 1)

2. Becky wants to buy some fish for her aquarium. She has $20 to spend and the fish cost $2.50 each. Determine how many fish Becky can afford. Then interpret the solution. (Example 1)

3. Sadie wants to make several batches of rolls. She has 13 tablespoons of yeast left in the jar and each batch of rolls takes $3\frac{1}{4}$ tablespoons. Determine the number of batches of rolls Sadie can make. Then interpret the solution. (Example 1)

4. Trini needs more than 51 cubic feet of soil to fill her raised garden. Each bag of soil contains 1.5 cubic feet. Determine how many bags of soil Trini needs. Then interpret the solution. (Example 1)

5. A teacher is making tutus for the school play. She wants to make at least 24 tutus and needs 1.25 yards of tulle for each tutu. Determine the amount of tulle she needs to buy. Then interpret the solution. (Example 2)

6. Paul is making picture frames. He wants to make at least 8 picture frames and needs 24.5 inches of materials for each frame. Determine how much of the materials Paul should buy. Then interpret the solution. (Example 2)

7. Chase is making bookmarks. He wants to make no more than 12 bookmarks and needs 4.25 inches of fabric for each bookmark. Determine the amount of fabric he needs to buy. Then interpret the solution. (Example 2)

Test Practice

8. **Open Response** Mae wants to make more than 6 gift baskets for the school raffle. Each gift basket costs $15.50. Determine the amount of money she will spend to make the gift baskets. Then interpret the solution.

Apply

9. Greg's grandmother is knitting scarves for charity. The table shows the number of yards needed for different types of scarves. She plans to make bulky scarves and has no more than 2,250 yards of yarn. If the charity plans to sell the scarves for $24.50, how much money will the charity make?

Yarn Type	Number of Yards
Bulky	150
Light	250
Medium	200

10. At a school outing, a group decides to go rafting. Each raft holds 8 people and costs $25 for the day. If at least 70 people go rafting, how much money will they need for the rafts?

11. Write a real-world problem involving multiplication properties of an inequality that would have a solution $x \geq 25$.

12. MP **Reason Abstractly** You are asked to draw a rectangle with a width of 5 inches and an area less than 55 square inches. Can the length of the rectangle be 11 inches? Explain your reasoning.

13. **Create** Write, solve, and interpret the solution to a real-world problem that involves a one-step division inequality.

14. MP **Persevere with Problems** Solve each inequality.

a. $\frac{x}{3\frac{4}{7}} \leq 19\frac{3}{5}$

b. $\frac{x}{4\frac{8}{9}} > \frac{3}{4}$

Write and Solve Two-Step Inequalities

I Can... write two-step inequalities from real-world situations and use inverse operations to solve the inequalities.

Learn Solve Two-Step Inequalities

A **two-step inequality** is an inequality that contains two operations.

Go Online Watch the animation to learn how to solve a two-step inequality.

The animation shows the steps used to solve the two-step inequality $-2x + 6 \geq 12$.

Steps	Example
1. Undo the addition or subtraction.	$-2x + 6 \geq 12$ $\underline{\quad -6 \quad -6}$ $-2x \qquad \geq \ 6$
2. Undo the multiplication or division. Reverse the inequality symbol when multiplying or dividing by a negative number.	$\frac{-2x}{-2} \leq \frac{6}{-2}$ $x \leq -3$
3. Check the solution.	$-2x + 6 \geq 12$ $-2(-4) + 6 \overset{?}{\geq} 12$ $14 \geq 12$ ✓

Pause and Reflect

Compare and contrast solving a two-step inequality and a two-step equation. How are they similar? How are they different?

Record your observations here

What Vocabulary Will You Learn?
two-step inequality

Talk About It!
A student substituted the value −9 for x into the inequality $2x + 6 \geq 12$. Is this acceptable? Explain.

Your Notes

Think About It!
What steps do you need to take in order to solve the inequality?

Talk About It!
Suppose when Jesse solved the inequality, he claimed the solution is $x \leq -4$. Find his error and explain how to correct it.

Example 1 Solve Two-Step Inequalities

Solve $-5x - 12 \leq 8$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$-5x - 12 \leq 8$	Write the inequality.
$\underline{+12 \quad +12}$	Addition Property of Inequality
$-5x \leq 20$	Simplify.
$\frac{-5x}{-5} \geq \frac{20}{-5}$	Division Property of Inequality
$x \geq -4$	Simplify.

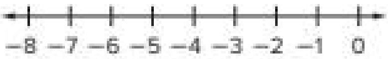
The solution of the inequality $-5x - 12 \leq 8$ is ________.

You can check the solution $x \geq -4$ by substituting a number greater than or equal to -4 into the original inequality.

$-5x - 12 \leq 8$	Write the inequality.
$-5(-1) - 12 \overset{?}{\leq} 8$	Replace x with -1.
$5 - 12 \overset{?}{\leq} 8$	Multiply $-5(-1)$.
$-7 \leq 8$ ✓	Simplify.

Part B Graph the solution set on a number line.

To graph $x \geq -4$ place a closed dot at -4 and an arrow to the right.



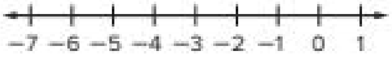
Check

Solve $-6x - 4 > 14$ and graph the solution set.

Part A Solve $-6x - 4 > 14$.

Show your work here

Part B Graph the solution set.



Go Online You can complete an Extra Example online.

Example 2 Solve Two-Step Inequalities

Solve $4.7x - 3.25 \leq 10.85$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$4.7x - 3.25 \leq 10.85$ Write the inequality.

$\underline{+3.25 \quad +3.25}$ Addition Property of Inequality

$4.7x \leq 14.1$ Simplify.

$\frac{4.7x}{4.7} \leq \frac{14.1}{4.7}$ Division Property of Inequality

$x \leq 3$ Simplify.

The solution of the inequality $4.7x - 3.25 \leq 10.85$ is ________.

You can check the solution $x \leq 3$ by substituting a number less than or equal to 3 into the original inequality.

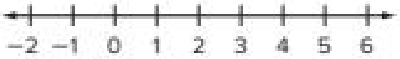
$4.7x - 3.25 \leq 10.85$ Write the inequality.

$4.7(1) - 3.25 \overset{?}{\leq} 10.85$ Replace x with 1.

$4.7 - 3.25 \overset{?}{\leq} 10.85$ Multiply.

$1.45 \leq 10.85$ ✓ Simplify.

Part B Graph the solution set on a number line.



Think About It!
What steps do you need to take in order to solve the inequality?

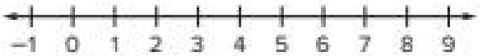
Check

Solve $1.3x - 3.2 \geq 4.6$ and graph the solution set.

Part A Solve $1.3x - 3.2 \geq 4.6$.

Show your work here

Part B Graph the solution set.



Go Online You can complete an Extra Example online.

Think About It!
What steps do you need to take in order to solve the inequality?

Example 3 Solve Two-Step Inequalities

Solve $\frac{3}{4}x - \frac{1}{2} > \frac{3}{8}$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$\frac{3}{4}x - \frac{1}{2} > \frac{3}{8}$ Write the inequality.

$\frac{3}{4}x - \frac{4}{8} > \frac{3}{8}$ Add $\frac{1}{2}$ to each side. Rewrite $\frac{1}{2}$ as $\frac{4}{8}$.

$\underline{\quad + \frac{4}{8} + \frac{4}{8}}$ Addition Property of Inequality

$\frac{3}{4}x > \frac{7}{8}$ Simplify.

$\frac{4}{3} \cdot \left(\frac{3}{4}x\right) > \frac{7}{8} \cdot \frac{4}{3}$ Multiply each side by the reciprocal.

$x > \frac{7}{6}$ or $1\frac{1}{6}$ Simplify.

The solution of the inequality $\frac{3}{4}x - \frac{1}{2} > \frac{3}{8}$ is ________.

You can check the solution $x > 1\frac{1}{6}$ by substituting a number greater than $1\frac{1}{6}$ into the original inequality.

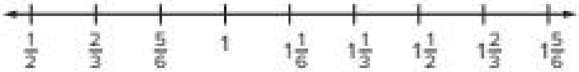
$\frac{3}{4}x - \frac{1}{2} > \frac{3}{8}$ Write the inequality.

$\frac{3}{4}(4) - \frac{1}{2} \overset{?}{>} \frac{3}{8}$ Replace x with 4.

$3 - \frac{1}{2} \overset{?}{>} \frac{3}{8}$ Multiply.

$2\frac{1}{2} > \frac{3}{8}$ ✓ Simplify.

Part B Graph the solution set on a number line.



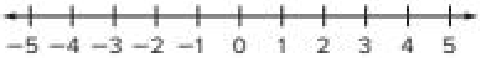
Check

Solve $\frac{2}{3}x - \frac{5}{6} < \frac{1}{2}$ and graph the solution set.

Part A Solve $\frac{2}{3}x - \frac{5}{6} < \frac{1}{2}$.

Show your work here

Part B Graph the solution set.



Go Online
You can complete an Extra Example online.

Example 4 Write and Solve Two-Step Inequalities

Halfway through the bowling league season, Stewart has 34 strikes. He averages 2 strikes per game. He needs at least 61 strikes to beat the league record.

Write and solve an inequality to determine the possible number of additional games Stewart should bowl to have at least 61 strikes. Then interpret the solution.

Think About It!

What is the unknown in the problem?

Part A Write an inequality.

Words	The number of strikes Stewart has plus two strikes per game is at least the total number of strikes needed.
Variable	Let g represent the number of games.
Inequality	$34 + 2g \geq 61$

Part B Solve the inequality.

$34 + 2g \geq 61$	Write the inequality.
$\underline{-34 \qquad -34}$	Subtraction Property of Inequality
$2g \geq 27$	Simplify.
$\frac{2g}{2} \geq \frac{27}{2}$	Division Property of Inequality
$g \geq 13.5$	Simplify.

The solution of the inequality $34 + 2g \geq 61$ is $g \geq$ __________

Talk About It!

When interpreting the solution of the inequality, why is the least number of games Stewart needs to bowl equal to 14, and not 13 or 13.5?

Part C Interpret the solution.

Graph the solution set of $g \geq 13.5$ on the number line.

Use the graph to interpret the solution.

So, in order to beat the record, Stewart will have to bowl __________ games.

Check

Peter can spend no more than \$100 on new clothes for school. He spends \$35 on a new pair of shoes. Shirts cost \$15.

Write and solve an inequality to determine how many shirts Peter can purchase. Then interpret the solution.

Part A Which inequality can be used to determine the number of shirts Peter can buy?

Ⓐ $35 + 15x \le 100$

Ⓑ $35x + 15 \le 100$

Ⓒ $35 + 15x \ge 100$

Ⓓ $35x + 15 \ge 100$

Part B What is the solution of the inequality in Part A?

Ⓐ $x \ge 2.4$

Ⓑ $x \le 4.\overline{3}$

Ⓒ $x \le 2.4$

Ⓓ $x \ge 4.\overline{3}$

Show your work here

Part C Interpret the solution.

Peter can buy ____________ shirts.

Go Online You can complete an Extra Example online.

Example 5 Write and Solve Two-Step Inequalities

Meredith is given a $50 monthly allowance to buy lunch at school. Any remaining money can be spent on entertainment. Meredith would like to have at least $12 left at the end of the month to go to the movies with her friends. It costs Meredith $2.50 per lunch that she buys at school.

Write and solve an inequality to determine the number of lunches Meredith can buy and have at least $12 left. Then interpret the solution.

Part A Write an inequality to determine the number of lunches Meredith can buy.

Words	Monthly allowance minus the cost per lunch is at least the amount remaining.
Variable	Let x represent the number of lunches.
Inequality	$50 - 2.50x \geq 12$

Part B Solve the inequality.

$$\begin{array}{rl}
50 - 2.50x \geq 12 & \text{Write the inequality.} \\
\underline{-50 \qquad\quad -50} & \text{Subtraction Property of Inequality} \\
-2.50x \geq -38 & \text{Simplify.} \\
\frac{-2.50x}{-2.50} \leq \frac{-38}{-2.50} & \text{Division Property of Inequality} \\
x \leq 15.2 & \text{Simplify.}
\end{array}$$

The solution of the inequality $50 - 2.50x \geq 12$ is ____________.

Part C Interpret the solution.

Graph the solution set of $x \leq 15.2$ on the number line.

Use the graph to interpret the solution.

So, Meredith can buy ________________ lunches in order to have at least $12 remaining to go to the movies.

Think About It!

What inequality symbol will you use to write the inequality?

Talk About It!

If you forgot to reverse the inequality symbol when you divided each side by −2.50, how can you know that your solution is incorrect?

Check

Sylvia was given $25 for her birthday and would like to use some of the money to purchase music from a music streaming website. It costs $1.20 per song she downloads. She would like to have at least $10 left.

Write and solve an inequality to determine the number of songs Sylvia can purchase and have at least $10 left. Then interpret the solution.

Part A Which inequality can be used to determine the number of songs that Sylvia can buy?

Ⓐ $25 - 1.20x \geq 10$

Ⓑ $10 - 1.20x \leq 25$

Ⓒ $10 - 1.20x \geq 25$

Ⓓ $25 - 1.20x \leq 10$

Part B What is the solution of the inequality in Part A?

Ⓐ $x \leq 12.5$

Ⓑ $x \geq 12.5$

Ⓒ $x \geq -12.5$

Ⓓ $x \leq -12.5$

Show your work here

Part C Interpret the solution.

Sylvia can buy ________________ songs to have at least $10 left.

Go Online You can complete an Extra Example online.

Apply School

To earn the grade she wants in her English class, Elspeth needs an average of 85% from her quiz scores. Each quiz is worth 20 points. The scores of her first four quizzes are shown in the table. If there is one more quiz, what score can she receive to earn at least an 85% average in English class?

Quiz	Score
1	18
2	16
3	19
4	14

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It!

In the inequality $\frac{67 + x}{100} \geq 0.85$ that represents Elspeth's average score, why is 100 in the denominator?

Check

In order for Ainsley to earn the grade she wants in science class, she needs an average of 85% on her quiz scores. Each quiz is worth 30 points. The scores of her first 5 quizzes are shown in the table. There will be one more quiz. What score can she receive to earn at least an 85% average in science class?

Quiz	Score
1	25
2	26
3	25
4	24
5	28

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

Name ______________________ Period __________ Date __________

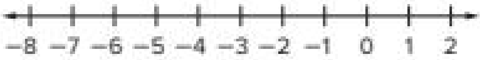
Practice

Go Online You can complete your homework online.

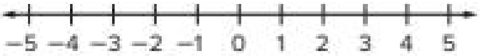
Solve each inequality. Graph the solution set on a number line. (Examples 1–3)

1. $-3x - 3 > 12$

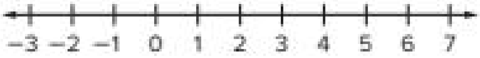
2. $-4 \leq 4x + 8$



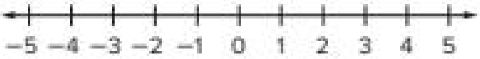
3. $6.5x - 11.3 \leq 8.2$



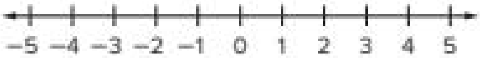
4. $-2.45x + 3.2 < -6.6$



5. $\frac{1}{2}x - \frac{1}{4} < \frac{5}{8}$



6. $\frac{x}{10} + \frac{1}{4} \geq \frac{1}{5}$



7. A rental company charges \$15 plus \$4 per hour to rent a bicycle. If Margie wants to spend no more than \$27 for her rental, write and solve an inequality to determine how many hours she can rent the bicycle. Then interpret the solution. (Example 4)

8. Matilda needs at least \$112 to buy a new dress. She has already saved \$40. She earns \$9 an hour babysitting. Write and solve an inequality to determine how many hours she will need to babysit to buy the dress. Then interpret the solution. (Example 4)

9. Douglas bought a \$20 game card at a game center. The go-karts cost \$3.50 each time you race. He wants to have at least \$7.75 left on his card to play arcade games. Write and solve an inequality to determine how many times Douglas can race the go-karts. Then interpret the solution. (Example 5)

Test Practice

10. Open Response Robin was given a \$40 monthly allowance. She wants to go to the movies as many times as possible and have at least \$12.50 left at the end of the month to go to a concert. A movie ticket costs \$5. Write and solve an inequality to determine how many times Robin can go to the movies this month. Then interpret the solution.

Apply

11. To earn the score he wants in a trivia game, Jet needs an average of 80% after five rounds. Each round is worth 50 points. The scores of his first four rounds are shown in the table. If there is one more round, what is the minimum score he can receive to earn at least an 80% average in the trivia game?

Round	Score
1	49
2	46
3	44
4	45

12. Eden needs an average of 92% on her quiz scores to earn the grade she wants in science class. Each quiz is worth 20 points. The scores of her first four quizzes are shown in the table. There will be one more quiz. What score she can receive to earn at least a 92% average in science class?

Quiz	Score
1	20
2	18
3	19
4	17

13. Solve $-2(x + 2) < x + 8$. Then graph the solution set on a number line.

14. MP **Identify Structure** Write a two-step inequality that can be solved by first adding 4 to each side.

15. MP **Identify Structure** Explain how you can solve the inequality $-2x + 4 < 16$ without multiplying or dividing by a negative coefficient.

16. **Create** Write, solve, and interpret the solution to a real-world problem that involves a two-step inequality.

Module 7 • Write and Solve Inequalities

Review

Foldables Use your Foldable to help review the module.

Solve Inequalities

$x + 7 > -9$

$8x < 72$

$-9x > 99$

$-4x + 2 < 26$

Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Reflect on the Module

Use what you learned about writing and solving inequalities to complete the graphic organizer.

Essential Question

How are the solutions to inequalities different from the solutions to equations?

Symbol	Phrases
$<$	
$>$	
$\leq$	
$\geq$	

How are solving equations and inequalities similar?

How are solving equations and inequalities different?

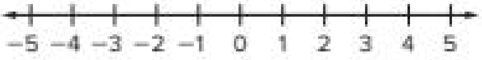
Name ______________________ Period __________ Date __________

Test Practice

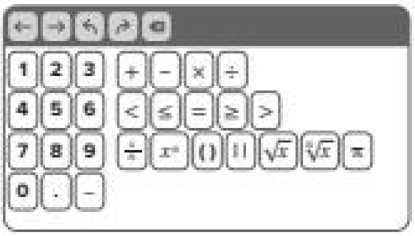
1. **Grid** Consider the inequality $x + 8 \geq 5$. (Lesson 1)

   A. Solve the inequality.

   B. Graph the solution of the inequality.



2. **Equation Editor** Write the correct symbol and value that represents the solution to the inequality $0.8 + n \leq -2.5$. (Lesson 1)



3. **Open Response** Jessica has a department store gift card worth $75. The table shows the items that she has picked out so far. (Lesson 2)

Item	Cost ($)
bracelet	8
sunglasses	15
beach towel	12
hat	10
sandals	14

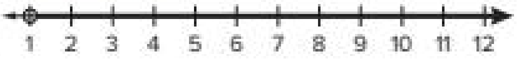
Jessica also wants to buy two T-shirts. Write and solve an inequality to represent how much she can spend on the T-shirts. Interpret the solution.

4. **Grid** The difference between the high and low temperatures on Sunday was at least 35°F. The low temperature on Sunday was −8°F. (Lesson 2)

   A. Write and solve an inequality to find t, the high temperature on Sunday.

   B. Graph the solution of the inequality.

5. **Multiple Choice** Which number line represents the solution to the inequality $\frac{c}{2} > 3$? (Lesson 3)

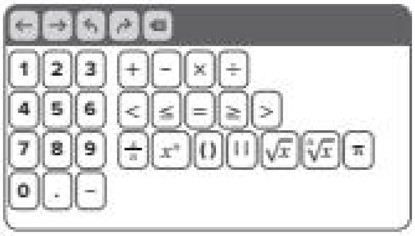
Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

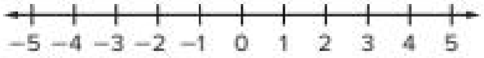
6. **Equation Editor** Write the correct symbol and value that represents the solution to the inequality $9x < 27$. (Lesson 3)



7. Grid Consider the inequality $-11x \geq 22$. (Lesson 4)

A. Solve the inequality. Explain why or why not the direction of the inequality symbol is reversed.

B. Graph the solution of the inequality.



8. Multiple Choice Which of the following represents the solution to the inequality $-1.25b < -6$? (Lesson 4)

Ⓐ $b > -4.8$

Ⓑ $b < -4.8$

Ⓒ $b > 4.8$

Ⓓ $b < 4.8$

9. Multiselect Tom is making birdhouses to sell at a farmer's market. Each birdhouse requires 4.5 feet of wood planks to build, and he sells them for $19.75 each. If he has no more than 80 feet of planks to make birdhouses, how much money can Tom earn at the farmer's market? Select all that apply. (Lesson 5)

☐ no more than $335.75

☐ ≥ $335.75

☐ at least $335.75

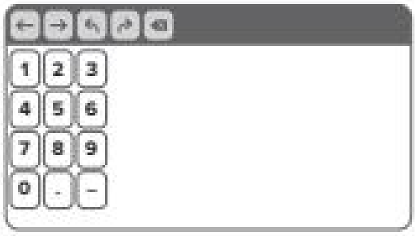
☐ ≤ $335.75

☐ exactly $335.75

10. Equation Editor Ling makes bracelets and necklaces and sells them at different craft fairs. The table shows how much string is used to make each item. (Lesson 5)

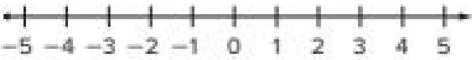
Piece	String Needed (in.)
bracelet	$7\frac{1}{2}$
small necklace	15
large necklace	$19\frac{1}{4}$

Ling plans to make at least 20 bracelets. Write and solve an inequality to determine the minimum length of string that she needs to buy. What is the least number of inches of string Ling will need?



11. Open Response Solve the inequality $6.6z - 2.25 \leq -10.5$. Explain why or why not the direction of the inequality symbol is reversed. (Lesson 6)

12. Grid Graph the solution of the inequality $\frac{1}{3}x - \frac{5}{6} < \frac{1}{2}$. (Lesson 6)



Module 8

Geometric Figures

Essential Question

How does geometry help to describe objects?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY

■ — I don't know. ◆ — I've heard of it. ★ — I know it!

	Before			After		
	■	◆	★	■	◆	★
classifying and naming angles						
identifying and using vertical angles and adjacent angles to solve problems						
identifying and using complementary and supplementary angles to solve problems						
classifying triangles						
drawing triangles freehand, using tools, or with technology						
using scale drawings to solve problems						
creating scale drawings						
describing three-dimensional figures						
describing cross sections of three-dimensional figures						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about geometric figures.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ adjacent angles
☐ bases
☐ complementary angles
☐ cone
☐ congruent
☐ cross section
☐ cylinder
☐ edge
☐ face
☐ plane
☐ polyhedron
☐ prism
☐ pyramid
☐ scale
☐ scale drawings
☐ scale factor
☐ scale model
☐ scalene triangle
☐ straight angle
☐ supplementary angles
☐ vertex
☐ vertical angles
☐ vertices
☐ zero angle

Are You Ready?

Study the Quick Review to see if you are ready to start this module.
Then complete the Quick Check.

Quick Review

Example 1

Name segments and rays.

Name the figures.

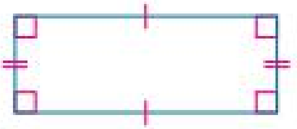
The figure on the left has two endpoints, *A* and *B*. The name of the figure is segment *AB* or segment *BA*.

The figure on the right has one endpoint, *B*, and goes on forever in the other direction. The name of the figure is ray *BC*.

Example 2

Identify two-dimensional figures.

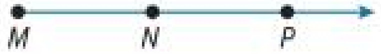
Identify the figure by its sides and angles.



The figure has 4 sides and 4 angles. The 4 angles are right angles. Both pairs of opposite sides are parallel. The figure is a rectangle.

Quick Check

1. Name one segment and one ray.



2. Identify the figure that represents the top of the table shown.

How Did You Do?

Which exercises did you answer correctly in the Quick Check?
Shade those exercise numbers at the right.

① ②

Vertical and Adjacent Angles

I Can... identify vertical and adjacent angles, and use them to write and solve equations to find unknown angle measures.

Learn Angles

The hands on a clock form an angle with the **vertex** at the center of the clock where the hands meet. At different times of day, the angle formed by the hands could be **obtuse**, **acute**, **right**, **straight**, or **zero**.

Draw the hands of each clock to represent each type of angle.

Types of Angles		
	obtuse	greater than 90°, less than 180°
	acute	less than 90°, greater than 0°
	right	exactly 90°
	straight	exactly 180°
	zero	exactly 0°

Learn Name Angles

An angle can be named using three capital letters. These letters come from three points labeled on the angle—one point from the vertex and one point from each ray. The middle letter must be the vertex of the angle.

The symbol for angle is ∠. An angle named ∠*XYZ* is read *angle XYZ*.

An angle can be named using only one letter, the vertex. An angle can also be named by placing a number in the interior of the angle near the vertex.

(continued on next page)

What Vocabulary Will You Learn?
acute angle
adjacent angles
congruent
obtuse angle
right angle
straight angle
vertex
vertical angles
zero angle

Talk About It!

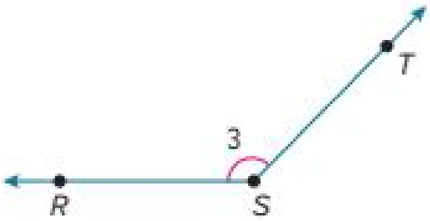
Is it possible for an angle to have a measure greater than 180°? Explain.

Your Notes

The angle can be named in four ways.

$\angle RST$, $\angle S$,

$\angle 3$, $\angle TSR$



Talk About It!

A classmate states that the angle is named $\angle RTS$. Explain why this is incorrect.

Example 1 Name Angles

Name the angle in four ways.

Select all of the correct names for the given angle.

☐ $\angle 1$

☐ $\angle XYZ$

☐ $\angle XZY$

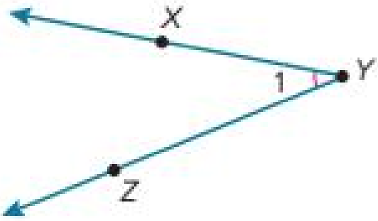
☐ $\angle ZXY$

☐ $\angle ZYX$

☐ $\angle X$

☐ $\angle Y$

☐ $\angle Z$



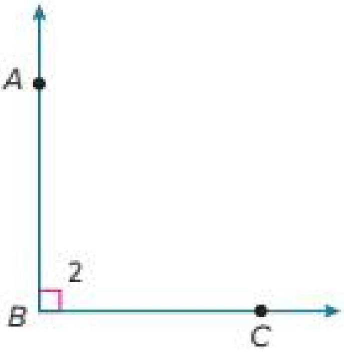
Talk About It!

Why is $\angle XZY$ not a correct name for the angle?

So, the angle can be named by the vertex, three points on the angle with a specified order, and a number in the interior of the angle.

Check

Name the angle in four ways.



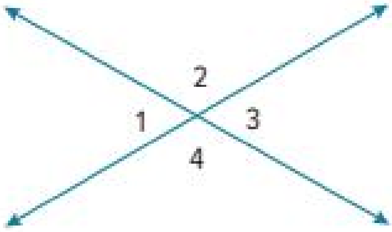
Go Online You can complete an Extra Example online.

Explore Vertical and Adjacent Angle Pairs

Online Activity You will use Web Sketchpad to explore attributes of vertical and adjacent angles.

Learn Identify Vertical Angles

Two angles are **vertical angles** if they are opposite angles formed by the intersection of two lines. Vertical angles are **congruent,** or have the same measure.



Angle 1 is congruent to angle 3.

$\angle 1 \cong \angle 3$

Angle 2 is congruent to angle 4.

$\angle 2 \cong \angle 4$

The measure of angle 1 is equal to the measure of angle 3.

$m\angle 1 = m\angle 3$

The measure of angle 2 is equal to the measure of angle 4.

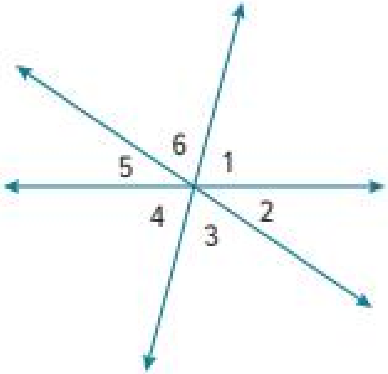
$m\angle 2 = m\angle 4$

Talk About It!

Vertical angles share a common point. How can you name or describe that point to a classmate?

Example 2 Identify Vertical Angles

Identify the vertical angle pairs in the figure.



$\angle 1$ is vertical to $\angle$ ________ .

$\angle 2$ is vertical to $\angle$ ________ .

$\angle 3$ is vertical to $\angle$ ________ .

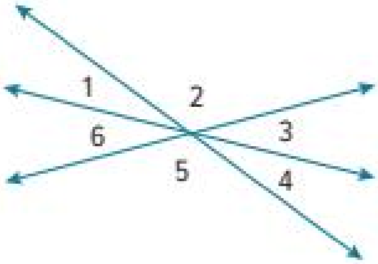
So, the vertical angle pairs are $\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 5$, and $\angle 3$ and $\angle 6$.

Talk About It!

A classmate stated that $\angle 2$ and $\angle 6$ are vertical angles since they share the same vertex and are on opposite sides of the horizontal line. Make an argument that shows why this reasoning is incorrect.

Check

Identify the vertical angle pairs by writing each angle label from the diagram by its corresponding vertical angle.



∠1 is vertical to ∠ ________

∠2 is vertical to ∠ ________

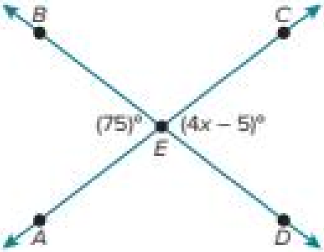
∠3 is vertical to ∠ ________

Go Online You can complete an Extra Example online.

Learn Use Vertical Angles to Find Missing Values

Go Online Watch the animation to see how to find missing values using vertical angles.

The animation shows how to write and solve an equation to find the value of *x*.



Angle *AEB* and angle *CED* are vertical angles.

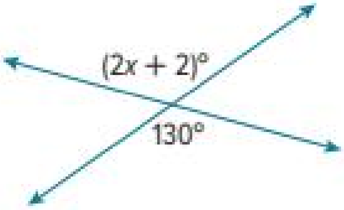
$\angle AEB \cong \angle CED$	Vertical angles are congruent.
$m\angle AEB = m\angle CED$	Definition of congruence
$75 = 4x - 5$	$m\angle AEB = 75°$, $m\angle CED = (4x - 5)°$
$\underline{+5 \qquad +5}$	Add 5 to each side.
$80 = 4x$	Simplify.
$\frac{80}{4} = \frac{4x}{4}$	Divide each side by 4.
$20 = x$	Simplify.

So, the value of *x* is 20.

Talk About It!
How can you check your solution?

Example 3 Use Vertical Angles to Find Missing Values

Write and solve an equation to find the value of *x*.



Part A Write an equation.

Because the two angles are vertical angles, they are congruent. Write an equation showing that the two angle measures are equivalent.

☐ = ☐

Part B Solve the equation.

$2x + 2 = 130$ Write the equation.

$-2 \quad -2$ Subtract 2 from each side.

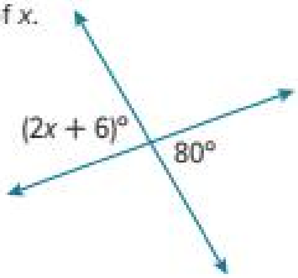
$2x = 128$ Simplify.

$x = \boxed{}$ Divide each side by 2.

So, $x = 64$.

Check

Write and solve an equation to find the value of *x*.



Part A Write an equation.

Part B Solve the equation.

Show your work here

Go Online You can complete an Extra Example online.

Think About It!

What is the relationship between the two angles shown?

Talk About It!

How can you use the value of *x* to check your solution?

Talk About It!
Where have you heard the term *adjacent* before? How can you remember what it means in geometry?

Learn Identify Adjacent Angles

Two angles are **adjacent angles** if they share a common vertex, a common side, and do not overlap.

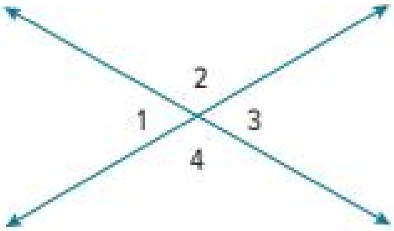
The diagram shows four pairs of adjacent angles.

∠1 and ∠2

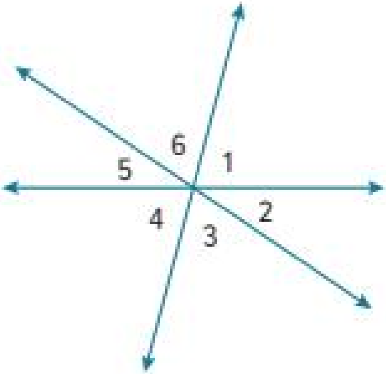
∠2 and ∠3

∠3 and ∠4

∠4 and ∠1



The diagram below shows three intersecting lines.



Which angles are adjacent to ∠2? __________

Which angles are adjacent to ∠5? __________

Talk About It!
A classmate stated that ∠4 and ∠5 are adjacent. Do you agree? Justify your reasoning.

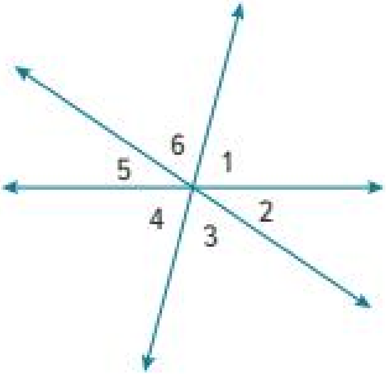
Example 4 Identify Adjacent Angles

Name the angles that are adjacent to ∠1.

Because ∠1 shares a common side and vertex with ∠2, they are adjacent angles.

What other angle shares a side and vertex with ∠1? ________

So, ∠2 and ∠6 are adjacent to ∠1.



Talk About It!
A classmate stated that ∠4 and ∠2 are also adjacent. Do you agree? Justify your reasoning.

Check

Select all of the angles that are adjacent to ∠3.

☐ ∠1

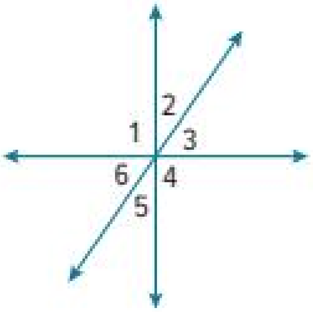
☐ ∠2

☐ ∠3

☐ ∠4

☐ ∠5

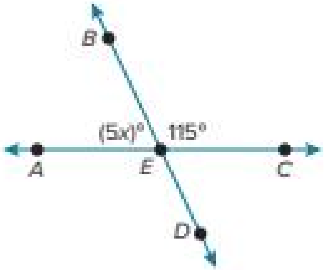
☐ ∠6



Go Online You can complete an Extra Example online.

Learn Use Adjacent Angles to Find Missing Values

Go Online Watch the animation to see how to use adjacent angles to find a missing value.



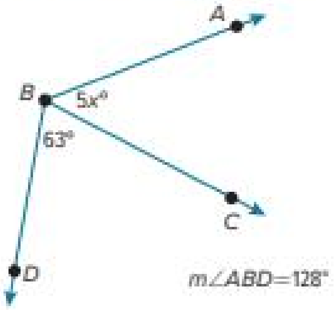
Angle *AEB* and angle *BEC* are adjacent angles.

$m\angle AEB + m\angle BEC = 180$	The adjacent angles form a straight angle. The sum is 180°.
$5x + 115 = 180$	$m\angle AEB = (5x)°$, $m\angle BEC = 115°$
$\underline{-115 \quad -115}$	Subtract 115 from each side.
$5x = 65$	Simplify.
$\frac{5x}{5} = \frac{65}{5}$	Divide each side by 5.
$x = 13$	Simplify.

So, the value of *x* is 13.

Example 5 Use Adjacent Angles to Find Missing Values

Write and solve an equation to find the value of x.



The diagram shows that $m\angle ABC + m\angle CBD = m\angle ABD$.

$m\angle ABC + m\angle CBD = m\angle ABD$	Write the equation.
$5x + 63 = 128$	$m\angle ABC = 5x°$, $m\angle CBD = 63°$, $m\angle ABD = 128°$
$\underline{-63 \quad -63}$	Subtract 63 from each side.
$5x = 65$	Simplify.
$\frac{5x}{5} = \frac{65}{5}$	Divide each side by 5.
$x = 13$	Simplify.

So, $x = 13$.

Talk About It!

How can you use the value of x to find the measure of $\angle ABC$?

Talk About It!

A classmate found the value of x by setting the sum of the angle measures equal to 180. Explain your classmate's error.

Check

Write and solve an equation to find the value of x.

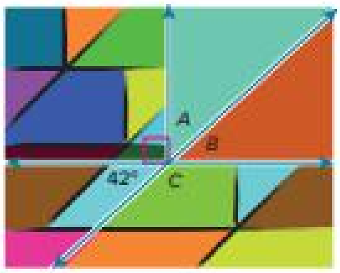
Part A Write an equation.

Part B Solve the equation.

Show your work here

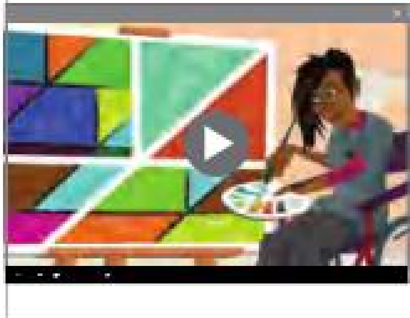
Go Online You can complete an Extra Example online.

Apply Art



Tamika is using lines and angles to create abstract art. She needs to find the measure of ∠A to continue the pattern in the art. What is the measure of ∠A?

Go Online Watch the animation.



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

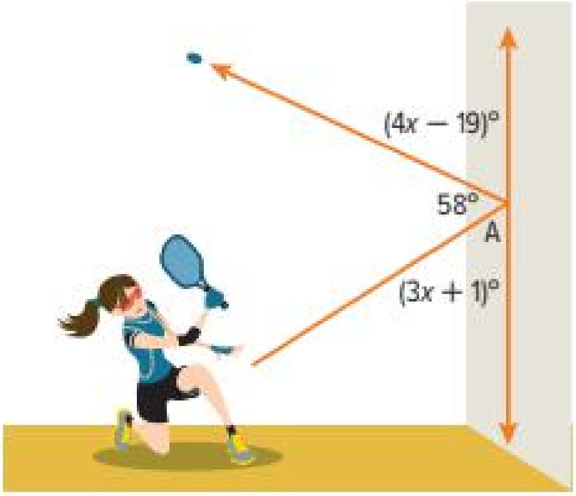
4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It!
How can you solve the problem another way?

Check

While playing racquetball, Tia bounced the ball off the wall at the angle shown. Determine the measure of ∠A.

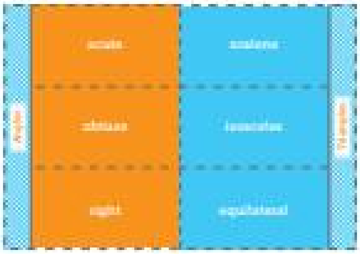


The measure of ∠A is ________.

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

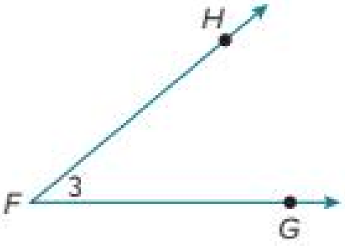


Name ______________________ Period __________ Date __________

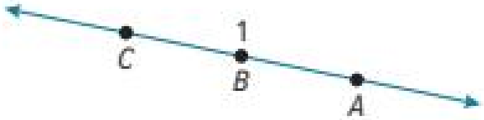
Practice

Go Online You can complete your homework online.

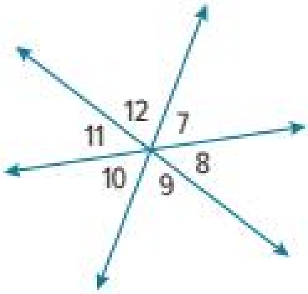
1. Name the angle in four ways. (Example 1)



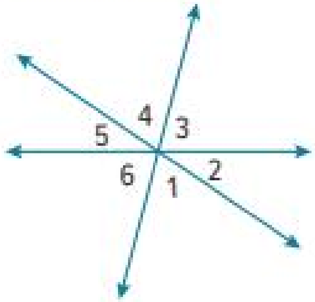
2. Name the angle in four ways. (Example 1)



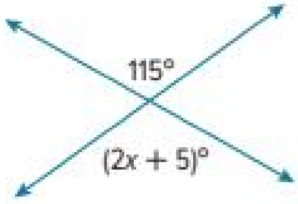
3. Refer to the diagram below. Identify three pairs of vertical angles. Name all the angles that are adjacent to ∠10. (Examples 2 and 4)



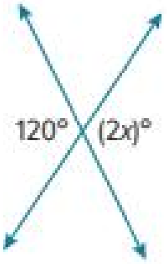
4. Identify three pairs of vertical angles. Name all the angles that are adjacent to ∠3. (Examples 2 and 4)



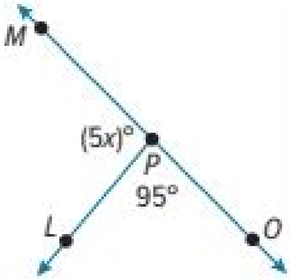
5. Write and solve an equation to find the value of x. (Example 3)



6. Write and solve an equation to find the value of x. (Example 3)

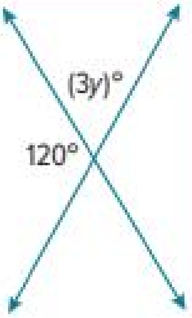


7. Write and solve an equation to find the value of x. (Example 5)



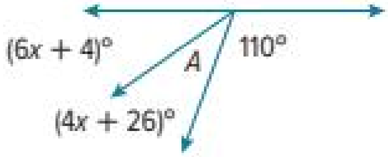
Test Practice

8. **Open Response** Write and solve an equation to find the value of y.

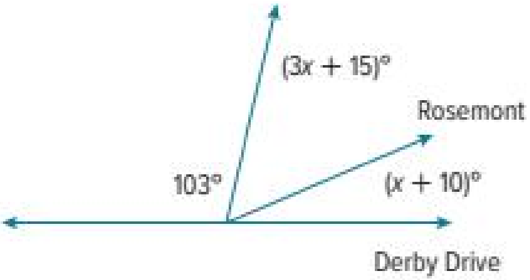


Apply

9. Levi was designing a kite. He needs to determine the measure of ∠A before cutting the fabric. What is the measure of angle A?

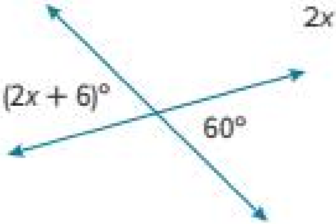


10. Jess was drawing a map of her neighborhood. What is the measure of the angle of the intersection between Derby Drive and Rosemont?



11. Draw and label a pair of vertical angles.

12. **Find the Error** A student was finding the value of x. Identify the student's error and correct it.



$$\begin{aligned} 2x + 6 + 60 &= 180 \\ 2x + 66 &= 180 \\ 2x &= 114 \\ x &= 57 \end{aligned}$$

13. **Be Precise** A student said that the sum of the measures of a pair of adjacent angles must equal 180°. Is the student correct? Write an argument that can be used to defend your solution.

14. **Reason Abstractly** Determine if the following statement is *true* or *false*. If true, provide a diagram. If false, explain.

A pair of acute angles can also be adjacent angles.

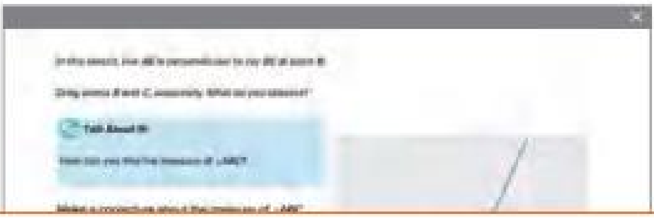
Complementary and Supplementary Angles

I Can... identify complementary and supplementary angles, and use them to write and solve equations to find unknown angle measures.

What Vocabulary Will You Learn?
complementary angles
supplementary angles

Explore Complementary and Supplementary Angle Pairs

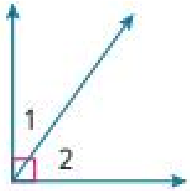
Online Activity You will use Web Sketchpad to explore the properties of complementary and supplementary angle pairs.



Learn Identify Complementary Angles

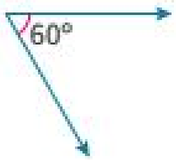
Two angles are **complementary angles** if the sum of their measures is 90°.

Words	The measure of angle 1 plus the measure of angle 2 equals 90 degrees.
Symbols	$m\angle 1 + m\angle 2 = 90°$



Example 1 Identify Complementary Angles

Give the measure of the angle that is complementary to the given angle.



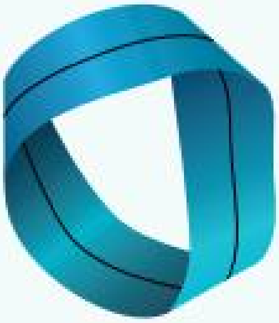
Complementary angles have a sum of 90°.

The equation $60 + x = 90$ can be used to find the measure of the angle that is complementary to the given angle.

Because $x = 30$, the measure of the angle complementary to the 60 degree angle is ________.

Talk About It!
Trevor stated that all complementary angles are adjacent. Draw a diagram that supports his claim. Then draw a diagram that illustrates a counterexample. Is Trevor correct?

Your Notes



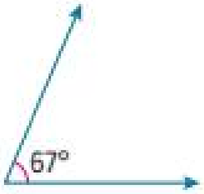
Math History Minute

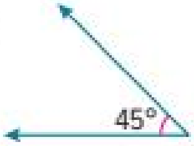
German mathematician and astronomer **August Ferdinand Möbius (1790–1868)** created the Möbius strip which has fascinated mathematicians worldwide since. It is created by twisting a strip of paper one time and then joining the two ends. The Möbius strip only has one edge and one side. You can take a pencil and draw a single line in a continuous loop without ever crossing an edge.

Check

Select the angle that is complementary to the given angle.

Ⓐ


Ⓑ


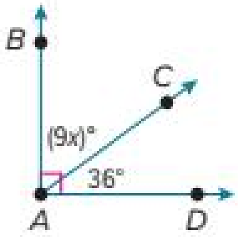
Ⓒ


Ⓓ


Go Online You can complete an Extra Example online.

Learn Use Complementary Angles to Find Missing Values

Go Online Watch the animation to see how to use complementary angles to find a missing value.



Step 1 Identify the complementary angles.

∠*BAC* and ∠ ________ are complementary and have a sum of 90°.

Step 2 Write the relationship between the angles.

$m\angle BAC + m\angle CAD = 90°$

Step 3 Write an equation by substituting for each angle measure.

$(9x) + 36 = 90$

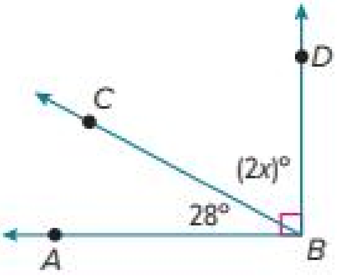
Step 4 Solve the equation.

$$\begin{aligned} 9x + 36 &= 90 && \text{Write the equation.}\\ \underline{-36\ \ -36} & && \text{Subtract 36 from each side.}\\ 9x &= 54 && \text{Simplify.}\\ \frac{9x}{9} &= \frac{54}{9} && \text{Divide each side by 9.}\\ x &= \boxed{} && \text{Simplify.} \end{aligned}$$

So, the value of *x* is 6.

Example 2 Use Complementary Angles to Find Missing Values

Write and solve an equation to find the value of *x*.



Part A Write an equation.

Because $m\angle ABC$ and $m\angle CBD$ have a sum of 90°, write an equation showing that the sum of the two angle measures is 90°.

$\boxed{} + \boxed{} = 90$

Part B Solve the equation.

$2x + 28 = 90$ Write the equation.

$\underline{-28 \quad -28}$ Subtract 28 from each side.

$2x = 62$ Simplify.

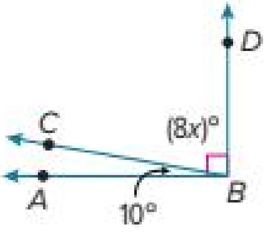
$\frac{2x}{2} = \frac{62}{2}$ Divide each side by 2.

$x = \boxed{}$ Divide by 2.

So, $x = 31$.

Check

Write and solve an equation to find the value of *x*.



Part A Write an equation.

Part B Solve the equation.

Show your work here

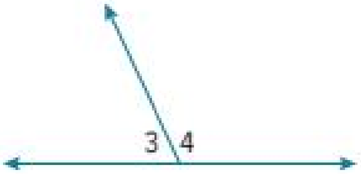
Think About It!

What is the relationship between the two angles shown?

Go Online You can complete an Extra Example online.

Learn Identify Supplementary Angles

Two angles are **supplementary angles** if the sum of their measures is 180°.

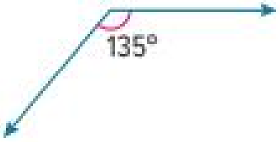


Words	The measure of angle 3 plus the measure of angle 4 equals 180 degrees.
Symbols	$m\angle 3 + m\angle 4 = 180°$

Think About It!
What do you know about two supplementary angles?

Example 3 Identify Supplementary Angles

What is the measure of the angle that is supplementary to the given angle?



What is sum of the angle measures of supplementary angles?

____

Let *x* represent the measure of the angle that is supplementary to the given angle. The equation $135 + x = 180$ can be used to represent this situation.

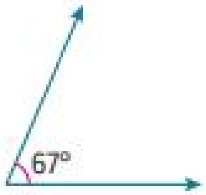
Solve the equation for *x*.

$135 + x = 180$	Write the equation.
$\underline{-135 \qquad -135}$	Subtract 135 from each side.
$x = 45$	Simplify.

So, the measure of the angle that is supplementary to the given angle is 45°.

Check

What is the angle measure of the angle that is supplementary to the given angle?

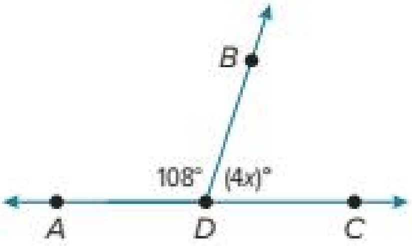


Go Online You can complete an Extra Example online.

Learn Use Supplementary Angles to Find Missing Values

You can use the properties of supplementary angles to find missing measures.

Go Online Watch the animation to see how to use supplementary angles to find a missing value.



Step 1 Identify the supplementary angles.

∠ADB and *∠BDC*

Step 2 Write the relationship between the angles. Because supplementary angles have measures with a sum of 180°, set the sum of the angle measures equal to 180°.

$m\angle ADB + m\angle BDC = 180°$

Step 3 Write an equation by substituting for each angle measure.

$108 + 4x = 180$

Step 4 Solve the equation.

$$\begin{array}{rcll} 108 + 4x & = & 180 & \text{Write the equation.} \\ \underline{-108} & & \underline{-108} & \text{Subtract 108 from each side.} \\ 4x & = & 72 & \text{Simplify.} \\ \frac{4x}{4} & = & \frac{72}{4} & \text{Divide each side by 4.} \\ x & = & 18 & \text{Simplify.} \end{array}$$

Think About It!

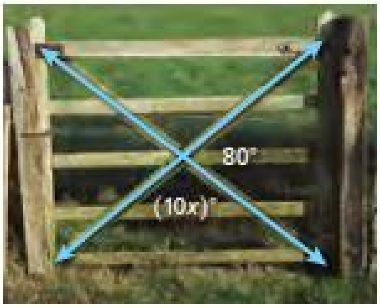
What is the relationship between the two angles shown?

Talk About It!

Why were the expressions for the angle measures not set equal to each other ($10x = 80$)?

Example 4 Use Supplementary Angles to Find Missing Values

Write and solve an equation to find the value of *x*.



Part A Write an equation.

Because the angles are supplementary angles, set the sum of the two angle measures equal to 180°.

$10x + 80 = 180$

Part B Solve the equation.

$10x + 80 = 180$ Write the equation.

$\underline{\quad - 80 \quad -80}$ Subtract 80 from each side.

$10x = 100$ Simplify.

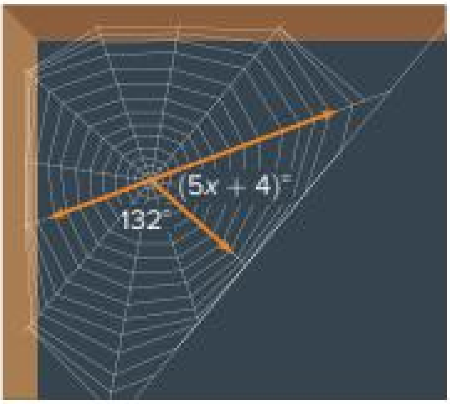
$\frac{10x}{10} = \frac{100}{10}$ Divide each side by 10.

$x = \boxed{}$ Simplify.

So, $x = 10$.

Check

Write and solve an equation to find the value of *x*.



Part A Write an equation.

Part B Solve the equation.

$x =$ ____________

Show your work here

Go Online You can complete an Extra Example online.

Apply Engineering

A space shuttle scaffold has the angles shown. Engineers determined that the measure of angle *x* needs to be about 7% less to be more supportive. What is the measure of the new angle?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It!
How do you calculate the 7% decrease in the measure of the angle labeled $x°$?

Check

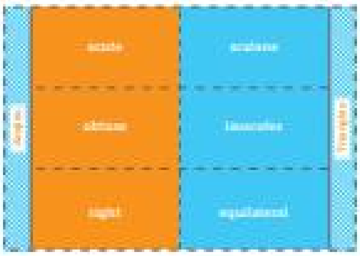
In the winter, a solar panel is set with the angles shown. In the summer, the measure of angle *x* is reduced by about 46.2%. What is the new measure of the angle in the summer? Round to the nearest degree.

The new measure of angle *x* is about ________.

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



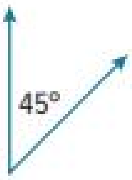
Name ________________________ Period ____________ Date ____________

Practice

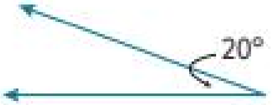
Go Online You can complete your homework online.

Give the measure of the angle that is complementary to the given angle. (Example 1)

1.



2.



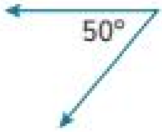
3.

Give the measure of the angle that is supplementary to the given angle. (Example 3)

4.

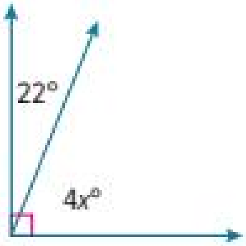
5.

6.



Write and solve an equation to find the value of *x* in each figur . (Examples 2 and 4)

7.



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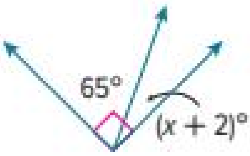

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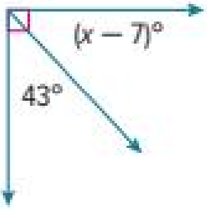
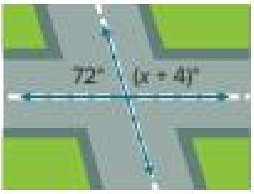
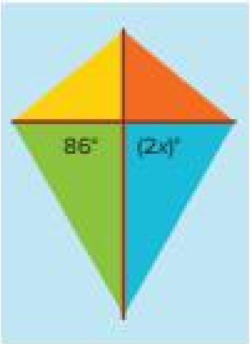


10.

11.

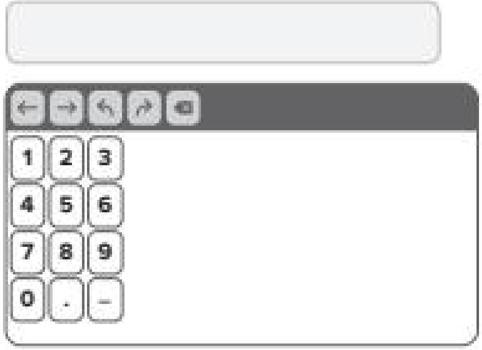


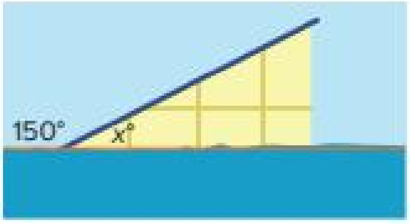
12.



Test Practice

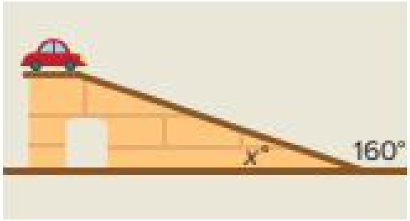
13. Equation Editor An adjustable water ski ramp is set at the angles shown. An instructor wants to decrease angle *x* by 8%. What is the new measure of the angle, to the nearest tenth of a degree?





Apply

14. Truman's father is designing a toy car ramp for him. His dad determined that the measure of angle *x* needs to be increased by 20%. What is the measure of the new angle?



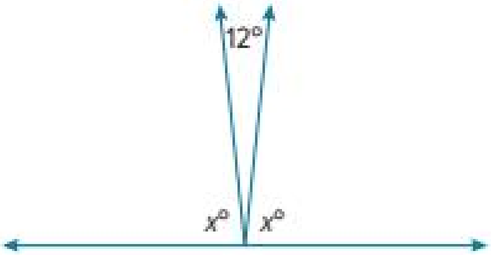
15. Draw a pair of supplementary, adjacent angles. Label the measures of the angles.

16. **Persevere with Problems** Find the measure of angle *A* and angle *B* for the given situation.

complementary angles *A* and *B*, where $m\angle A = (y - 16)^\circ$ and $m\angle B = (y + 4)^\circ$

17. **Justify Conslusions** A student said that a pair of complementary angles cannot also be adjacent angles. Is the student correct? Explain. Support your answer with a drawing.

18. What is the value of *x*? Write an argument that can be used to defend your solution.



Lesson 8-3

Triangles

I Can... classify and draw triangles, freehand, with tools, and with technology given certain conditions, such as angle measures or side lengths.

What Vocabulary Will You Learn?
acute triangle
equilateral triangle
isosceles triangle
obtuse triangle
right triangle
scalene triangle

Explore Create Triangles

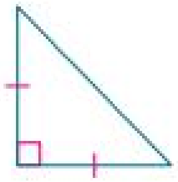
Online Activity You will use Web Sketchpad to explore the relationships among the side lengths or angle measures in a triangle.

Learn Classify Triangles

A triangle is a figure with three sides and three angles. The sum of the measures of the angles is 180°.

A triangle can be classified by its angle measures.	
acute triangle	three acute angles
obtuse triangle	one obtuse angle
right triangle	one 90° angle
A triangle can also be classified by its sides.	
scalene triangle	three unequal sides
isosceles triangle	at least two equal sides
equilateral triangle	three equal sides

Classify each triangle by its angles and sides.



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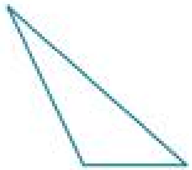


____

Talk About It!
Explain to a classmate why right equilateral and obtuse equilateral triangles are not possible.

Your Notes

Learn Draw Triangles Freehand

You can draw a triangle freehand given the angle and side length descriptions.

Go Online Watch the video to learn how to draw triangles freehand.

The video shows how to draw an obtuse scalene and a right isosceles triangle freehand. Use the spaces below for your drawings.

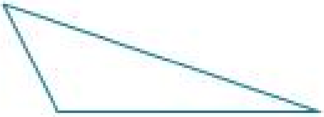
Draw an obtuse scalene triangle. Start by drawing an obtuse angle. The two segments of the angle should have different lengths. Connect the two segments.	**Draw a right isosceles triangle.** Draw a right angle. Draw the line segments so they appear to be the same length. Connect the two segments to form a triangle. Label the right angle. Draw tick marks on the two congruent segments.

Think About It!
What is an obtuse angle? What are congruent sides?

Example 1 Draw Triangles Freehand

Draw a triangle with one obtuse angle and no congruent sides. Classify the triangle by its sides and angles. Then determine if these characteristics create a unique triangle or more than one triangle.

Part A Draw a triangle with one obtuse angle and no congruent sides.



Step 1 Draw an obtuse angle. The angle should be greater than 90° and less than 180°.

Step 2 Draw the sides. None of the sides should appear be congruent.

(continued on next page)

Part B Classify the triangle by its sides and angles. The triangle has one obtuse angle and no congruent sides.

Part C Determine if these characteristics create a unique triangle or more than one triangle.

If the triangle is facing a different way or turned, it does not create a new triangle. You can draw a different triangle with the same characteristics by drawing an angle with a different measure but still obtuse. The sides can be many different lengths as long as they are not congruent. So, these characteristics create more than one triangle.

Talk About It!

Make a conjecture about three characteristics that would create a unique triangle. Draw an example to support your conjecture. Then find a counterexample, if one exists.

Check

A triangle has one right angle and two congruent sides.

Part A Draw the triangle.

Show your work here

Part B Classify the triangle by its sides and angles.

Part C Is it possible to draw a different triangle with those same characteristics? If so, explain what would be different. If not, explain why not.

Go Online You can complete an Extra Example online.

Learn Draw Triangles Using Tools

You can draw figures with greater precision if you use tools such as a ruler or a protractor.

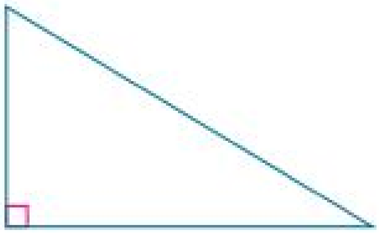
Go Online Watch the video to see how to draw the following triangles with the given conditions, using tools.

Draw a triangle that has a 90° angle and a side that measures 5 centimeters.

Use a ruler to draw a line segment that is 5 centimeters long.

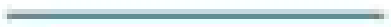
Use a protractor to draw a 90° angle from one endpoint. Because you are only given one length, the second side can be any length.

Connect the endpoints to draw the third side.

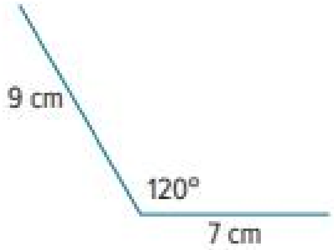


Draw a triangle that has a 120° angle and sides that measure 7 centimeters and 9 centimeters.

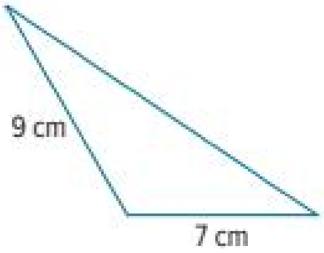
Use a ruler to draw a line segment that is 7 centimeters long.



Use a protractor to mark a 120° angle at one endpoint. Draw the side that is 9 centimeters long.



Connect the endpoints of the two segments to draw the third side of the triangle.



Talk About It!
How do tools help you draw a triangle with greater precision?

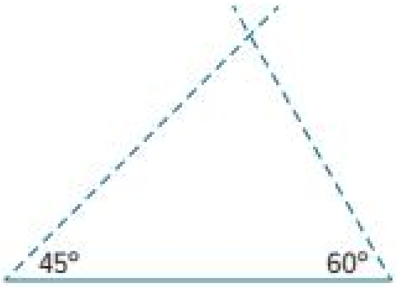
Example 2 Draw Triangles Using Tools

Use a ruler and a protractor to determine whether or not it is possible to draw a triangle with a 45° angle, a 60° angle, and a 75° angle. If so, draw the triangle. If not, explain why.

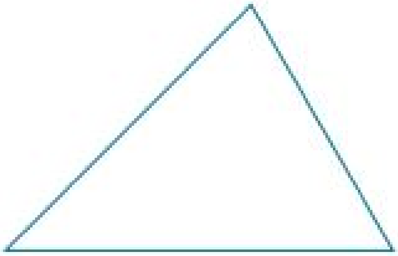
Step 1 Draw a line segment. Because side lengths are not given, the segment can be any length. This will be the base of the triangle.

Step 2 Draw the first angle. Use the protractor to draw a 45° angle from one end of the segment.

Step 3 Draw the second angle. Use the protractor to draw a 60° angle from the other end of the segment.



Step 4 Extend the sides of the angles to determine whether they intersect. Use a protractor to measure the third angle. Because the third angle measures 75°, a triangle with the given angle measures is possible.



So, a triangle with angle measures of 45°, 60°, and 75° is possible.

Talk About It!

Without drawing the triangle, how do you know a triangle with a 45° angle, a 60° angle, and a 75° angle is possible?

Check

Use a ruler and a protractor to determine if it is possible to draw a triangle with a 32° angle, an 82° angle, and a 67° angle. If it is possible, draw the triangle. If not, explain why.

Show your work here

Talk About It!

Is it possible to draw a different triangle with those same characteristics? If so, draw the triangle. If not, explain.

Go Online You can complete an Extra Example online.

Example 3 Draw Triangles Using Tools

Use a ruler and a protractor to determine whether or not it is possible to draw a triangle with a 63° angle, a 127° angle, and a side of 4 inches between the two angles. If so, draw the triangle. If not, explain why.

Step 1 Use the ruler to draw a line segment that measures 4 inches. This can be the base of your triangle.

Step 2 Use the protractor to draw a 63° angle from one end of the segment.

Step 3 Use the protractor to draw a 127° angle from the other end of the segment.

Step 4 Determine if the sides intersect.

Use this space below for your drawing.

So, because the sides do not intersect, a triangle with angle measures of 63°, 127°, and a side of 4 inches between the two angles is not possible.

Talk About It!

Without trying to draw the triangle, how do you know a triangle with a 63° angle, a 127° angle, and a 4-inch side is not possible?

Check

Use a ruler and a protractor to determine if it is possible to draw a triangle with a 30° angle, a 60° angle, and a side that measures 7 centimeters between the two angles. If it is possible, draw the triangle. If not, explain why.

Show your work here

Go Online You can complete an Extra Example online.

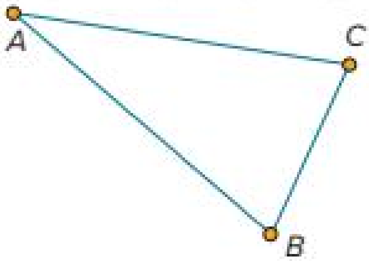
Learn Draw Triangles with Technology

You can create triangles with even greater precision by using technology. If you are given a set of conditions, you can use geometry software, such as Web Sketchpad, to determine if the conditions determine a unique triangle, more than one triangle, or no triangle.

Go Online Use Web Sketchpad to complete the activity.

Web Sketchpad was used to create a triangle with a 32° angle and two 74° angles. Do these conditions determine a unique triangle, more than one triangle, or no triangle?

$m\angle BAC = 32.0°$ $AB = 3.26$ in.
$m\angle CBA = 74.0°$ $AC = 3.26$ in.
$m\angle ACB = 74.0°$ $BC = 1.80$ in.



The side lengths can vary, but the angle measures stay the same. So, the given conditions create more than one triangle.

Talk About It!
Without using technology, is there another way to determine how many triangles can be drawn?

Pause and Reflect

Use Web Sketchpad to verify that more than one triangle can be drawn that satisfies the given conditions of having one 32° angle and two 74° angles. Sketch and label at least 2 triangles below that meet these conditions.

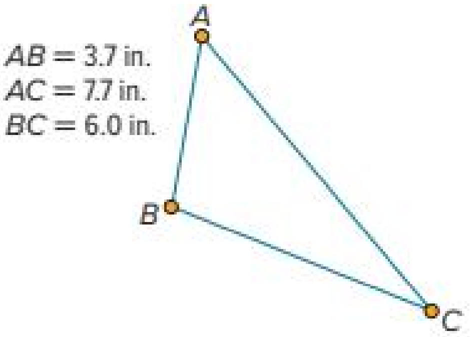
Record your observations here

Example 4 Draw Triangles with Technology

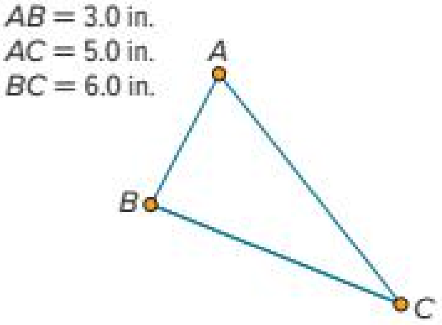
Use technology to determine whether or not it is possible to draw a triangle with side lengths of 3, 5, and 6 inches. If so, draw the triangle. If not, explain why.

Go Online Use Web Sketchpad to complete the example.

Step 1 One side must have a length of 6 inches. Let $BC = 6$ inches.



Step 2 Using Web Sketchpad, drag the vertices to create a triangle so that $AC = 5$ inches and $AB = 3$ inches.



So, a triangle with side lengths of 3, 5, and 6 inches is possible.

Talk About It!

Can you create more than one triangle with these conditions? Explain.

Check

Determine whether or not it is possible to draw a triangle with side lengths 3, 4, and 5 inches. If so, use a sketch to draw the triangle. If not, explain why.

Show your work here

Go Online You can complete an Extra Example online.

Name ______________________ Period ____________ Date ____________

Practice

Go Online You can complete your homework online.

1. Draw a triangle with three acute angles and two congruent sides. Classify the triangle by its sides and angles. Then determine if these characteristics create a unique triangle or more than one triangle. (Example 1)

2. Draw a triangle with one right angle and two congruent sides. Classify the triangle by its sides and angles. Then determine if these characteristics create a unique triangle or more than one triangle. (Example 1)

3. Use a ruler and a protractor to determine whether or not it is possible to draw a triangle with a 50° angle, a 60° angle, and an 80° angle. If so, draw the triangle. If not, explain why. (Examples 2 and 3)

4. Use a ruler and a protractor to determine whether or not it is possible to draw a triangle with a 60° angle, a 60° angle, and a 60° angle. If so, draw the triangle. If not, explain why. (Examples 2 and 3)

5. Use a ruler and a protractor to determine whether or not it is possible to draw a triangle with a 6 millimeter side, an 8 millimeter side, and a 90° angle between them. If so, draw the triangle. If not, explain why. (Examples 2 and 3)

6. Use a ruler and a protractor to determine whether or not it is possible to draw a triangle with a 75° angle, a 115° angle, and a side of 4 inches between the two angles. If so, draw the triangle. If not, explain why. (Examples 2 and 3)

7. Use Web Sketchpad or other geometry software to determine whether or not it is possible to draw a triangle with side lengths of 2, 2, and 5 inches. If so, draw the triangle. If not, explain why. (Example 4)

Test Practice

8. Multiselect Select all of the sets of measurements that can form a triangle.

☐ 35°, 15°, 130°

☐ 90°, 3 inches, 7 inches

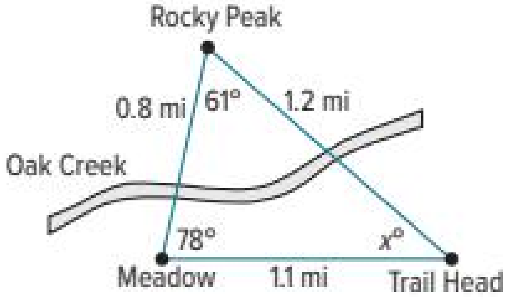
☐ 70°, 70°, 70°

☐ 17 inches, 8 inches, 2 inches

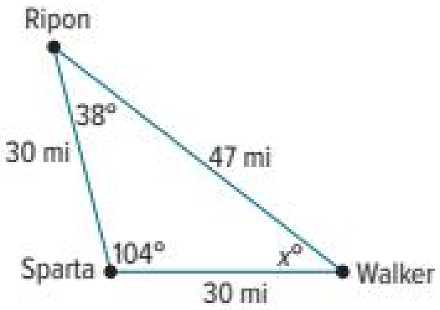
☐ 5 inches, 6 inches, 7 inches

Apply

9. The figure shows the Oak Creek trail, which is shaped like a triangle. Solve the equation $61 + 78 + x = 180$ to find the value of x in the figure. Then classify the triangle by its angles and by its sides.

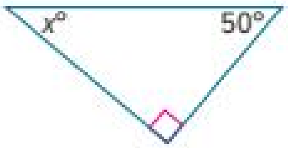


10. The three towns of Ripon, Sparta, and Walker form a triangle as shown. Solve the equation $38 + 104 + x = 180$ to find the value of x in the triangle. Then classify the triangle by its angles and by its sides.



11. **Reason Abstractly** Without drawing the triangle, how do you know a triangle with a 95° angle, a 95° angle, and a 5-inch side is not possible?

12. Find the value of x in the diagram. Then, find the supplement of the missing angle.



13. **Justify Conclusions** Construct an argument to explain why it is possible for a triangle to contain three acute angles.

14. Draw a triangle with one angle greater than 90° and no congruent sides. Then classify the triangle.

Scale Drawings

I Can... use ratio reasoning to find actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale.

What Vocabulary Will You Learn?

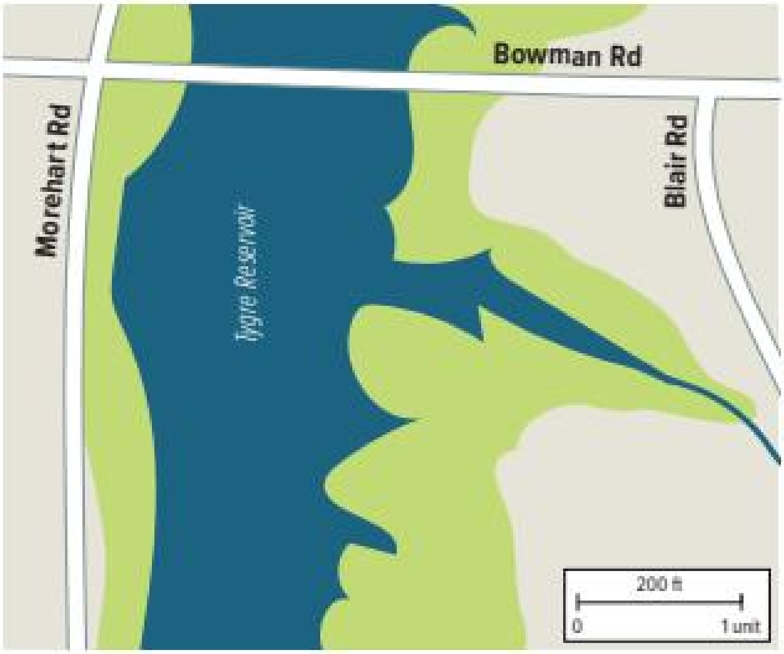
scale
scale drawings
scale factor
scale models

Learn Use Scale Drawings to Find Length

Scale drawings, or **scale models**, are used to represent objects that are too large or too small to be drawn or built at actual size.

The **scale** gives the ratio that compares the measurements of the drawing or model to the measurements of the real object. The measurements on a drawing or model are proportional to the measurements on the object.

You can use a scale drawing to find the actual length of an object or the actual distance between two points.



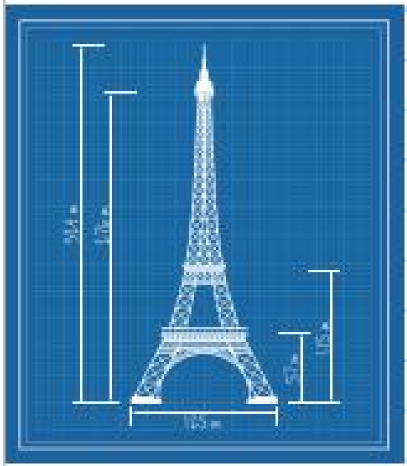
Because the scale is 200 feet per unit, you can estimate that the distance along Bowman Road from the intersection of Morehart Road to the intersection of Blair Road is about 4 units, or 800 feet.

Scale **Length**

$$\begin{array}{rcl} \text{map} \rightarrow & \overset{\times 4}{\dfrac{1\text{ unit}}{200\text{ ft}} = \dfrac{4\text{ units}}{800\text{ ft}}} & \leftarrow \text{map} \\ \text{actual} \rightarrow & \underset{\times 4}{} & \leftarrow \text{actual} \end{array}$$

Talk About It!

What might be a good scale to use for the scale drawing of the Eiffel Tower, if the actual height of the tower is 324 meters?



Your Notes

Think About It!

How can you use equivalent ratios to solve the problem?

Example 1 Use Scale Drawings to Find Length

Use the scale of the map to find the actual distance between Hagerstown and Annapolis.

The distance between the two cities on the map is 4 units.

Step 1 Write an equation involving equivalent ratios, using the scale as one of the ratios. Let *d* represent the actual distance between the cities.

$$\begin{array}{rcccl} & \textbf{Scale} & & \textbf{Length} & \\ \begin{matrix}\text{map}\rightarrow\\ \text{actual}\rightarrow\end{matrix} & \dfrac{1\text{ unit}}{24\text{ miles}} & = & \dfrac{4\text{ units}}{d\text{ miles}} & \begin{matrix}\leftarrow\text{map}\\ \leftarrow\text{actual}\end{matrix}\end{array}$$

Step 2 Use scaling to find the missing value.

$$\begin{tikzcd} \dfrac{1\text{ unit}}{24\text{ miles}} \arrow[r, phantom, "="] \arrow[r, bend left, "\times 4"] \arrow[r, bend right, "\times 4"'] & \dfrac{4\text{ units}}{d\text{ miles}} \end{tikzcd}$$

So, the actual distance between the cities is about 24 × 4, or ________ miles.

Check

On the map, the distance between Akron and Cleveland is $1\frac{1}{2}$ units. What is the actual distance between the cities?

Show your work here

Go Online You can complete an Extra Example online.

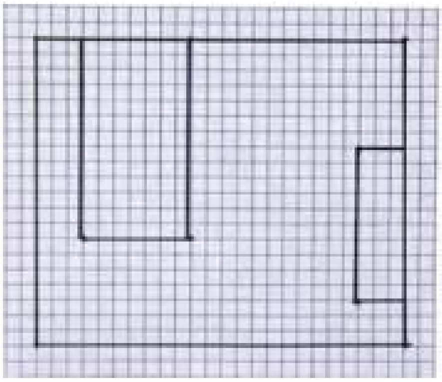
Learn Create Scale Drawings

Go Online Watch the video to see how you can make your own scale drawing if you know the actual measurements and the scale.

The video shows that, to make a scale drawing, first measure the lengths and widths of the actual objects. Record the measurements in a table. The table shows the measurements for the bedroom shown in the video.

Object	Length (ft)	Width (ft)
Room	10	12
Bed	3.5	6.5
Dresser	1.5	5

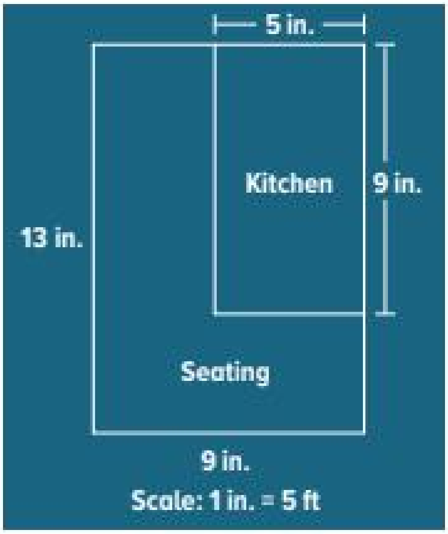
Choose a scale for the drawing and convert each measurement using the scale. On grid paper, use the scale to draw the measurements. One unit on the grid paper equals 0.5 foot of actual length.



Learn Use Scale Drawings to Find Area

You can use scale drawings to find the actual area of a space. First, write an equation involving equivalent ratios to find the actual length and width of the space. Then use the area formula to find the area.

The drawing shows the map of a restaurant, drawn to scale. On the map, 1 inch represents 5 feet. What is the actual area of the kitchen?
The kitchen is a rectangle.
To find the area of the kitchen, first find the length and width of the kitchen.



Step 1 Find the actual length of the kitchen.

$$\underset{\times 9}{\overset{\times 9}{\frac{1\text{ in.}}{5\text{ ft}} = \frac{9\text{ in.}}{\ell\text{ ft}}}}$$

Because 1 × 9 = 9, multiply 5 × 9.

Actual length: ________ ft

Step 2 Find the actual width of the kitchen.

$$\underset{\times 5}{\overset{\times 5}{\frac{1\text{ in.}}{5\text{ ft}} = \frac{5\text{ in.}}{w\text{ ft}}}}$$

Because 1 × 5 = 5, multiply 5 × 5.

Actual width: ________ ft

Step 3 Find the actual area of the kitchen.

$$\begin{aligned} \text{Area} &= \text{length} \times \text{width} \\ &= 45 \times 25 \\ &= 1{,}125\text{ ft}^2 \end{aligned}$$

So, the actual area of the kitchen is 1,125 square feet.

Example 2 Use Scale Drawings to Find Area

The scale of the floor plan is 1 inch = 3 feet.

What is the actual area of Bedroom 3?

Bedroom 3 is a rectangle. To find the area of Bedroom 3, first find the length and width of Bedroom 3.

Think About It!

What unit of measure will you use in your answer?

Step 1 Find the actual length of Bedroom 3.

$$\begin{tikzcd}\frac{1\text{ in.}}{3\text{ ft}} \arrow[rr, bend left, "\times 4\frac{1}{2}"] \arrow[rr, bend right, "\times 4\frac{1}{2}"'] & = & \frac{4\frac{1}{2}\text{ in.}}{x\text{ ft}}\end{tikzcd}$$

The actual length of Bedroom 3 is 3 × $4\frac{1}{2}$, or $13\frac{1}{2}$ feet.

Step 2 Find the actual width of Bedroom 3.

$$\begin{tikzcd}\frac{1\text{ in.}}{3\text{ ft}} \arrow[rr, bend left, "\times 3"] \arrow[rr, bend right, "\times 3"'] & = & \frac{3\text{ in.}}{w\text{ ft}}\end{tikzcd}$$

The actual width of Bedroom 3 is 3 × 3, or 9 feet.

Step 3 Find the area.

The area is 13.5 × 9 or ________ square feet.

Check

Find the actual area of Bedroom 1.

Show your work here

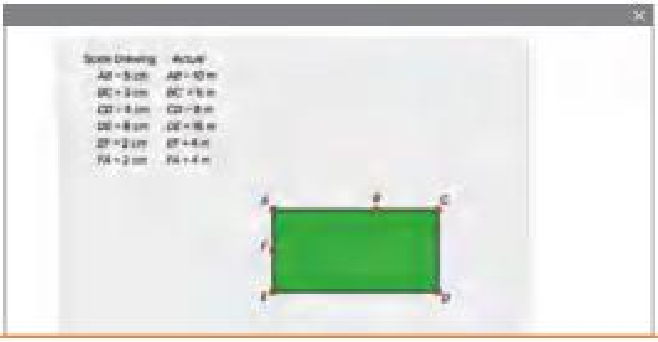
Go Online You can complete an Extra Example online.

Learn Reproduce Scale Drawings

Artists use scale drawings to create wall murals. An artist might draw the mural on a piece of grid paper. Then, he or she would draw a much larger grid on the wall before painting or drawing the mural. You can use a scale to reproduce a drawing that is similar to the original but a different size.

Explore Scale Drawings

Online Activity You will use Web Sketchpad to explore how to reproduce scale drawings using different scales.



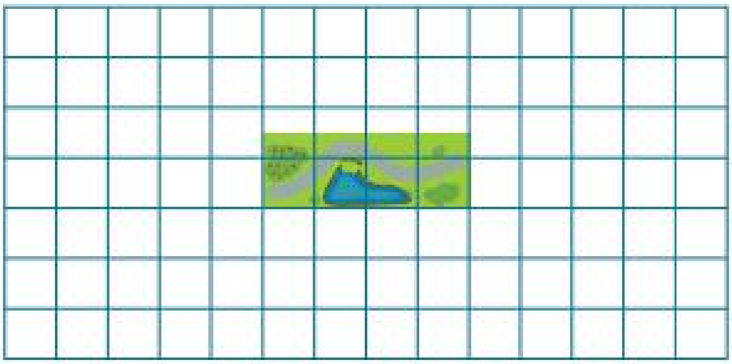
Think About It!

Will your drawing be smaller or larger than the one shown?

Example 3 Reproduce Scale Drawings

The diagram represents a city park. The scale is 1 unit = 30 meters.

Reproduce the drawing with a scale of 1 unit = 10 meters.



The length of the drawing of the park is 4 units.

The width of the drawing of the park is 1.5 units.

The actual dimensions of the park are 4 • 30 or ________ meters and 1.5 • 30 or ________ meters.

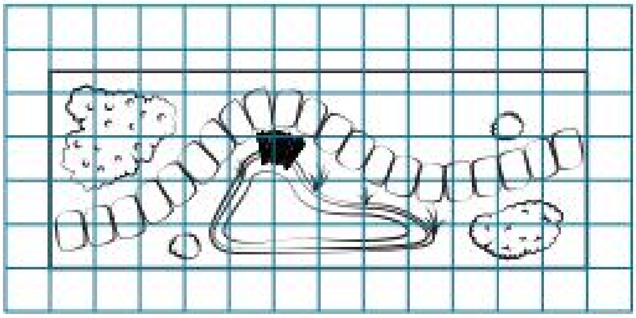
Using the new scale of 1 unit = 10 meters, the new drawing will have a length of 120 ÷ 10 or 12 units.

The new drawing will have a width of 45 ÷ 10 or 4.5 units.

(continued on next page)

So, to reproduce a scale drawing, use the grid to count the number of squares for the length and width of the original drawing, use the new scale to find the length and width of the new drawing, and create the new drawing on the grid, counting the squares for the new length and width.

Using the new dimensions, the drawing is created with a length of 12 units and width of 4.5 units.

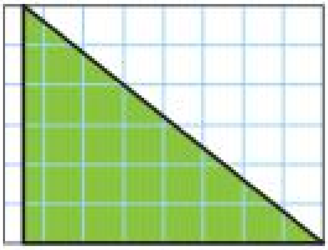


Talk About It!

In the second scale drawing, why are the drawing dimensions multiplied by 10 instead of 30?

Check

The drawing shown uses a scale of 1 unit = 2 feet. Choose the drawing that is reproduced using the new scale, 1 unit = 6 feet.



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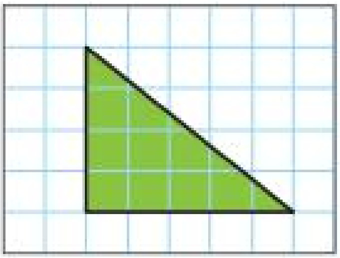

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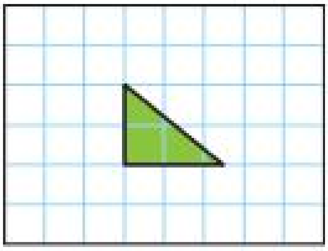

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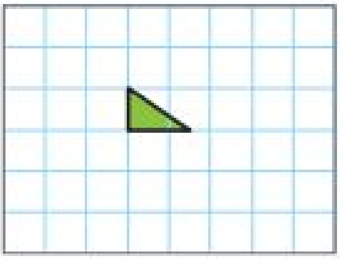

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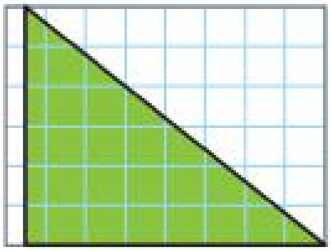
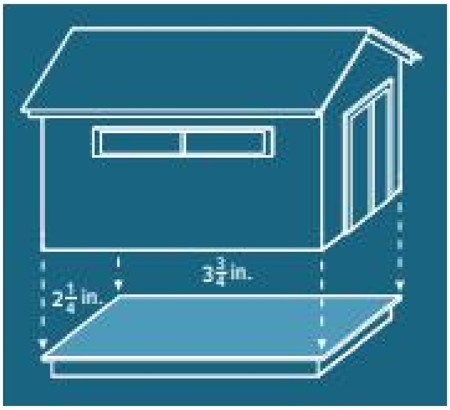


Go Online You can complete an Extra Example online.

Apply Construction

William is laying new flooring in a storage shed. The blueprint of the floor shown uses a scale of 1 inch = 3 feet. If the building material costs $1.09 per square foot, how much will it cost for the new flooring? Round to the nearest cent if necessary.



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

Talk About It!

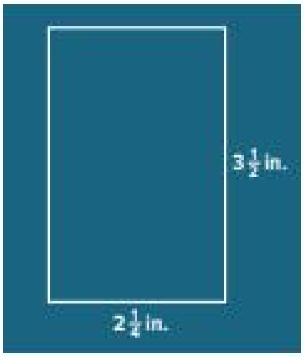
What do you need to determine before you calculate the cost of the flooring?

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Check

Chantele is buying wallpaper for one wall of her living room. The blueprint of the wall uses a scale of 1 inch = 4 feet. If the wallpaper costs $1.79 per square foot, how much will it cost to buy wallpaper for the actual wall of the living room? Round to the nearest cent if necessary.



Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare what you learned about scales and scale drawings in this lesson to concepts you learned in an earlier module or grade. How did knowing those concepts help you in this lesson?

Record your observations here

Talk About It!
How is scale factor different from scale?

Learn Find a Scale Factor

A scale written as a ratio without units in simplest form is called the **scale factor**.

Find the scale factor of a model sailboat if the scale is 1 inch = 6 feet.

$$\frac{1\text{ inch}}{6\text{ feet}}$$ Write the ratio as a fraction.

$$\frac{1\text{ inch}}{6\text{ feet}} = \frac{1\text{ inch}}{72\text{ inches}}$$ Multiply 6 feet by 12 to convert feet to inches.

$$\frac{1\text{ inch}}{72\text{ inches}} = \frac{1}{72}$$ Divide out the common units.

So, the scale factor is $\frac{1}{72}$.

Pause and Reflect

Using the grid paper shown, create a scale drawing of a room in your school or home. Include the scale in your drawing. Trade your drawings with a classmate. Find the actual length or area of the room of your classmate's drawing.

Name ______________________ Period ____________ Date ____________

Practice

Go Online You can complete your homework online.

Refer to the map of Florida. (Example 1)

1. What is the actual distance between Daytona Beach and Orlando? Use a ruler to measure the map.

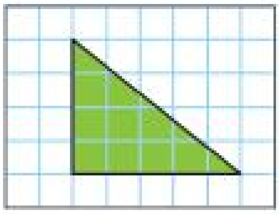
2. What is the actual distance between Tampa and Orlando? Use a ruler to measure the map.

Refer to the floor plan. The scale of the floor plan is 1 inch = 6 feet. (Example 2)

3. Find the actual area of the hallway.

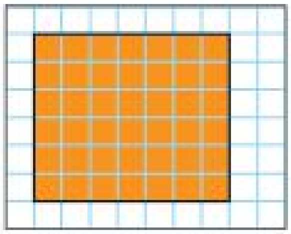
4. Find the actual area of the kitchen.

5. The drawing of a vegetable garden uses a scale of 1 unit = 10 feet. Reproduce the drawing with a scale of 1 unit = 5 feet. (Example 3)



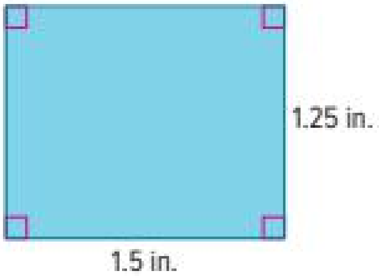
Test Practice

6. Grid The drawing of a sandbox uses a scale of 1 unit = 12 inches. Reproduce the drawing with a scale of 1 unit = 24 inches.



Apply

7. Mr. Miller is tiling his shower floor. The blueprint of the shower floor shown uses a scale of 1 inch = 3 feet. If the tile costs $5.99 per square foot, how much will it cost to tile the bathroom? Round to the nearest cent if necessary.



8. Raul is drawing a plan for his bedroom. He needs to determine the material costs for his flooring. The blueprint of the bedroom uses a scale of 1 inch = 4 feet. If the flooring material costs $2.55 per square foot, how much will it cost to buy the flooring for Raul's bedroom?

9. MP **Reason Abstractly** Determine if the following statement is *true* or *false*. Write an argument that can be used to defend your solution.

If the scale factor of a scale drawing is greater than one, the scale drawing is smaller than the object.

10. Conduct brief research to find what careers use scale drawings.

11. Two cities are 64 miles apart. If the distance on the map is $3\frac{1}{4}$ inches, what is the scale of the map?

12. A rectangle has an area of 24 square inches. The rectangle is reduced by a scale factor of $\frac{1}{2}$. What is the area of the new rectangle?

Three-Dimensional Figures

I Can... describe three-dimensional figures and determine the shapes resulting from horizontal, vertical, and angled cross sections.

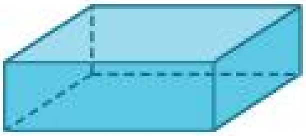
What Vocabulary Will You Learn?

bases
cone
cross section
cylinder
edge
face
plane
polyhedron
prism
pyramid
vertices

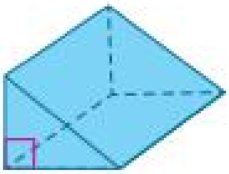
Learn Describe Three-Dimensional Figures

A **polyhedron** is a three-dimensional figure, or solid, with flat surfaces that are polygons. Prisms and pyramids are types of polyhedra. Polyhedra is the plural of polyhedron.

A **prism** is a three-dimensional figure with at least two congruent parallel faces called **bases** that are polygons. Prisms are named by the shape of their base.

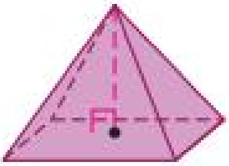


Rectangular Prism

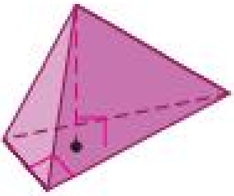


Triangular Prism

A **pyramid** is a three-dimensional figure with one base that is a polygon and other faces that are triangles. Pyramids are also named by the shape of their base.



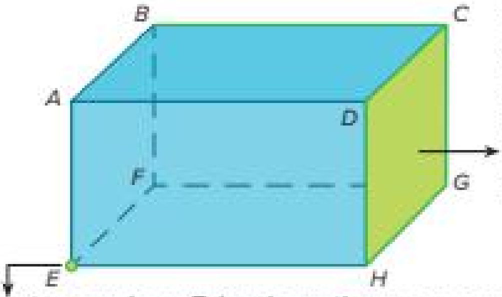
Rectangular Pyramid



Triangular Pyramid

The diagram shows the parts of a prism: the **faces**, the **edges**, and the **vertices**.

An **edge** is the line segment where two faces of a polyhedron meet. $\overline{BC}$ is an edge.



A **face** is a flat surface of a polyhedron. Rectangle *DCGH* is a face.

A **vertex,** such as *E*, is where three or more faces of a polyhedron intersect. **Vertices** is the plural of vertex.

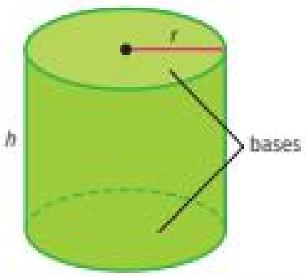
(continued on next page)

Your Notes

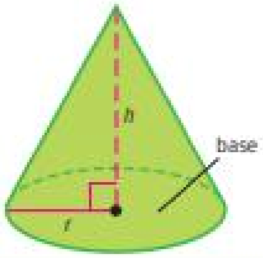
There are also solids that are *not* polyhedra. A **cylinder** is a three-dimensional figure with two parallel and congruent circular bases connected by a curved surface. A **cone** has one circular base connected by a curved side to a single point, called an *apex*.

Talk About It!
Why are cylinders and cones not polyhedra?

cylinder

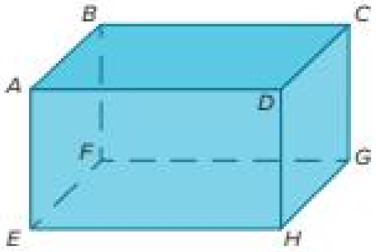


cone



Example 1 Describe Three-Dimensional Figures

The figure shown is a rectangular prism.



Find the number of faces, edges, and vertices.

The faces are the flat surfaces of the prism. The prism has a top face, a bottom face, two side faces, a front face, and a back face.

The edges are the line segments where two faces meet.

Edges: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$, $\overline{CG}$, $\overline{GH}$, $\overline{DH}$, $\overline{FG}$, $\overline{EH}$, $\overline{EF}$, $\overline{AE}$, $\overline{BF}$

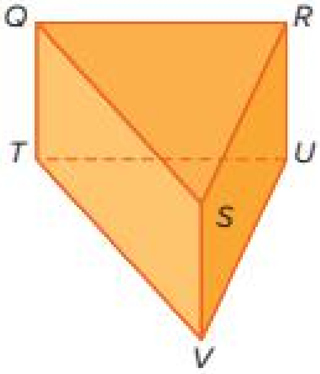
The vertices are *A*, *B*, *C*, *D*, *E*, *F*, *G*, and *H*.

So, a rectangular prism has 6 faces, 12 edges, and 8 vertices.

Talk About It!
Any rectangular prism has six faces. Consider other prisms, such as triangular prisms and pentagonal prisms. How can you find the number of faces, if the base is a polygon with *n* sides?

Check

The figure shown is a triangular prism. Find the number of faces, edges, and vertices.



faces: ☐

edges: ☐

vertices: ☐

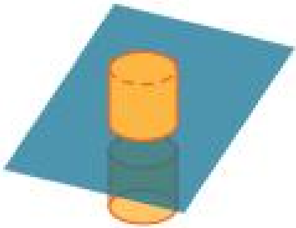
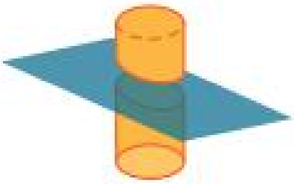
Go Online You can complete an Extra Example online.

Learn Describe Cross Sections of Three-Dimensional Figures

A **plane** is a flat surface that extends forever in all directions. The intersection of a solid and a plane is called a **cross section** of the solid.

Go Online Watch the video to see the cross sections of different three-dimensional figures.

The video shows various three-dimensional figures and their cross sections. The table shows the three cross sections of a cylinder.

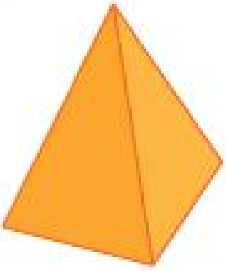
Horizontal	Vertical	Angled
A horizontal cross section results in a circle. 	A vertical cross section results in a rectangle. 	An angled cross section results in an oval. 

Talk About It!

Cross sections are sometimes used to show the interiors of buildings, cars, airplanes, and even bugs! Research to find examples of cross sections and explain how they might be used.

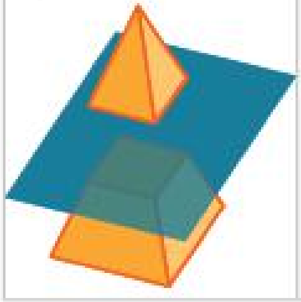
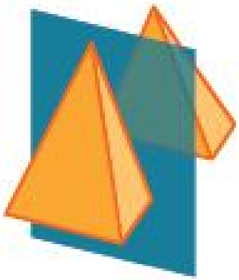
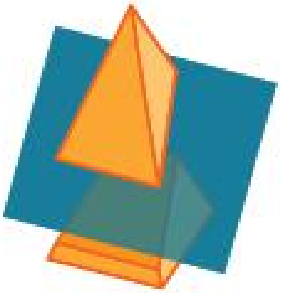
Example 2 Describe Cross Sections of Three-Dimensional Figures

A square pyramid is shown.



Describe the shape resulting from a horizontal cross section, a vertical cross section, and an angled cross section.

The table shows the result of each cross section.

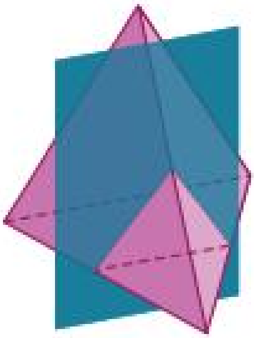
Horizontal	Vertical	Angled
A horizontal cross section results in a square. 	A vertical cross section results in a triangle. 	An angled cross section results in a trapezoid. 

Talk About It!

Why is the cross section through the center of a sphere always a circle, no matter which way it is sliced?

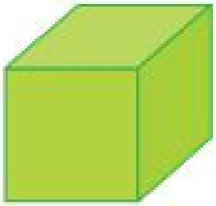
Check

Describe the shape resulting from the vertical cross section of the triangular pyramid.

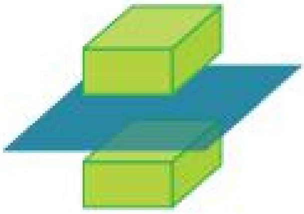
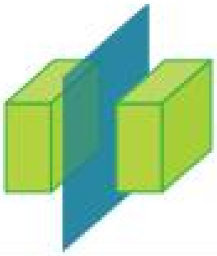


Example 3 Describe Cross Sections of Three-Dimensional Figures

A rectangular prism is shown.



Describe the shape resulting from a vertical cross section, a horizontal cross section, and an angled cross section.

Horizontal	Vertical	Angled
A horizontal cross section results in a __________. 	A vertical cross section results in a __________. 	An angled cross section results in a __________. 

So, a rectangular prism has a rectangular vertical cross section, a rectangular horizontal cross section, and a rectangular angled cross section.

Check

Describe the shape resulting from the cross section of the triangular prism.

Go Online You can complete an Extra Example online.

Think About It!

How will you visualize how the cross sections will look?

Talk About It!

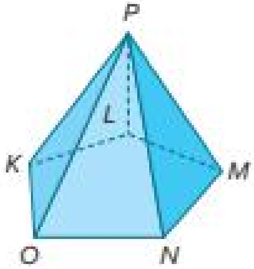
Mario states that all of the faces of a cube are squares. For this reason, the shape resulting from the cross section of a cube is always a square, no matter which way the cube is sliced. Explain why Mario is incorrect and draw a counterexample.

Name ______________________________ Period ____________ Date ____________

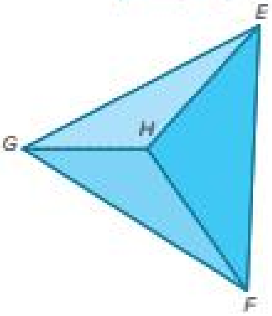
Practice

Go Online You can complete your homework online.

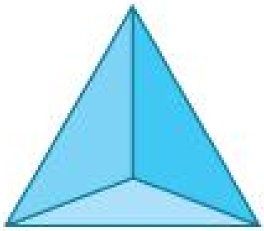
1. The figure shown is a pentagonal pyramid. Find the number of faces, edges, and vertices. (Example 1)



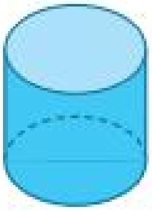
2. The figure shown is a triangular pyramid. Find the number of faces, edges, and vertices. (Example 1)



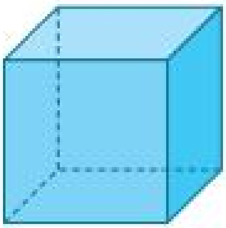
3. A triangular pyramid is shown. Describe the shape resulting from a horizontal cross section, a vertical cross section, and an angled cross section. (Examples 2 and 3)



4. A cylinder is shown. Describe the shape resulting from a horizontal cross section, a vertical cross section, and an angled cross section. (Examples 2 and 3)

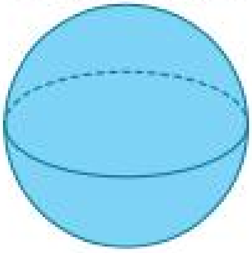


5. A cube is shown. Describe the shape resulting from a horizontal cross section, a vertical cross section, and an angled cross section. (Examples 2 and 3)



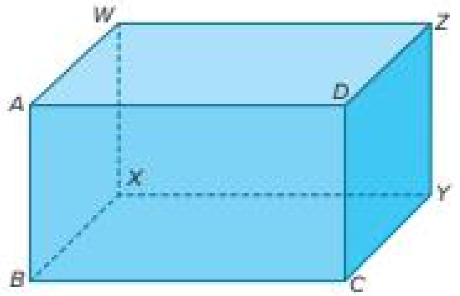
Test Practice

6. Open Response A sphere is shown. Describe the shape resulting from a horizontal cross section, a vertical cross section, and an angled cross section.

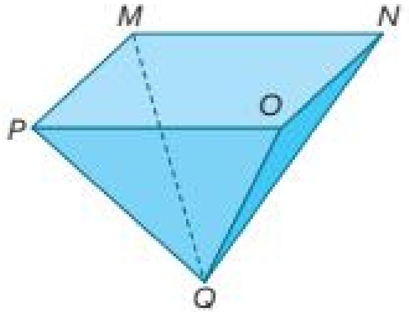


Apply

7. Refer to the figure. Identify the figure. Find the number of faces, edges, and vertices. Then describe a real-world object that resembles this figure.



8. Refer to the figure. Identify the figure. Find the number of faces, edges, and vertices. Then describe a real-world object that resembles this figure.



9. Select a real-world three-dimensional object. Draw the object and describe the resulting shape from a horizontal cross section of the object.

10. MP **Reason Abstractly** Determine if the following statements are *always*, *sometimes*, or *never* true. Explain your reasoning.

a. A prism has a rectangular base.

b. The lateral faces of a pyramid (the faces that are not the base) are triangles.

11. Determine if the following statement is *true* or *false*. If false, provide a counterexample.

A prism always has an even number of vertices.

12. Determine if the following statement is *true* or *false*. If false, provide a counterexample.

A prism always has 2 bases and 4 faces.

Review

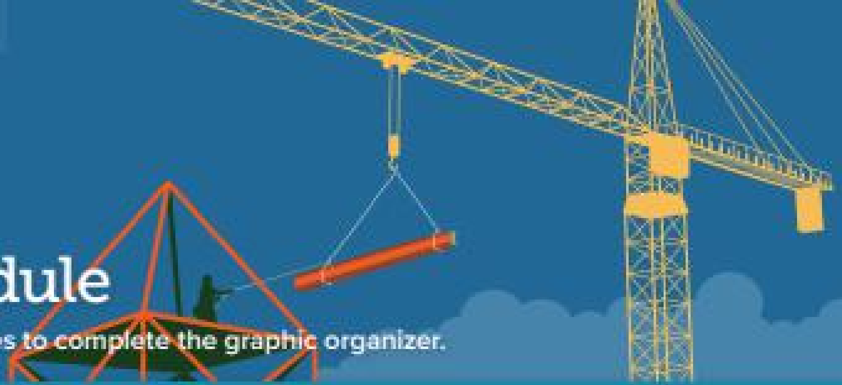
Foldables Use your Foldable to help review the module.

Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.

Write about a question you still have.



Reflect on the Module

Use what you learned about geometric figures to complete the graphic organizer.

Essential Question

How does geometry help to describe objects?

What are angles? How do angles help describe real-world objects?

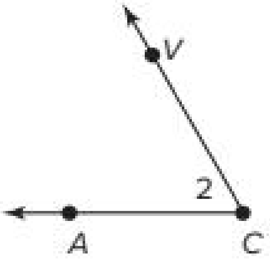
What are triangles? How do triangles help describe real-world objects?

What are polyhedrons? How do polyhedrons help describe real-world objects?

Name _______________ Period _______ Date _______

Test Practice

1. **Multiselect** Which of the following correctly names the angle shown? Select all that apply. (Lesson 1)



☐ ∠2

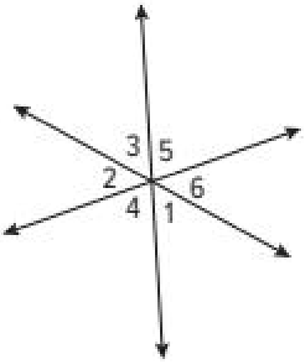
☐ ∠C

☐ ∠ACV

☐ ∠AVC

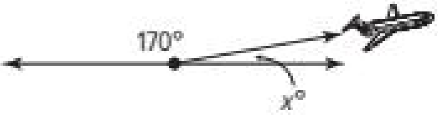
☐ ∠VCA

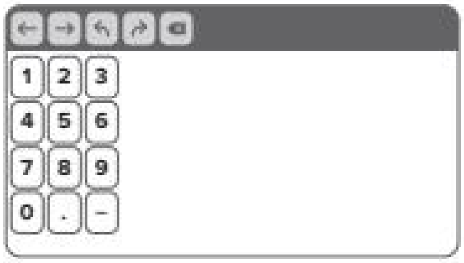
2. **Table Item** Place an X in each cell to indicate whether each pair of angles represents a pair of vertical angles, adjacent angles, or neither. (Lesson 1)



	vertical	adjacent	neither
∠1 and ∠3			
∠5 and ∠6			
∠5 and ∠4			
∠1 and ∠2			

3. **Equation Editor** The diagram represents the trajectory of an airplane at take off. The angle that represents the trajectory of a jet is 15% greater than the trajectory of the airplane. How many degrees does the trajectory angle of the jet measure? (Lesson 2)





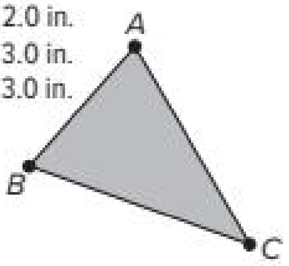
4. **Open Response** A tree is leaning as shown in the figure. (Lesson 2)

A. Write an equation that can be used to find the value of *x*. Explain your reasoning.

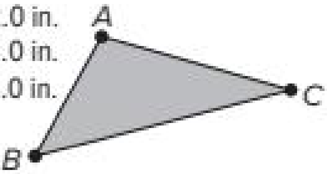
B. What is the value of *x*? What is the measure of the acute angle formed by the tree and the ground?

5. **Multiple Choice** Is it possible to draw a triangle with side lengths of 2, 3, and 5 inches? If yes, select the triangle that meets the given conditions. If not, select the answer that explains why it is not possible. (Lesson 3)

Ⓐ $AB = 2.0$ in.
$AC = 3.0$ in.
$BC = 3.0$ in.



Ⓑ $AB = 2.0$ in.
$AC = 3.0$ in.
$BC = 4.0$ in.



Ⓒ It is not possible to draw the triangle because the sum of two of the side lengths *is* greater than the third side.

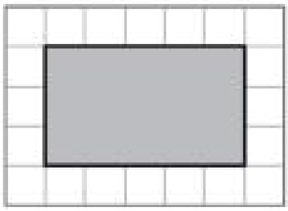
Ⓓ It is not possible to draw the triangle because the sum of two of the side lengths *is not* greater than the third side.

6. **Open Response** Use a ruler and a protractor to determine whether or not it is possible to draw a triangle with a 50° angle, an 80° angle, and a 40° angle. Then complete the sentence below. (Lesson 3)

A triangle with angle measures of 50°, 80°, and 40° ______ possible.

7. **Open Response** The distance between Franklin City and Williamsville on a map is 3.2 inches. If the scale of the map is 1 inch = 15 miles, how many miles apart are the two cities? (Lesson 4)

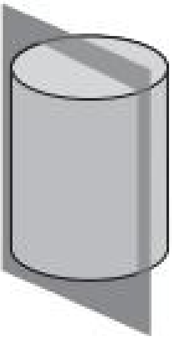
8. **Open Response** The shaded figure in the diagram below represents a rectangular parking lot. The scale of the drawing is 1 unit = 40 yards. (Lesson 4)



A. What is the length and width of the parking lot in units?

B. Suppose the scale drawing is reproduced using a scale of 1 unit = 20 yards. What is the new length and width, in units, of the drawing of the parking lot?

9. **Multiple Choice** Which of the following describes the cross section of a cylinder and a vertical plane as shown? (Lesson 5)



Ⓐ circle
Ⓑ ellipse
Ⓒ rectangle
Ⓓ square

10. **Multiple Choice** Which three-dimensional figure has 6 faces, 12 edges, and 8 vertices? (Lesson 5)

Ⓐ rectangular prism
Ⓑ square pyramid
Ⓒ triangular prism
Ⓓ triangular pyramid

Module 9

Measure Figures

Essential Question

How can we measure objects to solve problems?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY	Before			After		
🟥 — I don't know. 🔶 — I've heard of it. ⭐ — I know it!	🟥	🔶	⭐	🟥	🔶	⭐
finding circumferences of circles						
using circumferences of circles to find missing dimensions						
finding areas of circles						
using circumferences of circles to find area						
finding areas of composite figures						
finding volumes of prisms and pyramids						
using volumes of prisms and pyramids to find missing dimensions						
finding surface areas of prisms and pyramids						
finding volumes and surface areas of composite figures						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about measuring figures.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ area
- ☐ center
- ☐ circle
- ☐ circumference
- ☐ composite figure
- ☐ diameter
- ☐ face
- ☐ lateral face
- ☐ pi
- ☐ prism
- ☐ pyramid
- ☐ radius
- ☐ regular pyramid
- ☐ semicircle
- ☐ slant height
- ☐ surface area
- ☐ volume

Are You Ready?

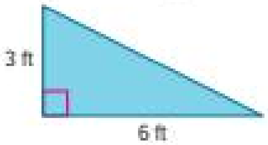
Study the Quick Review to see if you are ready to start this module.
Then complete the Quick Check.

Quick Review

Example 1

Find area of triangles.

Find the area of the triangle.



$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(6)(3)$ Replace b with 6 and h with 3.

$A = 9$ Simplify.

The area of the triangle is 9 square feet.

Example 2

Find area of parallelograms.

Find the area of the parallelogram.

$A = bh$ Area of a parallelogram

$A = (8)(4)$ Replace b with 8 and h with 4.

$A = 32$ Simplify.

The area of the parallelogram is 32 square meters.

Quick Check

1. A road sign in the shape of a triangle has a base length of 18 inches and a height of 16 inches. What is the area of the road sign?

2. A banner in the shape of a parallelogram has a length of 3.5 feet and a height of 2.5 feet. What is the area of the banner?

How Did You Do?

Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②

Lesson 9-1

Circumference of Circles

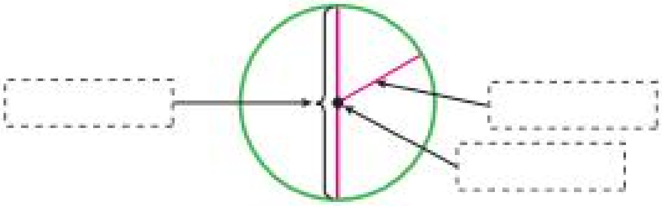
I Can... find the circumferences of circles, given the radius or diameter, using the formulas for the circumference of a circle, and find the radius or diameter of a circle, given its circumference.

Learn Radius and Diameter

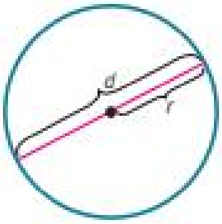
A **circle** is the set of all points in a plane that are the same distance from a point, called the **center**. The **diameter** is the distance across a circle through its center. The **radius** is the distance from the center to any point on the circle.

Label the parts of the circle with the correct terms.

center diameter radius



Because the radius of a circle is the distance from the center to any point on the circle, the length of the diameter is always twice the radius. It also means that the radius is half the diameter.



$$d = 2r$$

$$r = \frac{1}{2}d$$

What Vocabulary Will You Learn?

- center
- circle
- circumference
- diameter
- pi (π)
- radius

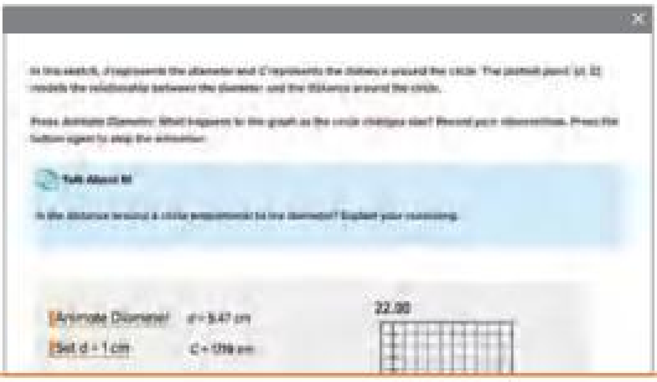
Talk About It!
What equation can be used to find the diameter d of a circle given the radius r?

Talk About It!
What equation can be used to find the radius r of a circle given the diameter d?

Your Notes

Explore The Distance Around a Circle

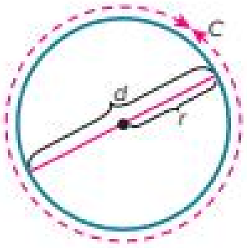
Online Activity You will explore the relationship between the distance around a circle and its diameter.



Learn Circumference of Circles

Circumference is the distance around a circle. The circumference of a circle is proportional to its diameter. The exact ratio of $\frac{C}{d}$ is represented by the Greek letter **π (pi)**. The value of π is 3.1415926... The decimal never ends, but is often approximated to **3.14**. Another approximation for π is $\frac{22}{7}$.

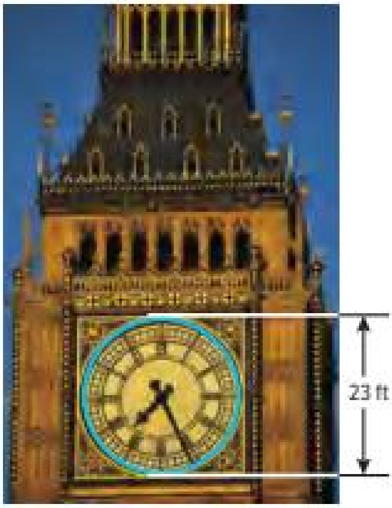
The table shows the use of two formulas to find the circumference of a circle.

Words	Model
To find the circumference *C* of a circle, multiply π by its diameter, *d* or π by two times its radius, *r*.	
Symbols	
$C = \pi d$ or $C = 2\pi r$	

Talk About It!
When you use 3.14 or $\frac{22}{7}$ to find the circumference of a circle, will it be the exact circumference or an approximation? Justify your response.

Example 1 Find the Circumference Given the Diameter

Big Ben is a famous clock tower in London, England. The diameter of the clock face is 23 feet.



Find the circumference of the clock face. Use 3.14 for π. Round to the nearest hundredth if necessary.

Because you are given the diameter, use the formula $C = \pi d$.

$C = \pi d$	Circumference of a circle
$C = \pi(23)$	Replace *d* with 23.
$C = 23\pi$	Simplify. This is the *exact* circumference.
$C \approx 23(3.14)$	Replace π with 3.14.
$C \approx 72.22$	Simplify. This is the *approximate* circumference.

So, the distance around the clock face is about ________ feet.

Think About It!

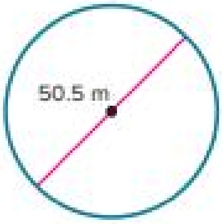
What formula can you use to find the circumference if you know the diameter?

Talk About It!

In the fourth line of the solution, why was the equal sign (=) changed to an approximately equal to symbol (≈)?

Check

The Niagara SkyWheel, which overlooks Niagara Falls, Canada, has a diameter of 50.5 meters. Find the circumference of the Niagara SkyWheel. Use 3.14 for π. Round to the nearest hundredth if necessary.



Show your work here

Go Online You can complete an Extra Example online.

Example 2 Find the Circumference Given the Radius

Find the circumference of a circle with a radius of 21.2 inches. Use 3.14 for π. Round to the nearest hundredth if necessary.

Because you are given the radius, use the formula $C = 2\pi r$.

$C = 2\pi r$	Circumference of a circle
$C = 2\pi\left(\boxed{}\right)$	Replace *r* with 21.2.
$C = \boxed{}\ \pi$	Simplify. This is the *exact* circumference.
$C \approx 42.4\left(\boxed{}\right)$	Replace π with 3.14.
$C \approx \boxed{}$	Simplify. This is the *approximate* circumference.

So, the circumference of a circle with a radius of 21.2 inches is about 133.14 inches.

Think About It!

What formula can you use to find the circumference if you know the radius?

Check

Find the circumference of a circle with a radius of 0.9 centimeter. Use 3.14 for π. Write your answer as a decimal rounded to the nearest hundredth.

Show your work here

Talk About It!

Why are the units of the circumference given as inches and not square inches or cubic inches?

Go Online You can complete an Extra Example online.

Pause and Reflect

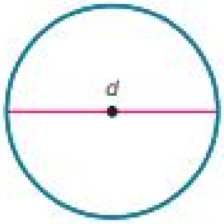
Compare and contrast the concepts of perimeter and circumference.

Record your observations here

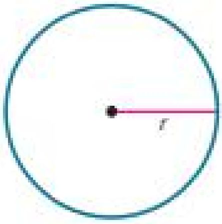
Learn Use Circumference to Find Missing Dimensions

You can use the formula for the circumference of a circle to find the diameter or radius, given the circumference. Rewrite the circumference formula in terms of *d* or *r* using the properties of equality.

$C = \pi d$



$C = 2\pi r$



$C = \pi d$	Circumference of a circle	$C = 2\pi r$
$\frac{C}{\pi} = \frac{\pi d}{\pi}$	Division Property of Equality	$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$
$\frac{C}{\pi} = d$	Simplify.	$\frac{C}{2\pi} = r$
$d = \frac{C}{\pi}$		$r = \frac{C}{2\pi}$

Talk About It!

Why is there a 2 in the denominator for the equation to find the radius, but not in the equation to find the diameter?

Example 3 Find the Diameter Given the Circumference

One of the largest water fountains in the world, Singapore's Fountain of Wealth, consists of a circular bronze ring that has a circumference of 66 meters.

Find the approximate diameter of the fountain's bronze ring. Use 3.14 for π. Round to the nearest hundredth.

Because you need to find the diameter, use the formula $d = \frac{C}{\pi}$.

$d = \frac{C}{\pi}$	Diameter of a circle
$d \approx \frac{66}{3.14}$	Replace π with 3.14 and *C* with 66.
$d \approx 21.02$	Simplify.

So, the approximate diameter of the fountain's bronze ring is about 21.02 meters.

Think About It!

What is a good estimate for the diameter? Explain how you calculate that estimate.

Talk About It!

How does the solution compare to your estimate?

Check

Find the approximate diameter of a basketball hoop that has a circumference of 56.52 inches. Use 3.14 for π. Round to the nearest hundredth if necessary.

Show your work here

Think About It!

What is a good estimate for the radius? Explain how you calculate that estimate.

Example 4 Find the Radius Given the Circumference

Find the approximate radius of a circle with a circumference of 70.82 inches. Use 3.14 for π. Round to the nearest hundredth.

Because you need to find the radius, use the formula $r = \frac{C}{2\pi}$.

$r = \frac{C}{2\pi}$	Radius of a circle
$r \approx \frac{70.82}{2(3.14)}$	Replace π with 3.14 and C with 70.82.
$r \approx 11.28$	Simplify.

So, the approximate length of the radius of a circle that has a 70.82-inch circumference is about _______ inches.

Talk About It!

How does your solution compare to the estimate?

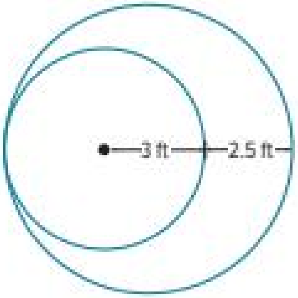
Check

Find the approximate radius of a circle with a circumference of 7.92 centimeters. Use 3.14 for π. Round to the nearest hundredth.

Show your work here

Go Online You can complete an Extra Example online.

Apply Gardening



Kamma has a circular garden with a radius of 3 feet. The diameter of her neighbor's circular garden is 2.5 feet longer than the diameter of Kamma's garden. How much landscape edging does her neighbor need to border her garden? Use 3.14 for π. Round to the nearest hundredth.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

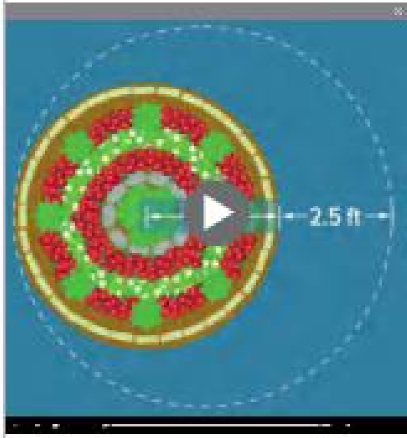
Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Go Online Watch the animation.



Talk About It!

Is it possible to find the circumference of the neighbor's garden using only radius measurements rather than finding the diameters? Explain your reasoning.

Check

A nickel has a diameter that is 2.16 millimeters longer than the diameter of a penny. If the radius of a penny is 9.525 millimeters, what is the circumference of a nickel? Use 3.14 for π. Round to the nearest tenth.

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Create a graphic organizer that will help you choose when to use the diameter or radius to find the circumference.

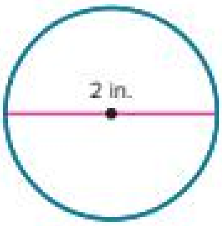
Record your observations here

Name ______________________ Period ____________ Date ____________

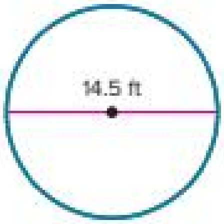
Practice

Go Online You can complete your homework online.

1. Find the circumference of the watch face. Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 1)



2. A circular fence is being used to surround a dog house. How much fencing is needed to build the fence? Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 1)



3. Find the circumference of a circle with a radius of $31\frac{1}{2}$ yards. Use 3.14 for π. Write your answer as a decimal rounded to the nearest hundredth. (Example 2)

4. Find the circumference of a circle with a radius of 4.4 inches. Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 2)

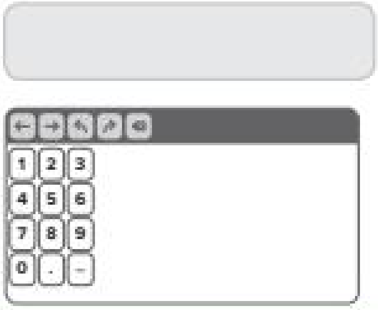
5. The world's largest flower, the Rafflesia, has a circumference of 286 centimeters. Find the approximate diameter of the flower. Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 3)

6. A helicopter pad has a circumference of $47\frac{1}{2}$ yards. Find the approximate diameter of the helicopter pad. Use 3.14 for π. Write your answer as a decimal rounded to the nearest hundredth if necessary. (Example 3)

7. Find the approximate radius of a circle with a circumference of 34.48 inches. Use 3.14 for π. Round to the nearest hundredth. (Example 4)

Test Practice

8. **Equation Editor** Find the approximate radius of a circle with a circumference of 198 centimeters. Use 3.14 for π. Round to the nearest hundredth.

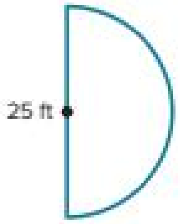


Apply

9. Poppy is using wire to make metal wall hangings that have the radius shown for her friends. Her older sister is making her wall hangings with a diameter that is $1\frac{3}{4}$ inches longer than Poppy's. How much more wire did her sister use per wall hanging than Poppy? Use 3.14 for π. Write your answer as a decimal rounded to the nearest hundredth.

10. Arun is making a bubble wand out of wire. The circular part of the wand has the radius shown. The diameter of his friend's wand is 4.5 millimeters shorter than the diameter of Arun's wand. How much wire did his friend need to make the circular part of his wand? Use 3.14 for π. Round to the nearest hundredth.

11. **Persevere with Problems** Find the distance around the figure. Use 3.14 for π.



12. Draw and label a circle with a circumference between 10 and 15 centimeters. Label the length of the diameter.

13. **Reason Abstractly** How would the circumference of a circle change if its radius was doubled? Provide an example to support your reasoning.

14. **Justify Conclusions** Use mental math to determine if the circumference of a circle with a radius of 5 inches will be greater than or less than 30 inches. Write an argument that can be used to justify your solution.

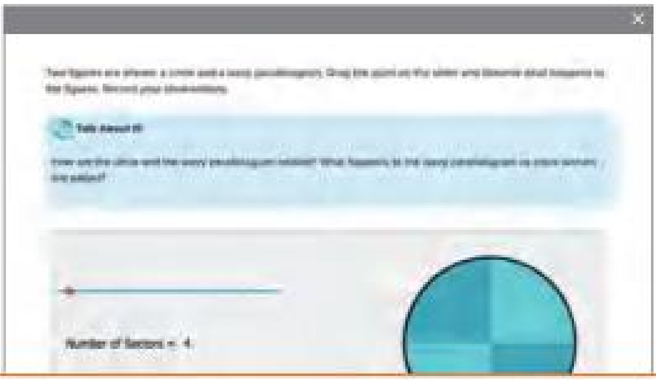
Area of Circles

I Can... find the areas of circles, given the radius or diameter, using the formula for the area of a circle.

What Vocabulary Will You Learn?
area
semicircle

Explore Area of Circles

Online Activity You will explore how to use the formula for the area of a parallelogram to help determine the formula for the area of a circle.



Learn Derive the Formula for the Area of a Circle

When a circle is divided into sections, you can rearrange the sections to make a figure that resembles a parallelogram. You can then use the formula for the area of a parallelogram to derive the formula for the area of a circle.

The process of writing the formula for the area of a circle based on the formula for the area of a parallelogram is shown.

$A = bh$ Area of a parallelogram

$A = br$ Because $h \approx r$, replace h with r.

$A = \left(\frac{1}{2}C\right)r$ Because $b \approx \frac{1}{2}C$, replace b with $\frac{1}{2}C$.

$A = \frac{1}{2}(2\pi r)r$ Because $C = 2\pi r$, replace C with $2\pi r$.

The formula $A = \frac{1}{2}(2\pi r)r$ can be simplified to obtain $A = \pi r^2$. So, the formula for the area of a circle $A = \pi r^2$.

Your Notes

Zu Chongzhi

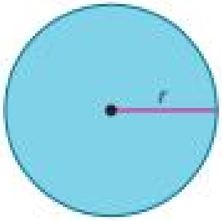
Math History Minute

Various values for π have been used throughout history. Chinese mathematician **Zu Chongzhi (429–500 B.C.E.)** calculated π as $\frac{355}{113}$, or 3.1415929... Later, Hindu mathematician **Aryabhata (476–550 C.E.)** calculated π as $\frac{62,843}{20,000}$. In 1150, another Hindu mathematician **Bhāskara II (1114–1185)** obtained a value of $\frac{3,927}{1,250}$, or 3.1416.

Learn Area of Circles

Area is the measure of the interior surface of a two-dimensional figure. As with the area of polygons, the area of a circle is expressed in square units.

The table shows the use of the formula to find the area of a circle, given the radius.

Words	Model
The area A of a circle equals the product of π and the square of the radius r.	
Symbols	
$A = \pi r^2$	

Example 1 Find the Area Given the Radius

Find the area of the circle. Use 3.14 for π. Round to the nearest hundredth if necessary.

$A = \pi r^2$	Area of a circle
$A = \pi(14.2)^2$	Replace r with 14.2.
$A = 201.64\pi$	Simplify. This is the *exact* area.
$A \approx 201.64(3.14)$	Replace π with 3.14.
$A \approx 633.1496$	Simplify. This is the *approximate* area.

So, the approximate area of the circle is ________ square inches.

Check

Find the area of the circle. Use 3.14 for π. Write your answer as a decimal rounded to the nearest hundredth.

Show your work here

Go Online You can complete an Extra Example online.

Example 2 Find the Area Given the Diameter

The city of Wellington is commissioning a statue to honor their former mayor. The circular base of the statue will be 26 feet in diameter.

What is the area of the space needed to fit the base of the statue? Use 3.14 for π. Round to the nearest hundredth if necessary.

Step 1 Find the radius of the circle.

Because the diameter of the base of the statue is 26 feet, the radius of the base is 26 ÷ 2 or

_________ feet.

Step 2 Calculate the area of the circle.

$A = \pi r^2$	Area of a circle
$A = \pi(\boxed{})^2$	Replace *r* with 13.
$A = \boxed{}\,\pi$	Simplify. This is the *exact* area.
$A \approx 169(\boxed{})$	Replace π with 3.14.
$A \approx 530.66$	Simplify. This is the *approximate* area.

So, the area of the space needed to fit the base of the statue is about 530.66 square feet.

Check

The circular area covered by a lawn sprinkler has a 24.25-foot diameter. What is the area of the space covered by the sprinkler? Use 3.14 for π. Round to the nearest hundredth if necessary.

Show your work here

Go Online You can complete an Extra Example online.

Think About It!

What is a good estimate for the area of the base of the statue? Explain how you calculated that estimate.

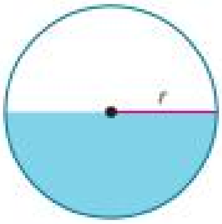
Talk About It!

How does the solution compare to your estimate?

Learn Area of Semicircles

A **semicircle** is half of a circle.

The table shows the use of the formula to find the area of a semicircle, given the radius.

Words	Model
The area *A* of a semicircle equals half the product of π and the square of the radius *r*.	
Symbols	
$A = \frac{1}{2}\pi r^2$	

Think About It!

What is a good estimate for the area of the space the dog has to roam? Explain how you calculated that estimate.

Example 3 Find Area of Semicircles

A wireless fence transmitter at the back door of a house allows a dog to roam freely within a semicircle that has a radius of 30 feet.

What is the area of the space the dog has to roam? Use 3.14 for π. Round to the nearest hundredth if necessary.

$A = \frac{1}{2}\pi r^2$	Area of a semicircle
$A = \frac{1}{2}\pi(\boxed{})^2$	Replace *r* with 30.
$A = \boxed{}\,\pi$	Simplify. This is the *exact* area.
$A \approx 450(\boxed{})$	Replace π with 3.14.
$A \approx 1{,}413$	Simplify. This is the *approximate* area.

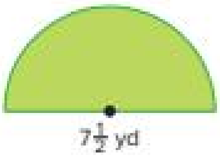
So, the dog has an approximate roaming area of 1,413 square feet.

Talk About It!

How does the solution compare to your estimate?

Check

What is the area of the semicircle? Use 3.14 for π. Write your estimate as a decimal rounded to the nearest hundredth.

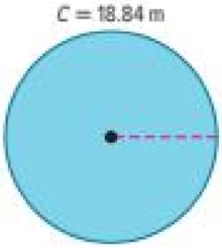


Show your work here

Go Online You can complete an Extra Example online.

Learn Use Circumference to Find Area

When you know the circumference of a circle, you can work backward to find the area of the circle.



Go Online Watch the animation to learn how to find the area of the circle, given its circumference.

Step 1 Find the radius of the circle.

$C = 2\pi r$ Circumference of a circle

$18.84 \approx 2 \cdot 3.14 \cdot r$ Replace C with 18.84 and π with 3.14.

$18.84 \approx 6.28r$ Simplify.

$\frac{18.84}{6.28} \approx \frac{6.28r}{6.28}$ Division Property of Equality

$3 \approx r$ Simplify.

Step 2 Find the area of the circle.

$A = \pi r^2$ Area of a circle

$A \approx 3.14 \cdot 3^2$ Replace π with 3.14 and r with 3.

$A \approx 28.26$ Simplify.

The area of the circle is about 28.26 m^2.

Example 4 Use Circumference to Find Area

The exact circumference of a circle is 32π inches.

What is the approximate area of the circle? Use 3.14 for π. Round to the nearest hundredth if necessary.

Think About It!
What information do you need to find the area of a circle?

Step 1 Use the circumference formula to find the radius of the circle.

$C = 2\pi r$	Circumference of a circle
$\boxed{} = 2\pi r$	Replace C with 32π.
$\dfrac{32\pi}{\boxed{}} = \dfrac{2\pi r}{\boxed{}}$	Division Property of Equality; Divide each side by 2π.
$16 = r$	Simplify.

The radius of the circle is 16 inches.

Talk About It!
What does the number 32 represent when the circumference is 32π?

Step 2 Find the area.

$A = \pi r^2$	Area of a circle
$A \approx 3.14 \cdot \boxed{}^2$	Replace π with 3.14 and r with 16.
$A \approx \boxed{}$	Simplify.

So, the approximate area of the circle is 803.84 square inches.

Check

The exact circumference of a circle is 13π feet. What is the approximate area of the circle? Use 3.14 for π. Round to the nearest hundredth.

Show your work here

Go Online You can complete an Extra Example online.

Apply Crafting

A square scrapbook page has an area of 144 square inches. Jillian wants to cut the largest circle possible from the page to create a layered background for a new page. What is the approximate area of the paper circle? Use 3.14 for π. Round to the nearest hundredth if necessary.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

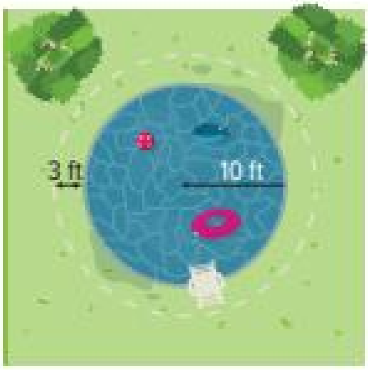
Talk About It!
How did you know that the radius of the circle is one-half the side length of the square?

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Check

The Blackwells have a circular pool with a radius of 10 feet. They want to install a 3-foot wide sidewalk around the pool.



What will be the area of the sidewalk? Use 3.14 for π. Round to the nearest hundredth if necessary.

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Describe when it is beneficial to use 3.14 instead of π, and when it is beneficial to use π instead of 3.14 when calculating the circumference or area of a circle.

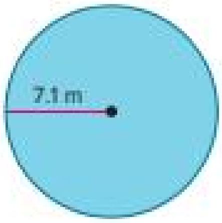
Record your observations here

Name ______________________ Period __________ Date __________

Practice

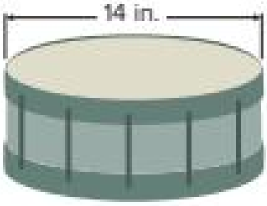
Go Online You can complete your homework online.

1. Find the area of the circle. Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 1)



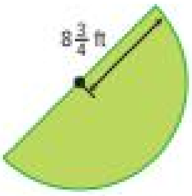
2. Find the area of the circle. Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 1)

3. What is the area of the drumhead on the drum? Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 2)

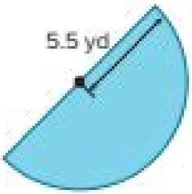


4. What is the area of one side of the penny? Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 2)

5. Mr. Ling is adding a pond in the shape of a semicircle in his backyard. What is the area of the pond? Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 3)



6. Vidur needs to buy mulch for his garden. What is the area of his garden? Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 3)



7. The exact circumference of a circle is 18π inches. What is the approximate area of the circle? Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 4)

Test Practice

8. Open Response The exact circumference of a circle is 34π meters. What is the approximate area of the circle? Use 3.14 for π. Round to the nearest hundredth if necessary.

Apply

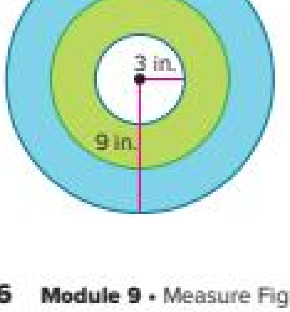
9. Tye has a square piece of yellow felt that has an area of 81 square inches. She wants to cut the largest circle possible from the material to create a sun for her art project. What is the area of the felt circle? Use 3.14 for π. Round to the nearest hundredth if necessary.

10. Tarek has 72 feet of plastic fencing to make a flower garden in his backyard. The garden shape can either be circular or square. If he uses all of the fencing, what is the difference between the area of the circular garden and the square garden? Use 3.14 for π. Round to the nearest hundredth if necessary.

11. **Reason Inductively** Explain how you could find the area of the three-quarter circle shown. Then write a formula that could be used to find the area of the three-quarter circle and use the formula to find the area of the figure. Use 3.14 for π.

12. Draw and label a circle with an area between 50 and 60 square inches.

13. **Persevere with Problems** The bullseye on an archery target has a radius of 3 inches. The entire target has a radius of 9 inches. To the nearest hundredth, find the area of the target outside of the bullseye. Use 3.14 for π.



14. **Justify Conclusions** Determine if the following statement is *true* or *false*. Support your answer with an example or counterexample.

If the length of a radius is doubled, the area of the circle is also doubled.

Lesson 9-3

Area of Composite Figures

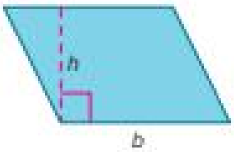
I Can... find areas of composite figures by decomposing the figures into known shapes, and then adding the areas of those shapes.

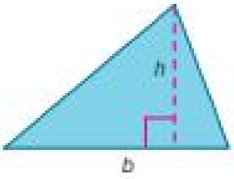
What Vocabulary Will You Learn?
composite figure

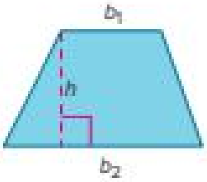
Learn Area of Composite Figures

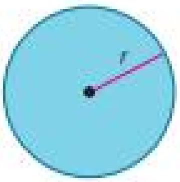
A **composite figure** is made up of two or more shapes. To find the area of a composite figure, decompose the figure into shapes with areas you know how to find. Then find the sum of those areas.

Label each shape with its correct name and corresponding area formula.

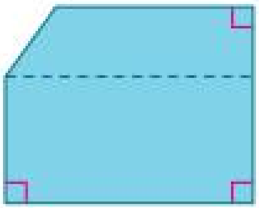








When analyzing the structure of a composite figure, such as the one shown, look for shapes like the ones above into which you can decompose the composite figure.



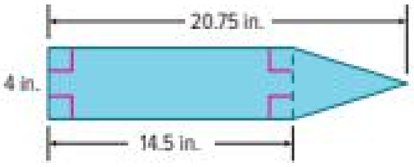
Your Notes

Think About It!
What dimensions/measurements do you need in order to find the area of this composite figure?

Talk About It!
Describe another method you can use to find the area of the composite figure.

Example 1 Area of Composite Figures

Ayanna is painting a sign made from a piece of reclaimed wood with the dimensions shown.



What is the area of the sign?

Step 1 Decompose the figure into smaller figures.

The figure is a pentagon that is composed of a rectangle and a triangle.

Step 2 Find the area of each figure.

Complete the steps.

Find the area of the rectangle.	Find the area of the triangle.
$A = \ell \cdot w$	$A = \frac{1}{2}bh$
$= 14.5 \cdot 4$	$= \frac{1}{2} \cdot 4 \cdot 6.25$
= ☐	= ☐

The area of the rectangle is 58 square inches and the area of the triangle is 12.5 square inches.

Step 3 Find the area of the composite figure.

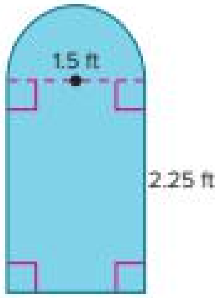
58 + 12.5 = ☐

So, the area of the composite figure is about 58 + 12.5, or ________ square inches.

Check

Find the area of the figure. Use 3.14 for π. Round to the nearest hundredth if necessary.

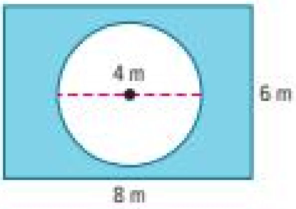
Show your work here



Go Online You can complete an Extra Example online.

Learn Area of Shaded Regions

Use area formulas to find the area of a shaded region. First find the area of the entire figure. Then subtract to find the area of the shaded region.



Go Online Watch the animation to see how to find the area of shaded regions.

The animation shows the following steps.

Step 1 Find the area of the entire figure.

$A = \ell w$ Area of entire figure, a rectangle

$A = 8 \cdot 6$ Replace ℓ with 8 and w with 6.

$A = \boxed{\quad}$ Simplify.

Step 2 Find the area of the unshaded region.

$A = \pi r^2$ Area of unshaded region, a circle

$A \approx 3.14 \cdot 2^2$ Replace π with 3.14 and r with 2.

$A \approx \boxed{\quad}$ Simplify.

Step 3 Subtract to find the area of the shaded region.

entire figure − unshaded region = shaded region

$48 \quad - \quad 12.6 \quad \approx \boxed{\quad} \text{ m}^2$

The area of the shaded region is about 35.4 square meters.

Pause and Reflect

If the area of the unshaded region was a triangle, what dimensions of the triangle would keep the area of the shaded region about the same?

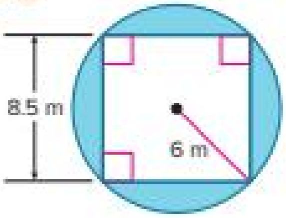
Record your observations here

Think About It!
Which area formulas will you need to use to solve the problem?

Talk About It!
Why is the side length of the square 8.5 meters and not 12 meters?

Example 2 Area of Shaded Regions

Find the area of the shaded region. Use 3.14 for π. Round to the nearest hundredth if necessary.



Step 1 Find the area of the entire figure.

$A = \pi r^2$ Area of entire figure, a circle

$A \approx 3.14 \cdot 6^2$ Replace π with 3.14 and r with 6.

$A \approx \boxed{}$ Simplify.

The area of the circle is approximately 113.04 square meters.

Step 2 Find the unshaded area.

$A = s^2$ Area of unshaded region, a square

$A = 8.5^2$ Replace s with 8.5.

$A = \boxed{}$ Simplify.

The unshaded area is 72.25 square meters.

Step 3 Find the area of the shaded region.

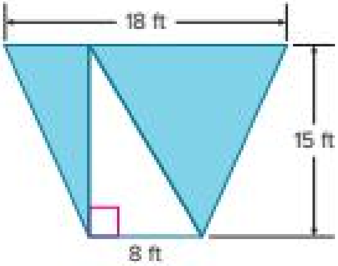
The area of the circle is about 113.04 square meters. The area of the square is 72.25 square meters. Subtract the area of the square from the area of the circle to find the approximate area of the shaded region.

Because 113.04 − 72.25 = ________, the area of the shaded region is approximately 40.79 square meters.

Check

Find the area of the shaded region.

Show your work here



Go Online You can complete an Extra Example online.

Apply Art

The members of the local community center are planning on using ceramic tiles to create a mosaic on the side of the building. One tile covers 2.5 square feet. How many tiles are needed to make the mosaic?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

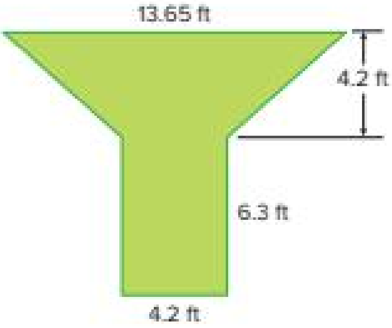
4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It!
How can you solve the problem another way?

Check

The Jamesons hired a landscaper to create the walkway shown.



If one case of decorative stone costs $25 and covers 6 square feet, how much will it cost to cover the walkway?

Go Online You can complete an Extra Example online.

Pause and Reflect

How well do you understand the concepts from today's lesson? What questions do you still have? How can you get those questions answered?

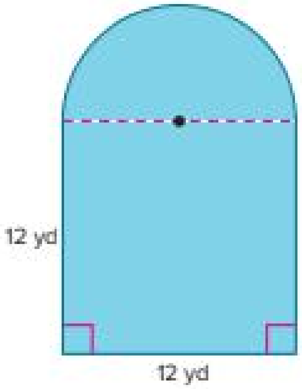
Name ______________________ Period ____________ Date ____________

Practice

Go Online You can complete your homework online.

Find the area of each figure. If necessary, use 3.14 for π and round to the nearest hundredth. (Example 1)

1.



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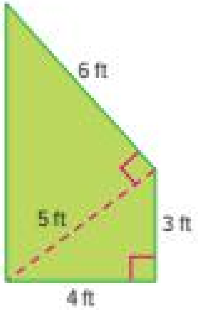

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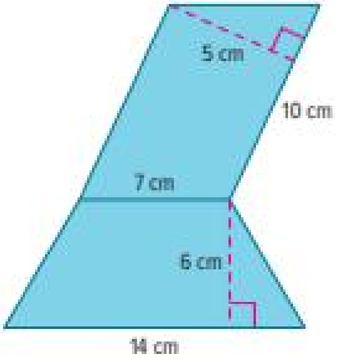

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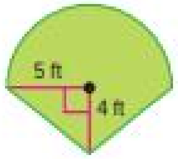

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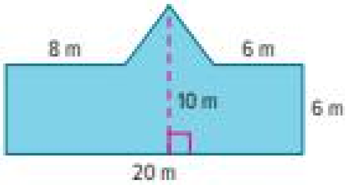

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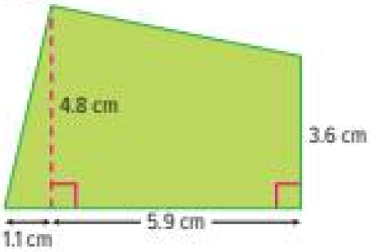
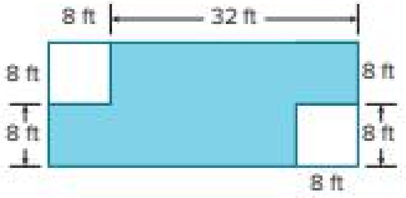
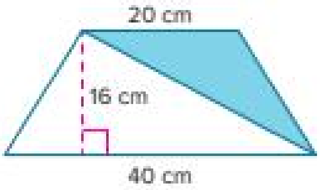


7. Find the area of the shaded region. (Example 2)

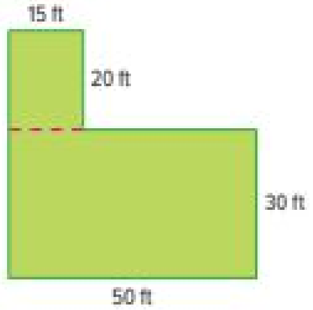


Test Practice

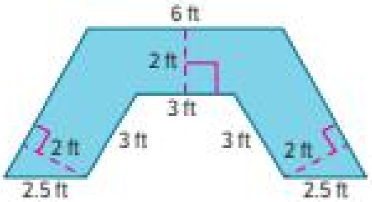
8. Open Response Find the area of the shaded region.



Apply

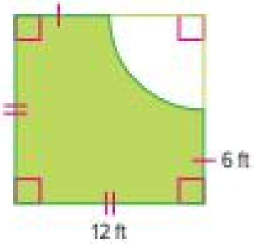


9. Alonzo needs to sod his backyard. The figure shows the measurements of the area of his yard which he intends to sod. One pallet of sod covers 400 square feet. How many full pallets of sod will Alonzo need to have enough for his entire yard?



10. Ward is planning to install a new countertop in his kitchen, as shown in the figure. The new countertop costs $42.50 per square foot. What will be the cost of the new countertop?

11. **Reason Inductively** Write an argument explaining how you can find the area of the shaded figure.



12. **Create** Write and solve a real-world problem that involves finding the area of a composite figure.

13. **Reason Abstractly** Suppose a swimming pool is in the shape of a composite figure that has a curved side that is not a semicircle. Explain how you could estimate the area of the swimming pool.

14. Draw and label a composite figure that involves a rectangle and triangle. Then find the area of the figure.

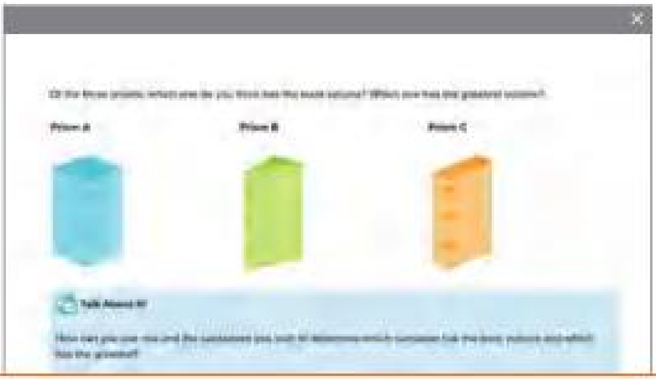
Volume

I Can... find volumes of prisms and pyramids by using formulas for volume of prisms and pyramids.

What Vocabulary Will You Learn?
cubic units
pyramid
rectangular prism
triangular prism
volume

Explore Volume of Prisms

Online Activity You will investigate the relationship between the base area of a prism and the volume of a prism.

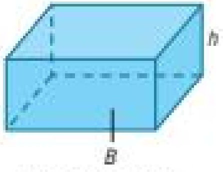
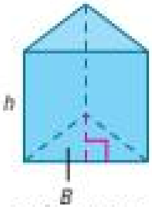


Learn Volume of Prisms

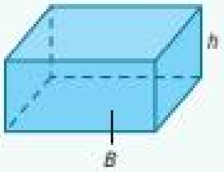
The **volume** of a three-dimensional figure is the measure of space it occupies. It is measured in cubic units such as cubic centimeters (cm^3) or cubic inches (in^3).

The table shows the use of the formula to find the volume of a prism.

A **rectangular prism** has two parallel congruent bases that are rectangles. A **triangular prism** has two parallel congruent bases that are triangles.

Words	Model
The volume V of a prism is the product of the area of the base B and the height h.	 **Rectangular Prism**  **Triangular Prism**
Symbols	
$V = Bh$	

Talk About It!
The shape of the base of a rectangular prism is a rectangle. How can you replace the B in the formula $V = Bh$ to write a formula specifically for the volume of a rectangular prism?

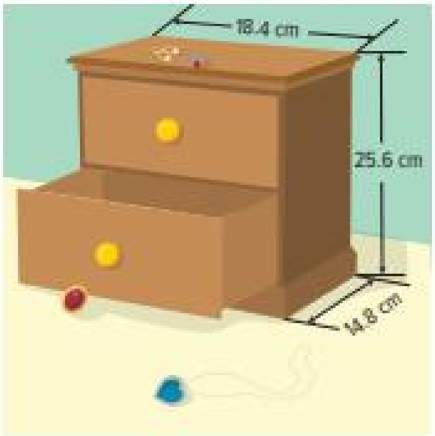


Your Notes

Example 1 Volume of Rectangular Prisms

A jewelry box is in the approximate shape of a rectangular prism.

What is the approximate volume of the jewelry box? Round to the nearest tenth if necessary.



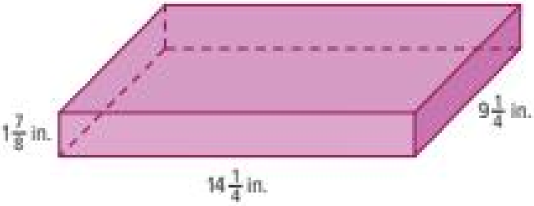
Use the formula $V = Bh$ to find the volume of the jewelry box.

$V = Bh$	Volume of a prism
$V = (\ell w)h$	The base is a rectangle, so $B = \ell w$.
$V = (18.4 \cdot 14.8)25.6$	Replace ℓ with 18.4, w with 14.8, and h with 25.6.
$V = 6{,}971.392$	Simplify.

So, the volume of the jewelry box is about ________ cubic centimeters.

Check

A gift box has the dimensions shown. What is the volume of the gift box? Write your answer as a mixed number in simplest form.

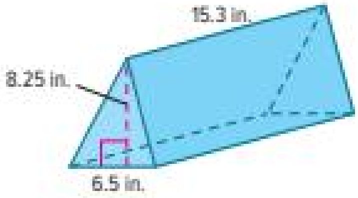


Show your work here

Go Online You can complete an Extra Example online.

Example 2 Volume of Triangular Prisms

Find the volume of the prism. Round to the nearest hundredth if necessary.



Use the formula $V = Bh$ to find the volume of the prism. The prism is a triangular prism, because the two parallel bases are triangles.

$V = Bh$ Volume of a prism

$V = \left(\frac{1}{2} \cdot 6.5 \cdot 8.25\right)h$ The base is a triangle, so $B = \frac{1}{2}bh$, where $b = 6.5$ and $h = 8.25$.

$V = \boxed{}\ h$ Simplify.

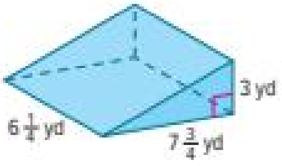
$V = 26.8125\left(\boxed{}\right)$ Replace h with 15.3.

$V = 410.23125$ Simplify.

So, the volume of the prism is about ________ cubic inches.

Check

What is the volume of the prism? Write your answer as a mixed number in simplest form.



Show your work here

Go Online You can complete an Extra Example online.

Think About It!

Which faces are the bases of the prism?

Talk About It!

When using the formula $V = Bh$ to find the volume of a triangular prism, you replace B with $\frac{1}{2}bh$. What is the difference between B and b?

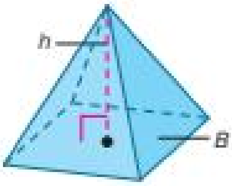
Explore Volume of Pyramids

Online Activity You will explore the relationship between the volume of a prism and the volume of a pyramid with the same base area and height.

Learn Volume of Pyramids

A **pyramid** is a polyhedron with one base that is a polygon and three or more triangular faces that meet at a common vertex. In the Explore activity, you learned that a pyramid has one-third the volume of a prism with the same base and height. The height of a pyramid is the perpendicular distance from the vertex of the pyramid to the base.

The table shows the use of the formula to find the volume of a pyramid.

Words	Model
The volume *V* of a prism is one third the area of the base *B* times the height of the pyramid *h*.	
Symbols	
$V = \frac{1}{3}Bh$	

Pause and Reflect

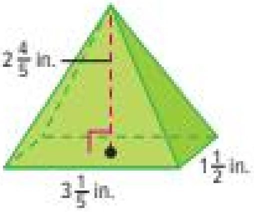
A rectangular prism and a rectangular pyramid each have a base area of 150 square inches. The prism and the pyramid have the same height. If the volume of the rectangular pyramid is 600 cubic inches, what is the volume of the rectangular prism? Write an argument to justify your solution.

Record your observations here

Example 3 Volume of Pyramids

Find the volume of the rectangular pyramid.

Use the formula $V = \frac{1}{3}Bh$ to find the volume of the pyramid.



$V = \frac{1}{3}Bh$ Volume of a pyramid

$V = \frac{1}{3}(\ell w)h$ The base is a rectangle, so $B = \ell w$.

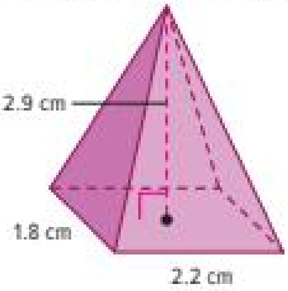
$V = \frac{1}{3}\left(3\frac{1}{5} \cdot 1\frac{1}{2}\right)\left(2\frac{4}{5}\right)$ Replace ℓ with $3\frac{1}{5}$, w with $1\frac{1}{2}$, and h with $2\frac{4}{5}$.

$V = \boxed{}$ Simplify.

So, the volume of the pyramid is about $4\frac{12}{25}$ cubic inches.

Check

Find the volume of the pyramid. Write your answer as a decimal rounded to the nearest hundredth.

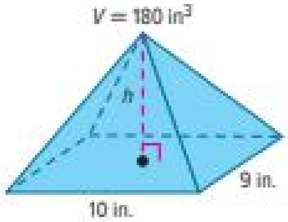


Show your work here

Go Online You can complete an Extra Example online.

Learn Use Volume to Find Missing Dimensions

Go Online Watch the animation to learn how you can use the volume formula to find missing dimensions if you know the volume.



The animation shows the steps to using the volume formula to find the unknown height for the pyramid shown.

Step 1 Write the volume formula.

$$V = \frac{1}{3}Bh$$

Step 2 Substitute the known values into the formula.

$$180 = \frac{1}{3}(10 \cdot 9)h \qquad V = 180,\ B = 10 \cdot 9$$

Step 3 Solve the equation.

$$180 = \frac{1}{3}(10 \cdot 9)h \qquad \text{Write the equation.}$$

$$180 = \boxed{}\,h \qquad \text{Multiply.}$$

$$\frac{180}{\boxed{}} = \frac{30h}{\boxed{}} \qquad \text{Divide by 30.}$$

$$\boxed{} = h \qquad \text{Simplify.}$$

The height of the pyramid is 6 inches.

Pause and Reflect

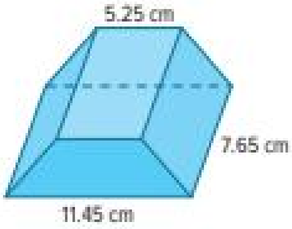
If the volume of a pyramid is given, what else must be given in order to solve for B?

Record your observations here.

Example 4 Use Volume to Find Missing Dimensions

The prism has a volume of 195.075 cubic centimeters.

What is the area of the base of the prism?



The figure is a trapezoidal prism because the shape of the two parallel and congruent bases are trapezoids. You know the volume and height of the prism, and you need to find the area of the base.

Use the formula $V = Bh$ to find the area of the base of the prism.

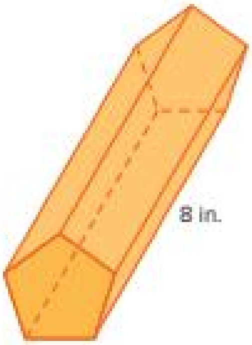
$V = Bh$	Volume of a prism
$\boxed{} = B(\boxed{})$	Replace V with 195.075 and h with 7.65.
$195.075 = 7.65B$	Simplify.
$\frac{195.075}{7.65} = \frac{7.65B}{7.65}$	Division Property of Equality
$25.5 = B$	Simplify.

So, the area of the base of the prism is 25.5 square centimeters.

Check

The pentagonal prism shown has a volume of about $124\frac{4}{5}$ cubic inches. What is the area of the base of the prism?

Show your work here



Think About It!
Which faces are the bases of the prism?

Talk About It!
Once you find the area of the base, can you find the height of the base? Explain your reasoning.

Go Online You can complete an Extra Example online.

Think About It!

What formula will you use to solve the problem?

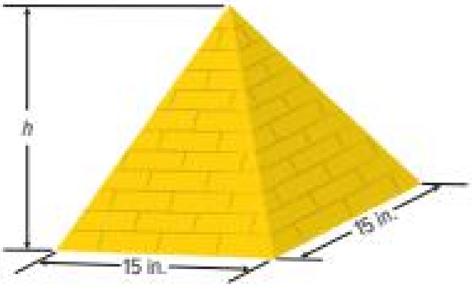
Talk About It!

In the formula for the volume of a pyramid, why do you need to multiply the product of the area of the base and the height by one third?

Example 5 Use Volume to Find Missing Dimensions

A model of the Great Pyramid of Giza has a square base with sides that are 15 inches long.

If the volume of the model is 675 square inches, what is the height of the model?



You know the volume of the model and the side lengths of the base. Use the formula $V = \frac{1}{3}Bh$ to find the height of the pyramid.

$V = \frac{1}{3}Bh$ Volume of a pyramid

$V = \frac{1}{3}s^2h$ The base is a square, so $B = s^2$.

$675 = \frac{1}{3}(15^2)h$ Replace V with 675 and s with 15.

$675 = \frac{1}{3}(225)h$ Simplify.

$675 = 75h$ Multiply.

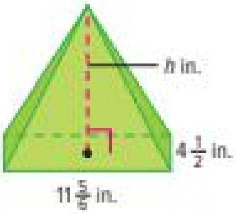
$\frac{675}{75} = \frac{75h}{75}$ Division Property of Equality

$9 = h$ Simplify.

So, the height of the model is 9 inches.

Check

The pyramid shown has a volume of $266\frac{1}{4}$ cubic inches.



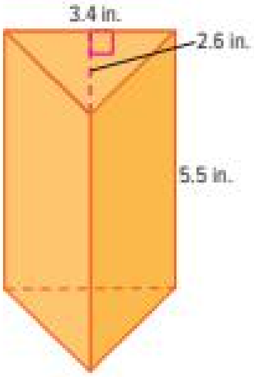
What is the height of the pyramid?

Show your work here

Go Online You can complete an Extra Example online.

Apply Packaging

Li is mailing a candle that has the dimensions shown in a rectangular box that is 4.2 inches long, 5.8 inches wide, and 7.6 inches tall. If one bag of packing material holds 25 cubic inches of material, how many bags does Li need to buy to fill the space around the candle?



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It!
How can you solve this problem if the volume of the rectangular box was unknown and the number of bags of packing material was given?

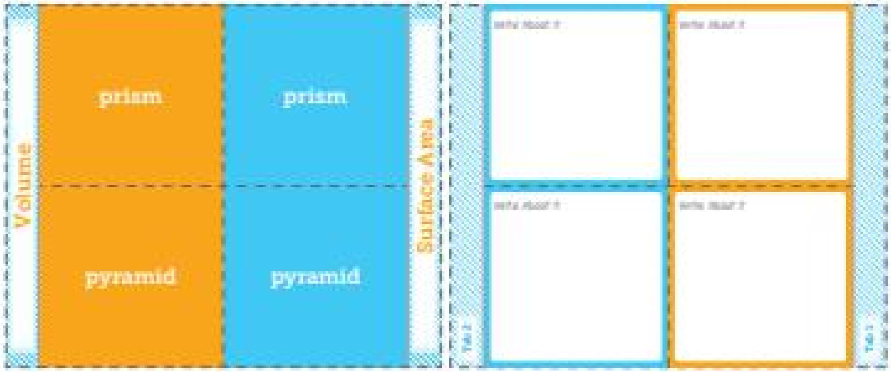
Check

Thema has a raised garden bed in her backyard that is shaped like a rectangular prism. It is 6 feet long, 3 feet wide, and $\frac{2}{3}$ foot deep. If a bag of garden soil holds 960 cubic inches of soil, how many bags will Thema need to fill the bed?

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

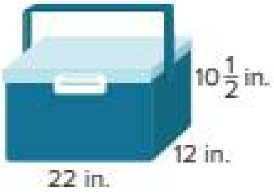


Name ______________________ Period __________ Date __________

Practice

Go Online You can complete your homework online.

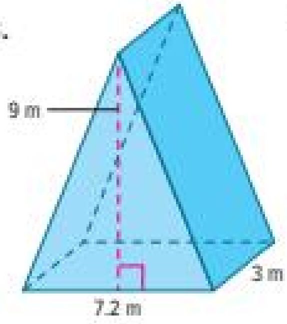
1. A cooler is in the shape of a rectangular prism. What is the volume of the cooler? Round to the nearest tenth if necessary. (Example 1)



2. A cereal box is in the shape of a rectangular prism. What is the volume of the cereal box? Express your answer as a decimal rounded to the nearest tenth if necessary. (Example 1)

Find the volume of each figure. Round to the nearest tenth if necessary. (Examples 2 and 3)

3.



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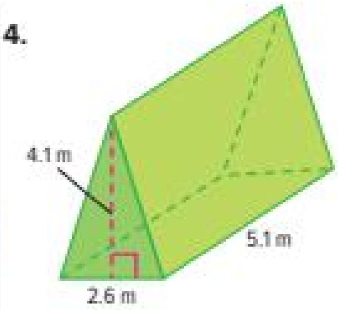

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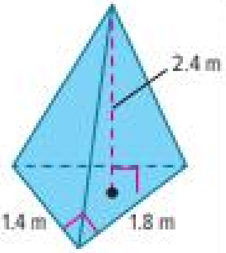

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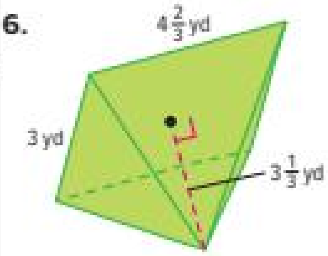
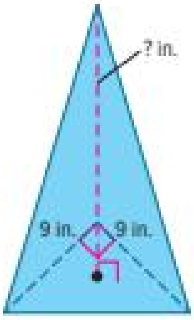


7. A triangular prism has a height of 5.9 meters and volume of 86.376 cubic meters. What is the area of the base of the prism? (Example 4)

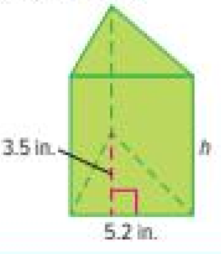
8. A rectangular pyramid has a height of 9.5 centimeters and a volume of 494 cubic centimeters. What is the area of the base of the pyramid? (Example 5)

9. A glass stand to display a doll is in the shape of a right triangular pyramid as shown. The volume of the stand is 202.5 cubic inches. What is the height of the stand? (Example 5)



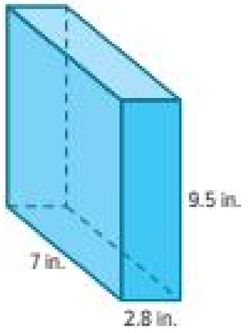
Test Practice

10. Open Response A triangular box of sticky notes is shown. The volume of the box of sticky notes is 54.6 cubic inches. What is the height of the box of sticky notes?

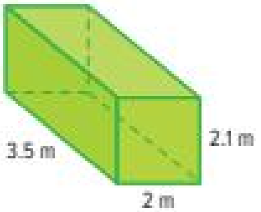


Apply

11. Sasha is mailing a photo box that has the dimensions shown in a rectangular box that is 12.5 inches long, 4.2 inches wide, and 12.5 inches tall. If one bag of packing material holds 75 cubic inches of material, how many bags does Sasha need to buy to fill the space around the photo box?



12. The cargo bed of a commercial truck is shaped like a rectangular prism. The dimensions are shown. Billy has 80 cubic meters of mulch to take to his house. How many trips will he have to make until all the mulch is at his house?



13. Create Write and solve a real-world problem that involves finding the volume of a rectangular prism or triangular prism.

14. MP **Reason Abstractly** Determine if the statement is *true* or *false*. Write an argument to justify your solution.

If a square pyramid and a cube have the same bases and volumes, then the height of the cube is three times the height of the pyramid.

15. A rectangular prism has a volume of 96 cubic inches. Find two possible measurements for the base area and height of the prism.

16. Draw and label a real-world object that is in the shape of a square pyramid or triangular pyramid. Then find the volume of the object.

Surface Area

I Can... find the surface areas of solids by relating the nets of those solids to the formulas for surface area.

What Vocabulary Will You Learn?
- face
- lateral face
- regular pyramid
- slant height
- surface area

Explore Surface Area of Prisms and Pyramids

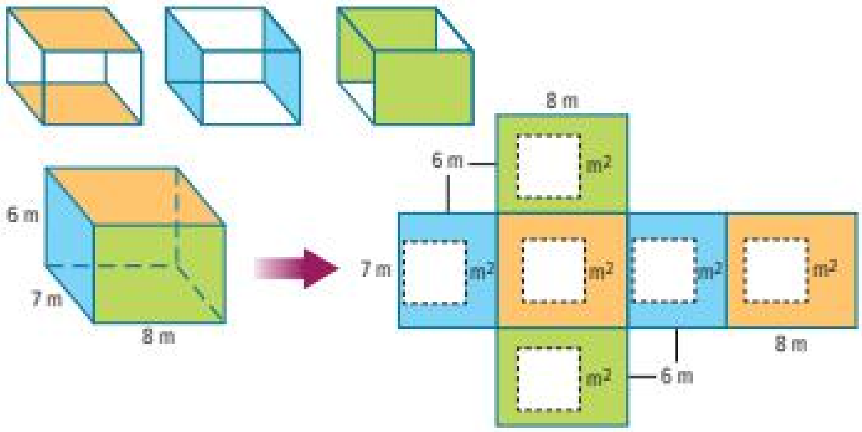
Online Activity You will investigate how to find the surface area of prisms and pyramids without using nets.

Learn Surface Area of Prisms

The sum of the areas of all the surfaces, or **faces**, of a three-dimensional figure is the **surface area**. When you find the surface area of a three-dimensional figure, the units are square units.

To find the surface area of a rectangular prism or a triangular prism, find the area of each face and then calculate the sum of all of the areas of the faces. Recall that you used nets to find surface area in an earlier grade.

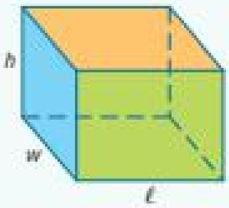
Label the area of each of the faces of the prism in the net.



Total Surface Area $= 56\text{ m}^2 + 56\text{ m}^2 + 42\text{ m}^2 + 42\text{ m}^2 + 48\text{ m}^2 + 48\text{ m}^2$
$= 292\text{ m}^2$

Talk About It!

Suppose the dimensions of the prism were unknown. Can you write a formula that would help you find the surface area of any rectangular prism with length ℓ, height h, and width w? Share your formulas with other classmates. Compare and contrast your formulas.



Your Notes

Think About It!

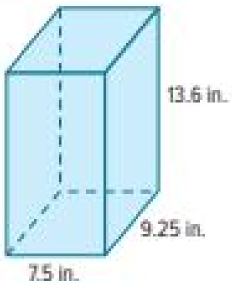
How can you determine how many pairs of congruent faces make up the figure?

Talk About It!

Suppose you have trouble remembering that the forumula $S.A. = 2\ell h + 2\ell w + 2hw$ can be used to find the surface area of a rectangular prism. How can you use the structure of a rectangular prism to relate the formula to the area of each face?

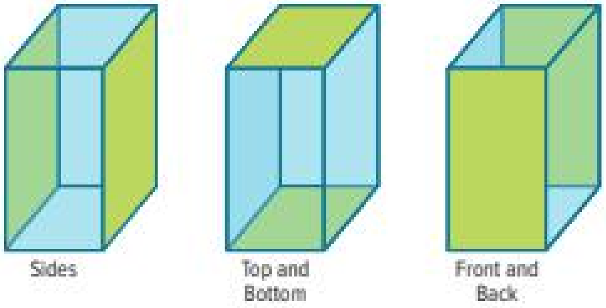
Example 1 Surface Area of Rectangular Prisms

Find the surface area of the rectangular prism.



Step 1 Find the area of each pair of opposite faces.

In the rectangular prism, opposite faces are congruent. To find the area of each pair of faces, multiply the area of one face by 2.



Area of sides: 2(9.25 · 13.6) or 251.6 in^2

Area of top and bottom: 2(7.5 · 9.25) or 138.75 in^2

Area of front and back: 2(7.5 · 13.6) or 204 in^2

Step 2 Find the sum of the areas of the faces.

So, the total surface area of the prism is 251.6 + 138.75 + 204, or 594.35 square inches.

By doing this, you are using the formula for the surface area of a rectangular prism, $S.A. = 2\ell h + 2\ell w + 2hw$.

$S.A. = 2\ell h + 2\ell w + 2hw$ Write the formula.

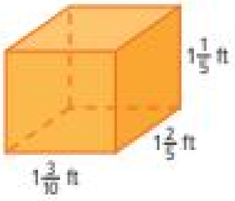
$= 2(7.5 \cdot 13.6) + 2(7.5 \cdot 9.25) + 2(13.6 \cdot 9.25)$ Substitute.

$= 204 + 138.75 + 251.6$ Simplify.

$= 594.35$ Add.

Check

Find the surface area of the prism. Write your answer as a mixed number in simplest form.

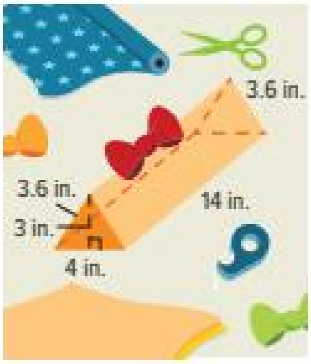


Show your work here

Go Online You can complete an Extra Example online.

Example 2 Surface Area of Triangular Prisms

How much paper is needed to cover the gift box shown?

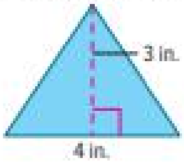


Step 1 Find the area of the bases and faces.

In any triangular prism, the bases are congruent, but the faces are not always congruent.

In this triangular prism, there are two congruent triangular bases. There are three rectangular faces, two of which are congruent.

Area of the Bases



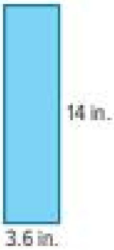
$\text{Area} = 2\left(\frac{1}{2} \cdot 4 \cdot 3\right)$ There are 2 triangular bases, each with an area of $\frac{1}{2} \cdot 4 \cdot 3$.

$= 2(6)$ Multiply.

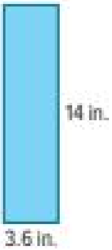
$= 12$ Simplify.

The combined area of the two triangular bases is 12 square inches.

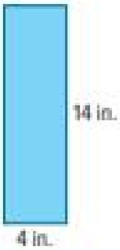
Area of Face 1



Area of Face 2



Area of Face 3



$A =$ ☐ · ☐
$=$ ☐

$A =$ ☐ · ☐
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$A =$ ☐ · ☐
$=$ ☐

The areas of the rectangular faces are 50.4 square inches, 50.4 square inches, and 56 square inches.

Step 2 Find the sum of the areas of the faces.

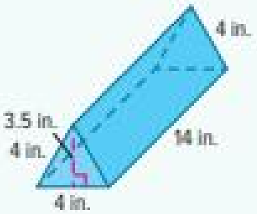
So, 12 + 50.4 + 50.4 + 56, or ________ square inches of paper is needed to cover the gift box.

Think About It!

How many faces make up the figure?

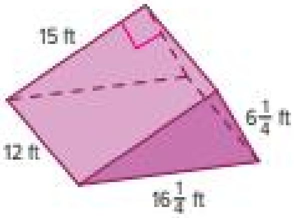
Talk About It!

If the base of the prism was an equilateral triangle, would the three rectangular faces be congruent? Explain.



Check

Find the surface area of the prism.



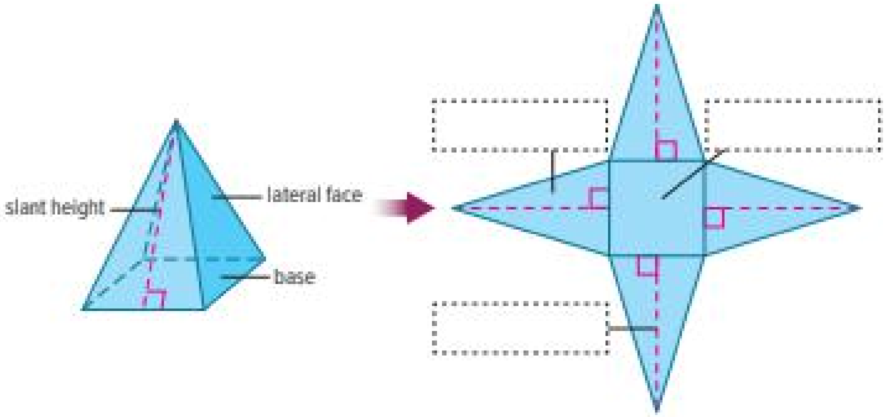
Show your work here

Go Online You can complete an Extra Example online.

Learn Surface Area of Pyramids

A **regular pyramid** is a pyramid whose base is a regular polygon. The **lateral faces**, the faces that are not the base, of a regular pyramid are congruent isosceles triangles that meet at the vertex. The height of each lateral face is called the **slant height** of the pyramid.

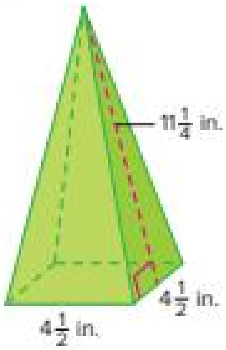
Label the parts of the pyramid in the net.



You can use the areas of the lateral faces and the base to find the surface area of a pyramid.

Example 3 Surface Area of Pyramids

Find the surface area of the pyramid.



Step 1 Find the area of the base.

The base of the pyramid is a square with $4\frac{1}{2}$-inch sides. Use the formula $A = s^2$ to find the area of the base.

$A = s^2$ Area of a square

$= 4\frac{1}{2} \cdot 4\frac{1}{2}$ Each side is $4\frac{1}{2}$ in.

$= \frac{9}{2} \cdot \frac{9}{2}$ Multiply.

$= \frac{81}{4}$ or $20\frac{1}{4}$ Simplify.

The area of the base is $4\frac{1}{2} \cdot 4\frac{1}{2}$ or ☐ square inches.

Step 2 Find the area of the lateral faces.

The lateral faces are four congruent triangles with a base length of $4\frac{1}{2}$ inches and a height of $11\frac{1}{4}$ inches. Use the formula $A = 4\left(\frac{1}{2}bh\right)$ to find the total area of the lateral faces.

$A = 4\left(\frac{1}{2}bh\right)$ There are 4 lateral faces with an area of $\frac{1}{2}bh$.

$= 4\left(\frac{1}{2} \cdot 4\frac{1}{2} \cdot 11\frac{1}{4}\right)$ Replace b with $4\frac{1}{2}$ and h with $11\frac{1}{4}$.

$= \frac{4}{1} \cdot \frac{1}{2} \cdot \frac{9}{2} \cdot \frac{45}{4}$ Multiply.

$= \frac{405}{4}$ or $101\frac{1}{4}$ Simplify.

The area of the lateral faces is $4\left(\frac{1}{2} \cdot 4\frac{1}{2} \cdot 11\frac{1}{4}\right)$ or ☐ square inches.

Step 3 Find the total surface area.

The area of the base is $20\frac{1}{4}$ square inches. The area of the lateral faces is $101\frac{1}{4}$ square inches.

So, the total surface area of the pyramid is $20\frac{1}{4} + 101\frac{1}{4}$, or $121\frac{1}{2}$ square inches.

Think About It!

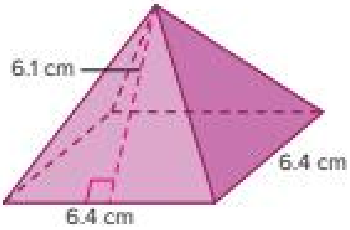
How can you determine how many pairs of congruent faces make up the figure?

Talk About It!

If the base was not a square, would the four triangular faces be congruent? Explain.

Check

Find the surface area of the pyramid.



Go Online You can complete an Extra Example online.

Pause and Reflect

How will you study the concepts in today's lesson? Describe some steps you can take.

Apply Painting

Domingo built a toy box for his little brother that is 42 inches long, 21 inches wide, and 24 inches tall. He has 1 quart of paint that covers about 87 square feet. Does he have enough paint to cover the outside of the toy box with two coats of paint?

Go Online Watch the animation.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

Talk About It!
How can you solve this problem another way?

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

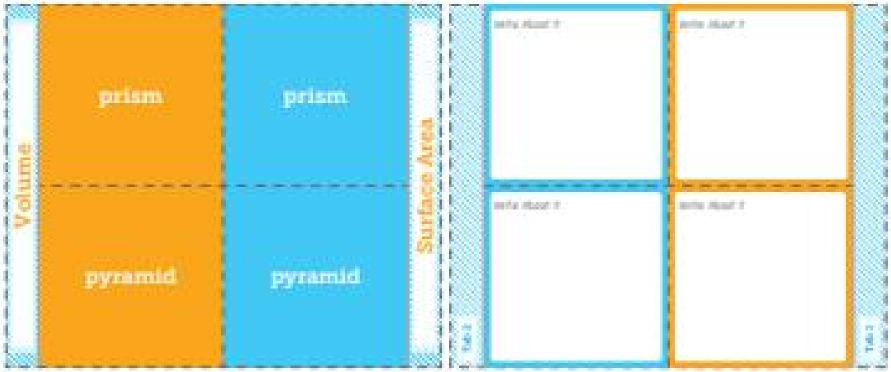
Check

Lucia is covering boxes with fabric to sell at a craft fair. The boxes are shaped like rectangular prisms and measure $10\frac{1}{2}$ inches wide, $14\frac{1}{2}$ inches long, and 3 inches tall. If she has 100 square feet of fabric, how many boxes can she cover?

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



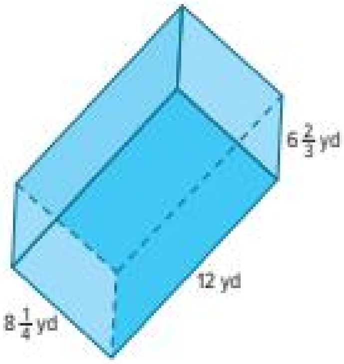
Name ______________________ Period __________ Date __________

Practice

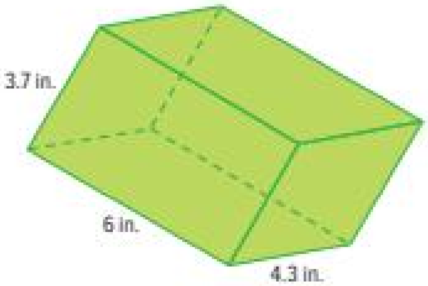
Go Online You can complete your homework online.

Find the surface area of each prism. Round to the nearest tenth if necessary. (Example 1)

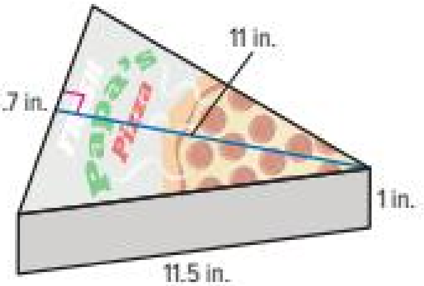
1.



2.

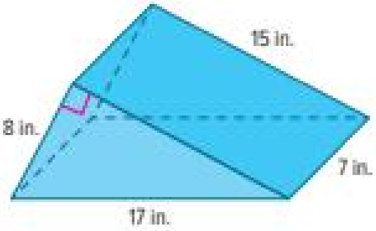


3. How much cardboard is needed to make the single slice of pizza box shown? (Example 2)



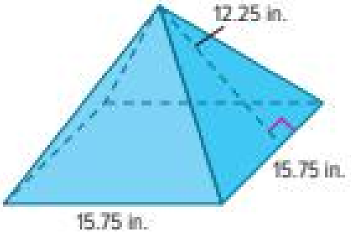
Test Practice

4. Open Response What is the surface area of the triangular prism-shaped toy car ramp shown?

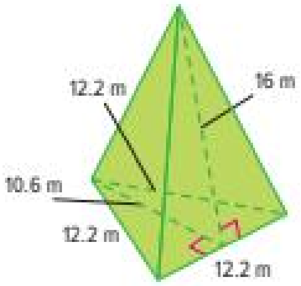


Find the surface area of each pyramid. Round to the nearest tenth if necessary. (Example 3)

5.

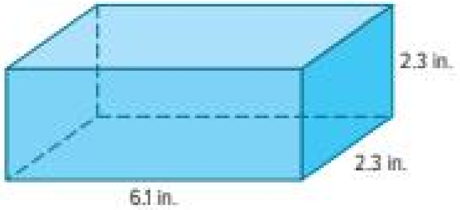


6.



Apply

7. Oscar is making a play block for his baby sister by gluing fabric over the entire surface of a foam block. Is 65 square inches of fabric enough? If so, how much fabric will remain? If not, how much more fabric will he need?



8. When wrapping a birthday gift in the shape of a rectangular prism for his mother, Kenji adds an additional 2.5 square feet of gift wrap to allow for overlap. How many square feet of gift wrap will Kenji use to wrap a gift 3.5 feet long, 18 inches wide, and 2 feet high?

9. Find the surface area of a rectangular prism with a height of $4\frac{1}{3}$ yards, a length of 6.2 yards, and a width of 3.15 yards.

10. Draw and label a square pyramid with a surface area between 200 and 300 square inches. Include the surface area.

11. MP **Reason Abstractly** The side measures of a rectangular prism are tripled. What is the relationship between the surface area of the original prism and the surface area of the new prism? Support your answer with an example.

12. **Create** Write and solve a real-world problem where you have to find the surface area of a rectangular prism.

Lesson 9-6

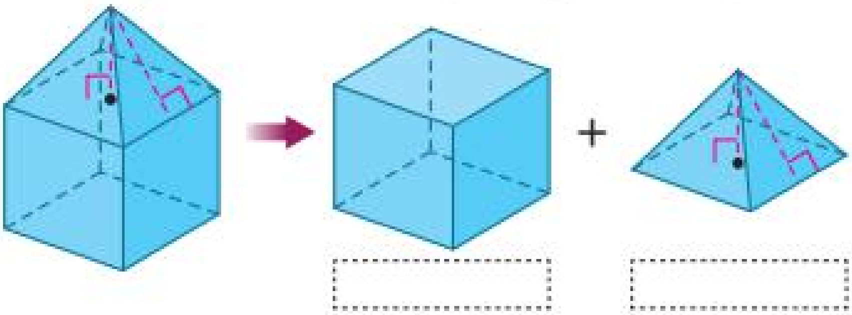
Volume and Surface Area of Composite Figures

I Can... find volumes and surface areas of composite figures by decomposing the figures into common solids and using the formulas for volume and surface area of those solids.

Learn Volume of Composite Figures

The volume of a composite figure can be found by decomposing the figure into solids whose volumes you know how to find.

Label the solids into which this composite figure is decomposed.



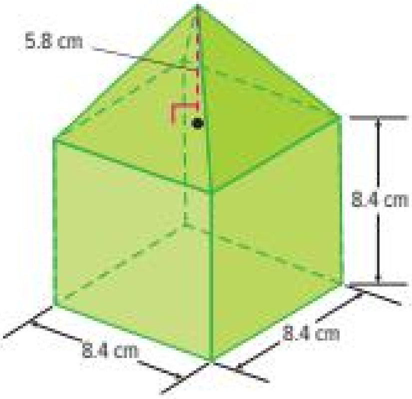
Pause and Reflect

Have you ever wondered when you might use the concepts you learn in math class? What are some everyday scenarios in which you might see composite figures? Draw some real-world composite figures you may have seen in your everyday life.

Think About It!
What are the different three-dimensional shapes that make up the toy block?

Example 1 Volume of Composite Figures

A toy block has the dimensions shown.



What is the volume of the block? Round to the nearest hundredth if necessary.

Step 1 Identify the figures that compose the toy block.

The toy block is composed of a cube and pyramid.

Step 2 Find the volume of each figure.

Volume of the cube	**Volume of the pyramid**
$V = s^3$	$V = \frac{1}{3}Bh$
$= (8.4)^3$	$= \frac{1}{3}(s^2)h$
≈ 592.70	$= \frac{1}{3}(8.4^2)5.8$
	≈ 136.42

So, the volume of the cube is about 592.70 cubic centimeters and the volume of the pyramid is about 136.42 cubic centimeters.

Step 3 Find the volume of the toy block.

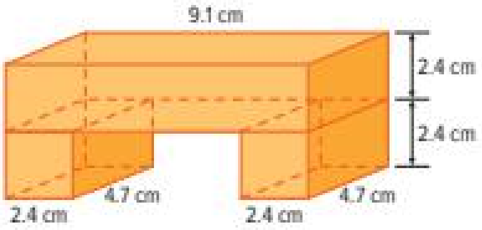
☐ + ☐ = ☐

So, the total volume of the toy block is about 592.70 + 136.42, or 729.12 square centimeters.

Check

What is the volume of the composite figure? Round to the nearest hundredth if necessary.

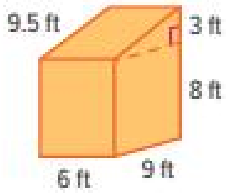
Show your work here



Go Online You can complete an Extra Example online.

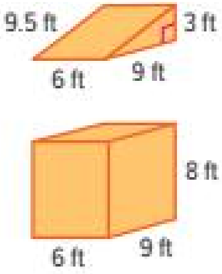
Learn Surface Area of Composite Figures

Go Online Watch the animation to learn how to find the surface area of a composite figure.



Step 1 Decompose the figure into simpler solids.

This figure is composed of a rectangular prism and a triangular prism, as shown below.



Step 2 Find the surface area of the simpler solids.

Add the surface areas of all of the faces except the top face of the rectangular prism because it is not exposed in the original composite figure.

Bottom		Left		Right		Front		Back	
54 ft²	+	☐ ft²	+	72 ft²	+	☐ ft²	+	48 ft²	= ☐ ft²

Add the surface areas of all of the faces except the bottom face of the triangular prism because it is not exposed in the original composite figure.

Top		Left		Right		Back	
☐ ft²	+	13.5 ft²	+	☐ ft²	+	18 ft²	= ☐ ft²

Step 3 Find the total surface area.

$294 + 102 = 396$ ft²

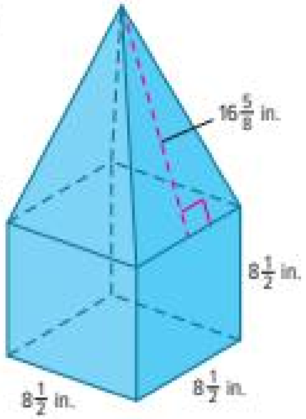
So, the total surface area of the composite figure is 396 square feet.

Talk About It!
One of the faces of each figure is not included when finding the surface area of the composite figure. Explain why.

Example 2 Surface Area of Composite Figures

A gift box from a stuffed animal store has the dimensions shown.

What is the surface area of the gift box?



Think About It!

The box is made up of a pyramid and a cube. Do any of the faces overlap?

Step 1 Identify the shapes of the faces of the figure.

In the figure, there are five square faces and four triangular faces.

Step 2 Find the areas of the faces.

The squares are all congruent, and the triangles are all congruent. Find the areas of the squares and triangles.

Complete the steps.

Area of the five squares	**Area of the four triangles**
$A = 5s^2$	$A = 4 \cdot \left(\frac{1}{2}bh\right)$
$= 5\left(\square\right)^2$	$= 4\left(\frac{1}{2} \cdot \square \cdot \square\right)$
$= \square$	$= \square$

Step 3 Find the total surface area of the figure.

Find the sum of the areas of the square faces and triangular faces to find the total surface area of the figure.

So, the surface area of the figure is $361\frac{1}{4} + 282\frac{5}{8}$, or $643\frac{7}{8}$ square inches.

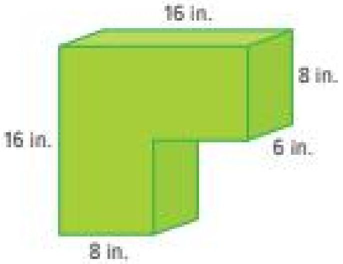
Talk About It!

Emilia added the individual surface areas of the pyramid and cube, and claimed that the total surface area is $788\frac{3}{8}$ square inches. Explain why this is incorrect.

Check

What is the surface area of the figure?

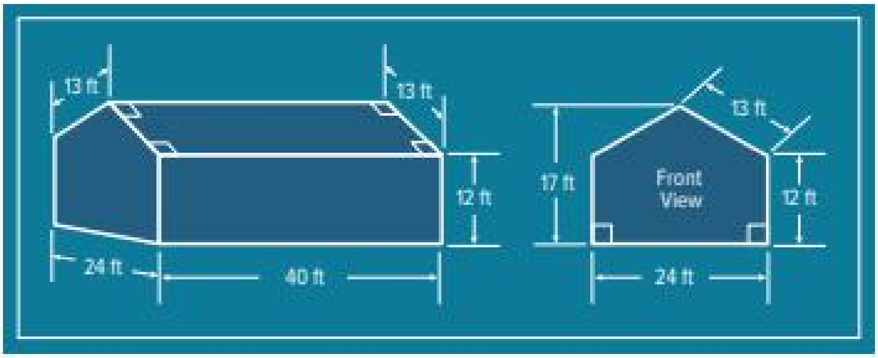
Show your work here



Go Online You can complete an Extra Example online.

Apply Construction

A new home builder has several different styles of homes for sale. They want to construct scale models of each home that have a scale of 1 inch = 2 feet to display in their showroom. The drawing shows two views of one of their homes. If there is no bottom to the model, would 800 square inches of plywood be enough to build the model?



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

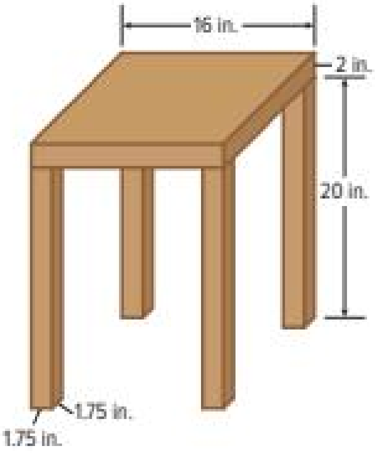
4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It!
If the bottom was added to the model, would there be enough plywood? Explain.

Check

Pam builds square-topped tables with the dimensions shown.



If cherry weighs 0.02 pound per cubic inch, about how much will a table made from cherry weigh? Round to the nearest pound.

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Describe the similarities and differences between finding the volume of a composite figure and finding the surface area of a composite figure.

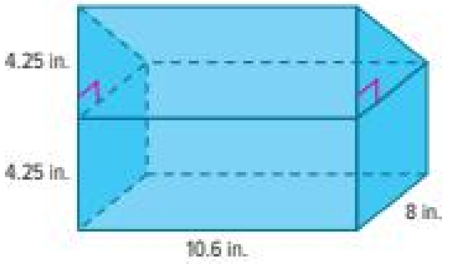
Record your observations here

Name ______________________ Period __________ Date __________

Practice

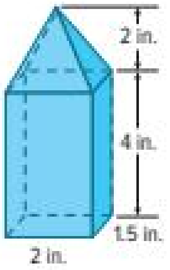
Go Online You can complete your homework online.

1. Mya's lunchbox is shown. What is the volume of the lunchbox? Round to the nearest tenth if necessary. (Example 1)

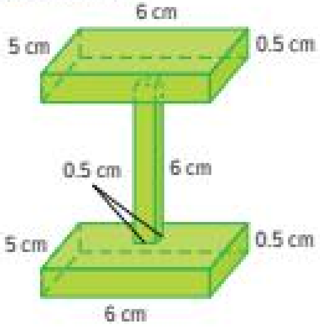


2. Anson's toy rocket is shown. What is the volume of the rocket? Round to the nearest tenth if necessary. (Example 1)

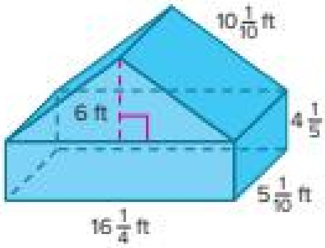
3. What is the volume of the birdfeeder? Round to the nearest tenth if necessary. (Example 1)



4. Zahir made this wooden perch for his pet bird. What is the volume of the bird perch? Round to the nearest tenth if necessary. (Example 1)

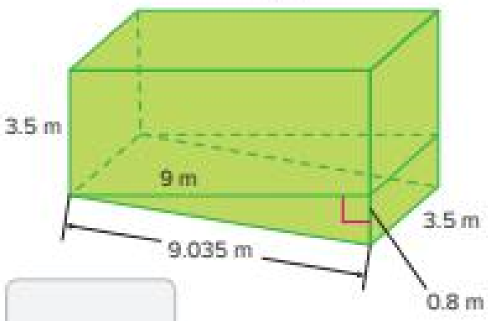


5. Find the surface area of the composite figure. Round to the nearest tenth if necessary. (Example 2)



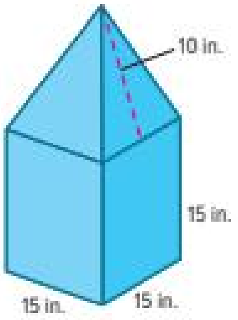
Test Practice

6. Open Response Find the surface area of the composite figure. Round to the nearest tenth if necessary.

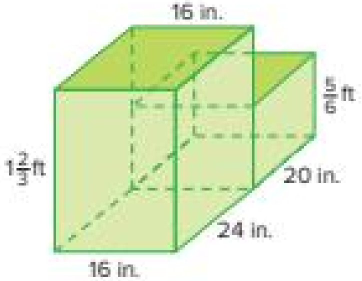


Apply

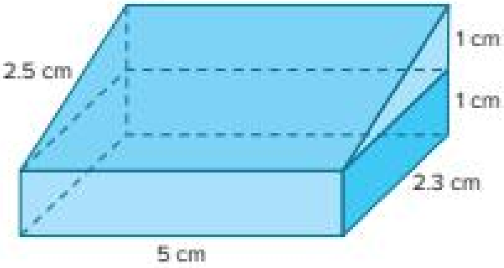
7. For a charity drive, each classroom is given a coin box made of cardboard like the one shown. The student council wants to construct a version of the coin box that has a scale factor of 3 times the classroom coin box. Is 100 square feet of cardboard enough to build the new coin box? Write an argument that can be used to defend your solution.



8. Jake wants to buy the foam gymnastic block shown. If the foam used to make the gymnastic block costs $24.99 per cubic foot, what is the cost of this block, to the nearest dollar?



9. **Reason Inductively** A student said that the surface area of the figure below was 57.4 square centimeters. Is the student correct? Explain.



10. Draw and label a composite figure made up of two prisms. Then find the volume of the figure.

11. **Be Precise** Explain how finding the volume and surface area of composite figures is similar.

12. **Create** Write and solve a real-world problem where you find the volume of a composite figure.

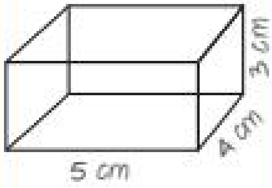
Review

Foldables Use your Foldable to help review the module.

Volume

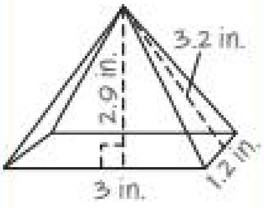
Tab 1

Volume =



Surface area =

Volume =



height = 2.8 in.

Surface area =

Surface Area

Tab 2

Rate Yourself! ☐ ◇ ☆

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.

Write about a question you still have.

Reflect on the Module

Use what you learned about measuring figures to complete the graphic organizer.

Essential Question

How can we measure objects to solve problems?

Circumference

Area

Volume

Surface Area

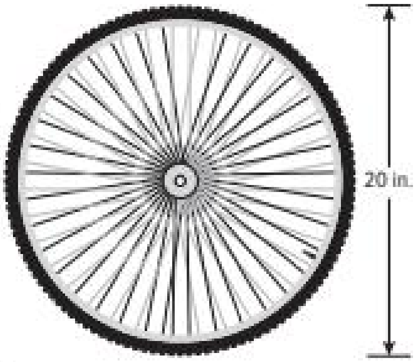
Name ______________________ Period __________ Date __________

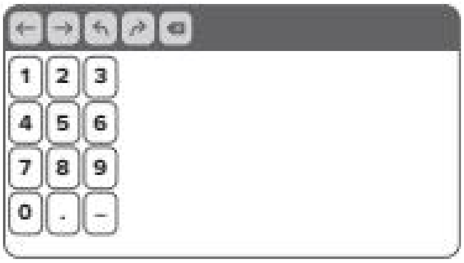
Test Practice

1. **Multiple Choice** What is the circumference of a circle with a radius of 7.5 centimeters? Use 3.14 for π. Round to the nearest hundredth if necessary. (Lesson 1)

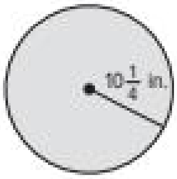
Ⓐ 23.55 cm
Ⓑ 38.4 cm
Ⓒ 47.1 cm
Ⓓ 176.63 cm

2. **Equation Editor** Jaime's bicycle tires have the diameter shown. How many inches in length is the circumference of each tire? Use 3.14 for π. Round to the nearest hundredth if necessary. (Lesson 1)





3. **Open Response** What is the area of the circle? Use 3.14 for π. Round to the nearest tenth if necessary. (Lesson 2)

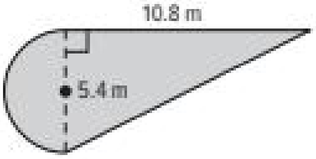


4. **Open Response** Collin has 100 feet of fencing to enclose a pen for his puppy. He is trying to decide whether to make the pen circular or square. He plans to use all of the fencing. (Lesson 2)

A. If Collin uses all of the fencing, what would be the area of each pen? Use 3.14 for π. Round to the nearest hundredth if necessary.

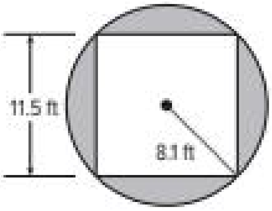
B. To have the largest possible area for the pen, which pen should Collin build?

5. **Multiple Choice** What is the area of the figure? Use 3.14 for π. Round to the nearest hundredth if necessary. (Lesson 3)

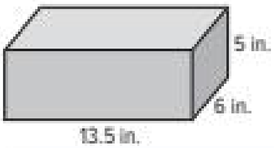


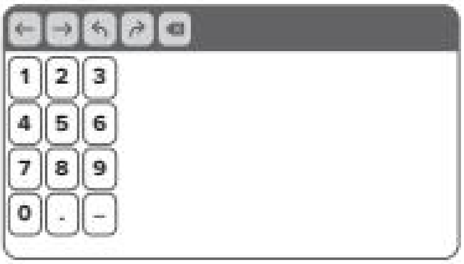
Ⓐ 40.61 m^2
Ⓑ 46.12 m^2
Ⓒ 52.06 m^2
Ⓓ 81.21 m^2

6. **Open Response** Find the area of the shaded region. Use 3.14 for π. Round to the nearest hundredth if necessary. (Lesson 3)

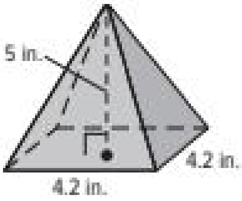


7. **Equation Editor** A shoebox has the dimensions shown. How many cubic inches is the volume of the shoebox? (Lesson 4)

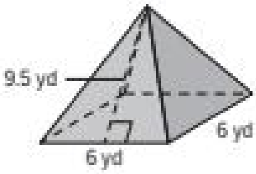




8. **Open Response** Nikki makes wax candles that are shaped like regular pyramids as shown. How much wax is needed to make each candle? Explain your reasoning. (Lesson 4)



9. **Multiple Choice** What is the total surface area of the regular pyramid? (Lesson 5)



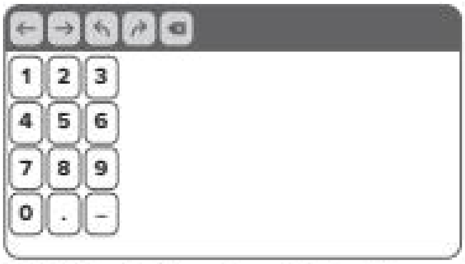
Ⓐ 114 yd^2

Ⓑ 120 yd^2

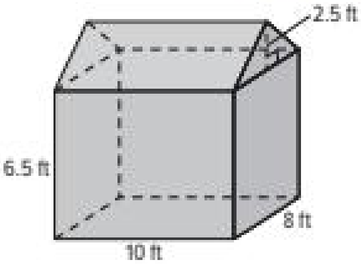
Ⓒ 138 yd^2

Ⓓ 150 yd^2

10. **Equation Editor** Jessie is building a wooden box for a stage prop to use in the school play. The dimensions of the box are 40 inches long, 24 inches wide, and 16 inches tall. She plans to cover the box with two coats of paint. If each can of paint covers about 16.5 square feet, what is the minimum number of cans needed in order for Jessie to complete the job? (Lesson 5)



11. **Multiple Choice** Jacob is building a shed with the dimensions shown. What is the volume of the shed? (Lesson 6)



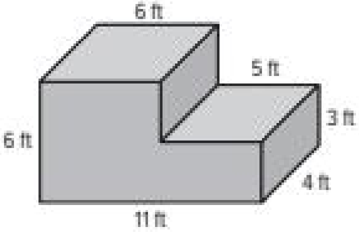
Ⓐ 540 cubic feet

Ⓑ 558 cubic feet

Ⓒ 620 cubic feet

Ⓓ 720 cubic feet

12. **Open Response** Find the surface area of the figure shown. (Lesson 6)



Module 10

Probability

Essential Question

How can probability be used to predict future events?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY

■ — I don't know. ◆ — I've heard of it. ★ — I know it!

	Before			After		
	■	◆	★	■	◆	★
finding likelihoods of probability events						
finding relative frequencies						
finding experimental probabilities						
making predictions using relative frequency						
finding sample spaces of probability events						
finding theoretical probabilities of simple events						
finding complements of simple events						
comparing relative frequencies to theoretical probabilities						
finding theoretical probabilities of compound events						
designing simulations of simple and compound events						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about probability.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ complementary event
- ☐ compound event
- ☐ event
- ☐ experimental probability
- ☐ likelihood
- ☐ outcome
- ☐ probability
- ☐ probability experiment
- ☐ relative frequency
- ☐ relative frequency bar graph
- ☐ relative frequency table
- ☐ sample space
- ☐ simple event
- ☐ simulation
- ☐ theoretical probability
- ☐ theoretical probability of a compound event
- ☐ tree diagram
- ☐ uniform probability model

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1

Write fractions in simplest form.

Write $\frac{20}{36}$ in simplest form.

First, identify the GCF of the numerator and denominator. The GCF of 20 and 36 is 4.

Then divide the numerator and denominator by the GCF.

$$\frac{20}{36} = \frac{20 \div 4}{36 \div 4} = \frac{5}{9}$$

Example 2

Multiply whole numbers.

Find $5 \cdot 4 \cdot 3 \cdot 2$.

$5 \cdot 4 \cdot 3 \cdot 2$ Write the problem.

$= 20 \cdot 3 \cdot 2$ Multiply 5 and 4.

$= 60 \cdot 2$ Multiply 20 and 3.

$= 120$ Multiply 60 and 2.

Quick Check

1. Write $\frac{18}{20}$ in simplest form.

2. Find $7 \cdot 8 \cdot 2$.

How Did You Do?

Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right.

① ②

Lesson 10-1

Find Likelihoods

I Can... describe the likelihood of an event as impossible, unlikely, equally likely to happen as not to happen, likely, or certain.

What Vocabulary Will You Learn?
event
likelihood
outcome

Explore Chance Events

Online Activity You will use Web Sketchpad to explore how to describe the likelihood of events.

Learn Likelihood of Events

Suppose you toss a quarter into the air. There are two sides to the quarter, and it can only land on one side at a time. Each of these results is called an **outcome**. The desired outcome or set of outcomes is called an **event**.

Both outcomes are *equally likely* because they both have the same chance of occurring. Each outcome is *equally likely* to happen as not to happen. You can describe an event's **likelihood** in different ways.

The table shows descriptions of likelihoods from *impossible* to *certain*.

	Impossible	Unlikely	Equally Likely	Likely	Certain
Description	not possible	having a poor chance of success	same chance of happening as not happening	having a good chance of success	sure to happen

The phrase *equally likely* is used to describe the likelihood of an event that is *equally likely* to happen as not to happen.

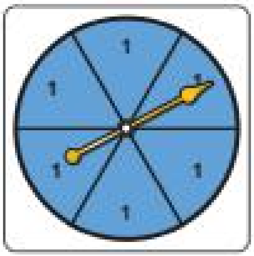
Talk About It!
Describe an event in everyday life that is unlikely to happen. Then describe an event that is likely to happen.

Your Notes

Example 1 Classify Likelihoods

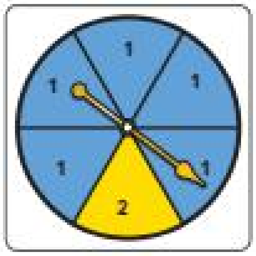
For each likelihood, select the spinner that best classifies the likelihood of the spinner landing on the number 2. Assume each spinner is spun once.

A



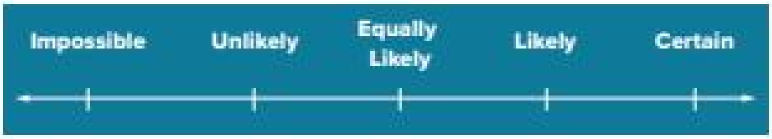
B

C



D

E



__________ __________ __________ __________ __________

Check

Classify the likelihood of each event as *impossible, unlikely, equally likely, likely,* or *certain*.

____________ spinning a number less than 5 on a spinner divided into 4 equal sections labeled 1 through 4

____________ choosing a weekday when randomly selecting dates from a given year

____________ it rains, given the chance of rain is 25%

____________ drawing a red marble from a bag containing only 10 blue marbles

____________ flipping a coin and it landing on heads

Go Online You can complete an Extra Example online.

Name ______________________ Period ____________ Date ____________

Practice

Go Online You can complete your homework online.

The spinner shown is spun once. Classify the likelihood of each event as *impossible*, *unlikely*, *equally likely*, *likely*, or *certain*. (Example 1)

1. the spinner landing on *dog*

2. the spinner landing on *hamster*

3. the spinner landing on *dog* or *cat*

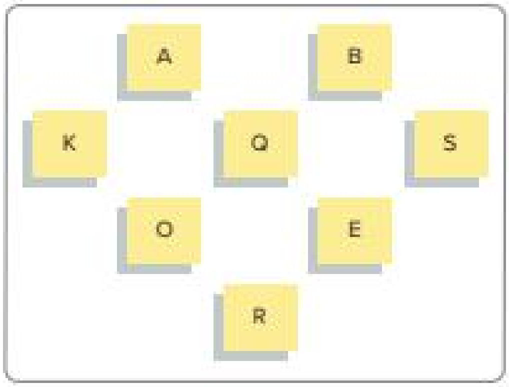
4. the spinner landing on *bird*

5. the spinner landing on an animal

6. the spinner landing on *cat* or *hamster*

Test Practice

For Exercises 7 and 8, a card is randomly selected from the ones shown.



7. Multiselect Select all events that are unlikely to happen.

- ☐ selecting the letter B
- ☐ selecting the letter T
- ☐ selecting a vowel or S
- ☐ selecting a consonant or vowel
- ☐ selecting a consonant or A
- ☐ selecting the letter Q or R

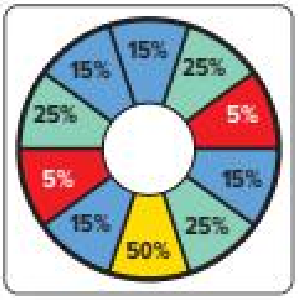
8. Multiselect Select all of the following events that are equally likely to happen as not to happen.

- ☐ selecting the letter B
- ☐ selecting the letter E
- ☐ selecting a vowel or S
- ☐ selecting a consonant or vowel
- ☐ selecting a consonant or A
- ☐ selecting the letter Q, R, B, or K

Apply

9. The spinner shows the prizes a person can win at a festival. The spinner shown is spun once. Order the prizes a person can win based on the likelihood of spinning that prize from least likely to most likely.

10. The spinner shows the amount of discount a shopper will receive on one item when they check out. Order the amount of the discounts based on the likelihood of spinning that discount from least likely to most likely.



11. Describe a real-world event that is equally likely to happen as not to happen.

12. **Persevere with Problems** Theresa is taking a multiple-choice test and does not know an answer. She can guess answer A, B, C, D, or E. Is the chance of her randomly selecting any of the answer choices equally likely of being the correct answer? Explain.

13. **Reason Abstractly** About 5% of Americans are vegetarians. If you ask a random person whether he or she is a vegetarian, is it likely or unlikely the person is not a vegetarian? Explain.

14. **Create** Write about a real-world event in which you need to find the likelihood of the event. Then find the likelihood of that event.

Relative Frequency of Simple Events

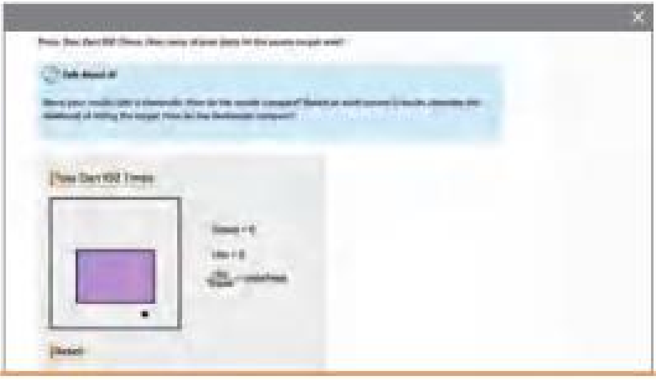
I Can... find the relative frequency of an event and use it to predict the chance of that event occurring in the future.

What Vocabulary Will You Learn?

experimental probability
probability
probability experiment
relative frequency
relative frequency bar graph
relative frequency table
simple event

Explore Experiments and Likelihood

Online Activity You will use Web Sketchpad to explore how running an experiment helps to classify the likelihood of an event.



Learn Relative Frequency

Probability is the chance that an event will occur. A **simple event** is one outcome or a collection of outcomes. For example, an event can occur when tossing a coin, spinning a spinner, or choosing a card at random from a stack of cards. When you perform one of these tasks, you are conducting one trial of a **probability experiment**.

You can use the results of a probability experiment to compare the number of favorable outcomes to the total number of outcomes. This is the **relative frequency** of the event.

Words	**Relative frequency** is the ratio of the number of favorable outcomes to the total number of outcomes in an experiment.
Ratio	$\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

(continued on next page)

Your Notes

Go Online Watch the animation to learn how to find the relative frequency of an event.

The animation shows that Lewis has a bag of fruit stars. He randomly selects a star, records its flavor, and returns it to the bag. He repeats these steps 80 times. Based on Lewis' results, what is the relative frequency of selecting a strawberry star?

Lewis' Fruit Star Data

Flavor	Number
Watermelon	25
Strawberry	36
Mango	19

Step 1 Write a ratio. The relative frequency of an event is the part-to-whole ratio of the number of times an event occurs to the total number of outcomes in the experiment.

$$\text{relative frequency} = \frac{\text{number of strawberry stars drawn}}{\text{total fruit stars drawn}}$$

$$= \frac{36}{80}$$

Step 2 Simplify the ratio by finding an equivalent ratio.

$$\overset{\div 4}{\underset{\div 4}{\frac{36}{80} = \frac{9}{20}}}$$

Divide both 36 and 80 by 4.

Talk About It!
Where do you see probability and relative frequency in everyday life?

Step 3 Write the relative frequency as a fraction, decimal, and percent.

As a fraction, the relative frequency ratio is $\frac{9}{20}$.

To write the ratio as a decimal and a percent, find an equivalent ratio with a denominator of 100.

$$\overset{\times 5}{\underset{\times 5}{\frac{9}{20} = \frac{45}{100}}}$$

Because 20 × 5 = 100, multiply 9 by 5.

$= 0.45$ $\frac{45}{100}$ means *forty-five hundredths*, which is 0.45

$= 45\%$ $\frac{45}{100} = 45\%$ by the definition of percent

As a decimal, the relative frequency is 0.45. As a percent, the relative frequency is 45%.

So, based on the results of Lewis' experiment, the relative frequency of getting a strawberry star is $\frac{9}{20}$, or 0.45, or 45%.

Example 1 Find Relative Frequencies

A number cube with sides labeled 1, 2, 3, 4, 5, and 6 is rolled 20 times. The number 5 is rolled four times.

What is the relative frequency of rolling a 5?

Step 1 Write the ratio. The number 5 was rolled four times.

$$\begin{aligned}\text{relative frequency of rolling a 5} &= \frac{\text{number of times 5 occured}}{\text{total number of rolls}} \\ &= \frac{4}{20}\end{aligned}$$

Step 2 Simplify the ratio by writing an equivalent ratio.

$$\underset{\div 4}{\overset{\div 4}{\frac{4}{20} = \frac{1}{5}}}$$

Divide both 4 and 20 by 4.

Step 3 Write the relative frequency ratio as a fraction, decimal, and percent.

As a fraction, the relative frequency ratio is $\frac{1}{5}$.

To write the ratio as a decimal and a percent, find an equivalent ratio with a denominator of 100.

$$\underset{\times 20}{\overset{\times 20}{\frac{1}{5} = \frac{20}{100}}}$$

Because 5 × 20 = 100, multiply 1 by 20.

$= 0.2$ $\frac{20}{100}$ means *twenty hundredths*, which is 0.20 or 0.2.

$= 20\%$ $\frac{20}{100} = 20\%$ by the definition of percent.

As a decimal, the relative frequency is 0.2. As a percent, the relative frequency is 20%.

So, the relative frequency of rolling the number 5 is $\frac{1}{5}$, 0.2, or 20%.

Check

A spinner with equal-sized sections labeled A, B, C, D, E, and F is spun 24 times. A vowel was spun 9 times. What is the relative frequency of spinning a vowel?

Show your work here

Go Online You can complete an Extra Example online.

Think About It!

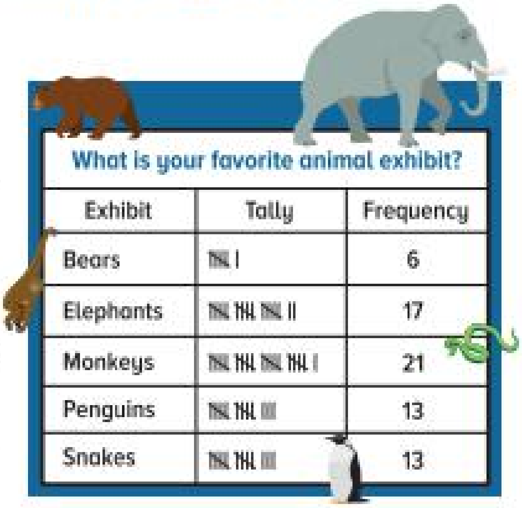
How will you set up the ratio to find the relative frequency?

Talk About It!

Because the relative frequency of rolling a 5 is 20%, describe the likelihood that this event will occur. Explain.

Example 2 Find Relative Frequencies from Tables

A group of students went on a field trip to the zoo. The frequency table shows the results of a survey about their favorite exhibit.



What is your favorite animal exhibit?

Exhibit	Tally	Frequency
Bears	𝍸 I	6
Elephants	𝍸 𝍸 𝍸 II	17
Monkeys	𝍸 𝍸 𝍸 𝍸 I	21
Penguins	𝍸 𝍸 III	13
Snakes	𝍸 𝍸 III	13

What is the relative frequency of the favorite exhibit being either penguins or bears?

Step 1 Find the total number of students surveyed.

6 + 17 + 21 + 13 + 13 = ☐ students

Step 2 Find how many students chose penguins or bears as their favorite exhibit.

6 + 13 = ☐ students

Step 3 Find the relative frequency by writing a ratio.

$$\frac{\text{number of students that chose either animal}}{\text{total number of students surveyed}} = \frac{19}{70}$$

So, the relative frequency is $\frac{19}{70}$, about 0.27, or about ________%.

Talk About It!

Is there another way to find the relative frequency?

Talk About It!

Why might it be advantageous to refer to the relative frequency as opposed to referring to just the frequency of an outcome?

Check

Students at a junior high school were asked the question, "What is your favorite activity at school?" The results are shown in the table.

Sport	Frequency
Football	21
Basketball	13
Track	4
Volleyball	17
Student Council	7
Band	13

According to the results, what is the relative frequency of a student's favorite activity being either track or football? Express the ratio as a fraction, decimal, and percent. Round to the nearest hundredth.

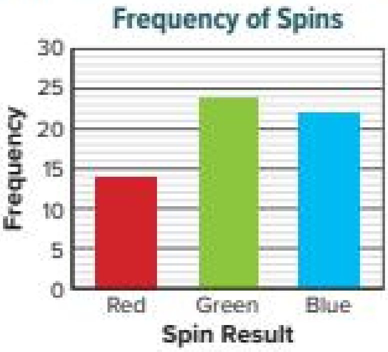
Show your work here

Go Online You can complete an Extra Example online.

Example 3 Find Relative Frequencies from Graphs

The graph shows the results of an experiment in which a spinner with three equal-size sections is spun a number of times.

Find the relative frequency of spinning green or blue for this experiment. Express the ratio as a fraction.



Complete the table of frequency values.

Red	Green	Blue
14		

Find the relative frequency by writing the ratio.

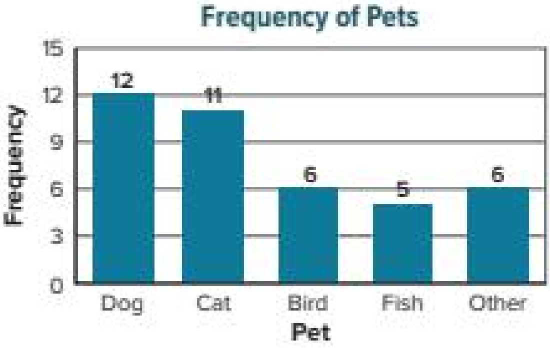
$$\frac{\text{number of green or blue spins}}{\text{total number of spins}} = \frac{24 + 22}{60}$$

$$= \frac{46}{60} \text{ or } \frac{\square}{\square}$$ Add. Then simplify.

So, the relative frequency of spinning green or blue is $\frac{23}{30}$.

Check

The graph shows the results of the question "What is your favorite household pet?" when asked of a group of students.



What was the relative frequency of a student's favorite animal being a dog or a cat? Express the ratio as a percent, rounded to the nearest tenth.

Show your work here

Go Online You can complete an Extra Example online.

Think About It!

What are the favorable outcomes in this problem?

Talk About It!

The spinner has three equal-size sections. Based on the description of the spinner, what might you expect the results of the graph to show after 60 spins? How do the relative frequency results compare to what you might expect?

Learn Relative Frequency Tables and Bar Graphs

Suppose 100 randomly selected people are asked their blood type. The results are shown.

Blood Type	Frequency
A	40
B	10
AB	5
O	45

To find the relative frequency ratio for each blood type, find the ratio of the frequency (number of people) for each blood type to the total number of people surveyed, 100.

Blood Type A: $\frac{40}{100}$

Blood Type AB: $\frac{5}{100}$

Blood Type B: $\frac{10}{100}$

Blood Type O: $\frac{45}{100}$

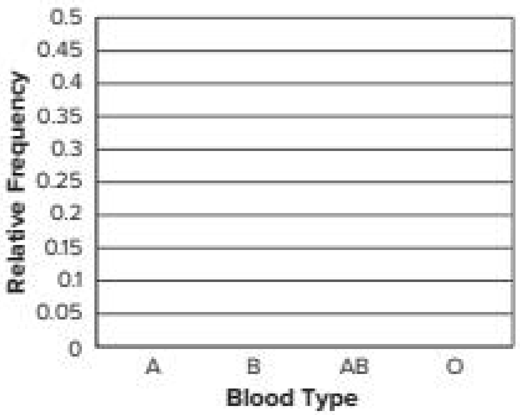
Talk About It!

Compare and contrast the relative frequency table with the frequency bar graph.

You can show the relative frequency of data in a **relative frequency table**. This kind of table lists both the frequency and relative frequency of data.

Blood Type	Frequency	Relative Frequency
A	40	$\frac{40}{100} = 0.4$
B	10	$\frac{10}{100} = 0.1$
AB	5	$\frac{5}{100} = 0.05$
O	45	$\frac{45}{100} = 0.45$

You can also graph the relative frequency values from the table on a **relative frequency bar graph**. This kind of bar graph shows the relative frequency of the data. Graph the relative frequencies.



Use a scale of 0 to 0.5 for the relative frequency values, because the greatest value is 0.45, which is less than 0.5.

The graph provides a visual representation for how the relative frequencies compare. The graph shows that the relative frequencies for Blood Types A and O are about the same, and much greater than the relative frequencies for Blood Types B and AB.

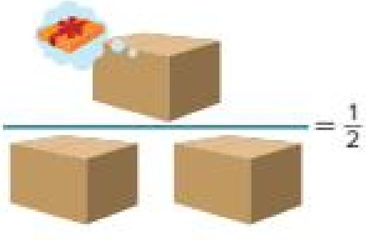
Learn Experimental Probability from Relative Frequency

The relative frequency of an event can be used to predict the chance of that event occurring in the future. The chance that the future event will occur, based on the experiment's results, is called the **experimental probability**. It has the same ratio as the relative frequency.

Words	**Experimental probability** is the ratio of the number of favorable outcomes to the total number of outcomes.
Ratio	$P\text{ (event)} = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

Suppose Emilia is presented with two unmarked boxes of the same size. Emilia opens the first box in which there is a present. She opens the second box in which there is no present. The relative frequency of selecting a box with a present inside is $\frac{1}{2}$, 0.5, or 50%.

The table compares and contrasts relative frequency and experimental probability for this experiment.

Relative Frequency	Experimental Probability
Similarities	
the ratio of the number of favorable outcomes to the total number of outcomes in an experiment $\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{1}{2}$	
Difference	
describing an event that has already occurred 	chance of an event happening in the future based on what has already happened 

Talk About It!

In your own words, compare and contrast experimental probability and relative frequency, including the process used to find each.

Think About It!

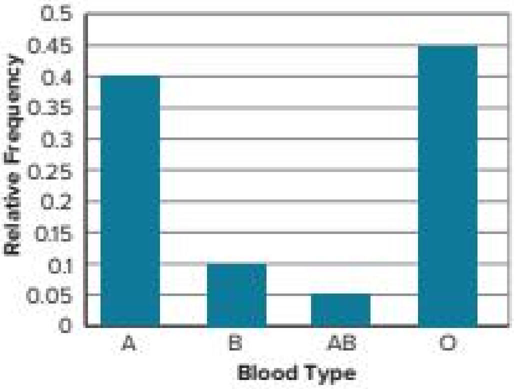
How will you set up the ratio to find the experimental probability?

Talk About It!

The relative frequency and experimental probability of type A or type B blood have the same value, 0.5. What is the difference between the two terms?

Example 4 Find Experimental Probabilities

Refer to the relative frequency bar graph shown that you saw earlier in this lesson. What is the experimental probability that a person chosen at random from the group will have type A or type B blood?



Find the relative frequency of type A or type B. The relative frequency of type A or type B is the sum of the relative frequency of type A and the relative frequency of type B.

(relative frequency of type A) + (relative frequency of type B)

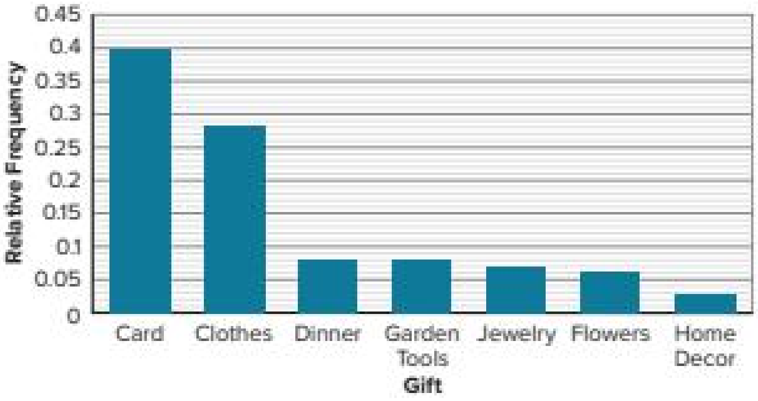
0.40 + ☐

☐

The experimental probability has the same ratio as the relative frequency. So, the experimental probability that a randomly chosen donor will have type A or type B blood is 0.5 or 50%.

Check

A random selection of adults was asked the question "What was the last gift you received?" The results are shown in the relative frequency bar graph.



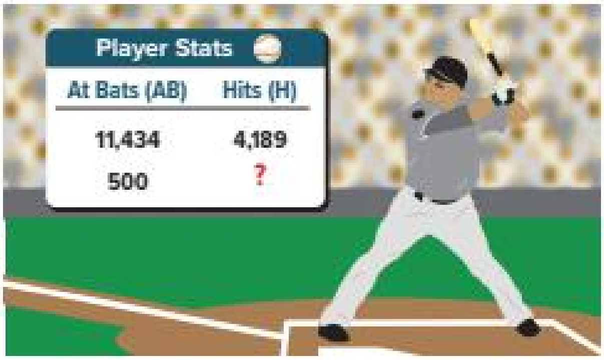
What is the experimental probability that an adult chosen at random will receive a card or clothes?

Show your work here

Go Online You can complete an Extra Example online.

Example 5 Estimate to Make Predictions

In baseball, a player's batting average is found by writing the ratio of a player's hits to their total at-bats, and then writing the ratio as a decimal. The player with the highest career batting average in history had 4,189 hits in 11,434 at-bats.



Based on this relative frequency, how many hits can be expected in a season where the player has 500 total at-bats?

Step 1 Find the relative frequency ratio of getting a hit.

$$\frac{\text{number of hits}}{\text{number of at-bats}} = \frac{4{,}189}{11{,}434}$$

Step 2 Use the relative frequency to make a prediction.

Use the relative frequency to find an equivalent ratio in order to predict the number of hits for 500 at-bats. Let *h* represent the number of hits for 500 at-bats.

$$\underset{\div 23}{\overset{\div 23}{\frac{4{,}189}{11{,}434} \approx \frac{h}{500}}}$$

Round 11,434 to 11,500. 11,500 ÷ 23 = 500.

$$\frac{4{,}189}{11{,}434} \approx \frac{182}{500}$$

$4{,}189 \div 23 \approx 182$

In each step, the values were rounded in order to find an approximate equivalent ratio. By using ratio reasoning, another possible equivalent ratio could be $\frac{183}{500}$.

So, in a season where the player has 500 total at-bats, it can be expected that he will have about 182 or 183 hits.

Think About It!

How can you use the relative frequency to make the prediction?

Talk About It!

Suppose the player had 200 hits in 500 at-bats. Does this mean your prediction was not accurate? Explain.

Check

Over the past five years, it rained on $\frac{1}{5}$ of the days in April. How many days can you expect it to rain in the upcoming April if the weather is expected to be consistent with the past five years?

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare what you learned today with something similar you learned in an earlier module or grade. How are they similar? How are they different?

Record your observations here

Apply Sales

Last year, a DVD store sold 670 action DVDs, 580 comedy DVDs, 450 drama DVDs, and 300 science fiction DVDs. The store owner makes $3.31 profit on each DVD sold and expects to sell 5,000 DVDs this year. Based on last year's results, how much profit can she expect to make on comedy DVDs for this year?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It!

If more than 1,125 drama DVDs are sold this year, how does that compare to the store owner's prediction of the total number of DVDs sold?

Check

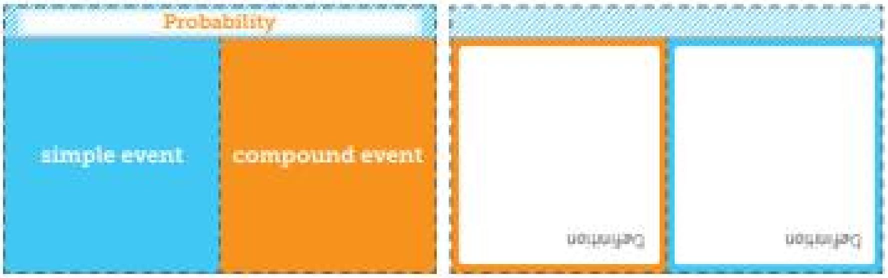
A local pizza shop sold 100 pizzas last week. The number of each type of pizza sold is shown in the table. The store makes $5.15 profit on each sausage pizza sold. Based on the relative frequency, how much profit can they expect to make on sausage pizzas next month if they plan to sell 525 total pizzas in that month?

Type of Pizza	Frequency
Pepperoni	43
Sausage	28
Mushroom	22
Onion	7

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Name ______________________ Period ____________ Date ____________

Practice

Go Online You can complete your homework online.

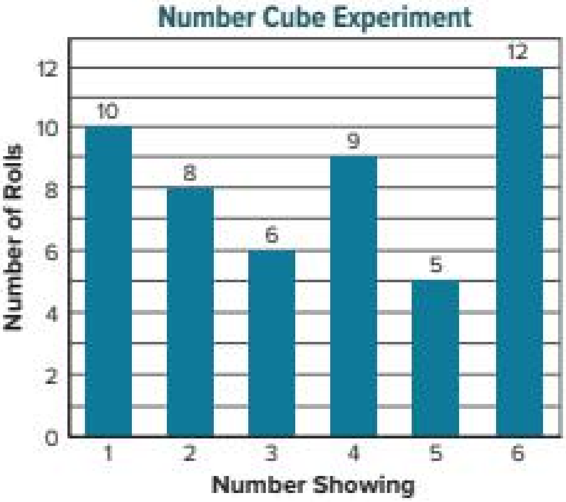
1. A spinner with four equal sections of blue, green, yellow, and red is spun 100 times. It lands on blue 14 times, green 10 times, yellow 8 times, and red 68 times. What is the relative frequency of landing on red? green? (Example 1)

2. The frequency table shows the results of a survey about favorite exhibits. (Example 2)

Exhibit	Frequency
Butterfly	12
Dinosaurs	25
Planets	17
Trains	6

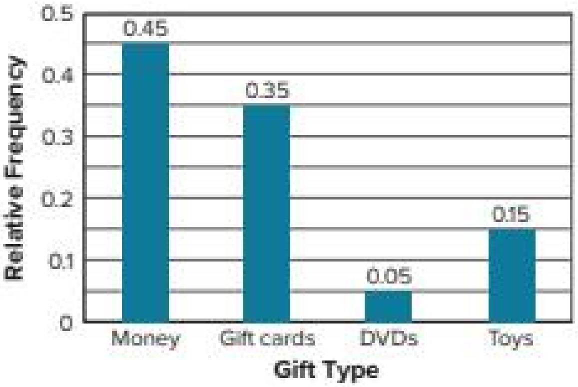
Find the relative frequency that a randomly selected student's favorite exhibit was either butterflies or trains, as a percent.

3. The graph shows the results of an experiment in which a number cube labeled 1 through 6 is rolled a number of times.



Find the relative frequency of rolling a number greater than 3. (Example 3)

4. A random selection of students was asked the question "What type of gift did you last receive?" and the results were recorded in the relative frequency bar graph.



What is the experimental probability that a student chosen at random received a gift card or money? (Example 4)

Test Practice

5. Open Response Based on previous orders, the manager of an ice cream shop determines the probability that a customer will order chocolate sauce is 85%. If there are 240 sundaes ordered in one weekend, how many sundaes are expected to be ordered with chocolate sauce?

Apply

6. The table shows the number of each type of snack bag that was sold this month at lunch. The school makes \$0.75 profit on each bag sold and expects to sell 1,200 bags next month. Based on last month's results, how much profit can the school expect to make on potato chips next month?

Snack Bag	Number Sold
Cheese Curls	250
Popcorn	125
Potato Chips	340
Pretzels	85

7. A laundry detergent company's 32-ounce bottles pass inspection $\frac{98}{100}$ of the time. If the bottle does not pass inspection, the company loses the unit cost for each bottle of laundry detergent that does not pass inspection, which is \$3.45. If 800 bottles of laundry detergent are produced, about how much money can the company expect to lose?

8. **Make Use of Structure** A spinner with three sections marked orange, yellow, and purple is spun 32 times. Purple is spun 24 times, orange is spun 4 times, and yellow is spun 4 times. Draw what the spinner might look like based on the relative frequencies.

9. **Create** Write and solve a problem where you use probability to estimate and make predictions.

10. **Persevere with Problems** A number cube is rolled 24 times and lands on 6 three times. Find the experimental probability of *not* landing on a 6. Express your answer as fraction, decimal, and percent.

11. **Persevere with Problems** The experimental probability of flipping a red-yellow counter and landing on yellow is $\frac{9}{16}$. If the counter landed on red 35 times, find the number of tosses.

Lesson 10-3

Theoretical Probability of Simple Events

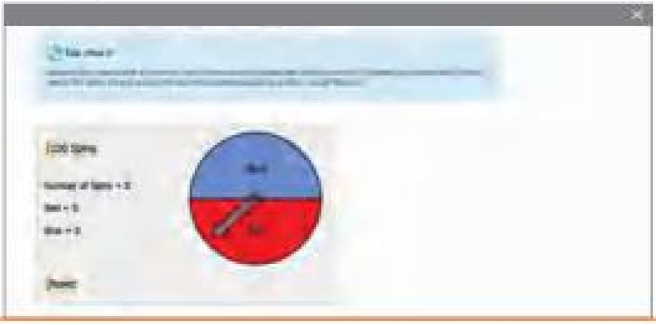
I Can... find the theoretical probability of a simple event and its complement, and understand the relationship between them.

What Vocabulary Will You Learn?

complementary event
sample space
theoretical probability
uniform probability model

Explore Long-Run Relative Frequencies

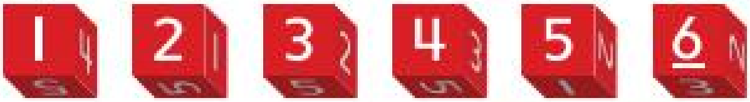
Online Activity You will use Web Sketchpad to explore the relationship between long-run relative frequency and theoretical probability.



Learn Sample Space of Simple Events

In a probability experiment, the set of all possible outcomes is called the **sample space**. To find the sample space of a simple event, you can make a list of each unique outcome.

When rolling a number cube once, the sample space is the outcome of each face: 1, 2, 3, 4, 5, and 6.



Suppose you rolled a number cube ten times and recorded the results as shown.

Number Rolled	Frequency
1	0
2	3
3	2
4	1
5	4
6	0

The relative frequency ratios of rolling a 1 or rolling a 6 are both $\frac{0}{6}$, or 0, because neither of those numbers were rolled.

The sample space is 1, 2, 3, 4, 5, and 6, even though rolling a 1 or a 6 did not happen. All possible outcomes are included in the sample space. The sample space is not dependent upon actual results. Each new time you roll the number cube, there are 6 possible outcomes.

Your Notes

Think About It!
What do you notice about the letters in the word MATHEMATICS?

Talk About It!
Why are there only 8 letters in the sample space out of the 11 letters in the word MATHEMATICS?

Example 1 Find the Sample Space of Simple Events

Each letter in the word MATHEMATICS is written on a piece of paper and placed into a bag. A letter is drawn at random.

What is the sample space?

The sample space is the set of all unique possible outcomes. In this example, because some letters repeat, the outcomes in the sample space are each *unique* letter that appears in the word. Record the letters that are in the sample space in the diagram below.

Sample Space

The sample space consists of the unique letters of the word MATHEMATICS. Each unique letter is only listed once.

Check

In a seventh grade math class, there are 5 students with blue eyes, 4 students with hazel eyes, and 2 students with green eyes. One student is selected at random. What is the sample space for eye color?

Ⓐ blue, hazel, brown

Ⓑ brown, blue, green

Ⓒ blue, hazel, green

Ⓓ brown, hazel, green

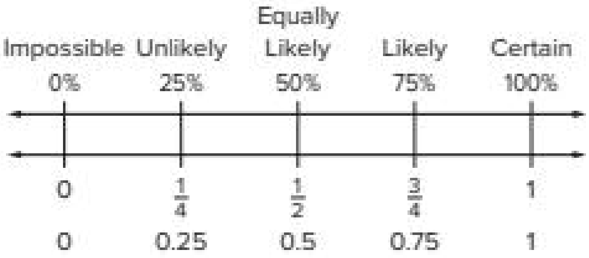
Go Online You can complete an Extra Example online.

Learn Theoretical Probability of Simple Events

Experiments, such as rolling a number cube or tossing a coin, in which all of the outcomes are equally likely are known as **uniform probability models**. **Theoretical probability** is based on uniform probability, or what should happen in a probability experiment.

Words	**Theoretical probability** is the ratio of the number of favorable outcomes to the total number of outcomes.
Ratio	$P\text{ (event)} = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

The probability number line shows sample probabilities that correspond with each likelihood.



Suppose you tossed a coin twenty times and recorded the results as shown.

Outcome	Frequency
Heads	17
Tails	3

The relative frequency ratio of tossing heads is $\frac{17}{20}$. The relative frequency ratio of tossing tails is $\frac{3}{20}$.

Because each outcome when tossing a coin is equally likely to happen as not to happen, the theoretical probability of tossing heads is $\frac{1}{2}$. The theoretical probability of tossing tails is also $\frac{1}{2}$.

Why are the relative frequency ratios so different from the theoretical probabilities? Theoretical probability is what you *should* expect when conducting an experiment, while the relative frequency ratio describes what *actually* happened.

Talk About It!

The probability number line shows that a probability of $\frac{1}{4}$ is unlikely. What are some other probability ratios that are unlikely?

Talk About It!

A classmate said that because heads turned up almost every time, out of the 20 tosses, that it has to land on tails on the next toss. Is this reasoning correct? Why or why not?

Example 2 Find Theoretical Probabilities of Simple Events

Eight discs are marked 3, 4, 5, 6, 7, 8, 9, and 10, such that each disc is marked with exactly one of these numbers. A disc is selected from the bag at random.

What is the theoretical probability of selecting a disc marked with a prime number on it?

$P(\text{prime}) = \frac{3}{8}$ There are three prime numbers: 3, 5, 7. There are 8 numbers total.

$= \frac{3}{8}$ or 0.375 or $\boxed{}$ % Simplify.

So, the theoretical probability that a disc with a prime number on it is selected is $\frac{3}{8}$, 0.375, or 37.5%.

Talk About It!

Suppose you designed an experiment where each trial consists of selecting a disc, recording its number, and replacing it. If you conducted 10 trials, how do you think your results might compare to the theoretical probability?

Check

A number cube, with sides labeled 1-6, is rolled. Which is the theoretical probability of rolling a number less than 6, in simplest form?

Ⓐ $\frac{1}{6}$ Ⓑ $\frac{1}{2}$

Ⓒ $\frac{1}{3}$ Ⓓ $\frac{5}{6}$

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

How is finding the sample space helpful when finding theoretical probability?

Record your observations here

Learn Complements of Simple Events

Suppose you are rolling a number cube and the desired outcome is an even number. A *success* is defined as rolling an even number. Rolling an odd number is *not a success*. These two events are known as **complementary events** because they cannot happen at the same time.

Words	**Complementary events** are two events in which either one or the other must happen, but they cannot happen at the same time. The sum of the probability of an event and its complement is 1, or 100%.
Symbols	If the probability of an event is $P(A)$, then the complement of an event is written as $P(\textit{not } A)$ or $P(A')$.
Equation	$P(A) + P(\textit{not } A) = 1$ $P(A) + P(A') = 1$

Consider the following scenarios.

Scenario 1: You roll a number cube. The desired outcome is a 3.

The complement of this event is *not rolling a 3*. In other words, the complement is rolling a 1, 2, 4, 5, or 6. Notice the relationship between the probabilities of an event and its complement.

$P(3) = \frac{1}{6}$ There is one number 3 and a total of six numbers on the number cube.

$P(\textit{not } 3) = \frac{5}{6}$ There are five numbers that are not 3 on the number cube: 1, 2, 4, 5, and 6.

The sum of the probabilities is equal to 1.

$$P(3) + P(\textit{not } 3) = \frac{1}{6} + \frac{5}{6}$$
$$= \frac{6}{6}, \text{ or } 1$$
Add. Then simplify.

Scenario 2: You spin a spinner that is divided into eight equal-size sections, numbered 1 through 8. The desired outcome is an even number.

The complement of spinning an even number is *spinning an odd number*, or *spinning a 1, 3, 5, or 7.*

$$P(\text{even}) + P(\textit{not } \text{even}) = \frac{4}{8} + \frac{4}{8}$$
$$= \frac{8}{8}, \text{ or } 1$$
Add. Then simplify.

Talk About It!

If you know the probability of an event, how can you find the probability of its complement?

Example 3 Find Complements of Simple Events

Think About It!

What do you know about an event and its complement?

Rafael is going to ride a roller coaster chosen at random and wants to find the probability of choosing a roller coaster with a height less than 250 feet.

What is the probability of the complement of the event?

Roller Coaster	Height (ft)
Thunder Dragon	345
Screamin' Spyder	410
Zipster	185
Maniac	230
Flying Eagle	255
Twister Wave	277
Triple Tornado	455
Ultra Loop	196

Step 1 Identify the complement of the event. List all of the outcomes that make up the event's complement. These are the roller coasters with a height *not* less than 250 feet. In other words, find the roller coasters with a height greater than or equal to 250 feet.

________________, ________________, ________________,

________________ and ________________.

There are 5 outcomes that make up the complement.

Step 2 Find the probability of the complement.

P(*not* less than 250 ft)

$$= \frac{\text{number of outcomes in the complement}}{\text{number of total outcomes}}$$ Write the probability ratio.

$$= \frac{5}{8}$$ There are 5 outcomes in the complement.

So, the probability of the complement is $\frac{5}{8}$. This is the same as saying the probability of choosing a roller coaster with a height greater than or equal to 250 feet is $\frac{5}{8}$.

Talk About It!

If you know the probability of an event is 37.5%, how can you find the probability of its complement?

Check

A bag contains 25 marbles, 10 of which are red. The other marbles are blue or green. A marble is selected at random. What is the probability of drawing a marble that is *not* red?

Show your work here

Go Online You can complete an Extra Example online.

Apply Probability

A spinner with eight equal-sized sections labeled 1 through 8 is spun 600 times. How many spins of a number less than 4 can be expected?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It!

The expected number is 225 times. Does this mean that you will always spin a number less than four 225 times if the spinner is spun 600 times? Explain.

Check

A number cube labeled 1–6 is rolled 1,200 times. How many times can it be expected to roll a multiple of three?

Show your work here

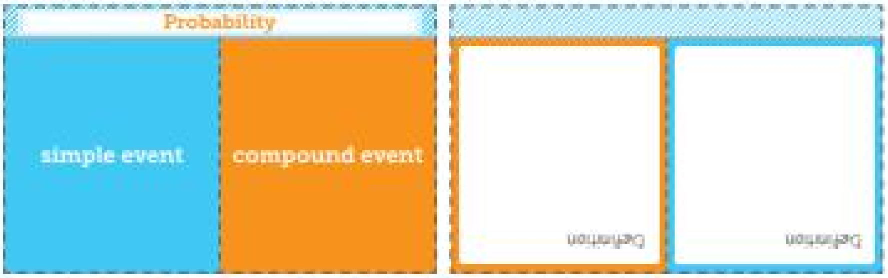
Go Online You can complete an Extra Example online.

Pause and Reflect

How can you use your knowledge of fractions, ratios, and proportions to help you in this lesson?

Record your observations here

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Name ________________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

1. The spinner shown is spun once. What is the sample space? (Example 1)

2. Each letter in the word MISSISSIPPI is written on a piece of paper and placed into a bag. A letter is drawn at random. What is the sample space? (Example 1)

3. A teacher placed the letter cards E, L, O, R, U, and W in a bag. A card is drawn at random. Determine the theoretical probability for drawing a card that has a vowel on it. (Example 2)

4. A player in a board game rolls a six-sided number cube labeled 1 through 6 once. Determine the theoretical probability of rolling a 1 or 2. (Example 2)

5. The table shows the lengths of time for rides at a fair. Zane will choose a ride at random and wants to find the probability of choosing a ride that lasts less than 200 seconds. What is the probability of the complement of the event? Describe the complement. (Example 3)

Ride	Time (seconds)
Barrel	150
Bumper Cars	195
Circus Carousel	210
Log Ride	120
Roller Coaster	55
Swings	225
Train	300
Zero Gravity Spinner	65

6. Red is spun on a spinner with five equal-size sections labeled red, yellow, blue, green, and purple. What is the probability of the complement of the event? Describe the complement. (Example 3)

Test Practice

7. Multiselect A sportscaster predicted that the local high school baseball team has a 75% chance of winning tonight. Select all of the values that represent the probability of the team *not* winning.

☐ 0.75

☐ 0.25

☐ 75%

☐ 25%

☐ $\frac{3}{4}$

☐ $\frac{1}{4}$

Apply

8. A pet store is having a prize give-away. The spinner shows the type of toy a customer can win for their pet. If a customer spins the spinner and it lands on *cat*, they will win a free cat toy. If the spinner is spun 540 times throughout the day, about how many dog or cat toys are expected to be given away?

9. The letters from the word FOOTBALL are written on 8 cards with one letter on each card. One card will be drawn randomly and then placed back into the stack. If this experiment is repeated 840 times, about how many times should you expect to draw a consonant?

10. Describe a real-world situation that involves a sample space. Then describe the sample space.

11. MP **Find the Error** The spinner shown has 8 equal-size sections. A student said that the theoretical probability of spinning a multiple of 3 on the spinner is $\frac{5}{8}$. Find the student's error and correct it.

12. MP **Reason Inductively** The weather reporter says that there is an 88% chance that it will *not* be windy tomorrow. Will tomorrow be a good day to fly a kite? Explain.

13. Create Write a real-world problem that involves finding the complement of the event. Then find the complement.

Lesson 10-4

Compare Probabilities of Simple Events

I Can... understand what happens to the long-run relative frequency as the number of trials increases, and compare relative frequencies to theoretical probabilities.

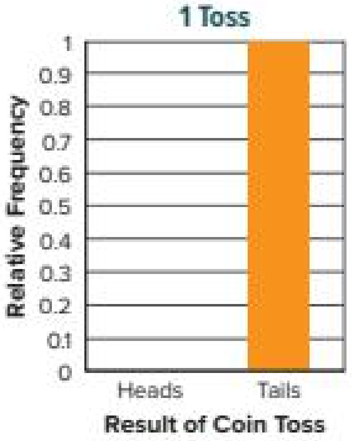
Learn Compare Relative Frequency to Theoretical Probability

The relative frequency of an event is what actually happens in a probability experiment, while the theoretical probability is what should happen based on the experiment's design. The theoretical probability and relative frequency of an event may or may not have the same value.

For example, the theoretical probability of a coin landing on heads is $\frac{1}{2}$.

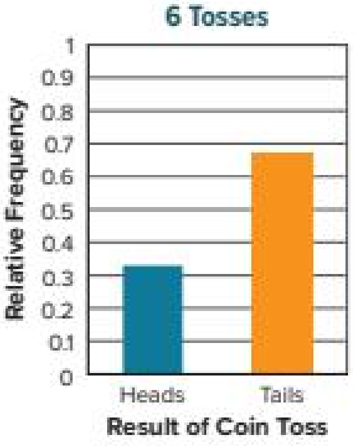
Suppose you toss a coin once and it lands on tails. The relative frequency ratio of tossing heads is $\frac{0}{1}$, 0, or 0%. The number of times the coin landed on heads is 0, and there was 1 toss.

In this case, the relative frequency, $\frac{0}{1}$, is not equal to the theoretical probability, $\frac{1}{2}$.



What happens if you increase the number of tosses? Suppose you toss the coin six times and it lands on heads twice and tails four times. The relative frequency ratio of tossing heads is $\frac{2}{6}$ or $\frac{1}{3}$.

When compared to the theoretical probability, the relative frequency is still not the same, but it is closer to $\frac{1}{2}$ than it was for the one toss.



Talk About It!

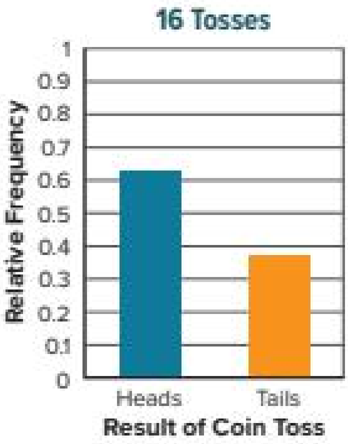
If you toss a coin 6 times, will it always land on heads twice? Explain. How many times do you expect it to land on heads?

(continued on next page)

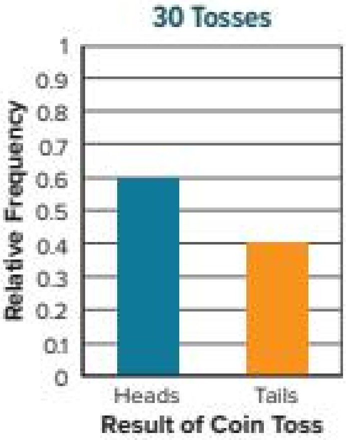
Your Notes

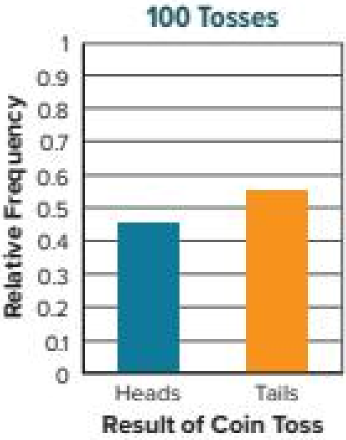
What happens if you continue to increase the number of tosses? Suppose you toss the coin sixteen times and it lands on heads ten times and tails six times. The relative frequency ratio of tossing heads is $\frac{10}{16}$ or $\frac{5}{8}$.

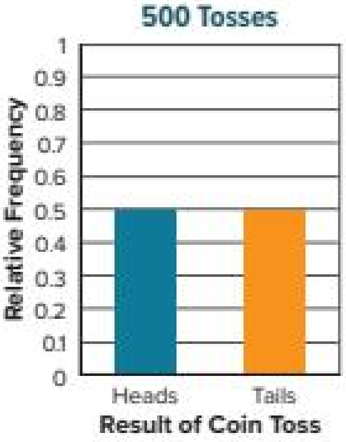
When compared to the theoretical probability, the relative frequency is still not the same as the theoretical probability, but it is closer to $\frac{1}{2}$ than it was for the one toss or for the six tosses.



Suppose the number of tosses continues to increase as shown in the relative frequency bar graphs below.







As the number of tosses increase, the long-run relative frequency ratio of tossing heads becomes closer to the theoretical probability of $\frac{1}{2}$. The long-run relative frequency of an event will approach the value of the theoretical probability as the number of trials increases. This is known as the *Law of Large Numbers*.

Pause and Reflect

Have each student in your class toss a coin 20 times and record whether the coin lands on heads or tails each time. Compile the results. Do the results demonstrate the Law of Large Numbers? Why or why not?

Record your observations here

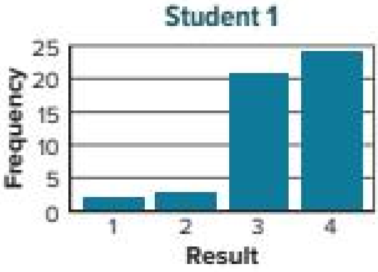
Example 1 Compare Relative Frequencies to Probabilities

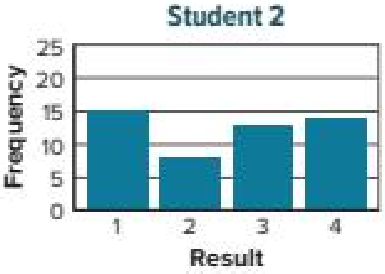
A tetrahedron is a three-dimensional figure with four equally-likely outcomes, 1 through 4, identified by the number showing at the top vertex. Maribel tosses the tetrahedron shown 50 times, while Dalton spins the spinner shown 50 times. Each student records their results in a frequency bar graph.

Tetrahedron

Spinner

Neither Maribel nor Dalton wrote their name on their graph. Which graph best represents the results that can be expected from Maribel's experiment? Dalton's experiment?





In Maribel's experiment, the tetrahedron has outcomes that are equally likely. The observed results are likely to be more evenly distributed across each possible outcome. The graph for Student 2 best represents Maribel's experiment.

In Dalton's experiment, the spinner's sections labeled 3 and 4 are each greater in size than the sections labeled 1 or 2. The observed results are likely to have a greater frequency for these two outcomes. The graph for Student 1 best represents Dalton's experiment.

Think About It!

Are the outcomes on the spinner equally likely? Explain.

Talk About It!

The number of trials in this example was 50. Suppose the number of trials was 10, but the structure of the graphs were similar. How might your confidence be affected in choosing the graph that best represents each experiment?

Check

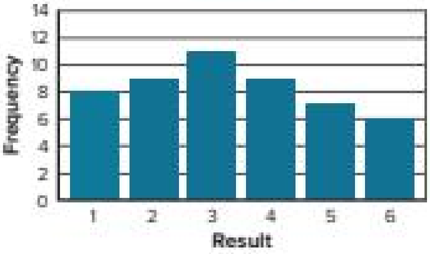
Jaden and Nikhil each spin their respective spinners shown 50 times. They each record their results in a frequency bar graph.

Jaden

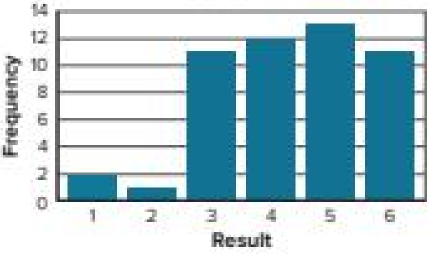
Nikhil

Neither Jaden nor Nikhil wrote their name on their graph. Which graph best represents the results that can be expected from Jaden's experiment? Nikhil's experiment? Explain your reasoning.

Student 1



Student 2

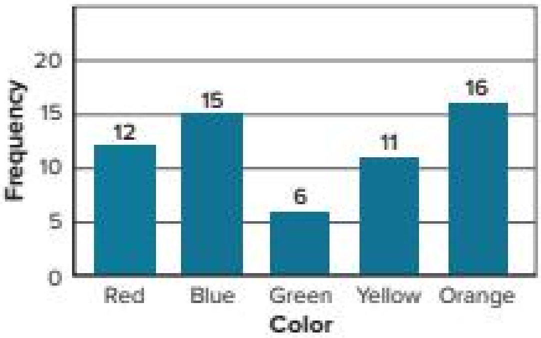


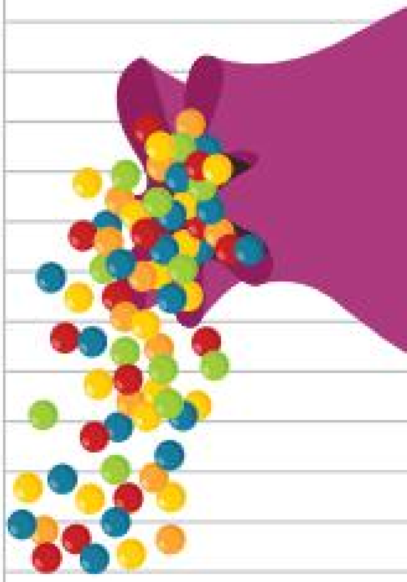
Show your work here

Go Online You can complete an Extra Example online.

Apply Experiments

Blaze randomly selects one marble from a bag that contains red, blue, green, yellow, and orange marbles. He replaces the marble and selects again. Blaze repeats this experiment 60 times. He then spins a spinner with five equal-size sections labeled red, blue, green, yellow, and orange 60 times. Which experiment can be best represented by the graph shown?





1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

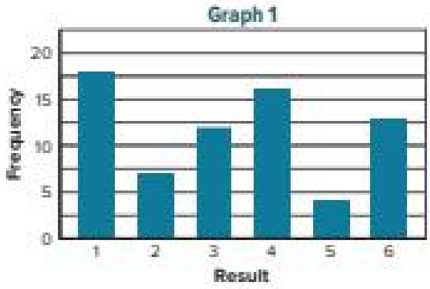
Write About It! Write an argument that can be used to defend your solution.

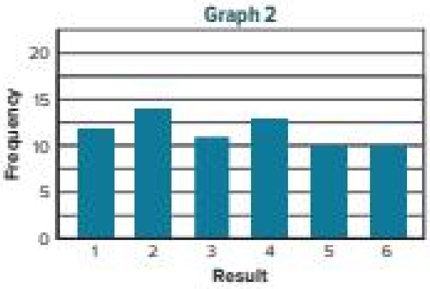
Talk About It!
Why does the graph shown not represent both experiments?

Check

Two experiments are conducted and their results are recorded in frequency bar graphs.

Experiment 1	Experiment 2
A number cube with numbers 1 through 6 is rolled 70 times.	A card is randomly selected from a bag containing the following: six cards labeled 1 two cards labeled 2 three cards labeled 3 five cards labeled 4 one card labeled 5 four cards labeled 6 There are 70 trials.





Which graph best represents the results that can be expected from Experiment 1? Experiment 2?

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

How well do you understand the concepts from today's lesson? What questions do you still have? How can you get those questions answered?

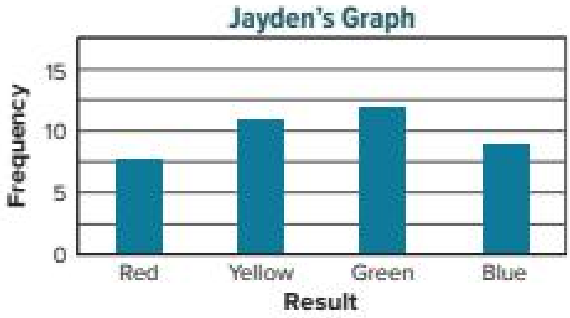
Record your observations here

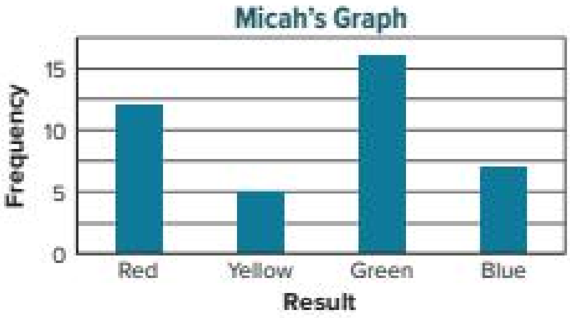
Name ____________________ Period __________ Date __________

Practice

Go Online You can complete your homework online.

1. Jayden spins a spinner with four equal-size sections labeled red, yellow, green, and blue, 40 times. Micah randomly selects one marble from a bag that contains an equal number each of red, yellow, green, and blue marbles. He replaces the marble and selects again. Micah repeats this experiment 40 times. Each student records their results in a frequency bar graph. Which student's graph best represents the results that can be expected from each experiment? (Example 1)

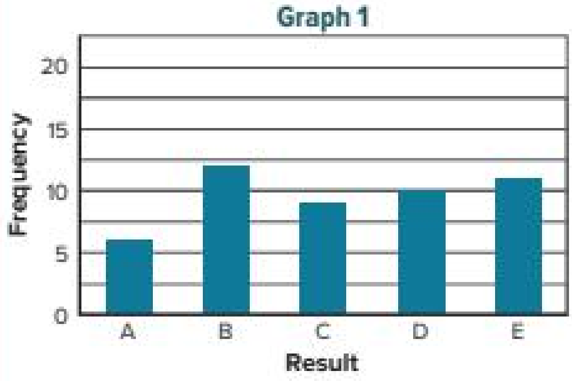


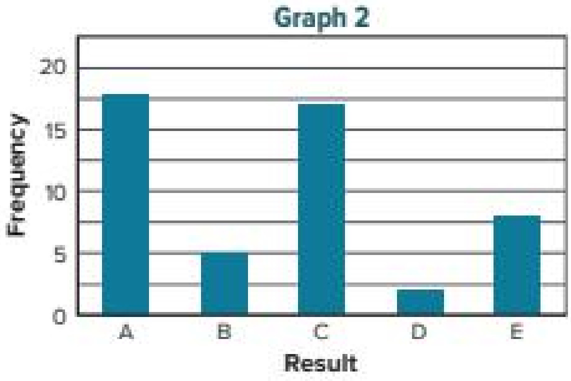


Test Practice

2. Open Response Two experiments are conducted and their results are recorded in frequency bar graphs.

Experiment 1	Experiment 2
A spinner with equal-size sections of A, B, C, D, and E is spun 50 times.	A card is randomly selected from a bag containing five A cards, three B cards, four C cards, one D card, and two E cards. The card is then placed back in the bag. There are 50 trials.

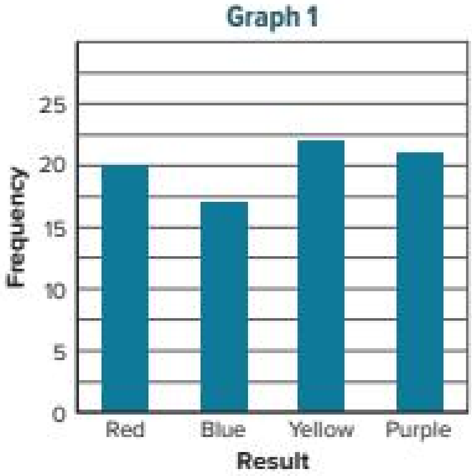


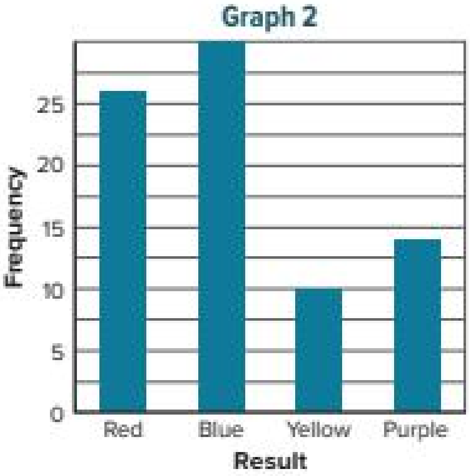


Which graph best represents the results that can be expected from Experiment 1? Experiment 2?

Apply

3. Suppose the spinner shown is spun 80 times. Another spinner with four equal-size sections labeled red, blue, yellow, and purple is spun 80 times. The results are recorded in the following frequency bar graphs. Which graph best represents the results that can be expected from the first spinner? the second spinner?


Graph 1


Graph 2

4. **Be Precise** Compare and contrast relative frequency and theoretical probability.

5. **Reason Inductively** A coin is tossed 30 times. It lands on heads 10 times. Find the experimental probability and theoretical probability of tossing heads. Are the probabilities close? If not, give a possible reason for the discrepancy.

6. Use the Internet, or another source, to research the Law of Large Numbers. Describe this law in your own words.

7. Refer to Exercise 1. Describe what *should* be expected for Jayden's experiment, based on the theoretical probability.

Probability of Compound Events

I Can... use organized lists, tables, or tree diagrams to find the sample space and probability of a compound event.

What Vocabulary Will You Learn?

compound event

theoretical probability of a compound event

tree diagram

Explore Sample Space of Repeated Simple Events

Online Activity You will use Web Sketchpad to explore how to find the sample space of repeated simple events.

Learn Sample Space for Compound Events

A **compound event** consists of two or more simple events.

As with simple events, the sample space for a compound event is the set of all possible outcomes.

Rolling a number cube labeled 1 through 6 followed by tossing a coin is an example of a compound event. The compound event consists of the two simple events of rolling a number cube and tossing a coin.

To find the sample space of a compound event, first find the sample space of each simple event.

- Rolling a number cube has six possible outcomes: 1, 2, 3, 4, 5, or 6.
- Tossing a coin has two possible outcomes: heads or tails.

(continued on next page)

Your Notes

You can use an organized list, such as a table or a tree diagram, to find the sample space of this compound event. The table shows the possible outcomes of rolling a number cube first, followed by tossing a coin.

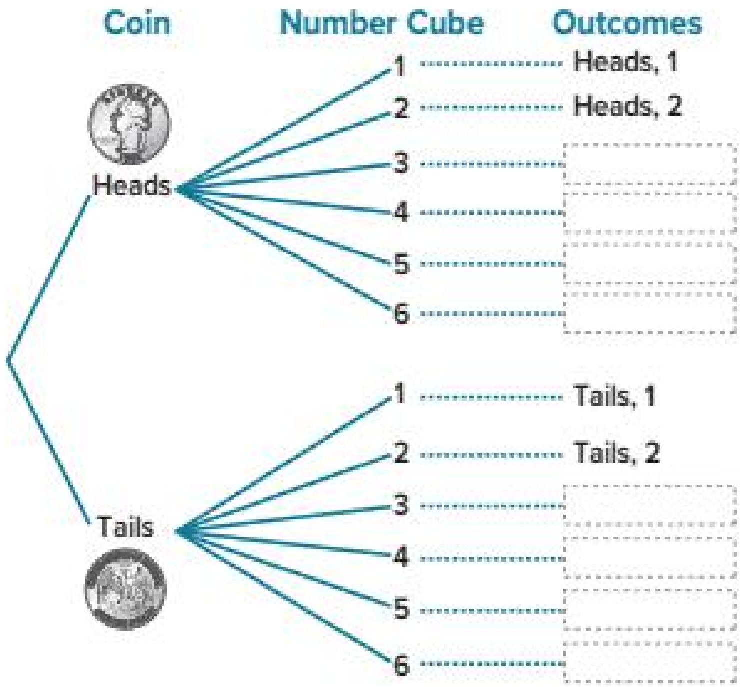
Outcomes		
Number Cube	Coin	Result
1	Heads	1, Heads
	Tails	1, Tails
2	Heads	2, Heads
	Tails	2, Tails
3	Heads	3, Heads
	Tails	3, Tails
4	Heads	4, Heads
	Tails	4, Tails
5	Heads	5, Heads
	Tails	5, Tails
6	Heads	6, Heads
	Tails	6, Tails

Talk About It!

Does the order of events in a compound event affect the sample space? Explain.

The table shows that there are 12 outcomes in the sample space for the compound event.

Complete the **tree diagram** that can be used to organize the possible outcomes of tossing a coin first, followed by rolling a number cube.



So, regardless of the order of events, the sample space in the compound event consists of 12 outcomes.

Example 1 Find the Sample Space of Compound Events

Suppose a penny and a nickel are tossed.

What is the sample space of possible outcomes?

Each coin can land on heads or tails for any toss.

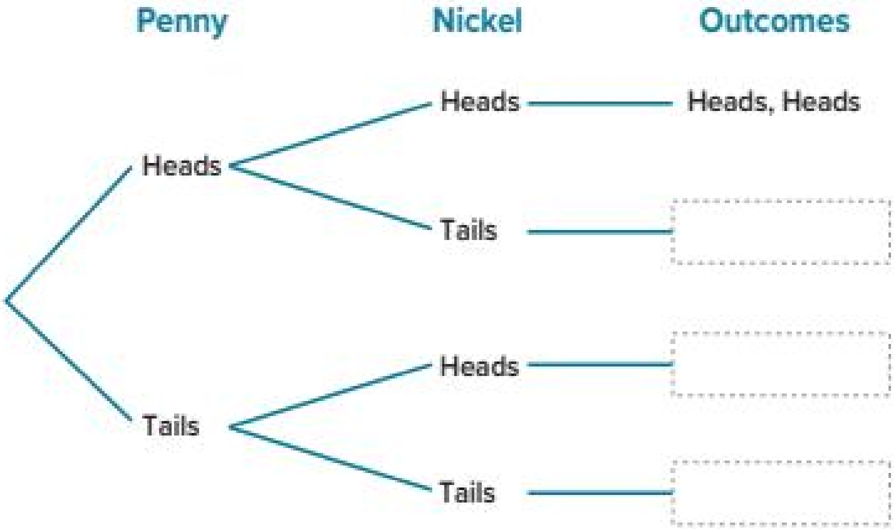
Method 1 Use an organized list, such as a table.

Complete the table to show the possible outcomes in the sample space.

Outcomes		
Penny	Nickel	Result
Heads	Heads	Heads, Heads
Tails	Heads	Tails, Heads

Method 2 Use a tree diagram.

Complete the outcomes for the tree diagram shown.



Using either method, there are 4 possible outcomes in the sample space.

Check

Your math teacher owns three pairs of pants and three pairs of shoes. He owns a pair of gray pants, green pants, and blue pants. He owns black shoes, brown shoes, and tan shoes. If he randomly chooses one pair of pants and one pair of shoes, what is the sample space of possible combinations of pants and shoes that he could wear on a typical school day?

Go Online You can complete an Extra Example online.

Think About It!

How many outcomes are there for each event?

Talk About It!

What do you notice about the relationship between the number of outcomes in the sample space and the number of outcomes in each event?

Think About It!

How can you start a tree diagram to model the situation?

Example 2 Find the Sample Space of Compound Events

A pizza shop sells pizzas with pan or thin crust, red or white sauce, and toppings of pepperoni, mushroom, or plain cheese.

How many possible outcomes are in the sample space for a randomly chosen type of pizza?

Method 1 Use an organized list, such as a table.

Complete the table to show the possible outcomes in the sample space.

Outcomes			
Crust	Sauce	Topping	Result
Pan	Red (R)	Pepperoni (P)	Pan, R, P
		Mushroom (M)	
		Cheese (C)	
	White (W)		Pan, W, P
Thin	Red (R)		

Talk About It!

Think about the number of outcomes you found using the tree diagram. What is the relationship between that value and the number of options in each category?

Method 2 Use a tree diagram.

Construct a tree diagram.

Show your work here

Using either method, there are 12 different pizzas that can be made.

Check

At a picnic, white, wheat, rye, or sourdough bread is available to make a sandwich. Guests can select turkey, ham, roast beef, or chicken. Guests can also select cheddar or Swiss cheese. If a guest randomly selects one type of bread, one type of meat, and one type of cheese, how many possible outcomes are in the sample space?

Show your work here

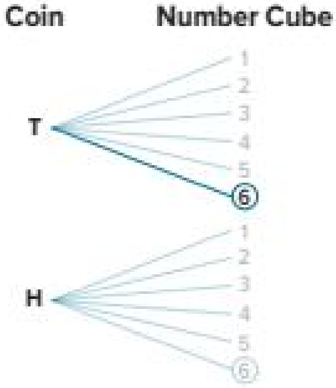
Go Online You can complete an Extra Example online.

Learn Theoretical Probability of Compound Events

When conducting a probability experiment, the **theoretical probability of a compound event** is the ratio of the number of possible favorable outcomes to the number of total possible outcomes in the sample space.

Words	The **theoretical probability of a compound event** is the ratio of the number of favorable outcomes to the total number of outcomes in the sample space.
Ratio	$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

A coin is tossed. Then a number cube labeled 1–6 is rolled. The tree diagram shows the sample space and identifies the outcome of tossing tails followed by rolling a 6. The tree diagram can be used to find the theoretical probability of this compound event.



There are 12 possible outcomes.

There is 1 favorable outcome.

$$\frac{\text{outcomes with tails and a 6}}{\text{total possible outcomes}} = \frac{1}{12}$$

So, the theoretical probability of tossing tails followed by rolling a 6 is $\frac{1}{12}$.

Math History Minute

Grace Murray Hopper (1906–1992) graduated with a Ph.D. in mathematics from Yale University and worked for the Naval Reserve as a computer programmer. In the late 1950s, she developed a computer language written in English rather than symbols. Later, she developed a program that translated the English words into code, which led to the creation of the programming language COBOL (Common Business Oriented Language).

Think About It!

How would you begin solving the problem?

Talk About It!

How can you classify the likelihood of rolling a sum of 9?

Example 3 Find Probabilities of Compound Events

Two number cubes, each labeled 1 through 6, are rolled.

What is the probability of rolling a sum of 9?

Step 1 Find the sample space and the favorable outcomes.

Shade or circle the cells that contain two rolls with a sum of 9.

Roll 2

		1	2	3	4	5	6
	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
Roll 1	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

There are 36 possible outcomes. The table shows 4 possible outcomes that result in a sum of 9 when the two number cubes are rolled.

Step 2 Find the probability.

$P(\text{sum of 9}) = \frac{\text{number of outcomes with sum of 9}}{\text{number of total outcomes}}$ Write the ratio.

$= \frac{4}{36}$ Substitute.

$= \frac{1}{9} \approx 0.111$ or about 11.1% Simplify.

So, the probability of rolling a sum of 9 is $\frac{1}{9}$, or about 11.1%.

Check

A coin is tossed and then a number cube labeled 1 through 6 is rolled. What is the probability of tossing tails and landing on an odd number?

Show your work here

Go Online You can complete an Extra Example online.

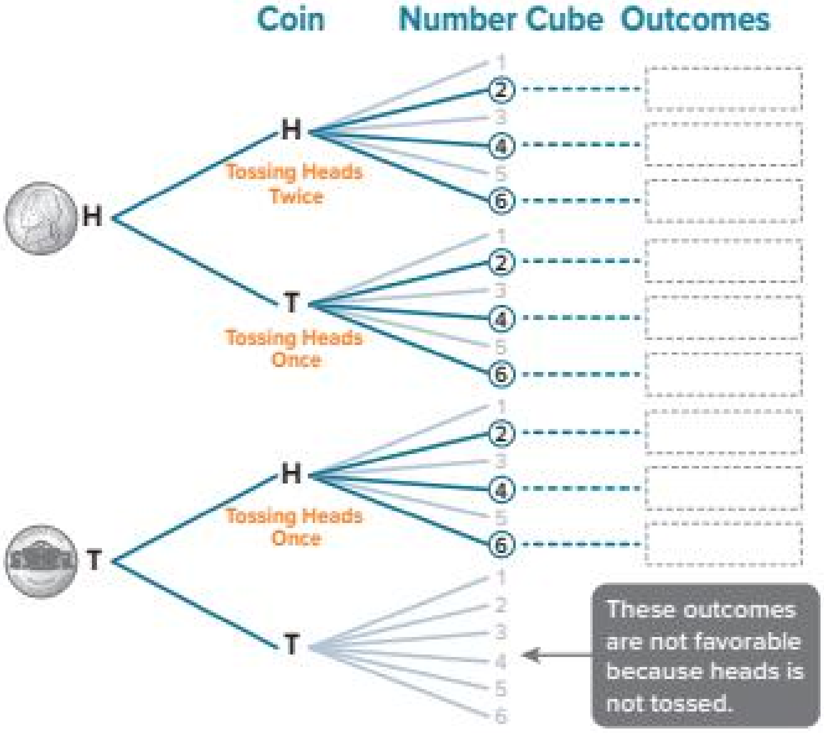
Example 4 Find Probabilities of Compound Events

Two coins are tossed and a number cube labeled 1 through 6 is rolled.

What is the probability of tossing heads at least once and rolling an even number?

Step 1 Find the sample space and the favorable outcomes.

Construct a tree diagram to identify the favorable outcomes.



These outcomes are not favorable because heads is not tossed.

There are 24 total possible outcomes. The diagram shows 9 possible outcomes that result in tossing heads at least once and rolling an even number.

Step 2 Find the probability.

There are 9 out of 24 possible outcomes that are favorable.

$$P(\text{heads} \geq 1 \text{ and even}) = \frac{9}{24} = \frac{\boxed{}}{\boxed{}}$$

Simplify the ratio.

So, the theoretical probability of tossing at least one heads and rolling an even number is $\frac{3}{8}$, 0.375, or 37.5%.

Think About It!

What tool(s) can you use to find the sample space and favorable outcomes?

Talk About It!

How can you classify the likelihood of tossing heads at least once and rolling an even number?

Check

A spinner with three equal-size sections labeled red, green, and yellow is spun once. Then a coin is tossed and one of two cards labeled with a 1 or a 2 is selected. What is the probability of spinning yellow, tossing heads, and selecting the number 2?

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

When finding probability, the language used is important. Describe the difference between *rolling a 3 or 6* in a simple event, and *rolling a 3 and 6* in a compound event.

Record your observations here

Apply Outcomes

Two number cubes were rolled together 60 times. The relative frequency for rolling a sum of 10 was $\frac{1}{6}$. What is the difference between the number of expected outcomes for 60 trials and the number of actual outcomes?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It!

How can you solve the problem another way?

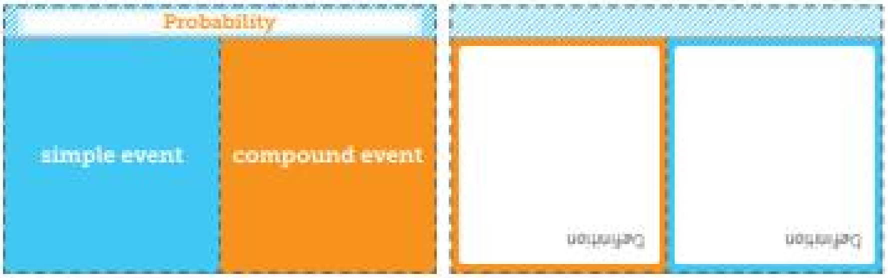
Check

John tosses a quarter and then spins a spinner with eight equal-size sections labeled 1 through 8. He performs this experiment 80 times and finds the relative frequency of getting heads and a number greater than six is $\frac{3}{20}$. What is the difference between the number of expected outcomes and the number of actual outcomes?

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Name ______________________ Period __________ Date __________

Practice

Go Online You can complete your homework online.

1. An Italian ice shop sells Italian ice in four flavors: lime, cherry, blueberry, and watermelon. The ice can be served plain, mixed with ice cream, or as a drink. Using an organized list or table, what is the sample space of possible outcomes? (Example 1)

2. A deli offers a lunch consisting of a soup, salad, and sandwich from the menu shown in the table. A customer randomly chooses lunch consisting of a soup, salad, and sandwich. Construct and use a tree diagram to determine the sample space of the event. How many possible outcomes are in the sample space? (Example 2)

Show your work here

Soup	Salad	Sandwich
Tortellini	Caesar	Roast Beef
Lentil	Macaroni	Ham
		Turkey

3. The spinner shown has six equal-size sections and is spun twice. What is the probability that the product of the numbers spun is 12? (Example 3)

4. A number from 0 to 9 is randomly selected and then a letter from A to D is randomly selected. What is the probability that the number 3 and a consonant are selected? (Example 4)

Test Practice

5. Open Response Lorelei tosses a coin 4 times. What is the probability of tossing four heads? Express as a percent. Round to the nearest tenth, if necessary.

Apply

6. A number cube labeled 1 through 6 is rolled and the spinner shown is spun once. The spinner has four equal-size sections. This experiment is repeated 60 times. The relative frequency for getting a sum of 5 was $\frac{1}{5}$. What is the difference between the number of expected outcomes and the number of actual outcomes?

7. Olivia tosses a two-sided counter and then spins a spinner with six equal-size sections labeled 1 through 6. One side of the counter is red. The other side is yellow. She performs this experiment 80 times. The relative frequency of tossing red and spinning a number greater than three was $\frac{2}{5}$. What is the difference between the number of expected outcomes and the number of actual outcomes?

8. MP **Justify Conclusions** Natalie has a choice of a black, blue, or tan skirt to wear with a red, blue, or white sweater. Without calculating the number of possible outcomes, how many more outfits can she create if she adds a yellow sweater to her collection? Explain.

9. MP **Persevere with Problems** Kimiko and Miko are playing a game in which each person rolls a number cube. If the sum of the numbers is a prime number, then Miko wins. Otherwise, Kimiko wins. Is this game fair? Write an argument to defend your response.

10. Does the algebraic expression x^{10} represent the number of possible outcomes if the spinner shown is spun x times? Explain.

11. Describe a real-world compound event that has a sample space with four possible outcomes. Show the sample space.

Simulate Chance Events

I Can... design a simulation to represent a simple or compound event and use the results of a simulation to find the experimental probability.

What Vocabulary Will You Learn?
simulation

Explore Simulations

Online Activity You will use Web Sketchpad to explore using a random number generator to model a simulation.

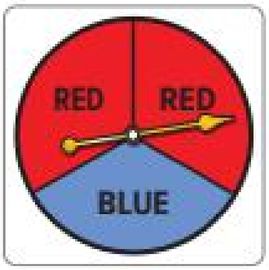
Learn Simulate Simple Events

A **simulation** is an experiment that is designed to model one or more events. Simulations often model events that can be difficult, time consuming, or impractical to perform in real life.

Suppose a cereal company places a prize in 1 out of every 3 of its cereal boxes. You can design a simulation that models whether or not a box of cereal you buy will contain a prize.

The event consists of randomly selecting a cereal box. To simulate the event, you can design an experiment that has the same probability of success. In this case, the probability of success is $\frac{1}{3}$, because 1 out of every 3 boxes contains a prize.

One way you can design a simulation is to design a spinner that has a probability of a successful outcome as $\frac{1}{3}$. In this case, the spinner will have three equal-size sections.



In this simulation, a success is defined as the spinner landing on blue and represents selecting a box with a prize. A failure is defined as the spinner landing on red and represents selecting a box without a prize.

(continued on next page)

Your Notes

Another way you can design a simulation is to use a number cube. Because a number cube has six sides, rewrite the probability, $\frac{1}{3}$, as an equivalent fraction with a denominator of 6.

$$\underset{\times 2}{\overset{\times 2}{\frac{1}{3} = \frac{2}{6}}}$$

In this simulation, success is defined by rolling 2 of the 6 faces and represents selecting a box with a prize. Failure is defined by rolling 4 of the 6 faces and represents selecting a box without a prize. You can determine which 2 faces are successes. Suppose you determine that rolling a 1 or a 2 represents a cereal box with a prize. What rolls represent a cereal box with no prize?

____

Five different events are shown in the table. Choose the model that can be used to correctly simulate each event by placing an X in that column.

Event	Spinner with Four Equal-Size Sections	One Coin Toss
your favorite book out of four books being randomly assigned for a book report		
your favorite baseball team has $\frac{3}{4}$ probability of winning		
a $\frac{1}{2}$ chance a girls' soccer team wins its first game		
forecast shows a 50% chance of rain		
a marble is randomly chosen from a bag containing four different color marbles		

Talk About It!

For the first event, how can a success be defined? How can a failure be defined?

Learn Simulate Compound Events

As with simple events, you can design a simulation to simulate a compound event. Coins, number cubes, and spinners are often used to simulate events. To design a simulation, you need to do each of the following.

- Define what each outcome represents, and determine if it is a success or failure.
- Define what each trial represents.

Go Online Watch the animation to learn how to use a simulation to estimate the probability of the following compound event.

Suppose each tiger cub born in a litter of cubs has an equal chance of being female or male. In a litter of 3 tiger cubs, estimate the probability that all 3 cubs will be female.

Step 1 Design a simulation.

For each cub, there are 2 possible outcomes, female or male.

One way to design the simulation is to toss a coin, because a coin has 2 possibe outcomes. Because there are 3 cubs in the litter, each trial represents tossing 3 coins. You can choose to let "Heads" represent a female cub, and "Tails" represent a male cub.

Step 2 Perform the simulation.

Suppose the table shows the results of 100 trials of the simulation.

Simulation Data: 100 Trials	
Outcome	Frequency
3 females, 0 males	14
2 females, 1 male	33
1 female, 2 males	41
0 females, 3 males	12

Talk About It!

Use a tree diagram to find the theoretical probability of having three female cubs. How does the simulated probability compare to the theoretical probability? Explain.

Step 3 Find the experimental probability, which has the same ratio as the relative frequency.

$P(\text{3 females}) = \frac{14}{100}$

Step 4 Simplify the ratio.

$\frac{14}{100} = \frac{\square}{\square}$

Based on the simulation, the estimated probability that all 3 tiger cubs will be female is $\frac{7}{50}$, 0.14, or 14%.

(continued on next page)

There are many ways to simulate compound events. Some examples are shown below.

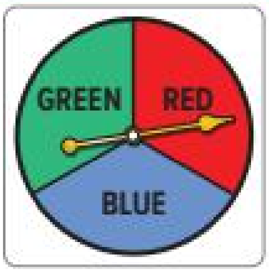
Weather

Suppose that, during the springtime, it rains 50% of the days. What is the chance that it will rain two days in a row this spring?

Coin Toss	Spinner
Let heads represent *rain*. Let tails represent *no rain*. Each trial consists of two tosses of a coin and a successful event is represented by tossing 2 heads.	Each trial consists of 2 spins, and a successful trial is the pointer landing on *RAIN* 2 times in a row. 

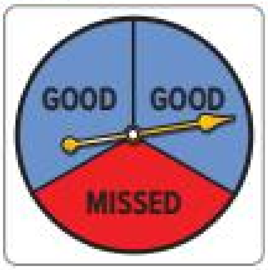
Marbles

Suppose you have a bag with an equal number of red, blue, and green marbles. What is the probability of randomly selecting a red marble from the bag 3 times in a row with replacement?

Number Cube	Spinner
Assign 2 unique numbers to represent *red*. Each trial consists of rolling the number cube 3 times. A success is landing on the 2 specified numbers 3 times in a row.	Each trial consists of 3 spins, and a successful trial is the pointer landing on *RED* 3 times in a row. 

Football

Suppose that, on average, a professional football kicker makes 2 out of 3 of his field goals from the 40-yard line. What is the probability that he makes two field goals in a row?

Number Cube	Spinner
Assign 4 of the numbers on the number cube to represent the success rate of $\frac{2}{3}$. Each trial consists of rolling the cube twice. A success is represented by landing on any of those assigned numbers 2 times in a row.	Each trial consists of 2 spins, and a successful trial is the pointer landing on *GOOD* 2 times in a row. 

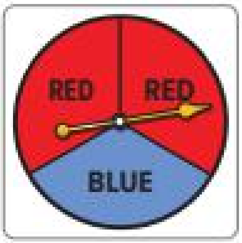
Example 1 Simulate Compound Events

A local grocery store sells cereal in two-packs for a special price. The probability of a box containing a prize is $\frac{1}{3}$. Design and simulate an event that estimates the probability of randomly selecting a two-pack that contains a prize in both boxes. Run the simulation 10 times.

What is the simulated probability of getting a prize in both boxes of cereal?

Part A Design a simulation.

The event is a compound event. The success of the first event represents one box in the two-pack containing a prize. The success of the second event represents the other box in the two-pack containing a prize.



One way to design a simulation is to use a spinner to represent each event. Because the probability of each box containing a prize is $\frac{1}{3}$, design a spinner with three equal-size sections. You can let one section, such as *blue*, represent the success of selecting a box that contains a prize. The other two sections represent the failure of selecting a box that does not contain a prize.

One trial consists of spinning the spinner twice.

Suppose the sample results are shown in the table.

Pack	Box 1	Box 2	Both Prizes
1	✓	✗	No
2	✗	✓	No
3	✗	✗	No
4	✓	✓	Yes
5	✓	✗	No
6	✓	✓	Yes
7	✗	✗	No
8	✗	✗	No
9	✓	✗	No
10	✗	✗	No

Part B Find the probability.

How many packs had prizes in both boxes? ________

So, the estimated probability of selecting a two-pack containing a prize in both boxes, based on the 10 simulated trials is $\frac{2}{10}$, 0.2, or 20%.

Think About It!

What tools can you use to simulate the situation?

Talk About It!

Based on the simulation, estimate the probability that you purchase a package of cereal where neither box contains a prize.

Talk About It!

If 100 people each bought a pack of cereal, use these results to predict how many customers would receive one prize, two prizes, or no prize.

Check

A store randomly gives a gift card to 5 out of every 8 customers that enter the store for a weekend grand opening event. The store owner wants to estimate the probability that a repeat customer receives a gift card two days in a row.

Part A

Select the appropriate way you can simulate the event, where a success is receiving a gift card and a failure is not receiving a gift card.

Ⓐ Each trial consists of rolling a number cube twice. A success represents both tosses landing on any of the numbers 1 through 4. A failure represents one or both tosses landing on the numbers 5 or 6.

Ⓑ Each trial consists of tossing a coin twice. A success represents both tosses landing on heads. A failure represents one or both tosses landing on tails.

Ⓒ Each trial consists of spinning a spinner with 8 equal-size sections twice. Label 5 of the sections with a "C", and label the 3 remaining sections with an "X". A success represents both spins landing on "C". A failure represents one or both spins landing on "X".

Ⓓ Each trial consists of spinning a spinner with 8 equal-size sections twice. Label 5 of the sections with a "C", and label the 3 remaining sections with an "X". A failure represents both spins landing on "C". A success represents one or both spins landing on "X".

Part B

The table shows the results of simulating 10 trials of the compound event. A "C" represents that they received a gift card that day and an "X" represents that they did not receive a gift card that day. According to the results, what is the experimental probability that a repeat customer receives gift cards two days in a row?

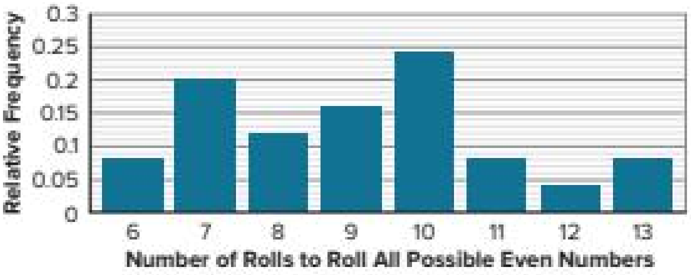
Trial	1	2	3	4	5	6	7	8	9	10
Day 1	C	C	C	X	C	X	X	X	X	C
Day 2	C	C	X	C	C	C	X	X	C	X

Show your work here

Example 2 Interpret Simulations of Compound Events

A computer simulation was designed to simulate rolling a number cube multiple times until all of the possible even numbers were rolled. The relative frequency bar graph shows the number of rolls needed for the computer to roll all of the even numbers.

What is the simulated probability that eight or fewer rolls are needed to obtain all of the even numbers on a number cube?



Find the sum of the relative frequencies that indicate that six, seven, or eight rolls were needed to obtain all of the even numbers.

$P(\leq 8 \text{ rolls}) = P(6) + P(7) + P(8)$ The numbers 6, 7, and 8 are each less than or equal to 8.

$= 0.08 + 0.20 + 0.12$ Substitute the relative frequencies.

$= 0.40$ or 40% Simplify.

The relative frequency ratio has the same value as the experimental probability.

So, the simulated probability that it takes eight or fewer rolls to obtain all of the even numbers on a number cube is 0.4 or 40%.

Pause and Reflect

How would you explain to a new student what each bar in a relative frequency bar graph means?

Think About It!

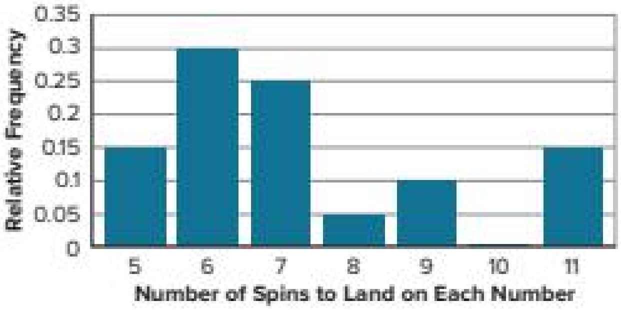
How can you begin solving the problem?

Talk About It!

If this graph represents 25 trials, how many trials did it take to roll all even numbers in eight or fewer rolls? Explain.

Check

Jake designs and conducts a computer simulation with 20 trials and uses the data from the simulation to create the relative frequency bar graph shown. The graph shows the relative frequency of the number of spins needed for a four-section spinner labeled 1 through 4 to land on each number at least once. Using the graph, what is the experimental probability that more than 7 spins are needed to land on each number at least once?



Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

How will you study the concepts in today's lesson? Describe some steps you can take.

Record your observations here

Name ____________________ Period __________ Date __________

Practice

Go Online You can complete your homework online.

1. Suppose the chance of rain on Saturday is $\frac{2}{5}$ and the chance of rain on Sunday is also $\frac{2}{5}$. A student wants to run a simulation to estimate the probability that it will rain on both days. (Example 1)

Part A How can the student model the chance of it raining on each day? Design a simulation.

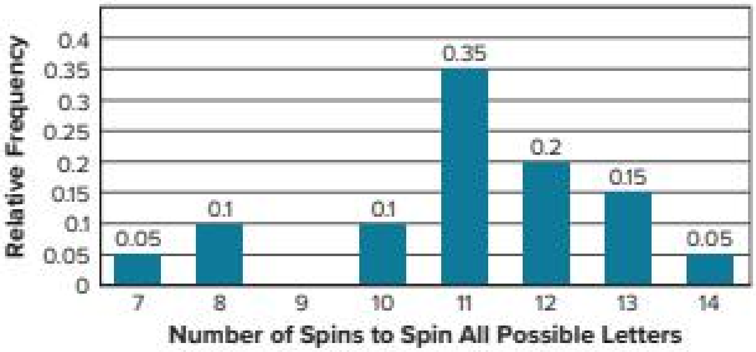
Part B Suppose the table shows the results of 10 trials of a simulation. An "R" represents a day that it rained and an "N" represents a day that it did not rain.

Trial	1	2	3	4	5	6	7	8	9	10
Saturday	N	R	R	N	N	R	R	N	R	N
Sunday	N	N	R	R	N	R	N	R	R	N

According to the results of the simulation, what is the experimental probability of having rain on both days?

Test Practice

2. Open Response Leigh designs and conducts a computer simulation with 30 trials and uses the data from the simulation to create the relative frequency bar graph shown. The graph shows the relative frequency of the number of spins needed for a spinner divided into 6 equal sections labeled A through F to land on each letter at least once. (Example 2)

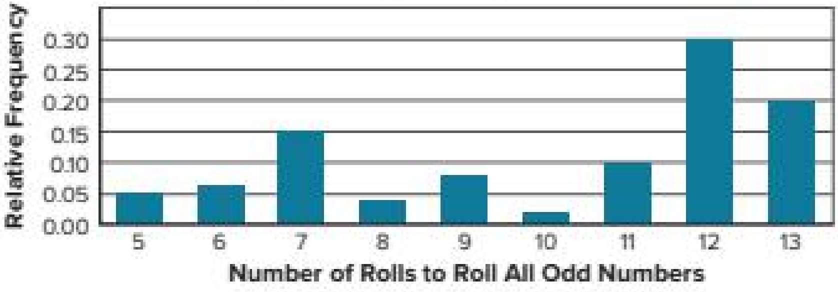


Using the graph, what is the experimental probability that more than 10 spins are needed to land on each letter at least once? Write the probability as a percent.

Apply

For Exercises 3 and 4, refer to the following information.

Nelly designs and conducts a computer simulation with 50 trials and uses the data from the simulation to create the frequency bar graph shown. The graph shows the relative frequency of the number of rolls needed for a number cube labeled 7 through 12 to roll all of the possible odd numbers.



3. How much greater is the probability that 7 or 11 rolls are needed than 13 rolls?

4. Is the probability that 7 or 12 rolls are needed greater than the probability that all of the other rolls are needed? Explain.

5. Use the Internet, or another source, to look up the term *fair game*. Describe a real-world scenario in which a game is fair. Then describe a real-world scenario in which a game is not fair.

6. **Model with Mathematics** Describe a real-world situation that can be simulated by tossing a coin and rolling a number cube. Be sure to include the number of outcomes in your description.

7. Why is it important to define each of the following when designing a simulation?

what each trial represents
what each outcome represents
what a success and a failure each represent

8. **Use Math Tools** Suppose the players at a certain carnival game win about 40% of the time. Describe a model that can be used to simulate the outcomes of playing this game.

Module 10 • Probability

Review

Foldables Use your Foldable to help review the module.

Probability

Example

Picture

Example

Picture

Rate Yourself! ■ ◆ ★

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.

____

____

____

____

____

____

____

Write about a question you still have.

____

____

____

____

____

____

____

Reflect on the Module

Use what you learned about probability to complete the graphic organizer.

Essential Question

How can probability be used to predict future events?

Theoretical Probability

Experimental Probability

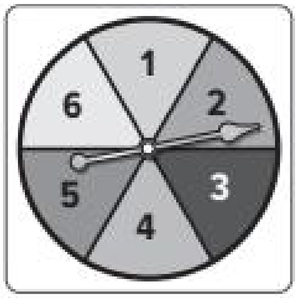
Sample Space

Simulation

Name ____________________ Period __________ Date __________

Test Practice

1. **Multiple Choice** The spinner shown is divided into 6 equal-size sections. Which of the following best describes the likelihood of the spinner landing on an odd number? (Lesson 1)



Ⓐ impossible

Ⓑ unlikely

Ⓒ equally likely

Ⓓ likely

2. **Multiple Choice** A jar contains 4 yellow marbles, 11 green marbles, and 9 blue marbles. Which is the best description of the likelihood of selecting a red marble from the jar? (Lesson 1)

Ⓐ impossible

Ⓑ unlikely

Ⓒ equally likely

Ⓓ likely

3. **Multiselect** A number cube with sides labeled 1, 2, 3, 4, 5, and 6 is rolled 50 times. The number 6 is rolled 10 times. What is the relative frequency of rolling a 6? Select all that apply. (Lesson 2)

☐ 0.2

☐ $\frac{1}{5}$

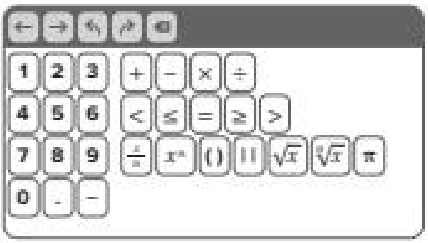
☐ 0.6

☐ $\frac{3}{5}$

☐ 20%

4. **Equation Editor** The table shows the number of items bought by different students at the school bookstore this morning. What is the relative frequency that a student bought one item? Express your answer as a fraction in simplest form. (Lesson 2)

Number of Items	Frequency
1	6
2	5
3	7
4	6



5. **Open Response** A spinner with 8 equal-sized sections labeled 1 through 8 is spun 400 times. How many times can you expect to spin a number greater than 3? Explain your reasoning. (Lesson 3)

6. Multiselect The number 4 is rolled on a number cube with sides numbered 1, 2, 3, 4, 5, and 6. (Lesson 3)

A. Which outcomes make up the complement of the event? Select all that apply.

☐ rolling a 1

☐ rolling a 2

☐ rolling a 3

☐ rolling a 4

☐ rolling a 5

☐ rolling a 6

B. What is the probability of the complement?

☐ $\frac{1}{6}$ ☐ $\frac{5}{6}$

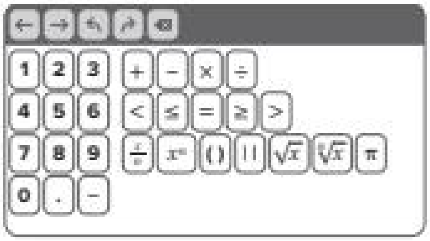
☐ $\frac{1}{3}$ ☐ $\frac{1}{2}$

☐ $\frac{2}{3}$ ☐ 0

7. Open Response The table shows the number of each color pen in Mrs. Devon's desk drawer. Suppose she selects a pen at random from the drawer. How much greater is the theoretical probability of selecting a black pen than selecting a red or a green pen? Express your answer as a percent. (Lesson 4)

Color	Number of Pens
Black	9
Blue	6
Green	1
Red	4

8. Equation Editor Suppose Julio tosses a coin four times. What is the theoretical probability of tossing heads at least two times? Express your answer as a fraction in simplest form. (Lesson 5)



9. Open Response Olivia flips a coin and rolls a number cube with sides labeled 1, 2, 3, 4, 5, and 6. After 90 trials of the experiment, the relative frequency of flipping heads and rolling a number less than 3 is $\frac{2}{15}$. What is the difference between the number of expected outcomes and the number of actual outcomes? (Lesson 5)

10. Open Response A weather forecast calls for a 60% chance of rain today and a 60% chance of rain tomorrow. The table shows the results of 10 simulated trials, where "R" represents rain and "N" represents no rain. (Lesson 6)

Trial	1	2	3	4	5	6	7	8	9	10
Today	N	R	R	N	R	R	N	R	R	R
Tomorrow	N	R	R	N	R	R	R	N	N	R

According to the results of the simulation, what is the experimental probability of having rain on both days?

Module 11

Sampling and Statistics

Essential Question

How can you use a sample to gain information about a population?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY

■ — I don't know. ◆ — I've heard of it. ★ — I know it!

	Before			After		
	■	◆	★	■	◆	★
identifying valid sampling methods						
identifying biased samples						
identifying valid inferences						
making predictions using sample data						
understanding the benefit of taking multiple samples						
making comparative inferences about two populations						
making inferences about the variability between two populations						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about sampling and statistics.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ asymmetric
- ☐ biased sample
- ☐ convenience sample
- ☐ distribution
- ☐ double box plot
- ☐ double line plot
- ☐ inferences
- ☐ invalid inference
- ☐ population
- ☐ sample
- ☐ simple random sample
- ☐ statistics
- ☐ stratified random sample
- ☐ survey
- ☐ symmetric
- ☐ systematic random sample
- ☐ unbiased sample
- ☐ valid inference
- ☐ valid sampling method
- ☐ variability
- ☐ visual overlap
- ☐ voluntary response sample

Are You Ready?

Study the Quick Review to see if you are ready to start this module.
Then complete the Quick Check.

Quick Review

Example 1

Find the mean of a data set.

What is the mean of the data shown?

The sum of the data values is 2(14) + 2(15) + 16 + 2(17) + 2(18), or 144.

Divide the sum, 144, by the number of data values, 9. Because 144 ÷ 9 = 16, the mean is 16.

Example 2

Find the percent of a number.

Find 8% of 350.

$$\underset{\times 3.5}{\overset{\times 3.5}{\frac{28}{350} = \frac{8}{100}}}$$

Write 8% as $\frac{8}{100}$. Find an equivalent ratio.

So, 8% of 350 is 28.

Quick Check

1. Find the mean of the data set.

2. Find 22% of 500.

How Did You Do?

Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right.

① ②

Biased and Unbiased Samples

I Can... identify biased and unbiased sampling methods and understand that inferences made are only valid if the sampling method is unbiased.

What Vocabulary Will You Learn?

- biased sample
- convenience sample
- inferences
- invalid inference
- population
- sample
- simple random sample
- statistics
- stratified random sample
- survey
- systematic random sample
- unbiased sample
- valid inference
- valid sampling method
- voluntary response sample

Learn Populations and Samples

Statistics is the practice of collecting and analyzing data. It can be used to gain information about a **population**, or the group being studied.

A sample is often used instead of an entire population. A **sample** is part of a population and should be representative of the population. A *statistic* refers to a single measure of some attribute of the sample. A statistic that is used is the percent of a sample that shares the same attribute or response.

Data from the sample are often collected in the form of a **survey**, which is a question or set of questions designed to gain information about the population as a whole.

For each of the three survey topics shown, determine whether each phrase describes a *population* or a *sample*.

Survey Topic 1: How many hours a night do you study?

25 randomly selected 7th graders ____________

all of the students in the middle school ____________

Survey Topic 2: What is the town's favorite flavor of ice cream?

residents of the town ____________

customers at a local ice cream shop ____________

Survey Topic 3: Who should be the mayor of our city?

350 residents outside of city hall ____________

all of the residents of the city ____________

Your Notes

Learn Valid Sampling Methods

When choosing a sample from a population, it is important to use a valid sample method. A **valid sampling method** is one that is:

- representative of the population
- selected at random, where each member has an equal chance of being selected, and
- large enough to provide accurate data

The table shows various valid sampling methods.

	Definitio	Example
Simple Random Sample	Each item or person in the population is as likely to be chosen as any other.	Twenty-five student names are written on slips of paper and placed in a basket. One name is randomly selected.
Stratified Random Sample	The population is divided into groups with similar traits that do not overlap. A simple random sample is then selected from each group.	Students of a school are divided into 6th, 7th, and 8th grade. A random sample of 25 students from each grade is chosen.
Systematic Random Sample	The sample is selected from the population according to a specific item or time interval.	Every 10th customer at a store is given a survey, or a customer is chosen to complete a survey every half hour.

Talk About It!

Why do the three names of the valid sampling methods all contain the word *random*?

Pause and Reflect

Where have you seen surveys in everyday life? What type of sample was the survey using?

Example 1 Identify Valid Sampling Methods

The astronomy association wants to take a survey to decide on the theme for their annual celebration. They are presented with three valid sampling descriptions as options to take the survey.

For each sampling description, select the valid sampling method that best represents it. Circle your selection.

a computer randomly chooses 500 people from a list of members

Stratified Random Sample	Simple Random Sample	Systematic Random Sample

members are separated by state and 10 people are randomly chosen from each state

Stratified Random Sample	Simple Random Sample	Systematic Random Sample

from a list of each member in the association, every 200th is surveyed

Stratified Random Sample	Simple Random Sample	Systematic Random Sample

Check

For each sampling description, identify the valid sampling method that best describes it.

To determine which passengers' carry-on bags are to be inspected, every eighth person to check in will have his or her bag inspected.

To test the accuracy of a biometric scanner, a scientist uses a computer to generate a sample of 20 subjects from a population.

The principal of a high school wants to use a survey to decide on the theme for their winter formal dance. She separates the students by grade – 9th, 10th, 11th, and 12th – and then takes a sample of 50 students from each grade.

Go Online You can complete an Extra Example online.

Think About It!

What are the different types of valid sampling methods?

Talk About It!

Suppose the astronomy association used a sampling method that did not select members at random. How might the results of the survey be affected?

Learn Biased Samples

An **unbiased sample** is obtained using a valid sampling method that is random and is representative of the population.

When a sample is *not* representative of the population, it is a **biased sample**. A biased sample usually favors one or more parts of the population over another.

The table shows two types of biased samples: **convenience sample** and **voluntary response sample** and the reasons why each is biased.

	Convenience Sample	Voluntary Response Sample
Definitio	This sample includes members of the population that are easily accessed.	This sample involves only those who want to, or can, participate in the sampling.
Example	You give a survey to the students that eat lunch with you to find out information about middle school students.	A school principal sends out a survey on a social networking site asking middle school students to vote for their favorite restaurant.
Why is it biased?	The sample is not randomly chosen and not representative of the population as a whole.	The sample involves only those who choose to participate. The responses will likely favor opinions that come only from people who feel very strongly about that topic.

Talk About It!

If you want to determine the favorite pizza shop of middle school students in your school, explain how a voluntary response sample might influence the results of the survey.

Suppose you want to determine the favorite pizza shop of middle school students in your city. Select all of the samples that are biased.

Ⓐ All of the middle school students that rode bicycles to school are surveyed.

Ⓑ A social media poll is sent to all middle school students. A winner is chosen from the participants.

Ⓒ Every ninth student that walks through the cafeteria door is surveyed.

Ⓓ Every person in the culinary section of the book store on a Monday evening is surveyed.

Example 2 Identify Biased Sampling Methods

Identify the type of biased sample for each situation.

To gather information about the favorite animal of middle school students, a student surveys everyone in his 7th grade language arts class.

Because the sample is everyone that is available in one class, it is a convenience sample.

A news organization posts a survey on its website that asks its readers to vote on a current topic.

Because only those people that choose to respond will, the sample is a voluntary response sample.

Check

Identify the type of biased sample for each situation.

You survey a sample of 10 households on your street to determine how often households in the state cut their grass.

The manager of a retail store mails out a survey to 500 potential customers. She uses the results of the surveys that are mailed back to determine which items to keep in stock for the spring season.

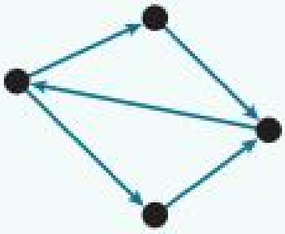
Go Online You can complete an Extra Example online.

Pause and Reflect

Describe some biased sampling methods you may have encountered in your everyday life. Why were they biased?

Math History Minute

Victor Neumann-Lara (1933-2004) was a Mexican mathematician and a pioneer in the field of graph theory. In 1982, he introduced the idea of a *digraph*. A digraph, also known as a directed graph, is a graph consisting of a set of vertices connected by edges, in which the edges have a direction associated with them.



Learn Valid Inferences

If you have used a valid sampling method and obtained an unbiased sample that is representative of the population, you can use the results to make **inferences**, or predictions, about the population. This is called a **valid inference**. An **invalid inference** is an inference that is based on a biased sample. An invalid inference makes a conclusion not supported by the results of the sample.

The table describes inferences by each sampling method, sample, and conclusion.

	Valid Inference	Invalid Inference
Sampling Method	uses a valid sampling method	may *not* use a valid sampling method
Sample	drawn from an unbiased, representative sample	drawn from a biased, or unrepresentative sample
Conclusion	makes a conclusion that is supported by the data	makes a conclusion that is *not* supported by the data

Talk About It!

Suppose you have used a valid sampling method to conduct a survey. Is it possible that you can still obtain a sample that is not representative of the population?

Example 3 Identify Valid Inferences

On a social networking app, a burrito company asked all of its followers to vote on their favorite style of food. The choices were Italian, Mexican, and Indian. The results are shown in the table. The company infers that the most popular style of cuisine is Mexican.

Style of Food	Percent of Sample
Italian	21%
Mexican	46%
Indian	33%

Identify the type of sampling method used. Then determine whether the inference is valid.

Part A Identify the type of sampling method used.

This is a voluntary response sample and is biased.

Part B Determine whether the inference is valid.

Because the company used a biased sampling method, they cannot make valid inferences based on the sample. The inference made by the company is not a valid inference.

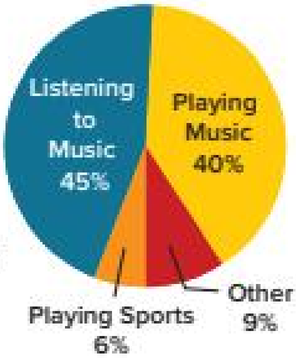
Talk About It!

Is it possible to have used a biased sample and still arrive at a claim that is valid for the population?

Check

The customers in a music store one morning are surveyed to determine their favorite activity in their free time. The results are shown in the graph. The store manager concludes that most Americans either play music or listen to music in their free time.

Identify the type of sample method used. Then determine whether the inference is valid.



Go Online You can complete an Extra Example online.

Example 4 Identify Valid Inferences

The manager of a shoe store wants to determine its customers' favorite shoe color. Every third customer is surveyed. The results are shown in the table. The store manager infers that their customers' most preferred shoe color is red.

Shoe Color Preference	
Red	40%
Blue	34%
White	16%
Black	8%
Multicolored	2%

Identify the type of sampling method used. Then determine whether the inference is valid.

Part A Identify the sampling method used.

Every third customer is surveyed. This is a systematic random sample. It is not biased.

Part B Determine whether the inference is valid.

Because the store used an unbiased sampling method, they can make inferences based on the sample. The inference made by the store manager is a valid inference.

Think About It!

What phrase or sentence tells you about the sampling method?

Talk About It!

Could the store manager use the survey results to infer that the favorite shoe color of the population of the United States is red? Justify your response.

Check

Twenty customers in a grocery store are randomly selected and surveyed about their juice preference. The results are shown in the table. After reviewing the data, the store manager decided that about half of the store's juice stock should be orange juice.

Juice Preference	
Orange	51%
Apple	32%
Pineapple	14%
Cranberry	3%

Part A Identify the sampling method used.

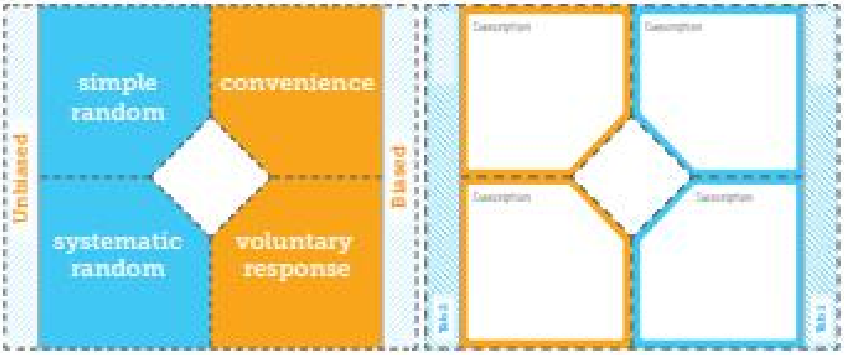
Show your work here

Part B Determine whether the inference is valid.

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Name ______________________ Period __________ Date __________

Practice

Go Online You can complete your homework online.

1. For each sampling description, identify the valid sampling method that best describes it. Choose from *simple random sample, stratified random sample*, or *systematic random sample*. (Example 1)

   a. To determine if a candidate for state senator is popular with voters, 25% of voters in 160 counties are surveyed.

   b. To determine whether students think a new school library is needed, a computer generates a list of 100 random students and they are surveyed.

   c. To determine the freshness of doughnuts, a baker selects a doughnut every 30 minutes and checks it.

2. Identify the type of biased sample for each situation. Choose from *convenience sample* or *voluntary response sample*. (Example 2)

   a. A physical education teacher posts an online survey about whether students would be interested in a 5K race. The responses received determine whether there will be a 5K race.

   b. To determine the theme of the school dance, the student council president surveys his homeroom class.

Identify the sample method used and whether it is biased or unbiased. Then determine whether the inference is valid. (Examples 3 and 4)

3. To evaluate customer satisfaction, a grocery store manager gives double coupons to anyone who completes a survey as they enter the store. The store manager determines that customers are very satisfied with their shopping experience in his store.

4. A member of the cafeteria staff asks every fifth student leaving the cafeteria to rank 5 entrees from most favorite to least favorite. She finds that pizza is one of the favorite entrees.

Test Practice

5. **Multiselect** To evaluate the defect rate of its lenses, a camera lens manufacturer tests every 100th lens off the production line. Out of 1,000 lenses tested, one lens is found to be defective. The manufacturer concludes that 3 lenses out of 3,000 will be defective. Select all of the statements that are true about the sampling method.

   ☐ This scenario is a systematic random sample.

   ☐ The sampling method is biased.

   ☐ The inference is valid.

   ☐ This scenario is a convenience sample.

   ☐ The sampling method is unbiased.

Apply

6. Members of the drama club plan to sell popcorn as a fundraiser for their spring play. To determine what flavor to sell, the members survey every 15th student from an alphabetical listing of all students. The table shows the results of their survey. Was the sample obtained using a valid sampling method? If so, find what percent of students prefer each type of popcorn. If not, explain why.

Flavor	Number
Butter	33
Cheese	15
Caramel	27

7. As people leave a restaurant one evening, 20 people are surveyed at random. Eight people say they usually order dessert when they eat out. Was the sample obtained using a valid sampling method? If so, what percent of those surveyed say they usually do not order dessert when they eat out? If not, explain why.

8. Give an example of a convenience sample.

9. MP **Reason Abstractly** Marc wants to determine how many students plan to attend the school's walk-a-thon. He decides to post an online survey. Of the survey responses, 80% plan to attend the walk-a-thon. Marc infers most students will attend the walk-a-thon. Is Marc's inference valid? Explain.

10. MP **Justify Conclusions** Determine if the statement is *true* or *false*. Explain.

A stratified random sample's results are never valid.

11. Suppose you wanted to know how many students brought their lunch to school. Describe a valid sampling method you could use.

Lesson 11-2

Make Predictions

I Can... make predictions about a population based on data from a random sample.

Learn Make Predictions

If a survey is conducted about a population using an unbiased sample, valid inferences can be made about the population. You can use those inferences to make predictions about the population.

Suppose you want to predict the percent of fans at a sporting event that are fans of the green team and the percent that are fans of the blue team. Because surveying everyone in the stadium might take too long, you can use a sample that is representative of the population to help make a prediction about the population.

Suppose two samples taken at the sporting event are shown.

One sample is taken from outside the stadium. Suppose that the ratio of fans of the blue team to fans of the green team from outside the stadium is 2 to 8 or 2 : 8.


Sample from Outside Stadium

This means that, for every 2 fans of the blue team, there are 8 fans of the green team for this sample.

A second sample is taken from the stadium seating sections. Suppose that the ratio of fans of the blue team to fans of the green team in the stadium seating is also 2 to 8 or 2 : 8.


Sample from Stadium Seating

This also means that, for every 2 fans of the blue team, there are 8 fans of the green team for this sample.

Talk About It!
Describe a sampling method you can use to ensure the sample you choose is not biased in favor of one team or another.

(continued on next page)

Your Notes

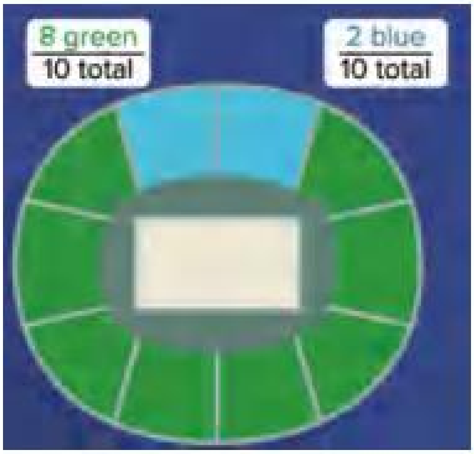
For each sample, the ratio of fans of the blue team to fans of the green team is 2 to 8 or 2 : 8. This is a *part-to-part ratio* because the ratio compares one part of the group (fans of the blue team) to another part of the same group (fans of the green team). You learned about part-to-part ratios in a previous grade.

Sample from Outside Stadium

Sample from Stadium Seating

You can also write a part-to-whole ratio. A *part-to-whole ratio* compares one part of the group (fans of the blue team, for example) to the whole group (total fans). You also learned about part-to-whole ratios in a previous grade.

The part-to-whole ratio of fans of the blue team to total fans is 2 to 10, 2 : 10, or $\frac{2}{10}$. In other words, 20% of the fans included in the samples are fans of the blue team.



If unbiased sampling methods were used to obtain the sample data, you can make inferences about the population. Some inferences that you can make are shown below.

- Twenty percent of the total fans attending the event are fans of the blue team.
- Eighty percent of the total fans attending the event are fans of the green team.
- One-fifth, or one out of every 5 fans, attending the event are fans of the blue team.

These inferences are only valid if unbiased sampling methods are used. If biased samples are used, then these inferences are not valid.

Talk About It!

Suppose an unbiased sample was taken. The ratio of fans of the green team to fans of the blue team was 8 to 2. Predict what percent of the fans inside the stadium are fans of the green team. Explain.

Example 1 Make Predictions

A high school athletic director is purchasing equipment for the athletic department in the coming year. In order to determine how much equipment is needed, the director randomly surveys 150 students who plan to participate in athletics in the coming year. The table shows the results.

Sport	Students
Baseball/Softball	36
Basketball	30
Football	45
Gymnastics	12
Tennis	18
Volleyball	9

How many volleyball uniforms should the director purchase if 500 total students plan to participate in athletics?

Step 1 Write the ratio of students who plan to play volleyball to the total number of students surveyed.

$$\frac{\text{volleyball players}}{\text{total students surveyed}} \begin{matrix}\longrightarrow \\ \longrightarrow\end{matrix} \frac{9}{150}$$

Step 2 Set up and solve a proportion. Let *v* represent the number of volleyball uniforms the director should order.

$$\begin{matrix}\text{volleyball players} \longrightarrow \\ \text{students surveyed} \longrightarrow\end{matrix} \quad \frac{9}{150} = \frac{v}{500} \quad \begin{matrix}\longleftarrow \text{volleyball uniforms} \\ \longleftarrow \text{total number of students}\end{matrix}$$

$$\frac{3}{50} = \frac{v}{500} \quad \text{Write } \tfrac{9}{150} \text{ as the equivalent ratio } \tfrac{3}{50}.$$

$$\frac{3}{50} = \frac{30}{500} \quad \text{Because } 50(10) = 500\text{, multiply 3 by 10 to obtain 30.}$$

So, the director should purchase 30 volleyball uniforms.

Think About It!

Without calculating, should the director order less than, greater than, or equal to 50 volleyball uniforms? Why?

Talk About It!

Suppose the school orders 30 uniforms based on this prediction. Does this mean that exactly 30 students will sign up to participate in volleyball? Explain.

Check

A local dentist wants to know how many adults in a town receive regular cleanings. The dentist surveys 120 random adults living in the town and finds 84 people receive regular cleanings. If there are 8,500 adults in the town, how many can be expected to receive regular cleanings?

Show your work here

Go Online You can complete an Extra Example online.

Think About It!

Without calculating, should the number of available positions for students who volunteer once a week be less than, greater than, or equal to 500? Why?

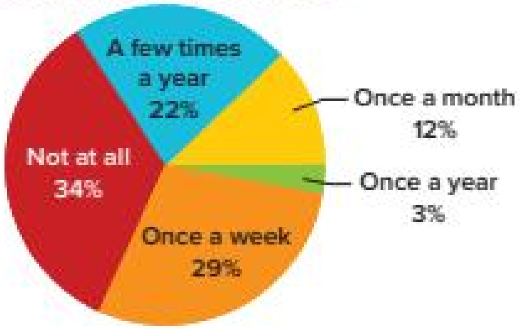
Talk About It!

Use number sense to estimate the number of volunteer positions the district should have available for students who volunteer once a month. Explain your estimate.

Example 2 Make Predictions

The superintendent of a school district wants to determine the number of volunteer positions to have available for students. The graph shows the results of a survey where randomly selected teenagers within the district were asked, "How often do you volunteer?"

How Often Teens Volunteer



If the district has 2,000 teenage students, about how many positions should the superintendent have available for students who volunteer once a week?

While you do not know the number of teens in the sample, the circle graph shows the percent of teens who volunteer. This percent is the ratio that can be used for the sample. The graph shows that 29% of students volunteer once a week.

Find 29% of 2,000. Let *n* represent the unknown part.

$$\frac{29}{100} = \frac{n}{2,000}$$ Write the proportion.

$$\underset{\times 20}{\overset{\times 20}{\frac{29}{100} = \frac{n}{2,000}}}$$ Find an equivalent ratio.

So, the superintendent should have about 29(20), or 580 volunteer positions available for students who volunteer once a week.

Check

The manager of a movie theater wants to better predict how much popcorn to prepare each day. Every 15th customer was surveyed as to whether or not they buy popcorn and 63% said they buy popcorn. If the theater expects to have 3,200 customers during a weekend, how many people are expected to buy popcorn?

Show your work here

Go Online You can complete an Extra Example online.

Apply Profit

A store sells 3 types of pants: jeans, capris, and athletic pants. The store employees randomly survey every 10th customer about their preferred type of pants. A total of 50 customers are surveyed. Their responses are shown in the table.

Type of Pants	Survey Response Frequency	Profit per Pair Sold ($)
Jeans	27	9.00
Capris	9	10.50
Athletic Pants	14	8.25

A total of 1,500 pairs of pants are expected to be sold in one month. The store manager uses the results of the survey to determine how many of each type are expected to be sold that month. What is the profit the store manager can expect to make?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It!
About what percent of the profit is expected to come from jeans?

Check

A manufacturer samples every 200th tablet computer produced in a batch as part of its quality control program. It is found that, of the 50 computers sampled, 1 was defective. Each defective computer costs the company $107 to repair. How much money can the company expect to pay for repairs on defective computers from a batch of 200,000 computers?

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

How will you study the concepts in today's lesson? Describe some steps you can take.

Record your observations here

Name ____________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

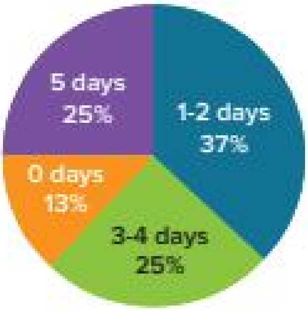
1. A school librarian is purchasing new books for her book clubs in the coming year. In order to determine how many books she needs, she randomly surveys 25 students who plan to participate in one of her book clubs in the coming year. The table shows the results. Predict how many science fiction books she will need to purchase if 125 students participate in book club next year. (Example 1)

Book Club Type	Number of Students
Autobiography	2
Graphic Novel	7
Mystery	10
Science Fiction	6

2. A smart tablet manufacturer tests 1 out of every 25 screens for flaws. Out of 125 tablets tested, 2 had defective screens. How many defective screens should the manufacturer expect out of 45,000 smart tablets? (Example 1)

3. The superintendent of a school district wants to predict next year's middle school lunch count. The graph shows the results of a survey of randomly selected middle school students. If the district has 5,000 middle school students next year, about how many students plan to buy lunch 1-2 days a week? (Example 2)

How Many Days Will You Buy Lunch?



4. The guidance department conducted a random survey of the student body and found that 16% of the students plan to volunteer at the school festival. Predict how many volunteer positions they should plan for a population of 950 students. (Example 2)

5. The owner of a travel agency randomly surveyed its customers. The survey showed that 55% of the agency's customers were planning an overseas vacation the following year. Predict how many of the travel agency's 12,400 travelers will vacation overseas the following year. (Example 2)

Test Practice

6. Open Response Every 30 minutes, a box of crayons is pulled from the assembly line to check the quality. Of 240 checked boxes of crayons, 2 did not pass inspection. How many boxes out of 12,000 should the crayon company expect to not pass inspection?

Apply

7. A bike shop surveys every 20th customer about future bike purchases. The responses of 50 customers are shown in the table. If the store uses the results of the survey to determine how many of each type of bike are purchased in an order of 600, how much profit can the store expect to make on comfort bikes?

Bicycle Type	Survey Response	Profit per Bike Sold ($)
Mountain	11	87.98
Touring	8	66.45
Comfort	9	32.50
Youth	19	34.50
Road	3	29.95

8. For an upcoming field trip to the science center, the school will allow students to select one extra activity. The school surveys a random sample of 25 students to determine about how many tickets of each kind they will need to buy. If there are 1,200 students going on the field trip, how much should the school expect to spend on all the activities?

Activity Type	Survey Response	Cost ($)
Movie	14	1.55
Planetarium	7	1.05
Backstage Tour	4	1.10

9. **Create** Write and solve a real-world problem where you use survey results to make a prediction.

10. **Find the Error** A student was solving the problem below and found the answer to be 92,500 customers. Find the student's error and correct it.

An unbiased survey showed that 74% of a pet supply's online customers spent at least $100 on their pets each year. Predict how many of the 125,000 online customers will spend less than $100 on their pets next year.

11. **Justify Conclusions** Determine if the following statement is *true* or *false*. Explain. *Survey results can always be used to make predictions.*

12. **Reason Abstractly** When making predictions from valid survey results, are the predictions exact answers or estimates? Explain your reasoning.

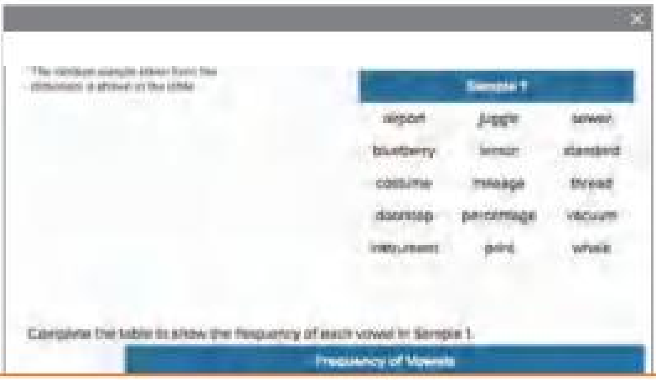
Generate Multiple Samples

I Can... understand how collecting multiple samples of data can help me determine how my predictions about a population might vary.

What Vocabulary Will You Learn?
variability

Explore Generate Multiple Samples

Online Activity You will explore how taking multiple samples can help you when making inferences about a population.



Learn Analyze Means of Multiple Samples

The mean calculated from a sample is called a sample mean. The sample mean is used to estimate the mean of the population. It is important to understand that a sample mean is rarely equal to the population mean. However, if the sample is properly conducted, the sample mean should be close to the population mean.

Talk About It!
Explain to a partner how to find the mean of a set of data. You learned about the mean in a previous grade.

If multiple samples are collected, the means of those samples can help you determine the reliability of a sample mean as an estimate for the population mean. Look for the place on the graph with the highest concentration of data values, or where the points seem to pile up. The closer the sample means are to this value, the better the estimate a sample mean is likely to provide.

Go Online Watch the animation to see how collecting multiple samples of a given size can help you determine how "far off" a mean from a sample of that size might be from the actual mean. The lesser the variation among the samples, the more likely a mean from a sample of that size will reflect the population mean.

(continued on next page)

Your Notes

The animation shows how multiple samples can help answer the statistical question *What is the length of a randomly selected word in a book?*

Step 1 Generate a random sample.

The table shows the results of a sample collected by counting the number of letters in 10 randomly selected words from a book.

Word	Number of Letters in Word
1	7
2	6
3	4
4	5
5	7
6	6
7	6
8	8
9	3
10	1

Step 2 Find the mean.

$$\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}}$$

$$= \frac{7+6+4+5+7+6+6+8+3+1}{10}$$

$$= \frac{53}{10} \text{ or } 5.3$$

The mean of the sample is 5.3 letters per word. How confident are you that this sample is representative of the population, even though it is a random sample?

Step 3 Gather data on multiple samples of the same type and size.

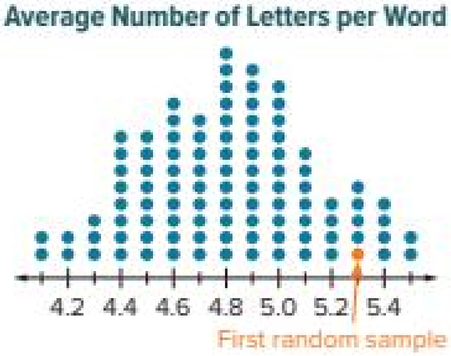
Suppose you take 100 samples and record the mean of each sample in a table like the one shown.

Sample	Average Number of Letters per Word
1	5.3
2	4.8
3	5.4
4	4.2
5	4.8
...	...
100	4.9

Step 4 Graph the data from the multiple samples using a dot plot. Each dot represents the mean of a sample.

Step 5 Make an inference based on the graph.

The highest concentration of data values is around 4.8 letters per word. The data seem to pile up around this value, with many sample means between 4.4 and 5.1 letters per word. Notice that the first random sample had an average of 5.3 letters per word. By collecting multiple samples and graphing the means of each sample, you can visually see the variation among the means.



Talk About It!

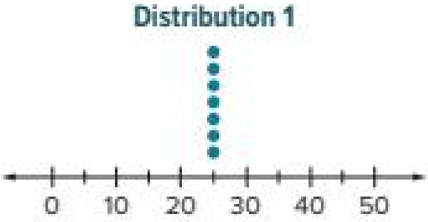
Why can you be more confident in making inferences about a population when you use multiple samples, as opposed to one?

(continued on next page)

Variability describes how the data vary within a sample or set of samples. Taking multiple samples of the same size helps you to understand the variability among the samples. You can see how "far off" your predictions might be had you only used one or two samples.

A graph of multiple samples will only report the mean value in each sample as a single data point. The amount of variability can be informally described based on the visual distribution of values as having high, low, or no variability.

Consider the distribution shown.



Do you think the distribution has high, low, or no variability among the samples? Recall that you learned about the mean absolute deviation in a previous grade.

The mean absolute deviation, which is a measure of variability, is the average distance each data value is from the mean of the samples. To find the mean absolute deviation, first find the mean of the samples.

$\text{mean} = \frac{25 + 25 + 25 + 25 + 25 + 25 + 25}{7}$ mean = $\frac{\text{sum of data values}}{\text{number of data values}}$

$= \frac{175}{7}$ Simplify.

$= 25$ Divide. The mean is 25.

Talk About It!

How can you know the mean is 25 without calculating it? How can you know the mean absolute deviation is 0 without calculating it?

To find the MAD, find the mean distance each data value is from the mean.

$\text{MAD} = \frac{0 + 0 + 0 + 0 + 0 + 0 + 0}{7}$ Because each data value is also 25, the distance between each data value and the mean is 0.

$= \frac{0}{7}$, or 0 Simplify. The MAD is 0.

Because the mean absolute deviation is 0, the distribution has no variability. You can visually see this on the graph because the data values do not vary.

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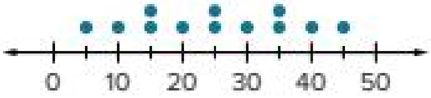
Consider the distribution shown. Do you think the distribution has high, low, or no variability among the samples?

Distribution 2



Find the mean absolute deviation.

First find the mean of the samples.

$$\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}}$$

$$= \frac{5 + 10 + 2(15) + 20 + 2(25) + 30 + 2(35) + 40 + 45}{12}$$

$$= \frac{300}{12} \qquad \text{Simplify.}$$

$$= 25 \qquad \text{Divide. The mean is 25.}$$

To find the MAD, find the mean distance each data value is from the mean. Remember that distance is always positive.

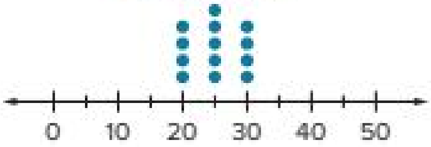
$$\text{MAD} = \frac{20 + 15 + 2(10) + 5 + 2(0) + 5 + 2(10) + 15 + 20}{12} \qquad \text{Find each distance from the mean.}$$

$$= \frac{120}{12}\text{, or } 10 \qquad \text{Simplify. The MAD is 10.}$$

This distribution has a greater mean absolute deviation than Distribution 1, because 10 > 0. So, it has a greater variability. You can see this by visually comparing the distributions.

Study the distribution shown. Do you think this distribution has a greater variability among the samples than Distribution 1 or Distribution 2?

Distribution 3



Find the mean absolute deviation. Then compare this distribution's variability to Distribution 1 and Distribution 2.

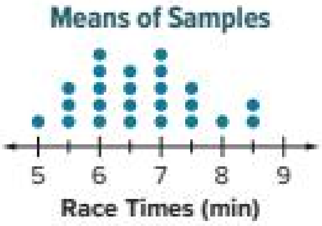
Show your work here

Talk About It!

Without calculating the mean absolute deviation, how do you know that Distribution 2 has a greater variability than Distribution 1?

Example 1 Analyze Means of Multiple Samples

The dot plot shows the means of 24 random samples of 20 runners' times, across local high schools, for a one-mile race. Each dot represents the mean of one random sample.



Which race time is the best estimate of the population mean? Find and interpret the variability in the distribution.

Part A Which race time is the best estimate of the population mean?

The sample means seem to pile up between 6 and 7 minutes, or about 6.5 minutes. Find the mean of the distribution.

mean

$$= \frac{5 + 3(5.5) + 5(6) + 4(6.5) + 5(7) + 3(7.5) + 8 + 2(8.5)}{24}$$ mean $= \frac{\text{sum of data values}}{\text{number of data values}}$

$$= \frac{160}{24}$$ Simplify.

$$= 6.\overline{6} \text{ or about } 6.7$$ Divide. Round to the nearest tenth.

The mean of the distribution is 6.7 minutes, which is near where the sample means seem to pile up. This means the mean of the population is likely to be close to 6.7.

Part B Find and interpret the variability in the distribution.

To find the MAD, find the mean distance each data value is from the mean of 6.7 minutes. Remember that distance is always positive.

MAD

$$= \frac{1.7 + 3(1.2) + 5(0.7) + 4(0.2) + 5(0.3) + 3(0.8) + 1.3 + 2(1.8)}{24}$$ Find each distance from the mean.

$$= \frac{18.4}{24}\text{, or } 0.8$$ Simplify. Round to the nearest tenth.

The mean absolute deviation is 0.8 minute. This means that the average distance each data value is from the mean is 0.8 minute. This distribution has a relatively low variability. Because the distribution has a relatively low variability, the estimate of 6.7 minutes as the population mean is not far off from what the true population mean may be.

Think About It!

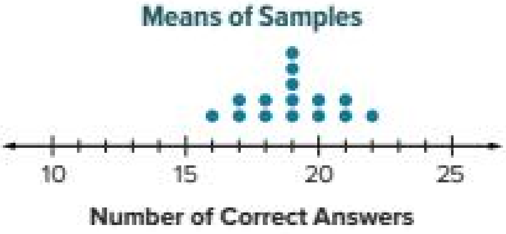
What does each dot on the dot plot represent?

Talk About It!

Do you think there is high, low, or no variability in the means of the samples? Justify your selection by describing the shape of the distribution.

Check

The scoring team of a national quiz bowl championship is analyzing the results of a 50-question assessment recently completed by all members. They collect 15 random samples of 20 assessments in each sample. The mean number of correct answers in each sample is shown on the dot plot.



How many correct answers is an appropriate estimate of the population mean? Find and interpret the variability in the distribution.

Part A How many correct answers is an appropriate estimate of the population mean?

Show your work here

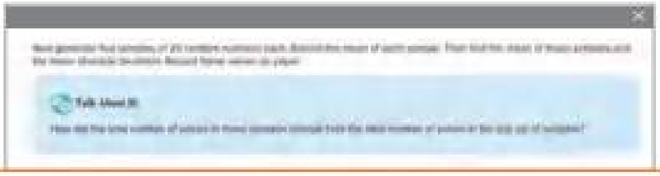
Part B Find and interpret the variability in the distribution of sample means.

Show your work here

Go Online You can complete an Extra Example online.

Explore Sample Size in Multiple Samples

Online Activity You will use Web Sketchpad to explore how increasing the sample size allows you to make more accurate predictions.



Apply Animal Science

Ten years ago, researchers randomly gathered 8 samples of 100 manatees each, and recorded their weights. This year, they repeated the experiment with 8 different samples of the same size. The table shows the mean weights of these samples. Can the researchers infer that the weight of the manatee population has less variation this year than from ten years ago? Explain your reasoning.

Sample	Ten Years Ago Mean Weight (lb)	This Year Mean Weight (lb)
1	944	937
2	980	943
3	1,025	897
4	962	1,000
5	886	963
6	872	985
7	1,052	964
8	975	999

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Talk About It!
How could you solve this problem another way?

Check

Five years ago, researchers randomly gathered six samples of 50 Walleye from a lake, and recorded their weights. This year, they repeated the experiment with six different samples of the same size. The table shows the mean weights of these samples. Can the researchers infer that the weight of the Walleye population in the lake has more variation this year than from five years ago? Explain.

Sample	Five Years Ago Mean Weight (lb)	This Year Mean Weight (lb)
1	23	30
2	26	21
3	22	24
4	24	27
5	24	25
6	25	17

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

What are some instances of sample size that you have seen in everyday life? How reliable do you think surveys are when the sample sizes are relatively small?

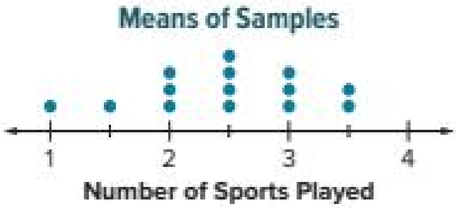
Record your observations here

Name ______________________ Period __________ Date __________

Practice

Go Online You can complete your homework online.

1. The dot plot displays data from 14 random samples, each consisting of 30 middle school students. Each dot represents the mean number of sports played per year by students in the sample. (Example 1)

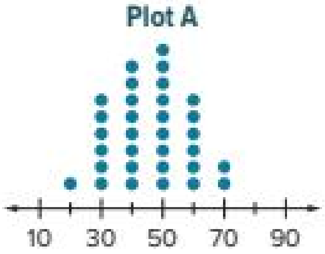

Means of Samples

a. Which number best represents the mean number of sports played by middle school students?

b. Find and interpret the variability in the distribution.

Test Practice

2. Open Response Below are two dot plots containing sample means from the same population.


Plot A


Plot B

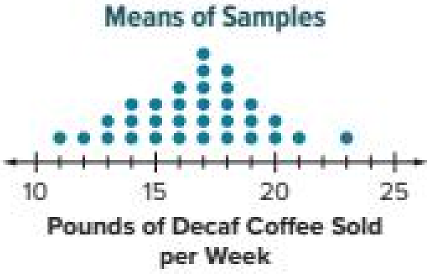
A. How many samples are represented in each plot? How do you know?

B. Which dot plot has higher variability? Defend your answer.

C. One plot contains samples of size 25, and the other plot contains samples of size 60. Which dot plot contains the samples of size 60? How do you know?

Apply

3. A large company is trying to determine the mean number of pounds of decaf coffee sold per week in its stores. The dot plot shows the mean pounds of decaf coffee sold per week from 32 samples of 50 stores each.



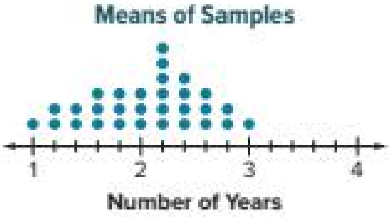
a. Describe the variability of the dot plot.

b. How might the dot plot be different if each of the 32 samples contained data from 200 stores?

c. The company randomly samples 50 of its stores and records the pounds of decaf sold per week for each store. A mean sale of 18 pounds of decaf coffee per week is calculated from this sample. Based on the sample mean of 18 and the variability observed in the dot plot, what range of values could be used to describe the population mean?

d. The company samples 200 stores and finds a mean of 17 pounds of decaf coffee sold per week. Based on your answer to Part B, what range of values might describe the mean for all stores in the company? Justify your answer.

4. MP **Find the Error** A student examines the dot plot below and states that it contains samples of size 30. Find the student's mistake and correct it.



5. Draw a dot pot with low variability. Write an argument to support why your dot plot has low variability.

Lesson 11-4

Compare Two Populations

I Can... use the measures of center and measures of variation to compare two samples and make comparative inferences about two populations.

What Vocabulary Will You Learn?

asymmetric distribution

distribution

double box plot

double line plot

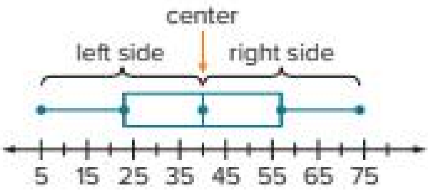
symmetric distribution

Learn Shape of Data Distributions

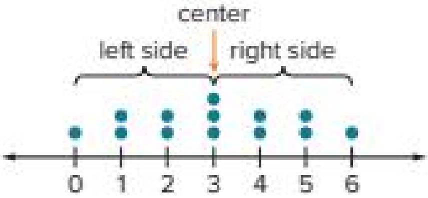
Data collected from a sample can be organized and displayed in a graph, such as a box plot or a dot plot. The shape of a graph is often referred to as its **distribution**, which shows the arrangement of data values.

In **symmetric distributions**, the shape of the graph to the left of the center is the same as the shape of the graph to the right of the center.

In symmetric box plots, the lengths of each box and whisker are similar.

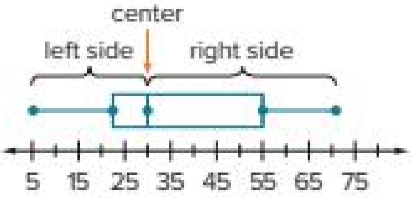


In symmetric dot plots, the frequencies of data values on either side of the center are similar.

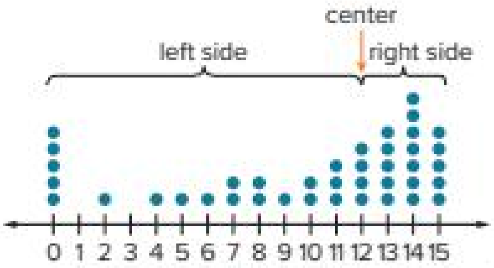


In **asymmetric distributions**, the shape of the graph on one side of the center is very different from the other side. The data may contain one or more outliers.

In asymmetric box plots, the lengths of the boxes and whiskers vary. Recall that a shorter length indicates the data are clustered together, while a longer length indicates the data are more spread out.



In asymmetric dot plots, the frequencies of data values on either side of the center vary.



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Your Notes

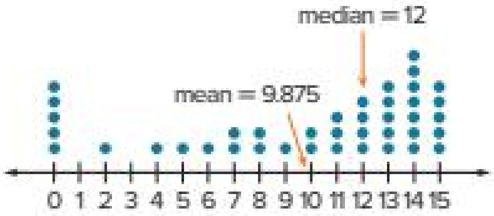
Talk About It!

Explain to a partner how to find the mean and median of a set of data. You learned these measures of center in a previous grade.

Talk About It!

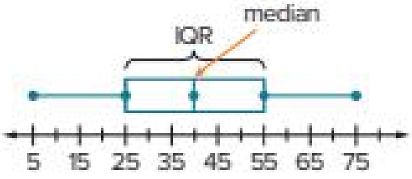
Explain to a partner how to find the mean absolute deviation and interquartile range of a set of data. You learned these measures of center in a previous grade.

In the distribution shown, the mean is less than the median because the mean is affected by the five data values of 0. Because of this, the median is the most appropriate measure of center for asymmetric data. For symmetric data, you can use the mean or the median.



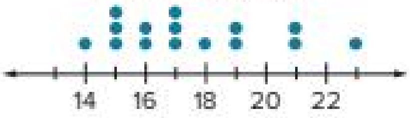
After selecting the appropriate measure of center, select the corresponding measure of variation. If you select the mean to describe the center, select the mean absolute deviation (MAD) to describe how the data vary from the mean. If you select the median to describe the center, select the interquartile range (IQR) to describe how the data vary.

For symmetric or asymmetric box plots, select the median and interquartile range because box plots are constructed to display these values.

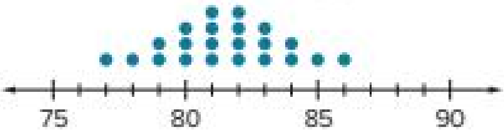


Select the appropriate measure of center and variation based on the shape of each distribution. Write either *mean and mean absolute deviation* or *median and interquartile range*. If you can use either, write *either*.

Race Times (min)



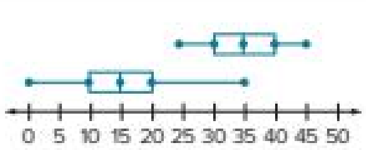
Temperature (°F)



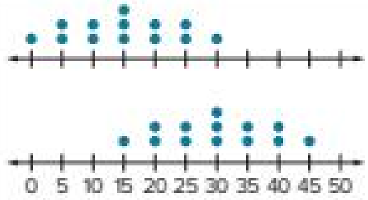
Learn Compare Two Populations

A **double box plot** consists of two box plots graphed on the same number line. A **double dot plot** consists of two dot plots that are constructed so that the values on each number line align. You can draw inferences about two populations represented by a double box plot or a double dot plot by comparing their centers and variations.

Sample Double Box Plot



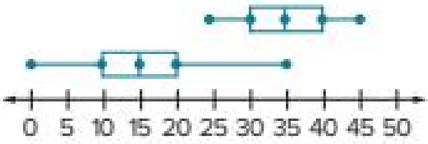
Sample Double Dot Plot



To select the appropriate measures of center and variability to compare populations in double box plots or double dot plots, check for symmetry in each data set.

Symmetry	Measure of Center	Measure of Variation
both sets of data are symmetric	mean or median	mean absolute deviation or interquartile range
neither set of data is symmetric	median	interquartile range
only one set of data is symmetric	median	interquartile range

Use the measures of center and variation to compare the two populations. Refer to the double box plot shown.



What is the median of the top box plot? ______

What is the median of the bottom box plot? ______

How do the centers compare? The median of the top box plot is more than twice the median of the bottom box plot.

What is interquartile range of the top box plot? ____________

What is interquartile range of the bottom box plot? ____________

How do the data in the populations vary around the median? Because the IQRs are the same, the data are similarly clustered around each median, although the medians are different.

Talk About It!

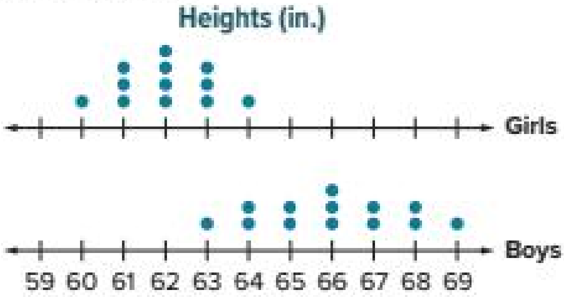
What else do you notice about the two populations?

Example 1 Compare Two Populations

Think About It!

What does each dot on the graph represent?

The double dot plot shows the heights, in inches, for the girls and boys in Imani's math class.



Use the measures of center and variability of this sample to make an inference about the heights of students in Imani's grade at school.

Step 1 Compare the measures of center and variation.

Both dot plots are symmetric. You can use either the mean and mean absolute deviation, or the median and interquartile range. For this example, the mean and mean absolute deviation (MAD) are selected.

Find each mean.

Girls

$$\text{mean} = \frac{60 + 3(61) + 4(62) + 3(63) + 64}{12}$$

$\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}}$

$$= \frac{744}{12} \text{ or } 62$$

Simplify. The mean height for girls is 62 in.

Boys

$$\text{mean} = \frac{63 + 2(64) + 2(65) + 3(66) + 2(67) + 2(68) + 69}{13}$$

$\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}}$

$$= \frac{858}{13} \text{ or } 66$$

Simplify. The mean height for boys is 66 in.

Find each mean absolute deviation (MAD). To find the MAD, find the mean distance from each data value to the mean.

Girls

$$\text{MAD} = \frac{2 + 3(1) + 4(0) + 3(1) + 2}{12}$$

The mean is 62 in. Find each distance from the mean.

$$= \frac{10}{12} \text{ or about } 0.83$$

Simplify. The MAD for girls is about 0.83 in.

Boys

$$\text{MAD} = \frac{3 + 2(2) + 2(1) + 3(0) + 2(1) + 2(2) + 3}{13}$$

The mean is 66 in. Find each distance from the mean.

$$= \frac{18}{13} \text{ or about } 1.38$$

Simplify. The MAD for boys is about 1.38 in.

(continued on next page)

The mean height for boys is greater than the mean height for girls. There is a greater variation around the mean height for boys than for girls, because 1.38 > 0.83. The girls' heights are more closely clustered together than the boys' heights.

Step 2 Make an inference about the heights of students in Imani's grade at school.

Based on these samples, you can infer that the boys in Imani's grade are generally taller than the girls.

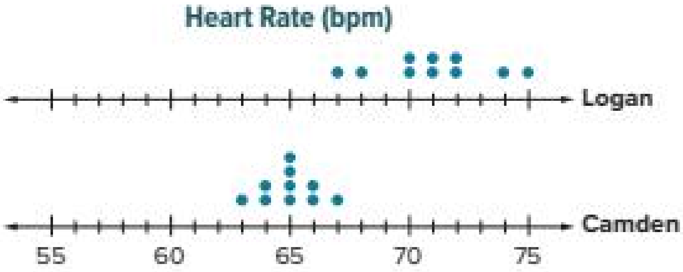
This inference is based on these samples alone. Different samples may lead to different inferences about the populations.

Talk About It!

Can you make any other inferences about the heights of students in Imani's grade at school, based on these samples?

Check

Camden and Logan record their resting heart rates each morning for ten days. The double dot plot shows their heart rates in beats per minute. Use the measures of center and variability of these samples to select the person(s) to which each statement applies.



	Logan	Camden
The mean heart rate is 65 bpm.		
The dot plot is symmetric.		
This person is likely to have a higher heart rate on a randomly selected day.		
The data have greater variability.		
This person is more likely to have a heart rate of 65 bpm on a randomly selected day.		

Show your work here

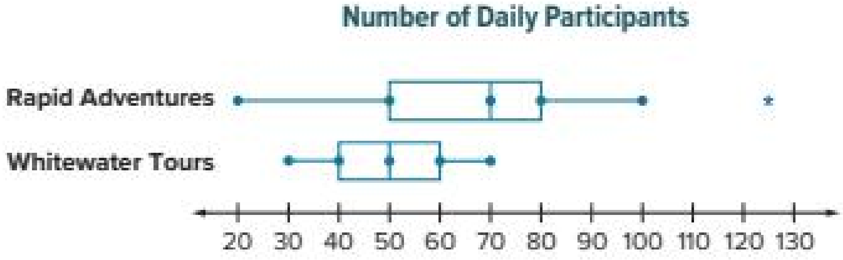
Go Online You can complete an Extra Example online.

Example 2 Compare Two Populations

Think About It!

Without calculating, which box plot has a greater spread of data? How do you know?

The double box plot shows the number of daily participants for two adventure companies.



Use the measures of center and variation of this sample to make an inference about the daily participants for each adventure company.

Step 1 Compare the measures of center and variation.

The distribution for one company, Rapid Adventures, is asymmetric and contains an outlier, indicated by the asterisk (*). So, the median and interquartile range are the most appropriate measures.

Find each median.

Rapid Adventures

The median is 70 daily participants.

Whitewater Tours

The median is 50 daily participants.

Find each interquartile range (IQR).

Rapid Adventures

IQR = 80 − 50

= 30

IQR = $Q_3 - Q_1$

Subtract.

Whitewater Tours

IQR = 60 − 40

= 20

The median number of daily participants is greater for Rapid Adventures than Whitewater Tours. There is greater variability among the data for Rapid Adventures than for Whitewater Tours. The data are more closely clustered around the center for Whitewater Tours.

Step 2 Make an inference about the population of daily participants for the two adventure companies.

Based on these samples, you can infer that, on any randomly selected day, it is likely that Rapid Adventures will have a greater number of daily participants. However, the number of daily participants for Whitewater Tours is more likely to be consistent.

This inference is based on these samples alone. Different samples may lead to different inferences about the populations.

Check

The students of a middle school start a blog for their English class and contribute to it all year long. The double box plot shows the results for how often 7th- and 8th- grade students contribute to the blog.

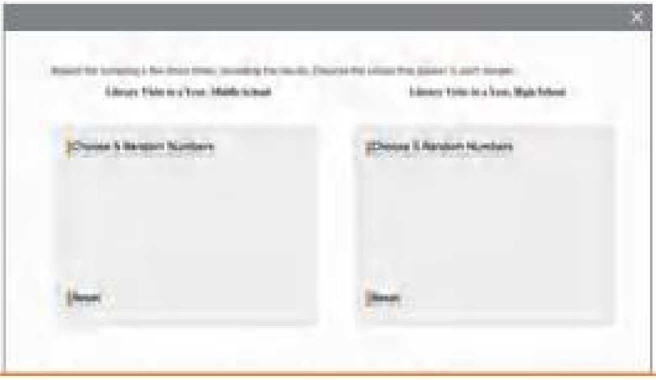
Use the measures of center and variability of these samples to select all of the statements that can be inferred about the data.

- ☐ The students in the 8th grade posted blogs more often than students in 7th grade.
- ☐ The students in the 7th grade posted blogs more often than students in 8th grade.
- ☐ The amount of variability for 7th graders is greater than that for 8th graders.
- ☐ Every student in 8th grade posted more blogs than every student in 7th grade.
- ☐ 25% of 8th graders posted at least 25 blogs throughout the year.

Go Online You can complete an Extra Example online.

Explore Compare Means of Two Populations

Online Activity You will use Web Sketchpad to explore how you can use samples from different populations to make comparative inferences about the population means.



Pause and Reflect

Create a graphic organizer that will help you study the concepts you learned today in class.

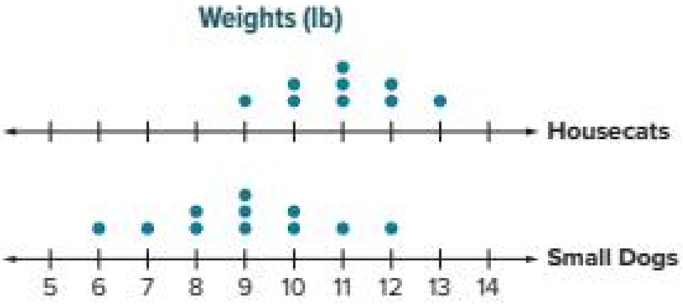
Record your observations here

Name ______________________ Period __________ Date __________

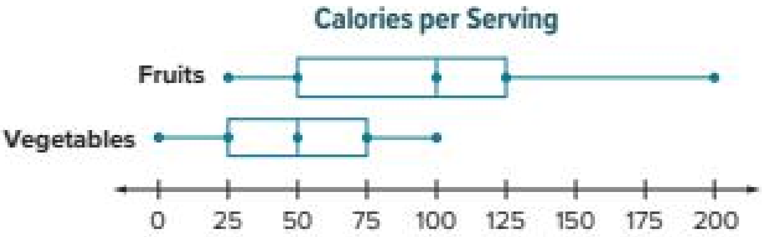
Practice

Go Online You can complete your homework online.

1. The double dot plot shows the weights in pounds of several housecats and small dogs. Compare their centers and variability. What are some appropriate inferences you can make about the data? (Example 1)

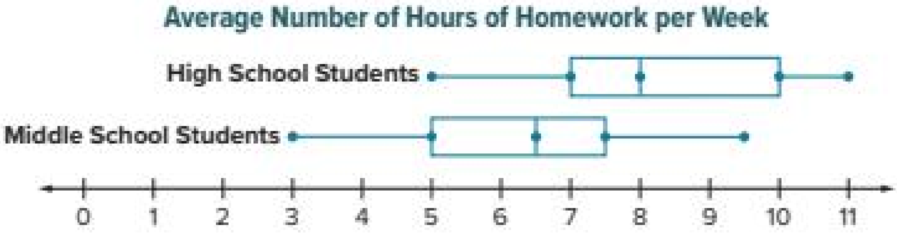


2. The double box plot shows the number of Calories per serving for various fruits and vegetables. What are some appropriate inferences you can make about the data? (Example 1)



Test Practice

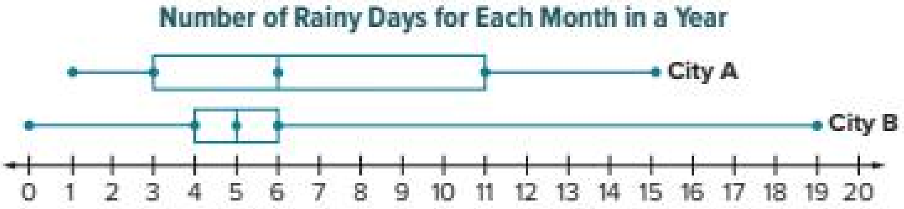
3. **Table Item** The double box plot represents the average number of hours of homework each week for high school students and middle school students. Use the measures of center and variability of these samples to select the age group(s) to which each statement applies.



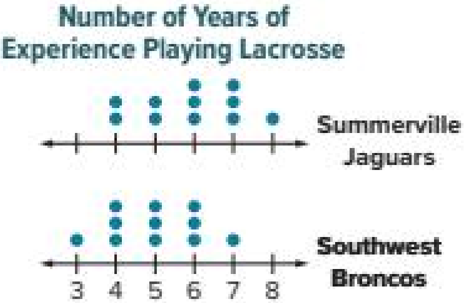
	Middle School	High School
The median is greater.		
The IQR is 2.5.		
The data have greater variability.		
A person from this sample is more likely to have more than 7 hours of homework a week.		
The data are more symmetric.		

Apply

4. The double box plot shows the number of rainy days for each month in a year for two different cities. For which city is it more likely that a randomly selected month will have 6 or more rainy days?


Number of Rainy Days for Each Month in a Year

5. The double dot plot shows the number of years of experience playing lacrosse for members of two high school lacrosse teams. A player with six years of experience is on a lacrosse team. On which team is the player more likely to be? Write an argument that can be used to defend your solution.


Number of Years of Experience Playing Lacrosse

6. **Find the Error** A student claims that 50% of a sample of data is less than the median and 50% of data is greater than the median, therefore the data is symmetric. Explain the student's error and correct it.

7. **Create** Create a double box plot in which both sets of data have the same median, but the IQR for one set is twice that of the other.

8. Explain when it would be appropriate to use the mean and mean absolute deviation to compare two populations.

Assess Visual Overlap

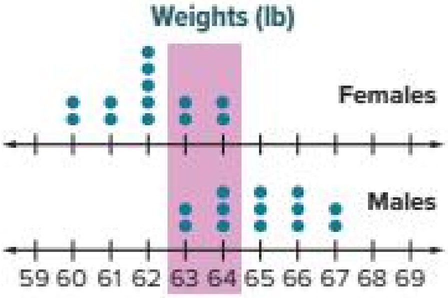
I Can... assess the amount of visual overlap between two distributions to make comparative inferences about two populations.

What Vocabulary Will You Learn?
visual overlap

Learn Interpret Visual Overlap

You have learned how to make inferences about two populations by comparing the centers and variations of the sample distributions. You can also make inferences about how likely it is that the means of the populations are similar or different.

A local veterinarian, Dr. Gibson, weighed a random sample of male and female adult dogs. The results are shown in the double box plot. Dr. Gibson wants to know how likely it is that the mean weights of female and male adult dogs are different.



To do so, Dr. Gibson can informally assess the degree of visual overlap between the two distributions. The **visual overlap** shows the distance between the two means of the distributions, as seen in the shaded section of the double dot plot.

To informally assess the degree of visual overlap, express the difference in the means as a multiple of the measure of variability. In other words, find and analyze the ratio $\frac{\text{difference in means}}{\text{MAD}}$.

Step 1 Both distributions are symmetric. Find the means and mean absolute deviations.

Find each mean.

Females

$$\text{mean} = \frac{2(60) + 2(61) + 5(62) + 2(63) + 2(64)}{13}$$
$$= \frac{806}{13} \text{ or } 62$$

Males

$$\text{mean} = \frac{2(63) + 3(64) + 3(65) + 3(66) + 2(67)}{13}$$
$$= \frac{845}{13} \text{ or } 65$$

The mean weights for males and females differ by about 65 − 62, or 3 pounds.

(continued on next page)

Talk About It!
Just by looking at the samples in the double dot plot, how likely do you think it is that the population means are different? Explain.

Your Notes

Talk About It!
The MADs were not exactly the same. Provide an argument for why they can be viewed as similar.

Talk About It!
Make sense of the phrase *the means are separated by about 3 MADs.* Describe this in your own words.

Talk About It!
Draw a double dot plot in which both data sets are symmetric, have similar variability, and a noticeable separation between the samples.

Find each mean absolute deviation (MAD), by finding the mean distance from each data value to the mean.

Females

$$\text{MAD} = \frac{2(2) + 2(1) + 5(0) + 2(1) + 2(2)}{13} = \frac{12}{13} \text{ or about } 0.92$$

Males

$$\text{MAD} = \frac{2(2) + 3(1) + 3(0) + 3(1) + 2(2)}{13} = \frac{14}{13} \text{ or about } 1.08$$

To the nearest whole pound, the variability, MAD, of each distribution is 1 pound. This means the average distance each dog's weight in the sample is from the mean is 1 pound. The distributions have similar variability, but different means.

Step 2 Find the ratio $\frac{\text{difference in means}}{\text{MAD}}$.

$\frac{\text{difference in means}}{\text{MAD}} = \frac{3}{1}$ Write the ratio of the difference in the means, 3, to the mean absolute deviation, 1.

$= 3$ Simplify.

The means are separated by $\frac{3}{1}$, or 3 MADs.

Step 3 Analyze the ratio.

When the variability is small, such as 1 pound, the data are less spread out and are more consistent. Dr. Gibson can have a higher confidence in making an inference that it is likely the means are different, because the sample means are different *and* the variability is low. The ratio $\frac{\text{difference in means}}{\text{MAD}}$ takes both of these into account.

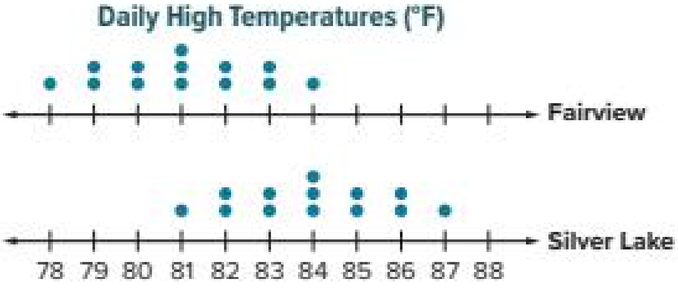
The greater the ratio, the more likely it is the means of the populations are different. In this lesson, you will use the following conventions to informally assess how likely it is that the means of the populations are similar or different based on this ratio.

Sample Distributions		
$\frac{\text{difference in means}}{\text{MAD}}$	Separation Between the Samples	Means of the Population
less than 2	should be less noticeable	more likely to be the same
greater than 2	should be more noticeable	more likely to be different

The ratio in this scenario is $\frac{3}{1}$ or 3. Because $3 > 2$, the separation between the samples for males and females is noticeable. The mean weights of male and female dogs are likely to be different.

Example 1 Measure Variability Between Populations

The double dot plot shows the daily high temperatures of two cities from thirteen randomly chosen days in July. The table gives the mean and mean absolute deviation for each city.



	Fairview	Silver Lake
Mean (°F)	82	84
MAD (°F)	1.4	1.4

How many measures of variability separate the means of the samples? Make an inference as to whether or not the average high temperature in July is likely to be different for each city.

Part A Find the number of measures of variability that separate the means of the samples.

Find the ratio $\frac{\text{difference in means}}{\text{MAD}}$.

$\frac{\text{difference in means}}{\text{MAD}} = \frac{84 - 82}{1.4}$ Write the ratio of the difference of the means to the MAD.

$= \frac{2}{1.4}$ Simplify.

≈ 1.43 Divide. Round to the nearest hundredth.

The means are separated by about 1.43 measures of variability.

Part B Make an inference as to whether or not the means of the two populations are likely to be different.

Using the conventions provided earlier in the Learn, a ratio that is less than 2 indicates the means of the population are likely to be the same. Because 1.43 < 2, it is likely that the average high temperature in July is the same for each city.

Think About It!

Describe the visual overlap in your own words.

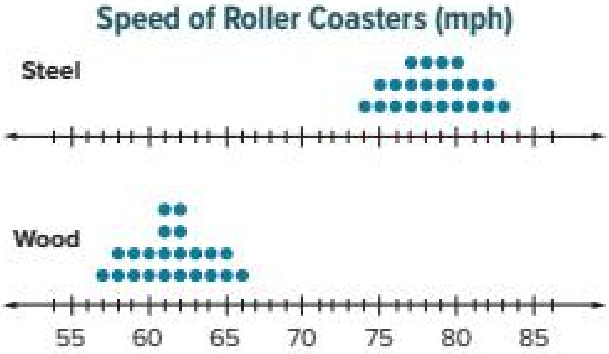
Talk About It!

Is it possible to infer that the means of two populations are likely different when their dot plots overlap? Explain.

Check

The double dot plot shows the results of a random sample of the top speeds of both wood and steel coasters. The table gives the mean and mean absolute deviation for each type of material.

How many measures of variability separate the means of the samples? Then make an inference that compares the means of the populations.



	Roller Coaster	
	Wood	Steel
Mean (mph)	61.5	78.5
MAD (mph)	2.0	2.0

Part A How many measures of variability separate the means of the samples?

Part B Make an inference as to whether or not the average top speeds of wood and steel coasters are likely to be different.

Show your work here

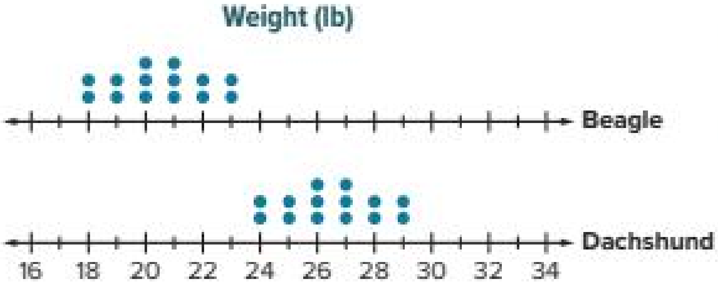
Go Online You can complete an Extra Example online.

Name ________________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

1. The double dot plot shows sample weights of two breeds of dogs. The table gives the mean and mean absolute deviation for each breed. (Example 1)



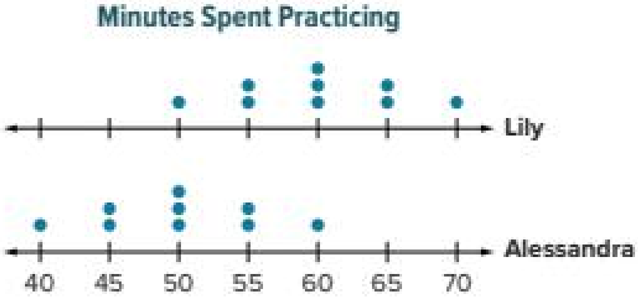
	Beagle	Dachshund
Mean (lb)	20.5	26.5
MAD (lb)	1.36	1.36

a. How many measures of variability separate the means of the samples?

b. Make an inference as to whether or not the means of the two populations are likely to be different.

Test Practice

2. Multiselect The double dot plot shows the number of minutes two students spent practicing the piano on random days this month. The table gives the mean and mean absolute deviation for each student. Select each statement that is true about the data.



	Lily	Alessandra
Mean (min)	60	50
MAD (min)	4.4	4.4

☐ The means are separated by about 2.3 measures of variability.

☐ On a randomly selected day, it is likely that Lily practices playing the piano more than Alessandra.

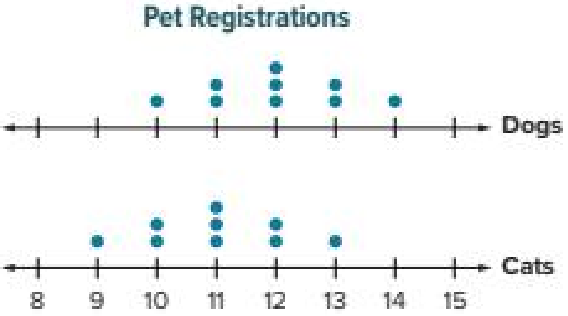
☐ The means are separated by 0 measures of variability because the shapes of the data sets are equal.

☐ Because the mean absolute deviations are the same, there is no difference in means of the data sets.

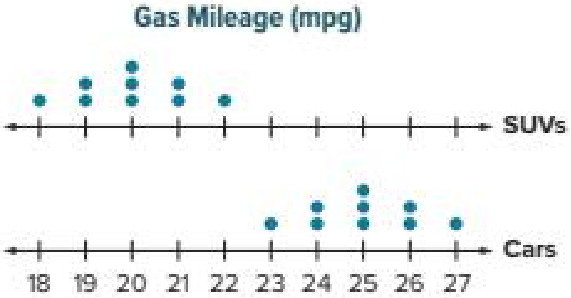
☐ On a randomly selected day, it is likely that Alessandra practices playing the piano more than Lily.

Apply

3. The double dot plot shows the number of city pet registrations for randomly selected days this month. How many measures of variability separate the means of the samples? Make an inference about the means of each population.


Pet Registrations

4. The double dot plot shows the gas mileage, in miles per gallon, for several cars and SUVs. How many measures of variability separate the means of the samples? Make an inference about the means of each population.


Gas Mileage (mpg)

5. **Reason Abstractly** Suppose the measures of variability between the mean of two samples is 1.05. Explain the meaning of this ratio.

6. **Justify Conclusions** Determine if the following statement is *true* or *false*. Explain.

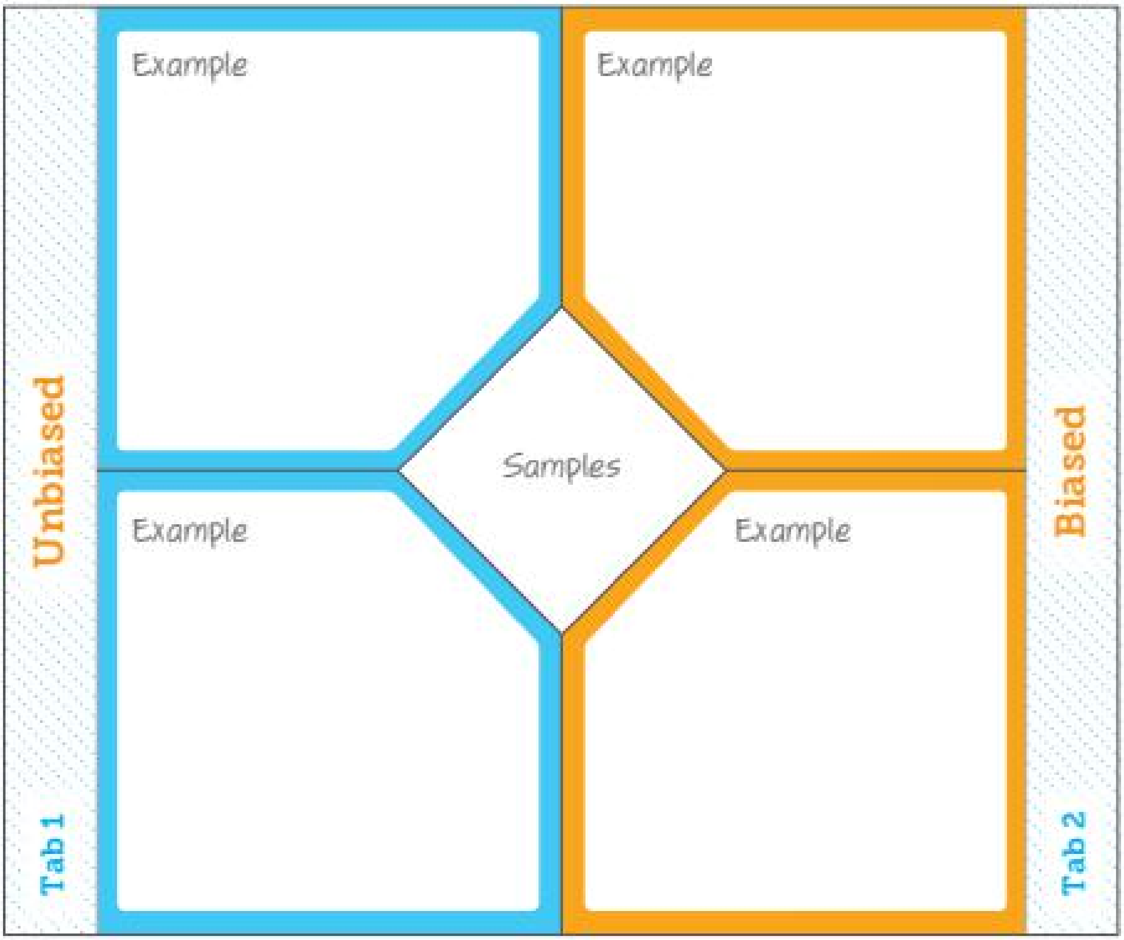
   The greater the ratio of the difference in centers to the greater variability, the more likely it is that the means of their populations are the same.

7. Give an example of a double box plot where neither set of data displayed is symmetric.

Module 11 • Sampling and Statistics

Review

Foldables Use your Foldable to help review the module.



Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.

Write about a question you still have.

Reflect on the Module

Use what you learned about sampling and statistics to complete the graphic organizer.

Essential Question

How can you use a sample to gain information about a population?

Simple Random Sample	Stratified Random Sample	Systematic Random Sample
Definition	Definition	Definition
Example	Example	Example

Name ______________________ Period ____________ Date ____________

Test Practice

1. **Open Response** Consider the sampling method described: From a production run of 15,600 LED monitors, every 120th is tested. (Lesson 1)

   A. What sampling method best describes this scenario?

   B. Identify the population of the study. Is the sample likely to be representative of the population? Explain why or why not.

2. **Open Response** On a social networking app, an amusement park asked all of its followers to vote on their favorite park attraction. The results are shown in the table.

Attraction	Percent of Votes
rollercoasters	42%
water slides	30%
games	15%
shows	13%

Based on the results, the amusement park infers that the most popular attraction is rollercoasters. (Lesson 1)

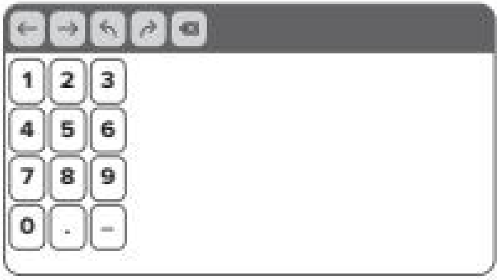
   A. What sampling method best describes this scenario?

   B. Determine whether the inference made by the amusement park is valid. Explain your reasoning.

3. **Equation Editor** A food truck owner is ordering ingredients for their grand opening weekend. In order to determine how much is needed, he conducts a survey of 75 randomly selected potential customers asking which item they are likely to order. The results are shown in the table. (Lesson 2)

Item	Frequency
Tacos	23
Burritos	18
Chimichangas	13
Fajitas	9
Tamales	12

The owner expects to have 400 customers over the weekend. Based on the survey results, predict the number of burritos that will be sold.



4. **Multiple Choice** A grocery store conducts a random survey of its customers and finds that 78% would sign up for a rewards program if it were offered. If the rewards program is rolled out this weekend and there are 1,550 customers, how many of them would you expect to sign up for the program? (Lesson 2)

Ⓐ 341 customers
Ⓑ 429 customers
Ⓒ 1,145 customers
Ⓓ 1,209 customers

5. Open Response In order to analyze how much time students typically spend doing homework each night, a teacher takes 15 random samples of 20 students each. The graph shows the mean of each sample (rounded to the nearest half hour). (Lesson 3)

A. What is the best estimate of the mean number of hours spent per night doing homework of the population?

B. Describe the variability in the distribution.

6. Multiple Choice Wildlife conservationists want to estimate the deer population in a region. They place an identifying tag on 220 deer. Several months later, researchers investigate 8 samples of 50 deer and record how many have a tag. Based on these results, how many deer should they estimate live in the region? (Lesson 3)

Sample	1	2	3	4	5	6	7	8
Number of Tagged Deer	3	1	5	4	2	2	1	4

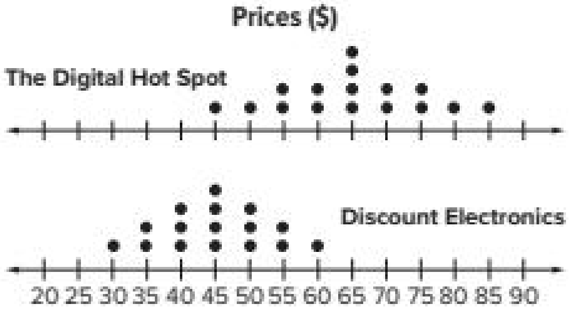
Ⓐ 2,750 deer

Ⓑ 3,175 deer

Ⓒ 3,500 deer

Ⓓ 4,000 deer

7. Multiple Choice The double dot plot shows the costs, in dollars, of MP3 players at two different stores. (Lesson 5)



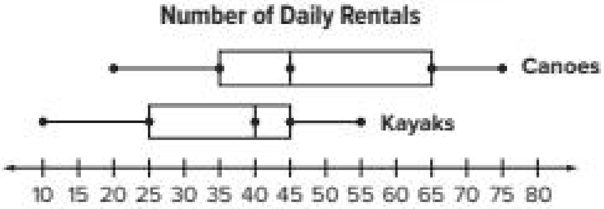
Which of the following are appropriate inferences that can be made about the data?

Ⓐ Both distributions are asymmetric.

Ⓑ The mean price at the Digital Hot Spot is higher than the mean price at Discount Electronics.

Ⓒ The prices at Discount Electronics have a higher variability than the prices at the Digital Hot Spot.

8. Table Item The double box plot shows the number of daily canoe and kayak rentals at Riverside Adventures. Based on the data, identify the type of equipment rental to which each statement applies. (Lesson 4)



	Canoes	Kayaks
This data set has an interquartile range of 20.		
This data set has the day with the highest number of rentals.		
This data set has a smaller displayed measure of variation.		

Selected Answers

Lesson 5-1 Simplify Algebraic Expressions, Practice Pages 241–242

1. $1.25c$ **3.** $-17y - 16z + 4$ **5.** $-7c - 2d - 6$
7. $\frac{3}{10}y + 1\frac{1}{8}$ **9.** $-6x + 10$ **11.** $15y - 10z$
13. $-7x + 14$

15.

Triangle Side	Length (units)
1	$2(a + 3)$
2	$3a - 1$
3	$a - 2$

17. The perimeter of Quadrilateral 1 is $13x + 6$. The perimeter of Quadrilateral 2 is $14x - 5$. Quadrilateral 1 has the greater perimeter if $x = 3$. This will not be true if x is greater than or equal to 11. **19.** Sample answer: The student multiplied 4 by 3 instead of -3. $5x - 3(x + 4) = 2x - 12$

Lesson 5-2 Add Linear Expressions, Practice Pages 249–250

1. $2x + 7$ **3.** $-3x - 4$ **5.** $-14x + 1$
7. $\frac{7}{16}x + 2$ **9.** $x + \frac{1}{3}$ **11.** $-\frac{5}{12}x - 8$
13. $10x + 4$ units or $2(5x + 2)$ units **15.** $258
17. $5x - 3$ **19.** The sum will be zero when the coefficients of th x-terms are opposites.

Lesson 5-3 Subtract Linear Expressions, Practice Pages 257–258

1. $-3x + 6$ **3.** $4x + 8$ **5.** $2x + 1$ **7.** $-8x - 9$
9. $\frac{4}{15}x + \frac{7}{8}$ **11.** $-\frac{1}{2}x - \frac{1}{3}$ **13.** $(3x + 4)$ points
15. $374 **17.** $6x + 3$ **19.** Sample answer: The student only added the additive inverse of $-x$ and not 5. $(6x - 2) - (-x + 5) = 7x - 7$

Lesson 5-4 Factor Linear Expressions, Practice Pages 265–266

1. $4y$ **3.** $8m$ **5.** $14ab$ **7.** $5(x + 7)$
9. cannot be factored **11.** $18x(4 - y)$
13. cannot be factored **15.** $\frac{1}{2}(x + 1)$
17. $9 **19.** Sample answer: $3x$, $9x$
21. c. $7x + 3$; All the other expressions can be factored but $7x + 3$ cannot be factored.

Lesson 5-5 Combine Operations with Linear Expressions, Practice Pages 271–272

1. $4(2x + 3)$ **3.** $15(x + 2)$ **5.** $\frac{13}{24}x - 3$
7. $\frac{3}{4}x + 5$ **9.** $2(7x + 11)$ **11.** $12(x - 1)$
13. $2(9x + 4)$ **15.** $5(3x + 13)$ **17.** Sample answer: The student forgot to distribute -3 to both terms inside the parentheses. The student only distributed it to the first term. The correct answer is $3(x - 2)$. **19a.** Sample answer: $\frac{1}{2}(x + 12)$ **19b.** Sample answer: $\frac{3}{4}(x - 24)$

Module 5 Review Pages 275–276

1. $1.15c$ **3A.** Triangle 1: $-6x + 105$; Triangle 2: $20x - 30$ **3B.** Triangle 1 **5.** C **7.** $-\frac{1}{2}x - 2$; Sample answer: I used the Distributive Property for the subtraction sign through the second parentheses. Then I used the Commutative Property to group like terms. Then I combined like terms. **9.** C **11.** A **13.** $2x - 3$

Lesson 6-1 Write and Solve One-Step Equations, Practice Pages 287–288

1. $y = -14$ **3.** $p = 6$ **5.** $x = -8$ **7.** $d = 54$
9. $x = -\frac{25}{56}$ **11.** $b = -30$ **13.** $c = -1.1$
15. $n = 1.5$ **17.** $d + 10\frac{3}{4} = -56\frac{1}{2}$; $-67\frac{1}{4}$ ft
19. $2x = 48.50$; Samuel earned $24.25; $x + 7 = 24.25$; Kara earned $17.25; The difference is $31.25. **21.** The student divided each side by 2 instead of multiplying each side by 2. $x = -80$ **23.** The value of b decreases by 1. Sample answer: $(a + 1) + (b - 1) = 10$ if $a + b = 10$. If 1 is added, then 1 must be subtracted for the sum to remain the same.

Lesson 6-2 Solve Two-Step Equations: $px + q = r$, Practice Pages 297–298

1. $x = 3$ **3.** $x = -4$ **5.** $x = -4$ **7.** $x = 7.5$ **9.** $x = 3$ **11.** $x = -50$ **13.** 5 fruit baskets **15.** 25.7°C **17.** The student added −5 to both sides instead of adding +5 to both sides. The solution is $x = 10$. **19.** Sample answer: The problems both involve multiplying by 4 and subtracting 1. They are different because $4(3) - 1$ is an expression, while $4x - 1 = 13$ is an equation in which you solve by adding 1 then dividing by 4.

Lesson 6-3 Write and Solve Two-Step Equations: $px + q = r$, Practice Pages 305–306

1. $29.50p + 15 = 133$; 4 people **3.** $0.60m + 1.50 = 13.50$; 20 mi **5.** $100\frac{1}{5} - 10\frac{4}{5}m = 57$; 4 min **7.** $2c - 4 = 24.50$; $14.25 **9.** 6 hours **11.** Sample answer: You and your friend spent a total of $35 on dinner. Your dinners were the same cost and you ordered a $5 dessert. What was the cost of your dinner? $15

13a.

Number of Hours	1	2	3
Gallons Left	22	16	10

13b. $28 - 6x = 4$; 4 h

Lesson 6-4 Solve Two-Step Equations: $p(x + q) = r$, Practice Pages 315–316

1. $x = 3$ **3.** $x = -13$ **5.** $x = 17$ **7.** $x = 52$ **9.** $x = 55$ **11.** $x = 18$ **13.** 3.7 **15.** 15.88 m **17.** Sample answer: The student only distributed the −2 to the first term inside the parentheses instead of both terms. The correct solution is $x = -1$. **19a.** $x = 8$ **19b.** $x = 9$

Lesson 6-5 Write and Solve Two-Step Equations: $p(x + q) = r$, Practice Pages 323–324

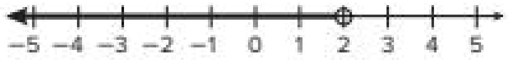
1. $6(1\frac{1}{4} + b) = 10$; $\frac{5}{12}$ yd **3.** $8(a + 12) = 168$; 9 apples **5.** $3(b - 8) = 21$; 15 balloons **7.** $6(p - 0.55) = 8.7$; $2 **9.** 6.75 in. **11.** Sample answer: You and 11 friends go bowling. Shoe rental costs $2.50. The total cost of one game and one shoe rental for everyone is $78. What is the cost of one game of bowling for one person? $4 **13.** Sample answer: You can estimate the solution of the equation by rounding each constant and coefficient. $x - 4) = 36$, which has a solution of 8. So, the solution of the given equation should be close to 8. Therefore, the student's solution must be incorrect.

Module 6 Review Pages 327–328

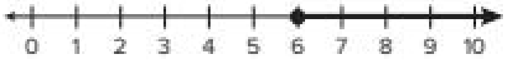
1A. $w = -12$ **1B.** I can check my solution by substituting the result back into the original equation; $-7 = w + 5 - 7 = -12 + 5$; $-7 = -7$ **3.** B **5A.** C **5B.** $x = 2$ **7.** 6 **9.** Let h = number of hours; $73 - 3.5h = 59$; $h = 4$; So, it will take 4 hours for the temperature to reach 59°F. **11.** 134.8 meters

Lesson 7-1 Solve One-Step Addition and Subtraction Inequalities, Practice Pages 339–340

1. $x < 2$

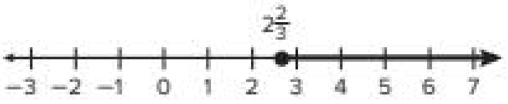


3. $x \geq 6$

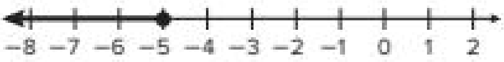


5. $x < -7$

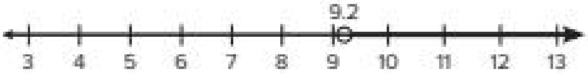
7. $x \geq 2\frac{2}{3}$



9. $x \leq -5$



11. $x > 9.2$



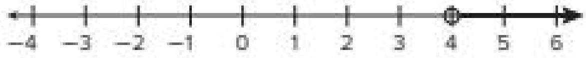
13. $x > 2\frac{3}{4}$ pounds; more than 44 ounces of brown sugar **15.** Sample answer: $x + 7 \leq 12$; $x - 4 \leq 1$ **17.** $x < z + y$

Lesson 7-2 Write and Solve One-Step Addition and Subtraction Inequalities, Practice Pages 347–348

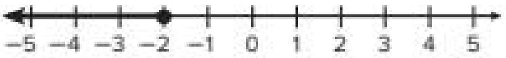
1. $x + 26.5 \leq 40$; $x \leq 13.50$; The most Gabe can spend on souvenirs and snacks is \$13.50. **3.** $-50 + x > -35$; $x > 15$; The dolphin ascended more than 15 feet. **5.** $x - 8\frac{3}{4} \geq 6$; $x \geq 14\frac{3}{4}$; The weight of the Maine Coon is at least $14\frac{3}{4}$ pounds. **7.** $x - (-2) < 27$; $x < 25$; The monthly high temperature was less than 25° Fahrenheit. **9.** Sample answer: one popcorn ball and one candy stick **11.** Sample answer: A school bus can hold at most 40 students and there are currently 30 students on the bus. How many more students can board the bus? **13.** Sample answer: Petra must write a report with more than 1,000 words for her history class. So far, she has written 684 words. Write and solve an inequality to find how many more words Petra needs to write for her report. $684 + x > 1{,}000$; $x > 316$ words.

Lesson 7-3 Solve One-Step Multiplication and Division Inequalities with Positive Coefficients, Practice Pages 355–356

1. $x > 4$

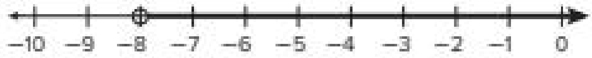


3. $x \leq -2$

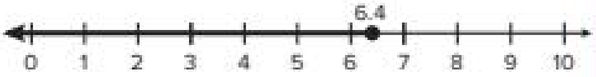


5. $x < -4$

7. $x > -8$



9. $x \leq 6.4$

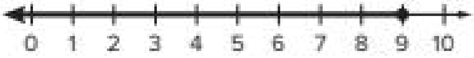


11. \$2.00 **13.** Sample answer: $4x < 40$; $\frac{x}{2} < 5$ **15.** Sample answer: $27x < 9$

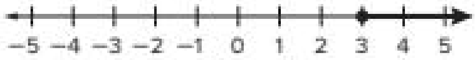
Lesson 7-4 Solve One-Step Multiplication and Division Inequalities with Negative Coefficients, Practice Pages 363–364

1. $x < -11$

3. $x \leq 9$

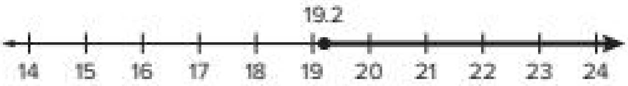


5. $x \geq 3$



7. $x \leq 15$

9. $x \geq 19.2$



11. $-3x \leq -27$; $x \geq 9$; at least 9 bananas; at least 18 seconds were lost in the race

13. Sample answer: $-5x \geq -20$; $\frac{x}{-2} \geq -2$

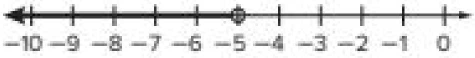
15. D. $\frac{x}{-3} \geq -2$; This inequality's solution is $x \leq 6$. The other three inequalities solutions are $x \geq 6$.

Lesson 7-5 Write and Solve One-Step Multiplication and Division Inequalities, Practice Pages 371–372

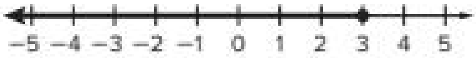
1. $6x \geq 168$; $x \geq 28$; Hermes needs to babysit 28 or more hours. **3.** $3\frac{1}{4}x \leq 13$; $x \leq 4$; Sadie can make at most 4 batches. **5.** $\frac{x}{1.25} \geq 24$; $x \geq 30$; The teacher needs at least 30 yards of tulle. **7.** $\frac{x}{4.25} \leq 12$; $x \leq 51$; Chase should buy at most 51 inches of fabric. **9.** at most \$367.50 **11.** Sample answer: Kail earns \$4 for every dog he walks. He needs to earn at least \$100 for new ski boots. Write and solve an inequality to determine the number of dogs he must walk to earn enough for the ski boots. **13.** Sample answer: Sara is buying colored sand to fill star sand art bottles. She wants to make at least 9 star sand art bottles and needs 5 ounces of colored sand for each bottle. Write and solve an inequality to determine the amount of sand she needs to buy. Then interpret the solution. $\frac{x}{5} \geq 9$; $x \geq 45$; Sara needs at least 45 ounces of colored sand.

Lesson 7-6 Write and Solve Two-Step Inequalities, Practice Pages 383–384

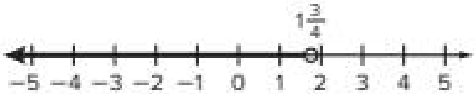
1. $x < -5$



3. $x \leq 3$

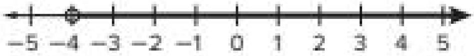


5. $x < 1\frac{3}{4}$



7. $15 + 4x \leq 27, x \leq 3$; Margie can rent the bicycle for up to 3 hours. **9.** $20 - 3.5x \geq 7.75$, $x \leq 3.5$; Douglas can race the go–karts no more than 3 times. **11.** a minimum score of 16 points

13. $x > -4$

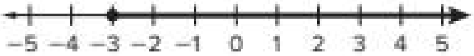


15. Sample answer: Add $2x$ to each side of the inequality before solving.

Module 7 Review Pages 387–388

1A. $x \geq -3$

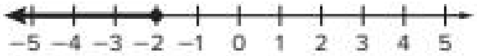
1B.



3. Let x = cost of T–shirt; $59 + 2x \leq 75$; $x \leq 8$; Sample answer: Based upon the solution, each T–shirt can cost \$8 or less. **5.** D

7A. $x \leq -2$; Sample answer: The direction of the inequality symbol is reversed because I divided each side of the inequality by a negative number in order to solve for x.

7B.



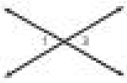
9. no more than \$335.75; ≤ \$335.75

11. $z \leq -1.25$; Sample answer: The direction of the inequality symbol is not reversed because I did not have to multiply or divide each side of the inequality by a negative number in order to solve for z.

Lesson 8-1 Vertical and Adjacent Angles, Practice Pages 401–402

1. $\angle 3$, $\angle F$, $\angle HFG$, $\angle GFH$ **3.** $\angle 8$ and $\angle 11$, $\angle 7$ and $\angle 10$, $\angle 9$ and $\angle 12$; $\angle 9$ and $\angle 11$ **5.** $115 = 2x + 5$; $x = 55$ **7.** $5x + 95 = 180$; $x = 17$ **9.** 42°

11. Sample answer:

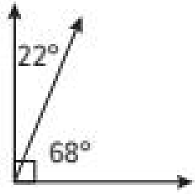


13. no; Sample answer: A pair of adjacent angles must share a common vertex, share a common side, and not overlap. They may equal 180° but do not have to.

Lesson 8-2 Complementary and Supplementary Angles, Practice Pages 411–412

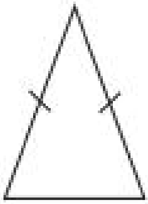
1. 45° **3.** 36° **5.** 20° **7.** $22 + 4x = 90$; $x = 17$ **9.** $43 + (x - 7) = 90$; $x = 54$ **11.** $72 + (x + 4) = 180$; $x = 104$ **13.** 27.6 **15.** Sample answer:

17. no; Sample answer: A pair of complementary angles must equal 90°. Adjacent angles share a common side and vertex. A pair of complementary angles can also be adjacent.



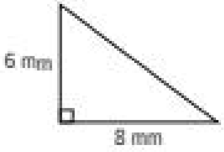
Lesson 8-3 Triangles, Practice Pages 421–422

1. acute, isosceles triangle; more than one; Sample answer:



3. no; Sample answer: The sum of the angle measures is greater than 180°, so the endpoints of the sides cannot meet.

5. Sample answer:

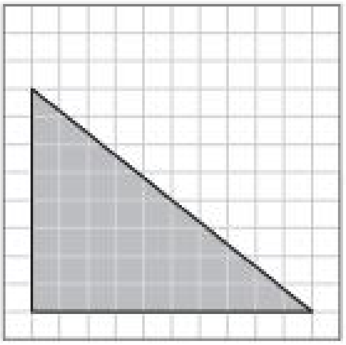


7. no; Sample answer: The sum of the two sides is not greater than the third side. **9.** 41; acute, scalene **11.** Sample answer: The sum of the two given angle measures is greater than 180° and the sum of the measures of the angles of a triangle is 180°. **13.** Sample answer: The sum of the interior angles of a triangle equal 180°. Three acute angles can have a sum of 180°. For example, 60°, 60°, and 60° are all acute angles and $60° + 60° + 60°$ is 180°.

Lesson 8-4 Scale Drawings, Practice Pages 433–434

1. about 52 mi **3.** 108 ft^2

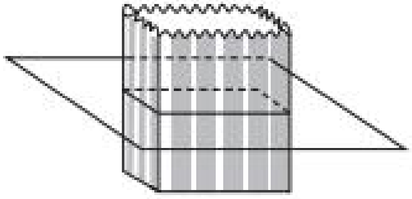
5.



7. $101.08 **9.** false; Sample answer: The scale drawing will be greater than the object. For example, if the scale factor is 2 this means that 2 units on the drawing equal 1 unit of the object. This makes the scale drawing greater than the object. **11.** 1 inch is about 19.7 mi

Lesson 8-5 Three-Dimensional Figures, Practice Pages 439–440

1. 6; 10; 6 **3.** triangle; triangle; triangle **5.** square; square; rectangle **7.** rectangular prism; 6, 12, 8; Sample answer: a tissue box **9.** rectangle; Sample answer:



11. true

Module 8 Review Pages 443–444

1. $\angle 2, \angle C, \angle ACV, \angle VCA$ **3.** 11.5 **5.** D **7.** 48 **9.** C

Lesson 9-1 Circumference of Circles, Practice Pages 455–456

1. 6.28 in. **3.** 197.82 yd **5.** 91.08 cm **7.** 5.49 in. **9.** 5.50 in. **11.** 64.25 ft **13.** Sample answer: The circumference would also double. For example, a circle with a radius of 3 feet would have a circumference that is about 18 feet. When the radius doubles to 6 feet, the circumference is about 36 feet. $18 \times 2 = 36$

Lesson 9-2 Area of Circles, Practice Pages 465–466

1. 158.29 m^2 **3.** 153.86 in^2 **5.** 120.20 ft^2 **7.** 254.34 in^2 **9.** 63.59 in^2 **11.** Sample answer: To find the area, multiply the area of the entire circle by $\frac{3}{4}$. $A = \frac{3}{4}\pi r^2$; 84.78cm^2 **13.** 226.08 in^2

Lesson 9-3 Area of Composite Figures, Practice Pages 473–474

1. 200.52 yd^2 **3.** 113 cm^2 **5.** 132 m^2 **7.** 512 ft^2 **9.** 5 pallets **11.** Sample answer: First find the area of the squar . $A = 12 \times 12$ or 144 ft^2. Then find the area of the quarter circl . $A = \frac{1}{4}(3.14 \times 6 \times 6)$ or 28.26 ft^2. Subtract the area of the quarter circle from the area of the square. $144 - 28.26 = 115.74$ ft^2 **13.** Sample answer: Use polygons to approximate the shape of the curved side of the swimming pool.

Lesson 9-4 Volume, Practice Pages 485–486

1. 2,772 in^3 **3.** 97.2 m^3 **5.** 1.0 m^3 **7.** 14.64m^2 **9.** 15 in. **11.** 7 bags **13.** Sample answer: Alexia's bathroom has a tub in the shape of a rectangular prism with a length of 1.5 meters, a width of 0.8 meter, and a height of 0.4 meter. How many cubic meters of water can it hold?; 0.48 m^3 **15.** Sample answer: First prism: area of the base: 24 in^2 and height: 4 in.; Second prism: area of the base: 16 in^2 and height: 6 in.

Lesson 9-5 Surface Area, Practice Pages 495–496

1. 468 yd^2 **3.** 103.4 in^2 **5.** 633.9 in^2 **7.** no; He needs an additional 1.7 in^2 of fabric. **9.** $120\frac{7}{75}$ yd^2 or about 120.09 yd^2 **11.** The surface area of the original prism is the surface area of the new prism. Sample answer: If the original prism has a length of 4 m, a width of 3 m, and a height of 2 m, the S.A. of the prism is 52 m^2. The new prism would have a length of 12 m, a width of 9 m, and a height of 6 m. The S.A. is 468 m^2. $\frac{52}{468} = \frac{1}{9}$

Lesson 9-6 Volume and Surface Area of Composite Figures, Practice Pages 512–513

1. 540.6 in^3 **3.** 14 in^3 **5.** 462.7 ft^2 **7.** yes; The surface area of the box is 1,425 square inches or 9.89 square feet. The new version of the coin box has a surface area of 89.0625 square feet, so there is enough cardboard available. **9.** no; Sample answer: The student included the shared portion of the figur . The correct surface area is 45.9 cm^2. **11.** Sample answer: When finding the volume and surfac area, you decompose the composite figure int solids/figures whose volumes/areas you kno how to find

Module 9 Review Pages 516–517

1. C **3.** 329.9 square inches **5.** A **7.** 405 **9.** D **11.** C

Lesson 10-1 Find Likelihoods, Practice Pages 513–514

1. equally likely **3.** likely **5.** certain **7.** selecting the letter B, selecting the letter Q or R **9.** key ring, yo–yo, cap **11.** Sample answer: likelihood of flipping a red–yellow counter and it landing on yellow **13.** likely; Sample answer: About 95% of Americans are not vegetarians. The likelihood of 95% means that is very likely the person is not a vegetarian.

Lesson 10-2 Relative Frequency of Simple Events, Practice Pages 527–528

1. 68%; 10% **3.** $\frac{13}{25}$ **5.** 204 sundaes **7.** $55.20 **9.** Sample answer: Based on last year's class, a teacher determines that if a student plays a sport, the probability that they are also in a club is 75%. If there are 24 students who play a sport in this year's class, how many students would you expect to also be in a club?; about 18 students **11.** 80 tosses

Lesson 10-3 Theoretical Probability of Simple Events, Practice Pages 537–538

1. 1, 2, 3, 4, 5 **3.** $\frac{1}{2}$, 0.5, 50% **5.** $\frac{3}{8}$, 0.375, 37.5%; The complement is choosing a ride that lasts at least 200 seconds. **7.** 0.25, 25%, $\frac{1}{4}$ **9.** 525 times **11.** The student found the complement of spinning a multiple of 3. The correct probability is $\frac{3}{8}$. **13.** Sample answer: The weather reporter says that there is a 65% chance that it will rain tomorrow. What is the chance it will not rain tomorrow? 35%

Lesson 10-4 Compare Probabilities of Simple Events, Practice Pages 545–546

1. Jayden **3.** Graph 2; Graph 1 **5.** $\frac{1}{3}$; $\frac{1}{2}$; They are not close. Sample answer: There were not enough trials performed. **7.** Sample answer: The spinner is expected to land on each section a total of 10 times.

Lesson 10-5 Probability of Compound Events, Practice Pages 557–558

1. Sample space: lime plain, lime ice cream, lime drink, cherry plain, cherry ice cream, cherry drink, blueberry plain, blueberry ice cream, blueberry drink, watermelon plain, watermelon ice cream, watermelon drink. **3.** $\frac{1}{9}$; 11.1% **5.** 6.3% **7.** 12 more **9.** no; Sample answer: The probability that Kimiko will win is $\frac{7}{12}$. Because $\frac{7}{12}$ is greater than $\frac{5}{12}$, Kimiko has a greater chance of winning. **11.** Sample answer: Choosing a hamburger or hot dog and then potato salad or macaroni salad. Sample space: hamburger, potato salad; hamburger, macaroni salad; hot dog, potato salad; hot dog, macaroni salad

Lesson 10-6 Simulate Chance Events, Practice Pages 567–568

1A. Sample answer: Use a spinner with five equal-size sections. Label two sections "R" for rain and three sections "N" for no rain. The spinner is spun twice for each trial. **1B.** $\frac{3}{10}$, 0.3, 30% **3.** $\frac{1}{20}$, 0.05, 5% **5.** Sample answer: A fair game could consist of tossing a coin, and winning the game is represented by tossing heads. A game that is not fair could consist of rolling a number cube labeled 1–6, and winning the game is represented by landing on the numbers 1 or 2. **7.** Sample answer: If you do not clearly define each of these, it can be difficult to interpret the results of the simulation in order to find the experimental probability of a favorable outcome.

Module 10 Review Pages 571–572

1. C **3.** 0.2, $\frac{1}{5}$, 20% **5.** I would expect a number greater than 3 to be spun 250 times, because $\frac{5}{8}$ of the numbers on the spinner are greater than 3, and $\frac{5}{8} \cdot 400 = 250$. **7.** 20% **9.** 3

Lesson 11-1 Biased and Unbiased Samples, Practice Pages 583–584

1a. stratified random sample **1b.** simple random sample **1c.** systematic random sample **3.** voluntary response sample; biased; The conclusion is not valid. **5.** This scenario is a systematic random sample; The conclusion is valid; The sampling method is unbiased. **7.** yes; 60% **9.** no; Sample answer: Marc used a voluntary response sample. The results are biased and therefore invalid because only those that wanted to respond were included. **11.** Sample answer: Every 25th student is chosen from an alphabetical listing of all students. The chosen students are then surveyed.

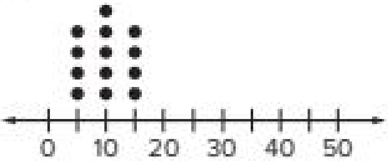
Lesson 11-2 Make Predictions, Practice Pages 591–592

1. 30 science fiction books **3.** about 1,850 students **5.** 6,820 customers **7.** $3,510 **9.** Sample answer: A random survey of high school students with jobs were asked whether they saved some of the money they earned. 82% of the students said they saved some money. Out of 340 students, predict how many would save some of their earnings.; about 279 students **11.** false; The survey's sample must be unbiased.

Lesson 11-3 Generate Multiple Samples, Practice Pages 601–602

1a. 2.5 sports **1b.** 0.54 sports; Sample answer: The majority of the sample means are within 0.5 sport of the mean. This means our estimate is likely not far off from the true mean. **3a.** The majority of the data are clustered between 14 and 19 pounds. **3b.** Sample answer: The data would be more tightly clustered between 15 and 18 pounds. **3c.** Sample answer: The majority of the data appear to be within 3 pounds of the center, so the company can expect the sample mean of 18 pounds to be within 3 pounds of the population mean. The mean decaf sales for stores in this company is likely to be between 15 and 21 pounds per week. **3d.** Sample answer: Due to the increased sample size, there will be less discrepancy between the sample mean and the population mean. The store might expect to sell between 16 and 18 pounds of decaf coffee.

5.



Sample answer: The mean is 10; the MAD is about 3, which means that the average distance each data value is from the mean is 3.

Lesson 11-4 Compare Two Populations, Practice Pages 611–612

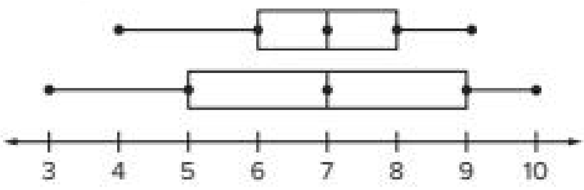
1. Sample answer: The mean for the housecat data is 11 lb with a variation of about 0.9 lb. The mean for the small dog data is 9 lb with a variation of 1.3 lb. Overall, the housecats weigh more with less variation. You can infer that a randomly selected housecat is likely to weigh more than a randomly selected small dog.

3.

	Middle School	**High School**
The median is greater.		X
The IQR is 2.5.	X	
The data has greater variability.		X
A person from this sample is more likely to have more than 7 hours of homework a week.		X
The data are more symmetric.	X	

5. Summerville Jaguars; The median number of years of experience for Southwest Broncos is 5 years while the median number of years of experience for Summerville Jaguars is 6 years. It is more likely that the player belongs to the Jaguars.

7. Sample answer:



Lesson 11-5 Assess Visual Overlap, Practice Pages 617–618

1a. about 4.4 measures of variability **1b.** Sample answer: A ratio greater than two suggests the population means are likely to be different Because 4.4 > 2, the populations in this situation are likely to be different **3.** about 1.1 measures of variability; This means that a randomly selected registered pet could either be a dog or cat. **5.** Sample answer: Because the ratio is less than 2, it is likely that the populations could have the same mean.

7. Sample answer:

Module 11 Review Pages 621–622

1A. systemic random sample **1B.** The population is the 15,600 LED monitors. The sample is representative of the population because it is selected randomly, and it is large enough to provide accurate data. **3.** 96 **5A.** 2 **5B.** The data vary as low as 1 and as high as 4. To the left, the data vary 1 off of the center, and to the right, the data vary 2 off o the center. **7.** The mean price at the Digital Hot Spot is higher than the mean price at Discount Electronics.

Glossary

The Multilingual eGlossary contains words and definitions in the following 14 languages:

- Arabic
- Bengali
- Brazilian Portuguese
- English
- French
- Haitian Creole
- Hmong
- Korean
- Mandarin
- Russian
- Spanish
- Tagalog
- Urdu
- Vietnamese

A

English	Español
absolute value (Lesson 3-1) The distance the number is from zero on a number line.	**valor absoluto** Distancia a la que se encuentra un número de cero en la recta numérica.
acute angle (Lesson 8-1) An angle with a measure greater than 0° and less than 90°.	**ángulo agudo** Ángulo que mide más de 0° y menos de 90°.
acute triangle (Lesson 8-3) A triangle having three acute angles.	**triángulo acutángulo** Triángulo con tres ángulos agudos.
Addition Property of Equality (Lesson 6-1) If you add the same number to each side of an equation, the two sides remain equal.	**propiedad de adición de la igualdad** Si sumas el mismo número a ambos lados de una ecuación, los dos lados permanecen iguales.
Addition Property of Inequality (Lesson 7-1) If you add the same number to each side of an inequality, the inequality remains true.	**propiedad de desigualdad en la suma** Si se suma el mismo número a cada lado de una desigualdad, la desigualdad sigue siendo verdadera.
Additive Identity Property (Lesson 3-1) The sum of any number and zero is the number.	**propiedad de identidad de la suma** La suma de cualquier número y cero es el mismo número.
additive inverse (Lesson 3-1) Two integers that are opposites. The sum of an integer and its additive inverse is zero.	**inverso aditivo** Dos enteros opuestos. La suma de un entero y su inverso aditiva es cero.
Additive Inverse Property (Lesson 3-1) The sum of any number and its additive inverse is zero.	**propiedad inversa aditiva** La suma de cualquier número y su inversa aditiva es cero.
adjacent angles (Lesson 8-1) Angles that have the same vertex, share a common side, and do not overlap.	**ángulos adyacentes** Ángulos que comparten el mismo vértice y un común lado, pero no se sobreponen.
algebra (Lesson 3-1) A branch of mathematics that involves expressions with variables.	**álgebra** Rama de las matemáticas que trata de las expresiones con variables.
algebraic expression (Lesson 5-1) A combination of variables, numbers, and at least one operation.	**expresión algebraica** Combinación de variables, números y por lo menos una operación.

amount of error (Lesson 2-4) The positive difference between the estimate and the actual amount.	**cantidad de error** La diferencia positiva entre la estimación y la cantidad real.
angle (Lesson 8-1) Two rays with a common endpoint form an angle. The rays and vertex are used to name the angle.	**ángulo** Dos rayos con un extremo común forman un ángulo. Los rayos y el vértice se usan para nombrar el ángulo.
area (Lesson 9-2) The measure of the interior surface of a two-dimensional figure.	**área** La medida de la superficie interior de una figura bidimensional.
asymmetric distribution (Lesson 11-4) A distribution in which the shape of the graph on one side of the center is very different than the other side, or it has outliers that might affect the average.	**distribución asimétrica** Una distribución en la que la forma del gráfico en un lado del centro es muy diferente del otro lado, o tiene valores atípicos que pueden afectar al promedio.

B

bar notation (Lesson 4-1) In repeating decimals, the line or bar placed over the digits that repeat. For example, $2.\overline{63}$ indicates that the digits 63 repeat.	**notación de barra** Línea o barra que se coloca sobre los dígitos que se repiten en decimales periódicos. Por ejemplo, $2.\overline{63}$ indica que los dígitos 63 se repiten.
base (Lesson 8-5) One of the two parallel congruent faces of a prism.	**base** Una de las dos caras paralelas congruentes de un prisma.
biased sample (Lesson 11-1) A sample drawn in such a way that one or more parts of the population are favored over others.	**muestra sesgada** Muestra en que se favorece una o más partes de una población.
box plot (Lesson 11-4) A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.	**diagrama de caja** Un método de mostrar visualmente una distribución de valores usando la mediana, cuartiles y extremos del conjunto de datos. Una caja muestra el 50% del medio de los datos.

C

center (Lesson 9-1) The point from which all points on a circle are the same distance.	**centro** El punto desde el cual todos los puntos en una circunferencia están a la misma distancia.
circle (Lesson 9-1) The set of all points in a plane that are the same distance from a given point called the center.	**círculo** Conjunto de todos los puntos de un plano que están a la misma distancia de un punto dado denominado centro.
circumference (Lesson 9-1) The distance around a circle.	**circunferencia** Distancia en torno a un círculo.
coefficient (Lesson 5-1) The numerical factor of a term that contains a variable.	**coeficiente** El factor numérico de un término que contiene una variable.
commission (Lesson 2-5) A payment equal to a percent of the amount of goods or services that an employee sells for the company.	**comisión** Un pago igual a un porcentaje de la cantidad de bienes o servicios que un empleado vende para la empresa.

common denominator (Lesson 4-1) A common multiple of the denominators of two or more fractions. 24 is a common denominator for $\frac{1}{3}$, $\frac{5}{8}$, and $\frac{3}{4}$ because 24 is the LCM of 3, 8, and 4.	**común denominador** El múltiplo común de los denominadores de dos o más fracciones. 24 es un denominador común para $\frac{1}{3}$, $\frac{5}{8}$, y $\frac{3}{4}$ porque 24 es el mcm de 3, 8 y 4.
complementary angles (Lesson 8-2) Two angles are complementary if the sum of their measures is 90°.	**ángulos complementarios** Dos ángulos son complementarios si la suma de sus medidas es 90°.
complementary events (Lesson 10-3) Two events in which either one or the other must happen, but they cannot happen at the same time. The sum of the probability of an event and its complement is 1 or 100%.	**eventos complementarios** Dos eventos en los cuales uno o el otro debe suceder, pero no pueden ocurrir al mismo tiempo. La suma de la probabilidad de un evento y su complemento es 1 o 100%.
complex fraction (Lesson 1-1) A fraction $\frac{A}{B}$ where A and/or B are fractions and B does not equal zero.	**fracción compleja** Una fracción $\frac{A}{B}$ en la cual A y/o B son fracciones y B no es igual a cero.
composite figure (Lesson 9-3) A figure that is made up of two or more figures.	**figura compuesta** Figura formada por dos o más figuras.
compound event (Lesson 10-5) An event consisting of two or more simple events.	**evento compuesto** Un evento que consiste en dos o más eventos simples.
cone (Lesson 8-5) A three-dimensional figure with one circular base connected by a curved surface to a single point.	**cono** Una figura tridimensional con una base circular conectada por una superficie curva para un solo punto.
congruent (Lesson 8-1) Having the same measure.	**congruente** Que tiene la misma medida.
congruent angles (Lesson 8-1) Angles that have the same measure.	**ángulos congruentes** Ángulos que tienen la misma medida.
congruent figures (Lesson 8-3) Figures that have the same size and same shape and corresponding sides and angles with equal measure.	**figuras congruentes** Figuras que tienen el mismo tamaño y la misma forma y los lados y los ángulos correspondientes tienen igual medida.
congruent segments (Lesson 8-3) Sides with the same length.	**segmentos congruentes** Lados con la misma longitud.
constant (Lesson 5-1) A term that does not contain a variable.	**constante** Término que no contiene ninguna variable.
constant of proportionality (Lesson 1-3) A constant ratio or unit rate of two variable quantities. It is also called the constant of variation.	**constante de proporcionalidad** Una razón constante o tasa por unidad de dos cantidades variables. También se llama constante de variación.
constant of variation (Lesson 1-3) The constant ratio in a direct variation. It is also called the constant of proportionality.	**constante de variación** Una razón constante o tasa por unidad de dos cantidades variables. También se llama constante de proporcionalidad.
constant rate of change (Lesson 1-3) The rate of change in a linear relationship.	**razón constante de cambio** Tasa de cambio en una relación lineal.

convenience sample (Lesson 11-1) A sample which consists of members of a population that are easily accessed.	**muestra de conveniencia** Muestra que incluye miembros de una población fácilmente accesibles.
cross section (Lesson 8-5) The intersection of a solid and a plane.	**sección transversal** Intersección de un sólido con un plano.
cylinder (Lesson 8-5) A three-dimensional figure with two parallel congruent circular bases connected by a curved surface.	**cilindro** Una figura tridimensional con dos paralelas congruentes circulares bases conectados por una superficie curva.

D

defining a variable (Lesson 6-1) Choosing a variable and a quantity for the variable to represent in an expression or equation.	**definir una variable** El eligir una variable y una cantidad que esté representada por la variable en una expresión o en una ecuacion.
degrees (Lesson 8-1) The most common unit of measure for angles. If a circle were divided into 360 equal-sized parts, each part would have an angle measure of 1 degree.	**grados** La unidad más común para medir ángulos. Si un círculo se divide en 360 partes iguales, cada parte tiene una medida angular de 1 grado.
diameter (Lesson 9-1) The distance across a circle through its center.	**diámetro** Segmento que pasa por el centro de un círculo y lo divide en dos partes iguales.
dimensional analysis (Lesson 1-2) The process of including units of measurement when you compute.	**análisis dimensional** Proceso que incluye las unidades de medida al hacer cálculos.
discount (Lesson 2-8) The amount by which the regular price of an item is reduced.	**descuento** Cantidad que se le rebaja al precio regular de un artículo.
distribution (Lesson 11-4) The shape of a graph of data.	**distribución** La forma de un gráfico de datos.
Distributive Property (Lesson 3-3) To multiply a sum by a number, multiply each addend of the sum by the number outside the parentheses. For any numbers a, b, and c, $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$. Example: $2(5 + 3) = (2 \cdot 5) + (2 \cdot 3)$ and $2(5 - 3) = (2 \cdot 5) - (2 \cdot 3)$	**propiedad distributiva** Para multiplicar una suma por un número, multiplíquese cada sumando de la suma por el número que está fuera del paréntesis. Sean cuales fuere los números a, b, y c, $a(b + c) = ab + ac$ y $a(b - c) = ab - ac$. Ejemplo: $2(5 + 3) = (2 \cdot 5) + (2 \cdot 3)$ y $2(5 - 3) = (2 \cdot 5) - (2 \cdot 3)$
Division Property of Equality (Lesson 6-1) If you divide each side of an equation by the same nonzero number, the two sides remain equal.	**propiedad de igualdad de la división** Si divides ambos lados de una ecuación entre el mismo número no nulo, los lados permanecen iguales.
Division Property of Inequality (Lesson 7-3) When you divide each side of an inequality by a negative number, the inequality symbol must be reversed for the inequality to remain true.	**propiedad de desigualdad en la división** Cuando se divide cada lado de una desigualdad entre un número negativo, el símbolo de desigualdad debe invertirse para que la desigualdad siga siendo verdadera.

double box plot (Lesson 11-4) Two box plots graphed on the same number line.	**doble diagrama de caja** Dos diagramas de caja sobre la misma recta numérica.
double line plot (Lesson 11-4) A method of visually displaying a distribution of two sets of data values where each value is shown as a dot above a number line.	**doble diagrama de línea** Un método de mostrar visualmente una distribución de dos conjuntos de valores donde cada valor se muestra como un punto arriba de una recta numérica.

E

edge (Lesson 8-5) The line segment where two faces of a polyhedron intersect.	**borde** El segmento de línea donde se cruzan dos caras de un poliedro.
enlargement (Lesson 8-4) An image larger than the original.	**ampliación** Imagen más grande que la original.
equation (Lesson 6-1) A mathematical sentence that contains an equals sign, =, stating that two quantities are equal.	**ecuación** Enunciado matemático que contiene el signo de igualdad = indicando que dos cantidades son iguales.
equiangular (Lesson 8-3) In a polygon, all of the angles are congruent.	**equiangular** En un polígono, todos los ángulos son congruentes.
equilateral (Lesson 8-3) In a polygon, all of the sides are congruent.	**equilátero** En un polígono, todos los lados son congruentes.
equilateral triangle (Lesson 8-3) A triangle having three congruent sides.	**triángulo equilátero** Triángulo con tres lados congruentes.
equivalent equations (Lesson 6-1) Two or more equations with the same solution.	**ecuaciones equivalentes** Dos o más ecuaciones con la misma solución.
equivalent expressions (Lesson 5-1) Expressions that have the same value.	**expresiones equivalentes** Expresiones que tienen el mismo valor.
equivalent ratios (Lesson 1-2) Two ratios that have the same value.	**razones equivalentes** Dos razones que tienen el mismo valor.
evaluate (Lesson 6-1) To find the value of an expression.	**evaluar** Calcular el valor de una expresión.
event (Lesson 10-1) The desired outcome or set of outcomes in a probability experiment.	**evento** El resultado deseado o conjunto de resultados en un experimento de probabilidad.
experimental probability (Lesson 10-2) An estimated probability based on the relative frequency of positive outcomes occurring during an experiment. It is based on what *actually* occurred during such an experiment.	**probabilidad experimental** Probabilidad estimada que se basa en la frecuencia relativa de los resultados positivos que ocurren durante un experimento. Se basa en lo que en *realidad* ocurre durante dicho experimento.

F

face **(Lesson 8-5)** A flat surface of a polyhedron.	**cara** Una superficie plana de un poliedro.
factor **(Lesson 5-4)** To write a number as a product of its factors.	**factorizar** Escribir un número como el producto de sus factores.
factored form **(Lesson 5-4)** An expression expressed as the product of its factors.	**forma factorizada** Una expresión expresada como el producto de sus factores.
factors **(Lesson 5-1)** Two or more numbers that are multiplied together to form a product.	**factores** Dos o más números que se multiplican entre sí para formar un producto.
fee **(Lesson 2-5)** A payment for a service. It can be a fixed amount, a percent of the charge, or both.	**cuota** Un pago por un servicio. Puede ser una cantidad fija, un porcentaje del cargo, o ambos.

G

graph **(Lesson 1-4)** The process of placing a point on a number line or on a coordinate plane at its proper location.	**graficar** Proceso de dibujar o trazar un punto en una recta numérica o en un plano de coordenadas en su ubicación correcta.
gratuity **(Lesson 2-7)** Also known as a tip. It is a small amount of money in return for a service.	**gratificación** También conocida como propina. Es una cantidad pequeña de dinero en retribución por un servicio.
greatest common factor (GCF) of two monomials **(Lesson 5-4)** The greatest monomial that is a factor of both monomials. The greatest common factor also includes any variables that the monomials have in common.	**mayor factor común (GCF) de dos monomios** El monomio más grande que es un factor de ambos monomios. El factor común más grande también incluye las variables que los monomios tienen en común.

I

inequality **(Lesson 7-1)** An open sentence that uses $<, >, \neq, \leq$, or $\geq$ to compare two quantities.	**desigualdad** Enunciado abierto que usa $<, >, \neq, \leq$, o $\geq$ para comparar dos cantidades.
inference **(Lesson 11-1)** A prediction made about a population.	**inferencia** Una predicción hecha sobre una población.
integer **(Lesson 3-1)** Any number from the set $\{..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$, where ... means continues without end.	**entero** Cualquier número del conjunto $\{..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$, donde ... significa que continúa sin fin.
interest **(Lesson 2-9)** The amount paid or earned for the use of the principal.	**interés** La cantidad pagada o ganada por el uso del principal.

interquartile range (IQR) (Lesson 11-4) A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set.	**rango intercuartil (RIQ)** El rango intercuartil, una medida de la variacion en un conjunto de datos numéricos, es la distancia entre el primer y el tercer cuartil del conjunto de datos
invalid inference (Lesson 11-1) An inference that is based on a biased sample or makes a conclusion not supported by the results of the sample.	**inferencia inválida** Una inferencia que se basa en una muestra sesgada o hace una conclusión no apoyada por los resultados de la muestra.
Inverse Property of Multiplication (Lesson 6-1) The product of a number and its multiplicative inverse is 1.	**propiedad inversa de la multiplicación** El producto de un número y su inverso multiplicativo es 1.
isosceles triangle (Lesson 8-3) A triangle having at least two congruent sides.	**triángulo isósceles** Triángulo que tiene por lo menos dos lados congruentes.

L

lateral face (Lesson 9-5) In a polyhedron, a face that is not a base.	**cara lateral** En un poliedro, las caras que no forman las bases.
lateral surface area (Lesson 9-5) The sum of the areas of all of the lateral faces of a solid.	**área de superficie lateral** Suma de las áreas de todas las caras de un sólido.
least common denominator (LCD) (Lesson 4-1) The least common multiple of the denominators of two or more fractions. You can use the LCD to compare fractions.	**mínimo común denominador (mcd)** El menor de los múltiplos de los denominadores de dos o más fracciones. Puedes usar el mínimo común denominador para comparar fracciones.
like terms (Lesson 5-1) Terms that contain the same variable(s) raised to the same power. Example: $5x$ and $6x$ are like terms.	**términos semejante** Términos que contienen las mismas variable(s) elevadas a la misma potencia. Ejemplo: $5x$ y $6x$ son términos semejante.
likelihood (Lesson 10-1) The chance of an event occurring.	**probabilidad** La probabilidad de que ocurra un evento.
linear expression (Lesson 5-2) An algebraic expression in which the variable is raised to the first power, and variables are neither multiplied nor divided.	**expresión lineal** Expresión algebraica en la cual la variable se eleva a la primera potencia.
linear relationship (Lesson 1-4) A relationship for which the graph is a straight line.	**relación lineal** Una relación para la cual la gráfica es una línea recta.

M

markdown (Lesson 2-8) An amount by which the regular price of an item is reduced.	**rebaja** Una cantidad por la cual el precio regular de un artículo se reduce.
markup (Lesson 2-7) The amount the price of an item is increased above the price the store paid for the item.	**margen de utilidad** Cantidad de aumento en el precio de un artículo por encima del precio que paga la tienda por dicho artículo.

mean (Lesson 11-4) The sum of the data divided by the number of items in the data set.	**media** La suma de los datos dividida entre el número total de artículos en el conjunto de datos.
mean absolute deviation (MAD) (Lesson 11-4) A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values.	**desviación media absoluta** Una medida de variación en un conjunto de datos numéricos que se calcula sumando las distancias entre el valor de cada dato y la media, y luego dividiendo entre el número de valores.
measures of center (Lesson 11-4) Numbers that are used to describe the center of a set of data. These measures include the mean, median, and mode.	**medidas del centro** Números que se usan para describir el centro de un conjunto de datos. Estas medidas incluyen la media, la mediana y la moda.
measures of variation (Lesson 11-4) A measure used to describe the distribution of data.	**medidas de variación** Medida usada para describir la distribución de los datos.
median (Lesson 11-4) A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values.	**mediana** Una medida del centro en un conjunto de dados númericos. La mediana de una lista de valores es el valor que aparace en el centro de una versíon ordenada de la lista, o la media de dos valores centrales si la lista contiene un número par de valores.
monomial (Lesson 5-4) A number, variable, or product of a number and one or more variables.	**monomio** Número, variable o producto de un número y una o más variables.
Multiplication Property of Equality (Lesson 6-1) If you multiply each side of an equation by the same nonzero number, the two sides remain equal.	**propiedad de multiplicación de la igualdad** Si multiplicas ambos lados de una ecuación por el mismo número no nulo, lo lados permanecen iguales.
Multiplication Property of Inequality (Lesson 7-3) When you multiply each side of an inequality by a negative number, the inequality symbol must be reversed for the inequality to remain true.	**propiedad de desigualdad en la multiplicación** Cuando se multiplica cada lado de una desigualdad por un número negativo, el símbolo de desigualdad debe invertirse para que la desigualdad siga siendo verdadera.
Multiplicative Identity Property (Lesson 3-3) The product of any number and one is the number.	**propiedad de identidad de la multiplicación** El producto de cualquier número y uno es el mismo número.
multiplicative inverse (Lesson 4-5) Two numbers with a product of 1. For example, the multiplicative inverse of $\frac{2}{3}$ is $\frac{3}{2}$.	**inverso multiplicativo** Dos números cuyo producto es 1. Por ejemplo, el inverso multiplicativo de $\frac{2}{3}$ es $\frac{3}{2}$.
Multiplicative Property of Zero (Lesson 3-3) The product of any number and zero is zero.	**propiedad del cero en la multiplicación** El producto de cualquier número y cero es cero.

N

negative integer (Lesson 3-1) An integer that is less than zero. Negative integers are written with a — sign.	**entero negativo** Número menor que cero. Se escriben con el signo —.

net **(Lesson 9-5)** A two-dimensional figure that can be used to build a three-dimensional figure.	**red** Figura bidimensional que sirve para hacer una figura tridimensional.
nonproportional **(Lesson 1-3)** The relationship between two ratios with a rate or ratio that is not constant.	**no proporcional** Relación entre dos razones cuya tasa o razón no es constante.
numerical expression **(Lesson 5-1)** A combination of numbers and operations.	**expresión numérica** Combinación de números y operaciones.

O

obtuse angle **(Lesson 8-1)** Any angle that measures greater than 90° but less than 180°.	**ángulo obtuso** Cualquier ángulo que mide más de 90° pero menos de 180°.
obtuse triangle **(Lesson 8-3)** A triangle having one obtuse angle.	**triángulo obtusángulo** Triángulo que tiene un ángulo obtuso.
opposites **(Lesson 3-1)** Two integers are opposites if they are represented on the number line by points that are the same distance from zero, but on opposite sides of zero. The sum of two opposites is zero.	**opuestos** Dos enteros son opuestos si, en la recta numérica, están representados por puntos que equidistan de cero, pero en direcciones opuestas. La suma de dos opuestos es cero.
order of operations **(Lesson 4-6)** The rules to follow when more than one operation is used in a numerical expression. 1. Evaluate the expressions inside grouping symbols. 2. Evaluate all powers. 3. Multiply and divide in order from left to right. 4. Add and subtract in order from left to right.	**orden de las operaciones** Reglas a seguir cuando se usa más de una operación en una expresión numérica. 1. Primero, evalúa las expresiones dentro de los símbolos de agrupación. 2. Evalúa todas las potencias. 3. Multiplica y divide en orden de izquierda a derecha. 4. Suma y resta en orden de izquierda a derecha.
outcome **(Lesson 10-1)** Any one of the possible results of an action. For example, 4 is an outcome when a number cube is rolled.	**resultado** Cualquiera de los resultados posibles de una acción. Por ejemplo, 4 puede ser un resultado al lanzar un cubo numerado.

P

parallelogram **(Lesson 9-3)** A quadrilateral with opposite sides parallel and opposite sides congruent.	**paralelogramo** Cuadrilátero cuyos lados opuestos son paralelos y congruentes.
percent error **(Lesson 2-4)** A ratio that compares the inaccuracy of an estimate (amount of error) to the actual amount.	**porcentaje de error** Una razón que compara la inexactitud de una estimación (cantidad del error) con la cantidad real.
percent of change **(Lesson 2-3)** A ratio that compares the change in a quantity to the original amount. $\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}} \cdot 100$	**porcentaje de cambio** Razón que compara el cambio en una cantidad a la cantidad original. $\text{porcentaje de cambio} = \frac{\text{cantidad del cambio}}{\text{cantidad original}} \cdot 100$

percent of decrease (Lesson 2-3) A negative percent of change.	**porcentaje de disminución** Porcentaje de cambio negativo.
percent of increase (Lesson 2-3) A positive percent of change.	**porcentaje de aumento** Porcentaje de cambio positivo.
pi (Lesson 9-1) The ratio of the circumference of a circle to its diameter. The Greek letter π represents this number. The value of pi is 3.1415926.... Approximations for pi are 3.14 and $\frac{22}{7}$.	**pi** Relación entre la circunferencia de un círculo y su diámetro. La letra griega π representa este número. El valor de pi es 3.1415926.... Las aproximaciones de pi son 3.14 y $\frac{22}{7}$.
plane (Lesson 8-5) A two-dimensional flat surface that extends in all directions.	**plano** Superficie bidimensional que se extiende en todas direcciones.
polygon (Lesson 9-3) A simple closed figure formed by three or more straight line segments.	**polígono** Figura cerrada simple formada por tres o más segmentos de recta.
polyhedron (Lesson 8-5) A three-dimensional figure with faces that are polygons.	**poliedro** Una figura tridimensional con caras que son polígonos.
population (Lesson 11-1) The entire group of items or individuals from which the samples under consideration are taken.	**población** El grupo total de individuos o de artículos del cual se toman las muestras bajo estudio.
positive integer (Lesson 3-1) An integer that is greater than zero. They are written with or without a + sign.	**entero positivo** Entero que es mayor que cero; se escribe con o sin el signo +.
principal (Lesson 2-9) The amount of money deposited or borrowed.	**capital** Cantidad de dinero que se deposita o se toma prestada.
prism (Lesson 8-5) A polyhedron with two parallel congruent faces called bases.	**prisma** Un poliedro con dos caras congruentes paralelas llamadas bases.
probability (Lesson 10-2) The chance that some event will happen. It is the ratio of the number of favorable outcomes to the number of possible outcomes.	**probabilidad** La posibilidad de que suceda un evento. Es la razón del número de resultados favorables al número de resultados posibles.
probability experiment (Lesson 10-2) When you perform an event to find the likelihood of an event.	**experimento de probabilidad** Cuando realiza un evento para encontrar la probabilidad de un evento.
probability model (Lesson 10-3) A model used to assign probabilities to outcomes of a chance process by examining the nature of the process.	**modelo de probabilidad** Un modelo usado para asignar probabilidades a resultados de un proceso aleatorio examinando la naturaleza del proceso.
properties (Lesson 3-1) Statements that are true for any number or variable.	**propiedades** Enunciados que son verdaderos para cualquier número o variable.
proportion (Lesson 1-5) An equation stating that two ratios or rates are equivalent.	**proporción** Ecuación que indica que dos razones o tasas son equivalentes.

proportional **(Lesson 1-3)** The relationship between two ratios with a constant rate or ratio.	**proporcional** Relación entre dos razones con una tasa o razón constante.
pyramid **(Lesson 8-5)** A polyhedron with one base that is a polygon and three or more triangular faces that meet at a common vertex.	**pirámide** Un poliedro con una base que es un polígono y tres o más caras triangulares que se encuentran en un vértice común.

Q

quadrilateral **(Lesson 9-3)** A closed figure having four sides and four angles.	**cuadrilátero** Figura cerrada que tiene cuatro lados y cuatro ángulos.

R

radius **(Lesson 9-1)** The distance from the center of a circle to any point on the circle.	**radio** Distancia desde el centro de un círculo hasta cualquiera de sus puntos.
random **(Lesson 10-2)** Outcomes occur at random if each outcome occurs by chance. For example, rolling a number on a number cube occurs at random.	**azar** Los resultados ocurren aleatoriamente si cada resultado ocurre por casualidad. Por ejemplo, sacar un número en un cubo numerado ocurre al azar.
rate **(Lesson 1-1)** A special kind of ratio in which the units are different.	**tasa** Un tipo especial de relación en el que las unidades son diferentes.
ratio **(Lesson 1-1)** A comparison between two quantities, in which for every *a* units of one quantity, there are *b* units of another quantity.	**razón** Una comparación entre dos cantidades, en la que por cada *a* unidades de una cantidad, hay unidades *b* de otra cantidad.
rational numbers **(Lesson 4-1)** The set of numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.	**números racionales** Conjunto de números que puede escribirse en la forma $\frac{a}{b}$ donde a y b son números enteros y $b \neq 0$.
Examples: $1 = \frac{1}{1}, \frac{2}{9}, -2.3 = \frac{-23}{10}$	Ejemplos: $1 = \frac{1}{1}, \frac{2}{9}, -2.3 = \frac{-23}{10}$
reciprocal **(Lesson 4-5)** The multiplicative inverse of a number.	**recíproco** El inverso multiplicativo de un número.
rectangular prism **(Lesson 9-4)** A prism that has two parallel congruent bases that are rectangles.	**prisma rectangular** Un prisma con dos bases paralelas congruentes que son rectángulos.
reduction **(Lesson 8-4)** An image smaller than the original.	**reducción** Imagen más pequeña que la original.
regular polygon **(Lesson 9-3)** A polygon that has all sides congruent and all angles congruent.	**polígono regular** Polígono con todos los lados y todos los ángulos congruentes.
regular pyramid **(Lesson 9-5)** A pyramid whose base is a regular polygon and in which the segment from the vertex to the center of the base is the altitude.	**pirámide regular** Pirámide cuya base es un polígono regular y en la cual el segmento desde el vértice hasta el centro de la base es la altura.

relative frequency (Lesson 10-2) A ratio that compares the frequency of each category to the total.	**frecuencia relativa** Razón que compara la frecuencia de cada categoría al total.
relative frequency graph (Lesson 10-2) A graph used to organize occurrences compared to a total.	**gráfico de frecuencia relativa** Gráfico utilizado para organizar las ocurrencias en comparación con un total.
relative frequency table (Lesson 10-2) A table used to organize occurrences compared to a total.	**tabla de frecuencia relativa** Una tabla utilizada para organizar las ocurrencias en comparación con un total.
repeating decimal (Lesson 4-1) A decimal in which 1 or more digits repeat.	**decimal periódico** Un decimal en el que se repiten 1 o más dígitos.
rhombus (Lesson 9-3) A parallelogram having four congruent sides.	**rombo** Paralelogramo que tiene cuatro lados congruentes.
right angle (Lesson 8-1) An angle that measures exactly 90°.	**ángulo recto** Ángulo que mide exactamente 90°.
right triangle (Lesson 8-3) A triangle having one right angle.	**triángulo rectángulo** Triángulo que tiene un ángulo recto.

S

sales tax (Lesson 2-6) An additional amount of money charged on items that people buy.	**impuesto sobre las ventas** Cantidad de dinero adicional que se cobra por los artículos que se compran.
sample (Lesson 11-1) A randomly selected group chosen for the purpose of collecting data.	**muestra** Grupo escogido al azar o aleatoriamente que se usa con el propósito de recoger datos.
sample space (Lesson 10-3) The set of all possible outcomes of a probability experiment.	**espacio muestral** Conjunto de todos los resultados posibles de un experimento probabilístico.
scale (Lesson 8-4) The scale that gives the ratio that compares the measurements of a drawing or model to the measurements of the real object.	**escala** Razón que compara las medidas de un dibujo o modelo a las medidas del objeto real.
scale drawing (Lesson 8-4) A drawing that is used to represent objects that are too large or too small to be drawn at actual size.	**dibujo a escala** Dibujo que se usa para representar objetos que son demasiado grandes o demasiado pequeños como para dibujarlos de tamaño natural.
scale factor (Lesson 8-4) A scale written as a ratio without units in simplest form.	**factor de escala** Escala escrita como una razón sin unidades en forma simplificada.
scale model (Lesson 8-4) A model used to represent objects that are too large or too small to be built at actual size.	**modelo a escala** Réplica de un objeto real, el cual es demasiado grande o demasiado pequeño como para construirlo de tamaño natural.
scalene triangle (Lesson 8-3) A triangle having no congruent sides.	**triángulo escaleno** Triángulo sin lados congruentes.

selling price **(Lesson 2-7)** The amount the customer pays for an item.	**precio de venta** Cantidad de dinero que paga un consumidor por un artículo.
semicircle **(Lesson 9-2)** Half of a circle. The formula for the area of a semicircle is $A = \frac{1}{2}\pi r^2$.	**semicírculo** Medio círculo. La fórmula para el área de un semicírculo es $A = \frac{1}{2}\pi r^2$.
simple event **(Lesson 10-2)** One outcome or a collection of outcomes.	**eventos simples** Un resultado o una colección de resultados.
simple interest **(Lesson 2-9)** The amount paid or earned for the use of money. The formula for simple interest is $I = prt$.	**interés simple** Cantidad que se paga o que se gana por el uso del dinero. La fórmula para calcular el interés simple es $I = prt$.
simple random sample **(Lesson 11-1)** An unbiased sample where each item or person in the population is as likely to be chosen as any other.	**muestra aleatoria simple** Muestra de una población que tiene la misma probabilidad de escogerse que cualquier otra.
simplest form **(Lesson 5-1)** An expression is in simplest form when it is replaced by an equivalent expression having no like terms or parentheses.	**expresión mínima** Expresión en su forma más simple cuando es reemplazada por una expresión equivalente que no tiene términos similares ni paréntesis.
simplify **(Lesson 5-5)** Write an expression in simplest form.	**simplificar** Escribir una expresión en su forma más simple.
simulation **(Lesson 10-6)** An experiment that is designed to model the action in a given situation.	**simulación** Un experimento diseñado para modelar la acción en una situación dada.
slant height **(Lesson 9-5)** The height of each lateral face.	**altura oblicua** Altura de cada cara lateral.
solution **(Lesson 6-1)** A replacement value for the variable in an open sentence. A value for the variable that makes an equation true. Example: The *solution* of $12 = x + 7$ is 5.	**solución** Valor de reemplazo de la variable en un enunciado abierto. Valor de la variable que hace que una ecuación sea verdadera. Ejemplo: La *solución* de $12 = x + 7$ es 5.
statistics **(Lesson 11-1)** The study of collecting, organizing, and interpreting data.	**estadística** Estudio que consiste en recopilar, organizar e interpretar datos.
straight angle **(Lesson 8-1)** An angle that measures exactly 180°.	**ángulo llano** Ángulo que mide exactamente 180°.
stratified random sample **(Lesson 11-1)** A sample in which the population is divided into groups with similar traits that do not overlap. A simple random sample is then selected from each group.	**muestra aleatoria estratificada** Una muestra en la que la población se divide en grupos con rasgos similares que no se superponen. A continuación, se selecciona una muestra aleatoria simple de cada grupo.
Subtraction Property of Equality **(Lesson 6-1)** If you subtract the same number from each side of an equation, the two sides remain equal.	**propiedad de sustracción de la igualdad** Si restas el mismo número de ambos lados de una ecuación, los dos lados permanecen iguales.

Subtraction Property of Inequality (Lesson 7-1) If you subtract the same number from each side of an inequality, the inequality remains true.	**propiedad de desigualdad en la resta** Si se resta el mismo número a cada lado de una desigualdad, la desigualdad sigue siendo verdadera.
supplementary angles (Lesson 8-2) Two angles are supplementary if the sum of their measures is 180°.	**ángulos suplementarios** Dos ángulos son suplementarios si la suma de sus medidas es 180°.
surface area (Lesson 9-5) The sum of the areas of all the surfaces (faces) of a three-dimensional figure.	**área de superficie** La suma de las áreas de todas las superficies (caras) de una figura tridimensional.
survey (Lesson 11-1) A question or set of questions designed to collect data about a specific group of people, or population.	**encuesta** Pregunta o conjunto de preguntas diseñadas para recoger datos sobre un grupo específico de personas o población.
symmetric distribution (Lesson 11-4) A distribution in which the shape of the graph on each side of the center is similar.	**distribución simétrica** Distribución en la que la forma de la gráfica en cada lado del centro es similar.
systematic random sample (Lesson 11-1) A sample where the items or people are selected according to a specific time or item interval.	**muestra aleatoria sistemática** Muestra en que los elementos o personas se eligen según un intervalo de tiempo o elemento específico.

T

term (Lesson 5-1) A number, a variable, or a product or quotient of numbers and variables.	**término** Número, variable, producto o cociente de números y de variables.
terminating decimal (Lesson 4-1) A decimal with a repeating digit of 0.	**decimal finito** Un decimal que tiene un dígito que se repite que es 0.
theoretical probability (Lesson 10-3) The ratio of the number of ways an event can occur to the number of possible outcomes in the sample space. It is based on what *should* happen when conducting a probability experiment.	**probabilidad teórica** Razón del número de maneras en que puede ocurrir un evento al número de resultados posibles en el espacio muestral. Se basa en lo que *debería* pasar cuando se conduce un experiment probabilístico.
theoretical probability of a compound event (Lesson 10-5) The ratio of the number of ways an event can occur to the number of possible outcomes in the sample space. It is based on what *should* happen when conducting a probability experiment.	**probabilidad teórica de un evento compuesto** Razón del número de maneras en que puede ocurrir un evento al número de resultados posibles en el espacio muestral. Se basa en lo que *debería* pasar cuando se conduce un experimento probabilístico.
three-dimensional figure (Lesson 8-5) A figure with length, width, and height.	**figura tridimensional** Figura que tiene largo, ancho y alto.
tip (Lesson 2-7) Also known as a gratuity, it is a small amount of money in return for a service.	**propina** También conocida como gratificación; es una cantidad pequeña de dinero en recompensa por un servicio.
trapezoid (Lesson 9-3) A quadrilateral with one pair of parallel sides.	**trapecio** Cuadrilátero con un único par de lados paralelos.

tree diagram **(Lesson 10-5)** A diagram used to show the sample space.	**diagrama de árbol** Diagrama que se usa para mostrar el espacio muestral.
triangle **(Lesson 8-3)** A figure with three sides and three angles.	**triángulo** Figura con tres lados y tres ángulos.
triangular prism **(Lesson 9-4)** A prism that has two parallel congruent bases that are triangles.	**prisma triangular** Un prisma que tiene dos bases congruentes paralelas que triángulos.
two-step equation **(Lesson 6-2)** An equation having two different operations.	**ecuación de dos pasos** Ecuación que contiene dos operaciones distintas.
two-step inequality **(Lesson 7-6)** An inequality that contains two operations.	**desigualdad de dos pasos** Desigualdad que contiene dos operaciones.

U

unbiased sample **(Lesson 11-1)** A sample representative of the entire population.	**muestra no sesgada** Muestra que se selecciona de modo que se representativa de la población entera.
uniform probability model **(Lesson 10-3)** A probability model which assigns equal probability to all outcomes.	**modelo de probabilidad uniforme** Un modelo de probabilidad que asigna igual probabilidad a todos los resultados.
unit rate **(Lesson 1-1)** A rate in which the first quantity is compared to 1 unit of the second quantity.	**tasa unitaria** Una tasa en la que la primera candidad se compara con 1 unidad de la segunda candidad.
unit ratio **(Lesson 1-2)** A ratio in which the first quantity is compared to every 1 unit of the second quantity.	**razón unitaria** Una relación en la que la primera cantidad se compara con cada 1 unidad de la segunda cantidad.

V

valid inference **(Lesson 11-1)** A prediction, made about a population, based on an unbiased sample that is representative of the population.	**inferencia válida** Una predicción, hecha sobre una población, basada en una muestra imparcial que es representativa de la población.
valid sampling method **(Lesson 11-1)** A sampling method that is: representative of the population selected at random, where each member has an equal chance of being selected, and large enough to provide accurate data.	**método de muestreo válido** Un método de muestreo que es: representativo de la población seleccionada al azar, donde cada miembro tiene la misma oportunidad de ser seleccionado y suficientemente grande para proporcionar datos precisos.
variability **(Lesson 11-3)** A measure that describes the amount of diversity in values within a sample or samples.	**variabilidad** Medida que describe la cantidad de diversidad en valores dentro de una muestra o muestras.
variable **(Lesson 5-1)** A symbol, usually a letter, used to represent a number in mathematical expressions or sentences.	**variable** Símbolo, por lo general una letra, que se usa para representar un número en expresiones o enunciados matemáticos.

vertex (Lesson 8-1) A vertex of an angle is the common endpoint of the rays forming the angle.	**vértice** El vértice de un ángulo es el extremo común de los rayos que lo forman.
vertex (Lesson 8-5) The point where three or more faces of a polyhedron intersect.	**vértice** El punto donde tres o más caras de un poliedro se cruzan.
vertical angles (Lesson 8-1) Opposite angles formed by the intersection of two lines. Vertical angles are congruent.	**ángulos opuestos por el vértice** Ángulos opuestos formados por la intersección de dos rectas. Los ángulos opuestos por el vértice son congruentes.
vertices (Lesson 8-5) Plural of vertex.	**vértices** Plural de verticé.
visual overlap (Lesson 11-4) A visual demonstration that compares the centers of two distributions with their variation, or spread.	**superposición visual** Una demostración visual que compara los centros de dos distribuciones con su variación, o magnitud.
volume (Lesson 9-4) The number of cubic units needed to fill the space occupied by a solid.	**volumen** Número de unidades cúbicas que se requieren para llenar el espacio que ocupa un sólido.
voluntary response sample (Lesson 11-1) A sample which involves only those who want to participate in the sampling.	**muestra de respuesta voluntaria** Muestra que involucra sólo aquellos que quieren participar en el muestreo.

W

wholesale cost (Lesson 2-7) The amount the store pays for an item.	**coste al por mayor** La cantidad que la tienda paga por un artículo.

Z

zero angle (Lesson 8-1) An angle that measures exactly 0 degrees.	**ángulo cero** Un ángulo que mide exactamente 0 grados.
zero pair (Lesson 3-1) The result when one positive counter is paired with one negative counter. The value of a zero pair is 0.	**par nulo** Resultado de hacer coordinar una ficha positiva con una negativa. El valor de un par nulo es 0.

Index

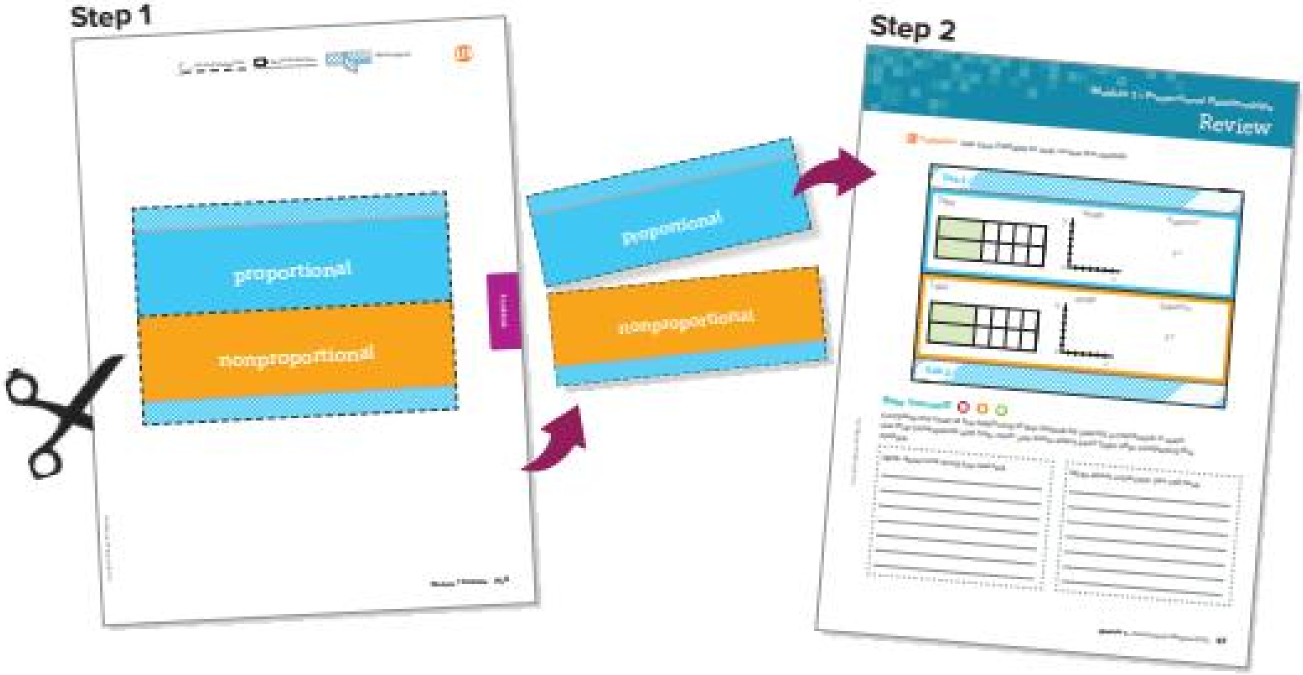
Foldables Study Organizers

What Are Foldables and How Do I Create Them?

Foldables are three-dimensional graphic organizers that help you create study guides for each module in your book.

Step 1 Go to the back of your book to find the Foldable for the module you are currently studying. Follow the cutting and assembly instructions at the top of the page.

Step 2 Go to the Module Review at the end of the module you are currently studying. Match up the tabs and attach your Foldable to this page. Dotted tabs show where to place your Foldable. Striped tabs indicate where to tape the Foldable.



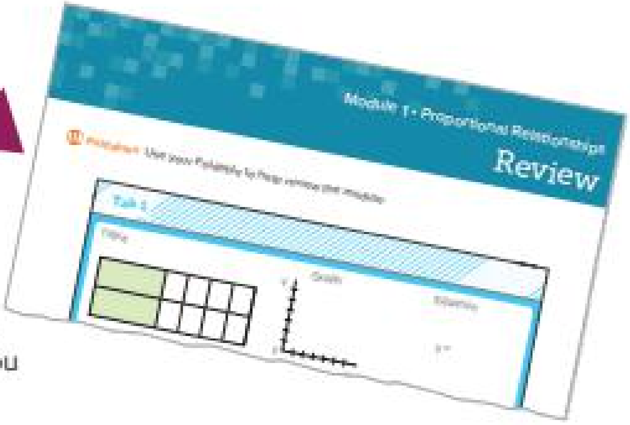
How Will I Know When to Use My Foldable?

You will be directed to work on your Foldable at the end of selected lessons. This lets you know that it is time to update it with concepts from that lesson. Once you've completed your Foldable, use it to study for the module test.

How Do I Complete My Foldable?

No two Foldables in your book will look alike. However, some will ask you to fill in similar information. Below are some of the instructions you'll see as you complete your Foldable. **HAVE FUN** learning math using Foldables!

Instructions and What They Mean



Best Used to...	Complete the sentence explaining when the concept should be used.
Definition	Write a definition in your own words.
Description	Describe the concept using words.
Equation	Write an equation that uses the concept. You may use one already in the text or you can make up your own.
Example	Write an example about the concept. You may use one already in the text or you can make up your own.
Formulas	Write a formula that uses the concept. You may use one already in the text.
How do I ...?	Explain the steps involved in the concept.
Models	Draw a model to illustrate the concept.
Picture	Draw a picture to illustrate the concept.
Solve Algebraically	Write and solve an equation that uses the concept.
Symbols	Write or use the symbols that pertain to the concept.
Write About It	Write a definition or description in your own words.
Words	Write the words that pertain to the concept.

Meet Foldables Author Dinah Zike

Dinah Zike is known for designing hands-on manipulatives that are used nationally and internationally by teachers and parents. Dinah is an explosion of energy and ideas. Her excitement and joy for learning inspires everyone she touches.

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Linear Expressions

add or subtract

factor

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Examples

Examples

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Solve Two-Step Equations

$-3x + 6 = 21$

$-4(x + 9) = 24$

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Write About It

Write About It

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Solve Inequalities

One-Step Addition and Subtraction

One-Step Multiplication and Division with Positive Coefficients

One-Step Multiplication and Division with Negative Coefficients

Two-Step

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How do I solve one-step addition and subtraction inequalities?

How do I solve one-step multiplication and division inequalities with positive coefficients?

How do I solve one-step multiplication and division inequalities with negative coefficients?

How do I solve two-step inequalities?

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Angles	Triangles
acute	scalene
obtuse	isosceles
right	equilateral

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Picture

Picture

Picture

Picture

Picture

Picture

Tab 2

Tab 1

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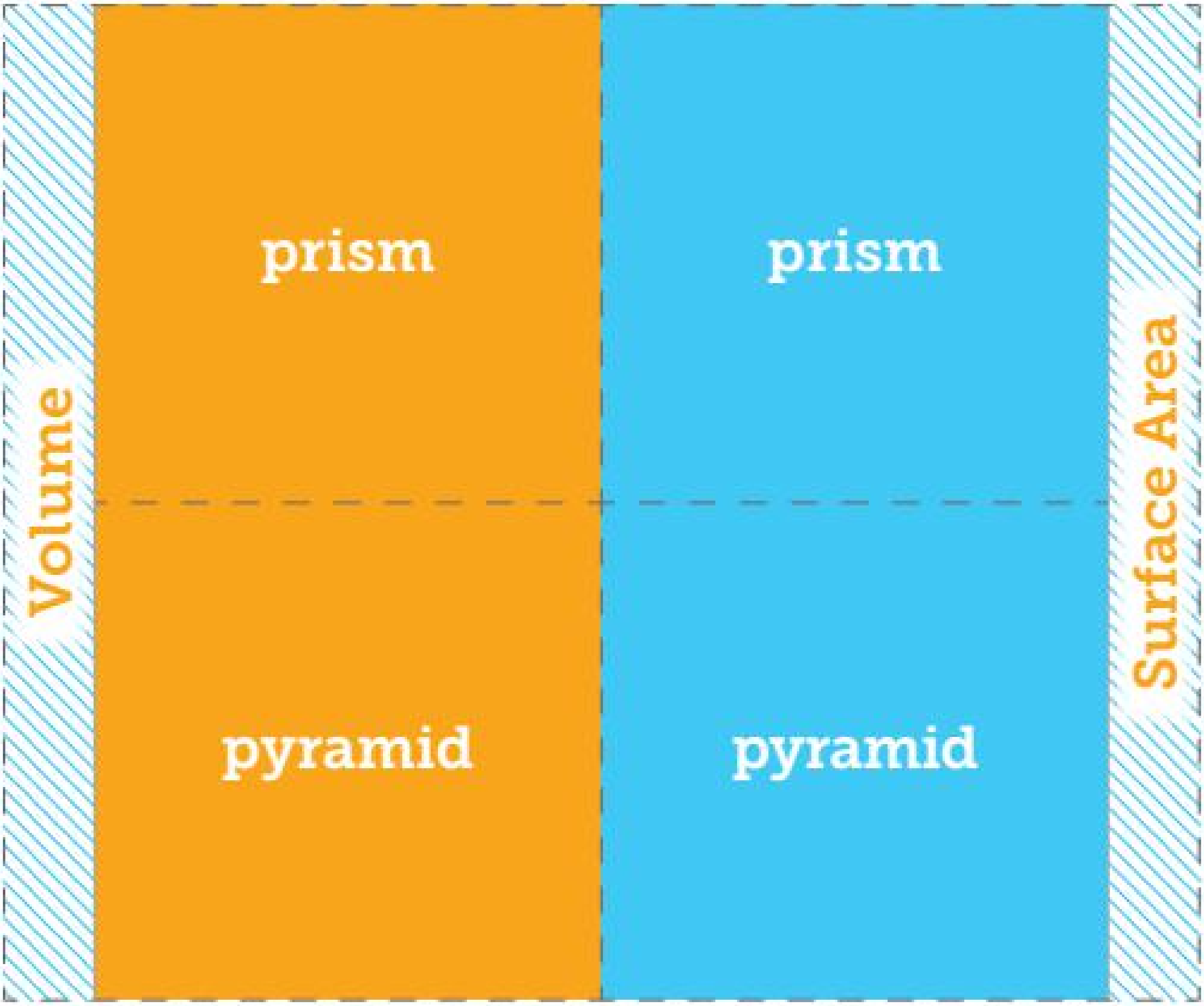

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Tab 2

Write About It

Write About It

Write About It

Write About It

Tab 1

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Probability

simple event

compound event

Foldables

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Definition

Definition

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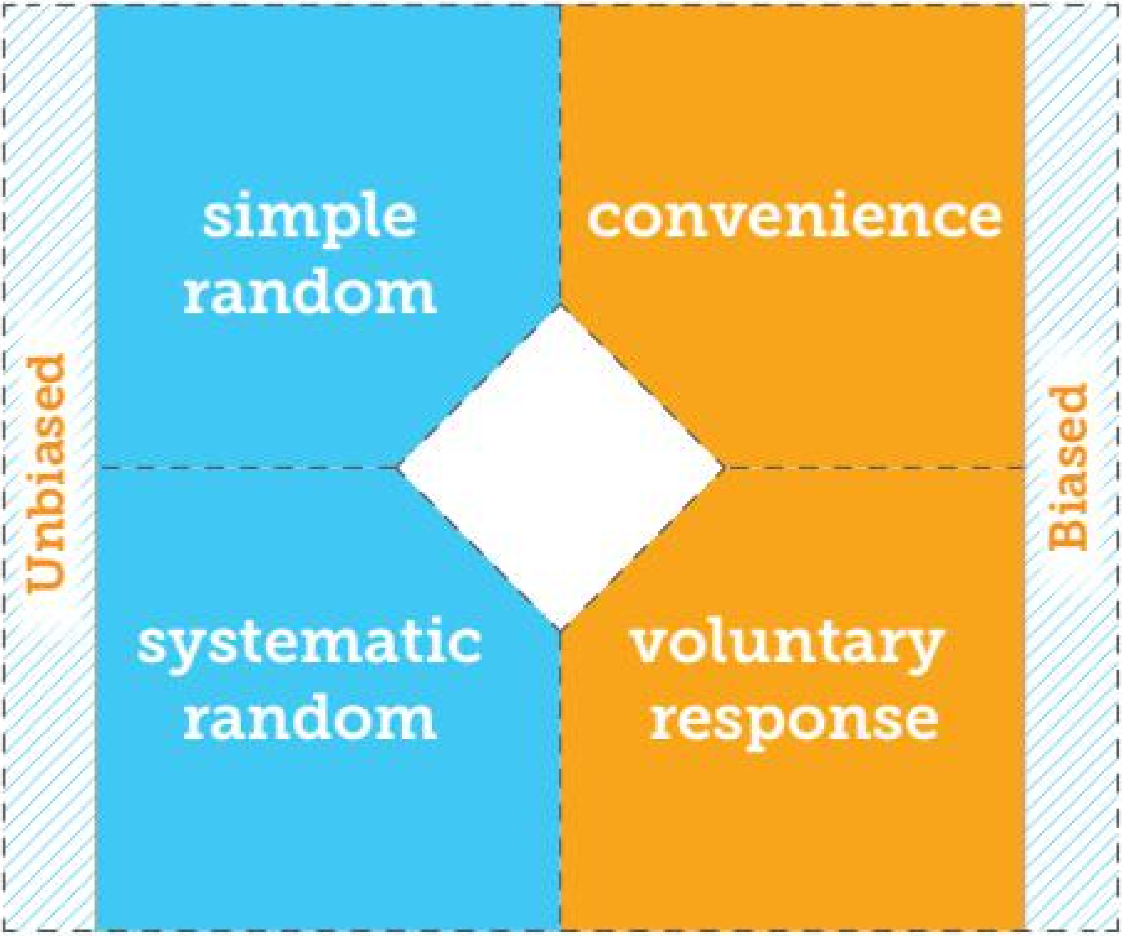

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