Find the area bounded by the graphs of $y = -x$, $y = \sqrt{x}$ *and* $y = 2$. Choose the variable of integration to write the area as a single integral.

أوجد مساحة المنطقة المحصورة بين منحنيات
$y = 2$ و $y = -x$, $y = \sqrt{x}$
اختر متغير التكامل لكتابة المساحة على صورة تكامل واحد.

$$A = \int_0^2 (y^2 + y)\,dy$$

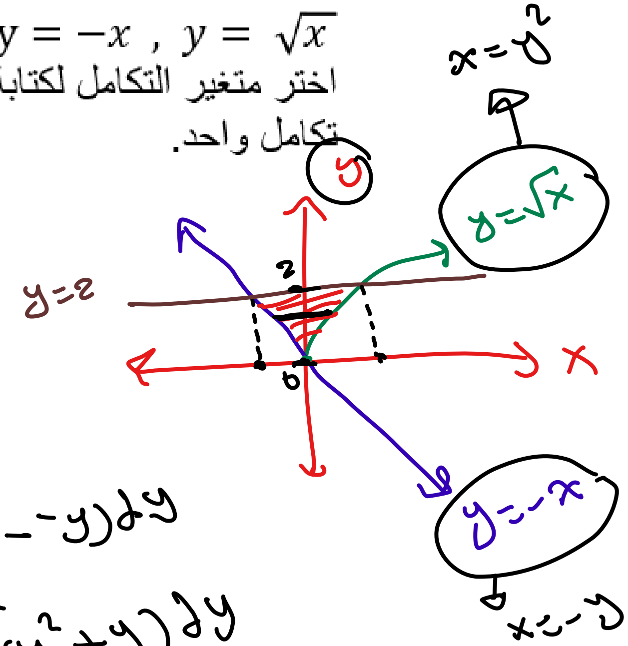
$$A = \int_0^2 (y^2 - y)\,dy$$

$$A = \int_{-2}^4 (\sqrt{x} + x)\,dx$$

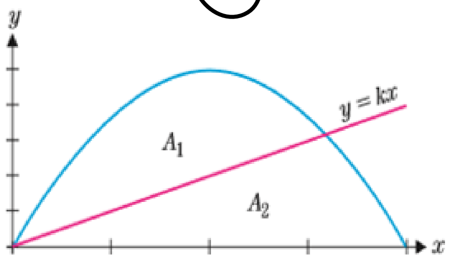
$$A = \int_{-2}^4 (\sqrt{x} - x)\,dx$$

$$A = \int_0^2 (y^2 - -y)\,dy$$

$$= \int_0^2 (y^2 + y)\,dy$$



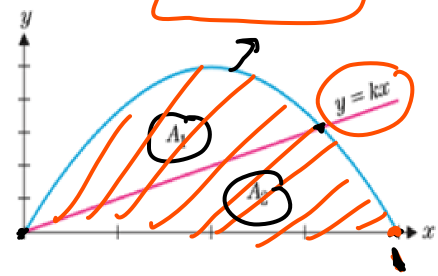
For $y = x - x^2$ and $y = k\,x$ as shown, find A_1 such that A1 = A2.



حيث $y = x - x^2$ و $y = k\,x$

كما يظهر بالرسم أدناه ، ما قيمة A_1

حيث $A_1 = A_2$.



$$A_1 + A_2 = \int_0^1 (x - x^2)\,dx$$

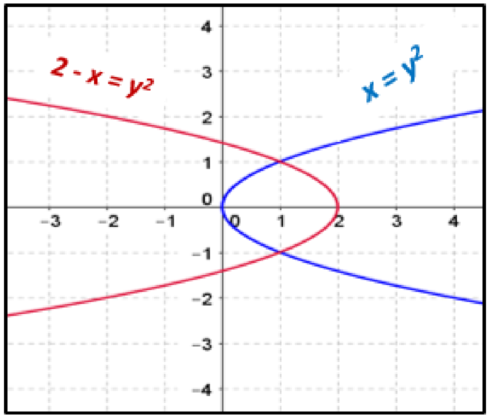
$$= \boxed{\frac{1}{6}} \checkmark\checkmark$$

$$A_1 = \frac{1}{2}\left(\frac{1}{6}\right) = \boxed{\frac{1}{12}}$$

$$x - x^2 = 0 \rightarrow \boxed{x = 0} \quad \boxed{x = 1}$$

$\frac{1}{12}$ $\frac{1}{6}$ $\frac{1}{8}$ $\frac{1}{10}$

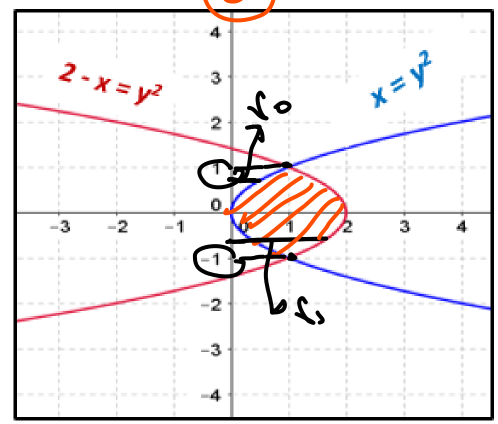
Compute the volume of the solid formed by revolving the region between $x = y^2$ and $2 - x = y^2$ about the y – axis .



أوجد حجم المجسم الناتج عن دوران المنطقة المحصورة
بين منحنيي $2 - x = y^2$ و $x = y^2$
حول محور y .

x=0 y

outer (الخارجي)



$r_1 = 2 - y^2 - 0$
$= 2 - y^2$

$r_0 = y^2 - 0$
$= y^2$

$2 - x = y^2$

$x = 2 - y^2$

$v = \pi \int_{-1}^{1} [(2-y^2)^2 - (y^2)^2] dy$

$$v = \pi \int_{-1}^{1} [(2 - y^2)^2 - (y^2)^2] \, dy$$

$$v = \pi \int_{0}^{2} [(2 - y^2)^2 - (y^2)^2] \, dy$$

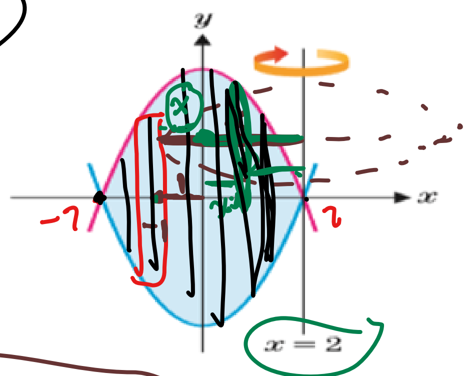
$$v = \pi \int_{-1}^{1} [(2 - 2 y^2)^2] \, dy$$

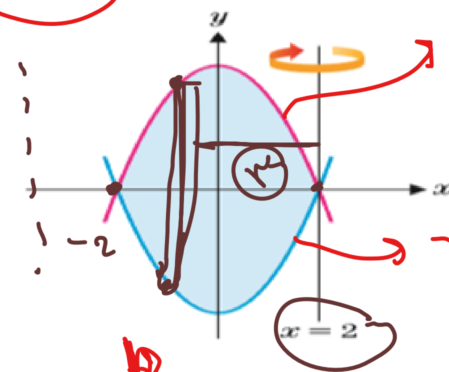
$$v = \pi \int_{-1}^{1} [(y^2)^2 - (2 - y^2)^2] \, dy$$

$r = a - x$
$r = x - a$
$r = y - c$
$r = c - y$

Use the method of cylindrical shells to find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 - 4$ and $y = 4 - x^2$ about the the line $x = 2$

مستخدماً طريقة الأصداف الأسطوانية احسب حجم المجسم الناتج عن دوران المنطقة المحصورة بين منحنيي $y = x^2 - 4$ و $y = 4 - x^2$ حول المستقيم $x = 2$





$4 - x^2$

$x^2 - 4$

$$V = 2\pi \int_a^b r\,h\,dx$$

$$\cancel{\Delta x =}$$

$$h = 4 - x^2 - (x^2 - 4)$$
$$= 4 - x^2 - x^2 + 4$$
$$= 8 - 2x^2$$

$$V = 2\pi \int_{-2}^{2} (2 - x)(8 - 2x^2)\,dx$$

$$r = 2 - x$$

$$v = 2\pi \int_{-2}^{2} [\,(2 - x)(8 - 2x^2)\,dx$$

$$v = 2\pi \int_{0}^{2} [\,(2 - x)(8 - 2x^2)\,dx$$

$$v = 2\pi \int_{-4}^{4} [\,(2 - y)(8 - 2y^2)\,dy$$

$$v = 2\pi \int_{-2}^{2} [\,(x - 2)(8 - 2x^2)\,dx$$

Find the arc length of the portion of the curve $y = lnx$ with $1 \leq x \leq 3$

أوجد طول قوس لجزء من منحنى الدالة

$y = lnx$ مع $1 \leq x \leq 3$.

$$s = \int_1^3 \frac{\sqrt{1+x^2}}{x}\,dx$$

$$s = \int_1^3 \frac{\sqrt{1-x^2}}{x}\,dx$$

$$s = \pi\int_1^3 \frac{\sqrt{1+x^2}}{x}\,dx$$

$$s = \int_1^3 \sqrt{1+x^2}\,dx$$

$$y' = \frac{1}{x} \longrightarrow (y')^2 = \frac{1}{x^2}$$

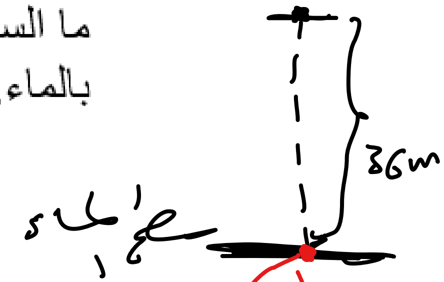
$$S = \int_1^3 \sqrt{\frac{1}{1} + \frac{1}{x^2}}\,dx$$

$$= \int_1^3 \sqrt{\frac{x^2+1}{x^2}}\,dx = \int_1^3 \frac{\sqrt{x^2+1}}{\sqrt{x^2}}\,dx$$

$$= \int_1^3 \frac{\sqrt{x^2+1}}{x}\,dx$$

A diver drops from 36 meters above the water. What is the diver's velocity at impact?

يسقط غطاس من ارتفاع 36 متر فوق الماء.
ما السرعة المتجهة للغطاس لحظة الاصطدام بالماء.



- $-26.6\ m/s$
- $-36\ m/s$
- $36\ m/s$
- $26.6\ m/s$

$$\boxed{h(0) = 36\text{ m}}$$

$$\boxed{h'(0) = 0}$$

$$h''(t) = -9.8$$

$$h'(t) = -9.8t + h'(0) = -9.8t$$

$$h(t) = -4.9t^2 + h'(0)t + h(0)$$

$$= -4.9t^2 + 36$$

$$h'(2.71) = -9.8(2.71) = -26.6\ m/s$$

$h'(t) = ??$

$t = ??$

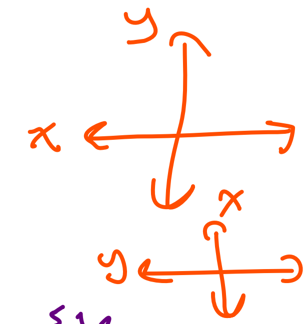
$$\boxed{h(t) = 0}$$

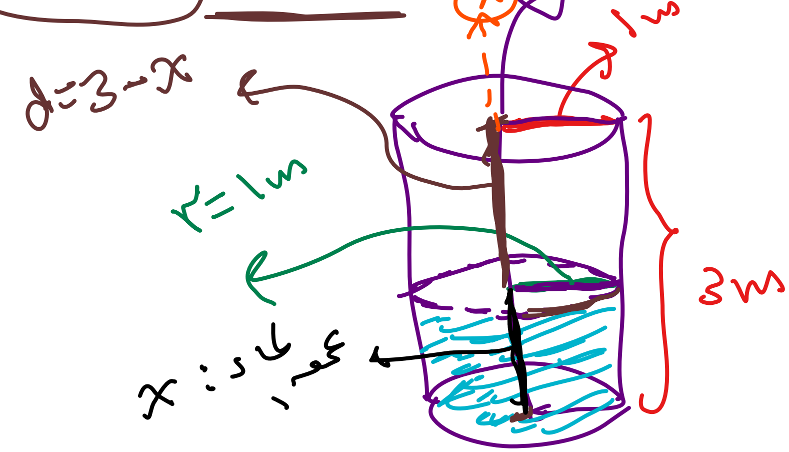
$$-4.9t^2 + 36 = 0$$

$$\boxed{t = 2.71}$$

Aright circular cylinder of radius 1m and height 3m is filled with water. Compute the work done pumping all of the water out of the top of the cylinder if the cylinder stands up right (the circular cross-sections are parallel to the ground).
The weight density of water is 9800 N/m^3.

أسطوانة دائرية قائمة طول نصف قطرها 1 m وارتفاعها 3 m ممتلئة بالماء. احسب الشغل المبذول عند ضخ كل الماء إلى الخارج انطلاقاً من الجزء العلوي للأسطوانة إذا كانت الأسطوانة في وضع قائم (المقاطع العرضية الدائرية موازية للأرض).
تبلغ كثافة وزن الماء 9800 N/m^3 .





$$W = 9800\,\pi \int_0^3 (3 - x)\,dx$$

مساحة الدائرة = $\pi r^2 = \pi$

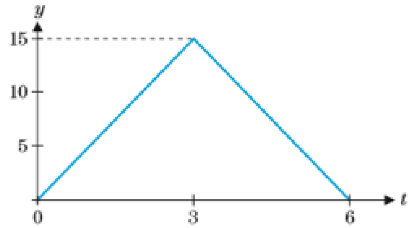
$$W = 9800\,\pi \int_0^3 (3 - x)^2\,dx$$

$$W = 9800\,\pi \int_0^3 x\,(3 - x)\,dx$$

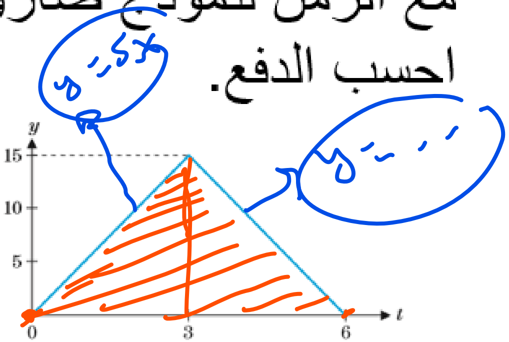
$$W = 9800 \int_0^3 (3 - x)\,dx$$

$$W = 9800 \int_0^3 \pi \cdot (3 - x)\,dx$$

A thrust-time curve for a model rocket is shown. Compute the impulse.



يبين الشكل أدناه منحنى الضغط مع الزمن لنموذج صاروخ. احسب الدفع.



45   90   22.5   52.5

$$J = \int_a^b F(t)\,dt = \int_0^6 F(t)\,dt = \text{Area} = \frac{1}{2}(6)(15) = \boxed{45}$$

For $f(x) = ce^{-4x}$, find c so that $f(x)$ is a pdf on the interval $[0, b]$ for $b > 0$.

لكل $f(x) = ce^{-4x}$ ، أوجد c بحيث تكون $f(x)$ دالة كثافة احتمالية (pdf) على الفترة $[0,b]$ لكل $b > 0$.

$c = \dfrac{4}{1 - e^{-4b}}$

$c = \dfrac{-4}{1 + e^{-4b}}$

$c = \dfrac{4}{1 - e^{4b}}$

$c = 4(1 + e^{4b})$

$$\int_0^b c e^{-4x}\, dx = 1$$

$$c \frac{e^{-4x}}{-4} \Big|_0^b = 1$$

$$c e^{-4x} \Big|_0^b = -4 \Rightarrow c\left[e^{-4b} - 1\right] = -4$$

$$\Rightarrow c = \frac{-4}{e^{-4b} - 1} = \frac{4}{1 - e^{-4b}}$$

Find the mean of a random variable with pdf $f(x) = x + 2x^3$ on the interval [0, 1].

أوجد المتوسط الحسابي لمتغير عشوائي دالة كثافته الاحتمالية (pdf) $f(x)$ $= x + 2x^3$ على الفترة [0, 1] .

$\frac{11}{15}$ $\qquad$ $\frac{7}{15}$ $\qquad$ $\frac{3}{4}$ $\qquad$ $\frac{1}{2}$

$$\mu = \int_0^1 x(x + 2x^3)\,dx = \frac{11}{15}$$

Evaluate $\int \frac{x^3}{1+x^8}\,dx$ أوجد قيمة $\int \frac{x^3}{1+x^8}\,dx$

$\frac{1}{4}\tan^{-1}(x^4)+c$

$\tan^{-1}(x^4)+c$

$4\tan^{-1}(x^4)+c$

$\sin^{-1}(x^4)+c$

$$= \int \frac{x^3}{1+(x^4)^2}\,dx$$

$$\boxed{u = x^4}$$

$$du = 4x^3\,dx$$

$$dx = \frac{du}{4x^3}$$

$$I = \int \frac{\cancel{x^3}}{1+u^2}\cdot\frac{du}{4\cancel{x^3}}$$

$$= \frac{1}{4}\int \frac{1}{1+u^2}\,du = \boxed{\frac{1}{4}\tan^{-1}(u) + c}$$

$$= \frac{1}{4}\tan^{-1}(x^4)+c$$

Evaluate

$$\int \cot(1-2x)\csc(1-2x)\,dx$$

أوجد قيمة

$$\int \cot(1-2x)\csc(1-2x)\,dx$$

$\frac{1}{2}\csc(1-2x)+c$

$-\frac{1}{2}\csc(1-2x)+c$

$2\csc(1-2x)+c$

$-2\csc(1-2x)+c$

$$= \frac{1}{-2} \cdot -\csc(1-2x) + c$$

$$= \frac{1}{2}\csc(1-2x) + c$$

$$* \int \csc(ax+b)\cot(ax+b)\,dx$$

$$= \frac{-1}{a} \cdot \csc(ax+b) + c$$

Evaluate

$$\int_{-0.5}^{0.5} cos^{-1}x\, dx$$

أوجد قيمة

$$\int_{-0.5}^{0.5} cos^{-1}x\, dx$$

$$x cos^{-1}x \Big|_{-0.5}^{0.5} + \int_{-0.5}^{0.5} \frac{x}{\sqrt{1-x^2}}\, dx$$

$$2x cos^{-1}x \Big|_{-0.5}^{0.5} - \int_{-0.5}^{0.5} \frac{x}{\sqrt{1-x^2}}\, dx$$

$$x cos^{-1}x \Big|_{-0.5}^{0.5} - \int_{-0.5}^{0.5} \frac{x}{\sqrt{1-x^2}}\, dx$$

$$x cos^{-1}x \Big|_{-0.5}^{0.5} + \int_{-0.5}^{0.5} \frac{1}{\sqrt{1-x^2}}\, dx$$

$u = \cos^{-1} x \rightarrow du = \frac{-1}{\sqrt{1-x^2}} dx$

$dv = dx \rightarrow v = x$

$I = x \cos^{-1} x \Big|_{-0.5}^{0.5} - \int_{-0.5}^{0.5} x \cdot \frac{-1}{\sqrt{1-x^2}} dx$

Evaluate

$$\int x\,tan^3(x^2+1)\,sec^2(x^2+1)\,dx$$

أوجد قيمة

$$\int x\,tan^3(x^2+1)\,sec^2(x^2+1)\,dx$$

$\frac{1}{8}tan^4(x^2+1)+c$

$\frac{1}{4}tan^4(x^2+1)+c$

$\frac{1}{2}tan^4(x^2+1)+c$

$2tan^4(x^2+1)+c$

$u = \tan(x^2+1) \rightarrow du = \sec^2(x^2+1)\cdot 2x\,dx$

$\rightarrow dx = \frac{du}{2x\sec^2(x^2+1)}$

$I = \int \cancel{x}\cdot u^3\cdot \cancel{\sec^2(x^2+1)}\cdot \frac{du}{2\cancel{x}\,\cancel{\sec^2(x^2+1)}}$

$= \frac{1}{2}\cdot\frac{u^4}{4}+c = \frac{1}{8}\tan^4(x^2+1)+c$

$* \frac{1}{2}\int 2x\tan^3(x^2+1)\sec^2(x^2+1)\,dx$

$2x\sec^2(x^2+1)$

$\int f^n\cdot f'\cdot dx = \frac{f^{n+1}}{n+1}+c$

$= \frac{1}{2}\cdot\frac{\tan^4(x^2+1)}{4}+c = \frac{1}{8}\tan^4(x^2+1)+c$

Evaluate

$$\int \frac{1}{\sqrt{4+x^2}}\,dx$$

substitute $x = 2tan\theta$

$, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

أوجد قيمة

$$\int \frac{1}{\sqrt{4+x^2}}\,dx$$

عوض $x = 2tan\theta$

$, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\sqrt{4+4\tan^2\theta}$

$= \sqrt{4(1+\tan^2\theta)}$

$= 2\sqrt{\sec^2\theta}$

$= 2\sec\theta$

- ✓ $\ln|sec\theta + tan\theta| + c \quad , were\ \theta = tan^{-1}\left(\frac{x}{2}\right)$
- ✗ $2\ln|sec\theta + tan\theta| + c \quad , were\ \theta = tan^{-1}\left(\frac{x}{2}\right)$
- $\ln|sec\theta + tan\theta| + c \quad , were\ \theta = tan^{-1}(2x)$
- ✗ $\ln|sec\theta - tan\theta| + c \quad , were\ \theta = tan^{-1}\left(\frac{x}{2}\right)$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$I = \int \frac{1}{\sqrt{4+4\tan^2\theta}} \cdot 2\sec^2\theta\, d\theta$

$= \int \frac{1}{\cancel{2\sec\theta}} \cdot \cancel{2}\sec^{\cancel{2}}\theta\, d\theta$

$= \int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$

$x = 2\tan\theta \rightarrow \tan\theta = \frac{x}{2}$

$\rightarrow \theta = \tan^{-1}\left(\frac{x}{2}\right)$

Find the partial fraction decomposition of

$$f(x) = \frac{x+4}{x^3+4x^2+4x}$$

أوجد تفكيك الكسور الجزئية للدالة

$$f(x) = \frac{x+4}{x^3+4x^2+4x}$$

$$f(x) = \frac{1}{x} - \frac{1}{x+2} - \frac{1}{(x+2)^2}$$

$$f(x) = \frac{1}{x} + \frac{1}{x+2} + \frac{1}{(x+2)^2}$$

$$f(x) = \frac{1}{x} - \frac{1}{x+2} + \frac{1}{(x+2)^2}$$

$$f(x) = \frac{1}{x} + \frac{1}{x+2} - \frac{1}{(x+2)^2}$$

$$\frac{x+4}{x(x^2+4x+4)} = \frac{x+4}{x(x+2)^2}$$

$$= \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$= \frac{A(x+2)^2 + Bx(x+2) + Cx}{x(x+2)^2}$$

$$x+4 = A(x+2)^2 + Bx(x+2) + Cx$$

$x=0 \Rightarrow 4 = 4A \Rightarrow \boxed{A=1}$

$x=-2 \Rightarrow 2 = -2C \Rightarrow \boxed{C=-1}$

$x=-1 \Rightarrow 3 = 1 - B + 1 \Rightarrow \boxed{B=-1}$

$\int \frac{\frac{1}{2}}{1-u}\,du = \frac{-1}{2}\ln|1-u| + C$

$= \frac{-1}{2}\int \frac{-1}{1-u}\,du$

Evaluate

$$\int \frac{e^x}{1-e^{2x}}\,dx$$

أوجد قيمة

$$\int \frac{e^x}{1-e^{2x}}\,dx$$

$u = e^x$

$\to du = e^x\,dx$

$\to dx = \frac{du}{e^x}$

$I = \int \frac{\cancel{u}}{1-u^2}\cdot\frac{du}{\cancel{u}}$

$= \int \frac{1}{1-u^2}\,du$

$\frac{1}{1-u^2} = \frac{A}{1-u} + \frac{B}{1+u}$

$= \frac{A(1+u)+B(1-u)}{1-u^2}$

$1 = A(1+u) + B(1-u)$

$u=-1 \Rightarrow 1 = 2B \Rightarrow \boxed{B = \frac{1}{2}}$

$u = 1 \Rightarrow 1 = 2A \Rightarrow \boxed{A = \frac{1}{2}}$

$$\frac{1}{2}\ln\left|\frac{1+e^x}{1-e^x}\right|$$

$$\frac{1}{2}\ln\left|\frac{1-e^x}{1+e^x}\right|$$

$$2\ln\left|\frac{1+e^x}{1-e^x}\right|$$

$$\frac{1}{2}\ln|1-e^x| + \frac{1}{2}\ln|1+e^x|$$

$I = \int \frac{\frac{1}{2}}{1-u}\,du + \int \frac{\frac{1}{2}}{1+u}\,du$

$= \frac{-1}{2}\ln|1-u| + \frac{1}{2}\ln|1+u| + C$

$= \frac{1}{2}\ln|1+e^x| - \frac{1}{2}\ln|1-e^x| + C$

$= \frac{1}{2}\ln\left|\frac{1+e^x}{1-e^x}\right| + C$

$-\frac{1}{2}\ln\left|\frac{1-e^x}{1+e^x}\right|+c$

Solve the differential equation

حل المعادلة التفاضلية

$$y' = 3y\ , \qquad y(0) = -2$$

$$y' = 3y \quad y(0) = -2$$

$$\frac{dy}{dx} = 3y$$

$$\int \frac{dy}{y} = \int 3\,dx \Rightarrow \ln|y| = 3x + c$$

$$y(0) = -2 \Rightarrow \ln 2 = 0 + c \Rightarrow c = \ln 2$$

$$\ln|y| = 3x + \ln 2$$

$$|y| = e^{3x+\ln 2} \Rightarrow |y| = e^{3x} \cdot e^{\ln 2}$$

$$\Rightarrow |y| = e^{3x} \cdot 2$$

$$\Rightarrow y = \pm 2e^{3x}$$

$y(0) = -2$

$y = -2e^{0}$

$= \boxed{-2}$

$$y = -2e^{3x}$$

$$y = 2e^{3x}$$

$$y = -2e^{\frac{1}{3}x}$$

$$y = 2e^{3x} - 4$$

$y(t) = Ae^{kt}$; $y' = ky$

$y = Ae^{3t}$

$y(0) = -2 \Rightarrow -2 = Ae^{0} \Rightarrow \boxed{A = -2}$

$\therefore y = -2e^{3x}$

Strontium-90 is a dangerous radio active isotope. Because of its similarity to calcium, it is easily absorbed into human bones. The half-life of strontium-90 is 28 years. If a certain amount is absorbed into the bones due to exposure to a nuclear explosion, what percentage will remain after 84 years?

يعتبر الأسترنتيوم – 90 أحد النظائر المشعة الخطيرة. وبسبب تشابهه مع الكالسيوم تمتصه العظام البشرية بكل سهولة. ويبلغ نصف العمر للأسترونتيوم – 90 حوالي 28 عاماً. إذا امتصت العظام كمية منه نتيجة تعرضها لانفجار نووي. فما النسبة المئوية التي ستبقى بعد مرور 84 عاماً؟

12.5% 25% 14.5% 29%

$y(28) = \frac{1}{2}A$

$y(t) = Ae^{kt}$

$\frac{1}{2}\cancel{A} = \cancel{A}e^{k(28)}$

$\Rightarrow -\ln 2 = 28k$

$\Rightarrow \boxed{k = \frac{-\ln 2}{28}}$

$y(t) = Ae^{\frac{-\ln 2}{28}t}$

$y(84) = ?? \Rightarrow \sim \%$

$y(84) = Ae^{\frac{-\ln 2}{28}(84)} = \boxed{0.125}A$

$0.125 \times 100\% = 12.5\%$

Which differential equation is **not** separable ?

أي المعادلات التفاضلية التالية **غير قابلة** للفصل؟

$$y' = 1 - \frac{3x}{y}$$

$y' = \frac{y - 3x}{y}$

$$y' = x^2 y^2$$

$$y' = x^2 + 5$$ ✓

$$y' = x^2 \sin(2y)$$ *

0551424288

https://bit.ly/3fVc7ts