



Grade 11 Elite Stream Mathematics Scheme of Work, Term 2, Academic Year 2022-2023

Purpose

- to define the **required** Elite Stream Mathematics content to be completed during the term for this grade;
- to **recommend** the pace at which the content is to be covered. The term's content is broken down into eight weeks, but the coverage of the content is flexible, with the actual pacing and deadlines to be determined by the teacher.

Assessment

- Assessment details for Term 3 will be communicated separately.

AP Calculus Mathematical Practices

The AP Calculus mathematical practices describe what a student should be able to do while exploring course concepts. Students should develop these practices during the AP Calculus AB and AP Calculus BC courses. The mathematical practices are

1. Implementing Mathematical Processes: Determine expressions and values using mathematical procedures and rules;
2. Connecting Representations: Translate mathematical information from a single representation or across multiple representations;
3. Justification: Justify reasoning and solutions; and
4. Communication and Notation: Use correct notation, language, and mathematical conventions to communicate results or solutions.

The mathematical practices are embedded throughout the course, providing teachers with a way to integrate these skills in the course content with sufficient repetition to prepare students to transfer those skills when taking the AP Exam.

For a complete breakdown of the specific skills into which these mathematical practices are categorized, see page 14 of the [AP Calculus AB and BC Course and Exam Description](#).

Additional Resources

- Dawkins, Paul. [Calculus 1](#) (Class Notes), 2018.
- Guichard, David et al. [Single Variable Calculus: Early Transcendentals](#), 2020.
- OpenStax, [Calculus Volume 1](#), [Calculus Volume 2](#), and [Calculus Volume 3](#).
- Schlieper, Jared and Tiemeyer, Michael. [Armstrong Calculus](#), 2017.
- Strang, Gilbert. [Calculus](#). Wellesley, MA. Wellesley-Cambridge Press (MIT), n.d.

Week 1: Jan. 2 – 6, 2023

Chapter 5 – Preliminaries

This week should be used to review topics required by students getting ready for the AP Calculus AB course. The material in Chapter 5 of the textbook can be used, along with other resources such as Khan Academy, to review and master any Precalculus topics for which teachers feel their students require some extra practice.

Topics which may need to be reviewed include, but are not limited to, the following:

- Linear and Quadratic Functions and Equations
- Rational and Radical Functions and Equations
- Trigonometric Functions, Equations, and Identities
- Exponential and Logarithmic Functions and Equations
- Inverse Functions
- Transformations of Functions
- Polynomials
- Inequalities

Teachers and students are encouraged to become very well acquainted with the wealth of material available on the [College Board website](#). In particular, teachers should refer to [AP Classroom](#) on a regular basis throughout the course. This website, along with the material in this Scheme of Work, is the primary source of information and resources for AP Calculus teachers. Textbook references are provided as a source of valuable supplementary material. As well, links to Khan Academy are included for almost every topic.

Week 2: Jan. 9 – 13, 2023

AP Calculus AB Unit 1 – Limits and Continuity

College Board recommended pacing: 22-23 periods; Actual: 30-31 periods

Topic	Learning Objectives	Essential Knowledge	Textbook	Khan Academy
1-1 Introducing Calculus: Can Change Occur at an Instant? AP Classroom	CHA-1.A Interpret the rate of change at an instant in terms of average rates of change over intervals containing that instant.	CHA-1.A.1 Calculus uses limits to understand and model dynamic change. CHA-1.A.2 Because an average rate of change divides the change in one variable by the change in another, the average rate of change is undefined at a point where the change in the independent variable would be zero. CHA-1.A.3 The limit concept allows us to define instantaneous rate of change in terms of average rates of change.	6.1 All	
1-2 Defining Limits and Using Limit Notation AP Classroom	LIM-1.A Represent limits analytically using correct notation.	LIM-1.A.1 Given a function f , the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x \rightarrow c} f(x) = R$. <i>Exclusion: The epsilon-delta definition of a limit is not assessed on the AP Calculus AB or BC Exam. However, teachers may include this topic in the course if time permits.</i>	6.2 Intro	Defining Limits and Using Limit Notation
	LIM-1.B Interpret limits expressed in analytic notation.	LIM-1.B.1 A limit can be expressed in multiple ways, including graphically, numerically, and analytically.		
1-3 Estimating Limit Values from Graphs AP Classroom	LIM-1.C Estimate limits of functions.	LIM-1.C.1 The concept of a limit includes one-sided limits. LIM-1.C.2 Graphical information about a function can be used to estimate limits. LIM-1.C.3 Because of issues of scale, graphical representations of functions may miss important function behavior. LIM-1.C.4 A limit might not exist for some functions at particular values of x . Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.	6.2 Ex. 1-8 6.3 Ex. 7, 8	Estimating Limit Values from Graphs

Week 3: Jan. 16 – 20, 2023

Topic	Learning Objectives	Essential Knowledge	Textbook	Khan Academy
1-4 Estimating Limit Values from Tables AP Classroom	LIM-1.C Estimate limits of functions.	LIM-1.C.5 Numerical information can be used to estimate limits.	6.2 Ex. 1, 2, 4-6, 8 6.3 Ex. 7, 8	Estimating Limit Values from Tables
1-5 Determining Limits Using Algebraic Properties of Limits AP Classroom	LIM-1.D Determine the limits of functions using limit theorems.	LIM-1.D.1 One-sided limits can be determined analytically or graphically. LIM-1.D.2 Limits of sums, differences, products, quotients, and composite functions can be found using limit theorems.	6.3 Ex. 1-4, 6, 7	Determining Limits Using Algebraic Properties of Limits: Direct Substitution Determining Limits Using Algebraic Properties of Limits: Limit Properties
1-6 Determining Limits Using Algebraic Manipulation AP Classroom	LIM-1.E Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.	LIM-1.E.1 It may be necessary or helpful to rearrange expressions into equivalent forms before evaluating limits.	6.2 Ex. 1, 4 6.3 Ex. 3, 5	Determining Limits Using Algebraic Manipulation
1-7 Selecting Procedures for Determining Limits AP Classroom	This topic is intended to focus on the skill of selecting an appropriate procedure for determining limits. Students should be given opportunities to practice when and how to apply all learning objectives relating to determining limits.		Discussed in many of the examples in Chapter 6	Selecting Procedures for Determining Limits

Week 4: Jan. 23 – 27, 2023

Topic	Learning Objectives	Essential Knowledge	Textbook	Khan Academy
1-8 Determining Limits Using the Squeeze Theorem AP Classroom	LIM-1.E Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.	LIM-1.E.2 The limit of a function may be found by using the squeeze theorem.	6.3 Ex. 8	Determining Limits Using the Squeeze Theorem
1-9 Connecting Multiple Representations of Limits AP Classroom	This topic is intended to focus on connecting representations. Students should be given opportunities to practice when and how to apply all learning objectives relating to limits and translating mathematical information from a single representation or across multiple representations.		6.3 All	
1-10 Exploring Types of Discontinuities AP Classroom	LIM-2.A Justify conclusions about continuity at a point using the definition.	LIM-2.A.1 Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.	6.4 Intro	Exploring Types of Discontinuities
1-11 Defining Continuity at a Point AP Classroom		LIM-2.A.2 A function f is continuous at $x = c$ provided that $f(c)$ exists, $\lim_{x \rightarrow c} f(x)$ exists, and $\lim_{x \rightarrow c} f(x) = f(c)$.	6.4 Intro	Defining Continuity at a Point

Week 5: Jan. 30 – Feb. 3, 2023

Topic	Learning Objectives	Essential Knowledge	Textbook	Khan Academy
1-12 Confirming Continuity over an Interval AP Classroom	LIM-2.B Determine intervals over which a function is continuous.	LIM-2.B.1 A function is continuous on an interval if the function is continuous at each point in the interval. LIM-2.B.2 Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous on all points in their domains.	6.4 Ex. 1-8	Confirming Continuity over an Interval
1-13 Removing Discontinuities AP Classroom	LIM-2.C Determine values of x or solve for parameters that make discontinuous functions continuous, if possible.	LIM-2.C.1 If the limit of a function exists at a discontinuity in its graph, then it is possible to remove the discontinuity by defining or redefining the value of the function at that point, so it equals the value of the limit of the function as x approaches that point. LIM-2.C.2 In order for a piecewise-defined function to be continuous at a boundary to the partition of its domain, the value of the expression defining the function on one side of the boundary must equal the value of the expression defining the other side of the boundary, as well as the value of the function at the boundary.	6.4 Ex. 2	Removing Discontinuities
1-14 Connecting Infinite Limits and Vertical Asymptotes AP Classroom	LIM-2.D Interpret the behavior of functions using limits involving infinity.	LIM-2.D.1 The concept of a limit can be extended to include infinite limits. LIM-2.D.2 Asymptotic and unbounded behavior of functions can be described and explained using limits.	6.5 Ex. 1-5, 9	Connecting Infinite Limits and Vertical Asymptotes
1-15 Connecting Limits at Infinity and Horizontal Asymptotes AP Classroom		LIM-2.D.3 The concept of a limit can be extended to include limits at infinity. LIM-2.D.4 Limits at infinity describe end behavior. LIM-2.D.5 Relative magnitudes of functions and their rates of change can be compared using limits.	6.5 Ex. 6, 7, 10, 11	Connecting Limits at Infinity and Horizontal Asymptotes

Week 6: Feb. 6 – 10, 2023

Topic	Learning Objectives	Essential Knowledge	Textbook	Khan Academy
1-16 Working with the Intermediate Value Theorem AP Classroom	FUN-1.A Explain the behavior of a function on an interval using the Intermediate Value Theorem.	FUN-1.A.1 If f is a continuous function on the closed interval $[a, b]$ and d is a number between $f(a)$ and $f(b)$, then the Intermediate Value Theorem guarantees that there is at least one number c between a and b , such that $f(c)$ equals d .	6.4 Ex. 9	Working with the Intermediate Value Theorem Optional Videos for Unit 1
AP Calculus AB Unit 2 – Differentiation: Definition and Basic Derivative Rules College Board recommended pacing: 13-14 periods; Actual: 25-26 periods				
2-1 Defining Average and Instantaneous Rates of Change at a Point AP Classroom	CHA-2.A Determine average rates of change using difference quotients.	CHA-2.A.1 The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.	7.1 All	Defining Average and Instantaneous Rates of Change at a Point
	CHA-2.B Represent the derivative of a function as the limit of a difference quotient.	CHA-2.B.1 The instantaneous rate of change of a function at $x = a$ can be expressed by $\lim_{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$ or $\lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$, provided the limit exists. These are equivalent forms of the definition of the derivative and are denoted $f'(a)$.		
2-2 Defining the Derivative of a Function and Using Derivative Notation AP Classroom	CHA-2.B Represent the derivative of a function as the limit of a difference quotient.	CHA-2.B.2 The derivative of f is the function whose value at x is $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$, provided this limit exists.	7.2 Intro, Ex. 1-6	Defining the Derivative of a Function and Using Derivative Notation
		CHA-2.B.3 For $y = f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, and y' . CHA-2.B.4 The derivative can be represented graphically, numerically, analytically, and verbally.		
	CHA-2.C Determine the equation of a line tangent to a curve at a given point.	CHA-2.C.1 The derivative of a function at a point is the slope of the line tangent to a graph of the function at that point.		

Week 7: Feb. 13 – 17, 2023

Topic	Learning Objectives	Essential Knowledge	Textbook	Khan Academy
2-3 Estimating Derivatives of a Function at a Point AP Classroom	CHA-2.D Estimate derivatives.	CHA-2.D.1 The derivative at a point can be estimated from information given in tables or graphs. CHA-2.D.2 Technology can be used to calculate or estimate the value of a derivative of a function at a point.	7.1 Ex. 3, 4 7.2 Ex. 8, 9	Estimating Derivatives of a Function at a Point
2-4 Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist AP Classroom	FUN-2.A Explain the relationship between differentiability and continuity.	FUN-2.A.1 If a function is differentiable at a point, then it is continuous at that point. In particular, if a point is not in the domain of f , then it is not in the domain of f' . FUN-2.A.2 A continuous function may fail to be differentiable at a point in its domain.	7.2 Ex. 7	Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist
2-5 Applying the Power Rule AP Classroom	FUN-3.A Calculate derivatives of familiar functions.	FUN-3.A.1 Direct application of the definition of the derivative and specific rules can be used to calculate the derivative for functions of the form $f(x) = x^n$.	7.3 Ex. 1-5	Applying the Power Rule
2-6 Derivative Rules: Constant, Sum, Difference, and Constant Multiple AP Classroom	FUN-3.A Calculate derivatives of familiar functions.	FUN-3.A.2 Sums, differences, and constant multiples of functions can be differentiated using derivative rules. FUN-3.A.3 The power rule combined with sum, difference, and constant multiple properties can be used to find the derivatives for polynomial functions.	7.3 Ex. 3-5	Derivative Rules: Constant, Sum, Difference, and Constant Multiple: Introduction Derivative Rules: Constant, Sum, Difference, and Constant Multiple: Connecting with the Power Rule

Week 8: Feb. 20 – 24, 2023

Topic	Learning Objectives	Essential Knowledge	Textbook	Khan Academy
2-8 The Product Rule AP Classroom	FUN-3.B Calculate derivatives of products and quotients of differentiable functions.	FUN-3.B.1 Derivatives of products of differentiable functions can be found using the product rule.	7.4 Ex. 1, 2, 4, 5	The Product Rule
2-9 The Quotient Rule AP Classroom		FUN-3.B.2 Derivatives of quotients of differentiable functions can be found using the quotient rule.	7.4 Ex. 3, 4, 6	The Quotient Rule
3-1 The Chain Rule AP Classroom	FUN-3.C Calculate derivatives of compositions of differentiable functions.	FUN-3.C.1 The chain rule provides a way to differentiate composite functions.	7.5 All	The Chain Rule: Introduction The Chain Rule: Further Practice

Week 9: Feb. 27 – March 3, 2023

Topic	Learning Objectives	Essential Knowledge	Textbook	Khan Academy
2-10 Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions AP Classroom	FUN-3.B Calculate derivatives of products and quotients of differentiable functions.	FUN-3.B.3 Rearranging tangent, cotangent, secant, and cosecant functions using identities allows differentiation using derivative rules.	8.1 Intro, Ex. 2, 5	Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions
2-7 Derivatives of $\cos x$, $\sin x$, e^x , and $\ln x$ AP Classroom	FUN-3.A Calculate derivatives of familiar functions.	FUN-3.A.4 Specific rules can be used to find the derivatives for sine, cosine, exponential, and logarithmic functions.	8.1 All 8.2 All	Derivatives of $\cos x$, $\sin x$, e^x, and $\ln x$ Optional Videos for Unit 2
	LIM-3.A Interpret a limit as a definition of a derivative.	LIM-3.A.1 In some cases, recognizing an expression for the definition of the derivative of a function whose derivative is known offers a strategy for determining a limit.		

Week 10: March 6 – 10, 2023

Week 11: March 13 – 17, 2023

Week 12: March 20 – 24, 2023

Term 2 Revision and End-of-Term Exam
Exam date to be determined by the Assessment Directorate