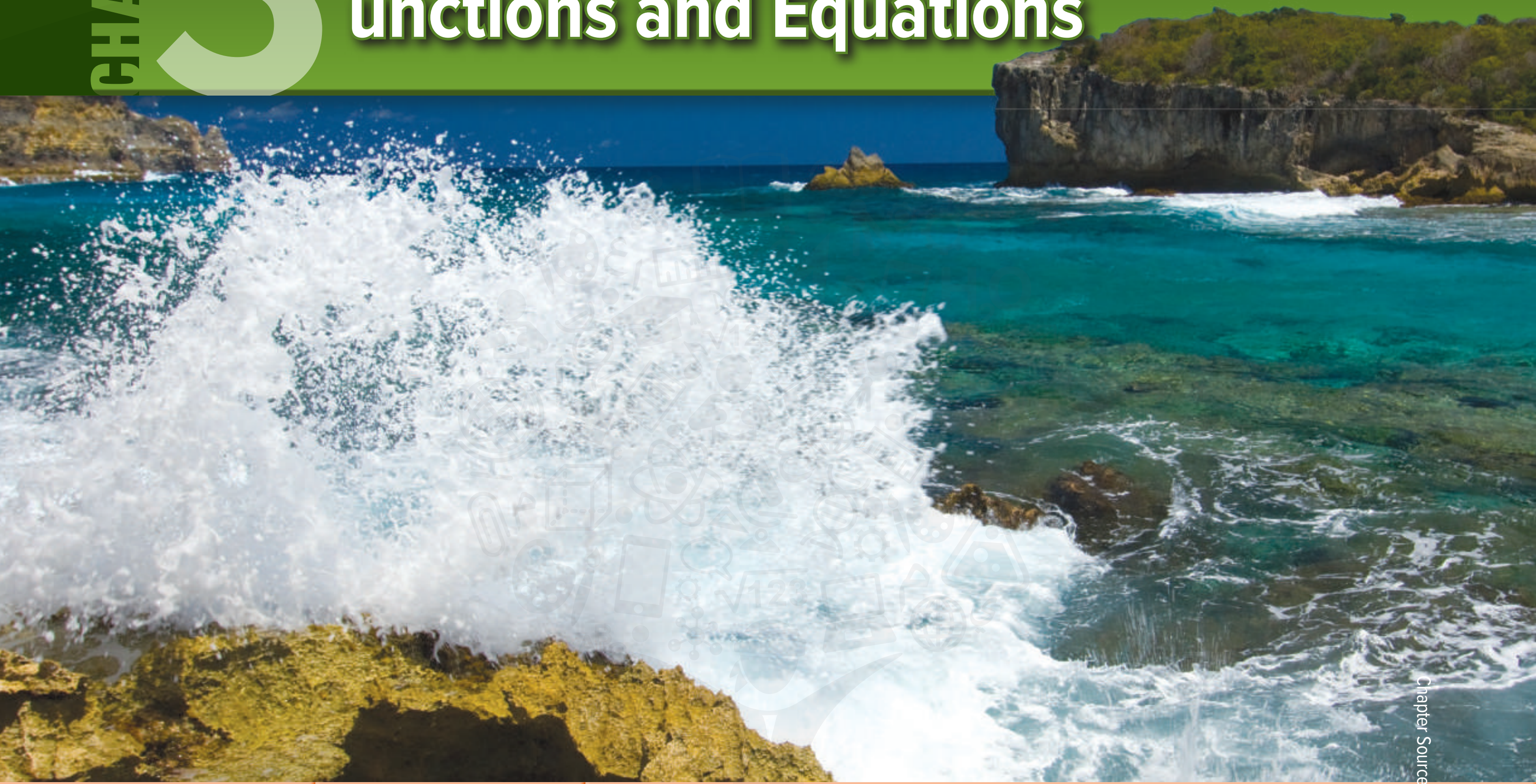
CHAPTER 3

Radical and Rational f unctions and Equations



Then

You solved quadratic and exponential equations.

Now

In this chapter, you will:

- Graph and transform radical functions.
- Simplify, add, subtract, and multiply radical expressions.
- Solve radical equations.
- Use the Pythagorean Theorem.
-

Why?

OCEANS Tsunamis, or large waves, are generated by undersea earthquakes. A radical equation can be used to find the speed of a tsunami in meters per second or the depth of the ocean in meters.

Get Ready for the Chapter

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Find each square root. If necessary, round to the nearest hundredth.

1. $\sqrt{82}$
2. $\sqrt{26}$
3. $\sqrt{15}$
4. $\sqrt{99}$
5. **SANDBOX** Eissa is making a square sandbox with an area of 100 square meters. How long is a side of the sandbox?

QuickReview

Example 1

Find the square root of $\sqrt{50}$. If necessary, round to the nearest hundredth.

$\sqrt{50} = 7.071067812....$ Use a calculator.

To the nearest hundredth, $\sqrt{50} = 7.07$.

Simplify each expression.

6. $(21x + 15y) - (9x - 4y)$
7. $13x - 5y + 2y$
8. $(10a - 5b) + (6a + 5b)$
9. $6m + 5n + 4 - 3m - 2n + 6$
10. $x + y - 3x - 4y + 2x - 8y$

Example 2

Simplify $3x + 7y - 4x - 8y$.

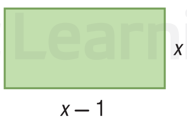
$3x + 7y - 4x - 8y$

$= (3x - 4x) + (7y - 8y)$ Combine like terms.

$= -x - y$ Simplify.

Solve each equation.

11. $2x^2 - 4x = 0$
12. $6x^2 - 5x - 4 = 0$
13. $x^2 - 7x + 10 = 0$
14. $2x^2 + 7x - 5 = -1$
15. **GEOMETRY** The area of the rectangle is 90 square meters. Find x.



Example 3

Solve $x^2 - 5x + 6 = 0$.

$x^2 - 5x + 6 = 0$ Original equation

$(x - 3)(x - 2) = 0$ Factor.

$x - 3 = 0$ or $x - 2 = 0$ Zero Product Property

$x = 3$ $\quad x = 2$ Solve each equation.

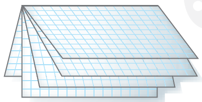
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 3. To get ready, identify important terms and organize your resources.

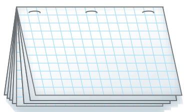
FOLDABLES® StudyOrganizer

Radical Functions and Geometry Make this Foldable to help you organize your Chapter 3 notes about radical functions and geometry. Begin with four sheets of grid paper.

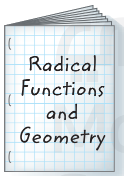
1 **Fold** in half along the width.



2 **Staple** along the fold.



3 **Turn** the fold to the left and write the title of the chapter on the front. On each left-hand page of the booklet, write the title of a lesson from the chapter.



NewVocabulary

- square root function
- radicand
- radical function
- radical expression
- conjugate
- rationalize the denominator
- closed
- radical equations
- extraneous solutions
- inverse variation
- product rule
- rational function
- excluded values
- asympote
- rational equation
- work problem
- rate problem

ReviewVocabulary

FOIL method to multiply two binomials, find the sum of the products of the First terms, Outer terms, Inner terms, and Last terms

perfect square a number with a square root that is a rational number

proportion an equation of the form $\frac{a}{b} = \frac{c}{d}$, $b \neq 0$, $d \neq 0$ stating that two ratios are equivalent

$$\frac{a}{b} \times \frac{c}{d}$$

$$ad = bc$$

LESSON 3-1 Square Root Functions

Then

- You graphed and analyzed linear, exponential, and quadratic functions.

Now

1. Graph and analyze dilations of radical functions.
2. Graph and analyze reflections and translations of radical functions.

Why?

- Scientists use sounds of whales to track their movements. The distance to a whale can be found by relating time to the speed of sound in water.

The speed of sound in water can be described by the *square root function* $c = \sqrt{\frac{E}{d}}$, where E represents the bulk modulus elasticity of the water and d represents the density of the water.

NewVocabulary
square root function
radical function
radicand

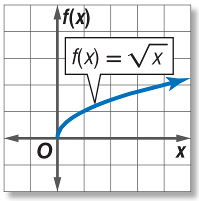
Mathematical Practices
Attend to precision.

1 Dilations of Radical Functions

A **square root function** contains the square root of a variable. Square root functions are a type of **radical function**. The expression under the radical sign is called the **radicand**. For a square root to be a real number, the radicand cannot be negative. Values that make the radicand negative are not included in the domain.

KeyConcept Square Root Function

Parent Function:	$f(x) = \sqrt{x}$
Type of Graph:	curve
Domain:	$\{x \mid x \geq 0\}$
Range:	$\{y \mid y \geq 0\}$



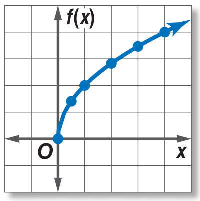
Example 1 Dilation of the Square Root Function

Graph $f(x) = 2\sqrt{x}$. State the domain and range.

Step 1 Make a table.

x	0	0.5	1	2	3	4
$f(x)$	0	≈1.4	2	≈2.8	≈3.5	4

Step 2 Plot points. Draw a smooth curve.



The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \geq 0\}$. Notice that the graph is increasing on the entire domain, the minimum value is 0, and there is no symmetry.

GuidedPractice

1A. $g(x) = 4\sqrt{x}$

1B. $h(x) = 6\sqrt{x}$

2 Reflections and Translations of Radical Functions

Recall that when the value of a is negative in the quadratic function $f(x) = ax^2$, the graph of the parent function is reflected across the x-axis.

> **StudyTip**
> **Graphing Radical Functions** Choose perfect squares for x-values that will result in coordinates that are easy to plot.

KeyConcept Graphing $y = a\sqrt{x + h} + k$

Step 1 Draw the graph of $y = a\sqrt{x}$. The graph starts at the origin and passes through (1, a). If $a > 0$, the graph is in quadrant I. If $a < 0$, the graph is reflected across the x-axis and is in quadrant IV.

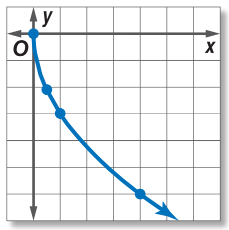
Step 2 Translate the graph k units up if $k > 0$ and $|k|$ units down if $k < 0$.

Step 3 Translate the graph h units left if $h > 0$ and $|h|$ units right if $h < 0$.

Example 2 Reflection of the Square Root Function

Graph $y = -3\sqrt{x}$. Compare to the parent graph. State the domain and range.

Make a table of values. Then plot the points on a coordinate system and draw a smooth curve that connects them.



x	0	0.5	1	4
y	0	≈−2.1	−3	−6

Notice that the graph is in the 4th quadrant. It is obtained by stretching the graph of $y = \sqrt{x}$ vertically and then reflecting across the x-axis. The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \leq 0\}$.

GuidedPractice

2A. $y = -2\sqrt{x}$

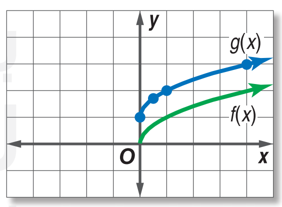
2B. $y = -4\sqrt{x}$

> **StudyTip**
> **Translating Radical Functions** If $h > 0$, a radical function $f(x) = \sqrt{x - h}$ is a horizontal translation h units to the right. $f(x) = \sqrt{x + h}$ is a horizontal translation h units to the left.

Example 3 Translation of the Square Root Function

Graph each function. Compare to the parent graph. State the domain and range.

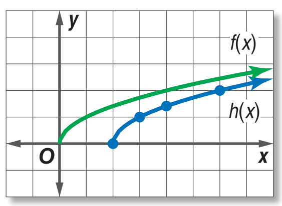
a. $g(x) = \sqrt{x} + 1$



x	0	0.5	1	4	9
y	0	≈1.7	2	3	4

Notice that the values of $g(x)$ are 1 greater than those of $f(x) = \sqrt{x}$. This is a vertical translation 1 unit up from the parent function. The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \geq 1\}$.

b. $h(x) = \sqrt{x - 2}$



x	2	3	4	6
y	0	1	≈1.4	2

This is a horizontal translation 2 units to the right of the parent function. The domain is $\{x \mid x \geq 2\}$, and the range is $\{y \mid y \geq 0\}$.

GuidedPractice

3A. $g(x) = \sqrt{x} - 4$

3B. $h(x) = \sqrt{x + 3}$

Physical phenomena such as motion can be modeled by radical functions. Often these functions are transformations of the parent square root function.

Real-WorldLink

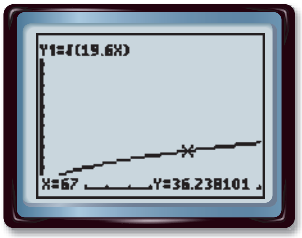
Approximately 39 million cars cross the Golden Gate Bridge in San Francisco each year.

Source: San Francisco Convention and Visitors Bureau

Real-World Example 4 Analyze a Radical Function

BRIDGES The Golden Gate Bridge is about 67 meters above the water. The velocity v of a freely falling object that has fallen h meters is given by $v = \sqrt{2gh}$, where g is the constant 9.8 meters per second squared. Graph the function. If an object is dropped from the bridge, what is its velocity when it hits the water?

Use a graphing calculator to graph the function. To find the velocity of the object, substitute 67 meters for h.



$v = \sqrt{2gh}$	Original function
$= \sqrt{2(9.8)(67)}$	$g = 9.8$ and $h = 67$
$= \sqrt{1313.2}$	Simplify.
≈ 36.2 m/s	Use a calculator.

The velocity of the object is about 36.2 meters per second after dropping 67 meters.

GuidedPractice

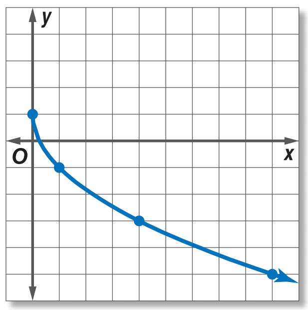
4. Use the graph above to estimate the initial height of an object if it is moving at 20 meters per second when it hits the water.

Transformations such as reflections, translations, and dilations can be combined in one equation.

Example 5 Transformations of the Square Root Function

Graph $y = -2\sqrt{x} + 1$, and compare to the parent graph. State the domain and range.

x	0	1	4	9
y	1	−1	−3	−5



This graph is the result of a vertical stretch of the graph of $y = \sqrt{x}$ followed by a reflection across the x-axis, and then a translation 1 unit up. The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \leq 1\}$.

GuidedPractice

5A. $y = \frac{1}{2}\sqrt{x} - 1$

5B. $y = -2\sqrt{x - 1}$

Check Your Understanding

Examples 1–3 **Graph each function. Compare to the parent graph. State the domain and range.**

1. $y = 3\sqrt{x}$
2. $y = -5\sqrt{x}$
3. $y = \frac{1}{3}\sqrt{x}$
4. $y = -\frac{1}{2}\sqrt{x}$
5. $y = \sqrt{x} + 3$
6. $y = \sqrt{x} - 2$
7. $y = \sqrt{x + 2}$
8. $y = \sqrt{x - 3}$

Example 4

9. **FREE FALL** The time t, in seconds, that it takes an object to fall a distance d, in meters, is given by the function $t = \frac{5}{11}\sqrt{d}$ (assuming zero air resistance). Graph the function, and state the domain and range.

Example 5 **Graph each function, and compare to the parent graph. State the domain and range.**

10. $y = \frac{1}{2}\sqrt{x} + 2$
11. $y = -\frac{1}{4}\sqrt{x} - 1$
12. $y = -2\sqrt{x + 1}$
13. $y = 3\sqrt{x - 2}$

Practice and Problem Solving

Examples 1–3 **Graph each function. Compare to the parent graph. State the domain and range.**

14. $y = 5\sqrt{x}$
15. $y = \frac{1}{2}\sqrt{x}$
16. $y = -\frac{1}{3}\sqrt{x}$
17. $y = 7\sqrt{x}$
18. $y = -\frac{1}{4}\sqrt{x}$
19. $y = -\sqrt{x}$
20. $y = -\frac{1}{5}\sqrt{x}$
21. $y = -7\sqrt{x}$
22. $y = \sqrt{x} + 2$
23. $y = \sqrt{x} + 4$
24. $y = \sqrt{x} - 1$
25. $y = \sqrt{x} - 3$
26. $y = \sqrt{x} + 1.5$
27. $y = \sqrt{x} - 2.5$
28. $y = \sqrt{x + 4}$
29. $y = \sqrt{x - 4}$
30. $y = \sqrt{x + 1}$
31. $y = \sqrt{x - 0.5}$
32. $y = \sqrt{x + 5}$
33. $y = \sqrt{x - 1.5}$

Example 4

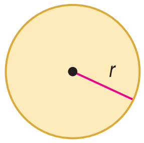
34. **GEOMETRY** The perimeter of a square is given by the function $P = 4\sqrt{A}$, where A is the area of the square.
    a. Graph the function.
    b. Determine the perimeter of a square with an area of 225 m^2.
    c. When will the perimeter and the area be the same value?

Example 5 **Graph each function, and compare to the parent graph. State the domain and range.**

35. $y = -2\sqrt{x} + 2$
36. $y = -3\sqrt{x} - 3$
37. $y = \frac{1}{2}\sqrt{x + 2}$
38. $y = -\sqrt{x - 1}$
39. $y = \frac{1}{4}\sqrt{x - 1} + 2$
40. $y = \frac{1}{2}\sqrt{x - 2} + 1$

41. **ENERGY** An object has kinetic energy when it is in motion. The velocity in meters per second of an object of mass m kilograms with an energy of E joules is given by the function $v = \sqrt{\frac{2E}{m}}$. Use a graphing calculator to graph the function that represents the velocity of a basketball with a mass of 0.6 kilogram.

42. **GEOMETRY** The radius of a circle is given by $r = \sqrt{\frac{A}{\pi}}$, where A is the area of the circle.



a. Graph the function.

b. Use a graphing calculator to determine the radius of a circle that has an area of 27 cm^2.

43 **SPEED OF SOUND** The speed of sound in air is determined by the temperature of the air. The speed c in meters per second is given by $c = 331.5\sqrt{1 + \frac{t}{273.15}}$, where t is the temperature of the air in degrees Celsius.

a. Use a graphing calculator to graph the function.

b. How fast does sound travel when the temperature is 55°C?

c. How is the speed of sound affected when the temperature increases to 65°C?

44. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between the graphs of square root functions and parabolas.

a. **Graphical** Graph $y = x^2$ on a coordinate system.

b. **Algebraic** Write a piecewise-defined function to describe the graph of $y^2 = x$ in each quadrant.

c. **Graphical** On the same coordinate system, graph $y = \sqrt{x}$ and $y = -\sqrt{x}$.

d. **Graphical** On the same coordinate system, graph $y = x$. Plot the points (2, 4), (4, 2), and (1, 1).

e. **Analytical** Compare the graph of the parabola to the graphs of the square root functions.

H.O.T. Problems Use Higher-Order Thinking Skills

CHALLENGE **Determine whether each statement is *true* or *false*. Provide an example or counterexample to support your answer.**

45. Numbers in the domain of a radical function will always be nonnegative.

46. Numbers in the range of a radical function will always be nonnegative.

47. **WRITING IN MATH** Why are there limitations on the domain and range of square root functions?

48. **TOOLS** Write a radical function with a domain of all real numbers greater than or equal to 2 and a range of all real numbers less than or equal to 5.

49. **WHICH DOES NOT BELONG?** Identify the equation that does not belong. Explain.

$y = 3\sqrt{x}$	$y = 0.7\sqrt{x}$	$y = \sqrt{x} + 3$	$y = \frac{\sqrt{x}}{6}$

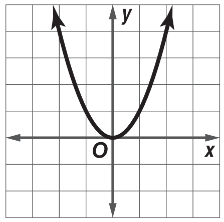
50. **OPEN ENDED** Write a function that is a reflection, translation, and a dilation of the parent graph $y = \sqrt{x}$.

51. **REASONING** If the range of the function $y = a\sqrt{x}$ is $\{y \mid y \leq 0\}$, what can you conclude about the value of a? Explain your reasoning.

52. **WRITING IN MATH** Compare and contrast the graphs of $f(x) = \sqrt{x} + 2$ and $g(x) = \sqrt{x + 2}$.

Standardized Test Practice

53.



Which function *best* represents the graph?

A $y = x^2$ **C** $y = \sqrt{x}$

B $y = 2^x$ **D** $y = x$

54. The statement "$x < 10$ and $3x - 2 \geq 7$" is true when x is equal to what?

F 0 **H** 8

G 2 **J** 12

55. Which of the following is the equation of a line parallel to $y = -\frac{1}{2}x + 3$ and passing through $(-2, -1)$?

A $y = \frac{1}{2}x$ **C** $y = -\frac{1}{2}x + 2$

B $y = 2x + 3$ **D** $y = -\frac{1}{2}x - 2$

56. SHORT RESPONSE A landscaper needs to mulch 6 rectangular flower beds that are 8 meters by 4 meters and 4 circular flower beds each with a radius of 3 meters. One bag of mulch covers 25 square meters. How many bags of mulch are needed to cover the flower beds?

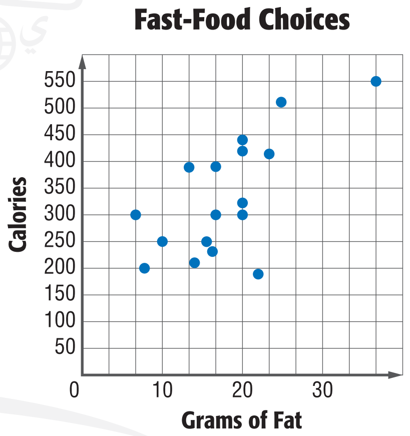
Spiral Review

57. HEALTH Khawla exercises every day by walking and jogging at least 3 kilometers. Khawla walks at a rate of 4 kilometers per hour and jogs at a rate of 8 kilometers per hour. Suppose she has at most one half-hour to exercise today.

a. Draw a graph showing the possible amounts of time she can spend walking and jogging today.

b. List three possible solutions.

58. NUTRITION Determine whether the graph shows a *positive*, *negative*, or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation.



Skills Review

Factor each monomial completely.

59. $28n^3$

60. $-33a^2b$

61. $150rt$

62. $-378nq^2r^2$

63. $225a^3b^2c$

64. $-160x^2y^4$

EXTEND 3-1

Graphing Technology Lab
Graphing Square Root Functions

For a square root to be a real number, the radicand cannot be negative. When graphing a radical function, determine when the radicand would be negative and exclude those values from the domain.

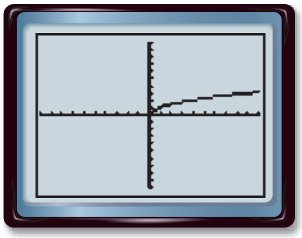
Mathematical Practices
Use appropriate tools strategically.

Activity 1 Parent Function

Graph $y = \sqrt{x}$.

Enter the equation in the **Y=** list, and graph in the standard viewing window.

KEYSTROKES: [Y=] [2nd] [[√‾]] [X,T,θ,n] [ )] [ZOOM] 6



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1A. Examine the graph. What is the domain of the function?

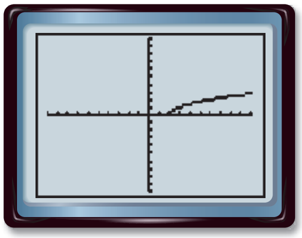
1B. What is the range of the function?

Activity 2 Translation of Parent Function

Graph $y = \sqrt{x - 2}$.

Enter the equation in the **Y=** list, and graph in the standard viewing window.

KEYSTROKES: [Y=] [2nd] [[√‾]] [X,T,θ,n] [−] 2 [ )] [ZOOM] 6



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2A. What are the domain and range of the function?

2B. How does the graph of $y = \sqrt{x - 2}$ compare to the graph of the parent function $y = \sqrt{x}$?

Exercises

Graph each equation, and sketch the graph on your paper. State the domain and range. Describe how the graph differs from that of the parent function $y = \sqrt{x}$.

1. $y = \sqrt{x - 1}$

2. $y = \sqrt{x + 3}$

3. $y = \sqrt{x} - 2$

4. $y = \sqrt{-x}$

5. $y = -\sqrt{x}$

6. $y = \sqrt{2x}$

7. $y = \sqrt{2 - x}$

8. $y = \sqrt{x - 3} + 2$

Solve each equation for y. Does the equation represent a function? Explain your reasoning.

9. $x = y^2$

10. $x^2 + y^2 = 4$

11. $x^2 + y^2 = 2$

Write a function with a graph that translates $y = \sqrt{x}$ in each way.

12. shifted 4 units to the left

13. shifted up 7 units

14. shifted down 6 units

15. shifted 5 units to the right and up 3 units

LESSON 3-2 Radical Equations

Then

- You added, subtracted, and multiplied radical expressions.

Now

- **1** Solve radical equations.
- **2** Solve radical equations with extraneous solutions.

Why?

- The waterline length of a sailboat is the length of the line made by the water's edge when the boat is full. A sailboat's hull speed is the fastest speed that it can travel.

  You can estimate hull speed h by using the formula $h = 1.34\sqrt{\ell}$, where ℓ is the length of the sailboat's waterline.

NewVocabulary
radical equations
extraneous solutions

Mathematical Practices
Construct viable arguments and critique the reasoning of others.
Model with mathematics.

1 Radical Equations

Equations that contain variables in the radicand, like $h = 1.34\sqrt{\ell}$, are called **radical equations**. To solve, isolate the desired variable on one side of the equation first. Then square each side of the equation to eliminate the radical.

KeyConcept Power Property of Equality

Words	If you square both sides of a true equation, the resulting equation is still true.
Symbols	If $a = b$, then $a^2 = b^2$.
Examples	If $\sqrt{x} = 4$, then $(\sqrt{x})^2 = 4^2$.

Real-World Example 1 Variable as a Radicand

SAILING Usama and Ismail are sailing in a friend's sailboat. They measure the hull speed at 9 kilometers per hour. Find the length of the sailboat's waterline. Round to the nearest meters.

Understand You know how fast the boat will travel and that it relates to the length.

Plan The boat travels at 9 nautical kilometers per hour. The formula for hull speed is $h = 1.34\sqrt{\ell}$.

Solve

$h = 1.34\sqrt{\ell}$ — Formula for hull speed

$9 = 1.34\sqrt{\ell}$ — Substitute 9 for h.

$\frac{9}{1.34} = \frac{1.34\sqrt{\ell}}{1.34}$ — Divide each side by 1.34.

$6.72 \approx \sqrt{\ell}$ — Simplify.

$(6.72)^2 \approx (\sqrt{\ell})^2$ — Square each side of the equation.

$45.16 \approx \ell$ — Simplify.

The sailboat's waterline length is about 45 meters.

Check Check by substituting the estimate into the original formula.

$h = 1.34\sqrt{\ell}$ — Formula for hull speed

$9 \stackrel{?}{=} 1.34\sqrt{45}$ — $h = 9$ and $\ell = 45$

$9 \approx 8.98899327$ ✓ — Multiply.

GuidedPractice

1. **DRIVING** The equation $v = \sqrt{21.4r}$ represents the maximum velocity that a car can travel safely on an unbanked curve when v is the maximum velocity in kilometers and r is the radius of the turn in meters. If a road is designed for a maximum speed of 505 kilometers per hour, what is the radius of the turn?

To solve a radical equation, isolate the radical first. Then square both sides of the equation.

Example 2 Expression as a Radicand

Solve $\sqrt{a+5} + 7 = 12$.

$\sqrt{a+5} + 7 = 12$	Original equation
$\sqrt{a+5} = 5$	Subtract 7 from each side.
$(\sqrt{a+5})^2 = 5^2$	Square each side.
$a + 5 = 25$	Simplify.
$a = 20$	Subtract 5 from each side.

GuidedPractice

Solve each equation.

2A. $\sqrt{c-3} - 2 = 4$

2B. $4 + \sqrt{h+1} = 14$

> **WatchOut!**
> **Squaring Each Side** Remember that when you square each side of the equation, you must square the entire side of the equation, even if there is more than one term on the side.

2 Extraneous Solutions

Squaring each side of an equation sometimes produces a solution that is not a solution of the original equation. These are called **extraneous solutions**. Therefore, you must check all solutions in the original equation.

Example 3 Variable on Each Side

Solve $\sqrt{k+1} = k - 1$. Check your solution.

$\sqrt{k+1} = k - 1$	Original equation
$(\sqrt{k+1})^2 = (k-1)^2$	Square each side.
$k + 1 = k^2 - 2k + 1$	Simplify.
$0 = k^2 - 3k$	Subtract k and 1 from each side.
$0 = k(k-3)$	Factor.
$k = 0$ or $k - 3 = 0$	Zero Product Property
$k = 3$	Solve.

CHECK

$\sqrt{k+1} = k - 1$	Original equation	$\sqrt{k+1} = k - 1$	Original equation
$\sqrt{0+1} \stackrel{?}{=} 0 - 1$	$k = 0$	$\sqrt{3+1} \stackrel{?}{=} 3 - 1$	$k = 3$
$\sqrt{1} \stackrel{?}{=} -1$	Simplify.	$\sqrt{4} \stackrel{?}{=} 2$	Simplify.
$1 \neq -1$ ✗	False	$2 = 2$ ✓	True

Since 0 does not satisfy the original equation, 3 is the only solution.

GuidedPractice

Solve each equation. Check your solution.

3A. $\sqrt{t+5} = t + 3$

3B. $x - 3 = \sqrt{x-1}$

> **StudyTip**
> **Extraneous Solutions** When checking solutions for extraneous solutions, we are only interested in principal roots.

Check Your Understanding

Example 1

1. **GEOMETRY** The surface area of a basketball is x square centimeters. What is the radius of the basketball if the formula for the surface area of a sphere is $SA = 4\pi r^2$?

Examples 2–3 **Solve each equation. Check your solution.**

2. $\sqrt{10h} + 1 = 21$
3. $\sqrt{7r + 2} + 3 = 7$
4. $5 + \sqrt{g - 3} = 6$
5. $\sqrt{3x - 5} = x - 5$
6. $\sqrt{2n + 3} = n$
7. $\sqrt{a - 2} + 4 = a$

Practice and Problem Solving

Example 1

8. **EXERCISE** Suppose the function $S = \pi\sqrt{\frac{9.8\ell}{1.6}}$, where S represents speed in meters per second and ℓ is the leg length of a person in meters, can approximate the maximum speed that a person can run.
   a. What is the maximum running speed of a person with a leg length of 1.1 meters to the nearest tenth of a meter?
   b. What is the leg length of a person with a running speed of 6.7 meters per second to the nearest tenth of a meter?
   c. As leg length increases, does maximum speed increase or decrease? Explain.

Examples 2–3 **Solve each equation. Check your solution.**

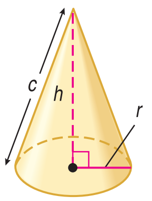
9. $\sqrt{a} + 11 = 21$
10. $\sqrt{t} - 4 = 7$
11. $\sqrt{n - 3} = 6$
12. $\sqrt{c + 10} = 4$
13. $\sqrt{h - 5} = 2\sqrt{3}$
14. $\sqrt{k + 7} = 3\sqrt{2}$
15. $y = \sqrt{12 - y}$
16. $\sqrt{u + 6} = u$
17. $\sqrt{r + 3} = r - 3$
18. $\sqrt{1 - 2t} = 1 + t$
19. $5\sqrt{a - 3} + 4 = 14$
20. $2\sqrt{x - 11} - 8 = 4$

21. **RIDES** The amount of time t, in seconds, that it takes a simple pendulum to complete a full swing is called the *period*. It is given by $t = 2\pi\sqrt{\frac{\ell}{9.8}}$, where ℓ is the length of the pendulum, in meters.
   a. The Giant Swing completes a period in about 8 seconds. About how long is the pendulum's arm? Round to the nearest meter.
   b. Does increasing the length of the pendulum increase or decrease the period? Explain.

Solve each equation. Check your solution.

22. $\sqrt{6a - 6} = a + 1$
23. $\sqrt{x^2 + 9x + 15} = x + 5$
24. $6\sqrt{\frac{5k}{4}} - 3 = 0$
25. $\sqrt{\frac{5y}{6}} - 10 = 4$
26. $\sqrt{2a^2 - 121} = a$
27. $\sqrt{5x^2 - 9} = 2x$

28. **REASONING** The formula for the slant height c of a cone is $c = \sqrt{h^2 + r^2}$, where h is the height of the cone and r is the radius of its base. Find the height of the cone if the slant height is 4 units and the radius is 2 units. Round to the nearest tenth.



29 **MULTIPLE REPRESENTATIONS** Consider $\sqrt{2x - 7} = x - 7$.

a. **Graphical** Clear the **Y=** list. Enter the left side of the equation as **Y1** $= \sqrt{2x - 7}$. Enter the right side of the equation as **Y2** $= x - 7$. Press [GRAPH].

b. **Graphical** Sketch what is shown on the screen.

c. **Analytical** Use the **intersect** feature on the **CALC** menu to find the point of intersection.

d. **Analytical** Solve the radical equation algebraically. How does your solution compare to the solution from the graph?

30. **PACKAGING** A cylindrical container of chocolate drink mix has a volume of 162 cubic centimeters. The radius r of the container can be found by using the formula $r = \sqrt{\frac{V}{\pi h}}$, where V is the volume of the container and h is the height.

a. If the radius is 2.5 centimeters, find the height of the container. Round to the nearest hundredth.

b. If the height of the container is 10 centimeters, find the radius. Round to the nearest hundredth.

H.O.T. Problems Use Higher-Order Thinking Skills

31. **CRITIQUE** Asma and Eiman solved $\sqrt{6 - b} = \sqrt{b + 10}$. Is either of them correct? Explain.

Asma

$\sqrt{6 - b} = \sqrt{b + 10}$

$(\sqrt{6 - b})^2 = (\sqrt{b + 10})^2$

$6 - b = b + 10$

$-2b = 4$

$b = -2$

Check $\sqrt{6 - (-2)} \stackrel{?}{=} \sqrt{(-2) + 10}$

$\sqrt{8} = \sqrt{8}$ ✓

Eiman

$\sqrt{6 - b} = \sqrt{b + 10}$

$(\sqrt{6 - b})^2 = (\sqrt{b + 10})^2$

$6 - b = b + 10$

$2b = 4$

$b = 2$

Check $\sqrt{6 - (2)} \stackrel{?}{=} \sqrt{(2) + 10}$

$\sqrt{4} \neq \sqrt{12}$ ✗

no solution

32. **REASONING** Which equation has the same solution set as $\sqrt{4} = \sqrt{x + 2}$? Explain.

A. $\sqrt{4} = \sqrt{x} + \sqrt{2}$ **B.** $4 = x + 2$ **C.** $2 - \sqrt{2} = \sqrt{x}$

33. **REASONING** Explain how solving $5 = \sqrt{x} + 1$ is different from solving $5 = \sqrt{x + 1}$.

34. **OPEN ENDED** Write a radical equation with a variable on each side. Then solve the equation.

35. **REASONING** Is the following equation *sometimes*, *always* or *never* true? Explain.

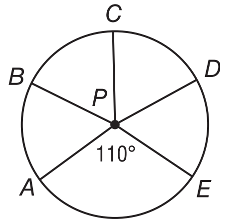
$$\sqrt{(x - 2)^2} = x - 2$$

36. **CHALLENGE** Solve $\sqrt{x + 9} = \sqrt{3} + \sqrt{x}$.

37. **WRITING IN MATH** Write some general rules about how to solve radical equations. Demonstrate your rules by solving a radical equation.

Standardized Test Practice

38. SHORT RESPONSE Hassan needs to drill a hole at A, B, C, D, and E on circle P.



If Hassan drills holes so that $m\angle APE = 110^\circ$ and the other four angles are congruent, what is $m\angle CPD$?

39. Which expression is undefined when $w = 3$?

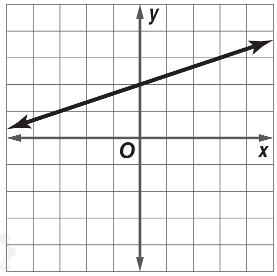
A $\dfrac{w-3}{w+1}$

B $\dfrac{w^2-3w}{3w}$

C $\dfrac{w+1}{w^2-3w}$

D $\dfrac{3w}{3w^2}$

40. What is the slope of a line that is parallel to the line?



F -3

G $-\dfrac{1}{3}$

H $\dfrac{1}{3}$

J 3

41. What are the solutions of $\sqrt{x+3} - 1 = x - 4$?

A 1, 6

B $-1, -6$

C 1

D 6

Spiral Review

42. ELECTRICITY The voltage V required for a circuit is given by $V = \sqrt{PR}$, where P is the power in watts and R is the resistance in ohms. How many more volts are needed to light a 100-watt light bulb than a 75-watt light bulb if the resistance of both is 110 ohms? (Lesson 9-3)

Simplify each expression. (Lesson 9-2)

43. $\sqrt{6} \cdot \sqrt{8}$

44. $\sqrt{3} \cdot \sqrt{6}$

45. $7\sqrt{3} \cdot 2\sqrt{6}$

46. $\sqrt{\dfrac{27}{a^2}}$

47. $\sqrt{\dfrac{5c^5}{4d^5}}$

48. $\dfrac{\sqrt{9x^3y}}{\sqrt{16x^2y^2}}$

Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

49. 12

50. $4x^3$

51. $a - 2b$

52. $4n + 5p$

53. $\dfrac{x}{y^2}$

54. $\dfrac{1}{5}$

Skills Review

Simplify.

55. 9^2

56. 10^6

57. 4^5

58. $(8v)^2$

59. $\left(\dfrac{w^3}{9}\right)^2$

60. $(10y^2)^3$

Mid-Chapter Quiz

Lessons 3-1 through 3-2

Graph each function. Compare to the parent graph. State the domain and range. (Lesson 3-1)

1. $y = 2\sqrt{x}$
2. $y = -4\sqrt{x}$
3. $y = \frac{1}{2}\sqrt{x}$
4. $y = \sqrt{x} - 3$
5. $y = \sqrt{x - 1}$
6. $y = 2\sqrt{x - 2}$

7. **MULTIPLE CHOICE** The length of the side of a square is given by the function $s = \sqrt{A}$, where A is the area of the square. What is the length of the side of a square that has an area of 121 square centimeters? (Lesson 3-1)

   A 121 centimeters
   B 11 centimeters
   C 44 centimeters
   D 10 centimeters

Simplify each expression. (Lesson 3-2)

8. $2\sqrt{25}$
9. $\sqrt{12} \cdot \sqrt{8}$
10. $\sqrt{72xy^5z^6}$
11. $\frac{3}{1 + \sqrt{5}}$
12. $\frac{1}{5 - \sqrt{7}}$

13. **SATELLITES** A satellite is launched into orbit 200 kilometers above Earth. The orbital velocity of a satellite is given by the formula $v = \sqrt{\frac{Gm_E}{r}}$. v is velocity in meters per second, G is a given constant, m_E is the mass of Earth, and r is the radius of the satellite's orbit in meters. (Lesson 3-2)

    a. The radius of Earth is 6,380,000 meters. What is the radius of the satellite's orbit in meters?

    b. The mass of Earth is 5.97×10^{24} kilograms, and the constant G is $6.67 \times 10^{-11}\ N \cdot \frac{m^2}{kg^2}$ where N is in Newtons. Use the formula to find the orbital velocity of the satellite in meters per second.

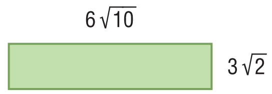
14. **MULTIPLE CHOICE** Which expression is equivalent to $\sqrt{\frac{16}{32}}$? (Lesson 3-2)

    F $\frac{1}{2}$
    G $\frac{\sqrt{2}}{2}$
    H 2
    J 4

Simplify each expression. (Lesson 3-2)

15. $3\sqrt{2} + 5\sqrt{2}$
16. $\sqrt{11} - 3\sqrt{11}$
17. $6\sqrt{2} + 4\sqrt{50}$
18. $\sqrt{27} - \sqrt{48}$
19. $4\sqrt{3}\left(2\sqrt{6}\right)$
20. $3\sqrt{20}(2\sqrt{5})$
21. $(\sqrt{5} + \sqrt{7})(\sqrt{20} + \sqrt{3})$

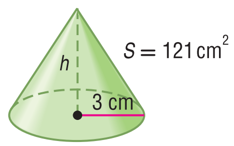
22. **GEOMETRY** Find the area of the rectangle. (Lesson 3-2)



Solve each equation. Check your solution. (Lesson 3-2)

23. $\sqrt{5x} - 1 = 4$
24. $\sqrt{a - 2} = 6$
25. $\sqrt{15 - x} = 4$
26. $\sqrt{3x^2 - 32} = x$
27. $\sqrt{2x - 1} = 2x - 7$
28. $\sqrt{x + 1} + 2 = 4$

29. **GEOMETRY** The lateral surface area S of a cone can be found by using the formula $S = \pi r\sqrt{r^2 + h^2}$, where r is the radius of the base and h is the height of the cone. Find the height of the cone. (Lesson 3-2)



LESSON 3-3 Inverse Variation

Then

- You solved problems involving direct variation.

Now

1. Identify and use inverse variations.
2. Graph inverse variations.

Why?

- The time it takes a runner to finish a race is inversely proportional to the average pace of the runner. The runner's time decreases as the pace of the runner increases. So, these quantities are *inversely proportional*.

NewVocabulary
inverse variation
product rule

Mathematical Practices
Make sense of problems and persevere in solving them.

1 Identify and Use Inverse Variations

An **inverse variation** can be represented by the equation $y = \frac{k}{x}$ or $xy = k$.

KeyConcept Inverse Variation

y varies inversely as x if there is some nonzero constant k such that $y = \frac{k}{x}$ or $xy = k$, where $x, y \neq 0$.

In an inverse variation, the product of two values remains constant. Recall that a relationship of the form $y = kx$ is a *direct variation*. The constant k is called the *constant of variation* or the *constant of proportionality*.

Example 1 Identify Inverse and Direct Variations

Determine whether each table or equation represents an *inverse* or a *direct variation*. Explain.

a.

x	y
1	16
2	8
4	4

In an inverse variation, xy equals a constant k. Find xy for each ordered pair in the table.

$1 \cdot 16 = 16 \quad 2 \cdot 8 = 16 \quad 4 \cdot 4 = 16$

The product is constant, so the table represents an inverse variation.

b.

x	y
1	3
2	6
3	9

Notice that xy is not constant. So, the table does not represent an indirect variation.

$3 = k(1) \quad 6 = k(2) \quad 9 = k(3)$
$3 = k \quad 3 = k \quad 3 = k$

The table of values represents the direct variation $y = 3x$.

c. $x = 2y$

The equation can be written as $y = \frac{1}{2}x$. Therefore, it represents a direct variation.

d. $2xy = 10$

$2xy = 10$ Write the equation.
$xy = 5$ Divide each side by 2.

The equation represents an inverse variation.

GuidedPractice

1A.

x	1	2	5
y	10	5	2

1B. $-2x = y$

You can use $xy = k$ to write an inverse variation equation that relates x and y.

ReadingMath

Variation Equations For direct variation equations, you say that y *varies directly* as x. For inverse variation equations, you say that y *varies inversely* as x.

Example 2 Write an Inverse Variation

Assume that y varies inversely as x. If $y = 18$ when $x = 2$, write an inverse variation equation that relates x and y.

$xy = k$	Inverse variation equation
$2(18) = k$	$x = 2$ and $y = 18$
$36 = k$	Simplify.

The constant of variation is 36. So, an equation that relates x and y is $xy = 36$ or $y = \frac{36}{x}$.

GuidedPractice

2. Assume that y varies inversely as x. If $y = 5$ when $x = -4$, write an inverse variation equation that relates x and y.

If (x_1, y_1) and (x_2, y_2) are solutions of an inverse variation, then $x_1y_1 = k$ and $x_2y_2 = k$.

$x_1y_1 = k$ and $x_2y_2 = k$	
$x_1y_1 = x_2y_2$	Substitute x_2y_2 for k.

The equation $x_1y_1 = x_2y_2$ is called the **product rule** for inverse variations.

KeyConcept Product Rule for Inverse Variations

Words	If (x_1, y_1) and (x_2, y_2) are solutions of an inverse variation, then the products x_1y_1 and x_2y_2 are equal.
Symbols	$x_1y_1 = x_2y_2$ or $\frac{x_1}{x_2} = \frac{y_2}{y_1}$

Example 3 Solve for x or y

Assume that y varies inversely as x. If $y = 3$ when $x = 12$, find x when $y = 4$.

$x_1y_1 = x_2y_2$	Product rule for inverse variations
$12 \cdot 3 = x_2 \cdot 4$	$x_1 = 12$, $y_1 = 3$, and $y_2 = 4$
$36 = x_2 \cdot 4$	Simplify.
$\frac{36}{4} = x_2$	Divide each side by 4.
$9 = x_2$	Simplify.

So, when $y = 4$, $x = 9$.

GuidedPractice

3. If y varies inversely as x and $y = 4$ when $x = -8$, find y when $x = -4$.

The product rule for inverse variations can be used to write an equation to solve real-world problems.

Real-WorldLink

A standard hockey puck is 2.5 centimeters thick and 7.6 centimeters in diameter. Its mass is between approximately 156 and 170 grams.

Source: *NHL Rulebook*

Real-World Example 4 Use Inverse Variations

PHYSICS The acceleration a of a hockey puck is inversely proportional to its mass m. Suppose a hockey puck with a mass of 164 grams is hit so that it accelerates 122 m/s^2. Find the acceleration of a 159-gram hockey puck if the same amount of force is applied.

Make a table to organize the information.

Let $m_1 = 164$, $a_1 = 122$, and $m_2 = 164$. Solve for a_2.

Puck	Mass	Acceleration
1	164 g	122 m/s^2
2	158 g	a_2

$m_1a_1 = m_2a_2$ — Use the product rule to write an equation.

$164 \cdot 122 = 158a_2$ — $m_1 = 164$, $a_1 = 122$, and $m_2 = 158$

$20{,}008 = 158a_2$ — Simplify.

$126.6 \approx a_2$ — Divide each side by 158 and simplify.

The 159-gram puck has an acceleration of approximately 126.6 m/s^2.

GuidedPractice

4. **RACING** Abdulaziz runs an average of 8 kilometers per hour and finishes a race in 0.39 hour. Huda finished the race in 0.35 hour. What was her average pace?

2 Graph Inverse Variations

The graph of an inverse variation is not a straight line like the graph of a direct variation.

Example 5 Graph an Inverse Variation

Graph an inverse variation equation in which $y = 8$ when $x = 3$.

Problem-SolvingTip

Sense-Making Sometimes it is necessary to break a problem into parts, solve each part, and then combine them to find the solution to the problem.

Step 1 Write an inverse variation equation.

$xy = k$ — Inverse variation equation

$3(8) = k$ — $x = 3, y = 8$

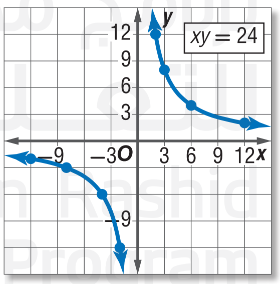
$24 = k$ — Simplify.

The inverse variation equation is $xy = 24$ or $y = \frac{24}{x}$.

Step 2 Choose values for x and y that have a product of 24.

Step 3 Plot each point and draw a smooth curve that connects the points.

x	y
−12	−2
−8	−3
−4	−6
−2	−12
0	undefined
2	12
3	8
6	4
12	2



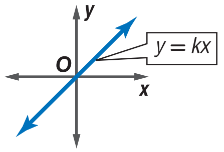
Notice that since y is undefined when $x = 0$, there is no point on the graph when $x = 0$. This graph is called a hyperbola.

GuidedPractice

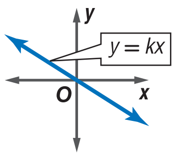
5. Graph an inverse variation equation in which $y = 16$ when $x = 4$.

Concept Summary Direct and Inverse Variations

Direct Variation



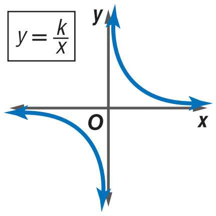
$k > 0$



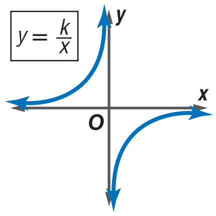
$k < 0$

- $y = kx$
- y varies directly as x.
- The ratio $\frac{y}{x}$ is a constant.

Inverse Variation



$k > 0$



$k < 0$

- $y = \frac{k}{x}$
- y varies inversely as x.
- The product xy is a constant.

Check Your Understanding

Example 1 **Determine whether each table or equation represents an *inverse* or a *direct* variation. Explain.**

1.

x	1	4	8	12
y	2	8	16	24

2.

x	1	2	3	4
y	24	12	8	6

3. $xy = 4$

4. $y = \frac{x}{10}$

Examples 2, 5 **Assume that y varies inversely as x. Write an inverse variation equation that relates x and y. Then graph the equation.**

5. $y = 8$ when $x = 6$

6. $y = 2$ when $x = 5$

7. $y = 3$ when $x = -10$

8. $y = -1$ when $x = -12$

Example 3 **Solve. Assume that y varies inversely as x.**

9. If $y = 8$ when $x = 4$, find x when $y = 2$.

10. If $y = 7$ when $x = 6$, find y when $x = -21$.

11. If $y = -5$ when $x = 9$, find y when $x = 6$.

Example 4

12. **RACING** The time it takes to complete a go-cart race course is inversely proportional to the average speed of the go-cart. One rider has an average speed of 22.3 meters per second and completes the course in 30 seconds. Another rider completes the course in 25 seconds. What was the average speed of the second rider?

13. **OPTOMETRY** When a person does not have clear vision, an optometrist can prescribe lenses to correct the condition. The power P of a lens, in a unit called diopters, is equal to 1 divided by the focal length f, in meters, of the lens.

a. Graph the inverse variation $P = \frac{1}{f}$.

b. Find the powers of lenses with focal lengths +0.2 to −0.4 meters.

Practice and Problem Solving

Example 1 **Determine whether each table or equation represents an *inverse* or a *direct* variation. Explain.**

14.

x	y
1	30
2	15
5	6
6	5

15.

x	y
2	−6
3	−9
4	−12
5	−15

16.

x	y
−4	−2
−2	−1
2	1
4	2

17.

x	y
−5	8
−2	20
4	−10
8	−5

18. $5x - y = 0$

19. $xy = \frac{1}{4}$

20. $x = 14y$

21. $\frac{y}{x} = 9$

Examples 2, 5 **Assume that y varies inversely as x. Write an inverse variation equation that relates x and y. Then graph the equation.**

22. $y = 2$ when $x = 20$

23. $y = 18$ when $x = 4$

24. $y = -6$ when $x = -3$

25. $y = -4$ when $x = -3$

26. $y = -4$ when $x = 16$

27. $y = 12$ when $x = -9$

Example 3 **Solve. Assume that y varies inversely as x.**

28. If $y = 12$ when $x = 3$, find x when $y = 6$.

29. If $y = 5$ when $x = 6$, find x when $y = 2$.

30. If $y = 4$ when $x = 14$, find x when $y = -5$.

31. If $y = 9$ when $x = 9$, find y when $x = -27$.

32. If $y = 15$ when $x = -2$, find y when $x = 3$.

33. If $y = -8$ when $x = -12$, find y when $x = 10$.

Example 4

34. **EARTH SCIENCE** The water level in a river varies inversely with air temperature. When the air temperature was 32° Celsius, the water level was 3.35 meters. If the air temperature was 43° Celsius, what was the level of water in the river?

35 **MUSIC** When under equal tension, the frequency of a vibrating string in a piano varies inversely with the string length. If a string that is 420 millimeters in length vibrates at a frequency of 523 cycles a second, at what frequency will a 707-millimeter string vibrate?

Determine whether each situation is an example of an *inverse* or a *direct* variation. Justify your reasoning.

36. The drama club can afford to purchase 10 wigs at AED 2 each or 5 wigs at AED 4 each.

37. Salama's family buys several lemonades for AED 1.50 each.

38. Amal earns AED 14 for providing child care for 2 hours, and AED 21 for providing child care for 3 hours.

39. Thirty video game tokens are divided evenly among a group of friends.

Determine whether each table or graph represents an *inverse* or a *direct* variation. Explain.

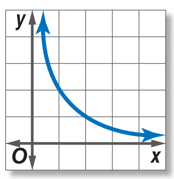
40.

x	y
5	1
8	1.6
11	2.2

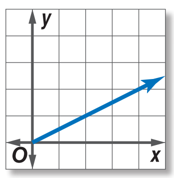
41.

x	y
−3	−7
−2	−10.5
4	5.25

42.



43.



44. **PHYSICAL SCIENCE** When two people are balanced on a seesaw, their distances from the center of the seesaw are inversely proportional to their weights. If a 53.5 kilogram person sits 1.8 meters from the center of the seesaw, how far should a 56.7 kilogram person sit from the center to balance the seesaw?

Solve. Assume that y varies inversely as x.

45. If $y = 9.2$ when $x = 6$, find x when $y = 3$.

46. If $y = 3.8$ when $x = 1.5$, find x when $y = 0.3$.

47. If $y = \frac{1}{5}$ when $x = -20$, find y when $x = -\frac{8}{5}$.

48. If $y = -6.3$ when $x = \frac{2}{3}$, find y when $x = 8$.

49. **SWIMMING** Badr and Hessa each bought a pool membership. Their average cost per day is inversely proportional to the number of days that they go to the pool. Badr went to the pool 25 days for an average cost per day of AED 5.60. Hessa went to the pool 35 days. What was her average cost per day?

50. **PHYSICAL SCIENCE** The amount of force required to do a certain amount of work in moving an object is inversely proportional to the distance that the object is moved. Suppose 90 N of force is required to move an object 10 meters. Find the force needed to move another object 15 meters if the same amount of work is done.

51. **DRIVING** Buthaina must practice driving 40 hours with a parent or guardian before she is allowed to take the test to get her driver's license. She plans to practice the same number of hours each week.

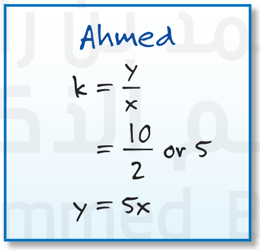
a. Let h represent the number of hours per week that she practices driving. Make a table showing the number of weeks w that she will need to practice for the following values of h: 1, 2, 4, 5, 8, and 10.

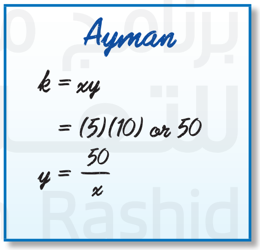
b. Describe how the number of weeks changes as the number of hours per week increases.

c. Write and graph an equation that shows the relationship between h and w.

H.O.T. Problems Use Higher-Order Thinking Skills

52. **CRITIQUE** Ahmed and Ayman found an equation such that x and y vary inversely, and $y = 10$ when $x = 5$. Is either of them correct? Explain.





53. **CHALLENGE** Suppose f varies inversely with g, and g varies inversely with h. What is the relationship between f and h?

54. **REASONING** Does $xy = -k$ represent an inverse variation when $k \neq 0$? Explain.

55. **OPEN ENDED** Give a real-world situation or phenomena that can be modeled by an inverse variation equation. Use the correct terminology to describe your example and explain why this situation is an inverse variation.

56. **WRITING IN MATH** Compare and contrast direct and inverse variation. Include a description of the relationship between slope and the graphs of a direct and inverse variation.

Standardized Test Practice

57. Given a constant force, the acceleration of an object varies inversely with its mass. Assume that a constant force is acting on an object with a mass of 6 kilograms resulting in an acceleration of 10 m/s^2. The same force acts on another object with a mass of 12 kilograms. What would be the resulting acceleration?

A 4 m/s^2
B 5 m/s^2
C 6 m/s^2
D 7 m/s^2

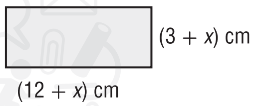
58. Houriyya had an average of 56% on her first seven tests. What would she have to make on her eighth test to average 60% on 8 tests?

F 82%
G 88%
H 98%
J 100%

59. Hareb takes a picture of a 1-meter snake beside a brick wall. When he develops the pictures, the 1-meter snake is 2 centimeters long and the wall is 4.5 centimeters high. What was the actual height of the brick wall?

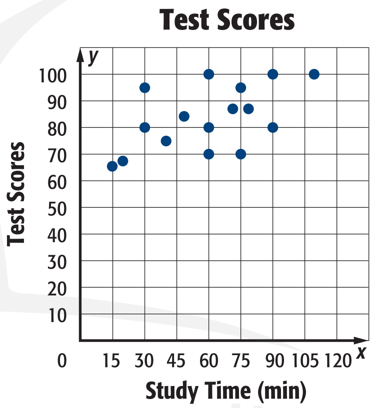
A 2.25 cm
B 22.5 cm
C 225 cm
D 2250 cm

60. **SHORT RESPONSE** Find the area of the rectangle.



Spiral Review

61. **TESTS** Determine whether the graph at the right shows a *positive*, *negative*, or *no* correlation. If there is a correlation, describe its meaning.



Suppose y varies directly as x.

62. If $y = 2.5$ when $x = 0.5$, find y when $x = 20$.

63. If $y = -6.6$ when $x = 9.9$, find y when $x = 6.6$.

64. If $y = 2.6$ when $x = 0.25$, find y when $x = 1.125$.

65. If $y = 6$ when $x = 0.6$, find x when $y = 12$.

66. **FINANCIAL LITERACY** A salesperson is paid AED 32,000 a year plus 5% of the amount in sales made. What is the amount of sales needed to have an annual income greater than AED 45,000?

Skills Review

Simplify. Assume that no denominator is equal to zero.

67. $\frac{7^8}{7^6}$

68. $\frac{x^8y^{12}}{x^2y^7}$

69. $\frac{5pq^7}{10p^6q^3}$

70. $\left(\frac{2c^3d}{7z^2}\right)^3$

71. $\left(\frac{4a^2b}{2c^3}\right)^2$

72. $y^0(y^5)(y^{-9})$

73. $\frac{(4m^{-3}n^5)^0}{mn}$

74. $\frac{(3x^2y^5)^0}{(21x^5y^2)^0}$

EXPLORE 3-4

Graphing Technology Lab
Family of Rational Functions

You can use a graphing calculator to analyze how changing the parameters a and b in $y = \frac{a}{x - b} + c$ affects the graphs in the family of rational functions.

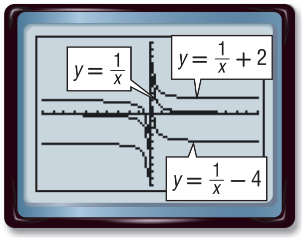
Activity Change Parameters

Graph each set of equations on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

a. $y = \frac{1}{x}, y = \frac{1}{x} + 2, y = \frac{1}{x} - 4$

Enter the equations in the Y= list and graph in the standard viewing window.

The graphs have the same shape. Each graph approaches the y-axis on both sides. However, the graphs have different vertical positions.



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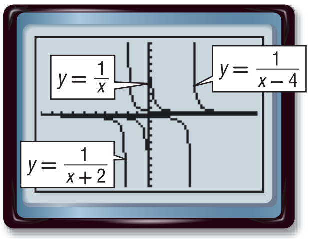
b. $y = \frac{1}{x}, y = \frac{1}{x + 2}, y = \frac{1}{x - 4}$



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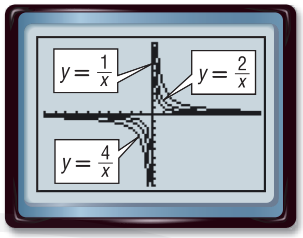
The graphs have the same shape, and all approach the x-axis from both sides. However, the graphs have different horizontal positions.

c. $y = \frac{1}{x}, y = \frac{2}{x}, y = \frac{4}{x}$



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The graphs all approach the x-axis and the y-axis from both sides. However, the graphs have different shapes.

Model and Analyze

1. How do a, b, and c affect the graph of $y = \frac{a}{x - b} + c$? Give examples.

Examine each pair of equations and predict the similarities and differences in their graphs. Use a graphing calculator to confirm your predictions. Write a sentence or two comparing the two graphs.

2. $y = \frac{1}{x}, y = \frac{1}{x} + 2$

3. $y = \frac{1}{x}, y = \frac{1}{x + 5}$

4. $y = \frac{1}{x}, y = \frac{3}{x}$

LESSON 3-4 Rational Functions

Then

- You wrote inverse variation equations.

Now

1. Identify excluded values from the domain of a function.
2. Identify and use asymptotes to graph rational functions.

Why?

- Halima is reading a 300-page book. The average number of pages she reads each day y is given by $y = \frac{300}{x}$, where x is the number of days that she reads.

NewVocabulary
rational function
excluded value
asymptote

Mathematical Practices
Construct viable arguments and critique the reasoning of others.
Look for and make use of structure.

1 Identify Excluded Values

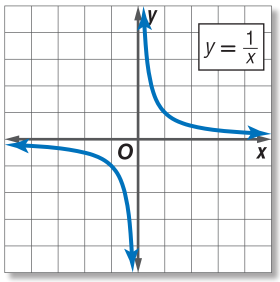
The function $y = \frac{300}{x}$ is an example of a **rational function**. This function is nonlinear.

KeyConcept Rational Functions

Words A rational function can be described by an equation of the form $y = \frac{p}{q}$, where p and q are polynomials and $q \neq 0$.

Parent function:	$f(x) = \frac{1}{x}$
Type of graph:	hyperbola
Domain:	$\{x \mid x \neq 0\}$
Range:	$\{y \mid y \neq 0\}$

Graph



Since division by zero is undefined, any value of a variable that results in a denominator of zero in a rational function is excluded from the domain of the function. These are called **excluded values** for the rational function.

Example 1 Find Excluded Values

State the excluded value for each function.

a. $y = -\frac{2}{x}$

The denominator cannot equal 0. So, the excluded value is $x = 0$.

b. $y = \frac{2}{x + 1}$

$x + 1 = 0$ Set the denominator equal to 0.

$x = -1$

The excluded value is $x = -1$.

c. $y = \frac{5}{4x - 8}$

$4x - 8 = 0$

$4x = 8$

$x = 2$

The excluded value is $x = 2$.

GuidedPractice

1A. $y = \frac{5}{2x}$ **1B.** $y = \frac{x}{x - 7}$ **1C.** $y = \frac{4}{3x + 9}$

Depending on the real-world situation, in addition to excluding x-values that make a denominator zero from the domain of a rational function, additional values might have to be excluded from the domain as well.

Real-WorldLink

As the temperature of the gas inside a hot air balloon increases, the density of the gas decreases. A hot air balloon rises because the density of the air inside it is less than the density of the air outside.

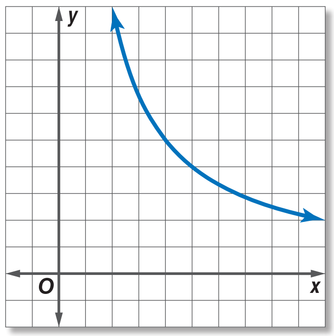
Source: Goddard Space Flight Center

Real-World Example 2 Graph Real-Life Rational Functions

BALLOONS If there are x people in the basket of a hot air balloon, the function $y = \frac{20}{x}$ represents the number of square meters y per person. Graph this function.

Since the number of people cannot be zero or less, it is reasonable to exclude negative values and only use positive values for x.

Number of People x	2	4	5	10
Square Meters per Person y	10	5	4	2



Notice that as x increases y approaches 0. This is reasonable since as the number of people increases, the space per person gets closer to 0.

GuidedPractice

2. GEOMETRY A rectangle has an area of 18 square centimeters. The function $\ell = \frac{18}{w}$ shows the relationship between the length and width. Graph the function.

2 Identify and Use Asymptotes

In Example 2, an excluded value is $x = 0$. Notice that the graph approaches the vertical line $x = 0$, but never touches it.

The graph also approaches but never touches the horizontal line $y = 0$. The lines $x = 0$ and $y = 0$ are called *asymptotes*. An **asymptote** is a line that the graph of a function approaches.

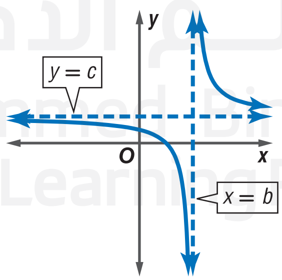
StudyTip

Use Asymptotes Asymptotes are helpful for graphing rational functions. However, they are not part of the graph.

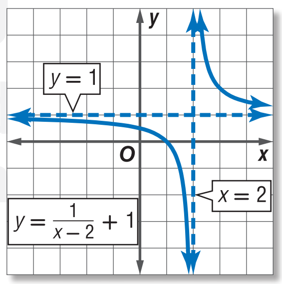
KeyConcept Asymptotes

Words A rational function in the form $y = \frac{a}{x-b} + c$, $a \neq 0$, has a vertical asymptote at the x-value that makes the denominator equal zero, $x = b$. It has a horizontal asymptote at $y = c$.

Model



Example



The domain of $y = \frac{a}{x-b} + c$ is all real numbers except $x = b$. The range is all real numbers except $y = c$. Rational functions cannot be traced with a pencil that never leaves the paper, so choose x-values on both sides of the vertical asymptote to graph both portions of the function.

Example 3 Identify and Use Asymptotes to Graph Functions

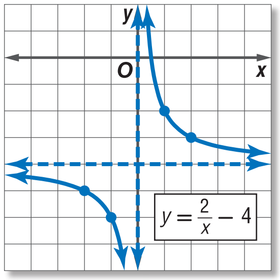
Identify the asymptotes of each function. Then graph the function.

a. $y = \frac{2}{x} - 4$

Step 1 Identify and graph the asymptotes using dashed lines.

vertical asymptote: $x = 0$

horizontal asymptote: $y = -4$



Step 2 Make a table of values and plot the points. Then connect them.

x	−2	−1	1	2
y	−5	−6	−2	−3

b. $y = \frac{1}{x+1}$

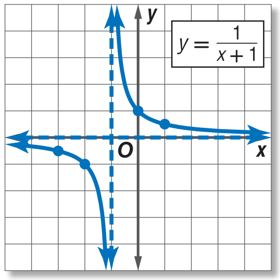
Step 1 To find the vertical asymptote, find the excluded value.

$x + 1 = 0$ Set the denominator equal to 0.

$x = -1$ Subtract 1 from each side.

vertical asymptote: $x = -1$

horizontal asymptote: $y = 0$



Step 2

x	−3	−2	0	1
y	−0.5	−1	1	0.5

GuidedPractice

3A. $y = -\frac{6}{x}$ **3B.** $y = \frac{1}{x-3}$ **3C.** $y = \frac{2}{x+2} + 1$

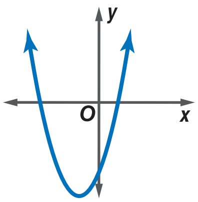
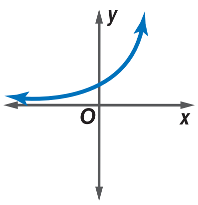
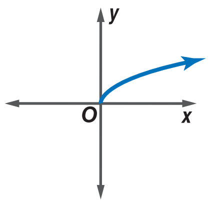
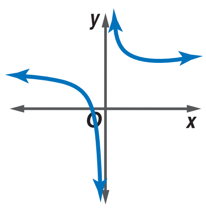
Math HistoryLink

Evelyn Boyd Granville

(1924–) Granville majored in mathematics and physics at Smith College in 1945, where she graduated summa cum laude. She earned an M.A. in mathematics and physics and a Ph.D. in mathematics from Yale University. Granville's doctoral work focused on functional analysis.

Four types of nonlinear functions are shown below.

ConceptSummary Families of Functions

Quadratic	Exponential	Radical	Rational
Parent function: $y = x^2$	Parent function: varies	Parent function: $y = \sqrt{x}$	Parent function: $y = \frac{1}{x}$
General form: $y = ax^2 + bx + c$	General form: $y = ab^x$	General form: $y = \sqrt{x - b} + c$	General form: $y = \frac{a}{x - b} + c$
			

Check Your Understanding

Example 1 **State the excluded value for each function.**

1. $y = \frac{5}{x}$ **2.** $y = \frac{1}{x+3}$ **3.** $y = \frac{x+2}{x-1}$ **4.** $y = \frac{x}{2x-8}$

Example 2 **5. PARTY PLANNING** The cost of decorations for a dinner party is AED 32. This is split among a group of friends. The amount each person pays y is given by $y = \frac{32}{x}$, where x is the number of people. Graph the function.

Example 3 **Identify the asymptotes of each function. Then graph the function.**

6. $y = \frac{2}{x}$ **7.** $y = \frac{3}{x} - 1$ **8.** $y = \frac{1}{x-2}$

9. $y = \frac{-4}{x+2}$ **10.** $y = \frac{3}{x-1} + 2$ **11.** $y = \frac{2}{x+1} - 5$

Practice and Problem Solving

Example 1 **State the excluded value for each function.**

12. $y = \frac{-1}{x}$ **13.** $y = \frac{8}{x-8}$ **14.** $y = \frac{x}{x+2}$ **15.** $y = \frac{4}{x+6}$

16. $y = \frac{x+1}{x-3}$ **17.** $y = \frac{2x+5}{x+5}$ **18.** $y = \frac{7}{5x-10}$ **19.** $y = \frac{x}{2x+14}$

Example 2 **20. ANTELOPES** A pronghorn antelope can run 40 kilometers without stopping. The average speed is given by $y = \frac{40}{x}$, where x is the time it takes to run the distance.

a. Graph $y = \frac{40}{x}$.

b. Describe the asymptotes.

21. CYCLING A cyclist rides 10 kilometers each morning. Her average speed y is given by $y = \frac{10}{x}$, where x is the time it takes her to ride 10 kilometers. Graph the function.

Example 3 **Identify the asymptotes of each function. Then graph the function.**

22. $y = \frac{5}{x}$ **23** $y = \frac{-3}{x}$ **24.** $y = \frac{2}{x} + 3$

25. $y = \frac{1}{x} - 2$ **26.** $y = \frac{1}{x+3}$ **27.** $y = \frac{1}{x-2}$

28. $y = \frac{-2}{x+1}$ **29.** $y = \frac{4}{x-1}$ **30.** $y = \frac{1}{x-2} + 1$

31. $y = \frac{3}{x-1} - 2$ **32.** $y = \frac{2}{x+1} - 4$ **33.** $y = \frac{-1}{x+4} + 3$

34. READING Refer to the application at the beginning of the lesson.

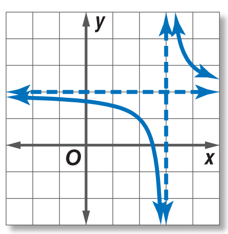
a. Graph the function. Interpret key features of the graph in terms of the situation.

b. Choose a point on the graph, and describe what it means in the context of the situation.

35. STRUCTURE The graph shows a translation of the graph of $y = \frac{1}{x}$.

a. Describe the asymptotes.

b. Write a possible function for the graph.



36. **BIRDS** A long-tailed jaeger is a sea bird that can migrate 5000 kilometers or more each year. The average rate in kilometers per hour r can be given by the function $r = \frac{5000}{t}$, where t is the time in hours. Use the function to determine the average rate of the bird if it spends 250 hours flying.

37. **CLASS TRIP** Students are going to a science museum. As part of the trip, each person is also contributing an equal amount of money to name a star.

a. Write a verbal description for the cost per person.

b. Write an equation to represent the total cost y per person if p people go to the museum.

c. Use a graphing calculator to graph the equation. Interpret key features of the graph in terms of the situation.

d. Estimate the number of people needed for the total cost of the trip to be about AED 15.

Graph each function. Identify the asymptotes.

38. $y = \frac{4x + 3}{2x - 4}$ **39.** $y = \frac{x^2}{x^2 - 1}$ **40.** $y = \frac{x}{x^2 - 9}$

41 **GEOMETRY** The equation $h = \frac{2(64)}{b_1 + 8}$ represents the height h of a trapezoid with an area of 64 square units. The trapezoid has two opposite sides that are parallel and h units apart; one is b_1 units long and another is 8 units long.

a. Describe a reasonable domain and range for the function.

b. Graph the function in the first quadrant.

c. Use the graph to estimate the value of h when $b_1 = 10$.

H.O.T. Problems Use Higher-Order Thinking Skills

42. **CHALLENGE** Graph $y = \frac{1}{x^2 - 4}$. State the domain and the range of the function.

43. **REASONING** Without graphing, describe the transformation that takes place between the graph of $y = \frac{1}{x}$ and the graph of $y = \frac{1}{x + 5} - 2$.

44. **OPEN ENDED** Write a rational function if the asymptotes of the graph are at $x = 3$ and $y = 1$. Explain how you found the function.

45. **ARGUMENTS** Is the following statement *true* or *false*? If false, give a counterexample.

The graph of a rational function will have at least one intercept.

46. **WHICH ONE DOESN'T BELONG** Identify the function that does not belong with the other three. Explain your reasoning.

$y = \frac{4}{x}$	$y = \frac{6}{x + 1}$	$y = \frac{8}{x} + 1$	$y = \frac{10}{2x}$

47. **WRITING IN MATH** How are the properties of a rational function reflected in its graph?

Standardized Test Practice

48. Simplify $\frac{2a^2d}{3bc} \cdot \frac{9b^2c}{16ad^2}$.

A $\frac{abd}{c}$

B $\frac{ab}{d}$

C $\frac{6a}{4bd}$

D $\frac{3ab}{8d}$

49. SHORT RESPONSE One day Suha ran 100 meters in 15 seconds, 200 meters in 45 seconds, and 300 meters over low hurdles in one and a half minutes. How many more seconds did it take her to run 300 meters over low hurdles than the 200-meter dash?

50. Khalid and Khalaf started a T-shirt printing business. The total start-up costs were AED 450. It costs AED 5.50 to print one T-shirt. Write a rational function $A(x)$ for the average cost of producing x T-shirts.

F $A(x) = \frac{450 + 5.5x}{x}$

G $A(x) = \frac{450}{x} + 5.5$

H $A(x) = 450x + 5.5$

J $A(x) = 450 + 5.5x$

51. GEOMETRY Which of the following is a quadrilateral with exactly one pair of parallel sides?

A parallelogram

B rectangle

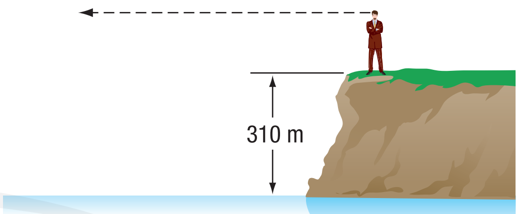
C square

D trapezoid

Spiral Review

52. TRAVEL Khamis' family can drive to the beach, which is 352 km away, in 4 hours if they drive 88 kilometers per hour. Sumayya says that they would save at least a half an hour if they were to drive 105 kilometers per hour. Is Sumayya correct? Explain. (Lesson 3-2)

53. SIGHT The formula $d = \sqrt{\frac{3h}{2}}$ represents the distance d in kilometers that a person h meters high can see. Saeed is standing on a cliff that is 310 meters above sea level. How far can Saeed see from the cliff? Write a simplified radical expression and a decimal approximation. (Lesson 9-2)



Skills Review

Factor each trinomial.

54. $x^2 + 11x + 24$

55. $w^2 + 13w - 48$

56. $p^2 - 2p - 35$

57. $72 + 27a + a^2$

58. $c^2 + 12c + 35$

59. $d^2 - 7d + 10$

60. $g^2 - 19g + 60$

61. $n^2 + 3n - 54$

62. $5x^2 + 27x + 10$

63. $24b^2 - 14b - 3$

64. $12a^2 - 13a - 35$

65. $6x^2 - 14x - 12$

LESSON 3-5

Rational Equations

Then

- You solved proportions.

Now

1. Solve rational equations.
2. Use rational equations to solve problems.

Why?

- Oceanic species of dolphins can swim 8 kilometers per hour faster than coastal species of dolphins. An oceanic dolphin can swim 4.8 kilometers in the same time that it takes a coastal dolphin to swim 3.2 kilometers.

Dolphins			
Species	**Distance**	**Rate**	**Time**
coastal		x kmph	t hours
oceanic		$x + 8$ kmph	t hours

Since time $= \frac{\text{distance}}{\text{rate}}$, the equation below represents this situation.

Time an oceanic dolphin swims 4.8 kilometers | equals | time a coastal dolphin swims 3.2 kilometers.

$$\frac{4.8}{x+8} = \frac{3.2}{x}$$

(distance → 4.8, rate → $x + 8$; 3.2 ← distance, x ← rate)

NewVocabulary
rational equation
extraneous solution
work problem
rate problem

Mathematical Practices
Reason abstractly and quantitatively.
Model with mathematics.

1 Solve Rational Equations

A **rational equation** contains one or more rational expressions. When a rational equation is a proportion, you can use cross products to solve it.

Real-World Example 1 Use Cross Products to Solve Equations

DOLPHINS **Refer to the information above. Solve $\frac{4.8}{x+8} = \frac{3.2}{x}$ to find the speed of a coastal dolphin. Check the solution.**

$\frac{4.8}{x+8} = \frac{3.2}{x}$	Original equation
$4.8x = 3.2x + 25.6$	Find the cross products.
$16x = 25.6$	Distributive Property
$x = 16$	Subtract $2x$ from each side.

So, a coastal dolphin can swim 16 kilometers per hour.

CHECK

$\frac{4.8}{x+8} = \frac{3.2}{x}$	Original equation
$\frac{4.8}{16+8} = \frac{3.2}{16}$	Replace x with 10.
$\frac{4.8}{24} = \frac{3.2}{16}$	Simplify.
$\frac{1}{5} = \frac{1}{5}$ ✓	Simplify.

GuidedPractice

Solve each equation. Check the solution.

1A. $\frac{7}{y-3} = \frac{3}{y+1}$

1B. $\frac{13}{10} = \frac{2f+0.2}{7}$

Another method that can be used to solve any rational equation is to find the LCD of all the fractions in the equation. Then multiply each side of the equation by the LCD to eliminate the fractions.

Example 2 Use the LCD to Solve Rational Equations

Solve $\frac{4}{y} + \frac{5y}{y+1} = 5$. Check the solution.

Step 1 Find the LCD.

The LCD of $\frac{4}{y}$ and $\frac{5y}{y+1}$ is $y(y + 1)$.

Step 2 Multiply each side of the equation by the LCD.

$$\frac{4}{y} + \frac{5y}{y+1} = 5$$ Original equation

$$y(y+1)\left(\frac{4}{y} + \frac{5y}{y+1}\right) = y(y+1)(5)$$ Multiply each side by the LCD, $y(y + 1)$.

$$\left(\frac{\overset{1}{\cancel{y}}(y+1)}{1} \cdot \frac{4}{\underset{1}{\cancel{y}}}\right) + \left(\frac{y\overset{1}{\cancel{(y+1)}}}{1} \cdot \frac{5y}{\underset{1}{\cancel{y+1}}}\right) = y(y+1)(5)$$ Distributive Property

$$(y+1)4 + y(5y) = y(y+1)(5)$$ Simplify.

$$4y + 4 + 5y^2 = 5y^2 + 5y$$ Multiply.

$$4y + 4 + 5y^2 - 5y^2 = 5y^2 - 5y^2 + 5y$$ Subtract $5y^2$ from each side.

$$4y + 4 = 5y$$ Simplify.

$$4y - 4y + 4 = 5y - 4y$$ Subtract $4y$ from each side.

$$4 = y$$ Simplify.

CHECK

$$\frac{4}{y} + \frac{5y}{y+1} = 5$$ Original equation

$$\frac{4}{4} + \frac{5(4)}{4+1} \stackrel{?}{=} 5$$ Replace y with 4.

$$1 + 4 \stackrel{?}{=} 5$$ Simplify.

$$5 = 5 \checkmark$$ Simplify.

GuidedPractice

Solve each equation. Check your solutions.

2A. $\frac{2b-5}{b-2} - 2 = \frac{3}{b+2}$

2B. $1 + \frac{1}{c+2} = \frac{28}{c^2+2c}$

2C. $\frac{y+2}{y-2} - \frac{2}{y+2} = -\frac{7}{3}$

2D. $\frac{n}{3n+6} - \frac{n}{5n+10} = \frac{2}{5}$

StudyTip

Solutions It is important to check the solutions of rational equations to be sure that they satisfy the original equation.

VocabularyLink

extraneous
Everyday Use irrelevant or unimportant

extraneous solution
Math Use a result that is not a solution of the original equation

Recall that any value of a variable that makes the denominator of a rational expression zero must be excluded from the domain.

In the same way, when a solution of a rational equation results in a zero in the denominator, that solution must be excluded. Such solutions are also called **extraneous solutions**.

$$\frac{4+x}{x-5} + \frac{1}{x} = \frac{2}{x+1}$$ 5, 0, and −1 cannot be solutions.

Example 3 Extraneous Solutions

Solve $\frac{2n}{n-5} + \frac{4n-30}{n-5} = 5$. State any extraneous solutions.

$$\frac{2n}{n-5} + \frac{4n-30}{n-5} = 5 \quad \text{Original equation}$$

$$(n-5)\left(\frac{2n}{n-5} + \frac{4n-30}{n-5}\right) = (n-5)5 \quad \text{Multiply each side by the LCD, } n-5.$$

$$\left(\frac{\cancel{n-5}^{1}}{1} \cdot \frac{2n}{\cancel{n-5}_{1}}\right) + \left(\frac{\cancel{n-5}^{1}}{1} \cdot \frac{4n-30}{\cancel{n-5}_{1}}\right) = (n-5)5 \quad \text{Distributive Property}$$

$$2n + 4n - 30 = 5n - 25 \quad \text{Simplify.}$$

$$6n - 30 = 5n - 25 \quad \text{Add like terms.}$$

$$6n - 5n - 30 = 5n - 5n - 25 \quad \text{Subtract } 5n \text{ from each side.}$$

$$n - 30 = -25 \quad \text{Simplify.}$$

$$n - 30 + 30 = -25 + 30 \quad \text{Add 30 to each side.}$$

$$n = 5 \quad \text{Simplify.}$$

Since $n = 5$ results in a zero in the denominator of the original equation, it is an extraneous solution. So, the equation has no solution.

GuidedPractice

3. Solve $\frac{n^2 - 3n}{n^2 - 4} - \frac{10}{n^2 - 4} = 2$. State any extraneous solutions.

StudyTip

Solutions It is possible to get both a valid solution and an extraneous solution when solving a rational equation.

2 Use Rational Equations to Solve Problems

You can use rational equations to solve **work problems**, or problems involving work rates.

Real-World Example 4 Work Problem

JOBS At his part-time job at the zoo, Rasheed can clean the bird area in 2 hours. Reham can clean the same area in 1 hour and 15 minutes. How long would it take them if they worked together?

Understand It takes Rasheed 2 hours to complete the job and Reham $1\frac{1}{4}$ hours.

You need to find the rate that each person works and the total time t that it will take them if they work together.

Plan Find the fraction of the job that each person can do in an hour.

Rasheed's rate ⟶ $\frac{1 \text{ job}}{2 \text{ hours}} = \frac{1}{2}$ job per hour

Reham's rate ⟶ $\frac{1 \text{ job}}{1\frac{1}{4} \text{ hours}}$ or $\frac{1 \text{ job}}{\frac{5}{4} \text{ hours}} = \frac{4}{5}$ job per hour

Since rate • time = fraction of job done, multiply each rate by the time t to represent the amount of the job done by each person.

Study Tip

Reasoning When solving work problems, remember that each term should represent the portion of a job completed in one unit of time.

Solve Fraction of job Rasheed completes plus fraction of job Reham completes equals 1 job.

$$\frac{1}{2}t \quad + \quad \frac{4}{5}t \quad = \quad 1$$

$$10\left(\frac{1}{2}t + \frac{4}{5}t\right) = 10(1)$$ Multiply each side by the LCD, 10.

$$10\left(\frac{1}{2}t\right) + 10\left(\frac{4}{5}t\right) = 10$$ Distributive Property

$$5t + 8t = 10$$ Simplify.

$$t = \frac{10}{13}$$ Add like terms and divide each side by 13.

So, it would take them $\frac{10}{13}$ hour or about 46 minutes to complete the job if they work together.

Check In $\frac{10}{13}$ hour, Rasheed would complete $\frac{1}{2} \cdot \frac{10}{13}$ or $\frac{5}{13}$ of the job and Reham would complete $\frac{4}{5} \cdot \frac{10}{13}$ or $\frac{8}{13}$ of the job. Together, they complete $\frac{5}{13} + \frac{8}{13}$ or 1 whole job. So, the answer is reasonable. ✓

Guided Practice

4. RAKING Alia can rake the leaves in 2 hours. It takes her brother Zayed 3 hours. How long would it take them if they worked together?

Rational equations can also be used to solve **rate problems**.

Real-World Example 5 Rate Problem

AIRPLANES An airplane takes off and flies an average of 772 kilometers per hour. Another plane leaves 15 minutes later and flies to the same city traveling 900 kilometers per hour. How long will it take the second plane to pass the first plane?

Record the information that you know in a table.

Plane	Distance	Rate	Time
1	*d* kilometers	772 kilometers	*t* hours
2	*d* kilometers	900 kilometers	$t - \frac{1}{4}$ hours

Plane 2 took off 15 minutes, or $\frac{1}{4}$ hour, after Plane 1

Since both planes will have traveled the same distance when Plane 2 passes Plane 1, you can write the following equation.

Distance for Plane 1 = Distance for Plane 2

$$772 \cdot t = 900 \cdot \left(t - \frac{1}{4}\right)$$ distance = rate • time

$$772t = (900 \cdot t) - \left(900 \cdot \frac{1}{4}\right)$$ Distributive Property

$$772t = 900t - 225$$ Simplify.

$$-80t = -225$$ Subtract 560*t* from each side.

$$t = 1.75$$ Divide each side by −80.

So, the second plane passes the first plane after 1.75 hours.

Real-World Link

The longest nonstop commercial flight was 21,600 kilometers from Hong Kong Airport in China to London Heathrow in the United Kingdom. It took 22 hours and 42 minutes.

Source: *Guinness Book of World Records*

Guided Practice

5. Huda leaves the house walking at 3 kilometers per hour. After 10 minutes, her mother leaves the house riding a bicycle at 10 kilometers per hour. In how many minutes will Huda's mother catch her?

Check Your Understanding

Examples 1–3 **Solve each equation. State any extraneous solutions.**

1. $\frac{2}{x+1} = \frac{4}{x}$

2. $\frac{t+3}{5} = \frac{2t+3}{9}$

3. $\frac{a+3}{a} - \frac{6}{5a} = \frac{1}{a}$

4. $4 - \frac{p}{p-1} = \frac{2}{p-1}$

5. $\frac{2t}{t+1} + \frac{4}{t-1} = 2$

6. $\frac{x+3}{x^2-1} - \frac{2x}{x-1} = 1$

Example 4

7. **WEEDING** Sultan can weed the garden in 45 minutes. His sister Abeer can weed the garden in 50 minutes. How long would it take them to weed the garden if they work together?

Example 5

8. **LANDSCAPING** Amer is filling a 13.2 liter bucket to water plants at a faucet that flows at a rate of 6.6 liters a minute. If he were to add a hose that flows at a rate of 5.4 liters per minute, how many minutes would it take him to fill the bucket? Round to the nearest tenth.

Practice and Problem Solving

Examples 1–3 **Solve each equation. State any extraneous solutions.**

9. $\frac{8}{n} = \frac{3}{n-5}$

10. $\frac{6}{t+2} = \frac{4}{t}$

11. $\frac{3g+2}{12} = \frac{g}{2}$

12. $\frac{5h}{4} + \frac{1}{2} = \frac{3h}{8}$

13. $\frac{2}{3w} = \frac{2}{15} + \frac{12}{5w}$

14. $\frac{c-4}{c+1} = \frac{c}{c-1}$

15. $\frac{x-1}{x+1} - \frac{2x}{x-1} = -1$

16. $\frac{y+4}{y-2} + \frac{6}{y-2} = \frac{1}{y+3}$

17. $\frac{a}{a+3} + \frac{a^2}{a+3} = 2$

18. $\frac{12}{a+3} + \frac{6}{a^2-9} = \frac{8}{a+3}$

19. $\frac{3n}{n-1} + \frac{6n-9}{n-1} = 6$

20. $\frac{n^2-n-6}{n^2-n} - \frac{n-5}{n-1} = \frac{n-3}{n^2-n}$

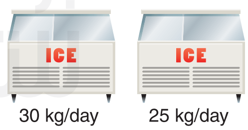
Example 4

21. **PAINTING** It takes Saeed 3 hours to paint one side of a fence. It takes Tarek 5 hours. How long would it take them if they worked together?

22. **DISHWASHING** Obaid works as a dishwasher and can wash 500 plates in two hours and 15 minutes. Abdulrahman can finish the 500 plates in 3 hours. About how long would it take them to finish all of the plates if they work together?

Example 5

23. **ICE** A hotel has two ice machines in its kitchen. How many hours would it take both machines to make 60 kg of ice? Round to the nearest tenth.



24. **CYCLING** Two cyclists travel in opposite directions around a 5.6-kilometer circular trail. They start at the same time. The first cyclist completes the trail in 22 minutes and the second in 28 minutes. At what time do they pass each other?

GRAPHING CALCULATOR **For each function, a) describe the shape of the graph, b) use factoring to simplify the function, and c) find the zeros of the function.**

25. $f(x) = \frac{x^2 - x - 30}{x-6}$

26. $f(x) = \frac{x^3 + x^2 - 2x}{x+2}$

27. $f(x) = \frac{x^3 + 6x^2 + 12x}{x}$

28. **REASONING** Abdulkarim can paint a standard-sized house in about 5 days. For his latest job, Abdulkarim hires two assistants. At what rate must these assistants work for Abdulkarim to meet a deadline of two days?

29. **AIRPLANES** Headwinds push against a plane and reduce its total speed, while tailwinds push on a plane and increase its total speed. Let w equal the speed of the wind, r equal the speed set by the pilot, and s equal the total speed.

a. Write an equation for the total speed with a headwind and an equation for the total speed with a tailwind.

b. Use the rate formula to write an equation for the distance traveled by a plane with a headwind and another equation for the distance traveled by a plane with a tailwind. Then solve each equation for time instead of distance.

30. **MIXTURES** A pitcher of fruit juice has 3 liters of pineapple juice and 2 liters of orange juice. Fatheya wants to add more orange juice so that the fruit juice mixture is 60% orange juice. Let x equal the liters of orange juice that she needs to add.

a. Copy and complete the table below.

Juice	Liters of Orange Juice	Total liters of Juice	Percent of Orange Juice
original mixture		5	
final mixture	$2 + x$		0.6

b. Write and solve an equation to find the liters of orange juice to add.

31. **DORMITORIES** The number of hours h it takes to clean a dormitory varies inversely with the number of people cleaning it c and directly with the number of people living there p.

a. Write an equation showing how h, c, and p are related. (*Hint*: Include the constant k.)

b. It takes 8 hours for 5 people to clean the dormitory when there are 100 people there. How long will it take to clean the dormitory if there are 10 people cleaning and the number of people living in the dorm stays the same?

Solve each equation. State any extraneous solutions.

32. $\frac{4b + 2}{b^2 - 3b} + \frac{b + 2}{b} = \frac{b - 1}{b}$

33. $\frac{x^2 - x - 6}{x + 2} + \frac{x^3 + x^2}{x} = 3$

34. $\frac{y^2 + 5y - 6}{y^3 - 2y^2} = \frac{5}{y} - \frac{6}{y^3 - 2y^2}$

35. $\frac{x - \frac{6}{5}}{x} - \frac{x - 10\frac{1}{2}}{x - 5} = \frac{x + 21}{x^2 - 5x}$

H.O.T. Problems Use Higher-Order Thinking Skills

36. **CHALLENGE** Solve $\frac{2x}{x - 2} + \frac{x^2 + 3x}{(x + 1)(x - 2)} = \frac{2}{(x + 1)(x - 2)}$.

37. **REASONING** How is an excluded value of a rational expression related to an extraneous solution of a corresponding rational equation? Explain.

38. **WRITING IN MATH** Why should you check solutions of rational equations?

39. **ARGUMENTS** Find a counterexample for the following statement.

The solution of a rational equation can never be zero.

40. **WRITING IN MATH** Describe the steps for solving a rational equation that is not a proportion.

Standardized Test Practice

41. It takes Ali 4 hours to build a fence. If he hires Omar to help him, they can do the job in 3 hours. If Omar built the same fence alone, how long would it take him?

A $1\frac{5}{7}$ hours **C** 8 hours

B $3\frac{2}{3}$ hours **D** 12 hours

42. In the 1000-meter race, Adnan finished 35 meters ahead of Mansour and 53 meters ahead of Ayoub. How far was Mansour ahead of Ayoub?

F 18 m **G** 35 m **H** 53 m **J** 88 m

43. Twenty liters of lemonade were poured into two containers of different sizes. Express the amount of lemonade poured into the smaller container in terms of g, the amount poured into the larger container.

A $g + 20$ **C** $g - 20$

B $20 + g$ **D** $20 - g$

44. **GRIDDED RESPONSE** The gym has 2-kilogram and 5-kilogram disks for weight lifting. They have fourteen disks in all. The total weight of the 2-kilogram disks is the same as the total weight of the 5-kilogram disks. How many 2-kilogram disks are there?

Spiral Review

45. **POPULATION** The country of Latvia has been experiencing a 1.1% annual decrease in population. In 2009, its population was 2,261,294. If the trend continues, predict Latvia's population in 2019.

46. **TOMATOES** There are more than 10,000 varieties of tomatoes. One seed company produces seed packages for 200 varieties of tomatoes. For how many varieties do they not provide seeds?

47. **DRIVING** Tires should be kept within 2 kilograms per square centimeter (psi) of the manufacturer's recommended tire pressure. If the recommendation for a tire is 30 psi, what is the range of acceptable pressures?

Express each number in scientific notation.

48. 12,300 **49.** 0.0000375 **50.** 1,255,000

51. **FINANCIAL LITERACY** Mohammad has AED 13 to order pizza. The pizza costs AED 7.50 plus AED 1.25 per topping. He plans to tip 15% of the total cost. Write and solve an inequality to find out how many toppings he can order.

Solve each inequality. Check your solution.

52. $\frac{b}{10} \leq 5$ **53.** $-7 > -\frac{r}{7}$ **54.** $\frac{5}{8}y \geq -15$

Skills Review

Determine the probability of each event if you randomly select a marble from a bag containing 9 red marbles, 6 blue marbles, and 5 yellow marbles.

55. P(blue) **56.** P(red) **57.** P(not yellow)

EXTEND 3-5

Graphing Technology Lab
Solving Rational Equations

You can use a graphing calculator to solve rational equations by graphing, by using tables, and by using a computer algebra system (CAS).

Mathematical Practices
Use appropriate tools strategically.

To solve by graphing, graph both sides of the equation and locate the point(s) of intersection.

Activity 1 Solve a Rational Equation by Graphing

Solve $\frac{5}{x+2} = \frac{3}{x}$ by graphing.

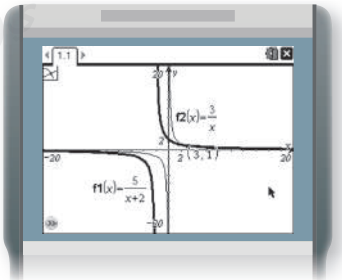
Step 1 Add a new **Graphs** page.

Step 2 Use the **Window Settings** option from the **Window/Zoom** menu to adjust the window to -20 to 20 for both x and y. Set both scales to 2.

Step 3 Enter $\frac{5}{x+2}$ into **f1(*x*)** and $\frac{3}{x}$ into **f2(*x*)**.

Step 4 Change the thickness of the graph of **f1(*x*)** by selecting the graph of **f1(*x*)** and the **ctrl menu Attributes** option.

Step 5 Use the **Intersection Point(s)** tool from the **Points & Lines** menu to find the intersection of the two graphs. Select the graph of **f1(*x*)** **enter** and then the graph of **f2(*x*)** **enter**.



$[-20, 20]$ scl: 2 by $[-20, 20]$ scl: 2

The graphs intersect at (3, 1). This means that $\frac{5}{x+2}$ and $\frac{3}{x}$ both equal 1 when $x = 3$. Thus, the solution of $\frac{5}{x+2} = \frac{3}{x}$ is $x = 3$.

Exercises

Use a graphing calculator to solve each equation.

1. $\frac{5}{x} + \frac{4}{x} = 10$

2. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$

3. $\frac{6}{x} + \frac{3}{2x} = 12$

4. $\frac{4}{x} + \frac{3}{4x} = \frac{1}{8}$

5. $\frac{4}{x} + \frac{x-2}{2x} = x$

6. $\frac{3}{3x-2} + \frac{5}{x} = 0$

7. $\frac{2x+1}{2} + \frac{3}{2x} = \frac{2}{x}$

8. $\frac{x}{x+2} + x = \frac{5x+8}{x+2}$

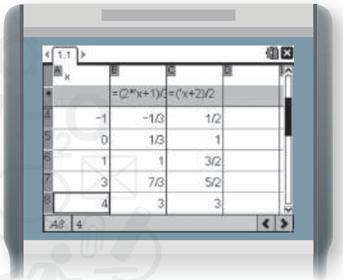
9. $\frac{1}{2x} + \frac{5}{x} = \frac{3}{x-1}$

10. $\frac{4x-3}{x-2} + \frac{2x+5}{x-2} = 6$

Solving Rational Equations *Continued*

Activity 2 Solve a Rational Equation by Using a Table

Solve $\frac{2x+1}{3} = \frac{x+2}{2}$ using a table.



Step 1 Add a new **Lists & Spreadsheet** page.

Step 2 Label column A as x. Enter values from -4 to 4 in cells A1 to A9.

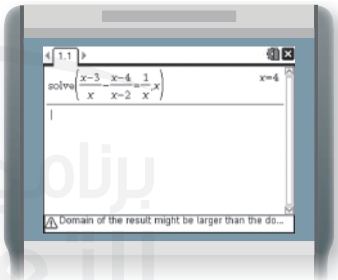
Step 3 In column B in the formula row, enter the left side of the rational equation, with parenthesis around the binomials. In column C in the formula row, enter the right side of the rational equation, with parenthesis around the binomials. Specify **Variable Reference** when prompted.

Scroll until you see where the values in Columns B and C are equal. This occurs at $x = 4$. Therefore the solution of $\frac{2x+1}{3} = \frac{x+2}{2}$ is 4.

You can also use a computer algebra system (CAS) to solve rational equations.

Activity 3 Solve a Rational Equation by Using a CAS

Solve $\frac{x-3}{x} - \frac{x-4}{x-2} = \frac{1}{x}$ using a CAS.



Step1 Add a new **Calculator** page.

Step 2 To solve, select the **Solve** tool from the **Algebra** menu. Enter the left side of the equation with parenthesis around the binomials. Enter = and the right side of the equation. Then type a comma, followed by x, and then **enter**.

The solution of 4 is displayed.

Exercises

Use a table or CAS to solve each equation.

11. $\frac{2}{x} + \frac{2+x}{2} = \frac{x+3}{2}$

12. $\frac{4}{x-2} = -\frac{1}{x+3}$

13. $\frac{3}{x+2} + \frac{4}{x-1} = 0$

14. $\frac{1}{x+1} + \frac{2}{x-1} = 0$

15. $\frac{2}{x+4} + \frac{4}{x-1} = 0$

16. $\frac{1}{x-2} + \frac{x+2}{4} = 2x$

17. $\frac{2x}{x+3} + \frac{x+1}{2} = x$

18. $\frac{2}{x-3} + \frac{3}{x-2} = \frac{4}{x}$

19. $\frac{x^2}{x+1} + \frac{x}{x-1} = x$

CHAPTER 3

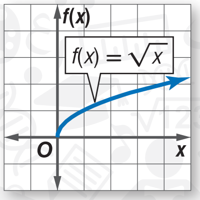
Study Guide and Review

Study Guide

KeyConcepts

Square Root Functions (Lesson 3-1)

- A square root function contains the square root of a variable.
- The parent function of the family of square root functions is $f(x) = \sqrt{x}$.



Operations with Radical Expressions and Equations (Lessons 3-2)

- Radical expressions with like radicals can be added or subtracted.
- Use the FOIL method to multiply radical expressions.

Inverse Variation (Lesson 3-3)

- You can use $\frac{x_1}{x_2} = \frac{y_2}{y_1}$ to solve problems involving inverse variation.

Rational Functions (Lesson 3-4)

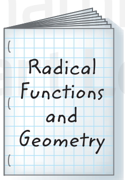
- Excluded values are values of a variable that result in a denominator of zero.
- If vertical asymptotes occur, it will be at excluded values.

Solving Rational Equations (Lesson 3-5)

- Use cross products to solve rational equations with a single fraction on each side of the equals sign.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

asymptote	radical function
closed	radicand
conjugate	rate problem
excluded value	rationalize the denominator
extraneous solution	rational function
inverse variation	rational equation
product rule	square root function
radical equations	work problem
radical expression	

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined word, phrase, expression, or number to make a true sentence.

1. The expressions $12\sqrt{4}$ and $\sqrt{288}$ are equivalent.
2. The expressions $2 + \sqrt{5}$ and $2 - \sqrt{5}$ are conjugates.
3. In the expression $-5\sqrt{2}$, the radicand is 2.
4. If the product of two variables is a nonzero constant, the relationship is an inverse variation.
5. If the line $x = a$ is a vertical asymptote of a rational function, then a is an excluded value.
6. The excluded values for $\frac{x}{x^2 + 5x + 6}$ are -2 and -3.
7. The equation $\frac{3x}{x - 2} = \frac{6}{x - 2}$ has an extraneous solution, 2.

Lesson-by-Lesson Review

3-1 Square Root Functions

Graph each function. Compare to the parent graph. State the domain and range.

8. $y = \sqrt{x} - 3$

9. $y = \sqrt{x} + 2$

10. $y = -5\sqrt{x}$

11. $y = \sqrt{x} - 6$

12. $y = \sqrt{x - 1}$

13. $y = \sqrt{x} + 5$

14. **GEOMETRY** The function $s = \sqrt{A}$ can be used to find the length of a side of a square given its area. Use this function to determine the length of a side of a square with an area of 90 square centimeters. Round to the nearest tenth if necessary.

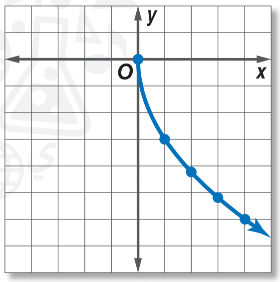
Example 1

Graph $y = -3\sqrt{x}$. Compare to the parent graph. State the domain and range.

Make a table. Choose nonnegative values for *x*.

x	0	1	2	3	4
y	0	−3	≈−4.2	≈−5.2	−6

Plot points and draw a smooth curve.



The graph of $y = \sqrt{x}$ is stretched vertically and is reflected across the *x*-axis.

The domain is $\{x | x \geq 0\}$.

The range is $\{y | y \leq 0\}$.

3-2 Radical Equations

Solve each equation. Check your solution.

34. $10 + 2\sqrt{x} = 0$

35. $\sqrt{5 - 4x} - 6 = 7$

36. $\sqrt{a + 4} = 6$

37. $\sqrt{3x} = 2$

38. $\sqrt{x + 4} = x - 8$

39. $\sqrt{3x - 14} + x = 6$

40. **FREE FALL** Assuming no air resistance, the time t in seconds that it takes an object to fall h meters can be determined by $t = \sqrt{\frac{2h}{4}}$. If a skydiver jumps from an airplane and free falls for 10 seconds before opening the parachute, how many meters does she free fall?

Example 2

Solve $\sqrt{7x + 4} - 18 = 5$.

$\sqrt{7x + 4} - 18 = 5$	Original equation
$\sqrt{7x + 4} = 23$	Add 18 to each side.
$(\sqrt{7x + 4})^2 = 23^2$	Square each side.
$7x + 4 = 529$	Simplify.
$7x = 525$	Subtract 4 from each side.
$x = 75$	Divide each side by 7.

CHECK

$\sqrt{7x + 4} - 18 = 5$	Original equation
$\sqrt{7(75) + 4} - 18 \stackrel{?}{=} 5$	$x = 75$
$\sqrt{525 + 4} - 18 \stackrel{?}{=} 5$	Multiply.
$\sqrt{529} - 18 \stackrel{?}{=} 5$	Add.
$23 - 18 \stackrel{?}{=} 5$	Simplify.
$5 = 5$ ✓	True.

3-3 Inverse Variation

Solve. Assume that y varies inversely as x.

41. If $y = 4$ when $x = 1$, find x when $y = 12$

42. If $y = -1$ when $x = -3$, find y when $x = -9$

43. If $y = 1.5$ when $x = 6$, find y when $x = -16$

44. **PHYSICS** A 61 kilograms person sits 1.5 m from the center of a seesaw. How far from the center should a 49 kilograms person sit to balance the seesaw?

Example 3

If y varies inversely as x and $y = 28$ when $x = 42$, find y when $x = 56$.

Let $x_1 = 42$, $x_2 = 56$, and $y_1 = 28$. Solve for y_2.

$\frac{x_1}{x_2} = \frac{y_2}{y_1}$	Proportion for inverse variation
$\frac{42}{56} = \frac{y_2}{28}$	Substitution
$1176 = 56y_2$	Cross multiply.
$21 = y_2$	

Thus, $y = 21$ when $x = 56$.

Lesson-by-Lesson Review

3-4 Rational Functions

State the excluded value for each function.

45. $y = \frac{1}{x - 3}$

46. $y = \frac{2}{2x - 5}$

47. $y = \frac{3}{3x - 6}$

48. $y = \frac{-1}{2x + 8}$

49. PIZZA PARTY Hiyam ordered pizza and soda for her study group for AED 38. The cost per person y is given by $y = \frac{38}{x}$, where x is the number of people in the study group. Graph the function and describe the asymptotes.

Example 4

State the excluded value for the function $y = \frac{1}{4x + 16}$.

Set the denominator equal to zero.

$$4x + 16 = 0$$

$$4x + 16 - 16 = 0 - 16 \quad \text{Subtract 16 from each side.}$$

$$4x = -16 \quad \text{Simplify.}$$

$$x = -4 \quad \text{Divide each side by 4.}$$

3-5 Rational Equations

Solve each equation. State any extraneous solutions.

50. $\frac{5n}{6} + \frac{1}{n - 2} = \frac{n + 1}{3(n - 2)}$

51. $\frac{4x}{3} + \frac{7}{2} = \frac{7x}{12} - 14$

52. $\frac{11}{2x} + \frac{2}{4x} = \frac{1}{4}$

53. $\frac{1}{x + 4} - \frac{1}{x - 1} = \frac{2}{x^2 + 3x - 4}$

54. $\frac{1}{n - 2} = \frac{n}{8}$

55. PAINTING Wafa can paint a room in 6 hours. Hana can paint a room in 4 hours. How long will it take them to paint the room working together?

Example 5

Solve $\frac{3}{x^2 + 3x} + \frac{x + 2}{x + 3} = \frac{1}{x}$.

$$\frac{3}{x^2 + 3x} + \frac{x + 2}{x + 3} = \frac{1}{x}$$

$$x(x + 3)\left(\frac{3}{x(x + 3)}\right) + x(x + 3)\left(\frac{x + 2}{x + 3}\right) = x(x + 3)\left(\frac{1}{x}\right)$$

$$3 + x(x + 2) = 1(x + 3)$$

$$3 + x^2 + 2x = x + 3$$

$$x^2 + x = 0$$

$$x(x + 1) = 0$$

$$x = 0 \text{ or } x = -1$$

The solution is −1, and there is an extraneous solution of 0.

CHAPTER 3 Practice Test

Graph each function, and compare to the parent graph. State the domain and range.

1. $y = -\sqrt{x}$
2. $y = \frac{1}{4}\sqrt{x}$
3. $y = \sqrt{x} + 5$
4. $y = \sqrt{x + 4}$

5. **MULTIPLE CHOICE** The length of the side of a square is given by the function $s = \sqrt{A}$, where A is the area of the square. What is the perimeter of a square that has an area of 64 square centimeters?

 A 64 centimeters

 B 8 centimeters

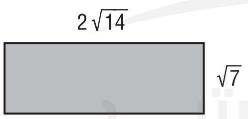
 C 32 centimeters

 D 16 centimeters

Simplify each expression.

6. $5\sqrt{36}$
7. $\frac{3}{1 - \sqrt{2}}$
8. $2\sqrt{3} + 7\sqrt{3}$
9. $3\sqrt{6}(5\sqrt{2})$

10. **MULTIPLE CHOICE** Find the area of the rectangle.



 F $7\sqrt{2}$

 G 14

 H $14\sqrt{2}$

 J $98\sqrt{2}$

Solve each equation. Check your solution.

11. $\sqrt{10x} = 20$

12. $\sqrt{4x - 3} = 6 - x$

13. **PACKAGING** A cylindrical container of chocolate drink mix has a volume of about 2564.7 m^3. The radius of the container can be found by using the formula $r = \sqrt{\frac{V}{\pi h}}$, where r is the radius and h is the height. If the height is 21 centimeters, find the radius of the container.

Determine whether each table represents an inverse variation. Explain.

14.

x	y
2	10
4	12
8	14

Solve. Assume that y varies inversely as x.

15. If $y = 3$ when $x = 9$, find x when $y = 1$.

16. If $y = 2$ when $x = 0.5$, find y when $x = 3$.

Assume that y varies inversely as x. Write an inverse variation equation that relates x and y.

17. $y = 2$ when $x = 8$

18. $y = -3$ when $x = 1$

19. **MULTIPLE CHOICE** Humaid can shovel the driveway in 3 hours, and Hamad can shovel the driveway in 2 hours. How long will it take them working together?

 F 6 hours

 G 5 hours

 H $\frac{3}{2}$ hours

 J $\frac{6}{5}$ hours

20. **PAINTING** Jassim can paint a 60-square meter wall in 40 minutes. Working with his friend Jamal, the two of them can paint the wall in 25 minutes. How long would it take Jamal to do the job himself?

Preparing for Standardized Tests

Draw a Picture

Sometimes it is easier to visualize how to solve a problem if you draw a picture first. You can sketch your picture on scrap paper or in your test booklet (if allowed). Be careful not make any marks on your answer sheet other than your answers.

Strategies for Drawing a Picture

Step 1

Read the problem statement carefully.

Ask yourself:

- What am I being asked to solve?
- What information is given in the problem?
- What is the unknown quantity for which I need to solve?

Step 2

Sketch and label your picture.

- Draw your picture as clearly and accurately as possible.
- Label the picture carefully. Be sure to include all of the information given in the problem statement.

Step 3

Solve the problem.

- Use your picture to help you model the problem situation with an equation. Then solve the equation.
- Check your answer to make sure it is reasonable.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

An 5.5 meter ladder is leaning against a building. For stability, the base of the ladder must be 100 centimeters away from the wall. How far up the wall does the ladder reach?

Read the problem statement carefully. You know the height of the ladder leaning against the building and you know that the base of the ladder must be 100 centimeters away from the wall. You need to find how far up the wall the ladder reaches.

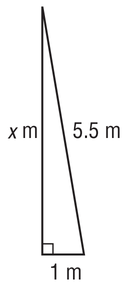
Scoring Rubric	
Criteria	**Score**
Full Credit: The answer is correct and a full explanation is provided that shows each step.	2
Partial Credit: • The answer is correct, but the explanation is incomplete. • The answer is incorrect, but the explanation is correct.	1
No Credit: Either an answer is not provided or the answer does not make sense.	0

Example of a 2-point response:

First convert all measurements to meters.

100 centimeters = 1 meter

Use a right triangle to find how high the ladder reaches. Draw and label a triangle to represent the situation.



You know the measures of a leg and the hypotenuse, and need to know the length of the other leg. So you can use the Pythagorean Theorem.

$$5.5^2 = 1^2 + b^2$$
$$30.25 = 1 + b^2$$
$$29.25 = b^2$$
$$\pm 5.4 = b$$
$$5.4 \approx b$$

100 centimeters = 1 meter

The ladder reaches about 5.4 meters.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. A building casts a 4.6 meter shadow, while a billboard casts a 1.4 meter shadow. If the billboard is 7.9 meters high, what is the height of the building? Round to the nearest tenth if necessary.

2. A space shuttle is directed toward the Moon, but drifts 1.2° from its intended course. The distance from Earth to the Moon is about 386,200 kilometers. If the pilot doesn't get the shuttle back on course, how far will the shuttle have drifted from its intended landing position?

CHAPTER 3

Standardized Test Practice

Cumulative, Chapters 1 through 3

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Each year a local country club sponsors a tennis tournament. Play starts with 256 participants. During each round, half of the players are eliminated. How many players remain after 6 rounds?

   A 128

   B 64

   C 16

   D 4

2. Evaluate $\frac{5^6 - 5^5}{4}$.

   F 5^6

   G 5^5

   H $\frac{5}{4}$

   J $\frac{25}{4}$

3. Which of the following numbers is less than zero?

   A 1.03×10^{-21}

   B 7.5×10^2

   C 8.21543×10^{10}

   D none of the above

4. Write an equation in slope-intercept form with a slope of $\frac{9}{10}$ and y-intercept of 3.

   F $y = 3x + \frac{9}{10}$

   G $y = \frac{9}{10}x + 3$

   H $y = \frac{9}{10}x - 3$

   J $y = 3x - \frac{9}{10}$

5. Bilal is playing games at a family fun center. So far he has won 38 prize tickets. How many more tickets would he need to win to place him in the gold prize category?

Number of Tickets	Prize Category
1–20	bronze
21–40	silver
41–60	gold
61–80	platinum

   F $2 \leq t \leq 22$

   G $3 \leq t \leq 22$

   H $1 \leq t \leq 20$

   J $3 \leq t \leq 20$

6. Which of the following is an equation of the line perpendicular to $4x - 2y = 6$ and passing through $(4, -4)$?

   F $y = -\frac{3}{4}x + 3$

   G $y = -\frac{3}{4}x - 1$

   H $y = -\frac{1}{2}x - 4$

   J $y = -\frac{1}{2}x - 2$

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

7. **GRIDDED RESPONSE** Mr. Abdalla bought a total of 9 tickets to the zoo. He bought children tickets at the rate of AED 6.50 and adult tickets for AED 9.25 each. If he spent AED 69.50 altogether, how many adult tickets did Mr. Abdalla purchase?

8. What is the domain of the following relation? $\{(2, -1), (4, 3), (7, 6)\}$

9. Rashid just added 15 more songs to his digital media player, making the total number of songs more than 84. Draw a number line that represents the original number of songs he had on his digital media player.

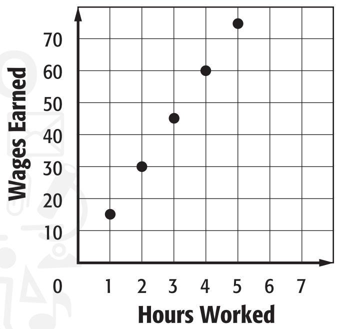
10. Khalifa bought a rare painting in 1995 for AED 14,200. By 2003, the painting was worth AED 17,120. Assuming that a linear relationship exists, write an equation in slope-intercept form that represents the value V of the painting after t years.

11. Ahmed spent AED 24.50 on peanuts and walnuts for a dinner party. He bought 1.5 kilograms more peanuts than walnuts. How many kilograms of peanuts and walnuts did he buy?

Product	Price per kilograms
Peanuts p	AED 3.80
Cashews c	AED 6.90
Walnuts w	AED 5.60

12. **GRIDDED RESPONSE** Moza purchased a car several years ago for AED 21,459. The value of the car depreciated at a rate of 15% annually. What was the value of the car after 5 years? Round your answer to the nearest whole dirham.

13. **GRIDDED RESPONSE** The amount of money that Nasser earns varies directly as the number of hours that he works as shown in the graph. How much money will he earn for working 40 hours next week? Express your answer in dirhams.



Extended Response

Record your answers on a sheet of paper. Show your work.

14. The fare charged by a taxi drive is a AED 3 fixed charge plus AED 0.35 per kilometer. Maysoun pays AED 10 for a ride of m kilometers.

Part A Write an equation that can be used to find m. Show your work.

Part B Use the equation in Part A to find how many kilometers Maysoun rode. Show your work.