CHAPTER 1

Quadratic Functions and Equations

Then

You solved quadratic equations by factoring and by using the Square Root Property.

Now

In this chapter, you will:

- Solve quadratic equations by graphing, completing the square, and using the Quadratic Formula.
- Analyze functions with successive differences and ratios.
- Identify and graph special functions.

Why?

FINANCE The value of a certain company's stock can be modeled by the function $f(x) = x^2 - 12x + 75$. By graphing this quadratic function, we can make an educated guess as to how the stock will perform in the near future.

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Get Ready for the Chapter

Diagnose Readiness Take the Quick Check below to check your prerequisite skills. Refer to the Quick Review for help.

QuickCheck	**QuickReview**

QuickCheck

Use a table of values to graph each equation.

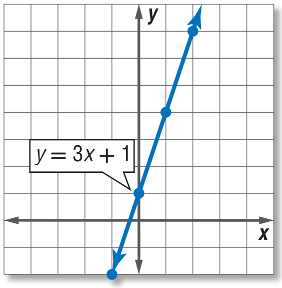
1. $y = x + 3$
2. $y = 2x + 2$
3. $y = -2x - 3$
4. $y = 0.5x - 1$
5. $4x - 3y = 12$
6. $3y = 6 + 9x$
7. **SAVINGS** Ahmed has AED 100 to buy a game system. He plans to save AED 10 each week. Graph an equation to show the total amount T Ahmed will have in w weeks.

QuickReview

Example 1

Use a table of values to graph $y = 3x + 1$.

x	$y = 3x + 1$	y
-1	$3(-1) + 1$	-2
0	$3(0) + 1$	1
1	$3(1) + 1$	4
2	$3(2) + 1$	7



QuickCheck

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

8. $a^2 + 12a + 36$
9. $x^2 + 5x + 25$
10. $x^2 - 12x + 32$
11. $x^2 + 20x + 100$
12. $4x^2 + 28x + 49$
13. $k^2 - 16k + 64$
14. $a^2 - 22a + 121$
15. $5t^2 - 12t + 25$

QuickReview

Example 2

Determine whether $x^2 - 10x + 25$ is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

1. Is the first term a perfect square? yes
2. Is the last term a perfect square? yes
3. Is the middle term equal to $-2(1x)(5)$? yes

$x^2 - 10x + 25 = (x - 5)^2$

QuickCheck

Evaluate each expression if $a = -2$, $b = -1$, $c = 0$, and $d = 2.5$.

16. $|a - 3|$
17. $|2a + 1|$
18. $|4 - b|$
19. $\left|\frac{1}{2}b - 2\right|$
20. $|12 - 4c|$
21. $|2c - 3| + 1$
22. $|4d - 6|$
23. $|3d - 2| - 8$

QuickReview

Example 3

Evaluate $|2x + 1| - 7$ if $x = -1$.

$|2x + 1| - 7 = |2(-1) + 1| - 7$	$x = -1$
$= |-2 + 1| - 7$	Multiply.
$= |-1| - 7$	Add.
$= 1 - 7$	$|-1| = 1$
$= -6$	Subtract.

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Get Started on the Chapter

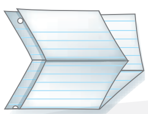
You will learn several new concepts, skills, and vocabulary terms as you study this chapter. To get ready, identify important terms and organize your resources. You may wish to refer to previous chapters to review prerequisite skills.

FOLDABLES StudyOrganizer

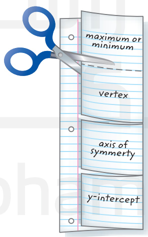
Quadratic Functions and Equations Make this Foldable to help you organize your notes about quadratic functions. Begin with a sheet of notebook paper.

1 Fold the sheet of paper along the length so that the edge of the paper aligns with the margin rule on the paper.

2 Fold the sheet twice widthwise to form four sections.



3 Unfold the sheet, and cut along the folds on the front flap only.



4 Label each section as shown.

NewVocabulary

quadratic function
parabola
axis of symmetry
vertex
minimum
maximum
double root
transformation
completing the square
Quadratic Formula
discriminant
step function
greatest integer function
absolute value function

ReviewVocabulary

domain all the possible values of the independent variable, *x*

leading coefficient the coefficient of the first term of a polynomial written in standard form

range all the possible values of the dependent variable, *y*

In the function represented by the table, the domain is {0, 2, 4, 6}, and the range is {3, 5, 7, 9}.

x	y
0	3
2	5
4	7
6	9

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LESSON 1-1 Graphing Quadratic Functions

::Then	::Now	::Why?
You graphed linear and exponential functions.	**1** Analyze the characteristics of graphs of quadratic functions. **2** Graph quadratic functions.	The Innovention Fountain in Epcot's Futureworld in Orlando, Florida, is an elaborate display of water, light, and music. The sprayers shoot water in shapes that can be modeled by quadratic equations. You can use graphs of these equations to show the path of the water.

NewVocabulary

quadratic function
standard form
parabola
axis of symmetry
vertex
minimum
maximum

Mathematical Practices
Reason abstractly and quantitatively.

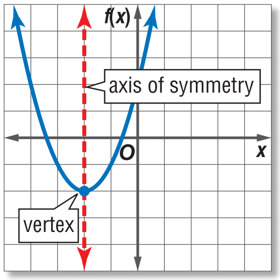
1 Characteristics of Quadratic Functions

Quadratic functions are nonlinear and can be written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. This form is called the **standard form** of a quadratic function.

The shape of the graph of a quadratic function is called a **parabola**. Parabolas are symmetric about a central line called the **axis of symmetry**. The axis of symmetry intersects a parabola at only one point, called the **vertex**.

KeyConcept Quadratic Functions

Parent Function:	$f(x) = x^2$
Standard Form:	$f(x) = ax^2 + bx + c$
Type of Graph:	parabola
Axis of Symmetry:	$x = -\frac{b}{2a}$
y-intercept:	c

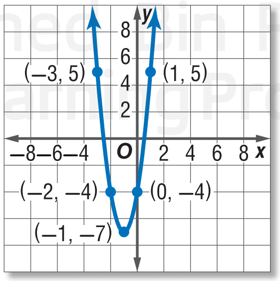


When $a > 0$, the graph of $y = ax^2 + bx + c$ opens upward. The lowest point on the graph is the **minimum**. When $a < 0$, the graph opens downward. The highest point is the **maximum**. The maximum or minimum is the vertex.

Example 1 Graph a Parabola

Use a table of values to graph $y = 3x^2 + 6x - 4$. State the domain and range.

x	y
1	5
0	−4
−1	−7
−2	−4
−3	5



Graph the ordered pairs, and connect them to create a smooth curve. The parabola extends to infinity. The domain is all real numbers. The range is $\{y \mid y \geq -7\}$, because −7 is the minimum.

GuidedPractice

1. Use a table of values to graph $y = x^2 + 3$. State the domain and range.

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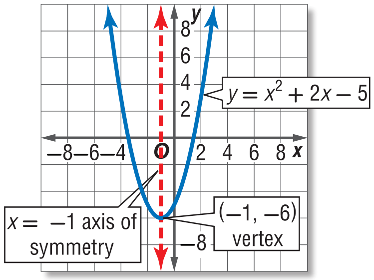
Recall that figures with symmetry are those in which each half of the figure matches exactly.

A parabola is symmetric about the axis of symmetry. Every point on the parabola to the left of the axis of symmetry has a corresponding point on the other half. The function is increasing on one side of the axis of symmetry and decreasing on the other side.

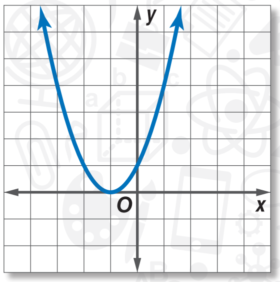


When identifying characteristics from a graph, it is often easiest to locate the vertex first. It is either the maximum or minimum point of the graph.

Example 2 Identify Characteristics from Graphs

Find the vertex, the equation of the axis of symmetry, and the *y*-intercept of each graph.

a.

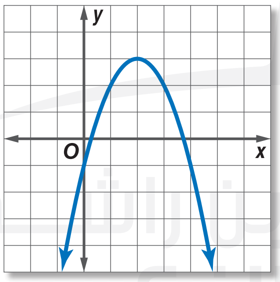


Step 1 Find the vertex.
Because the parabola opens upward, the vertex is located at the minimum point of the parabola. It is located at $(-1, 0)$.

Step 2 Find the axis of symmetry.
The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at $x = -1$.

Step 3 Find the y-intercept.
The y-intercept is the point where the graph intersects the y-axis. It is located at $(0, 1)$, so the y-intercept is 1.

b.



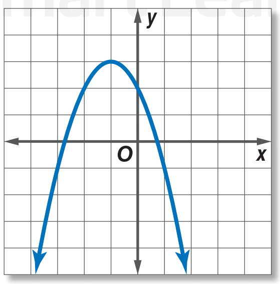
Step 1 Find the vertex.
The parabola opens downward, so the vertex is located at its maximum point, $(2, 3)$.

Step 2 Find the axis of symmetry.
The axis of symmetry is located at $x = 2$.

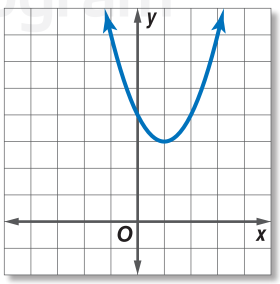
Step 3 Find the y-intercept.
The y-intercept is where the parabola crosses the y-axis. It is located at $(0, -1)$, so the y-intercept is -1.

Guided Practice

2A.



2B.



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StudyTip

Function Characteristics When identifying characteristics of a function, it is often easiest to locate the axis of symmetry first.

Example 3 Identify Characteristics from Functions

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each function.

a. $y = 2x^2 + 4x - 3$

$x = -\frac{b}{2a}$ — Formula for the equation of the axis of symmetry

$x = -\frac{4}{2 \cdot 2}$ — $a = 2$ and $b = 4$

$x = -1$ — Simplify.

The equation for the axis of symmetry is $x = -1$.

To find the vertex, use the value you found for the axis of symmetry as the x-coordinate of the vertex. Find the y-coordinate using the original equation.

$y = 2x^2 + 4x - 3$ — Original equation

$= 2(-1)^2 + 4(-1) - 3$ — $x = -1$

$= -5$ — Simplify.

The vertex is at $(-1, -5)$.

The y-intercept always occurs at $(0, c)$. So, the y-intercept is -3.

b. $y = -x^2 + 6x + 4$

$x = -\frac{b}{2a}$ — Formula for the equation of the axis of symmetry

$x = -\frac{6}{2(-1)}$ — $a = -1$ and $b = 6$

$x = 3$ — Simplify.

The equation of the axis of symmetry is $x = 3$.

$y = -x^2 + 6x + 4$ — Original equation

$= -(3)^2 + 6(3) + 4$ — $x = 3$

$= 13$ — Simplify.

The vertex is at $(3, 13)$.

The y-intercept is 4.

StudyTip

y-intercept The y-coordinate of the y-intercept is also the constant term (c) of the quadratic function in standard form.

GuidedPractice

3A. $y = -3x^2 + 6x - 5$

3B. $y = 2x^2 + 2x + 2$

Next you will learn how to identify whether the vertex is a maximum or a minimum.

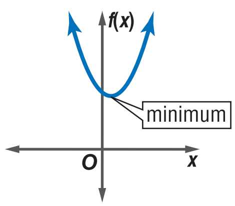
KeyConcept Maximum and Minimum Values

Words The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$:

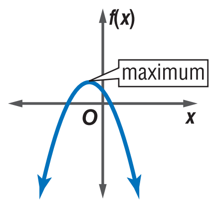
- opens upward and has a minimum value when $a > 0$, and
- opens downward and has a maximum value when $a < 0$.
- The range of a quadratic function is all real numbers greater than or equal to the minimum, or all real numbers less than or equal to the maximum.

Examples

a is positive.



a is negative.



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Example 4 Maximum and Minimum Values

Consider $f(x) = -2x^2 - 4x + 6$.

a. Determine whether the function has a *maximum* or *minimum* value.

For $f(x) = -2x^2 - 4x + 6$, $a = -2$, $b = -4$, and $c = 6$.

Because a is negative the graph opens down, so the function has a maximum value.

b. State the maximum or minimum value of the function.

The maximum value is the y-coordinate of the vertex.

The x-coordinate of the vertex is $\frac{-b}{2a}$ or $\frac{4}{2(-2)}$ or -1.

$f(x) = -2x^2 - 4x + 6$	Original function
$f(-1) = -2(-1)^2 - 4(-1) + 6$	$x = -1$
$f(-1) = 8$	Simplify.

The maximum value is 8.

c. State the domain and range of the function.

The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or $\{y \mid y \leq 8\}$.

WatchOut!

Minimum and Maximum Values Don't forget to find both coordinates of the vertex (x, y). The minimum or maximum value is the y-coordinate.

ReviewVocabulary

Domain and Range The domain is the set of all of the possible values of the independent variable x. The range is the set of all of the possible values of the dependent variable y.

GuidedPractice

Consider $g(x) = 2x^2 - 4x - 1$.

4A. Determine whether the function has a *maximum* or *minimum* value.

4B. State the maximum or minimum value.

4C. State the domain and range of the function.

2 Graph Quadratic Functions

You have learned how to find several important characteristics of quadratic functions.

KeyConcept Graph Quadratic Functions

Step 1 Find the equation of the axis of symmetry.

Step 2 Find the vertex, and determine whether it is a maximum or minimum.

Step 3 Find the y-intercept.

Step 4 Use symmetry to find additional points on the graph, if necessary.

Step 5 Connect the points with a smooth curve.

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StudyTip

Symmetry and Points When locating points that are on opposite sides of the axis of symmetry, not only are the points equidistant from the axis of symmetry, they are also equidistant from the vertex.

Example 5 Graph Quadratic Functions

Graph $f(x) = x^2 + 4x + 3$.

Step 1 Find the equation of the axis of symmetry.

$x = \frac{-b}{2a}$ Formula for the equation of the axis of symmetry

$x = \frac{-4}{2 \cdot 1}$ or -2 $a = 1$ and $b = 4$

Step 2 Find the vertex, and determine whether it is a maximum or minimum.

$f(x) = x^2 + 4x + 3$ Original equation

$= (-2)^2 + 4(-2) + 3$ $x = -2$

$= -1$ Simplify.

The vertex lies at $(-2, -1)$. Because a is positive the graph opens up, and the vertex is a minimum.

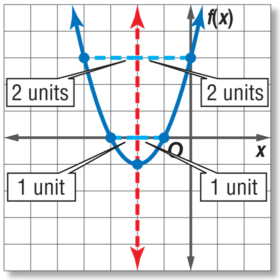
Step 3 Find the y-intercept.

$f(x) = x^2 + 4x + 3$ Original equation

$= (0)^2 + 4(0) + 3$ $x = 0$

$= 3$ The y-intercept is 3.

Step 4 The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same y-value.



Step 5 Connect the points with a smooth curve.

GuidedPractice **Graph each function.**

5A. $f(x) = -2x^2 + 2x - 1$

5B. $f(x) = 3x^2 - 6x + 2$

There are general differences between linear, exponential, and quadratic functions.

	Linear Functions	Exponential Functions	Quadratic Functions
Equation	$y = mx + b$	$y = ab^x$, $a \neq 0$, $b > 0$, $b \neq 1$	$y = ax^2 + bx + c$, $a \neq 0$
Degree	1	x	2
Graph	line	curve	parabola
Increasing / Decreasing	$m > 0$: y is increasing on the entire domain. $m < 0$: y is decreasing on the entire domain.	$a > 0$, $b > 1$ or $a < 0$, $0 < b < 1$: y is increasing on the entire domain. $a > 0$, $0 < b < 1$ or $a < 0$, $b > 1$: y is decreasing on the entire domain.	$a > 0$: y is decreasing to the left of the axis of symmetry and increasing on the right. $a < 0$: y is increasing to the left of the axis of symmetry and decreasing on the right.
End Behavior	$m > 0$: as x increases, y increases; as x decreases, y decreases $m < 0$: as x increases, y decreases; as x decreases, y increases	$b > 1$: as x decreases, y approaches 0; $a > 0$, as x increases, y increases; $a < 0$, as x increases, y decreases. $0 < b < 1$: as x increases, y approaches 0; $a > 0$, as x decreases, y increases; $a < 0$, as x decreases, y decreases.	$a > 0$: as x increases, y increases; as x decreases, y increases. $a < 0$: as x increases, y decreases; as x decreases, y decreases

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You have used what you know about quadratic functions, parabolas, and symmetry to create graphs. You can analyze these graphs to solve real-world problems.

Real-WorldLink

About 1 in 17 high school seniors playing football will go on to play football at an NCAA school.

Source: National Collegiate Athletic Association

Real-World Example 6 Use a Graph of a Quadratic Function

SCHOOL SPIRIT **The student council at Ara High School launch T-shirts into the crowd every time the Lakers score a touchdown. The height of the T-shirt can be modeled by the function $h(x) = -16x^2 + 48x + 6$, where $h(x)$ represents the height in feet of the T-shirt after x seconds.**

a. Graph the function.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

$x = -\frac{48}{2(-16)}$ or $\frac{3}{2}$ $a = -16$ and $b = 48$

The equation of the axis of symmetry is $x = \frac{3}{2}$. Thus, the x-coordinate for the vertex is $\frac{3}{2}$.

$y = -16x^2 + 48x + 6$ Original equation

$= -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 6$ $x = \frac{3}{2}$

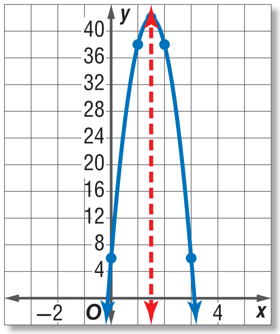
$= -16\left(\frac{9}{4}\right) + 48\left(\frac{3}{2}\right) + 6$ $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$= -36 + 72 + 6$ or 42 Simplify.

The vertex is at $\left(\frac{3}{2}, 42\right)$.

Let's find another point. Choose an x-value of 0 and substitute. Our new point is at (0, 6). The point paired with it on the other side of the axis of symmetry is (3, 6).

Repeat this and choose an x-value of 1 to get (1, 38) and its corresponding point (2, 38). Connect these points and create a smooth curve.



b. At what height was the T-shirt launched?

The T-shirt is launched when time equals 0, or at the y-intercept.

So, the T-shirt was launched 6 feet from the ground.

c. What is the maximum height of the T-shirt? When was the maximum height reached?

The maximum height of the T-shirt occurs at the vertex.

So the T-shirt reaches a maximum height of 42 feet. The time was $\frac{3}{2}$ or 1.5 seconds after launch.

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6. **TRACK** Adnan is competing in the javelin throw. The height of the javelin can be modeled by the equation $y = -16x^2 + 64x + 6$, where y represents the height in feet of the javelin after x seconds.

A. Graph the path of the javelin.

B. At what height is the javelin thrown?

C. What is the maximum height of the javelin?

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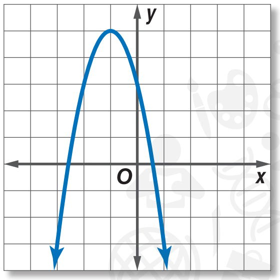
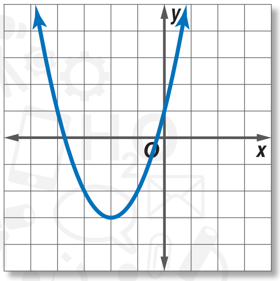
Check Your Understanding

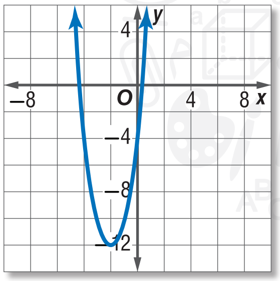
Example 1 **Use a table of values to graph each equation. State the domain and range.**

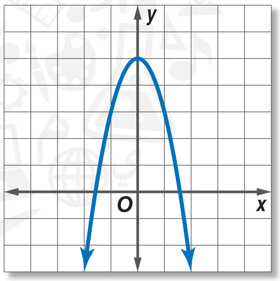
1. $y = 2x^2 + 4x - 6$
2. $y = x^2 + 2x - 1$
3. $y = x^2 - 6x - 3$
4. $y = 3x^2 - 6x - 5$

Example 2 **Find the vertex, the equation of the axis of symmetry, and the y-intercept of each graph.**

5. 

6. 

7. 

8. 

Example 3 **Find the vertex, the equation of the axis of symmetry, and the y-intercept of the graph of each function.**

9. $y = -3x^2 + 6x - 1$
10. $y = -x^2 + 2x + 1$
11. $y = x^2 - 4x + 5$
12. $y = 4x^2 - 8x + 9$

Example 4 **Consider each function.**

a. **Determine whether the function has *maximum* or *minimum* value.**

b. **State the maximum or minimum value.**

c. **What are the domain and range of the function?**

13. $y = -x^2 + 4x - 3$
14. $y = -x^2 - 2x + 2$
15. $y = -3x^2 + 6x + 3$
16. $y = -2x^2 + 8x - 6$

Example 5 **Graph each function.**

17. $f(x) = -3x^2 + 6x + 3$
18. $f(x) = -2x^2 + 4x + 1$
19. $f(x) = 2x^2 - 8x - 4$
20. $f(x) = 3x^2 - 6x - 1$

Example 6

21. **REASONING** A juggler is tossing a ball into the air. The height of the ball in feet can be modeled by the equation $y = -16x^2 + 16x + 5$, where y represents the height of the ball at x seconds.

a. Graph this equation.

b. At what height is the ball thrown?

c. What is the maximum height of the ball?

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Practice and Problem Solving

Example 1 Use a table of values to graph each equation. State the domain and range.

22. $y = x^2 + 4x + 6$

23. $y = 2x^2 + 4x + 7$

24. $y = 2x^2 - 8x - 5$

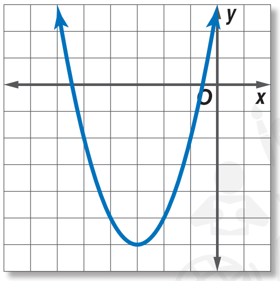
25. $y = 3x^2 + 12x + 5$

26. $y = 3x^2 - 6x - 2$

27. $y = x^2 - 2x - 1$

Example 2 Find the vertex, the equation of the axis of symmetry, and the y-intercept of each graph.

28.



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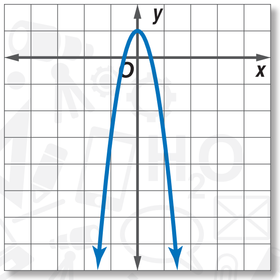

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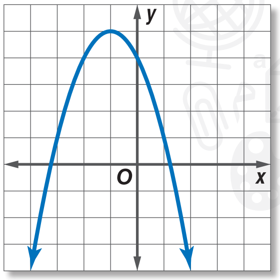

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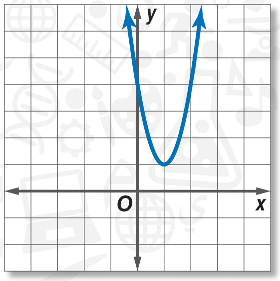

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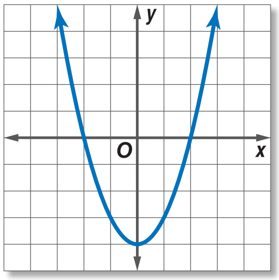

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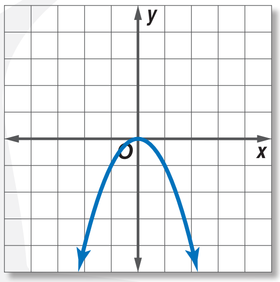

Example 3 Find the vertex, the equation of the axis of symmetry, and the y-intercept of each function.

34. $y = x^2 + 8x + 10$

35. $y = 2x^2 + 12x + 10$

36. $y = -3x^2 - 6x + 7$

37. $y = -x^2 - 6x - 5$

38. $y = 5x^2 + 20x + 10$

39. $y = 7x^2 - 28x + 14$

40. $y = 2x^2 - 12x + 6$

41. $y = -3x^2 + 6x - 18$

42. $y = -x^2 + 10x - 13$

Example 4 Consider each function.

a. Determine whether the function has a *maximum* or *minimum* value.

b. State the maximum or minimum value.

c. What are the domain and range of the function?

43. $y = -2x^2 - 8x + 1$

44. $y = x^2 + 4x - 5$

45. $y = 3x^2 + 18x - 21$

46. $y = -2x^2 - 16x + 18$

47. $y = -x^2 - 14x - 16$

48. $y = 4x^2 + 40x + 44$

49. $y = -x^2 - 6x - 5$

50. $y = 2x^2 + 4x + 6$

51. $y = -3x^2 - 12x - 9$

Example 5 Graph each function.

52. $y = -3x^2 + 6x - 4$

53. $y = -2x^2 - 4x - 3$

54. $y = -2x^2 - 8x + 2$

55. $y = x^2 + 6x - 6$

56. $y = x^2 - 2x + 2$

57. $y = 3x^2 - 12x + 5$

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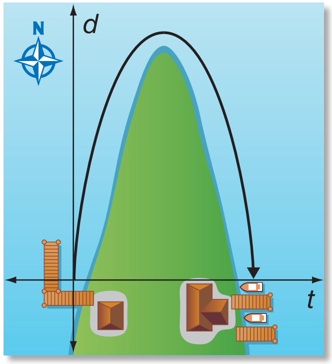
Example 6

58. **BOATING** Hidaya has her boat docked on the west side of khor Dubai. She is boating over to Dubai Marina. The distance traveled by Hidaya over time can be modeled by the equation $d = -16t^2 + 66t$, where d is the number of feet she travels in t minutes.



a. Graph this equation.

b. What is the maximum number of feet north that she traveled?

c. How long did it take her to reach Casper Marina?

GRAPHING CALCULATOR **Graph each equation. Use the** TRACE **feature to find the vertex on the graph. Round to the nearest thousandth if necessary.**

59. $y = 4x^2 + 10x + 6$

60. $y = 8x^2 - 8x + 8$

61. $y = -5x^2 - 3x - 8$

62. $y = -7x^2 + 12x - 10$

63. **GOLF** The average amateur golfer can hit a ball with an initial upward velocity of 31.3 meters per second. The height can be modeled by the equation $h = -4.9t^2 + 31.3t$, where h is the height of the ball, in meters, after t seconds.

a. Graph this equation. What do the portions of the graph where $h > 0$ represent in the context of the situation? What does the end behavior of the graph represent?

b. At what height is the ball hit?

c. What is the maximum height of the ball?

d. How long did it take for the ball to hit the ground?

e. State a reasonable range and domain for this situation.

64. **FUNDRAISING** The marching band is selling flowers to buy new uniforms. Last year the band charged AED 5 each, and they sold 150. They want to increase the price this year, and they expect to lose 10 sales for each AED 1 increase. The sales revenue R, in dirhams, generated by selling the flowers is predicted by the function $R = (5 + p)(150 - 10p)$, where p is the number of AED 1 price increases.

a. Write the function in standard form.

b. Find the maximum value of the function.

c. At what price should the flowers be sold to generate the most sales revenue? Explain your reasoning.

65. **FOOTBALL** A football is kicked up from ground level at an initial upward velocity of 90 feet per second. The equation $h = -16t^2 + 90t$ gives the height h of the football after t seconds.

a. What is the height of the ball after one second?

b. When is the ball 126 feet high?

c. When is the height of the ball 0 feet? What do these points represent in the context of the situation?

66. **STRUCTURE** Let $f(x) = x^2 - 9$.

a. What is the domain of $f(x)$?

b. What is the range of $f(x)$?

c. For what values of x is $f(x)$ negative?

d. When x is a real number, what are the domain and range of $f(x) = \sqrt{x^2 - 9}$?

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67. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate solving quadratic equations using tables.

a. Algebraic Determine the related function for each equation. Copy and complete the first two columns of the table below.

Equation	Related Function	Zeros	y-Values
$x^2 - x = 12$			
$x^2 + 8x = 9$			
$x^2 = 14x - 24$			
$x^2 + 16x = -28$			

b. Graphical Graph each related function with a graphing calculator.

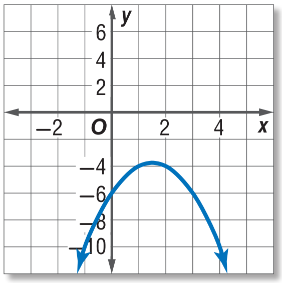
c. Analytical The number of zeros is equal to the degree of the related function. Use the table feature on your calculator to determine the zeros of each related function. Record the zeros in the table above. Also record the values of the function one unit less than and one unit more than each zero.

d. Verbal Examine the function values for x-values just before and just after a zero. What happens to the sign of the function value before and after a zero?

H.O.T. Problems Use Higher-Order Thinking Skills

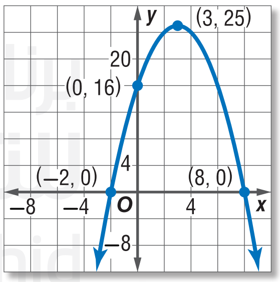
68. **OPEN ENDED** Write a quadratic function for which the graph has an axis of symmetry of $x = -\frac{3}{8}$. Summarize your steps.

69. **ERROR ANALYSIS** Sendeyah thinks that the parabolas represented by the graph and the description have the same axis of symmetry. Hamdan disagrees. Who is correct? Explain your reasoning.



a parabola that opens downward, passing through (0, 6) and having a vertex at (2, 2)

70. **CHALLENGE** Using the axis of symmetry, the y-intercept, and one x-intercept, write an equation for the graph shown.



71. **STRUCTURE** The graph of a quadratic function has a vertex at (2, 0). One point on the graph is (5, 9). Find another point on the graph. Explain how you found it.

72. **OPEN ENDED** Describe a real-world situation that involves a quadratic equation. Explain what the vertex represents.

73. **REASONING** Provide a counterexample that is a specific case to show that the following statement is false. *The vertex of a parabola is always the minimum of the graph.*

74. **WRITING IN MATH** Use tables and graphs to compare and contrast an exponential function $f(x) = ab^x + c$, where $a \neq 0$, $b > 0$, and $b \neq 1$, a quadratic function $g(x) = ax^2 + c$, and a linear function $h(x) = ax + c$. Include intercepts, portions of the graph where the functions are increasing, decreasing, positive, or negative, relative maximums and minimums, symmetries, and end behavior. Which function eventually exceeds the others?

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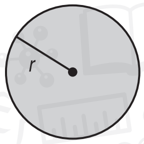
Standardized Test Practice

75. Which of the following is an equation for the line that passes through (2, −5) and is perpendicular to $2x + 4y = 8$?

A $y = 2x + 10$
B $y = -\frac{1}{2}x - 4$
C $y = 2x - 9$
D $y = -2x - 1$

76. GEOMETRY The area of the circle is 36π square units. If the radius is doubled, what is the area of the new circle?



$A = 36\pi$

F 1296π units^2
G 144π units^2
H 72π units^2
J 9π units^2

77. What is the range of the function $f(x) = -4x^2 - \frac{1}{2}$?

A $\left\{\text{all integers less than or equal to } \frac{1}{2}\right\}$
B {all nonnegative integers}
C {all real numbers}
D $\left\{\text{all real numbers less than or equal to } -\frac{1}{2}\right\}$

78. SHORT RESPONSE Khamis is delivers newspapers for extra money. He starts delivering the newspapers at 3:15 P.M. and finishes at 5:05 P.M. How long does it take Khamis to complete his route?

Spiral Review

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it. (Lesson 1-9)

79. $4x^2 + 4x + 1$
80. $4x^2 - 20x + 25$
81. $9x^2 + 8x + 16$

Factor each polynomial if possible. If the polynomial cannot be factored, write *prime*. (Lesson 1-8)

82. $n^2 - 16$
83. $x^2 + 25$
84. $9 - 4a^2$

Find each product. (Lesson 1-3)

85. $(b - 7)(b + 3)$
86. $(c - 6)(c - 5)$
87. $(2x - 1)(x + 9)$

88. MULTIPLE BIRTHS The number of quadruplet births Q in recent years can be modeled by $Q = -0.5t^3 + 11.7t^2 - 21.5t + 218.6$, where t represents the number of years since 2002. What is the expected number of quadruplet births in 2017? (Lesson 1-1)

89. FORESTRY The number of board feet B that a log will yield can be estimated by using the formula $B = \frac{L}{16}\left(D^2 - 8D + 16\right)$, where D is the diameter in inches and L is the log length in feet. For logs that are 16 feet long, what diameter will yield approximately 256 board feet? (Lesson 1-9)

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Skills Review

Find the *x*-intercept of the graph of each equation.

90. $x + 2y = 10$
91. $2x - 3y = 12$
92. $3x - y = -18$

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EXTEND

1-1 Algebra Lab
Rate of Change of a Quadratic Function

A model rocket is launched from the ground with an upward velocity of 144 feet per second. The function $y = -16x^2 + 144x$ models the height y of the rocket in feet after x seconds. Using this function, we can investigate the rate of change of a quadratic function.

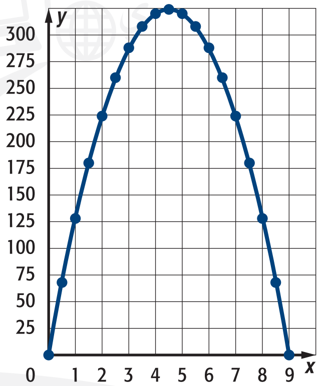
Activity

Step 1 Copy the table below.

x	0	0.5	1.0	1.5	...	9.0
y	0					
Rate of Change	–					

Step 2 Find the value of y for each value of x from 0 through 9.

Step 3 Graph the ordered pairs (x, y) on grid paper. Connect the points with a smooth curve. Notice that the function *increases* when $0 < x < 4.5$ and *decreases* when $4.5 < x < 9$.



Step 4 Recall that the *rate of change* is the change in y divided by the change in x. Find the rate of change for each half second interval of x and y.

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Exercises

Use the quadratic function $y = x^2$.

1. Make a table, similar to the one in the Activity, for the function using $x = -4, -3, -2, -1, 0, 1, 2, 3,$ and 4. Find the values of y for each x-value.

2. Graph the ordered pairs on grid paper. Connect the points with a smooth curve. Describe where the function is increasing and where it is decreasing.

3. Find the rate of change for each column starting with $x = -3$. Compare the rates of change when the function is increasing and when it is decreasing.

4. **CHALLENGE** If an object is dropped from 100 feet in the air and air resistance is ignored, the object will fall at a rate that can be modeled by the function $f(x) = -16x^2 + 100$, where $f(x)$ represents the object's height in feet after x seconds. Make a table like that in Exercise 1, selecting appropriate values for x. Fill in the x-values, the y-values, and rates of change. Compare the rates of change. Describe any patterns that you see.

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LESSON 1-2 Solving Quadratic Equations by Graphing

:: Then

- You solved quadratic equations by factoring.

:: Now

1. Solve quadratic equations by graphing.
2. Estimate solutions of quadratic equations by graphing.

:: Why?

- Dorton Arena at the state fairgrounds in Raleigh, North Carolina, has a shape created by two intersecting parabolas. The shape of one of the parabolas can be modeled by $y = -x^2 + 127x$, where x is the width of the parabola in meters, and y is the length of the parabola in meters. The x-intercepts of the graph of this function can be used to find the distance between the points where the parabola meets the ground.

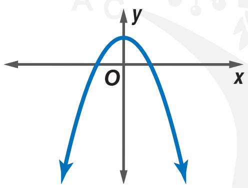
NewVocabulary
double root

Mathematical Practices
Construct viable arguments and critique the reasoning of others.
Attend to precision.

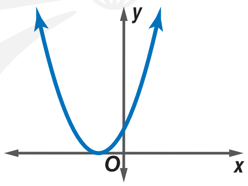
1 Solve by Graphing

A quadratic equation can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$. To write a quadratic function as an equation, replace y or $f(x)$ with 0. Recall that the solutions or roots of an equation can be identified by finding the x-intercepts of the related graph.

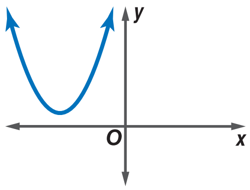
KeyConcept Solutions of Quadratic Equations



two unique real solutions



one unique real solution



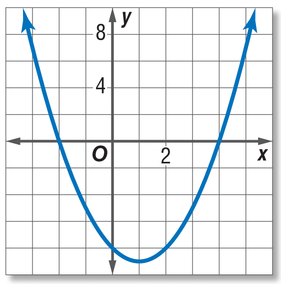
no real solutions

Example 1 Two Roots

Solve $x^2 - 2x - 8 = 0$ by graphing.

Graph the related function $f(x) = x^2 - 2x - 8$.

The x-intercepts of the graph appear to be at -2 and 4, so the solutions are -2 and 4.



CHECK Check each solution in the original equation.

$x^2 - 2x - 8 = 0$	Original equation	$x^2 - 2x - 8 = 0$
$(-2)^2 - 2(-2) - 8 \stackrel{?}{=} 0$	$x = -2$ or $x = 4$	$(4)^2 - 2(4) - 8 \stackrel{?}{=} 0$
$0 = 0$ ✓	Simplify.	$0 = 0$ ✓

GuidedPractice Solve each equation by graphing.

1A. $-x^2 - 3x + 18 = 0$

1B. $x^2 - 4x + 3 = 0$

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The solutions in Example 1 were two distinct numbers. Sometimes the two roots are the same number, called a **double root**.

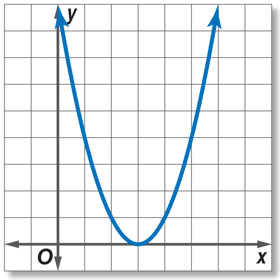
Example 2 Double Root

Solve $x^2 - 6x = -9$ by graphing.

Step 1 Rewrite the equation in standard form.

$$x^2 - 6x = -9 \quad \text{Original equation}$$
$$x^2 - 6x + 9 = 0 \quad \text{Add 9 to each side.}$$

Step 2 Graph the related function $f(x) = x^2 - 6x + 9$.



Step 3 Locate the x-intercepts of the graph. Notice that the vertex of the parabola is the only x-intercept. Therefore, there is only one solution, 3.

CHECK Solve by factoring.

$$x^2 - 6x + 9 = 0 \quad \text{Original equation}$$
$$(x - 3)(x - 3) = 0 \quad \text{Factor.}$$
$$x - 3 = 0 \quad \text{or} \quad x - 3 = 0 \quad \text{Zero Product Property}$$
$$x = 3 \qquad\qquad x = 3 \quad \text{Add 3 to each side.}$$

The only solution is 3.

WatchOut!

Precision Solutions found from the graph of an equation may appear to be exact. Check them in the original equation to be sure.

GuidedPractice

Solve each equation by graphing.

2A. $x^2 + 25 = 10x$

2B. $x^2 = -8x - 16$

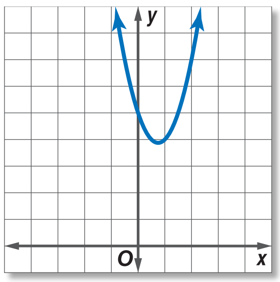
Sometimes the roots are not real numbers. Quadratic equations may have two, one, or no real solutions. Quadratic equations with solutions that are not real numbers lead us to extend the number system to allow for solutions of these equations. These numbers are called *complex numbers*. You will study complex numbers in Algebra 2.

Example 3 No Real Roots

Solve $2x^2 - 3x + 5 = 0$ by graphing.

Step 1 Rewrite the equation in standard form.
This equation is written in standard form.

Step 2 Graph the related function $f(x) = 2x^2 - 3x + 5$.



Step 3 Locate the x-intercepts of the graph. This graph has no x-intercepts. Therefore, this equation has no real number solutions. The solution set is $\varnothing$.

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GuidedPractice

Solve each equation by graphing.

3A. $-x^2 - 3x = 5$

3B. $-2x^2 - 8 = 6x$

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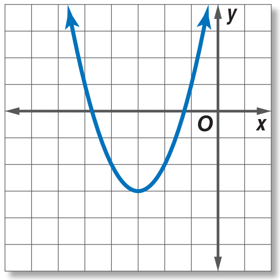
2 Estimate Solutions

The real roots found thus far have been integers. However, the roots of quadratic equations are usually not integers. In these cases, use estimation to approximate the roots of the equation.

Example 4 Approximate Roots with a Table

Solve $x^2 + 6x + 6 = 0$ by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.



Graph the related function $f(x) = x^2 + 6x + 6$.

The x-intercepts are located between −5 and −4 and between −2 and −1.

Make a table using an increment of 0.1 for the x-values located between −5 and −4 and between −2 and −1.

Look for a change in the signs of the function values. The function value that is closest to zero is the best approximation for a zero of the function.

x	−4.9	−4.8	−4.7	−4.6	−4.5	−4.4	−4.3	−4.2	−4.1
y	0.61	0.24	−0.11	−0.44	−0.75	−1.04	−1.31	−1.56	−1.79

x	−1.9	−1.8	−1.7	−1.6	−1.5	−1.4	−1.3	−1.2	−1.1
y	−1.79	−1.56	−1.31	−1.04	−0.75	−0.44	−0.11	0.24	0.61

For each table, the function value that is closest to zero when the sign changes is −0.11. Thus, the roots are approximately −4.7 and −1.3.

Guided Practice

4. Solve $2x^2 + 6x - 3 = 0$ by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

Study Tip

Location of Zeros Since quadratic functions are continuous, there must be a zero between two x-values for which the corresponding y-values have opposite signs.

Real-World Link

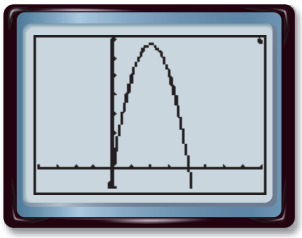
The game of soccer, called "football" outside of North America, began in 1863 in Britain when the Football Association was founded. Soccer is played on every continent of the world.

Source: Sports Know How

Approximating the x-intercepts of graphs is helpful for real-world applications.

Real-World Example 5 Approximate Roots with a Calculator

FOOTBALL A goalie kicks a ball with an upward velocity of 65 feet per second, and his foot meets the ball 1 foot off the ground. The quadratic function $h = -16t^2 + 65t + 1$ represents the height of the ball h in feet after t seconds. Approximately how long is the ball in the air?



[−4, 7] scl: 1 by [−10, 70] scl: 10

You need to find the roots of the equation $-16t^2 + 65t + 1 = 0$. Use a graphing calculator to graph the related function $f(x) = -16t^2 + 65t + 1$.

The positive x-intercept of the graph is approximately 4. Therefore, the ball is in the air for approximately 4 seconds.

Guided Practice

5. If the goalie kicks the ball with an upward velocity of 55 feet per second and his foot meets the ball 2 feet off the ground, approximately how long is the ball in the air?

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Check Your Understanding

Examples 1–3 **Solve each equation by graphing.**

1. $x^2 + 3x - 10 = 0$

2. $2x^2 - 8x = 0$

3. $x^2 + 4x = -4$

4. $x^2 + 12 = -8x$

Example 4 **Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.**

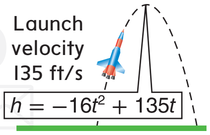
5. $-x^2 - 5x + 1 = 0$

6. $-9 = x^2$

7. $x^2 = 25$

8. $x^2 - 8x = -9$

Example 5 **9.** **SCIENCE FAIR** Zayed built a model rocket. Its flight can be modeled by the equation shown, where h is the height of the rocket in feet after t seconds. About how long was Zayed's rocket in the air?



Practice and Problem Solving

Examples 1–3 **Solve each equation by graphing.**

10. $x^2 + 7x + 14 = 0$

11 $x^2 + 2x - 24 = 0$

12. $x^2 - 16x + 64 = 0$

13. $x^2 - 5x + 12 = 0$

14. $x^2 + 14x = -49$

15. $x^2 = 2x - 1$

16. $x^2 - 10x = -16$

17. $-2x^2 - 8x = 13$

18. $2x^2 - 16x = -30$

19. $2x^2 = -24x - 72$

20. $-3x^2 + 2x = 15$

21. $x^2 = -2x + 80$

Example 4 **Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.**

22. $x^2 + 2x - 9 = 0$

23. $x^2 - 4x = 20$

24. $x^2 + 3x = 18$

25. $2x^2 - 9x = -8$

26. $3x^2 = -2x + 7$

27. $5x = 25 - x^2$

Example 5 **28.** **SOFTBALL** The equation $h = -16t^2 + 47t + 3$ models the height h, in feet, of a ball that Amani hits after t seconds. How long is the ball in the air?

29. **RIDES** The Ajman Tower launches riders straight up and returns straight down. The equation $h = -16t^2 + 122t$ models the height h, in feet, of the riders from their starting position after t seconds. How long is it until the riders return to the bottom?

Use factoring to determine how many times the graph of each function intersects the x-axis. Identify each zero.

30. $y = x^2 - 8x + 16$

31. $y = x^2 + 4x + 4$

32. $y = x^2 + 2x - 24$

33. $y = x^2 + 12x + 32$

34. **NUMBER THEORY** Use a quadratic equation to find two numbers that have a sum of 9 and a product of 20.

35. **NUMBER THEORY** Use a quadratic equation to find two numbers that have a sum of 1 and a product of −12.

36. **MODELING** The height of a golf ball in the air can be modeled by the equation $h = -16t^2 + 76t$, where h is the height in feet of the ball after t seconds.

a. How long was the ball in the air?

b. What is the ball's maximum height?

c. When will the ball reach its maximum height?

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37. **SKIING** Ayesha is in a freestyle aerial competition. The equation $h = -16t^2 + 30t + 10$ models Ayesha's height h, in feet, t seconds after leaving the ramp.

a. How long is Ayesha in the air?

b. When will Ayesha reach a height of 15 feet?

c. To earn bonus points in the competition, you must reach a height of 20 feet. Will Ayesha earn bonus points?

38. **MULTIPLE REPRESENTATIONS** In this problem, you will explore how to further interpret the relationship between quadratic functions and graphs.

a. **Graphical** Graph $y = x^2$.

b. **Analytical** Name the vertex and two other points on the graph.

c. **Graphical** Graph $y = x^2 + 2$, $y = x^2 + 4$, and $y = x^2 + 6$ on the same coordinate plane as the previous graph.

d. **Analytical** Name the vertex and two points from each of these graphs that have the same x-coordinates as the first graph.

e. **Analytical** What conclusion can you draw from this?

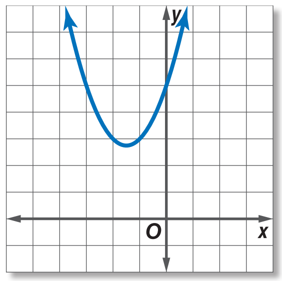
GRAPHING CALCULATOR **Solve each equation by graphing.**

39. $x^3 - 3x^2 - 6x + 8 = 0$

40. $x^3 - 8x^2 + 15x = 0$

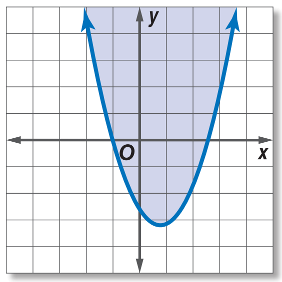
H.O.T. Problems Use Higher-Order Thinking Skills

41. **CRITIQUE** Ismail and Osama are finding the number of real zeros of the function graphed at the right. Ismail says that the function has no real zeros because there are no x-intercepts. Osama says that the function has one real zero because the graph has a y-intercept. Is either of them correct? Explain your reasoning.



42. **OPEN ENDED** Describe a real-world situation in which a thrown object travels in the air. Write an equation that models the height of the object with respect to time, and determine how long the object travels in the air.

43. **REASONING** The graph shown is that of a *quadratic inequality*. Analyze the graph, and determine whether the y-value of a solution of the inequality is *sometimes*, *always*, or *never* greater than 2. Explain.



44. **CHALLENGE** Write a quadratic equation that has the roots described.

a. one double root

b. one rational (nonintegral) root and one integral root

c. two distinct integral roots that are additive opposites.

45. **CHALLENGE** Find the roots of $x^2 = 2.25$ without using a calculator. Explain your strategy.

46. **WRITING IN MATH** Explain how to approximate the roots of a quadratic equation when the roots are not integers.

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Standardized Test Practice

47. Khalaf earned 50 out of 80 points on a test. What percentage did Khalaf score on the test?

A 62.5% **C** 6.25%
B 16% **D** 1.6%

48. Badr needs to loosen a bolt. He needs a wrench that is smaller than a $\frac{7}{8}$ cm wrench, but larger than a $\frac{3}{4}$ cm wrench. Which of the following sizes should Badr use?

F $\frac{3}{8}$ cm **H** $\frac{13}{16}$ cm
G $\frac{5}{8}$ cm **J** $\frac{15}{16}$ cm

49. EXTENDED RESPONSE Two boats leave a dock. One boat travels 4 km east and then 5 km north. The second boat travels 12 km south and 9 km west. Draw a diagram that represents the paths traveled by the boats.
How far apart are the boats in km?

50. The formula $s = \frac{1}{2}at^2$ represents the distance s in meters that a free-falling object will fall on a planet or moon in a given time t in seconds. Solve the formula for a, the acceleration due to gravity.

A $a = \frac{1}{2}t^2 - s$ **C** $a = s - \frac{1}{2}t^2$
B $a = 2s - t^2$ **D** $a = \frac{2s}{t^2}$

Spiral Review

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function. (Lesson 1-1)

51. $y = 3x^2$ **52.** $y = -4x^2 - 5$ **53.** $y = -x^2 + 4x - 7$
54. $y = x^2 - 6x - 8$ **55.** $y = 3x^2 + 2x + 1$ **56.** $y = -4x^2 - 8x + 5$

Solve each equation. Check the solutions. (Lesson 1-9)

57. $2x^2 = 32$ **58.** $(x - 4)^2 = 25$ **59.** $4x^2 - 4x + 1 = 16$
60. $2x^2 + 16x = -32$ **61.** $(x + 3)^2 = 5$ **62.** $4x^2 - 12x = -9$

Find each sum or difference. (Lesson 1-1)

63. $(3n^2 - 3) + (4 + 4n^2)$ **64.** $(2d^2 - 7d - 3) - (4d^2 + 7)$
65. $(2b^3 - 4b^2 + 4) - (3b^4 + 5b^2 - 9)$ **66.** $(8 - 4h^2 + 6h^4) + (5h^2 - 3 + 2h^3)$

Skills Review

Graph each function.

67. $y = x^2 + 5$ **68.** $y = x^2 - 8$ **69.** $y = 2x^2 - 7$
70. $y = -x^2 + 2$ **71.** $y = -0.5x^2 - 3$ **72.** $y = (-x)^2 + 1$

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EXTEND 1-2

Graphing Technology Lab
Quadratic Inequalities

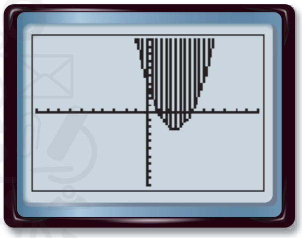
Recall that the graph of a linear inequality consists of the boundary and the shaded half plane. The solution set of the inequality lies in the shaded region of the graph. Graphing quadratic inequalities is similar to graphing linear inequalities.

Activity 1 Shade Inside a Parabola

Graph $y \geq x^2 - 5x + 4$ in a standard viewing window.

First, clear all functions from the **Y=** list.

To graph $y \geq x^2 - 5x + 4$, enter the equation in the **Y=** list. Then use the left arrow to select =. Press [ENTER] until shading above the line is selected.



[−10, 10] scl: 1 by [−10, 10] scl: 1

KEYSTROKES: [◀] [◀] [ENTER] [ENTER] [◀] [▶] [X,T,θ,n] [x^2] [−] 5 [X,T,θ,n] [+] 4 [ZOOM] 6

All ordered pairs for which y is *greater than or equal* to $x^2 - 5x + 4$ lie *above or on* the line and are solutions.

A similar procedure will be used to graph an inequality in which the shading is outside of the parabola.

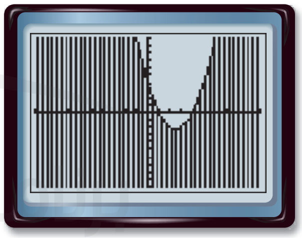
Activity 2 Shade Outside a Parabola

Graph $y - 4 \leq x^2 - 5x$ in a standard viewing window.

First, clear the graph that is displayed.

KEYSTROKES: [Y=] [CLEAR]

Then rewrite $y - 4 \leq x^2 - 5x$ as $y \leq x^2 - 5x + 4$, and graph it.



[−10, 10] scl: 1 by [−10, 10] scl: 1

KEYSTROKES: [◀] [◀] [ENTER] [ENTER] [ENTER] [▶] [▶] [X,T,θ,n] [x^2] [−] 5 [X,T,θ,n] [+] 4 [GRAPH]

All ordered pairs for which y is *less than or equal* to $x^2 - 5x + 4$ lie *below or on* the line and are solutions.

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Exercises

1. Compare and contrast the two graphs shown above.
2. Graph $y - 2x + 6 \geq 5x^2$ in the standard viewing window. Name three solutions of the inequality.
3. Graph $y - 6x \leq -x^2 - 3$ in the standard viewing window. Name three solutions of the inequality.

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LESSON 1-3 Solving Quadratic Equations by Completing the Square

Then

- You solved quadratic equations by using the square root property.

Now

- **1** Complete the square to write perfect square trinomials.
- **2** Solve quadratic equations by completing the square.

Why?

- In competitions, skateboarders may launch themselves from a half pipe into the air to perform tricks. The equation $h = -16t^2 + 20t + 12$ can be used to model their height, in feet, after t seconds.

  To find how long a skateboarder is in the air if he is 25 feet above the half pipe, you can solve $25 = -16t^2 + 20t + 12$ by using a method called completing the square.

NewVocabulary
completing the square

Mathematical Practices
Model with mathematics.

1 Complete the Square

You have previously solved equations by taking the square root of each side. This method worked only because the expression on the left-hand side was a perfect square. In perfect square trinomials in which the leading coefficient is 1, there is a relationship between the **coefficient of the x-term** and the **constant term**.

$$(x + 5)^2 = x^2 + 2(5)(x) + 5^2$$
$$= x^2 + 10x + 25$$

Notice that $\left(\frac{10}{2}\right)^2 = 25$. To get the constant term, divide the coefficient of the x-term by 2 and square the result. Any quadratic expression in the form $x^2 + bx$ can be made into a perfect square by using a method called **completing the square**.

KeyConcept Completing the Square

Words To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below.

Step 1 Find one half of b, the coefficient of x.

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

Symbols $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

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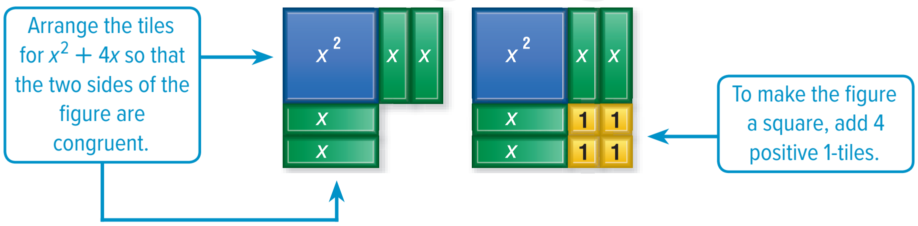
Example 1 Complete the Square

Find the value of c that makes $x^2 + 4x + c$ a perfect square trinomial.

Method 1 Use algebra tiles.



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StudyTip

Algorithms An algorithm is a series of steps for carrying out a procedure or solving a problem.

Method 2 Use complete the square algorithm.

Step 1	Find $\frac{1}{2}$ of 4.	$\frac{4}{2} = 2$
Step 2	Square the result in Step 1.	$2^2 = 4$
Step 3	Add the result of Step 2 to $x^2 + 4x$.	$x^2 + 4x + 4$

Thus, $c = 4$. Notice that $x^2 + 4x + 4 = (x + 2)^2$.

GuidedPractice

1. Find the value of c that makes $r\,2 - 8r + c$ a perfect square trinomial.

2 Solve Equations by Completing the Square

You can complete the square to solve quadratic equations. First, you must isolate the x^2- and bx-terms.

Example 2 Solve an Equation by Completing the Square

Solve $x^2 - 6x + 12 = 19$ by completing the square.

$x^2 - 6x + 12 = 19$	Original equation
$x^2 - 6x = 7$	Subtract 12 from each side.
$x^2 - 6x + 9 = 7 + 9$	Since $\left(\frac{-6}{2}\right)^2 = 9$, add 9 to each side.
$(x - 3)^2 = 16$	Factor $x^2 - 6x + 9$.
$x - 3 = \pm 4$	Take the square root of each side.
$x = 3 \pm 4$	Add 3 to each side.
$x = 3 + 4$ or $x = 3 - 4$	Separate the solutions.
$= 7$ $\quad$ $= -1$	The solutions are 7 and -1.

GuidedPractice

2. Solve $x^2 - 12x + 3 = 8$ by completing the square.

To solve a quadratic equation in which the leading coefficient is not 1, divide each term by the coefficient. Then isolate the x^2- and x-terms and complete the square.

Example 3 Equation with $a \neq 1$

Solve $-2x^2 + 8x - 18 = 0$ by completing the square.

$-2x^2 + 8x - 18 = 0$	Original equation
$\frac{-2x^2 + 8x - 18}{-2} = \frac{0}{-2}$	Divide each side by -2.
$x^2 - 4x + 9 = 0$	Simplify.
$x^2 - 4x = -9$	Subtract 9 from each side.
$x^2 - 4x + 4 = -9 + 4$	Since $\left(\frac{-4}{2}\right)^2 = 4$, add 4 to each side.
$(x - 2)^2 = -5$	Factor $x^2 - 4x + 4$.

No real number has a negative square. So, this equation has no real solutions.

WatchOut!

Leading Coefficient Remember that the leading coefficient has to be 1 before you can complete the square.

GuidedPractice

3. Solve $3x^2 - 9x - 3 = 21$ by completing the square.

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Real-WorldLink

The oldest public high school rivalry takes place between Wellesley High School and Needham Heights High School in Massachusetts. The first football game between them took place in 1882 in football.

Source: USA Mball

Real-World Example 4 Use a Graph of a Quadratic Function

JERSEYS The senior class at a High School buys jerseys to wear to the mball games. The cost of the jerseys can be modeled by the equation $C = 0.1x^2 + 2.4x + 25$, where C is the amount it costs to buy x jerseys. How many jerseys can they purchase for AED 430?

The seniors have AED 430, so set the equation equal to 430 and complete the square.

$0.1x^2 + 2.4x + 25 = 430$	Original equation
$\frac{0.1x^2 + 2.4x + 25}{0.1} = \frac{430}{0.1}$	Divide each side by 0.1.
$x^2 + 24x + 250 = 4300$	Simplify.
$x^2 + 24x + 250 - 250 = 4300 - 250$	Subtract 250 from each side.
$x^2 + 24x = 4050$	Simplify.
$x^2 + 24x + 144 = 4050 + 144$	Since $\left(\frac{24}{2}\right)^2 = 144$, add 144 to each side.
$x^2 + 24x + 144 = 4194$	Simplify.
$(x + 12)^2 = 4194$	Factor $x^2 + 24x + 144$.
$x + 12 = \pm\sqrt{4194}$	Take the square root of each side.
$x = -12 \pm\sqrt{4194}$	Subtract 12 from each side.

Use a calculator to approximate each value of x.

$x = -12 + \sqrt{4194}$ or $x = -12 - \sqrt{4194}$ — Separate the solutions.

≈ 52.8 ≈ -76.8 — Evaluate.

Since you cannot buy a negative number of jerseys, the negative solution is not reasonable. The seniors can afford to buy 52 jerseys.

GuidedPractice

4. If the senior class were able to raise AED 620, how many jerseys could they buy?

Check Your Understanding

Example 1 **Find the value of c that makes each trinomial a perfect square.**

1. $x^2 - 18x + c$ **2.** $x^2 + 22x + c$

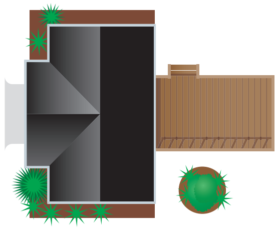
3. $x^2 + 9x + c$ **4.** $x^2 - 7x + c$

Examples 2–3 **Solve each equation by completing the square. Round to the nearest tenth if necessary.**

5. $x^2 + 4x = 6$ **6.** $x^2 - 8x = -9$

7. $4x^2 + 9x - 1 = 0$ **8.** $-2x^2 + 10x + 22 = 4$

Example 4 **9. MODELING** Tarek is building a deck on the back of his family's house. He has enough lumber for the deck to be 144 square meters. The length should be 10 m more than its width. What should the dimensions of the deck be?



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Practice and Problem Solving

Example 1 **Find the value of c that makes each trinomial a perfect square.**

10. $x^2 + 26x + c$ **11.** $x^2 - 24x + c$ **12.** $x^2 - 19x + c$

13. $x^2 + 17x + c$ **14.** $x^2 + 5x + c$ **15.** $x^2 - 13x + c$

16. $x^2 - 22x + c$ **17.** $x^2 - 15x + c$ **18.** $x^2 + 24x + c$

Examples 2–3 **Solve each equation by completing the square. Round to the nearest tenth if necessary.**

19. $x^2 + 6x - 16 = 0$

20. $x^2 - 2x - 14 = 0$

21. $x^2 - 8x - 1 = 8$

22. $x^2 + 3x + 21 = 22$

23. $x^2 - 11x + 3 = 5$

24. $5x^2 - 10x = 23$

25. $2x^2 - 2x + 7 = 5$

26. $3x^2 + 12x + 81 = 15$

27. $4x^2 + 6x = 12$

28. $4x^2 + 5 = 10x$

29. $-2x^2 + 10x = -14$

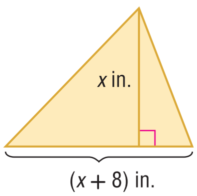
30. $-3x^2 - 12 = 14x$

Example 4

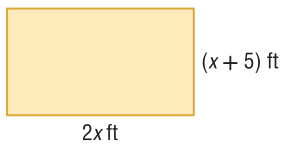
31. **FINANCIAL LITERACY** The price p in dirhams for a particular stock can be modeled by the quadratic equation $p = 3.5t - 0.05t^2$, where t represents the number of days after the stock is purchased. When is the stock worth AED 60?

GEOMETRY **Find the value of x for each figure. Round to the nearest tenth if necessary.**

32. $A = 45 \text{ in}^2$



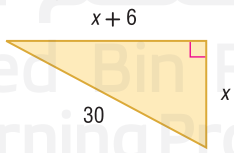
33. $A = 110 \text{ ft}^2$



34. **NUMBER THEORY** The product of two consecutive even integers is 224. Find the integers.

35. **PRECISION** The product of two consecutive negative odd integers is 483. Find the integers.

36. **GEOMETRY** Find the area of the triangle below.



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Solve each equation by completing the square. Round to the nearest tenth if necessary.

37. $0.2x^2 - 0.2x - 0.4 = 0$

38. $0.5x^2 = 2x - 0.3$

39. $2x^2 - \frac{11}{5}x = -\frac{3}{10}$

40. $\frac{2}{3}x^2 - \frac{4}{3}x = \frac{5}{6}$

41. $\frac{1}{4}x^2 + 2x = \frac{3}{8}$

42. $\frac{2}{5}x^2 + 2x = \frac{1}{5}$

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43 **ASTRONOMY** The height of an object t seconds after it is dropped is given by the equation $h = -\frac{1}{2}gt^2 + h_0$, where h_0 is the initial height and g is the acceleration due to gravity. The acceleration due to gravity near the surface of Mars is 3.73 m/s^2, while on Earth it is 9.8 m/s^2. Suppose an object is dropped from an initial height of 120 meters above the surface of each planet.

a. On which planet would the object reach the ground first?

b. How long would it take the object to reach the ground on each planet? Round each answer to the nearest tenth.

c. Do the times that it takes the object to reach the ground seem reasonable? Explain your reasoning.

44. Find all values of c that make $x^2 + cx + 100$ a perfect square trinomial.

45. Find all values of c that make $x^2 + cx + 225$ a perfect square trinomial.

46. **PAINTING** Before she begins painting a picture, Shaima stretches her canvas over a wood frame. The frame has a length of 60 cm and a width of 4 cm. She has enough canvas to cover 480 square cm. Shaima decides to increase the dimensions of the frame. If the increase in the length is 10 times the increase in the width, what will the dimensions of the frame be?

47. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a property of quadratic equations.

a. Tabular Copy the table shown and complete the second column.

b. Algebraic Set each trinomial equal to zero, and solve the equation by completing the square. Complete the last column of the table with the number of roots of each equation.

Trinomial	$b^2 - 4ac$	Number of Roots
$x^2 - 8x + 16$	0	1
$2x^2 - 11x + 3$		
$3x^2 + 6x + 9$		
$x^2 - 2x + 7$		
$x^2 + 10x + 25$		
$x^2 + 3x - 12$		

c. Verbal Compare the number of roots of each equation to the result in the $b^2 - 4ac$ column. Is there a relationship between these values? If so, describe it.

d. Analytical Predict how many solutions $2x^2 - 9x + 15 = 0$ will have. Verify your prediction by solving the equation.

H.O.T. Problems Use Higher-Order Thinking Skills

48. **PERSEVERANCE** Given $y = ax^2 + bx + c$ with $a \neq 0$, derive the equation for the axis of symmetry by completing the square and rewriting the equation in the form $y = a(x - h)^2 + k$.

49. **REASONING** Determine the number of solutions $x^2 + bx = c$ has if $c < -\left(\frac{b}{2}\right)^2$. Explain.

50. **WHICH ONE DOESN'T BELONG?** Identify the expression that does not belong with the other three. Explain your reasoning.

$n^2 - n + \frac{1}{4}$	$n^2 + n + \frac{1}{4}$	$n^2 - \frac{2}{3}n + \frac{1}{9}$	$n^2 + \frac{1}{3}n + \frac{1}{9}$

51. **OPEN ENDED** Write a quadratic equation for which the only solution is 4.

52. **WRITING IN MATH** Compare and contrast the following strategies for solving $x^2 - 5x - 7 = 0$: completing the square, graphing, and factoring.

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Standardized Test Practice

53. The length of a rectangle is 3 times its width. The area of the rectangle is 75 square meters. Find the length of the rectangle in meters.

A 25 **B** 15 **C** 10 **D** 5

54. **PROBABILITY** At a festival, winners of a game draw a token for a prize. There is one token for each prize. The prizes include 9 movie passes, 8 stuffed toys, 5 hats, 10 jump ropes, and 4 glow necklaces. What is the probability that the first person to draw a token will win a movie pass?

F $\frac{1}{36}$ **G** $\frac{1}{9}$ **H** $\frac{9}{61}$ **J** $\frac{1}{4}$

55. **GRIDDED RESPONSE** The population of a town can be modeled by $P = 22{,}000 + 125t$, where P represents the population and t represents the number of years from 2000. How many years after 2000 will the population be 26,000?

56. Abdul Karim delivers pizzas for Pizza King. He is paid AED 6 an hour plus AED 2.50 for each pizza he delivers. Abdul Karim earned AED 280 last week. If he worked a total of 30 hours, how many pizzas did he deliver?

A 250 pizzas
B 184 pizzas
C 40 pizzas
D 34 pizzas

Spiral Review

Describe how the graph of each function is related to the graph of $f(x) = x^2$.
(Lesson 1-3)

57. $g(x) = -12 + x^2$ **58.** $h(x) = (x + 2)^2$ **59.** $g(x) = 2x^2 + 5$

60. $h(x) = \frac{2}{3}(x - 6)^2$ **61.** $g(x) = 6 + \frac{4}{3}x^2$ **62.** $h(x) = -1 - \frac{3}{2}x^2$

63. **RIDES** A popular amusement park ride whisks riders to the top of a 250-foot tower and drops them. A function for the height of a rider is $h = -16t^2 + 250$, where h is the height and t is the time in seconds. The ride stops the descent of the rider 40 feet above the ground. Write an equation that models the drop of the rider. How long does it take to fall from 250 feet to 40 feet? (Lesson 1-2)

Describe how the graph of each function is related to the graph of $f(x) = x^2$. (Lesson 1-3)

64. $g(x) = x^2 - 8$ **65.** $h(x) = \frac{1}{4}x^2$ **66.** $h(x) = -x^2 + 5$

67. $g(x) = (x + 10)^2$ **68.** $g(x) = -2x^2$ **69.** $h(x) = -x^2 - \frac{4}{3}$

Skills Review

Evaluate $\sqrt{b^2 - 4ac}$ for each set of values. Round to the nearest tenth if necessary.

70. $a = 2, b = -5, c = 2$ **71.** $a = 1, b = 12, c = 11$ **72.** $a = -9, b = 10, c = -1$

73. $a = 1, b = 7, c = -3$ **74.** $a = 2, b = -4, c = -6$ **75.** $a = 3, b = 1, c = 2$

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EXTEND 1-3

Algebra Lab
Finding the Maximum or Minimum Value

In Lesson 9-3, we learned about the vertex form of the equation of a quadratic function. You will now learn how to write equations in vertex form and use them to identify key characteristics of the graphs of quadratic functions.

Activity 1 Find a Minimum

Write $y = x^2 + 4x - 10$ in vertex form. Identify the axis of symmetry, extrema, and zeros. Then graph the function.

Step 1 Complete the square to write the function in vertex form.

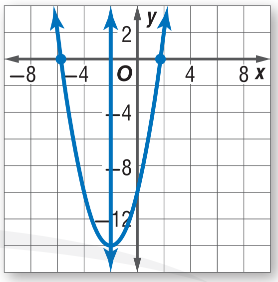
$y = x^2 + 4x - 10$	Original function
$y + 10 = x^2 + 4x$	Add 10 to each side.
$y + 10 + 4 = x^2 + 4x + 4$	Since $\left(\frac{4}{2}\right)^2 = 4$, add 4 to each side.
$y + 14 = (x + 2)^2$	Factor $x^2 + 4x + 4$.
$y = (x + 2)^2 - 14$	Subtract 14 from each side to write in vertex form.

Step 2 Identify the axis of symmetry and extrema based on the equation in vertex form. The vertex is at (h, k) or $(-2, -14)$. Since there is no negative sign before the x^2-term, the parabola opens up and has a minimum at $(-2, -14)$. The equation of the axis of symmetry is $x = -2$.

Step 3 Solve for x to find the zeros.

$(x + 2)^2 - 14 = 0$	Vertex form, $y = 0$
$(x + 2)^2 = 14$	Add 14 to each side.
$x + 2 = \pm\sqrt{14}$	Take square root of each side.
$x \approx -5.74$ or 1.74	Subtract 2 from each side.

The zeros are approximately -5.74 and 1.74.



Step 4 Use the key features to graph the function.

There may be a negative coefficient before the quadratic term. When this is the case, the parabola will open down and have a maximum.

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Activity 2 Find a Maximum

Write $y = -x^2 + 6x - 5$ in vertex form. Identify the axis of symmetry, extrema, and zeros. Then graph the function.

Step 1 Complete the square to write the equation of the function in vertex form.

$y = -x^2 + 6x - 5$	Original function
$y + 5 = -x^2 + 6x$	Add 5 to each side.
$y + 5 = -(x^2 - 6x)$	Factor out −1.
$y + 5 - 9 = -(x^2 - 6x + 9)$	Since $\left(\frac{6}{2}\right)^2 = 9$, add −9 to each side.
$y - 4 = -(x - 3)^2$	Factor $x^2 - 6x + 9$.
$y = -(x - 3)^2 + 4$	Add 4 to each side to write in vertex form.

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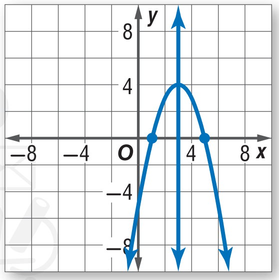
Step 2 Identify the axis of symmetry and extrema based on the equation in vertex form. The vertex is at (h, k) or $(3, 4)$. Since there is a negative sign before the x^2-term, the parabola opens down and has a maximum at $(3, 4)$. The equation of the axis of symmetry is $x = 3$.

Step 3 Solve for x to find the zeros.

$0 = -(x - 3)^2 + 4$	Vertex form, $y = 0$
$(x - 3)^2 = 4$	Add $(x - 3)^2$ to each side.
$x - 3 = \pm 2$	Take the square root of each side.
$x = 5$ or 1	Add 3 to each.

Step 4 Use the key features to graph the function.



Analyze the Results

1. Why do you need to complete the square to write the equation of a quadratic function in vertex form?

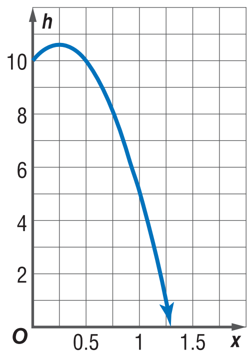
Write each function in vertex form. Identify the axis of symmetry, extrema, and zeros. Then graph the function.

2. $y = x^2 + 6x$
3. $y = x^2 - 8x + 6$
4. $y = x^2 + 2x - 12$
5. $y = x^2 + 6x + 8$
6. $y = x^2 - 4x + 3$
7. $y = x^2 - 2.4x - 2.2$
8. $y = -4x^2 + 16x - 11$
9. $y = 3x^2 - 12x + 5$
10. $y = -x^2 + 6x - 5$

Activity 3 Use Extrema in the Real World

DIVING **Leila jumps from a diving platform upward and outward before diving into the pool. The function $h = -9.8t^2 + 4.9t + 10$, where h is the height of the diver in meters above the pool after t seconds approximates Leila's dive. Graph the function, then find the maximum height that she reaches and the equation of the axis of symmetry.**

Step 1 Graph the function.



Step 2 Complete the square to write the eqution of the function in vertex form.
$h = -9.8t^2 + 4.9t + 10$
$h = -9.8(t - 0.25)^2 + 10.6125$

Step 3 The vertex is at $(0.25, 10.6125)$, so the maximum height is 10.6125 meters. The equation of the axis of symmetry is $x = 0.25$.

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Exercise

11. **SOFTBALL** Maha throws a ball in the air. The function $h = -16t^2 + 40t + 5$, where h is the height in m and t represents the time in seconds, approximates Maha's throw. Graph the function, then find the maximum height of the ball and the equation of the axis of symmetry. When does the ball hit the ground?

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CHAPTER 1

Chapter Quiz I

Lessons 1-1 through 1-3

Use a table of values to graph each equation. State the domain and range. (Lesson 1-1)

1. $y = x^2 + 3x + 1$
2. $y = 2x^2 - 4x + 3$
3. $y = -x^2 - 3x - 3$
4. $y = -3x^2 - x + 1$

Consider $y = x^2 - 5x + 4$. (Lesson 1-1)

5. Write the equation of the axis of symmetry.
6. Find the coordinates of the vertex. Is it a maximum or minimum point?
7. Graph the function.

8. **FOOTBALL** A football is kicked from ground level with an initial upward velocity of 90 meters per second. The equation $h = -16t^2 + 90t$ gives the height h of the ball after t seconds. (Lesson 1-1)

   a. What is the height of the ball after one second?

   b. How many seconds will it take for the ball to reach its maximum height?

   c. When is the height of the ball 0 meter? What do these points represent in this situation?

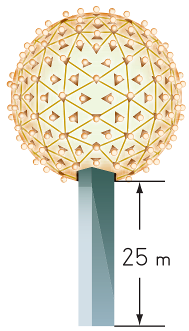
Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth. (Lesson 1-2)

9. $x^2 + 5x + 6 = 0$
10. $x^2 + 8 = -6x$
11. $-x^2 + 3x - 1 = 0$
12. $x^2 = 12$

13. **BASEBALL** Jamal hits a baseball. The equation $h = -16t^2 + 120t$ models the height h, in meters, of the ball after t seconds. How long is the ball in the air? (Lesson 1-2)

14. **CONSTRUCTION** Kareem is repairing the roof on a shed. He accidentally dropped a box of nails from a height of 14 m. This is represented by the equation $h = -16t^2 + 14$, where h is the height in meters and t is the time in seconds. Describe how the graph is related to $h = t^2$. (Lesson 1-3)

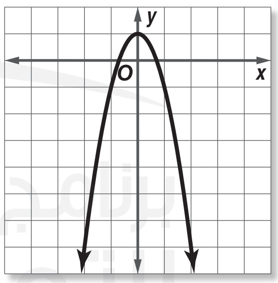
15. **PARTIES** Abir's parents are throwing a graduation party for her. At 10:00, a ball will slide 25 m down a pole and light up. A function that models the drop is $h = -t^2 + 5t + 25$, where h is height in meters of the ball after t seconds. How many seconds will it take for the ball to reach the bottom of the pole? (Lesson 1-2)



Describe how the graph of each function is related to the graph of $f(x) = x^2$. (Lesson 1-3)

16. $g(x) = x^2 + 3$
17. $h(x) = 2x^2$
18. $g(x) = x^2 - 6$
19. $h(x) = \frac{1}{5}x^2$
20. $g(x) = -x^2 + 1$
21. $h(x) = -\frac{5}{8}x^2$

22. **MULTIPLE CHOICE** Which is an equation for the function shown in the graph? (Lesson 1-2)



   A $y = -2x^2$

   B $y = 2x^2 + 1$

   C $y = x^2 - 1$

   D $y = -2x^2 + 1$

Solve each equation by completing the square. Round to the nearest tenth. (Lesson 1-3)

23. $x^2 + 4x + 2 = 0$
24. $x^2 - 2x - 10 = 0$
25. $2x^2 + 4x - 5 = 7$

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EXTEND 1-4

Graphing Technology Lab
Modeling Real-World Data

You can use a TI-83/84 Plus graphing calculator to model data points for which a curve of best fit is a quadratic function.

WATER A bottle is filled with water. The water is allowed to drain from a hole made near the bottom of the bottle. The table shows the level of the water *y* measured in centimeters from the bottom of the bottle after *x* seconds.

Time (s)	0	20	40	60	80	100	120	140	160	180	200	220
Water level (cm)	42.6	40.7	38.9	37.2	35.8	34.3	33.3	32.3	31.5	30.8	30.4	30.1

Find and graph a linear regression equation and a quadratic regression equation. Determine which equation is a better fit for the data.

Activity

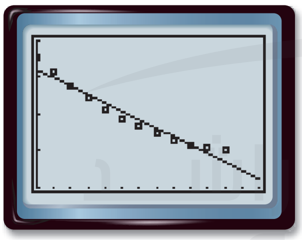
Step 1 **Find and graph a linear regression equation.**

- Enter the times in **L1** and the water levels in **L2**. Then find a linear regression equation.

  KEYSTROKES: *Refer to Lesson 2-5.*

- Use **STAT PLOT** to graph a scatter plot. Copy the equation to the **Y=** list and graph.

  KEYSTROKES: *Review statistical plots and graphing a regression equation in Lesson 2-5.*



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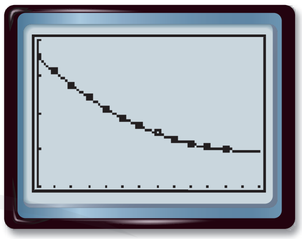
Step 2 **Find and graph a quadratic regression equation.**

- Find the quadratic regression equation. Then copy the equation to the **Y=** list and graph.

  KEYSTROKES: STAT ▶ 5 ENTER Y= VARS 5 ▶ ▶ ENTER GRAPH



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Notice that the graph of the linear regression equation appears to pass through just two data points. However, the graph of the quadratic regression equation fits the data very well.

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Exercises

Refer to the table.

Height of Player's Feet above Floor

Time (s)	Height (cm)
0.1	3.04
0.2	5.76
0.3	8.16
0.4	10.24
0.5	12
0.6	13.44
0.7	14.56

1. Find and graph a linear regression equation and a quadratic regression equation for the data. Determine which equation is a better fit for the data.
2. Estimate the height of the player's feet after 1 second and 1.5 seconds. Use mental math to check the reasonableness of your estimates.
3. Compare and contrast the estimates you found in Exercise 2.
4. How might choosing a regression equation that does not fit the data well affect predictions made by using the equation?

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LESSON 1-4 Solving Quadratic Equations by Factoring

Then

- You found the greatest common factors of sets of numbers.

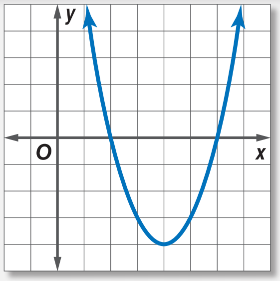
Now

1. Write quadratic equations in standard form.
2. Solve quadratic equations by factoring.

Why?

- The **factored form** of a quadratic equation is $0 = a(x - p)(x - q)$. In the equation, p and q represent the x-intercepts of the graph of the equation.

The x-intercepts of the graph at the right are 2 and 6. In this lesson, you will learn how to change a quadratic equation in factored form into standard form and vice versa.



Standard Form	Factored Form
$0 = x^2 - 8x + 12$	$0 = (x - 6)(x - 2)$

Factors

Related Graph
2 and 6 are
x-intercepts.

NewVocabulary
factored form
FOIL method

Mathematical Practices
Reason abstractly and quantitatively.

1 Standard Form

You can use the FOIL method to write a quadratic equation that is in factored form in standard form. The **FOIL method** uses the Distributive Property to multiply binomials.

KeyConcept FOIL Method for Multiplying Binomials

Words To multiply two binomials, find the sum of the products of ***F*** the *First terms*, ***O*** the *Outer terms*, ***I*** the *Inner terms*, and ***L*** the *Last terms*.

Examples

		Product of First Terms		Product of Outer Terms		Product of Inner Terms		Product of Last Terms
F O I L: $(x - 6)(x - 2)$	$=$	$(x)(x)$	$+$	$(x)(-2)$	$+$	$(-6)(x)$	$+$	$(-6)(-2)$

$= x^2 - 2x - 6x + 12$ or $x^2 - 8x + 12$

Example 1 Translate Sentences into Equations

Write a quadratic equation in standard form with $-\frac{1}{3}$ and 6 as its roots.

$(x - p)(x - q) = 0$ Write the pattern.

$\left[x - \left(-\frac{1}{3}\right)\right](x - 6) = 0$ Replace p with $-\frac{1}{3}$ and q with 6.

$\left(x + \frac{1}{3}\right)(x - 6) = 0$ Simplify.

$x^2 - \frac{17}{3}x - 2 = 0$ Multiply.

$3x^2 - 17x - 6 = 0$ Multiply each side by 3 so that b and c are integers.

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1. Write a quadratic equation in standard form with $\frac{3}{4}$ and -5 as its roots.

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2 Solve Equations by Factoring

Solving quadratic equations by factoring is an application of the Zero Product Property.

KeyConcept Zero Product Property

Words	For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal zero.
Example	If $(x + 3)(x - 5) = 0$, then $x + 3 = 0$ or $x - 5 = 0$.

Example 2 Factor the GCF

Solve $16x^2 + 8x = 0$.

$16x^2 + 8x = 0$ — Original equation.

$8x(2x) + 8x^2(1) = 0$ — Factor the GCF.

$8x(2x + 1) = 0$ — Distributive Property

$8x = 0$ or $2x + 1 = 0$ — Zero Product Property

$x = 0 \qquad 2x = -1$ — Solve both equations.

$x = -\frac{1}{2}$

GuidedPractice Solve each equation.

2A. $20x^2 + 15x = 0$ **2B.** $4y^2 + 16y = 0$ **2C.** $6a^5 + 18a^4 = 0$

ReviewVocabulary

perfect square a number with a positive square root that is a whole number

Trinomials and binomials that are perfect squares have special factoring rules. In order to use these rules, the first and last terms need to be perfect squares and the middle term needs to be twice the product of the square roots of the first and last terms.

Example 3 Perfect Squares and Differences of Squares

Solve each equation.

a. $x^2 + 16x + 64 = 0$

$x^2 = (x)^2$; $64 = (8)^2$ — First and last terms are perfect squares.

$16x = 2(x)(8)$ — Middle term equals $2ab$.

$x^2 + 16x + 64$ is a perfect square trinomial.

$x^2 + 16x + 64 = 0$ — Original equation

$(x + 8)^2 = 0$ — Factor using the pattern.

$x + 8 = 0$ — Take the square root of each side.

$x = -8$ — Solve.

b. $x^2 = 64$

$x^2 = 64$ — Original equation

$x^2 - 64 = 0$ — Subtract 64 from each side.

$x^2 - (8)^2 = 0$ — Write in the form $a^2 - b^2$.

$(x + 8)(x - 8) = 0$ — Factor the difference of squares.

$x + 8 = 0$ or $x - 8 = 0$ — Zero Product Property

$x = -8 \qquad x = 8$ — Solve.

StudyTip

Square Roots By inspection, notice that the square root of 64 is −8 and 8. Also, for $x^2 = 4$, the solutions would be −2 and 2.

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3A. $4x^2 - 12x + 9 = 0$ **3B.** $81x^2 - 9x = 0$ **3C.** $6a^2 - 3a = 0$

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StudyTip

Structure If values for m and p exist, then the trinomial can always be factored.

A special pattern is used when factoring trinomials of the form $ax^2 + bx + c$. First, multiply the values of a and c. Then, find two values, m and p, such that their product equals ac and their sum equals b.

Consider $6x^2 + 13x - 5$: $ac = 6(-5) = -30$.

Factors of −30	Sum	Factors of −30	Sum
1, −30	−29	−1, 30	29
2, −15	−13	**−2, 15**	**13**
3, −10	−7	−3, 10	7
5, −6	−1	−5, 6	1

Now the middle term, $13x$, can be rewritten as $-2x + 15x$.

This polynomial can now be factored by grouping.

$6x^2 + 13x - 5 = 6x^2 + mx + px - 5$ Write the pattern.
$= 6x^2 - 2x + 15x - 5$ $m = -2$ and $p = 15$
$= (6x^2 - 2x) + (15x - 5)$ Group terms.
$= 2x(3x - 1) + 5(3x - 1)$ Factor the GCF.
$= (2x + 5)(3x - 1)$ Distributive Property

Example 4 Factor Trinomials

Solve each equation.

a. $x^2 + 9x + 20 = 0$

$ac = 20$ $a = 1, c = 20$

Factors of 20	Sum	Factors of 20	Sum
1, 20	21	−1, −20	−21
2, 10	12	−2, −10	−12
4, 5	**9**	−4, −5	−9

$x^2 + 9x + 20 = 0$ Original expression
$x^2 + mx + px + 20 = 0$ Write the pattern.
$x^2 + 4x + 5x + 20 = 0$ $m = 4$ and $p = 5$
$(x^2 + 4x) + (5x + 20) = 0$ Group terms with common factors.
$x(x + 4) + 5(x + 4) = 0$ Factor the GCF from each grouping.
$(x + 5)(x + 4) = 0$ Distributive Property
$x + 5 = 0$ or $x + 4 = 0$ Zero Product Property
$x = -5$ $x = -4$ Solve each equation.

StudyTip

Trinomials It does not matter if the values of m and p are switched when grouping.

b. $6y^2 - 23y + 20 = 0$

$ac = 120$ $a = 6, c = 20$
$m = -8, p = -15$ $-8(-15) = 120; -8 + (-15) = -23$
$6y^2 - 23y + 20 = 0$ Original equation
$6y^2 + my + py + 20 = 0$ Write the pattern.
$6y^2 - 8y - 15y + 20 = 0$ $m = -8$ and $p = -15$
$(6y^2 - 8y) + (-15y + 20) = 0$ Group terms with common factors.
$2y(3y - 4) - 5(3y - 4) = 0$ Factor the GCF from each grouping.
$(2y - 5)(3y - 4) = 0$ Distributive Property
$2y - 5 = 0$ or $3y - 4 = 0$ Zero Product Property
$2y = 5$ $3y = 4$ Solve both equations.
$y = \frac{5}{2}$ $y = \frac{4}{3}$

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4A. $x^2 - 11x + 30 = 0$

4B. $x^2 - 4x - 21 = 0$

4C. $15x^2 - 8x + 1 = 0$

4D. $-12x^2 + 8x + 15 = 0$

Real-WorldLink

Cuba's Osleidys Menendez broke the javelin world record in 2002 with a distance of 71.52 meters.

Source: *New York Times*

Real-World Example 5 Solve Equations by Factoring

TRACK AND FIELD **The height of a javelin in feet is modeled by $h(t) = -16t^2 + 79t + 5$, where t is the time in seconds after the javelin is thrown. How long is it in the air?**

To determine how long the javelin is in the air, we need to find when the height equals 0. We can do this by solving $-16t^2 + 79t + 5 = 0$.

$-16t^2 + 79t + 5 = 0$	Original equation
$m = 80;\ p = -1$	$-16(5) = -80,\ 80 \cdot (-1) = -80,\ 80 + (-1) = 79$
$-16t^2 + 80t - t + 5 = 0$	Write the pattern.
$(-16t^2 + 80t) + (-t + 5) = 0$	Group terms with common factors.
$16t(-t + 5) + 1(-t + 5) = 0$	Factor GCF from each group.
$(16t + 1)(-t + 5) = 0$	Distributive Property
$16t + 1 = 0$ or $-t + 5 = 0$	Zero Product Property
$16t = -1$ $\quad -t = -5$	Solve both equations.
$t = -\frac{1}{16}$ $\quad t = 5$	Solve.

CHECK We have two solutions.

- The first solution is negative and since time cannot be negative, this solution can be eliminated.
- The second solution of 5 seconds seems reasonable for the time a javelin spends in the air.
- The answer can be confirmed by substituting back into the original equation.

$$-16t^2 + 79t + 5 = 0$$
$$-16(5)^2 + 79(5) + 5 \stackrel{?}{=} 0$$
$$-400 + 395 + 5 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

The javelin is in the air for 5 seconds.

GuidedPractice

5. BUNGEE JUMPING Jamal recorded his brother bungee jumping from a height of 300 feet. At the time the cord lifted his brother back up, he was 44 feet above the ground. If Jamal started recording as soon as his brother fell, how much time elapsed when the cord snapped back? Use $f(t) = -16t^2 + c$, where c is the height in feet.

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Check Your Understanding

Example 1 Write a quadratic equation in standard form with the given root(s).

1. $-8, 5$
2. $\frac{3}{2}, \frac{1}{4}$
3. $-\frac{2}{3}, \frac{5}{2}$

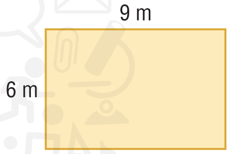
Examples 2–4 Factor each polynomial.

4. $35x^2 - 15x$
5. $18x^2 - 3x + 24x - 4$
6. $x^2 - 12x + 32$
7. $x^2 - 4x - 21$
8. $2x^2 + 7x - 30$
9. $16x^2 - 16x + 3$

Example 5 Solve each equation.

10. $x^2 - 36 = 0$
11. $12x^2 - 18x = 0$
12. $12x^2 - 2x - 2 = 0$
13. $x^2 - 9x = 0$
14. $x^2 - 3x - 28 = 0$
15. $2x^2 - 24x = -72$

16. **SENSE-MAKING** Huriah wants to double the area of her garden by increasing the length and width by the same amount. What will be the dimensions of her garden then?



Practice and Problem Solving

Example 1 Write a quadratic equation in standard form with the given root(s).

17. 7
18. $-5, \frac{1}{2}$
19. $\frac{1}{5}, 6$

Examples 2–4 Factor each polynomial.

20. $40a^2 - 32a$
21. $51c^3 - 34c$
22. $32xy + 40bx - 12ay - 15ab$
23. $3x^2 - 12$
24. $15y^2 - 240$
25. $48cg + 36cf - 4dg - 3df$
26. $x^2 + 13x + 40$
27. $x^2 - 9x - 22$
28. $3x^2 + 12x - 36$
29. $15x^2 + 7x - 2$
30. $4x^2 + 29x + 30$
31. $18x^2 + 15x - 12$
32. $8x^2z^2 - 4xz^2 - 12z^2$
33. $9x^2 - 25$
34. $18x^2y^2 - 24xy^2 + 36y^2$

Example 3 Solve each equation.

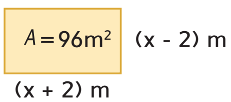
35. $15x^2 - 84x - 36 = 0$
36. $12x^2 + 13x - 14 = 0$
37. $12x^2 - 108x = 0$
38. $x^2 + 4x - 45 = 0$
39. $x^2 - 5x - 24 = 0$
40. $x^2 = 121$
41. $x^2 + 13 = 17$
42. $-3x^2 - 10x + 8 = 0$
43. $-8x^2 + 46x - 30 = 0$

44. **GEOMETRY** The hypotenuse of a right triangle is 1 centimeter longer than one side and 4 centimeters longer than three times the other side. Find the dimensions of the triangle.

45. **NUMBER THEORY** Find two consecutive even integers with a product of 624.

GEOMETRY Find x and the dimensions of each rectangle.

46.



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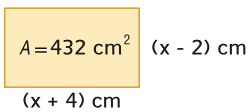

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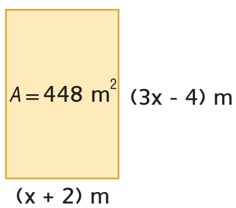

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Solve each equation by factoring.

49. $12x^2 - 4x = 5$

50. $5x^2 = 15x$

51. $16x^2 + 36 = -48x$

52. $75x^2 - 60x = -12$

53. $4x^2 - 144 = 0$

54. $-7x + 6 = 20x^2$

55. **MOVIE THEATER** A company plans to build a large multiplex theater. The financial analyst told her manager that the profit function for their theater was $P(x) = -x^2 + 48x - 512$, where x is the number of movie screens, and $P(x)$ is the profit earned in thousands of dirhams. Determine the range of production of movie screens that will guarantee that the company will not lose money.

Write a quadratic equation in standard form with the given root(s).

56. $-\frac{4}{7}, \frac{3}{8}$

57. 3.4, 0.6

58. $\frac{2}{11}, \frac{5}{9}$

Solve each equation by factoring.

59. $10x^2 + 25x = 15$

60. $27x^2 + 5 = 48x$

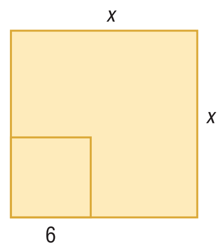
61. $x^2 + 0.25x = 1.25$

62. $48x^2 - 15 = -22x$

63. $3x^2 + 2x = 3.75$

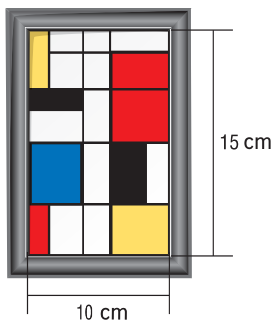
64. $-32x^2 + 56x = 12$

65. **DESIGN** A square is cut out of the figure at the right. Write an expression for the area of the figure that remains, and then factor the expression.



66. **PERSEVERANCE** After analyzing the market, a company that sells Web sites determined the profitability of their product was modeled by $P(x) = -16x^2 + 368x - 2035$, where x is the price of each Web site and $P(x)$ is the company's profit. Determine the price range of the Web sites that will be profitable for the company.

67. **PAINTINGS** Asma wants to add a border to her painting, distributed evenly, that has the same area as the painting itself. What are the dimensions of the painting with the border included?



68. **MULTIPLE REPRESENTATIONS** In this problem, you will consider $a(x - p)(x - q) = 0$.

a. Graphical Graph the related function for $a = 1$, $p = 2$, and $q = -3$.

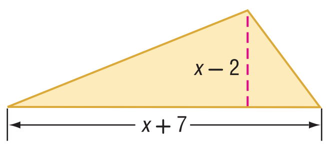
b. Analytical What are the solutions of the equation?

c. Graphical Graph the related functions for $a = 4, -3$, and $\frac{1}{2}$ on the same graph.

d. Verbal What are the similarities and differences between the graphs?

e. Verbal What conclusion can you make about the relationship between the factored form of a quadratic equation and its solutions?

69. **GEOMETRY** The area of the triangle is 26 square centimeters. Find the length of the base.



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70. **SOCCER** When a ball is kicked in the air, its height in meters above the ground can be modeled by $h(t) = -4.9t^2 + 14.7t$ and the distance it travels can be modeled by $d(t) = 16t$, where t is the time in seconds.

a. How long is the ball in the air?

b. How far does it travel before it hits the ground? (*Hint*: Ignore air resistance.)

c. What is the maximum height of the ball?

Factor each polynomial.

71. $18a - 24ay + 48b - 64by$

72. $3x^2 + 2xy + 10y + 15x$

73. $6a^2b^2 - 12ab^2 - 18b^3$

74. $12a^2 - 18ab + 30ab^3$

75. $32ax + 12bx - 48ay - 18by$

76. $30ac + 80bd + 40ad + 60bc$

77. $5ax^2 - 2by^2 - 5ay^2 + 2bx^2$

78. $12c^2x + 4d^2y - 3d^2x - 16c^2y$

H.O.T. Problems Use Higher-Order Thinking Skills

79. **ERROR ANALYSIS** Huriah and Khadijah are solving $-12x^2 + 5x + 2 = 0$. Is either of them correct? Explain your reasoning.

Huriah

$$-12x^2 + 5x + 2 = 0$$
$$-12x^2 + 8x - 3x + 2 = 0$$
$$4x(-3x + 2) - (3x + 2) = 0$$
$$(4x - 1)(3x + 2) = 0$$
$$x = \frac{1}{4} \text{ or } -\frac{2}{3}$$

Khadijah

$$-12x^2 + 5x + 2 = 0$$
$$-12x^2 + 8x - 3x + 2 = 0$$
$$4x(-3x + 2) + (-3x + 2) = 0$$
$$(4x + 1)(-3x + 2) = 0$$
$$x = -\frac{1}{4} \text{ or } \frac{2}{3}$$

80. **CHALLENGE** Solve $3x^6 - 39x^4 + 108x^2 = 0$ by factoring.

81. **CHALLENGE** The rule for factoring a difference of cubes is shown below. Use this rule to factor $40x^5 - 135x^2y^3$.

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

82. **OPEN ENDED** Choose two integers. Then write an equation in standard form with those roots. How would the equation change if the signs of the two roots were switched?

83. **CHALLENGE** For a quadratic equation of the form $(x - p)(x - q) = 0$, show that the axis of symmetry of the related quadratic function is located halfway between the x-intercepts p and q.

84. **WRITE A QUESTION** A classmate is using the guess-and-check strategy to factor trinomials of the form $x^2 + bx + c$. Write a question to help him think of a way to use that strategy for $ax^2 + bx + c$.

85. **ARGUMENTS** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

In a quadratic equation in standard form where a, b, and c are integers, if b is odd, then the quadratic cannot be a perfect square trinomial.

86. **WRITING IN MATH** Explain how to factor a trinomial in standard form with $a > 1$.

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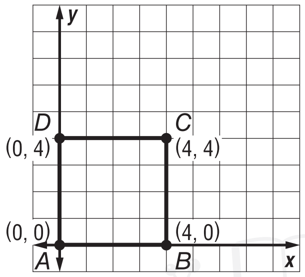
Standardized Test Practice

87. SHORT RESPONSE If *ABCD* is transformed by $(x, y) \rightarrow (3x, 4y)$, determine the area of $A'B'C'D'$.



88. For $y = 2|6 - 3x| + 4$, which set describes x when $y < 6$?

A $\left\{x \,\middle|\, \frac{5}{3} < x < \frac{7}{3}\right\}$

B $\left\{x \,\middle|\, x < \frac{5}{3} \text{ or } x > \frac{7}{3}\right\}$

C $\left\{x \,\middle|\, x < \frac{5}{3}\right\}$

D $\left\{x \,\middle|\, x > \frac{7}{3}\right\}$

89. PROBABILITY A 5-character password can contain the numbers 0 through 9 and 26 letters of the alphabet. None of the characters can be repeated. What is the probability that the password begins with a consonant?

F $\frac{21}{26}$

G $\frac{21}{35}$

H $\frac{21}{36}$

J $\frac{5}{36}$

90. SAT/ACT If $c = \frac{8a^3}{b}$, what happens to the value of c when both a and b are doubled?

A c is unchanged.

B c is halved.

C c is doubled.

D c is multiplied by 4.

E c is multiplied by 8.

Spiral Review

Graph each function. (Lesson 1-7)

91. $f(x) = |3x + 2|$

92. $f(x) = \begin{cases} x - 2 \text{ if } x > -1 \\ x + 3 \text{ if } x \leq -1 \end{cases}$

93. $f(x) = [\![x + 1]\!]$

94. $f(x) = \left|\frac{1}{4}x - 1\right|$

Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function. (Lesson 1-6)

95. $\{(-2, 5), (-1, 3), (0, 1), (1, -1), (2, -3)\}$

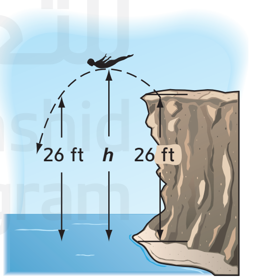
96. $\{(0, 0), (1, 3), (2, 4), (3, 3), (4, 0)\}$

97. $\left\{\left(-2, \frac{1}{4}\right), (0, 1), (1, 2), (2, 4), (3, 8)\right\}$

98. $\{(-3, 1), (-2, -5), (-1, -7), (0, -5), (1, 1)\}$

99. FINANCIAL LITERACY Determine the amount of an investment if AED 250 is invested at an interest rate of 7.3% compounded quarterly for 40 years. (Lesson 1-3)

100. DIVING To avoid hitting any rocks below, a cliff diver jumps up and out. The equation $h = -16t^2 + 4t + 26$ describes her height h in feet t seconds after jumping. Find the time at which she returns to a height of 26 feet. (1-2)



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Skills Review

Simplify.

101. $\sqrt{5} \cdot \sqrt{15}$

102. $\sqrt{8} \cdot \sqrt{32}$

103. $2\sqrt{3} \cdot \sqrt{27}$

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LESSON 1-5

Complex Numbers

Then

- You simplified square roots.

Now

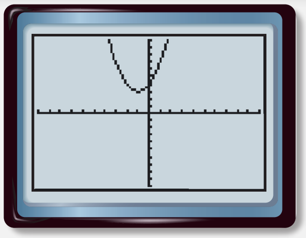
1. Perform operations with pure imaginary numbers.
2. Perform operations with complex numbers.

Why?

Consider the graph of $y = x^2 + 2x + 4$ at the right. Notice how this graph has no x-intercepts and therefore does not have any roots. Does this mean there are no solutions to $0 = x^2 + 2x + 4$?

Use the **Solver** function located in the MATH menu of a graphing calculator. Enter the equation and select $x = 2$ as your guess to a solution.

Press [ALPHA] [ENTER] and the calculator will attempt to solve the equation. The calculator indicates there is no solution with the error message. So there are no real solutions. However, there are imaginary solutions.



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NewVocabulary

imaginary unit
pure imaginary number
complex number
complex conjugates

Mathematical Practices
Attend to precision.

1 Pure Imaginary Numbers

In your math studies so far, you have worked with real numbers. Equations like the one above led mathematicians to define imaginary numbers. The **imaginary unit i** is defined to be $i^2 = -1$. The number i is the principal square root of -1; that is, $i = \sqrt{-1}$.

Numbers of the form $6i$, $-2i$, and $i\sqrt{3}$ are called **pure imaginary numbers**. Pure imaginary numbers are square roots of negative real numbers. For any positive real number b, $\sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1}$ or bi.

Example 1 Square Roots of Negative Numbers

Simplify.

a. $\sqrt{-27}$

$$\sqrt{-27} = \sqrt{-1 \cdot 3^2 \cdot 3}$$
$$= \sqrt{-1} \cdot \sqrt{3^2} \cdot \sqrt{3}$$
$$= i \cdot 3 \cdot \sqrt{3} \text{ or } 3i\sqrt{3}$$

b. $\sqrt{-216}$

$$\sqrt{-216} = \sqrt{-1 \cdot 6^2 \cdot 6}$$
$$= \sqrt{-1} \cdot \sqrt{6^2 \cdot 6}$$
$$= i \cdot 6 \cdot \sqrt{6} \text{ or } 6i\sqrt{6}$$

GuidedPractice

1A. $\sqrt{-18}$

1B. $\sqrt{-125}$

The Commutative and Associative Properties of Multiplication hold true for pure imaginary numbers. The first few powers of i are shown below.

$i^1 = i$	$i^2 = -1$	$i^3 = i^2 \cdot i$ or $-i$	$i^4 = (i^2)^2$ or 1
$i^5 = i^4 \cdot i$ or i	$i^6 = i^4 \cdot i^2$ or -1	$i^7 = i^4 \cdot i^3$ or $-i$	$i^8 = (i^2)^4$ or 1

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Example 2 Products of Pure Imaginary Numbers

Simplify.

a. $-5i \cdot 3i$

$$-5i \cdot 3i = -15i^2 \quad \text{Multiply.}$$
$$= -15(-1) \quad i^2 = -1$$
$$= 15 \quad \text{Simplify.}$$

b. $\sqrt{-6} \cdot \sqrt{-15}$

$$\sqrt{-6} \cdot \sqrt{-15} = i\sqrt{6} \cdot i\sqrt{15} \quad i = \sqrt{-1}$$
$$= i^2\sqrt{90} \quad \text{Multiply.}$$
$$= -1 \cdot \sqrt{9} \cdot \sqrt{10} \quad \text{Simplify.}$$
$$= -3\sqrt{10} \quad \text{Multiply.}$$

GuidedPractice

2A. $3i \cdot 4i$ **2B.** $\sqrt{-20} \cdot \sqrt{-12}$ **2C.** i^{31}

You can solve some quadratic equations by using the **Square Root Property**. Similar to a difference of squares, the sum of squares can be factored over the complex numbers.

Real-WorldCareer

Electrical Engineer
Electrical engineers design, develop, test, and supervise the making of electrical equipment such as digital music players, electric motors, lighting, and radar and navigation systems. A bachelor's degree in engineering is required for almost all entry-level engineering jobs.

Example 3 Equation with Pure Imaginary Solutions

Solve $x^2 + 64 = 0$.

Method 1 Square Root Property

$$x^2 + 64 = 0$$
$$x^2 = -64$$
$$x = \pm\sqrt{-64}$$
$$x = \pm 8i$$

Method 2 Factoring

$$x^2 + 64 = 0$$
$$x^2 + 8^2 = 0$$
$$x^2 - (-8^2) = 0$$
$$(x + 8i)(x - 8i) = 0$$
$$(x + 8i) = 0 \text{ or } (x - 8i) = 0$$
$$x = -8i \qquad x = 8i$$

GuidedPractice

Solve each equation.

3A. $4x^2 + 100 = 0$ **3B.** $x^2 + 4 = 0$

2 Operations with Complex Numbers

Consider $2 + 3i$. Since 2 is a real number and $3i$ is a pure imaginary number, the terms are not like terms and cannot be combined. This type of expression is called a **complex number**.

KeyConcept Complex Numbers

Words	A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit. a is called the real part, and b is called the imaginary part.
Examples	$5 + 2i$ $\qquad$ $1 - 3i = 1 + (-3)i$

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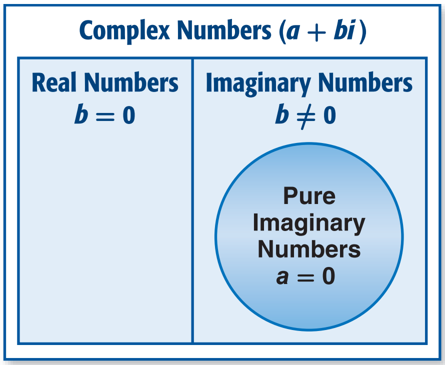
The Venn diagram shows the set of complex numbers.

- If $b = 0$, the complex number is a real number.
- If $b \neq 0$, the complex number is imaginary.
- If $a = 0$, the complex number is a pure imaginary number.



Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. That is, $a + bi = c + di$ if and only if $a = c$ and $b = d$.

StudyTip

Complex Numbers Whereas all real numbers are also complex, the term *complex number* usually refers to a number that is not real.

Example 4 Equate Complex Numbers

Find the values of x and y that make $3x - 5 + (y - 3)i = 7 + 6i$ true.

Set the real parts equal to each other and the imaginary parts equal to each other.

$3x - 5 = 7$	Real parts	$y - 3 = 6$	Imaginary parts
$3x = 12$	Add 5 to each side.	$y = 9$	Add 3 to each side.
$x = 4$	Divide each side by 3.		

GuidedPractice

4. Find the values of x and y that make $5x + 1 + (3 + 2y)i = 2x - 2 + (y - 6)i$ true.

The Commutative, Associative, and Distributive Properties of Multiplication and Addition hold true for complex numbers. To add or subtract complex numbers, combine like terms. That is, combine the real parts, and combine the imaginary parts.

Example 5 Add and Subtract Complex Numbers

Simplify.

a. $(5 - 7i) + (2 + 4i)$

$(5 - 7i) + (2 + 4i) = (5 + 2) + (-7 + 4)i$ — Commutative and Associative Properties

$= 7 - 3i$ — Simplify.

b. $(4 - 8i) - (3 - 6i)$

$(4 - 8i) - (3 - 6i) = (4 - 3) + [-8 - (-6)]i$ — Commutative and Associative Properties

$= 1 - 2i$ — Simplify.

GuidedPractice

5A. $(-2 + 5i) + (1 - 7i)$

5B. $(4 + 6i) - (-1 + 2i)$

StudyTip

Reading Math Electrical engineers use j as the imaginary unit to avoid confusion with the i for current.

Complex numbers are used with electricity. In these problems, j usually represents the imaginary unit. In a circuit with alternating current, the voltage, current, and impedance, or hindrance to current, can be represented by complex numbers. To multiply these numbers, use the FOIL method.

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Real-World Link

An example of a series circuit is a string of holiday lights. The number of bulbs on a circuit affects the strength of the current, which in turn affects the brightness of the lights.

Source: *Popular Science*

Real-World Example 6 Multiply Complex Numbers

ELECTRICITY **In an AC circuit, the voltage V, current C, and impedance I are related by the formula $V = C \cdot I$. Find the voltage in a circuit with current $2 + 4j$ amps and impedance $9 - 3j$ ohms.**

$V = C \cdot I$	Electricity formula
$= (2 + 4j) \cdot (9 - 3j)$	$C = 2 + 4j$ and $I = 9 - 3j$
$= 2(9) + 2(-3j) + 4j(9) + 4j(-3j)$	FOIL Method
$= 18 - 6j + 36j - 12j^2$	Multiply.
$= 18 + 30j - 12(-1)$	$j^2 = -1$
$= 30 + 30j$	Add.

The voltage is $30 + 30j$ volts.

GuidedPractice

6. Find the voltage in a circuit with current $2 - 4j$ amps and impedance $3 - 2j$ ohms.

Two complex numbers of the form $a + bi$ and $a - bi$ are called **complex conjugates**. The product of complex conjugates is always a real number. You can use this fact to simplify the quotient of two complex numbers.

Example 7 Divide Complex Numbers

Simplify.

a. $\dfrac{2i}{3 + 6i}$

$\dfrac{2i}{3 + 6i} = \dfrac{2i}{3 + 6i} \cdot \dfrac{3 - 6i}{3 - 6i}$	$3 + 6i$ and $3 - 6i$ are complex conjugates.
$= \dfrac{6i - 12i^2}{9 - 36i^2}$	Multiply.
$= \dfrac{6i - 12(-1)}{9 - 36(-1)}$	$i^2 = -1$
$= \dfrac{6i + 12}{45}$	Simplify.
$= \dfrac{4}{15} + \dfrac{2}{15}i$	$a + bi$ form

b. $\dfrac{4 + i}{5i}$

$\dfrac{4 + i}{5i} = \dfrac{4 + i}{5i} \cdot \dfrac{i}{i}$	Multiply by $\frac{i}{i}$.
$= \dfrac{4i + i^2}{5i^2}$	Multiply.
$= \dfrac{4i - 1}{-5}$	$i^2 = -1$
$= \dfrac{1}{5} - \dfrac{4}{5}i$	$a + bi$ form

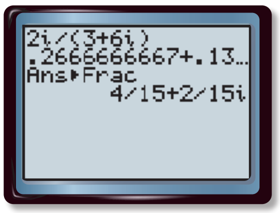
GuidedPractice

7A. $\dfrac{-2i}{3 + 5i}$

7B. $\dfrac{2 + i}{1 - i}$

StudyTip

Technology Operations with complex numbers can be preformed with a TI-83/84 Plus graphing calculator. Use the [2nd] [i] function to enter the expression. Then press [MATH] [ENTER] [ENTER] to view the answer.



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Check Your Understanding

Examples 1–2 Simplify.

1. $\sqrt{-81}$
2. $\sqrt{-32}$
3. $(4i)(-3i)$
4. $3\sqrt{-24} \cdot 2\sqrt{-18}$
5. i^{40}
6. i^{63}

Example 3 Solve each equation.

7. $4x^2 + 32 = 0$
8. $x^2 + 1 = 0$

Example 4 Find the values of a and b that make each equation true.

9. $3a + (4b + 2)i = 9 - 6i$
10. $4b - 5 + (-a - 3)i = 7 - 8i$

Examples 5 and 7 Simplify.

11. $(-1 + 5i) + (-2 - 3i)$
12. $(7 + 4i) - (1 + 2i)$
13. $(6 - 8i)(9 + 2i)$
14. $(3 + 2i)(-2 + 4i)$
15. $\frac{3 - i}{4 + 2i}$
16. $\frac{2 + i}{5 + 6i}$

Example 6

17. **ELECTRICITY** The current in one part of a series circuit is $5 - 3j$ amps. The current in another part of the circuit is $7 + 9j$ amps. Add these complex numbers to find the total current in the circuit.

Practice and Problem Solving

Examples 1–2 **STRUCTURE** Simplify.

18. $\sqrt{-121}$
19. $\sqrt{-169}$
20. $\sqrt{-100}$
21. $\sqrt{-81}$
22. $(-3i)(-7i)(2i)$
23. $4i(-6i)^2$
24. i^{11}
25. i^{25}
26. $(10 - 7i) + (6 + 9i)$
27. $(-3 + i) + (-4 - i)$
28. $(12 + 5i) - (9 - 2i)$
29. $(11 - 8i) - (2 - 8i)$
30. $(1 + 2i)(1 - 2i)$
31. $(3 + 5i)(5 - 3i)$
32. $(4 - i)(6 - 6i)$
33. $\frac{2i}{1 + i}$
34. $\frac{5}{2 + 4i}$
35. $\frac{5 + i}{3i}$

Example 3 Solve each equation.

36. $4x^2 + 4 = 0$
37. $3x^2 + 48 = 0$
38. $2x^2 + 50 = 0$
39. $2x^2 + 10 = 0$
40. $6x^2 + 108 = 0$
41. $8x^2 + 128 = 0$

Example 4 Find the values of x and y that make each equation true.

42. $9 + 12i = 3x + 4yi$
43. $x + 1 + 2yi = 3 - 6i$
44. $2x + 7 + (3 - y)i = -4 + 6i$
45. $5 + y + (3x - 7)i = 9 - 3i$
46. $a + 3b + (3a - b)i = b + bi$
47. $(2a - 4b)i + a + 5b = 15 + 58i$

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Examples 5 and 7

Simplify.

48. $\sqrt{-10} \cdot \sqrt{-24}$

49. $4i\left(\frac{1}{2}i\right)^2(-2i)^2$

50. i^{41}

51. $(4 - 6i) + (4 + 6i)$

52. $(8 - 5i) - (7 + i)$

53. $(-6 - i)(3 - 3i)$

54. $\frac{(5 + i)^2}{3 - i}$

55. $\frac{6 - i}{2 - 3i}$

56. $(-4 + 6i)(2 - i)(3 + 7i)$

57. $(1 + i)(2 + 3i)(4 - 3i)$

58. $\frac{4 - i\sqrt{2}}{4 + i\sqrt{2}}$

59. $\frac{2 - i\sqrt{3}}{2 + i\sqrt{3}}$

Example 6

60. **ELECTRICITY** The impedance in one part of a series circuit is $7 + 8j$ ohms, and the impedance in another part of the circuit is $13 - 4j$ ohms. Add these complex numbers to find the total impedance in the circuit.

ELECTRICITY **Use the formula $V = C \cdot I$.**

61 The current in a circuit is $3 + 6j$ amps, and the impedance is $5 - j$ ohms. What is the voltage?

62. The voltage in a circuit is $20 - 12j$ volts, and the impedance is $6 - 4j$ ohms. What is the current?

63. Find the sum of $ix^2 - (4 + 5i)x + 7$ and $3x^2 + (2 + 6i)x - 8i$.

64. Simplify $[(2 + i)x^2 - ix + 5 + i] - [(-3 + 4i)x^2 + (5 - 5i)x - 6]$.

65. **MULTIPLE REPRESENTATIONS** In this problem, you will explore quadratic equations that have complex roots.

a. Algebraic Write a quadratic equation in standard form with $3i$ and $-3i$ as its roots.

b. Graphical Graph the quadratic equation found in part **a** by graphing its related function.

c. Algebraic Write a quadratic equation in standard form with $2 + i$ and $2 - i$ as its roots.

d. Graphical Graph the quadratic equation found in part **c** by graphing its related function.

e. Analytical How do you know when a quadratic equation will have only complex solutions?

H.O.T. Problems Use Higher-Order Thinking Skills

66. **CRITIQUE** Amina and Maysoon are simplifying $(2i)(3i)(4i)$. Is either of them correct? Explain your reasoning.

Amina	Maysoon
$24i^3 = -24$	$24i^3 = -24i$

67. **CHALLENGE** Simplify $(1 + 2i)^3$.

68. **REASONING** Determine whether the following statement is *always, sometimes,* or *never* true. Explain your reasoning.

Every complex number has both a real part and an imaginary part.

69. **OPEN ENDED** Write two complex numbers with a product of 20.

70. **WRITING IN MATH** Explain how complex numbers are related to quadratic equations.

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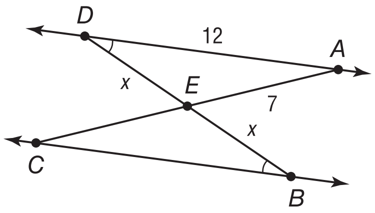
Standardized Test Practice

71. EXTENDED RESPONSE Refer to the figure to answer the following.



a. Name two congruent triangles with vertices in correct order.

b. Explain why the triangles are congruent.

c. What is the length of $\overline{EC}$? Explain your procedure.

72. $(3 + 6)^2 =$

A $2 \times 3 + 2 \times 6$
B 9^2
C $3^2 + 6^2$
D $3^2 \times 6^2$

73. SAT/ACT A store charges AED 49 for a pair of pants. This price is 40% more than the amount it costs the store to buy the pants. After a sale, any employee is allowed to purchase any remaining pairs of pants at 30% off the store's cost. How much would it cost an employee to purchase the pants after the sale?

F AED 10.50
G AED 12.50
H AED 13.72
J AED 24.50
K AED 35.00

74. What are the values of x and y when $(5 + 4\boldsymbol{i}) - (x + y\boldsymbol{i}) = (-1 - 3\boldsymbol{i})$?

A $x = 6, y = 7$
B $x = 4, y = \boldsymbol{i}$
C $x = 6, y = \boldsymbol{i}$
D $x = 4, y = 7$

Spiral Review

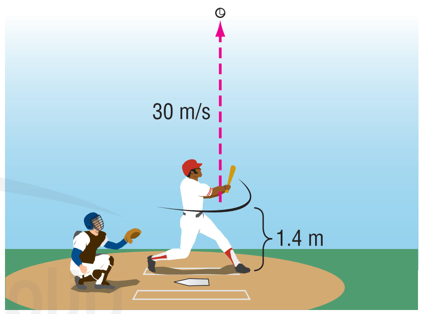
Solve each equation by factoring. (Lesson 1-2)

75. $2x^2 + 7x = 15$

76. $4x^2 - 12 = 22x$

77. $6x^2 = 5x + 4$

78. BASEBALL A baseball player hits a high pop-up with an initial upward velocity of 30 meters per second, 1.4 meters above the ground. The height $h(t)$ of the ball in meters t seconds after being hit is modeled by $h(t) = -4.9t^2 + 30t + 1.4$. How long does an opposing player have to get under the ball if he catches it 1.7 meters above the ground? Does your answer seem reasonable? Explain. (Lesson 1-2)



79. ELECTRICITY The impedance in one part of a series circuit is $3 + 4j$ ohms, and the impedance in another part of the circuit is $2 - 6j$ ohms. Add these complex numbers to find the total impedance of the circuit. (Lesson 1-3)

Simplify. (Lesson 1-3)

80. $(8 + 5\boldsymbol{i})^2$

81. $4(3 - \boldsymbol{i}) + 6(2 - 5\boldsymbol{i})$

82. $\frac{5 - 2\boldsymbol{i}}{6 + 9\boldsymbol{i}}$

Write a quadratic equation in standard form with the given root(s). (Lesson 1-2)

83. $\frac{4}{5}, \frac{3}{4}$

84. $-\frac{2}{5}, 6$

85. $-\frac{1}{4}, -\frac{6}{7}$

Skills Review

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*.

86. $x^2 + 16x + 64$

87. $x^2 - 12x + 36$

88. $x^2 + 8x - 16$

89. $x^2 - 14x - 49$

90. $x^2 + x + 0.25$

91. $x^2 + 5x + 6.25$

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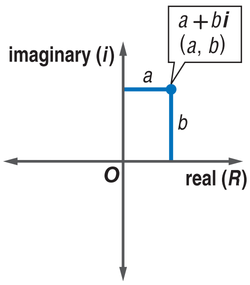
EXTEND 1-5

Algebra Lab
The Complex Plane

A complex number $a + bi$ can be graphed in the **complex plane** by representing it with the point (a, b). Similar to a coordinate plane, the complex plane is comprised of two axes. The real component is plotted on the **real axis**, which is horizontal. The imaginary component is plotted on the **imaginary axis**, which is vertical. The complex plane may also be referred to as the **Argand (ar GON) plane**.



Example 1 Graph in the Complex Plane

Graph $z = 3 + 4i$ in the complex plane.

Step 1 Represent z with the point (a, b).

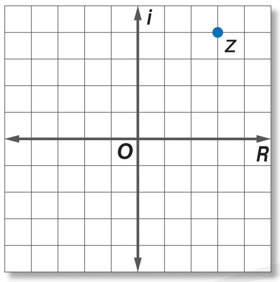
The real component a of z is 3.

The imaginary component bi of z is $4i$.

z can be represented by the point (a, b) or $(3, 4)$.

Step 2 Graph z in the complex plane.

Construct the complex plane and plot the point $(3, 4)$.



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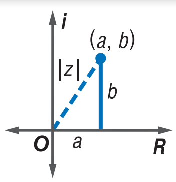
Recall that for a real number, the absolute value is its distance from zero on the number line. Similarly, the **absolute value of a complex number** is its distance from the origin in the complex plane. When $a + bi$ is graphed in the complex plane, the absolute value of $a + bi$ is the distance from (a, b) to the origin. This can be found by using the Distance Formula.

$$\sqrt{(a - 0)^2 + (b - 0)^2} \text{ or } \sqrt{a^2 + b^2}$$

KeyConcept Absolute Value of a Complex Number

The absolute value of the complex number $z = a + bi$ is

$$|z| = |a + bi| = \sqrt{a^2 + b^2}.$$



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The Complex Plane *Continued*

Example 2 Absolute Value of a Complex Number

Find the absolute value of $z = -5 + 12i$.

Step 1 Determine values for a and b.

The real component a of z is -5. The imaginary component bi of z is $12i$.
Thus, $a = -5$ and $b = 12$.

Step 2 Find the absolute value of z.

$$|z| = \sqrt{a^2 + b^2} \quad \text{Absolute value of a complex number}$$

$$= \sqrt{(-5)^2 + 12^2} \quad a = -5 \text{ and } b = 12$$

$$= \sqrt{169} \text{ or } 13 \quad \text{Simplify.}$$

The absolute value of $z = -5 + 12i$ is 13.

Addition and subtraction of complex numbers can be performed graphically.

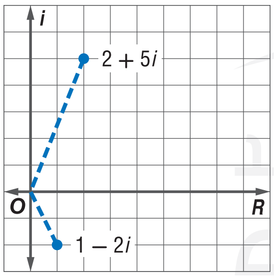
Example 3 Simplify by Graphing

Simplify $(1 - 2i) - (-2 - 5i)$ by graphing.

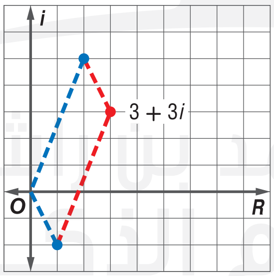
Step 1 Write $(1 - 2i) - (-2 - 5i)$ as $(1 - 2i) + (2 + 5i)$.

Step 2 Graph $1 - 2i$ and $2 + 5i$ on the same complex plane. Connect each point with the origin using a dashed segment.

Step 3 Complete the parallelogram that has the two segments as two of its sides. Plot a point where the two additional sides meet. The solution of $(1 - 2i) - (-2 - 5i)$ is $3 + 3i$.



Step 2



Step 3

Exercises

Graph each number in the complex plane.

1. $z = 3 + i$
2. $z = -4 - 2i$
3. $z = 2 - 2i$

Find the absolute value of each complex number.

4. $z = -4 - 3i$
5. $z = 7 - 2i$
6. $z = -6 - i$

Simplify by graphing.

7. $(6 + 5i) + (-2 - 3i)$
8. $(8 - 2i) - (4 + 7i)$
9. $(5 + 6i) + (-4 + 3i)$

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EXTEND 1-5B

Graphing Technology Lab: Solving Quadratic Equations

You can use a graphing calculator with CAS technology to solve quadratic equations.

Activity Finding Roots

Solve each equation.

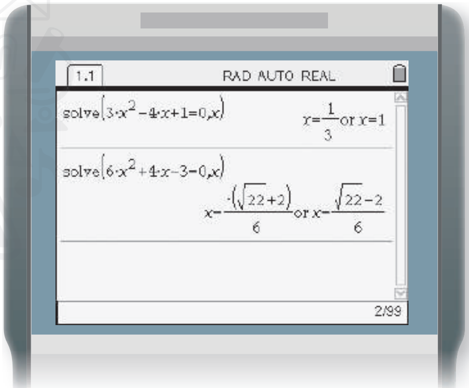
a. $3x^2 - 4x + 1 = 0$

Step 1 Add a new **Calculator** page.

Step 2 Select the **Solve** tool from the **Algebra** menu.

Step 3 Type $3x^2 - 4x + 1 = 0$ followed by a comma, x, and then **enter**.

The solutions are $x = \frac{1}{3}$ or $x = 1$.



b. $6x^2 + 4x - 3 = 0$

Step 1 Select the **Solve** tool from the **Algebra** menu.

Step 2 Type $6x^2 - 4x - 3 = 0$ followed by a comma, x, and then **enter**.

The solutions are $x = \frac{-2 \pm \sqrt{22}}{6}$.

c. $x^2 - 6x + 10 = 0$.

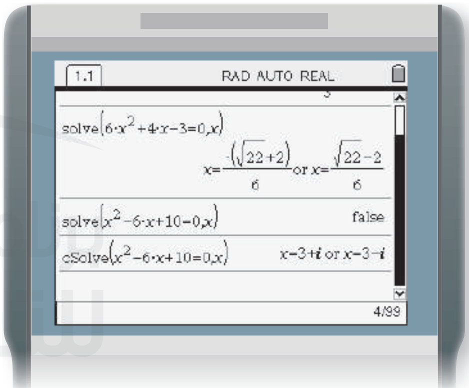
Step 1 Select the **Solve** tool from the **Algebra** menu.

Step 2 Type $x^2 - 6x + 10 = 0$ followed by a comma, x, and then **enter**.

The calculator returns a value of *false*, meaning that there are no real solutions.

Step 3 Under menu, select **Algebra**, **Complex**, then **Solve**. Reenter the equation.

The solutions are $x = 3 \pm i$.



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Exercises

Solve each equation.

1. $x^2 - 2x - 24 = 0$

2. $-x^2 + 4x - 1 = 0$

3. $0 = -3x^2 - 6x + 9$

4. $x^2 - 2x + 5 = 0$

5. $0 = 4x^2 - 8$

6. $0 = 2x^2 - 4x + 1$

7. $x^2 + 3x + 8 = 5$

8. $25 + 4x^2 = -20x$

9. $x^2 - x = -6$

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CHAPTER 1

Chapter Quiz II

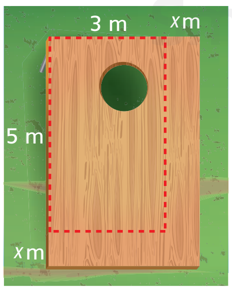
Lessons 1-4 through 1-5

Write a quadratic equation in standard form with the given root(s). (Lesson 1-1)

1. 7, 2
2. 0, 3
3. $-5, 8$
4. $-7, -8$
5. $-6, -3$
6. $3, -4$
7. $1, \frac{1}{2}$
8. **NUMBER THEORY** Find two consecutive even positive integers whose product is 624. (Lesson 1-4)
9. **GEOMETRY** The length of a rectangle is 2 meters more than its width. Find the dimensions of the rectangle if its area is 63 square meters. (Lesson 1-4)

Solve each equation by factoring. (Lesson 1-3)

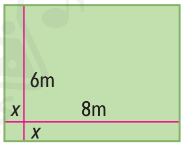
10. $x^2 - x - 12 = 0$
11. $3x^2 + 7x + 2 = 0$
12. $x^2 - 2x - 15 = 0$
13. $2x^2 + 5x - 3 = 0$
14. Write a quadratic equation in standard form with roots -6 and $\frac{1}{4}$. (Lesson 1-4)
15. **GAMES** Jasim constructed a platform for a bean bag toss game. The plans for the original platform had dimensions of 3 meters by 5 meters. He made his platform larger by adding x meters to each side. The area of the new platform is 35 square meters. (Lesson 1-4)



a. Write a quadratic equation that represents the area of his platform.

b. Find the dimensions of the platform Jasim made.

16. **TRIANGLES** Find the dimensions of a triangle if the base is $\frac{2}{3}$ the measure of the height and the area is 12 square centimeters. (Lesson 1-4)
17. **PATIO** Ali is putting a cement slab in his backyard. The original slab was going to have dimensions of 8 meters by 6 meters. He decided to make the slab larger by adding x meters to each side. The area of the new slab is 120 square meters. (Lesson 1-4)



a. Write a quadratic equation that represents the area of the new slab.

b. Find the new dimensions of the slab.

Simplify. (Lesson 1-5)

18. $\sqrt{-81}$
19. $\sqrt{-25x^4y^5}$
20. $(15 - 3i) - (4 - 12i)$
21. i^{37}
22. $(5 - 3i)(5 + 3i)$
23. $\frac{3 - i}{2 + 5i}$
24. The impedance in one part of a series circuit is $3 + 4j$ ohms and the impedance in another part of the circuit is $6 - 7j$ ohms. Add these complex numbers to find the total impedance in the circuit. (Lesson 1-5)

Simplify. (Lesson 1-5)

25. $(3 - 4i) - (9 - 5i)$
26. $\frac{4i}{4 - i}$

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LESSON 1-6 The Quadratic Formula and the Discriminant

Then	Now	Why?
You solved equations by completing the square.	**1** Solve quadratic equations by using the Quadratic Formula. **2** Use the discriminant to determine the number and type of roots of a quadratic equation.	Pumpkin catapult is an event in which a contestant builds a catapult and launches a pumpkin at a target. The path of the pumpkin can be modeled by the quadratic function $h = -4.9t^2 + 117t + 42$, where h is the height of the pumpkin and t is the number of seconds. To predict when the pumpkin will hit the target, you can solve the equation $0 = -4.9t^2 + 117t + 42$. This equation would be difficult to solve using factoring, graphing, or completing the square.

NewVocabulary
Quadratic Formula
discriminant

Mathematical Practices
Look for and express regularity in repeated reasoning.

1 Quadratic Formula

You have found solutions of some quadratic equations by graphing, by factoring, and by using the Square Root Property. There is also a formula that can be used to solve any quadratic equation. This formula can be derived by solving the standard form of a quadratic equation.

General Case		Specific Case
$ax^2 + bx + c = 0$	Standard quadratic equation	$2x^2 + 8x + 1 = 0$
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divide each side by a.	$x^2 + 4x + \frac{1}{2} = 0$
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Subtract $\frac{c}{a}$ from each side.	$x^2 + 4x = -\frac{1}{2}$
$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$	Complete the square.	$x^2 + 4x + \left(\frac{4}{2}\right)^2 = -\frac{1}{2} + \left(\frac{4}{2}\right)^2$
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$	Factor the left side.	$(x + 2)^2 = -\frac{1}{2} + \left(\frac{4}{2}\right)^2$
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	Simplify the right side.	$(x + 2)^2 = \frac{7}{2}$
$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$	Square Root Property	$x + 2 = \pm\sqrt{\frac{7}{2}}$
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$	Subtract $\frac{b}{2a}$ from each side.	$x = -2 \pm \sqrt{\frac{7}{2}}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Simplify.	$x = \frac{-4 \pm \sqrt{14}}{2}$

The equation $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is known as the **Quadratic Formula**.

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> **StudyTip**
> **Quadratic Formula** Although factoring may be an easier method to solve some of the equations, the Quadratic Formula can be used to solve any quadratic equation.

KeyConcept Quadratic Formula

Words The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example $x^2 + 5x + 6 = 0 \rightarrow x = \dfrac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)}$

Example 1 Two Rational Roots

Solve $x^2 - 10x = 11$ by using the Quadratic Formula.

First, write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$\begin{array}{ccc} & & ax^2 + bx + c = 0 \\ & & \downarrow \quad \downarrow \quad \downarrow \\ x^2 - 10x = 11 & \rightarrow & 1x^2 - 10x - 11 = 0 \end{array}$$

Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-11)}}{2(1)}$$ Replace a with 1, b with −10, and c with −11.

$$= \frac{10 \pm \sqrt{100 + 44}}{2}$$ Multiply.

$$= \frac{10 \pm \sqrt{144}}{2}$$ Simplify.

$$= \frac{10 \pm 12}{2}$$ $\sqrt{144} = 12$

$$x = \frac{10 + 12}{2} \quad \text{or} \quad x = \frac{10 - 12}{2}$$ Write as two equations.

$$= 11 \qquad\qquad = -1$$ Simplify.

The solutions are −1 and 11.

CHECK Substitute both values into the original equation.

$$\begin{array}{rr} x^2 - 10x = 11 & x^2 - 10x = 11 \\ (-1)^2 - 10(-1) \stackrel{?}{=} 11 & (11)^2 - 10(11) \stackrel{?}{=} 11 \\ 1 + 10 \stackrel{?}{=} 11 & 121 - 110 \stackrel{?}{=} 11 \\ 11 = 11 \checkmark & 11 = 11 \checkmark \end{array}$$

GuidedPractice

Solve each equation by using the Quadratic Formula.

1A. $x^2 + 6x = 16$

1B. $2x^2 + 25x + 33 = 0$

> **ReviewVocabulary**
> **radicand** the value underneath the radical symbol

When the value of the radicand in the Quadratic Formula is 0, the quadratic equation has exactly one rational root.

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Math HistoryLink

Brahmagupta (598–668) Indian mathematician Brahmagupta offered the first general solution of the quadratic equation $ax^2 + bx = c$, now known as the Quadratic Formula.

Example 2 One Rational Root

Solve $x^2 + 8x + 16 = 0$ by using the Quadratic Formula.

Identify a, b, and c. Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

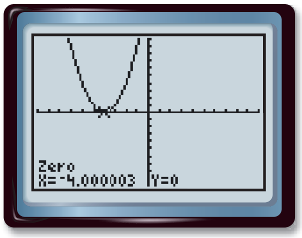
$$= \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(16)}}{2(1)}$$ Replace a with 1, b with 8, and c with 16.

$$= \frac{-8 \pm \sqrt{0}}{2}$$ Simplify.

$$= \frac{-8}{2} \text{ or } -4$$ $\sqrt{0} = 0$

The solution is -4.

CHECK A graph of the related function shows that there is one solution at $x = -4$.



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GuidedPractice

Solve each equation by using the Quadratic Formula.

2A. $x^2 - 16x + 64 = 0$

2B. $x^2 + 34x + 289 = 0$

You can express irrational roots exactly by writing them in radical form.

Example 3 Irrational Roots

Solve $2x^2 + 6x - 7 = 0$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

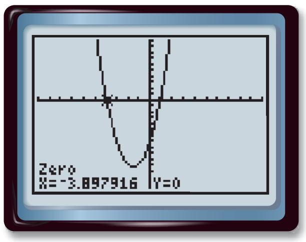
$$= \frac{-(6) \pm \sqrt{(6)^2 - 4(2)(-7)}}{2(2)}$$ Replace a with 2, b with 6, and c with -7.

$$= \frac{-6 \pm \sqrt{92}}{4}$$ Simplify.

$$= \frac{-6 \pm 2\sqrt{23}}{4} \text{ or } \frac{-3 \pm \sqrt{23}}{2}$$ $\sqrt{92} = \sqrt{4 \cdot 23}$ or $2\sqrt{23}$

The approximate solutions are -3.9 and 0.9.

CHECK Check these results by graphing the related quadratic function, $y = 2x^2 + 6x - 7$. Using the **ZERO** function of a graphing calculator, the approximate zeros of the related function are -3.9 and 0.9.



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GuidedPractice

Solve each equation by using the Quadratic Formula.

3A. $3x^2 + 5x + 1 = 0$

3B. $x^2 - 8x + 9 = 0$

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> **StudyTip**
> **Complex Numbers** Remember to write your solutions in the form $a + bi$, sometimes called the *standard form* of a complex number.

When using the Quadratic Formula, if the value of the radicand is negative, the solutions will be complex. Complex solutions always appear in conjugate pairs.

Example 4 Complex Roots

Solve $x^2 - 6x = -10$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

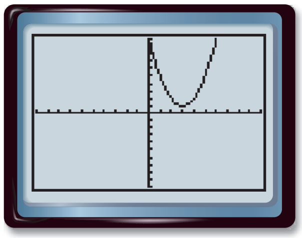
$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$ Replace a with 1, b with −6, and c with 10.

$$= \frac{6 \pm \sqrt{-4}}{2}$$ Simplify.

$$= \frac{6 \pm 2i}{2}$$ $\sqrt{-4} = \sqrt{4 \cdot (-1)}$ or $2i$

$$= 3 \pm i$$ Simplify.

The solutions are the complex numbers $3 + i$ and $3 - i$.



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CHECK A graph of the related function shows that the solutions are complex, but it cannot help you find them. To check complex solutions, substitute them into the original equation.

$x^2 - 6x = -10$	Original equation
$(3 + i)^2 - 6(3 + i) \stackrel{?}{=} -10$	$x = 3 + i$
$9 + 6i + i^2 - 18 - 6i \stackrel{?}{=} -10$	Square of a sum; Distributive Property
$-9 + i^2 \stackrel{?}{=} -10$	Simplify.
$-9 - 1 = -10$ ✓	$i^2 = -1$

$x^2 - 6x = -10$	Original equation
$(3 - i)^2 - 6(3 - i) \stackrel{?}{=} -10$	$x = 3 - i$
$9 - 6i + i^2 - 18 + 6i \stackrel{?}{=} -10$	Square of a sum; Distributive Property
$-9 + i^2 \stackrel{?}{=} -10$	Simplify.
$-9 - 1 = -10$ ✓	$i^2 = -1$

GuidedPractice

Solve each equation by using the Quadratic Formula.

4A. $3x^2 + 5x + 4 = 0$

4B. $x^2 - 4x = -13$

2 Roots and the Discriminant

In the previous examples, observe the relationship between the value of the expression under the radical and the roots of the quadratic equation. The expression $b^2 - 4ac$ is called the **discriminant**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \leftarrow \text{discriminant}$$

The value of the discriminant can be used to determine the number and type of roots of a quadratic equation. The table on the following page summarizes the possible types of roots.

The discriminant can also be used to confirm the number and type of solutions after you solve the quadratic equation.

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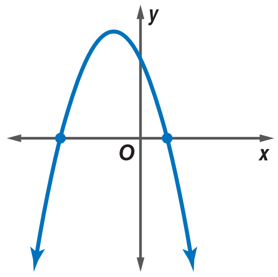
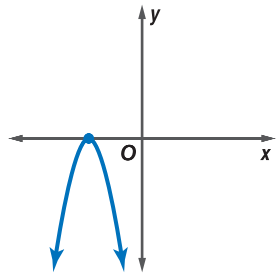
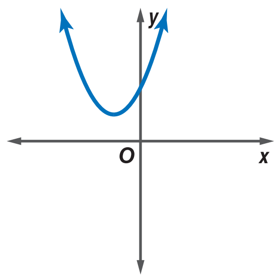
StudyTip

Roots Remember that the solutions of an equation are called *roots* or *zeros* and are the value(s) where the graph crosses the x-axis.

KeyConcept Discriminant

Consider $ax^2 + bx + c = 0$, where a, b, and c are rational numbers and $a \neq 0$.

Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is *not* a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real rational root	
$b^2 - 4ac < 0$	2 complex roots	

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Example 5 Describe Roots

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

a. $7x^2 - 11x + 5 = 0$

$a = 7, b = -11, c = 5$

$$b^2 - 4ac = (-11)^2 - 4(7)(5)$$
$$= 121 - 140$$
$$= -19$$

The discriminant is negative, so there are two complex roots.

b. $x^2 + 22x + 121 = 0$

$a = 1, b = 22, c = 121$

$$b^2 - 4ac = (22)^2 - 4(1)(121)$$
$$= 484 - 484$$
$$= 0$$

The discriminant is 0, so there is one rational root.

GuidedPractice

5A. $-5x^2 + 8x - 1 = 0$

5B. $-7x + 15x^2 - 4 = 0$

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You have studied a variety of methods for solving quadratic equations. The table below summarizes these methods.

StudyTip

Study Notebook You may wish to copy this list of methods to your math notebook or Foldable to keep as a reference as you study.

ConceptSummary Solving Quadratic Equations

Method	Can be Used	When to Use
graphing	sometimes	Use only if an exact answer is not required. Best used to check the reasonableness of solutions found algebraically.
factoring	sometimes	Use if the constant term is 0 or if the factors are easily determined. **Example** $x^2 - 7x = 0$
Square Root Property	sometimes	Use for equations in which a perfect square is equal to a constant. **Example** $(x - 5)^2 = 18$
completing the square	always	Useful for equations of the form $x^2 + bx + c = 0$, where b is even. **Example** $x^2 + 6x - 14 = 0$
Quadratic Formula	always	Useful when other methods fail or are too tedious. **Example** $2.3x^2 - 1.8x + 9.7 = 0$

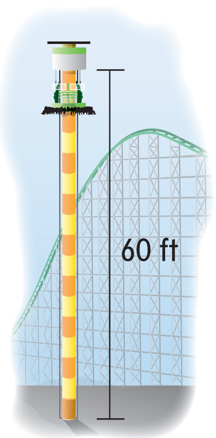
Check Your Understanding

Examples 1–4 **Solve each equation by using the Quadratic Formula.**

1. $x^2 + 12x - 9 = 0$
2. $x^2 + 8x + 5 = 0$
3. $4x^2 - 5x - 2 = 0$
4. $9x^2 + 6x - 4 = 0$
5. $10x^2 - 3 = 13x$
6. $22x = 12x^2 + 6$
7. $-3x^2 + 4x = -8$
8. $x^2 + 3 = -6x + 8$

Examples 3–4

9. **MODELING** An amusement park ride takes riders to the top of a tower and drops them at speeds reaching 80 feet per second. A function that models this ride is $h = -16t^2 - 64t + 60$, where h is the height in feet and t is the time in seconds. About how many seconds does it take for riders to drop from 60 feet to 0 feet?



Example 5 **Complete parts a and b for each quadratic equation.**

a. Find the value of the discriminant.

b. Describe the number and type of roots.

10. $3x^2 + 8x + 2 = 0$
11. $2x^2 - 6x + 9 = 0$
12. $-16x^2 + 8x - 1 = 0$
13. $5x^2 + 2x + 4 = 0$

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Practice and Problem Solving

Examples 1–4 **Solve each equation by using the Quadratic Formula.**

14. $x^2 + 45x = -200$

15. $4x^2 - 6 = -12x$

16. $3x^2 - 4x - 8 = -6$

17. $4x^2 - 9 = -7x - 4$

18. $5x^2 - 9 = 11x$

19. $12x^2 + 9x - 2 = -17$

20. **DIVING** Competitors in the 10-meter platform diving competition jump upward and outward before diving into the pool below. The height h of a diver in meters above the pool after t seconds can be approximated by the equation $h = -4.9t^2 + 3t + 10$.

a. Determine a domain and range for which this function makes sense.

b. When will the diver hit the water?

Example 5 **Complete parts a–c for each quadratic equation.**

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

21 $2x^2 + 3x - 3 = 0$

22. $4x^2 - 6x + 2 = 0$

23. $6x^2 + 5x - 1 = 0$

24. $6x^2 - x - 5 = 0$

25. $3x^2 - 3x + 8 = 0$

26. $2x^2 + 4x + 7 = 0$

27. $-5x^2 + 4x + 1 = 0$

28. $x^2 - 6x = -9$

29. $-3x^2 - 7x + 2 = 6$

30. $-8x^2 + 5 = -4x$

31. $x^2 + 2x - 4 = -9$

32. $-6x^2 + 5 = -4x + 8$

33. **VIDEO GAMES** While Tarek is grounded his friend Khalid brings him a video game. Tarek stands at his bedroom window, and Khalid stands directly below the window. If Khalid tosses a game cartridge to Tarek with an initial velocity of 35 feet per second, an equation for the height h feet of the cartridge after t seconds is $h = -16t^2 + 35t + 5$.

a. If the window is 25 feet above the ground, will Tarek have 0, 1, or 2 chances to catch the video game cartridge?

b. If Tarek is unable to catch the video game cartridge, when will it hit the ground?

34. **SENSE-MAKING** Civil engineers are designing a section of road that is going to dip below sea level. The road's curve can be modeled by the equation $y = 0.00005x^2 - 0.06x$, where x is the horizontal distance in meters between the points where the road is at sea level and y is the elevation. The engineers want to put stop signs at the locations where the elevation of the road is equal to sea level. At what horizontal distances will they place the stop signs?

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

35. $5x^2 + 8x = 0$

36. $8x^2 = -2x + 1$

37. $4x - 3 = -12x^2$

38. $0.8x^2 + 2.6x = -3.2$

39. $0.6x^2 + 1.4x = 4.8$

40. $-4x^2 + 12 = -6x - 8$

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41 **SMOKING** A decrease in smoking has resulted in lower death rates caused by lung cancer. The number of deaths per 100,000 people y can be approximated by $y = -0.26x^2 - 0.55x + 91.81$, where x represents the number of years after 2000.

Year	Deaths per 100,000
2000	91.8
2002	89.7
2004	85.5
2010	60.3
2015	?
2017	?

a. Calculate the number of deaths per 100,000 people for 2015 and 2017.

b. Use the Quadratic Formula to solve for x when $y = 50$.

c. According to the quadratic function, when will the death rate be 0 per 100,000? Do you think that this prediction is reasonable? Why or why not?

42. **NUMBER THEORY** The sum S of consecutive integers 1, 2, 3, …, n is given by the formula $S = \frac{1}{2}n(n + 1)$. How many consecutive integers, starting with 1, must be added to get a sum of 666?

H.O.T. Problems Use Higher-Order Thinking Skills

43. **CRITIQUE** Abdullah and Abdulaziz are determining the number of solutions of $3x^2 - 5x = 7$. Is either of them correct? Explain your reasoning.

Abdullah

$3x^2 - 5x = 7$

$b^2 - 4ac = (-5)^2 - 4(3)(7)$

$= -59$

Since the discriminant is negative, there are no real solutions.

Abdulaziz

$3x^2 - 5x = 7$

$3x^2 - 5x - 7 = 0$

$b^2 - 4ac = (-5)^2 - 4(3)(-7)$

$= 109$

Since the discriminant is positive, there are two real roots.

44. **CHALLENGE** Find the solutions of $4ix^2 - 4ix + 5i = 0$ by using the Quadratic Formula.

45. **REASONING** Determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

a. In a quadratic equation in standard form, if a and c are different signs, then the solutions will be real.

b. If the discriminant of a quadratic equation is greater than 1, the two roots are real irrational numbers.

46. **OPEN ENDED** Sketch the corresponding graph and state the number and type of roots for each of the following.

a. $b^2 - 4ac = 0$

b. A quadratic function in which $f(x)$ never equals zero.

c. A quadratic function in which $f(a) = 0$ and $f(b) = 0$; $a \neq b$.

d. The discriminant is less than zero.

e. a and b are both solutions and can be represented as fractions.

47. **CHALLENGE** Find the value(s) of m in the quadratic equation $x^2 + x + m + 1 = 0$ such that it has one solution.

48. **WRITING IN MATH** Describe three different ways to solve $x^2 - 2x - 15 = 0$. Which method do you prefer, and why?

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Standardized Test Practice

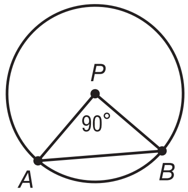
49. A company determined that its monthly profit P is given by $P = -8x^2 + 165x - 100$, where x is the selling price for each unit of product. Which of the following is the best estimate of the maximum price per unit that the company can charge without losing money?

A AED 10 **B** AED 20 **C** AED 30 **D** AED 40

50. SAT/ACT For which of the following sets of numbers is the mean greater than the median?

F {4, 5, 6, 7, 8}
G {4, 6, 6, 6, 8}
H {4, 5, 6, 7, 9}
J {3, 5, 6, 7, 8}
K {2, 6, 6, 6, 6}

51. SHORT RESPONSE In the figure below, P is the center of the circle with radius 15 centimeters. What is the area of $\triangle APB$?



52. 75% of 88 is the same as 60% of what number?

A 100 **B** 101 **C** 108 **D** 110

Spiral Review

Simplify. (Lesson 2-3)

53. i^{26}

54. $\sqrt{-16}$

55. $4\sqrt{-9} \cdot 2\sqrt{-25}$

56. HIGHWAY SAFETY Engineers can use the formula $d = 0.05v^2 + 1.1v$ to estimate the minimum stopping distance d in meters for a vehicle traveling v km per hour. If a car is able to stop after 20 m, what is the fastest it could have been traveling when the driver first applied the brakes? (Lesson 2-4)

57. BRIDGES The supporting cables of the Golden Gate Bridge approximate the shape of a parabola. The parabola can be modeled by the quadratic function $y = 0.00012x^2 + 6$, where x represents the distance from the axis of symmetry and y represents the height of the cables. The related quadratic equation is $0.00012x^2 + 6 = 0$. (Lesson 2-4)

a. Calculate the value of the discriminant.

b. What does the discriminant tell you about the supporting cables of the Golden Gate Bridge?

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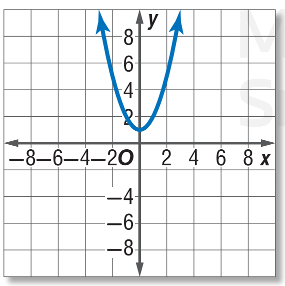
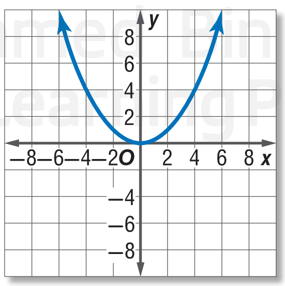
Skills Review

Write an equation for each graph.

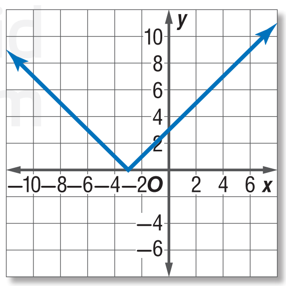
58.



59.



60.



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LESSON 1-7 Transformations of Quadratic Functions

Then

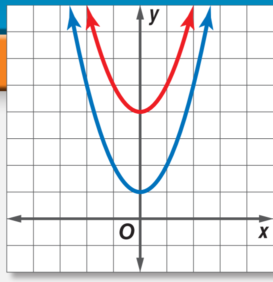
You graphed quadratic functions by using the vertex and axis of symmetry.

Now

1. Apply translations to quadratic functions.
2. Apply dilations and reflections to quadratic functions.
3. Write a quadratic function in the form $y = a(x - h)^2 + k$.
4. Transform graphs of quadratic functions of the form $y = a(x - h)^2 + k$.

Why?

The graphs of the parabolas shown at the right are the same size and shape, but notice that the vertex of the red parabola is higher on the y-axis than the vertex of the blue parabola. Shifting a parabola up and down is an example of a transformation.



NewVocabulary

- transformation
- translation
- dilation
- reflection
- vertex form

Mathematical Practices

Make sense of problems and persevere in solving them.
Look for and express regularity in repeated reasoning.
Look for and make use of structure.

1 Translations

A **transformation** changes the position or size of a figure. One transformation, a **translation**, moves a figure up, down, left, or right. When a constant k is added to or subtracted from the parent function, the graph of the resulting function $f(x) \pm k$ is the graph of the parent function translated up or down.

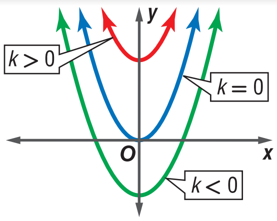
The parent function of the family of quadratics is $f(x) = x^2$. All other quadratic functions have graphs that are transformations of the graph of $f(x) = x^2$.

KeyConcept Vertical Translations

The graph of $f(x) = x^2 + k$ is the graph of $f(x) = x^2$ translated vertically.

If $k > 0$, the graph of $f(x) = x^2$ is translated $|k|$ units **up**.

If $k < 0$, the graph of $f(x) = x^2$ is translated $|k|$ units **down**.



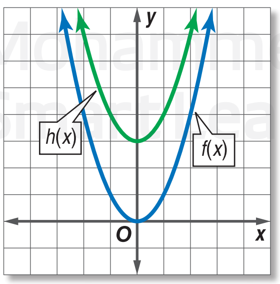
Example 1 Describe and Graph Translations

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

a. $h(x) = x^2 + 3$

$k = 3$ and $3 > 0$

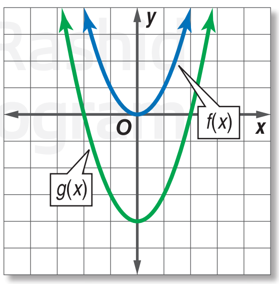
$h(x)$ is a translation of the graph of $f(x) = x^2$ up 3 units.



b. $g(x) = x^2 - 4$

$k = -4$ and $-4 < 0$

$g(x)$ is a translation of the graph of $f(x) = x^2$ down 4 units.



GuidedPractice

1A. $f(x) = x^2 - 7$ **1B.** $g(x) = 5 + x^2$ **1C.** $h(x) = -5 + x^2$ **1D.** $f(x) = x^2 + 1$

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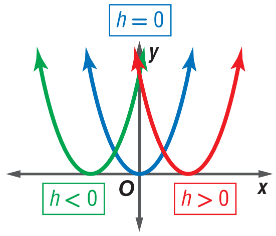
A quadratic graph can be translated horizontally by subtracting an h term from x.

KeyConcept Horizontal Translations

The graph of $g(x) = (x - h)^2$ is the graph of $f(x) = x^2$ translated horizontally.

If $h > 0$, the graph of $f(x) = x^2$ is translated h units to the **right**.

If $h < 0$, the graph of $f(x) = x^2$ is translated $|h|$ units to the **left**.



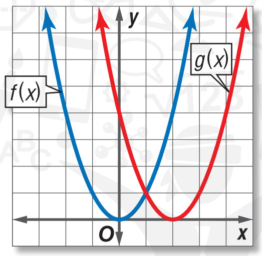
Example 2 Horizontal Translations

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

a. $g(x) = (x - 2)^2$

$k = 0$, $h = 2$ and $2 > 0$

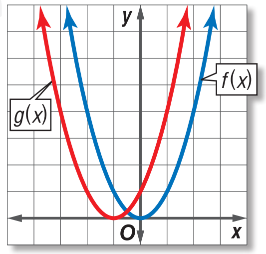
$g(x)$ is a translation of the graph of $f(x) = x^2$ to the right 2 units.



b. $g(x) = (x + 1)^2$

$k = 0$, $h = -1$ and $-1 < 0$

$g(x)$ is a translation of the graph of $f(x) = x^2$ to the left 1 unit.



GuidedPractice

2A. $g(x) = (x - 3)^2$

2B. $g(x) = (x + 2)^2$

A quadratic graph can be translated both horizontally and vertically.

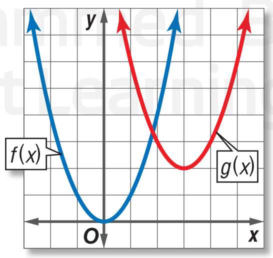
Example 3 Horizontal and Vertical Translations

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

a. $g(x) = (x - 3)^2 + 2$

$k = 2$, $h = 3$ and $3 > 0$

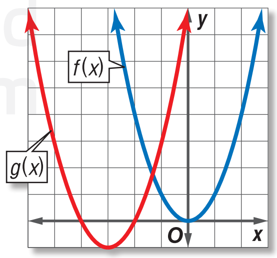
$g(x)$ is a translation of the graph of $f(x) = x^2$ to the right 3 units and up 2 units.



b. $g(x) = (x + 3)^2 - 1$

$k = -1$, $h = -3$ and $-3 < 0$

$g(x)$ is a translation of the graph of $f(x) = x^2$ to the left 3 units and down 1 unit.



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3A. $g(x) = (x + 2)^2 + 3$

3B. $g(x) = (x - 4)^2 - 4$

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2 Dilations and Reflections

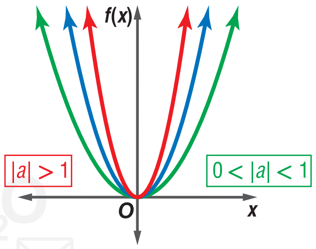
Another type of transformation is a dilation. A **dilation** makes the graph narrower than the parent graph or wider than the parent graph. When the parent function $f(x) = x^2$ is multiplied by a constant a, the graph of the resulting function $f(x) = ax^2$ is either stretched or compressed vertically.

KeyConcept Dilations

The graph of $g(x) = ax^2$ is the graph of $f(x) = x^2$ stretched or compressed vertically.

If $|a| > 1$, the graph of $f(x) = x^2$ is stretched vertically.

If $0 < |a| < 1$, the graph of $f(x) = x^2$ is compressed vertically.



StudyTip

Sense-Making When the graph of a quadratic function is stretched vertically, the shape of the graph is narrower than that of the parent function. When it is compressed vertically, the graph is wider than the parent function.

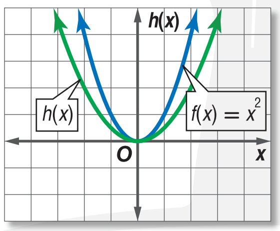
Example 4 Describe and Graph Dilations

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

a. $h(x) = \frac{1}{2}x^2$

$a = \frac{1}{2}$ and $0 < \frac{1}{2} < 1$

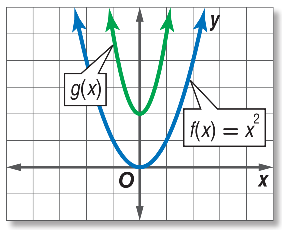
$h(x)$ is a dilation of the graph of $f(x) = x^2$ that is compressed vertically.



b. $g(x) = 3x^2 + 2$

$a = 3$ and $3 > 1$, $k = 2$ and $2 > 0$

$g(x)$ is a dilation of the graph of $f(x) = x^2$ that is stretched vertically and translated up 2 units.



GuidedPractice

4A. $j(x) = 2x^2$

4B. $h(x) = 5x^2 - 2$

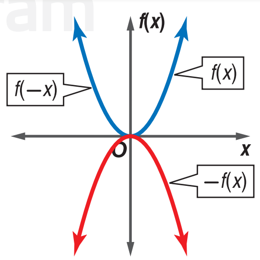
4C. $g(x) = \frac{1}{3}x^2 + 2$

A **reflection** flips a figure across a line.

KeyConcept Reflections

The graph of $-f(x)$ is the reflection of the graph of $f(x) = x^2$ across the x-axis.

The graph of $f(-x)$ is the reflection of the graph of $f(x) = x^2$ across the y-axis.



StudyTip

Reflection A reflection of $f(x) = x^2$ across the y-axis results in the same function, because $f(-x) = (-x)^2 = x^2$.

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WatchOut!

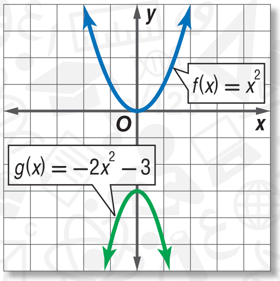
Transformations The graph of $f(x) = -ax^2$ can result in two transformations of the graph of $f(x) = x^2$: a reflection across the x-axis if $a > 0$ and either a compression or expansion depending on the absolute value of a.

Example 5 Describe and Graph Transformations

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

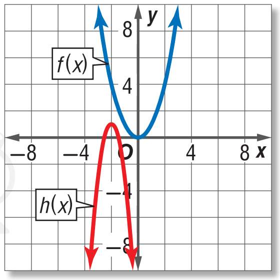
a. $g(x) = -2x^2 - 3$

- $a = -2$, $-2 < 0$, and $|-2| > 1$, so there is a reflection across the x-axis and the graph is vertically stretched.
- $k = -3$ and $-3 < 0$, so there is a translation down 3 units.



b. $h(x) = -4(x + 2)^2 + 1$

- $a = -4$, $-4 < 0$, and $|-4| > 1$, so there is a reflection across the x-axis and the graph is vertically stretched.
- $h = -2$ and $-2 < 0$, so there is a translation 2 units to the left.
- $k = 1$ and $1 > 0$, so there is a translation up 1 unit.



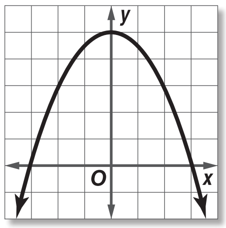
GuidedPractice

5A. $h(x) = 2(-x)^2 - 9$ **5B.** $g(x) = \frac{1}{5}x^2 + 3$ **5C.** $j(x) = -2(x - 1)^2 - 2$

You can use what you know about the characteristics of graphs of quadratic equations to match an equation with a graph.

Standardized Test Example 6 Identify an Equation for a Graph

Which is an equation for the function shown in the graph?



A $y = \frac{1}{2}x^2 - 5$ **C** $y = -\frac{1}{2}x^2 + 5$

B $y = -2x^2 - 5$ **D** $y = 2x^2 + 5$

Read the Test Item

You are given a graph. You need to find its equation.

Solve the Test Item

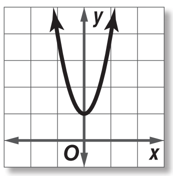
The graph opens downward, so the graph of $y = x^2$ has been reflected across the x-axis. The leading coefficient should be negative, so eliminate choices A and D.

The parabola is translated up 5 units, so $k = 5$. Look at the equations. Only choices C and D have $k = 5$. The answer is C.

GuidedPractice

6. Which is the graph of $y = -3x^2 + 1$?

F



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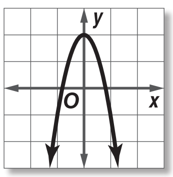

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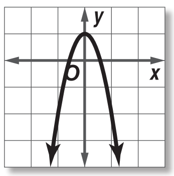

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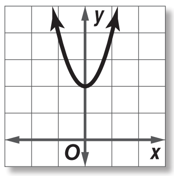

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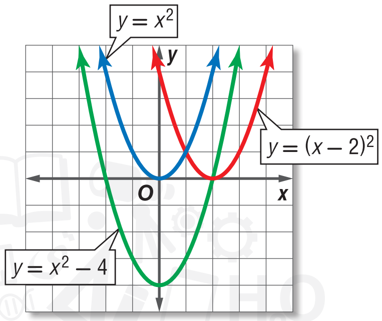
Recall that a family of graphs is a group of graphs that display one or more similar characteristics. The parent graph is the simplest graph in the family. For the family of quadratic functions, y = x 2 is the parent graph. Other graphs in the family of quadratic functions, such as y = (x - 2) 2 and y = x 2 - 4, can be drawn by transforming the graph of y = x 2.



3 Write Quadratic Functions in Vertex Form

Each function above is written in **vertex form**, $y = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola, $x = h$ is the axis of symmetry, and a determines the shape of the parabola and the direction in which it opens.

When a quadratic function is in the form $y = ax^2 + bx + c$, you can complete the square to write the function in vertex form. If the coefficient of the quadratic term is not 1, then factor the coefficient from the quadratic and linear terms *before* completing the square. After completing the square and writing the function in vertex form, the value of k indicated a minimum value if $a < 0$ or a maximum value if $a > 0$.

Example 7 Write Functions in Vertex Form

Write each function in vertex form.

a. $y = x^2 + 6x - 5$

$y = x^2 + 6x - 5$	Original function
$y = (x^2 + 6x + 9) - 5 - 9$	Complete the square.
$y = (x + 3)^2 - 14$	Simplify.

b. $y = -2x^2 + 8x - 3$

$y = -2x^2 + 8x - 3$	Original function
$y = -2(x^2 - 4x) - 3$	Group $ax^2 + bx$ and factor, dividing by a.
$y = -2(x^2 - 4x + 4) - 3 - (-2)(4)$	Complete the square.
$y = -2(x - 2)^2 + 5$	Simplify.

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1A. $y = x^2 + 4x + 6$

1B. $y = 2x^2 - 12x + 17$

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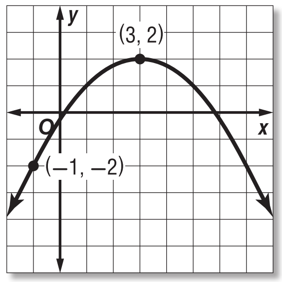
If the vertex and one additional point on the graph of a parabola are known, you can write the equation of the parabola in vertex form.

Standardized Test Example 8 Write an Equation Given a Graph

Which is an equation of the function shown in the graph?



A $y = -4(x - 3)^2 + 2$

B $y = -\frac{1}{4}(x - 3)^2 + 2$

C $y = \frac{1}{4}(x + 3)^2 - 2$

D $y = 4(x + 3)^2 - 2$

Read the Test Item

You are given a graph of a parabola with the vertex and a point on the graph labeled. You need to find an equation of the parabola.

Solve the Test Item

The vertex of the parabola is at (3, 2), so $h = 3$ and $k = 2$. Since $(-1, -2)$ is a point on the graph, let $x = -1$ and $y = -2$. Substitute these values into the vertex form of the equation and solve for a.

$y = a(x - h)^2 + k$	Vertex form
$-2 = a(-1 - 3)^2 + 2$	Substitute −2 for *y*, −1 for *x*, 3 for *h* and 2 for *k*.
$-2 = a(16) + 2$	Simplify.
$-4 = 16a$	Subtract 2 from each side.
$-\frac{1}{4} = a$	Divide each side by 16.

The equation of the parabola in vertex form is $y = -\frac{1}{4}(x - 3)^2 + 2$.

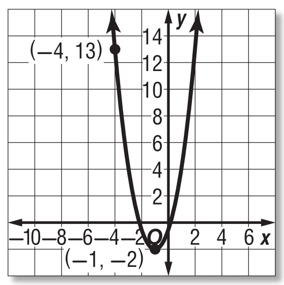
The answer is B.

Test-Taking Tip

The Meaning of *a* The sign of *a* in vertex form does not determine the width of the parabola. The sign indicates whether the parabola opens up or down. The width of a parabola is determined by the absolute value of *a*.

Guided Practice

2. Which is an equation of the function shown in the graph?



F $y = \frac{9}{25}(x - 1)^2 + 2$

G $y = \frac{3}{5}(x + 1)^2 - 2$

H $y = \frac{5}{3}(x + 1)^2 - 2$

J $y = \frac{25}{9}(x - 1)^2 + 2$

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4 Transformations of Quadratic Graphs

You learned how different transformations affect the graphs of parent functions. The following summarizes these transformations for quadratic functions.

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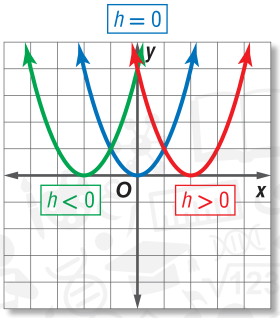
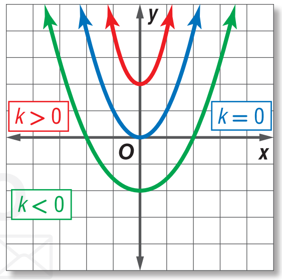
ConceptSummary Transformations of Quadratic Functions

$f(x) = a(x - h)^2 + k$

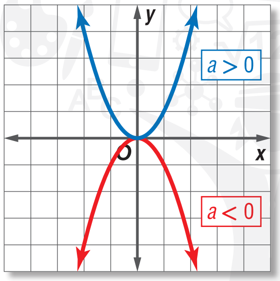
h, Horizontal Translation
h units to the right if h is positive
$|h|$ units to the left if h is negative



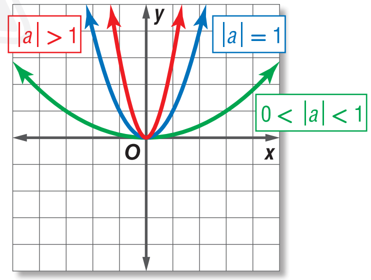
k, Vertical Translation
k units up if k is positive
$|k|$ units down if k is negative



a, Reflection
If $a > 0$, the graph opens up.
If $a < 0$, the graph opens down.



a, Dilation
If $|a| > 1$, the graph is stretched vertically. If $0 < |a| < 1$, the graph is compressed vertically.



StudyTip

Absolute Value

$0 < |a| < 1$ means that a is a rational number between 0 and 1, such as $\frac{3}{4}$, or a rational number between −1 and 0, such as −0.3.

Example 9 Graph Equations in Vertex Form

Graph $y = 4x^2 - 16x - 40$.

Step 1 Rewrite the equation in vertex form.

$y = 4x^2 - 16x - 40$ Original equation

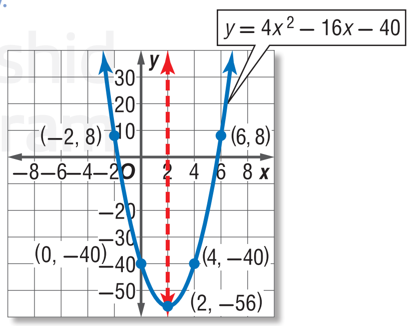
$y = 4(x^2 - 4x) - 40$ Distributive Property

$y = 4(x - 4x + 4) - 40 - 4(4)$ Complete the square.

$y = 4(x - 2)^2 - 56$ Simplify.

Step 2 The vertex is at $(2, -56)$. The axis of symmetry is $x = 2$. Because $a = 4$, the graph is narrower than the graph of $y = x^2$.

Step 3 Plot additional points to help you complete the graph.



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3A. $y = (x - 3)^2 - 2$

3B. $y = 0.25(x + 1)^2$

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Check Your Understanding

Example 7 Write each function in vertex form.

1. $y = x^2 + 6x + 2$ **2.** $y = -2x^2 + 8x - 5$ **3.** $y = 4x^2 + 24x + 24$

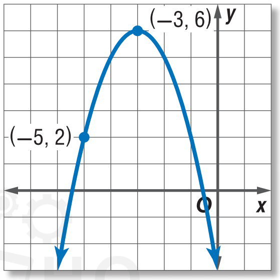
Example 8 **4.** **MULTIPLE CHOICE** Which function is shown in the graph?

A $y = -(x + 3)^2 + 6$

B $y = -(x - 3)^2 - 6$

C $y = -2(x + 3)^2 + 6$

D $y = -2(x - 3)^2 - 6$



Example 9 Graph each function.

5. $y = (x - 3)^2 - 4$ **6.** $y = -2x^2 + 5$ **7.** $y = \frac{1}{2}(x + 6)^2 - 8$

Practice and Problem Solving

Example 7 Write each function in vertex form.

8. $y = x^2 + 9x + 8$ **9.** $y = x^2 - 6x + 3$ **10.** $y = -2x^2 + 5x$

11. $y = x^2 + 2x + 7$ **12.** $y = -3x^2 + 12x - 10$ **13.** $y = x^2 + 8x + 16$

14. $y = 2x^2 - 4x - 3$ **15.** $y = 3x^2 + 10x$ **16.** $y = x^2 - 4x + 9$

17. $y = -4x^2 - 24x - 15$ **18.** $y = x^2 - 12x + 36$ **19.** $y = -x^2 - 4x - 1$

Example 8 **20.** **FIREWORKS** During a National Day fireworks show, the height h in meters of a specific rocket after t seconds can be modeled by $h = -4.9(t - 4)^2 + 80$. Graph the function.

21. **FINANCIAL LITERACY** A bicycle rental shop rents an average of 120 bicycles per week and charges AED 25 per day. The manager estimates that there will be 15 additional bicycles rented for each AED 1 reduction in the rental price. The maximum income the manager can expect can be modeled by $y = -15x^2 + 255x + 3000$, where y is the weekly income and x is the number of bicycles rented. Write this function in vertex form. Then graph.

Example 9 Graph each function.

22. $y = (x - 5)^2 + 3$ **23.** $y = 9x^2 - 8$ **24.** $y = -2(x - 5)^2$

25. $y = \frac{1}{10}(x + 6)^2 + 6$ **26.** $y = -3(x - 5)^2 - 2$ **27.** $y = -\frac{1}{4}x^2 - 5$

28. $y = 2x^2 + 10$ **29.** $y = -(x + 3)^2$ **30.** $y = \frac{1}{6}(x - 3)^2 - 10$

31. $y = (x - 9)^2 - 7$ **32.** $y = -\frac{5}{8}x^2 - 8$ **33.** $y = -4(x - 10)^2 - 10$

34. **MODELING** A sailboard manufacturer uses an automated process to manufacture the masts for its sailboards. The function $f(x) = \frac{1}{250}x^2 + \frac{3}{5}x$ is programmed into a computer to make one such mast.

a. Write the quadratic function in vertex form. Then graph the function.

b. Describe how the manufacturer can adjust the function to make its masts with a greater or smaller curve.

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Write an equation in vertex form for each parabola.

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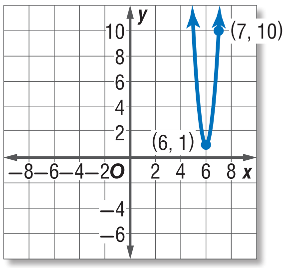

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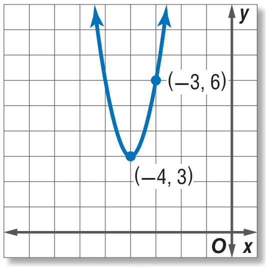

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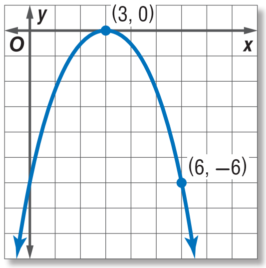

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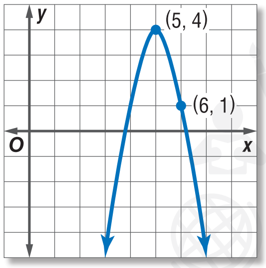

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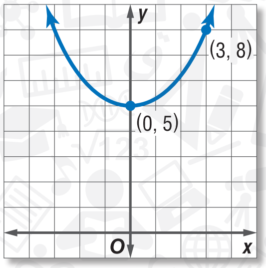

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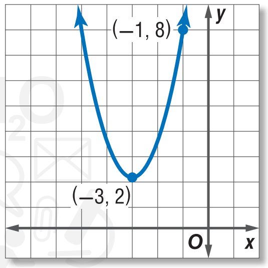

Write each function in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

41. $3x^2 - 4x = 2 + y$

42. $-2x^2 + 7x = y - 12$

43. $-x^2 - 4.7x = y - 2.8$

44. $x^2 + 1.4x - 1.2 = y$

45. $x^2 - \frac{2}{3}x - \frac{26}{9} = y$

46. $x^2 + 7x + \frac{49}{4} = y$

47 **CARS** The formula $S(t) = \frac{1}{2}at^2 + v_0t$ can be used to determine the position $S(t)$ of an object after t seconds at a rate of acceleration a with initial velocity v_0. Nabila's car can accelerate 0.002 km per second squared.

a. Express $S(t)$ in vertex form as she accelerates from 35 km per hour to enter highway traffic.

b. How long will it take Nabila to match the average speed of highway traffic of 68 km per hour? (*Hint*: Use *acceleration* • *time* = *velocity*.)

c. If the entrance ramp is $\frac{1}{8}$-km long, will Nabila have sufficient time to match the average highway speed? Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

48. **OPEN ENDED** Write an equation for a parabola that has been translated, compressed, and reflected in the x-axis.

49. **CHALLENGE** Explain how you can find an equation of a parabola using the coordinates of three points on the graph.

50. **CHALLENGE** Write the standard form of a quadratic function $ax^2 + bx + c = y$ in vertex form. Identify the vertex and the axis of symmetry.

51. **REASONING** Describe the graph of $f(x) = a(x - h)^2 + k$ when $a = 0$. Is the graph the same as that of $g(x) = ax^2 + bx + c$ when $a = 0$? Explain.

52. **ARGUMENTS** Explain how the graph of $y = x^2$ can be used to graph any quadratic function. Include a description of the effects produced by changing a, h, and k in the equation $y = a(x - h)^2 + k$, and a comparison of the graph of $y = x^2$ and the graph of $y = a(x - h)^2 + k$ using values you choose for a, h, and k.

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Standardized Test Practice

53. Flowering bushes need a mixture of 70% soil and 30% vermiculite. About how many buckets of vermiculite should you add to 20 buckets of soil?

A 6.0 **C** 14.0
B 8.0 **D** 24.0

54. **SAT/ACT** The sum of the integers x and y is 495. The units digit of x is 0. If x is divided by 10, the result is equal to y. What is the value of x?

F 40 **J** 250
G 45 **K** 450
H 245

55. What is the solution set of the inequality $|4x - 1| < 9$?

A $\{x \mid 2.5 < x \text{ or } x < -2\}$
B $\{x \mid x < 2.5\}$
C $\{x \mid x > -2\}$
D $\{x \mid -2 < x < 2.5\}$

56. **SHORT RESPONSE** At your store, you buy wrenches for AED 30.00 a dozen and sell them for AED 3.50 each. What is the percent markup for the wrenches?

Spiral Review

Solve each equation by using the method of your choice. Find exact solutions. (Lesson 1-4)

57. $4x^2 + 15x = 21$ **58.** $-3x^2 + 19 = 5x$ **59.** $6x - 5x^2 + 9 = 3$

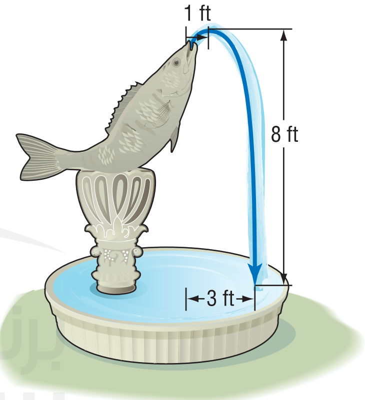
Simplify. (Lesson 1-3)

60. $(3 + 4i)(5 - 2i)$ **61.** $(\sqrt{6} + i)(\sqrt{6} - i)$ **62.** $\frac{1 + i}{1 - i}$ **63.** $\frac{4 - 3i}{1 + 2i}$

64. **FOUNTAINS** The height of a fountain's water stream can be modeled by a quadratic function. Suppose the water from a jet reaches a maximum height of 8 feet at a distance 1 foot away from the jet. (Lesson 1-5)

a. If the water lands 3 feet away from the jet, find a quadratic function that models the height $h(d)$ of the water at any given distance d feet from the jet. Then compare the graph of the function to the parent function.

b. Suppose a worker increases the water pressure so that the stream reaches a maximum height of 12.5 feet at a distance of 15 inches from the jet. The water now lands 3.75 feet from the jet. Write a new quadratic function for $h(d)$. How do the changes in h and k affect the shape of the graph?



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Skills Review

Determine whether the given value satisfies the inequality.

65. $3x^2 - 5 > 6;\ x = 2$ **66.** $-2x^2 + x - 1 < 4;\ x = -2$ **67.** $4x^2 + x - 3 \leq 36;\ x = 3$

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EXTEND 1-7

Algebra Lab
Quadratics and Rate of Change

You have learned that a linear function has a constant rate of change. In this lab, you will investigate the rate of change for quadratic functions.

Activity Determine Rate of Change

Consider $f(x) = 0.1875x^2 - 3x + 12$.

Step 1 Make a table like the one below. Use values from 0 through 16 for x.

x	0	1	2	3	...	16
y	12	9.1875	6.75			
First-Order Differences						
Second-Order Differences						

Step 2 Find each y-value. For example, when $x = 1$, $y = 0.1875(1)^2 - 3(1) + 12$ or 9.1875.

Step 3 Graph the ordered pairs (x, y). Then connect the points with a smooth curve. Notice that the function *decreases* when $0 < x < 8$ and *increases* when $8 < x < 16$.

Step 4 The rate of change from one point to the next can be found by using the slope formula. From (0, 12) to (1, 9.1875), the slope is $\frac{9.1875 - 12}{1 - 0}$ or -2.8125. This is the first-order difference at $x = 1$. Complete the table for all the first-order differences. Describe any patterns in the differences.

Step 5 The second-order differences can be found by subtracting consecutive first-order differences. For example, the second-order difference at $x = 2$ is found by subtracting the first order difference at $x = 1$ from the first-order difference at $x = 2$. Describe any patterns in the differences.

Exercises

For each function make a table of values for the given x-values. Graph the function. Then determine the first-order and second-order differences.

1. $y = -x^2 + 2x - 1$ for $x = -3, -2, -1, 0, 1, 2, 3$

2. $y = 0.5x^2 + 2x - 2$ for $x = -5, -4, -3, -2, -1, 0, 1$

3. $y = -3x^2 - 18x - 26$ for $x = -6, -5, -4, -3, -2, -1, 0$

4. **MAKE A CONJECTURE** Repeat the activity for a cubic function. At what order difference would you expect $g(x) = x^4$ to be constant? $h(x) = x^n$?

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LESSON 1-8

Quadratic Inequalities

Then

- You solved linear inequalities.

Now

- **1** Graph quadratic inequalities in two variables.
- **2** Solve quadratic inequalities in one variable.

Why?

- A water balloon launched from a slingshot can be represented by several different quadratic equations and inequalities.

Suppose the height of a water balloon $h(t)$ in meters above the ground t seconds after being launched is modeled by the quadratic function $h(t) = -4.9t^2 + 32t + 1.2$. You can solve a quadratic inequality to determine how long the balloon will be a certain distance above the ground.

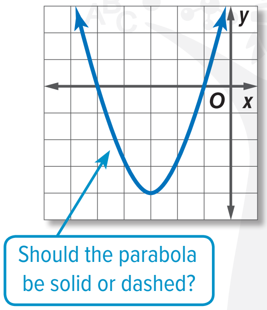
NewVocabulary
quadratic inequality

Mathematical Practices
Make sense of problems and persevere in solving them.

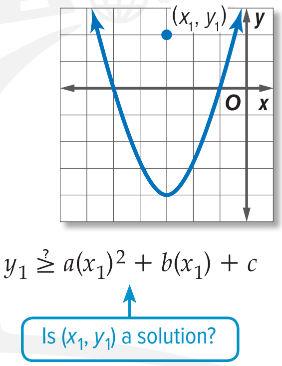
1 Graph Quadratic Inequalities

You can graph **quadratic inequalities** in two variables by using the same techniques used to graph linear inequalities in two variables.

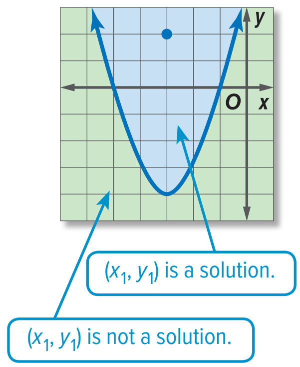
Step 1 Graph the related function.



Step 2 Test a point not on the parabola.



Step 3 Shade accordingly.



Example 1 Graph a Quadratic Inequality

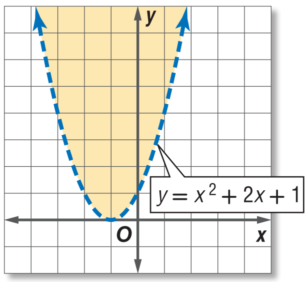
Graph $y > x^2 + 2x + 1$.

Step 1 Graph the related function, $y = x^2 + 2x + 1$. The parabola should be dashed.

Step 2 Test a point not on the graph of the parabola.

$$y > x^2 + 2x + 1$$
$$-1 \overset{?}{>} 0^2 + 2(0) + 1$$
$$-1 \not> 1$$

So, $(0, -1)$ is *not* a solution of the inequality.



Step 3 Shade the region that does not contain the point $(0, -1)$.

GuidedPractice

Graph each inequality.

1A. $y \leq x^2 + 2x + 4$

1B. $y < -2x^2 + 3x + 5$

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2 Solve Quadratic Inequalities

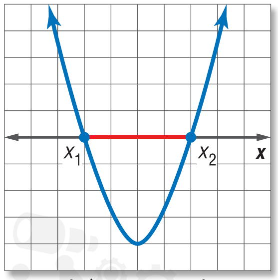
Quadratic inequalities in one variable can be solved using the graphs of the related quadratic functions.

$ax^2 + bx + c < 0$

Graph $y = ax^2 + bx + c$ and identify the x-values for which the graph lies *below* the x-axis.

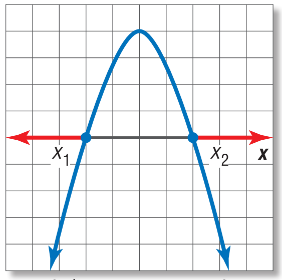
For $\leq$, include the x-intercepts in the solution.

$a > 0$



$\{x \mid x_1 < x < x_2\}$

$a < 0$



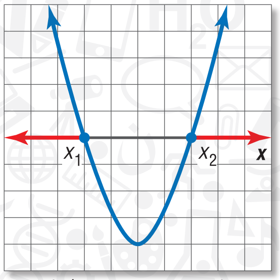
$\{x \mid x < x_1 \text{ or } x > x_2\}$

$ax^2 + bx + c > 0$

Graph $y = ax^2 + bx + c$ and identify the x-values for which the graph lies *above* the x-axis.

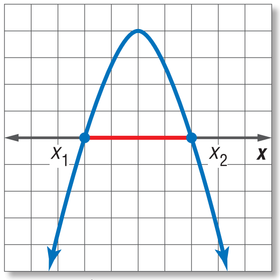
For $\geq$, include the x-intercepts in the solution.

$a > 0$



$\{x \mid x < x_1 \text{ or } x > x_2\}$

$a < 0$



$\{x \mid x_1 < x < x_2\}$

StudyTip

Solving Quadratic Inequalities by Graphing
A precise graph of the related quadratic function is not necessary since the zeros of the function were found algebraically.

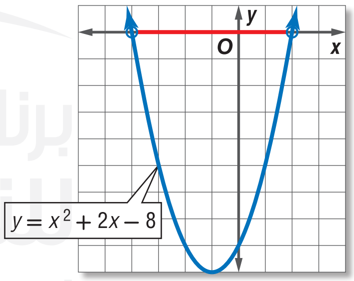
Example 2 Solve $ax^2 + bx + c < 0$ by Graphing

Solve $x^2 + 2x - 8 < 0$ by graphing.

The solution consists of x-values for which the graph of the related function lies *below* the x-axis. Begin by finding the roots of the related function.

$x^2 + 2x - 8 = 0$	Related equation
$(x - 2)(x + 4) = 0$	Factor.
$x - 2 = 0$ or $x + 4 = 0$	Zero Product Property
$x = 2$ $\quad$ $x = -4$	Solve each equation.

Sketch the graph of a parabola that has x-intercepts at -4 and 2. The graph should open up because $a > 0$.



The graph lies below the x-axis between $x = -4$ and $x = 2$. Thus, the solution set is $\{x \mid -4 < x < 2\}$ or $(-4, 2)$.

CHECK Test one value of x less than -4, one between -4 and 2, and one greater than 2 in the original inequality.

Test $x = -6$.	Test $x = 0$.	Test $x = 5$.
$x^2 + 2x - 8 < 0$	$x^2 + 2x - 8 < 0$	$x^2 + 2x - 8 < 0$
$(-6)^2 + 2(-6) - 8 \stackrel{?}{<} 0$	$0^2 + 2(0) - 8 \stackrel{?}{<} 0$	$5^2 + 2(5) - 8 \stackrel{?}{<} 0$
$16 < 0$ ✗	$-8 < 0$ ✓	$27 < 0$ ✗

GuidedPractice

Solve each inequality by graphing.

2A. $0 > x^2 + 5x - 6$

2B. $-x^2 + 3x + 10 \leq 0$

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Example 3 Solve $ax^2 + bx + c \geq 0$ by Graphing

Solve $2x^2 + 4x - 5 \geq 0$ by graphing.

The solution consists of x-values for which the graph of the related function lies *on and above* the x-axis. Begin by finding the roots of the related function.

$2x^2 + 4x - 5 = 0$ — Related equation

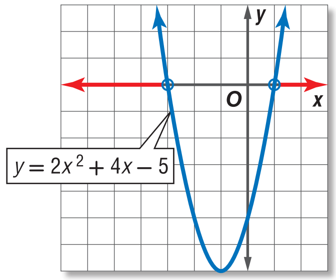
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Use the Quadratic Formula

$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-5)}}{2(2)}$ — Replace a with 4, b with 2, and c with −5.

$x = \frac{-4 + \sqrt{56}}{4}$ or $x = \frac{-4 - \sqrt{56}}{4}$ — Simplify and write as two equations.

≈ 0.87 $\qquad \approx -2.87$ — Simplify.

Sketch the graph of a parabola with x-intercepts at -2.87 and 0.87. The graph opens up since $a > 0$. The graph lies on and above the x-axis at about $x \leq -2.87$ and $x \geq 0.87$. Therefore, the solution is approximately $\{x \mid x \leq -2.87 \text{ or } x \geq 0.87\}$ or $(-\infty, -2.87] \cup [0.87, \infty)$.



GuidedPractice

Solve each inequality by graphing.

3A. $x^2 - 6x + 2 > 0$

3B. $-4x^2 + 5x + 7 \geq 0$

Real-world problems can be solved by graphing quadratic inequalities.

Real-WorldLink

It takes just milliseconds for a water balloon to break. A high-speed camera can capture the impact on the fluid before gravity makes it fall.

Source: NASA

RubberBall/Alamy

Real-World Example 4 Solve a Quadratic Inequality

WATER BALLOONS Refer to the beginning of the lesson. At what time will a water balloon be within 3 meters of the ground after it has been launched?

The function $h(t) = -4.9t^2 + 32t + 1.2$ describes the height of the water balloon. Therefore, you want to find the values of t for which $h(t) \leq 3$.

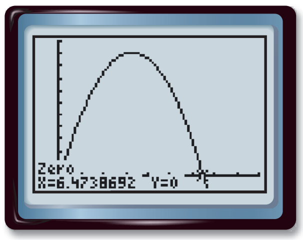
$h(t) \leq 3$ — Original inequality

$-4.9t^2 + 32t + 1.2 \leq 3$ — $h(t) = -4.9t^2 + 32t + 1.2$

$-4.9t^2 + 32t - 1.8 \leq 0$ — Subtract 3 from each side.

Graph the related function $y = -4.9x^2 + 32x - 1.8$ using a graphing calculator. The zeros of the function are about 0.06 and 6.47, and the graph lies below the x-axis when $x < 0.06$ and $x > 6.47$.

So, the water balloon is within 3 meters of the ground during the first 0.06 second after being launched and again after about 6.47 seconds until it hits the ground.



[−1, 9] scl: 1 by [−5, 55] scl: 5

GuidedPractice

4. ROCKETS The height $h(t)$ of a model rocket in feet t seconds after its launch can be represented by the function $h(t) = -16t^2 + 82t + 0.25$. During what interval is the rocket at least 100 feet above the ground?

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StudyTip

Solving Quadratic Inequalities Algebraically The solution set of a quadratic inequality is all real numbers when all three test points satisfy the inequality. It is the empty set when none of the test points satisfy the inequality.

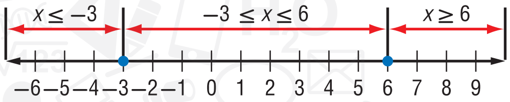
Example 5 Solve a Quadratic Inequality Algebraically

Solve $x^2 - 3x \le 18$ algebraically.

Step 1 Solve the related quadratic equation $x^2 - 3x = 18$.

$x^2 - 3x = 18$	Related quadratic equation
$x^2 - 3x - 18 = 0$	Subtract 18 from each side.
$(x + 3)(x - 6) = 0$	Factor.
$x + 3 = 0$ or $x - 6 = 0$	Zero Product Property
$x = -3$ $\quad$ $x = 6$	Solve each equation.

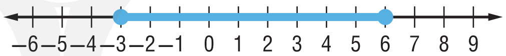
Step 2 Plot −3 and 6 on a number line. Use dots since these values are solutions of the original inequality. Notice that the number line is divided into three intervals.



Step 3 Test a value from each interval to see if it satisfies the original inequality.

$x \le -3$	$-3 \le x \le 6$	$x \ge 6$
Test $x = -5$.	Test $x = 0$.	Test $x = 8$.
$x^2 - 3x \le 18$	$x^2 - 3x \le 18$	$x^2 - 3x \le 18$
$(-5)^2 - 3(-5) \overset{?}{\le} 18$	$(0)^2 - 3(0) \overset{?}{\le} 18$	$(8)^2 - 3(8) \overset{?}{\le} 18$
$40 \not\le 18$	$0 \le 18$	$40 \not\le 18$

The solution set is $\{x \mid -3 \le x \le 6\}$ or $[-3, 6]$.



GuidedPractice

Solve each inequality algebraically.

5A. $x^2 + 5x < -6$ $\qquad$ **5B.** $x^2 + 11x + 30 \ge 0$

Check Your Understanding

Example 1 **Graph each inequality.**

1. $y \le x^2 - 8x + 2$ $\qquad$ **2.** $y > x^2 + 6x - 2$ $\qquad$ **3.** $y \ge -x^2 + 4x + 1$

Examples 2–3 **SENSE-MAKING** **Solve each inequality by graphing.**

4. $0 < x^2 - 5x + 4$ $\qquad$ **5.** $x^2 + 8x + 15 < 0$

6. $-2x^2 - 2x + 12 \ge 0$ $\qquad$ **7.** $0 \ge 2x^2 - 4x + 1$

Example 4 **8. FOOTBALL** A midfielder kicks a ball toward the goal during a match. The height of the ball in feet above the ground $h(t)$ at time t can be represented by $h(t) = -0.1t^2 + 2.4t + 1.5$. If the height of the goal is 8 feet, at what time during the kick will the ball be able to enter the goal?

Example 5 **Solve each inequality algebraically.**

9. $x^2 + 6x - 16 < 0$ $\qquad$ **10.** $x^2 - 14x > -49$

11. $-x^2 + 12x \ge 28$ $\qquad$ **12.** $x^2 - 4x \le 21$

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Practice and Problem Solving

Example 1 **Graph each inequality.**

13. $y \geq x^2 + 5x + 6$
14. $x^2 - 2x - 8 < y$
15. $y \leq -x^2 - 7x + 8$
16. $-x^2 + 12x - 36 > y$
17. $y > 2x^2 - 2x - 3$
18. $y \geq -4x^2 + 12x - 7$

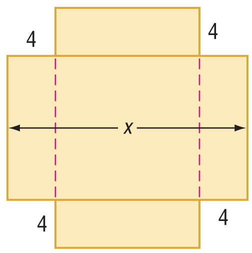
Examples 2–3 **Solve each inequality by graphing.**

19. $x^2 - 9x + 9 < 0$
20. $x^2 - 2x - 24 \leq 0$
21. $x^2 + 8x + 16 \geq 0$
22. $x^2 + 6x + 3 > 0$
23. $0 > -x^2 + 7x + 12$
24. $-x^2 + 2x - 15 < 0$
25. $4x^2 + 12x + 10 \leq 0$
26. $-3x^2 - 3x + 9 > 0$
27. $0 > -2x^2 + 4x + 4$
28. $3x^2 + 12x + 36 \leq 0$
29. $0 \leq -4x^2 + 8x + 5$
30. $-2x^2 + 3x + 3 \leq 0$

Example 4

31. **ARCHITECTURE** An arched entry of a room is shaped like a parabola that can be represented by the equation $f(x) = -x^2 + 6x + 1$. How far from the sides of the arch is its height at least 7 feet?

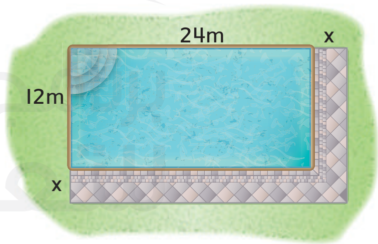
32. **MANUFACTURING** A box is formed by cutting 4 cm squares from each corner of a square piece of cardboard and then folding the sides. If $V(x) = 4x^2 - 64x + 256$ represents the volume of the box, what should the dimensions of the original piece of cardboard be if the volume of the box cannot exceed 750 cubic cm?



Example 5 **Solve each inequality algebraically.**

33. $x^2 - 9x < -20$
34. $x^2 + 7x \geq -10$
35. $2 > x^2 - x$
36. $-3 \leq -x^2 - 4x$
37. $-x^2 + 2x \leq -10$
38. $-6 > x^2 + 4x$
39. $2x^2 + 4 \geq 9$
40. $3x^2 + x \geq -3$
41. $-4x^2 + 2x < 3$
42. $-11 \geq -2x^2 - 5x$
43. $-12 < -5x^2 - 10x$
44. $-3x^2 - 10x > -1$

45. **PERSEVERANCE** A family is adding a deck along two sides of their swimming pool. The deck width will be the same on both sides and the total area of the pool and deck cannot exceed 750 square meters.

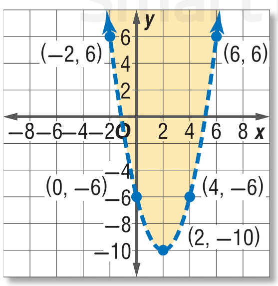


a. Graph the quadratic inequality.

b. Determine the possible widths of the deck.

Write a quadratic inequality for each graph.

46.



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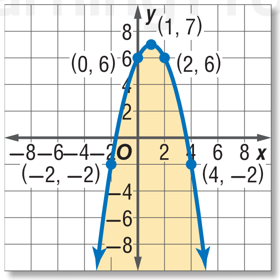

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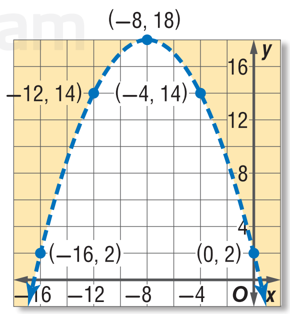

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Solve each quadratic inequality by using a graph, a table, or algebraically.

49. $-2x^2 + 12x < -15$

50. $5x^2 + x + 3 \geq 0$

51 $11 \leq 4x^2 + 7x$

52. $x^2 - 4x \leq -7$

53. $-3x^2 + 10x < 5$

54. $-1 \geq -x^2 - 5x$

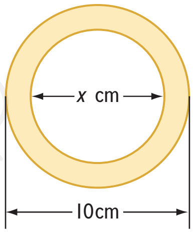
55. **BUSINESS** An electronics manufacturer uses the function $P(x) = x(-27.5x + 3520) + 20{,}000$ to model their monthly profits when selling x thousand digital audio players.

a. Graph the quadratic inequality for a monthly profit of at least AED 100,000.

b. How many digital audio players must the manufacturer sell to earn a profit of at least AED 100,000 in a month?

c. Suppose the manufacturer has an additional monthly expense of AED 25,000. Explain how this affects the graph of the profit function. Then determine how many digital audio players the manufacturer needs to sell to have at least AED 100,000 in profits.

56. **UTILITIES** A contractor is installing drain pipes for a shopping center's parking lot. The outer diameter of the pipe is to be 10 cm. The cross sectional area of the pipe must be at least 35 square cm and should not be more than 42 square cm.



a. Graph the quadratic inequalities.

b. What thickness of drain pipe can the contractor use?

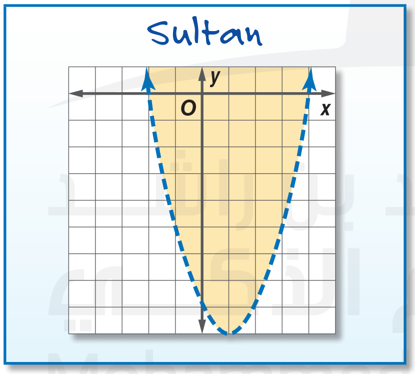
H.O.T. Problems Use Higher-Order Thinking Skills

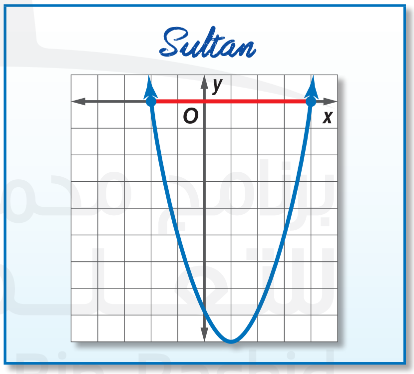
57. **OPEN ENDED** Write a quadratic inequality for each condition.

a. The solution set is all real numbers.

b. The solution set is the empty set.

58. **CRITIQUE** Sultan and Adnan used a graph to solve the quadratic inequality $x^2 - 2x - 8 > 0$. Is either of them correct? Explain.





59. **REASONING** Are the boundaries of the solution set of $x\ 2 + 4x - 12 \leq 0$ twice the value of the boundaries of $\frac{1}{2}x^2 + 2x - 6 \leq 0$? Explain.

60. **REASONING** Determine if the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

The intersection of $y \leq -ax^2 + c$ *and* $y \geq ax^2 - c$ *is the empty set.*

61. **CHALLENGE** Graph the intersection of the graphs of $y \leq -x^2 + 4$ and $y \geq x^2 - 4$.

62. **WRITING IN MATH** How are the techniques used when solving quadratic inequalities and quadratic equations similar? different?

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Standardized Test Practice

63. GRIDDED RESPONSE You need to seed an area that is 80 feet by 40 feet. Each bag of seed can cover 25 square yards of land. How many bags of seed will you need?

64. SAT/ACT The product of two integers is between 107 and 116. Which of the following *cannot* be one of the integers?

A 5
B 10
C 12
D 15
E 23

65 PROBABILITY Five students are to be arranged side by side with the tallest student in the center and the two shortest students on the ends. If no two students are the same height, how many different arrangements are possible?

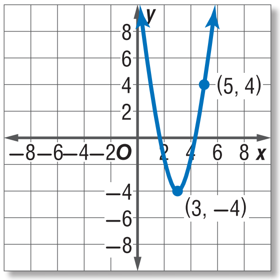
F 2
G 4
H 5
J 6

66. SHORT RESPONSE Simplify $\frac{5+i}{6-3i}$.

Spiral Review

Write an equation in vertex form for each parabola. (Lesson 1-8)

67.



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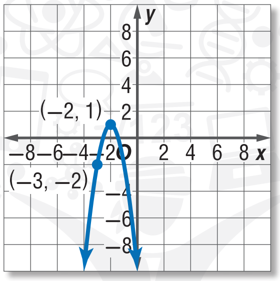

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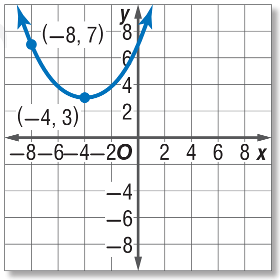

Complete parts a and b for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots. (Lesson 1-7)

70. $4x^2 + 7x - 3 = 0$

71. $-3x^2 + 2x - 4 = 9$

72. $6x^2 + x - 4 = 12$

73. GAS MILEAGE The gas mileage y in km per liter for a certain vehicle is given by the equation $y = 10 + 0.9x - 0.01x^2$, where x is the speed of the vehicle between 10 and 75 km per hour. Find the range of speeds that would give a gas mileage of at least 25 km per liter. (Lesson 1-6)

Write each equation in vertex form, if not already in that form. Identify the vertex, axis of symmetry, and direction of opening. Then graph the function. (Lesson 1-8)

74. $y = -6(x + 2)^2 + 3$

75. $y = -\frac{1}{3}x^2 + 8x$

76. $y = (x - 2)^2 - 2$

77. $y = 2x^2 + 8x + 10$

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Skills Review

Use the Distributive Property to find each product.

78. $-6(x - 4)$

79. $8(w + 3x)$

80. $-4(-2y + 3z)$

81. $-1(c - d)$

82. $0.5(5x + 6y)$

83. $-3(-6y - 4z)$

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EXTEND 1-8

Graphing Technology Lab
More Quadratic Inequalities

Complete the following for more practice with graphing inequalities.

Activity 1 Shade Outside a Parabola

Graph $y \geq -4x^2 + 12x - 7$ in a standard viewing window.

First, clear all functions from the **Y=** list.

To graph $y \geq -4x^2 + 12x - 7$, enter the equation in the **Y=** list. Then use the left arrow to select =. Press [ENTER] until shading above the line is selected.

KEYSTROKES: [◀] [◀] [ENTER] [ENTER] [▶] [▶] [—] 4 [X,T,θ,*n*] [x^2] [+] 12 [X,T,θ,*n*] [—] 7 [ZOOM] 6

All ordered pairs for which y is *greater than or equal* to $-4x^2 + 12x - 7$ lie *above or on* the line and are solutions.

Activity 2 Shade Inside a Parabola

Graph $y \leq -x^2 - 7x + 8$ in a standard viewing window.

First, clear the graph that is displayed.

KEYSTROKES: [Y=] [CLEAR]

KEYSTROKES: [◀] [◀] [ENTER] [ENTER] [ENTER] [▶] [▶] [—] [X,T,θ,*n*] [x^2] [—] 7 [X,T,θ,*n*] [+] 8 [GRAPH]

All ordered pairs for which y is *less than or equal* to $-x^2 - 7x + 8$ lie *below or on* the line and are solutions.

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Exercises

1. Compare and contrast the two graphs shown above.
2. For each inequality in the Activities above, write and graph a new inequality that will have the reverse portions of the parabola shaded.

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EXTEND 1-8

Graphing Technology Lab
Modeling Motion

Set Up the Lab

- Place a board on a stack of books to create a ramp.
- Connect the data collection device to the graphing calculator. Place at the top of the ramp so that the data collection device can read the motion of the car on the ramp.
- Hold the car still about 15 cm up from the bottom of the ramp and zero the collection device.

Mathematical Practices
Model with mathematics.

Activity

Step 1 One group member should press the button to start collecting data.

Step 2 Another group member places the car at the bottom of the ramp. After data collection begins, gently but quickly push the car so it travels up the ramp toward the motion detector.

Step 3 Stop collecting data when the car returns to the bottom of the ramp. Save the data as Trial 1.

Step 4 Remove one book from the stack. Then repeat the experiment. Save the data as Trial 2. For Trial 3, create a steeper ramp and repeat the experiment.

Analyze the Results

1. What type of function could be used to represent the data? Justify your answer.
2. Use the **CALC** menu to find the vertex of the graph. Record the coordinates in a table like the one at the right.
3. Use the **TRACE** feature of the calculator to find the coordinates of another point on the graph. Then use the coordinates of the vertex and the point to find an equation of the graph.
4. Find an equation for each of the graphs of Trials 2 and 3.
5. How do the equations for Trials 1, 2, and 3 compare? Which graph is widest and which is most narrow? Explain what this represents in the context of the situation. How is this represented in the equations?
6. What do the x-intercepts and vertex of each graph represent?
7. Why were the values of h and k different in each trial?

Trial	Vertex (h, k)	Point (x, y)	Equation
1			
2			
3			

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CHAPTER 1

Study Guide and Review

Study Guide

KeyConcepts

Solving Quadratic Equations (Lessons 1-1)

- Roots of a quadratic equation are the zeros of the related quadratic function. You can find the zeros of a quadratic function by finding the *x*-intercepts of the graph.

Complex Numbers (Lesson 1-2)

- i is the imaginary unit; $i^2 = -1$ and $i = \sqrt{-1}$.

Solving Quadratic Equations (Lessons 1-3)

- Completing the square: **Step 1** Find one half of b, the coefficient of x. **Step 2** Square the result in Step 1. **Step 3** Add the result of Step 2 to $x^2 + bx$.
- Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Transformations of Quadratic Graphs (Lesson 1-4)

- The graph of $y = (x - h)^2 + k$ is the graph of $y = x^2$ translated $|h|$ units left if h is negative or h units right if h is positive and k units up if k is positive or $|k|$ units down if k is negative.
- Consider $y = a(x - h)^2 + k$, $a \neq 0$. If $a > 0$, the graph opens up; if $a < 0$ the graph opens down. If $|a| > 1$, the graph is narrower than the graph of $y = x^2$. If $|a| < 1$, the graph is wider than the graph of $y = x^2$.

Complex Numbers (Lesson 1-5)

- i is the imaginary unit; $i^2 = -1$ and $i = \sqrt{-1}$.

Transformations of Quadratic Graphs (Lesson 1-7)

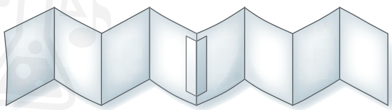
- The graph of $y = (x - h)^2 + k$ is the graph of $y = x^2$ translated $|h|$ units left if h is negative or h units right if h is positive and k units up if k is positive or $|k|$ units down if k is negative.
- Consider $y = a(x - h)^2 + k$, $a \neq 0$. If $a > 0$, the graph opens up; if $a < 0$ the graph opens down. If $|a| > 1$, the graph is narrower than the graph of $y = x^2$. If $|a| < 1$, the graph is wider than the graph of $y = x^2$.

Quadratic Inequalities (Lesson 1-8)

- Graph the related function, test a point *not* on the parabola and determine if it is a solution, and shade accordingly.

FOLDABLES® StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



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KeyVocabulary

complex conjugates	imaginary unit
complex number	pure imaginary number
discriminant	quadratic inequality
factored form	Square Root Property
FOIL method	vertex form
piecewise-defined function	absolute value function
piecewise-linear function	axis of symmetry
Quadratic Formula	completing the square
quadratic function	dilation
reflection	discriminant
standard form	double root
step function	greatest integer function
transformation	maximum
translation	minimum
vertex	parabola

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The axis of symmetry will intersect a parabola in one point called the vertex.
2. A method called FOIL method is used to make a quadratic expression a perfect square in order to solve the related equation.
3. The number $6i$ is called a pure imaginary number.
4. The two numbers $2 + 3i$ and $2 - 3i$ are called complex conjugates.
5. The axis of symmetry of a quadratic function can be found by using the equation $x = -\frac{b}{2a}$.
6. The vertex is the maximum or minimum point of a parabola.
7. The graph of a quadratic function is a straight line.
8. The graph of a quadratic function has a maximum if the coefficient of the x^2-term is positive.
9. A quadratic equation with a graph that has two x-intercepts has one real root.
10. The expression $b^2 - 4ac$ is called the discriminant.
11. A function that is defined differently for different parts of its domain is called a piecewise-defined function.
12. The range of the greatest integer function is the set of all real numbers.
13. The solutions of a quadratic equation are called roots.
14. The graph of the parent function is translated down to form the graph of $f(x) = x^2 + 5$.

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CHAPTER 1

Study Guide and Review *Continued*

1-1 Graphing Quadratic Functions

Consider each equation.

a. **Determine whether the function has a *maximum* or *minimum* value.**

b. **State the maximum or minimum value.**

c. **What are the domain and range of the function?**

11. $y = x^2 - 4x + 4$

12. $y = -x^2 + 3x$

13. $y = x^2 - 2x - 3$

14. $y = -x^2 + 2.$

15. **ROCKET** A toy rocket is launched with an upward velocity of 32 feet per second. The equation $h = -16t^2 + 32t$ gives the height of the ball t seconds after it is launched.

a. Determine whether the function has a *maximum* or *minimum* value.

b. State the maximum or minimum value.

c. State a reasonable domain and range for this situation.

Example 1

Consider $f(x) = x^2 + 6x + 5$.

a. **Determine whether the function has a *maximum* or *minimum* value.**

For $f(x) = x^2 + 6x + 5$, $a = 1$, $b = 6$, and $c = 5$.

Because a is positive, the graph opens up, so the function has a minimum value.

b. **State the *maximum* or *minimum* value of the function.**

The minimum value is the y-coordinate of the vertex.

The x-coordinate of the vertex is $\frac{-b}{2a}$ or $\frac{-6}{2(1)}$ or -3.

$f(x) = x^2 + 6x + 5$	Original function
$f(-3) = (-3)^2 + 6(-3) + 5$	$x = -3$
$f(-3) = -4$	Simplify.

The minimum value is -4.

c. **State the domain and range of the function.**

The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or $\{y|y \geq -4\}$.

1-2 Solving Quadratic Equations by Graphing

Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

16. $x^2 - 3x - 4 = 0$

17. $-x^2 + 6x - 9 = 0$

18. $x^2 - x - 12 = 0$

19. $x^2 + 4x - 3 = 0$

20. $x^2 - 10x = -21$

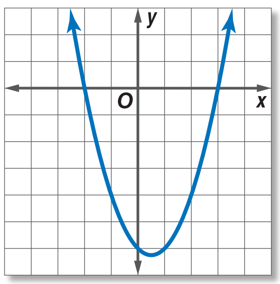
21. $6x^2 - 13x = 15$

22. **NUMBER THEORY** Find two numbers that have a sum of 2 and a product of -15.

Example 2

Solve $x^2 - x - 6 = 0$ by graphing.

Graph the related function $f(x) = x^2 - x - 6$.



The x-intercepts of the graph appear to be at -2 and 3, so the solutions are -2 and 3.

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1-3 Solving Quadratic Equations by Completing the Square

Solve each equation by completing the square. Round to the nearest tenth if necessary.

31. $x^2 + 6x + 9 = 16$

32. $-a^2 - 10a + 25 = 25$

33. $y^2 - 8y + 16 = 36$

34. $y^2 - 6y + 2 = 0$

35. $n^2 - 7n = 5$

36. $-3x^2 + 4 = 0$

37. **NUMBER THEORY** Find two numbers that have a sum of -2 and a product of -48.

Example 3

Solve $x^2 - 16x + 32 = 0$ by completing the square. Round to the nearest tenth if necessary.

Isolate the x^2- and x-terms. Then complete the square and solve.

$x^2 - 16x + 32 = 0$	Original equation
$x^2 - 16x = -32$	Isolate the x^2- and x-terms.
$x^2 - 16x + 64 = -32 + 64$	Complete the square.
$(x - 8)^2 = 32$	Factor.
$x - 8 = \pm\sqrt{32}$	Take the square root.
$x = 8 \pm \sqrt{32}$	Add 8 to each side.
$x = 8 \pm 4\sqrt{2}$	Simplify.

The solutions are about 2.3 and 13.7.

1-4 Solving Quadratic Equations by Factoring

Write a quadratic equation in standard form with the given roots.

5. 5, 6

6. $-3, -7$

7. $-4, 2$

8. $-\frac{2}{3}, 1$

9. $\frac{1}{6}, 5$

10. $-\frac{1}{4}, -1$

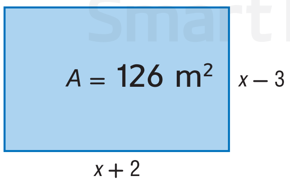
Solve each equation by factoring.

11. $2x^2 - 2x - 24 = 0$

12. $2x^2 - 5x - 3 = 0$

13. $3x^2 - 16x + 5 = 0$

14. Find x and the dimensions of the rectangle below.



Example 4

Write a quadratic equation in standard form with $-\frac{1}{2}$ and 4 as its roots.

$(x - p)(x - q) = 0$	Write the pattern.
$\left[x - \left(-\frac{1}{2}\right)\right](x - 4) = 0$	Replace p with $-\frac{1}{2}$ and q with 4.
$\left(x + \frac{1}{2}\right)(x - 4) = 0$	Simplify.
$x^2 - \frac{7}{2}x - 2 = 0$	Multiply.
$2x^2 - 7x - 4 = 0$	Multiply each side by 2 so that b and c are integers.

Example 5

Solve $2x^2 - 3x - 5 = 0$ by factoring.

$2x^2 - 3x - 5 = 0$	Original equation
$(2x - 5)(x + 1) = 0$	Factor the trinomial.
$2x - 5 = 0$ or $x + 1 = 0$	Zero Product Property
$x = \frac{5}{2}$ $\quad$ $x = -1$	

The solution set is $\left\{-1, \frac{5}{2}\right\}$ or $\left\{x \mid x = -1, \frac{5}{2}\right\}$.

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1-5 Complex Numbers

Simplify.

15. $\sqrt{-8}$

16. $(2 - i) + (13 + 4i)$

17. $(6 + 2i) - (4 - 3i)$

18. $(6 + 5i)(3 - 2i)$

19. **ELECTRICITY** The impedance in one part of a series circuit is $3 + 2j$ ohms, and the impedance in the other part of the circuit is $4 - 3j$ ohms. Add these complex numbers to find the total impedance in the circuit.

Solve each equation.

20. $2x^2 + 50 = 0$

21. $4x^2 + 16 = 0$

22. $3x^2 + 15 = 0$

23. $8x^2 + 16 = 0$

24. $4x^2 + 1 = 0$

Example 6

Simplify $(12 + 3i) - (-5 + 2i)$.

$(12 + 3i) - (-5 + 2i)$

$= [12 - (-5)] + (3 - 2)i$ — Group the real and imaginary parts.

$= 17 + i$ — Simplify.

Example 7

Solve $3x^2 + 12 = 0$.

$3x^2 + 12 = 0$ — Original equation

$3x^2 = -12$ — Subtract 12 from each side.

$x^2 = -4$ — Divide each side by 3.

$x = \pm\sqrt{-4}$ — Square Root Property

$x = \pm 2i$ — $\sqrt{-4} = \sqrt{4} \cdot \sqrt{-1}$

1-6 The Quadratic Formula and the Discriminant

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

25. $x^2 - 10x + 25 = 0$

26. $x^2 + 4x - 32 = 0$

27. $2x^2 + 3x - 18 = 0$

28. $2x^2 + 19x - 33 = 0$

29. $x^2 - 2x + 9 = 0$

30. $4x^2 - 4x + 1 = 0$

31. $2x^2 + 5x + 9 = 0$

32. **PHYSICAL SCIENCE** Layla throws a ball with an initial velocity of 40 feet per second. The equation for the height of the ball is $h = -16t^2 + 40t + 5$, where h represents the height in feet and t represents the time in seconds. When will the ball hit the ground?

Example 8

Solve $x^2 - 4x - 45 = 0$ by using the Quadratic Formula.

In $x^2 - 4x - 45 = 0$, $a = 1$, $b = -4$, and $c = -45$.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Quadratic Formula

$= \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-45)}}{2(1)}$

$= \dfrac{4 \pm 14}{2}$

Write as two equations.

$x = \dfrac{4 + 14}{2}$ or $x = \dfrac{4 - 14}{2}$

$= 9$ $\quad$ $= -5$

The solution set is $\{-5, 9\}$ or $\{x \mid x = -5, 9\}$.

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Practice Test

1-7 Transformations of Quadratic Graphs

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening. Then graph the function.

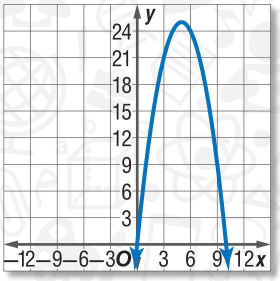
33. $y = -3(x - 1)^2 + 5$

34. $y = 2x^2 + 12x - 8$

35. $y = -\frac{1}{2}x^2 - 2x + 12$

36. $y = 3x^2 + 36x + 25$

37. The graph at the right shows a product of 2 numbers with a sum of 10. Find a function that models this product and use it to determine the two numbers that would give a maximum product.



Example 9

Write the quadratic function $y = 3x^2 + 24x + 15$ in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

$y = 3x^2 + 24x + 15$	Original equation
$y = 3(x^2 + 8x) + 15$	Group and factor.
$y = 3(x^2 + 8x + 16) + 15 - 3(16)$	Complete the square.
$y = 3(x + 4)^2 - 33$	Rewrite $x^2 + 8x + 16$ as a perfect square.

So, $a = 3$, $h = -4$, and $k = -33$. The vertex is at $(-4, -33)$ and the axis of symmetry is $x = -4$. Since a is positive, the graph opens up.

1-8 Quadratic Inequalities

Graph each quadratic inequality.

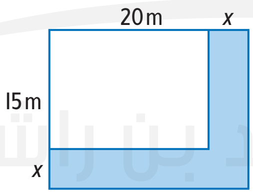
38. $y \geq x^2 + 5x + 4$

39. $y < -x^2 + 5x - 6$

40. $y > x^2 - 6x + 8$

41. $y \leq x^2 + 10x - 4$

42. Omar wants to put a deck along two sides of his garden. The deck width will be the same on both sides and the total area of the garden and deck cannot exceed 500 square meters. How wide can the deck be?



Solve each inequality using a graph or algebraically.

43. $x^2 + 8x + 12 > 0$

44. $6x + x^2 \geq -9$

45. $2x^2 + 3x - 20 > 0$

46. $4x^2 - 3 < -5x$

47. $3x^2 + 4 > 8x$

Example 10

Graph $y > x^2 + 3x + 2$.

Step 1 Graph the related function, $y > x^2 + 3x + 2$. Because the inequality symbol > is used, the parabola should be dashed.

Step 2 Test a point not on the graph of the parabola such as (0, 0).

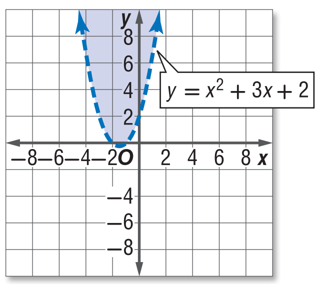
$$y > x^2 + 3x + 2$$

$$(0) \overset{?}{>} (0)^2 + 3(0) + 2$$

$$0 \not> 2$$

So, (0, 0) is not a solution of the inequality.

Step 3 Shade the region that does not contain the point (0, 0).



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CHAPTER 1 Practice Test

Use a table of values to graph the following functions. State the domain and range.

1. $y = x^2 + 2x + 5$

2. $y = 2x^2 - 3x + 1$

Consider $y = x^2 - 7x + 6$.

3. Determine whether the function has a *maximum* or *minimum* value.

4. State the maximum or minimum value.

5. What are the domain and range?

Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

6. $x^2 + 7x + 10 = 0$

7. $x^2 - 5 = -3x$

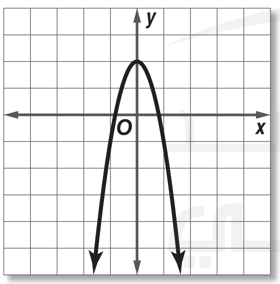
Describe how the graph of each function is related to the graph of $f(x) = x^2$.

8. $g(x) = x^2 - 5$

9. $g(x) = -3x^2$

10. $h(x) = \frac{1}{2}x^2 + 4$

11. **MULTIPLE CHOICE** Which is an equation for the function shown in the graph?



A $y = -3x^2$

B $y = 3x^2 + 1$

C $y = x^2 + 2$

D $y = -3x^2 + 2$

Solve each equation by completing the square.

12. $x^2 + 2x + 5 = 0$

13. $x^2 - x - 6 = 0$

14. $2x^2 - 36 = -6x$

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

15. $x^2 - x - 30 = 0$

16. $x^2 - 10x = -15$

17. $2x^2 + x - 15 = 0$

18. **BASEBALL** Elias hits a baseball into the air. The equation $h = -16t^2 + 60t + 3$ models the height h in feet of the ball after t seconds. How long is the ball in the air?

19. Graph {(−2, 4), (−1, 1), (0, 0), (1, 1), (2, 4)}. Determine whether the ordered pairs represent a *linear function*, a *quadratic function*, or an *exponential function*.

20. Look for a pattern in the table to determine which kind of model best describes the data.

x	0	1	2	3	4
y	1	3	5	7	9

21. **CAR CLUB** The table shows the number of car club members for four consecutive years after it began.

Time (years)	0	1	2	3	4
Members	10	20	40	80	160

a. Determine which model best represents the data.

b. Write a function that models the data.

c. Predict the number of car club members after 6 years.

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Practice Test

22. **MULTIPLE CHOICE** Which equation below has roots at -6 and $\frac{1}{5}$?

   A $0 = 5x^2 - 29x - 6$

   B $0 = 5x^2 + 31x + 6$

   C $0 = 5x^2 + 29x - 6$

   D $0 = 5x^2 - 31x + 6$

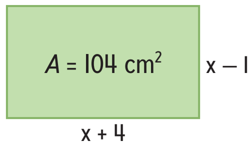
23. **PHYSICS** A ball is thrown into the air vertically with a velocity of 112 feet per second. The ball was released 6 feet above the ground. The height above the ground t seconds after release is modeled by $h(t) = -16t^2 + 112t + 6$.

   a. When will the ball reach 130 feet?

   b. Will the ball ever reach 250 feet? Explain.

   c. In how many seconds after its release will the ball hit the ground?

24. The rectangle below has an area of 104 square cm. Find the value of x and the dimensions of the rectangle.



25. **MULTIPLE CHOICE** Which value of c makes the trinomial $x^2 - 12x + c$ a perfect square trinomial?

   F 6

   G 12

   H 36

   J 144

Solve each inequality by using a graph or algebraically.

26. $x^2 + 6x > -5$

27. $4x^2 - 19x \leq -12$

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Use a Graph

Using a graph can help you solve many different kinds of problems on standardized tests. Graphs can help you solve equations, evaluate functions, and interpret solutions to real-world problems.

Strategies for Using a Graph

Step 1

Read the problem statement carefully.

Ask yourself:

- What am I being asked to solve?
- What information is given in the problem?
- How could a graph help me solve the problem?

Step 2

Create your graph.

- Sketch your graph on scrap paper if appropriate.
- If allowed, you can also use a graphing calculator to create the graph.

Step 3

Solve the problem.

- Use your graph to help you model and solve the problem.
- Check to be sure your answer makes sense.

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Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

The students in Mr. Humaid's physics class built a model rocket. The rocket is launched in a large field with an initial upward velocity of 128 feet per second. The function $h(t) = -16t^2 + 128t$ models the height of the rocket above the ground (in feet) t seconds after it is launched. How long will it take for the rocket to reach its maximum height?

A 4 seconds

B 5 seconds

C 6 seconds

D 8 seconds

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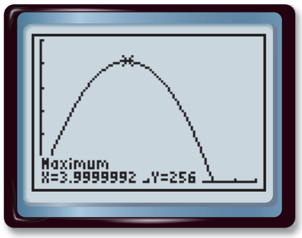
Graphing the quadratic function will allow you to determine the peak height of the rocket and when it occurs. A graphing calculator can help you quickly graph the function and analyze it.

KEYSTROKES: [Y=] [(–)] 16 [X,T,θ,n] [x^2] [+] 128 [X,T,θ,n] [GRAPH]

After graphing the equation, use **maximum** under the **CALC** menu.

Press [2nd] [CALC] 4. Then use [◀] to place the cursor to the left of the maximum point and press [ENTER]. Use [▶] to place the cursor to the right of the maximum point and press [ENTER] [ENTER].

The graph shows that the rocket takes 4 seconds to reach its maximum height of 256 feet. The correct answer is A.



[0, 10] scl: 1 by [0, 300] scl: 50

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

1. What are the roots of $y = 2x^2 + 10x - 48$?

A $-5, 4$

B $-6, 1$

C $-8, 3$

D $2, 3$

2. How many times does the graph of $f(x) = 2x^2 - 3x + 2$ cross the x-axis?

F 0 **H** 2

G 1 **J** 3

3. Which statement best describes the graphs of the two equations?

$$16x - 2y = 24$$
$$12x = 3y - 36$$

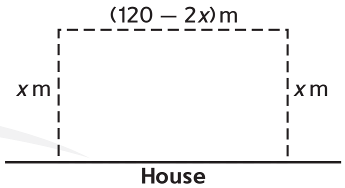
A The lines are parallel.

B The lines are the same.

C The lines intersect in only one point.

D The lines intersect in more than one point, but are not the same.

4. Ahmad is using 120 meters of fencing to enclose a rectangular area for his pet. One side of the enclosure will be her house.



The function $f(x) = x(120 - 2x)$ represents the area of the enclosure. What is the greatest area Ahmad can enclose with the fencing?

F $1650\ m^2$ **H** $1980\ m^2$

G $1800\ m^2$ **J** $2140\ m^2$

5. For which equation is the x-coordinate of the vertex at 4?

A $f(x) = x^2 - 8x + 15$ **C** $f(x) = x^2 + 6x + 8$

B $f(x) = -x^2 - 4x + 12$ **D** $f(x) = -x^2 - 2x + 2$

6. For what value of x does $f(x) = x^2 + 5x + 6$ reach its minimum value?

F -5 **H** $-\frac{5}{2}$

G -3 **J** -2

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CHAPTER 1

Standardized Test Practice

Cumulative and Chapter 1

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. The graph of $g(x) = \frac{2}{5}x^2 - 4x + 2$ is translated down 5 units to produce the graph of the function $h(x)$. Which of the following could be $h(x)$?

F $h(x) = \frac{2}{5}x^2 - 4x + 7$

G $h(x) = \frac{2}{5}x^2 - 4x - 3$

H $h(x) = \frac{2}{5}x^2 - 9x + 2$

J $h(x) = \frac{2}{5}x^2 + x + 2$

2. The function $P(t) = -0.068t^2 + 7.85t + 56$ can be used to approximate the population, in thousands, of a city between 1960 and 2000. The domain t of the function is the number of years since 1960. According to the model, in what year did the population of the city reach 200,000 people?

F 1974

G 1977

H 1981

J 1983

3. What is the effect on the graph of the equation $y = x^2 + 4$ when it is changed to $y = x^2 - 3$?

F The slope of the graph changes.

G The graph widens.

H The graph is the same shape, and the vertex of the graph is moved down.

J The graph is the same shape, and the vertex of the graph is shifted to the left.

4. Which equation will produce the narrowest parabola when graphed?

A $y = 3x^2$

B $y = \frac{3}{4}x^2$

C $y = -6x^2$

D $y = -\frac{3}{4}x^2$

5. Which of the following does *not* accurately describe the graph $y = -2x^2 + 4$?

A The parabola is symmetric about the y-axis.

B The parabola opens downward.

C The parabola has the origin as its vertex.

D The parabola crosses the x-axis in two different places.

6. Which of the following is *not* a factor of $x^4 - 6x^2 - 27$?

A $x^2 + 3$

B $x - 3$

C $x + 3$

D $x^2 - 3$

7. Graph $f(x) \geq |x - 2|$ on a coordinate grid.

8. **GRIDDED RESPONSE** How many times does the graph of $y = x^2 - 4x + 10$ cross the x-axis?

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Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. **GRIDDED RESPONSE** Simplify $-2i \cdot 5i$.

10. Describe the translation of the graph of $y = (x + 5)^2 - 1$ to the graph of $y = (x - 1)^2 + 3$.

Extended Response

Record your answers on a sheet of paper. Show your work.

11. For a given quadratic equation $y = ax^2 + bx + c$, describe what the discriminant $b^2 - 4ac$ tells you about the roots of the equation.

12. Khalifa's father is building a tool chest that is shaped like a rectangular prism. He wants the tool chest to have a surface area of 62 square meters. The height of the chest will be 1 meter shorter than the width. The length will be 3 meters longer than the height.

    a. Sketch a model to represent the problem.

    b. Write a polynomial that represents the surface area of the tool chest.

    c. What are the dimensions of the tool chest?

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