

6 Systems of Linear Equations and Inequalities



Then

- You solved linear equations in one variable.

Now

- In this chapter, you will:
 - Solve systems of linear equations by graphing, substitution, and elimination.
 - Solve systems of linear inequalities by graphing.

Why? ▲

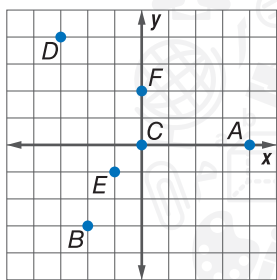
- MUSIC** AED 1500 worth of tickets were sold for a marching band competition. Adult tickets were AED 12 each, and student tickets were AED 8 each. If you knew how many total tickets were sold, you could use a system of equations to determine how many adult tickets and how many student tickets were sold.

Get Ready for the Chapter

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Name the ordered pair for each point on the coordinate plane.

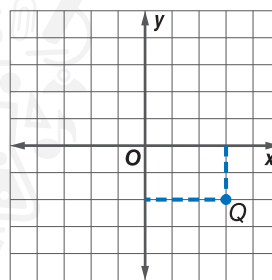


- | | |
|------|------|
| 1. A | 2. D |
| 3. B | 4. C |
| 5. E | 6. F |

QuickReview

Example 1

Name the ordered pair for Q on the coordinate plane.



Follow a vertical line from the point to the x -axis. This gives the x -coordinate, 3.

Follow a horizontal line from the point to the y -axis. This gives the y -coordinate, -2 .

The ordered pair is $(3, -2)$.

Solve each equation or formula for the variable specified.

- $2x + 4y = 12$, for x
- $x = 3y - 9$, for y
- $m - 2n = 6$, for m
- $y = mx + b$, for x
- $P = 2\ell + 2w$, for ℓ
- $5x - 10y = 40$, for y
- GEOMETRY** The formula for the area of a triangle is $A = \frac{1}{2}bh$, where A represents the area, b is the base, and h is the height of the triangle. Solve the equation for b .

Example 2

Solve $12x + 3y = 36$ for y .

$$12x + 3y = 36$$

Original equation

$$12x + 3y - 12x = 36 - 12x$$

Subtract $12x$ from each side.

$$3y = 36 - 12x$$

Simplify.

$$\frac{3y}{3} = \frac{36 - 12x}{3}$$

Divide each side by 3.

$$y = 12 - 4x$$

Simplify.

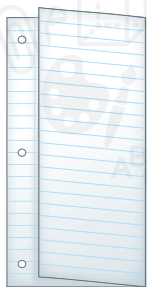
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 6. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

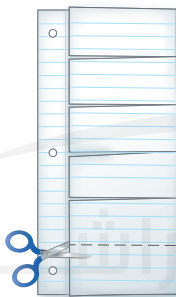
FOLDABLES StudyOrganizer

Linear Functions Make this Foldable to help you organize your Chapter 6 notes about solving systems of equations and inequalities. Begin with a sheet of notebook paper.

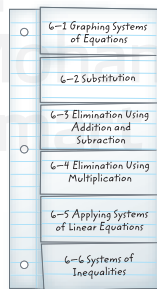
- 1** Fold lengthwise to the holes.



- 2** Cut 6 tabs.



- 3** Label the tabs using the lesson titles.



New Vocabulary

system of equations
consistent
independent
dependent
inconsistent
substitution
elimination
matrix
element
dimension
augmented matrix
row reduction
identity matrix
system of inequalities

Review Vocabulary

domain the set of the first numbers of the ordered pairs in a relation
intersection the graph of a compound inequality containing *and*; the solution is the set of elements common to both graphs
proportion an equation stating that two ratios are equal

Proportion

$$\frac{24}{30} = \frac{4}{5}$$

$\div 6$
 $\div 6$

Graphing Systems of Equations

Then

- You graphed linear equations.

Now

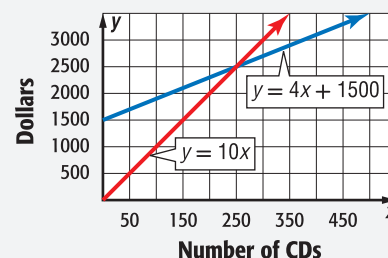
- Determine the number of solutions to a system of linear equations, if any.
- Solve systems of linear equations by graphing and classify them according to the number of solutions.

Why?

- The cost to begin production on a band's CD is AED 1500. Each CD costs AED 4 to produce and will sell for AED 10. The band wants to know how many CDs they will have to sell to earn a profit.

Graphing a system can show when a company makes a profit. The cost of producing the CD can be modeled by the equation $y = 4x + 1500$, where y represents the cost of production and x is the number of CDs produced.

Band's CD Sales



New Vocabulary

system of equations
consistent
independent
dependent
inconsistent

Mathematical Practices

Construct viable arguments and critique the reasoning of others.

Look for and express regularity in repeated reasoning.

- Possible Number of Solutions** The income from the CDs sold can be modeled by the equation $y = 10x$, where y represents the total income of selling the CDs, and x is the number of CDs sold.

If we graph these equations, we can see at which point the band begins making a profit. The point where the two graphs intersect is where the band breaks even. This happens when the band sells 250 CDs. If the band sells more than 250 CDs, they will make a profit.

The two equations, $y = 4x + 1500$ and $y = 10x$, form a **system of equations**. The ordered pair that is a solution of both equations is the solution of the system. A system of two linear equations can have one solution, an infinite number of solutions, or no solution.

- If a system has at least one solution, it is said to be **consistent**. The graphs intersect at one point or are the same line.
- If a consistent system has exactly one solution, it is said to be **independent**. If it has an infinite number of solutions, it is **dependent**. This means that there are unlimited solutions that satisfy both equations.
- If a system has no solution, it is said to be **inconsistent**. The graphs are parallel.

Concept Summary Possible Solutions

Number of Solutions	exactly one	infinite	no solution
Terminology	consistent and independent	consistent and dependent	inconsistent
Graph			

StudyTip

Number of Solutions

When both equations are of the form $y = mx + b$, the values of m and b can determine the number of solutions.

Compare m and b	Number of Solutions
different m values	one
same m value, but different b values	none
same m value, and same b value	infinite

Example 1 Number of Solutions

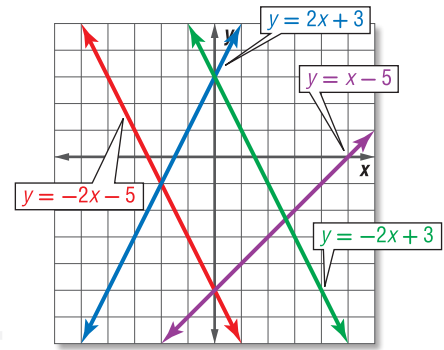
Use the graph at the right to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*.

a. $y = -2x + 3$
 $y = x - 5$

Since the graphs of these two lines intersect at one point, there is exactly one solution. Therefore, the system is consistent and independent.

b. $y = -2x - 5$
 $y = -2x + 3$

Since the graphs of these two lines are parallel, there is no solution of the system. Therefore, the system is inconsistent.



GuidedPractice

1A. $y = 2x + 3$
 $y = -2x - 5$

1B. $y = x - 5$
 $y = -2x - 5$

2 Solve by Graphing One method of solving a system of equations is to graph the equations carefully on the same coordinate grid and find their point of intersection. This point is the solution of the system.

Example 2 Solve by Graphing

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

a. $y = -3x + 10$
 $y = x - 2$

The graphs appear to intersect at the point (3, 1).
 You can check this by substituting 3 for x and 1 for y .

CHECK $y = -3x + 10$ Original equation

$1 \stackrel{?}{=} -3(3) + 10$ Substitution

$1 \stackrel{?}{=} -9 + 10$ Multiply.

$1 = 1$ ✓

$y = x - 2$ Original equation

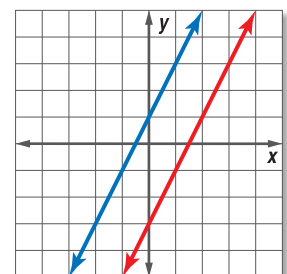
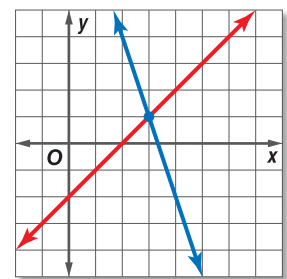
$1 \stackrel{?}{=} 3 - 2$ Substitution

$1 = 1$ ✓ Multiply.

The solution is (3, 1).

b. $2x - y = -1$
 $4x - 2y = 6$

The lines have the same slope but different y -intercepts, so the lines are parallel. Since they do not intersect, there is no solution of this system. The system is inconsistent.



ReviewVocabulary

parallel lines never intersect and have the same slope

GuidedPractice

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

2A. $x - y = 2$
 $3y + 2x = 9$

2B. $y = -2x - 3$
 $6x + 3y = -9$

We can use what we know about systems of equations to solve many real-world problems involving constraints that are modeled by two or more different functions.

Real-World Example 3 Write and Solve a System of Equations

SPORTS The number of girls participating in high school soccer and track and field has steadily increased over the past few years. Use the information in the table to predict the approximate year when the number of girls participating in these two sports will be the same.

High School Sport	Number of Girls Participating in 2008 (thousands)	Average rate of increase (thousands per year)
soccer	345	8
track and field	458	3

Source: National Federation of State High School Associations

Words	Number of girls participating	equals	rate of increase	times	number of years after 2008	plus	number participating in 2008.
Variables	Let y = number of girls competing. Let x = number of years after 2008.						
Equations	Soccer: y	=	8	•	x	+	345
	Track and field: y	=	3	•	x	+	458

Graph $y = 8x + 345$ and $y = 3x + 458$. The graphs appear to intersect at approximately (22.5, 525).

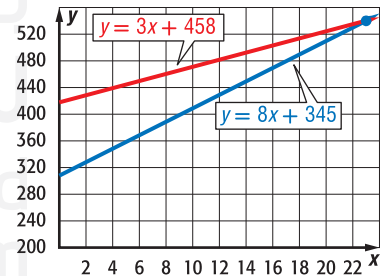
CHECK Use substitution to check this answer.

$$y = 8x + 345 \qquad y = 3x + 458$$

$$525 \stackrel{?}{=} 8(22.5) + 345 \qquad 525 \stackrel{?}{=} 3(22.5) + 458$$

$$525 = 525 \checkmark \qquad 525 \approx 525.5 \checkmark$$

The solution means that approximately 22 years after 2008, or in 2030, the number of girls participating in high school soccer and track and field will be the same, about 525,000.

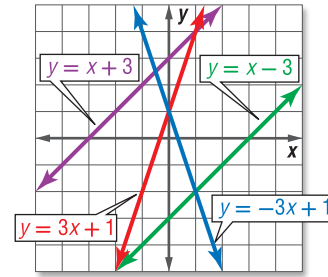


GuidedPractice

3. **VIDEO GAMES** Ibrahim and Ahmed each want to buy a video game. Ibrahim has AED 14 and saves AED 10 a week. Ahmed has AED 26 and saves AED 7 a week. In how many weeks will they have the same amount?

Check Your Understanding

Example 1 Use the graph at the right to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*.

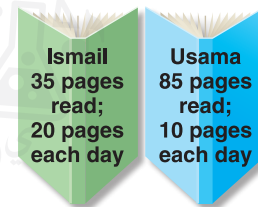


1. $y = -3x + 1$
 $y = 3x + 1$
2. $y = 3x + 1$
 $y = x - 3$
3. $y = x - 3$
 $y = x + 3$
4. $y = x + 3$
 $x - y = -3$
5. $x - y = -3$
 $y = -3x + 1$
6. $y = -3x + 1$
 $y = x - 3$

Example 2 Graph each system and determine the number of solutions that it has. If it has one solution, name it.

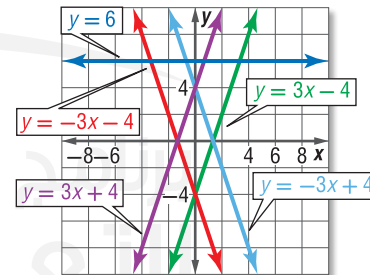
7. $y = x + 4$
 $y = -x - 4$
8. $y = x + 3$
 $y = 2x + 4$

- Example 3** 9. **MODELING** Ismail and Usama are reading a graphic novel.
- a. Write an equation to represent the pages each boy has read.
 - b. Graph each equation.
 - c. How long will it be before Ismail has read more pages than Usama? Check and interpret your solution.



Practice and Problem Solving

Example 1 Use the graph at the right to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*.



10. $y = 6$
 $y = 3x + 4$
11. $y = 3x + 4$
 $y = -3x + 4$
12. $y = -3x + 4$
 $y = -3x - 4$
13. $y = -3x - 4$
 $y = 3x - 4$
14. $3x - y = -4$
 $y = 3x + 4$
15. $3x - y = 4$
 $3x + y = 4$

Example 2 Graph each system and determine the number of solutions that it has. If it has one solution, name it.

16. $y = -3$
 $y = x - 3$
17. $y = 4x + 2$
 $y = -2x - 3$
18. $y = x - 6$
 $y = x + 2$
19. $x + y = 4$
 $3x + 3y = 12$
20. $x - y = -2$
 $-x + y = 2$
21. $x + 2y = 3$
 $x = 5$
22. $2x + 3y = 12$
 $2x - y = 4$
23. $2x + y = -4$
 $y + 2x = 3$
24. $2x + 2y = 6$
 $5y + 5x = 15$

Example 3

- 25. SCHOOL FUNDRAISER** Ayman and Mahmoud are competing to see who can sell the most tickets for the Winter Fundraiser. On Monday, Ayman sold 22 and then sold 30 per day after that. Mahmoud sold 53 on Monday and then sold 20 per day after that.
- Write equations for the number of tickets each person has sold.
 - Graph each equation.
 - Solve the system of equations. Check and interpret your solution.

- 26. MODELING** If x is the number of years since 2000 and y is the percent of people using travel services, the following equations represent the percent of people using travel agents and the percent of people using the Internet to plan travel.

Travel agents: $y = -2x + 30$

Internet: $y = 6x + 41$

- Graph the system of equations.
- Estimate the year travel agents and the Internet were used equally.

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

27. $y = \frac{1}{2}x$

$y = x + 2$

28. $y = 6x + 6$

$y = 3x + 6$

29. $y = 2x - 17$

$y = x - 10$

30. $8x - 4y = 16$

$-5x - 5y = 5$

31. $3x + 5y = 30$

$3x + y = 18$

32. $-3x + 4y = 24$

$4x - y = 7$

33. $2x - 8y = 6$

$x - 4y = 3$

34. $4x - 6y = 12$

$-2x + 3y = -6$

35. $2x + 3y = 10$

$4x + 6y = 12$

36. $3x + 2y = 10$

$2x + 3y = 10$

37. $3y - x = -2$

$y - \frac{1}{3}x = 2$

38. $\frac{8}{5}y = \frac{2}{5}x + 1$

$\frac{2}{5}y = \frac{1}{10}x + \frac{1}{4}$

39. $\frac{1}{3}x + \frac{1}{3}y = 1$

$x + y = 1$

40. $\frac{3}{4}x + \frac{1}{2}y = \frac{1}{4}$

$\frac{2}{3}x + \frac{1}{6}y = \frac{1}{2}$

41. $\frac{5}{6}x + \frac{2}{3}y = \frac{1}{2}$

$\frac{2}{5}x + \frac{1}{5}y = \frac{3}{5}$

- 42. PHOTOGRAPHY** Suppose x represents the number of cameras sold and y represents the number of years since 2000. Then the number of digital cameras sold each year since 2000, in millions, can be modeled by the equation $y = 12.5x + 10.9$. The number of film cameras sold each year since 2000, in millions, can be modeled by the equation $y = -9.1x + 78.8$.

- Graph each equation.
- In which year did digital camera sales surpass film camera sales?
- In what year did film cameras stop selling altogether?
- What are the domain and range of each of the functions in this situation?

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

43. $2y = 1.2x - 10$

$4y = 2.4x$

44. $x = 6 - \frac{3}{8}y$

$4 = \frac{2}{3}x + \frac{1}{4}y$

- 45. WEB SITES** Personal publishing site *Lookatme* had 2.5 million visitors in 2009. Each year after that, the number of visitors rose by 13.1 million. Online auction site *Buyyourstuff* had 59 million visitors in 2009, but each year after that the number of visitors fell by 2 million.
- Write an equation for each of the companies.
 - Make a table of values for 5 years for each of the companies.
 - Graph each equation.
 - When will *Lookatme* and *Buyyourstuff*'s sites have the same number of visitors?
 - Name the domain and range of these functions in this situation.
- 46. MULTIPLE REPRESENTATIONS** In this problem, you will explore different methods for finding the intersection of the graphs of two linear equations.
- Algebraic** Use algebra to solve the equation $\frac{1}{2}x + 3 = -x + 12$.
 - Graphical** Use a graph to solve $y = \frac{1}{2}x + 3$ and $y = -x + 12$.
 - Analytical** How is the equation in part **a** related to the system in part **b**?
 - Verbal** Explain how to use the graph in part **b** to solve the equation in part **a**.

H.O.T. Problems Use Higher-Order Thinking Skills

- 47. ERROR ANALYSIS** Store A is offering a 10% discount on the purchase of all electronics in their store. Store B is offering AED 10 off all the electronics in their store. Asma and Badr are deciding which offer will save them more money. Is either of them correct? Explain your reasoning.

Asma

You can't determine which store has the better offer unless you know the price of the items you want to buy.

Badr

Store A has the better offer because 10% of the sale price is a greater discount than AED 10.

- 48. CHALLENGE** Use graphing to find the solution of the system of equations $2x + 3y = 5$, $3x + 4y = 6$, and $4x + 5y = 7$.
- 49. ARGUMENTS** Determine whether a system of two linear equations with $(0, 0)$ and $(2, 2)$ as solutions *sometimes*, *always*, or *never* has other solutions. Explain.
- 50. WHICH ONE DOESN'T BELONG?** Which one of the following systems of equations doesn't belong with the other three? Explain your reasoning.

$$\begin{aligned} 4x - y &= 5 \\ -2x + y &= -1 \end{aligned}$$

$$\begin{aligned} -x + 4y &= 8 \\ 3x - 6y &= 6 \end{aligned}$$

$$\begin{aligned} 4x + 2y &= 14 \\ 12x + 6y &= 18 \end{aligned}$$

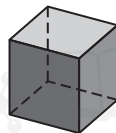
$$\begin{aligned} 3x - 2y &= 1 \\ 2x + 3y &= 18 \end{aligned}$$

- 51. OPEN ENDED** Write three equations such that they form three systems of equations with $y = 5x - 3$. The three systems should be inconsistent, consistent and independent, and consistent and dependent, respectively.
- 52. WRITING IN MATH** Describe the advantages and disadvantages to solving systems of equations by graphing.

Standardized Test Practice

- 53. SHORT RESPONSE** Certain bacteria can reproduce every 20 minutes, doubling the population. If there are 450,000 bacteria in a population at 9:00 A.M., how many bacteria will be in the population at 2:00 P.M.?

- 54. GEOMETRY** An 84-centimeter piece of wire is cut into equal segments and then attached at the ends to form the edges of a cube. What is the volume of the cube?



- A 294 cm^3 C 1158 cm^3
B 343 cm^3 D 2744 cm^3

- 55.** What is the solution of the inequality $-9 < 2x + 3 < 15$?

- F $-x \geq 0$ H $-6 < x < 6$
G $x \leq 0$ J $-5 < x < 5$

- 56.** What is the solution of the system of equations?

$$\begin{aligned} x + 2y &= -1 \\ 2x + 4y &= -2 \end{aligned}$$

- A $(-1, -1)$ C no solution
B $(2, 1)$ D infinitely many solutions

Spiral Review

Graph each inequality. (Lesson 5-6)

57. $3x + 6y > 0$ 58. $4x - 2y < 0$
59. $3y - x \leq 9$ 60. $4y - 3x \geq 12$
61. $y < -4x - 8$ 62. $3x - 1 > y$

- 63. LIBRARY** To get a grant from the city's historical society, the number of history books must be within 25 of 1500. What is the range of the number of historical books that must be in the library? (Lesson 5-5)

- 64. SCHOOL** Amani's scores on three math tests are shown in the table. The fourth and final test of the grading period is tomorrow. She needs an average of at least 92 to receive an A for the grading period. (Lesson 5-3)

- a. If m represents her score on the fourth math test, write an inequality to represent this situation.
b. If Amani wants an A in math, what must she score on the test?
c. Is your solution reasonable? Explain.

Test	Score
1	91
2	95
3	88

Write the slope-intercept form of an equation for the line that passes through the given point and is perpendicular to the graph of the equation.

65. $(-3, 1)$, $y = \frac{1}{3}x + 2$ 66. $(6, -2)$, $y = \frac{3}{5}x - 4$
67. $(2, -2)$, $2x + y = 5$ 68. $(-3, -3)$, $-3x + y = 6$

Skills Review

Find the solution of each equation using the given replacement set.

69. $f - 14 = 8$; $\{12, 15, 19, 22\}$ 70. $15(n + 6) = 165$; $\{3, 4, 5, 6, 7\}$
71. $23 = \frac{d}{4}$; $\{91, 92, 93, 94, 95\}$ 72. $36 = \frac{t-9}{2}$; $\{78, 79, 80, 81\}$

Evaluate each expression if $a = 2$, $b = -3$, and $c = 11$.

73. $a + 6b$ 74. $7 - ab$ 75. $(2c + 3a) \div 4$ 76. $b^2 + (a^3 - 8)5$



You can use a graphing calculator to graph and solve a system of equations.

Activity 1 Solve a System of Equations

Solve the system of equations. State the decimal solution to the nearest hundredth.

$$5.23x + y = 7.48$$

$$6.42x - y = 2.11$$

Mathematical Practices

Use appropriate tools strategically.

Step 1 Solve each equation for y to enter them into the calculator.

$$5.23x + y = 7.48$$

First equation

$$5.23x + y - 5.23x = 7.48 - 5.23x$$

Subtract $5.23x$ from each side.

$$y = 7.48 - 5.23x$$

Simplify.

$$6.42x - y = 2.11$$

Second equation

$$6.42x - y - 6.42x = 2.11 - 6.42x$$

Subtract $6.42x$ from each side.

$$-y = 2.11 - 6.42x$$

Simplify.

$$(-1)(-y) = (-1)(2.11 - 6.42x)$$

Multiply each side by -1 .

$$y = -2.11 + 6.42x$$

Simplify.

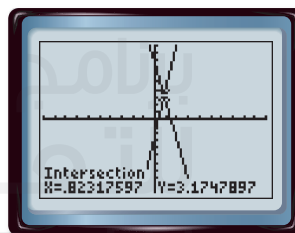
Step 2 Enter these equations in the $Y=$ list and graph in the standard viewing window.

KEYSTROKES: $Y=$ 7.48 $-$ 5.23 X,T,θ,n
 ENTER $(-)$ 2.11 $+$
 6.42 X,T,θ,n ZOOM 6



Step 3 Use the **CALC** menu to find the point of intersection.

KEYSTROKES: 2^{nd} [CALC] 5 ENTER ENTER
 ENTER



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

The solution is approximately $(0.82, 3.17)$.

When you solve a system of equations with $y = f(x)$ and $y = g(x)$, the solution is an ordered pair that satisfies both equations. The solution always occurs when $f(x) = g(x)$. Thus, the x -coordinate of the solution is the value of x where $f(x) = g(x)$.

One method you can use to solve an equation with one variable is by graphing and solving a system of equations based on the equation. To do this, write a system using both sides of the equation. Then use a graphing calculator to solve the system.

Activity 2 Use a System to Solve a Linear Equation

Use a system of equations to solve $5x + 6 = -4$.

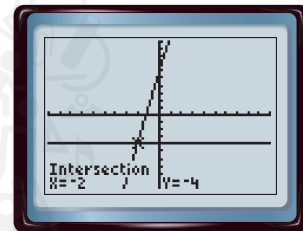
Step 1 Write a system of equations. Set each side of the equation equal to y .

$$\begin{array}{ll} y = 5x + 6 & \text{First equation} \\ y = -4 & \text{Second equation} \end{array}$$

Step 2 Enter these equations in the $Y=$ list and graph.



Step 3 Use the **CALC** menu to find the point of intersection.



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

The solution is -2 .

Exercises

Use a graphing calculator to solve each system of equations. Write decimal solutions to the nearest hundredth.

1. $y = 2x - 3$
 $y = -0.4x + 5$

3. $x + y = 9.35$
 $5x - y = 8.75$

5. $5.2x - y = 4.1$
 $1.5x + y = 6.7$

7. $7x - 2y = 16$
 $11x + 6y = 32.3$

9. $0.62x + 0.35y = 1.60$
 $-1.38x + y = 8.24$

2. $y = 6x + 1$
 $y = -3.2x - 4$

4. $2.32x - y = 6.12$
 $4.5x + y = -6.05$

6. $1.8 = 5.4x - y$
 $y = -3.8 - 6.2x$

8. $3x + 2y = 16$
 $5x + y = 9$

10. $75x - 100y = 400$
 $33x - 10y = 70$

Use a graphing calculator to solve each equation. Write decimal solutions to the nearest hundredth.

11. $4x - 2 = -6$

12. $3 = 1 + \frac{x}{2}$

13. $\frac{x+4}{-2} = -1$

14. $\frac{3}{2}x + \frac{1}{2} = 2x - 3$

15. $4x - 9 = 7 + 7x$

16. $-2 + 10x = 8x - 1$

17. **WRITING IN MATH** Explain why you can solve an equation like $r = ax + b$ by solving the system of equations $y = r$ and $y = ax + b$.

LESSON 6-2 Substitution

Then

- You solved systems of equations by graphing.

Now

- Solve various systems of linear equations by using substitution.
- Solve real-world problems involving systems of equations by using substitution.

Why?

- Two movies were released at the same time. Movie A earned AED 31 million in its opening week, but fell to AED 15 million the following week. Movie B opened earning AED 21 million and fell to AED 11 million the following week. If the earnings for each movie continue to decrease at the same rate, when will they earn the same amount?



New Vocabulary

substitution

Mathematical Practices
Reason abstractly and quantitatively.

1 Solve by Substitution You can use a system of equations to find when the movie earnings are the same. One method of finding an exact solution of a system of equations is called **substitution**.

Key Concept Solving by Substitution

- Step 1** When necessary, solve at least one equation for one variable.
- Step 2** Substitute the resulting expression from Step 1 into the other equation to replace the variable. Then solve the equation.
- Step 3** Substitute the value from Step 2 into either equation, and solve for the other variable. Write the solution as an ordered pair.

Example 1 Solve a System by Substitution

Use substitution to solve the system of equations.

$$\begin{aligned} y &= 2x + 1 \\ 3x + y &= -9 \end{aligned}$$

Step 1 The first equation is already solved for y .

Step 2 Substitute $2x + 1$ for y in the second equation.

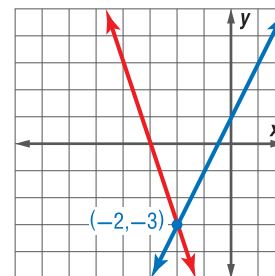
$$\begin{aligned} 3x + y &= -9 && \text{Second equation} \\ 3x + 2x + 1 &= -9 && \text{Substitute } 2x + 1 \text{ for } y. \\ 5x + 1 &= -9 && \text{Combine like terms.} \\ 5x &= -10 && \text{Subtract 1 from each side.} \\ x &= -2 && \text{Divide each side by 5.} \end{aligned}$$

Step 3 Substitute -2 for x in either equation to find y .

$$\begin{aligned} y &= 2x + 1 && \text{First equation} \\ &= 2(-2) + 1 && \text{Substitute } -2 \text{ for } x. \\ &= -3 && \text{Simplify.} \end{aligned}$$

The solution is $(-2, -3)$.

CHECK You can check your solution by graphing.



Guided Practice

1A.
$$\begin{aligned} y &= 4x - 6 \\ 5x + 3y &= -1 \end{aligned}$$

1B.
$$\begin{aligned} 2x + 5y &= -1 \\ y &= 3x + 10 \end{aligned}$$

StudyTip

Slope-Intercept Form

If both equations are in the form $y = mx + b$, they can simply be set equal to each other and then solved for x . The solution for x can then be used to find the value of y .

If a variable is not isolated in one of the equations in a system, solve an equation for a variable first. Then you can use substitution to solve the system.

Example 2 Solve and then Substitute

Use substitution to solve the system of equations.

$$\begin{aligned}x + 2y &= 6 \\ 3x - 4y &= 28\end{aligned}$$

Step 1 Solve the first equation for x since the coefficient is 1.

$$\begin{aligned}x + 2y &= 6 \\ x + 2y - 2y &= 6 - 2y \\ x &= 6 - 2y\end{aligned}$$

First equation

Subtract $2y$ from each side.

Simplify.

Step 2 Substitute $6 - 2y$ for x in the second equation to find the value of y .

$$\begin{aligned}3x - 4y &= 28 \\ 3(6 - 2y) - 4y &= 28 \\ 18 - 6y - 4y &= 28 \\ 18 - 10y &= 28 \\ 18 - 10y - 18 &= 28 - 18 \\ -10y &= 10 \\ y &= -1\end{aligned}$$

Second equation

Substitute $6 - 2y$ for x .

Distributive Property

Combine like terms.

Subtract 18 from each side.

Simplify.

Divide each side by -10 .

Step 3 Find the value of x .

$$\begin{aligned}x + 2y &= 6 \\ x + 2(-1) &= 6 \\ x - 2 &= 6 \\ x &= 8\end{aligned}$$

First equation

Substitute -1 for y .

Simplify.

Add 2 to each side.

Guided Practice

2A.
$$\begin{aligned}4x + 5y &= 11 \\ y - 3x &= -13\end{aligned}$$

2B.
$$\begin{aligned}x - 3y &= -9 \\ 5x - 2y &= 7\end{aligned}$$

Generally, if you solve a system of equations and the result is a false statement such as $3 = -2$, there is no solution. If the result is an identity, such as $3 = 3$, then there are an infinite number of solutions.

Example 3 No Solution or Infinitely Many Solutions

Use substitution to solve the system of equations.

$$\begin{aligned}y &= 2x - 4 \\ -6x + 3y &= -12\end{aligned}$$

Substitute $2x - 4$ for y in the second equation.

$$\begin{aligned}-6x + 3y &= -12 \\ -6x + 3(2x - 4) &= -12 \\ -6x + 6x - 12 &= -12 \\ -12 &= -12\end{aligned}$$

Second equation

Substitute $2x - 4$ for y .

Distributive Property

Combine like terms.

This statement is an identity. Thus, there are an infinite number of solutions.

StudyTip

Dependent Systems

There are infinitely many solutions of the system in Example 3 because the equations in slope-intercept form are equivalent, and they have the same graph.

GuidedPractice Use substitution to solve each system of equations.

3A. $2x - y = 8$
 $y = 2x - 3$

3B. $4x - 3y = 1$
 $6y - 8x = -2$

2 Solve Real-World Problems

You can use substitution to find the solution of a real-world problem involving constraints modeled by a system of equations.



Real-WorldLink

Sound Engineering Technician

Sound engineering technicians record, synchronize, mix, and reproduce music, voices, and sound effects in recording studios, sporting arenas, and theater, movie, or video productions. They need to have at least a 2-year associate's degree in electronics.

Real-World Example 4 Write and Solve a System of Equations

MUSIC A store sold a total of 125 car stereo systems and speakers in one week. The stereo systems sold for AED 104.95, and the speakers sold for AED 18.95. The sales from these two items totaled AED 6926.75. How many of each item were sold?

Number of Units Sold	c	t	125
Sales (AED)	$104.95c$	$18.95t$	6926.75

Let c = the number of car stereo systems sold, and let t = the number of speakers sold.

So, the two equations are $c + t = 125$ and $104.95c + 18.95t = 6926.75$.

Notice that $c + t = 125$ represents combinations of car stereo systems and speakers with a sum of 125.

The equation $104.95c + 18.95t = 6926.75$ represents the combinations of car stereo systems and speakers with a sales of AED 6926.75. The solution of the system of equations represents the option that meets both of the constraints.

Step 1 Solve the first equation for c .

$$c + t = 125$$

First equation

$$c + t - t = 125 - t$$

Subtract t from each side.

$$c = 125 - t$$

Simplify.

Step 2 Substitute $125 - t$ for c in the second equation.

$$104.95c + 18.95t = 6926.75$$

Second equation

$$104.95(125 - t) + 18.95t = 6926.75$$

Substitute $125 - t$ for c .

$$13,118.75 - 104.95t + 18.95t = 6926.75$$

Distributive Property

$$13,118.75 - 86t = 6926.75$$

Combine like terms.

$$-86t = -6192$$

Subtract 13118.75 from each side.

$$t = 72$$

Divide each side by -86 .

Step 3 Substitute 72 for t in either equation to find the value of c .

$$c + t = 125$$

First equation

$$c + 72 = 125$$

Substitute 72 for t .

$$c = 53$$

Subtract 72 from each side.

The store sold 53 car stereo systems and 72 speakers.

GuidedPractice

- 4. BASEBALL** As of 2009, the New York Yankees and the Cincinnati Reds together had won a total of 32 World Series. The Yankees had won 5.4 times as many as the Reds. How many World Series had each team won?

Check Your Understanding

Examples 1–3 Use substitution to solve each system of equations.

1. $y = x + 5$
 $3x + y = 25$

2. $x = y - 2$
 $4x + y = 2$

3. $3x + y = 6$
 $4x + 2y = 8$

4. $2x + 3y = 4$
 $4x + 6y = 9$

5. $x - y = 1$
 $3x = 3y + 3$

6. $2x - y = 6$
 $-3y = -6x + 18$

Example 4 7. **GEOMETRY** The sum of the measures of angles X and Y is 180° . The measure of angle X is 24° greater than the measure of angle Y.

- Define the variables, and write equations for this situation.
- Find the measure of each angle.

Practice and Problem Solving

Examples 1–3 Use substitution to solve each system of equations.

8. $y = 5x + 1$
 $4x + y = 10$

9. $y = 4x + 5$
 $2x + y = 17$

10. $y = 3x - 34$
 $y = 2x - 5$

11. $y = 3x - 2$
 $y = 2x - 5$

12. $2x + y = 3$
 $4x + 4y = 8$

13. $3x + 4y = -3$
 $x + 2y = -1$

14. $y = -3x + 4$
 $-6x - 2y = -8$

15. $-1 = 2x - y$
 $8x - 4y = -4$

16. $x = y - 1$
 $-x + y = -1$

17. $y = -4x + 11$
 $3x + y = 9$

18. $y = -3x + 1$
 $2x + y = 1$

19. $3x + y = -5$
 $6x + 2y = 10$

20. $5x - y = 5$
 $-x + 3y = 13$

21. $2x + y = 4$
 $-2x + y = -4$

22. $-5x + 4y = 20$
 $10x - 8y = -40$

Example 4 23. **ECONOMICS** In 2000, the demand for nurses was 2,000,000, while the supply was only 1,890,000. The projected demand for nurses in 2020 is 2,810,414, while the supply is only projected to be 2,001,998.

- Define the variables, and write equations to represent these situations.
- Use substitution to determine during which year the supply of nurses was equal to the demand.

24. **REASONING** The table shows the approximate number of tourists in two areas of the world during a recent year and the average rates of change in tourism.

Destination	Number of Tourists	Average Rates of Change in Tourists (millions per year)
South America and the Caribbean	40.3 million	increase of 0.8
Middle East	17.0 million	increase of 1.8

- Define the variables, and write an equation for each region's tourism rate.
- If the trends continue, in how many years would you expect the number of tourists in the regions to be equal?

- 25 SPORTS** The table shows the winning times for the Triathlon World Championship.

Year	Men's	Women's
2000	1:51:39	1:54:43
2009	1:44:51	1:59:14

- The times are in hours, minutes, and seconds. Rewrite the times rounded to the nearest minute.
 - Let the year 2000 be 0. Assume that the rate of change remains the same for years after 2000. Write an equation to represent each of the men's and women's winning times y in any year x .
 - If the trend continues, when would you expect the men's and women's winning times to be the same? Explain your reasoning.
- 26. CONCERT TICKETS** Bilal is buying tickets online for a concert. He finds tickets for himself and his friends for AED 65 each plus a one-time fee of AED 10. Amna is looking for tickets to the same concert. She finds them at another Web site for AED 69 and a one-time fee of AED 13.60.
- Define the variables, and write equations to represent this situation.
 - Create a table of values for 1 to 5 tickets for each person's purchase.
 - Graph each of these equations.
 - Use the graph to determine who received the better deal. Explain why.

H.O.T. Problems Use Higher-Order Thinking Skills

- 27. ERROR ANALYSIS** In the system $a + b = 7$ and $1.29a + 0.49b = 6.63$, a represents kilograms of apples and b represents kilograms of bananas. Jassim and Buthaina are finding and interpreting the solution. Is either of them correct? Explain.

Jassim

$$\begin{aligned}
 1.29a + 0.49b &= 6.63 \\
 1.29a + 0.49(a + 7) &= 6.63 \\
 1.29 + 0.49a + 3.43 &= 6.63 \\
 0.49a &= 3.2 \\
 a &= 1.9 \\
 a + b &= 7, \text{ so } b = 5. \text{ The solution } (2, 5) \\
 &\text{means that 2 kilograms of apples and} \\
 &\text{5 kilograms of bananas were bought.}
 \end{aligned}$$

Buthaina

$$\begin{aligned}
 1.29a + 0.49b &= 6.63 \\
 1.29(7 - b) + 0.49b &= 6.63 \\
 9.03 - 1.29b + 0.49b &= 6.63 \\
 -0.8b &= -2.4 \\
 b &= 3 \\
 \text{The solution } b = 3 \text{ means that} \\
 &\text{3 kilograms of apples and 3 kilograms} \\
 &\text{of bananas were bought.}
 \end{aligned}$$

- 28. PERSEVERANCE** A school fundraiser has 60 volunteers. The ratio of teenage boys to adults is 7:5. Find the number of teenage boy volunteers and the number of adult volunteers.
- 29. REASONING** Compare and contrast the solution of a system found by graphing and the solution of the same system found by substitution.
- 30. OPEN ENDED** Create a system of equations that has one solution. Illustrate how the system could represent a real-world situation and describe the significance of the solution in the context of the situation.
- 31. WRITING IN MATH** Explain how to determine what to substitute when using the substitution method of solving systems of equations.

Standardized Test Practice

32. The debate team plans to make and sell trail mix. They can spend AED 34.

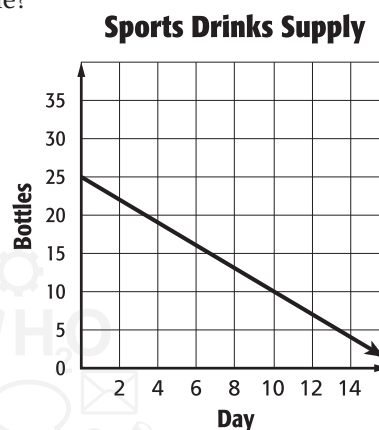
Item	Cost Per Kilogram
sunflower seeds	AED 4.00
raisins	AED 1.50

The kilograms of raisins in the mix are to be 3 times the kilograms of sunflower seeds. Which system can be used to find r , the kilograms of raisins, and p , kilograms of sunflower seeds, they should buy?

- A $3p = r$
 $4p + 1.5r = 34$
- C $3r = p$
 $4p + 1.5r = 34$
- B $3p = r$
 $4r + 1.5p = 34$
- D $3r = p$
 $4r + 1.5p = 34$

33. **GRIDDED RESPONSE** The perimeters of two similar polygons are 250 centimeters and 300 centimeters, respectively. What is the scale factor between the first and second polygons?

34. Based on the graph, which statement is true?



- F Badria started with 30 bottles.
 G On day 10, Badria will have 10 bottles left.
 H Badria will be out of sports drinks on day 14.
 J Badria drank 5 bottles the first two days.
35. If p is an integer, which of the following is the solution set for $2|p| = 16$?
- A $\{0, 8\}$
 B $\{-8, 0\}$
 C $\{-8, 8\}$
 D $\{-8, 0, 8\}$

Spiral Review

Graph each system and determine how many solutions it has. If it has one solution, name it. (Lesson 6-1)

36. $y = -5$
 $3x + y = 1$
37. $x = 1$
 $2x - y = 7$
38. $y = x + 5$
 $y = x - 2$
39. $x + y = 1$
 $3y + 3x = 3$

40. **ENTERTAINMENT** Coach Jamal wants to take the soccer team out for pizza after their game. His budget is at most AED 70.

- a. Using the sign, write an inequality that represents this situation.
 b. Are there any restrictions on the variables? Explain.

Solve each inequality. Check your solution.

41. $6v + 1 \geq -11$
 43. $-11 \geq \frac{2}{5}q + 5$
42. $24 > 18 + 2n$
 45. $-3t + 9 \leq 0$
44. $\frac{a}{8} - 10 > -3$
 46. $54 > -10 - 8n$



Skills Review

Rewrite each product using the Distributive Property. Then simplify.

47. $10b + 5(3 + 9b)$
 49. $7h^2 + 4(3h + h^2)$
48. $5(3t^2 + 4) - 8t$
 50. $-2(7a + 5b) + 5(2a - 7b)$

Elimination Using Addition and Subtraction

Then

- You solved systems of equations by using substitution.

Now

- Solve systems of equations by using elimination with addition.
- Solve systems of equations by using elimination with subtraction.

Why?

- In Chicago, Illinois, there are two more months a when the mean high temperature is below 21°C than there are months b when it is above 21°C . The system of equations, $a + b = 12$ and $a - b = 2$, represents this situation.



New Vocabulary

elimination

Mathematical Practices

Look for and make use of structure.

- Elimination Using Addition** If you add these equations, the variable b will be eliminated. Using addition or subtraction to solve a system is called **elimination**.

Key Concept Solving by Elimination

- Step 1** Write the system so like terms with the same or opposite coefficients are aligned.
- Step 2** Add or subtract the equations, eliminating one variable. Then solve the equation.
- Step 3** Substitute the value from Step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair.

Example 1 Elimination Using Addition

Use elimination to solve the system of equations.

$$4x + 6y = 32$$

$$3x - 6y = 3$$

← **Step 1** $6y$ and $-6y$ have opposite coefficients.

- Step 2** Add the equations.

$$4x + 6y = 32$$

$$(+)\ 3x - 6y = 3$$

$$7x = 35$$

$$\frac{7x}{7} = \frac{35}{7}$$

$$x = 5$$

The variable y is eliminated.

Divide each side by 7.

Simplify.

- Step 3** Substitute 5 for x in either equation to find the value of y .

$$4x + 6y = 32$$

$$4(5) + 6y = 32$$

$$20 + 6y = 32$$

$$20 + 6y - 20 = 32 - 20$$

$$6y = 12$$

$$\frac{6y}{6} = \frac{12}{6}$$

$$y = 2$$

First equation

Replace x with 5.

Multiply.

Subtract 20 from each side.

Simplify.

Divide each side by 6.

Simplify.

- ∴ The solution is $(5, 2)$.

GuidedPractice

1A. $-4x + 3y = -3$
 $4x - 5y = 5$

1B. $4y + 3x = 22$
 $3x - 4y = 14$

We can use elimination to find specific numbers that are described as being related to each other.

Example 2 Write and Solve a System of Equations

Negative three times one number plus five times another number is -11 .
 Three times the first number plus seven times the other number is -1 .
 Find the numbers.

Negative three times one number	plus	five times another number	is	-11 .
$-3x$	+	$5y$	=	-11
Three times the first number	plus	seven times the other number	is	-1 .
$3x$	+	$7y$	=	-1

Steps 1 and 2 Write the equations vertically and add.

$$\begin{array}{r}
 -3x + 5y = -11 \\
 (+) 3x + 7y = -1 \\
 \hline
 12y = -12 \\
 \frac{12y}{12} = \frac{-12}{12} \\
 y = -1
 \end{array}$$

The variable x is eliminated.

Divide each side by 12.

Simplify.

Step 3 Substitute -1 for y in either equation to find the value of x .

$3x + 7y = -1$	Second equation
$3x + 7(-1) = -1$	Replace y with -1 .
$3x + (-7) = -1$	Simplify.
$3x + (-7) + 7 = -1 + 7$	Add 7 to each side.
$3x = 6$	Simplify.
$\frac{3x}{3} = \frac{6}{3}$	Divide each side by 3.
$x = 2$	Simplify.

The numbers are 2 and -1 .

CHECK

$-3x + 5y = -11$	First equation
$-3(2) + 5(-1) \stackrel{?}{=} -11$	Substitute 2 for x and -1 for y .
$-11 = -11$ ✓	Simplify.
$3x + 7y = -1$	Second equation
$3(2) + 7(-1) \stackrel{?}{=} -1$	Substitute 2 for x and -1 for y .
$-1 = -1$ ✓	Simplify.

StudyTip

Coefficients When the coefficients of a variable are the same, subtracting the equations will eliminate the variable. When the coefficients are opposites, adding the equations will eliminate the variable.

Problem-SolvingTip

Perseverance Checking your answers in both equations of a system helps ensure there are no calculation errors.

GuidedPractice

2. The sum of two numbers is -10 . Negative three times the first number minus the second number equals 2. Find the numbers.

2 Elimination Using Subtraction

Sometimes we can eliminate a variable by subtracting one equation from another.

Standardized Test Example 3

Solve the system of equations.

$$2t + 5r = 6$$

$$9r + 2t = 22$$

A $(-7, 15)$

B $(7, \frac{8}{9})$

C $(4, -7)$

D $(4, -\frac{2}{5})$

Read the Test Item

Since both equations contain $2t$, use elimination by subtraction.

Solve the Test Item

Step 1 Subtract the equations.

$$\begin{array}{r} 5r + 2t = 6 \\ (-) 9r + 2t = 22 \\ \hline -4r \quad = -16 \\ r = 4 \end{array}$$

Write the system so like terms are aligned.

The variable t is eliminated.

Simplify.

Step 2 Substitute 4 for r in either equation to find the value of t .

$$5r + 2t = 6$$

First equation

$$5(4) + 2t = 6$$

$$r = 4$$

$$20 + 2t = 6$$

Simplify.

$$20 + 2t - 20 = 6 - 20$$

Subtract 20 from each side.

$$2t = -14$$

Simplify.

$$t = -7$$

Simplify.

The solution is $(4, -7)$. The correct answer is C.

Guided Practice

3. Solve the system of equations.

$$8b + 3c = 11$$

$$8b + 7c = 7$$

F $(1.5, -1)$

G $(1.75, -1)$

H $(1.75, 1)$

J $(1.5, 1)$

Real-World Example 4 Write and Solve a System of Equations

JOBS Hareb and Husam work at an ice cream shop. Hareb earns AED 8.50 per hour and Husam earns AED 7.50 per hour. During a typical week, Hareb and Husam earn AED 299.50 together. One week, Husam doubles his work hours, and the boys earn AED 412. How many hours does each boy work during a typical week?

Understand You know how much Hareb and Husam each earn per hour and how much they earned together.

Plan Let c = Hareb's hours and j = Husam's hours.

Hareb's pay	plus	Husam's pay	equals	AED 299.50.
$8.50c$	+	$7.50j$	=	299.50
Hareb's pay	plus	Husam's pay	equals	AED 412.
$8.50c$	+	$7.50(2j)$	=	412



Real-WorldLink

The five most dangerous jobs for teens are: delivery and other driving jobs, working alone in cash-based businesses, traveling youth crews, cooking, and construction.

Source: National Consumers League

StudyTip

Another Method Instead of subtracting the equations, you could also multiply one equation by -1 and then add the equations.

Solve Subtract the equations to eliminate one of the variables. Then solve for the other variable.

$$8.50c + 7.50j = 299.50$$

Write the equations vertically.

$$(-) 8.50c + 7.50(2)j = 412$$

$$8.50c + 7.50j = 299.50$$

$$(-) 8.50c + 15j = 412$$

Simplify.

$$-7.50j = -112.50$$

Subtract. The variable c is eliminated.

$$\frac{-7.50j}{-7.50} = \frac{-112.50}{-7.50}$$

Divide each side by -7.50 .

$$j = 15$$

Simplify.

Now substitute 15 for j in either equation to find the value of c .

$$8.50c + 7.50j = 299.50$$

First equation

$$8.50c + 7.50(15) = 299.50$$

Substitute 15 for j .

$$8.50c + 112.50 = 299.50$$

Simplify.

$$8.50c = 187$$

Subtract 112.50 from each side.

$$c = 22$$

Divide each side by 8.50.

Check Substitute both values into the other equation to see if the equation holds true. If $c = 22$ and $j = 15$, then $8.50(22) + 15(15)$ or 412.

Hareb works 22 hours, while Husam works 15 hours during a typical week.

GuidedPractice

4. **PARTIES** Hamdah and Houryya are throwing a dinner party for their friend. Hamdah invited 5 fewer friends than Houryya. Together they invited 47 guests. How many guests did each girl invite?

Check Your Understanding

Examples 1, 3 Use elimination to solve each system of equations.

1. $5m - p = 7$

$$7m - p = 11$$

2. $8x + 5y = 38$

$$-8x + 2y = 4$$

3. $7f + 3g = -6$

$$7f - 2g = -31$$

4. $6a - 3b = 27$

$$2a - 3b = 11$$

Example 2 5. **REASONING** The sum of two numbers is 24. Five times the first number minus the second number is 12. What are the two numbers?

Example 4 6. **RECYCLING** The recycling and reuse industry employs approximately 1,025,000 more workers than the waste management industry. Together they provide 1,275,000 jobs. How many jobs does each industry provide?

Practice and Problem Solving

Examples 1, 3 Use elimination to solve each system of equations.

7. $-v + w = 7$

$v + w = 1$

8. $y + z = 4$

$y - z = 8$

9. $-4x + 5y = 17$

$4x + 6y = -6$

10. $5m - 2p = 24$

$3m + 2p = 24$

11. $a + 4b = -4$

$a + 10b = -16$

12. $6r - 6t = 6$

$3r - 6t = 15$

13. $6c - 9d = 111$

$5c - 9d = 103$

14. $11f + 14g = 13$

$11f + 10g = 25$

15. $9x + 6y = 78$

$3x - 6y = -30$

16. $3j + 4k = 23.5$

$8j - 4k = 4$

17. $-3x - 8y = -24$

$3x - 5y = 4.5$

18. $6x - 2y = 1$

$10x - 2y = 5$

Example 2

19. The sum of two numbers is 22, and their difference is 12. What are the numbers?

20. Find the two numbers with a sum of 41 and a difference of 9.

21. Three times a number minus another number is -3 . The sum of the numbers is 11. Find the numbers.

22. A number minus twice another number is 4. Three times the first number plus two times the second number is 12. What are the numbers?

Example 4

23. **TOURS** The Hassans and Hamads are going to Hershey's Really Big 3D Show in Pennsylvania. Find the adult price and the children's price of the show.

Family	Number of Adults	Number of Children	Total Cost
Hassan	2	5	AED 31.65
Hamad	2	3	AED 23.75

Use elimination to solve each system of equations.

24. $4(x + 2y) = 8$

$4x + 4y = 12$

25. $3x - 5y = 11$

$5(x + y) = 5$

26. $4x + 3y = 6$

$3x + 3y = 7$

27. $6x - 7y = -26$

$6x + 5y = 10$

28. $\frac{1}{2}x + \frac{2}{3}y = 2\frac{3}{4}$

$\frac{1}{4}x - \frac{2}{3}y = 6\frac{1}{4}$

29. $\frac{3}{5}x + \frac{1}{2}y = 8\frac{1}{3}$

$-\frac{3}{5}x + \frac{3}{4}y = 8\frac{1}{3}$

30. **SENSE-MAKING** The total height of an office building b and the radio antenna that stands on top of it g is 326.6 meters. The difference in heights between the building and the antenna is 295.4 meters.

a. How tall is the antenna?

b. How tall is the building?

31. **BIKE RACING** Professional Mountain Bike Racing currently has 66 teams. The number of non-U.S. teams is 30 more than the number of U.S. teams.

a. Let x represent the number of non-U.S. teams and y represent the number of U.S. teams. Write a system of equations that represents the number of U.S. teams and non-U.S. teams.

b. Use elimination to find the solution of the system of equations.

c. Interpret the solution in the context of the situation.

d. Graph the system of equations to check your solution.

32. **SHOPPING** Let x represent the number of years since 2004 and y represent the number of catalogs.

Catalogs	Number in 2004	Growth Rate (number per year)
online	7440	1293
print	3805	-1364

Source: MediaPost Publications

- Write a system of equations to represent this situation.
- Use elimination to find the solution to the system of equations.
- Analyze the solution in terms of the situation. Determine the reasonableness of the solution.

33. **MULTIPLE REPRESENTATIONS** Collect 9 25fil coins and 9 paper clips. For this game, you use 9 objects to score points. Each paper clip is worth 1 point and each 25fil coins is worth 3 points. Let p represent the number of 25fil coins and c represent the number of paper clips.

$$9 \text{ points} = \begin{array}{c} \text{2 coins} \\ \text{3 clips} \end{array} + \begin{array}{c} \text{3 clips} \end{array} = 3p + c = 3(2) + 3$$

- Concrete** Choose a combination of 9 objects and find your score.
- Analytical** Write and solve a system of equations to find the number of paper clips and 25fil coins used for 15 points.
- Tabular** Make a table showing the number of paper clips used and the total number of points when the number of 25fil coins is 0, 1, 2, 3, 4, or 5.
- Verbal** Does the result in the table match the results in part **b**? Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

- REASONING** Describe the solution of a system of equations if after you added two equations the result was $0 = 0$.
- REASONING** What is the solution of a system of equations if the sum of the equations is $0 = 2$?
- OPEN ENDED** Create a system of equations that can be solved by using addition to eliminate one variable. Formulate a general rule for creating such systems.
- STRUCTURE** The solution of a system of equations is $(-3, 2)$. One equation in the system is $x + 4y = 5$. Find a second equation for the system. Explain how you derived this equation.
- CHALLENGE** The sum of the digits of a two-digit number is 8. The result of subtracting the units digit from the tens digit is -4 . Define the variables and write the system of equations that you would use to find the number. Then solve the system and find the number.
- WRITING IN MATH** Describe when it would be most beneficial to use elimination to solve a system of equations.

Standardized Test Practice

40. SHORT RESPONSE Khadija is on a train traveling at a speed of 188 km/h between two cities 1128 kilometers apart. If the train has been traveling for an hour, how many more hours is her train ride?

41. GEOMETRY Ms. Khawla wants to tile her rectangular kitchen floor. She knows the dimensions of the floor. Which formula should she use to find the area?

A $A = \ell w$

B $V = Bh$

C $P = 2\ell + 2w$

D $c^2 = a^2 + b^2$

42. If the pattern continues, what is the 8th number in the sequence?

$$2, 3, \frac{9}{2}, \frac{27}{4}, \frac{81}{8}, \dots$$

F $\frac{2187}{64}$

G $\frac{2245}{64}$

H $\frac{2281}{64}$

J $\frac{2445}{64}$

43. What is the solution of this system of equations?

$$x + 4y = 1$$

$$2x - 3y = -9$$

A $(2, -8)$

B $(-3, 1)$

C no solution

D infinitely many solutions

Spiral Review

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions. (Lesson 6-2)

44. $y = 6x$
 $2x + 3y = 40$

45. $x = 3y$
 $2x + 3y = 45$

46. $x = 5y + 6$
 $x = 3y - 2$

47. $y = 3x + 2$
 $y = 4x - 1$

48. $3c = 4d + 2$
 $c = d - 1$

49. $z = v + 4$
 $2z - v = 6$

50. FINANCIAL LITERACY Humaid and Khalid each want to buy a bicycle. Humaid has already saved AED 35 and plans to save AED 10 per week. Khalid has AED 26 and plans to save AED 13 per week. (Lesson 6-1)

- In how many weeks will Humaid and Khalid have saved the same amount of money?
- How much will each person have saved at that time?

51. GEOMETRY A *parallelogram* is a quadrilateral in which opposite sides are parallel. Determine whether ABCD is parallelogram. Explain your reasoning.

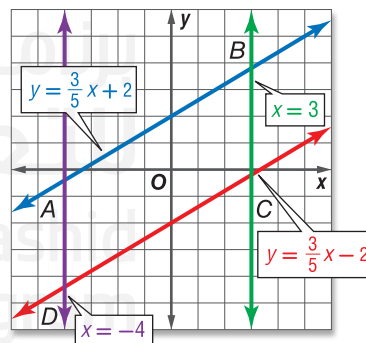
Solve each equation. Check your solution.

52. $6u = -48$

53. $75 = -15p$

54. $\frac{2}{3}a = 8$

55. $-\frac{3}{4}d = 15$



Skills Review

Simplify each expression. If not possible, write *simplified*.

56. $6q - 3 + 7q + 1$

57. $7w^2 - 9w + 4w^2$

58. $10(2 + r) + 3r$

59. $5y - 7(y + 5)$

Elimination Using Multiplication

Then

- You used elimination with addition and subtraction to solve systems of equations.

Now

- Solve systems of equations by using elimination with multiplication.
- Solve real-world problems involving systems of equations.

Why?

- The table shows the number of cars at Scott's Auto Repair Shop for each type of service.

The manager has allotted 1110 minutes for body work and 570 minutes for engine work. The system $3r + 4m = 1110$ and $2r + 2m = 570$ can be used to find the average time for each service.

Item	Repairs	Maintenance
body	3	4
engine	2	2

Mathematical Practices

Make sense of problems and persevere in solving them.

- Elimination Using Multiplication** In the system above, neither variable can be eliminated by adding or subtracting. You can use multiplication to solve.

KeyConcept Solving by Elimination

- Step 1** Multiply at least one equation by a constant to get two equations that contain opposite terms.
- Step 2** Add the equations, eliminating one variable. Then solve the equation.
- Step 3** Substitute the value from Step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair.

Example 1 Multiply One Equation to Eliminate a Variable

Use elimination to solve the system of equations.

$$5x + 6y = -8$$

$$2x + 3y = -5$$

Steps 1 and 2

$$5x + 6y = -8$$

$$2x + 3y = -5$$

Multiply each term by -2 .

$$\begin{array}{r} 5x + 6y = -8 \\ (+) -4x - 6y = 10 \\ \hline x = 2 \end{array}$$

Add.
 y is eliminated.

Step 3 $2x + 3y = -5$

$$2(2) + 3y = -5$$

$$4 + 3y = -5$$

$$3y = -9$$

$$y = -3$$

Second equation

Substitution, $x = 2$

Simplify.

Subtract 4 from each side and simplify.

Divide each side by 3 and simplify.

The solution is $(2, -3)$.

GuidedPractice

1A. $6x - 2y = 10$
 $3x - 7y = -19$

1B. $9r + q = 13$
 $3r + 2q = -4$

Sometimes you have to multiply each equation by a different number in order to solve the system.

Example 2 Multiply Both Equations to Eliminate a Variable

Use elimination to solve the system of equations.

$$4x + 2y = 8$$

$$3x + 3y = 9$$

Method 1 Eliminate x .

$$\begin{array}{rcl} 4x + 2y = 8 & \text{Multiply by 3.} & 12x + 6y = 24 \\ 3x + 3y = 9 & \text{Multiply by } -4. & (+) -12x - 12y = -36 \\ & & -6y = -12 \\ & & \frac{-6y}{-6} = \frac{-12}{-6} \\ & & y = 2 \end{array}$$

Add equations.
 x is eliminated.

Divide each side by -6 .

Simplify.

Now substitute 2 for y in either equation to find the value of x .

$$\begin{array}{rcl} 3x + 3y = 9 & \text{Second equation} & \\ 3x + 3(2) = 9 & \text{Substitute 2 for } y. & \\ 3x + 6 = 9 & \text{Simplify.} & \\ 3x = 3 & \text{Subtract 6 from each side and simplify.} & \\ \frac{3x}{3} = \frac{3}{3} & \text{Divide each side by 3.} & \\ x = 1 & \text{The solution is } (1, 2). & \end{array}$$

Method 2 Eliminate y .

$$\begin{array}{rcl} 4x + 2y = 8 & \text{Multiply by 3.} & 12x + 6y = 24 \\ 3x + 3y = 9 & \text{Multiply by } -2. & (+) -6x - 6y = -18 \\ & & 6x = 6 \\ & & \frac{6x}{6} = \frac{6}{6} \\ & & x = 1 \end{array}$$

Add equations.
 y is eliminated.

Divide each side by 6.

Simplify.

Now substitute 1 for x in either equation to find the value of y .

$$\begin{array}{rcl} 3x + 3y = 9 & \text{Second equation} & \\ 3(1) + 3y = 9 & \text{Substitute 1 for } x. & \\ 3 + 3y = 9 & \text{Simplify.} & \\ 3y = 6 & \text{Subtract 3 from each side and simplify.} & \\ \frac{3y}{3} = \frac{6}{3} & \text{Divide each side by 3.} & \\ y = 2 & \text{Simplify.} & \end{array}$$

The solution is $(1, 2)$, which matches the result obtained with Method 1.

CHECK Substitute 1 for x and 2 for y in the first equation.

$$\begin{array}{rcl} 4x + 2y = 8 & \text{Original equation} & \\ 4(1) + 2(2) \stackrel{?}{=} 8 & \text{Substitute } (1, 2) \text{ for } (x, y). & \\ 4 + 4 \stackrel{?}{=} 8 & \text{Multiply.} & \\ 8 = 8 \checkmark & \text{Add.} & \end{array}$$

Guided Practice

2A. $5x - 3y = 6$
 $2x + 5y = -10$

2B. $6a + 2b = 2$
 $4a + 3b = 8$

StudyTip

Choosing a Variable to Eliminate

Unless the problem is asking for the value of a specific variable, you may use multiplication to eliminate either variable.

Math HistoryLink

Leonardo Pisano

(1170–1250) Leonardo Pisano is better known by his nickname *Fibonacci*. His book introduced the Hindu-Arabic place-valued decimal system. Systems of linear equations are studied in this work.

2 Solve Real-World Problems

Sometimes it is necessary to use multiplication before elimination in real-world problem solving too.

Real-World Example 3 Solve a System of Equations

FLIGHT A personal aircraft traveling with the wind flies 520 kilometers in 4 hours. On the return trip, the airplane takes 5 hours to travel the same distance. Find the speed of the airplane if the air is still.

You are asked to find the speed of the airplane in still air.

Let a = the rate of the airplane if the air is still.

Let w = the rate of the wind.

	r	t	d	$r \cdot t = d$
With the Wind	$a + w$	4	520	$(a + w)4 = 520$
Against the Wind	$a - w$	5	520	$(a - w)5 = 520$

So, our two equations are $4a + 4w = 520$ and $5a - 5w = 520$.

$$\begin{array}{rcl}
 4a + 4w = 520 & \text{Multiply by 5.} & 20a + 20w = 2600 \\
 5a - 5w = 520 & \text{Multiply by 4.} & (+) 20a - 20w = 2080 \\
 \hline
 & & 40a = 4680 \\
 & & \frac{40a}{40} = \frac{4680}{40} \\
 & & a = 117
 \end{array}$$

w is eliminated.
Divide each side by 40.
Simplify.

The rate of the airplane in still air is 117 kilometers per hour.

Guided Practice

3. **CANOEING** A canoeist travels 4 kilometers downstream in 1 hour. The return trip takes the canoeist 1.5 hours. Find the rate of the boat in still water.

Check Your Understanding

Examples 1–2 Use elimination to solve each system of equations.

- $2x - y = 4$
 $7x + 3y = 27$
- $2x + 7y = 1$
 $x + 5y = 2$
- $4x + 2y = -14$
 $5x + 3y = -17$
- $9a - 2b = -8$
 $-7a + 3b = 12$

Example 3

5. **SENSE-MAKING** A kayaking group with a guide travels 16 kilometers downstream, stops for a meal, and then travels 16 kilometers upstream. The speed of the current remains constant throughout the trip. Find the speed of the kayak in still water.



6. **PODCASTS** Khalaf subscribed to 10 podcasts for a total of 340 minutes. He used his two favorite tags, Hobbies and Recreation and Soliloquies. Each of the Hobbies and Recreation episodes lasted about 32 minutes. Each Soliloquies episode lasted 42 minutes. To how many of each tag did Khalaf subscribe?

Practice and Problem Solving

Examples 1–2 Use elimination to solve each system of equations.

7. $x + y = 2$

$-3x + 4y = 15$

9. $x + 5y = 17$

$-4x + 3y = 24$

11. $2x + 5y = 11$

$4x + 3y = 1$

13. $3x + 4y = 29$

$6x + 5y = 43$

15. $8x + 3y = -7$

$7x + 2y = -3$

17. $12x - 3y = -3$

$6x + y = 1$

8. $x - y = -8$

$7x + 5y = 16$

10. $6x + y = -39$

$3x + 2y = -15$

12. $3x - 3y = -6$

$-5x + 6y = 12$

14. $8x + 3y = 4$

$-7x + 5y = -34$

16. $4x + 7y = -80$

$3x + 5y = -58$

18. $-4x + 2y = 0$

$10x + 3y = 8$

Example 3

- 19. NUMBER THEORY** Seven times a number plus three times another number equals negative one. The sum of the two numbers is negative three. What are the numbers?

- 20. FOOTBALL** A field goal is 3 points and the extra point after a touchdown is 1 point. In a recent post-season, Adam Vinatieri of the Indianapolis Colts made a total of 21 field goals and extra point kicks for 49 points. Find the number of field goals and extra points that he made.

Use elimination to solve each system of equations.

21. $2.2x + 3y = 15.25$

$4.6x + 2.1y = 18.325$

22. $-0.4x + 0.25y = -2.175$

$2x + y = 7.5$

23. $\frac{1}{4}x + 4y = 2\frac{3}{4}$

$3x + \frac{1}{2}y = 9\frac{1}{4}$

24. $\frac{2}{5}x + 6y = 24\frac{1}{5}$

$3x + \frac{1}{2}y = 3\frac{1}{2}$

- 25. MODELING** A staffing agency for in-home nurses and support staff places necessary personnel at locations on a daily basis. Each placed nurse works 240 minutes per day at a daily rate of AED 90. Each support staff employee works 360 minutes per day at a daily rate of AED 120.
- On a given day, 3000 total minutes are worked by the nurses and support staff that were placed. Write an equation that represents this relationship.
 - On the same day, earnings for placed nurses and support staff totaled AED 1050. Write an equation that represents this relationship.
 - Solve the system of equations, and interpret the solution in the context of the situation.
- 26. GEOMETRY** The graphs of $x + 2y = 6$ and $2x + y = 9$ contain two of the sides of a triangle. A vertex of the triangle is at the intersection of the graphs.
- What are the coordinates of the vertex?
 - Draw the graph of the two lines. Identify the vertex of the triangle.
 - The line that forms the third side of the triangle is the line $x - y = -3$. Draw this line on the previous graph.
 - Name the other two vertices of the triangle.

- 27. ENTERTAINMENT** At an entertainment center, two groups of people bought batting tokens and miniature golf games, as shown in the table.

Group	Number of Batting Tokens	Number of Miniature Golf Games	Total Cost
A	16	3	AED 30
B	22	5	AED 43

- Define the variables, and write a system of linear equations from this situation.
 - Solve the system of equations, and explain what the solution represents.
- 28. TESTS** Mrs. Rana discovered that she had accidentally reversed the digits of a test score and did not give a student 36 points. Mrs. Rana told the student that the sum of the digits was 14 and agreed to give the student his correct score plus extra credit if he could determine his actual score. What was his correct score?

H.O.T. Problems Use Higher-Order Thinking Skills

- 29. REASONING** Explain how you could recognize a system of linear equations with infinitely many solutions.
- 30. CRITIQUE** Khalifa and Reham are solving a system of equations. Is either of them correct? Explain your reasoning.

Khalifa

$$\begin{array}{r} 2r + 7t = 11 \\ r - 9t = -7 \\ \hline 2r + 7t = 11 \\ (-) 2r - 18t = -14 \\ \hline 25t = 25 \\ t = 1 \\ 2r + 7t = 11 \\ 2r + 7(1) = 11 \\ 2r + 7 = 11 \\ 2r = 4 \\ \frac{2r}{2} = \frac{4}{2} \\ r = 2 \end{array}$$

The solution is (2, 1).

Reham

$$\begin{array}{r} 2r + 7t = 11 \\ (-) r - 9t = -7 \\ \hline r = 18 \\ 2r + 7t = 11 \\ 2(18) + 7t = 11 \\ 36 + 7t = 11 \\ 7t = -25 \\ \frac{7t}{7} = \frac{-25}{7} \\ t = -3.6 \end{array}$$

The solution is (18, -3.6).

- 31. OPEN ENDED** Write a system of equations that can be solved by multiplying one equation by -3 and then adding the two equations together.
- 32. CHALLENGE** The solution of the system $4x + 5y = 2$ and $6x - 2y = b$ is $(3, a)$. Find the values of a and b . Discuss the steps that you used.
- 33. ? WRITING IN MATH** Why is substitution sometimes more helpful than elimination, and vice versa?

Standardized Test Practice

34. What is the solution of this system of equations?

$$\begin{aligned} 2x - 3y &= -9 \\ -x + 3y &= 6 \end{aligned}$$

- A (3, 3) C (-3, 1)
B (-3, 3) D (1, -3)

35. A buffet has one price for adults and another for children. The Rashid family has two adults and three children, and their bill was AED 40.50. The Rasheed family has three adults and one child. Their bill was AED 38. Which system of equations could be used to determine the price for an adult and for a child?

- F $x + y = 40.50$ H $2x + 3y = 40.50$
 $x + y = 38$ $x + 3y = 38$
G $2x + 3y = 40.50$ J $2x + 2y = 40.50$
 $3x + y = 38$ $3x + y = 38$

36. **SHORT RESPONSE** A customer at the paint store has ordered 12 liters of ivy green paint. Khamis mixes the paint in a ratio of 3 parts blue to one part yellow. How many liters of blue paint does she use?

37. **PROBABILITY** The table shows the results of a number cube being rolled. What is the experimental probability of rolling a 3?

Outcome	Frequency
1	4
2	8
3	2
4	0
5	5
6	1

- A $\frac{2}{3}$ B $\frac{1}{3}$ C 0.2 D 0.1

Spiral Review

Use elimination to solve each system of equations. (Lesson 6-3)

38. $f + g = -3$
 $f - g = 1$

39. $6g + h = -7$
 $6g + 3h = -9$

40. $5j + 3k = -9$
 $3j + 3k = -3$

41. $2x - 4z = 6$
 $x - 4z = -3$

42. $-5c - 3v = 9$
 $5c + 2v = -6$

43. $4b - 6n = -36$
 $3b - 6n = -36$

44. **JOBS** Sumaya and Sindiya work at an after-school child care center. Together they cared for 32 children this week. Sumaya cared for 0.6 times as many children as Sindiya. How many children did each girl care for? (Lesson 6-2)

Solve each inequality. Then graph the solution set.

45. $|m - 5| \leq 8$

46. $|q + 11| < 5$

47. $|2w + 9| > 11$

48. $|2r + 1| \geq 9$

Skills Review

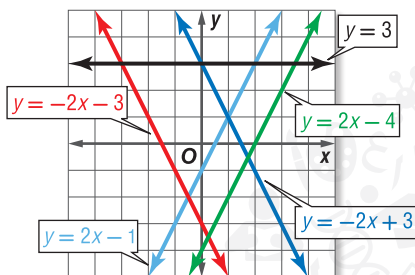
Translate each sentence into a formula.

49. The area A of a triangle equals one half times the base b times the height h .
50. The circumference C of a circle equals the product of 2, π , and the radius r .
51. The volume V of a rectangular box is the length ℓ times the width w multiplied by the height h .
52. The volume of a cylinder V is the same as the product of π and the radius r to the second power multiplied by the height h .
53. The area of a circle A equals the product of π and the radius r squared.
54. Acceleration A equals the increase in speed s divided by time t in seconds.

Mid-Chapter Quiz

Lessons 6-1 through 6-4

Use the graph to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*. (Lesson 6-1)



1. $y = 2x - 1$
 $y = -2x + 3$
2. $y = -2x + 3$
 $y = -2x - 3$

Graph each system and determine the number of solutions that it has. If it has one solution, name it. (Lesson 6-1)

3. $y = 2x - 3$
 $y = x + 4$
4. $x + y = 6$
 $x - y = 4$
5. $x + y = 8$
 $3x + 3y = 24$
6. $x - 4y = -6$
 $y = -1$
7. $3x + 2y = 12$
 $3x + 2y = 6$
8. $2x + y = -4$
 $5x + 3y = -6$

Use substitution to solve each system of equations. (Lesson 6-2)

9. $y = x + 4$
 $2x + y = 16$
10. $y = -2x - 3$
 $x + y = 9$
11. $x + y = 6$
 $x - y = 8$
12. $y = -4x$
 $6x - y = 30$

13. **FOOD** The cost of two meals at a restaurant is shown in the table below. (Lesson 6-2)

Meal	Total Cost
3 tacos, 2 burritos	AED 7.40
4 tacos, 1 burrito	AED 6.45

- a. Define variables to represent the cost of a taco and the cost of a burrito.
- b. Write a system of equations to find the cost of a single taco and a single burrito.
- c. Solve the systems of equations, and explain what the solution means.
- d. How much would a customer pay for 2 tacos and 2 burritos?

14. **AMUSEMENT PARKS** The cost of two groups going to an amusement park is shown in the table. (Lesson 6-3)

Group	Total Cost
4 adults, 2 children	AED 184
4 adults, 3 children	AED 200

- a. Define variables to represent the cost of an adult ticket and the cost of a child ticket.
- b. Write a system of equations to find the cost of an adult ticket and a child ticket.
- c. Solve the system of equations, and explain what the solution means.
- d. How much will a group of 3 adults and 5 children be charged for admission?

15. **MULTIPLE CHOICE** Suha spent AED 16 for 12 pieces of candy to take to a meeting. She has AED 16. Each chocolate bar costs AED 2, and each lollipop costs AED 1. Determine how many of each she bought. (Lesson 6-3)

- A 6 chocolate bars, 6 lollipops
- B 4 chocolate bars, 8 lollipops
- C 7 chocolate bars, 5 lollipops
- D 3 chocolate bars, 9 lollipops

Use elimination to solve each system of equations. (Lessons 6-3 and 6-4)

16. $x + y = 9$
 $x - y = -3$
17. $x + 3y = 11$
 $x + 7y = 19$
18. $9x - 24y = -6$
 $3x + 4y = 10$
19. $-5x + 2y = -11$
 $5x - 7y = 1$

20. **MULTIPLE CHOICE** The Blue Mountain High School Drama Club is selling tickets to their spring performance. Adult tickets are AED 4 and student tickets are AED 1. A total of 285 tickets are sold for AED 765. How many of each type of ticket are sold? (Lesson 6-4)

- F 145 adult, 140 student
- G 120 adult, 165 student
- H 180 adult, 105 student
- J 160 adult, 125 student

Applying Systems of Linear Equations

Then

- You solved systems of equations by using substitution and elimination.

Now

- Determine the best method for solving systems of equations.
- Apply systems of equations.

Why?

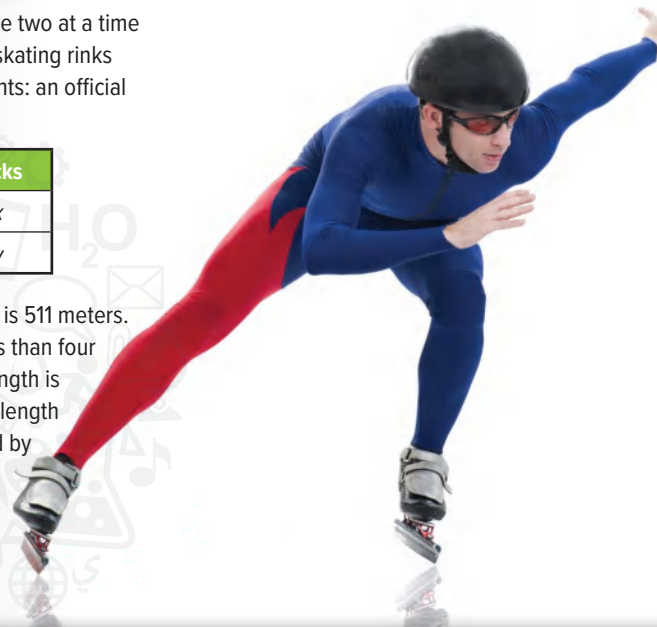
- In speed skating, competitors race two at a time on a double track. Indoor speed skating rinks have two track sizes for race events: an official track and a short track.

Speed Skating Tracks

official track	x
short track	y

The total length of the two tracks is 511 meters. The official track is 44 meters less than four times the short track. The total length is represented by $x + y = 511$. The length of the official track is represented by $x = 4y - 44$.

You can solve the system of equations to find the length of each track.



Mathematical Practices

Reason abstractly and quantitatively.
Model with mathematics.

- Determine the Best Method** You have learned five methods for solving systems of linear equations. The table summarizes the methods and the types of systems for which each method works best.

ConceptSummary Solving Systems of Equations

Method	The Best Time to Use
Graphing	To estimate solutions, since graphing usually does not give an exact solution.
Substitution	If one of the variables in either equation has a coefficient of 1 or -1 .
Elimination Using Addition	If one of the variables has opposite coefficients in the two equations.
Elimination Using Subtraction	If one of the variables has the same coefficient in the two equations.
Elimination Using Multiplication	If none of the coefficients are 1 or -1 and neither of the variables can be eliminated by simply adding or subtracting the equations.

Substitution and elimination are algebraic methods for solving systems of equations. An algebraic method is best for an exact solution. Graphing, with or without technology, is a good way to estimate a solution.

A system of equations can be solved using each method. To determine the best approach, analyze the coefficients of each term in each equation.

Example 1 Choose the Best Method

Determine the best method to solve the system of equations. Then solve the system.

$$\begin{aligned}4x - 4y &= 8 \\ -8x + y &= 19\end{aligned}$$

StudyTip

Reasoning The system of equations in Example 1 can also be solved by using elimination with multiplication. You can multiply the first equation by 2 and then add to eliminate the x -term.

Understand To determine the best method to solve the system of equations, look closely at the coefficients of each term.

Plan Neither the coefficients of x nor y are the same or additive inverses, so you cannot add or subtract to eliminate a variable. Since the coefficient of y in the second equation is 1, you can use substitution.

Solve First, solve the second equation for y .

$$-8x + y = 19 \quad \text{Second equation}$$

$$-8x + y + 8x = 19 + 8x \quad \text{Add } 8x \text{ to each side.}$$

$$y = 19 + 8x \quad \text{Simplify.}$$

Next, substitute $19 + 8x$ for y in the first equation.

$$4x - 4y = 8 \quad \text{First equation}$$

$$4x - 4(19 + 8x) = 8 \quad \text{Substitution}$$

$$4x - 76 - 32x = 8 \quad \text{Distributive Property}$$

$$-28x - 76 = 8 \quad \text{Simplify.}$$

$$-28x - 76 + 76 = 8 + 76 \quad \text{Add } 76 \text{ to each side.}$$

$$-28x = 84 \quad \text{Simplify.}$$

$$\frac{-28x}{-28} = \frac{84}{-28} \quad \text{Divide each side by } -28.$$

$$x = -3 \quad \text{Simplify.}$$

Last, substitute -3 for x in the second equation.

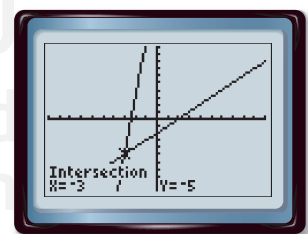
$$-8x + y = 19 \quad \text{Second equation}$$

$$-8(-3) + y = 19 \quad x = -3$$

$$y = -5 \quad \text{Simplify.}$$

The solution of the system of equations is $(-3, -5)$.

Check Use a graphing calculator to check your solution. If your algebraic solution is correct, then the graphs will intersect at $(-3, -5)$.



$[-10, 10]$ scl: 1 $[-10, 10]$ scl: 1

GuidedPractice

1A. $\begin{aligned}5x + 7y &= 2 \\ -2x + 7y &= 9\end{aligned}$

1C. $\begin{aligned}x - y &= 9 \\ 7x + y &= 7\end{aligned}$

1B. $\begin{aligned}3x - 4y &= -10 \\ 5x + 8y &= -2\end{aligned}$

1D. $\begin{aligned}5x - y &= 17 \\ 3x + 2y &= 5\end{aligned}$

2 Apply Systems of Linear Equations

When applying systems of linear equations to problems, it is important to analyze each solution in the context of the situation.



Real-World Example 2 Apply Systems of Linear Equations

PENGUINS Of the 17 species of penguins in the world, the largest species is the emperor penguin. One of the smallest is the Galapagos penguin. The total height of the two penguins is 169 centimeters. The emperor penguin is 22 centimeters more than twice the height of the Galapagos penguin. Find the height of each penguin.

The total height of the two species can be represented by $p + g = 169$, where p represents the height of the emperor penguin and g the height of the Galapagos penguin. Next write an equation to represent the height of the emperor penguin.

Words	The emperor penguin is 22 centimeters more than twice the height of the Galapagos penguin.
Variables	Let p = the height of the emperor penguin and g = the height of the Galapagos penguin.
Equation	$p = 22 + 2g$

First rewrite the second equation.

$$p = 22 + 2g \quad \text{Second equation}$$

$$p - 2g = 22 \quad \text{Subtract } 2g \text{ from each side.}$$

You can use elimination by subtraction to solve this system of equations.

$$\begin{array}{rcl} p + g & = & 169 \quad \text{First equation} \\ (-) p - 2g & = & 22 \quad \text{Subtract the second equation.} \\ \hline 3g & = & 147 \quad \text{Eliminate } p. \\ \frac{3g}{3} & = & \frac{147}{3} \quad \text{Divide each side by 3.} \\ g & = & 49 \quad \text{Simplify.} \end{array}$$

Next substitute 49 for g in one of the equations.

$$\begin{array}{rcl} p & = & 22 + 2g \quad \text{Second equation} \\ & = & 22 + 2(49) \quad g = 49 \\ & = & 120 \quad \text{Simplify.} \end{array}$$

The height of the emperor penguin is 120 centimeters, and the height of the Galapagos penguin is 49 centimeters.

Does the solution make sense in the context of the problem?

Check by verifying the given information. The penguins' heights added together would be $120 + 49$ or 169 centimeters and $22 + 2(49)$ is 120 centimeters.

Guided Practice

- VOLUNTEERING** Salem has volunteered 50 hours and plans to volunteer 3 hours in each coming week. Saeed is a new volunteer who plans to volunteer 5 hours each week. Write and solve a system of equations to find how long it will be before they will have volunteered the same number of hours.

Check Your Understanding

Example 1 Determine the best method to solve each system of equations. Then solve the system.

1. $2x + 3y = -11$
 $-8x - 5y = 9$
2. $3x + 4y = 11$
 $2x + y = -1$
3. $3x - 4y = -5$
 $-3x + 2y = 3$
4. $3x + 7y = 4$
 $5x - 7y = -12$

Example 2 5. **SHOPPING** At a sale, Sultan bought 4 T-shirts and 3 pairs of jeans for AED 181. At the same store, Shaikha bought 1 T-shirt and 2 pairs of jeans for AED 94. The T-shirts were all the same price, and the jeans were all the same price.

- a. Write a system of equations that can be used to represent this situation.
- b. Determine the best method to solve the system of equations.
- c. Solve the system.

Practice and Problem Solving

Example 1 Determine the best method to solve each system of equations. Then solve the system.

6. $-3x + y = -3$
 $4x + 2y = 14$
7. $2x + 6y = -8$
 $x - 3y = 8$
8. $3x - 4y = -5$
 $-3x - 6y = -5$
9. $5x + 8y = 1$
 $-2x + 8y = -6$
10. $y + 4x = 3$
 $y = -4x - 1$
11. $-5x + 4y = 7$
 $-5x - 3y = -14$

Example 2 12. **FINANCIAL LITERACY** For a Future Teachers of America fundraiser, Abdalla sold food as shown in the table. He sold 11 more subs than pizzas and earned a total of AED 233. Write and solve a system of equations to represent this situation. Then describe what the solution means.

Item	Selling Price
pizza	AED 5.00
sub	AED 3.00

13. **DVDs** Shaima has a total of 40 DVDs of movies and television shows. The number of movies is 4 less than 3 times the number of television shows. Write and solve a system of equations to find the numbers of movies and television shows that she has on DVD.
14. **CAVES** The Caverns of Sonora have two different tours: the Crystal Palace tour and the Horseshoe Lake tour. The total length of both tours is 3.25 kilometers. The Crystal Palace tour is a half-kilometer less than twice the distance of the Horseshoe Lake tour. Determine the length of each tour.
15. **MODELING** The *break-even point* is the point at which income equals expenses. Ridgmont High School is paying AED 13,200 for the writing and research of their yearbook plus a printing fee of AED 25 per book. If they sell the books for AED 40 each, how many will they have to sell to break even? Explain.
16. **PAINTBALL** Ayesha and her friends are planning a trip to a paintball park. Find the cost of lunch and the cost of each paintball. What would be the cost for 400 paintballs and lunch?

PAINTBALL
IN THE PARK

- AED 25 for 500 paintballs
- AED 16 for 200 paintballs

Lunch is included

- 17 RECYCLING** Abeer and Alia each recycled aluminum cans and newspaper, as shown in the table. Abeer earned AED 3.77, and Alia earned AED 4.65.

Materials	Kilograms Recycled	
	Abeer	Alia
aluminum cans	9	9
newspaper	26	114

- Define variables and write a system of linear equations from this situation.
 - What was the price per kilogram of aluminum? Determine the reasonableness of your solution.
- 18. BOOKS** The library is having a book sale. Hardcover books sell for AED 4 each, and paperback books are AED 2 each. If Ghaya spends AED 26 for 8 books, how many hardcover books did she buy?
- 19. MUSIC** An online music club offers individual songs for one price or entire albums for another. Saleh pays AED 14.90 to download 5 individual songs and 1 album. Tarek pays AED 21.75 to download 3 individual songs and 2 albums.
- How much does the music club charge to download a song?
 - How much does the music club charge to download an entire album?
- 20. CANOEING** Amer canoed against the current for 2 hours and then with the current for 1 hour before resting. Abdulrahman traveled against the current for 2.5 hours and then with the current for 1.5 hours before resting. If they traveled a total of 9.5 Kilometers against the current, 20.5 Kilometers with the current, and the current is 3 Kilometers per hour, how fast do Amer and Abdulrahman travel in still water?

H.O.T. Problems Use Higher-Order Thinking Skills

- 21. OPEN ENDED** Formulate a system of equations that represents a situation in your school. Describe the method that you would use to solve the system. Then solve the system and explain what the solution means.
- 22. REASONING** In a system of equations, x represents the time spent riding a bike, and y represents the distance traveled. You determine the solution to be $(-1, 7)$. Use this problem to discuss the importance of analyzing solutions in the context of real-world problems.
- 23. CHALLENGE** Solve the following system of equations by using three different methods. Show your work.

$$4x + y = 13$$

$$6x - y = 7$$

- 24. WRITE A QUESTION** A classmate says that elimination is the best way to solve a system of equations. Write a question to challenge his conjecture.
- 25. WHICH ONE DOESN'T BELONG?** Which system is different? Explain.

$$\begin{aligned} x - y &= 3 \\ x + \frac{1}{2}y &= 1 \end{aligned}$$

$$\begin{aligned} -x + y &= 0 \\ 5x &= 2y \end{aligned}$$

$$\begin{aligned} y &= x - 4 \\ y &= \frac{2}{x} \end{aligned}$$

$$\begin{aligned} y &= x + 1 \\ y &= 3x \end{aligned}$$

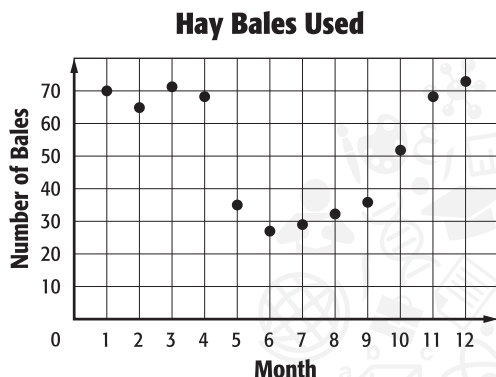
- 26. ? WRITING IN MATH** How do you know what method to use when solving a system of equations?

Standardized Test Practice

27. If $5x + 3y = 12$ and $4x - 5y = 17$, what is y ?

- A -1 B 3 C (-1, 3) D (3, -1)

28. **STATISTICS** The scatter plot shows the number of hay bales used on the Hassan farm during the last year.

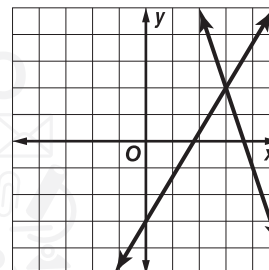


Which is an invalid conclusion?

- F The Hassans used less hay in the summer than they did in the winter.
 G The Hassans used about 629 bales of hay during the year.
 H On average, the Hassans used about 52 bales each month.
 J The Hassans used the most hay in February.

29. **SHORT RESPONSE** At noon, Abdulraheem cast a shadow 0.15 meter long. Next to him a streetlight cast a shadow 0.25 meter long. If Abdulraheem is 6 meters tall, how tall is the streetlight?

30. The graph shows the solution to which of the following systems of equations?



- A $y = -3x + 11$
 $3y = 5x - 9$
 B $y = 5x - 15$
 $2y = x + 7$
 C $y = -3x + 11$
 $2y = 4x - 5$
 D $y = 5x - 15$
 $3y = 2x + 18$

Spiral Review

Use elimination to solve each system of equations. (Lesson 6-4)

31. $x + y = 3$
 $3x - 4y = -12$

32. $-4x + 2y = 0$
 $2x - 3y = 16$

33. $4x + 2y = 10$
 $5x - 3y = 7$

34. **TRAVELING** A youth group is traveling in two vans to visit an aquarium. The number of people in each van and the cost of admission for that van are shown. What are the adult and student prices? (Lesson 6-3)

Van	Number of Adults	Number of Students	Total Cost
A	2	5	AED 77
B	2	7	AED 95

Graph each inequality.

35. $y < 4$

36. $x \geq 3$

37. $7x + 12y > 0$

38. $y - 3x \leq 4$

Skills Review

Find each sum or difference.

39. $(-3.81) + (-8.5)$

40. $12.625 + (-5.23)$

41. $21.65 + (-15.05)$

42. $(-4.27) + 1.77$

43. $(-78.94) - 14.25$

44. $(-97.623) - (-25.14)$

Algebra Lab Using Matrices to Solve Systems of Equations



A **matrix** is a rectangular arrangement of numbers, called **elements**, in rows and columns enclosed in brackets. Usually named using an uppercase letter, a matrix can be described by its **dimensions** or by the number of rows and columns in the matrix. A matrix with m rows and n columns is an $m \times n$ matrix (read “ m by n ”).

$$A = \begin{bmatrix} 7 & -9 & 5 & 3 \\ -1 & 3 & -3 & 6 \\ 0 & -4 & 8 & 2 \end{bmatrix}$$

A is a 3×4 matrix.

3 rows

4 columns

The element 2 is in Row 3, Column 4.

You can use an augmented matrix to solve a system of equations. An **augmented matrix** consists of the coefficients and the constant terms of a system of equations. Make sure that the coefficients of the x -terms are listed in one column, the coefficients of the y -terms are in another column, and the constant terms are in a third column. The coefficients and constant terms are usually separated by a dashed line.

Linear System

$$\begin{aligned} x - 3y &= 8 \\ -9x + 2y &= -4 \end{aligned}$$

Augmented Matrix

$$\left[\begin{array}{cc|c} 1 & -3 & 8 \\ -9 & 2 & -4 \end{array} \right]$$

Activity 1 Write an Augmented Matrix

Write an augmented matrix for each system of equations.

a. $-2x + 7y = 11$
 $6x - 4y = 2$

Place the coefficients of the equations and the constant terms into a matrix.

$$\begin{aligned} -2x + 7y &= 11 \\ 6x - 4y &= 2 \end{aligned} \quad \rightarrow \quad \left[\begin{array}{cc|c} -2 & 7 & 11 \\ 6 & -4 & 2 \end{array} \right]$$

b. $x - 2y = 5$
 $y = -4$

$$\begin{aligned} x - 2y &= 5 \\ y &= -4 \end{aligned} \quad \rightarrow \quad \left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & -4 \end{array} \right]$$

You can solve a system of equations by using an augmented matrix. By performing row operations, you can change the form of the matrix. The operations are the same as the ones used when working with equations.

KeyConcept Elementary Row Operations

The following operations can be performed on an augmented matrix.

- Interchange any two rows.
- Multiply all entries in a row by a nonzero constant.
- Replace one row with the sum of that row and a multiple of another row.

Row operations produce a matrix equivalent to the original system. **Row reduction** is the process of performing elementary row operations on an augmented matrix to solve a system.

The goal is to get the coefficients portion of the matrix to have the form $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, which is called the **identity matrix**. The first row will give you the solution for x , because the coefficient of y is 0. The second row will give you the solution for y , because the coefficient of x is 0.

Activity 2 Use Row Operations to Solve a System

Use an augmented matrix to solve the system of equations.

$$\begin{aligned} -5x + 3y &= 6 \\ x - y &= 4 \end{aligned}$$

Step 1 Write the augmented matrix: $\left[\begin{array}{cc|c} -5 & 3 & 6 \\ 1 & -1 & 4 \end{array} \right]$.

Step 2 Notice that the first element in the second row is 1. Interchange the rows so 1 can be in the upper left-hand corner.

$$\left[\begin{array}{cc|c} -5 & 3 & 6 \\ 1 & -1 & 4 \end{array} \right] \xrightarrow{\text{Interchange } R_1 \text{ and } R_2} \left[\begin{array}{cc|c} 1 & -1 & 4 \\ -5 & 3 & 6 \end{array} \right]$$

Step 3 To make the first element in the second row a 0, multiply the first row by 5 and add the result to row 2.

$$\left[\begin{array}{cc|c} 1 & -1 & 4 \\ -5 & 3 & 6 \end{array} \right] \xrightarrow{5R_1 + R_2} \left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & -2 & 26 \end{array} \right] \quad \begin{aligned} 1(5) + (-5) &= 0; -1(5) + 3 = -2; \\ 4(5) + 6 &= 26 \end{aligned}$$

Step 4 To make the second element in the second row a 1, multiply the second row by $-\frac{1}{2}$.

$$\left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & -2 & 26 \end{array} \right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & 1 & -13 \end{array} \right] \quad \begin{aligned} 0\left(-\frac{1}{2}\right) &= 0; -2\left(-\frac{1}{2}\right) = 1; \\ 26\left(-\frac{1}{2}\right) &= -13 \end{aligned}$$

Step 5 To make the second element in the first row a 0, add the rows together.

$$\left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & 1 & -13 \end{array} \right] \xrightarrow{R_2 + R_1} \left[\begin{array}{cc|c} 1 & 0 & -9 \\ 0 & 1 & -13 \end{array} \right] \quad \begin{aligned} 1 + 0 &= 1; -1 + 1 = 0; \\ 4 + (-13) &= -9 \end{aligned}$$

The solution is $(-9, -13)$.

Model and Analyze

Write an augmented matrix for each system of equations. Then solve the system.

1. $x + y = -3$
 $x - y = 1$

2. $x - y = -2$
 $2x + 2y = 12$

3. $3x - 4y = -27$
 $x + 2y = 11$

4. $x + 4y = -6$
 $2x - 5y = 1$

5. $x - 3y = -2$
 $4x + y = 31$

6. $x + 2y = 3$
 $-3x + 3y = 27$

6-6 Systems of Inequalities

Then

- You graphed and solved linear inequalities.

Now

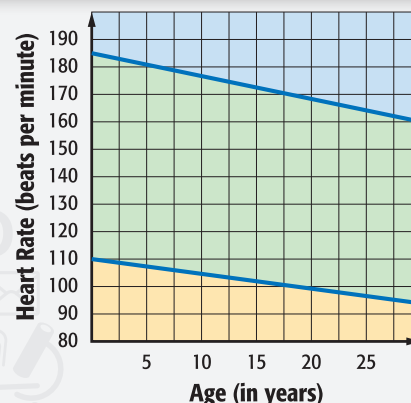
- Solve systems of linear inequalities by graphing.
- Apply systems of linear inequalities.

Why?

- Fatema is beginning an exercise program that involves an intense cardiovascular workout. Her trainer recommends that for a person her age, her heart rate should stay within the following range as she exercises.

- It should be higher than 102 beats per minute.
- It should not exceed 174 beats per minute.

The graph shows the maximum and minimum target heart rate for people ages 0 to 30 as they exercise. If the preferred range is in light green, how old do you think Fatema is?



New Vocabulary

system of inequalities

Mathematical Practices

Make sense of problems and persevere in solving them.
Attend to precision.

1 Systems of Inequalities

The graph above is a graph of two inequalities. A set of two or more inequalities with the same variables is called a **system of inequalities**.

The solution of a system of inequalities with two variables is the set of ordered pairs that satisfy all of the inequalities in the system. The solution set is represented by the overlap, or intersection, of the graphs of the inequalities.

Example 1 Solve by Graphing

Solve the system of inequalities by graphing.

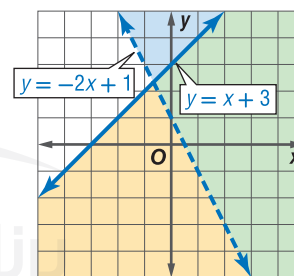
$$y > -2x + 1$$

$$y \leq x + 3$$

The graph of $y = -2x + 1$ is dashed and is not included in the graph of the solution. The graph of $y = x + 3$ is solid and is included in the graph of the solution.

The solution of the system is the set of ordered pairs in the intersection of the graphs of $y > -2x + 1$ and $y \leq x + 3$. This region is shaded in green.

When graphing more than one region, it is helpful to use two different colored pencils or two different patterns for each region. This will make it easier to see where the regions intersect and find possible solutions.



Guided Practice

1A. $y \leq 3$
 $x + y \geq 1$

1C. $y \geq -4$
 $3x + y \leq 2$

1B. $2x + y \geq 2$
 $2x + y < 4$

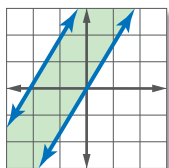
1D. $x + y > 2$
 $-4x + 2y < 8$

Sometimes the regions never intersect. When this happens, there is no solution because there are no points in common.

StudyTip

Parallel Boundaries

A system of equations represented by parallel lines does not have a solution. However, a system of inequalities with parallel boundaries can have a solution. For example:

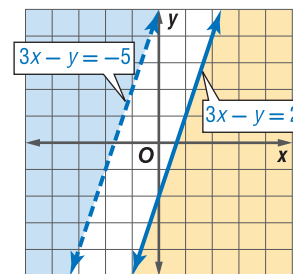


Example 2 No Solution

Solve the system of inequalities by graphing.

$$\begin{aligned} 3x - y &\geq 2 \\ 3x - y &< -5 \end{aligned}$$

The graphs of $3x - y = 2$ and $3x - y = -5$ are parallel lines. The two regions do not intersect at any point, so the system has no solution.



GuidedPractice

2A. $y > 3$
 $y < 1$

2B. $x + 6y \leq 2$
 $y \geq -\frac{1}{6}x + 7$

2 Apply Systems of Inequalities When using a system of inequalities to describe constraints on the possible combinations in a real-world problem, sometimes only whole-number solutions will make sense.

Real-World Example 3 Whole-Number Solutions

ELECTIONS Fatheya is running for student council. The election rules say that for the election to be valid, at least 80% of the 900 students must vote. Fatheya knows that she needs more than 330 votes to win.

a. Define the variables, and write a system of inequalities to represent this situation. Then graph the system.

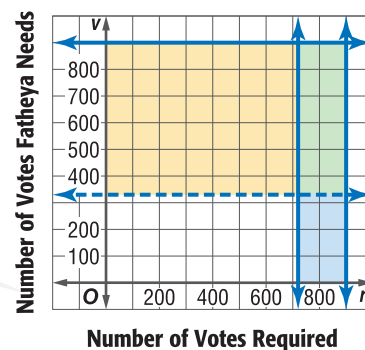
Let r = the number of votes required by the election rules; 80% of 900 students is 720 students. So $r \geq 720$.

Let v = the number of votes that Fatheya needs to win. So $v > 330$.

The system of inequalities is $r \geq 720$ and $v > 330$.

b. Name one viable option.

Only whole-number solutions make sense in this problem. One possible solution is (800, 400); 800 students voted and Fatheya received 400 votes.



GuidedPractice

3. **FUNDRAISING** The Theater Club is selling shirts. They have only enough supplies to print 120 shirts. They will sell sweatshirts for AED 22 and T-shirts for AED 15, with a goal of at least AED 2000 in sales.

A. Define the variables, and write a system of inequalities to represent this situation.

B. Then graph the system.

C. Name one possible solution.

D. Is (45, 30) a solution? Explain.

Real-WorldLink

Student government might be a good activity for you if you like to bring about change, plan events, and work with others.

Check Your Understanding

Examples 1–2 Solve each system of inequalities by graphing.

1. $x \geq 4$
 $y \leq x - 3$
2. $y > -2$
 $y \leq x + 9$
3. $y < 3x + 8$
 $y \geq 4x$
4. $3x - y \geq -1$
 $2x + y \geq 5$
5. $y \leq 2x - 7$
 $y \geq 2x + 7$
6. $y > -2x + 5$
 $y \geq -2x + 10$
7. $2x + y \leq 5$
 $2x + y \leq 7$
8. $5x - y < -2$
 $5x - y > 6$

Example 3

9. **AUTO RACING** At a racecar driving school there are safety requirements.
- a. Define the variables, and write a system of inequalities to represent the height and weight requirements in this situation. Then graph the system.
 - b. Name one possible solution.
 - c. Is $(50, 180)$ a solution? Explain.



Practice and Problem Solving

Examples 1–2 Solve each system of inequalities by graphing.

10. $y < 6$
 $y > x + 3$
11. $y \geq 0$
 $y \leq x - 5$
12. $y \leq x + 10$
 $y > 6x + 2$
13. $y < 5x - 2$
 $y > -6x + 2$
14. $2x - y \leq 6$
 $x - y \geq -1$
15. $3x - y > -5$
 $5x - y < 9$
16. $y \geq x + 10$
 $y \leq x - 3$
17. $y < 5x - 5$
 $y > 5x + 9$
18. $y \geq 3x - 5$
 $3x - y > -4$
19. $4x + y > -1$
 $y < -4x + 1$
20. $3x - y \geq -2$
 $y < 3x + 4$
21. $y > 2x - 3$
 $2x - y \geq 1$
22. $5x - y < -6$
 $3x - y \geq 4$
23. $x - y \leq 8$
 $y < 3x$
24. $4x + y < -2$
 $y > -4x$

Example 3

25. **ICE RINKS** Ice resurfacers are used for rinks of at least 100 square meters and up to 1700 square meters. The price ranges from as little as AED 10,000 to as much as AED 150,000.
- a. Define the variables, and write a system of inequalities to represent this situation. Then graph the system.
 - b. Name one possible solution.
 - c. Is $(1500, 30,000)$ a solution? Explain.
26. **MODELING** Abdulaziz works between 10 and 30 hours per week at a pizzeria. He earns AED 6.50 an hour, but can earn tips when he delivers pizzas.
- a. Write a system of inequalities to represent the AED d he could earn for working h hours in a week.
 - b. Graph this system.
 - c. If Abdulaziz received AED 17.50 in tips and earned a total of AED 180 for the week, how many hours did he work?

Solve each system of inequalities by graphing.

27. $x + y \geq 1$
 $x + y \leq 2$

28. $3x - y < -2$
 $3x - y < 1$

29. $2x - y \leq -11$
 $3x - y \geq 12$

30. $y < 4x + 13$
 $4x - y \geq 1$

31. $4x - y < -3$
 $y \geq 4x - 6$

32. $y \leq 2x + 7$
 $y < 2x - 3$

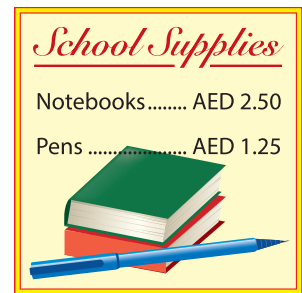
33. $y > -12x + 1$
 $y \leq 9x + 2$

34. $2y \geq x$
 $x - 3y > -6$

35. $x - 5y > -15$
 $5y \geq x - 5$

36. **CLASS PROJECT** An economics class formed a company to sell school supplies. They would like to sell at least 20 notebooks and 50 pens per week, with a goal of earning at least AED 60 per week.

- Define the variables, and write a system of inequalities to represent this situation.
- Graph the system.
- Name one possible solution.



37. **FINANCIAL LITERACY** Abdulkarim makes AED 15 per hour working for a photographer. He also coaches a competitive soccer team for AED 10 per hour. Abdulkarim needs to earn at least AED 90 per week, but he does not want to work more than 20 hours per week.

- Define the variables, and write a system of inequalities to represent this situation.
- Graph this system.
- Give two possible solutions to describe how Abdulkarim can meet his goals.
- Is (2, 2) a solution? Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

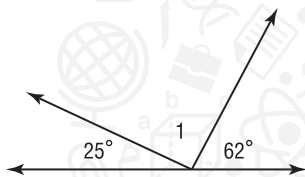
38. **CHALLENGE** Create a system of inequalities equivalent to $|x| \leq 4$.
39. **REASONING** State whether the following statement is *sometimes*, *always*, or *never* true. Explain your answer with an example or counterexample.
Systems of inequalities with parallel boundaries have no solutions.
40. **REASONING** Describe the graph of the solution of this system without graphing.
 $6x - 3y \leq -5$
 $6x - 3y \geq -5$
41. **OPEN ENDED** One inequality in a system is $3x - y > 4$. Write a second inequality so that the system will have no solution.
42. **PRECISION** Graph the system of inequalities. Estimate the area of the solution.
 $y \geq 1$
 $y \leq x + 4$
 $y \leq -x + 4$
43. **WRITING IN MATH** Refer to the beginning of the lesson. Explain what each colored region of the graph represents. Explain how shading in various colors can help to clearly show the solution set of a system of inequalities.

Standardized Test Practice

44. EXTENDED RESPONSE To apply for a scholarship, you must have a minimum of 20 hours of community service and a grade-point average of at least 3.75. Another scholarship requires at least 40 hours of community service and a minimum grade-point average of 3.0.

- Write a system of inequalities to represent the credentials you must have to apply for both scholarships.
- Graph the system of inequalities.
- If you are eligible for both scholarships, give one possible solution.

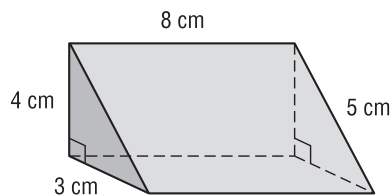
45. GEOMETRY What is the measure of $\angle 1$?



- A 83°
B 87°

- C 90°
D 93°

46. GEOMETRY What is the volume of the triangular prism?



- F 120 cm^3
G 96 cm^3

- H 48 cm^3
J 30 cm^3

47. Ten kilograms of fresh tomatoes make about 7.5 liters of cooked tomatoes. How many liters of cooked tomatoes does one kilogram of fresh tomatoes make?

- A 0.75 liters
B 1.25 liters
C 2.5 liters
D 5 liters

Spiral Review

48. CHEMISTRY Orion Labs needs to make 500 liters of 34% acid solution. The only solutions available are a 25% acid solution and a 50% acid solution. Write and solve a system of equations to find the number of liters of each solution that should be mixed to make the 34% solution. (Lesson 6-5)

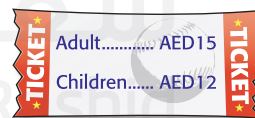
Use elimination to solve each system of equations. (Lesson 6-4)

49. $x + y = 7$
 $2x + y = 11$

50. $a - b = 9$
 $7a + b = 7$

51. $q + 4r = -8$
 $3q + 2r = 6$

52. ENTERTAINMENT A group of 11 adults and children bought tickets for the baseball game. If the total cost was AED 156, how many of each type of ticket did they buy? (Lesson 6-4)



Graph each inequality.

53. $4x - 2 \geq 2y$

54. $9x - 3y < 0$

55. $2y \leq -4x - 6$

Skills Review

Evaluate each expression.

56. 3^3

57. 2^4

58. $(-4)^3$



You can use TI-Nspire technology to explore systems of inequalities. To prepare your calculator, add a new **Graphs** page from the Home screen.

Activity Graph Systems of Inequalities

Mr. Obaid owns a car washing and detailing business. It takes 20 minutes to wash a car and 60 minutes to detail a car. He works at most 8 hours per day and does at most 4 details per day. Write a system of linear inequalities to represent this situation.

First, write a linear inequality that represents the time it takes for car washing and car detailing. Let x represent the number of car washes, and let y represent the number of car details. Then $20x + 60y \leq 480$.

To graph this using a graphing calculator, solve for y .

$$20x + 60y \leq 480$$

Original inequality

$$60y \leq -20x + 480$$

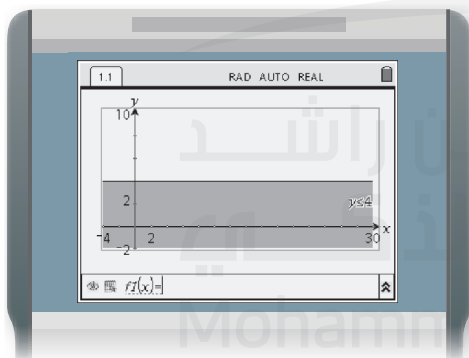
Subtract $20x$ from each side and simplify.

$$y \leq -\frac{1}{3}x + 8$$

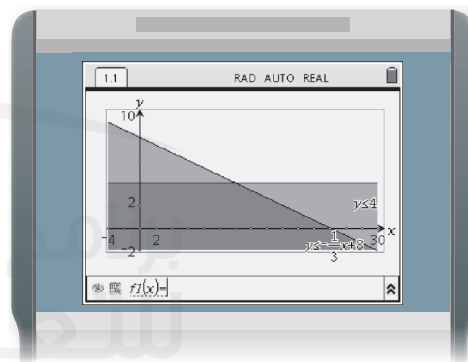
Divide each side by 60 and simplify.

Mr. Obaid does at most 4 details per day. This means that $y \leq 4$.

Step 1 Adjust the viewing window and then graph $y \leq 4$. Use the **Window Settings** option from the **Window/Zoom** menu to adjust the window to -4 to 30 for x and -2 to 10 for y . Keep the scales as **Auto**. Then enter $\text{del } \leq 4$ enter.



Step 2 Graph $y \leq -\frac{1}{3}x + 8$. Press **tab del** \leq and then enter $-\frac{1}{3}x + 8$.



The darkest shaded region of the graph represents the solutions.

Analyze the Results

- If Mr. Obaid charges AED 75 for each car he details and AED 25 for each car wash, what is the maximum amount of money he could earn in one day?
- What is the greatest number of car washes that Mr. Obaid could do in a day? Explain your reasoning.

Study Guide and Review

Study Guide

Key Concepts

Systems of Equations (Lessons 6-1 through 6-5)

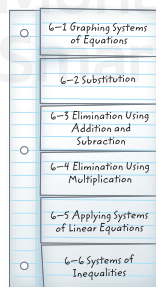
- A system with a graph of two intersecting lines has one solution and is *consistent and independent*.
- Graphing a system of equations can only provide approximate solutions. For exact solutions, you must use algebraic methods.
- In the substitution method, one equation is solved for a variable and the expression substituted into the second equation to find the value of another variable.
- In the elimination method, one variable is eliminated by adding or subtracting the equations.
- Sometimes multiplying one or both equations by a constant makes it easier to use the elimination method.
- The best method for solving a system of equations depends on the coefficients of the variables.

Systems of Inequalities (Lesson 6-6)

- A system of inequalities is a set of two or more inequalities with the same variables.
- The solution of a system of inequalities is the intersection of the graphs.

FOLDABLES® Study Organizer

Be sure the Key Concepts are noted in your Foldable.



Key Vocabulary

augmented matrix	inconsistent
consistent	independent
dependent	matrix
dimension	substitution
element	system of equations
elimination	system of inequalities

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

- If a system has at least one solution, it is said to be consistent.
- If a consistent system has exactly two solution(s), it is said to be independent.
- If a consistent system has an infinite number of solutions, it is said to be inconsistent.
- If a system has no solution, it is said to be inconsistent.
- Substitution involves substituting an expression from one equation for a variable in the other.
- In some cases, dividing two equations in a system together will eliminate one of the variables. This process is called elimination.
- A set of two or more inequalities with the same variables is called a system of equations.
- When the graphs of the inequalities in a system of inequalities do not intersect, there are no solutions to the system.

Lesson-by-Lesson Review

6-1 Graphing Systems of Equations

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

9. $x - y = 1$
 $x + y = 5$
10. $y = 2x - 4$
 $4x + y = 2$
11. $2x - 3y = -6$
 $y = -3x + 2$
12. $-3x + y = -3$
 $y = x - 3$
13. $x + 2y = 6$
 $3x + 6y = 8$
14. $3x + y = 5$
 $6x = 10 - 2y$
15. **MAGIC NUMBERS** Adnan is trying to find two numbers with a sum of 14 and a difference of 4. Define two variables, write a system of equations, and solve by graphing.

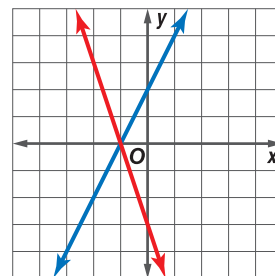
Example 1

Graph the system and determine the number of solutions it has. If it has one solution, name it.

$$y = 2x + 2$$

$$y = -3x - 3$$

The lines appear to intersect at the point $(-1, 0)$. You can check this by substituting -1 for x and 0 for y .



CHECK $y = 2x + 2$

$$0 \stackrel{?}{=} 2(-1) + 2$$

$$0 \stackrel{?}{=} -2 + 2$$

$$0 = 0 \checkmark$$

$$y = -3x - 3$$

$$0 \stackrel{?}{=} -3(-1) - 3$$

$$0 \stackrel{?}{=} 3 - 3$$

$$0 = 0 \checkmark$$

Original equation

Substitution

Multiply.

Original equation

Substitution

Multiply.

The solution is $(-1, 0)$.

6-2 Substitution

Use substitution to solve each system of equations.

16. $x + y = 3$
 $x = 2y$
17. $x + 3y = -28$
 $y = -5x$
18. $3x + 2y = 16$
 $x = 3y - 2$
19. $x - y = 8$
 $y = -3x$
20. $y = 5x - 3$
 $x + 2y = 27$
21. $x + 3y = 9$
 $x + y = 1$
22. **GEOMETRY** The perimeter of a rectangle is 48 centimeters. The length is 6 centimeters greater than the width. Define the variables, and write equations to represent this situation. Solve the system by using substitution.

Example 2

Use substitution to solve the system.

$$3x - y = 18$$

$$y = x - 4$$

$$3x - y = 18$$

First equation

$$3x - (x - 4) = 18$$

Substitute $x - 4$ for y .

$$2x + 4 = 18$$

Simplify.

$$2x = 14$$

Subtract 4 from each side.

$$x = 7$$

Divide each side by 2.

Use the value of x and either equation to find the value for y .

$$y = x - 4$$

Second equation

$$= 7 - 4 \text{ or } 3$$

Substitute and simplify.

The solution is $(7, 3)$.

Study Guide and Review *Continued*

6-3 Elimination Using Addition and Subtraction

Use elimination to solve each system of equations.

$$\begin{aligned} 23. \quad x + y &= 13 \\ x - y &= 5 \end{aligned}$$

$$\begin{aligned} 25. \quad x + 4y &= -4 \\ x + 10y &= -16 \end{aligned}$$

$$\begin{aligned} 27. \quad 6x + y &= 9 \\ -6x + 3y &= 15 \end{aligned}$$

$$\begin{aligned} 29. \quad 2x + 2y &= 4 \\ 2x - 8y &= -46 \end{aligned}$$

$$\begin{aligned} 24. \quad -3x + 4y &= 21 \\ 3x + 3y &= 14 \end{aligned}$$

$$\begin{aligned} 26. \quad 2x + y &= -5 \\ x - y &= 2 \end{aligned}$$

$$\begin{aligned} 28. \quad x - 4y &= 2 \\ 3x + 4y &= 38 \end{aligned}$$

$$\begin{aligned} 30. \quad 3x + 2y &= 8 \\ x + 2y &= 2 \end{aligned}$$

31. **BASEBALL CARDS** Ali bought 24 baseball cards for AED 50. One type cost AED 1 per card, and the other cost AED 3 per card. Define the variables, and write equations to find the number of each type of card he bought. Solve by using elimination.

Example 3

Use elimination to solve the system of equations.

$$\begin{aligned} 3x - 5y &= 11 \\ x + 5y &= -3 \end{aligned}$$

$$\begin{array}{r} 3x - 5y = 11 \\ (+) \quad x + 5y = -3 \\ \hline 4x \quad = 8 \end{array}$$

$$x = 2$$

The variable y is eliminated.

Divide each side by 4.

Now, substitute 2 for x in either equation to find the value of y .

$$3x - 5y = 11$$

$$3(2) - 5y = 11$$

$$6 - 5y = 11$$

$$-5y = 5$$

$$y = -1$$

First equation

Substitute.

Multiply.

Subtract 6 from each side.

Divide each side by -5 .The solution is $(2, -1)$.

6-4 Elimination Using Multiplication

Use elimination to solve each system of equations.

$$\begin{aligned} 32. \quad x + y &= 4 \\ -2x + 3y &= 7 \end{aligned}$$

$$\begin{aligned} 34. \quad 3x + 4y &= 1 \\ 5x + 2y &= 11 \end{aligned}$$

$$\begin{aligned} 36. \quad 8x - 3y &= -35 \\ 3x + 4y &= 33 \end{aligned}$$

$$\begin{aligned} 38. \quad -7x + 3y &= 12 \\ 2x - 8y &= -32 \end{aligned}$$

$$\begin{aligned} 33. \quad x - y &= -2 \\ 2x + 4y &= 38 \end{aligned}$$

$$\begin{aligned} 35. \quad -9x + 3y &= -3 \\ 3x - 2y &= -4 \end{aligned}$$

$$\begin{aligned} 37. \quad 2x + 9y &= 3 \\ 5x + 4y &= 26 \end{aligned}$$

$$\begin{aligned} 39. \quad 8x - 5y &= 18 \\ 6x + 6y &= -6 \end{aligned}$$

40. **BAKE SALE** On the first day, a total of 40 items were sold for AED 356. Define the variables, and write a system of equations to find the number of cakes and pies sold. Solve by using elimination.



Example 4

Use elimination to solve the system of equations.

$$\begin{aligned} 3x + 6y &= 6 \\ 2x + 3y &= 5 \end{aligned}$$

Notice that if you multiply the second equation by -2 , the coefficients of the y -terms are additive inverses.

$$\begin{array}{r} 3x + 6y = 6 \\ 2x + 3y = 5 \quad \xrightarrow{\text{Multiply by } -2} \quad (+) \quad -4x - 6y = -10 \\ \hline -x \quad = -4 \\ x = 4 \end{array}$$

Now, substitute 4 for x in either equation to find the value of y .

$$2x + 3y = 5$$

$$2(4) + 3y = 5$$

$$8 + 3y = 5$$

$$3y = -3$$

$$y = -1$$

Second equation

Substitution

Multiply.

Subtract 8 from both sides.

Divide each side by 3.

The solution is $(4, -1)$.

6-5 Applying Systems of Linear Equations

Determine the best method to solve each system of equations. Then solve the system.

41. $y = x - 8$
 $y = -3x$

43. $x + 3y = 12$
 $x = -6y$

45. $3x + 2y = -4$
 $5x + 2y = -8$

47. $3x + 4y = 26$
 $2x + 3y = 19$

42. $y = -x$
 $y = 2x$

44. $x + y = 10$
 $x - y = 18$

46. $6x + 5y = 9$
 $-2x + 4y = 14$

48. $11x - 6y = 3$
 $5x - 8y = -25$

49. **COINS** Laila has saved 10 fils and 25 fils in her cashbox. Define the variables, and write a system of equations to determine the number of 10 fils and 10 fils. Then solve the system using the best method for the situation.



50. **FAIR** At a county fair, the cost for 4 slices of pizza and 2 orders of French fries is AED 21.00. The cost of 2 slices of pizza and 3 orders of French fries is AED 16.50. To find out how much a single slice of pizza and an order of French fries costs, define the variables and write a system of equations to represent the situation. Determine the best method to solve the system of equations. Then solve the system. (Lesson 6-5)

Example 5

Determine the best method to solve the system of equations. Then solve the system.

$$\begin{aligned} 3x + 5y &= 4 \\ 4x + y &= -6 \end{aligned}$$

The coefficient of y is 1 in the second equation. So solving by substitution is a good method. Solve the second equation for y .

$$\begin{aligned} 4x + y &= -6 \\ y &= -6 - 4x \end{aligned}$$

Second equation
Subtract $4x$ from each side.

Substitute $-6 - 4x$ for y in the first equation.

$$\begin{aligned} 3x + 5(-6 - 4x) &= 4 \\ 3x - 30 - 20x &= 4 \\ -17x - 30 &= 4 \\ -17x &= 34 \\ x &= -2 \end{aligned}$$

Substitute.
Distributive Property
Simplify.
Add 30 to each side.
Divide by -17 .

Last, substitute -2 for x in either equation to find y .

$$\begin{aligned} 4x + y &= -6 \\ 4(-2) + y &= -6 \\ -8 + y &= -6 \\ y &= 2 \end{aligned}$$

Second equation
Substitute.
Multiply.
Add 8 to each side.

The solution is $(-2, 2)$.

6-6 Systems of Inequalities

Solve each system of inequalities by graphing.

51. $x > 3$
 $y < x + 2$

52. $y \leq 5$
 $y > x - 4$

53. $y < 3x - 1$
 $y \geq -2x + 4$

54. $y \leq -x - 3$
 $y \geq 3x - 2$

55. **JOBS** Omar makes AED 7 an hour working at the grocery store and AED 10 an hour delivering newspapers. He cannot work more than 20 hours per week. Graph two inequalities that Omar can use to determine how many hours he needs to work at each job if he wants to earn at least AED 90 per week.

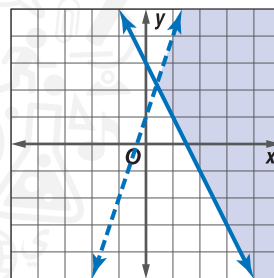
Example 6

Solve the system of inequalities by graphing.

$$y < 3x + 1$$

$$y \geq -2x + 3$$

The solution set of the system is the set of ordered pairs in the intersection of the two graphs. This portion is shaded in the graph below.



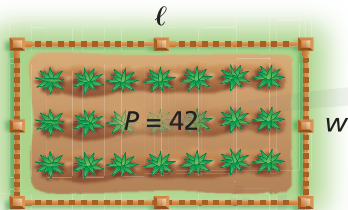
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Graph each system and determine the number of solutions that it has. If it has one solution, name it.

- $y = 2x$
 $y = 6 - x$
- $y = x - 3$
 $y = -2x + 9$
- $x - y = 4$
 $x + y = 10$
- $2x + 3y = 4$
 $2x + 3y = -1$

Use substitution to solve each system of equations.

- $y = x + 8$
 $2x + y = -10$
- $x = -4y - 3$
 $3x - 2y = 5$
- GARDENING** Eissa has 42 meters of fencing around his garden. The garden is rectangular in shape, and its length is equal to twice the width minus 3 meters. Define the variables, and write a system of equations to find the length and width of the garden. Solve the system by using substitution.



- MULTIPLE CHOICE** Use elimination to solve the system.

$$\begin{aligned} 6x - 4y &= 6 \\ -6x + 3y &= 0 \end{aligned}$$

- A (5, 6)
B (-3, -6)
C (1, 0)
D (4, -8)
- SHOPPING** Maysa has AED 175 to shop for jeans and sweaters. Each pair of jeans costs AED 25, each sweater costs AED 20, and she buys 8 items. Determine the number of pairs of jeans and sweaters Maysa bought.

Use elimination to solve each system of equations.

- $x + y = 13$
 $x - y = 5$
- $3x + 7y = 2$
 $3x - 4y = 13$
- $x + y = 8$
 $x - 3y = -4$
- $2x + 6y = 18$
 $3x + 2y = 13$

- MAGAZINES** Maha subscribes to a sports magazine and a fashion magazine. She received 24 issues this year. The number of fashion issues is 6 less than twice the number of sports issues. Define the variables, and write a system of equations to find the number of issues of each magazine.

Determine the best method to solve each system of equations. Then solve the system.

- $y = 3x$
 $x + 2y = 21$
- $x + y = 12$
 $y = x - 4$
- $x + y = 15$
 $x - y = 9$
- $3x + 5y = 7$
 $2x - 3y = 11$

- OFFICE SUPPLIES** At a sale, Faris bought 24 reams of paper and 4 inkjet cartridges for AED 320. Moza bought 2 reams of paper and 1 inkjet cartridge for AED 50. The reams of paper were all the same price and the inkjet cartridges were all the same price. Write a system of equations to represent this situation. Determine the best method to solve the system of equations. Then solve the system.

Solve each system of inequalities by graphing.

- $x > 2$
 $y < 4$
- $3x - y > 9$
 $y > -2x$
- $x + y \leq 5$
 $y \geq x + 2$
- $y \geq 2x + 3$
 $-4x - 3y > 12$

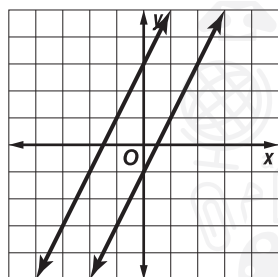
Standardized Test Practice

Cumulative, Chapters 1 through 6

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which of the following terms *best* describes the system of equations shown in the graph?



- A consistent
B consistent and dependent
C consistent and independent
D inconsistent

2. Use substitution to solve the system of equations below.

$$\begin{cases} y = 4x - 7 \\ 3x - 2y = -1 \end{cases}$$

- F (3, 5) H (5, -2)
G (4, -1) J (-6, 2)

3. Which ordered pair is the solution of the system of linear equations shown below?

$$\begin{cases} 3x - 8y = -50 \\ 3x - 5y = -38 \end{cases}$$

- A $(\frac{5}{8}, \frac{3}{2})$ C $(-\frac{2}{7}, \frac{4}{9})$
B (4, -9) D (-6, 4)

4. A home goods store received AED 881 from the sale of 4 table saws and 9 electric drills. If the receipts from the saws exceeded the receipts from the drills by AED 71, what is the price of an electric drill?

- F AED 45 H AED 108
G AED 59 J AED 119

5. A region is defined by this system.

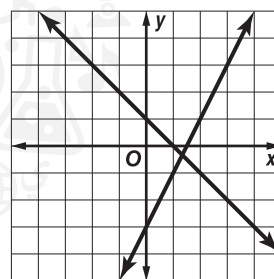
$$y > -\frac{1}{2}x - 1$$

$$y > -x + 3$$

In which quadrant(s) of the coordinate plane is the region located?

- A I and IV only C I, II, and IV only
B III only D II and III only

6. Which of the following terms *best* describes the system of equations shown in the graph?



- F consistent
G consistent and independent
H consistent and dependent
J inconsistent

7. Use elimination to solve the system of equations below.

$$3x + 2y = -2$$

$$2x - 2y = -18$$

- A (1, 3) C (-2, -3)
B (7, -4) D (-4, 5)

8. What is the solution of the following system of equations?

$$\begin{cases} y = 6x - 1 \\ y = 6x + 1 \end{cases}$$

- F (2, 11) H (7, 5)
G (-3, -14) J no solution

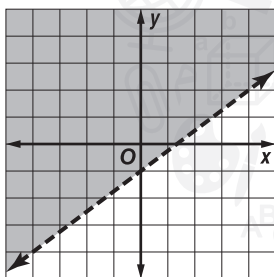
Test-Taking Tip

Question 8 You can subtract the second equation from the first equation to eliminate the x -variable. Then solve for y .

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. **GRIDDED RESPONSE** Najla and her sister have AED 15 to spend on pizza. A medium pizza costs AED 11.50 plus AED 0.75 per topping. What is the maximum number of toppings Najla and her sister can get on their pizza?
10. Write an inequality for the graph below.



11. **GRIDDED RESPONSE** Hala is taking a road trip. After she drives 12 more kilometers, she will have driven at least half of the 108-kilometers trip. What is the least number of kilometers she has driven so far?
12. Write an equation in slope-intercept form with a slope of $-\frac{2}{3}$ and a y -intercept of 6.
13. A rental company charges AED 9.50 per hour for a scooter plus a AED 15 fee. Write an equation in slope-intercept form for the total rental cost C of renting a scooter for h hours.
14. **GRIDDED RESPONSE** A computer supplies store is having a storewide sale this weekend. An inkjet printer that normally sells for AED 179.00 is on sale for AED 143.20. What is the percent discount of the sale price?

15. In 1980, the population of Kentucky was about 3.66 million people. By 2000, this number had grown to about 4.04 million people. What was the annual rate of change in population from 1980 to 2000?
16. Majed's cell phone service charges him AED 0.15 per text. Write an equation that represents the cost C of his cell phone service for t texts sent each month.
17. A store is offering a AED 15 mail-in-rebate on all printers. If Mohammad is looking at printers that range from AED 45 to AED 89, how much can he expect to pay?

Extended Response

Record your answers on a sheet of paper. Show your work.

18. The table shows how many canned goods were collected during the first day of a charity food drive.

Food Drive Day 1 Results	
Class	Number Collected
10 th graders	78
11 th graders	80
12 th graders	92

- a. Estimate how many canned goods will be collected during the 5-day food drive. Explain your answer.
- b. Is this estimate a reasonable expectation? Explain.

Preparing for Standardized Tests

Guess and Check

It is very important to pace yourself and keep track of how much time you have when taking a standardized test. If time is running short, or if you are unsure how to solve a problem, the guess and check strategy may help you determine the correct answer quickly.

Strategies for Guessing and Checking

Step 1

Carefully look over each possible answer choice, and evaluate for reasonableness. Eliminate unreasonable answers.

Ask yourself:

- Are there any answer choices that are clearly incorrect?
- Are there any answer choices that are not in the proper format?
- Are there any answer choices that do not have the proper units for the correct answer?

Step 2

For the remaining answer choices, use the guess and check method.

- **Equations:** If you are solving an equation, substitute the answer choice for the variable and see if this results in a true number sentence.
- **Inequalities:** Likewise, you can substitute the answer choice for the variable and see if it satisfies the inequality.
- **System of Equations:** Find the answer choice that satisfies both equations of the system.

Step 3

Choose an answer choice and see if it satisfies the constraints of the problem statement. Identify the correct answer.

- If the answer choice you are testing does not satisfy the problem, move on to the next reasonable guess and check it.
- When you find the correct answer choice, stop. You do not have to check the other answer choices.



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Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Solve $\begin{cases} 4x - 8y = 20 \\ -3x + 5y = -14 \end{cases}$

A (5, 0)

C (3, -1)

B (4, -2)

D (-6, -5)

The solution of a system of equations is an ordered pair, (x, y) . Since all four answer choices are of this form, they are all possible correct answers and must be checked. Begin with the first answer choice and substitute it in each equation. Continue until you find the ordered pair that satisfies both equations of the system.

	First Equation	Second Equation
Guess: (5, 0)	$4x - 8y = 20$ $4(5) - 8(0) = 20$ ✓	$-3x + 5y = -14$ $-3(5) + 5(0) \neq -14$ ✗

	First Equation	Second Equation
Guess: (4, -2)	$4x - 8y = 20$ $4(4) - 8(-2) \neq 20$ ✗	$-3x + 5y = -14$ $-3(4) + 5(-2) \neq -14$ ✗

	First Equation	Second Equation
Guess: (3, -1)	$4x - 8y = 20$ $4(3) - 8(-1) = 20$ ✓	$-3x + 5y = -14$ $-3(3) + 5(-1) = -14$ ✓

The ordered pair (3, -1) satisfies both equations of the system. So, the correct answer is C.

Exercises

Read each problem. Eliminate any unreasonable answers. Then use the information in the problem to solve.

- Nabila bought 5 sandwiches and 3 soft drinks at the ball game for AED 11.50. Faleh bought 4 sandwiches and 2 soft drinks for AED 8.50. How much does a single sandwich and a single drink cost?

A sandwiches: AED 1.25 C sandwiches: AED 1.50
 soft drinks: AED 1.50 soft drinks: AED 1.25

B sandwiches: AED 1.25 D sandwiches: AED 1.50
 soft drinks: AED 1.75 soft drinks: AED 1.75

- The bookstore hopes to sell at least 30 binders and calculators each week. The store also hopes to have sales revenue of at least AED 200 in binders and calculators. How many binders and calculators could be sold to meet both of these sales goals?

Store Prices	
Item	Price
binders	AED 3.65
calculators	AED 14.80

- F 25 binders, 5 calculators H 22 binders, 9 calculators
- G 12 binders, 15 calculators J 28 binders, 6 calculators