CHAPTER 4

Equations of Linear Functions

Then

- You graphed linear functions.

Now

- In this chapter, you will:
  - Write and graph linear equations in various forms.
  - Use scatter plots and lines of fit, and write equations of best-fit lines using linear regression.
  - Find inverse linear functions.

Why? ▲

- **TRAVEL** The number of trips people take changes from year to year. From the yearly data, patterns emerge. Rate of change can be applied to these data to determine a linear model. This can be used to predict the number of trips taken in future years.

Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Evaluate $3a^2 - 2ab + c$ for the values given.

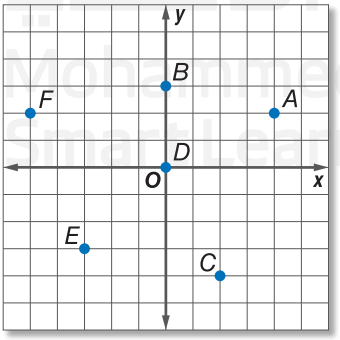
1. $a = 2, b = 1, c = 5$
2. $a = -3, b = -2, c = 3$
3. $a = -1, b = 0, c = 11$
4. $a = 5, b = -3, c = -9$
5. **CAR RENTAL** The cost of renting a car is given by $49x + 0.3y$. Let x represent the number of days rented, and let y represent the number of miles driven. Find the cost for a five-day rental over 125 miles.

Solve each equation for the given variable.

6. $x + y = 5$ for y
7. $2x - 4y = 6$ for x
8. $y - 2 = x + 3$ for y
9. $4x - 3y = 12$ for x
10. **GEOMETRY** The formula for the perimeter of a rectangle is $P = 2w + 2\ell$, where w represents width and ℓ represents length. Solve for w.

Write the ordered pair for each point.

11. A
12. B
13. C
14. D
15. E
16. F



QuickReview

Example 1

Evaluate $2(m - n)^2 + 3p$ for $m = 5$, $n = 2$, and $p = -3$.

$$\begin{aligned} &2(m - n)^2 + 3p \\ &= 2(5 - 2)^2 + 3(-3) \\ &= 2(3)^2 + 3(-3) \\ &= 2(9) + 3(-3) \\ &= 18 + (-9) \\ &= 9 \end{aligned}$$

Example 2

Solve $5x + 15y = 9$ for x.

$$\begin{aligned} 5x + 15y &= 9 \\ 5x + 15y - 15y &= 9 - 15y \\ 5x &= 9 - 15y \\ \frac{5x}{5} &= \frac{9 - 15y}{5} \\ x &= \frac{9}{5} - 3y \end{aligned}$$

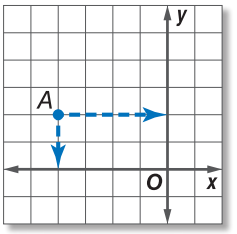
Example 3

Write the ordered pair for A.

Step 1 Begin at point A.

Step 2 Follow along a vertical line to the x-axis. The x-coordinate is -4.

Step 3 Follow along a horizontal line to the y-axis. The y-coordinate is 2.



The ordered pair for point A is $(-4, 2)$.

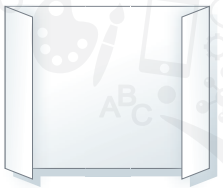
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study this chapter. To get ready, identify important terms and organize your resources. You may wish to refer to previous chapters to review prerequisite skills.

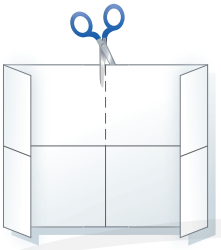
FOLDABLES StudyOrganizer

Linear Functions Make this Foldable to help you organize your notes about linear functions. Begin with one sheet of 11" by 17" paper.

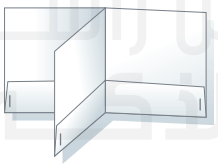
1 **Fold** each end of the paper in about 2 inches.



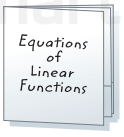
2 **Fold** along the width and the length. Unfold. Cut along the fold line from the top to the center.



3 **Fold** the top flaps down. Then fold in half and turn to form a folder. Staple the flaps down to form pockets.



4 **Label** the front with the chapter title.



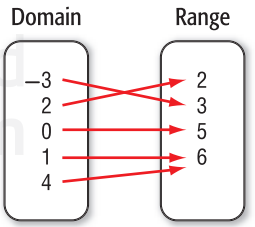
NewVocabulary

slope-intercept form
linear extrapolation
point-slope form
parallel lines
perpendicular lines
scatter plot
line of fit
linear interpolation
best-fit line
linear regression
correlation coefficient
median-fit line
inverse relation
inverse function

ReviewVocabulary

coefficient the numerical factor of a term

function a relation in which each element of the domain is paired with exactly one element of the range



ratio a comparison of two numbers by division

EXPLORE

4-1 Graphing Technology Lab: Investigating Slope-Intercept Form

Set Up the Lab

- Cut a small hole in a top corner of a plastic sandwich bag. Hang the bag from the end of the force sensor.
- Connect the force sensor to your data collection device.

Activity Collect Data

Step 1 Use the sensor to collect the weight with 0 washers in the bag. Record the data pair in the calculator.

Step 2 Place one washer in the plastic bag. Wait for the bag to stop swinging, then measure and record the weight.

Step 3 Repeat the experiment, adding different numbers of washers to the bag. Each time, record the number of washers and the weight.

Analyze the Results

1. The domain contains values of the independent variable, number of washers. The range contains values of the dependent variable, weight. Use the graphing calculator to create a scatter plot using the ordered pairs (washers, weight).
2. Write a sentence that describes the points on the graph.
3. Describe the position of the point on the graph that represents the trial with no washers in the bag.
4. The rate of change can be found by using the formula for slope.

$$\frac{\text{rise}}{\text{run}} = \frac{\text{change in weight}}{\text{change in number of washers}}$$

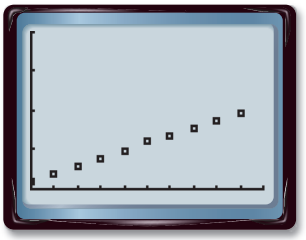
Find the rate of change in the weight as more washers are added.

5. Explain how the rate of change is shown on the graph.

Make a Conjecture

The graph shows sample data from a washer experiment. Describe the graph for each situation.

6. a bag that hangs weighs 0.8 N when empty and increases in weight at the rate of the sample
7. a bag that has the same weight when empty as the sample and increases in weight at a faster rate
8. a bag that has the same weight when empty as the sample and increases in weight at a slower rate



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LESSON 4-1

Graphing Equations in Slope-Intercept Form

Then

- You found rates of change and slopes.

Now

1. Write and graph linear equations in slope-intercept from.
2. Model real-world data with equations

Why?

- Jamil has 500 songs on his digital media player. He joins a music club that lets him download 30 songs per month for a monthly fee. The number of songs that Jamil could eventually have in his player if he does not delete any songs is represented by $y = 30x + 500$.

New Vocabulary

slope-intercept form
constant function

Mathematical Practices

Reason abstractly and quantitatively.

Look for and express regularity in repeated reasoning.

1 Slope-Intercept Form

An equation of the form $y = mx + b$, where m is the slope and b is the y-intercept, is in **slope-intercept form**. The variables m and b are called *parameters* of the equation. Changing either value changes the equation's graph.

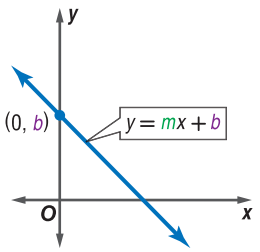
KeyConcept Slope-Intercept Form

Words The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.

Example $y = mx + b$

$y = 2x + 6$

slope — y-intercept



Example 1 Write and Graph an Equation

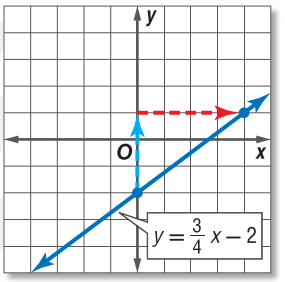
Write an equation in slope-intercept form for the line with a slope of $\frac{3}{4}$ and a y-intercept of -2. Then graph the equation.

$y = mx + b$

$y = \frac{3}{4}x + (-2)$

$y = \frac{3}{4}x - 2$

Now graph the equation.



Step 1 Plot the y-intercept $(0, -2)$.

Step 2 The slope is $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$. From $(0, -2)$, move up 3 units and right 4 units. Plot the point.

Step 3 Draw a line through the two points.

GuidedPractice

Write an equation of a line in slope intercept form with the given slope and y-intercept. Then graph the equation.

1A. slope: $-\frac{1}{2}$, y-intercept: 3

1B. slope: -3, y-intercept: -8

When an equation is not written in slope-intercept form, it may be easier to rewrite it before graphing.

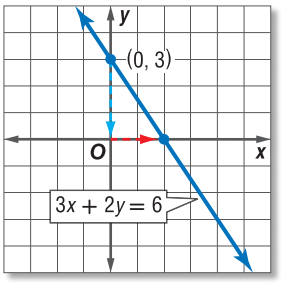
Example 2 Graph Linear Equations

Graph $3x + 2y = 6$.

Rewrite the equation in slope-intercept form.

$$3x + 2y = 6$$
$$3x + 2y - 3x = 6 - 3x$$
$$2y = 6 - 3x$$
$$2y = -3x + 6$$
$$\frac{2y}{2} = \frac{-3x + 6}{2}$$
$$y = -\frac{3}{2}x + 3$$

Now graph the equation. The slope is $-\frac{3}{2}$, and the y-intercept is 3.



Step 1 Plot the y-intercept $(0, 3)$.

Step 2 The slope is $\frac{\text{rise}}{\text{run}} = -\frac{3}{2}$. From $(0, 3)$, move down 3 units and right 2 units. Plot the point.

Step 3 Draw a line through the two points.

StudyTip

Counting and Direction

When counting rise and run, a negative sign may be associated with the value in the numerator or denominator. If with the numerator, begin by counting down for the rise. If with the denominator, count left when counting the run. The resulting line will be the same.

GuidedPractice

Graph each equation.

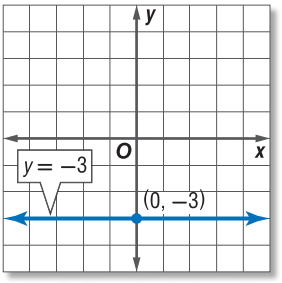
2A. $3x - 4y = 12$

2B. $-2x + 5y = 10$

Except for the graph of $y = 0$, which lies on the x-axis, horizontal lines have a slope of 0. They are graphs of **constant functions**, which can be written in slope-intercept form as $y = 0x + b$ or $y = b$, where b is any number. Constant functions do not cross the x-axis. Their domain is all real numbers, and their range is b.

Example 3 Graph Linear Equations

Graph $y = -3$.



Step 1 Plot the y-intercept $(0, -3)$.

Step 2 The slope is 0. Draw a line through the points with y-coordinate -3.

GuidedPractice

Graph each equation.

3A. $y = 5$

3B. $2y = 1$

Vertical lines have no slope. So, equations of vertical lines cannot be written in slope-intercept form.

There are times when you will need to write an equation when given a graph. To do this, locate the y-intercept and use the rise and run to find another point on the graph. Then write the equation in slope-intercept form.

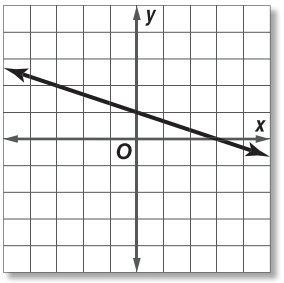
Test-Taking Tip

Eliminating Choices

Analyze the graph to determine the slope and the y-intercept. Then you can save time by eliminating answer choices that do not match the graph.

Standardized Test Example 4 Write an Equation in Slope-Intercept Form

Which of the following is an equation in slope-intercept form for the line shown?



A $y = -3x + 1$

B $y = -3x + 3$

C $y = -\frac{1}{3}x + 1$

D $y = -\frac{1}{3}x + 3$

Read the Test Item

You need to find the slope and y-intercept of the line to write the equation.

Solve the Test Item

Step 1 The line crosses the y-axis at (0, 1), so the y-intercept is 1. The answer is either A or C.

Step 2 To get from (0, 1) to (3, 0), go down 1 unit and 3 units to the right. The slope is $-\frac{1}{3}$.

Step 3 Write the equation.

$y = mx + b$

$y = -\frac{1}{3}x + 1$

CHECK The graph also passes through $(-3, 2)$. If the equation is correct, this should be a solution.

$y = -\frac{1}{3}x + 1$

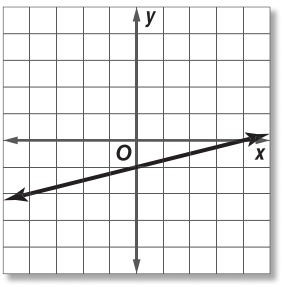
$2 \stackrel{?}{=} -\frac{1}{3}(-3) + 1$

$2 \stackrel{?}{=} 1 + 1$

$2 = 2$ ✔ The answer is C.

Guided Practice

4. Which of the following is an equation in slope-intercept form for the line shown?



F $y = \frac{1}{4}x - 1$

G $y = \frac{1}{4}x + 4$

H $y = 4x - 1$

J $y = 4x + 4$

2 Modeling Real-World Data Real-world data can be modeled by a linear equation if there is a constant rate of change. The rate of change represents the slope. The y-intercept is the point where the value of the independent variable is 0.

Real-WorldLink

Suppose that about 2.6 million students competed in high school sports in 1997. Then suppose that the number of students competing in high school sports has increased by an average of 0.06 million per year since 1997.

Source: National Federation of High School Associations

Real-World Example 5 Write and Graph a Linear Equation

SPORTS **Use the information at the left about high school sports.**

a. Write a linear equation to find the number of students in high school sports after 1997.

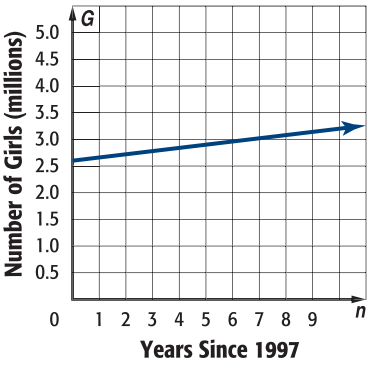
Words	Number of students competing	equals	rate of change	times	number of years	plus	amount at start.
Variables	Let G = number of students competing.		Let n = number of years since 1997.				
Equation	G	$=$	0.06	$\times$	n	$+$	2.6

The equation is $G = 0.06n + 2.6$.

b. Graph the equation.

The y-intercept is where the data begins. So, the graph passes through (0, 2.6).

The rate of change is the slope, so the slope is 0.06.



c. Estimate the number of students competing in 2017.

The year 2017 is 20 years after 1997.

$G = 0.06n + 2.6$

$= 0.06(20) + 2.6$

$= 3.8$

There will be about 3.8 million students competing in high school sports in 2017.

GuidedPractice

5. FUNDRAISERS The band boosters are selling sandwiches for AED 5 each. They bought AED 1160 in ingredients.

A. Write an equation for the profit P made on n sandwiches.

B. Graph the equation.

C. Find the total profit if 1400 sandwiches are sold.

Check Your Understanding

Example 1 **Write an equation of a line in slope-intercept form with the given slope and y-intercept. Then graph the equation.**

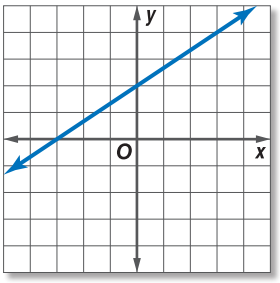
1. slope: 2, y-intercept: 4
2. slope: -5, y-intercept: 3
3. slope: $\frac{3}{4}$, y-intercept: -1
4. slope: $-\frac{5}{7}$, y-intercept: $-\frac{2}{3}$

Examples 2–3 **Graph each equation.**

5. $-4x + y = 2$
6. $2x + y = -6$
7. $-3x + 7y = 21$
8. $6x - 4y = 16$
9. $y = -1$
10. $15y = 3$

Example 4 **Write an equation in slope-intercept form for each graph shown.**

11.



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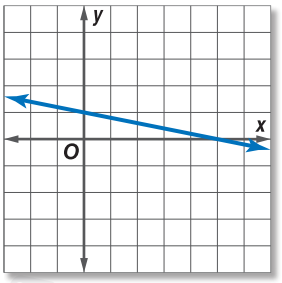

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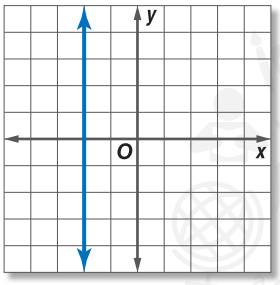

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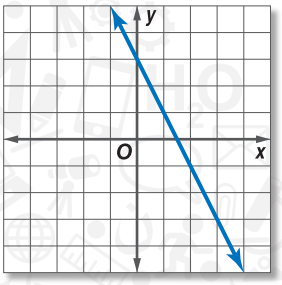


Example 5

15. **FINANCIAL LITERACY** Rashid is buying a new stereo system for his car using a layaway plan.

a. Write an equation for the total amount S that he has paid after w weeks.

b. Graph the equation.

c. Find out how much Rashid will have paid after 8 weeks.

16. **REASONING** Mariam is driving from her home in Abu Dhabi, to her grandmother's house in Ras Al Khaimah. On the first day, she will travel 240 km to Dubai, to pick up her cousin. Then they will travel 350 km each day.

a. Write an equation that models the total number of kilometers m Mariam has traveled, if d represents the number of days after she picks up her cousin.

b. Graph the equation.

c. How long will the drive take if the total length of the trip is 1343 km?

Practice and Problem Solving

Example 1 **Write an equation of a line in slope-intercept form with the given slope and y-intercept. Then graph the equation.**

17. slope: 5, y-intercept: 8

18. slope: 3, y-intercept: 10

19. slope: -4, y-intercept: 6

20. slope: -2, y-intercept: 8

21. slope: 3, y-intercept: -4

22. slope: 4, y-intercept: -6

Examples 2–3 **Graph each equation.**

23. $-3x + y = 6$

24. $-5x + y = 1$

25. $-2x + y = -4$

26. $y = 7x - 7$

27. $5x + 2y = 8$

28. $4x + 9y = 27$

29. $y = 7$

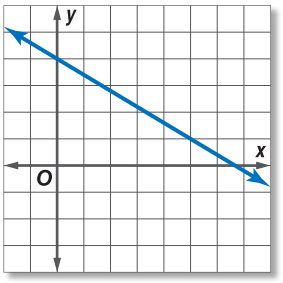
30. $y = -\frac{2}{3}$

31. $21 = 7y$

32. $3y - 6 = 2x$

Example 4 **Write an equation in slope-intercept form for each graph shown.**

33.



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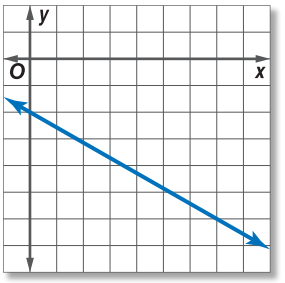

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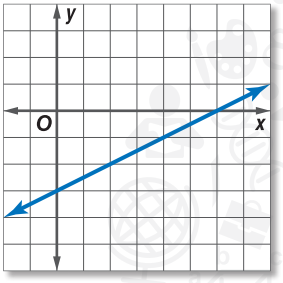

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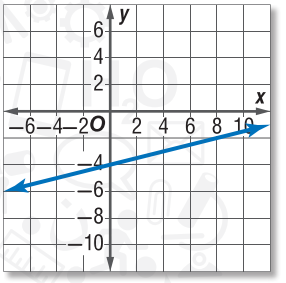


Example 5 **37** **MANATEES** In 1991, 1267 manatees inhabited Florida's waters. The manatee population has increased at a rate of 123 manatees per year.

a. Write an equation for the manatee population, P, t years since 1991.

b. Graph this equation.

c. In 2006, the manatee was removed from Florida's endangered species list. What was the manatee population in 2006?

Write an equation of a line in slope-intercept form with the given slope and y-intercept.

38. slope: $\frac{1}{2}$, y-intercept: -3

39. slope: $\frac{2}{3}$, y-intercept: -5

40. slope: $-\frac{5}{6}$, y-intercept: 5

41. slope: $-\frac{3}{7}$, y-intercept: 2

42. slope: 1, y-intercept: 4

43. slope: 0, y-intercept: 5

Graph each equation.

44. $y = \frac{3}{4}x - 2$

45. $y = \frac{5}{3}x + 4$

46. $3x + 8y = 32$

47. $5x - 6y = 36$

48. $-4x + \frac{1}{2}y = -1$

49. $3x - \frac{1}{4}y = 2$

50. **TRAVEL** A rental company charges AED 8 per hour for a mountain bike plus a AED 5 fee for a helmet.

a. Write an equation in slope-intercept form for the total rental cost C for a helmet and a bicycle for t hours.

b. Graph the equation.

c. What would the cost be for 2 helmets and 2 bicycles for 8 hours?

51. **REASONING** For Illinois residents, the average tuition at Chicago State University is AED 157 per credit hour. Fees cost AED 218 per year.

a. Write an equation in slope-intercept form for the tuition T for c credit hours.

b. Find the cost for a student who is taking 32 credit hours.

Write an equation of a line in slope-intercept form with the given slope and y-intercept.

52. slope: -1, y-intercept: 0

53. slope: 0.5, y-intercept: 7.5

54. slope: 0, y-intercept: 7

55. slope: -1.5, y-intercept: -0.25

56. Write an equation of a horizontal line that crosses the y-axis at $(0, -5)$.

57. Write an equation of a line that passes through the origin and has a slope of 3.

58. TEMPERATURE The temperature dropped rapidly overnight. Starting at 80°F, the temperature dropped 3° per minute.

a. Draw a graph that represents this drop from 0 to 8 minutes.

b. Write an equation that describes this situation. Describe the meaning of each variable as well as the slope and y-intercept.

59. FITNESS Refer to the information at the right.

a. Write an equation that represents the cost C of a membership for m months.

b. What does the slope represent?

c. What does the C-intercept represent?

d. What is the cost of a two-year membership?

60. MAGAZINES A teen magazine began with a circulation of 500,000 in its first year. Since then, the circulation has increased an average of 33,388 per year.

a. Write an equation that represents the circulation c after t years.

b. What does the slope represent?

c. What does the c-intercept represent?

d. If the magazine began in 1944, and this trend continues, in what year will the circulation reach 3,000,000?

61. SMART PHONES A telecommunications company sold 3305 smart phones in the first year of production. Suppose, on average, they expect to sell 25 phones per day.

a. Write an equation for the number of smart phones P sold t years after the first year of production, assuming 365 days per year.

b. If sales continue at this rate, how many years will it take for the company to sell 100,000 phones?

H.O.T. Problems Use Higher-Order Thinking Skills

62. OPEN ENDED Draw a graph representing a real-world linear function and write an equation for the graph. Describe what the graph represents.

63. REASONING Determine whether the equation of a vertical line can be written in slope-intercept form. Explain your reasoning.

64. CHALLENGE Summarize the characteristics that the graphs $y = 2x + 3$, $y = 4x + 3$, $y = -x + 3$, and $y = -10x + 3$ have in common.

65. REGULARITY If given an equation in standard form, explain how to determine the rate of change.

66. WRITING IN MATH Explain how you would use a given y-intercept and the slope to predict a y-value for a given x-value without graphing.

Standardized Test Practice

67. A music store has x CDs in stock. If 350 are sold and $3y$ are added to stock, which expression represents the number of CDs in stock?

A $350 + 3y - x$ **C** $x + 350 + 3y$

B $x - 350 + 3y$ **D** $3y - 350 - x$

68. PROBABILITY The table shows the result of a survey of favorite activities. What is the probability that a student's favorite activity is sports or drama club?

Extracurricular Activity	Students
art club	24
band	134
choir	37
drama club	46
mock trial	19
school paper	26
sports	314

F $\frac{3}{8}$ **G** $\frac{4}{9}$ **H** $\frac{3}{5}$ **J** $\frac{2}{3}$

69. A recipe for fruit punch calls for 2 oz of orange juice for every 8 oz of lemonade. If Hamda uses 64 oz of lemonade, which proportion can she use to find x, the number of ounces of orange juice needed?

A $\frac{2}{x} = \frac{64}{6}$ **C** $\frac{2}{8} = \frac{x}{64}$

B $\frac{8}{x} = \frac{64}{2}$ **D** $\frac{6}{2} = \frac{x}{64}$

70. EXTENDED RESPONSE The table shows the results of a canned food drive. 1225 cans were collected, and the 12th-grade class collected 55 more cans than the 10th-grade class. How many cans each did the 10th- and 12th-grade classes collect? Show your work.

Grade	Cans
9	340
10	x
11	280
12	y

Spiral Review

For each arithmetic sequence, determine the related function. Then determine if the function is *proportional* or *nonproportional*. (Lesson 3-6)

71. 3, 7, 11, ... **72.** 8, 6, 4, ... **73.** 0, 3, 6, ... **74.** 1, 2, 3, ...

75. GAME SHOWS Contestants on a game show win money by answering 10 questions. (Lesson 3-5)

a. Find the value of the 10th question.

b. If all questions are answered correctly, how much are the winnings?

Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve. (Lesson 3-4)

76. If $y = 10$ when $x = 5$, find y when $x = 6$.

77. If $y = -16$ when $x = 4$, find x when $y = 20$.

78. If $y = 6$ when $x = 18$, find y when $x = -12$.

79. If $y = 12$ when $x = 15$, find x when $y = -6$.

Skills Review

Find the slope of the line that passes through each pair of points.

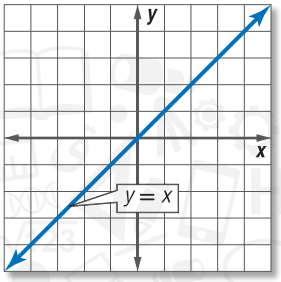
80. (2, 3), (9, 7) **81.** (−3, 6), (2, 4) **82.** (2, 6), (−1, 3) **83.** (−3, 3), (1, 3)

EXTEND

4-1 Graphing Technology Lab: The Family of Linear Graphs

A family of people is related by birth and marriage. Often people in families share characteristics. The graphs in a family share at least one characteristic. Graphs in the linear family are all lines, with the simplest graph in the family being that of the parent function $y = x$. This parent function is also known as the **identity function**. Its graph contains all points with coordinates (a, a). Its domain and range are all real numbers.

Parent Graph
Identity Function



Mathematical Practices
Look for and make use of structure.

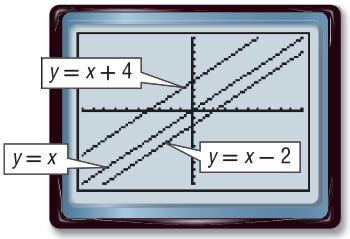
You can use a graphing calculator to investigate how changing the parameters m and b in $y = mx + b$ affects the graphs in the family of linear functions.

Activity 1 Changing b in $y = mx + b$

Graph $y = x$, $y = x + 4$, and $y = x - 2$ in the standard viewing window.

Enter the equations in the **Y=** list as **Y1**, **Y2**, and **Y3**. Then graph the equations.

KEYSTROKES: [Y=] [X,T,θ,n] [ENTER] [X,T,θ,n] [+] 4 [ENTER] [X,T,θ,n] [−] 2 [ENTER] [ZOOM] 6



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1A. How do the slopes of the graphs compare?

1B. Compare the graph of $y = x + 4$ and the graph of $y = x$. How would you obtain the graph of $y = x + 4$ from the graph of $y = x$?

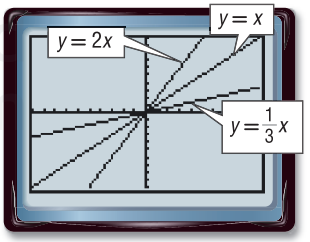
1C. How would you obtain the graph of $y = x - 2$ from the graph of $y = x$?

Changing the y-intercept, b, translates, or moves, a linear function up or down the y-axis. Changing m in $y = mx + b$ affects the graphs in a different way. First, investigate positive values of m.

Activity 2 Changing m in $y = mx + b$, Positive Values

Graph $y = x$, $y = 2x$, and $y = \frac{1}{3}x$ in the standard viewing window.

Enter the equations in the **Y=** list and graph.



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2A. How do the y-intercepts of the graphs compare?

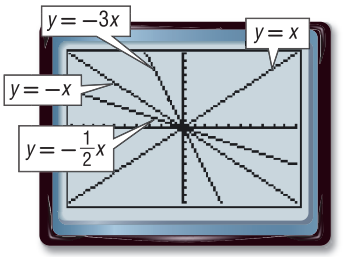
2B. Compare the graph of $y = 2x$ and the graph of $y = x$.

2C. Which is steeper, the graph of $y = \frac{1}{3}x$ or the graph of $y = x$?

Does changing m to a negative value affect the graph differently than changing it to a positive value?

Activity 3 Changing m in $y = mx + b$, Negative Values

Graph $y = x$, $y = -x$, $y = -3x$, and $y = -\frac{1}{2}x$ in the standard viewing window.



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Enter the equations in the Y= list and graph.

3A. How are the graphs with negative values of m different than graphs with a positive m?

3B. Compare the graphs of $y = -x$, $y = -3x$, and $y = -\frac{1}{2}x$. Which is steepest?

Analyze the Results

SENSE-MAKING AND PERSEVERANCE **Graph each set of equations on the same screen. Describe the similarities or differences.**

1. $y = 2x$
$y = 2x + 3$
$y = 2x - 7$

2. $y = x + 1$
$y = 2x + 1$
$y = \frac{1}{4}x + 1$

3. $y = x + 4$
$y = 2x + 4$
$y = \frac{3}{4}x + 4$

4. $y = 0.5x + 2$
$y = 0.5x - 5$
$y = 0.5x + 4$

5. $y = -2x - 2$
$y = -4.2x - 2$
$y = -\frac{1}{3}x - 2$

6. $y = 3x$
$y = 3x + 6$
$y = 3x - 7$

7. Families of graphs have common characteristics. What do the graphs of all equations of the form $y = mx + b$ have in common?

8. How does the value of b affect the graph of $y = mx + b$?

9. What is the result of changing the value of m on the graph of $y = mx + b$ if m is positive?

10. How can you determine which graph is steepest by examining the following equations?
$y = 3x$, $y = -4x - 7$, $y = \frac{1}{2}x + 4$

11. Explain how knowing about the effects of m and b can help you sketch the graph of an equation.

12. The equation $y = k$ can also be a parent graph. Graph $y = 5$, $y = 2$, and $y = -4$ on the same screen. Describe the similarities or differences among the graphs.

Extension

Nonlinear functions can also be defined in terms of a family of graphs. Graph each set of equations on the same screen. Describe the similarities or differences.

13. $y = x^2$
$y = -3x^2$
$y = (-3x)^2$

14. $y = x^2$
$y = x^2 + 3$
$y = (x - 2)^2$

15. $y = x^2$
$y = 2x^2 + 4$
$y = (3x)^2 - 5$

16. Describe the similarities and differences in the classes of functions $f(x) = x^2 + c$ and $f(x) = (x + c)^2$, where c is any real number.

LESSON 4-2

Writing Equations in Slope-Intercept Form

Then

- You graphed lines given the slope and the *y*-intercept.

Now

1. Write an equation of a line in slope-intercept form given the slope and one point.
2. Write an equation of a line in slope-intercept form given two points.

Why?

- In 2006, the attendance at the Columbus Zoo and Aquarium was about 1.6 million. In 2009, the zoo's attendance was about 2.2 million. You can find the average rate of change for these data. Then you can write an equation that would model the average attendance at the zoo for a given year.

New Vocabulary

constraint

linear extrapolation

Mathematical Practices

Construct viable arguments and critique the reasoning of others.

Attend to precision.

1 Write an Equation Given the Slope and a Point

The next example shows how to write an equation of a line if you are given a slope and a point other than the *y*-intercept.

Example 1 Write an Equation Given the Slope and a Point

Write an equation of the line that passes through (2, 1) with a slope of 3.

You are given the slope but not the *y*-intercept.

Step 1 Find the *y*-intercept.

$$y = mx + b$$
$$1 = 3(2) + b$$
$$1 = 6 + b$$
$$1 - 6 = 6 + b - 6$$
$$-5 = b$$

Step 2 Write the equation in slope-intercept form.

$$y = mx + b$$
$$y = 3x - 5$$

Therefore, the equation of the line is $y = 3x - 5$.

Guided Practice

Write an equation of a line that passes through the given point and has the given slope.

1A. $(-2, 5)$, slope 3

1B. $(4, -7)$, slope -1

2 Write an Equation Given Two Points

If you are given two points through which a line passes, you can use them to find the slope first. Then follow the steps in Example 1 to write the equation.

StudyTip

Choosing a point Given two points on a line, you may select either point to be (x_1, y_1). Be sure to remain consistent throughout the problem.

Example 2 Write an Equation Given Two Points

Write an equation of the line that passes through each pair of points.

a. (3, 1) and (2, 4)

Step 1 Find the slope of the line containing the given points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4 - 1}{2 - 3}$$
$$= \frac{3}{-1} \text{ or } -3$$

Step 2 Use either point to find the y-intercept.

$$y = mx + b$$
$$4 = (-3)(2) + b$$
$$4 = -6 + b$$
$$4 - (-6) = -6 + b - (-6)$$
$$10 = b$$

Step 3 Write the equation in slope-intercept form.

$$y = mx + b$$
$$y = -3x + 10$$

Therefore, the equation is $y = -3x + 10$.

b. (−4, −2) and (−5, −6)

StudyTip

Slope If the (x_1, y_1) coordinates are negative, be sure to account for both the negative signs and the subtraction symbols in the Slope Formula.

Step 1 Find the slope of the line containing the given points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-6 - (-2)}{-5 - (-4)}$$
$$= \frac{-4}{-1} \text{ or } 4$$

Step 2 Use either point to find the y-intercept.

$$y = mx + b$$
$$-2 = 4(-4) + b$$
$$-2 = -16 + b$$
$$-2 - (-16) = -16 + b - (-16)$$
$$14 = b$$

Step 3 Write the equation in slope-intercept form.

$$y = mx + b$$
$$y = 4x + 14$$

Therefore, the equation is $y = 4x + 14$.

GuidedPractice

Write an equation of the line that passes through each pair of points.

2A. (−1, 12), (4, −8)

2B. (5, −8), (−7, 0)

In mathematics, a **constraint** is a condition that a solution must satisfy. Equations can be viewed as constraints in a problem situation. The solutions of the equation meet the constraints of the problem.

Real-WorldCareer

Ground Crew

Airline ground crew responsibilities include checking tickets, helping passengers with luggage, and making sure that baggage is loaded properly and secure. This job usually requires a high school diploma or GED.

Source: Airline Jobs

Real-World Example 3 Use Slope-Intercept Form

FLIGHTS **The table shows the number of domestic flights from 2004 to 2008. Write an equation that could be used to predict the number of flights if it continues to decrease at the same rate.**

Year	Flights (millions)
2004	9.97
2005	10.04
2006	9.71
2007	9.84
2008	9.37

Understand You know the number of flights for 2004–2008.

Plan Let x represent the number of years since 2000, and let y represent the number of flights. Write an equation of the line that passes through (4, 9.97) and (8, 9.37).

Solve Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{9.37 - 9.97}{8 - 4}$$
$$= -\frac{0.6}{4} \text{ or } -0.15$$

Use (8, 9.37) to find the y-intercept of the line.

$$y = mx + b$$
$$9.37 = -0.15(8) + b$$
$$9.37 = -1.2 + b$$
$$10.57 = b$$

Write the equation using $m = -0.15$ and $b = 10.57$.

$$y = mx + b$$
$$y = -0.15x + 10.57$$

Check Check your result by using the coordinates of the other point.

$$y = -0.15x + 10.57$$
$$9.97 \stackrel{?}{=} -0.15(4) + 10.57$$
$$9.97 = 9.97 \checkmark$$

GuidedPractice

3. **FINANCIAL LITERACY** In addition to his weekly salary, Ahmad is paid AED 16 per delivery. Last week, he made 5 deliveries, and his total pay was AED 215. Write a linear equation to find Ahmad's total weekly pay T if he makes d deliveries.

You can use a linear equation to make predictions about values that are beyond the range of the data. This process is called **linear extrapolation**.

Problem-SolvingTip

Precision Deciding whether an answer is reasonable is useful when an exact answer is not neccessary.

Real-World Example 4 Predict from Slope-Intercept Form

FLIGHTS **Estimate the number of domestic flights in 2020.**

$$y = -0.15x + 10.57$$
$$= -0.15(20) + 10.57 \text{ or } 7.57 \text{ million}$$

GuidedPractice

4. **MONEY** Use the equation in Guided Practice 3 to predict how much money Ahmad will earn in a week if he makes 8 deliveries.

Check Your Understanding

Example 1 **Write an equation of the line that passes through the given point and has the given slope.**

1. (3, −3), slope 3

2. (2, 4), slope 2

3. (1, 5), slope −1

4. (−4, 6), slope −2

Example 2 **Write an equation of the line that passes through each pair of points.**

5. (4, −3), (2, 3)

6. (−7, −3), (−3, 5)

7. (−1, 3), (0, 8)

8. (−2, 6), (0, 0)

Examples 3, 4 **9.** **WHITEWATER RAFTING** Ten people from a local youth group went to Black Hills Whitewater Rafting Tour Company for a one-day rafting trip. The group paid AED 425.

a. Write an equation in slope-intercept form to find the total cost C for p people.

b. How much would it cost for 15 people?

Practice and Problem Solving

Example 1 **Write an equation of the line that passes through the given point and has the given slope.**

10. (3, 1), slope 2

11. (−1, 4), slope −1

12. (1, 0), slope 1

13. (7, 1), slope 8

14. (2, 5), slope −2

15. (2, 6), slope 2

Example 2 **Write an equation of the line that passes through each pair of points.**

16. (9, −2), (4, 3)

17. (−2, 5), (5, −2)

18. (−5, 3), (0, −7)

19. (3, 5), (2, −2)

20. (−1, −3), (−2, 3)

21. (−2, −4), (2, 4)

Examples 3, 4 **22.** **MODELING** Mahmoud is driving a remote control car at a constant speed. He starts the timer when the car is 5 ft away. After 2 s the car is 35 ft away.

a. Write a linear equation to find the distance d of the car from Mahmoud.

b. Estimate the distance the car has traveled after 10 s.

23. **ZOOS** Refer to the beginning of the lesson.

a. Write a linear equation to find the attendance (in millions) y after x years. Let x be the number of years since 2000.

b. Estimate the zoo's attendance in 2020.

24. **BOOKS** In 1904, a dictionary cost 30 fils. Since then the cost of a dictionary has risen an average of 6 fils per year.

a. Write a linear equation to find the cost C of a dictionary y years after 1904.

b. If this trend continues, what will the cost of a dictionary be in 2020?

Write an equation of the line that passes through the given point and has the given slope.

25. (4, 2), slope $\frac{1}{2}$

26. (3, −2), slope $\frac{1}{3}$

27. (6, 4), slope $-\frac{3}{4}$

28. (2, −3), slope $\frac{2}{3}$

29. (2, −2), slope $\frac{2}{7}$

30. (−4, −2), slope $-\frac{3}{5}$

31. CATS In 2008, there were about 56.1 thousand shorthair cats registered. In 2016, the number was 62.5 thousand.

a. Write a linear equation to find the number of thousands of shorthair cats G that will be registered in year t, where $t = 0$ is the year 2007.

b. Graph the equation.

c. Estimate the number of shorthair cats that will be registered in 2017.

32. GYM MEMBERSHIPS A local recreation center offers a yearly membership for AED 265. The center offers aerobics classes for an additional AED 5 per class.

a. Write an equation that represents the total cost of the membership.

b. Carly spent AED 500 one year. How many aerobics classes did she take?

33. SUBSCRIPTION A magazine offers an online subscription that allows you to view up to 25 archived articles free. To view 30 archived articles, you pay AED 49.15. To view 33 archived articles, you pay AED 57.40.

a. What is the cost of each archived article for which you pay a fee?

b. What is the cost of the magazine subscription?

Write an equation of the line that passes through the given points.

34. (5, −2), (7, 1)

35 (5, −3), (2, 5)

36. $\left(\frac{5}{4}, 1\right), \left(-\frac{1}{4}, \frac{3}{4}\right)$

37. $\left(\frac{5}{12}, -1\right), \left(-\frac{3}{4}, \frac{1}{6}\right)$

Determine whether the given point is on the line. Explain why or why not.

38. $(3, -1); y = \frac{1}{3}x + 5$

39. $(6, -2); y = \frac{1}{2}x - 5$

For Exercises 40–42, determine which equation best represents each situation. Explain the meaning of each variable.

A	B	C
$y = -\frac{1}{3}x + 72$	$y = 2x + 225$	$y = 8x + 4$

40. CONCERTS Tickets to a concert cost AED 8 each plus a processing fee of AED 4 per order.

41. FUNDRAISING The Grade 9 class has AED 225. They sell raffle tickets at AED 2 each to raise money for a field trip.

42. POOLS The current water level of a swimming pool, is 6 ft. The rate of evaporation is $\frac{1}{3}$ in per day.

43. SENSE-MAKING A manufacturer implemented a program to reduce waste. In 1998 they sent 946 tons of waste to landfills. Each year after that, they reduced their waste by an average 28.4 tons.

a. How many tons were sent to the landfill in 2010?

b. In what year will it become impossible for this trend to continue? Explain.

44. COMBINING FUNCTIONS The parents of a college student open an account for her with a deposit of AED 5000, and they set up automatic deposits of AED 100 to the account every week.

a. Write a function $d(t)$ to express the amount of money in the account t weeks after the initial deposit.

b. The student plans on spending AED 600 the first week and AED 250 in each of the following weeks for room and board and other expenses. Write a function $w(t)$ to express the amount of money taken out of the account each week.

c. Find $B(t) = d(t) - w(t)$. What does this new function represent?

d. Will the student run out of money? If so, when?

45 **CONCERT TICKETS** Rashid is ordering tickets for a concert online. There is a processing fee for each order, and the tickets are AED 52 each. Rashid ordered 5 tickets and the cost was AED 275.

a. Determine the processing fee. Write a linear equation to represent the total cost C for t tickets.

b. Make a table of values for at least three other numbers of tickets.

c. Graph this equation. Predict the cost of 8 tickets.

46. MUSIC A music store is offering a Frequent Buyers Club membership. The membership costs AED 22 per year, and then a member can buy CDs at a reduced price. If a member buys 17 CDs in one year, the cost is AED 111.25.

a. Determine the cost of each CD for a member.

b. Write a linear equation to represent the total cost y of a one year membership, if x CDs are purchased.

c. Graph this equation.

H.O.T. Problems Use Higher-Order Thinking Skills

47. ERROR ANALYSIS Khadija and Asma are writing an equation of the line through $(3, -2)$ and $(6, 4)$. Is either of them correct? Explain your reasoning.

Khadija

$m = \frac{4 - (-2)}{6 - 3} = \frac{6}{3}$ or 2

$y = mx + b$

$6 = 2(4) + b$

$6 = 8 + b$

$-2 = b$

$y = 2x - 2$

Asma

$m = \frac{4 - (-2)}{6 - 3} = \frac{6}{3}$ or 2

$y = mx + b$

$-2 = 2(3) + b$

$-2 = 6 + b$

$-8 = b$

$y = 2x - 8$

48. CHALLENGE Consider three points, $(3, 7)$, $(-6, 1)$ and $(9, p)$, on the same line. Find the value of p and explain your steps.

49. REASONING Consider the standard form of a linear equation, $Ax + By = C$.

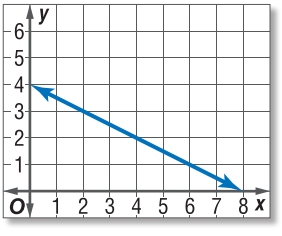
a. Rewrite the equation in slope-intercept form.

b. What is the slope?

c. What is the y-intercept?

d. Is this true for all real values of A, B, and C?

50. OPEN ENDED Create a real-world situation that fits the graph at the right. Define the two quantities and describe the functional relationship between them. Write an equation to represent this relationship and describe what the slope and y-intercept mean.

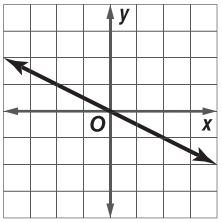


51. WRITING IN MATH Linear equations are useful in predicting future events. Describe some factors in real-world situations that might affect the reliability of the graph in making any predictions.

52. ARGUMENTS What information is needed to write the equation of a line? Explain.

Standardized Test Practice

53. Which equation *best* represents the graph?

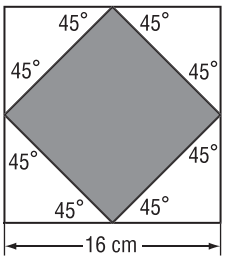


A $y = 2x$
B $y = -2x$
C $y = \frac{1}{2}x$
D $y = -\frac{1}{2}x$

54. Mustafa receives an employee discount of 12%. If he buys a AED 355 item at the store, what is his discount to the nearest dirham?

F AED 3
G AED 4
H AED 30
J AED 43

55. GEOMETRY The midpoints of the sides of the large square are joined to form a smaller square. What is the area of the smaller square?



A 64 cm^2
B 128 cm^2
C 248 cm^2
D 256 cm^2

56. SHORT RESPONSE If $\frac{5(x + 4)}{2} + 7 = 37$, what is the value of $3x - 9$?

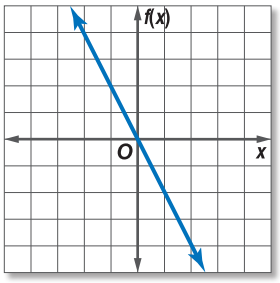
Spiral Review

Graph each equation. (Lesson 4-1)

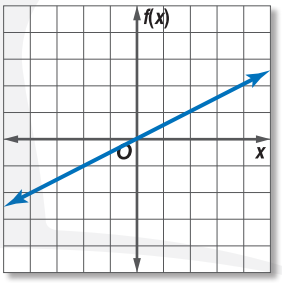
57. $y = 3x + 2$
58. $y = -4x + 2$
59. $3y = 2x + 6$
60. $y = \frac{1}{2}x + 6$
61. $3x + y = -1$
62. $2x + 3y = 6$

Write an equation in function notation for each relation. (Lesson 3-6)

63.



64.



65. METEOROLOGY The distance d in miles that the sound of thunder travels in t seconds is given by the equation $d = 0.21t$. (Lesson 3-4)

a. Graph the equation.

b. Use the graph to estimate how long it will take you to hear thunder from a storm 3 miles away.

Solve each equation. Check your solution. (Lesson 2-3)

66. $-5t - 2.2 = -2.9$
67. $-5.5a - 43.9 = 77.1$
68. $4.2r + 7.14 = 12.6$
69. $-14 - \frac{n}{9} = 9$
70. $\frac{-8b - (-9)}{-10} = 17$
71. $9.5x + 11 - 7.5x = 14$

Skills Review

Find the value of r so the line through each pair of points has the given slope.

72. $(6, -2)$, $(r, -6)$, $m = 4$
73. $(8, 10)$, $(r, 4)$, $m = 6$
74. $(7, -10)$, $(r, 4)$, $m = -3$
75. $(6, 2)$, $(9, r)$, $m = -1$
76. $(9, r)$, $(6, 3)$, $m = -\frac{1}{3}$
77. $(5, r)$, $(2, -3)$, $m = \frac{4}{3}$

LESSON 4-3

Writing Equations in Point-Slope Form

Then

- You wrote linear equations given either one point and the slope or two points.

Now

1. Write equations of lines in point-slope form.
2. Write linear equations in different forms.

Why?

- Most humane societies have foster homes for newborn puppies, kittens, and injured or ill animals. During the spring and summer, a large shelter can place 3000 animals in homes each month.

If a shelter had 200 animals in foster homes at the beginning of spring, the number of animals in foster homes at the end of the summer could be represented by $y = 3000x + 200$, where x is the number of months and y is the number of animals.

New Vocabulary
point-slope form

Mathematical Practices
Reason abstractly and quantitatively.

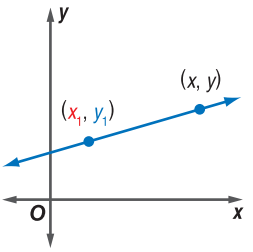
1 Point-Slope Form

An equation of a line can be written in **point-slope form** when given the coordinates of one known point on a line and the slope of that line.

KeyConcept Point-Slope Form

Words The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.

Symbols $y - y_1 = m(x - x_1)$

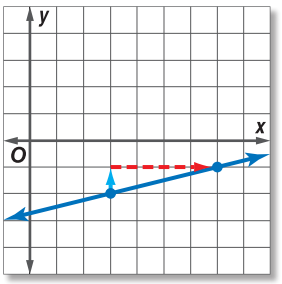


Example 1 Write and Graph an Equation in Point-Slope Form

Write an equation in point-slope form for the line that passes through $(3, -2)$ with a slope of $\frac{1}{4}$. Then graph the equation.

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = \frac{1}{4}(x - 3)$$
$$y + 2 = \frac{1}{4}(x - 3)$$

Plot the point at $(3, -2)$ and use the slope to find another point on the line. Draw a line through the two points.



GuidedPractice

1. Write an equation in point-slope form for the line that passes through $(-2, 1)$ with a slope of -6. Then graph the equation.

2 Forms of Linear Equations If you are given the slope and the coordinates of one or two points, you can write the linear equation in the following ways.

StudyTip

Slope The slope of the line remains unchanged throughout the line. You can go in either direction along the line using the same rise over run and you will always end at a point on the line.

ConceptSummary Writing Equations

Given the Slope and One Point

Step 1 Substitute the value of m and let the x and y coordinates be (x_1, y_1). Or, substitute the value of m, x, and y into the slope-intercept form and solve for b.

Step 2 Rewrite the equation in the needed form.

Given Two Points

Step 1 Find the slope.

Step 2 Choose one of the two points to use.

Step 3 Follow the steps for writing an equation given the slope and one point.

ReviewVocabulary

Standard form of a linear equation $Ax + By = C$, where $A \geq 0$, A and B are not both zero, and A, B, and C are integers with a greatest common factor of 1

Example 2 Standard Form

Write $y - 1 = -\frac{2}{3}(x - 5)$ in standard form.

$$y - 1 = -\frac{2}{3}(x - 5)$$

$$3(y - 1) = 3\left(-\frac{2}{3}\right)(x - 5)$$

$$3(y - 1) = -2(x - 5)$$

$$3y - 3 = -2x + 10$$

$$3y = -2x + 13$$

$$2x + 3y = 13$$

GuidedPractice

2. Write $y - 1 = 7(x + 5)$ in standard form.

To find the y-intercept of an equation, rewrite the equation in slope-intercept form.

Example 3 Slope-Intercept Form

Write $y + 3 = \frac{3}{2}(x + 1)$ in slope-intercept form.

$$y + 3 = \frac{3}{2}(x + 1)$$

$$y + 3 = \frac{3}{2}x + \frac{3}{2}$$

$$y = \frac{3}{2}x - \frac{3}{2}$$

GuidedPractice

3. Write $y + 6 = -3(x - 4)$ in slope-intercept form.

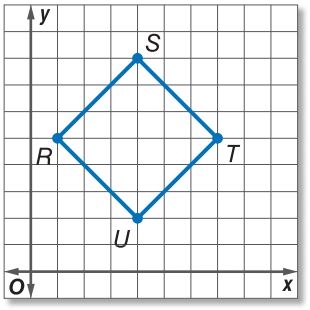
Being able to use a variety of forms of linear equations can be useful in other subjects as well.

StudyTip

Slopes in Squares
Nonvertical opposite sides of a square have equal slopes. If the coordinates for one of the vertices are unavailable, use the slope of the opposite side.

Example 4 Point-Slope Form and Standard Form

GEOMETRY **The figure shows square *RSTU*.**



a. Write an equation in point-slope form for the line containing side $\overline{TU}$.

Step 1 Find the slope of $\overline{TU}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - 2}{7 - 4} \text{ or } 1$$

Step 2 You can select either point for (x_1, y_1) in the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 1(x - 4)$$
$$y - 5 = 1(x - 7)$$

b. Write an equation in standard form for the same line.

$$y - 2 = 1(x - 4) \qquad y - 5 = 1(x - 7)$$
$$y - 2 = 1x - 4 \qquad y - 5 = 1x - 7$$
$$y = 1x - 2 \qquad y = 1x - 2$$
$$-1x + y = -2 \qquad -1x + y = -2$$
$$x - y = 2 \qquad x - y = 2$$

GuidedPractice

4A. Write an equation in point-slope form of the line containing side $\overline{ST}$.

4B. Write an equation in standard form of the line containing $\overline{ST}$.

Check Your Understanding

Example 1 **Write an equation in point-slope form for the line that passes through the given point with the slope provided. Then graph the equation.**

1 $(-2, 5)$, slope -6 **2.** $(-2, -8)$, slope $\frac{5}{6}$ **3.** $(4, 3)$, slope $-\frac{1}{2}$

Example 2 **Write each equation in standard form.**

4. $y + 2 = \frac{7}{8}(x - 3)$ **5.** $y + 7 = -5(x + 3)$ **6.** $y + 2 = \frac{5}{3}(x + 6)$

Example 3 **Write each equation in slope-intercept form.**

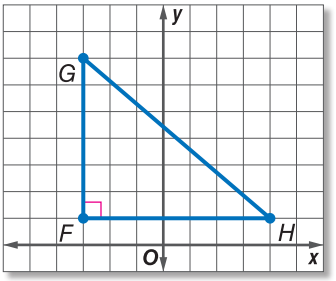
7. $y - 10 = 4(x + 6)$ **8.** $y - 7 = -\frac{3}{4}(x + 5)$ **9.** $y - 9 = x + 4$

Example 4

10. GEOMETRY Use right triangle FGH.

a. Write an equation in point-slope form for the line containing $\overline{GH}$.

b. Write the standard form of the line containing $\overline{GH}$.



Practice and Problem Solving

Example 1

Write an equation in point-slope form for the line that passes through each point with the given slope. Then graph the equation.

11. $(5, 3)$, $m = 7$

12. $(2, -1)$, $m = -3$

13. $(-6, -3)$, $m = -1$

14. $(-7, 6)$, $m = 0$

15. $(-2, 11)$, $m = \frac{4}{3}$

16. $(-6, -8)$, $m = -\frac{5}{8}$

17. $(-2, -9)$, $m = -\frac{7}{5}$

18. $(-6, 0)$, horizontal line

Example 2

Write each equation in standard form.

19. $y - 10 = 2(x - 8)$

20. $y - 6 = -3(x + 2)$

21. $y - 9 = -6(x + 9)$

22. $y + 4 = \frac{2}{3}(x + 7)$

23. $y + 7 = \frac{9}{10}(x + 3)$

24. $y + 7 = -\frac{3}{2}(x + 1)$

25. $2y + 3 = -\frac{1}{3}(x - 2)$

26. $4y - 5x = 3(4x - 2y + 1)$

Example 3

Write each equation in slope-intercept form.

27. $y - 6 = -2(x - 7)$

28. $y - 11 = 3(x + 4)$

29. $y + 5 = -6(x + 7)$

30. $y - 1 = \frac{4}{5}(x + 5)$

31. $y + 2 = \frac{1}{6}(x - 4)$

32. $y + 6 = -\frac{3}{4}(x + 8)$

33. $y + 3 = -\frac{1}{3}(2x + 6)$

34. $y + 4 = 3(3x + 3)$

Example 4

35. MOVIE RENTALS The number of copies of a movie rented at a video kiosk decreased at a constant rate of 5 copies per week. The 6th week after the movie was released, 4 copies were rented. How many copies were rented during the second week?

36. REASONING A company offers premium cable for AED 39.95 per month plus a one-time setup fee. The total cost for setup and 6 months of service is AED 264.70.

a. Write an equation in point-slope form to find the total price y for any number of months x. (*Hint*: The point (6, 264.70) is a solution to the equation.)

b. Write the equation in slope-intercept form.

c. What is the setup fee?

Write an equation for the line described in standard form.

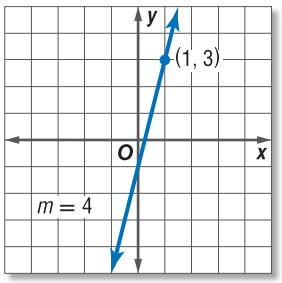
37. through $(-1, 7)$ and $(8, -2)$

38. through $(-4, 3)$ with y-intercept 0

39. with x-intercept 4 and y-intercept 5

Write an equation in point-slope form for each line.

40.



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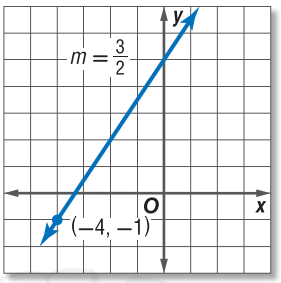

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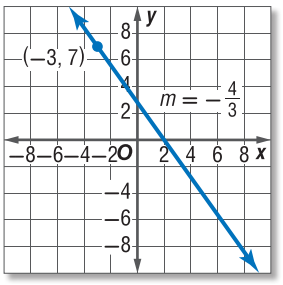


Write each equation in slope-intercept form.

43 $y + \frac{3}{5} = x - \frac{2}{5}$

44. $y - \frac{7}{2} = \frac{1}{2}(x - 4)$

45. $y + \frac{1}{3} = \frac{5}{6}\left(x + \frac{2}{5}\right)$

46. Write an equation in point-slope form, slope-intercept form, and standard form for a line that passes through $(-2, 8)$ with slope $\frac{8}{5}$.

47. Line ℓ passes through $(-9, 4)$ with slope $\frac{4}{7}$. Write an equation in point-slope form, slope-intercept form, and standard form for line ℓ.

48. **WEATHER** The barometric pressure is 598 mm of mercury (mmHg) at an altitude of 1.8 km and 577 mm of mercury at 2.1 km.

a. Write a formula for the barometric pressure as a function of the altitude.

b. What is the altitude if the pressure is 657 mm of mercury?

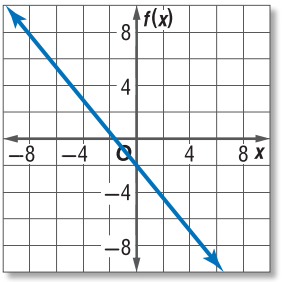
H.O.T. Problems Use Higher-Order Thinking Skills

49. **WHICH ONE DOESN'T BELONG?** Identify the equation that does not belong. Explain your reasoning.

$y - 5 = 3(x - 1)$	$y + 1 = 3(x + 1)$	$y + 4 = 3(x + 1)$	$y - 8 = 3(x - 2)$

50. **CRITIQUE** Alia thinks that $f(x)$ and $g(x)$ have the same slope but different intercepts. Suha thinks that $f(x)$ and $g(x)$ describe the same line. Is either of them correct? Explain your reasoning.

The graph of $g(x)$ is the line that passes through $(3, -7)$ and $(-6, 4)$.



51. **OPEN ENDED** Describe a real-life scenario that has a constant rate of change and a value of y for a particular value of x. Represent this situation using an equation in point-slope form, an equation in standard form, and an equation in slope-intercept form.

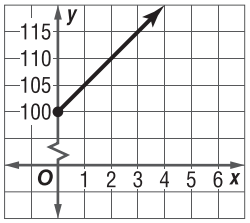
52. **REASONING** Write an equation for the line that passes through $(-4, 8)$ and $(3, -7)$. What is the slope? Where does the line intersect the x-axis? the y-axis?

53. **CHALLENGE** Write an equation in point-slope form for the line that passes through the points (f, g) and (h, j).

54. **WRITING IN MATH** Why do we represent linear equations in more than one form?

Standardized Test Practice

55. Which statement is *most* strongly supported by the graph?



A You have AED 100 and spend AED 5 weekly.

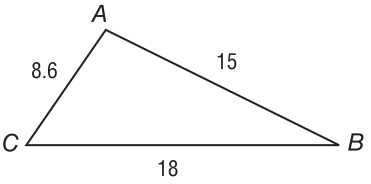
B You have AED 100 and save AED 5 weekly.

C You need AED 100 for a new CD player and save AED 5 weekly.

D You need AED 100 for a new CD player and spend AED 5 weekly.

56. SHORT RESPONSE A store offers customers a AED 5 gift certificate for every AED 75 they spend. How much would a customer have to spend to earn AED 35 worth of gift certificates?

57. GEOMETRY Which triangle is similar to $\triangle ABC$?



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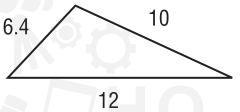

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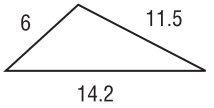

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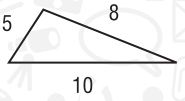

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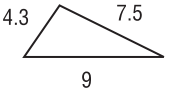


58. In a class of 25 students, 6 have blue eyes, 15 have brown hair, and 3 have blue eyes and brown hair. How many students have neither blue eyes nor brown hair?

A 4 **C** 10

B 7 **D** 22

Spiral Review

Write an equation of the line that passes through each pair of points. (Lesson 4-2)

59. (4, 2), (−2, −4)

60. (3, −2), (6, 4)

61. (−1, 3), (2, −3)

62. (2, −2), (3, 2)

63. (7, −2), (−4, −2)

64. (0, 5), (−3, 5)

Write an equation in slope-intercept form of the line with the given slope and *y*-intercept. (Lesson 4-1)

65. slope: −2, *y*-intercept: 6

66. slope: 3, *y*-intercept: −5

67. slope: $\frac{1}{2}$, *y*-intercept: 3

68. slope: $-\frac{3}{5}$, *y*-intercept: 12

69. slope: 0, *y*-intercept: 3

70. slope: −1, *y*-intercept: 0

71. THEATER The Coral Gables Actors' Playhouse has 7 rows of seats in the orchestra section. The number of seats in the rows forms an arithmetic sequence, as shown in the table. On opening night, 368 tickets were sold for the orchestra section. Was the section oversold? (Lesson 3-5)

Rows	Number of Seats
7	76
6	68
5	60

Skills Review

Solve each equation or formula for the variable specified.

72. $y = mx + b$, for m

73. $v = r + at$, for a

74. $km + 5x = 6y$, for m

75. $4b - 5 = -t$, for b

LESSON 4-4

Parallel and Perpendicular Lines

Then
- You wrote equations in point-slope form.

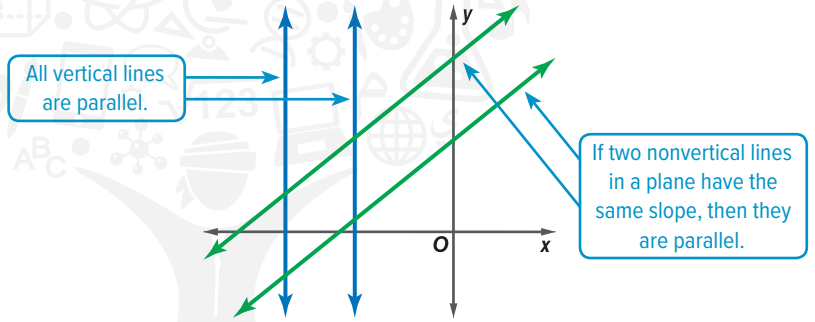
Now
1. Write an equation of the line that passes through a given point, parallel to a given line.
2. Write an equation of the line that passes through a given point, perpendicular to a given line.

Why?
- Notice the squares, rectangles and lines in the piece of art shown at the right. Some of the lines intersect forming right angles. Other lines do not intersect at all.

New Vocabulary
parallel lines
perpendicular lines

Mathematical Practices
Use appropriate tools strategically.

1 Parallel Lines Lines in the same plane that do not intersect are called **parallel lines**. Nonvertical parallel lines have the same slope.



You can write an equation of a line parallel to a given line if you know a point on the line and an equation of the given line. First find the slope of the given line. Then, substitute the point provided and the slope from the given line into the point-slope form.

Example 1 Parallel Line Through a Given Point

Write an equation in slope-intercept form for the line that passes through $(-3, 5)$ and is parallel to the graph of $y = 2x - 4$.

Step 1 The slope of the line with equation $y = 2x - 4$ is 2. The line parallel to $y = 2x - 4$ has the same slope, 2.

Step 2 Find the equation in slope-intercept form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= 2[x - (-3)] \\ y - 5 &= 2(x + 3) \\ y - 5 &= 2x + 6 \\ y - 5 + 5 &= 2x + 6 + 5 \\ y &= 2x + 11 \end{aligned}$$

Guided Practice

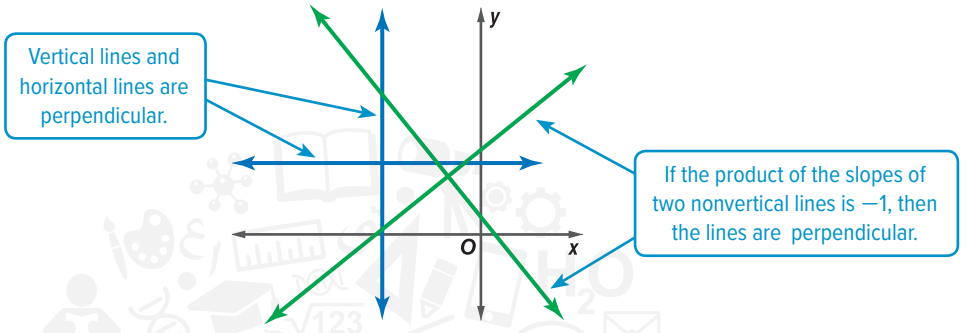
1. Write an equation in point-slope form for the line that passes through $(4, -1)$ and is parallel to the graph of $y = \frac{1}{4}x + 7$.

ReviewVocabulary

opposite reciprocals

The opposite reciprocal of $\frac{a}{b}$ is $-\frac{b}{a}$. Their product is -1.

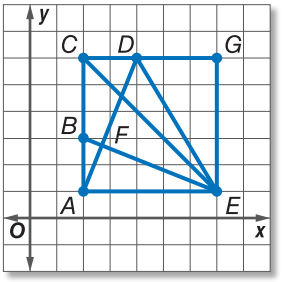
2 Perpendicular Lines Lines that intersect at right angles are called **perpendicular lines**. The slopes of nonvertical perpendicular lines are opposite reciprocals. That is, if the slope of a line is 4, the slope of the line perpendicular to it is $-\frac{1}{4}$.



You can use slope to determine whether two lines are perpendicular.

Real-World Example 2 **Slopes of Perpendicular Lines**

DESIGN **The outline of a company's new logo is shown on a coordinate plane.**



a. Is $\angle DFE$ a right angle in the logo?

If $\overline{BE}$ and $\overline{AD}$ are perpendicular, then $\angle DFE$ is a right angle. Find the slopes of $\overline{BE}$ and $\overline{AD}$.

slope of $\overline{BE}$: $m = \frac{1-3}{7-2}$ or $-\frac{2}{5}$

slope of $\overline{AD}$: $m = \frac{6-1}{4-2}$ or $\frac{5}{2}$

The line segments are perpendicular because $-\frac{2}{5} \times \frac{5}{2} = -1$. Therefore, $\angle DFE$ is a right angle.

b. Is each pair of opposite sides parallel?

If a pair of opposite sides are parallel, then they have the same slope.

slope of $\overline{AC}$: $m = \frac{6-1}{2-2}$ or undefined

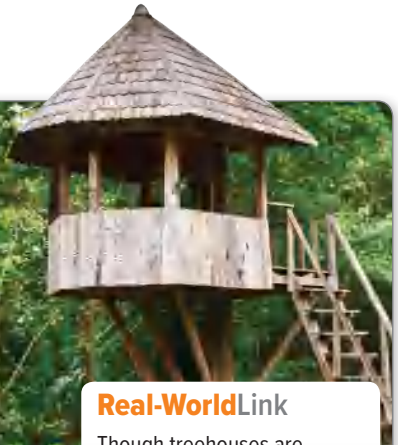
Since $\overline{AC}$ and $\overline{GE}$ are both parallel to the y-axis, they are vertical and are therefore parallel.

slope of $\overline{CG}$: $m = \frac{6-6}{7-2}$ or 0

Since $\overline{CG}$ and $\overline{AE}$ are both parallel to the x-axis, they are horizontal and are therefore parallel.

GuidedPractice

2. CONSTRUCTION On the plans for a treehouse, a beam represented by $\overline{QR}$ has endpoints $Q(-6, 2)$ and $R(-1, 8)$. A connecting beam represented by $\overline{ST}$ has endpoints $S(-3, 6)$ and $T(-8, 5)$. Are the beams perpendicular? Explain.



Real-WorldLink

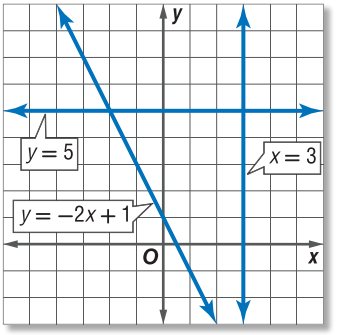
Though treehouses are typically built for recreational purposes, they were originally designed as a way to be protected from wild animals, dense population, and other threats.

Source: *The Treehouse Book*

You can determine whether the graphs of two linear equations are parallel or perpendicular by comparing the slopes of the lines.

Example 3 Parallel or Perpendicular Lines

Determine whether the graphs of $y = 5$, $x = 3$, and $y = -2x + 1$ are *parallel* or *perpendicular*. Explain.



Graph each line on a coordinate plane.

From the graph, you can see that $y = 5$ is parallel to the x-axis and $x = 3$ is parallel to the y-axis. Therefore, they are perpendicular. None of the lines are parallel.

GuidedPractice

3. Determine whether the graphs of $6x - 2y = -2$, $y = 3x - 4$, and $y = 4$ are *parallel* or *perpendicular*. Explain.

You can write the equation of a line perpendicular to a given line if you know a point on the line and the equation of the given line.

StudyTip

Tools Graph the given equation on a coordinate grid and plot the given point. Using a ruler, draw a line perpendicular to the given line that passes through the point.

Example 4 Perpendicular Line Through a Given Point

Write an equation in slope-intercept form for the line that passes through $(-4, 6)$ and is perpendicular to the graph of $2x + 3y = 12$.

Step 1 Find the slope of the given line by solving the equation for y.

$$2x + 3y = 12$$

$$2x - 2x + 3y = -2x + 12$$

$$3y = -2x + 12$$

$$\frac{3y}{3} = \frac{-2x + 12}{3}$$

$$y = -\frac{2}{3}x + 4$$

The slope is $-\frac{2}{3}$.

Step 2 The slope of the perpendicular line is the opposite reciprocal of $-\frac{2}{3}$ or $\frac{3}{2}$. Find the equation of the perpendicular line.

$$y - y_1 = m(x - x_1)$$

$$y - 6 = \frac{3}{2}[x - (-4)]$$

$$y - 6 = \frac{3}{2}(x + 4)$$

$$y - 6 = \frac{3}{2}x + 6$$

$$y - 6 + 6 = \frac{3}{2}x + 6 + 6$$

$$y = \frac{3}{2}x + 12$$

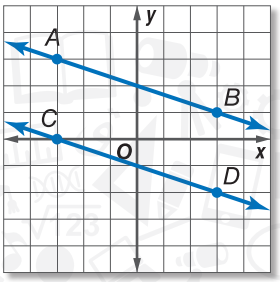
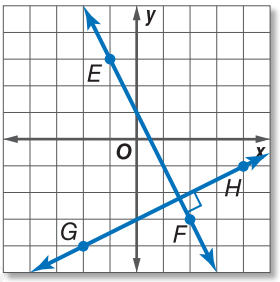
GuidedPractice

4. Write an equation in slope-intercept form for the line that passes through (4, 7) and is perpendicular to the graph of $y = \frac{2}{3}x - 1$.

ReadingMath

Parallel and Perpendicular Lines The symbol for parallel is ∥. The symbol for perpendicular is ⊥.

ConceptSummary Parallel and Perpendicular Lines

	Parallel Lines	Perpendicular Lines
Words	Two nonvertical lines are parallel if they have the same slope.	Two nonvertical lines are perpendicular if the product of their slopes is −1.
Symbols	$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	$\overleftrightarrow{EF} \perp \overleftrightarrow{GH}$
Models		

Check Your Understanding

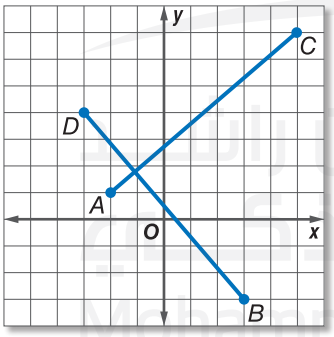
Example 1 **Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of the given equation.**

1. $(-1, 2)$, $y = \frac{1}{2}x - 3$

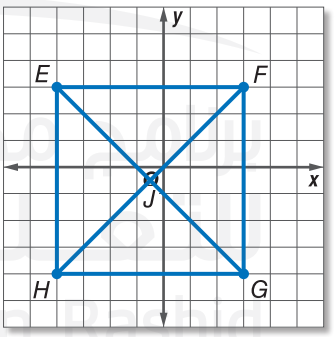
2. $(0, 4)$, $y = -4x + 5$

Example 2

3. GARDENS A garden is in the shape of a quadrilateral with vertices $A(-2, 1)$, $B(3, -3)$, $C(5, 7)$, and $D(-3, 4)$. Two paths represented by $\overline{AC}$ and $\overline{BD}$ cut across the garden. Are the paths perpendicular? Explain.



4. PRECISION A square is a quadrilateral that has opposite sides parallel, consecutive sides that are perpendicular, and diagonals that are perpendicular. Determine whether the quadrilateral is a square. Explain.



Example 3 **Determine whether the graphs of the following equations are *parallel* or *perpendicular*. Explain.**

5 $y = -2x$, $2y = x$, $4y = 2x + 4$

6. $y = \frac{1}{2}x$, $3y = x$, $y = -\frac{1}{2}x$

Example 4 **Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.**

7. $(-2, 3)$, $y = -\frac{1}{2}x - 4$

8. $(-1, 4)$, $y = 3x + 5$

9. $(2, 3)$, $2x + 3y = 4$

10. $(3, 6)$, $3x - 4y = -2$

Practice and Problem Solving

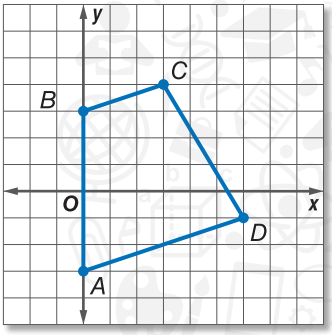
Example 1 **Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of the given equation.**

11. $(3, -2)$, $y = x + 4$ **12.** $(4, -3)$, $y = 3x - 5$ **13.** $(0, 2)$, $y = -5x + 8$

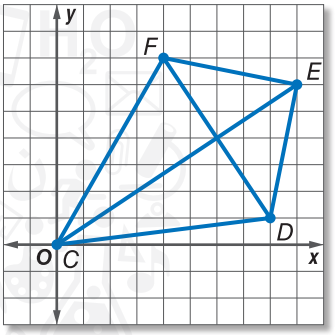
14. $(-4, 2)$, $y = -\frac{1}{2}x + 6$ **15.** $(-2, 3)$, $y = -\frac{3}{4}x + 4$ **16.** $(9, 12)$, $y = 13x - 4$

Example 2

17. **GEOMETRY** A trapezoid is a quadrilateral that has exactly one pair of parallel opposite sides. Is $ABCD$ a trapezoid? Explain your reasoning.



18. **GEOMETRY** $CDEF$ is a kite. Are the diagonals of the kite perpendicular? Explain your reasoning.



19. Determine whether the graphs of $y = -6x + 4$ and $y = \frac{1}{6}x$ are perpendicular. Explain.

20. **MAPS** On a map, Khalifa Drive passes through $R(4, -11)$ and $S(0, -9)$, and Sharjah Road passes through $J(6, -2)$ and $K(4, -5)$. If they are straight lines, are the two streets perpendicular? Explain.

Example 3 **PERSEVERANCE** **Determine whether the graphs of the following equations are *parallel* or *perpendicular*. Explain.**

21. $2x - 8y = -24$, $4x + y = -2$, $x - 4y = 4$

22. $3x - 9y = 9$, $3y = x + 12$, $2x - 6y = 12$

Example 4 **Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.**

23 $(-3, -2)$, $y = -2x + 4$ **24.** $(-5, 2)$, $y = \frac{1}{2}x - 3$ **25.** $(-4, 5)$, $y = \frac{1}{3}x + 6$

26. $(2, 6)$, $y = -\frac{1}{4}x + 3$ **27.** $(3, 8)$, $y = 5x - 3$ **28.** $(4, -2)$, $y = 3x + 5$

Write an equation in slope-intercept form for a line perpendicular to the graph of the equation that passes through the *x*-intercept of that line.

29. $y = -\frac{1}{2}x - 4$ **30.** $y = \frac{2}{3}x - 6$ **31.** $y = 5x + 3$

32. Write an equation in slope-intercept form for the line that is perpendicular to the graph of $3x + 2y = 8$ and passes through the y-intercept of that line.

Determine whether the graphs of each pair of equations are *parallel*, *perpendicular*, or *neither*.

33. $y = 4x + 3$
$4x + y = 3$

34. $y = -2x$
$2x + y = 3$

35. $3x + 5y = 10$
$5x - 3y = -6$

36. $-3x + 4y = 8$
$-4x + 3y = -6$

37. $2x + 5y = 15$
$3x + 5y = 15$

38. $2x + 7y = -35$
$4x + 14y = -42$

39. Write an equation of the line that is parallel to the graph of $y = 7x - 3$ and passes through the origin.

40. EXCAVATION Scientists excavating a dinosaur mapped the site on a coordinate plane. If one bone lies from $(-5, 8)$ to $(10, -1)$ and a second bone lies from $(-10, -3)$ to $(-5, -6)$, are the bones parallel? Explain.

41. ARCHAEOLOGY In the ruins of an ancient civilization, an archaeologist found pottery at $(2, 6)$ and hair accessories at $(4, -1)$. A pole is found with one end at $(7, 10)$ and the other end at $(14, 12)$. Is the pole perpendicular to the line through the pottery and the hair accessories? Explain.

42. GRAPHICS To create a design on a computer, Mariam must enter the coordinates for points on the design. One line segment she drew has endpoints of $(-2, 1)$ and $(4, 3)$. The other coordinates that Mariam entered are $(2, -7)$ and $(8, -3)$. Could these points be the vertices of a rectangle? Explain.

43. MULTIPLE REPRESENTATIONS In this problem, you will explore parallel and perpendicular lines.

a. Graphical Graph the points $A(-3, 3)$, $B(3, 5)$, and $C(-4, 0)$ on a coordinate plane.

b. Analytical Determine the coordinates of a fourth point D that would form a parallelogram. Explain your reasoning.

c. Analytical What is the minimum number of points that could be moved to make the parallelogram a rectangle? Describe which points should be moved, and explain why.

H.O.T. Problems Use Higher-Order Thinking Skills

44. CHALLENGE If the line through $(-2, 4)$ and $(5, d)$ is parallel to the graph of $y = 3x + 4$, what is the value of d?

45. REASONING Which key features of the graphs of two parallel lines are the same, and which are different? Which key features of the graphs of two perpendicular lines are the same, and which are different?

46. OPEN ENDED Graph a line that is parallel and a line that is perpendicular to $y = 2x - 1$.

Example 3

47. CRITIQUE Karimah and Amenah are finding an equation of the line that is perpendicular to the graph of $y = \frac{1}{3}x + 2$ and passes through the point $(-3, 5)$. Is either of them correct? Explain your reasoning.

Karimah

$y - 5 = -3[x - (-3)]$

$y - 5 = -3(x + 3)$

$y = -3x - 9 + 5$

$y = -3x - 4$

Amenah

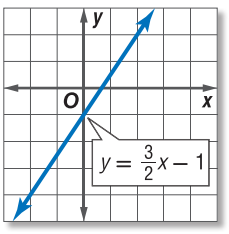
$y - 5 = 3[x - (-3)]$

$y - 5 = 3(x + 3)$

$y = 3x + 9 + 5$

$y = 3x + 14$

48. WRITING IN MATH Illustrate how you can determine whether two lines are parallel or perpendicular. Write an equation for the graph that is parallel and an equation for the graph that is perpendicular to the line shown. Explain your reasoning.



Standardized Test Practice

49. Which of the following is an algebraic translation of the following phrase?

5 less than the quotient of a number and 8

A $5 - \frac{n}{8}$ **C** $5 - \frac{8}{n}$

B $\frac{n}{8} - 5$ **D** $\frac{8}{n} - 5$

50. A line through which two points would be parallel to a line with a slope of $\frac{3}{4}$?

F (0, 5) and (−4, 2) **H** (0, 0) and (0, −2)

G (0, 2) and (−4, 1) **J** (0, −2) and (−4, −2)

51. Which equation best fits the data in the table?

x	y
1	5
2	7
3	9
4	11

A $y = x + 4$

B $y = 2x + 3$

C $y = 7$

D $y = 4x - 5$

52. SHORT RESPONSE Ahmad is filling his 6000-gallon pool at a constant rate. After 4 hours, the pool contained 800 gallons. How many total hours will it take to completely fill the pool?

Spiral Review

Write each equation in standard form. (Lesson 4-3)

53. $y - 13 = 4(x - 2)$

54. $y - 5 = -2(x + 2)$

55. $y + 3 = -5(x + 1)$

56. $y + 7 = \frac{1}{2}(x + 2)$

57. $y - 1 = \frac{5}{6}(x - 4)$

58. $y - 2 = -\frac{2}{5}(x - 8)$

59. CANOE RENTAL Latifah and her friends rented a canoe for 3 hours and paid a total of AED 45. (Lesson 4-2)

a. Write a linear equation to find the total cost C of renting the canoe for h hours.

b. How much would it cost to rent the canoe for 8 hours?

Write an equation of the line that passes through each point with the given slope. (Lesson 4-2)

60. (5, −2), $m = 3$

61. (−5, 4), $m = -5$

62. (3, 0), $m = -2$

63. (3, 5), $m = 2$

64. (−3, −1), $m = -3$

65. (−2, 4), $m = -5$

Simplify each expression. If not possible, write *simplified*. (Lesson 1-4)

66. $13m + m$

67. $14a^2 + 13b^2 + 27$

68. $3(x + 2x)$

69. FINANCIAL LITERACY At a Farmers' Market, merchants can rent a small table for AED 5.00 and a large table for AED 8.50. One time, 25 small and 10 large tables were rented. Another time, 35 small and 12 large were rented. (Lesson 1-2)

a. Write an algebraic expression to show the total amount of money collected.

b. Evaluate the expression.

Skills Review

Express each relation as a graph. Then determine the domain and range.

70. {(3, 8), (3, 7), (2, −9), (1, −9), (−5, −3)}

71. {(3, 4), (4, 3), (2, 2), (5, −4), (−4, 5)}

72. {(0, 2), (−5, 1), (0, 6), (−1, 9), (−4, −5)}

73. {(−7, 6), (−3, −4), (4, −5), (−2, 6), (−3, 2)}

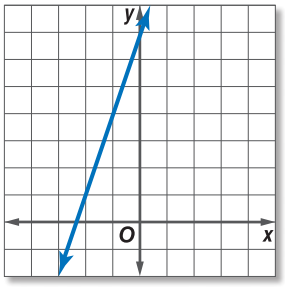
CHAPTER 4

Mid-Chapter Quiz

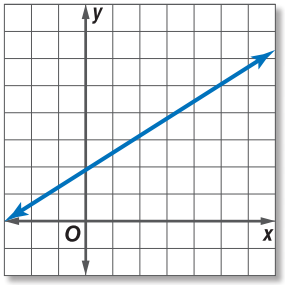
Lessons 4-1 through 4-4

Write an equation in slope-intercept form for each graph shown. (Lesson 4-1)

1.



2.



Graph each equation. (Lesson 4-1)

3. $y = 2x + 3$

4. $y = \frac{1}{3}x - 2$

5. BOATS Write an equation in slope-intercept form for the total rental cost C for a pontoon boat used for t hours. (Lesson 4-1)

Write an equation of the line with the given conditions. (Lesson 4-2)

6. (2, 5); slope 3

7. (−3, −1), slope $\frac{1}{2}$

8. (−3, 4), (1, 12)

9. (−1, 6), (2, 4)

10. (2, 1), slope 0

11. MULTIPLE CHOICE Write an equation of the line that passes through the point (0, 0) and has slope −4. (Lesson 4-2)

A $y = x - 4$

B $y = x + 4$

C $y = -4x$

D $y = 4 - x$

Write an equation in point-slope form for the line that passes through each point with the given slope. (Lesson 4-3)

12. (1, 4), $m = 6$

13. (−2, −1), $m = -3$

14. Write an equation in point-slope form for the line that passes through the point (8, 3), $m = -2$. (Lesson 4-3)

15. Write $y + 3 = \frac{1}{2}(x - 5)$ in standard form. (Lesson 4-3)

16. Write $y + 4 = -7(x - 3)$ in slope-intercept form. (Lesson 4-3)

Write each equation in standard form. (Lesson 4-3)

17. $y - 5 = -2(x - 3)$

18. $y + 4 = \frac{2}{3}(x - 3)$

Write each equation in slope-intercept form. (Lesson 4-3)

19. $y - 3 = 4(x + 3)$

20. $y + 1 = \frac{1}{2}(x - 8)$

21. MULTIPLE CHOICE Determine whether the graphs of the pair of equations are *parallel, perpendicular,* or *neither*. (Lesson 4-4)

$$y = -6x + 8$$
$$3x + \frac{1}{2}y = -3$$

F parallel

G perpendicular

H neither

J not enough information

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation. (Lesson 4-4)

22. (3, −4); $y = -\frac{1}{3}x - 5$

23. (0, −3); $y = -2x + 4$

24. (−4, −5); $-4x + 5y = -6$

25. (−1, −4); $-x - 2y = 0$

LESSON 4-5

Regression and Median-Fit Lines

Then

- You used lines of fit and scatter plots to evaluate trends and make predictions.

Now

1. Write equations of best-fit lines using linear regression.
2. Write equations of median-fit lines.

Why?

- The table shows the total attendance, in millions of people, at a festival from 2005 to 2009. You can use a graphing calculator to find the equation of a *best-fit line* and use it to make predictions about future attendance at the fair.

Year	Attendance (millions)
2005	1.633
2006	1.681
2007	1.682
2008	1.693
2009	1.790

New Vocabulary

- best-fit line
- linear regression
- correlation coefficient
- residual
- median-fit line

Mathematical Practices

Use appropriate tools strategically.

1 Best-Fit Lines

You have learned how to find and write equations for lines of fit by hand. Many calculators use complex algorithms that find a more precise line of fit called the **best-fit line**. One algorithm is called **linear regression**.

Your calculator may also compute a number called the **correlation coefficient**. This number will tell you if your correlation is positive or negative and how closely the equation is modeling the data. The closer the correlation coefficient is to 1 or −1, the more closely the equation models the data.

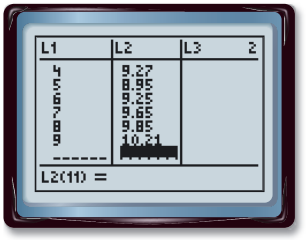
Real-World Example 1 Best-Fit Line

MOVIES **The table shows the amount of money made by movies in the United States. Use a graphing calculator to write an equation for the best-fit line for that data.**

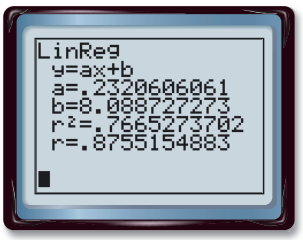
Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Income (AED billion)	7.48	8.13	9.19	9.35	9.27	8.95	9.25	9.65	9.85	10.21

Before you begin, make sure that your Diagnostic setting is on. You can find this under the **CATALOG** menu. Press **D** and then scroll down and click **DiagnosticOn**. Then press ENTER.

Step 1 Enter the data by pressing STAT and selecting the **Edit** option. Let the year 2000 be represented by 0. Enter the years since 2000 into List 1 (L1). These will represent the x-values. Enter the income (AED billion) into List 2 (L2). These will represent the y-values.



Step 2 Perform the regression by pressing STAT and selecting the **CALC** option. Scroll down to **LinReg (ax+b)** and press ENTER twice.



Step 3 Write the equation of the regression line by rounding the a and b values on the screen. The form that we chose for the regression was $ax + b$, so the equation is $y = 0.23x + 8.09$. The correlation coefficient is about 0.8755, which means that the equation models the data fairly well.

GuidedPractice

Write an equation of the best-fit line for the data in each table. Name the correlation coefficient. Round to the nearest ten-thousandth. Let x be the number of years since 2003.

1A. HOCKEY The table shows the number of goals of leading scorers for the Mustang Hockey Team.

Year	2003	2004	2005	2006	2007	2008	2009	2010
Goals	30	23	41	35	31	43	33	45

1B. HOCKEY The table gives the number of goals scored by the team each season.

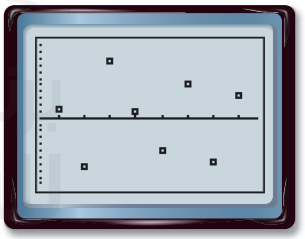
Year	2003	2004	2005	2006	2007	2008	2009	2010
Goals	63	44	55	63	81	85	93	84

We know that not all of the points will lie on the best-fit line. The difference between an observed y-value and its predicted y-value (found on the best-fit line) is called a **residual**. Residuals measure how much the data deviate from the regression line. When residuals are plotted on a scatter plot they can help to assess how well the best-fit line describes the data. If the best-fit line is a good fit, there is no pattern in the residual plot.

Real-World Example 2 Graph and Analyze a Residual Plot

HOCKEY Graph and analyze the residual plot for the data for Guided Practice 1A. Determine if the best-fit line models the data well.

After calculating the best-fit line in Guided Practice 1A, you can obtain the residual plot of the data. Turn on **Plot2** under the **STAT PLOT** menu and choose . Use **L1** for the **Xlist** and **RESID** for the **Ylist**. You can obtain **RESID** by pressing 2nd [**STAT**] and selecting **RESID** from the list of names. Graph the scatter plot of the residuals by pressing ZOOM and choosing **ZoomStat**.



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The residuals appear to be randomly scattered and centered about the line $y = 0$. Thus, the best-fit line seems to model the data well.

GuidedPractice

2. UNEMPLOYMENT Graph and analyze the residual plot for the following data comparing graduation rates and unemployment rates.

Graduation Rate	73	85	64	81	68	82
Unemployment Rate	6.9	4.1	3.2	5.5	4.3	5.1

A residual is positive when the observed value is above the line, negative when the observed value is below the line, and zero when it is on the line. One common measure of goodness of fit is the sum of squared vertical distances from the points to the line. The best-fit line, which is also called the *least-squares regression line*, minimizes the sum of the squares of those distances.

We can use points on the best-fit line to estimate values that are not in the data. Recall that when we estimate values that are between known values, this is called *linear interpolation*. When we estimate a number outside of the range of the data, it is called *linear extrapolation*.

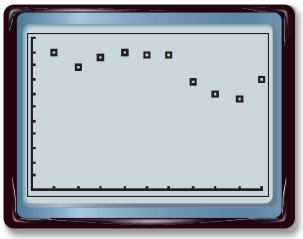
Real-World Example 3 Use Interpolation and Extrapolation

PAINTBALL **The table shows the points received by the top ten paintball teams at a tournament. Estimate how many points the 20th-ranked team received.**

Rank	1	2	3	4	5	6	7	8	9	10
Score	100	89	96	99	97	98	78	70	64	80

Write an equation of the best-fit line for the data. Then extrapolate to find the missing value.

Step 1 Enter the data from the table into the lists. Let the ranks be the x-values and the scores be the y-values. Then graph the scatter plot.



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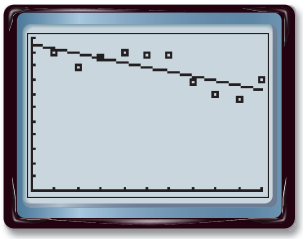
Step 2 Perform the linear regression using the data in the lists. Find the equation of the best-fit line.

The equation is about $y = -3.32x + 105.3$.



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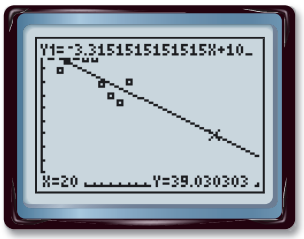
Step 3 Graph the best-fit line. Press Y= VARS and choose **Statistics**. From the **EQ** menu, choose **RegEQ**. Then press GRAPH.

Step 4 Use the graph to predict the points that the 20th-ranked team received. Change the viewing window to include the x-value to be evaluated. Press 2nd [CALC] ENTER 20 ENTER to find that when $x = 20$, $y \approx 39$. It is estimated that the 20th ranked team received 39 points.



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GuidedPractice

ONLINE GAMES **Use linear interpolation to estimate the percent of Americans that play online games for the following ages.**

Age	15	20	30	40	50
Percent	81	54	37	29	25

Source: Pew Internet & American Life Survey

3A. 35 years **3B.** 18 years

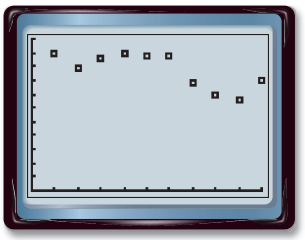
2 Median-Fit Lines

A second type of fit line that can be found using a graphing calculator is a **median-fit line**. The equation of a median-fit line is calculated using the medians of the coordinates of the data points.

Example 4 Median-Fit Line

PAINTBALL **Find and graph the equation of a median-fit line for the data in Example 3. Then predict the score of the 15th ranked team.**

Step 1 Reenter the data if it is not in the lists. Clear the Y= list and graph the scatter plot.



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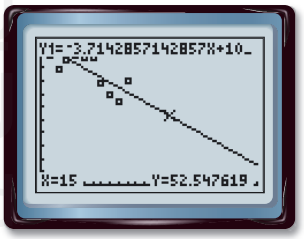
Step 2 To find the median-fit equation, press the STAT key and select the **CALC** option. Scroll down to the **Med-Med** option and press ENTER. The value of a is the slope, and the value of b is the y-intercept.

The equation for the median-fit line is about $y = -3.71x + 108.26$.

Step 3 Copy the equation to the Y= list and graph. Use the **value** option to find the value of y when $x = 15$.



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The 15th place team scored about 53 points.

Notice that the equations for the regression line and the median-fit line are very similar.

GuidedPractice

4. Use the data from Guided Practice 3 and a median-fit line to estimate the numbers of 18- and 35-year-olds who play online games. Compare these values with the answers from the regression line.

Check Your Understanding

Examples 1, 2

1. POTTERY A local university is keeping track of the number of art students who use the pottery studio each day.

Day	1	2	3	4	5	6	7
Students	10	15	18	15	13	19	20

a. Write an equation of the regression line and find the correlation coefficient.

b. Graph the residual plot and determine if the regression line models the data well.

Example 3

2. COMPUTERS The table below shows the percent of Americans with a broadband connection at home in a recent year. Use linear extrapolation and a regression equation to estimate the percentage of 60-year-olds with broadband at home.

Age	25	30	35	40	45	50
Percent	40	42	36	35	36	32

Example 4

3. VACATION Mohammad's family want to rent a house on the lake that sleeps eight people. The cost of the house per night is based on how close it is to the water.

Distance from Lake (mi)	0.0 (houseboat)	0.3	0.5	1.0	1.25	1.5	2.0
Price/Night (AED)	785	325	250	200	150	140	100

a. Find and graph an equation for the median-fit line.

b. What would you estimate is the cost of a rental 1.75 miles from the lake?

Practice and Problem Solving

Example 1

Write an equation of the regression line for the data in each table. Then find the correlation coefficient.

4. SKYSCRAPERS The table ranks the ten tallest buildings in the world.

Rank	1	2	3	4	5	6	7	8	9	10
Stories	101	88	110	88	88	80	69	102	78	70

5. MUSIC The table gives the number of annual violin auditions held by a youth symphony each year since 2004. Let x be the number of years since 2004.

Year	2004	2005	2006	2007	2008	2009	2010
Auditions	22	19	25	37	32	35	42

Example 2

6. RETAIL The table gives the sales at a clothing chain since 2004. Let x be the number of years since 2004.

Year	2004	2005	2006	2007	2008	2009	2010
Sales (Millions of dirhams)	6.84	7.6	10.9	15.4	17.6	21.2	26.5

a. Write an equation of the regression line.

b. Graph and analyze the residual plot.

Examples 3, 4 **7** **MARATHON** The number of entrants in the Boston Marathon every five years since 1975 is shown. Let x be the number of years since 1975.

Year	1975	1980	1985	1990	1995	2000	2005	2010
Entrants	2395	5417	5594	9412	9416	17,813	20,453	26,735

a. Find an equation for the median-fit line.

b. According to the equation, how many entrants were there in 2003?

8. **CAMPING** A campground keeps a record of the number of campsites rented the week of July 4 for several years. Let x be the number of years since 2000.

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010
Sites Rented	34	45	42	53	58	47	57	65	59

a. Find an equation for the regression line.

b. Predict the number of campsites that will be rented in 2012.

c. Predict the number of campsites that will be rented in 2020.

9. **ICE CREAM** An ice cream company keeps a count of the tubs of chocolate ice cream delivered to each of their stores in a particular area.

a. Find an equation for the median-fit line.

b. Graph the points and the median-fit line.

c. How many tubs would be delivered to a 1500-square-foot store? a 5000-square-foot store?

Store Size (ft^2)	2100	2225	3135	3569	4587
Tubs (hundreds)	110	102	215	312	265

10. **SENSE-MAKING** The prices of the eight top-selling brands of jeans at Jamila's Jeans are given in the table below.

Sales Rank	1	2	3	4	5	6	7	8
Price (AED)	43	44	50	61	64	135	108	78

a. Find the equation for the regression line.

b. According to the equation, what would be the price of a pair of the 12th best-selling brand?

c. Is this a reasonable prediction? Explain.

11. **STATE FAIRS** Refer to the beginning of the lesson.

a. Graph a scatter plot of the data, where $x = 1$ represents 2005. Then find and graph the equation for the best-fit line.

b. Graph and analyze the residual plot.

c. Predict the total attendance in 2020.

12. FIREFIGHTERS The table shows statistics from the U.S. Fire Administration.

a. Find an equation for the median-fit line.

b. Graph the points and the median-fit line.

c. Does the median-fit line give you an accurate picture of the number of firefighters? Explain.

Age	Number of Firefighters
18	40,919
25	245,516
35	330,516
45	296,665
55	167,087
65	54,559

13. ATHLETICS The table shows the number of participants in high school athletics.

Year Since 1970	1	10	20	30	35
Athletes	3,960,932	5,356,913	5,298,671	6,705,223	7,159,904

a. Find an equation for the regression line.

b. According to the equation, how many participated in 1988?

14. ART A count was kept on the number of paintings sold at an auction by the year in which they were painted. Let x be the number of years since 1950.

Year Painted	1950	1955	1960	1965	1970	1975
Paintings Solds	8	5	25	21	9	22

a. Find the equation for the linear regression line.

b. How many paintings were sold that were painted in 1961?

c. Is the linear regression equation an accurate model of the data? Explain why or why not.

H.O.T. Problems Use Higher-Order Thinking Skills

15. ARGUMENTS Below are the results of the World Superpipe Championships in 2008.

Men	Score	Rank	Women	Score
Shaun White	93.00	1	Torah Bright	96.67
Mason Aguirre	90.33	2	Kelly Clark	93.00
Janne Korpi	85.33	3	Soko Yamaoka	85.00
Luke Mitrani	85.00	4	Ellery Hollingsworth	79.33
Keir Dillion	81.33	5	Sophie Rodriguez	71.00

Find an equation of the regression line for each, and graph them on the same coordinate plane. Compare and contrast the men's and women's graphs.

16. REASONING For a class project, the scores that 10 randomly selected students earned on the first 8 tests of the school year are given. Explain how to find a line of best fit. Could it be used to predict the scores of other students? Explain your reasoning.

17. OPEN ENDED For 10 different people, measure their heights and the lengths of their heads from chin to top. Use these data to generate a linear regression equation and a median-fit equation. Make a prediction using both of the equations.

18. WRITING IN MATH How are lines of fit and linear regression similar? different?

Standardized Test Practice

19. GEOMETRY Saeed is putting a border around a poster. x represents the poster's width, and y represents the poster's length. Which equation represents how much border Saeed will use if he doubles the length and the width?

A $4xy$ **C** $4(x + y)$

B $(x + y)^4$ **D** $16(x + y)$

20. SHORT RESPONSE Fatima wants to run 5 miles at an average pace of 9 minutes per mile. After 4 miles, her average pace is 9 minutes 10 seconds. In how many minutes must she complete the final mile to reach her goal?

21. What is the slope of the line that passes through (1, 3) and (−3, 1)?

F -2 **H** $\frac{1}{2}$

G $-\frac{1}{2}$ **J** 2

22. What is an equation of the line that passes through (0, 1) and has a slope of 3?

A $y = 3x - 1$

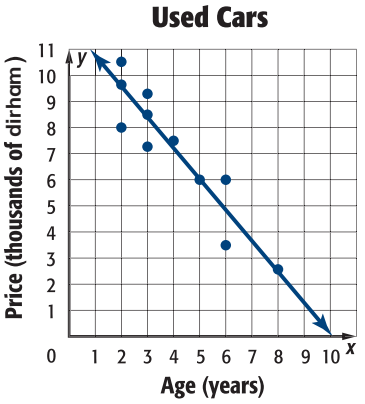
B $y = 3x - 2$

C $y = 3x + 4$

D $y = 3x + 1$

Spiral Review

23. USED CARS Ghaya wants to buy a specific make and model of a used car. She researched prices from dealers and private sellers and made the graph shown. (Lesson 4-5)



a. Describe the relationship in the data.

b. Use the line of fit to predict the price of a car that is 7 years old.

c. Is it reasonable to use this line of fit to predict the price of a 10-year-old car? Explain.

24. GEOMETRY A quadrilateral has sides with equations $y = -2x$, $2x + y = 6$, $y = \frac{1}{2}x + 6$, and $x - 2y = 9$. Is the figure a rectangle? Explain your reasoning. (Lesson 4-4)

Write each equation in standard form. (Lesson 4-3)

25. $y - 2 = 3(x - 1)$ **26.** $y - 5 = 6(x + 1)$

27. $y + 2 = -2(x - 5)$ **28.** $y + 3 = \frac{1}{2}(x + 4)$

29. $y - 1 = \frac{2}{3}(x + 9)$ **30.** $y + 3 = -\frac{1}{4}(x + 2)$

Find the slope of the line that passes through each pair of points. (Lesson 3-3)

31. (3, 4), (10, 8) **32.** (−4, 7), (3, 5) **33.** (3, 7), (−2, 4) **34.** (−3, 2), (−3, 4)

Skills Review

If $f(x) = x^2 - x + 1$, find each value.

35. $f(-1)$ **36.** $f(5) - 3$ **37.** $f(a)$ **38.** $f(b^2)$

Graph each equation.

39. $y = x + 2$ **40.** $x + 5y = 4$ **41.** $2x - 3y = 6$ **42.** $5x + 2y = 6$

LESSON 4-6

Inverse Linear Functions

Then

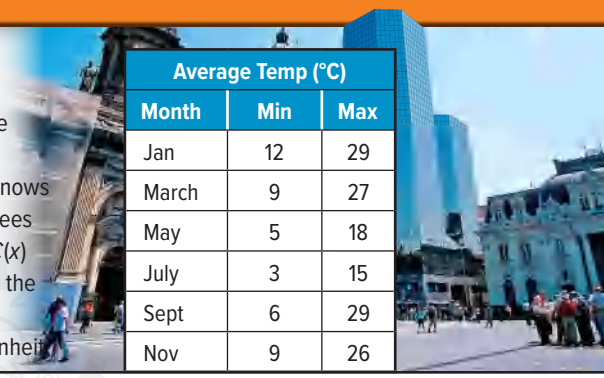
- You represented relations as tables, graphs, and mappings.

Now

1. Find the inverse of a function or relation.
2. Find the inverse of a linear function.

Why?

- Rasheed is writing a report on Santiago, Chile, and he wants to include a brief climate analysis. He found a table of temperatures recorded in degrees Celsius. He knows that a formula for converting degrees Fahrenheit to degrees Celsius is $C(x) = \frac{5}{9}(x - 32)$. He will need to find the *inverse* function to convert from degrees Celsius to degrees Fahrenheit.



Average Temp (°C)		
Month	Min	Max
Jan	12	29
March	9	27
May	5	18
July	3	15
Sept	6	29
Nov	9	26

New Vocabulary

inverse relation
inverse function

Mathematical Practices

Attend to precision.

1 Inverse Relations

An **inverse relation** is the set of ordered pairs obtained by exchanging the x-coordinates with the y-coordinates of each ordered pair in a relation. If (5, 3) is an ordered pair of a relation, then (3, 5) is an ordered pair of the inverse relation.

KeyConcept Inverse Relations

Words If one relation contains the element (a, b), then the inverse relation will contain the element (b, a).

Example A and B are inverse relations.

A		B
$(-3, -16)$	→	$(-16, -3)$
$(-1, 4)$	→	$(4, -1)$
$(2, 14)$	→	$(14, 2)$
$(5, 32)$	→	$(32, 5)$

Notice that the domain of a relation becomes the range of its inverse, and the range of the relation becomes the domain of its inverse.

Example 1 Inverse Relations

Find the inverse of each relation.

a. {(4, −10), (7, −19), (−5, 17), (−3, 11)}

To find the inverse, exchange the coordinates of the ordered pairs.

$(4, -10) \rightarrow (-10, 4)$ $\quad (-5, 17) \rightarrow (17, -5)$

$(7, -19) \rightarrow (-19, 7)$ $\quad (-3, 11) \rightarrow (11, -3)$

The inverse is $\{(-10, 4), (-19, 7), (17, -5), (11, -3)\}$.

b.

x	−4	−1	5	9
y	−13	−8.5	0.5	6.5

Write the coordinates as ordered pairs. Then exchange the coordinates of each pair.

$(-4, -13) \rightarrow (-13, -4)$ $\quad (5, 0.5) \rightarrow (0.5, 5)$

$(-1, -8.5) \rightarrow (-8.5, -1)$ $\quad (9, 6.5) \rightarrow (6.5, 9)$

The inverse is $\{(-13, -4), (-8.5, -1), (0.5, 5), (6.5, 9)\}$.

GuidedPractice

1A. $\{(-6, 8), (-15, 11), (9, 3), (0, 6)\}$

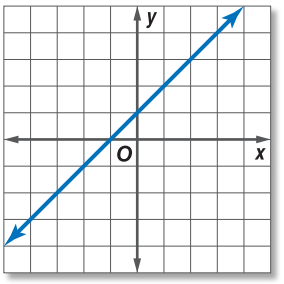
1B.

x	−10	−4	−3	0
y	5	11	12	15

The graphs of relations can be used to find and graph inverse relations.

Example 2 Graph Inverse Relations

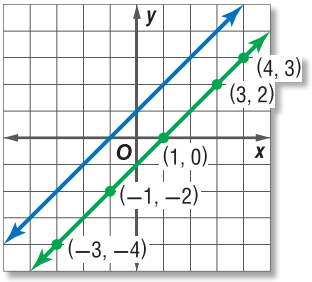
Graph the inverse of the relation.



StudyTip

Precision Only two points are necessary to graph the inverse of a line, but several should be used to avoid possible error.

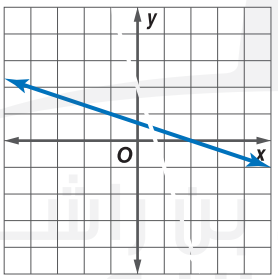
The graph of the relation passes through the points at $(-4, -3)$, $(-2, -1)$, $(0, 1)$, $(2, 3)$, and $(3, 4)$. To find points through which the graph of the inverse passes, exchange the coordinates of the ordered pairs. The graph of the inverse passes through the points at $(-3, -4)$, $(-1, -2)$, $(1, 0)$, $(3, 2)$, and $(4, 3)$. Graph these points and then draw the line that passes through them.



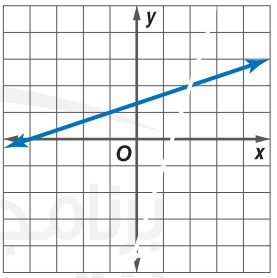
GuidedPractice

Graph the inverse of each relation.

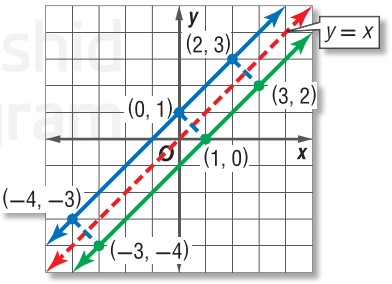
2A.



2B.



The graphs from Example 2 are graphed on the right with the line $y = x$. Notice that the graph of an inverse is the graph of the original relation reflected in the line $y = x$. For every point (x, y) on the graph of the original relation, the graph of the inverse will include the point (y, x).



2 Inverse Functions A linear relation that is described by a function has an **inverse function** that can generate ordered pairs of the inverse relation The inverse of the linear function $f(x)$ can be written as $f^{-1}(x)$ and is read *f of x inverse* or *the inverse of f of x.*

KeyConcept Finding Inverse Functions

To find the inverse function $f^{-1}(x)$ of the linear function $f(x)$, complete the following steps.

Step 1 Replace $f(x)$ with y in the equation for $f(x)$.

Step 2 Interchange y and x in the equation.

Step 3 Solve the equation for y.

Step 4 Replace y with $f^{-1}(x)$ in the new equation.

Example 3 Find Inverse Linear Functions

Find the inverse of each function.

a. $f(x) = 4x - 8$

Step 1 $f(x) = 4x - 8$

$y = 4x - 8$

Step 2 $x = 4y - 8$

Step 3 $x + 8 = 4y$

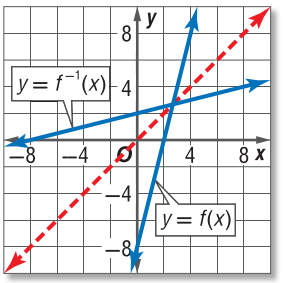
$\frac{x + 8}{4} = y$

Step 4 $\frac{x + 8}{4} = f^{-1}(x)$

The inverse of $f(x) = 4x - 8$ is $f^{-1}(x) = \frac{x + 8}{4}$ or $f^{-1}(x) = \frac{1}{4}x + 2$.

WatchOut!
Notation The -1 in $f^{-1}(x)$ is *not* an exponent.

CHECK Graph both functions and the line $y = x$ on the same coordinate plane. $f^{-1}(x)$ appears to be the reflection of $f(x)$ in the line $y = x$. ✓



b. $f(x) = -\frac{1}{2}x + 11$

Step 1 $f(x) = -\frac{1}{2}x + 11$

$y = -\frac{1}{2}x + 11$

Step 2 $x = -\frac{1}{2}y + 11$

Step 3 $x - 11 = -\frac{1}{2}y$

$-2(x - 11) = y$

$-2x + 22 = y$

Step 4 $-2x + 22 = f^{-1}(x)$

The inverse of $f(x) = -\frac{1}{2}x + 11$ is $f^{-1}(x) = -2x + 22$.

GuidedPractice

3A. $f(x) = 4x - 12$

3B. $f(x) = \frac{1}{3}x + 7$

Real-WorldLink

The winter months in Chile occur during the summer months in the U.S. due to Chile's location in the southern hemisphere. The average daily high temperature of Santiago during its winter months is about 60° F.

Source: World Weather Information Service

Real-World Example 4 Use an Inverse Function

TEMPERATURE **Refer to the beginning of the lesson. Rasheed wants to convert the temperatures from degrees Celsius to degrees Fahrenheit.**

a. Find the inverse function $C^{-1}(x)$.

Step 1 $C(x) = \frac{5}{9}(x - 32)$

$y = \frac{5}{9}(x - 32)$

Step 2 $x = \frac{5}{9}(y - 32)$

Step 3 $\frac{9}{5}x = y - 32$

$\frac{9}{5}x + 32 = y$

Step 4 $\frac{9}{5}x + 32 = C^{-1}(x)$

The inverse function of $C(x)$ is $C^{-1}(x) = \frac{9}{5}x + 32$.

b. What do x and $C^{-1}(x)$ represent in the context of the inverse function?

x represents the temperature in degrees Celsius. $C^{-1}(x)$ represents the temperature in degrees Fahrenheit.

c. Find the average temperatures for July in degrees Fahrenheit.

The average minimum and maximum temperatures for July are 3° C and 15° C, respectively. To find the average minimum temperature, find $C^{-1}(3)$.

$C^{-1}(x) = \frac{9}{5}x + 32$

$C^{-1}(3) = \frac{9}{5}(3) + 32$

$= 37.4$

To find the average maximum temperature, find $C^{-1}(15)$.

$C^{-1}(x) = \frac{9}{5}x + 32$

$C^{-1}(15) = \frac{9}{5}(15) + 32$

$= 59$

The average minimum and maximum temperatures for July are 37.4° F and 59° F, respectively.

GuidedPractice

4. RENTAL CAR Najla rents a car for the day. The total cost $C(x)$ in dirhams is given by $C(x) = 19.99 + 0.3x$, where x is the number of miles she drives.

A. Find the inverse function $C^{-1}(x)$.

B. What do x and $C^{-1}(x)$ represent in the context of the inverse function?

C. How many miles did Najla drive if her total cost was AED 34.99?

Check Your Understanding

Example 1 **Find the inverse of each relation.**

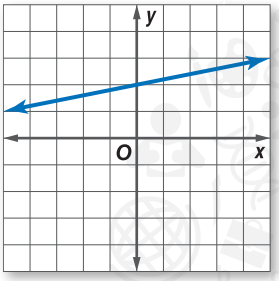
1. {(4, −15), (−8, −18), (−2, −16.5), (3, −15.25)}

2.

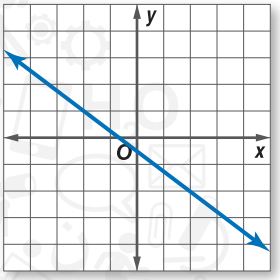
x	−3	0	1	6
y	11.8	3.7	1	−12.5

Example 2 **Graph the inverse of each relation.**

3.



4.



Example 3 **Find the inverse of each function.**

5. $f(x) = -2x + 7$

6. $f(x) = \frac{2}{3}x + 6$

Example 4

7. **REASONING** Mustafa and his brother purchase season tickets to the Cleveland Crusaders games. The ticket package requires a one-time purchase of a personal seat license costing AED 1200 for two seats. A ticket to each game costs AED 70. The cost $C(x)$ in dirhams for Mustafa for the first season is $C(x) = 600 + 70x$, where x is the number of games Mustafa attends.

a. Find the inverse function.

b. What do x and $C^{-1}(x)$ represent in the context of the inverse function?

c. How many games did Mustafa attend if his total cost for the season was AED 950?

Practice and Problem Solving

Example 1 **Find the inverse of each relation.**

8. {(−5, 13), (6, 10.8), (3, 11.4), (−10, 14)}

9 {(−4, −49), (8, 35), (−1, −28), (4, 7)}

10.

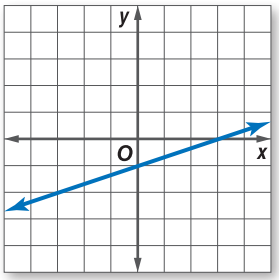
x	*y*
−8	−36.4
−2	−15.4
1	−4.9
5	9.1
11	30.1

11.

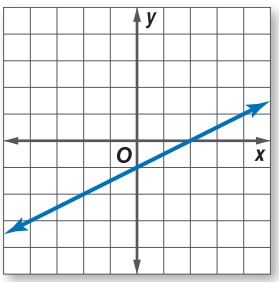
x	*y*
−3	7.4
−1	4
1	0.6
3	−2.8
5	−6.2

Example 2 **Graph the inverse of each relation.**

12.



13.



Example 3 **Find the inverse of each function.**

14. $f(x) = 25 + 4x$

15 $f(x) = 17 - \frac{1}{3}x$

16. $f(x) = 4(x + 17)$

17. $f(x) = 12 - 6x$

18. $f(x) = \frac{2}{5}x + 10$

19. $f(x) = -16 - \frac{4}{3}x$

Example 4

20. DOWNLOADS An online music subscription service allows members to download songs for AED 0.99 each after paying a monthly service charge of AED 3.99. The total monthly cost $C(x)$ of the service in dirhams is $C(x) = 3.99 + 0.99x$, where x is the number of songs downloaded.

a. Find the inverse function.

b. What do x and $C^{-1}(x)$ represent in the context of the inverse function?

c. How many songs were downloaded if a member's monthly bill is AED 27.75?

21. LANDSCAPING At the start of the mowing season, Ahmad collects a one-time maintenance fee of AED 10 from his customers. He charges Rashid's family AED 35 for each cut. The total amount collected from Rashid's family in dirhams for the season is $C(x) = 10 + 35x$, where x is the number of times Ahmad mows the lawn of Rashid's family.

a. Find the inverse function.

b. What do x and $C^{-1}(x)$ represent in the context of the inverse function?

c. How many times did Ahmad mow the lawn of Rashid's family if he collected a total of AED 780 from them?

Write the inverse of each equation in $f^{-1}(x)$ notation.

22. $3y - 12x = -72$

23. $x + 5y = 15$

24. $-42 + 6y = x$

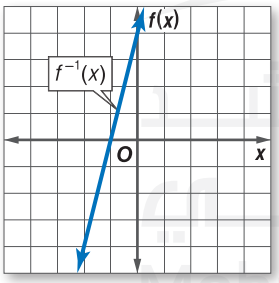
25. $3y + 24 = 2x$

26. $-7y + 2x = -28$

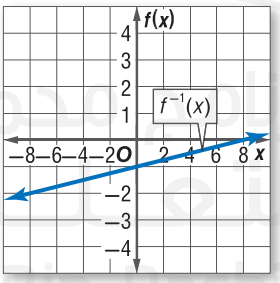
27. $3y - x = 3$

TOOLS **Match each function with the graph of its inverse.**

A.



B.



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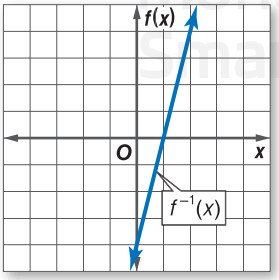

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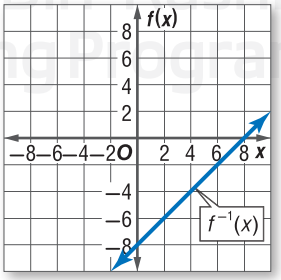


28. $f(x) = x + 4$

29. $f(x) = 4x + 4$

30. $f(x) = \frac{1}{4}x + 1$

31. $f(x) = \frac{1}{4}x - 1$

Write an equation for the inverse function $f^{-1}(x)$ that satisfies the given conditions.

32. slope of $f(x)$ is 7; graph of $f^{-1}(x)$ contains the point (13, 1)

33 graph of $f(x)$ contains the points $(-3, 6)$ and $(6, 12)$

34. graph of $f(x)$ contains the point (10, 16); graph of $f^{-1}(x)$ contains the point $(3, -16)$

35. slope of $f(x)$ is 4; $f^{-1}(5) = 2$

36. CELL PHONES Mona pays a monthly fee for her cell phone package which includes 700 minutes. She gets billed an additional charge for every minute she uses the phone past the 700 minutes. During her first month, Mona used 26 additional minutes and her bill was AED 37.79. During her second month, Mona used 38 additional minutes and her bill was AED 41.39.

a. Write a function that represents the total monthly cost $C(x)$ of Mona's cell phone package, where x is the number of additional minutes used.

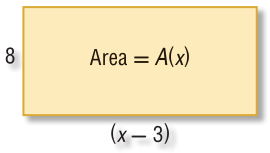
b. Find the inverse function.

c. What do x and $C^{-1}(x)$ represent in the context of the inverse function?

d. How many additional minutes did Mona use if her bill for her third month was AED 48.89?

37. MULTIPLE REPRESENTATIONS In this problem, you will explore the domain and range of inverse functions.

a. Algebraic Write a function for the area $A(x)$ of the rectangle shown.



b. Graphical Graph $A(x)$. Describe the domain and range of $A(x)$ in the context of the situation.

c. Algebraic Write the inverse of $A(x)$. What do x and $A^{-1}(x)$ represent in the context of the situation?

d. Graphical Graph $A^{-1}(x)$. Describe the domain and range of $A^{-1}(x)$ in the context of the situation.

e. Logical Determine the relationship between the domains and ranges of $A(x)$ and $A^{-1}(x)$.

H.O.T. Problems Use Higher-Order Thinking Skills

38. CHALLENGE If $f(x) = 5x + a$ and $f^{-1}(10) = -1$, find a.

39. CHALLENGE If $f(x) = \frac{1}{a}x + 7$ and $f^{-1}(x) = 2x - b$, find a and b.

ARGUMENTS Determine whether the following statements are *sometimes*, *always*, or *never* true. Explain your reasoning.

40. If $f(x)$ and $g(x)$ are inverse functions, then $f(a) = b$ and $g(b) = a$.

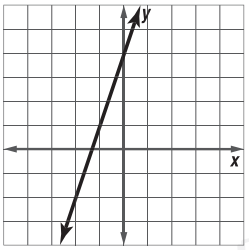
41. If $f(a) = b$ and $g(b) = a$, then $f(x)$ and $g(x)$ are inverse functions.

42. OPEN ENDED Give an example of a function and its inverse. Verify that the two functions are inverses by graphing the functions and the line $y = x$ on the same coordinate plane.

43. WRITING IN MATH Explain why it may be helpful to find the inverse of a function.

Standardized Test Practice

44. Which equation represents a line that is perpendicular to the graph and passes through the point at (2, 0)?



A $y = 3x - 6$

B $y = -3x + 6$

C $y = -\frac{1}{3}x + \frac{2}{3}$

D $y = \frac{1}{3}x - \frac{2}{3}$

45. A giant tortoise travels at a rate of 0.17 mile per hour. Which equation models the time t it would take the giant tortoise to travel 0.8 mile?

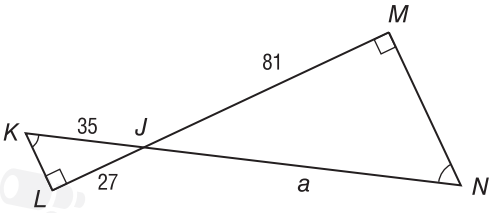
F $t = \frac{0.8}{0.17}$

G $t = (0.17)(0.8)$

H $t = \frac{0.17}{0.8}$

J $0.8 = \frac{0.17}{t}$

46. GEOMETRY If $\triangle JKL$ is similar to $\triangle JNM$ what is the value of a?



A 62.5

B 105

C 125

D 155.5

47. GRIDDED RESPONSE What is the difference in the value of $2.1(x + 3.2)$, when $x = 5$ and when $x = 3$?

Spiral Review

Write an equation of the regression line for the data in each table. (Lesson 4-6)

48.

x	1	3	5	7	9
y	3	8	15	18	21

49.

x	3	5	7	9	11
y	7.2	23.5	41.2	56.4	73.1

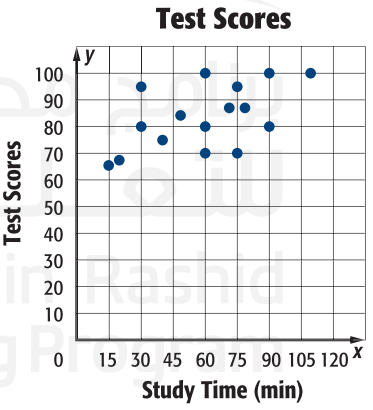
50.

x	1	2	3	4	5
y	21	33	39	54	64

51.

x	2	4	6	8	10
y	1.4	2.4	2.9	3.3	4.2

52. TESTS Determine whether the graph at the right shows a *positive*, *negative*, or *no* correlation. If there is a correlation, describe its meaning. (Lesson 4-5)



Suppose y varies directly as x. (Lesson 3-4)

53. If $y = 2.5$ when $x = 0.5$, find y when $x = 20$.

54. If $y = -6.6$ when $x = 9.9$, find y when $x = 6.6$.

55. If $y = 2.6$ when $x = 0.25$, find y when $x = 1.125$.

56. If $y = 6$ when $x = 0.6$, find x when $y = 12$.

Skills Review

Solve each equation.

57. $104 = k - 67$

58. $-4 + x = -7$

59. $\frac{m}{7} = -11$

60. $\frac{2}{3}p = 14$

61. $-82 = 18 - n$

62. $\frac{9}{t} = -27$

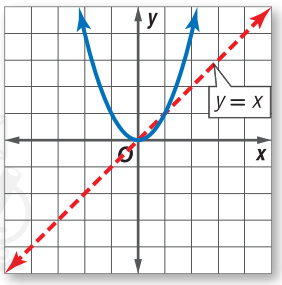
EXTEND

4-6 Algebra Lab: Drawing Inverses

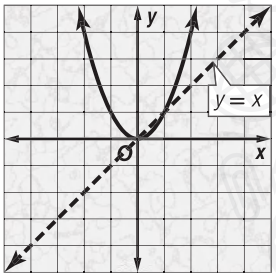
You can use patty paper to draw the graph of an inverse relation by reflecting the original graph in the line $y = x$.

Activity Draw an Inverse

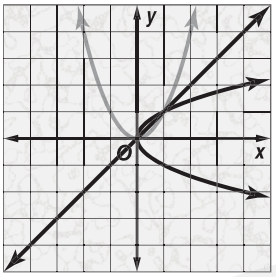
Consider the graphs shown.

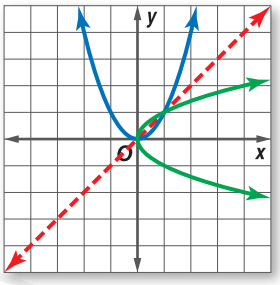


Step 1 Trace the graphs onto a square of patty paper, waxed paper, or tracing paper.



Step 2 Flip the patty paper over and lay it on the original graph so that the traced $y = x$ is on the original $y = x$.





Notice that the result is the reflection of the graph in the line $y = x$ or the inverse of the graph.

Analyze The Results

1. Is the graph of the original relation a function? Explain.
2. Is the graph of the inverse relation a function? Explain.
3. What are the domain and range of the original relation? of the inverse relation?
4. If the domain of the original relation is restricted to $D = \{x \mid x \geq 0\}$, is the inverse relation a function? Explain.
5. If the graph of a relation is a function, what can you conclude about the graph of its inverse?
6. **CHALLENGE** The vertical line test can be used to determine whether a relation is a function. Write a rule that can be used to determine whether a function has an inverse that is also a function.

LESSON 4-7

Special Linear Functions

Then
- You identified and graphed linear, exponential, and quadratic functions.

Now
1. Identify and graph step functions.
2. Identify and graph absolute value and piecewise-defined functions.

Why?
- Kim is ordering books online. The site charges for shipping based on the amount of the order. If the order is less than AED 10, shipping costs AED 3. If the order is at least AED 10 but less than AED 20, it will cost AED 5 to ship it.

New Vocabulary
- step function
- piecewise-linear function
- greatest integer function
- absolute value function
- piecewise-defined function

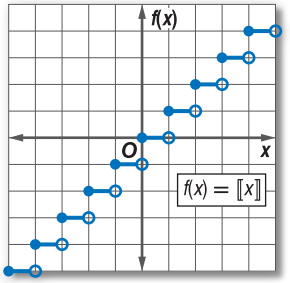
Mathematical Practices
Model with mathematics.

1 Step Functions

The graph of a **step function** is a series of line segments. Because each part of a step function is linear, this type of function is called a **piecewise-linear function**. One example of a step function is the **greatest integer function**, written as $f(x) = [\![x]\!]$, where $f(x)$ is the greatest integer not greater than x. For example, $[\![6.8]\!] = 6$ because 6 is the greatest integer that is not greater than 6.8.

KeyConcept Greatest Integer Function

Parent function:	$f(x) = [\![x]\!]$
Type of graph:	disjointed line segments
Domain:	all real numbers
Range:	all integers

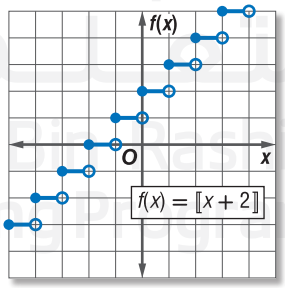


Example 1 Greatest Integer Function

Graph $f(x) = [\![x + 2]\!]$. State the domain and range.

First, make a table. Select a few values between integers. On the graph, dots represent included points. Circles represent points not included.

x	$x + 2$	$[\![x + 2]\!]$
0	2	2
0.25	2.25	2
0.5	2.5	2
1	3	3
1.25	3.25	3
1.5	3.5	3
2	4	4
2.25	4.25	4



Note that this is the graph of $f(x) = [\![x]\!]$ shifted 2 units to the left.

Because the dots and circles overlap, the domain is all real numbers. The range is all integers. Notice that the graph has no symmetry and no maximum or minimum values. As x increases, $f(x)$ increases, and as x decreases, $f(x)$ decreases.

Guided Practice

1. Graph $g(x) = 2[\![x]\!]$. State the domain and range.

Step functions can be used to represent many real-world situations involving money.

Real-WorldLink

North Americans are the most likely to have cell phones with 93.2% of the population owning phones.

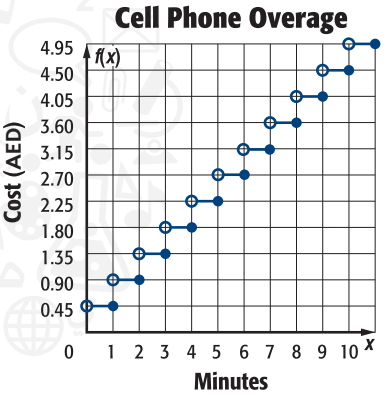
Source: IT Facts

Real-World Example 2 Step Function

CELL PHONE PLANS **Cell phone companies charge by the minute, not by the second. A cell phone company charges AED 0.45 per minute or any fraction thereof for exceeding the number of minutes allotted on each plan. Draw a graph that represents this situation.**

The total cost for the extra minutes will be a multiple of AED 0.45, and the graph will be a step function. If the time is greater than 0 but less than or equal to 1 minute, the charge will be AED 0.45. If the time is greater than 2 but is less than or equal to 3 minutes, you will be charged for 3 minutes or AED 1.35.

x	$f(x)$
$0 < x \leq 1$	0.45
$1 < x \leq 2$	0.90
$2 < x \leq 3$	1.35
$3 < x \leq 4$	1.80
$4 < x \leq 5$	2.25
$5 < x \leq 6$	2.70
$6 < x \leq 7$	3.15



GuidedPractice

2. PARKING A garage charges AED 4 for the first hour and AED 1 for each additional hour. Draw a graph that represents this situation.

ReviewVocabulary

absolute value the distance a number is from zero on a number line; written $|n|$

2 Absolute Value Functions

Another type of piecewise-linear function is the **absolute value function**. Recall that the absolute value of a number is always nonnegative. So in the absolute value parent function, written as $f(x) = |x|$, all of the values of the range are nonnegative.

KeyConcept Absolute Value Function

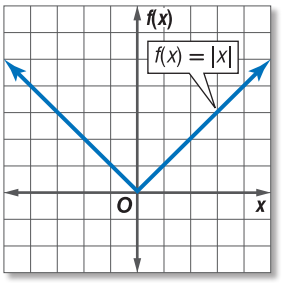
Parent function: $f(x) = |x|$, defined as

$$f(x) = \begin{cases} x \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -x \text{ if } x < 0 \end{cases}$$

Type of graph: V-shaped

Domain: all real numbers

Range: all nonnegative real numbers



The absolute value function is called a **piecewise-defined function** because it is defined using two or more expressions.

Example 3 Absolute Value Function

Graph $f(x) = |x - 4|$. State the domain and range.

Since $f(x)$ cannot be negative, the minimum point of the graph is where $f(x) = 0$.

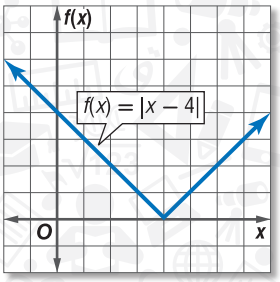
$f(x) = |x - 4|$ Original function

$0 = x - 4$ Replace $f(x)$ with 0 and $|x - 4|$ with $x - 4$.

$4 = x$ Add 4 to each side.

Next make a table of values. Include values for $x > 4$ and $x < 4$.

$f(x) = \|x - 4\|$	
x	$f(x)$
−2	6
0	4
2	2
4	0
5	1
6	2
7	3
8	4



The domain is all real numbers. The range is all real numbers greater than or equal to 0. Note that this is the graph of $f(x) = |x|$ shifted 4 units to the right. Notice that the graph is symmetric about the line $x = 4$, and the minimum value of the function is 0 at $x = 4$. As x increases, $f(x)$ increases, and as x decreases, $f(x)$ increases.

GuidedPractice

3. Graph $f(x) = 2x + 1$. State the domain and range.

Not all piecewise-defined functions are absolute value functions. Step functions are also piecewise-defined functions. In fact, all piecewise-linear functions are piecewise-defined.

StudyTip

Piecewise Functions

To graph a piecewise-defined function, graph each "piece" separately. There should be a dot or line that contains each member of the domain.

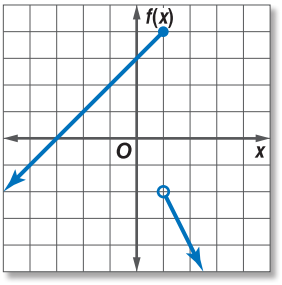
Example 4 Piecewise-Defined Function

Graph $f(x) = \begin{cases} -2x \text{ if } x > 1 \\ x + 3 \text{ if } x \leq 1 \end{cases}$. State the domain and range.

Graph the first expression. Create a table of values for when $x > 1$, $f(x) = -2x$ and draw the graph. Since x is not equal to 1, place a circle at $(1, -2)$.

Next, graph the second expression. Create a table of values for when $x \leq 1$, $f(x) = x + 3$ and draw the graph. If $x = 1$, then $f(x) = 4$; place a dot at $(1, 4)$.

The domain is all real numbers. The range is $y \leq 4$.



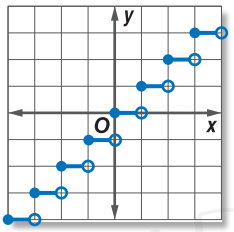
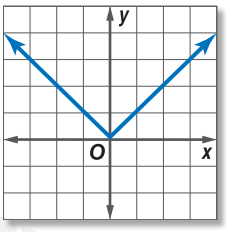
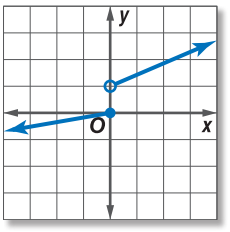
GuidedPractice

4. Graph $f(x) = \begin{cases} 2x + 1 \text{ if } x > 0 \\ 3 \text{ if } x \leq 0 \end{cases}$. State the domain and range.

StudyTip

Nonlinear Functions
Like exponential and quadratic functions, the greatest integer function, absolute value function, and piecewise defined functions are nonlinear functions.

ConceptSummary Special Functions

Step Function	Absolute Value Function	Piecewise-Defined Function
		

Check Your Understanding

Example 1 **Graph each function. State the domain and range.**

1. $f(x) = \frac{1}{2}[\![x]\!]$

2. $g(x) = -[\![x]\!]$

3. $h(x) = [\![2x]\!]$

Example 2

4. SHIPPING Eissa is ordering a gift for his dad online. The table shows the shipping rates. Graph the step function.

Order Total (AED)	Shipping Cost (AED)
0–15	3.99
15.01–30	5.99
30.01–50	6.99
50.01–75	7.99
75.01–100	8.99
Over AED 100	9.99

Examples 3–4 **Graph each function. State the domain and range.**

5. $f(x) = |x - 3|$

6. $g(x) = |2x + 4|$

7. $f(x) = \begin{cases} 2x - 1 \text{ if } x > -1 \\ -x \text{ if } x \leq -1 \end{cases}$

8. $g(x) = \begin{cases} -3x - 2 \text{ if } x > -2 \\ -x + 1 \text{ if } x \leq -2 \end{cases}$

Practice and Problem Solving

Example 1 **Graph each function. State the domain and range.**

9. $f(x) = 3[\![x]\!]$

10. $f(x) = [\![-x]\!]$

11. $g(x) = -2[\![x]\!]$

12. $g(x) = [\![x]\!] + 3$

13. $h(x) = [\![x]\!] - 1$

14. $h(x) = \frac{1}{2}[\![x]\!] + 1$

Example 2

15. CAB FARES Lamis wants to take a taxi from a hotel to a friend's house. The rate is AED 3 plus AED 1.50 per mile after the first mile. Every fraction of a mile is rounded up to the next mile.

a. Draw a graph to represent the cost of using a taxi cab.

b. What is the cost if the trip is 8.5 miles long?

16. MODELING The United States Postal Service increases the rate of postage periodically. The table shows the cost to mail a letter weighing 1 oz or less from 1995 through 2009. Draw a step graph to represent the data.

Year	1995	1999	2001	2002	2006	2007	2008	2009
Cost (AED)	0.32	0.33	0.34	0.37	0.39	0.41	0.42	0.44

Examples 3–4 Graph each function. State the domain and range.

17. $f(x) = |2x - 1|$

18. $f(x) = |x + 5|$

19. $g(x) = |-3x - 5|$

20. $g(x) = |-x - 3|$

21. $f(x) = \left|\frac{1}{2}x - 2\right|$

22. $f(x) = \left|\frac{1}{3}x + 2\right|$

23. $g(x) = |x + 2| + 3$

24. $g(x) = |2x - 3| + 1$

25. $f(x) = \begin{cases} \frac{1}{2}x - 1 \text{ if } x > 3 \\ -2x + 3 \text{ if } x \leq 3 \end{cases}$

26. $f(x) = \begin{cases} 2x - 5 \text{ if } x > 1 \\ 4x - 3 \text{ if } x \leq 1 \end{cases}$

27. $f(x) = \begin{cases} 2x + 3 \text{ if } x \geq -3 \\ -\frac{1}{3}x + 1 \text{ if } x < -3 \end{cases}$

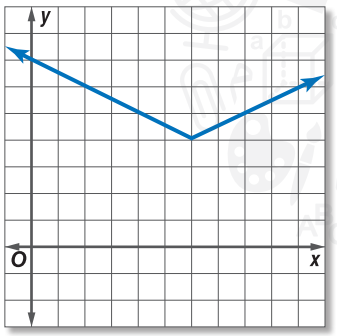
28. $f(x) = \begin{cases} 3x + 4 \text{ if } x \geq 1 \\ x + 3 \text{ if } x < 1 \end{cases}$

29. $f(x) = \begin{cases} 3x + 2 \text{ if } x > -1 \\ -\frac{1}{2}x - 3 \text{ if } x \leq -1 \end{cases}$

30. $f(x) = \begin{cases} 2x + 1 \text{ if } x < -2 \\ -3x - 1 \text{ if } x \geq -2 \end{cases}$

Determine the domain and range of each function.

31



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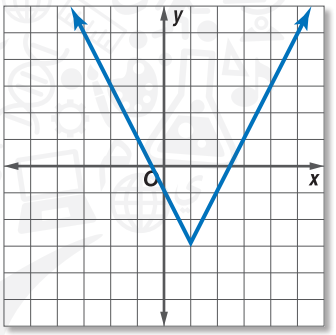

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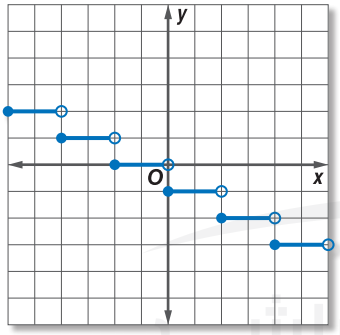

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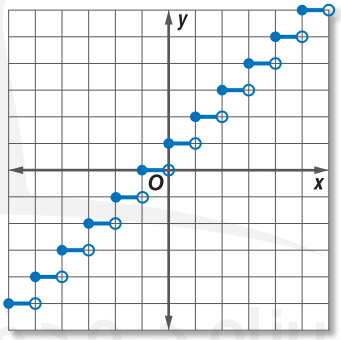
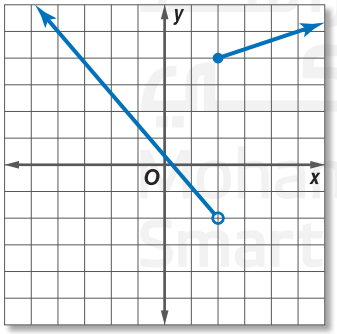


35.



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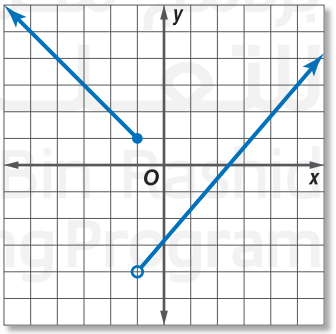


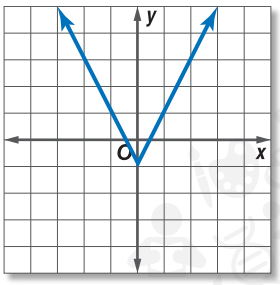
37. BOATING According to Boat Minnesota, the maximum number of people that can safely ride in a boat is determined by the boat's length and width. The table shows some guidelines for the length of a boat that is 6 ft wide. Graph this relation.

Length of Boat (ft)	18–19	20–22	23–24
Number of People	7	8	9

For Exercises 38–41, match each graph to one of the following equations.

A	B	C	D
$y = 2x - 1$	$y = [\![2x]\!] - 1$	$y = \lvert 2x \rvert - 1$	$y = \begin{cases} 2x + 1 \text{ if } x > 0 \\ -2x + 1 \text{ if } x \leq 0 \end{cases}$

38.



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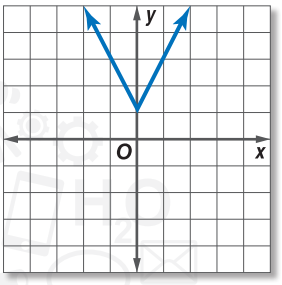

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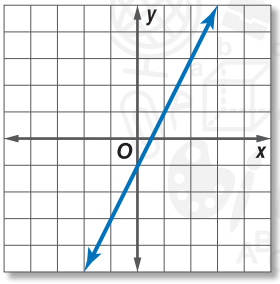

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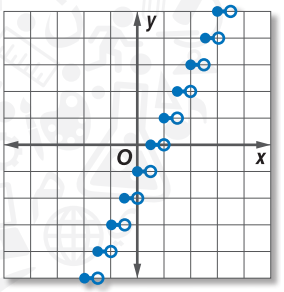


42. **CAR LEASE** As part of Majed's leasing agreement, he will be charged AED 0.20 per mile for each mile over 12,000. Any fraction of a mile is rounded up to the next mile. Make a step graph to represent the cost of going over the mileage.

43 **BASEBALL** A baseball team is ordering T-shirts with the team logo on the front and the players' names on the back. A graphic design store charges AED 10 to set up the artwork plus AED 10 per shirt, AED 4 each for the team logo, and AED 2 to print the last name for an order of 10 shirts or less. For orders of 11–20 shirts, a 5% discount is given. For orders of more than 20 shirts, a 10% discount is given.

a. Organize the information into a table. Include a column showing the total order price for each size order.

b. Write an equation representing the total price for an order of x shirts.

c. Graph the piecewise relation.

44. Consider the function $f(x) = \lvert 2x + 3 \rvert$.

a. Make a table of values where x is all integers from -5 to 5, inclusive.

b. Plot the points on a coordinate grid.

c. Graph the function.

45. Consider the function $f(x) = \lvert 2x \rvert + 3$.

a. Make a table of values where x is all integers from -5 to 5, inclusive.

b. Plot the points on a coordinate grid.

c. Graph the function.

d. Describe how this graph is different from the graph in Exercise 44.

46. DANCE A local studio owner will teach up to 4 students by herself. Her instructors can teach up to 5 students each. Draw a step function graph that best describes the number of instructors needed for the different numbers of students.

47. THEATERS A community theater will only perform a show if there are at least 250 pre-sale ticket requests. Additional performances will be added for each 250 requests after that. Draw a step function graph that best describes this situation.

Graph each function.

48. $f(x) = \frac{1}{2}|x| + 2$

49 $g(x) = \frac{1}{3}|x| + 4$

50. $h(x) = -2|x - 3| + 2$

51. $f(x) = -4|x + 2| - 3$

52. $g(x) = -\frac{2}{3}|x + 6| - 1$

53. $h(x) = -\frac{3}{4}|x - 8| + 1$

54. MULTIPLE REPRESENTATIONS In this problem, you will explore piecewise linear functions.

a. Tabular Copy and complete the table of values for $f(x) = |[\![x]\!]|$ and $g(x) = [\![|x|]\!]$.

x	$[\![x]\!]$	$f(x) = \lvert[\![x]\!]\rvert$	$\lvert x \rvert$	$g(x) = [\![\lvert x \rvert]\!]$
−3	−3	3	3	3
−2.5				
−2				
0				
0.5				
1				
1.5				

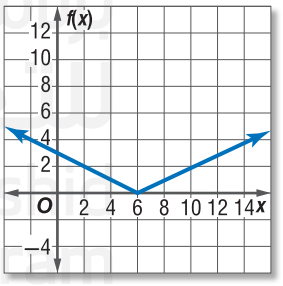
b. Graphical Graph each function on a coordinate plane.

c. Analytical Compare and contrast the graphs of $f(x)$ and $g(x)$.

H.O.T. Problems Use Higher-Order Thinking Skills

55. REASONING Does the piecewise relation $y = \begin{cases} -2x + 4 \text{ if } x \geq 2 \\ -\frac{1}{2}x - 1 \text{ if } x \leq 4 \end{cases}$ represent a function? Why or why not?

SENSE-MAKING **Refer to the graph.**



56. Write an absolute value function that represents the graph.

57. Write a piecewise function to represent the graph.

58. What are the domain and range?

59. WRITING IN MATH Compare and contrast the graphs of absolute value, step, and piecewise-defined functions with the graphs of quadratic and exponential functions. Discuss the domains, ranges, maxima, minima, and symmetry.

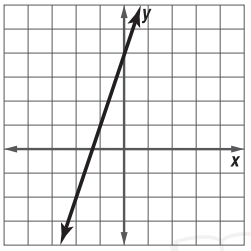
60. CHALLENGE A bicyclist travels up and down a hill with a vertical cross section that can be modeled by $y = -\frac{1}{4}|x - 400| + 100$, where x and y are measured in feet.

a. If $0 \leq x \leq 800$, find the slope for the uphill portion of the trip and downhill portion of the trip.

b. Graph this function. What are the domain and range?

Standardized Test Practice

61. Which equation represents a line that is perpendicular to the graph and passes through the point at (2, 0)?



A $y = 3x - 6$

B $y = -3x + 6$

C $y = -\frac{1}{3}x + \frac{2}{3}$

D $y = \frac{1}{3}x - \frac{2}{3}$

62. A giant tortoise travels at a rate of 0.17 mph. Which equation models the time t it would take the giant tortoise to travel 0.8 mi?

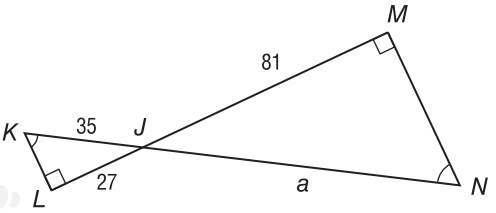
F $t = \frac{0.8}{0.17}$

G $t = (0.17)(0.8)$

H $t = \frac{0.17}{0.8}$

J $0.8 = \frac{0.17}{t}$

63. GEOMETRY If $\triangle JKL$ is similar to $\triangle JNM$ what is the value a?



A 62.5

B 105

C 125

D 155.5

64. GRIDDED RESPONSE What is the difference in the value of $2.1(x + 3.2)$, when $x = 5$ and when $x = 3$?

Spiral Review

Look for a pattern in each table of values to determine which model best describes the data. (Lesson 2-6)

65.

x	0	1	2	3	4
y	1	3	5	7	9

66.

x	−2	−1	0	1	2
y	5	2	1	2	5

67.

x	−1	0	1	2	3
y	1	2	4	8	16

68.

x	5	6	7	8	9
y	−2.5	−1.5	1.5	6.5	13.5

Determine the domain and range for each function. (Lesson 2-7)

69. $f(x) = |2x - 5|$

70. $h(x) = [\![x - 1]\!]$

71. $g(x) = \begin{cases} -3x + 4 \text{ if } x > 2 \\ x - 1 \text{ if } x \leq 2 \end{cases}$

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 2-5)

72. $x^2 - 25 = 0$

73. $r^2 + 25 = 0$

74. $4w^2 + 100 = 40w$

75. $2r^2 + r - 14 = 0$

76. $5v^2 - 7v = 1$

77. $11z^2 - z = 3$

Skills Review

Evaluate each expression. If necessary, round to the nearest hundredth.

78. $\sqrt{9}$

79. $\sqrt{12}$

80. $\sqrt{4.5}$

81. $3\sqrt{16}$

82. $2\sqrt{10}$

83. $\sqrt{5} - 2$

EXTEND

4-7 Graphing Technology Lab: Piecewise-Linear Functions

You can use a graphing calculator to graph and analyze various piecewise functions, including greatest integer functions and absolute value functions.

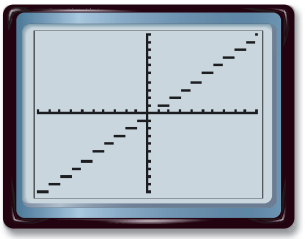
Activity 1 Greatest Integer Functions

Graph $f(x) = [\![x]\!]$ in the standard viewing window.

The calculator may need to be changed to dot mode for the function to graph correctly. Press MODE then use the arrow and ENTER keys to select **DOT**.

Enter the equation in the **Y=** list. Then graph the equation.

KEYSTROKES: Y= MATH ▶ 5 X,T,θ,n) ZOOM 6



[−10, 10] scl: 1 by [−10, 10] scl: 1

1A. How does the graph of $f(x) = [\![x]\!]$ compare to the graph of $f(x) = x$?

1B. What are the domain and range of the function $f(x) = [\![x]\!]$? Explain.

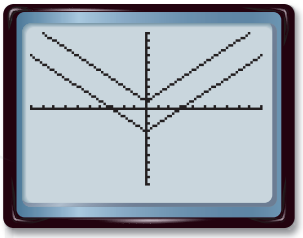
The graphs of piecewise functions are affected by changes in parameters.

Activity 2 Absolute Value Functions

Graph $y = |x| - 3$ and $y = |x| + 1$ in the standard viewing window.

Enter the equations in the **Y=** list. Then graph.

KEYSTROKES: Y= MATH ▶ 1 X,T,θ,n) − 3 ENTER MATH ▶ 1 X,T,θ,n) + 1 ZOOM 6



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2A. Compare and contrast the graphs to the graph of $y = |x|$.

2B. How does the value of k affect the graph of $y = |x| + k$?

Analyze the Results

1. A parking garage charges AED 4 for every hour or fraction of an hour. Is this situation modeled by a *linear* function or a *step* function? Explain your reasoning.

2. A maintenance technician is testing an elevator system. The technician starts the elevator at the fifth floor. It is sent to the ground floor, then back to the fifth floor. Assume the elevator travels at a constant rate. Should the height of the elevator be modeled by a step function or an absolute value function? Explain.

Because the points on a graph are solutions of its equation, the x-coordinates of points where $y = f(x)$ and $y = g(x)$ intersect are solutions of $f(x) = g(x)$. For example, the solution of $5x - 2 = |x|$ is the intersection of the graphs of $y = 5x - 2$ and $y = |x|$. Write each equation as a system of equations, and then use a graph or a table to solve it.

3. $5x - 2 = |x|$

4. $2|x - 2| = x - 1$

5. $|4x + 2| = -|x| + 3$

CHAPTER 4

Study Guide and Review

Study Guide

KeyConcepts

Slope-Intercept Form (Lessons 4-1 and 4-2)

- The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.
- If you are given two points through which a line passes, use them to find the slope first.

Point-Slope Form (Lesson 4-3)

- The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.

Parallel and Perpendicular Lines (Lesson 4-4)

- Nonvertical parallel lines have the same slope.
- Lines that intersect at right angles are called perpendicular lines. The slopes of perpendicular lines are opposite reciprocals.

Scatter Plots and Lines of Fit (Lesson 4-5)

- Data with two variables are called bivariate data.
- A scatter plot is a graph in which two sets of data are plotted as ordered pairs in a coordinate plane.

Regression and Median-Fit Lines (Lesson 4-6)

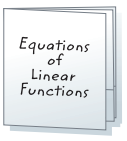
- A graphing calculator can be used to find regression lines and median-fit lines.

Inverse Linear Functions (Lesson 4-7)

- An inverse relation is the set of ordered pairs obtained by exchanging the x-coordinates with the y-coordinates of each ordered pair of a relation.
- A linear function $f(x)$ has an inverse function that can be written as $f^{-1}(x)$ and is read *f of x inverse* or *the inverse of f of x*.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

best-fit line	linear interpolation
bivariate data	linear regression
constant function	line of fit
constraint	median-fit line
correlation coefficient	parallel lines
identity function	perpendicular lines
inverse function	point-slope form
inverse relation	scatter plot
linear extrapolation	slope-intercept form

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The y-intercept is the y-coordinate of the point where the graph crosses the y-axis.
2. The process of using a linear equation to make predictions about values that are beyond the range of the data is called linear regression.
3. An inverse relation is the set of ordered pairs obtained by exchanging the x-coordinates with the y-coordinates of each ordered pair of a relation.
4. The correlation coefficient describes whether the correlation between the variables is positive or negative and how closely the regression equation is modeling the data.
5. Lines in the same plane that do not intersect are called parallel lines.
6. Lines that intersect at acute angles are called perpendicular lines.
7. A(n) constant function can generate ordered pairs for an inverse relation.
8. The range of a relation is the range of its inverse function.
9. An equation of the form $y = mx + b$ is in point-slope form.

Lesson-by-Lesson Review

4-1 Graphing Equations in Slope-Intercept Form

Write an equation of a line in slope-intercept form with the given slope and *y*-intercept. Then graph the equation.

10. slope: 3, *y*-intercept: 5

11. slope: −2, *y*-intercept: −9

12. slope: $\frac{2}{3}$, *y*-intercept: 3

13. slope: $-\frac{5}{8}$, *y*-intercept: −2

Graph each equation.

14. $y = 4x - 2$

15. $y = -3x + 5$

16. $y = \frac{1}{2}x + 1$

17. $3x + 4y = 8$

18. SKI RENTAL Write an equation in slope-intercept form for the total cost of skiing for *h* hours with one lift ticket.

Example 1

Write an equation of a line in slope-intercept form with slope −5 and *y*-intercept −3. Then graph the equation.

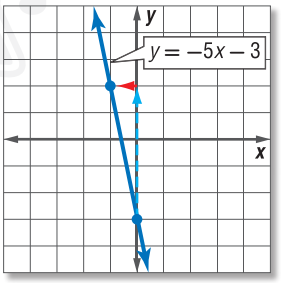
$y = mx + b$

$y = -5x + (-3)$

$y = -5x - 3$

To graph the equation, plot the *y*-intercept (0, −3).

Then move up 5 units and left 1 unit. Plot the point. Draw a line through the two points.



4-2 Writing Equations in Slope-Intercept Form

Write an equation of the line that passes through the given point and has the given slope.

19. (1, 2), slope 3

20. (2, −6), slope −4

21. (−3, −1), slope $\frac{2}{5}$

22. (5, −2), slope $-\frac{1}{3}$

Write an equation of the line that passes through the given points.

23. (2, −1), (5, 2)

24. (−4, 3), (1, 13)

25. (3, 5), (5, 6)

26. (2, 4), (7, 2)

27. CAMP In 2005, a camp had 450 campers. Five years later, the number of campers rose to 750. Write a linear equation that represents the number of campers that attend camp.

Example 2

Write an equation of the line that passes through (3, 2) with a slope of 5.

Step 1 Find the *y*-intercept.

$y = mx + b$

$2 = 5(3) + b$

$2 = 15 + b$

$-13 = b$

Step 2 Write the equation in slope-intercept form.

$y = mx + b$

$y = 5x - 13$

4-3 Writing Equations in Point-Slope Form

Write an equation in point-slope form for the line that passes through the given point with the slope provided.

28. (6, 3), slope 5

29. (−2, 1), slope −3

30. (−4, 2), slope 0

Write each equation in standard form.

31. $y - 3 = 5(x - 2)$

32. $y - 7 = -3(x + 1)$

33. $y + 4 = \frac{1}{2}(x - 3)$

34. $y - 9 = -\frac{4}{5}(x + 2)$

Write each equation in slope-intercept form.

35. $y - 2 = 3(x - 5)$

36. $y - 12 = -2(x - 3)$

37. $y + 3 = 5(x + 1)$

38. $y - 4 = \frac{1}{2}(x + 2)$

Example 3

Write an equation in point-slope form for the line that passes through (3, 4) with a slope of −2.

$y - y_1 = m(x - x_1)$

$y - 4 = -2(x - 3)$

Example 4

Write $y + 6 = -4(x - 3)$ in standard form.

$y + 6 = -4(x - 3)$

$y + 6 = -4x + 12$

$4x + y + 6 = 12$

$4x + y = 6$

4-4 Parallel and Perpendicular Lines

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

39. (2, 5), $y = x - 3$

40. (0, 3), $y = 3x + 5$

41. (−4, 1), $y = -2x - 6$

42. (−5, −2), $y = -\frac{1}{2}x + 4$

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the given equation.

43. (2, 4), $y = 3x + 1$

44. (1, 3), $y = -2x - 4$

45. (−5, 2), $y = \frac{1}{3}x + 4$

46. (3, 0), $y = -\frac{1}{2}x$

Example 5

Write an equation in slope-intercept form for the line that passes through (−2, 4) and is parallel to the graph of $y = 6x - 3$.

The slope of the line with equation $y = 6x - 3$ is 6. The line parallel to $y = 6x - 3$ has the same slope, 6.

$y - y_1 = m(x - x_1)$

$y - 4 = 6[x - (-2)]$

$y - 4 = 6(x + 2)$

$y - 4 = 6x + 12$

$y = 6x + 16$

4-5 Regression and Median-Fit Lines

47. SALE The table shows the number of purchases made at an outerwear store during a sale. Write an equation of the regression line. Then estimate the daily purchases on day 10 of the sale.

Days Since Sale Began	1	2	3	4	5	6	7
Daily Purchases	15	21	32	30	40	38	51

48. MOVIES The table shows ticket sales at a certain theater during the first week after a movie opened. Write an equation of the regression line. Then estimate the daily ticket sales on the 15th day.

Days Since Movie Opened	1	2	3	4	5	6	7
Daily Ticket Sales	85	92	89	78	65	68	55

Example 6

ATTENDANCE The table shows the annual attendance at an amusement park. Write an equation of the regression line for the data.

Years Since 2004	0	1	2	3	4	5	6
Attendance (thousands)	75	80	72	68	65	60	53

Step 1 Enter the data by pressing STAT and selecting the **Edit** option.

Step 2 Perform the regression by pressing STAT and selecting the **CALC** option. Scroll down to **LinReg (ax + b)** and press ENTER.

Step 3 Write the equation of the regression line by rounding the a- and b-values on the screen.

$y = -4.04x + 79.68$

4-6 Inverse Linear Functions

Find the inverse of each relation.

49. {(7, 3.5), (6.2, 8), (−4, 2.7), (−12, 1.4)}

50. {(1, 9), (13, 26), (−3, 4), (−11, −2)}

51.

X	Y
−4	2.7
−1	3.8
0	4.1
3	7.2

52.

X	Y
−12	4
−8	0
−4	−4
0	−8

Find the inverse of each function.

53. $f(x) = \frac{5}{11}x + 10$

54. $f(x) = 3x + 8$

55. $f(x) = -4x - 12$

56. $f(x) = \frac{1}{4}x - 7$

57. $f(x) = -\frac{2}{3}x + \frac{1}{4}$

58. $f(x) = -3x + 3$

Example 7

Find the inverse of the relation.

{(5, −3), (11, 2), (−6, 12), (4, −2)}

To find the inverse, exchange the coordinates of the ordered pairs.

$(5, -3) \rightarrow (-3, 5)$ $\quad$ $(-6, 12) \rightarrow (12, -6)$

$(11, 2) \rightarrow (2, 11)$ $\quad$ $(4, -2) \rightarrow (-2, 4)$

The inverse is {(−3, 5), (2, 11), (12, −6), (−2, 4)}.

Example 8

Find the inverse of $f(x) = \frac{1}{4}x + 9$.

$f(x) = \frac{1}{4}x + 9$

$y = \frac{1}{4}x + 9$

$x = \frac{1}{4}y + 9$

$x - 9 = \frac{1}{4}y$

$4(x - 9) = y$

$4x - 36 = y$

$4x - 36 = f^{-1}(x)$

4-7 Special Functions

Graph each function. State the domain and range.

59. $f(x) = [\![x]\!]$

60. $f(x) = [\![2x]\!]$

61. $f(x) = |x|$

62. $f(x) = |2x - 2|$

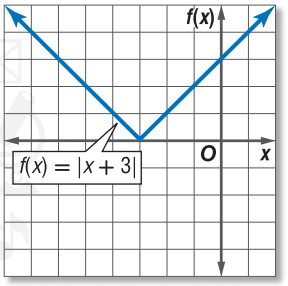
63. $f(x) = \begin{cases} x - 2 \text{ if } x < 1 \\ 3x \text{ if } x \geq 1 \end{cases}$

64. $f(x) = \begin{cases} 2x - 3 \text{ if } x \leq 2 \\ x + 1 \text{ if } x > 2 \end{cases}$

Example 8

Graph $f(x) = |x + 3|$. State the domain and range.

x	$f(x)$
−5	2
−4	1
−3	0
−2	1
−1	2



The domain is all real numbers, and the range is $f(x) \geq 0$.

CHAPTER 4 Practice Test

1. Graph $y = 2x - 3$.

2. **MULTIPLE CHOICE** A popular pizza parlor charges AED 12 for a large cheese pizza plus AED 1.50 for each additional topping. Write an equation in slope-intercept form for the total cost C of a pizza with t toppings.

   A $C = 12t + 1.50$

   B $C = 13.50t$

   C $C = 12 + 1.50t$

   D $C = 1.50t - 12$

Write an equation of a line in slope-intercept form that passes through the given point and has the given slope.

3. $(-4, 2)$; slope -3

4. $(3, -5)$; slope $\frac{2}{3}$

Write an equation of the line in slope-intercept form that passes through the given points.

5. $(1, 4)$, $(3, 10)$

6. $(2, 5)$, $(-2, 8)$

7. $(0, 4)$, $(-3, 0)$

8. $(7, -1)$, $(9, -4)$

9. **PAINTING** The data in the table show the size of a room in square feet and the time it takes to paint the room in minutes.

Room Size	100	150	200	400	500
Painting Time	160	220	270	500	680

   a. Use the points (100, 160) and (500, 680) to write an equation in slope-intercept form.

   b. Predict the amount of time required to paint a room measuring 750 ft^2.

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

10. $(2, -3)$, $y = 4x - 9$

11. $(-5, 1)$, $y = -3x + 2$

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.

12. $(1, 4)$, $y = -2x + 5$

13. $(-3, 6)$, $y = \frac{1}{4}x + 2$

14. **SPONSORSHIP** The table shows the number of children from Ethiopia sponsored by U.S. citizens.

Years Since 2000	5	6	7	8	9
Number of Children	442	731	1254	1724	2277

   a. Write the slope-intercept form of the equation for the line of fit.

   b. Predict the number of children from Ethiopia who will be sponsored in 2025.

Find the inverse of each function.

15. $f(x) = -5x - 30$

16. $f(x) = 4x + 10$

17. $f(x) = \frac{1}{6}x - 2$

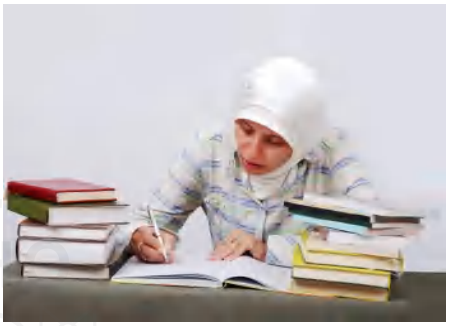
18. $f(x) = \frac{3}{4}x + 12$

CHAPTER 4

Preparing for Standardized Tests

Short Answer Questions

Short answer questions require you to provide a solution to the problem, along with a method, explanation, and/or justification used to arrive at the solution.



Strategies for Solving Short Answer Questions

Step 1

Short answer questions are typically graded using a **rubric**, or a scoring guide. The following is an example of a short answer question scoring rubric.

Scoring Rubric	
Criteria	**Score**
Full Credit: The answer is correct and a full explanation is provided that shows each etep.	2
Partial Credit: • The answer is correct, but the explanation is incomplete. • The answer is incorrect, but the explanation is correct.	1
No Credit: Either an answer is not provided or the answer does not make sense.	0

Step 2

In solving short answer questions, remember to...

- explain your reasoning or state your approach to solving the problem.
- show all of your work or steps.
- check your answer if time permits.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

The table shows production costs for building different numbers of skateboards. Determine the missing value, x, that will result in a linear model.

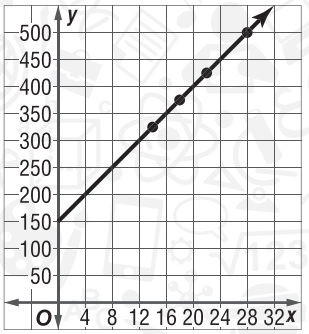
Skateboards Built	Production Costs
14	AED 325
28	AED 500
x	AED 375
22	AED 425

Read the problem carefully. You are given several data points and asked to find the missing value that results in a linear model.

Example of a 2-point response:

Set up a coordinate grid and plot the three given points:
(14, 325), (28, 500), (22, 425).

Then draw a straight line through them and find the x-value that produces a y-value of 375.



So, building 18 skateboards would result in production costs of AED 375. These data form a linear model.

The steps, calculations, and reasoning are clearly stated. The student also arrives at the correct answer. So, this response is worth the full 2 points.

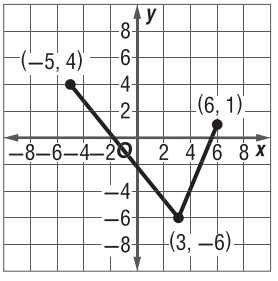
Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. Given points $M(-1, 7)$, $N(3, -5)$, $O(6, 1)$, and $P(-3, -2)$, determine two segments that are perpendicular to each other.

2. Write the equation of a line that is parallel to $4x + 2y = 8$ and has a y-intercept of 5.

3. Three vertices of a quadrilateral are shown on the coordinate grid. Determine a fourth vertex that would result in a trapezoid.



CHAPTER 4

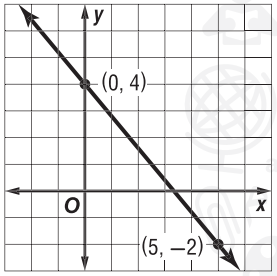
Standardized Test Practice

Cumulative, Chapters 1 through 4

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What is the rate of change represented in the graph?



A $-\frac{2}{5}$

B $-\frac{5}{6}$

C $-\frac{6}{5}$

D $-\frac{5}{2}$

2. The table below shows the cost for renting a bicycle at a bike shop located in Venice Beach. What is a function that can represent this sequence?

Number of Hours	Cost (AED)
1	10
2	14
3	18
4	22

F $f(n) = 4n + 10$

G $f(n) = 4n + 6$

H $f(n) = 10n + 4$

J $f(n) = 10n - 6$

3. Jasim bought a car in 2005 for AED 28,500. By 2008, the car was worth AED 23,700. Based on a linear model, what will the value of the car be in 2012?

A AED 17,300

B AED 17,550

C AED 18,100

D AED 18,475

4. If the graph of a line has a positive slope and a negative y-intercept, what happens to the x-intercept if the slope and the y-intercept are doubled?

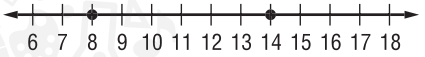
F The x-intercept becomes four times as great.

G The x-intercept becomes twice as great.

H The x-intercept becomes one-fourth as great.

J The x-intercept remains the same.

5. Which absolute value equation has the graph below as its solution?



A $|x - 3| = 11$

B $|x - 4| = 12$

C $|x - 11| = 3$

D $|x - 12| = 4$

6. The table below shows the relationship between certain temperatures in degrees Fahrenheit and degrees Celsius. Which of the following linear equations correctly models this relationship?

Celsius (C)	Fahrenheit (F)
10°	50°
15°	59°
20°	68°
25°	77°
30°	86°

F $F = \frac{8}{5}C + 35$

G $F = \frac{4}{5}C + 42$

H $F = \frac{9}{5}C + 32$

J $F = \frac{12}{5}C + 26$

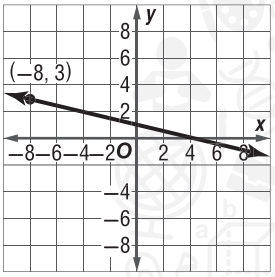
Test-Taking Tip

Question 3 Find the average annual depreciation between 2005 and 2008. Then extend the pattern to find the car's value in 2012.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

7. What is the equation of the line graphed below?



Express your answer in point slope form using the point (−8, 3).

8. GRIDDED RESPONSE The linear equation below is a best fit model for the peak depth of the Mad River when x inches of rain fall. What would you expect the peak depth of the river to be after a storm that produces $1\frac{3}{4}$ inches of rain? Round your answer to the nearest tenth of a foot if necessary.

$$y = 2.5x + 14.8$$

9. Khalifa formed an advertising company in 1992. Initially, the company only had 14 employees. In 2008, the company had grown to a total of 63 employees. Find the percent of change in the number of employees working at Khalifa's company. Round to the nearest tenth of a percent if necessary.

10. The table shows the total amount of rain during a storm.

Hours	1	2	3	4
Inches	0.45	0.9	1.35	1.8

a. Write an equation to fit the data in the table.

b. Describe the relationship between the hour and the amount of rain received.

11. An electrician charges a AED 25 consultation fee plus AED 35 per hour for labor.

a. Copy and complete the following table showing the charges for jobs that take 1, 2, 3, 4, or 5 hours.

Hours, h	Total Cost, C
1	
2	
3	
4	
5	

b. Write an equation in slope-intercept form for the total cost of a job that takes h hours.

c. If the electrician bills in quarter hours, how much would it cost for a job that takes 3 hours 15 minutes to complete?

Extended Response

Record your answer on a sheet of paper. Show your work.

12. Explain how you can determine whether two lines are parallel or perpendicular.