9-2 Graphs of Polar Equations

Graph each equation by plotting points.

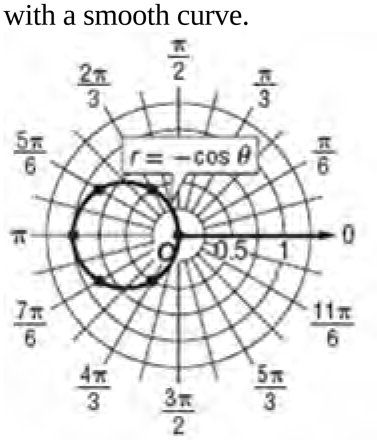
1. $r = -\cos\theta$

SOLUTION:

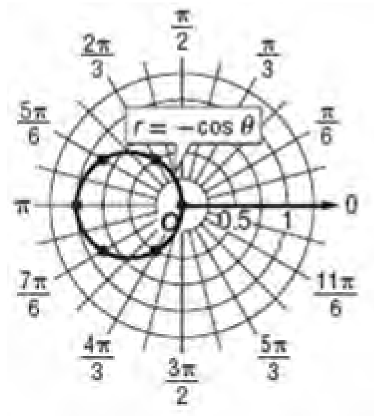
Make a table of values to find the r-values corresponding to various values of θ on the interval $[0, 2\pi]$. Round each r-value to the nearest tenth.

θ	$r = -\cos\theta$
0	–1
$\frac{\pi}{6}$	–0.9
$\frac{\pi}{3}$	–0.5
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	0.5
$\frac{5\pi}{6}$	0.9
π	1
$\frac{7\pi}{6}$	0.9
$\frac{4\pi}{3}$	0.5
$\frac{3\pi}{2}$	0
$\frac{5\pi}{3}$	–0.5
$\frac{11\pi}{6}$	–0.9
2π	–1

Graph the ordered pairs (r, θ) and connect them with a smooth curve.



ANSWER:



2. $r = \csc\theta$

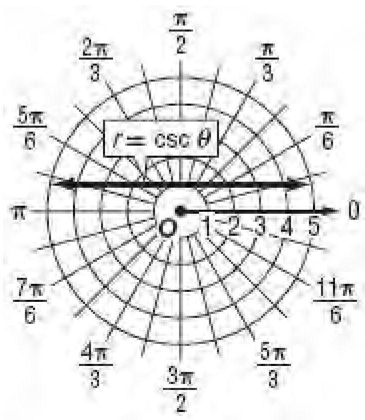
SOLUTION:

Make a table of values to find the r-values corresponding to various values of θ on the interval $[0, 2\pi]$. Round each r-value to the nearest tenth.

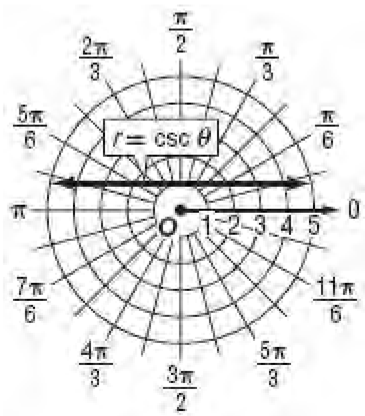
θ	$r = \csc\theta$
0	–
$\frac{\pi}{6}$	2
$\frac{\pi}{3}$	1.2
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	1.2
$\frac{5\pi}{6}$	2
π	–
$\frac{7\pi}{6}$	–2
$\frac{4\pi}{3}$	–1.2
$\frac{3\pi}{2}$	–2
$\frac{5\pi}{3}$	–1.2
$\frac{11\pi}{6}$	–2
2π	–

Graph the ordered pairs (r, θ) and connect them with a line.

9-2 Graphs of Polar Equations



ANSWER:



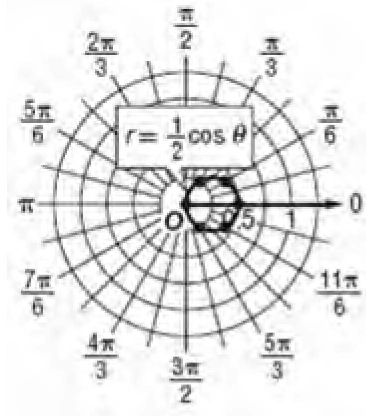
3. $r = \frac{1}{2}\cos\theta$

SOLUTION:

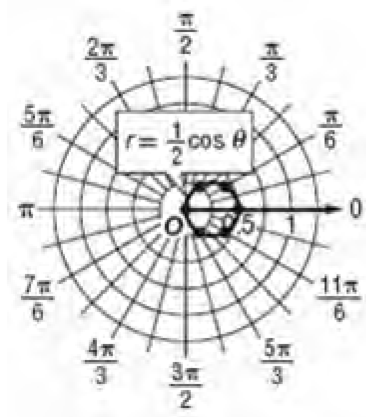
Make a table of values to find the *r*-values corresponding to various values of θ on the interval $[0, 2\pi]$. Round each *r*-value to the nearest tenth.

θ	$r = \frac{1}{2}\cos\theta$
0	0.5
$\frac{\pi}{6}$	0.4
$\frac{\pi}{3}$	0.3
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	–0.3
$\frac{5\pi}{6}$	–0.4
π	–0.5
$\frac{7\pi}{6}$	–0.4
$\frac{4\pi}{3}$	–0.3
$\frac{3\pi}{2}$	0
$\frac{5\pi}{3}$	0.3
$\frac{11\pi}{6}$	0.4
2π	0.5

Graph the ordered pairs (r, θ) and connect them with a smooth curve.



ANSWER:



4. $r = 3\sin\theta$

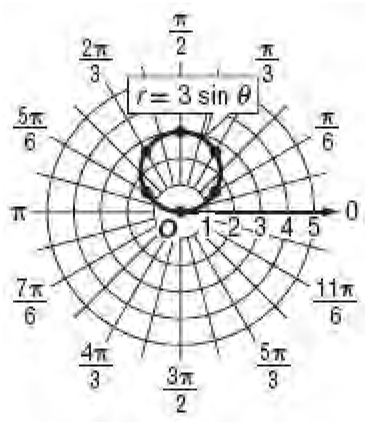
SOLUTION:

Make a table of values to find the *r*-values corresponding to various values of θ on the interval $[0, 2\pi]$. Round each *r*-value to the nearest tenth.

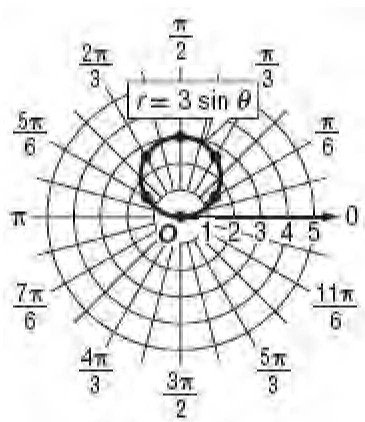
θ	$r = 3\sin\theta$
0	0
$\frac{\pi}{6}$	1.5
$\frac{\pi}{3}$	2.6
$\frac{\pi}{2}$	3

$\frac{2\pi}{3}$	2.6
$\frac{5\pi}{6}$	1.5
π	0
$\frac{7\pi}{6}$	−1.5
$\frac{4\pi}{3}$	−2.6
$\frac{3\pi}{2}$	−3
$\frac{5\pi}{3}$	−2.6
$\frac{11\pi}{6}$	−1.5
2π	0

Graph the ordered pairs (r, θ) and connect them with a smooth curve.



ANSWER:



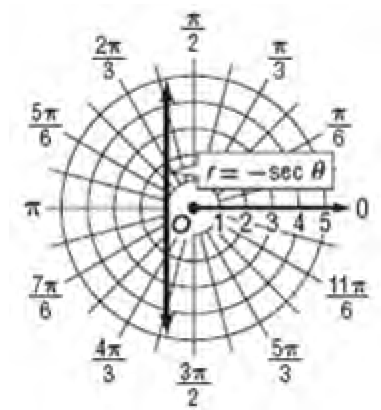
5. $r = -\sec\theta$

SOLUTION:

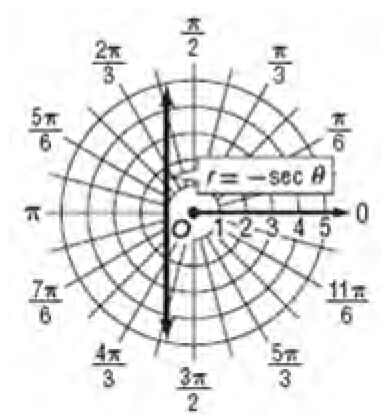
Make a table of values to find the *r*-values corresponding to various values of θ on the interval [0, 2π]. Round each *r*-value to the nearest tenth.

θ	$r = -\sec\theta$
0	−1
$\frac{\pi}{6}$	−1.2
$\frac{\pi}{3}$	−2
$\frac{\pi}{2}$	–
$\frac{2\pi}{3}$	2
$\frac{5\pi}{6}$	1.2
π	1
$\frac{7\pi}{6}$	1.2
$\frac{4\pi}{3}$	2
$\frac{3\pi}{2}$	–
$\frac{5\pi}{3}$	−2
$\frac{11\pi}{6}$	−1.2
2π	−1

Graph the ordered pairs (r, θ) and connect them with a line.



ANSWER:



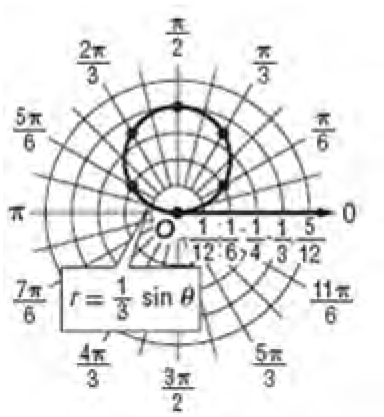
6. $r = \frac{1}{3} \sin \theta$

SOLUTION:

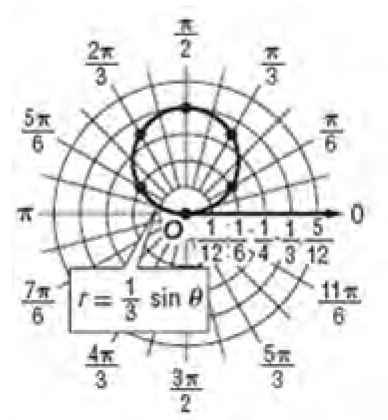
Make a table of values to find the r-values corresponding to various values of θ on the interval $[0, 2\pi]$. Round each r-value to the nearest tenth.

θ	$r = \frac{1}{3}\sin\theta$
0	0
$\frac{\pi}{6}$	0.2
$\frac{\pi}{3}$	0.3
$\frac{\pi}{2}$	0.3
$\frac{2\pi}{3}$	0.3
$\frac{5\pi}{6}$	0.2
π	0
$\frac{7\pi}{6}$	−0.2
$\frac{4\pi}{3}$	−0.3
$\frac{3\pi}{2}$	−0.3
$\frac{5\pi}{3}$	−0.3
$\frac{11\pi}{6}$	−0.2
2π	0

Graph the ordered pairs (r, θ) and connect them with a smooth curve.



ANSWER:



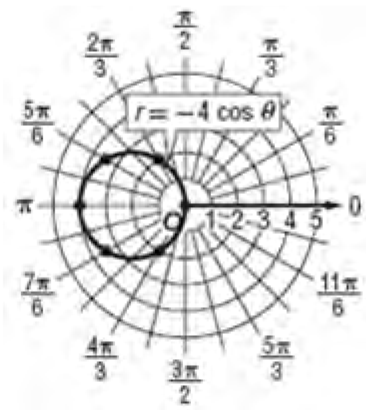
7. $r = -4 \cos \theta$

SOLUTION:

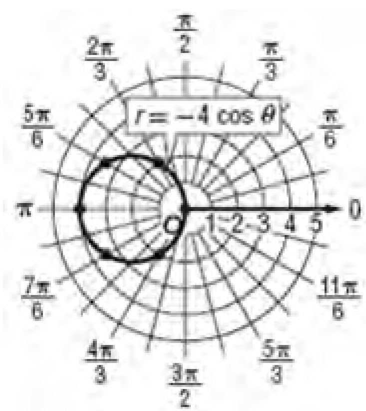
Make a table of values to find the r-values corresponding to various values of θ on the interval $[0, 2\pi]$. Round each r-value to the nearest tenth.

θ	$r = -4\cos\theta$
0	−4
$\frac{\pi}{6}$	−3.5
$\frac{\pi}{3}$	−2
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	2
$\frac{5\pi}{6}$	3.5
π	4
$\frac{7\pi}{6}$	3.5
$\frac{4\pi}{3}$	2
$\frac{3\pi}{2}$	0
$\frac{5\pi}{3}$	−2
$\frac{11\pi}{6}$	−3.5
2π	−4

Graph the ordered pairs (r, θ) and connect them with a smooth curve.



ANSWER:



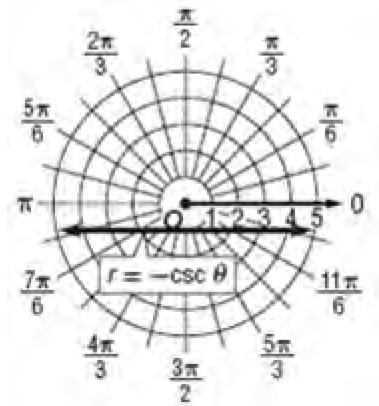
8. $r = -\csc\theta$

SOLUTION:

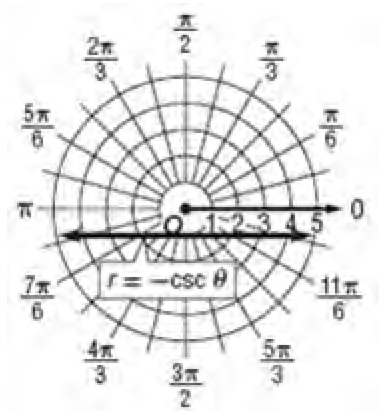
Make a table of values to find the *r*-values corresponding to various values of θ on the interval [0, 2π]. Round each *r*-value to the nearest tenth.

θ	$r = -\csc\theta$
0	–
$\frac{\pi}{6}$	−2
$\frac{\pi}{3}$	−1.2
$\frac{\pi}{2}$	−1
$\frac{2\pi}{3}$	−1.2
$\frac{5\pi}{6}$	−2
π	–
$\frac{7\pi}{6}$	2
$\frac{4\pi}{3}$	1.2
$\frac{3\pi}{2}$	1
$\frac{5\pi}{3}$	1.2
$\frac{11\pi}{6}$	2
2π	–

Graph the ordered pairs (r, θ) and connect them with a line.



ANSWER:



Use symmetry to graph each equation.

9. $r = 3 + 3\cos\theta$

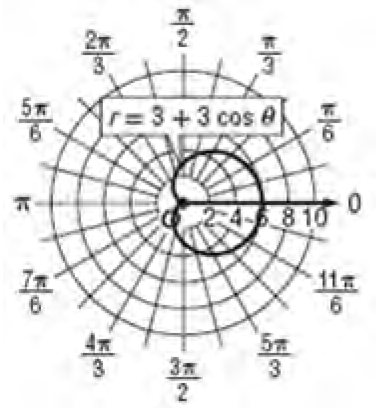
SOLUTION:

Because the polar equation is a function of the cosine function, it is symmetric with respect to the polar axis. Therefore, make a table and calculate the values of *r* on [0, π].

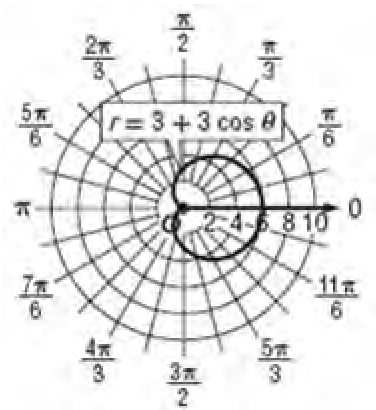
θ	$r = 3 + 3\cos\theta$
0	6
$\frac{\pi}{6}$	5.6
$\frac{\pi}{4}$	5.1
$\frac{\pi}{3}$	4.5
$\frac{\pi}{2}$	3
$\frac{2\pi}{3}$	1.5

$\frac{3\pi}{4}$	0.9
$\frac{5\pi}{6}$	0.4
π	0

Use these points and polar axis symmetry to graph the function.



ANSWER:



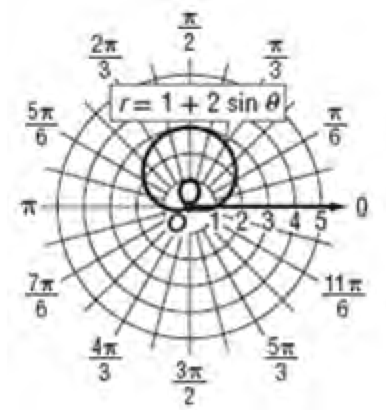
10. $r = 1 + 2\sin\theta$

SOLUTION:

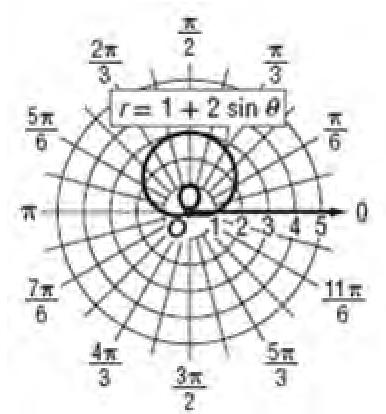
Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$. Therefore, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = 1 + 2\sin\theta$
$-\frac{\pi}{2}$	−1
$-\frac{\pi}{3}$	−0.7
$-\frac{\pi}{4}$	−0.4
$-\frac{\pi}{6}$	0
0	1
$\frac{\pi}{6}$	2
$\frac{\pi}{4}$	2.4
$\frac{\pi}{3}$	2.7
$\frac{\pi}{2}$	3

Use these points and symmetry with respect to the line $\theta = \frac{\pi}{2}$ to graph the function.



ANSWER:



11. $r = 4 - 3\cos\theta$

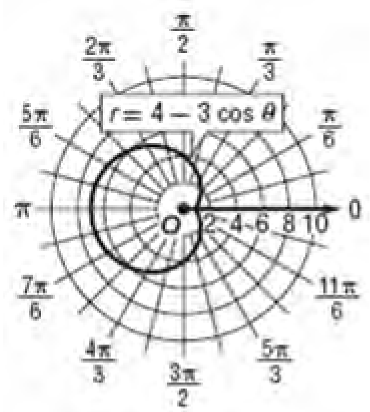
SOLUTION:

Because the polar equation is a function of the cosine function, it is symmetric with respect to the polar axis. Therefore, make a table and calculate the values of r on $[0, \pi]$.

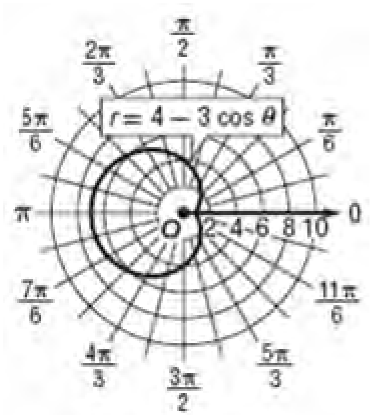
θ	$r = 4 - 3\cos\theta$
0	1
$\frac{\pi}{6}$	1.4

$\frac{\pi}{4}$	1.9
$\frac{\pi}{3}$	2.5
$\frac{\pi}{2}$	4
$\frac{2\pi}{3}$	5.5
$\frac{3\pi}{4}$	6.12
$\frac{5\pi}{6}$	6.6
π	7

Use these points and polar axis symmetry to graph the function.



ANSWER:



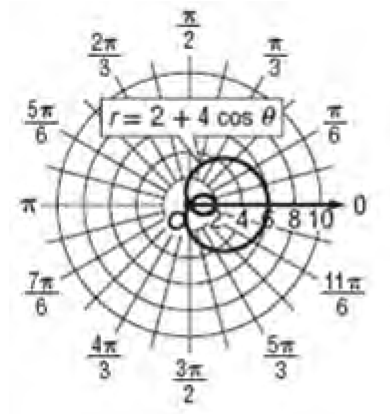
12. $r = 2 + 4 \cos \theta$

SOLUTION:

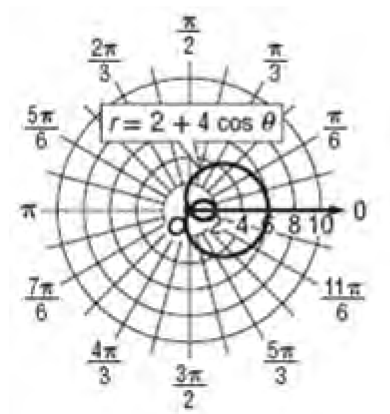
Because the polar equation is a function of the cosine function, it is symmetric with respect to the polar axis. Therefore, make a table and calculate the values of r on [0, π].

θ	$r = 2 + 4 \cos \theta$
0	6
$\frac{\pi}{6}$	5.5
$\frac{\pi}{4}$	4.8
$\frac{\pi}{3}$	4
$\frac{\pi}{2}$	2
$\frac{2\pi}{3}$	0
$\frac{3\pi}{4}$	−0.8
$\frac{5\pi}{6}$	−1.5
π	−2

Use these points and polar axis symmetry to graph the function.



ANSWER:



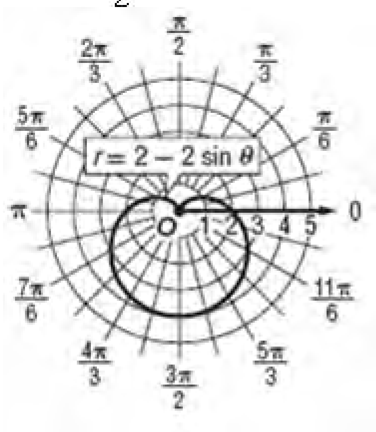
13. $r = 2 - 2 \sin \theta$

SOLUTION:

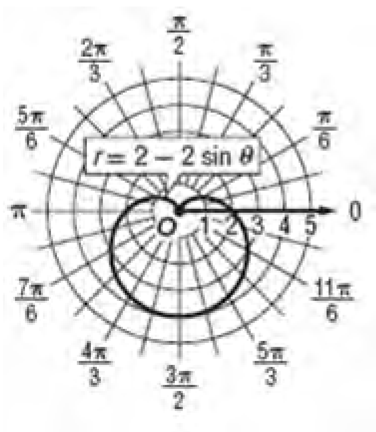
Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$. Therefore, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = 2 - 2 \sin \theta$
$-\frac{\pi}{2}$	4
$-\frac{\pi}{3}$	3.7
$-\frac{\pi}{4}$	3.4
$-\frac{\pi}{6}$	3
0	2
$\frac{\pi}{6}$	1
$\frac{\pi}{4}$	0.6
$\frac{\pi}{3}$	0.3
$\frac{\pi}{2}$	0

Use these points and symmetry with respect to the line $\theta = \frac{\pi}{2}$ to graph the function.



ANSWER:



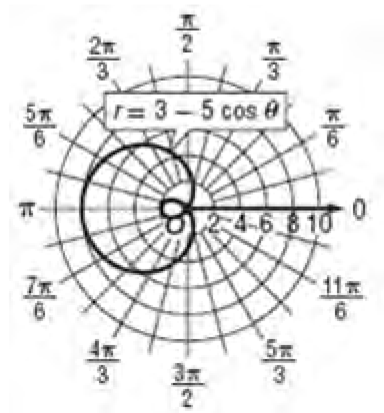
14. $r = 3 - 5 \cos \theta$

SOLUTION:

Because the polar equation is a function of the cosine function, it is symmetric with respect to the polar axis. Therefore, make a table and calculate the values of r on $[0, \pi]$.

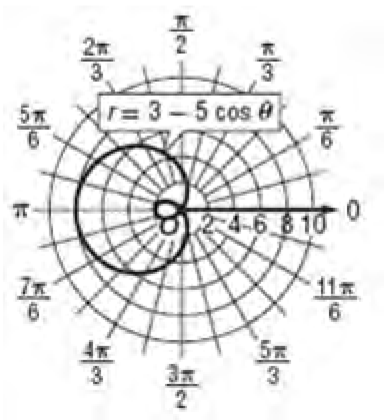
θ	$r = 3 - 5 \cos \theta$
0	–2
$\frac{\pi}{6}$	–1.3
$\frac{\pi}{4}$	–0.5
$\frac{\pi}{3}$	0.5
$\frac{\pi}{2}$	3
$\frac{2\pi}{3}$	5.5
$\frac{3\pi}{4}$	6.5
$\frac{5\pi}{6}$	7.3
π	8

Use these points and polar axis symmetry to graph the function.



ANSWER:

9-2 Graphs of Polar Equations



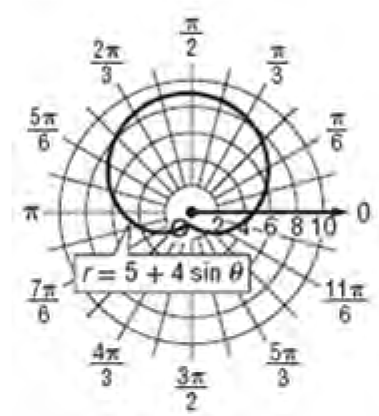
15. $r = 5 + 4\sin\theta$

SOLUTION:

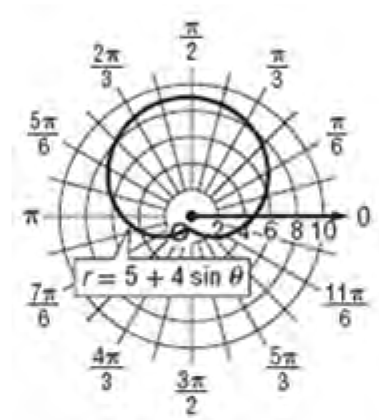
Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$. Therefore, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = 5 + 4\sin\theta$
$-\frac{\pi}{2}$	1
$-\frac{\pi}{3}$	1.5
$-\frac{\pi}{4}$	2.2
$-\frac{\pi}{6}$	3
0	5
$\frac{\pi}{6}$	7
$\frac{\pi}{4}$	7.8
$\frac{\pi}{3}$	8.5
$\frac{\pi}{2}$	9

Use these points and symmetry with respect to the line $\theta = \frac{\pi}{2}$ to graph the function.



ANSWER:



16. $r = 6 - 2\sin\theta$

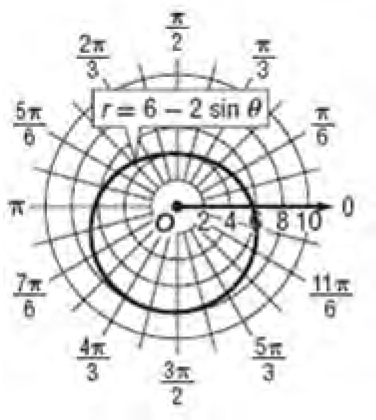
SOLUTION:

Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$. Therefore, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

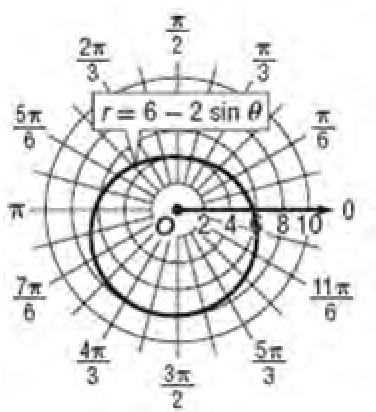
θ	$r = 6 - 2\sin\theta$
$-\frac{\pi}{2}$	8
$-\frac{\pi}{3}$	7.7
$-\frac{\pi}{4}$	7.4
$-\frac{\pi}{6}$	7
0	6
$\frac{\pi}{6}$	5
$\frac{\pi}{4}$	4.6
$\frac{\pi}{3}$	4.3

$\frac{\pi}{2}$	4

Use these points and symmetry with respect to the line $\theta = \frac{\pi}{2}$ to graph the function.



ANSWER:



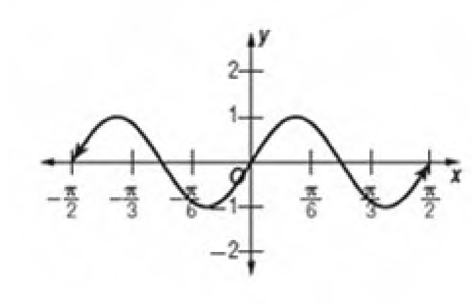
Use symmetry, zeros, and maximum *r*-values to graph each function.

17. $r = \sin 4\theta$

SOLUTION:

Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

Sketch the graph of the rectangular function $y = \sin 4x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. From the graph, you can see that $|y| = 1$ when $x = -\frac{3\pi}{8}, -\frac{\pi}{8}, \frac{\pi}{8}, \text{and } \frac{3\pi}{8}$ and $y = 0$ when $x = -\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \text{and } \frac{\pi}{2}$.

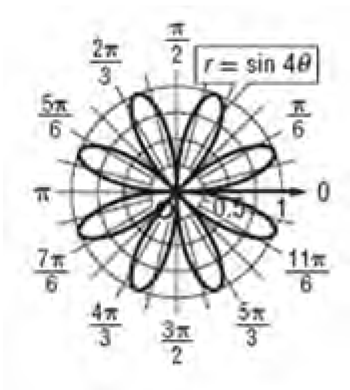


Interpreting these results in terms of the polar equation $r = \sin 4\theta$, we can say that $|r|$ has a maximum value of 1 when $\theta = -\frac{3\pi}{8}, -\frac{\pi}{8}, \frac{\pi}{8}, \text{or } \frac{3\pi}{8}$ and $r = 0$ when $\theta = -\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \text{or } \frac{\pi}{2}$.

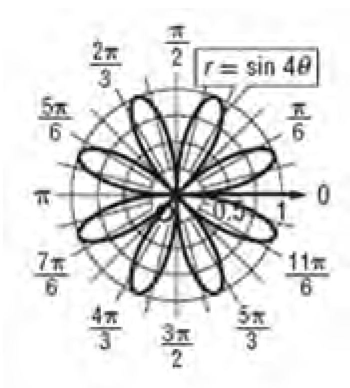
Since the function is symmetric with respect to the line $\theta = \frac{\pi}{2}$, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = \sin 4\theta$
$-\frac{\pi}{2}$	0
$-\frac{\pi}{3}$	0.9
$-\frac{\pi}{4}$	0
$-\frac{\pi}{6}$	−0.9
0	0
$\frac{\pi}{6}$	0.9
$\frac{\pi}{4}$	0
$\frac{\pi}{3}$	−0.9
$\frac{\pi}{2}$	0

Use these and a few additional points to sketch the graph of the function.



ANSWER:

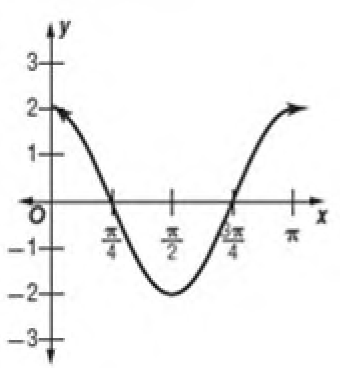


18. $r = 2 \cos 2\theta$

SOLUTION:

Because the polar equation is a function of the cosine function, it is symmetric with respect to the polar axis.

Sketch the graph of the rectangular function $y = 2 \cos 2x$ on the interval $[0, \pi]$. From the graph, you can see that $|y| = 2$ when $x = 0, \frac{\pi}{2}$, and π and $y = 0$ when $x = \frac{\pi}{4}$, and $\frac{3\pi}{4}$.

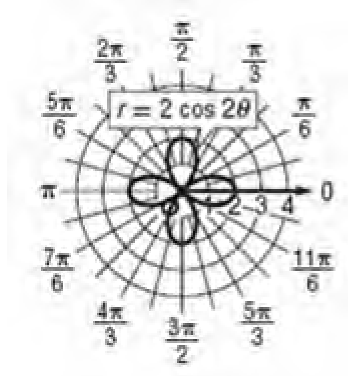


Interpreting these results in terms of the polar equation $r = 2 \cos 2\theta$, we can say that $|r|$ has a maximum value of 2 when $\theta = 0, \frac{\pi}{2}$, and π and $r = 0$ when $\theta = \frac{\pi}{4}$ and $\frac{3\pi}{4}$.

Since the function is symmetric with respect to the polar axis, make a table and calculate the values of r on $[0, \pi]$.

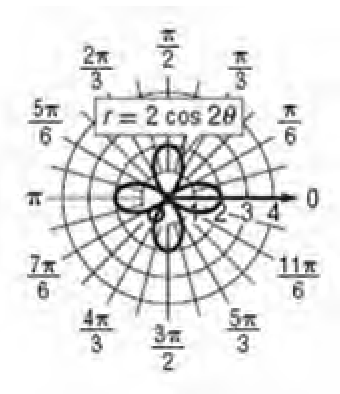
θ	$r = 2 \cos 2\theta$
0	2
$\frac{\pi}{6}$	1
$\frac{\pi}{4}$	0
$\frac{\pi}{3}$	−1
$\frac{\pi}{2}$	−2
$\frac{2\pi}{3}$	−1
$\frac{3\pi}{4}$	0
$\frac{5\pi}{6}$	1
π	2

Use these and a few additional points to sketch the graph of the function.



ANSWER:

9-2 Graphs of Polar Equations

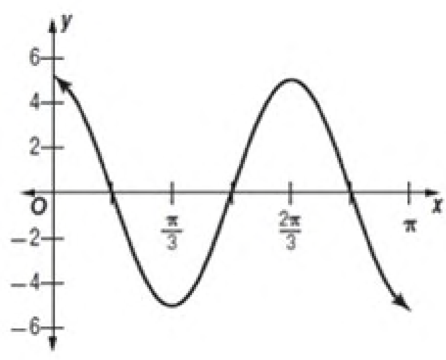


19. $r = 5 \cos 3\theta$

SOLUTION:

Because the polar equation is a function of the cosine function, it is symmetric with respect to the polar axis.

Sketch the graph of the rectangular function $y = 5 \cos 3x$ on the interval $[0, \pi]$. From the graph, you can see that $|y| = 5$ when $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \text{and } \pi$ and $y = 0$ when $x = \frac{\pi}{6}, \frac{\pi}{2}, \text{and } \frac{5\pi}{6}$.

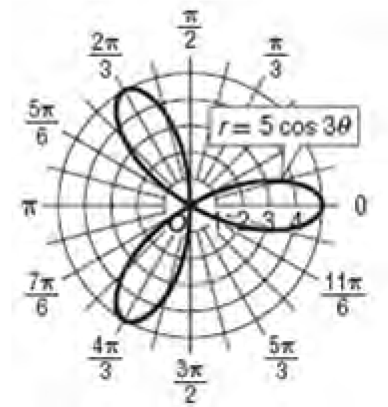


Interpreting these results in terms of the polar equation $r = 5 \cos 3\theta$, we can say that $|r|$ has a maximum value of 5 when $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \text{and } \pi$ and $r = 0$ when $\theta = \frac{\pi}{6}, \frac{\pi}{2} \text{and } \frac{5\pi}{6}$.

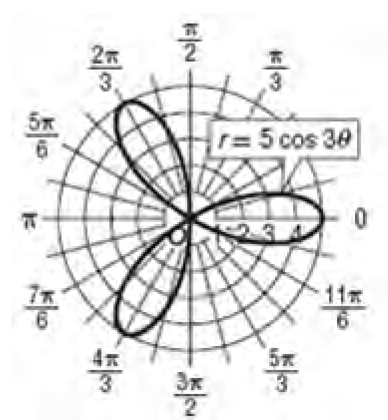
Since the function is symmetric with respect to the polar axis, make a table and calculate the values of r on $[0, \pi]$.

θ	$r = 5 \cos 3\theta$
0	5
$\frac{\pi}{12}$	3.5
$\frac{\pi}{6}$	0
$\frac{\pi}{4}$	−3.5
$\frac{\pi}{3}$	−5
$\frac{5\pi}{12}$	−3.5
$\frac{\pi}{2}$	0
$\frac{7\pi}{12}$	3.5
$\frac{2\pi}{3}$	5
$\frac{3\pi}{4}$	3.5
$\frac{5\pi}{6}$	0
$\frac{11\pi}{12}$	−3.5
π	−5

Use these and a few additional points to sketch the graph of the function.



ANSWER:

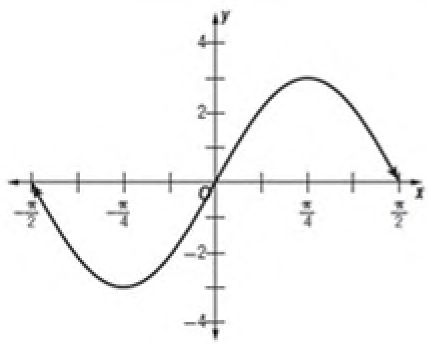


20. $r = 3 \sin 2\theta$

SOLUTION:

Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

Sketch the graph of the rectangular function $y = 3 \sin 2x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. From the graph, you can see that $|y| = 3$ when $x = -\frac{\pi}{4}$ and $\frac{\pi}{4}$ and $y = 0$ when $x = -\frac{\pi}{2}, 0,$ and $\frac{\pi}{2}$.

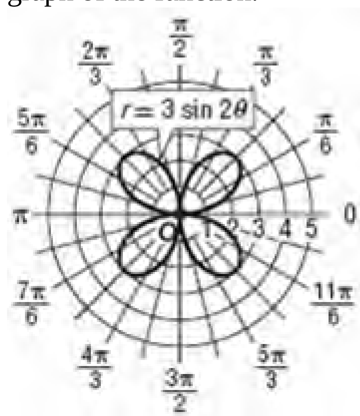


Interpreting these results in terms of the polar equation $r = 3 \sin 2\theta$, we can say that $|r|$ has a maximum value of 3 when $\theta = -\frac{\pi}{4}$ and $\frac{\pi}{4}$ and $r = 0$ when $\theta = -\frac{\pi}{2}, 0,$ and $\frac{\pi}{2}$.

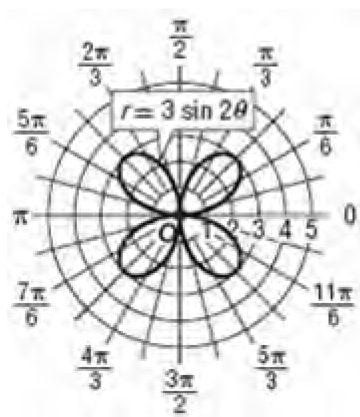
Since the function is symmetric with respect to the line $\theta = \frac{\pi}{2}$, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = 3 \sin 2\theta$
$-\frac{\pi}{2}$	0
$-\frac{\pi}{3}$	−2.6
$-\frac{\pi}{4}$	−3
$-\frac{\pi}{6}$	−2.6
0	0
$\frac{\pi}{6}$	2.6
$\frac{\pi}{4}$	3
$\frac{\pi}{3}$	2.6
$\frac{\pi}{2}$	0

Use these and a few additional points to sketch the graph of the function.



ANSWER:



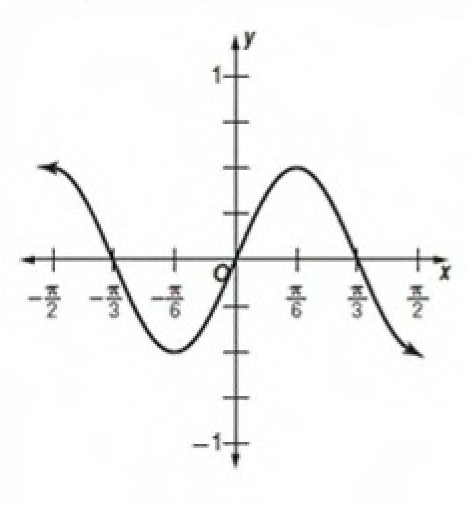
21. $r = \frac{1}{2} \sin 3\theta$

SOLUTION:

Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

Sketch the graph of the rectangular function $y = \frac{1}{2} \sin 3x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. From the graph, you can see that $|y| = \frac{1}{2}$ when $x = -\frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6},$ and $\frac{\pi}{2}$ and $y = 0$ when

$x = -\frac{\pi}{3}, 0,$ and $\frac{\pi}{3}$.



Interpreting these results in terms of the polar equation $r = \frac{1}{2}\sin 3\theta$, we can say that $|r|$ has a maximum value of $\frac{1}{2}$ when

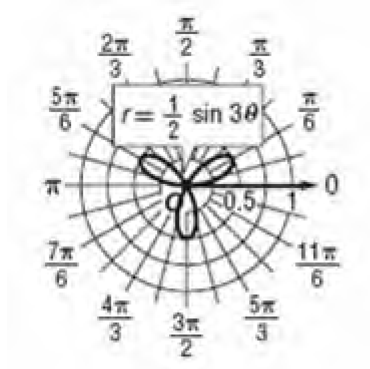
$\theta = -\frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6},$ and $\frac{\pi}{2}$ and $r = 0$ when

$\theta = -\frac{\pi}{4}$ and $\frac{\pi}{4}$.

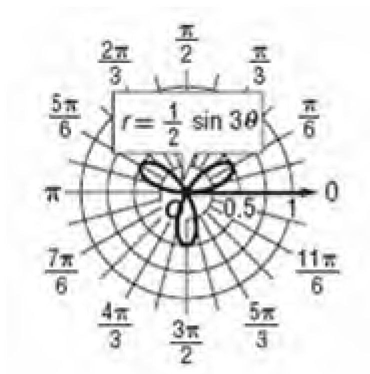
Since the function is symmetric with respect to the line $\theta = \frac{\pi}{2}$, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = \frac{1}{2}\sin 3\theta$
$-\frac{\pi}{2}$	0.5
$-\frac{5\pi}{12}$	0.4
$-\frac{\pi}{3}$	0
$-\frac{\pi}{4}$	−0.4
$-\frac{\pi}{6}$	−0.5
$-\frac{\pi}{12}$	−0.4
0	0
$\frac{\pi}{12}$	0.4
$\frac{\pi}{6}$	0.5
$\frac{\pi}{4}$	0.4
$\frac{\pi}{3}$	0
$\frac{5\pi}{12}$	−0.4
$\frac{\pi}{2}$	−0.5

Use these and a few additional points to sketch the graph of the function.



ANSWER:



22. $r = 4\cos 5\theta$

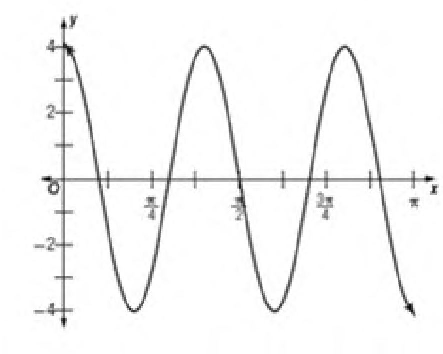
SOLUTION:

Because the polar equation is a function of the cosine function, it is symmetric with respect to the polar axis.

Sketch the graph of the rectangular function $y = 4\cos 5x$ on the interval $[0, \pi]$. From the graph, you can see that $|y| = 4$ when

$x = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5},$ and π and $y = 0$ when

$x = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10},$ and $\frac{9\pi}{10}$.

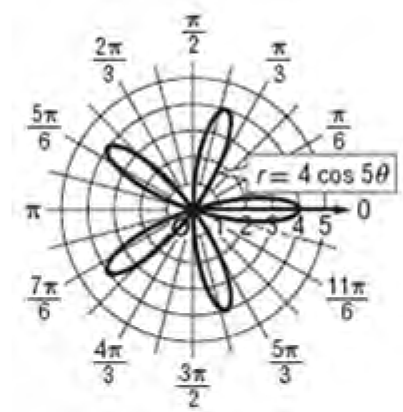


Interpreting these results in terms of the polar equation $r = 4 \cos 5\theta$, we can say that $|r|$ has a maximum value of 4 when $\theta = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \text{and } \pi$ and $r = 0$ when $\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \text{and } \frac{9\pi}{10}$.

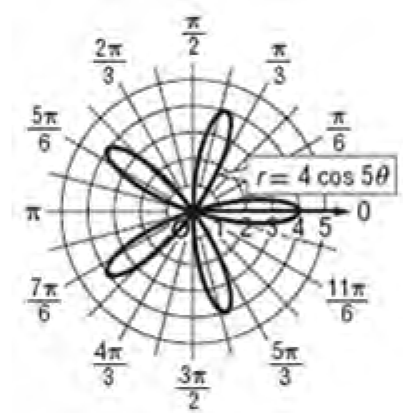
Since the function is symmetric with respect to the polar axis, make a table and calculate the values of r on $[0, \pi]$.

θ	$r = 4 \cos 5\theta$
0	4
$\frac{\pi}{10}$	0
$\frac{\pi}{5}$	–4
$\frac{3\pi}{10}$	0
$\frac{2\pi}{5}$	4
$\frac{\pi}{2}$	0
$\frac{3\pi}{5}$	–4
$\frac{7\pi}{10}$	0
$\frac{4\pi}{5}$	4
$\frac{9\pi}{10}$	0
π	–4

Use these and a few additional points to sketch the graph of the function.



ANSWER:

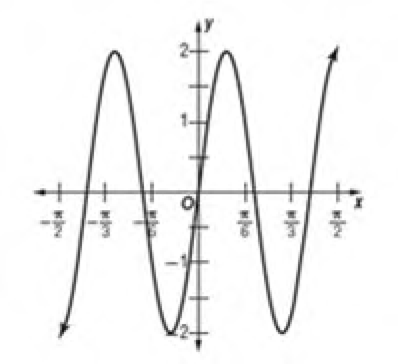


23. $r = 2 \sin 5\theta$

SOLUTION:

Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

Sketch the graph of the rectangular function $y = 2 \sin 5x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. From the graph, you can see that $|y| = 2$ when $x = -\frac{\pi}{2}, -\frac{3\pi}{10}, -\frac{\pi}{10}, \frac{\pi}{10}, \frac{3\pi}{10} \text{ and } \frac{\pi}{2}$ and $y = 0$ when $x = -\frac{2\pi}{5}, -\frac{\pi}{5}, 0, \frac{\pi}{5} \text{ and } \frac{2\pi}{5}$.

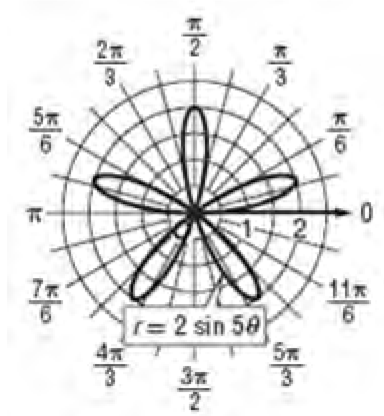


Interpreting these results in terms of the polar equation $r = 2 \sin 5\theta$, we can say that $|r|$ has a maximum value of 2 when $\theta = -\frac{\pi}{2}, -\frac{3\pi}{10}, -\frac{\pi}{10}, \frac{\pi}{10}, \frac{3\pi}{10}$ and $\frac{\pi}{2}$ and $r = 0$ when $\theta = -\frac{2\pi}{5}, -\frac{\pi}{5}, 0, \frac{\pi}{5}$ and $\frac{2\pi}{5}$.

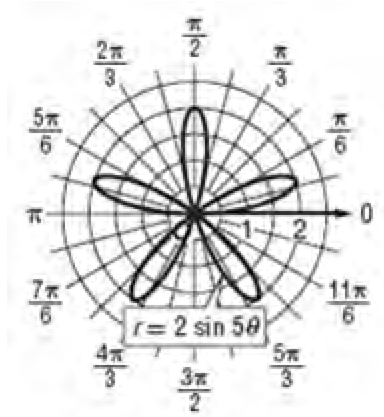
Since the function is symmetric with respect to the polar axis, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = 2 \sin 5\theta$
$-\frac{\pi}{2}$	−2
$-\frac{2\pi}{5}$	0
$-\frac{3\pi}{10}$	2
$-\frac{\pi}{5}$	0
$-\frac{\pi}{10}$	−2
0	0
$\frac{\pi}{10}$	2
$\frac{\pi}{5}$	0
$\frac{3\pi}{10}$	−2
$\frac{2\pi}{5}$	0
$\frac{\pi}{2}$	2

Use these and a few additional points to sketch the graph of the function.



ANSWER:

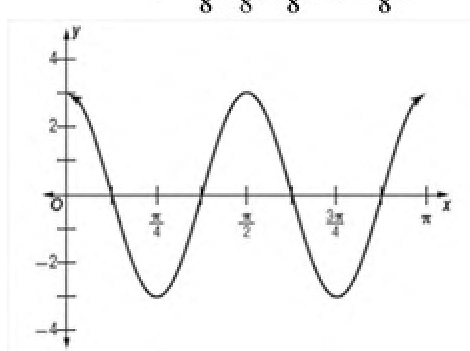


24. $r = 3 \cos 4\theta$

SOLUTION:

Because the polar equation is a function of the cosine function, it is symmetric with respect to the polar axis.

Sketch the graph of the rectangular function $y = 3 \cos 4x$ on the interval $[0, \pi]$. From the graph, you can see that $|y| = 3$ when $x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ and π and $y = 0$ when $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}$, and $\frac{7\pi}{8}$.

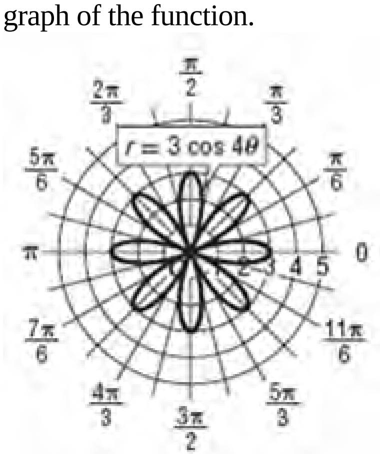


Interpreting these results in terms of the polar equation $r = 3 \cos 4\theta$, we can say that $|r|$ has a maximum value of 3 when $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ and π and $r = 0$ when $\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}$, and $\frac{7\pi}{8}$.

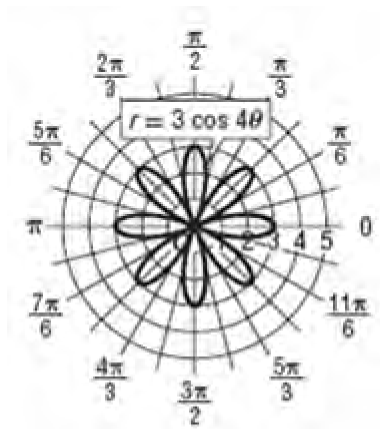
Since the function is symmetric with respect to the polar axis, make a table and calculate the values of r on $[0, \pi]$.

θ	$r = 3 \cos 4\theta$
0	3
$\frac{\pi}{12}$	1.5
$\frac{\pi}{6}$	−1.5
$\frac{\pi}{4}$	−3
$\frac{\pi}{3}$	−1.5
$\frac{5\pi}{12}$	1.5
$\frac{\pi}{2}$	3
$\frac{7\pi}{12}$	1.5
$\frac{2\pi}{3}$	−1.5
$\frac{3\pi}{4}$	−3
$\frac{5\pi}{6}$	−1.5
$\frac{11\pi}{12}$	1.5
π	3

Use these and a few additional points to sketch the graph of the function.



ANSWER:



25. **MARINE BIOLOGY** Rose curves can be observed in marine wildlife. Determine the symmetry, zeros, and maximum r-values of each function modeling a marine species for $0 \le \theta \le \pi$. Then use the information to graph the function.
a. The pores forming the petal pattern of a sand dollar (Refer to Figure 9.2.3 on Page 548) can be modeled by
$r = 3 \cos 5\theta$.
b. The outline of the body of a crown-of-thorns sea star (Refer to Figure 9.2.4 on Page 548) can be modeled by
$r = 20 \cos 8\theta$.

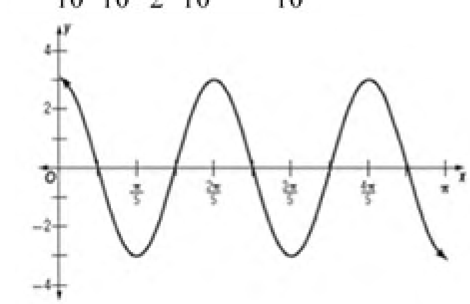
SOLUTION:

a. Because the polar equation is a function of the cosine function, it is symmetric with respect to the polar axis.

Sketch the graph of the rectangular function $y = 3 \cos 5x$ on the interval $[0, \pi]$. From the graph, you can see that $|y| = 3$ when

$x = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$ and π, and $y = 0$ when

$x = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}$, and $\frac{9\pi}{10}$.

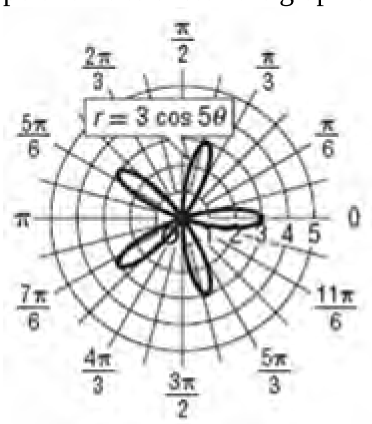


Interpreting these results in terms of the polar equation $r = 3 \cos 5\theta$, we can say that $|r|$ has a maximum value of 3 when $\theta = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$ and π and $r = 0$ when $\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \text{or } \frac{9\pi}{10}$.

Since the function is symmetric with respect to the polar axis, make a table and calculate a few additional values of r on $\left[0, \frac{\pi}{2}\right]$.

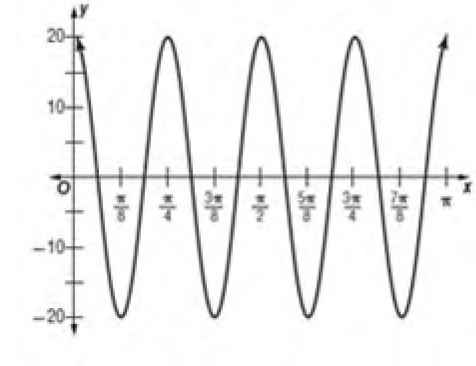
θ	$r = 3 \cos 5\theta$
0	3
$\frac{\pi}{12}$	0.8
$\frac{\pi}{6}$	−2.6
$\frac{\pi}{4}$	−2.1
$\frac{\pi}{3}$	1.5
$\frac{5\pi}{12}$	2.9
$\frac{\pi}{2}$	0

Use these points and symmetry with respect to the polar axis to sketch the graph of the function.



b. Because the polar equation is a function of the cosine function, it is symmetric with respect to the polar axis.

Sketch the graph of the rectangular function $y = 20 \cos 8x$ on the interval $[0, \pi]$. From the graph, you can see that $|y| = 20$ when $x = 0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{8}$, and π, and $y = 0$ when $x = \frac{\pi}{16}, \frac{3\pi}{16}, \frac{5\pi}{16}, \frac{7\pi}{16}, \frac{9\pi}{16}, \frac{11\pi}{16}, \frac{13\pi}{16}$, and $\frac{15\pi}{16}$.

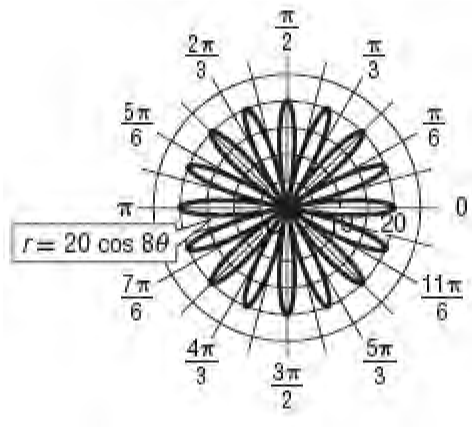


Interpreting these results in terms of the polar equation $r = 20 \cos 8\theta$, we can say that $|r|$ has a maximum value of 20 when $\theta = 0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{8}$, and π, and $r = 0$ when $\theta = \frac{\pi}{16}, \frac{3\pi}{16}, \frac{5\pi}{16}, \frac{7\pi}{16}, \frac{9\pi}{16}, \frac{11\pi}{16}, \frac{13\pi}{16}$, and $\frac{15\pi}{16}$

Make a table and calculate a few additional values of r on $\left[0, \frac{\pi}{2}\right]$.

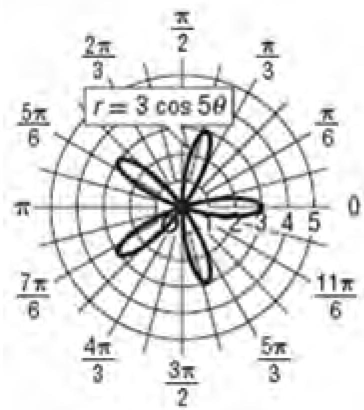
θ	$r = 20 \cos 8\theta$
0	20
$\frac{\pi}{12}$	−10
$\frac{\pi}{6}$	−10
$\frac{\pi}{4}$	20
$\frac{\pi}{3}$	−10
$\frac{5\pi}{12}$	−10
$\frac{\pi}{2}$	20

Use these points and symmetry with respect to the polar axis, line $\theta = \frac{\pi}{2}$, and pole to sketch the graph of the function.

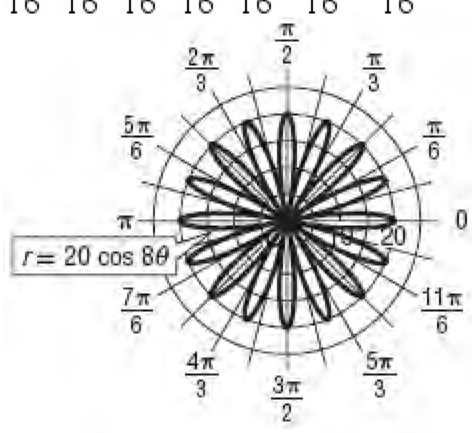


ANSWER:

a. symmetry with respect to the polar axis; $|r| = 3$ when $\theta = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$, and π; zeros for r when $\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}$, and $\frac{9\pi}{10}$



b. symmetry with respect to the polar axis, the line $\theta = \frac{\pi}{2}$, and the pole; $|r| = 20$ when $\theta = 0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{8}$, and π; zeros for r when $\theta = \frac{\pi}{16}, \frac{3\pi}{16}, \frac{5\pi}{16}, \frac{7\pi}{16}, \frac{9\pi}{16}, \frac{11\pi}{16}, \frac{13\pi}{16}$, and $\frac{15\pi}{16}$



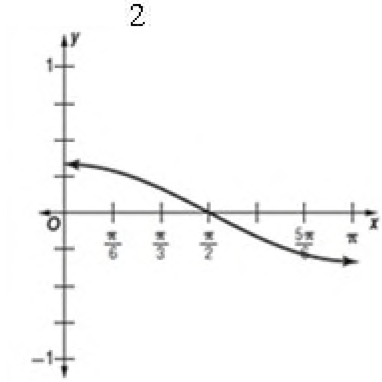
Identify the type of curve given by each equation. Then use symmetry, zeros, and maximum *r*-values to graph the function.

26. $r = \frac{1}{3}\cos\theta$

SOLUTION:

The equation is of the form $r = a\cos\theta$, so its graph is a circle. Because this polar equation is a function of the cosine function, it is symmetric with respect to the polar axis.

Sketch the graph of the rectangular function $y = \frac{1}{3}\cos x$ on the interval $[0, \pi]$. From the graph, you can see that $|y| = \frac{1}{3}$ when $x = 0$ and π and $y = 0$ when $x = \frac{\pi}{2}$.



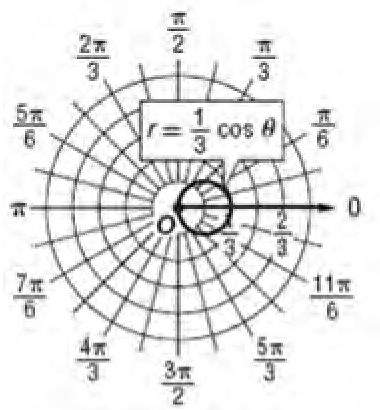
Interpreting these results in terms of the polar equation $r = \frac{1}{3}\cos\theta$, we can say that $|r|$ has a maximum value of $\frac{1}{3}$ when $\theta = 0$ and π and $r = 0$ when $\theta = \frac{\pi}{2}$

Since the function is symmetric with respect to the polar axis, make a table and calculate the values of r on $[0, \pi]$.

θ	$r = \frac{1}{3}\cos\theta$
0	0.33
$\frac{\pi}{6}$	0.29
$\frac{\pi}{4}$	0.24
$\frac{\pi}{3}$	0.17

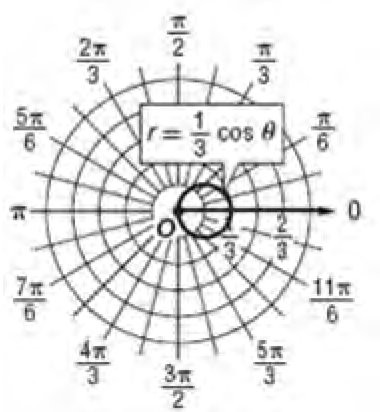
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	−0.17
$\frac{3\pi}{4}$	−0.24
$\frac{5\pi}{6}$	−0.29
π	−0.3

Use these and a few additional points to sketch the graph of the function.



ANSWER:

circle;



27. $r = 4\theta + 1$; $\theta > 0$

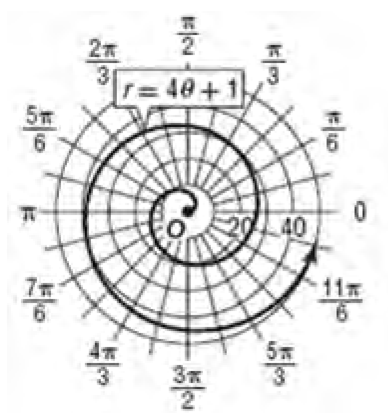
SOLUTION:

The equation is of the form $r = a\theta + b$, so its graph is a spiral of Archimedes. The function has no symmetry.
Spirals are unbounded. Therefore, the function has no maximum *r*-values and only one zero. To find the zero, substitute $r = 0$ into the function and solve for θ.

$$r = 4\theta + 1$$
$$0 = 4\theta + 1$$
$$-1 = 4\theta$$
$$-\frac{1}{4} = \theta$$

So, $r = 0$ when $\theta = -\frac{1}{4}$, which is not in the given domain.
Use points on the interval [0, 4π] to sketch the graph of the function.

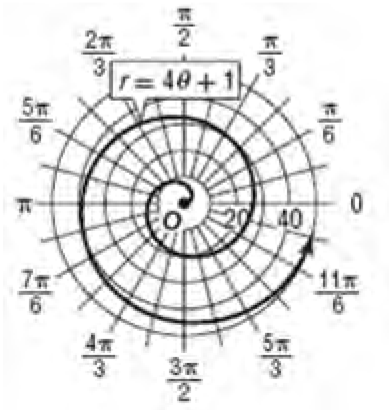
θ	$r = 4\theta + 1$
0	1
$\frac{\pi}{4}$	4.1
$\frac{\pi}{2}$	7.3
π	13.6
$\frac{3\pi}{2}$	19.9
2π	26.1
3π	38.7
4π	51.3



ANSWER:

spiral of Archimedes;

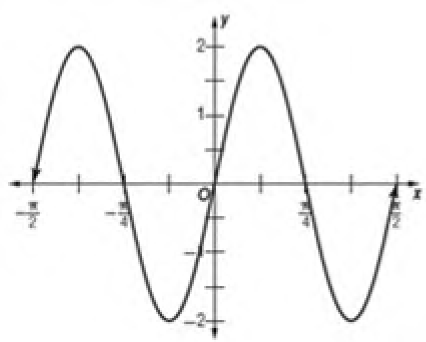
9-2 Graphs of Polar Equations



28. $r = 2 \sin 4\theta$

SOLUTION:

The equation is of the form $r = a \sin n\theta$, so its graph is a rose. Because this polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$. Sketch the graph of the rectangular function $y = 2 \sin 4x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. From the graph, you can see that $|y| = 2$ when $x = -\frac{3\pi}{8}, -\frac{\pi}{8}, \frac{\pi}{8}$, and $\frac{3\pi}{8}$ and $y = 0$ when $x = -\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}$, and $\frac{\pi}{2}$

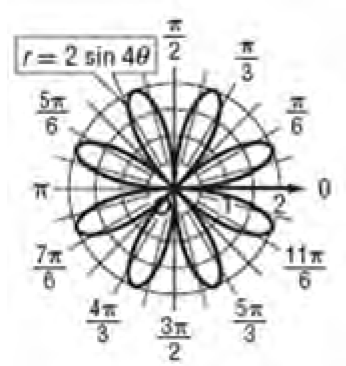


Interpreting these results in terms of the polar equation $r = 2 \sin 4\theta$, we can say that $|r|$ has a maximum value of 2 when $\theta = -\frac{3\pi}{8}, -\frac{\pi}{8}, \frac{\pi}{8}$, and $\frac{3\pi}{8}$ and $r = 0$ when $\theta = -\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}$, and $\frac{\pi}{2}$

Since the function is symmetric with respect to the polar axis, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = 2 \sin 4\theta$
$-\frac{\pi}{2}$	0
$-\frac{5\pi}{12}$	1.7
$-\frac{\pi}{3}$	1.7
$-\frac{\pi}{4}$	0
$-\frac{\pi}{6}$	−1.7
$-\frac{\pi}{12}$	−1.7
0	0
$\frac{\pi}{12}$	1.7
$\frac{\pi}{6}$	1.7
$\frac{\pi}{4}$	0
$\frac{\pi}{3}$	−1.7
$\frac{5\pi}{12}$	−1.7
$\frac{\pi}{2}$	0

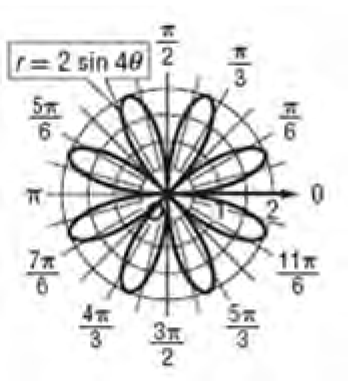
Use these and a few additional points to sketch the graph of the function.



ANSWER:

rose;

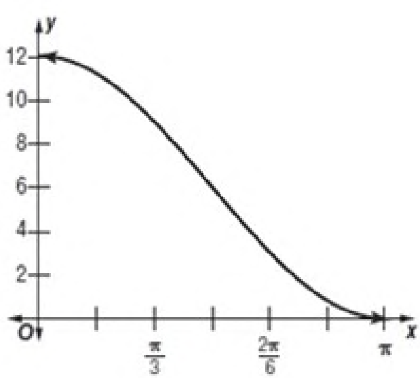
9-2 Graphs of Polar Equations



29. $r = 6 + 6 \cos \theta$

SOLUTION:

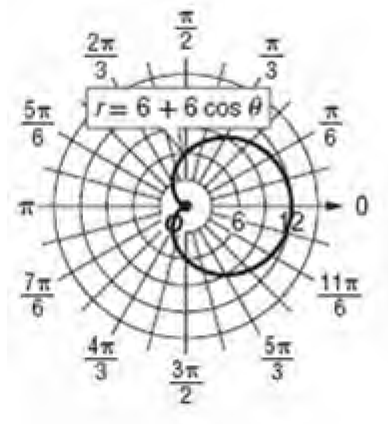
The equation is of the form $r = a + b \cos \theta$ and $a = b$, so its graph is a cardioid. Because this polar equation is a function of the cosine function, it is symmetric with respect to the polar axis.
Sketch the graph of the rectangular function $y = 6 + 6 \cos x$ on the interval $[0, \pi]$. From the graph, you can see that $|y| = 12$ when $x = 0$ and $y = 0$ when $x = \pi$.



Interpreting these results in terms of the polar equation $r = 6 + 6 \cos \theta$, we can say that $|r|$ has a maximum value of 12 when $\theta = 0$ and $r = 0$ when $\theta = \pi$.
Since the function is symmetric with respect to the polar axis, make a table and calculate the values of r on $[0, \pi]$.

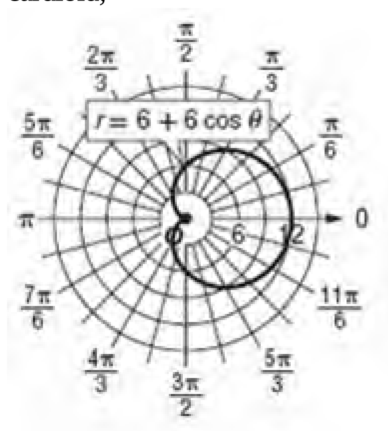
θ	$r = 6 + 6 \cos \theta$
0	12
$\frac{\pi}{6}$	11.2
$\frac{\pi}{3}$	9
$\frac{\pi}{2}$	6
$\frac{2\pi}{3}$	3
$\frac{5\pi}{6}$	0.8
π	0

Use these and a few additional points to sketch the graph of the function.



ANSWER:

cardioid;



30. $r^2 = 4 \cos 2\theta$

SOLUTION:

The equation is of the form $r^2 = a^2 \cos 2\theta$, so its graph is a lemniscate.

Replacing (r, θ) with $(r, -\theta)$ yields $r^2 = 4 \cos 2(-\theta)$. Since cosine is an even function, $r^2 = 4 \cos 2(-\theta)$ can be written as $r^2 = 4 \cos 2\theta$, and the function has symmetry with respect to the polar axis.

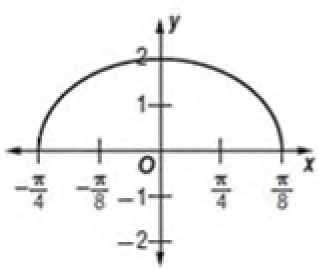
Replacing (r, θ) with $(-r, -\theta)$ yields $(-r)^2 = 4 \cos$

$2(-\theta)$ or $r^2 = 4\cos 2(-\theta)$. Again, since cosine is an even function, $r^2 = 4\cos 2(-\theta)$ can be written as $r^2 = 4\cos 2\theta$, and the function has symmetry with respect to the line $\theta = \frac{\pi}{2}$.

Finally, replacing (r, θ) with $(-r, \theta)$ yields $(-r)^2 = 4\cos 2\theta$ or $r^2 = 4\cos 2\theta$. Therefore, the function has symmetry with respect to the pole.

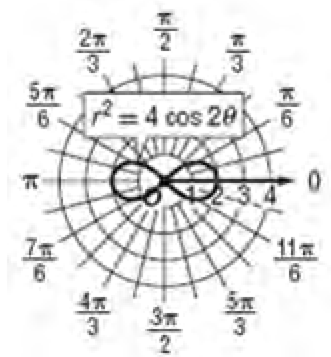
The equation $r^2 = 4\cos 2\theta$ is equivalent to $r = \pm\sqrt{4\cos 2\theta}$, which is undefined when $\cos 2\theta < 0$. Therefore, the domain of the function is restricted to the intervals $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ and $\left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$.

Sketch the graph of the rectangular function $y = \sqrt{4\cos 2x}$ on the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$. From the graph, you can see that $|y| = 2$ when $x = 0$ and $y = 0$ when $x = -\frac{\pi}{4}$ and $\frac{\pi}{4}$.



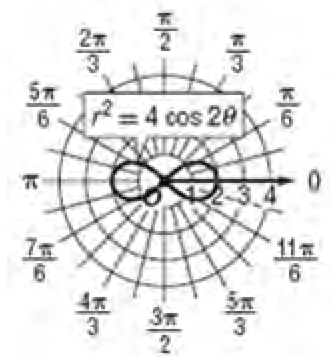
Interpreting these results in terms of the polar equation $r^2 = 4\cos 2\theta$, we can say that $|r|$ has a maximum value of 2 when $\theta = 0$ and $r = 0$ when $\theta = -\frac{\pi}{4}$ and $\frac{\pi}{4}$.
Use these points and the indicated symmetry to sketch the graph of the function.

θ	$r^2 = 4\cos 2\theta$
$-\frac{\pi}{4}$	0
$-\frac{\pi}{6}$	1.4
$-\frac{\pi}{8}$	1.7
0	2
$\frac{\pi}{8}$	1.7
$\frac{\pi}{6}$	1.4
$\frac{\pi}{4}$	0



ANSWER:

lemniscate;



31. $r = 5\theta + 2$; $\theta > 0$

SOLUTION:

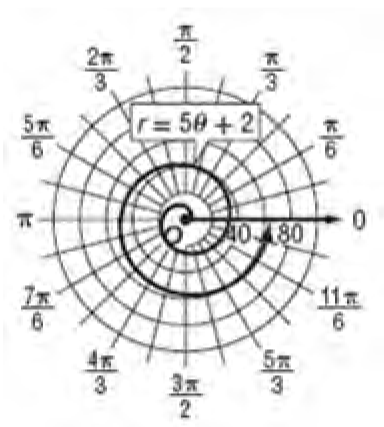
The equation is of the form $r = a\theta + b$, so its graph is a spiral of Archimedes. The function has no symmetry.
Spirals are unbounded. Therefore, the function has no maximum r-values and only one zero. To find the zero, substitute $r = 0$ into the function and solve for θ.

$$r = 5\theta + 2$$
$$0 = 5\theta + 2$$
$$-2 = 5\theta$$
$$-\frac{2}{5} = \theta$$

So, $r = 0$ when $\theta = -\frac{2}{5}$, which is not in the given domain.

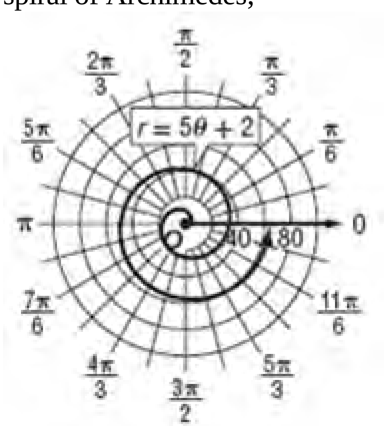
Use points on the interval [0, 4π] to sketch the graph of the function.

θ	$r = 5\theta + 2$
0	2
$\frac{\pi}{4}$	5.9
$\frac{\pi}{2}$	9.9
π	17.7
$\frac{3\pi}{2}$	25.6
2π	33.4
3π	49.2
4π	64.8



ANSWER:

spiral of Archimedes;

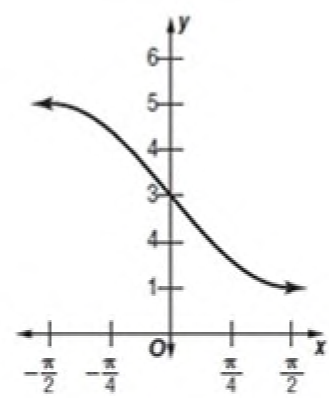


32. $r = 3 - 2 \sin \theta$

SOLUTION:

The equation is of the form $r = a - b \sin \theta$ and $b < a < 2b$, so its graph is a limaçon. More specifically, it is a dimpled limaçon. Because this polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

Sketch the graph of the rectangular function $y = 3 - 2 \sin x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. From the graph, you can see that $|y| = 5$ when $x = -\frac{\pi}{2}$. There are no zeros.



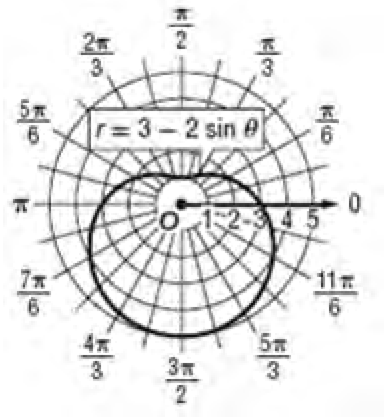
Interpreting these results in terms of the polar equation $r = 3 - 2 \sin \theta$, we can say that $|r|$ has a maximum value of 5 when $\theta = -\frac{\pi}{2}$.

Since the function is symmetric with respect to the polar axis, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = 3 - 2 \sin \theta$
$-\frac{\pi}{2}$	5
$-\frac{5\pi}{12}$	4.9
$-\frac{\pi}{3}$	4.7
$-\frac{\pi}{4}$	4.4
$-\frac{\pi}{6}$	4
$-\frac{\pi}{12}$	3.5
0	3
$\frac{\pi}{12}$	2.5
$\frac{\pi}{6}$	2
$\frac{\pi}{4}$	1.6
$\frac{\pi}{3}$	1.3

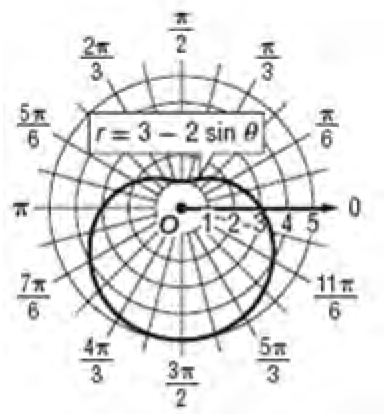
$\frac{5\pi}{12}$	1.1
$\frac{\pi}{2}$	1

Use these and a few additional points to sketch the graph of the function.



ANSWER:

limaçon ;



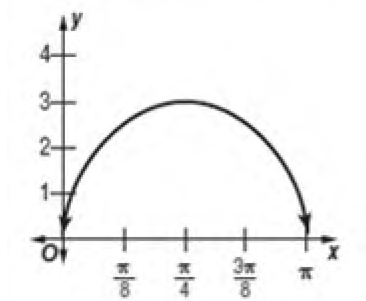
33. $r^2 = 9 \sin 2\theta$

SOLUTION:

The equation is of the form $r^2 = a^2 \sin 2\theta$, so its graph is a lemniscate. Replacing (r, θ) with $(-r, \theta)$ yields $(-r)^2 = 9 \sin 2\theta$ or $r^2 = 9 \sin 2\theta$. Therefore, the function has symmetry with respect to the pole.

The equation $r^2 = 9 \sin 2\theta$ is equivalent to $r = \pm\sqrt{9\sin 2\theta}$, which is undefined when $\sin 2\theta < 0$. Therefore, the domain of the function is restricted to the intervals $\left[0, \frac{\pi}{2}\right]$ and $\left[\pi, \frac{3\pi}{2}\right]$.

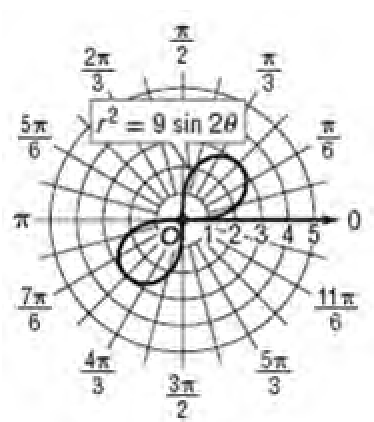
Sketch the graph of the rectangular function $y = \sqrt{9 sin 2x}$ on the interval $\left[0, \frac{\pi}{2}\right]$. From the graph, you can see that $|y| = 3$ when $x = \frac{\pi}{4}$ and $y = 0$ when $x = 0$ and $\frac{\pi}{2}$.



Interpreting these results in terms of the polar equation $r^2 = 9 \sin 2\theta$, we can say that $|r|$ has a maximum value of 3 when $\theta = \frac{\pi}{4}$ and $r = 0$ when $\theta = 0$ and $\frac{\pi}{2}$.

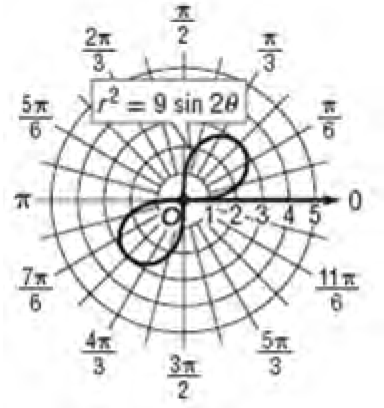
Use these points and the indicated symmetry to sketch the graph of the function.

θ	$r^2 = 9 \sin 2\theta$
0	0
$\frac{\pi}{12}$	2.1
$\frac{\pi}{6}$	2.8
$\frac{\pi}{4}$	3
$\frac{\pi}{3}$	2.8
$\frac{5\pi}{12}$	2.1
$\frac{\pi}{2}$	0



ANSWER:

lemniscate;



34. **FIGURE SKATING** The original focus of figure skating was to carve figures, known as *compulsory figures*, into the ice. The shape of one of these figures can be modeled by
$r^2 = 25 \cos 2\theta$.

a. Which classic curve does the figure model?

b. Graph the model.

SOLUTION:

a. The equation is of the form $r^2 = a^2 \cos 2\theta$, so its graph is a lemniscate.

b. Because this polar equation is a function of the cosine function, it is symmetric with respect to the polar axis.

Replace (r, θ) with $(r, \pi - \theta)$.

$$r^2 = 25\cos[2(\pi - \theta)]$$
$$r^2 = 25\cos(2\pi - 2\theta)$$
$$r^2 = 25(\cos 2\pi \cos 2\theta + \sin 2\pi \sin 2\theta)$$
$$r^2 = 25(1 \cdot \cos 2\theta + 0 \cdot \sin 2\theta)$$
$$r^2 = 25\cos 2\theta$$

Since $(r, \theta) = (r, \pi - \theta)$, $r^2 = 25 \cos 2\theta$ is symmetric with respect to the line $\frac{\pi}{2}$. Now replace (r, θ) with $(r, \pi + \theta)$.

$$r^2 = 25\cos[2(\pi + \theta)]$$
$$r^2 = 25\cos(2\pi + 2\theta)$$
$$r^2 = 25(\cos 2\pi \cos 2\theta - \sin 2\pi \sin 2\theta)$$
$$r^2 = 25(1 \cdot \cos 2\theta - 0 \cdot \sin 2\theta)$$
$$r^2 = 25\cos 2\theta$$

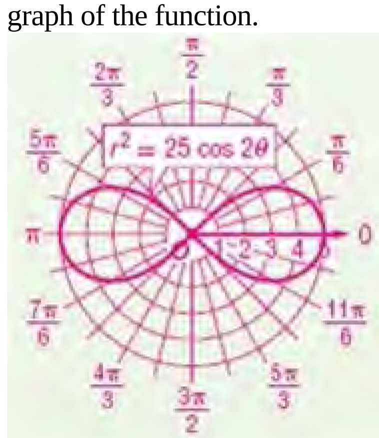
Since $(r, \theta) = (r, \pi + \theta)$, the $r^2 = 25 \cos 2\theta$ is symmetric with respect to the pole.

The equation $r^2 = 25 \cos 2\theta$ is equivalent to $r = \pm\sqrt{25\cos 2\theta}$, which is undefined when $\cos 2\theta < 0$. Therefore, the domain of the function is restricted to the intervals $\left[0, \frac{\pi}{4}\right]$ or $\left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$ or $\left[\frac{7\pi}{4}, 2\pi\right]$. Because you can use pole symmetry, you need only graph points in the interval $\left[0, \frac{\pi}{4}\right]$. The function attains a maximum r-value of $|a|$ or 5 when $\theta = 0$.

Make a table and calculate additional values for r on $\left[0, \frac{\pi}{4}\right]$.

θ	$r^2 = 25 \cos 2\theta$
0	25
$\frac{\pi}{12}$	±4.7
$\frac{\pi}{6}$	±3.5
$\frac{\pi}{4}$	0

Use these points and symmetry with respect to the polar axis, the line $\theta = \frac{\pi}{2}$, and the pole to sketch the graph of the function.

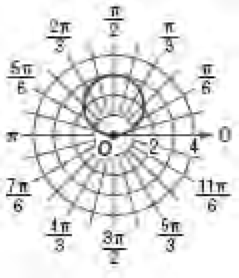


ANSWER:

a. lemniscate

b.

Write an equation for each graph.

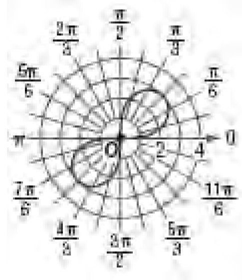
35. 

SOLUTION:

A circle whose center lies along the line $\theta = \frac{\pi}{2}$ and has one point at the pole has an equation of the form $r = a \sin \theta$, where $|a|$ is the length of the circle's diameter. If the circle is above the line $\theta = 0$, $a > 0$. The diameter of the circle is 3 units and the circle lies above the line $\theta = 0$, so $a = 3$. Therefore, an equation for this graph is $r = 3 \sin \theta$.

ANSWER:

$r = 3 \sin \theta$

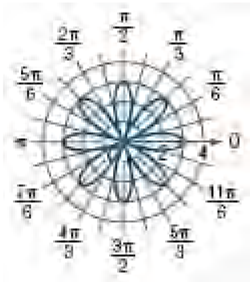
36. 

SOLUTION:

This is a lemniscates that is symmetric with respect to the pole and has a maximum r-value of 3. Lemniscates that are symmetric with respect to the pole have the form $r^2 = a^2 \sin 2\theta$, where a is the maximum r-value. Therefore, and equation for this graph is $r^2 = 3^2 \sin 2\theta$ or $r^2 = 9 \sin 2\theta$.

ANSWER:

$r^2 = 9 \sin 2\theta$

37. 

SOLUTION:

This is a rose curve with an even number of petals, 8, that is symmetric with respect to the line $\theta = \frac{\pi}{2}$ with petals 3 units long. This rose curve is symmetric with both the line $\theta = \frac{\pi}{2}$ and the pole, so it can have the form $r = a \sin n\theta$, where $n \geq 2$ is an integer.

The rose has 8 petals, so $n = 4$.
The length of each petal is $|a|$ so $|a| = 3$.

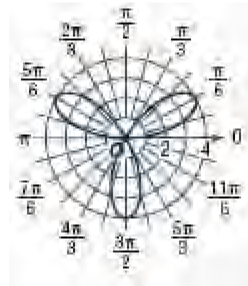
Use the fact that when $\theta = \frac{\pi}{4}$ on the graph $r = 3$ to determine if $a = -3$ or $a = 3$ and whether the equation is of the form $r = a \sin n\theta$ or $r = a \cos n\theta$.

$r = a\sin n\theta$	$r = a\cos n\theta$
$3 = a\sin\left(4 \cdot \frac{\pi}{2}\right)$	$3 = a\cos\left(4 \cdot \frac{\pi}{2}\right)$
$3 = a\sin 2\pi$	$3 = a\cos 2\pi$
$3 = a(0)$	$3 = a(1)$
$3 \neq 0$	$3 = a$

Since $\left(\frac{\pi}{2}, 3\right)$ is the solution of the equation $r = a \cos n\theta$, an equation for this graph is $r = 3 \cos 4\theta$.

ANSWER:

$r = 3 \cos 4\theta$

38. 

SOLUTION:

This is a rose curve with an odd number of petals, 3, that is symmetric with respect to the line $\theta = \frac{\pi}{2}$ with petals 4 units long.
A rose curve that is symmetric with respect to the line $\theta = \frac{\pi}{2}$ has the form $r = a \sin n\theta$ where $n \geq 2$ is an integer. The rose has n petals if n is odd, so $n = 3$ and the length of each petal is $|a|$ so $|a| = 4$.

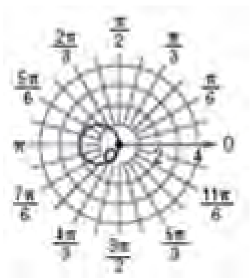
Use the fact that when $\theta = \frac{3\pi}{2}$ on the graph $r = 4$ to determine if $a = -4$ or $a = 4$.

$$
\begin{aligned}
r &= a\sin n\theta \\
4 &= a\sin\left(3 \cdot \frac{\pi}{2}\right) \\
4 &= a\sin\left(\frac{3\pi}{2}\right) \\
4 &= a(-1) \\
-4 &= a
\end{aligned}
$$

Therefore, an equation for this graph is $r = 4 \sin 3\theta$.

ANSWER:

$r = 4 \sin 3\theta$

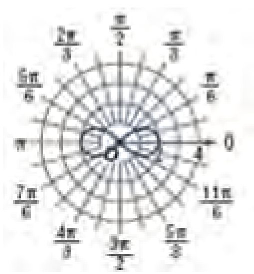
39. 

SOLUTION:

The circle is to the left of the line $\theta = \frac{\pi}{2}$, so $a < 0$. The graph is of a circle with one point on the circle at the pole and center along the line $\theta = 0$, so it's graph has the form $r = a \cos \theta$. The diameter of the circle is 2 units and the circle lies to the left the line $\theta = \frac{\pi}{2}$, so $a = -2$. Therefore, an equation for this graph is $r = -2 \cos \theta$.

ANSWER:

$r = -2 \cos \theta$

40. 

SOLUTION:

This is a lemniscates that is symmetric with respect to the the line $\theta = \frac{\pi}{2}$ and has a maximum r-value of 2. Lemniscates that are symmetric with respect to the line $\theta = \frac{\pi}{2}$ have the form $r^2 = a^2 \cos 2\theta$, where a is the maximum r-value. Therefore, and equation for this graph is $r^2 = 2^2 \cos 2\theta$ or $r^2 = 4 \cos 2\theta$.

ANSWER:

$r^2 = 4 \cos 2\theta$

41. **FAN** A ceiling fan has a central motor with five blades that each extend 4 units from the center. The shape of the fan can be represented by a rose curve.
a. Write two polar equations that can be used to represent the fan.

b. Sketch two graphs of the fan using the equations that you wrote.

SOLUTION:

a. A rose curve has the form $r = a \cos n\theta$ or $r = a \sin n\theta$, where $n \geq 2$ is an integer. For a rose with 5 petals (blades), $n = 5$. For a rose with petals that are 4 units long, $a = 4$. Therefore polar equations that can be used to represent this fan are $r = 4 \cos 5\theta$ and $r = 4 \sin 5\theta$.

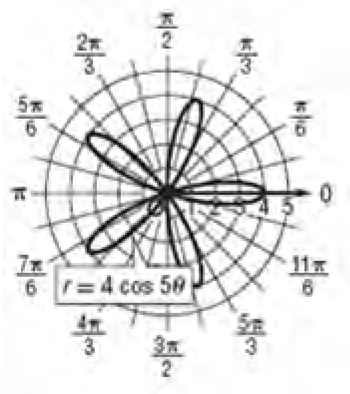
b. Because $r = 4 \cos 5\theta$ is a function of the cosine function, it is symmetric with respect to the polar axis. For the graph of $y = 4 \cos 5\theta$ on $[0, \pi]$, $|y| = 4$ when $x = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$ and π, and $y = 0$ when $x = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}$, and $\frac{9\pi}{10}$. Interpreting these results in terms of the polar equation $r = 4 \cos 5\theta$, we can say that $|r|$ has a maximum value of 4 when $\theta = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$ and π and $r = 0$ when $\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}$, or $\frac{9\pi}{10}$.

Since the function is symmetric with respect to the polar axis, make a table and calculate a few additional values of r on $\left[0, \frac{\pi}{2}\right]$.

θ	$r = 4 \cos 5\theta$
0	4
$\frac{\pi}{12}$	1.1
$\frac{\pi}{6}$	−3.5
$\frac{\pi}{4}$	−2.8
$\frac{\pi}{3}$	2
$\frac{5\pi}{12}$	3.9
$\frac{\pi}{2}$	0

Use these points and symmetry to sketch the graph of the function.

9-2 Graphs of Polar Equations

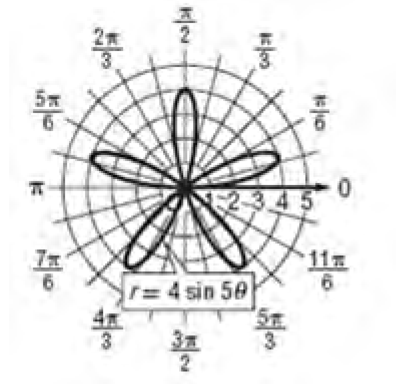


Because $r = 4 \sin 5\theta$ is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$. For the graph of $y = 4 \sin 5\theta$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $|y| = 4$ when $x = -\frac{\pi}{2}, -\frac{3\pi}{10}, -\frac{\pi}{10}, \frac{\pi}{10}, \frac{3\pi}{10}$, and $\frac{\pi}{2}$, and $y = 0$ when $x = -\frac{2\pi}{5}, -\frac{\pi}{5}, 0, \frac{\pi}{5}$, and $\frac{2\pi}{5}$. Interpreting these results in terms of the polar equation $r = 4 \sin 5\theta$, we can say that $|r|$ has a maximum value of 4 when $\theta = -\frac{\pi}{2}, -\frac{3\pi}{10}, -\frac{\pi}{10}, \frac{\pi}{10}, \frac{3\pi}{10}$, or $\frac{\pi}{2}$, and $r = 0$ when $\theta = 0 - \frac{2\pi}{5}, -\frac{\pi}{5}, 0, \frac{\pi}{5}$, or $\frac{2\pi}{5}$.

Since the function is symmetric with respect to the line $\theta = \frac{\pi}{2}$, make a table and calculate a few additional values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

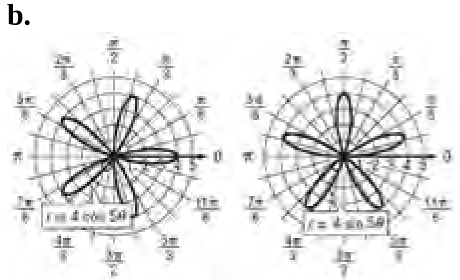
θ	$r = 4 \sin 5\theta$
$-\frac{\pi}{2}$	4
$-\frac{\pi}{3}$	−3.5
$-\frac{\pi}{4}$	−2.8
$-\frac{\pi}{6}$	2
$-\frac{\pi}{12}$	3.9
0	0
$\frac{\pi}{12}$	3.9
$\frac{\pi}{6}$	2
$\frac{\pi}{4}$	−2.8
$\frac{\pi}{3}$	−3.5
$\frac{\pi}{2}$	4

Use these points and symmetry to sketch the graph of the function.



ANSWER:

a. $r = 4 \cos 5\theta$; $r = 4 \sin 5\theta$
b.



9-2 Graphs of Polar Equations

Use one of the three tests to prove the specified symmetry.

42. $r = 3 + \sin\theta$, symmetric about the line $\theta = \frac{\pi}{2}$

SOLUTION:

Substitute $(r, \pi - \theta)$ for (r, θ).

$r = 3 + \sin\theta$	Original equation
$r = 3 + \sin(\pi - \theta)$	Substitute $(r, \pi - \theta)$ for (r, θ).
$r = 3 + \sin\pi\cos\theta - \cos\pi\sin\theta$	Sine Difference Identity
$r = 3 + [(0)\cos\theta - (-1)\sin\theta]$	$\sin\pi = 0$ and $\cos\pi = -1$
$r = 3 + \sin\theta$	Simplify.

Since this substitution produces an equivalent equation, $r = 3 + \sin\theta$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

ANSWER:

Substitute $(r, \pi - \theta)$ for (r, θ).

$$r = 3 + \sin\theta$$
$$r = 3 + \sin(\pi - \theta)$$
$$r = 3 + \sin\pi\cos\theta - \cos\pi\sin\theta$$
$$r = 3 + [(0)\cos\theta - (-1)\sin\theta]$$
$$r = 3 + \sin\theta$$

Since this substitution produces an equivalent equation, $r = 3 + \sin\theta$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

43. $r^2 = 4\sin 2\theta$, symmetric about the pole

SOLUTION:

Substitute $(-r, \theta)$ for (r, θ).

$r^2 = 4\sin 2\theta$	Original equation
$(-r)^2 = 4\sin 2\theta$	Substitute $(-r, \theta)$ for (r, θ).
$r = 4\sin 2\theta$	Simplify.

Since this substitution produces an equivalent equation, $r^2 = 4\sin 2\theta$ is symmetric with respect to the pole.

ANSWER:

Substitute $(-r, \theta)$ for (r, θ).

$$r^2 = 4\sin 2\theta$$
$$(-r)^2 = 4\sin 2\theta$$
$$r = 4\sin 2\theta$$

Since this substitution produces an equivalent equation, $r^2 = 4\sin 2\theta$ is symmetric with respect to the pole.

9-2 Graphs of Polar Equations

44. $r = 3 \sin 2\theta$, symmetric about the polar axis

SOLUTION:

Substitute $(-r, \pi - \theta)$ for (r, θ).

$r = 3\sin 2\theta$	Original equation
$-r = 3\sin 2(\pi - \theta)$	Substitute $(-r, \pi - \theta)$ for (r, θ).
$-r = 3\sin(2\pi - 2\theta)$	Distribute.
$-r = 3(\sin 2\pi\cos 2\theta - \cos 2\pi\sin 2\theta)$	Sine Difference Identity
$-r = 3[(0)\cos 2\theta - (1)\sin 2\theta]$	$\sin 2\pi = 0$ and $\cos 2\pi = 1$
$-r = 3(-\sin 2\theta)$	Simplify.
$r = 3\sin 2\theta$	Divide each side by -1.

Since this substitution produces an equivalent equation, $r = 3 \sin 2\theta$ is symmetric with respect to the polar axis.

ANSWER:

Substitute $(-r, \pi - \theta)$ for (r, θ).

$$r = 3\sin 2\theta$$
$$-r = 3\sin 2(\pi - \theta)$$
$$-r = 3\sin(2\pi - 2\theta)$$
$$-r = 3(\sin 2\pi\cos 2\theta - \cos 2\pi\sin 2\theta)$$
$$-r = 3[(0)\cos 2\theta - (1)\sin 2\theta]$$
$$-r = 3(-\sin 2\theta)$$
$$r = 3\sin 2\theta$$

Since this substitution produces an equivalent equation, $r = 3 \sin 2\theta$ is symmetric with respect to the polar axis.

45. $r = 5 \cos 8\theta$, symmetric about the line $\theta = \frac{\pi}{2}$

SOLUTION:

Substitute $(r, \pi - \theta)$ for (r, θ).

$r = 5\cos 8\theta$	Original equation
$r = 5\cos 8(\pi - \theta)$	Substitute $(r, \pi - \theta)$ for (r, θ).
$r = 5\cos(8\pi - 8\theta)$	Cosine Difference Identity
$r = 5(\cos 8\pi\cos 8\theta + \sin 8\pi\sin 8\theta)$	Distribute.
$r = 5[(1)\cos 8\theta - (0)\sin 8\theta]$	$\cos 8\pi = 1$ and $\sin 8\pi = 0$
$r = 5\cos 8\theta$	Simplify.

Since this substitution produces an equivalent equation, $r = 5 \cos 8\theta$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

ANSWER:

Substitute $(r, \pi - \theta)$ for (r, θ).

$$r = 5\cos 8\theta$$
$$r = 5\cos 8(\pi - \theta)$$
$$r = 5\cos(8\pi - 8\theta)$$
$$r = 5(\cos 8\pi\cos 8\theta + \sin 8\pi\sin 8\theta)$$
$$r = 5[(1)\cos 8\theta - (0)\sin 8\theta]$$
$$r = 5\cos 8\theta$$

Since this substitution produces an equivalent equation, $r = 5 \cos 8\theta$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

46. $r = 2 \sin 4\theta$, symmetric about the pole

SOLUTION:

Substitute $(r, \pi + \theta)$ for (r, θ).

$r = 2\sin 4\theta$	Original equation
$r = 2\sin 4(\pi + \theta)$	Substitute $(r, \pi + \theta)$ for (r, θ)
$r = 2\sin(4\pi + 4\theta)$	Distribute.
$r = 2(\sin 4\pi \cos 4\theta + \cos 4\pi \sin 4\theta)$	Sine Sum Identity
$r = 2[(0)\cos 4\theta + (1)\sin 4\theta]$	$\cos 4\pi = 0$ and $\sin 4\pi = 1$
$r = 2\sin 4\theta$	Simplify.

Since this substitution produces an equivalent equation, $r = 2 \sin 4\theta$ is symmetric with respect to the pole.

ANSWER:

Substitute $(r, \pi + \theta)$ for (r, θ).

$$r = 2\sin 4\theta$$
$$r = 2\sin 4(\pi + \theta)$$
$$r = 2\sin(4\pi + 4\theta)$$
$$r = 2(\sin 4\pi \cos 4\theta + \cos 4\pi \sin 4\theta)$$
$$r = 2[(0)\cos 4\theta + (1)\sin 4\theta]$$
$$r = 2\sin 4\theta$$

Since this substitution produces an equivalent equation, $r = 2 \sin 4\theta$ is symmetric with respect to the pole.

47. **FOUR-LEAF CLOVER** The shape of a certain type of clover can be represented using a rose curve. Write a polar equation for the clover if it has:
a. 5 petals with a length of 2 units each.
b. 4 petals with a length of 7 units each.
c. 8 petals with a length of 6 units each.

SOLUTION:

A rose curve has the form $r = a \cos n\theta$ or $r = a \sin n\theta$, where $n \geq 2$ is an integer. It has n petals if n is odd and $2n$ petals if n is even. The length of each petal is a units.
a. For a 5-petal clover, each 2 units in length, $n = 5$ and $a = 2$. Therefore a polar equation for the colver is $r = 2 \sin 5\theta$ or $r = 2 \cos 5\theta$.

b. For a 4-petal clover, each 7 units in length, $2n = 4$ so $n = 2$ and $a = 7$. Therefore a polar equation for the colver is $r = 7 \sin 2\theta$ or $r = 7 \cos 2\theta$.

c. For an 8-petal clover, each 6 units in length, $2n = 8$ so $n = 4$ and $a = 6$. Therefore a polar equation for the colver is $r = 6 \sin 4\theta$ or $r = 6 \cos 4\theta$.

ANSWER:

a. $r = 2 \sin 5\theta$ or $r = 2 \cos 5\theta$
b. $r = 7 \sin 2\theta$ or $r = 7 \cos 2\theta$
c. $r = 6 \sin 4\theta$ or $r = 6 \cos 4\theta$

48. **CONCERT** For a concert, a circular stage is constructed and placed in the center so fans can completely surround the musicians. To record the sound of the crowd, two directional microphones are placed next to each other on the stage, one facing due east and the other facing due west. The patterns of the microphones can be represented by the polar equations
$r = 2.5 + 2.5 \cos \theta$ and $r = -2.5 - 2.5 \cos \theta$.
a. Identify the type of curve given by each polar equation.
b. Sketch the graph of each microphone pattern on the same polar grid.
c. Describe what the graph tells you about the area covered by the microphones.

SOLUTION:

a. The equations $r = 2.5 + 2.5 \cos \theta$ and $r = -2.5 - 2.5 \cos \theta$ are of the form $r = a + b \cos\theta$, where $a = b = 2.5$ in each equation. Notice that since $r = -2.5$

$-2.5 \cos \theta$ is equivalent to $r = -(2.5 + 2.5 \cos \theta)$, the second equation is a reflection of the first in the line $\theta = \frac{\pi}{2}$. Therefore, each equation represents a special limaçon called a cardioid.

b. Because each polar equation is a function of the cosine function, each is symmetric with respect to the polar axis. Therefore, make a table and calculate the values of r on $[0, \pi]$.

θ	$r = 2.5 + 2.5 \cos \theta$	$r = -2.5 - 2.5 \cos \theta$
0	5	−5
$\frac{\pi}{6}$	4.7	−4.7
$\frac{\pi}{4}$	4.3	−4.3
$\frac{\pi}{3}$	3.75	−3.75
$\frac{\pi}{2}$	2.5	−2.5
$\frac{2\pi}{3}$	1.25	−1.25
$\frac{3\pi}{4}$	0.7	−0.7
$\frac{5\pi}{6}$	0.3	−0.3
π	0	0

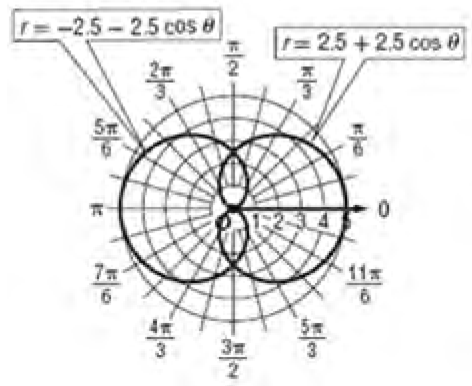
Use these points and polar axis symmetry to graph the functions on the same polar grid.

c. Sample answer: The overlapping areas of the two graphs indicate regions where sound will be detected by both microphones. For example, sounds made up to 2.5 units directly north or south of the microphones will be detected by both microphones. Also, each microphone will detect sound up to 5 units away directly in front it.

ANSWER:

a. Each is a cardioid.

b.

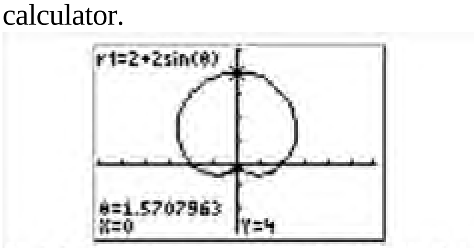


c. Sample answer: The overlapping areas of the two graphs indicate regions where sound will be detected by both microphones. For example, sounds made up to 2.5 units directly north or south of the microphones will be detected by both microphones. Also, each microphone will detect sound up to 5 units away directly in front it.

49. **CANDY** Write an equation that can model this lollipop in the shape of a limaçon if it is symmetric with respect to the line $\theta = \frac{\pi}{2}$ and measures 4 inches from the top of the lollipop to where the candy meets the stick.

SOLUTION:

A limaçon has a polar equation of the form $r = a \pm b \cos \theta$ or $r = a \pm b \sin \theta$, where a and b are both positive. Since the equation modeling this lollipop must be symmetric with respect to the line $\theta = \frac{\pi}{2}$, the equation must have the form $r = a \pm b \sin \theta$. Since the lollipop appears to have an upside-down heart shape, $a = b$ and the equation has the form $r = a + a \sin \theta$. The fact that the lollipop measures 4 inches from the top of the lollipop to where the candy meets the stick tells us that $2a = 4$, so $a = 2$. Therefore, an equation that models this lollipop is $r = 2 + 2 \sin \theta$.
Check this equation by graphing it on a graphing calculator.

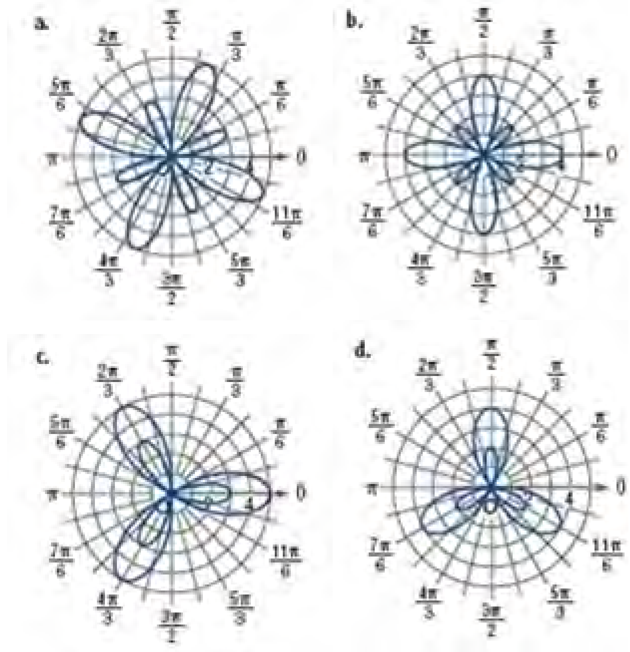


$[0, 2\pi]$ step: 0.5 by $[-6.1, 6.1]$ scl: 1 by $[-3, 5]$ scl: 1

ANSWER:

$r = 2 + 2 \sin \theta$

Match each equation with its graph.



50. $r = 1 + 4 \cos 3\theta$

SOLUTION:

Find two or three points on the graph of $r = 1 + 4 \cos 3\theta$.

θ	$r = 1 + 4 \cos 3\theta$
0	5
$\frac{\pi}{2}$	1
π	−3

The only graph that contains points $(0, 5)$, $\left(\frac{\pi}{2}, 1\right)$, and $(\pi, -3)$ is graph **c.**

ANSWER:

c

9-2 Graphs of Polar Equations

51. $r = 1 - 4 \sin 4\theta$

SOLUTION:

Find two or three points on the graph of $r = 1 - 4 \sin 4\theta$.

θ	$r = 1 - 4 \sin 4\theta$
$\frac{\pi}{8}$	−3
$\frac{3\pi}{8}$	5
π	0

The only graph that contains points $\left(\frac{\pi}{8}, -3\right)$, $\left(\frac{3\pi}{8}, 5\right)$, and $(\pi, 0)$ is graph **a.**

ANSWER:

a

52. $r = 1 - 3 \sin 3\theta$

SOLUTION:

Find two or three points on the graph of $r = 1 - 3 \sin 3\theta$.

θ	$r = 1 - 3 \sin 3\theta$
$\frac{\pi}{6}$	−2
$\frac{\pi}{2}$	4
π	1

The only graph that contains points $\left(\frac{\pi}{6}, -2\right)$, $\left(\frac{\pi}{2}, 4\right)$, and $(\pi, 1)$ is graph **d.**

ANSWER:

d

53. $r = 1 + 3 \cos 4\theta$

SOLUTION:

Find two or three points on the graph of $r = 1 + 3 \cos 4\theta$.

θ	$r = 1 + 3 \cos 4\theta$
0	4
$\frac{\pi}{4}$	−2
$\frac{\pi}{2}$	4

The only graph that contains points $(0, 4)$, $\left(\frac{\pi}{4}, -2\right)$, and $\left(\frac{\pi}{2}, 4\right)$ is graph **b.**

ANSWER:

b

Find *x* for the interval $0 \le \theta \le x$ so that *x* is a minimum and the graph is complete.

54. $r = 3 + 2 \cos \theta$

SOLUTION:

Make a table of values for $r = 3 + 2 \cos \theta$.

θ	r
0	5
$\frac{\pi}{6}$	4.7
$\frac{\pi}{4}$	4.4
$\frac{\pi}{3}$	4
$\frac{\pi}{2}$	3
$\frac{2\pi}{3}$	2
$\frac{3\pi}{4}$	1.6
$\frac{5\pi}{6}$	1.3

θ	r
π	1
$\frac{7\pi}{6}$	1.3
$\frac{5\pi}{4}$	1.6
$\frac{4\pi}{3}$	2
$\frac{3\pi}{2}$	3

$\frac{5\pi}{3}$	4
$\frac{7\pi}{4}$	4.4
$\frac{11\pi}{6}$	4.7

θ	r
2π	5
$\frac{13\pi}{6}$	4.7
$\frac{9\pi}{4}$	4.4
$\frac{7\pi}{3}$	4
$\frac{5\pi}{2}$	3
$\frac{8\pi}{3}$	2
$\frac{11\pi}{4}$	1.6
$\frac{17\pi}{6}$	1.3

Notice that the points in the first and third tables are equivalent polar coordinates. Therefore, for a complete polar graph, the function must be graphed on the interval $0 \le \theta \le 2\pi$. So $x = 2\pi$.

ANSWER:

2π

55. $r = 2 - \sin 2\theta$

SOLUTION:

Make a table of values for $r = 2 - \sin 2\theta$.

θ	r
0	2
$\frac{\pi}{6}$	1.13
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	1.13
$\frac{\pi}{2}$	2
$\frac{2\pi}{3}$	2.86
$\frac{3\pi}{4}$	3
$\frac{5\pi}{6}$	2.86

θ	r
π	2
$\frac{7\pi}{6}$	1.13
$\frac{5\pi}{4}$	1
$\frac{4\pi}{3}$	1.13
$\frac{3\pi}{2}$	2
$\frac{5\pi}{3}$	2.86
$\frac{7\pi}{4}$	3
$\frac{11\pi}{6}$	2.86

θ	r
2π	2
$\frac{13\pi}{6}$	1.13
$\frac{9\pi}{4}$	1
$\frac{7\pi}{3}$	1.13
$\frac{5\pi}{2}$	2
$\frac{8\pi}{3}$	2.86
$\frac{11\pi}{4}$	3
$\frac{17\pi}{6}$	2.86

Notice that the points in the first and third tables are equivalent polar coordinates. Therefore, for a complete polar graph, the function must be graphed on the interval $0 \le \theta \le 2\pi$. So $x = 2\pi$.

ANSWER:

2π

56. $r = 1 + \cos \frac{\theta}{3}$

SOLUTION:

Make a table of values for $r = 1 + \cos \frac{\theta}{3}$.

θ	r
0	2
$\frac{\pi}{6}$	1.98
$\frac{\pi}{4}$	1.97
$\frac{\pi}{3}$	1.94
$\frac{\pi}{2}$	1.87

$\frac{2\pi}{3}$	1.77
$\frac{3\pi}{4}$	1.71
$\frac{5\pi}{6}$	1.64

θ	r
π	1.5
$\frac{7\pi}{6}$	1.34
$\frac{5\pi}{4}$	1.26
$\frac{4\pi}{3}$	1.17
$\frac{3\pi}{2}$	1
$\frac{5\pi}{3}$	0.83
$\frac{7\pi}{4}$	0.74
$\frac{11\pi}{6}$	0.66

θ	r
2π	0.5
$\frac{13\pi}{6}$	0.36
$\frac{9\pi}{4}$	0.29
$\frac{7\pi}{3}$	0.23
$\frac{5\pi}{2}$	0.13
$\frac{8\pi}{3}$	0.06
$\frac{11\pi}{4}$	0.03
$\frac{17\pi}{6}$	0.02

θ	r
3π	0
$\frac{19\pi}{6}$	0.02
$\frac{13\pi}{4}$	0.03
$\frac{10\pi}{3}$	0.06
$\frac{7\pi}{2}$	0.13
$\frac{11\pi}{3}$	0.23
$\frac{15\pi}{4}$	0.29
$\frac{23\pi}{6}$	0.36

θ	r
4π	0.5
$\frac{25\pi}{6}$	0.66
$\frac{17\pi}{4}$	0.74
$\frac{13\pi}{3}$	0.83
$\frac{9\pi}{2}$	1
$\frac{14\pi}{3}$	1.17
$\frac{19\pi}{4}$	1.26
$\frac{29\pi}{6}$	1.34

θ	r
5π	1.5
$\frac{31\pi}{6}$	1.64
$\frac{21\pi}{4}$	1.71
$\frac{16\pi}{3}$	1.77
$\frac{11\pi}{2}$	1.87
$\frac{17\pi}{3}$	1.94
$\frac{23\pi}{4}$	1.97
$\frac{35\pi}{6}$	1.99

θ	r
6π	2
$\frac{31\pi}{6}$	1.98
$\frac{21\pi}{4}$	1.97
$\frac{16\pi}{3}$	1.94
$\frac{11\pi}{2}$	1.87
$\frac{17\pi}{3}$	1.77
$\frac{23\pi}{4}$	1.71
$\frac{35\pi}{6}$	1.64

Notice that the points in the first and sixth tables are equivalent polar coordinates. Therefore, for a complete polar graph, the function must be graphed on the interval $0 \le \theta \le 6\pi$. So $x = 6\pi$.

ANSWER:

6π

Match each equation with an equation that produces an equivalent graph.

a. $r = 5 + 4 \sin \theta$

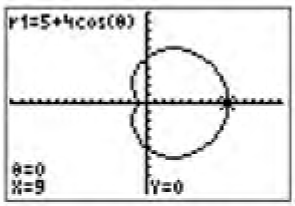
b. $r = -5 + 4 \cos \theta$

c. $r = 5 - 4 \cos \theta$

d. $r = -5 - 4 \sin \theta$

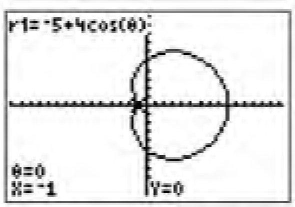
57. $r = 5 + 4 \cos \theta$

SOLUTION:



$[0, 2\pi]$ θstep: $\frac{\pi}{24}$ by $[-15, 15]$ scl: 1 by $[-10, 10]$ scl: 1

Then graph each of the equation choices given to find a match. Equation **b,** $r = -5 + 4 \cos \theta$, produces the same graph, as shown below.



$[0, 2\pi]$ θstep: $\frac{\pi}{24}$ by $[-15, 15]$ scl: 1 by $[-10, 10]$ scl: 1

The sign of a is irrelevant for these graphs. If $|a| = |a|$, and everything else is the same, then the equations will be equivalent.

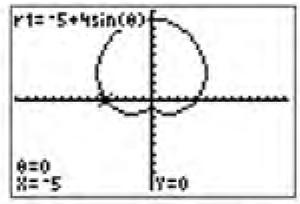
ANSWER:

b

58. $r = -5 + 4 \sin \theta$

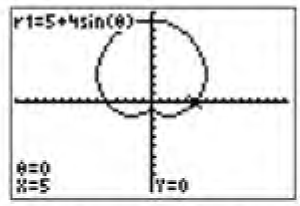
SOLUTION:

Use a graphing calculator to graph $r = -5 + 4 \sin \theta$.



$[0, 2\pi]$ θstep: $\frac{\pi}{24}$ by $[-15, 15]$ scl: 1 by $[-10, 10]$ scl: 1

Then graph each of the equation choices given to find a match. Equation **a,** $r = -5 + 4 \sin \theta$, produces the same graph, as shown below.



$[0, 2\pi]$ θstep: $\frac{\pi}{24}$ by $[-15, 15]$ scl: 1 by $[-10, 10]$ scl: 1

The sign of a is irrelevant for these graphs. If $|a| = |a|$, and everything else is the same, then the equations will be equivalent.

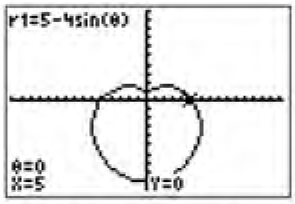
ANSWER:

a

59. $r = 5 - 4 \sin \theta$

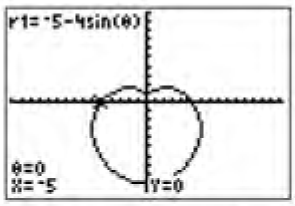
SOLUTION:

Use a graphing calculator to graph $r = 5 - 4 \sin \theta$.



$[0, 2\pi]$ θstep: $\frac{\pi}{24}$ by $[-15, 15]$ scl: 1 by $[-10, 10]$ scl: 1

Then graph each of the equation choices given to find a match. Equation **d,** $r = -5 - 4 \sin \theta$, produces the same graph, as shown below.



$[0, 2\pi]$ θstep: $\frac{\pi}{24}$ by $[-15, 15]$ scl: 1 by $[-10, 10]$ scl: 1

The sign of a is irrelevant for these graphs. If $|a| = |a|$, and everything else is the same, then the equations will be equivalent.

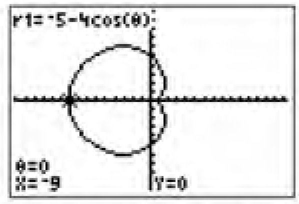
ANSWER:

d

60. $r = -5 - 4 \cos \theta$

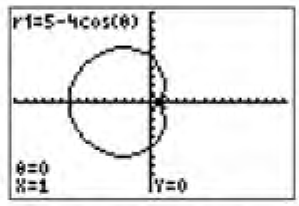
SOLUTION:

Use a graphing calculator to graph $r = -5 - 4 \cos \theta$.



$[0, 2\pi]$ θstep: $\frac{\pi}{24}$ by $[-15, 15]$ scl: 1 by $[-10, 10]$ scl: 1

Then graph each of the equation choices given to find a match. Equation **c,** $r = 5 - 4 \cos \theta$, produces the same graph, as shown below.



$[0, 2\pi]$ θstep: $\frac{\pi}{24}$ by $[-15, 15]$ scl: 1 by $[-10, 10]$ scl: 1

The sign of a is irrelevant for these graphs. If $b = b$ and $|a| = |a|$, and the +/- signs are the same, then the equations will be equivalent.

ANSWER:

c

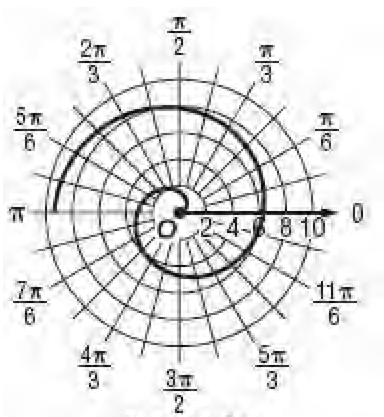
61. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a spiral of Archimedes.
a. GRAPHICAL Sketch separate graphs of $r = \theta$ for the intervals $0 \le \theta \le 3\pi$, $-3\pi \le \theta \le 0$, and $-3\pi \le \theta \le 3\pi$.
b. VERBAL Make a conjecture as to the symmetry of $r = \theta$. Explain your reasoning.
c. ANALYTICAL Prove your conjecture from part **b** by using one of the symmetry tests discussed in this lesson.
d. VERBAL How does changing the interval for θ affect the other classic curves? How does this differ from how the interval affects a spiral of Archimedes? Explain your reasoning.

SOLUTION:

a. Make tables of values for $r = \theta$ on $0 \le \theta \le 3\pi$, $-3\pi \le \theta \le 0$, and $-3\pi \le \theta \le 3\pi$. Then use each table of values to sketch each curve.

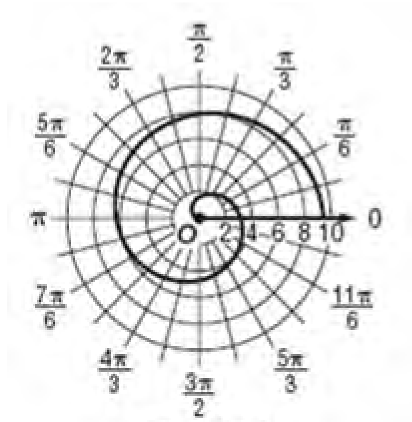
9-2 Graphs of Polar Equations

θ	$r = a\theta + b$
0	0
$\frac{\pi}{4}$	0.79
$\frac{\pi}{2}$	1.57
$\frac{3\pi}{4}$	2.36
π	3.14
$\frac{5\pi}{4}$	3.93
$\frac{3\pi}{2}$	4.71
2π	6.28
3π	9.42



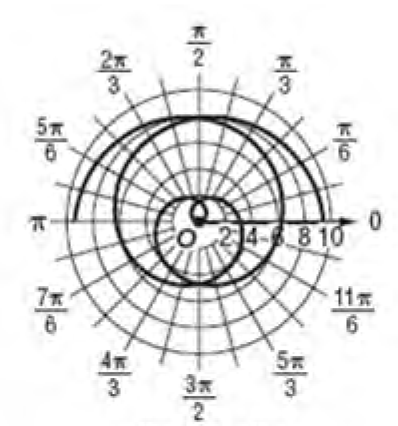
$0 \le \theta \le 3\pi$

θ	$r = a\theta + b$
-3π	–9.42
-2π	–6.28
$-\frac{3\pi}{2}$	–4.71
$-\frac{5\pi}{4}$	–3.93
$-\frac{3\pi}{4}$	–3.14
$-\pi$	–2.36
$-\frac{\pi}{2}$	– 1.57
$-\frac{\pi}{4}$	– 0.79
0	0



$-3\pi \le \theta \le 0$

θ	$r = a\theta + b$
-3π	–9.42
-2π	–6.28
$-\frac{3\pi}{2}$	–4.71
$-\pi$	–2.36
$-\frac{\pi}{2}$	– 1.57
0	0
$\frac{\pi}{2}$	1.57
π	2.36
$\frac{3\pi}{2}$	4.71
2π	6.28
3π	9.42



$-3\pi \le \theta \le 3\pi$

b. The equation $r = \theta$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$ when the interval for θ is $-a \le \theta \le a$, where a is any real number. .

c. Substitute$(-r, -\theta)$ for (r, θ).

$r = \theta$

$-r = -\theta$

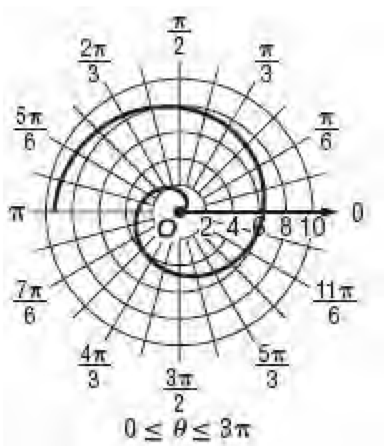
$r = \theta$

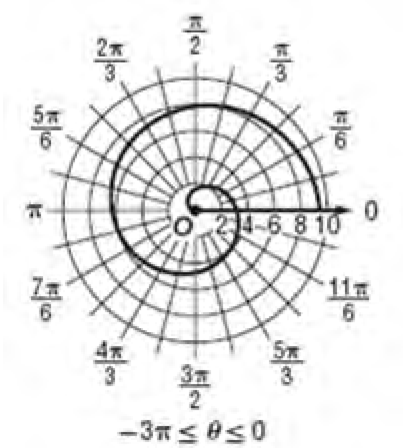
This substitution produces an equivalent equation, therefore $r = \theta$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

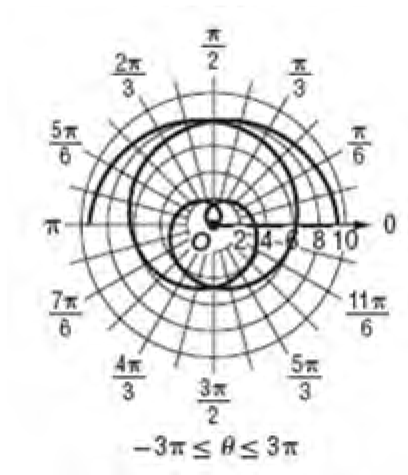
d. It does not affect the other classic curves. The classic curves all consist of either a sine or cosine function. Therefore, to achieve a complete graph, they just need to be graphed for all values of θ within their period. Extending the interval of θ to include additional values outside of the period will result in the graph repeating itself. Because the spiral of Archimedes does not contain a trigonometric function, additional values of θ result in different values of r.

ANSWER:

a.







b. Sample answer: The equation $r = \theta$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$ when the interval for θ is $-a \leq \theta \leq a$.

c. Substitute$(-r, -\theta)$ for (r, θ).

$r = \theta$

$-r = -\theta$

$r = \theta$

This substitution produces an equivalent equation, therefore $r = \theta$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

d. Sample answer: It does not affect the other classic curves. The classic curves all consist of either a sine or cosine function. Therefore, to achieve a complete graph, they just need to be graphed for all values of θ within their period. Extending the interval of θ to include additional values outside of the period will result in the graph repeating itself. Because the spiral of Archimedes does not contain a trigonometric function, additional values of θ result in different values of r.

62. **ERROR ANALYSIS** Haley and Ella are graphing polar equations. Ella says that $r = 7 \sin 2\theta$ is not a function because it does not pass the vertical line test. Haley says the vertical line test does not apply in a polar grid. Is either of them correct? Explain your reasoning.

SOLUTION:

Haley; sample answer: The vertical line test applies only to rectangular equations. If a table of values is created using the equation $r = 7 \sin 2\theta$, it is evident that each θ-value corresponds to only one r-value.

θ	$r = 7 \sin 2\theta$
0	0
$\frac{\pi}{6}$	6.1
$\frac{\pi}{3}$	6.1
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	−6.1
$\frac{5\pi}{6}$	−6.1
π	0

ANSWER:

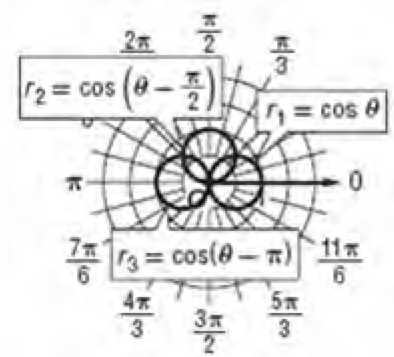
Haley; sample answer: The vertical line test applies only to rectangular equations. If a table of values is created using the equation $r = 7 \sin 2\theta$, it is evident that each θ-value corresponds to only one r-value.

63. **REASONING** Sketch the graphs of $r_1 = \cos \theta$, $r_2 = \cos\left(\theta - \frac{\pi}{2}\right)$, and $r_3 = \cos(\theta - \pi)$ on the same polar grid. Describe the relationship between the three graphs. Make a conjecture as to the change in a graph when a value d is subtracted from θ.

SOLUTION:

Make tables of values for $r_1 = \cos \theta$, $r_2 = \cos\left(\theta - \frac{\pi}{2}\right)$, and $r_3 = \cos(\theta - \pi)$ on $[0, 2\pi]$.

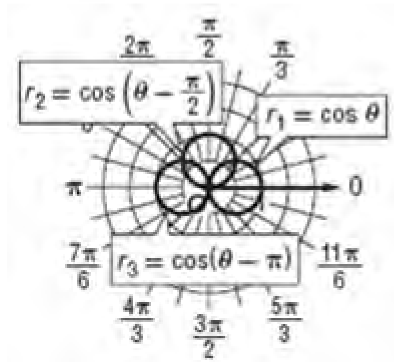
Then use the table of values to sketch the curves on the same polar grid.



r_2 and r_3 are the graphs of r_1 after a rotation about the pole of $\frac{\pi}{2}$ and π, respectively. The graph of $r = \cos(\theta - d)$ will be the graph of $r = \cos \theta$ after a rotation of d about the pole.

ANSWER:

Sample answer: r_2 and r_3 are the graphs of r_1 after a rotation about the pole of $\frac{\pi}{2}$ and π, respectively. The graph of $r = \cos(\theta - d)$ will be the graph of $r = \cos \theta$ after a rotation of d about the pole.



64. **CHALLENGE** Solve the following system of polar equations algebraically on $[0, 2\pi]$. Graph the system

and compare the points of intersection with the solutions that you found. Explain any discrepancies.

$r = 1 + 2\sin\theta$

$r = 4\sin\theta$

SOLUTION:

$r = 1 + 2\sin\theta$	First equation
$4\sin\theta = 1 + 2\sin\theta$	$r = 4\sin\theta$
$2\sin\theta = 1$	Subtract $2\sin\theta$ from each side.
$\sin\theta = \frac{1}{2}$	Divide each side by 2.
$\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$	Solve for θ on the interval $[0, 2\pi]$.

Find the corresponding r-values using either equation.

$r = 4\sin\theta$	$r = 4\sin\theta$
$r = 4\sin\frac{\pi}{6}$	$r = 4\sin\frac{5\pi}{6}$
$r = 4\left(\frac{1}{2}\right)$	$r = 4\left(\frac{1}{2}\right)$
$r = 2$	$r = 2$

Therefore, the solutions to the system on $[0, 2\pi]$ are $\left(\frac{\pi}{6}, 2\right)$ and $\left(\frac{5\pi}{6}, 2\right)$.

Check these solutions in both original equations.

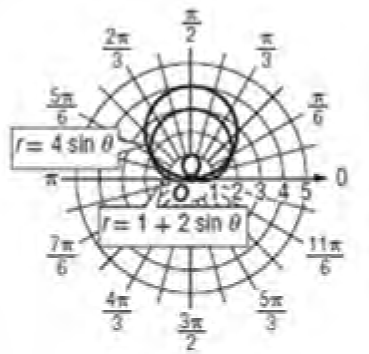
$$2 = 1 + 2\sin\frac{\pi}{6}$$
$$2 = 1 + 2\left(\frac{1}{2}\right)$$
$$2 = 1 + 1$$
$$2 = 2$$

$$2 = 4\sin\frac{\pi}{6}$$
$$2 = 4\left(\frac{1}{2}\right)$$
$$2 = 2$$

$$2 = 1 + 2\sin\frac{5\pi}{6}$$
$$2 = 1 + 2\left(\frac{1}{2}\right)$$
$$2 = 1 + 1$$
$$2 = 2$$

$$2 = 4\sin\frac{5\pi}{6}$$
$$2 = 4\left(\frac{1}{2}\right)$$
$$2 = 2$$

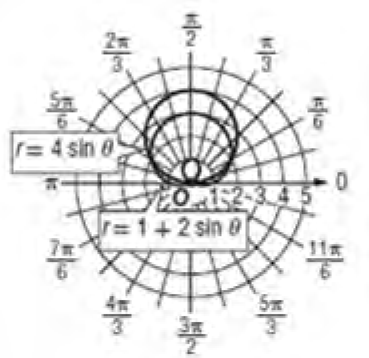
Graph the equations on the same polar grid.



Sample answer: The graphs appear to intersect at $\left(\frac{\pi}{6}, 2\right)$ and $\left(\frac{5\pi}{6}, 2\right)$. The graphs also appear to intersect at the pole. However, (0, 0) is not a solution of the equation $r = 1 + 2\sin\theta$, so it is not a solution of the system.

ANSWER:

$\left(\frac{\pi}{6}, 2\right)$ and $\left(\frac{5\pi}{6}, 2\right)$



Sample answer: The graphs appear to intersect at $\left(\frac{\pi}{6}, 2\right)$ and $\left(\frac{5\pi}{6}, 2\right)$. The graphs also appear to intersect at the pole. However, (0, 0) is not a solution of the equation $r = 1 + 2\sin\theta$, so it is not a solution of the system.

9-2 Graphs of Polar Equations

65. **PROOF** Prove that the graph of $r = a + b \cos 2\theta$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

SOLUTION:

To test for symmetry with respect to the line $\theta = \frac{\pi}{2}$, substitute $(r, \pi - \theta)$ for (r, θ).

$r = a + b\cos 2(\pi - \theta)$	Replace θ with $\pi - \theta$
$r = a + b\cos(2\pi - 2\theta)$	Distribute
$r = a + b(\cos 2\pi \cos 2\theta + \sin 2\pi \sin 2\theta)$	Cosine Difference Identity
$r = a + b[(1)\cos 2\theta + (0)\sin 2\theta]$	$\cos 2\pi = 1$ and $\sin 2\pi = 0$
$r = a + b\cos 2\theta$	Simplify

Since this substitution produces an equivalent equation, the graph of $r = a + b \cos 2\theta$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

ANSWER:

To test for symmetry with respect to the line $\theta = \frac{\pi}{2}$, substitute $(r, \pi - \theta)$ for (r, θ).

$$r = a + b\cos 2(\pi - \theta)$$
$$r = a + b\cos(2\pi - 2\theta)$$
$$r = a + b(\cos 2\pi \cos 2\theta + \sin 2\pi \sin 2\theta)$$
$$r = a + b[(1)\cos 2\theta + (0)\sin 2\theta]$$
$$r = a + b\cos 2\theta$$

Since this substitution produces an equivalent equation, the graph of $r = a + b \cos 2\theta$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

66. **PROOF** Prove that the graph of $r = a \sin 2\theta$ is symmetric with respect to the polar axis.

SOLUTION:

To test for symmetry with respect to the line $\theta = \frac{\pi}{2}$, substitute $(-r, \pi - \theta)$ for (r, θ).

$$-r = a\sin 2(\pi - \theta)$$
$$-r = a\sin(2\pi - 2\theta)$$
$$-r = a(\sin 2\pi \cos 2\theta - \cos 2\pi \sin 2\theta)$$
$$-r = a[(0)\cos 2\theta - (1)\sin 2\theta]$$
$$-r = -a\sin 2\theta$$
$$r = a\sin 2\theta$$

Since this substitution produces an equivalent equation, the graph of $r = a \sin 2\theta$ is symmetric with respect to the polar axis.

ANSWER:

To test for symmetry with respect to the line $\theta = \frac{\pi}{2}$, substitute $(-r, \pi - \theta)$ for (r, θ).

$$-r = a\sin 2(\pi - \theta)$$
$$-r = a\sin(2\pi - 2\theta)$$
$$-r = a(\sin 2\pi \cos 2\theta - \cos 2\pi \sin 2\theta)$$
$$-r = a[(0)\cos 2\theta - (1)\sin 2\theta]$$
$$-r = -a\sin 2\theta$$
$$r = a\sin 2\theta$$

Since this substitution produces an equivalent equation, the graph of $r = a \sin 2\theta$ is symmetric with respect to the polar axis.

67. **WRITING IN MATH** Describe the effect of a in the graph of $r = a \cos \theta$.

SOLUTION:

Sample answer: The value of a determines the diameter of the circle. If $a > 0$, the graph is located in Quadrants I and IV of the plane with the polar axis containing a diameter. If $a < 0$, the graph is located in Quadrants II and III of the plane with the polar axis containing a diameter.

ANSWER:

Sample answer: The value of a determines the diameter of the circle. If $a > 0$, the graph is located in Quadrants I and IV of the plane with the polar axis containing a diameter. If $a < 0$, the graph is located in Quadrants II and III of the plane with the polar axis containing a diameter.

68. **OPEN ENDED** Sketch the graph of a rose with 8 petals. Then write the equation for your graph.

SOLUTION:

You can describe a rose by either $r = a\cos n\theta$ or $r = a\sin n\theta$.

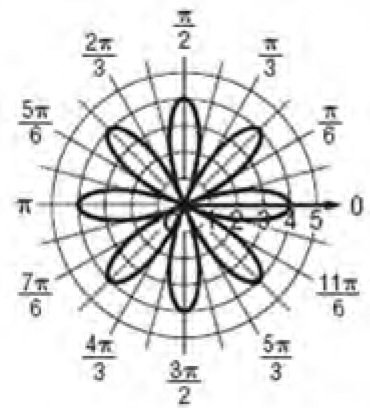
To have 8 petals, let $n = 4$.

a can be any value, so set a equal to 4. Then draw the petals 4 units from the center.

We can use sine or cosine. If we use cosine, then a *peak* of one of the petals where be at (4, 0).

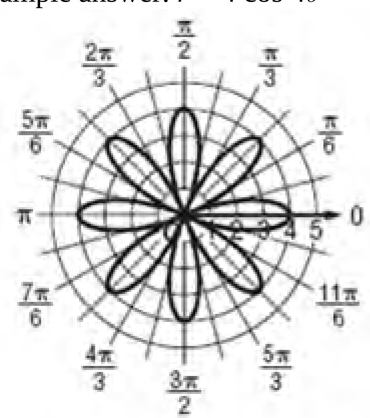
$4 \cos 4(0) = 4(1) = 4$

The equation is $r = 4 \cos 4\theta$.



ANSWER:

Sample answer: $r = 4 \cos 4\theta$



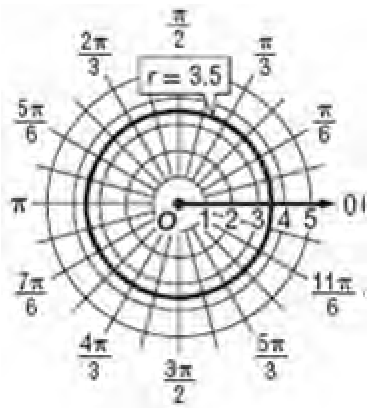
9-2 Graphs of Polar Equations

Graph each polar equation.

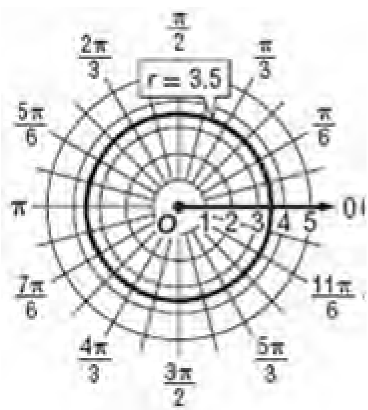
69. $r = 3.5$

SOLUTION:

The solutions of $r = 3.5$ are ordered pairs of the form $(3.5, \theta)$, where θ is any real number. The graph consists of all points that are 3.5 units from the pole, so the graph is a circle centered at the origin with radius 3.5.



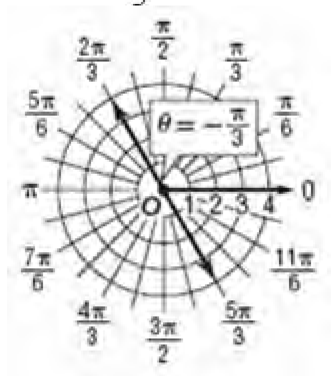
ANSWER:



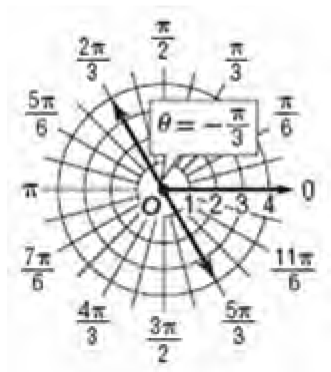
70. $\theta = -\frac{\pi}{3}$

SOLUTION:

The solutions of $\theta = -\frac{\pi}{3}$ are ordered pairs of the form $\left(r, -\frac{\pi}{3}\right)$, where r is any real number. The graph consists of all points on the line that make an angle of $-\frac{\pi}{3}$ with the positive polar axis.



ANSWER:

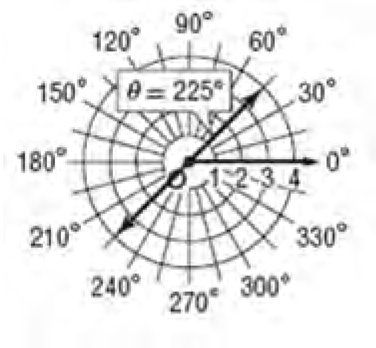


9-2 Graphs of Polar Equations

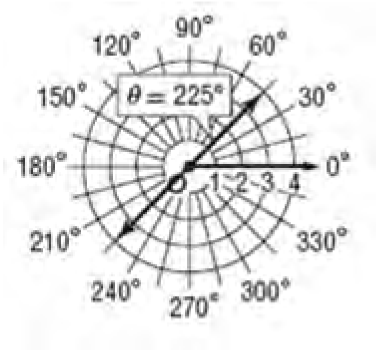
71. $\theta = 225°$

SOLUTION:

The solutions of $\theta = 225°$ are ordered pairs of the form $(r, 225°)$, where r is any real number. The graph consists of all points on the line that make an angle of 225° with the positive polar axis.



ANSWER:



Find the angle θ between vectors u and v to the nearest tenth of a degree.

72. $\mathbf{u} = \langle 4, -3, 5\rangle$, $\mathbf{v} = \langle 2, 6, -8\rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\langle 4, -3, 5\rangle \cdot \langle 2, 6, -8\rangle}{|\langle 4, -3, 5\rangle||\langle 2, 6, -8\rangle|}$$
$$\cos\theta = \frac{4(2) + (-3)6 + 5(-8)}{\sqrt{4^2 + (-3)^2 + 5^2}\sqrt{2^2 + 6^2 + (-8)^2}}$$
$$\cos\theta = \frac{8 + (-18) + (-40)}{\sqrt{16 + 9 + 25}\sqrt{4 + 36 + 64}}$$
$$\cos\theta = \frac{-50}{\sqrt{50}\sqrt{104}}$$
$$\cos\theta = \frac{-50}{\sqrt{5200}}$$
$$\cos\theta = \frac{-50}{12\sqrt{13}}$$
$$\cos\theta = \frac{-25}{6\sqrt{13}}$$
$$\theta \approx 133.9°$$

ANSWER:

133.9°

73. $\mathbf{u} = 2\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$, $\mathbf{v} = 5\mathbf{i} + 6\mathbf{j} - 11\mathbf{k}$

SOLUTION:

Write **u** and **v** in component form.
$\mathbf{u} = \langle 2, -4, 7\rangle$ and $\mathbf{v} = \langle 5, 6, -11\rangle$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\langle 2, -4, 7\rangle \cdot \langle 5, 6, -11\rangle}{|\langle 2, -4, 7\rangle||\langle 5, 6, -11\rangle|}$$
$$\cos\theta = \frac{2(5) + (-4)6 + 7(-11)}{\sqrt{2^2 + (-4)^2 + 7^2}\sqrt{5^2 + 6^2 + (-11)^2}}$$
$$\cos\theta = \frac{10 + (-24) + (-77)}{\sqrt{4 + 16 + 49}\sqrt{25 + 36 + 121}}$$
$$\cos\theta = \frac{-91}{\sqrt{69}\sqrt{182}}$$
$$\cos\theta = \frac{-91}{\sqrt{12,558}}$$
$$\theta \approx 144.3°$$

ANSWER:

144.3°

74. $\mathbf{u} = \langle -1, 1, 5\rangle$, $\mathbf{v} = \langle 7, -6, 9\rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\langle -1, 1, 5\rangle \cdot \langle 7, -6, 9\rangle}{|\langle -1, 1, 5\rangle||\langle 7, -6, 9\rangle|}$$
$$\cos\theta = \frac{(-1)7 + 1(-6) + 5(9)}{\sqrt{(-1)^2 + 1^2 + 5^2}\sqrt{7^2 + (-6)^2 + 9^2}}$$
$$\cos\theta = \frac{-7 + (-6) + (45)}{\sqrt{1 + 1 + 25}\sqrt{49 + 36 + 81}}$$
$$\cos\theta = \frac{32}{\sqrt{27}\sqrt{166}}$$
$$\cos\theta = \frac{32}{\sqrt{4482}}$$
$$\cos\theta = \frac{32}{3\sqrt{498}}$$
$$\theta \approx 61.4°$$

ANSWER:

61.4°

Let $\overline{DE}$ be the vector with the given initial and terminal points. Write $\overline{DE}$ as a linear combination of the vectors i and j.

75. $D\left(-5, \frac{2}{3}\right), E\left(-\frac{4}{5}, 0\right)$

SOLUTION:

First, find the component form of $\overline{DE}$.

$$\overline{DE} = \langle x_2 - x_1, y_2 - y_1\rangle$$
$$= \left\langle -\frac{4}{5} - (-5), 0 - \frac{2}{3}\right\rangle$$
$$= \left\langle \frac{21}{5}, -\frac{2}{3}\right\rangle$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\overline{DE} = \left\langle \frac{21}{5}, -\frac{2}{3}\right\rangle$$
$$= \frac{21}{5}\mathbf{i} - \frac{2}{3}\mathbf{j}$$

ANSWER:

$\frac{21}{5}\mathbf{i} - \frac{2}{3}\mathbf{j}$

76. $D\left(-\frac{1}{2}, \frac{4}{7}\right), E\left(-\frac{3}{4}, \frac{5}{7}\right)$

SOLUTION:

First, find the component form of $\overline{DE}$.

$$\overline{DE} = \langle x_2 - x_1, y_2 - y_1\rangle$$
$$= \left\langle -\frac{3}{4} - \left(-\frac{1}{2}\right), \frac{5}{7} - \frac{4}{7}\right\rangle$$
$$= \left\langle -\frac{1}{4}, \frac{1}{7}\right\rangle$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\overline{DE} = \left\langle -\frac{1}{4}, \frac{1}{7}\right\rangle$$
$$= -\frac{1}{4}\mathbf{i} + \frac{1}{7}\mathbf{j}$$

ANSWER:

$-\frac{1}{4}\mathbf{i} + \frac{1}{7}\mathbf{j}$

77. $D(9.7, -2.4)$, $E(-6.1, -8.5)$

SOLUTION:

First, find the component form of $\overline{DE}$.

$$\overline{DE} = \langle x_2 - x_1, y_2 - y_1\rangle$$
$$= \langle -6.1 - 9.7, -8.5 - (-2.4)\rangle$$
$$= \langle -15.8, -6.1\rangle$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\overline{DE} = \langle -15.8, -6.1\rangle$$
$$= -15.8\mathbf{i} - 6.1\mathbf{j}$$

ANSWER:

$-15.8\mathbf{i} - 6.1\mathbf{j}$

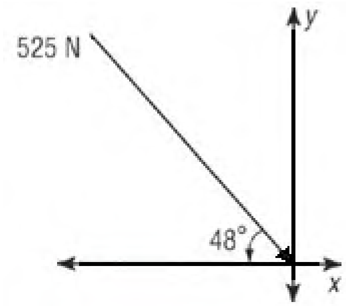
78. **YARDWORK** Kyle is pushing a wheelbarrow full of leaves with a force of 525 newtons at a 48° angle with the ground.

a. Draw a diagram that shows the resolution of the force that Kyle is exerting into its rectangular components.

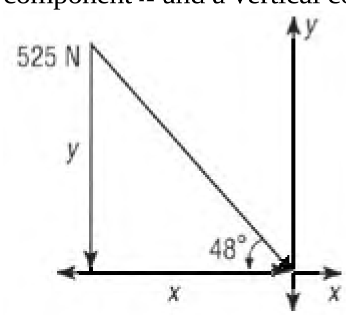
b. Find the magnitudes of the horizontal and vertical components of the force.

SOLUTION:

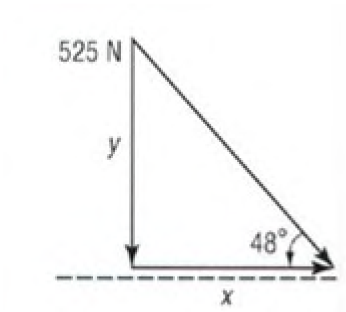
a. Kyle is pushing the handle of the wheelbarrow down with a force of 525 newtons at an angle of 48° with the ground. Draw a vector to represent the handle of the wheelbarrow.



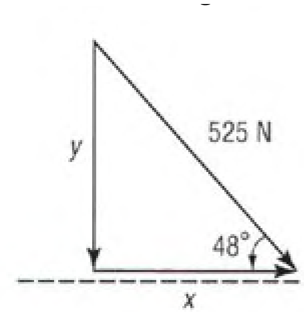
The vector can be resolved into a horizontal component **x** and a vertical component **y** as shown.



Remove the axes.



b. The horizontal and vertical components of the vector form a right triangle. Use the sine or cosine ratios to find the magnitude of each component.



$$\cos 48° = \frac{|\mathbf{x}|}{525} \qquad \sin 48° = \frac{|\mathbf{y}|}{525}$$

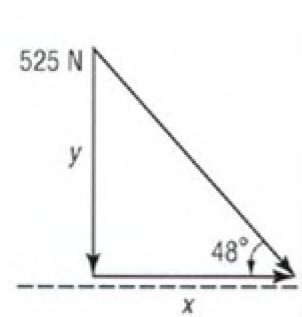
$$|\mathbf{x}| = 525\cos 48° \qquad |\mathbf{y}| = 525\sin 48°$$

$$|\mathbf{x}| \approx 351.3 \qquad |\mathbf{y}| \approx 390.2$$

The magnitude of the horizontal component is about 351.3 newtons and the magnitude of the vertical component is about 390.2 newtons.

ANSWER:

a.



b. horizontal component: 351.3 N, vertical component: 390.2 N

Graph the hyperbola given by each equation.

79. $\frac{x^2}{9} - \frac{y^2}{25} = 1$

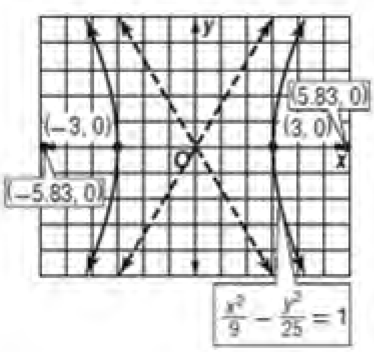
SOLUTION:

The equation is in standard form, where $h = 0$, $k = 0$, $a = 3$, $b = 5$, and $c = \sqrt{9+25}$ or about 5.83. In the standard form of the equation, the y-term is being subtracted. Therefore, the orientation of the hyperbola is horizontal.

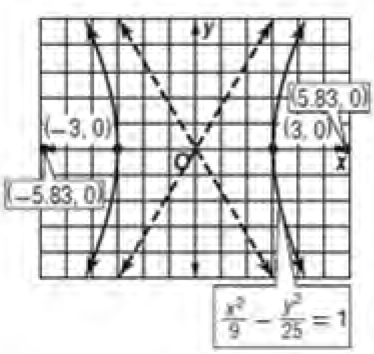
center: $(h, k) = (0, 0)$
vertices: $(h \pm a, k) = (3, 0)$ and $(-3, 0)$
foci: $(h \pm c, k) = (5.83, 0)$ and $(-5.83, 0)$
asymptotes:

$$\begin{aligned} y - k &= \pm\tfrac{b}{a}(x - h) \\ y - 0 &= \pm\tfrac{5}{3}(x - 0) \\ y &= \pm\tfrac{5}{3}x \\ y &= \tfrac{5}{3}x \quad OR \\ y &= -\tfrac{5}{3}x \end{aligned}$$

Graph the center, vertices, foci, and asymptotes. Then use a table of values to sketch the hyperbola.



ANSWER:



80. $\frac{(y-4)^2}{16} - \frac{(x+2)^2}{9} = 1$

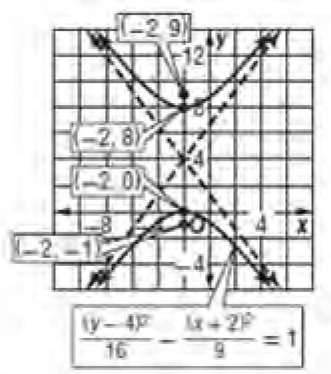
SOLUTION:

The equation is in standard form, where $h = -2$, $k = 4$, $a = 4$, $b = 3$, and $c = 5$. In the standard form of the equation, the x-term is being subtracted. Therefore, the orientation of the hyperbola is vertical.

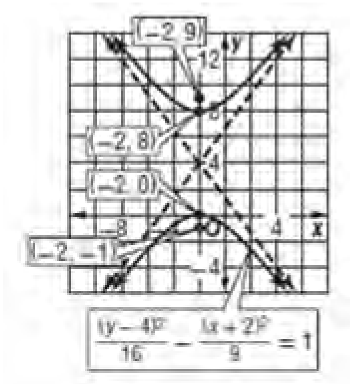
center: $(h, k) = (-2, 4)$
vertices: $(h, k \pm a) = (-2, 4 + 4)$ and $(-2, 4 - 4)$ or $(-2, 8)$ and $(-2, 0)$
foci: $(h, k \pm c) = (-2, 4 + 5)$ and $(-2, 4 - 5)$ or $(-2, 9)$ and $(-2, -1)$
asymptotes:

$$\begin{aligned} y - k &= \pm\tfrac{a}{b}(x - h) \\ y - 4 &= \pm\tfrac{4}{3}(x - [-2]) \\ y - 4 &= \pm\tfrac{4}{3}(x + 2) \\ y - 4 &= \pm\left[\tfrac{4}{3}x + \tfrac{8}{3}\right] \\ y &= \pm\left[\tfrac{4}{3}x + \tfrac{8}{3}\right] + 4 \\ y &= \pm\left[\tfrac{4}{3}x + \tfrac{8}{3}\right] + \tfrac{12}{3} \\ y &= \tfrac{4}{3}x + \tfrac{20}{3} \quad OR \\ y &= -\tfrac{4}{3}x + \tfrac{4}{3} \end{aligned}$$

Graph the center, vertices, foci, and asymptotes. Then use a table of values to sketch the hyperbola.



ANSWER:



81. $\dfrac{(x+1)^2}{4} - \dfrac{(y+3)^2}{9} = 1$

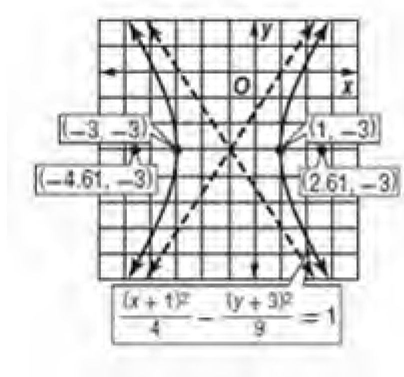
SOLUTION:

The equation is in standard form, where h = –1, k = –3, a = $\sqrt{4}$ or 2, b = $\sqrt{9}$ or 3, and c = $\sqrt{4+9}$ or about 3.61. In the standard form of the equation, the y-term is being subtracted. Therefore, the orientation of the hyperbola is horizontal.

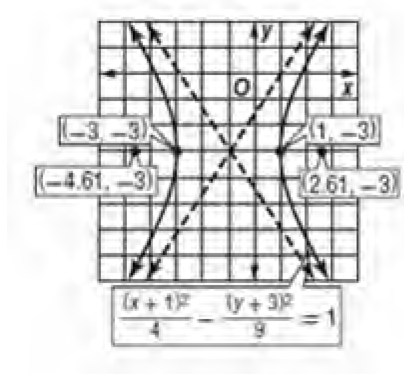
center: (h, k) = (–1, –3)
vertices: ($h \pm a$, k) = (–1 + 2, –3) and (−1 – 2, –3) or (1, –3) and (−3, –3)
foci: ($h \pm c$, k) = (–1 + 3.61, –3) and (–1 – 3.61, –3) or (2.61, –3) and (–4.61, –3)
asymptotes:

$$\begin{aligned} y - k &= \pm\tfrac{b}{a}(x - h) \\ y - (-3) &= \pm\tfrac{3}{2}(x - [-1]) \\ y + 3 &= \pm\tfrac{3}{2}(x + 1) \\ y + 3 &= \pm\left[\tfrac{3}{2}x + \tfrac{3}{2}\right] \\ y &= \pm\left[\tfrac{3}{2}x + \tfrac{3}{2}\right] - 3 \\ y &= \pm\left[\tfrac{3}{2}x + \tfrac{3}{2}\right] - \tfrac{6}{2} \\ y &= \tfrac{3}{2}x - \tfrac{3}{2} \quad OR \\ y &= -\tfrac{3}{2}x - \tfrac{9}{2} \end{aligned}$$

Graph the center, vertices, foci, and asymptotes. Then use a table of values to sketch the hyperbola.



ANSWER:



Write an equation for and graph each parabola with focus *F* and the given characteristics.

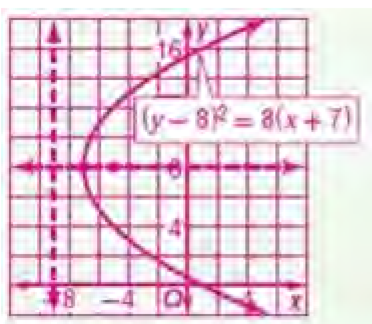
82. $F(-5, 8)$; opens right; contains $(-5, 12)$

SOLUTION:

Because the parabola opens to the right, the vertex is $(-5 - p, 8)$. Use the standard form of the equation of a horizontal parabola and the point $(-5, 12)$ to find the equation.

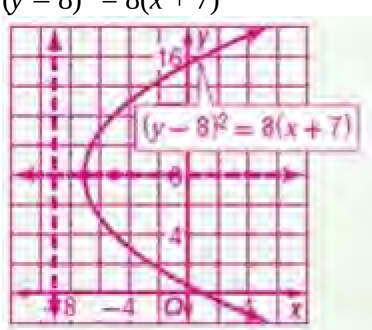
$$\begin{aligned}(y-k)^2 &= 4p(x-h)\\ (8-12)^2 &= 4p[-5-(-5-p)]\\ 16 &= 4p(p)\\ 16 &= 4p^2\\ 4 &= p^2\\ 2 \text{ or } -2 &= p\end{aligned}$$

Because the parabola opens to the right, the value of p must be positive. Therefore, $p = 2$. The vertex is $(-7, 8)$, and the standard form of the equation is $(y - 8)^2 = 4(2)(x + 7)$. Use a table of values to graph the parabola.



ANSWER:

$(y - 8)^2 = 8(x + 7)$



83. $F(-1, -5)$; opens left; contains $(-1, 5)$

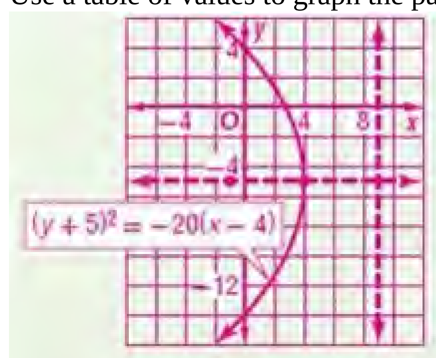
SOLUTION:

Because the parabola opens to the left, the vertex is $(-1 - p, -5)$. Use the standard form of the equation of a horizontal parabola and the point $(-1, 5)$ to find the equation.

$$\begin{aligned}(y-k)^2 &= 4p(x-h)\\ [5-(-5)]^2 &= 4p[-1-(-1-p)]\\ 10^2 &= 4p(p)\\ 100 &= 4p^2\\ 25 &= p^2\\ 5 \text{ or } -5 &= p\end{aligned}$$

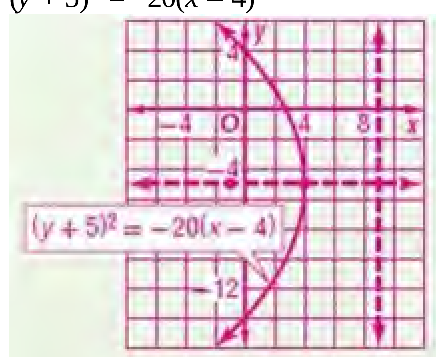
Because the parabola opens to the left, the value of p must be negative. Therefore, $p = -5$. The vertex is $(4, -5)$, and the standard form of the equation is $[y - (-5)]^2 = 4(-5)(x - 4)$, which simplifies to $(y + 5)^2 = -20(x - 4)$.

Use a table of values to graph the parabola.



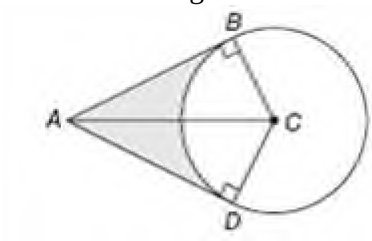
ANSWER:

$(y + 5)^2 = -20(x - 4)$



84. **SAT/ACT** In the figure, C is the center of the circle, $AC = 12$, and $m\angle BAD = 60°$. What is the perimeter

of the shaded region?



A 12 + 3π

B $6\sqrt{3}$ + 4π

C $6\sqrt{3}$ + 3π

D $12\sqrt{3}$ + 3π

E $12\sqrt{3}$ + 4π

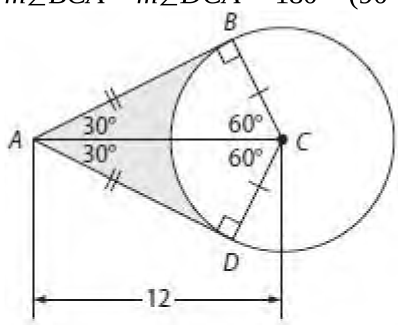
SOLUTION:

The perimeter of the shaded region is the sum of *AB*, *AD*, and arc *BD*. First find *AB* and *AC*.

Since $\overline{AB} \perp \overline{BC}$, $\overline{AB}$ is tangent to circle *C* at point *B*. Likewise, since $\overline{AD} \perp \overline{DC}$, $\overline{AD}$ is tangent to circle *C* at point *D*. From geometry, if two segments from the same exterior point are tangent to a circle, then they are congruent. Therefore, $\overline{AB} \cong \overline{AD}$.

Since all radii of a circle are congruent, $\overline{BC} \cong \overline{DC}$. By the Reflexive Property, $\overline{AC} \cong \overline{AC}$. Therefore, by SSS Triangle Congruence, *ABC* ≅ *ADC*. By CPCTC, ∠*BAC* ≅ ∠*DAC* and *m*∠*BAC* = *m*∠*DAC*.

Since *m*∠*BAD* = 60, *m*∠*BAC* = *m*□*DAC* = 30
m∠*BCA* = *m*∠*DCA* = 180 – (90 + 30) = 60.



Use right triangle *ABC* to find *AB*.

$$\begin{aligned}\sin 60^\circ &= \frac{AB}{12}\\ 12\sin 60^\circ &= AB\\ 12\left(\frac{\sqrt{3}}{2}\right) &= AB\\ 6\sqrt{3} &= AB\end{aligned}$$

Since $\overline{AB} \cong \overline{AD}$, *AB* = $6\sqrt{3}$ and *AD* = $6\sqrt{3}$.

To find the length of arc *BD*, find the measure of the central angle that intercepts this arc, ∠*BCD*. *m*∠*BCD* = *m*∠*BCA* + *m*∠*DCA* = 60 + 60 or 120. Then find the measure of the radius of circle *C* using right triangle *ABC* to find *BC*.

$$\begin{aligned}\sin 30^\circ &= \frac{BC}{12}\\ 12\sin 30^\circ &= BC\\ 12\left(\frac{1}{2}\right) &= BC\\ 6 &= BC\end{aligned}$$

The circumference of circle *C* is π time its twice its radius, 12π. The measure of arc *BC* is $\frac{120}{360}$ or $\frac{1}{3}$ the circumference of circle *C*, so the measure of arc *BC* is $\frac{1}{3}$(12π) = 4π.

Therefore, the perimeter of the shaded region is $6\sqrt{3}$ + $6\sqrt{3}$ + 4π or $12\sqrt{3}$ + 4π. The correct answer is E.

ANSWER:

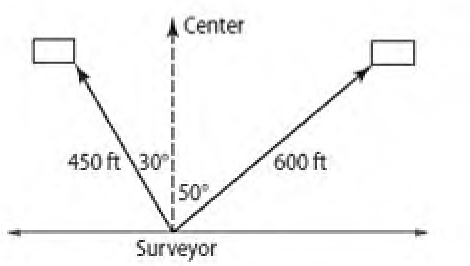
E

85. **REVIEW** While mapping a level site, a surveyor identifies a landmark 450 feet away and 30° left of center and another landmark 600 feet away and 50° right of center. What is the approximate distance between the two landmarks?

F 672 feet
G 685 feet
H 691 feet
J 703 feet

SOLUTION:

Place the surveyor at the origin of a polar coordinate system as shown.



The first landmark makes a 40°-angle with the polar axis and is located 600 ft along the angle's terminal side. The second landmark makes a 120°-angle with the polar axis and is located 450 ft along the angle's terminal side. Therefore, coordinates representing the first and second locations are (600, 40°) and (450, 120°), respectively.

Use the polar distance formula to find the distance between the two landmarks.

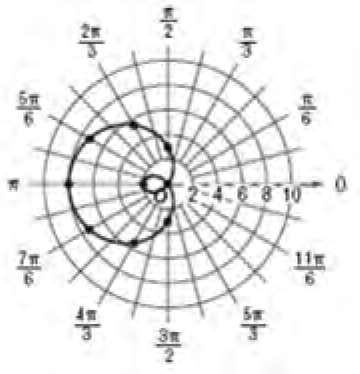
$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{600^2 + 450^2 - 2(600)(450)\cos(120° - 40°)}$$
$$\approx 685$$

The distance between the two landmarks is about 685 ft. The correct answer is G.

ANSWER:

G

86. Which type of curve does the figure represent?



A lemniscate
B limaçon
C rose
D cardioid

SOLUTION:

This graph shows a classic polar curve, a limaçon. Since the curve has an inner loop, we know that it is not a special limaçon called a cardioid. The correct answer is B.

ANSWER:

B

87. **REVIEW** An air traffic controller is tracking two jets at the same altitude. The coordinates of the jets are (5, 310°) and (6, 345°), with *r* measured in miles. What is the approximate distance between the jets?

F 2.97 miles
G 3.25 miles
H 3.44 miles
J 3.71 miles

SOLUTION:

Use the Polar Distance Formula.

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{5^2 + 6^2 - 2(5)(6)\cos(345° - 310°)}$$
$$= \sqrt{25 + 36 - 60\cos(35°)}$$
$$\approx 3.44$$

The approximate distance between the jets is 3.44 miles. The correct answer is H.

ANSWER:

H