13-4 Simulations

1. **GRADES** Clara got an A on 80% of her first semester Biology quizzes. Design and conduct a simulation using a geometric model to estimate the probability that she will get an A on a second semester Biology quiz. Report the results using appropriate numerical and graphical summaries.

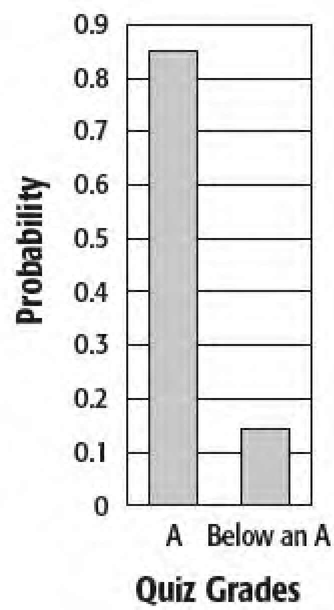
SOLUTION:

Sample answer: While a random number generator would work, a spinner will be easier. Divide the spinner into two sectors, one containing 80% or 288° and the other containing 20% or 72°. Another option would be to roll a six-sided die. Let 1-4 represent an *A*, let 5 represent a non *A*, and roll again for each 6.

Do 20 trials and record the results in a frequency table.

Outcome	Frequency
A	17
Below an A	3
Total	20

Divide the frequency by 20 to get the probabilities.

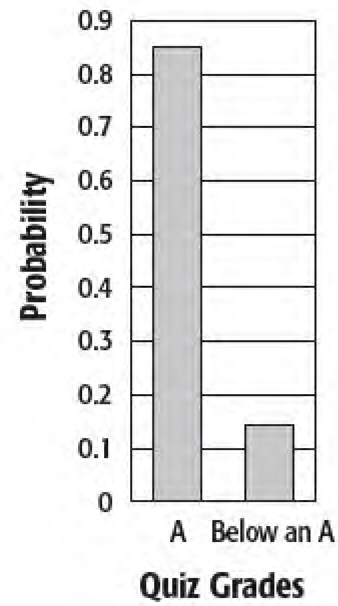


The probability of Clara getting an A on her next quiz is .85. The probability of earning any other grade is 1 – 0.85 or 0.15.

ANSWER:

Sample answer: Use a spinner that is divided into two sectors, one containing 80% or 288° and the other containing 20% or 72°. Do 20 trials and record the results in a frequency table.

Outcome	Frequency
A	17
Below an A	3
Total	20



The probability of Clara getting an A on her next quiz is .85. The probability of earning any other grade is 1 – 0.85 or 0.15.

2. **FITNESS** The table shows the percent of members participating in four classes offered at a gym. Design and conduct a simulation to estimate the probability that a new gym member will take each class. Report the results using appropriate numerical and graphical summaries.

Class	Sign-Up %
tae kwon do	45%
yoga	30%
swimming	15%
kick-boxing	10%

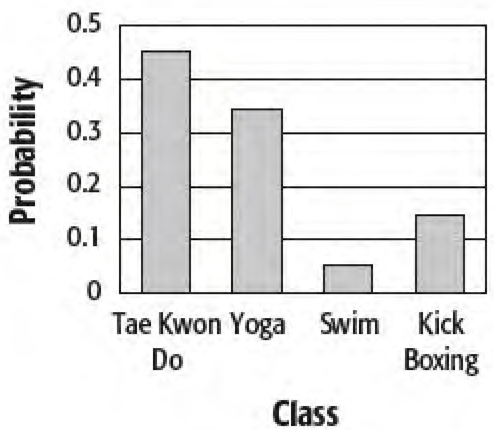
SOLUTION:

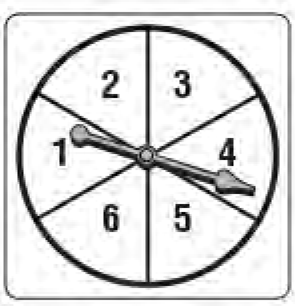
Use a random number generator to generate integers 1 through 20 where 1–9 represents tae kwon do, 10–15 represents yoga, 16–18 represents swimming, and 19–20 represents kick-boxing. Simply use the first 9 to represent 45%, the next 6, or 10-15, to represent 30%, and so on. We could also multiply all of these values by 5 and generate the integers 1-100.

Do 20 trials and record the results in a frequency table.

Outcome	Frequency
tae kwon do	9
yoga	7
swimming	1
kick-boxing	3
Total	20

Divide each frequency by 20 to get the corresponding probability.



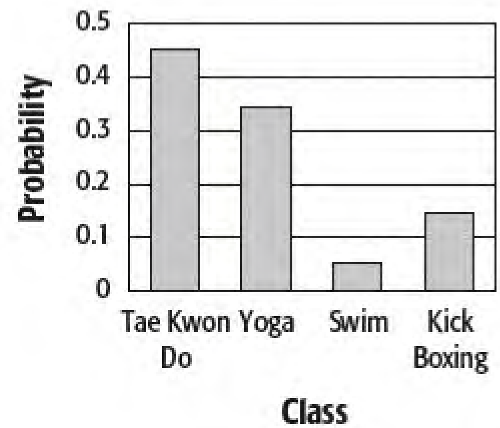


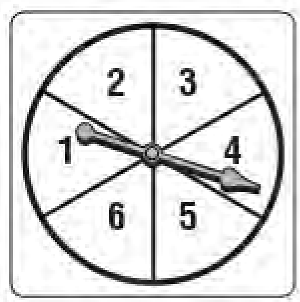
The probability of a customer taking the tae kwon do class is 0.45, taking yoga is 0.35, swimming is 0.05, and taking kickboxing is 0.15.

ANSWER:

Sample answer: Use a random number generator to generate integers 1 through 20 where 1–9 represents tae kwon do, 10–15 represents yoga, 16–18 represents swimming, and 19–20 represents kick-boxing. Do 20 trials and record the results in a frequency table.

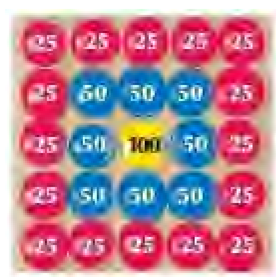
Outcome	Frequency
tae kwon do	9
yoga	7
swimming	1
kick-boxing	3
Total	20





The probability of a customer taking the tae kwon do class is 0.45, taking yoga is 0.35, swimming is 0.05, and taking kickboxing is 0.15.

3. **CARNIVAL GAMES** The object of the game shown is to accumulate points by using a dart to pop the balloons. Assume that each dart will hit a balloon.



a. Calculate the expected value from each throw.
b. Design a simulation and estimate the average value of this game.

c. How do the expected value and average value compare?

SOLUTION:

a. Find the sum of the products (outcome multiplied by the frequency) for each outcome. There are 25 total balloons, so there are 25 total outcomes.

$$\begin{aligned}&\text{Expected Value}\\&=\text{Sum of (Outcome} \times \text{Frequency)}\\&=\left(25\times\frac{16}{25}\right)+\left(50\times\frac{8}{25}\right)+\left(100\times\frac{1}{25}\right)\\&=16+16+4\\&=36\end{aligned}$$

b. With 25 outcomes, 16 will represent 25 points, 8 will represent 50 points, and 1 will represent 100 points.

Use a random number generator to generate integers 1 through 25 where 1–16 represents 25 points, 17–24 represents 50 points, and 25 represents 100 points. Do 50 trials and record the results in a frequency table.

Outcome	Frequency
25	29
50	21
100	0

The average value is calculated the same way as the expected value, except we use the simulation frequencies as opposed to the expected frequencies.

$$\begin{aligned}\text{average value}&=25\cdot\frac{29}{50}+50\cdot\frac{21}{50}+100\cdot\frac{0}{50}\\&=35.5\end{aligned}$$

c. Sample answer: The expected value and average value are very close. While this doesn't give much information about each individual outcome, it does tell us that the average for all of the trials was close to what was expected to happen.

ANSWER:

a. 36

b. Sample answer: Use a random number generator to generate integers 1 through 25 where 1–16 represents 25 points, 17–24 represents 50 points, and 25 represents 100 points. Do 50 trials and record the results in a frequency table.

Outcome	Frequency
25	29
50	21
100	0

The average value is 35.5.

c. Sample answer: The expected value and average value are very close.

Design and conduct a simulation using a geometric probability model. Then report the results using appropriate numerical and graphical summaries.

4. **BOWLING** Bridget is a member of the bowling club at her school. Last season she bowled a strike 60% of the time.

SOLUTION:

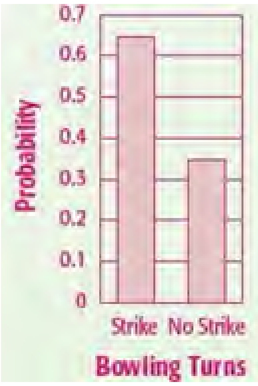
While we could use a random number generator for this, a spinner is also effective. We could also roll a 6-sided die, and let 1-3 represent a strike, 4-5 represent a non-strike, and roll again with each 6.

Use a spinner that is divided into two sectors, one containing 60% or 216° and the other containing 40% or 144°.

Sample answer: Do 20 trials and record the results in a frequency table.

Outcome	Frequency
Strike	13
Not a strike	7
Total	20

Divide the frequency by 20 to find the corresponding probability.

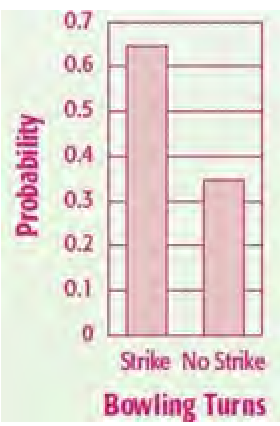


The probability of Bridget bowling a strike on her next turn is 0.65. The probability of bowling anything else is 1 – 0.65 or 0.35.

ANSWER:

Sample answer: Use a spinner that is divided into two sectors, one containing 60% or 216° and the other containing 40% or 144°. Do 20 trials and record the results in a frequency table.

Outcome	Frequency
Strike	13
Not a strike	7
Total	20



The probability of Bridget bowling a strike on her next turn is 0.65. The probability of bowling anything else is 1 – 0.65 or 0.35.

5. **VIDEO GAMES** Ian works at a video game store. Last year he sold 95% of the new-release video games.

SOLUTION:

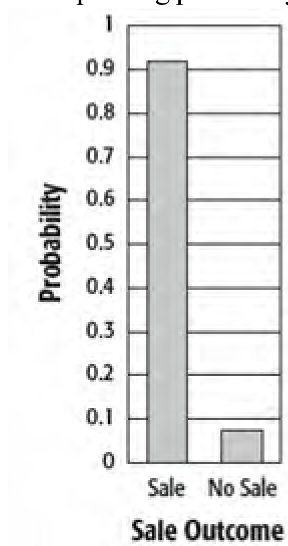
One option would be to create a spinner. Use a spinner that is divided into two sectors, one containing 95% or 342° and the other containing 5% or 18°.

Or you could use a random number generator to generate integers 1 through 20 where 1–19 represents a sale and 20 represents a no-sale. We could generate the integers 1-100 and let 96-100 represent a no-sale.

Do 50 trials and record the results in a frequency table.

Outcome	Frequency
Sale	46
No Sale	4
Total	50

Divide each frequency by 50 to find the corresponding probability.

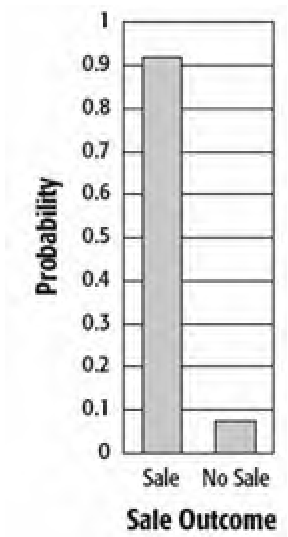


The probability of Ian selling a game is 0.92. The probability of not selling a game is 1 – 0.92 or 0.08.

ANSWER:

Sample answer: Use a spinner that is divided into two sectors, one containing 95% or 342° and the other containing 5% or 18°. Do 50 trials and record the results in a frequency table.

Outcome	Frequency
Sale	46
No Sale	4
Total	50



The probability of Ian selling a game is 0.92. The probability of not selling a game is 1 – 0.92 or 0.08.

6. **MUSIC** Kadisha is listening to a CD with her CD player set on the random mode. There are 10 songs on the CD.

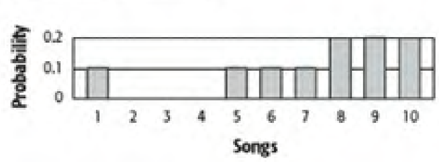
SOLUTION:

Sample answer: Use a spinner that is divided into 10 equal sectors, each 36°.

Or you could use a random number generator to generate integers 1 through 10 where each value represents a different song.

Do 10 trials and record the results in a frequency table.

Outcome	Frequency
Song 1	1
Song 2	0
Song 3	0
Song 4	0
Song 5	1
Song 6	1
Song 7	1
Song 8	2
Song 9	2
Song 10	2
Total	10

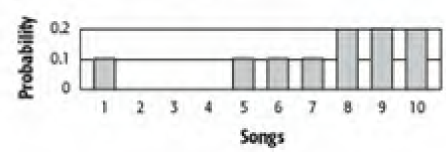


The probability of hearing song 1 is 0.1, songs 2–4 is 0, songs 5–7 is 0.1, and songs 8–10 is 0.2.

ANSWER:

Sample answer: Use a spinner that is divided into 10 equal sectors, each 36°. Do 10 trials and record the results in a frequency table.

Outcome	Frequency
Song 1	1
Song 2	0
Song 3	0
Song 4	0
Song 5	1
Song 6	1
Song 7	1
Song 8	2
Song 9	2
Song 10	2
Total	10



The probability of hearing song 1 is 0.1, songs 2–4 is 0, songs 5–7 is 0.1, and songs 8–10 is 0.2.

7. **BOARD GAMES** Pilar is playing a board game with eight different categories, each with questions that must be answered correctly in order to win.

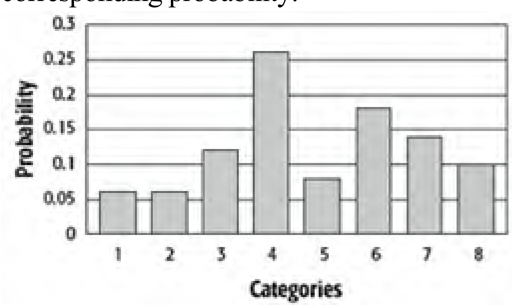
SOLUTION:

Sample answer: Use a spinner that is divided into 8 equal sectors, each 45°. Another option would be to use a random number generator from 1-8 where each value represents a different category.

Do 50 trials and record the results in a frequency table.

Outcome	Frequency
Category 1	3
Category 2	3
Category 3	6
Category 4	13
Category 5	4
Category 6	9
Category 7	7
Category 8	5
Total	50

Divide the frequency by 50 to find each corresponding probability.

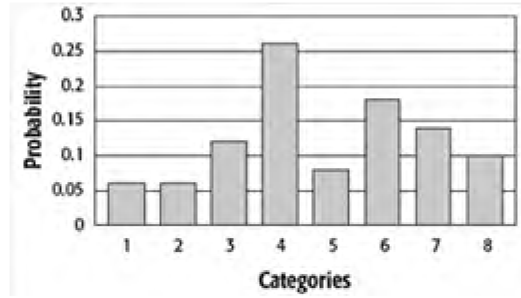


The probability of landing on Categories 1 and 2 is 0.06, Category 3 is 0.12, Category 4 is 0.26, Category 5 is 0.08, Category 6 is 0.18, Category 7 is 0.14, and Category 8 is 0.1.

ANSWER:

Sample answer: Use a spinner that is divided into 8 equal sectors, each 45°. Do 50 trials and record the results in a frequency table.

Outcome	Frequency
Category 1	3
Category 2	3
Category 3	6
Category 4	13
Category 5	4
Category 6	9
Category 7	7
Category 8	5
Total	50



The probability of landing on Categories 1 and 2 is 0.06, Category 3 is 0.12, Category 4 is 0.26, Category 5 is 0.08, Category 6 is 0.18, Category 7 is 0.14, and Category 8 is 0.1.

CCSS MODELING Design and conduct a simulation using a random number generator. Then report the results using appropriate numerical and graphical summaries.

8. **MOVIES** A movie theater reviewed sales from the previous year to determine which genre of movie sold the most tickets.

Genre	Ticket %
Drama	40%
Mystery	30%
Comedy	25%
Action	5%

SOLUTION:

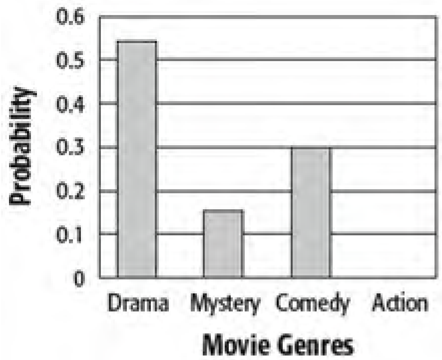
Use a random number generator.

Since the values are given in percents, we could generate the integers 1-100 and let 1-40 represent drama, 41-70 represent mystery, 71-95 represent comedy, and 96-100 represent action. Simply use the first 40 to represent 40%, the next 30, or 41-70, to represent 30%, and so on. We could also divide all of these values by 5 and generate the integers 1-20.

Do 20 trials and record the results in a frequency table.

Outcome	Frequency
drama	11
mystery	3
comedy	6
action	0
Total	20

Divide each frequency by 20 to get the corresponding probability.

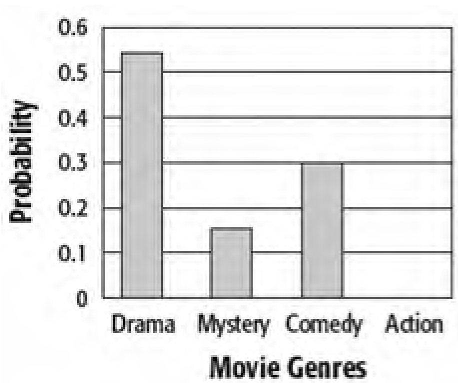


The probability of a customer choosing a drama is 0.55, choosing a mystery is 0.15, choosing a comedy is 0.3, and choosing an action film is 0.

ANSWER:

Sample answer: Use a random number generator to generate integers 1 through 20, where 1–8 represents drama, 9–14 represents mystery, 15–19 represents comedy, and 20 represents action. Do 20 trials and record the results in a frequency table.

Outcome	Frequency
drama	11
mystery	3
comedy	6
action	0
Total	20



The probability of a customer choosing a drama is 0.55, choosing a mystery is 0.15, choosing a comedy is 0.3, and choosing an action film is 0.

9. **BASEBALL** According to a baseball player's on-base percentages, he gets a single 60% of the time, a double 25% of the time, a triple 10% of the time, and a home run 5% of the time.

SOLUTION:

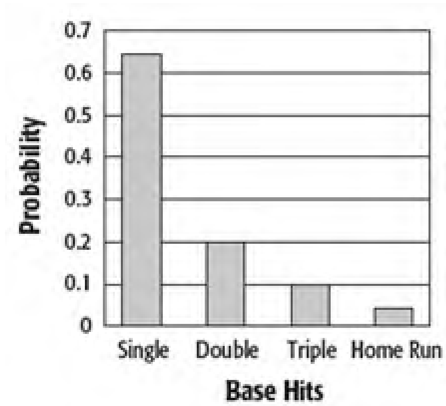
Use a random number generator.

Since the values are given in percents, we could generate the integers 1-100 and let 1-60 represent a single, 61-85 represent a double, 86-95 represent a triple, and 96-100 represent a home run. Simply use the first 60 to represent 60%, the next 25, or 61-85, to represent 25%, and so on. We could also divide all of these values by 5 and generate the integers 1-20.

Do 20 trials and record the results in a frequency table.

Outcome	Frequency
single	13
double	4
triple	2
home run	1
Total	20

Divide each frequency by 20 to get the corresponding probability.

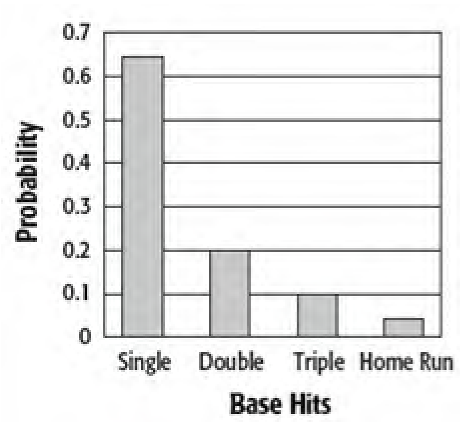


The probability of the baseball player hitting a single is 0.65, a double is 0.2, a triple is 0.1, and a home run is 0.05.

ANSWER:

Sample answer: Use a random number generator to generate integers 1 through 20, where 1–12 represents a single, 13–17 represents a double, 18–19 represents a triple, and 20 represents a home run. Do 20 trials and record the results in a frequency table.

Outcome	Frequency
single	13
double	4
triple	2
home run	1
Total	20



The probability of the baseball player hitting a single is 0.65, a double is 0.2, a triple is 0.1, and a home run is 0.05.

10. **VACATION** According to a survey done by a travel agency, 45% of their clients went on vacation to Europe, 25% went to Asia, 15% went to South America, 10% went to Africa, and 5% went to Australia.

SOLUTION:

Use a random number generator.

Since the values are given in percents, we could generate the integers 1-100 and let 1-45 represent Europe, 46-70 represent Asia, 71-85 represent South America, 86-95 represent Africa, and 96-100 represent Australia. Simply use the first 45 to represent 45%, the next 25, or 46-70, to represent 25%, and so on. We could also divide all of these values by 5 and generate the integers 1-20.

Do 20 trials and record the results in a frequency table.

Outcome	Frequency
Europe	7
Asia	6
South America	5
Africa	2
Australia	0
Total	20

Divide each frequency by 20 to get the corresponding probability.

The probability of a customer traveling to Europe is 0.35, to Asia is 0.3, to South America is 0.25, to Africa is 0.1, and to Australia is 0.

ANSWER:

Sample answer: Use a random number generator to generate integers 1 through 20 where 1-9 represents Europe, 10–14 represents Asia, 15–17 represents South America, and 18–19 represents Africa, and 20 represents Australia. Do 20 trials and record the results in a frequency table.

Outcome	Frequency
Europe	7
Asia	6
South America	5
Africa	2
Australia	0
Total	20

The probability of a customer traveling to Europe is 0.35, to Asia is 0.3, to South America is 0.25, to Africa is 0.1, and to Australia is 0.

11. **TRANSPORTATION** A car dealership's analysis indicated that 35% of the customers purchased a blue car, 30% purchased a red car, 15% purchased a white car, 15% purchased a black car, and 5% purchased any other color.

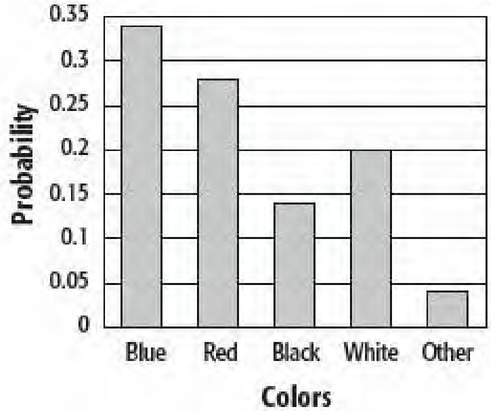
SOLUTION:
Use a random number generator.

Since the values are given in multiples of 5%, we could generate the integers 1-20 and let 1 - 7 represents blue, 8 -13 represents red, 14 - 16 represents white, 17 - 19 represents black, and 20 represents all other colors.

Do 50 trials and record the results in a frequency table.

Outcome	Frequency
blue	17
red	14
black	7
white	10
other	2
Total	50

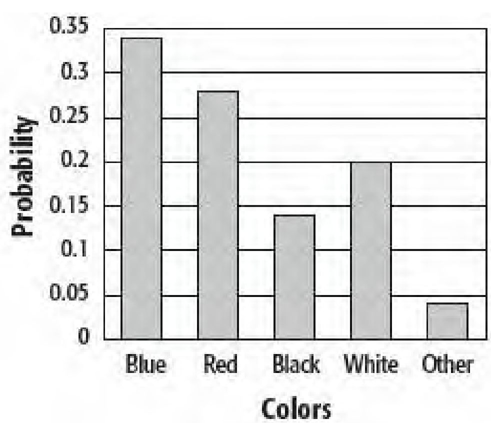
Divide each frequency by 50 to get the corresponding probability.



The probability of a customer buying a blue car is 0.34, buying a red car is 0.28, buying a black car is 0.14, buying a white car is 0.2, and any other color is 0.04.

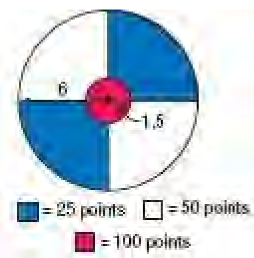
ANSWER:
Sample answer: Use a random number generator to generate integers 1 through 20, where 1 – 7 represents blue, 8 – 13 represents red, 14 – 16 represents white, 17 – 19 represents black, and 20 represents all other colors. Do 50 trials and record the results in a frequency table.

Outcome	Frequency
blue	17
red	14
black	7
white	10
other	2
Total	50



The probability of a customer buying a blue car is 0.34, buying a red car is 0.28, buying a black car is 0.14, buying a white car is 0.2, and any other color is 0.04.

DARTBOARDS The dimensions of each dartboard below are given in inches. There is only one shot per game. Calculate the expected value of each dart game. Then design a simulation to estimate each game's average value. Compare the average and expected values.

12. 

SOLUTION:

Find the geometric probability of red, blue and white regions. The area of the big circle is 36π . The area of the red circle is 2.25π .

$$P(\text{red}) = \frac{\text{area(red)}}{\text{area(circle)}} = \frac{2.25\pi}{36\pi} = 0.0625$$

$$P(\text{white}) = \frac{\left(\frac{\text{area(circle)} - \text{area(red)}}{2}\right)}{\text{area(circle)}} = \frac{\left(\frac{36\pi - 2.25\pi}{2}\right)}{36\pi} = 0.46875$$

$$P(\text{blue}) = \frac{\left(\frac{\text{area(circle)} - \text{area(red)}}{2}\right)}{\text{area(circle)}} = \frac{\left(\frac{36\pi - 2.25\pi}{2}\right)}{36\pi} = 0.46875$$

$$E = P(red) \times \text{value} + P(\text{white}) \times \text{value} + P(\text{blue}) \times \text{value}$$
$$E = (0.0625 \times 100) + (0.46875 \times 50) + (0.46875 \times 25) \approx 41.4$$

Now run a simulation and calculate the average value.

Sample answer:

Outcome	Frequency
red	7
blue	20
white	23
total	50

$$\text{average} = \left(100 \times \frac{7}{50}\right) + \left(25 \times \frac{20}{50}\right) + \left(50 \times \frac{23}{50}\right) = 47$$

So, the average value is greater than the expected value.

ANSWER:

$E(Y) = 41.4$

Sample answer:

Outcome	Frequency
red	7
blue	20
white	23
total	50

Average value = 47; the average value is greater than the expected value.

13. 

SOLUTION:

Find the geometric probability of red, blue and white regions. The area of the big circle is 25π. The area of the red circle is 0.25π.

$$P(\text{red}) = \frac{\text{area(red)}}{\text{area(circle)}} = \frac{0.25\pi}{25\pi} = 0.01$$

$$P(\text{white}) = \frac{\left(\frac{\text{area(circle)} - \text{area(red)}}{2}\right)}{\text{area(circle)}} = \frac{\left(\frac{25\pi - 0.25\pi}{2}\right)}{25\pi} = \frac{12.375\pi}{25\pi} = 0.495$$

$$P(\text{blue}) = \frac{\left(\frac{\text{area(circle)} - \text{area(red)}}{2}\right)}{\text{area(circle)}} = \frac{\left(\frac{25\pi - 0.25\pi}{2}\right)}{25\pi} = \frac{12.375\pi}{25\pi} = 0.495$$

$$E = P(\text{red}) \times \text{value} + P(\text{white}) \times \text{value} + P(\text{blue}) \times \text{value}$$
$$E = (0.01 \times 100) + (0.495 \times 50) + (0.495 \times 25) \approx 38.1$$

Now run a simulation and calculate the average value.

Sample answer:

Outcome	Frequency
red	0
blue	29
white	21
total	50

$$\text{average} = \left(100 \times \frac{0}{50}\right) + \left(25 \times \frac{29}{50}\right) + \left(50 \times \frac{21}{50}\right) = 35.5$$

The expected value is greater than the average value.

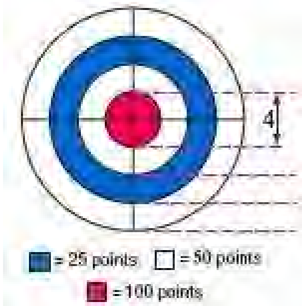
ANSWER:

$E(Y) \approx 38.1$

Sample answer:

Outcome	Frequency
red	0
blue	29
white	21
total	50

Average value = 35.5; the expected value is greater than the average value.

14. 

SOLUTION:

Find the geometric probability of red, blue and white regions. The area of the big circle is 64π. The area of the red circle is 4π.

$$P(\text{red}) = \frac{\text{area(red)}}{\text{area(circle)}} = \frac{4\pi}{64\pi} = 0.0625$$

For this section, find the area of the small white ring by subtracting the red circle from the circle that includes the ring. Do the same for the large white ring. Then add those values.

$$P(\text{white}) = \frac{\text{area(white)} - \text{area(red)} + \text{area(white)} - \text{area(blue)}}{\text{area(circle)}} = \frac{16\pi - 4\pi + 64\pi - 36\pi}{64\pi} = \frac{40\pi}{64\pi} = 0.625$$

$$P(\text{blue}) = \frac{\text{area(blue)} - \text{area(white)}}{\text{area(circle)}}$$
$$= \frac{36\pi - 16\pi}{64\pi}$$
$$= \frac{20\pi}{64\pi}$$
$$= 0.3125$$

$$E = P(red) \times \text{value} + P(\text{white}) \times \text{value} + P(\text{blue}) \times \text{value}$$
$$E = (0.0625 \times 100) + (0.625 \times 50) + (0.3125 \times 25)$$
$$\approx 45.3$$

Now run a simulation and calculate the average value.

Sample answer:

Outcome	Frequency
red	5
blue	16
white	29
total	50

$$\text{average} = \left(100 \times \frac{5}{50}\right) + \left(25 \times \frac{16}{50}\right) + \left(50 \times \frac{29}{50}\right)$$
$$= 10 + 8 + 29$$
$$= 47$$

The average value is greater than the expected value.

ANSWER:

$E(Y) = 45.3$

Sample answer:

Outcome	Frequency
red	5
blue	16
white	29
total	50

Average value = 47; the average value is greater than the expected value.

15. **CARDS** You are playing a team card game where a team can get 0 points, 1 point, or 3 points for a hand. The probability of your team getting 1 point for a hand is 60% and of getting 3 points for a hand is 5%.

a. Calculate your team's expected value for a hand.
b. Design a simulation and estimate your team's average value per hand.
c. Compare the values for parts a and b.

SOLUTION:

a. The expected value is the sum of the product of each point value and its corresponding probability.

$$E = \text{sum(point value} \times \text{probability)}$$
$$= 0 + 1 \cdot \frac{60}{100} + 3 \cdot \frac{5}{100}$$
$$= 0 + 0.6 + 0.15$$
$$= 0.75$$

b. Sample answer: Use a random number generator to generate integers 1 through 20, where 1–7 represents 0 points, 8–19 represents 1 point, and 20 represents 3 points. Do 50 trials and record the results in a frequency table.

Outcome	Frequency
0	16
1	32
3	2

The average value is the sum of the product of each point value and its corresponding frequency.

$$\text{average} = 0 \cdot \frac{16}{50} + 1 \cdot \frac{32}{50} + 3 \cdot \frac{2}{50}$$
$$= 0 + 0.64 + 0.12$$
$$= 0.76$$

c. Sample answer: The two values are almost equal.

ANSWER:

a. 0.75
b. Sample answer: Use a random number generator to generate integers 1 through 20, where 1–7 represents 0 points, 8–19 represents 1 point, and 20 represents 3 points. Do 50 trials and record the results in a frequency table.

Outcome	Frequency
0	16
1	32
3	2

Average value = 0. 76

c. Sample answer: The two values are almost equal.

16. **DECISION MAKING** The object of the game shown is to win money by rolling a ball up an incline into regions with different payoff values. The probability that Susana will get \$0 in a roll is 55%, \$1 is 20%, \$2 is 20%, and \$3 is 5%.

a. Suppose Susana pays \$1 to play. Calculate the expected payoff, which is the expected value minus the cost to play, for each roll.

b. Design a simulation to estimate Susana's average payoff for this game after she plays 10 times.

c. Should Susana play this game? Explain your reasoning.

SOLUTION:

a. The expected value is:

$0(0.55) + 1(0.20) + 2(0.20) + 3(0.05) = 0.75$

The expected payoff is thus:

\$1 – \$0.75 = –\$0.25

b. Sample answer: Use a random number generator to generate integers 1 through 20, where 1–11 represents \$0, 12–15 represents \$1, 16–19 represents \$2, and 20 represents \$3. Do 10 trials and record the results in a frequency table.

Outcome	Frequency
\$0	4
\$1	3
\$2	3
\$3	0

The total amount won was 4(\$0) + 3(\$1) + 3(\$2) = \$9, and therefore the average amount won per trial was \$9/10 = \$0.90.

The average payoff after 10 trials is \$0.90 – \$1 or – \$0.10.

c. For both the theoretical and expected payoff we calculated negative values. It's not recommended that Susana play the game.

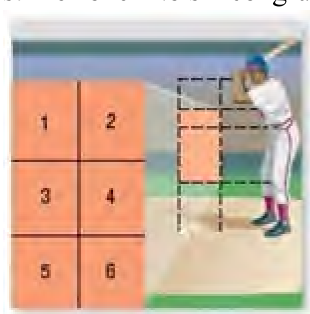
ANSWER:

a. –\$0.25

b. Sample answer: Use a random number generator to generate integers 1 through 20, where 1–11 represents \$0, 12–15 represents \$1, 16–19 represents \$2, and 20 represents \$3. The average payoff after 10 trials is \$0.90 – \$1 or –\$0.10.

c. Sample answer: No, both the theoretical and simulated expected payoff are less than what it costs to play the game.

17. **BASEBALL** Of his pitches thrown for strikes, a baseball pitcher wants to track which areas of the strike zone have a higher probability. He divides the strike zone into six congruent boxes as shown.



a. If a strike is equally likely to hit each box, what is the probability that he will throw a strike in each box?

b. Design a simulation to estimate the probability of a strike being thrown in each box.

c. Compare the values for parts a and b.

SOLUTION:

a. Total number of congruent boxes = 6

There is a $\frac{1}{6}$ or 16.7% probability of throwing a strike in each box.

b. Sample answer: Since there are six equal outcomes, a die with the numbers 1 through 6 can be rolled to simulate each strike that is thrown. Roll the die 100 times and record the number on the die to indicate in which of the six boxes each strike was thrown. Record the percent of times a strike is thrown in each area.

Strike Area	Accuracy (%)
1	15
2	17
3	19
4	22
5	19
6	8
Total	100

c. Sample answer: Some of the values are higher or lower, but most are very close to 16.7%.

ANSWER:

a. There is a $\frac{1}{6}$ or 16.7% probability of throwing a strike in each box.

b. Sample answer: Since there are six equal outcomes, a die with the numbers 1 through 6 can be rolled to simulate each strike that is thrown. Roll the die 100 times and record the number on the die to indicate in which of the six boxes each strike was thrown. Record the percent of times a strike is thrown in each area.

Strike Area	Accuracy (%)
1	15
2	17
3	19
4	22
5	19
6	8
Total	100

c. Sample answer: Some of the values are higher or lower, but most are very close to 16.7%.

18. **CCSS MODELING** Cynthia used her statistics from last season to design a simulation using a random number generator to predict what she would score each time she got possession of the ball.

Integer Values	Points Scored	Frequency
1–14	0	31
15	1	0
16–28	2	17
29–30	3	2

a. Based on the frequency table, what did she assume was the theoretical probability that she would score two points in a possession?
b. What is Cynthia's average value for a possession? her expected value?
c. Would you expect the simulated data to be different? If so, explain how. If not, explain why

SOLUTION:

a. Number of random integers (from 1 to 30) = 30
Number of random integer values for the score 2 (from 16 to 28) = 13
So, the theoretical probability that she will score two points in a possession is $\frac{13}{30}$ or 43.3%.

b. The average value of a possession is the sum of the products of the number of points scored (0, 1, 2, or 3) times the experimental probability.
The total number of trials is 31 + 0 + 17 + 2 = 50. So the probability of scoring 0 is $\frac{31}{50}$, the probability of scoring 1 is 0, the probability of scoring 2 is $\frac{17}{50}$, and the probability of scoring 3 is $\frac{31}{50}$.

$$\begin{aligned}\text{average value} &= 0\cdot\frac{31}{50}+1\cdot\frac{0}{50}+2\cdot\frac{17}{50}+3\cdot\frac{2}{50}\\ &= 0+0+0.68+0.12\\ &= 0.8\end{aligned}$$

The expected value is the sum of the products of the number of points scored (0, 1, 2, or 3) times the theoretical probability. The theoretical probability is determined by the number of random integers that are used to simulate each possible number of points. The theoretical probability of scoring 0 is 14 out of 30 or $\frac{14}{30}$, the probability of 1 point is $\frac{1}{30}$, the probability of 2 points is $\frac{13}{30}$, and the probability of 3 points is $\frac{14}{30}$.

$$\begin{aligned}\text{Expected value} &= 0\cdot\frac{14}{30}+1\cdot\frac{1}{30}+2\cdot\frac{13}{30}+3\cdot\frac{2}{30}\\ &= 0+\frac{33}{30}\\ &= 1.1\end{aligned}$$

c. Sample answer: Each time a simulation is run, the results are most likely different from the previous results. Usually they are very similar, but occasionally they can be quite different. Yes; I would have expected fewer values between 1 and 14 and more values between 16 and 28. The theoretical probability of scoring 2 points in a possession is about 9% higher than the experimental probability and the theoretical probability of scoring 0 points in a possession is about 15% lower than the experimental probability.

ANSWER:

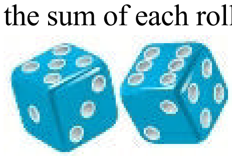
a. $\frac{13}{30}$ or 43.3%

b. 0.8; 1.1

c. Sample answer: Yes; I would have expected fewer values between 1 and 14 and more values between 16 and 28. The theoretical probability of scoring 2 points in a possession is about 9% higher than the experimental probability and the theoretical probability of scoring 0 points in a possession is about 15% lower than the experimental probability.

19. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate expected value.

a. CONCRETE Roll two dice 20 times and record the sum of each roll.



b. NUMERICAL Use the random number generator on a calculator to generate 20 pairs of integers between 1 and 6. Record the sum of each pair.

c. TABULAR Copy and complete the table below using your results from parts a and b.

Trial	Sum of Die Roll	Sum of Output from Random Number Generator
1		
2		
...		
20		

d. GRAPHICAL Use a bar graph to graph the number of times each possible sum occurred in the first 5 rolls. Repeat the process for the first 10 rolls and then all 20 outcomes.

e. VERBAL How does the shape of the bar graph change with each additional trial?

f. GRAPHICAL Graph the number of times each possible sum occurred with the random number generator as a histogram.

g. VERBAL How do the graphs of the die trial and the random number trial compare?

h. ANALYTICAL Based on the graphs, what do you think the expected value of each experiment would be? Explain your reasoning.

SOLUTION:

a. Sample answer: 9, 10, 6, 6, 7, 9, 5, 9, 7, 6, 5, 7, 3, 9, 7, 6, 7, 8, 7

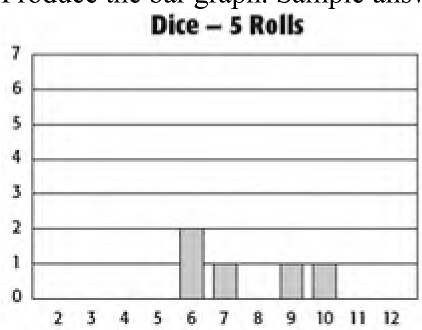
b. Sample answer: 4, 10, 5, 10, 6, 7, 12, 3, 7, 4, 7, 9, 3, 6, 4, 11, 5, 7, 5, 3

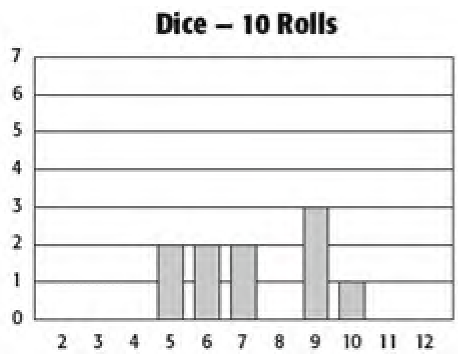
c. Transfer the values from parts a and b.

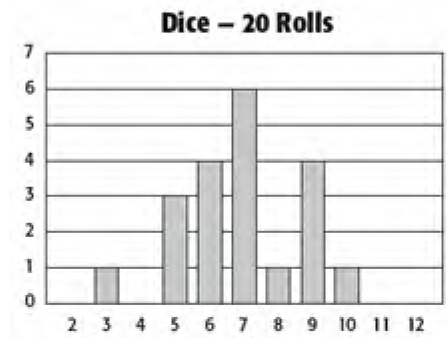
Sample answer:

Trial	Sum of Die Roll	Sum of Output from Random Number Generator
1	9	4
2	10	10
3	6	5
4	6	10
5	7	6
6	9	7
7	5	12
8	9	3
9	5	7
10	7	4
11	6	7
12	5	9
13	7	3
14	3	6
15	9	4
16	7	11
17	6	5
18	7	7
19	8	5
20	7	3

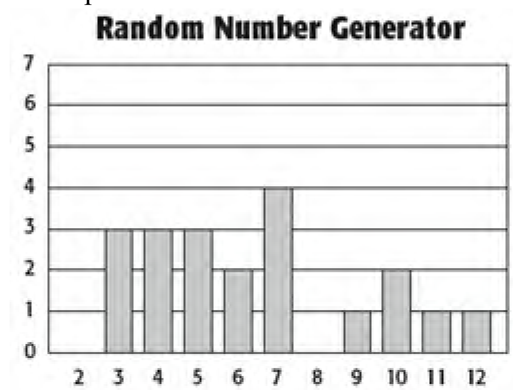
d. Enter the data values into L1 of your calculator. Produce the bar graph. Sample answer:







e. Sample answer: The bar graph has more data points at the middle sums as more trials are added.
f. Sample answer:



g. Sample answer: They both have the most data points at the middle sums.
h. Sample answer: The expected value in both experiments is 7 because it is the sum that occurs most frequently.

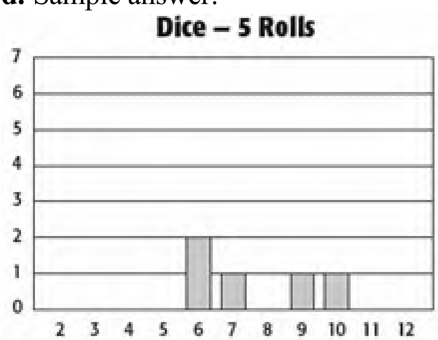
ANSWER:

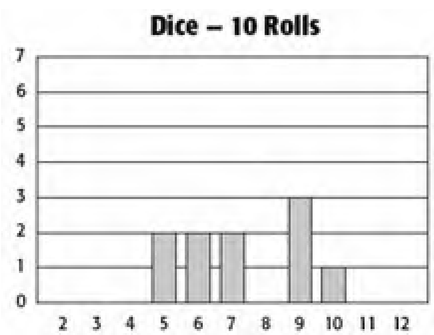
a. Sample answer: 9, 10, 6, 6, 7, 9, 5, 9, 7, 6, 5, 7, 3, 9, 7, 6, 7, 8, 7
b. Sample answer: 4, 10, 5, 10, 6, 7, 12, 3, 7, 4, 7, 9, 3, 6, 4, 11, 5, 7, 5, 3
c. Sample answer:

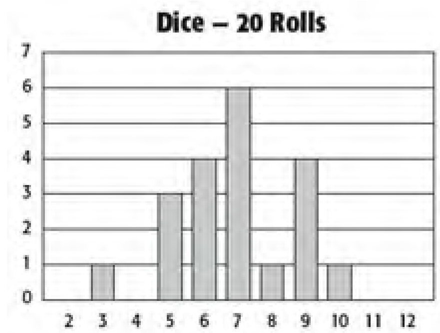
13-4 Simulations

Trial	Sum of Die Roll	Sum of Output from Random Number Generator
1	9	4
2	10	10
3	6	5
4	6	10
5	7	6
6	9	7
7	5	12
8	9	3
9	5	7
10	7	4
11	6	7
12	5	9
13	7	3
14	3	6
15	9	4
16	7	11
17	6	5
18	7	7
19	8	5
20	7	3

d. Sample answer:

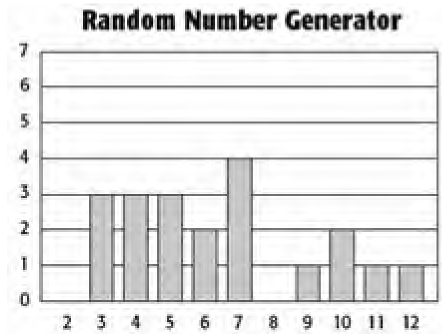






e. Sample answer: The bar graph has more data points at the middle sums as more trials are added.

f. Sample answer:



g. Sample answer: They both have the most data points at the middle sums.

h. Sample answer: The expected value in both experiments is 7 because it is the sum that occurs most frequently.

20. **CCSS ARGUMENTS** An experiment has three equally likely outcomes *A*, *B*, and *C*. Is it possible to use the spinner shown in a simulation to predict the probability of outcome *C*? Explain your reasoning.

SOLUTION:

There are 3 equally likely outcomes, so the probability of success of each individual outcome is one-third. The diagram is split into two pieces, one at 120 degrees. 120 degrees is one-third of the circle, so the diagram is broken into a one-third piece and a two-thirds piece. Therefore, the 120 degree area, or the pink sector, can represent one outcome while the other sector can represent the sum of the other two outcomes.

If the spinner were going to be divided equally into three outcomes, each sector would measure 120. Since you only want to know the probability of outcome C, you can record spins that end in the pink area as a success, or the occurrence of outcome C, and spins that end in the blue area as a failure, or an outcome of A or B.

ANSWER:

Yes; sample answer: If the spinner were going to be divided equally into three outcomes, each sector would measure 120. Since you only want to know the probability of outcome C, you can record spins that end in the pink area as a success, or the occurrence of outcome C, and spins that end in the blue area as a failure, or an outcome of A or B.

21. **REASONING** Can tossing a coin *sometimes*, *always*, or *never* be used to simulate an experiment with two possible outcomes? Explain.

SOLUTION:

Sometimes; sample answer: Flipping a coin can be used to simulate and experiment with two possible outcomes when both of the outcomes are equally likely. If the probabilities of the occurrence of the two outcomes are different, flipping a coin is not an appropriate simulation.

For example, flipping a coin can be used to simulate who will get the ball first at the beginning of a football game, while flipping a coin cannot be used to simulate who will win the game.

ANSWER:

Sometimes; sample answer: Flipping a coin can be used to simulate and experiment with two possible outcomes when both of the outcomes are equally likely. If the probabilities of the occurrence of the two outcomes are different, flipping a coin is not an appropriate simulation.

22. **DECISION MAKING** A lottery consists of choosing 5 winning numbers from 31 possible numbers (0–30). The person who matches all 5 numbers, in any sequence, wins $1 million.
a. If a lottery ticket costs $1, should you play? Explain your reasoning by computing the expected payoff value, which is the expected value minus the ticket cost.
b. Would your decision to play change if the winnings increased to $5 million? if the winnings were only $0.5 million, but you chose from 21 numbers instead of 31 numbers? Explain.

SOLUTION:

a. Since the order of the numbers does not matter, the total number of combinations of tickets is:

$$ {}_{31}C_5 = \frac{31!}{(31-5)!5!} $$
$$ = 169,911 $$

The expected value is then:

$$\$1,000,000\left(\frac{1}{169{,}911}\right) \approx \$5.89$$

And the expected payoff:
$5.89 – $1.00 = $4.89

The expected payoff is positive, but not very high, so you may not want to play the game.

b. If the winnings were increased to $5,000,000 then the expected value would be:
$$\$5,000,000\left(\frac{1}{169{,}911}\right) \approx \$29.43$$
And the expected payoff: $29.43 – $1.00 = $28.43.

If the winnings were only $0.5 million and you only chose from 21 numbers, the expected value would be:
$$\$500,000\left(\frac{1}{{}_{21}C_5}\right) = \$500,000\left(\frac{1}{20{,}349}\right)$$
$$= \$24.57$$

And the expected payoff: $24.57 – $1.00 = $23.57.

For both cases, the expected payoff is relatively high. I would play.

ANSWER:
a. Sample answer: The expected value is $\$1,000,000\left(\frac{1}{{}_{31}C_5}\right) \approx \$5.89$.
The expected payoff is $5.89 – $1 or $4.89, which means that on average, a person can expect to gain $4.89 for every $1 he or she spends to buy a ticket. This is a low payoff value, so I may not play.

b. Sample answer: The expected value is $\$5,000,000\left(\frac{1}{{}_{31}C_5}\right) \approx \$29.43$. The expected payoff is $29.43 – $1 or $28.43. This is a higher payoff, so I would play. The expected value from playing for $0.5 million but only having to choose from 21 numbers is $\$500,000\left(\frac{1}{{}_{21}C_5}\right) \approx \$24.57$.
The expected payoff is $24.57 – $1 or $23.57. This is still a higher payoff value, so I would play.

23. **REASONING** When designing a simulation where darts are thrown at targets, what assumptions need to be made and why are they needed?

SOLUTION:
Think about throwing darts. How would this be different than flipping a coin?

Sample answer: We assume that the object lands within the target area, and that it is equally likely that the object will land anywhere in the region. These are needed because in the real world it will not be equally likely to land anywhere in the region.

ANSWER:
Sample answer: We assume that the object lands within the target area, and that it is equally likely that the object will land anywhere in the region. These are needed because in the real world it will not be equally likely to land anywhere in the region.

24. **OPEN ENDED** Describe an experiment in which the expected value is not a possible outcome. Explain.

SOLUTION:
Sample answer: Rolling a die has an expected value that is not a possible outcome. Each of the six faces of the die is equally likely to occur, so the probability of each is $\frac{1}{6}$. The expected value is $\frac{1}{6}\cdot 1+\frac{1}{6}\cdot 2+\frac{1}{6}\cdot 3+\frac{1}{6}\cdot 4+\frac{1}{6}\cdot 5+\frac{1}{6}\cdot 6$ or 3.5. Since 3.5 is not a possible outcome, the expected value is not a possible outcome.

ANSWER:
Sample answer: Rolling a die has an expected value that is not a possible outcome. Each of the six faces of the die is equally likely to occur, so the probability of each is $\frac{1}{6}$. The expected value is $\frac{1}{6}\times 1+\frac{1}{6}\times 2+\frac{1}{6}\times 3+\frac{1}{6}\times 4+\frac{1}{6}\times 5+\frac{1}{6}\times 6 = 3.5$. Since 3.5 is not a possible outcome, the expected value is not a possible outcome.

13-4 Simulations

25. **WRITING IN MATH** How is expected value different from probability?

SOLUTION:

Consider an event such as rolling a die. Think of different probabilities and the expected value and compare the terms.

The probability of rolling any number is $\frac{1}{6}$. The probability of rolling an even number is the same as the probability of rolling an odd number. We also know that their sum must be 1, so the probability of rolling an even number is $\frac{1}{2}$.

The expected outcome of rolling a die is the sum of all possible outcomes multiplied by their probability:

$$E\text{(Rolling a die)} = \frac{1}{6}(1) + \frac{1}{6}(2) + \frac{1}{6}(3) + \frac{1}{6}(4) + \frac{1}{6}(5) + \frac{1}{6}(6)$$

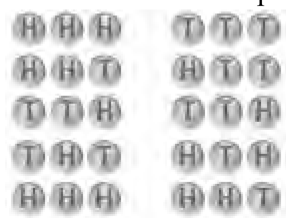
$$\begin{aligned} E\text{(Rolling a die)} &= \frac{1}{6}(1) + \frac{1}{6}(2) + \frac{1}{6}(3) + \frac{1}{6}(4) + \frac{1}{6}(5) + \frac{1}{6}(6) \\ &= \frac{1}{6}(1+2+3+4+5+6) \\ &= \frac{1}{6}(21) \\ &= 3.5 \end{aligned}$$

From this we can see that the probability of any given outcome is always less than or equal to 1, with the sum of probabilities for all possible outcomes equal to 1. The expected value can be greater than 1, and is the value we expect to get given the probabilities of each outcome.

ANSWER:

Sample answer: Expectation, or expected value, deals with all possible events, while probability typically deals with only one event. Probability is the likelihood that an event will happen, is between 0 and 1 inclusive, and is typically expressed as a decimal, fraction, or percent. Expectation is the weighted average of all possible events and can take on any value, even a value that is not a possible outcome. For example, the likelihood of rolling a "1" with a six-sided die is 1 out of every 6 rolls. So, the probability is $\frac{1}{6}$ or about 17%. When the same die is rolled, the expectation is 3.5, which is not a possible outcome.

26. **PROBABILITY** Kaya tosses three coins at the same time and repeats the process 9 more times. Her results are shown below where H represents heads and T represents tails. Based on Kaya's data, what is the probability that at least one of the group of 3 coins will land with heads up?



A 0.1
B 0.2
C 0.3
D 0.9

SOLUTION:

The probability is defined as the ratio of number of favorable outcomes to the number of possible outcomes. The number of possible outcomes is 10 and the number of favorable outcomes is 9.

$$\begin{aligned} P(\text{atleast one head}) &= \frac{9}{10} \\ &= 0.9 \end{aligned}$$

So, the correct choice is D.

ANSWER:

D

27. **ALGEBRA** Paul collects comic books. He has 20 books in his collection, and he adds 3 per month. In how many months will he have a total of 44 books in his collection?

F 5
G 6
H 8
J 15

SOLUTION:

Let x be the number of months required to collect 44 books.

$$\begin{aligned} 20 + 3x &= 44 \\ 3x &= 24 \\ x &= 8 \end{aligned}$$

The correct choice is H.

ANSWER:

H

28. **SHORT RESPONSE** Alberto designed a simulation to determine how many times a player would roll a number higher than 4 on a die in a board game with 5 rolls. The table below shows his results for 50 trials. What is the probability that a player will roll a number higher than 4 two or more times in 5 rolls?

Number of Rolls Greater Than 4	Frequency
0	8
1	15
2	18
3	9
4	0
5	0

SOLUTION:

$$\begin{aligned} P(\geq 2) &= P(2) + P(3) + P(4) + P5) \\ &= \frac{18}{50} + \frac{9}{50} + \frac{0}{50} + \frac{0}{50} \\ &= \frac{27}{50} \end{aligned}$$

ANSWER:

$\frac{27}{50}$

29. **SAT/ACT** If a jar contains 150 peanuts and 60 cashews, what is the approximate probability that a nut selected from the jar at random will be a cashew?

A 0.25
B 0.29
C 0.33
D 0.4
E 0.71

SOLUTION:

There are 60 outcomes in which a cashew can be chosen and 150 total cashews. The probability of selecting a cashew is 60 ÷ 150 = 0.33.

ANSWER:

B

Point X is chosen at random on $\overline{QT}$. Find the probability of each event.

30. $P(X \text{ is on } \overline{QS})$

SOLUTION:

$$\begin{aligned} P(X \text{ is on } \overline{QS}) &= \frac{\text{length of } QS}{\text{length of } QT} \\ &= \frac{6+3}{6+3+5} \\ &= \frac{9}{14} \end{aligned}$$

ANSWER:

$\frac{9}{14}$, 0.64, or 64%

31. $P(X \text{ is on } \overline{RT})$

SOLUTION:

$$\begin{aligned} P(X \text{ is on } \overline{RT}) &= \frac{\text{length of } RT}{\text{length of } QT} \\ &= \frac{3+5}{6+3+5} \\ &= \frac{8}{14} \\ &= \frac{4}{7} \end{aligned}$$

ANSWER:

$\frac{4}{7}$, 0.57, or 57%

32. **BOOKS** Paige is choosing between 10 books at the library. What is the probability that she chooses 3 particular books to check out from the 10 initial books?

SOLUTION:

Three books can be chosen from 10 books in ${}_{10}C_3$ ways and 3 particular books can be chosen in one way.

$${}_{10}C_3 = \frac{10!}{(10-3)!3!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 3 \cdot 2 \cdot 1} = 120$$

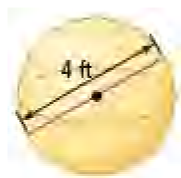
The three particular books can be chosen in only one way.

So, the number of possible outcomes is 120 and the number of favorable outcomes is 1. Therefore, the probability is $\frac{1}{120}$.

ANSWER:

$\frac{1}{120}$

Find the surface area of each figure. Round to the nearest tenth.

33. 

SOLUTION:

The surface area S of a sphere is $S = 4\pi r^2$, where r is the radius. The radius is 2 ft.

$$\begin{aligned} S &= 4\pi(2)^2 \\ &\approx 50.3 \text{ ft}^2 \end{aligned}$$

ANSWER:

50.3 ft^2

34.

SOLUTION:

To find the surface area of a hemisphere, find half the area of a sphere with a radius of 1.5 cm, and then add the area of the great circle.

The surface area S of a sphere is $S = 4\pi r^2$, where r is the radius.

$$\begin{aligned} S &= \frac{1}{2}\left(4\pi(1.5)^2\right) + \pi(1.5)^2 \\ &= 3\pi(1.5)^2 \\ &= 6.75\pi \\ &\approx 21.2 \text{ cm}^2 \end{aligned}$$

ANSWER:

21.2 cm^2

35.

SOLUTION:

The surface area S of a sphere is $S = 4\pi r^2$, where r is the radius.

$$\begin{aligned} S &= 4\pi(9)^2 \\ &\approx 1017.9 \text{ in}^2 \end{aligned}$$

ANSWER:

1017.9 in^2

36. **RECREATION** A group of 150 students was asked what they like to do during their free time.

a. How many students like going to the movies or shopping?
b. Which activity was mentioned by 37 students?
c. How many students did *not* say they like movies?

SOLUTION:

a. 20 + 3 + 4 + 10 + 5 + 75 or 117 students like going to the movies or shopping.
b. Find the sum of all of the values included in each bubble. Both the sports and shopping bubbles sum to 37.

c. 75 + 5 + 10 + 4 or 94 students like going to the movies.
So, 150 – 94 or 56 students didn't like movies.

ANSWER:

a. 117
b. sports or shopping
c. 56