13-3 Geometric Probability

Point X is chosen at random on $\overline{AD}$. Find the probability of each event.

1. $P(X \text{ is on } \overline{BD})$

SOLUTION:

$$\begin{aligned} P(X \text{ is on } \overline{BD}) &= \frac{\text{length of } BD}{\text{length of } AD} \\ &= \frac{5}{5+3+2} \\ &= \frac{5}{10} \\ &= \frac{1}{2} \end{aligned}$$

ANSWER:

$\frac{1}{2}$, 0.5, or 50%

2. $P(X \text{ is on } \overline{BC})$

SOLUTION:

$$\begin{aligned} P(X \text{ is on } \overline{BC}) &= \frac{\text{length of } BC}{\text{length of } AD} \\ &= \frac{3}{5+3+2} \\ &= \frac{3}{10} \end{aligned}$$

ANSWER:

$\frac{3}{10}$, 0.3, or 30%

3. **CARDS** In a game of cards, 43 cards are used, including one joker. Four players are each dealt 10 cards and the rest are put in a pile. If Greg doesn't have the joker, what is the probability that either his partner or the pile have the joker?

SOLUTION:

Greg has 10 cards, so there are 33 cards left. His partner has 10 cards and the pile has 3. Therefore, the probability is $\frac{10+3}{33} = \frac{13}{33}$.

ANSWER:

$\frac{13}{33}$, 0.39, or about 39%

4. **ARCHERY** An archer aims at a target that is 122 centimeters in diameter with 10 concentric circles whose diameters decrease by 12.2 centimeters as they get closer to the center. Find the probability that the archer will hit the center.

SOLUTION:

The diameter of the outermost circle is 122 cm. and the diameters decrease by 12.2 centimeters as they get closer to the center. So, the diameter of the inner most circle is 122 – 9(12.2) = 12.2.

$$P(\text{center}) = \frac{\text{area of center}}{\text{area of target}} = \frac{\pi(6.1)^2}{\pi(61)^2} = \frac{37.21\pi}{3721\pi} = \frac{1}{100}$$

ANSWER:

$\frac{1}{100}$, 0.01, or 1%

5. **NAVIGATION** A camper lost in the woods points his compass in a random direction. Find the probability that the camper is heading in the N to NE direction.

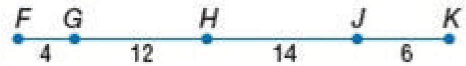
SOLUTION:

The compass is divided equally into 8 sectors of measure 45º each. The ratio of the area of a sector of a circle to the area of the entire circle is the same as the ratio of the sector's central angle to 360.The measure of the sector in N to NE direction is 45º. Therefore, the probability that the camper is heading in the N to NE direction is $\frac{45}{360} = \frac{1}{8} = 12.5\%$.

ANSWER:

$\frac{1}{8}$, 0.125, or 12.5%

CCSS REASONING Point X is chosen at random on $\overline{FK}$. Find the probability of each event.



6. $P(X \text{ is on } \overline{FH})$

SOLUTION:

$$\begin{aligned} P(X \text{ is on } \overline{FH}) &= \frac{\text{length of } FH}{\text{length of } FK} \\ &= \frac{16}{4+12+14+6} \\ &= \frac{16}{36} \\ &= \frac{4}{9} \end{aligned}$$

ANSWER:

$\frac{4}{9}$, 0.44, or 44%

7. $P(X \text{ is on } \overline{GJ})$

SOLUTION:

$$\begin{aligned} P(X \text{ is on } \overline{GJ}) &= \frac{\text{length of } GJ}{\text{length of } FK} \\ &= \frac{12+14}{4+12+14+6} \\ &= \frac{26}{36} \\ &= \frac{13}{18} \end{aligned}$$

ANSWER:

$\frac{13}{18}$, 0.72, or 72%

8. $P(X \text{ is on } \overline{HK})$

SOLUTION:

$$\begin{aligned} P(X \text{ is on } \overline{HK}) &= \frac{\text{length of } HK}{\text{length of } FK} \\ &= \frac{14+6}{4+12+14+6} \\ &= \frac{20}{36} \\ &= \frac{5}{9} \end{aligned}$$

ANSWER:

$\frac{5}{9}$, 0.56, or 56%

9. $P(X \text{ is on } \overline{FG})$

SOLUTION:

$$\begin{aligned} P(X \text{ is on } \overline{FG}) &= \frac{\text{length of } FG}{\text{length of } FK} \\ &= \frac{4}{4+12+14+6} \\ &= \frac{4}{36} \\ &= \frac{1}{9} \end{aligned}$$

ANSWER:

$\frac{1}{9}$, 0.11, or 11%

13-3 Geometric Probability

10. **BIRDS** Four birds are sitting on a telephone wire. What is the probability that a fifth bird landing at a randomly selected point on the wire will sit at some point between birds 3 and 4?

SOLUTION:

$$\begin{aligned} P(\text{between 3 and 4}) &= \frac{\text{distance from 3 to 4}}{\text{total distance}} \\ &= \frac{8}{6+10+8} \\ &= \frac{8}{24} \\ &= \frac{1}{3} \end{aligned}$$

ANSWER:

$\frac{1}{3}$, 0.33, or 33%

11. **TELEVISION** Julio is watching television and sees an ad for a CD that he knows his friend wants for her birthday. If the ad replays at a random time in each 3-hour interval, what is the probability that he will see the ad again during his favorite 30-minute sitcom the next day?

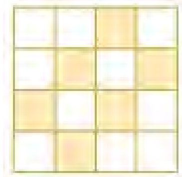
SOLUTION:

There are six 30-minute stretches in 3 hours. The ad can appear in any of the six. The probability that he will see the ad again during his favorite 30-minute sitcom the next day is $\frac{1}{6} \approx 17\%$.

ANSWER:

$\frac{1}{6}$, 0.17, or about 17%

Find the probability that a point chosen at random lies in the shaded region.

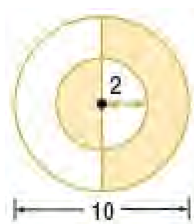
12. 

SOLUTION:

$$\begin{aligned} P(\text{shaded}) &= \frac{\text{area of shaded}}{\text{total area}} \\ &= \frac{6 \text{ squares}}{16 \text{ squares}} \\ &= \frac{6}{16} \\ &= \frac{3}{8} \end{aligned}$$

ANSWER:

$\frac{3}{8}$, 0.375 or 37.5%

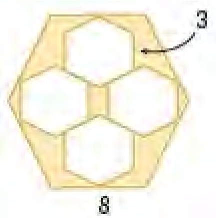
13. 

SOLUTION:

$$\begin{aligned} P(\text{shaded}) &= \frac{\text{area of shaded}}{\text{total area}} \\ &= \frac{\text{outer ring} + \text{inner ring}}{\text{circle}} \\ &= \frac{\text{half-circle}}{\text{circle}} \\ &= \frac{1}{2} \end{aligned}$$

ANSWER:

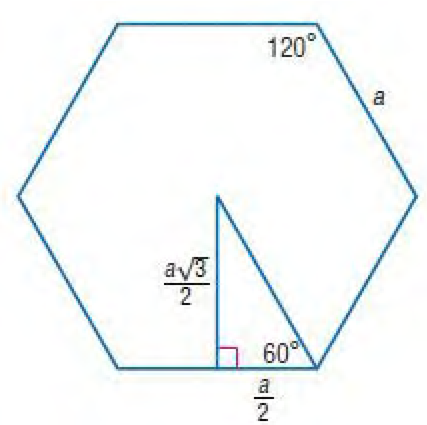
$\frac{1}{2}$, 0.5, or 50%

14. 

SOLUTION:

If a region A contains a region B and a point E in region A is chosen at random, then the probability that point E is in region B is $\frac{\text{Area of region A}}{\text{Area of region B}}$.

The area of a regular polygon is the half the product of the apothem and the perimeter. Find the apothem.



The length of the apothem of an hexagon of side length a units is $\frac{a\sqrt{3}}{2}$ units.

Area of the large hexagon:

$$\begin{aligned} A &= \tfrac{1}{2}Pa \\ &= \tfrac{1}{2}(6 \cdot 8)\left(\frac{8\sqrt{3}}{2}\right) \\ &= 24\left(\frac{8\sqrt{3}}{2}\right) \\ &= 96\sqrt{3} \end{aligned}$$

Area of the small hexagons:

$$\begin{aligned} A &= 4\left(\tfrac{1}{2}Pa\right) \\ &= 4\left[\tfrac{1}{2}(6 \cdot 3)\left(\frac{3\sqrt{3}}{2}\right)\right] \\ &= 4\left[9\left(\frac{3\sqrt{3}}{2}\right)\right] \\ &= 54\sqrt{3} \end{aligned}$$

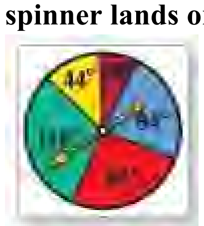
The area of the shaded region is $96\sqrt{3} - 54\sqrt{3} = 42\sqrt{3}$.

Therefore, the probability is $\frac{42\sqrt{3}}{96\sqrt{3}} = \frac{7}{16} = 43.75\%$.

ANSWER:

$\frac{7}{16}$, 0.4375, or 43.75%

Use the spinner to find each probability. If the spinner lands on a line it is spun again.



15. P(pointer landing on yellow)

SOLUTION:

The spinner is divided into 5 sectors. The ratio of the area of a sector of a circle to the area of the entire circle is the same as the ratio of the sector's central angle to 360.The measure of the sector colored in yellow is 44º. Therefore, the probability that the pointer will land on yellow is $\frac{44}{360} \approx 12.2\%$.

ANSWER:

12.2%

16. *P*(pointer landing on blue)

SOLUTION:

The spinner is divided into 5 sectors. The ratio of the area of a sector of a circle to the area of the entire circle is the same as the ratio of the sector's central angle to 360.The measure of the sector colored in blue is 84º. Therefore, the probability that the pointer will land on blue is $\frac{84}{360} \approx 23.3\%$.

ANSWER:

23.3%

17. *P*(pointer not landing on green)

SOLUTION:

The spinner is divided into 5 sectors. The ratio of the area of a sector of a circle to the area of the entire circle is the same as the ratio of the sector's central angle to 360.The measure of the sector colored in green is 110º. Therefore, the probability that the pointer will not land on green is $\frac{360-110}{360} = \frac{250}{360} \approx 69.4\%$.

ANSWER:

69.4%

18. *P*(pointer landing on red)

SOLUTION:

The spinner is divided into 5 sectors. The ratio of the area of a sector of a circle to the area of the entire circle is the same as the ratio of the sector's central angle to 360.The measure of the sector colored in red is 92º. Therefore, the probability that the pointer will land on red is $\frac{92}{360} \approx 25.6\%$.

ANSWER:

25.6%

19. *P*(pointer landing on neither red nor yellow)

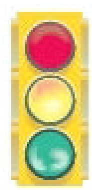
SOLUTION:

The spinner is divided into 5 sectors. The ratio of the area of a sector of a circle to the area of the entire circle is the same as the ratio of the sector's central angle to 360.The measure of the sector colored in red is 92º and that in yellow is 44º. Therefore, the probability that the pointer will land on neither red nor yellow is $\frac{360-(92+44)}{360} = \frac{224}{360} \approx 62.2\%$.

ANSWER:

62.2%

Describe an event with a 33% probability for each model.

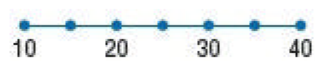
20. 

SOLUTION:

There are three possible outcomes red, yellow and green lights and the probability of each outcome is $\frac{1}{3} \approx 33\%$.

ANSWER:

Sample answer: getting a red light

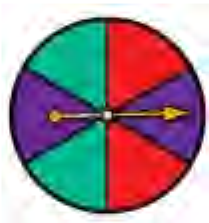
21. 

SOLUTION:

The line is divided into 6 equal segments and the alternate marked points are named. So, choosing a point between any two consecutive marked points, say 10 and 20 is $\frac{1}{3} \approx 33\%$.

ANSWER:

Sample answer: a point between 10 and 20

22. 

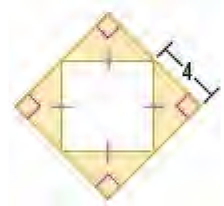
SOLUTION:

The spinner is divided into 6 equal sectors of two colors each. So, landing on any particular color has a probability of $\frac{2}{6} = \frac{1}{3} \approx 33\%$.

ANSWER:

Sample answer: landing on green

Find the probability that a point chosen at random lies in the shaded region.

23. 

SOLUTION:

The area of the large square is $8^2 = 64$.

The smaller square is formed by joining the midpoints of the sides of the larger square. The length of each side of the larger square is 8 units and that of the smaller square is $\sqrt{4^2 + 4^2} = 4\sqrt{2}$ units.

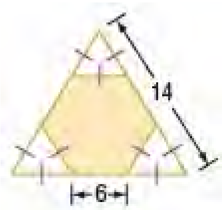
The area of the small square is $4\sqrt{2} \cdot 4\sqrt{2} = 32$.

The area of the shaded region is the difference of areas of the two squares. 64 – 32 = 32

Therefore, the probability that a point chosen at random lies in the shaded region is $\frac{32}{64} = \frac{1}{2} = 50\%$.

ANSWER:

$\frac{1}{2}$, 0.5, or 50%

24. 

SOLUTION:

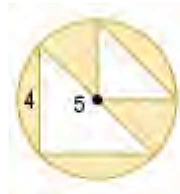
The length of each side of the large triangle is 14 units. The triangle is equilateral, so we can split it into two 30-60-90 triangles to find the height. The height is $7\sqrt{3}$.

The area of the large triangle is $0.5(14)(7\sqrt{3}) = 49\sqrt{3}$.

The length of each side of the small triangles is 4 units. The triangles are equilateral, so we can split them into two 30-60-90 triangles to find the height. The height is $2\sqrt{3}$.

The combined area of the small triangles is $3(0.5)(4)(2\sqrt{3}) = 12\sqrt{3}$.
The probability that a point chosen at random lies in the shaded region is $\frac{49\sqrt{3}-12\sqrt{3}}{49\sqrt{3}} \approx 75.5\%$.

ANSWER:
0.755 or 75.5%

25. 

SOLUTION:

The radius of the circle is 2.5 units, so the area of the circle is $(2.5)^2$**π = 6.25π.**

The length of each side of the leg of the smaller triangle is also 2.5 units. The area of the smaller triangle is 0.5(2.5)(2.5) = 3.125.

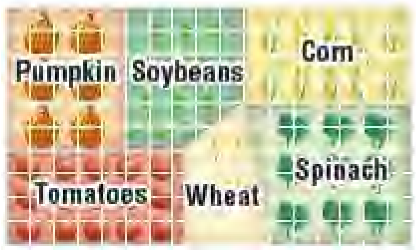
Use the Pythagorean Theorem to find the length of the other leg of the larger triangle.
$\sqrt{5^2 - 4^2} = \sqrt{9} = 3$
The area of the large triangle is 0.5(3)(4) = 6.

Then the area of the shaded region is **6.25π –** (3.125 + 6) ≈ 6.25**π – 9.125.**

Therefore, the probability that a point chosen at random lies in the shaded region is
$\frac{6.25\pi - 9.125}{6.25\pi} \approx 53.5\%$.

ANSWER:
53.5%

26. **FARMING** The layout for a farm is shown with each square representing a plot. Estimate the area of each field to answer each question.
a. What is the approximate combined area of the spinach and corn fields?
b. Find the probability that a randomly chosen plot is used to grow soybeans.



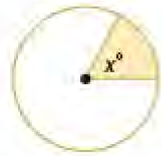
SOLUTION:
a. Count the number of squares to find the approximate area of the spinach and corn fields. There are 64 complete squares and 5 half squares. Therefore the approximate area is

$64+\frac{1}{2}(5)\approx 67$ sq. units.

b. The total area of the farm is 10(17) = 170 sq. units. and the area of the soybean field is about $25+\frac{1}{2}(4)=27$ sq. units. Therefore, the probability is $\frac{27}{170}\approx 0.16$ or about 16%.

ANSWER:
a. 67 square units
b. 0.16 or 16%

27. **ALGEBRA** Prove that the probability that a randomly chosen point in the circle will lie in the shaded region is equal to $\frac{x}{360}$.



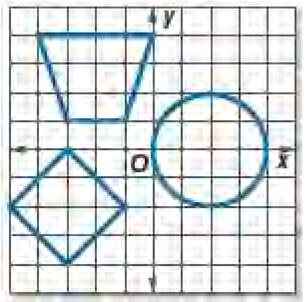
SOLUTION:
Sample answer: The probability that a randomly chosen point will lie in the shaded region is the ratio of the area of the sector to the area of the circle.

$$P(\text{lies in sector}) = \frac{\text{area of sector}}{\text{area of circle}}$$
$$\frac{x}{360} \stackrel{?}{=} \frac{\frac{x}{360}\pi r^2}{\pi r^2}$$
$$\frac{x}{360} = \frac{x}{360} \checkmark$$

ANSWER:
Sample answer: The probability that a randomly chosen point will lie in the shaded region is the ratio of the area of the sector to the area of the circle.

$$P(\text{lies in sector}) = \frac{\text{area of sector}}{\text{area of circle}}$$
$$\frac{x}{360} \stackrel{?}{=} \frac{\frac{x}{360}\pi r^2}{\pi r^2}$$
$$\frac{x}{360} = \frac{x}{360} \checkmark$$

28. **COORDINATE GEOMETRY** If a point is chosen at random in the coordinate grid, find each probability. Round to the nearest hundredth.



a. *P*(point inside the circle)
b. *P*(point inside the trapezoid)
c. *P*(point inside the trapezoid, square, or circle)

SOLUTION:
The total area of the coordinate grid is 100 sq. units.
a. The radius of the circle is 2 units. The area of the circle is $\pi(2)^2 = 4\pi$ sq. units.
Therefore, the probability that a point chosen is inside the circle is $\frac{4\pi}{100} = \frac{\pi}{25} \approx 13\%$.

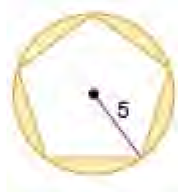
b. The lengths of the bases of the trapezoid are 2 and 4 units and the height is 3 units.
The area of the trapezoid is
$\frac{1}{2}(3)(2+4) = 9$ sq. units.
Therefore, the probability that a point chosen is inside the trapezoid is $\frac{9}{100} = 9\%$.

c. The length of each side of the square is $2\sqrt{2}$units.
The area of the square is 8 square units.

The sum of the area of the circle, trapezoid, and square is 4π + 8 + 9 ≈ 30 sq. units.
Therefore, the probability that a point chosen is inside the trapezoid, square, or circle is about $\frac{30}{100} = 30\%$.

ANSWER:
a. $\frac{\pi}{25}$, 0.13, or 13%
b. $\frac{9}{100}$, 0.09, or 9%
c. $\frac{3}{10}$, 0.30, or 30%

CCSS SENSE-MAKING Find the probability that a point chosen at random lies in a shaded region.

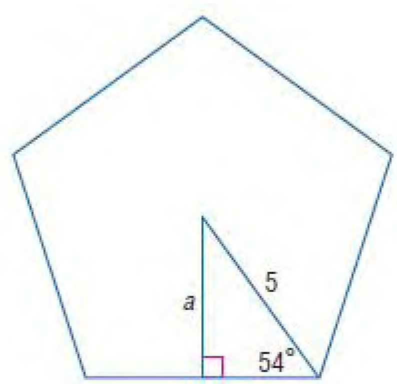
29. 

SOLUTION:
If a region *A* contains a region *B* and a point *E* in region *A* is chosen at random, then the probability that point *E* is in region *B* is
$\frac{\text{Area of region A}}{\text{Area of region B}}$.

The area of the circle is **$\pi(5)^2 = 25\pi$ units.**

The area of a regular polygon is the half the product of the apothem and the perimeter.

The measure of each interior angle of a regular pentagon is $\frac{(5-2)180}{5} = 108$. A line joining the center of the pentagon and one vertex will bisect this angle. The apothem will bisect the side.



The length of the apothem is 5(sin 54°) units.

The length of each side of the pentagon is 2(5 cos 54°) units.

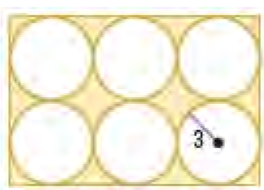
Area of pentagon:

$$A = \frac{1}{2}Pa$$
$$= \frac{1}{2}[5(10\cos 54)][5\sin 54]$$
$$\approx 59.4$$

The area of the shaded region is about 25π **– 59.4 ≈ 19.1 units^2.**

Therefore, the probability is about $\frac{19.1}{25\pi} \approx 24\%$.

ANSWER:
0.24 or 24%

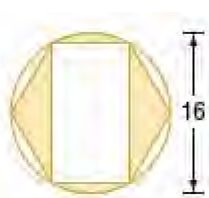
30. 

SOLUTION:
There are three circles with radius 3 units in a row and two in a column . The length of the rectangle is 18 units and the width is 6 units.
The area of each circle is $\pi(3)^2 = 9\pi$ units^2.
The area of the rectangle is 12(18) = 216 units^2.

The area of the shaded region is 216 – 6(9π**) = 46.4 units^2.**

Therefore, the probability is about $\frac{46.4}{216} \approx 21\%$.

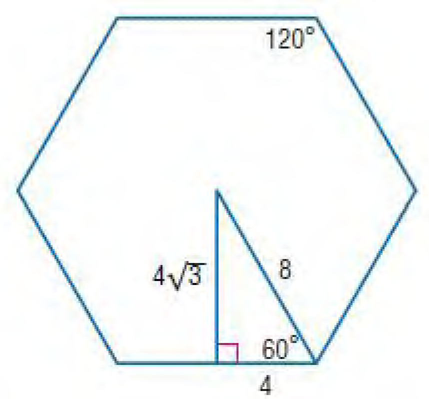
ANSWER:
0.21 or 21%

31. 

SOLUTION:
If a region *A* contains a region *B* and a point *E* in region *A* is chosen at random, then the probability that point *E* is in region *B* is $\frac{\text{Area of region A}}{\text{Area of region B}}$.
The area of the circle is $\pi(8)^2 = 64\pi$.

Find the area of the hexagon.

Find the apothem.
The measure of each interior angle of a regular hexagon is $\frac{(6-2)180}{6} = 120$. A line joining the center of the pentagon and one vertex will bisect this angle. The apothem will bisect the side.



Area of the hexagon:

$$A = \frac{1}{2}Pa$$
$$= \frac{1}{2}(6 \cdot 8)(4\sqrt{3})$$
$$= 96\sqrt{3}$$

The width of the rectangle is equal to the length of one side of the hexagon, or 8. The length of the rectangle is twice the apothem, or $8\sqrt{3}$.

Therefore, the area of the rectangle is $64\sqrt{3}$.

So far, we have each of the following areas:
Circle: 64π
Hexagon: $96\sqrt{3}$
Rectangle: $64\sqrt{3}$

The left and right pieces of the shaded region can be found by subtracting the area of the rectangle from the area of the hexagon:

$$96\sqrt{3} - 64\sqrt{3} = 32\sqrt{3}$$

Notice that the top and bottom sections of the shaded region are identical to 4 clear regions. These 6 regions represent the difference in area between the circle and the hexagon. So, the area of the top and bottom sections is

$$\frac{2}{6}(64\pi - 96\sqrt{3}) = \frac{64\pi}{3} - 32\sqrt{3}.$$

The combined area of the shaded regions is

$$32\sqrt{3} + \frac{64\pi}{3} - 32\sqrt{3} = \frac{64\pi}{3}.$$

Therefore, the probability that a point chosen at random lies in a shaded region is about

$$\frac{\frac{64\pi}{3}}{64\pi} \approx 33\%.$$

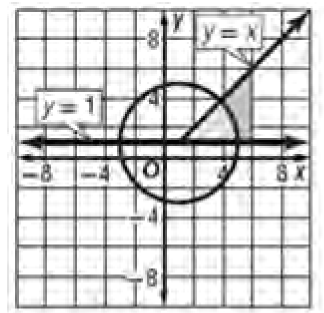
ANSWER:

0.33 or 33%

32. **COORDINATE GEOMETRY** Consider a system of inequalities, $1 \le x \le 6$, $y \le x$, and $y \ge 1$. If a point (x, y) in the system is chosen at random, what is the probability that $(x - 1)^2 + (y - 1)^2 \ge 16$?

SOLUTION:

Draw the inequalities in the same coordinate plane as shown.



The area of the shaded triangle with base 5 units and height 5 units is 0.5(5)(5) = 12.5 units^2.

Find the area of the shaded region that lies inside the circle. The shaded triangle is a right triangle with congruent legs, so it is a 45-45-90 triangle. So, the central angle of the circle is 45°.

$$\text{Area inside circle} = \frac{45°}{360°}(16\pi) = 2\pi \text{ units}^2$$

Therefore, the area of the shaded region outside the circle is the difference of the area of the triangle and the area of the shaded region that lies inside the circle.

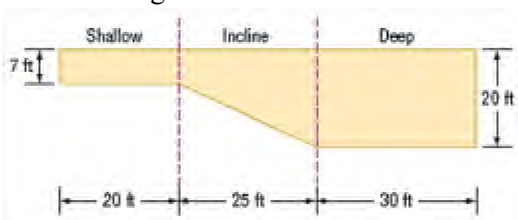
$$P(\text{satisfying point}) = \frac{\text{area(triangle)} - \text{area(inside circle)}}{\text{area(triangle)}} = \frac{12.5 - 2\pi}{12.5} \approx 0.50$$

Therefore, the probability that $(x-1)^2 + (y-1)^2 \ge 16$ is about 0.50 or 50%.

ANSWER:

0.50 or 50%

33. **VOLUME** The polar bear exhibit at a local zoo has a pool with the side profile shown. If the pool is 20 feet wide, what is the probability that a bear that is equally likely to swim anywhere in the pool will be in the incline region?



SOLUTION:

The required probability is ratio of the volume of the water in the inclined region to that of the pool. The shallow and deep areas are rectangular prisms and the inclined area is a trapezoidal prism. Find the corresponding volumes.

$$\begin{aligned} V_{\text{shallow}} &= 7 \times 20 \times 20 \\ &= 2800 \text{ ft}^2 \\ V_{\text{incline}} &= \frac{1}{2} \times 25 \times (7 + 20) \times 20 \\ &= 6750 \text{ft}^2 \\ V_{\text{deep}} &= 30 \times 20 \times 20 \\ &= 12000 \text{ ft}^2 \end{aligned}$$

Therefore, the probability that a bear that is equally likely to swim anywhere in the pool will be in the incline region is

$$\frac{6750}{2800 + 6750 + 12000} = \frac{6750}{21550} \approx 31\%.$$

ANSWER:

0.31 or 31%

34. **DECISION MAKING** Meleah's flight was delayed and she is running late to make it to a national science competition. She is planning on renting a car at the airport and prefers car rental company A over car rental company B. The courtesy van for car rental company A arrives every 7 minutes, while the courtesy van for car rental company B arrives every 12 minutes.

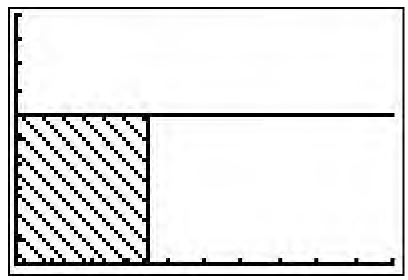
a. What is the probability that Meleah will have to wait 5 minutes or less to see each van? Explain your reasoning. (Hint: Use an area model.)

b. What is the probability that Meleah will have to wait 5 minutes or less to see one of the vans? Explain your reasoning.

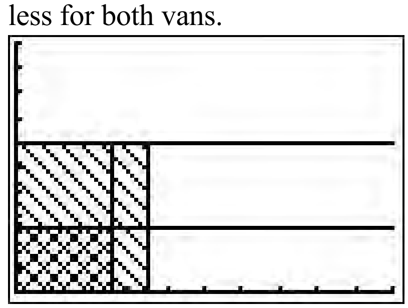
c. Meleah can wait no more than 5 minutes without risking being late for the competition. If the van from company B should arrive first, should she wait for the van from company A or take the van from company B? Explain your reasoning.

SOLUTION:

a. On a coordinate plane, graph $x = 7$ and shade between this line and the y-axis to represent the possible waiting times for the company A van. Graph $y = 12$ and shade between this line and the x-axis to represent the possible waiting times for the company B van. The area of the rectangle formed by the intersection is 84 units^2.

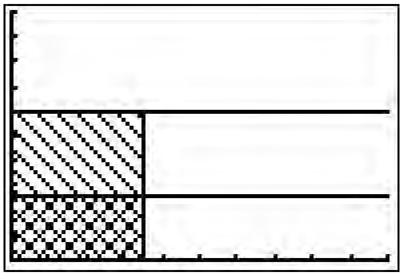


Then graph $x = 5$ and $y = 5$ and shade the region bounded by these lines and the axes to represent the possible waiting times of 5 minutes or less for both vans.

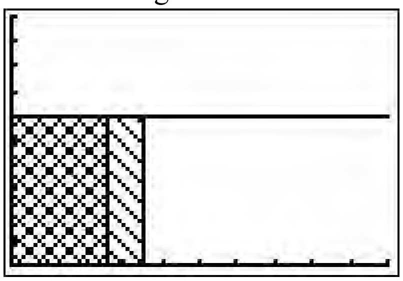


The area of the square is 25 square units. So the geometric probability is $\frac{25}{84}$ or about 30%.

b. On a coordinate plane, graph the lines $x = 7$ and $y = 12$ and shade as before. The area of this rectangle is 84 units^2. Then graph $x = 7$ and $y = 5$. Shade the region bounded by the lines $y = 5$, $x = 7$, and the axes to represent the possible waiting times of 5 minutes or less for the company A van. The area of this rectangle is 35 units^2.



Shade the region bounded by the lines $x = 5$, $y = 12$, and the axes to represent the possible waiting times of 5 minutes or less for the company B van. The area of this rectangle is 60 units^2.



In each of these rectangles, the waiting time of 5 minutes or less for both vans has been counted twice. So the geometric probability is $\frac{60}{84} + \frac{35}{84} - \frac{25}{84} = \frac{70}{84}$, or about 83%.

c. Sample answer: Since the chance of Meleah waiting 5 minutes or less to see the vans from both company A and B is only 30%, Meleah should take the van from company B.

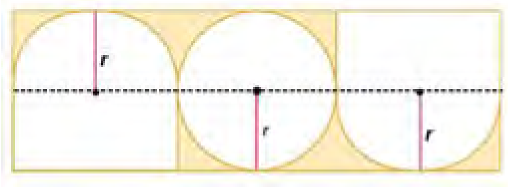
ANSWER:

a. On a coordinate plane, graph $x = 7$ and shade between this line and the y-axis to represent the possible waiting times for the company A van. Graph $y = 12$ and shade between this line and the x-axis to represent the possible waiting times for the company B van. The area of the rectangle formed by the intersection is 84 units^2. Then graph $x = 5$ and $y = 5$ and shade the region bounded by these lines and the axes to represent the possible waiting times of 5 minutes or less for both vans. The area of the square is 25 square units. So the geometric probability is $\frac{25}{84}$ or about 30%.

b. On a coordinate plane, graph the lines $x = 7$ and $y = 12$ and shade as before. The area of this rectangle is 84 units^2. Then graph $x = 7$ and $y = 5$. Shade the region bounded by the lines $y = 5$, $x = 7$, and the axes to represent the possible waiting times of 5 minutes or less for the company A van. The area of this rectangle is 35 units^2. Shade the region bounded by the lines $x = 5$, $y = 12$, and the axes to represent the possible waiting times of 5 minutes or less for the company B van. The area of this rectangle is 60 units^2. In each of these rectangles, the waiting time of 5 minutes or less for both vans has been counted twice. So the geometric probability is $\frac{60}{84} + \frac{35}{84} - \frac{25}{84} = \frac{70}{84}$, or about 83%.

c. Sample answer: Since the chance of Meleah waiting 5 minutes or less to see the vans from both company A and B is only 30%, Meleah should take the van from company B.

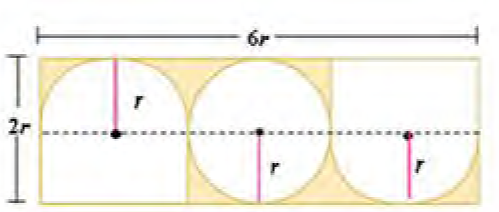
35. **CHALLENGE** Find the probability that a point chosen at random would lie in the shaded area of the figure. Round to the nearest tenth of a percent.



SOLUTION:

The probability that a point chosen at random is in the shaded area = $\frac{\text{area of shaded region}}{\text{area of big rectangle}}$

Area of the shaded region is equal to area of big rectangle – (area of the circle + area of two half circles + area of two small rectangles).



Find the area of all the parts.

Area of the big rectangle = $(6r)(2r)$ or $12r^2$.

Area of the circle = πr^2

Area of half circle = $\frac{\pi r^2}{2}$

Area of small rectangles = $r(2r)$ or $2r^2$

Find the probability.

$$
\begin{aligned}
P(\text{point chosen in the shaded region}) &= \frac{12r^2 - \left[\pi r^2 + 2\left(\frac{\pi r^2}{2}\right) + 2(2r^2)\right]}{12r^2} \\
&= \frac{12r^2 - 2\pi r^2 - 4r^2}{12r^2} \\
&= \frac{8r^2 - 2\pi r^2}{12r^2} \\
&= \frac{2r^2(4-\pi)}{12r^2} \\
&= \frac{4-\pi}{6} \text{ or about } 14.3\%
\end{aligned}
$$

ANSWER:

14.3%

36. **CCSS REASONING** An isosceles triangle has a perimeter of 32 centimeters. If the lengths of the sides of the triangle are integers, what is the probability that the area of the triangle is exactly 48 square centimeters? Explain.

SOLUTION:

Make a list of the possible integer sides for the isosceles triangle. Since two sides are equal, they must be less than half of the perimeter of 32. That means the two equal sides must have a measure of 1 to 15. Use the perimeter to find the third side of the triangle. Use the triangle inequality theorem to check if the possible sides could form a triangle.

Possible Leg, leg, base:

1 , 1, 30 (not possible since 30 > 1 + 1)
2, 2, 28 (not possible since 28 > 2 + 2)
3, 3, 26 (not possible since 26 > 3 + 3)
4, 4, 24 (not possible since 24 > 4 + 4)
5, 5, 22 (not possible since 22 > 5 + 5)
6, 6, 20 (not possible since 20 > 6 + 6)
7, 7, 18 (not possible since 18 > 7 + 7)
8, 8, 16 (not possible since 16 = 8 + 8)
9, 9, 14 (possible)
10, 10, 12 (possible)
11, 11, 10 (possible)
12, 12, 8 (possible)
13, 13, 6 (possible)
14, 14, 4 (possible)
15, 15, 2 (possible)

Using the Triangle Inequality Theorem, there are 7 possible isosceles triangles with integer side lengths and a perimeter of 32 centimeters.

Since the measure of the three sides are known, use Heron's formula $\left(A = \sqrt{s(s-a)(s-b)(s-c)}\right)$ to find the area of each triangle. The value of *s*, the semiperimeter, will be 16 for each triangle because the perimeter of each triangle is 32.

9, 9, 14:

$$\begin{aligned} A &= \sqrt{16(16-9)(16-9)(16-14)} \\ &= \sqrt{16(7)(7)(2)} \\ &= \sqrt{1568} \text{ or about } 39.6 \end{aligned}$$

10, 10, 12:

$$\begin{aligned} A &= \sqrt{16(16-10)(16-10)(16-12)} \\ &= \sqrt{16(6)(6)(4)} \\ &= \sqrt{2304} \text{ or } 48 \end{aligned}$$

11, 11, 10:

$$\begin{aligned} A &= \sqrt{16(16-11)(16-11)(16-10)} \\ &= \sqrt{16(5)(5)(6)} \\ &= \sqrt{2400} \text{ or about } 49.0 \end{aligned}$$

12, 12, 8:

$$\begin{aligned} A &= \sqrt{16(16-12)(16-12)(16-8)} \\ &= \sqrt{16(4)(4)(8)} \\ &= \sqrt{2048} \text{ or about } 45.3 \end{aligned}$$

13, 13, 6:

$$\begin{aligned} A &= \sqrt{16(16-13)(16-13)(16-6)} \\ &= \sqrt{16(3)(3)(10)} \\ &= \sqrt{1440} \text{ or about } 37.9 \end{aligned}$$

14, 14, 4:

$$\begin{aligned} A &= \sqrt{16(16-14)(16-14)(16-4)} \\ &= \sqrt{16(2)(2)(12)} \\ &= \sqrt{768} \text{ or about } 27.7 \end{aligned}$$

15, 15, 2:

$$\begin{aligned} A &= \sqrt{16(16-15)(16-15)(16-2)} \\ &= \sqrt{16(1)(1)(14)} \\ &= \sqrt{224} \text{ or about } 15.0 \end{aligned}$$

Of the possible triangles, only the one with side lengths 10, 10, and 12 has an area of exactly 48 square centimeters.
Therefore, the probability that an isosceles triangle with integer length sides and a perimeter of 32 cm has an area of 48 cm^2 is 1 in 7 or $\frac{1}{7}$.

ANSWER:

$\frac{1}{7}$; sample answer: Using the Triangle Inequality Theorem, there are 7 isosceles triangles with integer side lengths and a perimeter of 32 centimeters. Of those triangles, only the one with side lengths 10, 10, and 12 has an area of exactly 48 square centimeters. Therefore, the probability is 1 in 7.

37. **WRITING IN MATH** Can athletic events be considered random events? Explain.

SOLUTION:

No; Sample answer: Athletic events should not be considered random because there are factors involved, such as pressure and ability that have an impact on the success of the event.

For example, a basketball player may make 80% of her free throws throughout the season, but each individual free throw that she takes is not a random event. For instance, a foul shot taken at the beginning of the game is different than a foul shot taken with the game tied and only 3 seconds left on the clock.

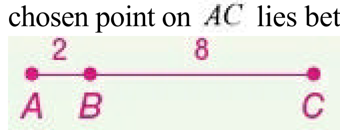
ANSWER:

No; Sample answer: Athletic events should not be considered random because there are factors involved, such as pressure and ability that have an impact on the success of the event.

38. **OPEN ENDED** Represent a probability of 20% using three different geometric figures.

SOLUTION:

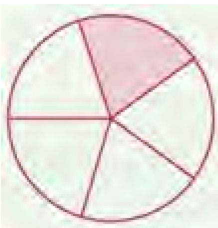
Sample answer: The probability that a randomly chosen point on $\overline{AC}$ lies between A and B is 20%.



13-3 Geometric Probability

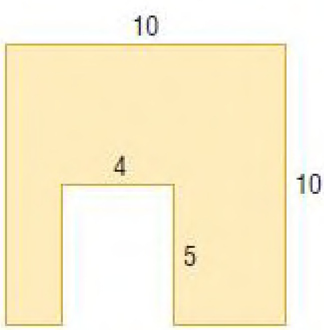
$$P(X \text{ is on } \overline{AB}) = \frac{\text{length of AB}}{\text{length of } AC} = \frac{2}{2+8} = \frac{2}{10} = 20\%$$

The probability that a randomly chosen point in the circle will lie in the shaded area is 20%.



$$P(\text{shaded}) = \frac{\text{area of shaded region}}{\text{area of figure}} = \frac{1 \text{ pie}}{5 \text{ pies}} = \frac{1}{5} = 20\%$$

The probability that a randomly chosen point in the square will lie in the unshaded area is 20%.

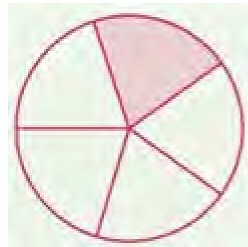


$$P(\text{unshaded}) = \frac{\text{area of unshaded region}}{\text{area of figure}} = \frac{4 \times 5}{10 \times 10} = \frac{20}{100} = 20\%$$

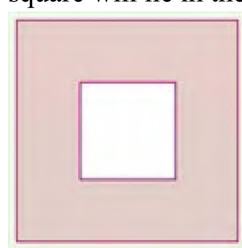
ANSWER:

Sample answer: The probability that a randomly chosen point on $\overline{AC}$ lies between A and B is 20%.

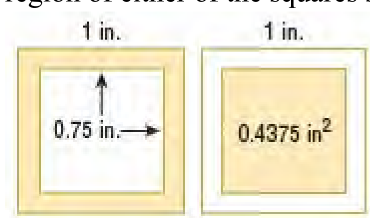
The probability that a randomly chosen point in the circle will lie in the shaded area is 20%.



The probability that a randomly chosen point in the square will lie in the unshaded area is 20%.



39. **WRITING IN MATH** Explain why the probability of a randomly chosen point falling in the shaded region of either of the squares shown is the same.



SOLUTION:

This is a geometric probability, so we will be comparing areas. Calculate the areas of each figure, as well as the shaded regions. Compare the ratio of the areas for the first figure with the ratio of the areas for the second figure.

Sample answer: The probability of a randomly chosen point lying in the shaded region of the square on the left is found by subtracting the area of the unshaded square from the area of the larger square and finding the ratio of the difference of the areas to the area of the larger square. The probability is $\frac{1^2 - 0.75^2}{1^2}$ or 43.75%. The probability of a randomly chosen point lying in the shaded region of the square on the left is the ratio of the area of the shaded square to the area of the larger square, which is $\frac{0.4375}{1}$ or 43.75%. Therefore, the probability of a randomly chosen point lying in the shaded area of either square is the same.

ANSWER:

Sample answer: The probability of a randomly chosen point lying in the shaded region of the square on the left is found by subtracting the area of the unshaded square from the area of the larger square and finding the ratio of the difference of the areas to the area of the larger square. The probability is $\frac{1^2 - 0.75^2}{1^2}$ or 43.75%. The probability of a randomly chosen point lying in the shaded region of the square on the left is the ratio of the area of the shaded square to the area of the larger square, which is $\frac{0.4375}{1}$ or 43.75%. Therefore, the probability of a randomly chosen point lying in the shaded area of either square is the same.

40. **PROBABILITY** A circle with radius 3 is contained in a square with side length 9. What is the probability that a randomly chosen point in the interior of the square will also lie in the interior of the circle?

A. $\frac{1}{9}$

B. $\frac{1}{3}$

C. $\frac{\pi}{9}$

D. $\frac{9}{\pi}$

SOLUTION:

The area of the square is 81 sq. units and the area of the circle is 9**π.** Therefore, the probability that a randomly chosen point in the interior of the square will also lie in the interior of the circle is $\frac{9\pi}{81} = \frac{\pi}{9}$.

The correct choice is C.

ANSWER:

C

41. **ALGEBRA** The area of Miki's room is $x^2 + 8x + 12$ square feet. A gallon of paint will cover an area of $x^2 + 6x + 8$ square feet. Which expression gives the number of gallons of paint that Miki will need to buy to paint her room?

F. $\frac{x+6}{x+4}$

G. $\frac{x-4}{x-6}$

H. $\frac{x+4}{x+6}$

J . $\frac{x-4}{x+6}$

SOLUTION:

The number of gallons of paint required to paint the room is $\frac{x^2+8x+12}{x^2+6x+8}$.

Factor the numerator and the denominator to simplify the fraction.

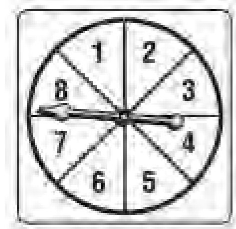
$$\frac{x^2+8x+12}{x^2+6x+8}=\frac{(x+6)(x+2)}{(x+4)(x+2)}=\frac{x+6}{x+4}$$

The correct choice is F.

ANSWER:

F

42. **EXTENDED RESPONSE** The spinner is divided into 8 equal sections.



a. If the arrow lands on a number, what is the probability that it will land on 3?

b. If the arrow lands on a number, what is the probability that it will land on an odd number?

SOLUTION:

a. The spinner is divided into 8 equal sections. So, there is an equal chance of landing on any number. Therefore, the chance of landing on 3 is $\frac{1}{8}$.

b. There are 4 odd numbers out of 8 numbers. Therefore, the chance of landing on an odd number is $\frac{4}{8}=\frac{1}{2}$.

ANSWER:

a. $\frac{1}{8}$

b. $\frac{1}{2}$

43. **SAT/ACT** A box contains 7 blue marbles, 6 red marbles, 2 white marbles, and 3 black marbles. If one marble is chosen at random, what is the probability that it will be red?
A 0.11
B 0.17
C 0.33
D 0.39
E 0.67

SOLUTION:
The probability is defined as the ratio of number of favorable outcomes to the number of possible outcomes. There are 6 red marbles out of 18 marbles. Therefore, the probability of choosing a red marble is $\frac{6}{18}=\frac{1}{3}$. The correct choice is C.

ANSWER:
C

44. **PROM** Four friends are sitting at a table together at the prom. What is the probability that a particular one of them will sit in the chair closest to the dance floor?

SOLUTION:
Since the samples are arranged with a fixed reference point, this is a linear permutation. So there are 4! or 24 ways in which the samples can be arranged. The number of favorable outcomes is the number of permutations of the other 3 people given that a particular one of them will sit in the chair closest to the dance floor, 3! or 6. Therefore, the probability is $\frac{6}{24}=\frac{1}{4}$..

Put more simply: There are 4 equally likely places to sit, so each person has a 1 in 4 chance in sitting at any particular chair.

ANSWER:
$\frac{1}{4}$

Represent the sample space for each experiment by making an organized list, a table, and a tree diagram.

45. Tito has a choice of taking music lessons for the next two years and playing drums or guitar.

SOLUTION:

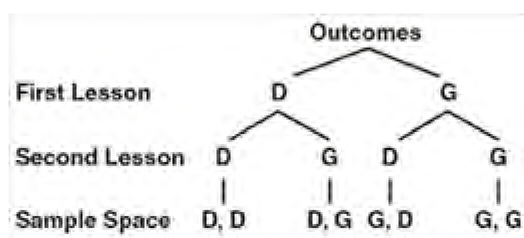
Organized List:
Pair each possible outcome for the first year with the possible outcomes for the second year.

D, D
G, G
D, G
G, D

Table:
List the outcomes of the first year in the left column and those of the second year in the top row.

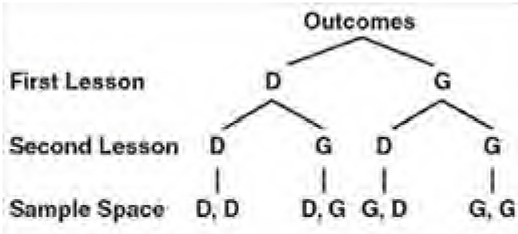
Outcomes	Drums	Guitar
Drums	D, D	D, G
Guitar	G, D	G, G

Tree Diagram:
The top group is all of the outcomes for the first year. The second group includes all of the outcomes for the second year. The last group shows the sample space.



ANSWER:
D, D
G, G
D, G
G, D

Outcomes	Drums	Guitar
Drums	D, D	D, G
Guitar	G, D	G, G



46. Denise can buy a pair of shoes in either flats or heels in black or navy blue.

SOLUTION:

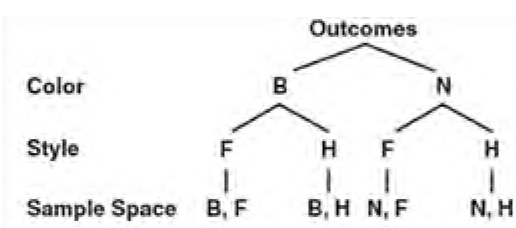
Organized List:
Pair each possible outcome for the color with the possible outcomes for the type.

B, F
N, F
B, H
N, H

Table:
List the outcomes of the color in the left column and those of the type in the top row.

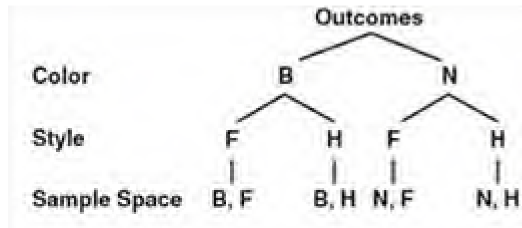
Outcomes	Flats	Heels
Black	B, F	B, H
Navy	N, F	N, H

Tree Diagram:
The top group is all of the outcomes for the color. The second group includes all of the outcomes for the type. The last group shows the sample space.

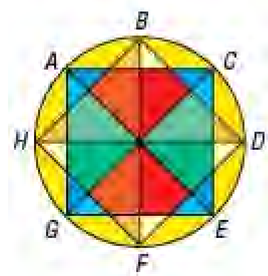


ANSWER:
B, F
N, F
B, H
N, H

Outcomes	Flats	Heels
Black	B, F	B, H
Navy	N, F	N, H



STAINED GLASS In the stained glass window design, all of the small arcs around the circle are congruent. Suppose the center of the circle is point *O*.



47. What is the measure of each of the small arcs?

SOLUTION:
The circle is divided into 8 equal arcs. The sum of the measures of the arcs is 360°. Therefore, the measure of each arc is $\frac{360}{8} = 45°$.

ANSWER:
45

48. What kind of figure is ΔAOC ? Explain.

SOLUTION:

Analyze the sides and angles of the triangle.

Sides: Two of the sides are radii, so they are congruent and the triangle is isosceles.

Angles: $\angle AOC$ is a central angle. The corresponding arc includes 2 of 8 congruent arcs around the circle. Each arc is 360 ÷ 8 = 45°. Two arcs = 90°, making $\angle AOC$ a right angle. Therefore, the triangle is a right triangle.

ANSWER:

Isosceles right triangle; the sides are congruent radii, making it isosceles, and $\angle AOC$ is a central angle for an arc of 90°, making it a right angle.

49. What kind of figure is quadrilateral *BDFH*? Explain.

SOLUTION:

Analyze the sides and angles of the quadrilateral.

Sides: Each side is a chord. Each chord corresponds with two arcs, so the chords are congruent. Therefore, the quadrilateral is either a rhombus or square.

Angles: Each angle intercepts a semicircle, making them 90° angles. Therefore, the quadrilateral must be a square.

ANSWER:

Square; each angle intercepts a semicircle, making them 90° angles. Each side is a chord of congruent arcs, so the chords are congruent.

50. What kind of figure is quadrilateral *ACEG*? Explain.

SOLUTION:

Analyze the sides and angles of the quadrilateral.

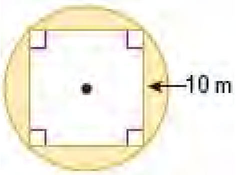
Sides: Each side is a chord. Each chord corresponds with two arcs, so the chords are congruent. Therefore, the quadrilateral is either a rhombus or square.

Angles: Each angle intercepts a semicircle, making them 90° angles. Therefore, the quadrilateral must be a square.

ANSWER:

Square; each angle intercepts a semicircle, making them 90° angles. Each side is a chord of congruent arcs, so the chords are congruent.

Find the area of the shaded region. Round to the nearest tenth.

51. 

SOLUTION:

The diagonal of the square goes through the center of the circle, so the diagonal is a diameter of the circle. The diagonal cuts the square into two congruent right triangles with 10 m sides. These triangles are 45-45-90 triangles. Since the side is 10 m, the hypotenuse is $10\sqrt{2}$ m.

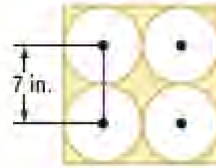
The hypotenuse is also the diagonal as well as the diameter. The radius is $\frac{10\sqrt{2}}{2} = 5\sqrt{2}$.

The area of the shaded region is the difference of areas of the circle and the square.

$$\begin{aligned}\text{Area} &= \text{Area(circle)} - \text{Area(square)} \\ &= \pi r^2 - s^2 \\ &= \pi\left(5\sqrt{2}\right)^2 - 10^2 \\ &= 50\pi - 100 \\ &= 57.1\end{aligned}$$

ANSWER:

57.1 m^2

52. 

SOLUTION:

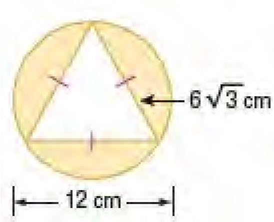
The four circles are all congruent. The radius of each circle is 3.5 in. So, the length of each side of the square is 14 in.

The area of the shaded region is the difference of areas of the square and the circles.

$$\begin{aligned}\text{Area} &= \text{Area(square)} - \text{Area(circles)} \\ &= s^2 - 4\pi r^2 \\ &= 14^2 - 4\pi(3.5)^2 \\ &= 196 - 49\pi \\ &\approx 42.1\end{aligned}$$

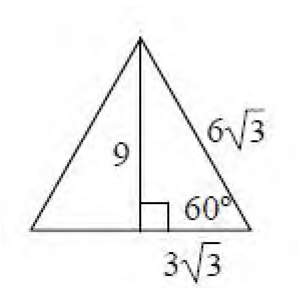
ANSWER:

42.1 in^2

53. 

SOLUTION:

The equilateral triangle can be split into two 30-60-90 triangles.



The hypotenuse is $6\sqrt{3}$, so the length of the small side is $6\sqrt{3} \div 2 = 3\sqrt{3}$. The height of the triangle is $3\sqrt{3} \times \sqrt{3} = 9$.

The base of the big triangle is $6\sqrt{3}$ and the radius of the circle is 6.

The area of the shaded region is the difference of areas of the circle and the triangle.

$$\begin{aligned} \text{Area} &= \text{Area(circle)} - \text{Area(triangle)} \\ &= \pi r^2 - \tfrac{1}{2}bh \\ &= \pi(6)^2 - \tfrac{1}{2}\left(6\sqrt{3}\right)(9) \\ &= 36\pi - 27\sqrt{3} \\ &\approx 66.3 \end{aligned}$$

ANSWER:

66.3 cm^2