13-1 Representing Sample Spaces

Represent the sample space for each experiment by making an organized list, a table, and a tree diagram.

1. For each at bat, a player can either get on base or make an out. Suppose a player bats twice.

SOLUTION:

Organized List:
Pair each possible outcome for the first at bat with the possible outcomes for the second at bat.

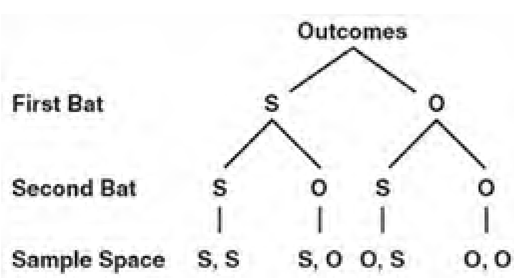
Table:
List the outcomes of the first at bat in the left column and those of the second at bat in the top row.

S, S   0, 0

S, 0   0, S

Outcomes	Safe	Out
Safe	S, S	S, 0
Out	0, S	0, 0

Tree Diagram:
The top group is all of the outcomes for the first at bat. The second group includes all of the outcomes for the second at bat. The last group shows the sample space.

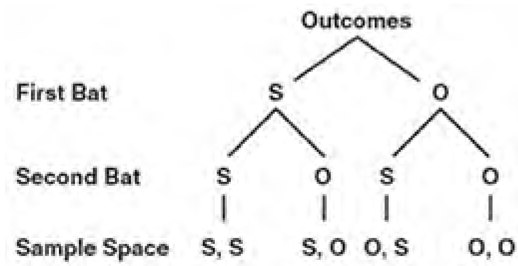


ANSWER:

S, S   0, 0

S, 0   0, S

Outcomes	Safe	Out
Safe	S, S	S, 0
Out	0, S	0, 0



2. Quinton sold the most tickets in his school for the annual Autumn Festival. As a reward, he gets to choose twice from a grab bag with tickets that say "free juice" or "free notebook."

SOLUTION:

Organized List:
Pair each possible outcome for the first choice with the possible outcomes for the second choice.

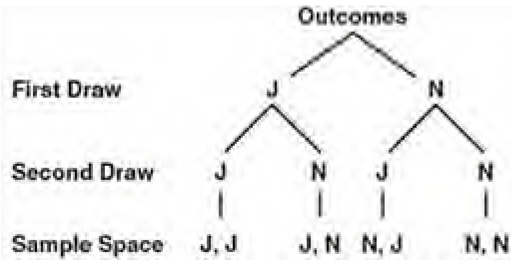
Table:
List the outcomes of the first choice in the left column and those of the second choice in the top row.

J, J   N, N

J, N   N, J

Outcomes	Juice	Notebook
Juice	J, J	J, N
Notebook	N, J	N, N

Tree Diagram:
The top group is all of the outcomes for the first choice. The second group includes all of the outcomes for the second choice. The last group shows the sample space.

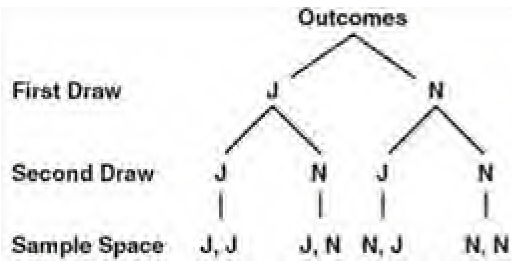


ANSWER:

J, J   N, N

J, N   N, J

Outcomes	Juice	Notebook
Juice	J, J	J, N
Notebook	N, J	N, N



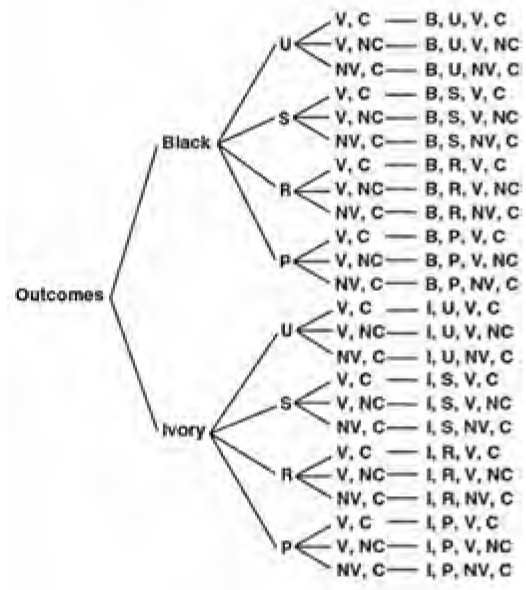
3. **TUXEDOS** Patrick is renting a prom tuxedo from the catalog shown. Draw a tree diagram to represent the sample space for this situation.

SOLUTION:

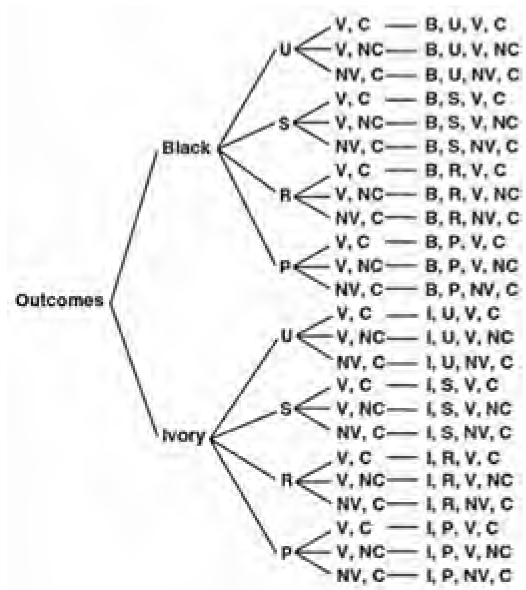
The sample space is the result of four stages. The suit color only has two options, so do it first.

• Suit Color (B or I)
• Tie Color (U, S, R, or P)
• Vest (V or NV)
• Cummerbund (C or NC)

Draw a tree diagram with four stages.



ANSWER:



Find the number of possible outcomes for each situation.

4. Marcos is buying a cell phone and must choose a plan. Assume one of each is chosen.

Cell Phone Options	Number of Choices
phone style	15
minutes package	5
Internet access	3
text messaging	4
insurance	2

SOLUTION:

By the Fundamental Counting Principle the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

The phone style can be chosen in 15 different ways, minute package in 5 different ways, internet access in 3 different ways, text messaging in 4 different ways, and the insurance in 2 different ways. Therefore, buying a cell phone and choosing a plan can be done in $15 \times 5 \times 3 \times 4 \times 2 = 1800$ ways.

ANSWER:

1800

5. Desirée is creating a new menu for her restaurant. Assume one of each item is ordered.

Menu Titles	Number of Choices
Appetizer	8
Soup	4
Salad	6
Entree	12
Dessert	9

SOLUTION:

By the Fundamental Counting Principle the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

An appetizer can be ordered in 8 different ways, soup in 4 different ways, salad in 6 different ways, entree in 12 different ways and the dessert in 9 different ways. Therefore, an order with one item from each set can be done in $8 \times 4 \times 6 \times 12 \times 9 = 20{,}736$ ways.

ANSWER:

20,736

CCSS REASONING Represent the sample space for each experiment by making an organized list, a table, and a tree diagram.

6. Gina is a junior and has a choice for the next two years of either playing volleyball or basketball during the winter quarter.

SOLUTION:

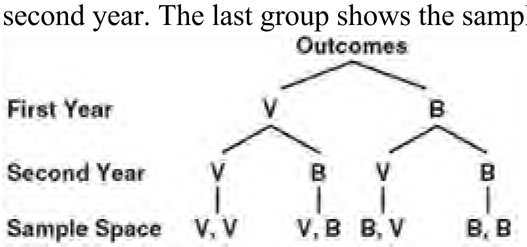
Organized List:
Pair each possible outcome for the first year with the possible outcomes for the second year.

Table:
List the outcomes of the first year in the left column and those of the second year in the top row.

V, V B, B
V, B B, V

Outcomes	Volleyball	Basketball
Volleyball	V, V	V, B
Basketball	B, V	B, B

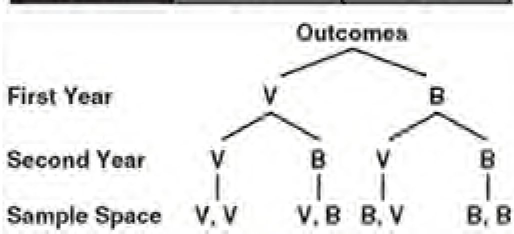
Tree Diagram:
The top group is all of the outcomes for the first year. The second group includes all of the outcomes for the second year. The last group shows the sample space.



ANSWER:

V, V B, B
V, B B, V

Outcomes	Volleyball	Basketball
Volleyball	V, V	V, B
Basketball	B, V	B, B



7. Two different history classes in New York City are taking a trip to either the Smithsonian or the Museum of Natural History.

SOLUTION:

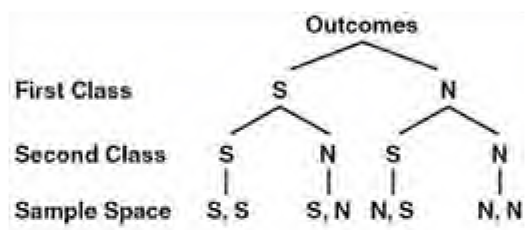
Organized List:
Pair each possible outcome for the first class with the possible outcomes for the second class.

Table:
List the outcomes of the first class in the left column and those of the second class in the top row.

S, S N, N
S, N N, S

Outcomes	Smithsonian	Natural
Smithsonian	S, S	S, N
Natural	N, S	N, N

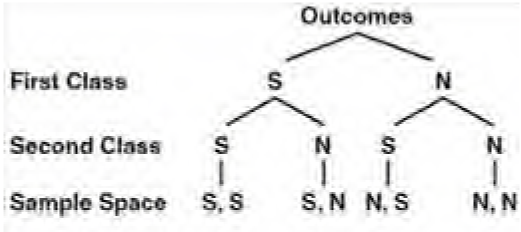
Tree Diagram:
The top group is all of the outcomes for the first class. The second group includes all of the outcomes for the second class. The last group shows the sample space.



ANSWER:

S, S N, N
S, N N, S

Outcomes	Smithsonian	Natural
Smithsonian	S, S	S, N
Natural	N, S	N, N



8. Simeon has an opportunity to travel abroad as a foreign exchange student during each of his last two years of college. He can choose between Ecuador or Italy.

SOLUTION:

Organized List:

Pair each possible outcome for the first year with the possible outcomes for the second year.

Table:

List the outcomes of the first year in the left column and those of the second year in the top row.

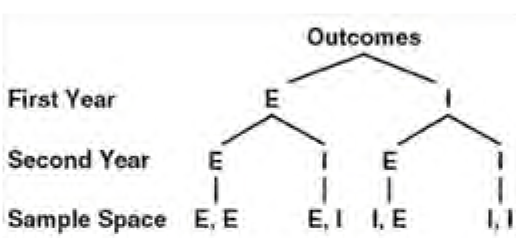
E, E I, I

E, I I, E

Outcomes	Italy	Ecuador
Italy	I, I	I, E
Ecuador	E, I	E, E

Tree Diagram:

The top group is all of the outcomes for the first year. The second group includes all of the outcomes for the second year. The last group shows the sample space.

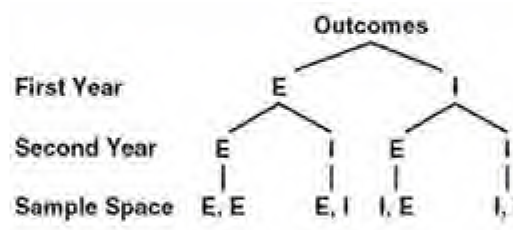


ANSWER:

E, E I, I

E, I I, E

Outcomes	Italy	Ecuador
Italy	I, I	I, E
Ecuador	E, I	E, E



9. A new club is formed, and a meeting time must be chosen. The possible meeting times are Monday or Thursday at 5:00 or 6:00 p.m.

SOLUTION:

Organized List:

Pair each possible outcome for the days with the possible outcomes for the times.

Table:

List the outcomes of the days in the left column and those of the times in the top row.

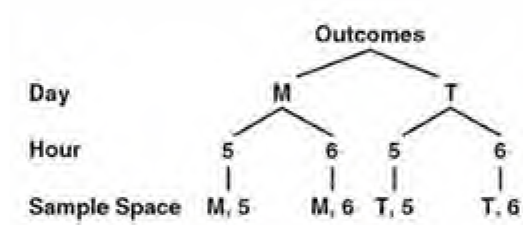
M, 5 T, 5

M, 6 T, 6

Outcomes	5	6
Monday	M, 5	M, 6
Thursday	T, 5	T, 6

Tree Diagram:

The top group is all of the outcomes for the days. The second group includes all of the outcomes for the times. The last group shows the sample space.

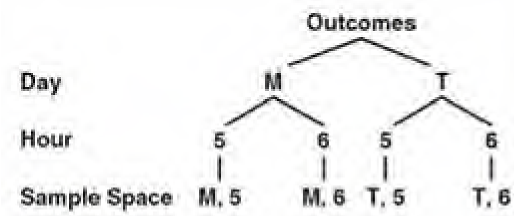


ANSWER:

M, 5 T, 5

M, 6 T, 6

Outcomes	5	6
Monday	M, 5	M, 6
Thursday	T, 5	T, 6



10. An exam with multiple versions has exercises with triangles. In the first exercise, there is an obtuse triangle or an acute triangle. In the second exercise, there is an isosceles triangle or a scalene triangle.

SOLUTION:

Organized List:

Pair each possible outcome for the angles with the possible outcomes for the sides.

Table:

List the outcomes of the angles in the left column and those of the sides in the top row.

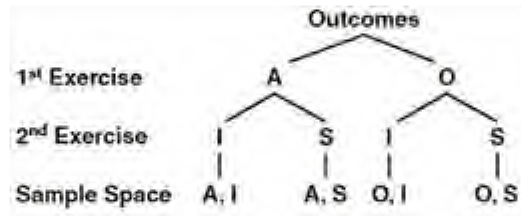
A, I A, S

O, I O, S

Outcomes	Isosceles	Scalene
Acute	A, I	A, S
Obtuse	O, I	O, S

Tree Diagram:

The top group is all of the outcomes for the angles. The second group includes all of the outcomes for the sides. The last group shows the sample space.

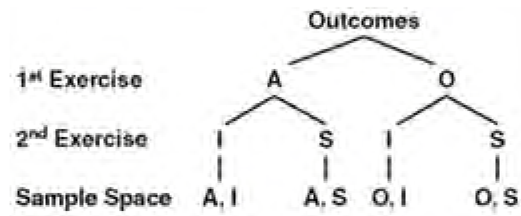


ANSWER:

A, I A, S

O, I O, S

Outcomes	Isosceles	Scalene
Acute	A, I	A, S
Obtuse	O, I	O, S



11. **PAINTING** In an art class, students are working on two projects where they can use one of two different types of paints for each project. Represent the sample space for this experiment by making an organized list, a table, and a tree diagram.

SOLUTION:

Organized List:

Pair each possible outcome for the first project with the possible outcomes for the second project.

Table:

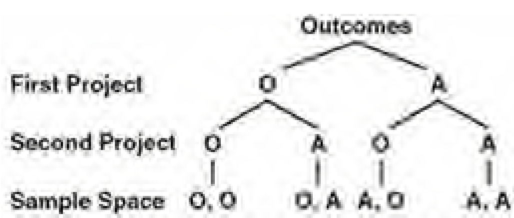
List the outcomes of the first project in the left column and those of the second project in the top row.

0, 0 A, A

0, A A, 0

Outcomes	Oil	Acrylic
Oil	0, 0	0, A
Acrylic	A, 0	A, A

Tree Diagram:
The top group is all of the outcomes for the first project. The second group includes all of the outcomes for the second project. The last group shows the sample space.

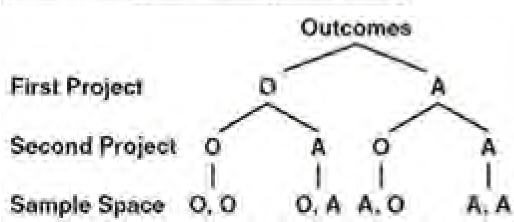


ANSWER:

0, 0 A, A

0, A A, 0

Outcomes	Oil	Acrylic
Oil	0, 0	0, A
Acrylic	A, 0	A, A



Draw a tree diagram to represent the sample space for each situation.

12. **BURRITOS** At a burrito stand, customers have the choice of beans, pork, or chicken with rice or no rice, and cheese and/or salsa.

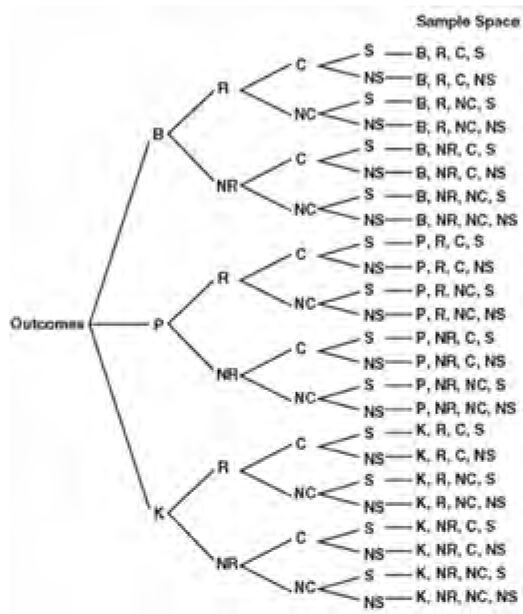
SOLUTION:

The sample space is the result of four stages.

- Choices (B, P, K)
- Rice(R or NR)
- Cheese (C or NC)
- Salsa (S or NS)

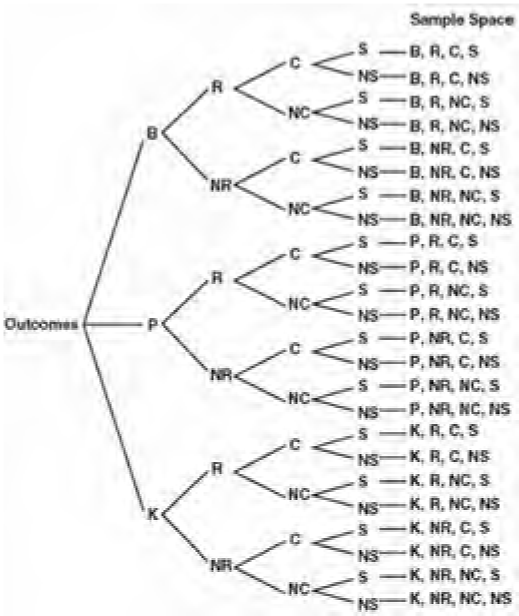
Draw a tree diagram with four stages.

B = beans, P = pork, K = chicken, R = rice, NR = no rice, C = cheese, NC = no cheese, S = salsa, and NS = no salsa.



ANSWER:

B = beans, P = pork, K = chicken, R = rice, NR = no rice, C = cheese, NC = no cheese, S = salsa, and NS = no salsa.



13. **TRANSPORTATION** Blake is buying a car and has a choice of sedan, truck, or van with leather or fabric interior, and a CD player and/or sunroof.

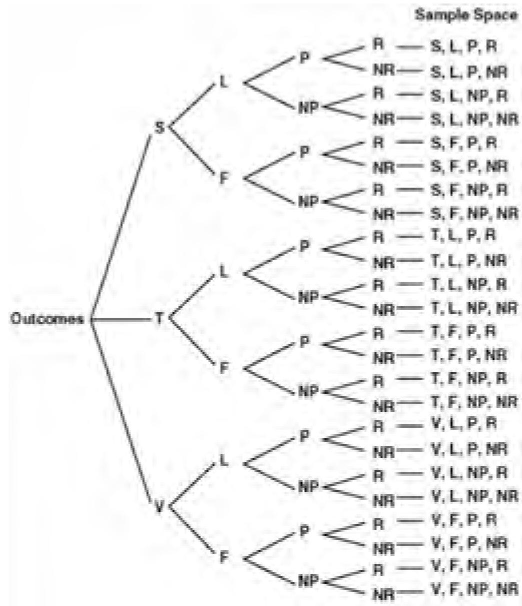
SOLUTION:

S = sedan, T = truck, V = van, L = leather, F = fabric, P = CD player, NP = no CD player, R = sunroof, NR = no sunroof

The sample space is the result of four stages.

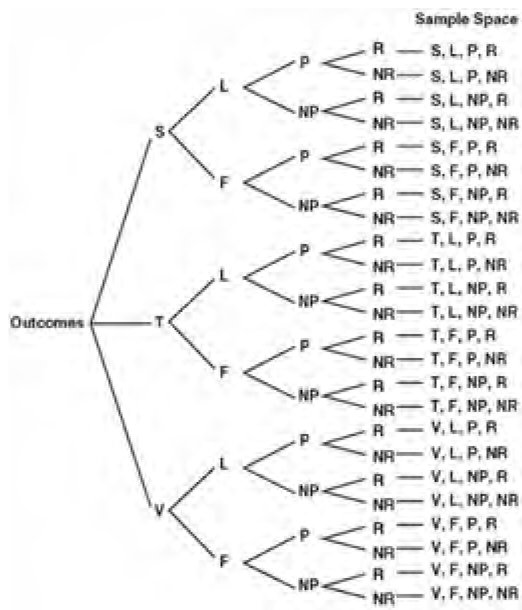
- Vehicle (S, T, or V)
- Interior (L or F)
- CD Player (C or NC)
- Sunroof (S or NS)

Draw a tree diagram with four stages.



ANSWER:

S = sedan, T = truck, V = van, L = leather, F = fabric, P = CD player, NP = no CD player, R = sunroof, NR = no sunroof



14. **TREATS** Ping and her friends go to a frozen yogurt parlor where they have a sign like the one at the right. Draw a tree diagram for all possible combinations of cones with peanuts and/or sprinkles.

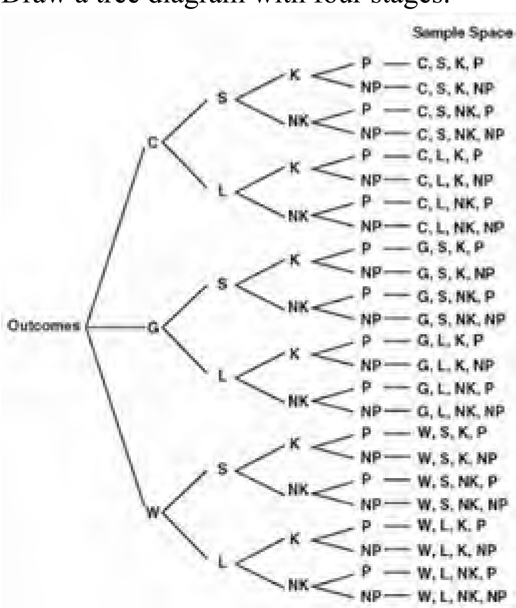
SOLUTION:

C = cake cone, G = sugar cone, W = waffle cone, S = strawberry, L = lime, P = peanuts, NP = no peanuts, K = sprinkles, NK = no sprinkles

The sample space is the result of four stages.

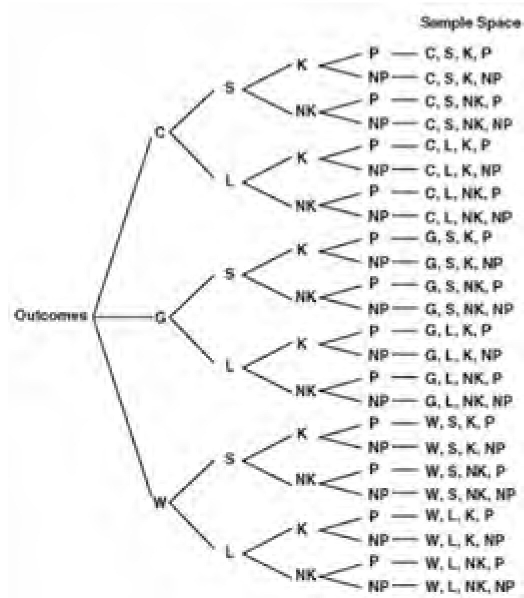
- Cone (C, G, or W)
- Flavor (S or L)
- Peanuts (P or NP)
- Sprinkles (S or NS)

Draw a tree diagram with four stages.



ANSWER:

C = cake cone, G = sugar cone, W = waffle cone, S = strawberry, L = lime, P = peanuts, NP = no peanuts, K = sprinkles, NK = no sprinkles



CCSS PERSEVERANCE In Exercises 15–18, find the number of possible outcomes for each situation.

15. In the Junior Student Council elections, there are 3 people running for secretary, 4 people running for treasurer, 5 people running for vice president, and 2 people running for class president.

SOLUTION:

By the Fundamental Counting Principle the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

A secretary can be chosen in 3 different ways, treasurer in 4 different ways, vice president in 5 different ways and the class president in 2 different ways. Therefore, the Junior Student Council can be formed in $3 \times 4 \times 5 \times 2 = 120$ ways.

ANSWER:

120

16. When signing up for classes during his first semester of college, Frederico has 4 class spots to fill with a choice of 4 literature classes, 2 math classes, 6 history classes, and 3 film classes.

SOLUTION:

By the Fundamental Counting Principle the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

Frederico can choose his literature classes in 4 different ways, math classes in 2 different ways, history classes in 6 different ways, and the film classes in 3 different ways. Therefore, he can choose his course in 4 × 2 × 6 × 3 = 144 ways.

ANSWER:

144

17. Niecy is choosing one each of 6 colleges, 5 majors, 2 minors, and 4 clubs.

SOLUTION:

By the Fundamental Counting Principle the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

Niecy can choose any of the 6 colleges, any of the 5 majors, any of the 2 minors, any of the 4 clubs. Therefore, she can choose her course in 6 × 5 × 2 × 4 = 240 ways.

ANSWER:

240

18. Evita works at a restaurant where she has to wear a white blouse, black pants or skirt, and black shoes. She has 5 blouses, 4 pants, 3 skirts, and 6 pairs of black shoes.

SOLUTION:

By the Fundamental Counting Principle the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

Evita can wear any of her 5 blouses and any of the 6 pairs of shoes. She can combine this either a pant of her choice from 4 pants or a skirt from 3 skirts. Therefore, she can choose her dressing in 5 × 6 × (3 + 4) = 210 ways.

ANSWER:

210

19. **ART** For an art class assignment, Mr. Green gives students their choice of two quadrilaterals to use as a base. One must have sides of equal length, and the other must have at least one set of parallel sides. Represent the sample space by making an organized list, a table, and a tree diagram.

SOLUTION:

Organized List:

Pair each possible outcome for the first quadrilateral with the possible outcomes for the second quadrilateral.

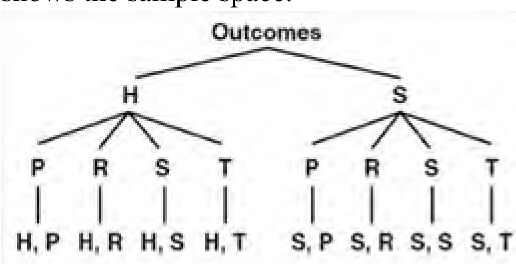
H = rhombus, P = parallelogram, R = rectangle, S = square, T = trapezoid;
(H, P), (H, R), (H, S), (H, T), (S, P), (S, R), (S, S), (S, T), (H, H), (S, H)

Table:

List the outcomes of the first quadrilateral in the left column and those of the second quadrilateral in the top row.

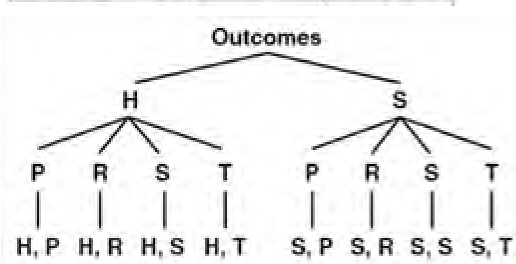
Outcomes	rhombus	square
parallelogram	H, P	S, P
rectangle	H, R	S, R
square	H, S	S, S
trapezoid	H, T	S, T
rhombus	H, H	S, H

Tree Diagram:
The top row is all of the outcomes for the first quadrilateral. The second row includes all of the outcomes for the second quadrilateral. The last row shows the sample space.



ANSWER:
H = rhombus, P = parallelogram, R = rectangle, S = square, T = trapezoid; H, P; H, R; H, S; H, T; S, P; S, R; S, S; S, T; H, H; S, H

Outcomes	rhombus	square
parallelogram	H, P	S, P
rectangle	H, R	S, R
square	H, S	S, S
trapezoid	H, T	S, T
rhombus	H, H	S, H



20. **BREAKFAST** A hotel restaurant serves omelets with a choice of vegetables, ham, or sausage that come with a side of hash browns, grits, or toast.

a. How many different outcomes of omelet and one side are there if a vegetable omelet comes with just one vegetable?
b. Find the number of possible outcomes for a vegetable omelet if you can get any or all vegetables on any omelet.

SOLUTION:
a. There are 4 different vegetables, and so 4 different types of vegetable omelets. Each has a choice of 3 different sides. So there are 4(3) = 12 different vegetable omelet orders.
There are another 3 outcomes for a ham omelet (with one of three different sides) and another 3 for a sausage omelette (with one of three different sides).
So, in all, there are 12 + 3 + 3 = 18 different outcomes.

b. There is $\binom{4}{4} = 1$ way to have all 4 vegetables.

There are $\binom{4}{3} = 4$ ways to have 3 vegetables.

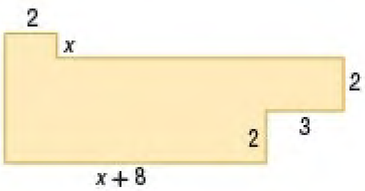
There are $\binom{4}{2} = 6$ ways to have 2 vegetables.

There are $\binom{4}{1} = 4$ ways to have 1 vegetable.

You must have 1 vegetable, or it is not a vegetable omelet!
So, there are 15 different possible vegetable omelets. Multiply by 3 for the different choices of sides. There are 45 different possible orders for a vegetable omelet.

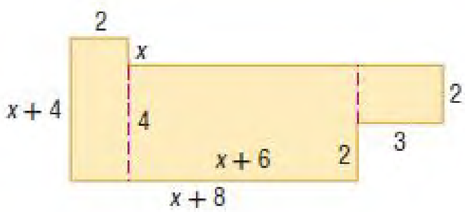
ANSWER:
a. 18
b. 45

21. **COMPOSITE FIGURES** Carlito is calculating the area of the composite figure at the right. List six different ways he can do this.

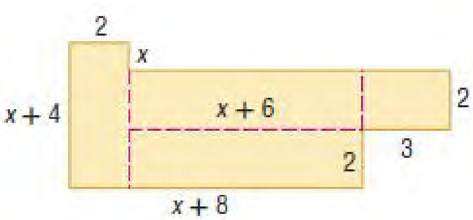


SOLUTION:

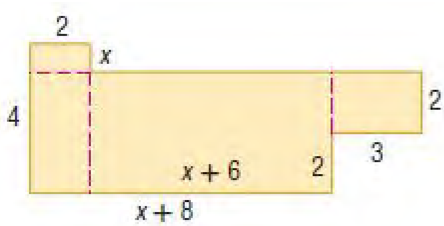
The figure can be divided into different sets of squares and rectangles. The area of the composite figure is the sum of the areas of the smaller squares and rectangles. Some of the ways the figure can be divided and the resulting sum of the areas are shown below.



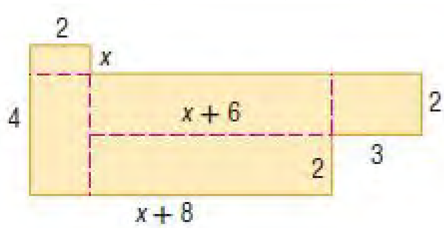
$A = 2(x + 4) + 4(x + 6) + 2(3)$;



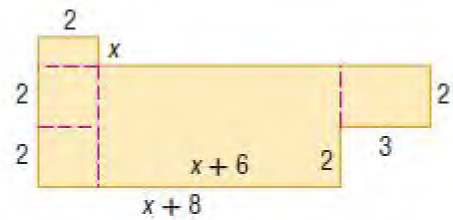
$A = 2(x + 4) + 2(x + 6) + 2(x + 6) + 2(3)$;



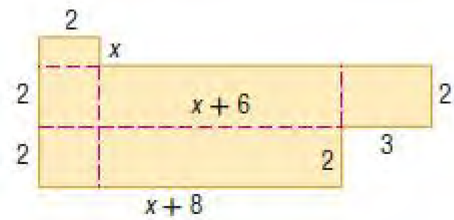
$A = 2(x) + 2(4) + 4(x + 6) + 2(3)$;



$A = 2(x) + 2(4) + 2(x + 6) + 2(x + 6) + 2(3)$;

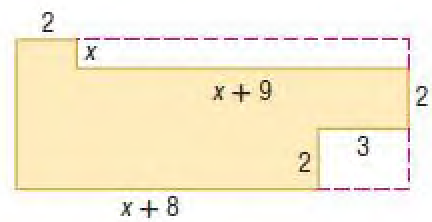


$A = 2(x) + 2(2) + 2(2) + 4(x + 6) + 2(3)$;



$A = 2(x) + 2(2) + 2(2) + 2(x + 6) + 2(x + 6) + 2(3)$;

The area of the figure can also be found by calculating the area of the whole rectangle and then subtracting the areas of the two rectangular holes.



$A = (x + 11)(x + 4) - [x(x + 9) + 2(3)]$

ANSWER:

Sample answer: 6 different ways
$2(x + 4) + 4(x + 6) + 2(3)$;
$2(x + 4) + 2(x + 6) + 2(x + 6) + 2(3)$;
$2(x) + 2(4) + 4(x + 6) + 2(3)$;
$2(x) + 2(4) + 2(x + 6) + 2(x + 6) + 2(3)$;
$2(x) + 2(2) + 2(2) + 4(x + 6) + 2(3)$;
$2(x) + 2(2) + 2(2) + 2(x + 6) + 2(x + 6) + 2(3)$

22. **TRANSPORTATION** Miranda got a new bicycle lock that has a four-number combination. Each number in the combination is from 0 to 9.

a. How many combinations are possible if there are no restrictions on the number of times Miranda can use each number?

b. How many combinations are possible if Miranda can use each number only once? Explain.

SOLUTION:

a. By the Fundamental Counting Principle the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

Miranda can use any of the 10 digits in the first, second, third, and fourth places. Therefore, total number of combinations are $10 \times 10 \times 10 \times 10 =$ 10,000.

b. Since Miranda can use each number only once, the combination cannot have any duplicates, like 9935 or 8888. Therefore, the number of combinations is not $10 \times 10 \times 10 \times 10$. We need to see how many possibilities there are for each number in the combination.

There are 10 possibilities for the first number in the combination. If the first number is 4, then the second number can be anything except 4. Therefore, there are only 9 possibilities for the second number in the combination. If the second number is 5, then the third number cannot be 4 or 5, so there are 8 possibilities for the third number in the combination. If the third number is 9, then the fourth number can be anything except 4, 5, or 9. Therefore, there are 7 possibilities for the fourth number in the combination.

The number of possible combinations is $10 \times 9 \times 8 \times 7$ or 5040.

ANSWER:

a. 10,000

b. 5040; Sample answer: There are 10 possibilities for the first number in the combination. Since Miranda can use each number only once, there are only 9 possibilities for the second number in the combination, 8 possibilities for the third number in the combination, and 7 possibilities for the fourth number in the combination. The number of possible combinations is $10 \times 9 \times 8 \times 7$ or 5040.

23. **GAMES** Cody and Monette are playing a board game in which you roll two dice per turn.

a. In one turn, how many outcomes result in a sum of 8?

b. How many outcomes in one turn result in an odd sum?

SOLUTION:

a. A sum of 8 can occur in 5 different ways, (2, 6), (6, 2), (3, 5), (5, 3), and (4, 4) where the first number in the ordered pair represent the outcome of the first roll and the second number represents that of the second roll.

b. A sum of an odd number can occur in 18 different ways as shown.

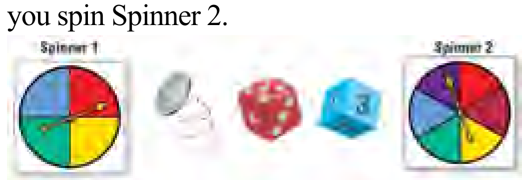
Outcome	Sum	Outcome	Sum
(1, 2)	3	(2, 1)	3
(1, 4)	5	(4, 1)	5
(1, 6)	7	(6, 1)	7
(2, 3)	5	(3, 2)	5
(2, 5)	7	(5, 2)	7
(3, 4)	7	(4, 3)	7
(3, 6)	9	(6, 3)	9
(4, 5)	9	(5, 4)	9
(5, 6)	11	(6, 5)	11

ANSWER:

a. 5

b. 18

24. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a sequence of events. In the first stage of a two-stage experiment, you spin Spinner 1 below. If the result is red, you flip a coin. If the result is yellow, you roll a die. If the result is green, you roll a number cube. If the result is blue, you spin Spinner 2.



a. GEOMETRIC Draw a tree diagram to represent the sample space for the experiment.

b. LOGICAL Draw a Venn diagram to represent

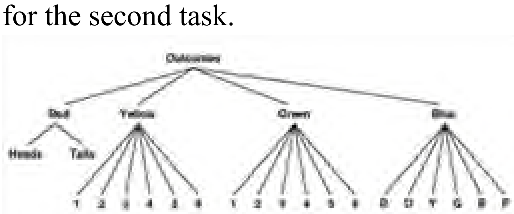
the possible outcomes of the experiment.
c. ANALYTICAL How many possible outcomes are there?
d. VERBAL Could you use the Fundamental Counting Principle to determine the number of outcomes? Explain.

SOLUTION:
a. There are four initial outcomes for the first spin. You can spin red, yellow, green, or blue. The outcome of the first spin determines what you do next.

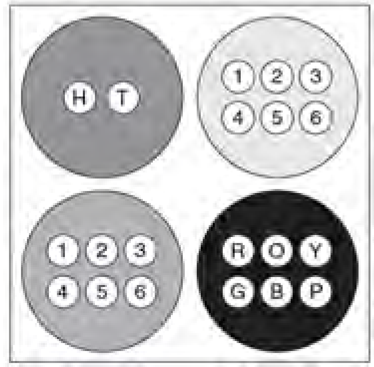
R: coin
Y: die
G: cube
B: spin

Organized List:
Pair each possible outcome for the first spin with the possible outcomes for the second task (spin, die, cube, spin).
R, H; R, T
Y, 1, Y, 2; Y, 3; Y, 4; Y, 5; Y, 6
G, 1, G, 2; G, 3; G, 4; G, 5; G, 6
B, R; B, O; B, Y; B, G; B, B; B, P

Tree Diagram:
The top group is all of the outcomes for the first task (spin). The second group includes all of the outcomes for the second task.



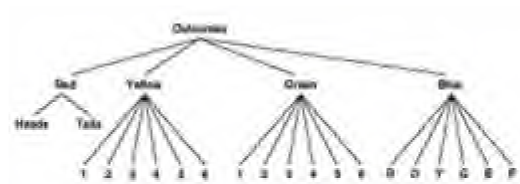
b. Each outcome of the first spin is independent, so the four circles representing these outcomes will not intersect. All of the outcomes for the second task shall be included in each respective circle because they cannot occur in any other circle (they are independent).



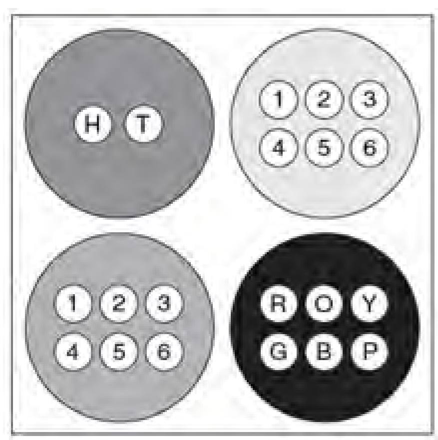
c. The total number of outcomes is 6 + 6 + 6 + 2 = 20. The complete list is provided in part **a** and each outcome is also shown in the Venn diagram in part **b**.

d. No; The Fundamental Counting Principle relies on independent outcomes. Since the second stage of the experiment depends on what happens in the first stage of the experiment, you cannot multiply the number of outcomes for each stage. You have to find the number of possible outcomes for each stage and add them.

ANSWER:
a.



b.



c. 20
d. Sample answer: No; since the second stage of the experiment depends on what happens in the first stage of the experiment, you cannot multiply the number of outcomes for each stage. You have to find

the number of possible outcomes for each stage and add them.

25. **CHALLENGE** A box contains n different objects. If you remove three objects from the box, one at a time, without putting the previous object back, how many possible outcomes exist? Explain your reasoning.

SOLUTION:
We have 3 stages: taking out each object three times.

Since we are not putting the objects back in the box, and there will be fewer objects in the box for each stage, these stages are not independent.

Each stage will have a different amount of possible outcomes. There are n objects in the box when you remove the first object, so after you remove one object, there are $n - 1$ possible outcomes. After you remove the second object, there are $n - 2$ possible outcomes.

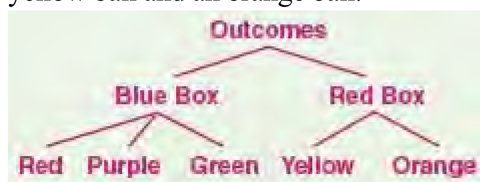
The number of possible outcomes is the product of the number of outcomes of each experiment or $n(n - 1)(n - 2) = n^3 - 3n^2 + 2n$.

ANSWER:
$n^3 - 3n^2 + 2n$; Sample answer: There are n objects in the box when you remove the first object, so after you remove one object, there are $n - 1$ possible outcomes. After you remove the second object, there are $n - 2$ possible outcomes. The number of possible outcomes is the product of the number of outcomes of each experiment or $n(n - 1)(n - 2)$.

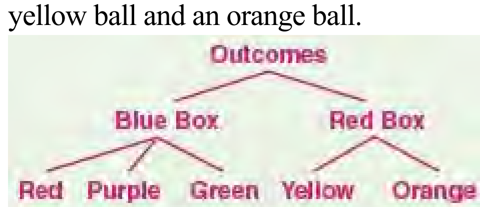
26. **OPEN ENDED** Sometimes a tree diagram for an experiment is not symmetrical. Describe a two-stage experiment where the tree diagram is asymmetrical. Include a sketch of the tree diagram. Explain.

SOLUTION:
In order to make a tree diagram asymmetrical, set up the events so that there are a different number of outcomes for each event. For example, rolling a die has 6 outcomes and flipping a coin has only 2 outcomes.

Sample answer: In an experiment, you choose between a blue box and a red box. You then remove a ball from the box that you chose without looking into the box. The blue box contains a red ball, a purple ball, and a green ball. The red box contains a yellow ball and an orange ball.



ANSWER:
Sample answer: In an experiment, you choose between a blue box and a red box. You then remove a ball from the box that you chose without looking into the box. The blue box contains a red ball, a purple ball, and a green ball. The red box contains a yellow ball and an orange ball.



27. **WRITING IN MATH** Explain why it is not possible to represent the sample space for a multi-stage experiment by using a table.

SOLUTION:

Sample answer: You can list the possible outcomes for one stage of an experiment in the columns and the possible outcomes for the other stage of the experiment in the rows. Since a table is two dimensional, it would be impossible to list the possible outcomes for three or more stages of an experiment. Therefore, tables can only used to represent the sample space for a two-stage experiment.

For example, consider two coin flips. All four outcomes can be shown in the table, with one stage in the column, and the other stage in the row.

	Heads	**Tails**
Heads	H, H	H, T
Tails	T, H	T, T

Now, consider three coin flips. Where do we place the third stage? It cannot be placed in the bottom row, the far right column, or anywhere inside the table. It would only work if the table were three-dimensional.

ANSWER:

Sample answer: You can list the possible outcomes for one stage of an experiment in the columns and the possible outcomes for the other stage of the experiment in the rows. Since a table is two dimensional, it would be impossible to list the possible outcomes for three or more stages of an experiment. Therefore, tables can only used to represent the sample space for a two-stage experiment.

28. **CCSS ARGUMENTS** Determine if the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.
When an outcome falls outside the sample space, it is a failure.

SOLUTION:

Sample answer: Never; the sample space is the set of all possible outcomes. An outcome cannot fall outside the sample space. A failure occurs when the outcome is in the sample space, but is not a favorable outcome.

For example, when you are flipping a coin, it can be heads or tails, and nothing else. If heads is the favorable outcome, then tails is the unfavorable outcome. However, tails is still in the sample space because it is a *possible* outcome.

ANSWER:

Sample answer: Never; the sample space is the set of all possible outcomes. An outcome cannot fall outside the sample space. A failure occurs when the outcome is in the sample space, but is not a favorable outcome.

29. **REASONING** A multistage experiment has n possible outcomes at each stage. If the experiment is performed with k stages, write an equation for the total number of possible outcomes P. Explain.

SOLUTION:

If there are n outcomes for each stage, then there are:

$n \times n$ or n^2 outcomes for 2 stages,

$n \times n \times n$ or n^3 outcomes for 3 stages, and

$n \times n \times n ... \times n$ or n^k outcomes for k stages.

$P = n^k$; Sample answer: The total number of possible outcomes is the product of the number of outcomes for each of the stages 1 through k. Since there are k stages, you are multiplying n by itself k times which is n^k.

ANSWER:

$P = n^k$; Sample answer: The total number of possible outcomes is the product of the number of outcomes for each of the stages 1 through k. Since there are k stages, you are multiplying n by itself k times which is n^k.

30. **WRITING IN MATH** Explain when it is necessary to show all of the possible outcomes of an experiment by using a tree diagram and when using the Fundamental Counting Principle is sufficient.

SOLUTION:

Sample answer: Drawing a tree diagram is necessary if you want to show the sample space for an experiment or if you want to know the number of times a certain outcome occurs. One example would be listing the different options of pairs to be chosen from a group of people.

Using the Fundamental Counting Principle only tells you how many possible outcomes there are, so it is only useful when you want to know how many outcomes there are. One example would be to find out the probably of rolling a 1, 1, 1, 1, 1 when rolling 5 six-sided dice. Listing all of the different outcomes would be time-consuming and irrelevant to finding the probability.

ANSWER:

Sample answer: Drawing a tree diagram is necessary if you want to show the sample space for an experiment or if you want to know the number of times a certain outcome occurs. Using the Fundamental Counting Principle only tells you how many possible outcomes there are, so it is only useful when you want to know how many outcomes there are.

31. **PROBABILITY** Alejandra can invite two friends to go out to dinner with her for her birthday. If she is choosing among four of her friends, how many possible outcomes are there?
A 4
B 6
C 8
D 9

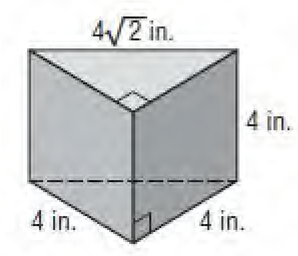
SOLUTION:

Let the friends be A, B, C, and D. Then she can choose 2 from the four in 6 different ways as AB, AC, AD, BC, BD, and CD. Therefore, the correct choice is B.

ANSWER:

B

32. **SHORT RESPONSE** What is the volume of the triangular prism shown below?



SOLUTION:

The volume V of a prism is $V = Bh$, where B is the area of a base and h is the height of the prism. The base is a right triangle with the legs 4 in. each long. The height of the prism is 4 in. Therefore, the volume is $\frac{1}{2}(4)(4)(4) = 32 \text{ in}^3$.

ANSWER:

32 in^3

33. Brad's password must be five digits long, use the numbers 0–9, and the digits must not repeat. What is the maximum number of different passwords that Brad can have?
F 15,120
H 59,049
G 30,240
J 100,000

SOLUTION:

There are 10 possibilities for the first number in the combination. Since Brad can use each number only once, there are only 9 possibilities for the second number in the combination, 8 possibilities for the third number in the combination, 7 possibilities for the fourth number in the combination, and 6 possibilities for the fourth number in the combination. The number of possible combinations is 10 × 9 × 8 × 7 × 6or 30,240. The correct choice is G.

ANSWER:

G

34. **SAT/ACT** A pizza shop offers 3 types of crust, 5 vegetable toppings, and 4 meat toppings. How many different pizzas could be ordered by choosing 1 crust, 1 vegetable topping, and 1 meat topping?

A 12
B 23
C 35
D 60
E infinite

SOLUTION:

By the Fundamental Counting Principle the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

A crust can be ordered in 3 different ways, vegetable topping in 5 different ways, and meat topping in 4 different ways. Therefore, an order with one item from each set can be done in 3 × 5 × 4 = 60 ways. The correct choice is D.

ANSWER:

D

35. **ARCHITECTURE** To encourage recycling, the people of Rome, Italy, built a model of Basilica di San Pietro from empty beverage cans. The model was built to a 1: 5 scale and was a rectangular prism that measured 26 meters high, 49 meters wide, and 93 meters long. Find the dimensions of the actual Basilica di San Pietro.

SOLUTION:

The scale of the model is 1 : 5. Let the length, width, and height of the actual building be l, w, and h respectively. Then we have the fractions,

$$\frac{1}{5}=\frac{93}{l}=\frac{49}{w}=\frac{26}{h}.$$

Solve the proportion to find the actual dimensions.

$$\frac{1}{5}=\frac{93}{l}=\frac{49}{w}=\frac{26}{h}.$$

$$\frac{1}{5}=\frac{93}{l}\Rightarrow l=93\cdot 5=465$$

$$\frac{1}{5}=\frac{49}{w}\Rightarrow w=49\cdot 5=245$$

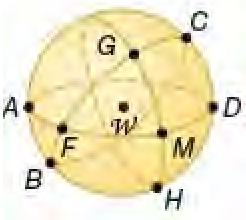
$$\frac{1}{5}=\frac{26}{h}\Rightarrow h=26\cdot 5=130$$

Therefore, the actual Basilica di San Pietro is 130 m high, 245 m wide, and 465 m long.

ANSWER:

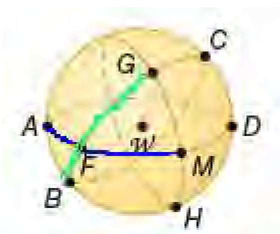
130 m high, 245 m wide, and 465 m long

Using spherical geometry, name each of the following on sphere *W*.



36. two lines containing point *F*

SOLUTION:



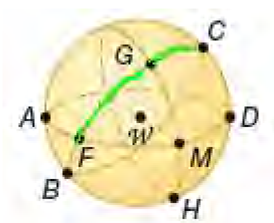
$\overleftrightarrow{BG}, \overleftrightarrow{AM}$

ANSWER:

$\overleftrightarrow{BG}, \overleftrightarrow{AM}$

37. a segment containing point *G*

SOLUTION:



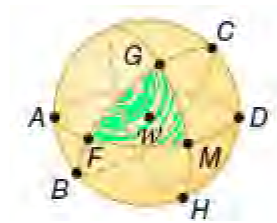
$\overline{FC}$

ANSWER:

$\overline{FC}$

38. a triangle

SOLUTION:



ΔFGM

ANSWER:

ΔFGM

Find the lateral area and surface area of each cylinder. Round to the nearest tenth.

39. 

SOLUTION:

The lateral area *L* of a right cylinder is *L* = 2π*rh*, where *r* is the radius of a base and *h* is the height.

The radius of the base is 7 ft and the height of the cylinder is 32.5 ft.

$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(7)(32.5) \\ &= 455\pi \\ &\approx 1429.4 \end{aligned}$$

The total surface area of the prism is the sum of the areas of the bases and the lateral surface area.

$$\begin{aligned} S &= 2\pi rh + 2\pi r^2 \\ &= 455\pi + 2\pi(7)^2 \\ &= 553\pi \\ &\approx 1737.3 \end{aligned}$$

ANSWER:

1429.4 ft^2

1737.3 ft^2

40. 

SOLUTION:

The lateral area *L* of a right cylinder is *L* = 2π*rh*, where *r* is the radius of a base and *h* is the height.

The radius of the base is 1.5 in and the height of the cylinder is 4 in.

$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(1.5)(4) \\ &= 12\pi \\ &\approx 37.7 \end{aligned}$$

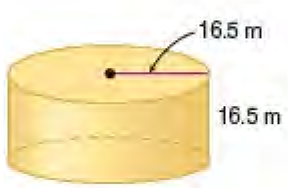
The total surface area of the prism is the sum of the areas of the bases and the lateral surface area.

$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(1.5)^2 + 12\pi \\ &= 16.5\pi \\ &\approx 51.8 \end{aligned}$$

ANSWER:

37.7 in^2

51.8 in^2

41. 

SOLUTION:

The lateral area L of a right cylinder is $L = 2\pi rh$, where r is the radius of a base and h is the height.

The radius of the base is 16.5 m. and the height of the cylinder is 16.5 m.

$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(16.5)(16.5) \\ &= 544.5\pi \\ &\approx 1710.6 \end{aligned}$$

The total surface area of the prism is the sum of the areas of the bases and the lateral surface area.

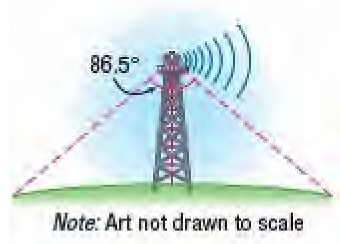
$$\begin{aligned} S &= 2\pi rh + 2\pi r^2 \\ &= 544.5\pi + 2\pi(16.5)^2 \\ &= 544.5\pi + 544.5\pi \\ &\approx 3421.2 \end{aligned}$$

ANSWER:

1710.6 m^2

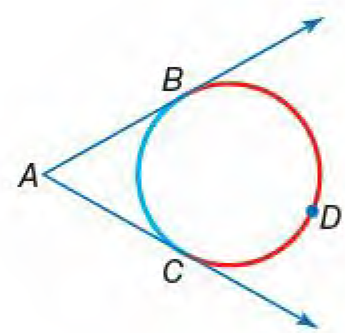
3421.2 m^2

42. **TELECOMMUNICATIONS** The signal from a tower follows a ray that has its endpoint on the tower and is tangent to Earth. Suppose a tower is located at sea level as shown. Determine the measure of the arc intercepted by the two tangents.



SOLUTION:

From Theorem 10.14, if two tangent lines intersect in the exterior of a circle, then the measure of the angle formed is one half the differences of the measures of the intercepted arcs.



$$m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$$

$$\begin{aligned} \angle A &= \frac{1}{2}(arcBDC - arcBC) \\ 86.5 &= \frac{1}{2}((360 - x) - x) \\ 173 &= 360 - x - x \\ 173 &= 360 - 2x \\ -187 &= -2x \\ 93.5 &= x \end{aligned}$$

ANSWER:

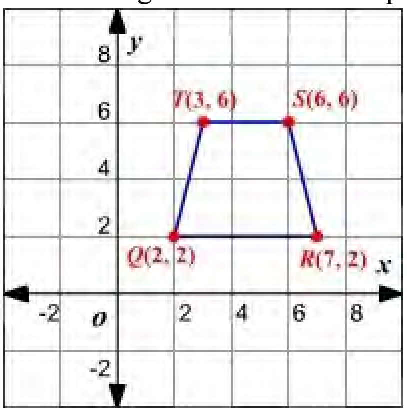
93.5

COORDINATE GEOMETRY Determine whether the figure with the given vertices has *line* symmetry and/or *rotational* symmetry.

43. $Q(2, 2)$, $R(7, 2)$, $S(6, 6)$, $T(3, 6)$

SOLUTION:

Draw the figure on a coordinate plane.



The figure has a line symmetry about the line $x = 4.5$ because if we were to cut the figure in half along that line, the two sides would be reflections of each other.

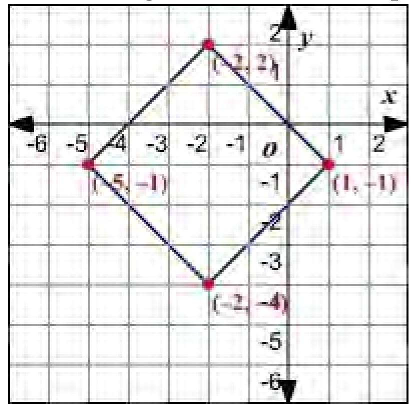
ANSWER:

line

44. $J(–2, 2)$, $K(–5, –1)$, $L(–2, –4)$, $M(1, –1)$

SOLUTION:

Draw the figure on a coordinate plane.



The figure has a line symmetry about the line $x = –2$ because if we were to cut the figure in half along that line, the two sides would be reflections of each other.

and rotational symmetry about an angle 90º because if we were to rotate the figure 90 degrees, the image would be identical to the original figure.

ANSWER:

line and rotational

Find each quotient.

45. $\frac{5^2}{2}$

SOLUTION:

$$\frac{5^2}{2} = \frac{5 \cdot 5}{2} = \frac{25}{2} = 12.5$$

ANSWER:

12.5

46. $\frac{3^3}{3 \cdot 2}$

SOLUTION:

$$\frac{3^3}{3 \cdot 2} = \frac{3 \cdot 3 \cdot 3}{3 \cdot 2} = \frac{27}{6} = 4.5$$

ANSWER:

4.5

47. $\frac{2^4 \cdot 6}{8}$

SOLUTION:

$$\frac{2^4 \cdot 6}{8} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 6}{8} = 12$$

ANSWER:

12

48. $\frac{2^3 \cdot 12}{6}$

SOLUTION:

$$\frac{2^3 \cdot 12}{6} = \frac{2 \cdot 2 \cdot 2 \cdot 12}{6} = 16$$

ANSWER:

16

49. $\frac{4^4 \cdot 3}{24}$

SOLUTION:

$$\frac{4^4 \cdot 3}{24} = \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 3}{24} = 32$$

ANSWER:

32