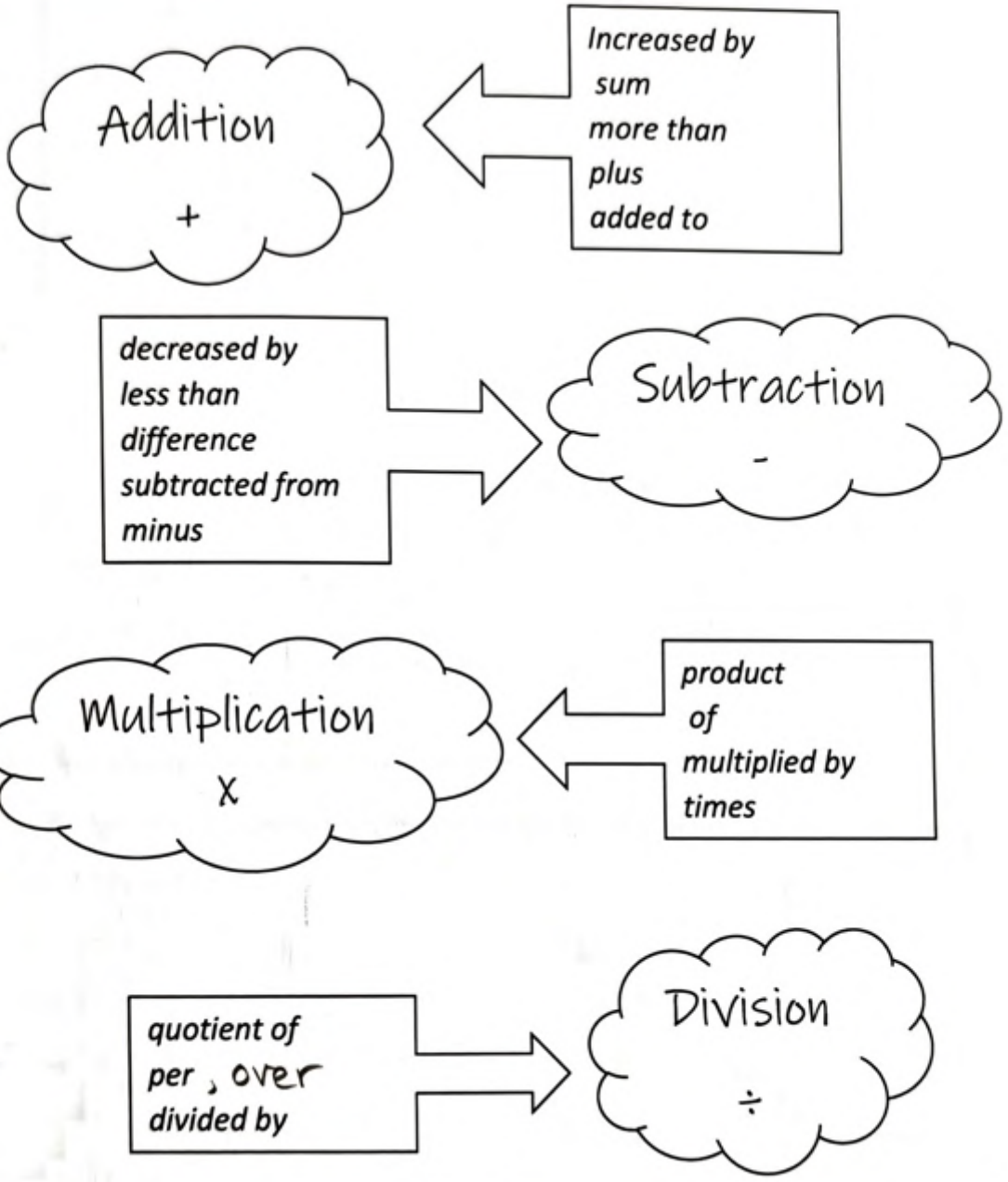
# Grade 9 Adv Revision

## Maths

- Read & learn the material provided
- Practice the questions in the book
- Practice the questions in the revision file
- Good luck!

## + − × ÷ Vocabulary




**Other vocabulary**

- Denominator-bottom of a fraction
- Numerator-top of a fraction
- Coefficient-number attached to a variable (eg coefficient of x in 2x is 2)
- Evaluate- find the answer
- Simplify-make it as simple as possible

**Powers**

$a^n$ is said `a to the power of n`

$a^2$ *is said `a squared`*

$a^3$ *is said `a cubed`*

Eg: $3^5$ *means* $3 \times 3 \times 3 \times 3 \times 3 = 243$

**Order of Operations: BODMAS**

**B**rackets

**O**ther ( eg powers)

**D**ivision

**M**ultiplication

**A**ddition

**S**ubtraction

eg Simplify $\frac{2^2+5}{15+9}$

when you have a fraction, treat the top & bottom like brackets

$\frac{(2^2+5)}{(15+9)} = \frac{(4+5)}{24} = \frac{9}{24} = \frac{3}{8}$

**Expanding Brackets (the distributive property)**

*Multiplying what's on the outside by each term on the inside to get rid of the brackets*

Eg 1: Expand $2(4 + 3x)$

$= 8 + 6x$

Eg 2: $x(y + 3)$

$= xy + 3x$

Eg 3: $x(x + 3)$

$= x^2 + 3x$

**Factorising**

*When you find a factor (something that goes into each term) and take it out and form a bracket*

Eg 1: Factorise $xy + 3x$

$= x(y + 3)$ *(Can check by expanding the bracket and seeing if you get what the question says)*

Eg 2: $4x + 6$

$= 2(2x + 3)$

Eg 3: $y^2 + 3y$

$= y(y + 3)$

**Consecutive integers: whole numbers in counting order**

Eg: -1, 0, 1, 2, 3... Or n, n+1, n+2, n+3...

- *Consecutive even integers:*

Eg: 2, 4, 6, 8... Or n, n+2, n+4...

- *Consecutive odd integers:*

Eg: 1, 3, 5.... Or n, n+2, n+4...

Eg find 3 consecutive odd integers with a sum of -51

$$(n) + (n + 2) + (n + 4) = -51$$

$$3n + 6 = -51$$

$$3n = -51 - 6$$

$$3n = -57$$

$$n = -\frac{57}{3}$$

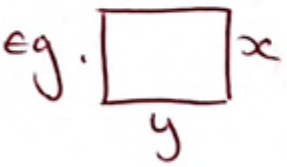
$$n = -19$$

$$n = -19, \quad n + 2 = -17, \quad n + 4 = -15$$

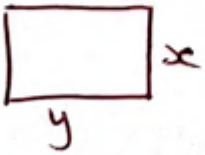
$\therefore$ *3 consecutive odd integers with a sum of* $-51$ *are* $-19, -17, -15$

**Perimeter and Area**

the space

eg.  Area = $xy$

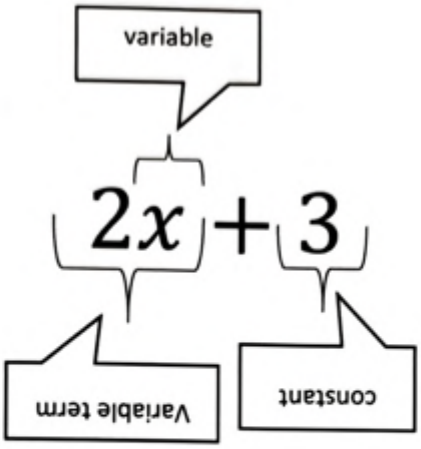
The length around the edge

eg  Perimeter $= 2x + 2y$

## Expressions

What is the difference between an equation and an expression? *An expression has no = sign*

Each part of an expression is called *a term.*




Like terms: *have the same variable* eg: $2x$ *and* $3x$ *are like terms*

Unlike terms: *do not have the same variable*

$eg: 2x^2 and\ 2x\ are\ unlike\ terms.$

$eg: 2x\ and\ 9\ are\ unlike\ terms$

**When solving, only like terms can be collected together.**

## No Solution

Eg: Solve $5n + 6 = 5n + 7$

$5n - 5n = 7 - 6$

$0 = 1$

This doesn't make sense as $0 \neq 1$ $\therefore$ *no solution or* $\emptyset$

## Identities

Eg: Is $6h + 12\ and\ (3h + 6)2$ an identity?

$6h + 12 \stackrel{?}{\equiv} (3h + 6)2$

$6h + 12 \equiv 6h + 12$

The left hand side (LHS) is equal to the right hand side (RHS) ∴ it's an identity and the solutions are all real numbers.

**Other points**

- $a(0) = 0$
- $a(1) = a$
- $\frac{0}{a} = 0$
- $\frac{a}{0} = undefined$ (denominator of a fraction cannot be 0)

Eg: Solve $x(3 - z) = 5 + 2y$ for $x$

$$x = \frac{5 + 2y}{3 - z}$$

But we know the denominator can't be 0 so we also have to say

$x = \frac{5+2y}{3-z}$ where $z \neq 3$ (because if z was 3 the denominator would be 0)

# Algebra

In algebra, letters (variables) can be used to represent unknown numerical values. For example, in the equation $3x + y = 16$, $x$ and $y$ are variables.

## Key Terminology

A **term** is a collection of numbers and letters. Terms are separated by mathematical symbols. $3x + 4xy = 18 + y$

An **expression** includes terms and operational (mathematical) symbols but not the equals symbol. $2x + 5y - 2$

An **equation** is made up of two expressions that are equal. $4x + 5y = 23$

An **identity** is an equation that is true for all values of the variables. $3x = 12$ is not an identity as it is only true when $x = 4$. $2y = y + y$

## Simplifying Algebraic Expressions

To simplify algebraic expressions, like terms can be collected together. Like terms contain the **same variable** raised to the **same power.**

### Addition and Subtraction

$a + a + a$ can be shortened to $3a$

$3a + 5a = 8a$

$4a + a = 5a$

$4b - b$ can be shortened to $3b$

$6b - b = 5b$

$5b - 3b = 2b$

### Multiplication

When multiplying (like or unlike) terms, the multiplication symbol is removed.

$a \times b$ can be shortened to $ab$

$3 \times a$ can be shortened to $3a$

$y \times y \times y \times y$ can be shortened to $y^4$ ← This is an **index** (power). It shows how many times y is multiplied by itself.

Remember $4y$ is not the same as $y^4$. $4y = y + y + y + y$ $y^4 = y \times y \times y \times y$

### Division

$a \div b$ is written as $\frac{a}{b}$

When dividing like terms, the variable can be removed from the answer.

$\frac{15b}{3b}$ can be shortened to $5$

$\frac{12b}{3b}$ can be shortened to $4$

# Solving Equations

Inverse operations are opposite operations that 'undo' each other.

**Addition and Subtraction are inverse operations.**

**Multiplication and Division are inverse operations.**

## How to Solve Equations With Inverse Operations

The aim when solving an equation is to get the variable by itself on one side of the equation with a number on the other side – for example, $x = 2$.

When there are other operations on the same side of an equation as the variable, they need to be removed. This is done by performing the inverse (opposite) of the operation acting upon the variable. **This must be done to both sides of the equation.**

**$n + 3 = 12$**

The inverse of addition is subtraction, so subtract 3 from both sides of the equation.

$n + 3 = 12$

$-3 \quad -3$

$n = 9$

**$x - 7 = 1$**

The inverse of subtraction is addition, so add 7 to both sides of the equation.

$x - 7 = 1$

$+7 \quad +7$

$x = 8$

**$5y = 20$**

The inverse of multiplication is division, so divide both sides of the equation by 5.

$5 \times y = 20$

$\div 5 \quad \div 5$

$y = 4$

**$\frac{m}{5} = 3$**

The inverse of division is multiplication, so multiply both sides of the equation by 5.

$m \div 5 = 3$

$\times 5 \quad \times 5$

$m = 15$

The above rules do not always work for division and subtraction.
When the variable is the divisor, or being subtracted, solve the problem in two steps.

**When the variable is being subtracted:**

$7 - d = 4$

Add $d$ to both sides of the equation. $+d \quad +d$

$7 = 4 + d$

Subtract 4 from both sides of the equation. $-4 \quad -4$

$3 = d$

**When the variable is the divisor:**

$\frac{12}{w} = 3$

Multiply both sides of the equation by $w$. $\times w \quad \times w$

$12 = 3w$

Divide both sides of the equation by 3. $\div 3 \quad \div 3$

$4 = w$

Max and Amy have 37 apps in total.
If Amy has 21 apps, how many does Max have?

**1. Turn the question into an equation.**

| Use $m$ to represent the unknown value (the number of apps Max has). | Max has _ apps | Amy has 21 apps | 37 apps in total |
|---|---|---|---|
| | $m$ | $+ 21$ | $= 37$ |

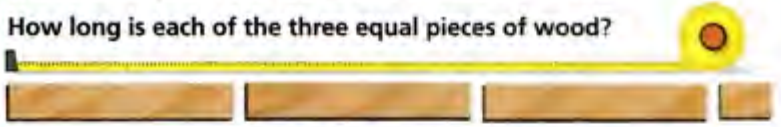
**2. Solve the equation to find the value of $m$.**

| Subtraction is the inverse operation of addition so **subtract 21 from both sides of the equation.** | $m + 21 = 37$ |
|---|---|
| | $-21 \quad -21$ |
| | $m = 16$ |

The equation is now solved. $m = 16$ so Max has 16 apps.

Rohan has three pieces of wood of equal length, and one 6 cm piece.
The total length of the four pieces of wood is 42 cm.

How long is each of the three equal pieces of wood?



**1. Turn the question into an equation.**

| Use $w$ to represent the unknown value (the length of the three equal pieces of wood). | 3 pieces of wood | Other piece of wood | Total length |
|---|---|---|---|
| | $3 \times w$ | $+ 6$ | $= 42$ |

**2. Solve the equation to find the value of $w$.**

| Subtraction is the inverse operation of addition so **subtract 6 from both sides of the equation.** | $3w + 6 = 42$ |
|---|---|
| | $-6 \quad -6$ |
| Division is the inverse operation of multiplication so **divide both sides of the equation by 3.** | $3w = 36$ |
| | $\div 3 \quad \div 3$ |
| | $w = 12$ |

The equation is now solved. $w = 12$ so each equal piece of wood is 12 cm long.

## Absolute Value Summary!

- Absolute value is also called Modulus
- The symbol used is $| \; |$
- Modulus makes its value positive, even if the inside is positive or negative

  Eg:

  $|4| = 4$

  $|-4| = 4$

- To solve modulus equations:

  *Take the inside of the modulus and equal it to ± of the answer*

  Eg: Solve $|x + 2| = 5$

  $x+2 = -5$ $\qquad$ $x+2 = 5$

  $x = -7$ $\qquad$ $x = 3$

  Solutions can also be drawn on a graph:

  Or written as a solution set:

  $\{-7, 3\}$

- The value of the modulus *cannot* be negative

  Eg: Solve $|x + 2| = -5$

  No solution

- Modulus helps with margin of error questions. Other words for margin of error are:

  ±

  give or take

  allowance

  above or below

- To write a modulus equation:

$$\left|x - \begin{matrix}\text{value you want} \\ \text{(the midpoint)}\end{matrix}\right| = \text{margin of error (distance from midpoint to min and max values)}$$

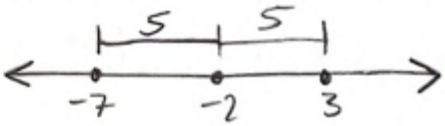
Eg: find the modulus equation for

*First find the midpoint:*

$$\text{midpoint} = \frac{\text{min} + \text{max}}{2}$$

$$= \frac{-7+3}{2} = -2$$

*Next, find the distance from the midpoint to the max and min values:*




*Then write the equation:*

$$|x - -2| = 5 \Rightarrow |x+2| = 5$$

*Check your answer:*

$$|x+2| = 5$$

$x+2 = -5$
$x = -7$ ✓

$x+2 = 5$
$x = 3$ ✓

- When solving unusual modulus questions treat the modulus as a variable

Eg: Solve $2|x| + 3 = 7$

$$2|x| = 4$$
$$|x| = 2$$
$$x = -2 \qquad x = 2$$

# Ratios and proportions

↳ allow us to compare 2 values.

* $x:y$
* $\frac{x}{y}$
* $x$ to $y$
* $x$ per $y$

↳ an equation stating 2 ratios (or fractions) are equal.

eg are 2:3 and 16:24 equivalent?

Consider $\frac{2}{3}$ and $\frac{16}{24}$

**Method 1**

$\frac{2}{3} = \frac{2}{3}$

$\frac{16}{24} = \frac{2}{3}$

Both simplify to the same so they are a proportion

**Method 2**

$\frac{2}{3} \stackrel{?}{=} \frac{16}{24}$ cross multiply

$2(24) \stackrel{?}{=} 16(3)$

$48 = 48$

Both sides are the same ∴ they are a proportion

## Scaling up ratios

eg Ali mixes paint in the ratio 5:4 for blue & red paint. If he has 20 pots of blue paint, how many pots of red paint does he need?

**Method 1**

Blue : Red

$5:4$ → $20:x$ (multiply both sides by $\times\frac{20}{5}$)

Scaling factor which is $\frac{\text{new}}{\text{old}} = \frac{20}{5}$

$x = 4 \times \frac{20}{5}$

$= \frac{80}{5}$

$= 16$

∴ 16 pots of red.

**Method 2** 5:4 and 20:x

$\frac{5}{4} = \frac{20}{x}$

$5x = 20(4)$

$5x = 80$

$x = \frac{80}{5}$

$= 16$

∴ 16 pots of red.

## To get 1:n or n:1

eg Make 4:30 in the form 1:n

$4:30$ → $1:n$ (multiply both sides by $\times\frac{1}{4}$)

$\frac{\text{new}}{\text{old}}$

$n = 30 \times \frac{1}{4} = 7.5$ ∴ 1:7.5

## Percentage summary

- Words used when we have a percent **increase**: *profit, growth, tax*
- Words when we have a percent **decrease**: *discount, loss, reduced, sale*

❖ To work out percent change:

$$percent\ change = \frac{new - original}{original} \times 100$$

*Note:*

**Positive answer means increase* **Negative answer means decrease*

Eg: The price of chocolate was AED 25 and has been reduced to AED 17. What is the percent decrease?

$$percent\ change = \frac{17 - 25}{25} \times 100$$

$$= -32 \quad \therefore percent\ decrease\ of\ 32\%$$

❖ To work out the new or original value when you know the percent change:

$$new = original \pm (\%\ of\ the\ original)$$

*Increase* (+) *Decrease* (−)

Eg 1: A dress has increased in price by 30%. It originally cost AED 40. What is the new price?

$$new = original + \left(\frac{30}{100} \times original\right)$$

$$new = 40 + \left(\frac{30}{100} \times 40\right)$$

$$= 52 \therefore the\ new\ price\ is\ AED\ 52$$

Eg 2: A car decreases in value by 40% to AED 72000. What was the original value?

$$new = original - \left(\frac{40}{100} \times original\right)$$

$$72000 = x - \frac{40}{100}x$$

$$72000 = x - 0.4x$$

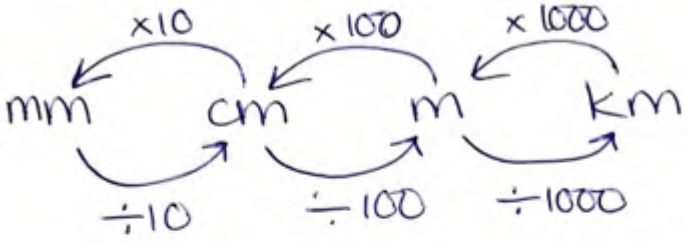
$$72000 = 0.6x$$

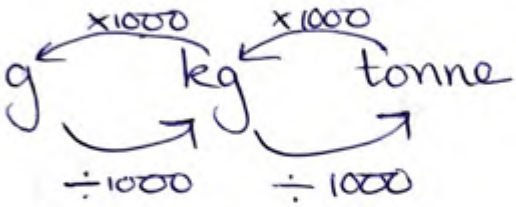
$$x = \frac{72000}{0.6}$$

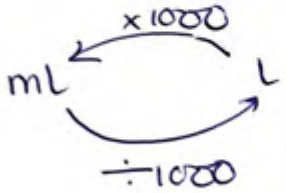
$$x = 12000\ AED$$

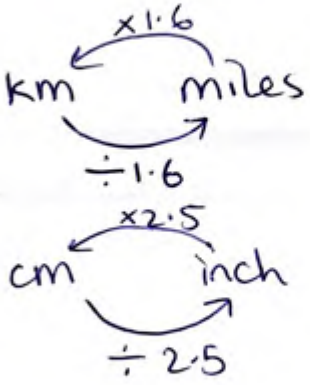
# Units (dimensional analysis)

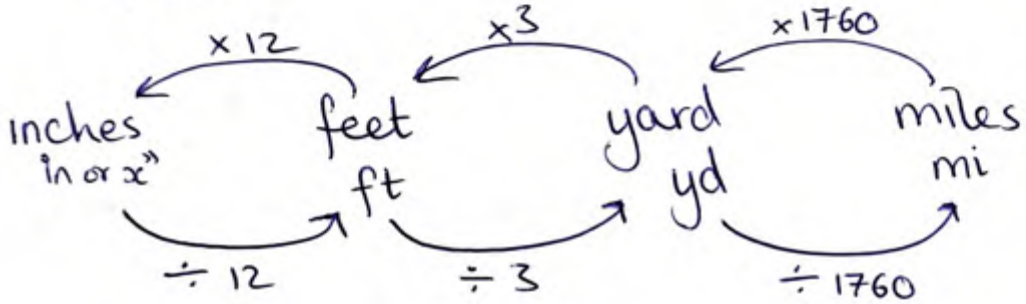
÷ if you want a smaller number
× if you want a bigger number




Eg1: Convert 2503 m into miles

write down the steps ↗

m → km → miles
÷1000 ÷1.6

∴ 2503 ÷ 1000 ÷ 1.6 ≈ 1.56 miles

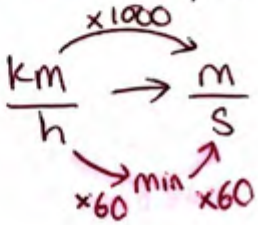
↙ don't forget to write the units.

Eg2: Convert 14 ft into cm.

ft → inch → cm
×12 ×2.5

14 × 12 × 2.5 = 420 cm

Eg3. Convert 50 km/h to m/s.




$50 \times \left(\frac{1000}{60 \times 60}\right)$

$= 13.89$ m/s.

# Linear equations (straight lines)

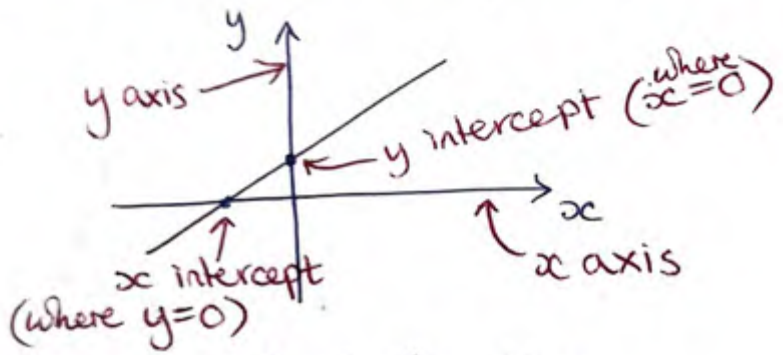
↳ have
- x term, y term, constant
- x term, y term
- x term, constant
- y term, constant

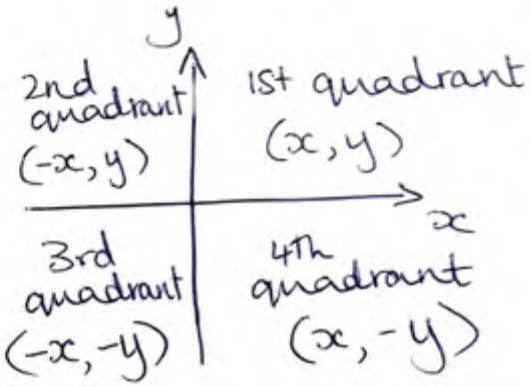
or $y-y_1 = m(x-x_1)$ point slope form

Can be written:

$ax+by+c=0$ all on one side — or — $ax+by=c$ standard form — or — $y=mx+b$ slope intercept form to be able to draw it.

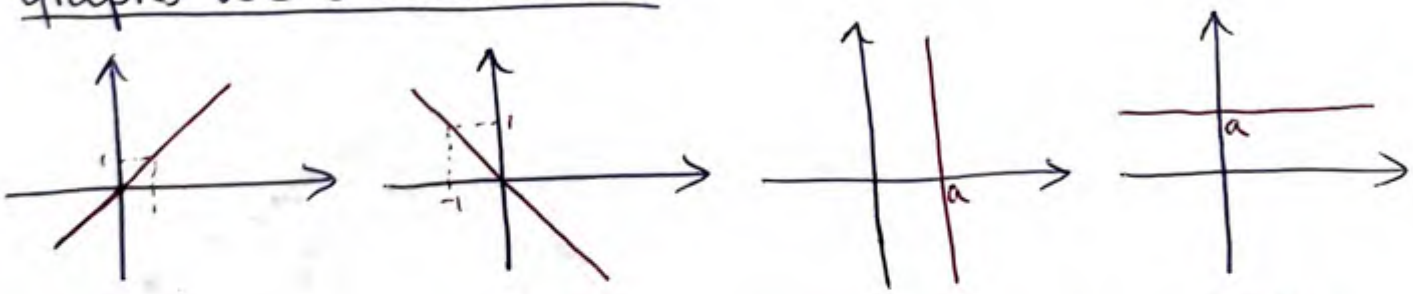
## Graph (coordinate plane)




Any point on the line is called 'coordinates'
$(x, y)$

## Graphs we should know




$y=x$ ($m=1$, $b=0$)

$y=-x$ ($m=-1$, $b=0$)

$x=a$ ($m$ is undefined, $b$ is none)

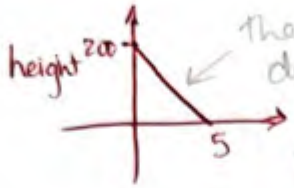
$y=a$ ($m=0$, $b=a$)

## Intercepts

* at x intercept, $y=0$
* at y intercept, $x=0$

* Can graph by:
  - → intercepts
  - → table
  - → $y=mx+b$

Eg A falcon starts flying down from 200m and reaches the ground 5 seconds later. Draw the situation & explain what it means.




the line doesn't go further as it's a real life situation

so this means when $t=0$, $h=200$ & when $t=5$, $h=0$

It takes him 5 seconds to reach the ground.

Eg2. Graph $2x+4y=16$ using the x & y intercepts.

we know at x int, $y=0$

$$2x+4(0)=16$$
$$2x+0=16$$
$$2x=16$$
$$x=\frac{16}{2}=8$$

We know at y int, $x=0$

$$2(0)+4y=16$$
$$0+4y=16$$
$$4y=16$$
$$y=\frac{16}{4}=4$$

## Graphing with a table

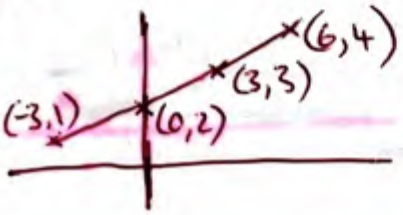
↳ pick random values & make a table

↳ if there is a fraction in the equation, pick x values that are multiples of the denominator

eg Sketch $y=\frac{1}{3}x+2$ using a table

↖ fraction so pick x values which are multiples of 3

| x | $y=\frac{1}{3}x+2$ | coordinates |
|---|---|---|
| -3 | $y=\frac{1}{3}(-3)+2=1$ | (-3, 1) |
| 0 | $y=\frac{1}{3}(0)+2=2$ | (0, 2) |
| 3 | $y=\frac{1}{3}(3)+2=3$ | (3, 3) |
| 6 | $y=\frac{1}{3}(6)+2=4$ | (6, 4) |



# Solving linear eqn by graphing

If you have a linear equation eg

$$2x - 8 = 0$$

you can write this as a linear function by replacing the 0 with $f(x)$ or $y$.

$2x - 8 = f(x)$ ← This means 'function of $x$' and is another way of writing $y$.

or $2x - 8 = y$

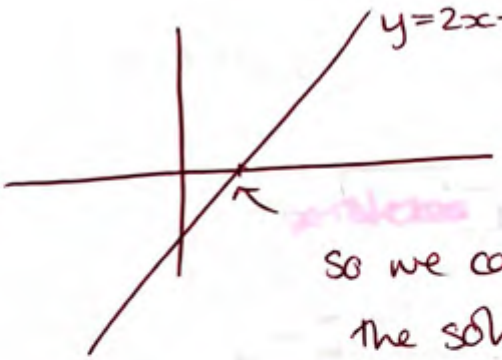
if $f(x) = 0$, it means

$y = 0$

When we solve $2x - 8 = 0$ we are solving when

$f(x) = 0$

ie when $y = 0$.




so we can say when $y = 0$, the solution is the x intercept.

when $f(x) = 0$,

we get the: **solution, x intercept, root, zero.**

A linear function $f(x)=0$ will only have maximum 1 solution as a line will only cross the x axis once.

Eg Solve $3x+1=-2$

**Method 1**
algebraically

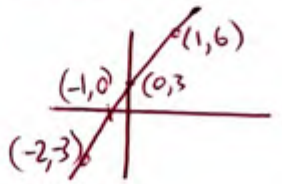
$3x+1=-2$
$3x=-2-1$
$3x=-3$
$x=\frac{-3}{3}=-1$

or

**Method 2**
graphically

* Put everything to one side
$3x+1+2=0$
$3x+3=0$
* Write $f(x)$ instead of 0
$f(x)=3x+3$
or $y=3x+3$
* Make a table

| x | y |
|---|---|
| -2 | -3 |
| -1 | 0 |
| 0 | 3 |
| 1 | 6 |



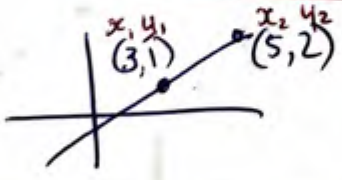

x int = $-1$
$\therefore$ Solution = $-1$
(root)

---

## Rate of change (how y changes with respect to x)

Rate of change = m = gradient = slope = steepness = ...

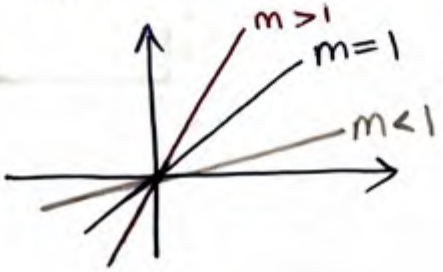
$$m=\frac{\text{change in } y}{\text{change in } x}=\frac{y_2-y_1}{x_2-x_1}$$

Eg 

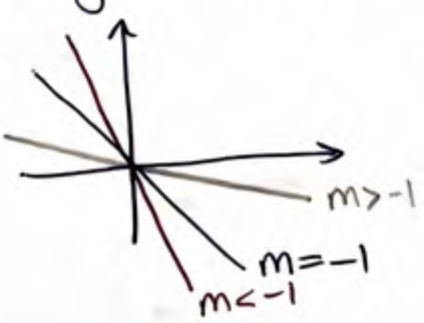

find $m=\frac{y_2-y_1}{x_2-x_1}=\frac{2-1}{5-3}=\frac{1}{2}$

---

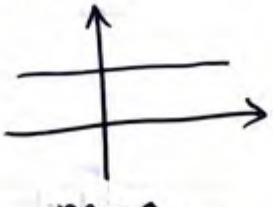
* Positive slope




* Negative slope




*



m=0

*

m is undefined
m=%

* for linear equations the slope is the same (constant) throughout.

## find coordinates when you know the gradient

Eg1 find the value of r when a line has coordinates (1,4) and (-5, r) and a slope of $\frac{1}{3}$.

Write what we know

$(1,4) = (x_1, y_1)$

$(-5, r) = (x_2, y_2)$

$m = \frac{1}{3}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{1}{3} = \frac{r-4}{-5-1}$$

$$\frac{1}{3} = \frac{r-4}{-6}$$

$$\frac{-6}{3} = r - 4$$

$$-2 = r - 4$$

$$-2 + 4 = r$$

$$2 = r$$

Real life example

| No. of games $x$ | cost $y$ |
|---|---|
| 2 | 78 |
| 4 | 156 |

a) What's the rate of change

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{156 - 78}{4 - 2} = 39$$

b) What is the real life meaning?

rate of change is $\frac{y}{x}$ which here is $\frac{\text{cost}}{\text{game}}$ so 39 is the cost per game.

# Equation of a straight line

✡ Slope intercept form: $\boxed{y = mx + b}$

↳ helps us draw the graph
↳ $m$ = gradient
↳ $b$ = y intercept.
↳ need $m$ & $b$ for this method

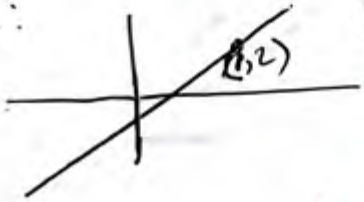
eg 1: $m = 2$
y intercept $= 3$
Write the equation:

$y = 2x + 3$.

eg 2: $y = -x + 3$
what is $m$ & $b$?

$m = -1$, $b = 3$

Eg 3:




Write the equation of the line when $m = 3$

$y = 3x + b$

to find $b$ use a point on the line

$(2) = 3(1) + b$
$2 = 3 + b$
$-1 = b$
∴ equation: $y = 3x - 1$

Eg 4. $2y - x = 3$
find $m$ & $c$
first get it into $y = mx + b$ form
$2y = x + 3$
$y = \frac{x}{2} + \frac{3}{2}$.
∴ $m = 1/2$
$b = 3/2$

Eg 5: Real life
Initially ($t=0$) only 5 students studied art. Since then the no. of students studying art has increased at a rate of 0.6 per year.

a) Draw a graph & write an equation for this.

* Underline important info from the question
* Write the information out: $x$ = time = $t$
$y$ = students studying art
when $t = 0$, $y = 5$
$m = 0.6$

∴ $y = 0.6x + 5$

b) Estimate the no. of students in 6 years

Means find $y$ when $x = 6$

$y = 0.6x + 5$

$y = 0.6(6) + 5$

$y = 8.6$ ∴ approximately 9 students in 6 years.

✱ Point slope form: $y - y_1 = m(x - x_1)$

↳ you need m & a point on the line

Eg find the equation of the straight line through $(3, 1)$ and $(2, 4)$ ($x_1, y_1$ and $x_2, y_2$)

Need m: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{2-3} = \frac{3}{-1} = -3$

Need a point: can use any of the points.

let's use $(3, 1)$

$y - y_1 = m(x - x_1)$

$y - 1 = -3(x - 3)$ point slope form.

can simplify it to standard form

or $y = mx + b$ form.

# Parallel and Perpendicular lines

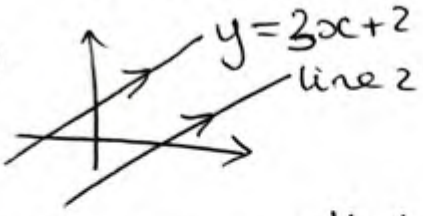
الخطوط المتوازية

الخطوط المتعامدة

or

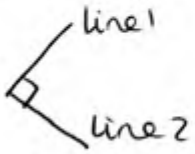


* same gradient

eg 


line 2 is parallel to $y=3x+2$. What is its gradient?

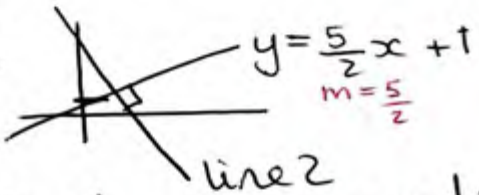
$m_{line2} = 3$ as the two lines are parallel




* gradients are the negative reciprocals of each other:
If gradient of line 1 = $m$
then gradient of line 2 = $\frac{-1}{m}$
(flip it & flip the sign)

* If you multiply the gradients you get $-1$

$m \times \frac{-1}{m} = -1$

eg 


What is the gradient of line 2?

Since they are perpendicular

$m_{line2} = -\frac{2}{5}$ (flip it and flip the sign)

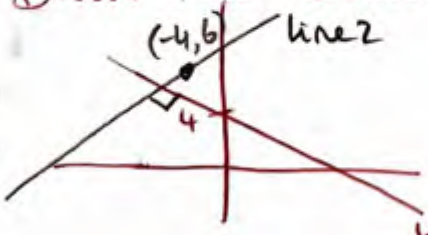
Eg: find the equation of the line that passes through $(-4,6)$ and is perpendicular to $2x+3y=12$

↑ this can be written as $y=mx+c$.

$3y = -2x + 12$

$y = \frac{-2}{3}x + 4$

Draw the situation:




since they are perpendicular

$m_{line2} = \frac{3}{2}$

$\therefore y - y_1 = m(x - x_1)$

$y - 6 = \frac{3}{2}(x - -4)$

then simplify

**Direct variation** ⇒ when y varies directly with x.

when $b = 0$, you get $y = mx$ which can be written as $\boxed{y = kx}$

↳ 'direct variation equation' or 'proportional relationship'

↳ $k = m$ and is called 'constant of variation' or 'constant of proportionality'

Eg y varies directly with x, and when $x = 8$, $y = 72$. Write an equation.

$y = kx$ Need to find k.

$(72) = k(8)$

$\frac{72}{8} = k$

$k = 9$

$\therefore y = 9x$.

**Real life**

One of most common real life uses is the formula: $\boxed{d = rt}$

- d: distance (y)
- r: rate or speed (k)
- t: time (x)

Eg The distance a plane travels varies directly as the no. of hours it flies. The plane flies 3420 km in 6 hours. Write an equation & graph it.

$y = kx$

$d = rt$ Need to find r

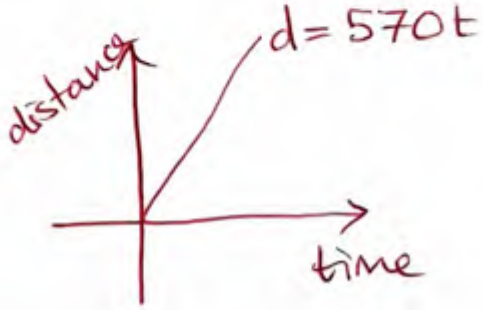
$(3420) = r(6)$

$r = 570$

$\therefore d = 570t$

$m = 570$

$c = 0$



# Inequalities

### < Less than

$x < 3$ means that
$x$ is less than 3

### > Greater than

$x > 7$ means that
$x$ is greater than 7

### ≤ Less than or equal to

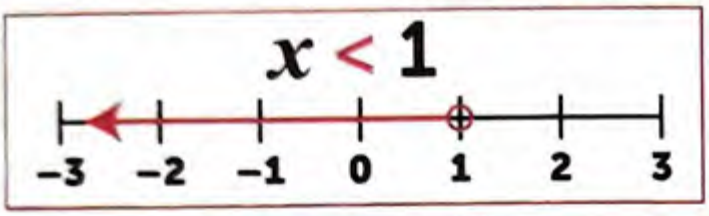
$y \leq 3$ means that
$y$ is less than or equal to 3

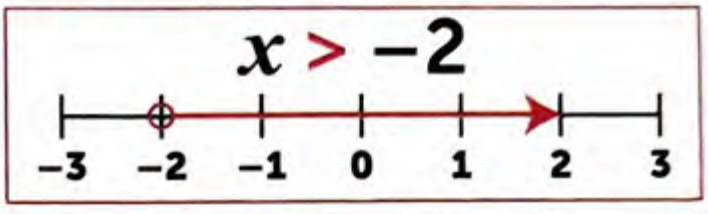
### ≥ Greater than or equal to

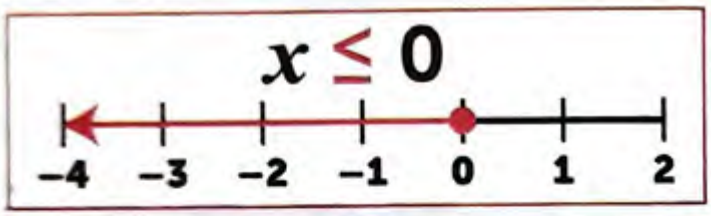
$y \geq 7$ means that
$y$ is greater than or equal to 7

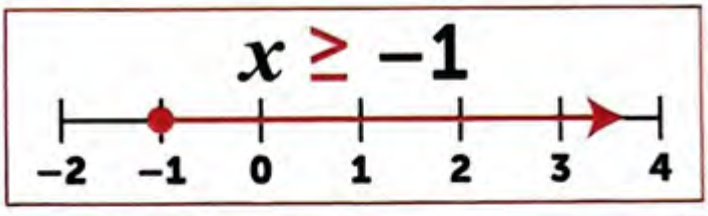
## Number Lines

You can represent inequalities on number lines.
Use ○ for < and >, and ● for ≤ and ≥.




## Solving Inequalities

**Most equations involving inequalities are solved in the same way as standard equations. However, when multiplying or dividing by a negative number, you must reverse the inequality.**

$$5 - 3x \leq 3$$
$$-5 \qquad -5$$
$$-3x \leq -2$$
$$\div -3 \qquad \div -3$$
$$x \geq \frac{-2}{-3}$$
$$x \geq \frac{2}{3}$$

Dividing by a negative number so flip the inequality

$$\frac{5x}{-3} - 2 > 8$$
$$+2 \qquad +2$$
$$\frac{5x}{-3} > 10$$
$$\times -3 \qquad \times -3$$
$$5x < -30$$
$$\div 5 \qquad \div 5$$
$$x < -6$$

Multiplying by a negative number so flip the inequality

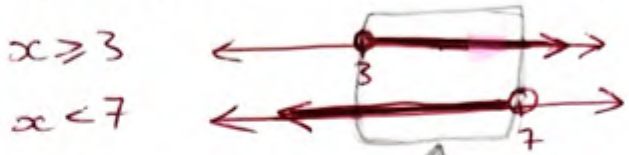
## Compound inequalities

↳ 2 or more inequalities together

### * AND

Eg $x \geq 3$ and $x < 7$

Graph each & look at the overlap (the intersection)



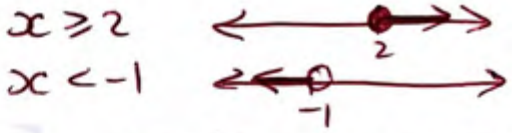

This is the overlap so the answer is

$3 \leq x < 7$

### * OR

Eg $x \geq 2$ or $x < -1$

Graph each & 'or' is all graphs. We call this the union.




so the answer is

$x \geq 2$ or $x < -1$

### Real life example

→ At least
→ Minimum
} ≥

→ At most
→ maximum
→ no more than
} ≤

Eg: A charity must raise at least 1000 AED. They already have 500 AED sadaqah and are selling tickets for 10 AED each. How many tickets do they need to sell?

$x$ = no of tickets

$10x + 500 \geq 1000$

$10x \geq 500$

$x \geq 50$

# Graphing inequalities on a coordinate plane

↳ will have shaded areas

* for $>$ and $<$ use ----
* for $\geq$ and $\leq$ use ——

Eg Graph $3x - y < 2$

Step 1: make $y$ the subject:

$$-y < 2 - 3x$$

flip the sign because $\div -1$

$$y > \frac{2 - 3x}{-1}$$

$$y > -2 + 3x$$

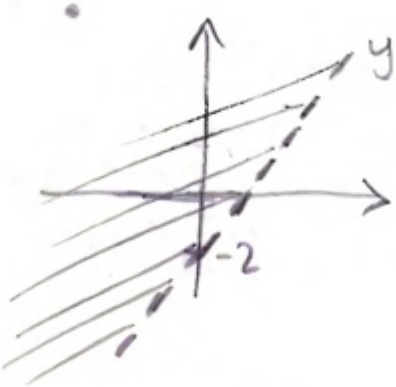
$$y > 3x - 2$$

Step 2: This is now in $y = mx + c$ form

so consider $y = 3x - 2$

$m = 3$, $c = -2$.

Step 3: Draw this graph & shade the inequality.




$y > 3x - 2$

$y$ is bigger than the graph

so shade <u>above</u> the line.

* for $y < mx + c$, shade <u>below</u> the line.

# Solving inequalities with absolute value

We know $|x|=3$ means $x=3$ or $-3$. inside of the mod with ± of the answer

so $|x|<3$ means $-3<x<3$

and $|x|>3$ means $-3>x>3$ but we can't leave the answer like this

which is $-3>x$ or $x>3$.

Eg1. Solve $|m+2| \leq 11$

$-11 \leq m+2 \leq 11$

$-11 \leq m+2$ and $m+2 \leq 11$

$-11-2 \leq m$ $\quad m \leq 11-2$

$-13 \leq m$ $\quad m \leq 9$

$-13 \leq m \leq 9$ or $\{m \mid -13 \leq m \leq 9\}$

or

Eg2: Solve $|3n+6|>12$

$-12 > 3n+6, > 12$ ← This is mathematically wrong but we write it just to help us put the correct symbols.

$-12 > 3n+6$ or $3n+6 > 12$

$-12-6 > 3n$ $\quad 3n > 12-6$

$-18 > 3n$ $\quad 3n > 6$

$\frac{-18}{3} > n$ $\quad n > \frac{6}{3}$

$-6 > n$ $\quad n > 2$

$\therefore -6 > n$ or $n > 2$

Eg3. Solve $|y-2| < -2$

no solutions as modulus can't be negative!

Eg4: Real life

A medicine has to be kept at 5°c with a margin of error has to be within 3°c. find the range of temperatures it can be.

$|x - \text{value you want}| \leq \text{margin of error}$

$|x-5| \leq 3 \Rightarrow -3 \leq x-5 \leq 3$

$-3 \leq x-5$ $\quad x-5 \leq 3$

$2 \leq x$ $\quad x \leq 8$

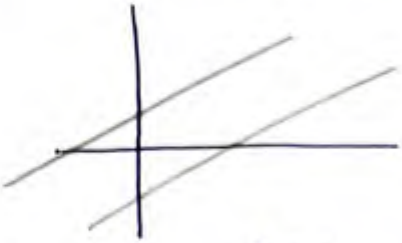
$2 \leq x \leq 8$

# Systems of equations (simultaneous equations)

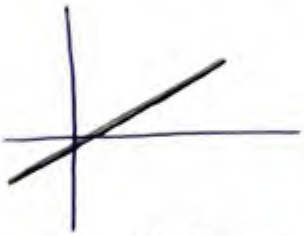
↳ when you have 2 or more equations.

↳ the number of solutions is how many times they cross each other ~~(the number of~~ (the point of intersection).

2 linear equations
→ 1 point of intersection
∴ 1 solution.



2 parallel linear equations
→ no point of intersection
∴ no solutions.



coincident lines

2 lines on top of each other
→ intersect at all points
∴ infinite solutions

## Solving simultaneous equations:

↳ to find $(x, y)$ of the point of intersection.

### * Method 1 : Substitution

Eg Solve $y - 2x = 1$ -①
$3x + y = -9$ -②

Step 1 : Label the equations

Step 2 : Rearrange one of the equations to make $x$ or $y$ the subject.

Rearrange ①: $y = 2x + 1$ -③

Step 3 : Substitute ③ into the other equation.

Substituting ③ into ② : $3x + (2x+1) = -9$

$3x + 2x + 1 = -9$

$5x + 1 = -9$

$5x = -9 - 1$

$5x = -10$

$x = -10/5 = -2$

Step 4 : find the other coordinate

substitute $x$ into ③

$y = 2(-2) + 1$

$= -4 + 1 = -3$

Step 5 : Write as coordinates

$(-2, -3)$ is the point of intersection

## * Method 2 : Elimination

If there are 2 terms the same you can use elimination.

Eg1. Solve $8y + 4x = 8$ → 'to get rid of'
$4y + 4x = 12$

There are 2 terms the same $4x$ and $4x$ so can use this method.

Next consider the signs of the terms:

DASS

different signs ↙ add, ↓ same signs ↘ substract

Here, the signs of $4x$ are the same (both are +) so we subtract

$$\begin{array}{r} 8y + 4x = 8 \\ -(4y + 4x = 12) \\ \hline 4y + 0x = -4 \end{array}$$

$$4y = -4$$
$$y = \frac{-4}{4} = -1$$

then find the other coordinate by substitution.

Using the first equation: $8(-1) + 4x = 8$

$$-8 + 4x = 8$$
$$4x = 8 + 8$$
$$4x = 16$$
$$\therefore x = \frac{16}{4} = 4$$

$\therefore$ The point of intersection is

$(4, -1)$

Eg 2: Solve $2x + 3y = 10$
$5x - 3y = 4$

- The same term is $3y$
- The signs are <u>different</u>, so <u>add</u>

$$2x + 3y = 10$$
$$+ (5x - 3y = 4)$$
$$7x + 0y = 14$$
$$7x = 14$$
$$x = \frac{14}{7} = 2$$

to find $y$: $2x + 3y = 10$
$2(2) + 3y = 10$
$4 + 3y = 10$
$3y = 10 - 4$
$3y = 6$
$y = \frac{6}{3} = 2$

coordinates are <u>(2, 2)</u>

# Systems of equations:

## Elimination method when there aren't 2 terms the same

Eg Solve $\begin{cases} 4x + 2y = 8 & ① \\ 3x + 3y = 9 & ② \end{cases}$ by elimination

There aren't 2 terms the same, so we need to make 2 terms the same:

We can make each equation have $6y$ in it if we do ① $\times 3$ and ② $\times 2$

① $\times 3$ $\Rightarrow$ $12x + 6y = 24$

② $\times 2$ $\Rightarrow$ $6x + 6y = 18$

Now we can do elimination as before:

Same signs, so subtract.

$$\begin{array}{r} 12x + 6y = 24 \\ -(6x + 6y = 18) \\ \hline 6x + 0y = 6 \end{array}$$

$$6x = 6$$

$$x = 1$$

substitute into one of the equations to find $y$: substitute $x = 1$ into ①

$$4(1) + 2y = 8$$

$$4 + 2y = 8$$

$$2y = 4$$

$$y = 2$$

$\therefore$ $(1, 2)$

# Real life of systems of equations

Eg In a bake sale pies were sold for AED 10 and cakes for AED 8. 40 items were sold for AED 356. Define the variables and write a system of equations to find the number of cakes & pies sold.

$x$ = no. of pies
$y$ = no. of cakes.

cost: $356 = 10x + 8y$
items sold: $40 = x + y$

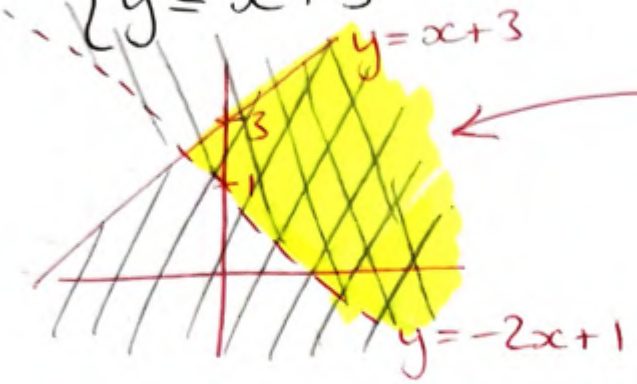
} system of equations.

# Systems of inequalities

Eg1. Solve the system of inequalities by graphing

$$\begin{cases} y > -2x + 1 \\ y \leq x + 3 \end{cases}$$

Draw and shade each equation




The solution will be the intersection (the overlap)

Eg2. Solve by graphing.

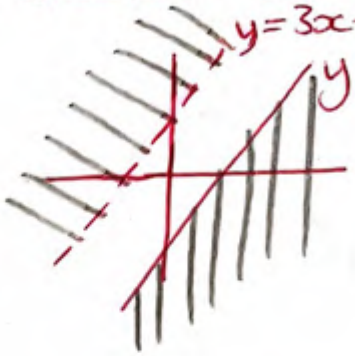
$$\begin{cases} y \leq 3x - 2 \\ 3x - y < -5 \end{cases}$$

first rearrange so both in the form $y = mx + c$.

$y \leq 3x - 2$

$y > 3x + 5$

then sketch & shade.




no intersection

$\therefore$ no solution

# Sequences

↳ a set of numbers that go up in a pattern

## Arithmetic sequence

↳ a sequence that goes up by adding.

eg 1, 3, 5, 7 ...

+2 +2 +2

first term = $a$ In the example $a = 1$

difference between each term = $d$ In the example $d = 2$

In general:

| $t_1$ (term 1) | $t_2$ (term 2) | $t_3$ | $t_4$ | ... | $t_n$ ($n^{th}$ term (sometimes called $a_n$)) |
|---|---|---|---|---|---|
| $a$ | $a+d$ | $a+2d$ | $a+3d$ | ... | $a+(n-1)d$ (helps us find any term) |

To find $d$: Take a term and subtract the previous term.

eg $t_3 - t_2 = 5-3 = 2 = d$.

Examples: Is this an arithmetic sequence?

a) 2, 5, 8, 11 ...

$d = t_2 - t_1 = 5-2 = 3$

$d = t_3 - t_2 = 8-5 = 3$

since $d$ is constant it is an arithmetic sequence.

b) 1, 4, 9, 16, ....

$d = t_2 - t_1 = 4-1 = 3$

$d = t_3 - t_2 = 9-4 = 5$

$d$ is not constant ∴ not an arithmetic series.

Examples: 7, 11, 15, 19 ....

a) What's the 6th term?

Write down all the information you know:

$a = 7$

$d = t_3 - t_2 = 15-11 = 4$

$n = 6$

→

$t_n = a + (n-1)d$

$t_6 = 7 + (6-1)4$

$= 7 + (5)4$

$= \underline{\underline{27}}$

b) find the nth term formula for this sequence

Write what you know

$a = 7$

$d = 4$

$t_n = a + (n-1)d$

$t_n = 7 + (n-1)4$

$t_n = 7 + 4n - 4$

$t_n = \underline{\underline{3 + 4n}}$

c) Which term is 35?

Write what you know:

for this question $t_n = 3 + 4n$ , $t_n = 35$

$t_n = 3 + 4n$

$35 = 3 + 4n$

$35 - 3 = 4n$

$32 = 4n$

$\frac{32}{4} = n$

$n = 8 \quad \therefore \quad t_8 = 35$

---

An arithmetic sequence can be a linear function:

$t_n = a + (n-1)d$

$t_n = a + dn - d$

$t_n = dn + (a-d)$

$y = mx + b$

$\therefore d = m$ and $(a-d) = b$

## Real life example

The chart shows the length of Mansour's jumps.

| Jump | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Length (ft) | 8 | 9.5 | 11 | 12.5 |

a) Write a function to represent this sequence.

$a = 8$

$d = t_3 - t_2 = 11 - 9.5 = 1.5$

$t_n = a + (n-1)d$

$t_n = 8 + (n-1)1.5$

$t_n = 8 + 1.5n - 1.5$

$t_n = 1.5n + 6.5$

b) Graph the function:

$t_n = 1.5n + 6.5$

$(y = mx + c)$

$\therefore m = 1.5, \; c = 6.5$

check:
for arithmetic sequence
$d = m = 1.5$ ✓
$c = (a - d) = 8 - 1.5$
$= 6.5$ ✓

**Properties**

**You should be able to know and use these properties. You will need to know the names of the properties for the exam.**

| Property | Example | Name of the property |
|---|---|---|
| If $a = b$, then $b = a$ | If $2 + 6 = 8$ *then* $8 = 2 + 6$ | Symmetric property |
| $a = a$ | $5=5$ | Reflexive property |
| If $a = b$ *and* $b = c$, *then* $a = c$ | If $6 + 9 = 3 + 12$, *and* $3 + 12 = 15$, *then* $6 + 9 = 15$ | Transitive property |
| If $a = b$, *then* $a$ *may be replaced by* $b$ *in any expression* | If $n = 11$, *then* $4n = 4(11)$ | Substitution property |
| $a(b + c) = ab + ac$ | $2(x - 3) = 2x - 6$ | Distributive property |
| $a + 0 = a$ | $4 + 0 = 4$ | Additive identity |
| $a + (-a) = 0$ | $3 + (-3) = 3 - 3 = 0$ | Additive inverse |
| $a \cdot 1 = a$ | $4 \cdot 1 = 4$ | Multiplicative identity |
| $a \cdot 0 = 0$ | $4 \cdot 0 = 0$ | Multiplicative property of 0 |
| $\frac{a}{a} = 1$ | $\frac{4}{4} = 1$ | |
| $\frac{a}{b} \cdot \frac{b}{a} = 1$ | $\frac{2}{3} \cdot \frac{3}{2} = 1$ | Multiplicative inverse |
| $ab = ba$ *and* $a + b = b + a$ | $7 \times 11 = 11 \times 7$<br>$4 + 8 = 8 + 4$ | Commutative property |
| The way you group 3 or more numbers when just adding or just multiplying doesn't change the answer<br>$(a + b) + c = a + (b + c)$<br>*and* $(ab)c = a(bc)$ | Eg $3 + 5 + 7 = 15$<br>You get the same answer if you do<br>$(3 + 5) + 7$ or $3 + (5 + 7)$<br><br>Eg $2 \cdot 6 \cdot 9 = 108$<br>You get the same answer if you do<br>$2 \cdot (6 \cdot 9)$ or $(2 \cdot 6) \cdot 9$ | Associative property |