UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

YEAR OF TOLERANCE

2019-2020

Mathematics

United Arab Emirates Edition

2019/2020
UNIT 1

Mc Graw Hill

Grade 10 Advanced

CHAPTER 1

Linear Systems and Matrices



Chapter Sourced From: 1. Systems of Equations and Inequalities, from Algebra 2 Chapter 3 © 2014 & Matrices, from Algebra 2 Chapter 4 lesson 1 © 2010 Copyright © McGraw-Hill Education gulfimages/Alamy Stock Photo

Then

- You graphed equations of lines, transformed functions, and solved equations.

Now

- You will:
  - Solve systems of linear equations and linear inequalities.
  - Solve problems by using linear programming.
  - Perform operations with matrices and determinants.

Why? ▲

- **BUSINESS** Most of the time, being successful in business means that you have to have good math skills. In this chapter, you will learn how to maximize your profits and minimize your costs. By doing this you will earn the most money possible.

Get Ready for the Chapter

Diagnose Readiness | You have two options for che

1 **Textbook Option** Take the Quick Check below. Refer to th

QuickCheck

Graph each equation.

1. $x = 4y$
2. $y = \frac{1}{3}x + 5$
3. $x + 2y = 4$
4. $y = -x + 6$
5. $3x + 5y = 15$
6. $3y - 2x = -12$
7. **BUSINESS** A museum charges AED 8.50 for adult tickets and AED 5.25 for children's tickets. On Friday they made AED 650.
   a. Write an equation that can be used to model the ticket sales.
   b. Graph the equation.

Graph each inequality.

8. $y < 3$
9. $x + y \geq 1$
10. $3x - y > 6$
11. $x + 2y \leq 5$
12. $y \geq 4x - 1$
13. $5x - 4y < 12$
14. **FUNDRAISER** The student council is selling T-shirts for AED 15 and sweatshirts for AED 25. They must make AED 2500 to cover expenses. Write and graph an inequality to show the number of T-shirts and sweatshirts that they must sell.

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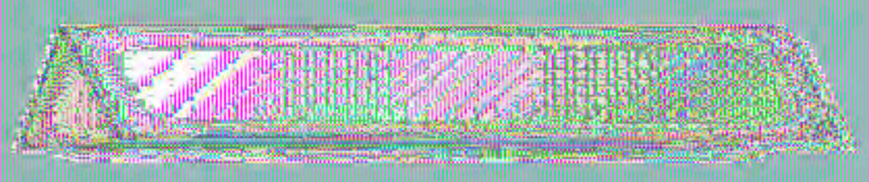
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms this chapter. To get ready, identify important terms and organize You may refer to previous chapters to review prerequisite skills.

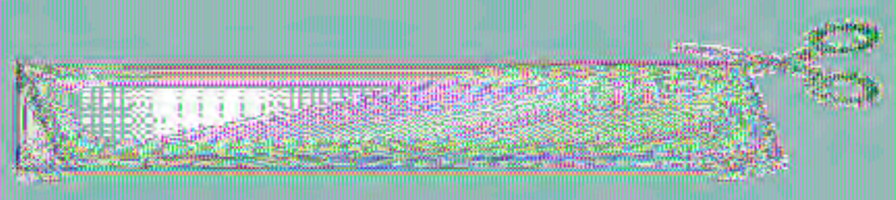
FOLDABLES StudyOrganizer

Systems of Equations and Inequalities Make this Foldable to help you organize your chapter notes about systems of equations and inequalities. Begin with a sheet of $8\frac{1}{2}$" by 11" paper.

1. **Fold** in half along the height.



2. **Cut** along the fold.



3. **Fold** each sheet along the width into fourths.



4. **Tape** the ends of two sheets together.



5. **Label** the tabs with *Solve by Graphing*, *Substitution Method*, *Elimination Method*, *Optimization*, *Systems in Three Variables*, *Matrix Multiplication*, *Cramer's Rule*, and *Inverse Matrices*.



Graphing Technology Lab: Intersections

You can use a graphing calculator to find where two graphs intersect.
You can use the Y= menu to graph each equation on the same set

Example: INTERSECTION OF TWO GRAPHS

Graph both equations in the standard viewing window.

$3x + y = 9$
$x - y = -1$

Step 1 Write each equation in the form $y = mx + b$.

$3x + y = 9$
$y = -3x + 9$

$x - y = 1$
$-y = -x - 1$
$y = x + 1$

Step 2 Enter $y = -3x + 9$ as Y1 and $y = x + 1$ as Y2. Th

KEYSTROKES: Y= (−) 3 X,T,θ,n + 9 ENTER
X,T,θ,n + 1 ENTER ZOOM 6

Step 3 Find the intersection of the lines.

KEYSTROKES: 2nd [CALC] 5 ENTER ENTER ENT

The intersection is at (2, 3).

Exercises

Use a graphing calculator to find where each pair of graph

1. $2x + 4y = 36$
   $10y - 5x = 0$

2. $2y - 3x = 7$

3. $4x - 2y = 16$
   $7x + 3y = 15$

5. $5x + y = 13$
   $3x = 15 - 3y$

4. $4x - 7y + 16 = 0$

7. $\frac{1}{4}x + y = \frac{11}{4}$
   $x - \frac{1}{2}y = 2$

8. $3x + 2y = -3$

9. $3x - 6y = 6$
   $2x - 4y = 4$

10. $6x + 8y = -16$

Holt McDougal Algebra 1

Solving Systems

Then | **Now** | **Why?**

You graphed and solved linear equations.

1. Solve systems of linear equations graphically.
2. Solve systems of linear equations algebraically.

Tarek borrowed A
lawn-mowing busi
AED 35 per lawn
operating costs pe
equations can be
the break-even po
point is the point
equals cost.

New Vocabulary
break-even point
system of equations
consistent
inconsistent
independent
dependent
substitution method
elimination method

Mathematical Practices
Reason abstractly and quantitatively.
Attend to precision.

1 Solve Systems Graphically

same variables. To solve a system
that satisfies all of the equations.

To solve a system of equations by us
form. Then substitute different valu
of use, choose 0 and 1 as your first

$y_1 = -2x + 8$
$y_2 = 4x - 7$

x	y_1	y_2
0	8	
1	6	
2	4	
3	2	

The solution is between 2 and 3.

Example 1 Solve by Using a Tab

Solve the system of equations.

$3x + 2y = -2$
$-4x + 5y = -28$

Write each equation in slope-intercept form.

3x
−4x

Use a table to find the solution that satisfies both equations.

x	
0	
1	
2	

The solution of the system is (2, −4).

Guided Practice

1A. $2x - 5y = 11$
$-3x + 4y = -13$

Another method for solving a
coordinate plane. The point of

Example 2 Solve by Graph

Solve the system of equatio

$2x - y = -1$ $2y$

Write each equation in slop

$2x - y = -1$ $\longrightarrow$ y

$2y + 5x = -16$ $\longrightarrow$ y

The graphs of the lines app

CHECK Substitute the coord

$2x - y = -1$

$2(-2) - (-3) \stackrel{?}{=} -1$

$-1 = -1$ ✓

The solution of the system

Study Tip

Checking Solutions Always check to see if the values work for **both** of the original equations.

Guided Practice

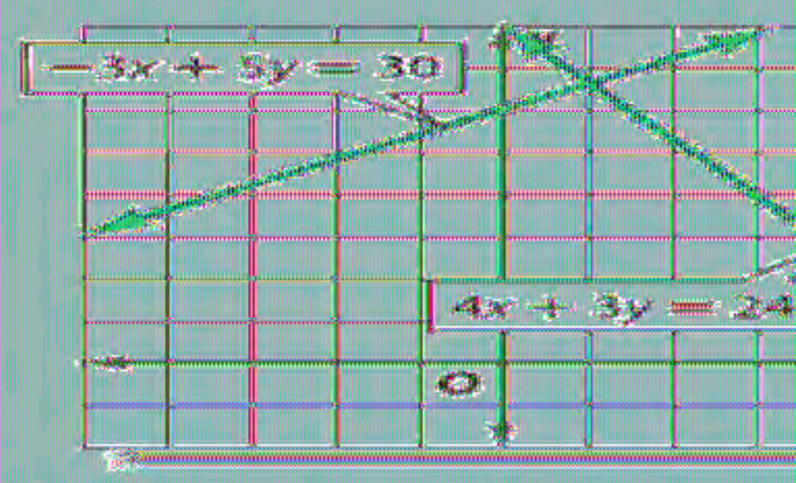
2A. $4x + 3y = 12$
$-6x + 4y = -1$

Systems of equations can be cl
has at least one solution and i
is **independent**, and if it has a

Example 3 Classify System

Graph each system of equa
consistent and dependent,

a. $4x + 3y = 24$ **b.** $-2x$
$-3x + 5y = 30$



The graphs of the lines
at one point, so there i
solution. The system is
and independent.

Study Tip

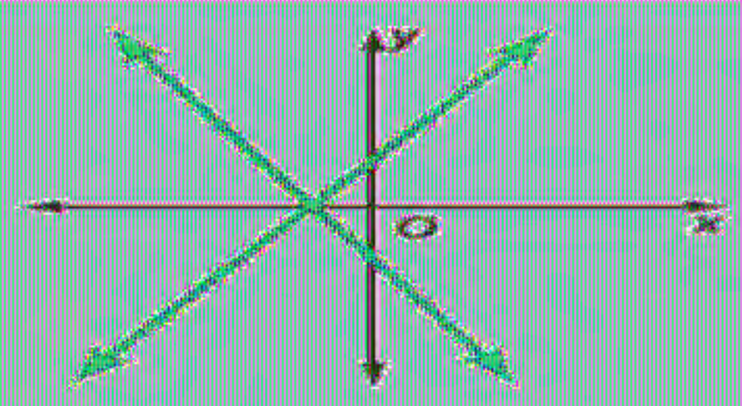
Slope and Classifying Systems If the equations have different slopes, then the system is consistent and independent.

Guided Practice

3A. $6x - 4y = 15$
$-6x + 4y = 18$

The relationship between the graphs

Concept Summary Characteristi

Consistent and Independent	C
 intersecting lines; one solution	

2 Solve Systems Algebraically

systems of equations. One alge

Key Concept Substitution Metho

Step 1 Solve one equation for one o

Step 2 Substitute the resulting expre
Then solve the equation.

Step 3 Substitute to solve for the oth

Systems of equations can be used to
modeled by two or more different fu

Real-World Example Use th

BUSINESS Ahmed has a compute
business can be represented by
also estimates that his income ca
customers will Ahmed need in o
60 customers?

$$y = 65x - 145$$

$$48x + 500 = 65x - 145$$

$$500 = 17x - 145$$

$$645 = 17x$$

$$37.9 \approx x$$

Ahmed needs 38 customers to br
60 customers, his income will be
and his costs will be 48(60) + 50
will be 3755 − 3380 or AED 375.

CHECK You can use a graphing
solution. The break-even
(37.9, 2321.2). Use the CA
cost and income for 60 c

Guided Practice

Use substitution to solve ea

4A. $5x - 3y = 23$
$2x + y = 7$

You can use the **elimination m**
coefficient in both equations.

Key Concept Elimination M

Step 1 Multiply one or both eq
opposite terms.

Step 2 Add the equations, elim

Step 3 Substitute to solve for t

Variables can be eliminated by

Example 3 Solve by Using

Use the elimination metho

$5x + 3y = -19$
$8x + 3y = -25$

Step 1 Multiply one equat

$8x + 3y = -25$

Step 2 Add the equations

$5x + 3y =$
$(+)\ -8x - 3y =$
$-3x =$
$x =$

Step 3 Substitute -2 for x

$8x + 3y =$
$8(-2) + 3y =$
$-16 + 3y =$
$3y =$
$y =$

The solution is $(-2, -3)$.

Guided Practice

5A. $4x - 3y = -22$
$2x + 3y = 16$

5B. 6

Study Tip

Perseverance
Remember when you add or subtract one equation from another to add or subtract every term, including the constant on the other side of the equal sign.

Review Vocabulary

Least Common Multiple
the least number that is a common multiple of two or more numbers

Sometimes, adding or subtrac
multiplication and least comm

Standardized Test Example 6

Solve the system of equations.

$5x + 3y = 52$

$15x + 9y = 54$

A (3, 1) B (8, 4)

Read the Test Item

You are given a system of two lin

Solve the Test Item

Neither variable has a common c
3 and 9 and their least common
value that will make the y-coeffic

$5x + 3y = 52$

$15x + 9y = 54$

Because $0 = 102$ is not true, this
The correct answer is C.

Guided Practice

6. Solve the system of equations

F (−2, 8) G (7, 3)

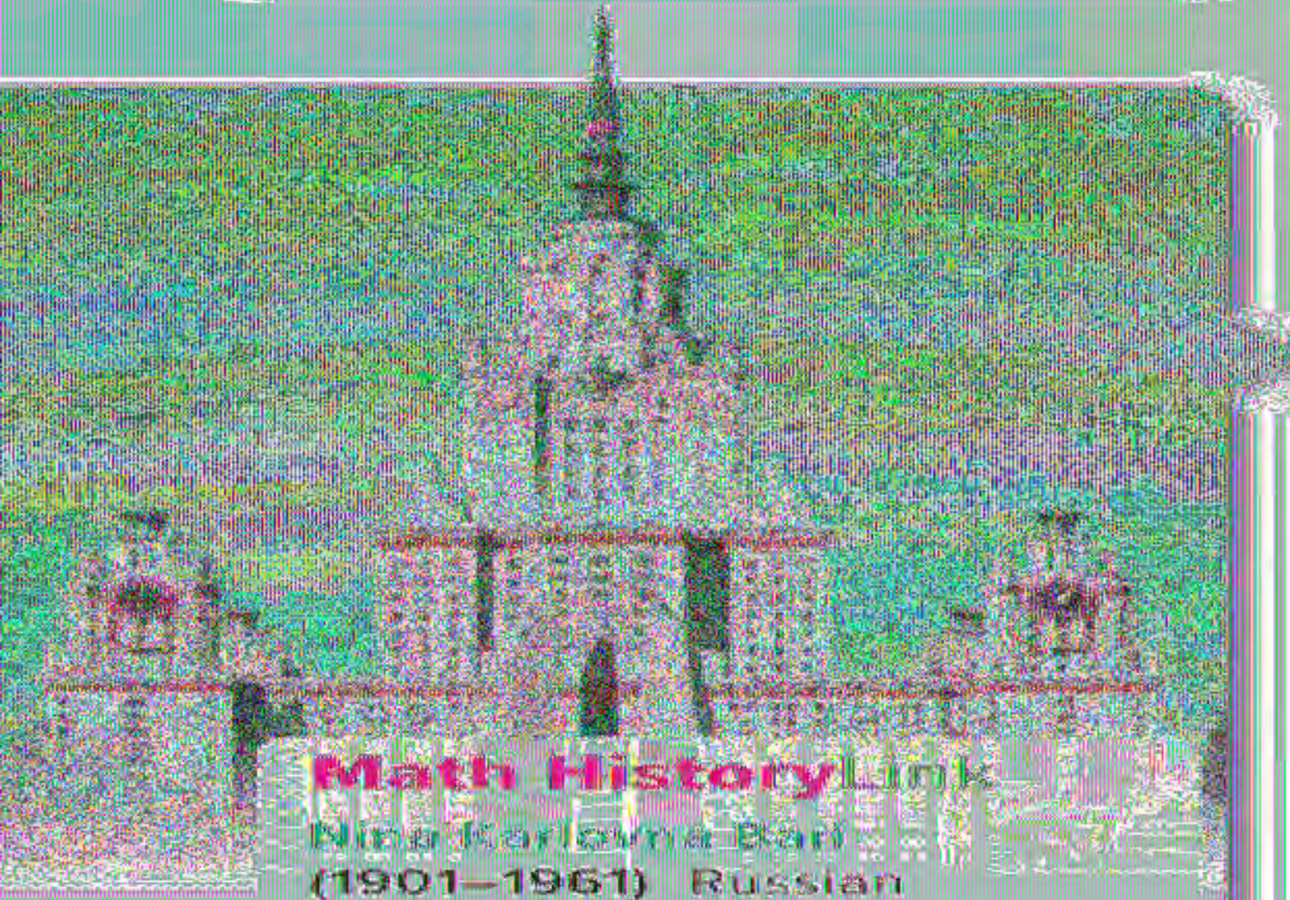
The following summarizes the variou

Concept Summary Solving Syste

Method
Table
Graphing
Substitution
Elimination Using Addition
Elimination Using Subtraction
Elimination Using Multiplication

Study Tip

Adding and Subtracting Equations If you add or subtract two equations in a system and the result is an equation that is never true, then the system is inconsistent. When you add or subtract two equations in a system and the result is an equation that is always true, then the system is dependent.



Math History Link

Nina Karlovna Bari
(1901–1961) Russian mathematician Nina Karlovna Bari was considered the principal leader of mathematics at Moscow State University, shown above. She is best known for her textbooks *Higher Algebra* and *The Theory of Series*.

Check Your Understanding

Example 1 **Solve each system of equations by using a**

1. $y = 3x - 4$
   $y = -2x + 11$

2. $4x - y = 1$
   $5x + 2y = 24$

Example 2 **Solve each system of equations by graphing**

3. $y = -3x + 6$
   $2y = 10x - 36$

4. $y = -x - 9$

5. $y = 0.5x + 4$
   $3y = 4x - 3$

6. $-3y = 4x + 11$

7. $4x + 5y = -41$
   $3y - 5x = 5$

8. $8x - y = 50$

9. **MODELING** Refer to the table at the right.

Digital Photos	
EZ Online	$0.25 per photo
Local Store	$0.10 per photo + $2.00 handling fee

   a. Write equations that represent the c
   of printing digital photos at each lab

   b. Under what conditions is the cost to print digital photos the same at both stores?

   c. When is it best to use EZ Online Digital Photos and when is it best to use the local store?

Example 3 **Graph each system of equations and descri**
***consistent and dependent*, or *inconsistent*.**

10. $y + 4x = 12$
    $3y = 8 - 12x$

11. $-2x - 3y = 9$

12. $9x -$
    $4x + 6$

Example 4 **Solve each system of equations by using su**

13. $x + 5y = 3$
    $3x - 2y = -8$

14. $y = 2x$
    $y = -4x + 8$

16. $a - 3b = -22$
    $4a + 2b = -4$

17. $6x - 7y = 23$
    $8x + 4y = 44$

18. 9

Examples 5–6 **Solve each system of equations by using eli**

19. $-6w - 8z = -44$
    $3w + 6z = 36$

20. $4x - 3y = 29$
    $4x + 3$

22. $8a - 3b = -11$
    $8a + 2b = -3$

23. $5a + 15$
    $-2a -$

25. **MULTIPLE CHOICE** What is the solution
    $4x +$
    $4x -$

    A $(8, -10)$

    B $(2, -2)$

Practice and Problem Solving

Example 1 **Solve each system of equations by using a table.**

26. $y = 5x + 3$
$y = x - 9$

27. $3x - 4y = 16$
$-6x + 5y =$

29. FUNDRAISER To raise money for new uniforms, the band boosters sell T-shirts and hats. The cost and sale price of each item is shown. The boosters spend a total of AED 8000 on T-shirts and hats. They sell all of the merchandise, and make AED 13500 How many T-shirts did they sell?

Example 2 **Solve each system of equations by graphing.**

30. $-3x + 2y = -6$
$-5x + 10y = 30$

31. $4x + 3y = -24$
$8x - 2y =$

33. $-3x - 8y = 12$
$12x + 32y = -48$

34. $y - 3x = -2$
$9x - 6y = 1$

36. MODELING Hidaya has a AED 10 coupon and store. The store has a policy that only one co When is it best for Hidaya to use the AED 1 use the 15% discount coupon?

Example 3 **Graph each system of equations and describe it *dependent*, or *inconsistent*.**

37. $y = 3x - 4$
$y = 6x - 8$

38. $y = 2x - 1$

39. $2x + 5y = 1$
$y = 2x + 6$

40. $x - 6y = 12$
$3x + 18y = 14$

41. $-5x - 6y =$
$12y + 10x =$

Example 4 **Solve each system of equations by using substitu**

43. $9y + 3x = 18$
$-3y - x = -6$

44. $5x - 20y =$
$6x + 5y =$

46. $-4a - 5b = 14$
$9a + 3b = -48$

47. $-9c - 4d =$
$6c + 6d = -24$

49. TENNIS At a park, there are 38 people playin are playing singles. There are 13 matches in and a singles match requires 2 players.

a. Write a system of two equations that repre matches going on.

b. How many matches of each kind are in pr

Examples 5–6 **Solve each system of equations by using elimina**

50. $8x + y = 27$
$-3x + 4y = 3$

51. $2a - 5b =$
$2a + 5b = 2$

53. $3x - 8y = 24$
$-12x + 32y = 96$

54. $5a - 2b =$
$8a + 5b =$

56. $6d + 5f = -32$
$5d - 9f = 26$

57. $11w = 8v + 35$
$8v = -6w +$

58. -1

Use a graphing calculator to solve each syst
intersection to the nearest hundredth.

59. $12y = 5x - 15$
$4.2y + 6.1x = 11$

60. $-3.8x$
$6.6x -$

Solve each system of equations.

62. $11p + 3q = 6$
$-0.75q - 2.75p = -1.5$

63. $8r - 5t = -60$

64.
$6r + 3t$

65. $6w = 12 - 4x$
$6x = -9w + 18$

66.
$\frac{3}{2}y + 2$
$-y -$

68. **ROWING** Alia can row a boat 1 kilomet
She can row the same distance downst
speed and the speed of the current are

a. Find the speed at which Alia is rowi

b. If Alia plans to meet her friends 3 ki
be on time? Explain.

69. **MODELING** The table shows the winnir
at the Olympics between 1964 and 200

Years Since 1964, x	Men's Gold Medal Tim
0	10.0
4	9.90
8	10.14
12	10.06
16	10.25
20	9.99
24	9.92
28	9.96
32	9.84
36	9.87
40	9.85
44	9.69

a. Write equations that represent the v
Assume that both times continue al

b. Graph both equations. Estimate whe
men's performance. Do you think th

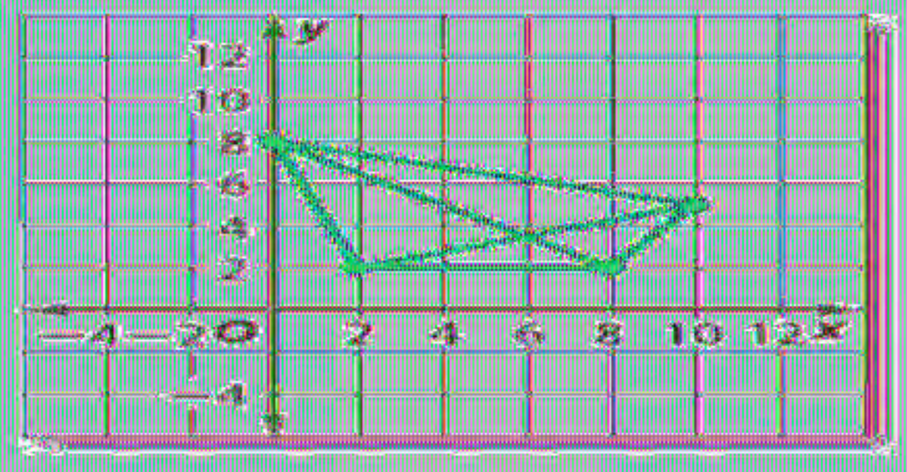
70. **JOBS** Kareem has a job offer in which
commission of 2% of the total price of t
AED 1200 per month plus a commissio
per month to make the new job a bette

71. **TRAVEL** A youth group went on a trip to an amusement park, travelling in two vans. The number of people in each van and the total cost of admission are shown in the table. Find the adult price and student price of admission.

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GEOMETRY Find the point at which the diagonals

72.



74. ELECTIONS In the election for student council while Candidate B received 1541 votes. If Ca Candidate A received, how many total votes

75. MULTIPLE REPRESENTATIONS In this probl equations and two variables.

$$3y + x = 16$$
$$y - 2x = -4$$
$$y + 5x = 10$$

a. **Tabular** Make a table of x- and y-values fo

b. **Analytical** Which values from the table in that satisfies all three equations?

c. **Graphical** Graph the three equations on a

d. **Verbal** What conditions must be met for a to have a solution? What conditions resul

H.O.T. Problems: Use Higher-Order Thinking Skills

76. CRITIQUE Ghaya and Huda are solving the s $-3x + 4y = -17$. Is either of them correct? E

Ghaya

$6x - 4y = 26$ $\quad 6(3) - 4y = 26$

$-3x + 4y = -17$ $\quad 18 - 4y = 26$

$3x = 9$ $\quad -4y = 8$

$x = 3$ $\quad y = -2$

The solution is (3, −2).

77. CHALLENGE Find values of a and b for which $(b - 1, b - 2)$.

$$-8ax + 4ay = -12a$$
$$2bx - by = 9$$

78. REASONING If a is consistent and dependent c is consistent and independent with d, then and independent with d. Explain your reason

79. OPEN ENDED Write a system of equations in 3 and the other needs to be multiplied by 4 i Then solve your system.

80. WRITING IN MATH Why is substitution so and vice versa?

Standardized Test Practice

81. SHORT RESPONSE Simplify $3y(4x + 6y - 5)$.

82. SAT/ACT Which of the following best describes the graph of the equations?

$$4y = 3x + 8$$
$$-6x = -8y + 24$$

A The lines are parallel.
B The lines are perpendicular.
C The lines have the same x-intercept.
D The lines have the same y-intercept.
E The lines are the same.

Spiral Review

85. CRAFTS Badria sells stuffed animals at a local craft sho
the small ones and AED 15 for the large ones. To cover
least AED 350 worth of animals.

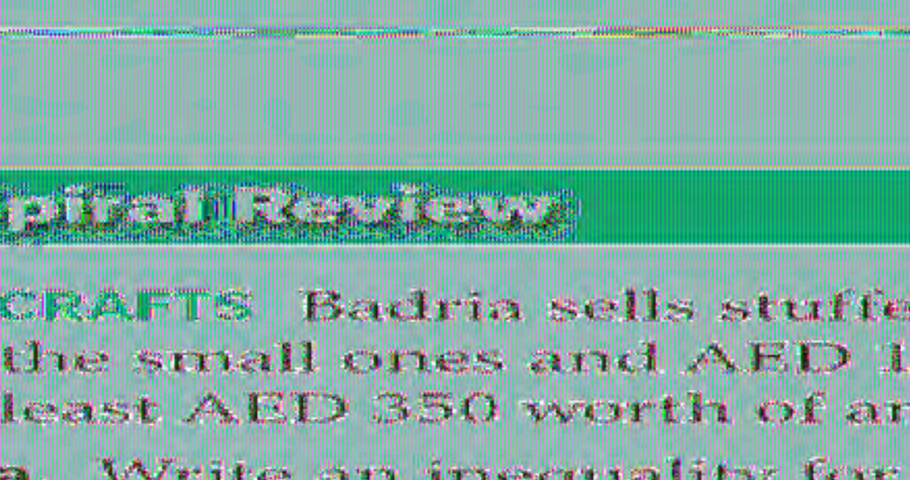
a. Write an inequality for this situation.

b. Graph the inequality.

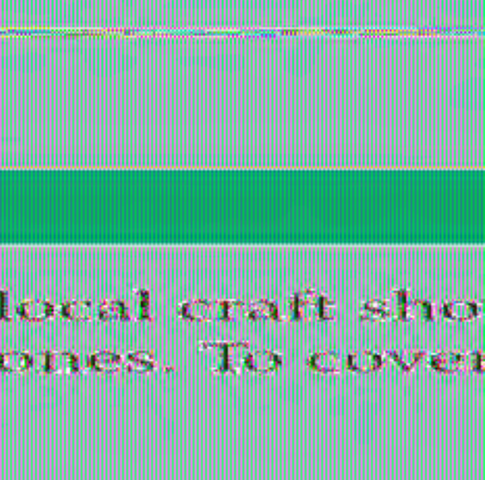
c. If she sells 10 small and 15 large animals, will she co

Write an equation for each function.

86.



87.



Solve each equation. Check your solution.

89. $2p = 14$

90. $-14 + n = -$

92. $x + 9x - 6x + 4x = 20$

93. $27 = -9(y +$

Skills Review

Determine whether the given point satisfies each inequali

95. $4x + 5y \leq 15$; $(2, -2)$

96. $3x + 5y \geq 8$; $(1, 1)$

Solving Systems of Inequalities by

Then | **Now** | **Why?**

You solved systems of linear equations graphically and algebraically.

1. Solve systems of inequalities by graphing.
2. Determine the coordinates of the vertices of a region formed by the graph of a system of inequalities.

Many weather con
met before a spac
The temperature r
-1.12°C and less th
wind speed cannc
A system of inequ
show these three

New Vocabulary
system of inequalities

Mathematical Practices
Make sense of problems and persevere in solving them.

1 Systems of Inequalities Solv
pairs that satisfy all of the inequ

Key Concept Solving Systems of

Step 1 Graph each inequality, shadin

Step 2 Identify the region that is sha

Example 1 Intersecting Regions

Solve the system of inequalities.
$y > 2x - 4$
$y \leq -0.5x + 3$

Solution of $y > 2x - 4$ → Region
Solution of $y \leq -0.5x + 3$ → Reg
Region 3 is part of the solution o
inequalities, so it is the solution

CHECK Notice that the origin is p
be used as a test point. Su
x and y in each equation.

$y > 2x - 4$
$0 \stackrel{?}{>} 2(0) - 4$
$0 \stackrel{?}{>} 0 - 4$
$0 > -4$ ✓

Guided Practice

1A. $y \leq -2x + 5$
$y > -\frac{1}{4}x - 6$

Reading Math

Empty Set The empty set is also called the *null* set. It can be represented by ∅ or { }.

It is possible that the regions c
system or the solution set is th

Example 2 Separate Regio

Solve the system of inequal
$y \geq x + 5$
$y < x - 4$

Graph both inequalities.

Since the graphs of the ine
do not overlap, there are no
in common and there is no
to the system.

The solution set is the empt

Guided Practice

2A. $y \geq -4x + 8$
$y < -4x + 4$



Real-World Link

Typical first year college students will spend more than 3 times as many hours studying in college as in high school.

Source: *National Survey of Student Engagement*

Real-World Examples 3

TIME MANAGEMENT Badriah
has up to 25 hours to study
She needs to spend at least
She hopes to spend betwee
of inequalities to represent

Calculus: at least 7 hours, b
$7 \leq c$

Physics: at least 8 hours, bu
$8 \leq p$

Badriah has 25 hours availa
will be spent on history. Sh
left for calculus and physics
$c + p$

Graph all of the inequalities
in the intersection is a solut
One solution is 10 hours on
12 hours on calculus.

Guided Practice

3. TRAVEL Mr. Yousef and
two children. They plan
Mr. Yousef wants to driv
a day. Mrs. Wata can dri
and graph a system of in

StudyTip

Boundaries If the inequality that forms the boundary is $<$ or $>$, then the boundary is not included in the solution, and the line should be dashed.

2 Find Vertices of an Enclosed

the graph of a system of inequa
enclosed region in the form of a pol
the vertices of the region, determine
of the points at which the boundarie

Example 4 **Find Vertices**

Find the coordinates of the vertic
$y \leq -\frac{1}{4}x + 6$, and $4y \geq -15x - 3$

Step 1 Graph each inequality.
The coordinates $(-4, 7)$
be determined from the
the coordinates of the th
solve the system of equat
and $y = -\frac{1}{4}x + 6$.

Step 2 Substitute for y in the se

$$2x - 8 = -\frac{1}{4}x + 6$$
$$2x = -\frac{1}{4}x + 14$$
$$\frac{9}{4}x = 14$$
$$x = \frac{56}{9} \text{ or } 6\frac{2}{9}$$

Step 3 Find y.

$$y = 2\left(6\frac{2}{9}\right) - 8$$
$$= 12\frac{4}{9} - 8$$
$$= 4\frac{4}{9}$$

CHECK Compare the coordinate
of the third vertex is betw
third vertex is between 4

The vertices of the triangle are at

Guided Practice

Find the coordinates of the vertic
of inequalities.

4A. $y \geq -3x - 6$
$2y \geq x - 16$
$11y + 7x \leq 12$

Check Your Understanding

Examples 1–2 **Solve each system of inequalities by graph**

1. $y \leq 6$
   $y > -3 + x$

2. $y \leq -3$
   $y \geq 2x$

Example 3

4. **REASONING** The most Alia can spend o
   is AED 35. A package of 10 sausages co
   and contains 8 buns. She needs to buy

   a. Graph the region that shows how m

   b. Give an example of three different p

Example 4 **Find the coordinates of the vertices of the t**
of inequalities.

5. $y \geq 2x + 1$
   $y \leq 8$
   $4x + 3y \geq 8$

Practice and Problem Solving

Examples 1–2 **Solve each system of inequalities by graphi**

7. $x < 3$
   $y \geq -4$

8. $y > 3x - 5$

9. $y < -3x + 4$
   $y \leq 4$

10. $y \geq 0$
    $y < x$

11. $6x - 2y \geq 12$

13. $5y < 2x + 10$
    $y - 4x > 8$

14. $3y - 2$
    $y \geq \frac{2}{3}x$

Example 3

16. **RECORDING** Kateb's band wants to spe
    The studio charges at least AED 35 per
    represent this situation.

17. **SUMMER TRIP** Salem has to save at leas
    8 weeks. He earns AED 9 an hour work
    working at a car wash. By law, he cann
    inequalities that Salem can use to deter
    at each job if he wants to make the trip

Example 4 **Find the coordinates of the vertices of the t**
of inequalities.

18. $x \geq 0$
    $y \geq 0$
    $x + 2y < 4$

19. $y \geq 3x - 7$

20. $x \leq 4$
    $y \leq 8$
    $x + y \geq$

21. $-3x + 4y \leq 15$
    $2y + 5x \geq -12$
    $10y + 60 \geq 27x$

22. $8y - 1$
    $38y +$
    $54y -$

24. **BAKING** Hessa wants to bake cookies a
    15 cookies at a time and 12 cupcakes at
    goods, but no more than 360, and she
    as cupcakes. What combination of bat

25. **CELL PHONES** Ayoub has a maximum c
each month. Daytime minutes cost AE
plans to use at least twice as many dayt
least 200 nighttime minutes and does r
minimum bill?

26. **TREES** Trees are divided into four categ
Data for the trees in a forest are descrit

Crown Class	domina
Height (in feet)	over 7
Trunk Circumference (in inches)	over 6

Source: USDA Forest Service

a. Write and graph the system of inequ
h and circumferences *c* for a co-dom

b. Determine the crown class of a bass
trunk circumference.

27. **REASONING** On a camping trip, Jamal
0.5 liters of water per day. Mazen need
water per day. Jamal's equipment weigh
20 kilograms. A liter of water weighs ap
their own supplies, and Jamal is capabl
carry 50 kilograms.

a. Graph the inequalities that represen

b. How many days can they camp, assu
at once?

c. Who will run out of supplies first?

Solve each system of inequalities by graphin

28. $y \geq |2x + 4| - 2$
$3y + x \leq 15$

29. $y \geq |6$
$|y| \leq 4$

31. $y > -3x + 1$
$4y \leq x - 8$
$3x - 5y < 20$

32. $6y + 2x \leq 9$

33. $2y + 1$
$y > -4$

34. $2x + 3y \geq 6$
$y \leq |x - 6|$

35. $8x + 4y < 10$
$y > |2$

37. **MUSIC** Khalid is trying to decide what
3 hours long and songs are 2.5 minutes
books on his MP3 player, but at least te
costs ABD 15.00 and each song costs A
music. Graph the inequalities to show
Khalid can have.

38. **JOBS** Khalifa has two jobs and can wor
hours per week. He wants to earn at le
Graph the inequalities to show possibl
worked at each job that will help him r

39. **TIME MANAGEMENT** Mansour uses his s
budgeted 35 hours per week. He wants
than 15. He also hopes to write betwee
a system of inequalities that represents

**Find the coordinates of the vertices of the f
inequalities.**

40. $y \geq 2x - 12$
$y \leq -4x + 20$
$4y - x \leq 8$
$y \geq -3x + 2$

41. $y \geq -x$
$2y \geq 3x$
$4y + x$
$y \leq 4x$

43. **FINANCIAL LITERACY** Mr. Hamad is inve
pay 6% interest, and a riskier second fu
amount he can invest in the risky fund

44. **DODGEBALL** A high school is selecting
exhibition against their rival. There can
and there must be more girls than boys

a. Write and graph a system of inequal
b. List all of the possible combinations
c. Explain why there is not an infinite

H.O.T. Problems Use Higher-Order Thinking Skills

45. **CHALLENGE** Find the area of the region
$y \geq -$
$4y \leq$
$3y +$
$4y - 2$

46. **OPEN ENDED** Write a system of two ine
a. lies only in the third quadrant.
b. does not exist.
c. lies only on a line.
d. lies on exactly one point.

47. **CHALLENGE** Write a system of inequaliti
solution shown at the right. How many
coordinates are solutions of the system

48. **ARGUMENTS** Determine whether the s
or *false*. If false, give a counterexample.

*A system of two linear inequalities has eith
infinitely many points in its solution.*

49. **WRITING IN MATH** Write a how-to manu
a system of inequalities.

50. **WRITING IN MATH** Explain how you wo
a system of inequalities.

Standardized Test Practice

51. To be a member of the marching band, a student must have a grade-point average of at least 2.0 and must have attended at least five after-school practices. Choose the system of inequalities that best represents this situation.

A $x \geq 2$, $y \geq 5$

B $x \leq 2$, $y \leq 5$

C $x < 2$, $y < 5$

D $x > 2$, $y > 5$

52. SAT/ACT The table at the right shows a relationship between x and y. Which equation represents this relationship?

x	y
1	5
2	8
3	11
4	14
5	17
6	20

F $y = 3x - 2$
G $y = 3x + 2$
H $y = 4x + 1$
J $y = 4x + 2$
K $y = 4x - 1$

Spiral Review

55. GEOMETRY Find the coordinates of the vertices of the parallelo
contained in the lines with equations $y = 3$, $y = 7$, $y = 2x$, a

Graph each inequality.

56. $x + y \geq 6$

57. $4x - 3y < 10$

Graph each function. Identify the domain and range.

59. $g(x) = \begin{cases} 0 \text{ if } x < 0 \\ -x + 2 \text{ if } x \geq 0 \end{cases}$

60. $h(x) = \begin{cases} x + 3 \text{ if } x \leq \\ 2x \text{ if } x > \end{cases}$

62. BOOK CLUB For each meeting of the High School book club, the activities account to buy snacks and materials. After the AED 350 left in the activities account.

a. If no money is put back into the account, what equation c much money is left in the activities account after having a

b. How much money was originally in the account?

c. After how many meetings will there be no money left in t

Skills Review

Find each value if $f(x) = 2x + 5$ and $g(x) = 3x - 4$.

63. $f(-3)$

64. $g(-2)$

65. $f(-1)$

66. $g(-0.5)$

67. $f(-0.25)$

68. $g(-0.75)$

Graphing Technology

Systems of Inequalities

You can graph systems of linear inequalities with a graphing calcula
Y= menu. You can choose different graphing styles to shade above

Example 6 INTERSECTION OF TWO GRAPHS

Graph the system of inequalities in the standard viewing

$y \geq -3x + 4$
$y \leq 2x - 1$

Step 1 Enter $-3x + 4$ as Y1. Because y is greater than $-3x + 4$, shade above the line.

KEYSTROKES: Y= ◄ ◄ ENTER ENTER ► ►
(−) 3 X,T,θ,n + 4 ENTER

Step 2 Enter $2x - 1$ as Y2. Because y is less than $2x - 1$, shade below the line.

KEYSTROKES: ◄ ◄ ENTER ENTER ENTER ►
X,T,θ,n − 1 ENTER

Step 3 Display the graphs in the standard viewing window.

KEYSTROKES: Zoom 6

Notice the shading pattern above the line $y = -3x + 4$ a
the line $y = 2x - 1$. The intersection of the graphs is the r
overlap. This region includes all the points that satisfy th
$y \leq 2x - 1$.

Exercises

Use a graphing calculator to solve each system of inequalit

1. $y \geq 3$
   $y \leq -x + 1$

2. $y \geq -4x$
   $y \leq -x - 1$

3. $y \geq 2 - x$
   $y \leq -5$

4. $y \geq 2x + 1$
   $y \leq -x - 1$

5. $2y \geq 3x - 1$

6. $y + 5x \geq 12$
   $3y \leq -x + 7$

7. $5y + 3x \geq 11$
   $3y - x \leq -8$

8. $10y - 7x \geq -$
   $7y - 5x \leq 11$

Optimization with Linear Programming

Then

You solved systems of linear inequalities by graphing.

Now

1. Find the maximum and minimum values of a function over a region.
2. Solve real-world optimization problems using linear programming.

Why?

An electronics co
digital audio playe
A sign on the com
board is shown.

If at least 2000 ite
produced per shift
each type should
minimize costs?

The company is ea
constraints, on pro
demand, shipping
factory. A system
represent these c

Keeping C

	Our co
Units	Minim
audio	60
phone	80

New Vocabulary
linear programming
feasible region
bounded
unbounded
optimize

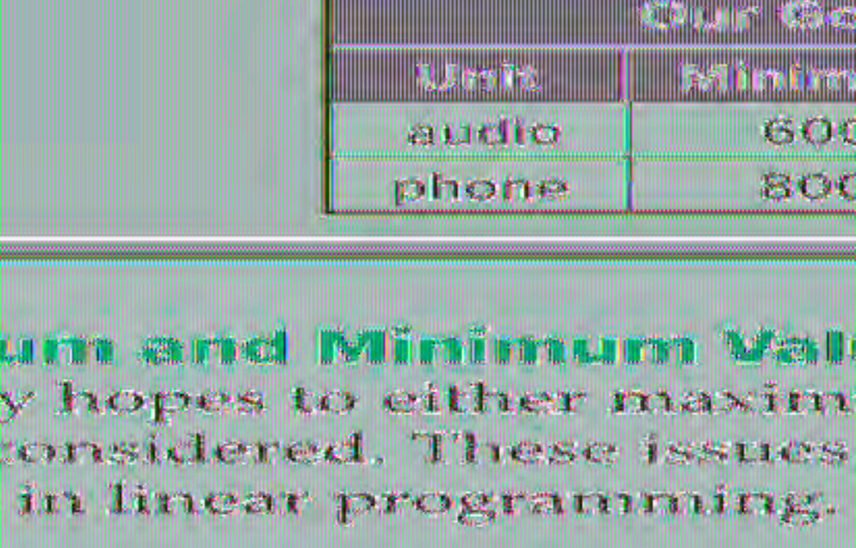
Mathematical Practices
Model with mathematics.
Look for and express regularity in repeated reasoning.

1 Maximum and Minimum Val

company hopes to either maxim
need to be considered. These issues
inequalities in linear programming.

Linear programming is a method fo
a given system of inequalities with e
is graphed and the vertices of the so
the function, you can determine the

Key Concept: Feasible Regions



The feasible region is enclosed, or bou
the constraints. The maximum or minim
of the related function *always* occurs at
of the feasible region.

Reading Math

Function Notation

The notation $f(x, y)$ is used to represent a function with two variables, x and y. It is read *f of x and y*.

Example 1 **Bounded Regi**

Graph the system of inequa region. Find the maximum

$3 \leq y \leq 6$
$y \leq 3x + 12$
$y \leq -2x + 6$
$f(x, y) = 4x - 2y$

Step 1 Graph the inequali

Step 2 Evaluate the functi

(x, y)	$4x - 2$
$(-3, 3)$	$4(-3) -$
$(1.5, 3)$	$4(1.5) -$
$(0, 6)$	$4(0) - 2$
$(-2, 6)$	$4(-2) -$

The maximum value is 0 at is -20 at $(-2, 6)$.

Guided Practice

1A. $-2 \leq x \leq 6$
$1 \leq y \leq 5$
$y \leq x + 3$
$f(x, y) = -5x + 2y$

When a system of inequalities

Example 2 **Unbounded Re**

Graph the system of inequa region. Find the maximum

$2y + 3x \geq -12$
$y \leq 3x + 12$
$y \geq 3x - 6$
$f(x, y) = 9x - 6y$

Evaluate the function at ea

(x, y)	$9x - 6y$	$f(x,$
$(-4, 0)$	$9(-4) - 6(0)$	$-$
$(0, -6)$	$9(0) - 6(-6)$	

The maximum value is 36 a minimum value. Notice tha feasible region, (0, 8), yield is less than -36.

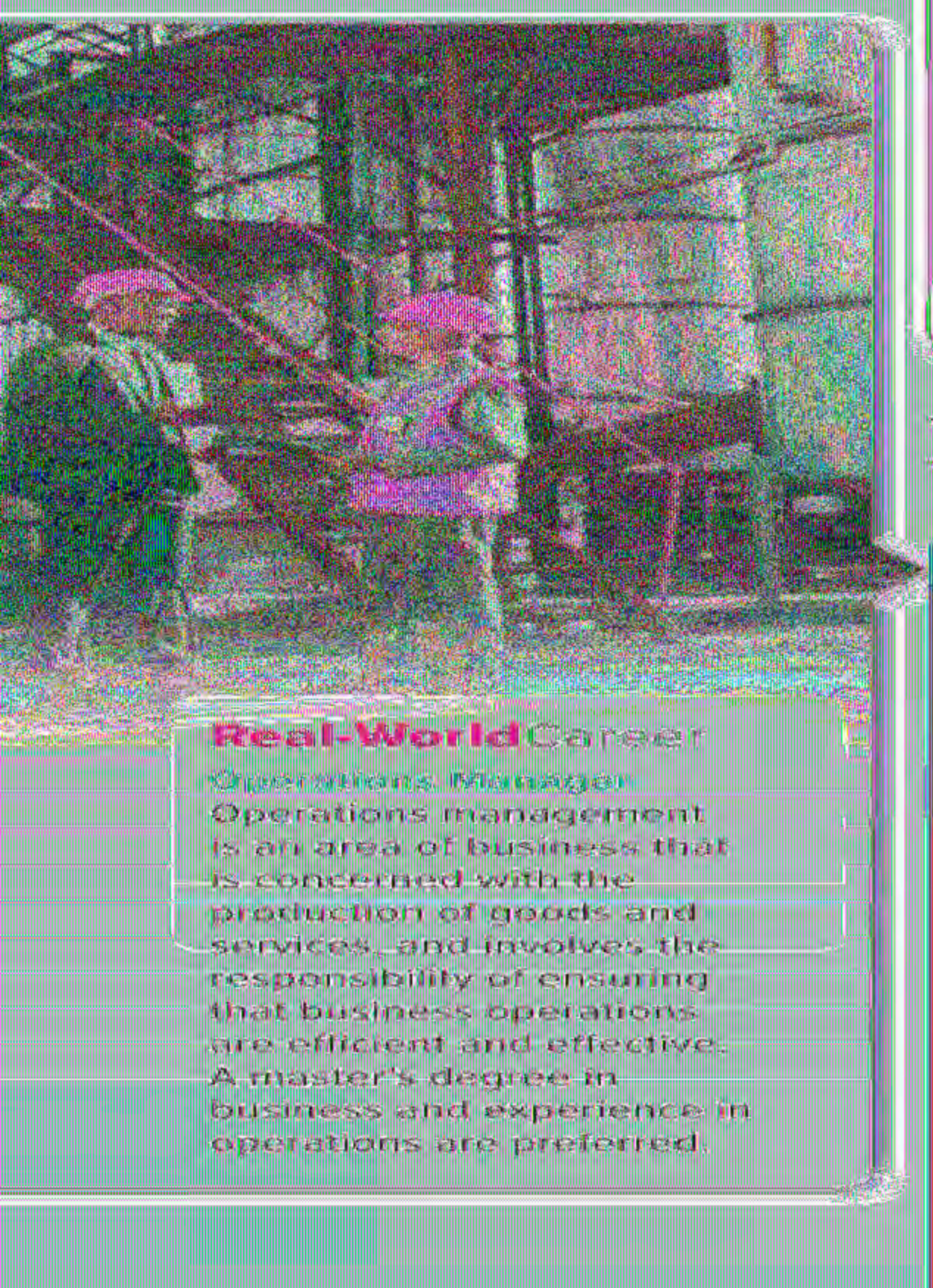
Guided Practice

2A. $y \leq 8$
$y \geq -x + 4$
$y \leq -x + 10$
$f(x, y) = -6x + 8y$

Watch Out!

Precision Do not assume that there is no maximum if the feasible region is unbounded above the vertices. Test points are needed to determine if there is a minimum or maximum.

2 **Optimization** To optimize means to maximize profits. This is often



Real-World Career

Operations Manager

Operations management is an area of business that is concerned with the production of goods and services, and involves the responsibility of ensuring that business operations are efficient and effective. A master's degree in business and experience in operations are preferred.

Study Tip

Reasonableness Check your solutions for reasonableness by thinking of the context of the problem.

Key Concept Optimization with L

Step 1 Define the variables.

Step 2 Write a system of inequalities

Step 3 Graph the system of inequalit

Step 4 Find the coordinates of the ve

Step 5 Write a linear function to be m

Step 6 Substitute the coordinates of

Step 7 Select the greatest or least re

When using a system of inequalities whole-number solutions will make s

Real-World Example Optim

BUSINESS Refer to the applicatio each type of device should be ma

Step 1 Let x = nu players produced Let p = number phones produced

Step 2 $600 \le x \le 1500$
$800 \le p \le 1700$
$x + p \ge 2000$

Steps 3 and 4 The system is gra right. Note the ve feasible region.

Step 5 The function to b

(x, p)	
(500, 1700)	55
(600, 1400)	55
(1500, 1700)	55
(1500, 800)	55
(1200, 800)	55

Step 6

Step 7 Produce 1200 and

Guided Practice

3. **JEWELRY** Each week, Nora ca If she earns profits of AED 3 o she plans to sell at least 30 pie

Check Your Understanding

Examples 1–2 **Graph each system of inequalities. Name t the feasible region. Find the maximum and function for this region.**

1. $y \leq 5$
$x \leq 4$
$y \geq -x$
$f(x, y) = 5x - 2y$

2. $y \leq -3x + 6$

3. $y \geq -3x +$
$-y \leq x$
$y \leq 3$
$f(x, y) =$

4. $-2 \leq y \leq 6$
$3y \leq 4x + 26$
$y \leq -2x + 2$
$f(x, y) = -3x - 6y$

5. $-3 \leq y \leq 7$

6. $y \leq 2$
$4y \geq 4$
$6y + 3$
$f(x, y) =$

Example 3

7. PRECISION The total number of worke in a skateboard factory is 85 hours. The and quality control each day. The table department for two different types of s

Boards Designed	
Board Type	Production
Pro Boards	1.5 hou
Specialty Boards	1 hour

a. Write a system of inequalities to repr
b. Draw the graph showing the feasible
c. List the coordinates of the vertices o
d. If the profit on a pro board is ABD $5 write a function for the total profit o
e. Determine the number of each type a maximum profit. What is the maxi

Practice and Problem Solving

Examples 1–2 **Graph each system of inequalities. Name t feasible region. Find the maximum and mi for this region.**

8. $1 \leq y \leq 4$
$4y - 6x \geq -32$
$2y \geq -x + 4$
$f(x, y) = -6x + 3y$

9. $2 \geq x \geq -3$

10. $-2 \leq x$
$y \geq -2$
$4y \leq 2$
$f(x, y) =$

11. $-8 \leq y \leq -2$
$y \leq x$
$y \leq -3x + 10$
$f(x, y) = 5x + 14y$

12. $x + 4y \geq 2$

13. $3 \leq$
$2x + 4$
$2 \leq x \leq$
$f(x, y) =$

Examples 1—2 Graph each system of inequalities. Name the coo
Find the maximum and minimum values of the

14. $-9 \leq x \leq -3$
$-9 \leq y \leq -5$
$3y + 12x \leq -75$
$f(x, y) = 20x + 8y$

15. $x \geq -8$

16. $y \geq$
$3x + 6y \leq 3$
$2y + 12 \geq 3x$
$f(x, y) = 10x$

17. $x \geq -6$
$y + x \leq -1$
$2x + 3y \geq -9$
$f(x, y) = -10x - 12y$

18. $-5 \geq y \geq -17$

19.
$y \leq 3x + 19$
$y \leq -4x + 1$
$f(x, y) = 8x -$

20. $y \leq x + 4$
$y \geq x - 4$
$y \leq -x + 10$
$y \geq -x - 10$
$f(x, y) = -10x + 9y$

21. $-4 \leq x \leq 8$

22. $y \geq$
$-8 \leq y \leq 6$
$y \geq x - 6$
$4y + 7x \leq 31$
$f(x, y) = 12x$

Example 3

23. COOKING Zainab's Bakery makes two types o
for AED 25, and strawberry cake, which sells
size, but the decorating and assembly time r
while the time is 3 hours for the strawberry c
available for production. How many of each
maximize revenue?

24. BUSINESS The manager of a travel agency is
advertise special discounts on vacation spots
costs AED 0.08 to print, and each flier costs
8 pages, and a flier requires 2 pages. The ma
600 pages, and she needs at least 50 brochur
she print to minimize the cost?

25. PRECISION Obed has 20 days to paint as ma
The sheds can be painted at a rate of 2.5 per
at a rate of 2 per day. He has 45 structures th

a. Write a system of inequalities to represent
the structures.

b. Draw a graph showing the feasible region
of the feasible region.

c. If the profit is AED 26 per shed and AED
he paint?

d. What is the maximum profit?

26. MOVIES Employees at a local movie theater w
or from 4 P.M. to midnight. The table below s
needed and their corresponding pay. Find th
night-shift workers that should be scheduled
minimal cost?

Time	noon–4 P.M.	4 P.M.–midnight
Number of Employees Needed	at least 5	at least 8
Rate per Hour	AED 8	AED 10

27. **BUSINESS** Each car on a freight train ca
of 480 cubic meters. The freight service
25 kilograms and are 3 cubic meters ea
5 cubic meters each. The freight service
for each large package.

a. Find the number of each type of pac
maximize revenue.

b. What is the maximum revenue per t

c. In this situation, is maximizing the r
to do? Explain.

28. **RECYCLING** A recycling plant processes
The plant processes up to 1200 tons of
processed for food containers, while at
containers. The profit is AED 64 per to
ton for processing drink containers. W

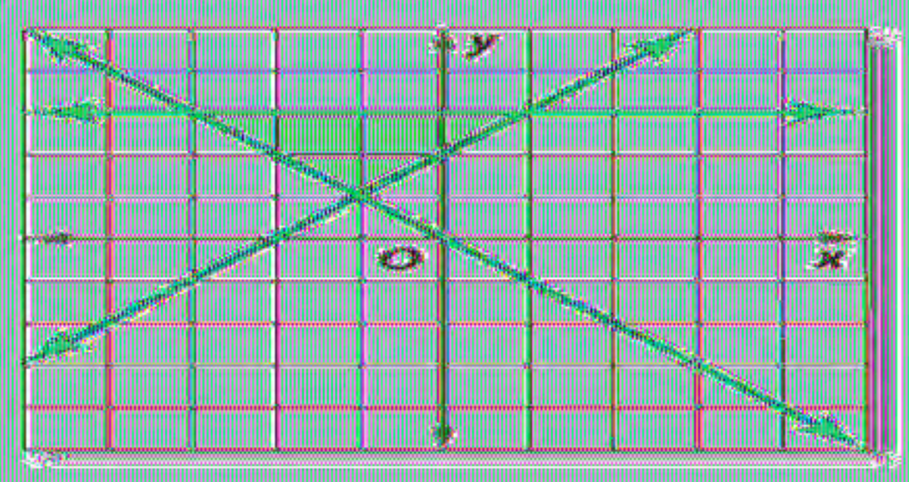
H.O.T. Problems Use Higher-Order Thinking Skills

29. **OPEN ENDED** Create a set of inequalitie
20 units^2 and lies only in the fourth qu

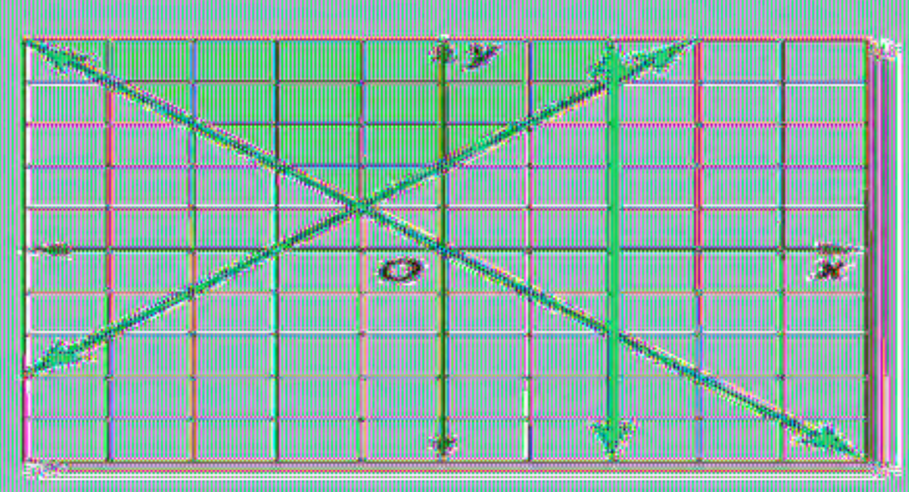
30. **CHALLENGE** Find the area of the bound
$y \geq |x| - 3$, $y \leq -|x| + 3$, and $x \geq \|y\|$

31. **ARGUMENTS** Identify the system of ine
Explain your reasoning.

a.



b.



32. **REASONING** Determine whether the fol
Explain your reasoning.

An unbounded region will not

33. **WRITING IN MATH** Upon determining a
vertices $A(-3, 4)$ and $B(5, 2)$ yielded th
Kelvin confirmed that his constraints w
Then he said that those two points wer
region. Explain how this could have ha

Standardized Test Practice

34. Hussain worked 350 hours during the summer and earned AED 2978.50. He earned AED 6.85 per hour when he worked at a video store and AED 11 per hour as an architectural intern. Let x represent the number of hours he worked at the video store and y represent the number of hours that he interned. Which system of equations represents this situation?

A $x + y = 350$
$11x + 6.85y = 2978.50$

B $x + y = 350$
$6.85x + 11y = 2978.50$

C $x + y = 2978.50$
$6.85x + 11y = 350$

D $x + y = 2978.50$
$11x + 6.85y = 350$

35. SHORT RESPONSE A family of four went out to dinner. Their bill, including tax, was AED 60. They left a 17% tip on the total cost of their bill. What is the total cost of the dinner including tip?

Spiral Review

Solve each system of inequalities by graphing. (Lesson 1-2)

38. $3x + 2y \geq 6$
$4x - y \geq 2$

39. $4x - 3y < 7$
$2y - x \leq -6$

40. $3y \leq 2x - 8$
$y \geq \frac{2}{}$

41. BUSINESS Last year the chess team paid AED 7 per hat and purchase of AED 330. This year they spent AED 360 to buy and hats because the hats now cost AED 8 and the shirts cos a system of two equations that represents the number of hat each year. (Lesson 1-1)

Write an equation in slope-intercept form for the line that satisf

42. passes through (5, 1) and (8, −4)

43. passes

Find the x-intercept and the y-intercept of the graph of each equ the equation.

44. $5x + 3y = 15$

45. $2x - 6y = 12$

47. $2x + 5y - 10 = 0$

48. $y = x$

49. $y =$

Skills Review

Evaluate each expression if $x = -1$, $y = 3$, and $z = 7$.

50. $x + y + z$

51. $2x - y + 2z$

52. $-x + 4y - 3z$

53. $4x + 2y - z$

54. $5x - y + 4z$

55. $-3x - 3y + 3z$

3.1 Systems of E
in Three Var

: • Then | **: • Now** | **: • Why?**

You solved systems of linear equations in two variables.

1. Solve systems of linear equations in three variables.
2. Solve real-world problems using systems of linear equations in three variables.

Seats close
stage cost
next sectio
seats are A
as many se
as in sectio
19,200 sea
amphithea

A system o
be used to
each sectio

New Vocabulary
ordered triple

Mathematical Practices
Construct viable arguments and critique the reasoning of others.

1 Systems in Three Variab

three variables can have or
a system is an **ordered triple** (

The graph of an equation in th
a plane. The graphs of a syste

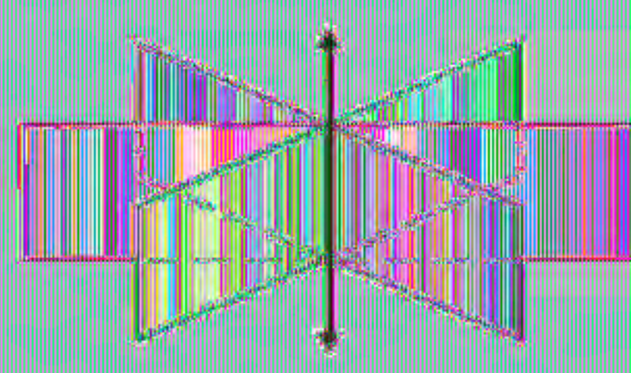
One Solution

The three individual planes in
a specific point.

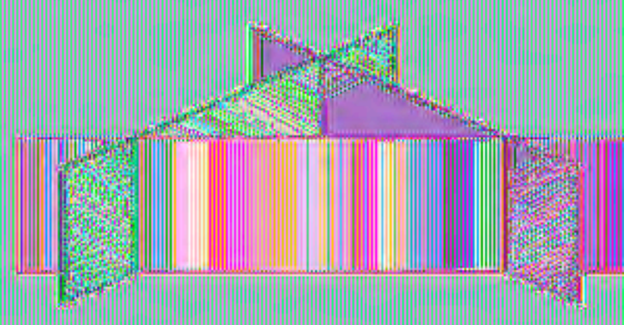
Infinitely Many Solutions

The planes intersect in a line.

Every coordinate on the line r
a solution of the system.



No Solution There are no poi



Solving systems of equations in thre
two variables. Use the strategies of s
that represents the solution of the s

Example 4 **A System with One S**

Solve the system of equations.

$$3x - 2y + 4z = 35$$
$$-4x + y - 5z = -36$$
$$5x - 3y + 3z = 31$$

Step 1 Eliminate one variable b

$$3x - 2y + 4z = 35$$
$$-4x + y - 5z = -36$$

$$-4x + y - 5z = -36$$
$$5x - 3y + 3z = 31$$

The y-terms in each equation hav
equations and two variables, x an

Step 2 Solve the system of two

$$-5x - 6z = -37$$
$$-7x - 12z = -77$$

Use substitution to solve for z.

$$-5x - 6z = -37$$
$$-5(-1) - 6z = -37$$
$$5 - 6z = -37$$
$$-6z = -42$$
$$z = 7$$

The result is $x = -1$ and $z = 7$.

Step 3 Substitute the two value

$$-4x + y - 5z = -36$$
$$-4(-1) + y - 5(7) = -36$$
$$4 + y - 35 = -36$$
$$y = -5$$

CHECK
$$-4x + y - 5z = -$$
$$-4(-1) + (-5) - 5(7) \stackrel{?}{=} -$$
$$4 + (-5) - 35 \stackrel{?}{=} -$$
$$-36 = -$$

The solution is $(-1, -5, 7)$.

Study Tip

Checking Solutions Always substitute your answer into all of the original equations to confirm your answer.

Guided Practice

1A. $2x + 4y - 5z = 18$
$-3x + 5y + 2z = -27$
$-5x + 3y - z = -17$

StudyTip

Infinite and No Solutions When solving a system of more than two equations, $0 = 5$ will always yield no solution, while $0 = 0$ may not always yield infinite solutions.

When solving a system of thre
your answer using all three of
for a solution to work for two

Example 2 No Solution an

Solve each system of equat

a. $5x + 4y - 5z = -10$
$-4x - 10y - 8z = -16$
$6x + 15y + 12z = 24$

Eliminate x in the secon

$-4x - 10y - 8z = -$

$6x + 15y + 12z = 24$

The equation $0 = 0$ is al
the same plane. Check t

$5x + 4y - 5z = -10$

$-4x - 10y - 8z = -$

The planes intersect in a

b. $-6a + 9b - 12c = 21$
$-2a + 3b - 4c = 7$
$10a - 15b + 20c = -30$

Eliminate a in the first t

$-6a + 9b - 12c = 21$
$-2a + 3b - 4c = 7$

The equation $0 = 0$ is al
the same plane. Check t

$-2a + 3b - 4c = 7$
$10a - 15b + 20c = -$

The equation $0 = 5$ is ne

Guided Practice

2A. $-4x - 2y - z = 15$
$12x + 6y + 3z = 45$
$2x + 5y + 7z = -29$

2 Real-World Problems T
four-step plan to help org
they represent. Then use the i
variables. Once you have thre
can solve the problem.



Real-World Career

Music Management

Music management involves acting as the talent manager to the artists. Other duties include negotiating with record labels, music promoters, and tour promoters. Managers usually earn a percentage of the artist's income. A bachelor's degree is usually required.

Real-World Example 3 Write

CONCERTS Refer to the beginnin
to determine how many seats are

Understand Define the variables

Plan There are 19,200 sea
$x + y + z = 19{,}200$
The total revenue is
$30x + 25y + 20z =$
There are twice as m
$y = 2x$

Solve Solve the system.

Step 1 Substitute y
$x + y +$
$x + 2x +$
$3x +$
$30x + 25y + 2$
$30x + 25(2x) + 2$
$80x + 2$

Step 2 Solve the sy
$3x +$
$80x + 20$

Step 3 Substitute t
$3x$
$3(3600)$
$10{,}800$

Step 4 Substitute t
$y = 2x$
$y = 2(3600)$
The solution is (360
7200 in section B, a

Check Substitute the value

Guided Practice

3. Ms. Chaya invested AED 50,0
as much money in an account
10% interest. The third accoun
in interest in a year, how much

Check Your Understanding

Examples 1–2 **Solve each system of equations.**

1. $-3a - 4b + 2c = 28$
   $a + 3b - 4c = -31$
   $2a + 3c = 11$

2. $3y - 5z = -2$
   $4x + 2y$
   $-2x -$

4. $-4r - s + 3t = -9$
   $3r + 2s - t = 3$
   $r + 3s - 5t = 29$

5. $3x + 5y - z =$
   $-2x -$
   $4x + 7y$

Example 3

7. **DOWNLOADING** Nabila downloaded sor
   0.3 gigabyte of memory; a drama, 0.6 g
   She downloaded 7 programs totaling 3
   many episodes of the drama as the site

   a. Write a system of equations for the

   b. How many episodes of each show d

Practice and Problem Solving

Examples 1–2 **Solve each system of equations.**

8. $-5x + y - 4z = 60$
   $2x + 4y + 3z = -12$
   $6x - 3y - 2z = -52$

9. $4a + 5b - 6c =$
   $-3a -$
   $-a + 4$

11. $4r + 6s - t = -18$
    $3r + 2s - 4t = -24$
    $-5r + 3s + 2t = 15$

12. $-2x +$
    $4x + 3y$
    $-3x +$

14. $8x + 3y + 6z = 43$
    $-3x + 5y + 2z = 32$
    $5x - 2y + 5z = 24$

15. $-6x - 5y + 4z$
    $5x + 3y$
    $8x - 6y$

17. $2x - y + z = 1$
    $x + 2y - 4z = 3$
    $4x + 3y - 7z = -8$

18. $x + 2y = 12$

19. x
    $3y - 4z$
    $x + 6y$

Example 3

20. **SENSE-MAKING** A friend e-mails you th
    meet. The e-mail states that 24 individu
    53 points. First place earned 3 points, s
    place earned 1 point. There were as ma
    third-place finishers combined.

    a. Write a system of three equations th
       each place.

    b. How many swimmers finished in fir

    c. Suppose the e-mail had said that the
       Explain why this statement is false a

21. **AMUSEMENT PARKS** Ayman goes to the
    cars, and water slides. The wait for the
    cars is 20 minutes long, and the wait fo
    rode 10 total rides during his visit. Beca
    of times he rode the roller coasters was
    If Ayman waited in line for a total of 6 h
    did he go on?

22. **BUSINESS** Rashid usually gets one of the routine maintenance options at Annie's Garage. Today however, he needs a different combination of work than what is listed.

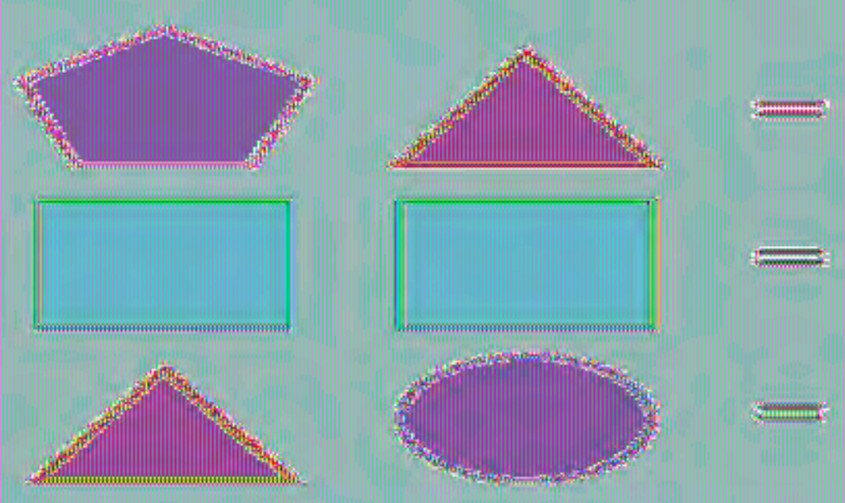
a. Assume that the price of an option is the same price as purchasing each item separately. Find the prices for an oil chang a radiator flush, and a brake pad replacement.

b. If Rashid wants his brake pads replaced a plan to spend?

23. **FINANCIAL LITERACY** Huda invested AED 100 three different accounts. If she invested AED 30,000 more in account A than accoun and is expected to earn AED 6300 in interest how much did she invest in each account?

H.O.T. Problems Use Higher-Order Thinking Skills

24. **REASONING** Write a system of equations to r below. Use the system to find the number of green circle.



25. **CHALLENGE** The general form of an equation where (x, y) is a point on the parabola. If the $(-5, -101)$, and $(6, -90)$, determine the valu form of the equation.

26. **PROOF** Consider the following system and p

$$rx + ty + uz = a$$

$$rx - ty + uz =$$
$$rx + ty - uz =$$

27. **OPEN ENDED** Write a system of three linear e $(-5, -2, 6)$. Show that the ordered triple sat

28. **REASONING** Use the diagrams of solutions of consider a system of inequalities in three var a system.

29. **WRITING IN MATH** Use your knowledge of sol with three variables to explain how to solve variables.

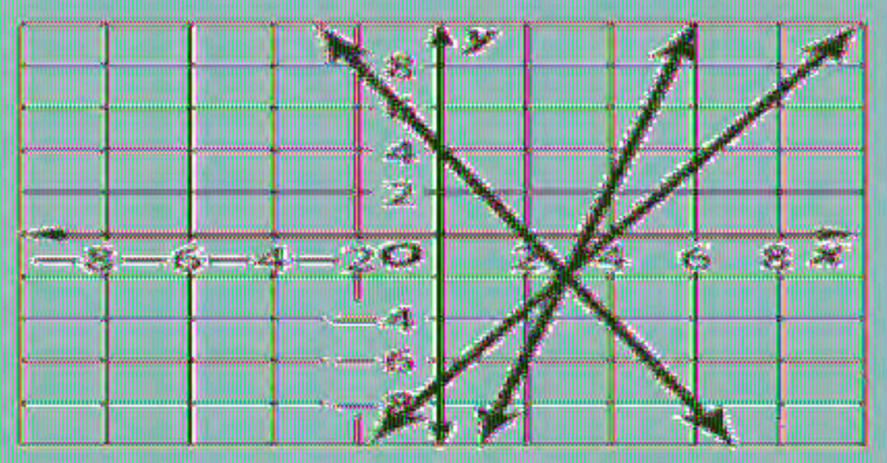
Standardized Test Practice

30. Solve the system of equations shown below.

$$\begin{cases} x - y + z = 0 \\ -5x + 3y - 2z = -1 \\ 2x - y + 4z = 11 \end{cases}$$

A (0, 3, 3)
B (2, 5, 3)
C no solution
D infinitely many solutions

31. SAT/ACT The graph shows which system of equations?



A $y + 14 = 4x$
$y = 4 - 2x$
$-7 = y - \frac{5}{3}x$

B $y - 14 = 4x$
$y = 4 + 2x$
$-7 = y + \frac{5}{3}x$

C $y + 14x = 4$
$-2y = 4 + y$
$-7 = y - \frac{5}{3}x$

D $y - 14x = 4$
$2x = 4 + y$
$7 = y - \frac{5}{3}x$

E $y - 4x = 14$
$y = 2x + 4$
$7 = y + \frac{5}{3}x$

Spiral Review

A feasible region has vertices at (−3, 2), (1, 3), (6, 1), and (minimum values of each function. (Lesson 1-3)

34. $f(x, y) = 2x - y$ **35.** $f(x, y) = x + 5y$ **36.** $f(x,$

38. SKI CLUB The ski club's budget for the year is AED 425 AED 75 per pair and boots for AED 40 per pair. They b than skis because the skis are adjustable to several size

a. Give an example of three different purchases that t

b. Suppose the ski club wants to spend all of its budge of skis and boots should they buy? Explain.

Skills Review

Solve each system of equations.

39. $x = y + 5$
$3x + y = 19$

40. $3x - 2y = 1$
$4x + 2y = 20$

41. $5x + 3y = 25$

42. $y =$
$4x + 7y =$

Mid-Chapter Quiz

Lessons 1-1 through 1-4

Solve each system of equations. (Lesson 1-1)

1. $2x - 3y = 9$ (3, 1)
   $4x + 3y = 9$

2. $x + 2y = 7$ (1, 3)
   $y = 5x - 2$

3. $-x + y = 2$ (3, 5)
   $4x - 3y = -3$

4. $\frac{1}{2}x + \frac{1}{3}y = 7$ (10, 6)
   $\frac{1}{5}x - \frac{2}{3}y = -2$

Solve each system of inequalities by graphing. (Lesson 1-2)

5. $x + y \leq 4$
   $y \geq x$

6. $2x + 3y > 12$
   $3x - y < 21$

7. $x - y > 0$
   $4 + y \leq 2x$

8. $2y - 5x \leq 6$
   $4x + y < -4$

9. **MULTIPLE CHOICE** Which statement best describes the graphs of the two equations? (Lesson 1-1)

   $x + 4y = 8$
   $3x + 12y = 2$

   A The lines are parallel.

   B The lines are the same.

   C The lines intersect in only one point.

   D The lines intersect in more than one point, but are not the same.

Solve each system of equations. (Lesson 1-4)

10. $x - 2y + 3z = 1$
    $4y - 4z = 12$
    $8y - 14z = 0$

16. **DRAMA** On opening night of the drama club's play, they made AED 1366. They sold a total of 199 tickets. They charged AED 8.50 for each adult ticket and AED 5.00 for each child's ticket. Write a system of equations that can be used to find the number of adult tickets and the number of children's tickets sold. (Lesson 1-1)

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and the minimum values of the given function. (Lesson 1-3)

17. $5 \geq y \geq -3$
$4x + y \leq 5$
$-2x + y \leq 5$
$f(x, y) = 4x - 3y$

18. $x \geq -10$
$1 \geq y \geq -6$
$3x + 4y \leq -8$
$2y \geq x - 10$
$f(x, y) = 2x + y$

19. **Geometry** An isosceles trapezoid has shorter base of measure a, longer base of measure c, and congruent legs of measure b. The perimeter of the trapezoid is 58 centimeters. The average of the bases is 19 centimeters and the longer base is twice the leg plus 7. (Lesson 1-4)

a. Find the lengths of the sides of the trapezoid.

b. Find the area of the trapezoid.

4 Matrices

Then

You solved a system of linear equations in many different ways.

Now

1. Identify the concept of a matrix.
2. Identify two equal matrices.

Why?

Matrices are used collection, which human or animal habits, etc.

New Vocabulary

matrix
element
entry
dimension

Mathematical Practices

TK

1 The concept of a matrix

A **matrix** and columns, usually enclosed so that each position in the matrix or **entry**. A matrix is usually named

The element -4 is in Row 2, Column 1, depicted by a_{21}.

$$A = \begin{bmatrix} 8 & \\ -1 & \\ 7 & \end{bmatrix}$$

A matrix can be described by its **dim** matrix (read "*m* by *n*"). Matrix *A* abo a_{ij} refers to an element of *A*, where

Special Matrices

Type of
Row ma
Column
Square
Zero ma

Example 1 Organize Data in a M

FOOTBALL Sharjah High School its season. Coach Walid wanted

Jamal: 11 games, 72 passes, 4390 meters, 6 goals, 8 fouls

Saad: 11 games, 82 passes, 542 goals, 7 fouls

a. Organize the data in a m

Player	Games	P
Karim	11	
Ahmed	10	
Ibrahim	9	
Saad	11	
jamal	11	

b. What are the dimension

There are five rows and
dimensions are 5×5. T
is in the third row and fo

2 Equal matrices Equal m
dimensions and their cor

Example 2 Equal Matrices

Matrices A and B are equal

$$A = \begin{bmatrix} 3 & 1 & 6 \\ 0 & 7 & 2 \end{bmatrix}$$

B

Matrices C and D are not e

$$C = \begin{bmatrix} 3 & 1 \\ 0 & 6 \\ 7 & 2 \end{bmatrix}$$

D

Guided Practice

We can use the equality of
Given that the following m

Solution:

Equate the corresponding e

$$\begin{aligned} x + 3 &= 6 \\ x &= 3 \\ y &= -1 \\ z - 3 &= 4 \\ z &= 7 \end{aligned}$$

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People in the workforce often use computer spr
data. Similar to a matrix, data in a spreadsheet
data can be used to create graphs or perform ca

Example 2 Organize Data in a Spreadsheet

The manager of a gourmet food store has g
of bulk coffees they have sold each week in
spreadsheet.

	Weekly Sales
Coffee	5/
Hawaiian Kona	17
Mocha Java	31
House Blend	55
Espresso	41
Decaf Espresso	23
Breakfast Blend	8
Decaf Breakfast Blend	22
Organic Italian Roast	26

Use Column A for the types of coffee, Colum
Column C for sales in the week starting 12/1, a
weeks starting 19/1 and 26/1.

January Weekly Sales

	A	B	C
1	Hawaiian Kona	17	22
2	Mocha Java	31	34
3	House Blend	55	61
4	Espresso	41	36
5	Decaf Espresso	23	29
6	Breakfast Blend	8	18
7	Decaf Breakfast Blend	22	18
8	Organic Italian Roast	26	16

Sheet 1 Sheet 2 Sheet 3

Exercises

1. State the dimensions of matrix A if

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 12 & 3 & 4 & 1 \\ 13 & 14 & 1 & 2 \end{bmatrix}$$?

a. 2×3 b. 3×2 c. 3×4

2. What is the a_{23} entry in matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 12 & 3 & 4 & 1 \\ 13 & 14 & 1 & 2 \end{bmatrix}$$?

a. 3 b. 4 c. 12

State the dimensions of each matrix.

3. $\begin{bmatrix} 6 & -1 & 5 \\ -2 & 3 & -4 \end{bmatrix}$

4. $\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$

6. $\begin{bmatrix} -3 & 17 & -22 \\ 9 & 31 & 16 \\ 20 & -15 & 4 \end{bmatrix}$

7. $\begin{bmatrix} 17 & -2 & 8 & -9 \\ 5 & 11 & 20 & -1 \end{bmatrix}$

9. Write an example for each type of the following Matrice
   a. Row matrix
   b. Column matrix
   c. Square matrix (4 x 4)

10. Analyze data in Matrices

The school distance coach summarized the number o
year
in the table.

a. Organize the data into a matrix.

b. Add the elements of each row and interpret the re

c. Add the elements of each column and interpret th

11. A charity sold tickets for a 3-day fund-raising event. Tic
individually and in gift packs, as shown in the table belo

A ticket sold for AED 3.00 and a gift-pack for AED 6.00
to determine the total revenue for the three days.

12. Find the value of a, b, and c if the following two matrice

$$\begin{bmatrix} a+3 & -1 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 6 & b \\ c-3 & 5 \end{bmatrix}$$

13. Use a matrix to organize and compare the following info
about some roller
coasters.

Organize the information in a matrix.

HOMEWORK HELP

For Exercises	See Examples
7–8	1
9–14	2
15–20	3

Roller Coaster	A
Speed (mph)	55
Height (feet)	90
Length (feet)	2300

14.

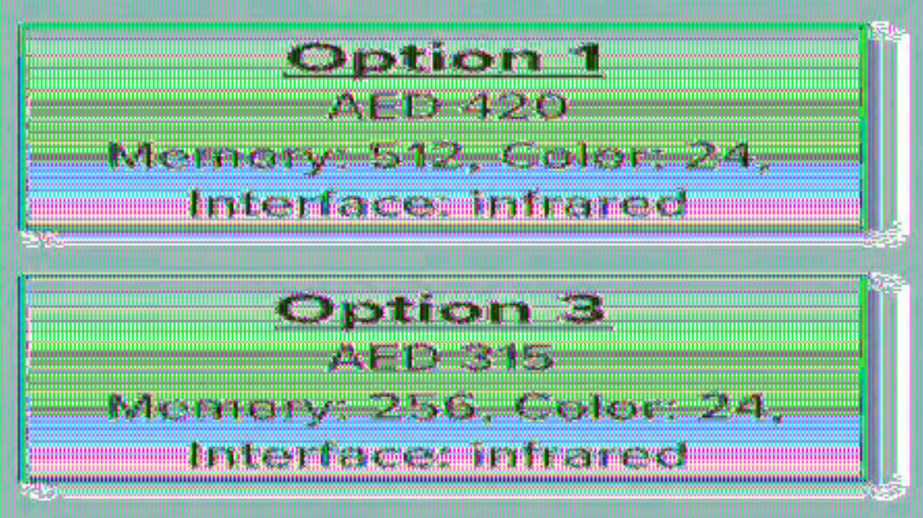
Ocean	Area (mi^2)	Ave Dep
Pacific	60,060,700	13,
Atlantic	29,637,900	12,
Indian	26,469,500	13,
Southern	7,848,300	16,
Arctic	5,427,000	3,

15.

Top Hockey Goalies			
Goalie	Games	Wins	Lo
Khaled	1029	551	3
Sultan	971	447	3
Saif	837	435	2
Humaid	886	423	3
Hani	906	407	3

16. Jamila is shopping for a new sma
options are available.

a. Organize the data in a mat
and label the columns opti
respectively.



b. What are the dimensions of
of mat?

Solve each equation.

17. $[4x \quad 3y] = [12 \quad -1]$

18. $[2x \quad 3 \quad 3z] = [5 \quad 3y \quad 9]$

19. $\begin{bmatrix} 4x \\ 5 \end{bmatrix} = \begin{bmatrix} 15 + x \\ 2y - 1 \end{bmatrix}$

20. $\begin{bmatrix} x + 3y \\ 3x + y \end{bmatrix} = \begin{bmatrix} -13 \\ 1 \end{bmatrix}$

21. $\begin{bmatrix} 2x + y \\ x - 3y \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \end{bmatrix}$

22. $\begin{bmatrix} 4x - 3 & 3y \\ 7 & 13 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix}$ 2z

23. **ACT/SAT** The results of a rec

Propos
Propos
Propos

Based on these results, wh
a. There were 771 votes cas
b. More people voted again
c. Proposition 2 has little ch
d. More people voted for Pr

24. **REVIEW** The chart shows an
different values of x.

A student concludes that f
produces a prime number
a counterexample to prove

f. -4 **h.** -2
g. -3 **j.** 4

25. ACTIVITY A SUM formula in a spreadsheet allows you to find the sum of the entries in a column or row.

a. The formula =SUM(B1:B8) finds the sum of column B. Enter formulas in cells B9, C9, D9, and E9 to find the sums of those columns. What do the sums of the columns represent in the situation?

b. Enter formulas in cells F1 through F8 to find the sums of rows 1 through 8. What do the sums of the rows represent in the situation?

c. Find the sum of row 9 and the sum of column F. What do you observe? Explain.

A. Enter the data on smartphones from Exercise 1 into a spreadsheet.

B. Compare and contrast how data are organized in a spreadsheet and in a matrix.

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Operations

Then | **Now** | **Why?**

You organized data into matrices.

1. Analyze data in matrices.
2. Perform algebraic operations with matrices.

Coastal Sal
the averag

Entry
Assist
Assoc

New Vocabulary
scalar
scalar multiplication

Analyze Data Data that
Sometimes, further analys

Real-World Example 1

BUSINESS The manager at
would like to use their mat
the representatives.

a. Add the elements in eac

The sum of the first colu
the three types of emplo
average annual sales of

b. The manager wants to de
Dubai location. He decid
of different positions. W

The average is 6000 ÷ 3

c. Is this an accurate avera

If there is the same num
If there is more of one ty
accurate and would nee

d. Would adding the rows

No. The sum of a row in

Guided Practice

1. **POPULATION** The table
   A. Organize the data in
   B. Add the elements in
   the results.
   C. Add the elements in
   D. Would finding the av
   provide any meaning

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2 Algebraic Operations Seven
organized in matrices. Matrices
same dimensions.

KeyConcept Adding and Sub

Words	To add or subtract two m corresponding element
Symbols	$A + B$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ $A - B$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix}$
Example	$\begin{bmatrix} 3 & -5 \\ 1 & 7 \end{bmatrix} + \begin{bmatrix} 2 & \\ -9 & \end{bmatrix}$

StudyTip

Corresponding Elements Elements are *corresponding* if they are in the exact same position in each matrix.

Example 2 **Add and Subtract Ma**

Find each of the following for A

a. $A + B$

$$A + B = \begin{bmatrix} 16 & 2 \\ -9 & 8 \end{bmatrix} + \begin{bmatrix} -4 & \\ -3 & \end{bmatrix}$$

$$= \begin{bmatrix} 16 + (-4) & 2 + (\\ -9 + (-3) & 8 + (\end{bmatrix}$$

$$= \begin{bmatrix} 12 & 1 \\ -12 & 1 \end{bmatrix}$$

b. $B - C$

$$B - C = \begin{bmatrix} -4 & -1 \\ -3 & -7 \end{bmatrix} - \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

Since the dimensions of B and

c. $B - A$

$$B - A = \begin{bmatrix} -4 & -1 \\ -3 & -7 \end{bmatrix} - \begin{bmatrix} 16 & \\ -9 & \end{bmatrix}$$

$$= \begin{bmatrix} -4 - 16 & -1 - \\ -3 - (-9) & -7 - \end{bmatrix}$$

$$= \begin{bmatrix} -20 & -3 \\ 6 & -15 \end{bmatrix}$$

Guided Practice

2A. $\begin{bmatrix} -3 & 4 \\ -9 & -5 \end{bmatrix} - \begin{bmatrix} -4 & 12 \\ 8 & -7 \end{bmatrix}$

2C. $\begin{bmatrix} 8 & -3 \\ -2 & 0 \\ 1 & 7 \end{bmatrix} - \begin{bmatrix} 5 & 1 \\ 10 & -6 \end{bmatrix}$

You can multiply any matrix b
each individual element by th
scalar multiplication.

Reading Math

Scalar Think of a scalar as a coefficient for a variable, but instead it is for a matrix.

Key Concept **Multiplying**

Words To multiply a mat

$k \cdot A$ —

Symbols $k\begin{bmatrix} a & b \\ c & d \end{bmatrix} =$ [

Example $-3\begin{bmatrix} 4 & 1 \\ 7 & -2 \end{bmatrix}$

Example 2 **Multiply a Matr**

If $R = \begin{bmatrix} -12 & 8 & 6 \\ -16 & 4 & 19 \end{bmatrix}$, fi

$5R = 5\begin{bmatrix} -12 & 8 & 6 \\ -16 & 4 & 19 \end{bmatrix}$

$= \begin{bmatrix} 5(-12) & 5(8) & 5 \\ 5(-16) & 5(4) & 5(\end{bmatrix}$

$= \begin{bmatrix} -60 & 40 & 30 \\ -80 & 20 & 95 \end{bmatrix}$

Guided Practice

3. If $T = \begin{bmatrix} 8 & 0 & 3 \\ -1 & -4 & -2 \end{bmatrix}$

Study Tip

Scalar Multiplication The bracket of a matrix is treated just like a regular grouping symbol. So when multiplying by a scalar, distribute the same way as with a grouping symbol.

Many properties of real numbe
properties is listed below.

Key Concept **Properties**

For any matrices A, B, and C for w
the following properties are true.

Commutative Property of Addition

Associative Property of Addition

Left Scalar Distributive Property

Right Scalar Distributive Property

Multi-step operations can be p
same as with real numbers.



Real-World Career

Financial Planner

Financial planners often use matrices to organize and describe the data they use. Financial planners need a bachelor's degree. Usually they have degrees in accounting, finance, economics, business, marketing, or commerce.

Study Tip

Corresponding Elements

When representing quantities with multiple matrices, make sure the corresponding elements represent corresponding quantities.

Example 4 **Multi-Step Operation**

If $A = \begin{bmatrix} -9 & 12 \\ 2 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -$

$-4B - 3A = -4\begin{bmatrix} -4 & -8 \\ 2 & -3 \end{bmatrix} - 3$

$= \begin{bmatrix} -4(-4) & -4(-8) \\ -4(2) & -4(-3) \end{bmatrix}$

$= \begin{bmatrix} 16 & 32 \\ -8 & 12 \end{bmatrix} - \begin{bmatrix} -2 \end{bmatrix}$

$= \begin{bmatrix} 16 - (-27) & 32 \\ -8 - 6 & 12 - \end{bmatrix}$

$= \begin{bmatrix} 43 & -4 \\ -14 & 30 \end{bmatrix}$

Guided Practice

4. If $A = \begin{bmatrix} -5 & 3 \\ 6 & -8 \\ 2 & 9 \end{bmatrix}$ and $B = \begin{bmatrix}$

Matrices can be used in many busin

Real-World Example 5 **Use**

BUSINESS **Refer to the applicatio average wages and sales for the**

To calculate the 5-day sales for th by 5 and the totals added togethe

$$5\begin{bmatrix} 900 & 145,000 \\ 2400 & 225,000 \\ 2700 & 290,000 \end{bmatrix} + 5\begin{bmatrix} 900 \\ 1800 \\ 1800 \end{bmatrix}$$

$$= \begin{bmatrix} 4500 & 725,000 \\ 12,000 & 1,125,000 \\ 13,500 & 1,450,000 \end{bmatrix} + \begin{bmatrix} 450 \\ 900 \\ 900 \end{bmatrix}$$

$$\begin{array}{r} \\ \text{Entry} \\ \text{Assistant} \\ \text{Associate} \end{array} \begin{array}{c} \begin{array}{cc} \text{Wages} & \text{Sales} \end{array} \\ \begin{bmatrix} 14,250 & 1,882,50 \\ 30,000 & 2,527,50 \\ 31,500 & 3,002,50 \end{bmatrix} \end{array}$$

The final matrix indicates the ave representatives of the company.

Guided Practice

5. Use the data above to calculat assuming 260 working days.

Check Your Understanding

Example 1

1. **MODELING** Use the table that shows th
different types of vehicles.

Vehicle	SUV	
City	23	
Highway	25	

Source: Auto Hoppers

a. Organize the gas mileages in a matri
b. Add the elements of each row and i
c. Add the elements of each column ar

Example 2

Perform the indicated operations. If the ma

2. $\begin{bmatrix} -8 & 2 & 6 \end{bmatrix} + \begin{bmatrix} 11 & -7 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 7 & -12 \\ 15 & 4 \end{bmatrix} - \begin{bmatrix} 9 & 6 \\ 4 & -9 \end{bmatrix}$ 5.

Example 3

Perform the indicated operations. If the ma

6. $3\begin{bmatrix} 6 & 4 & 0 \\ -2 & 14 & -8 \\ -4 & -6 & 7 \end{bmatrix}$

Example 4

Use matrices *A*, *B*, *C*, and *D* to find the foll

$A = \begin{bmatrix} 6 & -4 \\ 3 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 8 & -1 \\ -2 & 7 \end{bmatrix}$

8. $4B - 2A$ 9. $-8C + 3A$

10. $-5B - 2D$ 11. $-4C - 5B$

Example 5

12. **GRADES** Sultan, Tariq, and Samy have
the test grades for each student.

a. Write a matrix for the information fi
b. Find the sum of the scores from the
as a matrix.
c. Express the difference in scores from

Practice and Problem Solving

Example 1

13. **SHOES** A consumer service company r
look, and longevity using a scale of 1–5

a. Write a 4 × 4 matrix to organize this
b. Which shoe would you buy based on
information, and why?
c. Would finding the sum of the rows
provide any useful information? Exp
reasoning.

Example 2

Perform the indicated operations. If the ma

14. $\begin{bmatrix} 12 & -5 \\ -8 & -3 \end{bmatrix} + \begin{bmatrix} -6 & 11 \\ -7 & 2 \end{bmatrix}$ 15.

Examples 3–4

16. **BUSINESS** The drink menu from a fast-food restaurant is shown at the right. The store owner has decided that all of the prices must be increased by 10%.

a. Write matrix C to represent the current prices.

b. What scalar can be used to determine a n

c. Find N.

d. What is $N - C$? What does this represent

Use matrices A, B, C, and D to find the following

$$A = \begin{bmatrix} 0 & -7 \\ 8 & 12 \end{bmatrix} \qquad B = \begin{bmatrix} 11 & 4 \\ -3 & -17 \end{bmatrix}$$

17. $-3B + 2A$ 18. $9C - 4D$

19. $2C + 11A$ 20. $7A - 2B$

Example 5

21. **MODELING** Library A has 10,000 novels, 500 Library B has 15,000 novels, 10,000 biograph 4000 novels, 700 biographies, and 800 childr

a. Express each library's number of books as

b. Find the total number of each type of boo

c. How many more books of each type does

d. Find $A + B$. Does the matrix have meanin

Perform the indicated operations. If the matrix d

22. $-2\begin{bmatrix} -9.2 & -8.4 \\ 5.6 & -4.2 \end{bmatrix} - 4\begin{bmatrix} 4.1 & -2.9 \\ 7.2 & -8.2 \end{bmatrix}$ 23. —

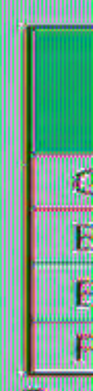
24. $-3\begin{bmatrix} 18 & -6 & -8 \\ -5 & -3 & 12 \\ 0 & 3x & -y \end{bmatrix}$ 25. $8\begin{bmatrix}$

26. $-4\begin{bmatrix} -7 \\ 4 \\ -3 \end{bmatrix} + 3\begin{bmatrix} -8 \\ 3x \\ -9 \end{bmatrix} - 5\begin{bmatrix} 4 \\ x - 6 \\ 12 \end{bmatrix}$ 27. —

28. **WEATHER** The table shows snowfall in inches.

a. Express the normal snowfall data and the 2007 data in two 4×3 matrices.

b. Subtract the matrix of normal data from the matrix of 2007 data. What does the difference represent in the context of the situation?



c. Explain the meaning of positive and nega trends do you see in the data?

29. **MODELING** The table shows some of th
swimming records.

Distance (m)	Wo
50	24.
100	53.9
200	1:56.5
800	8:16.2

Source: USA Swimming

a. Find the difference between the Am
as a matrix.

b. What is the meaning of each row in

c. In which events were the fastest tim

30. **MULTIPLE REPRESENTATIONS** In this
represent transformations.

a. **Algebraic** The matrix $\begin{bmatrix} -3 & -4 \\ 8 & 6 \end{bmatrix}$
represents a triangle with vertices at
$(-3, 8)$, $(-4, 6)$, and $(1, 0)$. Write a m
to represent $\triangle ABC$.

b. **Geometric** Multiply the matrix you w
Then graph the figure represented b
new matrix.

c. **Analytical** How do the figures compa
a conjecture about the result of mult
matrix by 0.5. Verify your conjecture

Higher-Order Problems — Use Higher-Order Thinking Skills

31. **PROOF** Prove that matrix addition is co

32. **PROOF** Prove that matrix addition is as

33. **CHALLENGE** Find the elements of C if:
$A = \begin{bmatrix} -3 & -4 \\ 8 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -1 \\ 2 & -4 \end{bmatrix}$, and

34. **REASONING** Determine whether each s
matrices A and B. Explain your reasoni

a. If $A + B$ exists, then $A - B$ exists.

b. If k is a real number, then kA and kB

c. If $A - B$ does not exist, then $B - A$ c

d. If A and B have the same number of

e. If kA exists and kB exists, then $kA +$

35. **OPEN ENDED** Give an example of matri

36. **WRITING IN MATH** Explain how to find
the same dimensions.

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Standardized Test Practice

37. What is the solution of the system of equations?

$$0.06p + 4q = 0.88$$
$$p - q = -2.25$$

A $(-0.912, -1.338)$
B $(0.912, -3.162)$
C $(-2, 0.25)$
D $(-2, -4.25)$

38. SHORT RESPONSE Find $A + B$ if $A = \begin{bmatrix} -7 & 3 \\ 2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ 0 & 1 \end{bmatrix}$.

Spiral Review

Solve each system of equations. (Lesson 1-4)

41. $2x + 3y - z = -1$
$5x + y + 4z = 30$
$-8x - 2y + 5z = -2$

42. $3x - 4y + 6z = 26$
$5x + 3y + 2z = 5$
$-2x + 5y - 3z = -$

44. PACKAGING The Cookie Factory sells chocolate chip and pea in combination packages that contain between six and twelv three of each type of cookie should be in each package. How of cookie should be in each package to maximize the profit?

Solve each system of inequalities by graphing. (Lesson 1-2)

45. $x - 2y \geq -4$
$y < -2x - 3$

46. $y \geq -4x + 6$

47. $4x + 2y > 8$
$3y < 2x + 9$

48. RAKING LEAVES A student can earn AED 20 plus an extra AE for each trash bag he or she completely fills with leaves. Write and solve an equation to determine how many bags the student will need to fill in order to earn AED 100.

49. SPORTS There are 15,991 more student athletes in New York than in Illinois. Write and solve an equation to find the number of student athletes in Illinois.

Skills Review

Simplify each expression.

50. $4(2x - 3y) + 2(5x - 6y)$

51. $-3(2a - 5b) - 4(4$

Multiplying M

Then

You multiplied matrices by a scalar.

Now

1. Multiply matrices.
2. Use the properties of matrix multiplication.

Why?

The table sh
summary fo
WNBA's all-
leader, duri
seasons. He
summarized
in the baske
values for e
basket mad
in the point

You can use

$$B = \begin{bmatrix} 19 \\ \\ 102 \end{bmatrix}$$

1 **Multiply Matrices** You c
of columns in A is equal to
$A_{m \times r}$ and $B_{r \times r}$ the resulting

$$A \times r$$

Example 1 Dimensions of

**Determine whether each m
the product.**

a. $A_{3 \times 4}$ and $B_{4 \times 2}$

$$A \cdot B = A$$

$$\times 4 \qquad 4 \times \qquad \times$$

The inner dimensions a

b. $A_{5 \times 3}$ and $B_{5 \times 4}$

$$A \cdot B$$

$$\times 3 \qquad 5 \times$$

The inner dimensions a

Guided Practice

1A. $A_{4 \times 6}$ and $B_{6 \times 2}$

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The product of two matrices is found

KeyConcept Multiplying Matr

Words The element in the *m*th
the corresponding elem

Symbols

$$\underset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \cdot \underset{B}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}$$

Example 2 Multiply Square Mat

Find XY if $X = \begin{bmatrix} 6 & -3 \\ -10 & -2 \end{bmatrix}$ and

$$XY = \begin{bmatrix} 6 & -3 \\ -10 & -2 \end{bmatrix} \cdot \begin{bmatrix} -5 & -4 \\ 3 & 3 \end{bmatrix}$$

Step 1 Multiply the numbers in
column of Y, add the pro
column of XY.

$$\begin{bmatrix} 6 & -3 \\ -10 & -2 \end{bmatrix} \cdot \begin{bmatrix} -5 & \\ 3 & \end{bmatrix}$$

Step 2 Follow the same procedu
numbers. Write the resu

$$\begin{bmatrix} 6 & -3 \\ -10 & -2 \end{bmatrix} \cdot \begin{bmatrix} -5 & \\ 3 & \end{bmatrix}$$

Step 3 Follow the same procedu
Write the result in the se

$$\begin{bmatrix} 6 & -3 \\ -10 & -2 \end{bmatrix} \cdot \begin{bmatrix} -5 & \\ 3 & \end{bmatrix}$$

Step 4 The procedure is the sam
column.

$$\begin{bmatrix} 6 & -3 \\ -10 & -2 \end{bmatrix} \cdot \begin{bmatrix} -5 & \\ 3 & \end{bmatrix}$$

Step 5 Simplify the product ma

$$\begin{bmatrix} 6(-5) + (-3)(3) & \\ -10(-5) + (-2)(3) & \end{bmatrix}$$

Guided Practice

2. Find UV if $U = \begin{bmatrix} 5 & 9 \\ -3 & -2 \end{bmatrix}$ a

WatchOut!

Losing Your Place It is easy to lose your place as you multiply matrices. It may help to cover rows or columns not being multiplied as you find elements of the product matrix.



Real-World Link

Swim meets consist of racing and diving competitions. There are more than 241,000 high schools that participate each year.

Source: National Federation of State High School Associations

Real-World Example

SWIM MEET At a particular
4 points for second, and 2
school. Which school won

Understand The final score
each school by
third-place fin

Plan Write the resu
Set up the mat
equals the num

Resul

$$R = \begin{bmatrix} 4 & 7 \\ 8 & 9 \\ 10 & 5 \\ 3 & 3 \end{bmatrix}$$

Solve Multiply the m

$$RP = \begin{bmatrix} 4 & 7 \\ 8 & 9 \\ 10 & 5 \\ 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4(7) \\ 8(7) \\ 10(7) \\ 3(7) \end{bmatrix}$$

$$= \begin{bmatrix} 62 \\ 94 \\ 96 \\ 45 \end{bmatrix}$$

The product m
Sharjah Model
meet with a to

Check R is a 4×3 ma
a 4×1 matrix

Guided Practice

3. **BASKETBALL** Refer to the
determine in which seas
she score that season?

Example 4 **Test of the Commuta**

Find each product if $G = \begin{bmatrix} 1 & \\ 4 & \end{bmatrix}$ —

a. GH

$$GH = \begin{bmatrix} 1 & 3 & -5 \\ 4 & -2 & 6 \end{bmatrix} \cdot \begin{bmatrix} & \\ -2 & \\ 1 & \end{bmatrix}$$

$$= \begin{bmatrix} 2 - 6 - 5 & 3 - 24 - \\ 8 + 4 + 0 & 12 + 16 \end{bmatrix}$$

b. HG

$$HG = \begin{bmatrix} 2 & 3 \\ -2 & -8 \\ 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 12 & 6 - 6 \\ -2 - 32 & -6 + 16 \\ 1 + 28 & 3 - 14 \end{bmatrix}$$

Guided Practice

4. Determine if $AB = BA$ is true

Example 4 demonstrates that the Co
for matrix multiplication. The order

Example 5 **Test of the Distributi**

Find each product if $J = \begin{bmatrix} 2 & \\ -5 & \end{bmatrix}$ —

a. $J(K + L)$

$$J(K + L) = \begin{bmatrix} 2 & 4 \\ -5 & -2 \end{bmatrix} \cdot \left(\begin{bmatrix} & \\ & \end{bmatrix} - \right.$$

$$= \begin{bmatrix} 2 & 4 \\ -5 & -2 \end{bmatrix} \cdot \begin{bmatrix} & \\ & \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 8 & 2 + 1 \\ 5 - 4 & -5 - \end{bmatrix}$$

b. $JK + JL$

$$JK + JL = \begin{bmatrix} 2 & 4 \\ -5 & -2 \end{bmatrix} \cdot \begin{bmatrix} 3 & \\ -1 & \end{bmatrix}$$

$$= \begin{bmatrix} 2(3) + 4(-1) & \\ -5(3) + (-2)(-1) & -5(2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 16 \\ -13 & -16 \end{bmatrix} + \begin{bmatrix} 4 & \\ 14 & \end{bmatrix}$$

Guided Practice

5. Use the matrices $R = \begin{bmatrix} 2 & \\ 1 & \end{bmatrix}$ —
if $(S + T)R = SR + TR$.

Study Tip

Proof and Counterexamples

To show that a property is *not* always true, you need to find only one counterexample.

The previous example suggest
Some properties of matrix mu

Key Concept Properties of

For any matrices A, B, and C for w
properties are true.

Associative Property of Matrix Mu
Associative Property of Scalar Mu
Left Distributive Property
Right Distributive Property

Check Your Understanding

Example 1 **Determine whether each matrix product is
product.**

1. $A_{2 \times 4} \cdot B_{4 \times 3}$ 2. $C_{5 \times 4} \cdot$

Examples 2–3 **Find each product, if possible.**

4. $\begin{bmatrix} 2 & 1 \\ 7 & -5 \end{bmatrix} \cdot \begin{bmatrix} -6 & 3 \\ -2 & -4 \end{bmatrix}$

6. $\begin{bmatrix} 9 & -2 \end{bmatrix} \cdot \begin{bmatrix} -2 & 4 \\ 6 & -7 \end{bmatrix}$ 7.

8. $\begin{bmatrix} -8 & 7 & 4 \\ -5 & -3 & 8 \end{bmatrix} \cdot \begin{bmatrix} 10 & 6 \\ 8 & 4 \end{bmatrix}$

10. $\begin{bmatrix} -4 & 3 & 2 \\ -1 & -5 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 6 \\ 8 & 4 & -1 \\ 5 & 3 & -2 \end{bmatrix}$ 11.

12. **SENSE-MAKING** The table shows
the number of people registered for
aerobics for the first quarter.

The Gym charges the following
registration fees: class-by-class, AED 16
11-class pass, AED 110; unlimited pass,

a. Write a matrix for the registration fe

b. Find the total amount of money the
registrations.

Examples 4–5 Use $X = \begin{bmatrix} -10 & -3 \\ 2 & -8 \end{bmatrix}$, $Y = \begin{bmatrix} -5 & 6 \\ -1 & 9 \end{bmatrix}$, an

following equations are true for the given

13. $XY = YX$

Practice and Problem Solving

Example 1 **Determine whether each matrix product is defin the product.**

15. $P_{2 \times 3} \cdot Q_{3 \times 4}$

16. $A_{5 \times 5}$ ·

18. $X_{2 \times 6} \cdot Y_{6 \times 3}$

19. $J_{2 \times 1}$ ·

Examples 2–3 **Find each product, if possible.**

21. $\begin{bmatrix} 1 & 6 \end{bmatrix} \cdot \begin{bmatrix} -10 \\ 6 \end{bmatrix}$

22.

23. $\begin{bmatrix} -3 & -7 \\ -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 4 \\ 9 & -3 \end{bmatrix}$

24.

25. $\begin{bmatrix} -1 & 0 & 6 \\ -4 & -10 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & -7 \\ -2 & -9 \end{bmatrix}$

26.

27. $\begin{bmatrix} 2 & 9 & -3 \\ 4 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 & 2 \\ -6 & 7 \\ -2 & 1 \end{bmatrix}$

28.

29. **TRAVEL** Zayed's family owns three bed and b with a single bed is AED 220 per night, a roo suite is AED 360.

a. Write a matrix for the number of each type of room at each bed and breakfast. Then write a room-cost matrix.

b. Write a matrix for total daily income, assuming that all the rooms are rented.

c. What is the total daily income from all thr rooms are rented?

Examples 4–5 **Use** $P = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$, $Q = \begin{bmatrix} 6 & 4 \\ -2 & -5 \end{bmatrix}$, $R = \begin{bmatrix} -$

the following equations are true for the given ma

30. $k(PQ) = P(kQ)$

31. $PQR = RQP$

32. $PR + QR = (P + Q)R$

33. $R(P + Q) = PR +$

34. **SENSE-MAKING** Student Council is selling flowers for Mother's Day. They bought 200 r 150 daffodils, and 100 orchids for the purcha prices shown. They sold all of the flowers for the sales prices shown.

a. Organize the data in two matrices, and us matrix multiplication to find the total amo that was spent on the flowers.

b. Write two matrices, and use matrix multip student council received for the flower sal

c. Use matrix operations to find how much their project.

35. **AUTO SALES** A car lot has four sales ass
At the end of the year, each sales assoc
a bonus of AED 1000 for every new car
sold and AED 500 for every used car th
a. Use a matrix to determine which sal
associate earned the most money.
b. What is the total amount of money t
lot spent on bonuses for the sales as
this year?

Use matrices $X = \begin{bmatrix} 2 & -6 \\ 3y & -4.5 \end{bmatrix}$, $Y = \begin{bmatrix} x + \end{bmatrix}$

the following. If the matrix does not exist,

36. XY 37. YX 38. ZY
40. $(YX)Z$ 41. $(XZ)X$ 42. $X(ZZ)$

44. **CAMERAS** Prices of digital cameras depend on features like optical zoom, digital zoom, and megapixels.
a. The 10-mp cameras are on sale for 20% off, and the other models are 10% off. Write a cost matrix.
b. Write a new matrix allowing for a 6.
c. Describe what the differences in the

45. **BUSINESS** The Photography Studio has available for senior portraits.
a. Use matrices to determine the total of each package.
b. The studio offers an early bird discou of 15% off any package. Find the ear bird price for each package.

H.O.T. Problems Use Higher-Order Thinking Skills

46. **REASONING** If the product matrix AB h 5×6, what are the dimensions of matr

47. **ARGUMENTS** Show that each property
a. Scalar Distributive Property
b. Matrix Distributive Property
c. Associative Property of Multiplicatio
d. Associative Property of Scalar Multip

48. **OPEN ENDED** Write two matrices A and

49. **CHALLENGE** Find the missing values in

50. **WRITING IN MATH** Use the data on Lisa to explain how matrices can be used in represents the total number of points s of a sport in which different point valu

Standardized Test Practice

51. GRIDDED RESPONSE The average (arithmetic mean) of r, w, x, and y is 8, and the average of x and y is 4. What is the average of r and w?

52. Rana, Suha, and Alia went shopping to get ready for college. Their purchases and total amounts spent are shown in the table below.

Person	Shirts	Pants	Shoes	Total Spent
Rana	3	4	2	AED 149.79
Suha	5	3	3	AED 183.19
Alia	6	5	1	AED 181.14

Assume that all of the shirts were the same price, all of the pants were the same price, and all of the shoes were the same price. What was the price of each item?

A shirt, AED 12.95; pants, AED 15.99; shoes, AED 23.49

B shirt, AED 15.99; pants, AED 12.95; shoes, AED 23.49

C shirt, AED 15.99; pants, AED 23.49; shoes, AED 12.95

D shirt, AED 23.49; pants, AED 15.99; shoes, AED 12.95

Spiral Review

Perform the indicated operations. If the matrix does not exist, w

55. $4\begin{bmatrix} 8 & -1 \\ -3 & -4 \end{bmatrix} - 5\begin{bmatrix} -2 & 4 \\ 6 & 3 \end{bmatrix}$

56. $5\left(2\begin{bmatrix} -2 & -5 \\ -1 & 3 \end{bmatrix} - 3\right.$

Solve each system of equations. (Lesson 1-6)

58. $2x - 4y + 3z = -3$
$-7x + 5y - 4z = 11$
$x - y - 2z = -21$

59. $-4x - 2y + 9z = -29$
$10x - 12y + 7z = 5$
$3x + 5y - 14z = 25$

60. $-7x +$

61. MEDICINE The graph shows how much Americans spent on doctors' visits in some recent years and a prediction for 2014.

a. Find a regression equation for the data without the predicted value.

b. Use your equation to predict the expenditures for 2014.

c. Compare your prediction to the one given in the graph.

62. How many different ways can the letters of the word *SAMPLING* be arranged?

Skills Review

Describe the transformation in each function. Then graph the f

63. $f(x) = |x - 4| + 3$

64. $f(x) = 2$

$|x + 3| - 5$

Graphing Technology Operations

A graphing calculator can be used to perform operations with matri

Activity 1 Perform Operations

Use A, B, and C to find the following.

$$A = \begin{bmatrix} -3.2 & 1.7 \\ 0.4 & -5.8 \end{bmatrix} \qquad B = \begin{bmatrix} 4.9 & 0.3 \\ -7.1 & 2.6 \end{bmatrix}$$

a. $3A + 2B$

Begin by entering matrix A into a graphing calcula

KEYSTROKES: 2nd [MATRIX] ▶ ▶ ENTER 2 ENTER ENTER (−) 3.2 ENTER 1.7 ENTER 0.4 ENTER (−) 5.8 ENTER 2nd [QUIT]

Enter matrix B into the graphing calculator using s keystrokes. Then, perform the indicated operation

KEYSTROKES: 3 2nd [MATRIX] ENTER + 2 2nd [MATR ENTER ENTER

$3A + 2B$ is equal to $\begin{bmatrix} 0.2 & 5.7 \\ -13 & -12.2 \end{bmatrix}$.

b. $4C + 3A$

Enter matrix C into the graphing calculator. Perfor indicated operations. Notice that the calculator di an error message when the dimensions do not allo operations to be performed.

Exercises

Use A, B, C, and D to find the following. If the matrix doe

$$A = \begin{bmatrix} 4.5 & -9.0 \\ -7.4 & 9.4 \end{bmatrix} \qquad B = \begin{bmatrix} 1.9 & -5.9 \\ 2.9 & 5.0 \end{bmatrix} \qquad C =$$

1. $CD + 4A$ 2. $-2B + 7A$ 3. $4(DC)$

4. $6B + DC$ 5. $2(AB) - 3B + 5A$ 6. $-3(CD)$

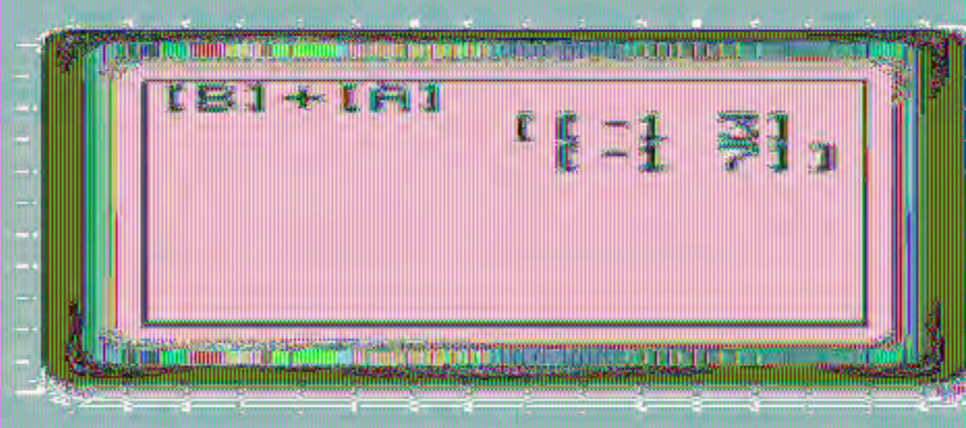
Activity 2 Explore Properties of Matrix Operations

Use A, B, and C to find the following.

$$A = \begin{bmatrix} -6 & 2 \\ 7 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 1 \\ -8 & 4 \end{bmatrix}$$

a. $A + B$ and $B + A$

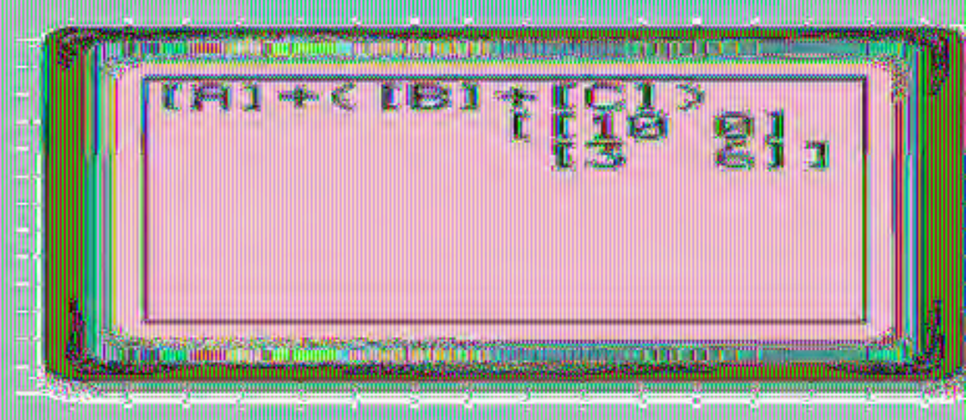




$A + B$ and $B + A$ are both equal to $\begin{bmatrix} -1 & 3 \\ -1 & 7 \end{bmatrix}$.

b. $(A + B) + C$ and $A + (B + C)$





$(A + B) + C$ and $A + (B + C)$ are both equal to $\begin{bmatrix} 10 & 0 \\ 3 & 6 \end{bmatrix}$.

Analyze the Results

What property is illustrated in each part of Activity 2?

7. part a
8. part b

Find each set of products. How are they similar or different? What

9. $(AB)C$, $A(BC)$
10. $3(AB)$, $(3A)B$, $A(3B)$
11. The zero matrix O is an $m \times n$ matrix with all elements equa verify the Additive Identity Property for Matrices, $A + O =$
12. Two matrices are additive inverses if their sum is the zero m that $A + E$ are additive inverses. Then verify that $A + E = O$
13. Find the additive inverses for matrix B and matrix C.
14. **CHALLENGE** What observations can be made about a matrix the additive inverse for $\begin{bmatrix} w & -x \\ -y & z \end{bmatrix}$.

Solving Systems Using Cramer's Rule

Then | **Now** | **Why?**

You solved systems of equations algebraically.

1. Evaluate determinants.
2. Solve systems of linear equations by using Cramer's Rule.

A zoologist a GPS track determine After severa determined was a triang the coordin this triangle and determ the size of t

New Vocabulary
determinant
second-order determinant
third-order determinant
diagonal rule
Cramer's Rule
coefficient matrix

Mathematical Practices
Look for and make use of structure.

1 Determinants

Every squ
2 × 2 matrix is called a **sec**

Key Concept Second-Or

Words The value of a se
of the products o

Symbols $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} =$

Example $\begin{vmatrix} 4 & 5 \\ -3 & 6 \end{vmatrix} = 4($

Example 1 Second-Order

Evaluate each determinant.

a. $\begin{vmatrix} 5 & -4 \\ 8 & 9 \end{vmatrix}$

$\begin{vmatrix} 5 & -4 \\ 8 & 9 \end{vmatrix} = 5(9) - (-4$
$= 45 + 32$
$= 77$

b. $\begin{vmatrix} 0 & 6 \\ 4 & -11 \end{vmatrix}$

$\begin{vmatrix} 0 & 6 \\ 4 & -11 \end{vmatrix} = 0(-11) -$
$= 0 - 24$
$= -24$

Guided Practice

1A. $\begin{vmatrix} -6 & -7 \\ 10 & 8 \end{vmatrix}$

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Study Tip

Diagonal Rule The diagonal rule can only be used for 3 × 3 matrices.

Determinants of 3 × 3 matrices are evaluated by using the **diagonal rule**.

Key Concept **Diagonal Rule**

Step 1 Rewrite the first two columns

Step 2 Draw diagonals, beginning wi elements in each diagonal. Re right-hand element.

Step 3 Find the sum of the products

Step 4 Subtract the second sum from

Example 2 **Use Diagonals**

Evaluate $\begin{vmatrix} 4 & -8 & 3 \\ -3 & 2 & 6 \\ -4 & 5 & 9 \end{vmatrix}$ using

Step 1 Rewrite the first two co

$$\begin{vmatrix} 4 & -8 & 3 \\ -3 & 2 & 6 \\ -4 & 5 & 9 \end{vmatrix} \begin{matrix} 4 \\ -3 \\ -4 \end{matrix}$$

Step 2 Find the products of th

$$\begin{vmatrix} 4 & -8 & 3 \\ -3 & 2 & 6 \\ -4 & 5 & 9 \end{vmatrix} \begin{matrix} 4 \\ -3 \\ -4 \end{matrix}$$

$4(2)(9) = 72$

$-8(6)(-4) = 192$

$3(-3)(5) = -45$

Step 3 Find the sum of each g

$72 + 192 + (-45) = 21$

Step 4 Subtract the sum of the

$219 - 312 = -93$

The value of the determ

Guided Practice

Evaluate each determinant.

2A. $\begin{vmatrix} -5 & 9 & 4 \\ -2 & -1 & 5 \\ -4 & 6 & 2 \end{vmatrix}$ **2B.**

Determinants can also be used
of the triangle are known, the
Because area cannot be negati

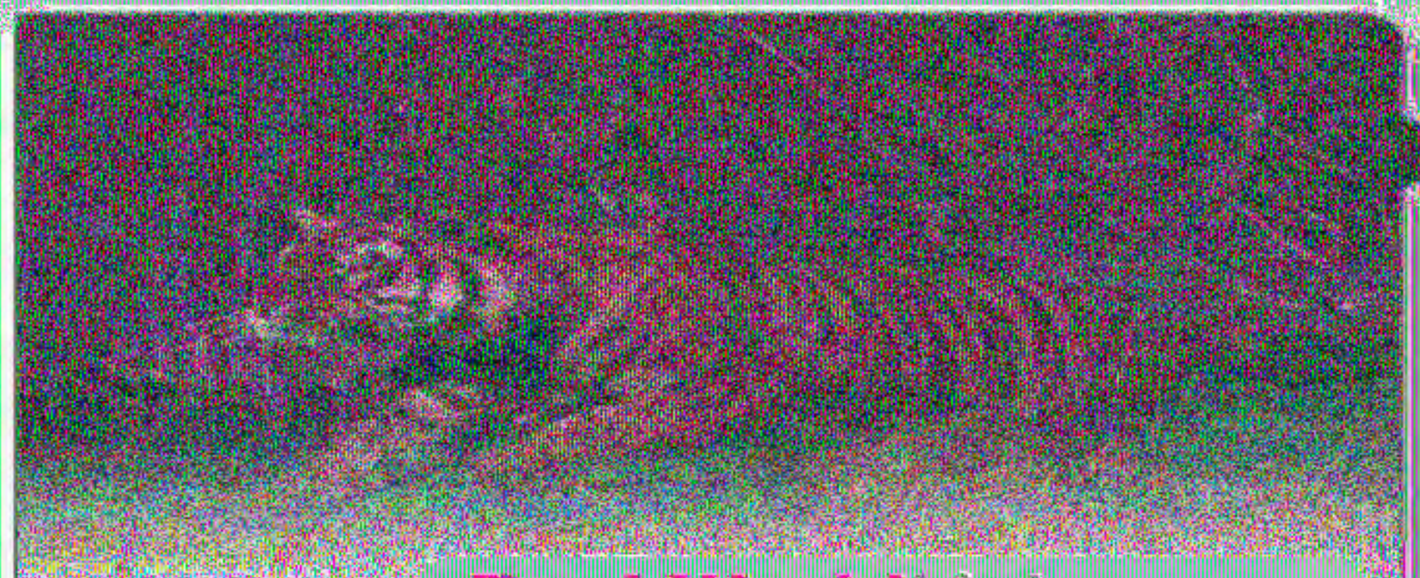
Key Concept Area of a T

Words The area of a tria
(a, b), (c, d), and

$$A = \frac{1}{2}\begin{vmatrix} a & b \\ c & d \\ e & f \end{vmatrix}$$

Example

$$A = \frac{1}{2}\begin{vmatrix} -4 & \\ 3 & \\ -2 & \end{vmatrix}$$



Real-World Link

Tigers are very territorial, solitary animals. Their territories can measure up to 100 square kilometers.

Source: *National Geographic*

Real-World Examples

ZOOLOGY Refer to the appl
beginning of the lesson. Th
of the vertices of the tiger's
shown to the right. Use det
find the area of the tiger's

$$A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 4 & 12 & 1 \\ -2 & 8 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 & 1 & 0 & 0 \\ 4 & 12 & 1 & 4 & 12 \\ -2 & 8 & 1 & -2 & 8 \end{vmatrix}$$

$0 + 0 + 32 = 32$

$$A = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 4 & 12 & 1 \\ -2 & 8 & 1 \end{vmatrix}$$

$= \left(\frac{1}{2}\right)[32 - (-24)]$ or 28

The area of the tiger's territ

Guided Practice

3. **CAR WASH** To raise mon
are advertising a car was
On a map, the coordinat
represents 0.5 kilometer
advertising?

Reading Math
Determinants The determinant is used to determine whether a system has a unique solution.

2 Cramer's Rule You can use d
determinant is nonzero, then t
then the system either has no soluti
uses the coefficient matrix. The coef
coefficients of the system.

Key Concept Cramer's Rule

Let C be the coefficient matrix of the sy

The solution of this system is $x = \frac{\begin{vmatrix} m & b \\ n & s \end{vmatrix}}{|C|}$

Test-Taking Tip
Cramer's Rule When the determinant of the coefficient matrix C is 0, the system does not have a unique solution.

Example 3 Solve a System of Tw

Solve the system by using Crame

$5x - 6y = 15$
$3x + 4y = -29$

$$x = \frac{\begin{vmatrix} m & b \\ n & s \end{vmatrix}}{|C|}$$

$$= \frac{\begin{vmatrix} 15 & -6 \\ -29 & 4 \end{vmatrix}}{\begin{vmatrix} 5 & -6 \\ 3 & 4 \end{vmatrix}}$$

$$= \frac{15(4) - (-29)(-6)}{5(4) - (3)(-6)}$$

$$= \frac{60 - 174}{20 + 18}$$

$$= \frac{-114}{38}$$

$$= -3$$

The solution of the system is (−3

CHECK $5(-3) - 6(-5) \stackrel{?}{=} 15$
$-15 + 30 \stackrel{?}{=} 15$
$15 = 15$

$3(-3) + 4(-5) \stackrel{?}{=} -$
$-9 - 20 \stackrel{?}{=} -$
$-29 = -$

Guided Practice

Solve each system using Cramer

4A. $7x + 3y = 37$
$-5x - 7y = -41$

4B. $8x -$

Cramer's Rule can also be used

Key Concept Cramer's Rule

Let C be the coefficient matrix of

The solution of this system is $x = \frac{|C_1|}{|C|}$, $y = \frac{|C_2|}{|C|}$, and $z = \frac{|C_3|}{|C|}$ if $|C| \neq 0$.

Example 3 Solve a System

Solve the system by using C

$$4x + 5y - 6z = -14$$
$$3x - 2y + 7z = 47$$
$$7x - 6y - 8z = 15$$

$$x = \frac{\begin{vmatrix} m & b & c \\ n & g & h \\ p & k & \ell \end{vmatrix}}{|C|}$$

$$= \frac{\begin{vmatrix} -14 & 5 & -6 \\ 47 & -2 & 7 \\ 15 & -6 & -8 \end{vmatrix}}{\begin{vmatrix} 4 & 5 & -6 \\ 3 & -2 & 7 \\ 7 & -6 & -8 \end{vmatrix}}$$

$$= \frac{3105}{621} \text{ or } 5$$

The solution of the system i

CHECK $4(5) + 5(-2) - 6(4)$

$20 - 10 - 24$

-14

$7(5) - 6(-2) - 8(4)$

$35 + 12 - 32$

15

Study Tip

Check for Accuracy
Always substitute your answers into the initial equations to confirm accuracy.

Guided Practice

Solve each system using Cr

5A. $3x + 5y + 2z = -7$
$-4x + 3y - 5z = -19$
$5x + 4y - 7z = -15$

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Check Your Understanding

Example 1

Evaluate each determinant.

1. $\begin{vmatrix} 8 & 6 \\ 5 & 7 \end{vmatrix}$

2.

3. $\begin{vmatrix} -4 & 12 \\ 9 & 5 \end{vmatrix}$

4.

Example 2

Evaluate each determinant using diagonals.

5. $\begin{vmatrix} 3 & -2 & 2 \\ -4 & 2 & -5 \\ -3 & 1 & 4 \end{vmatrix}$

6.

7. $\begin{vmatrix} 8 & 4 & 0 \\ -2 & -6 & -1 \\ 5 & -3 & 6 \end{vmatrix}$

8.

9. $\begin{vmatrix} 8 & 3 & 4 \\ 2 & 4 & 2 \\ 1 & 6 & 5 \end{vmatrix}$

10.

11. $\begin{vmatrix} 2 & -6 & -3 \\ 7 & 9 & -4 \\ -6 & 4 & 9 \end{vmatrix}$

12.

Example 4

Use Cramer's Rule to solve each system of equat

13. $4x - 5y = 39$
$3x + 8y = -6$

1

15. $-8a - 5b = -27$
$7a + 6b = 22$

16. $10c - 7d = -5$

Examples 3–5

17. **PERSEVERANCE** The "Bermuda Triangle" is a
located off the southeastern Atlantic coast of
United States, and is noted for reports of une
losses of ships, small boats, and aircraft.
a. Find the area of the triangle on the map.
b. Suppose each grid represents 175 miles. W
area of the Bermuda Triangle?

Use Cramer's Rule to solve each system of equat

18. $4x - 2y + 7z = 26$
$5x + 3y - 3z = -50$
$-7x - 8y - 3z = 49$

1

20. $6x - 5y + 2z = -49$
$-5x - 3y - 8z = -22$
$-3x + 8y - 5z = 55$

21. $-9x + 5$

2

24. $2w + 3p - 4w = 20$
$4w - p + 5w = 13$
$3w + 2p + 4w = 15$

2

Practice and Problem Solving

Examples 1–2 **Evaluate each determinant.**

26. $\begin{vmatrix} -7 & 12 \\ 5 & 6 \end{vmatrix}$

27. $\begin{vmatrix} -8 & \\ 11 & \end{vmatrix}$

29. $\begin{vmatrix} 3 & 5 & -2 \\ -1 & -4 & 6 \\ -6 & -2 & 5 \end{vmatrix}$

30. $\begin{vmatrix} 2 & \\ -3 & \\ -2 & \end{vmatrix}$

32. $\begin{vmatrix} 6 & -3 & -5 \\ 0 & -7 & 0 \\ 3 & -6 & -4 \end{vmatrix}$

33. $\begin{vmatrix} -8 & \\ 0 & \\ 8 & \end{vmatrix}$

35. $\begin{vmatrix} 1 & -8 & -9 \\ 6 & 5 & -6 \\ -2 & -8 & 10 \end{vmatrix}$

36. $\begin{vmatrix} 5 & \\ -8 & \\ -2 & \end{vmatrix}$

38. TRAVEL Mr. Osama's art class took a bu
65 kilometers per hour on the highway
museum is 375 kilometers away from t
Use Cramer's Rule to find how many h
hours it was driving in the city.

Examples 4–5 **Use Cramer's Rule to solve each system of**

39. $6x - 5y = 73$
$-7x + 3y = -71$

41. $-4c - 5d = -39$
$5c + 8d = 54$

42. $-6f - 8g =$

43. $9r + 4s = -55$
$-5r - 3s = 36$

44. $-11m -$

45. $5x - 4y + 6z = 58$
$-4x + 6y + 3z = -13$
$6x + 3y + 7z = 53$

47. DOUGHNUTS Khawla is ordering dough
class party. The box contains 2 dozen d
some of which are plain and some of w
jelly-filled. The plain doughnuts each c
and the jelly-filled cost AED 6.00. If the
is AED 126.00, use Cramer's Rule to fi
jelly-filled doughnuts Khawla ordered.

Examples 3–4 **48. PERSEVERANCE** The salary for each of
the supporting actors each receive AE
salaries of the actors and actresses is A
use Cramer's Rule to find the number

49. ARCHAEOLOGY Archaeologists found w
(5, 9). If the units of the coordinates ar
by these finds.

Use Cramer's Rule to solve each system of equat

50. $6a - 7b = -55$
$2a + 4b - 3c = 35$
$-5a - 3b + 7c = -37$

51. $3a - 5b - 9c$

52. $4x - 5y = -2$
$7x + 3z = -47$
$8y - 5z = -63$

53. $7x + 8y + 9z$

54. GARDENING Marwan wants to build a triangu
he uses a coordinate grid where each of the
coordinates for the vertices of his garden are
the garden.

55. FINANCIAL LITERACY A vendor sells small dri
1.75, and large drinks for AED 2.25. During
drinks as medium drinks, his total sales were

a. Use Cramer's Rule to determine how man

b. The vendor decided to increase the price
The next week, he sold 140 fewer small d
35 more large drinks. Calculate his sales

c. Was raising the price of the small drink a

H.O.T. Problems: Use Higher-Order Thinking Skills

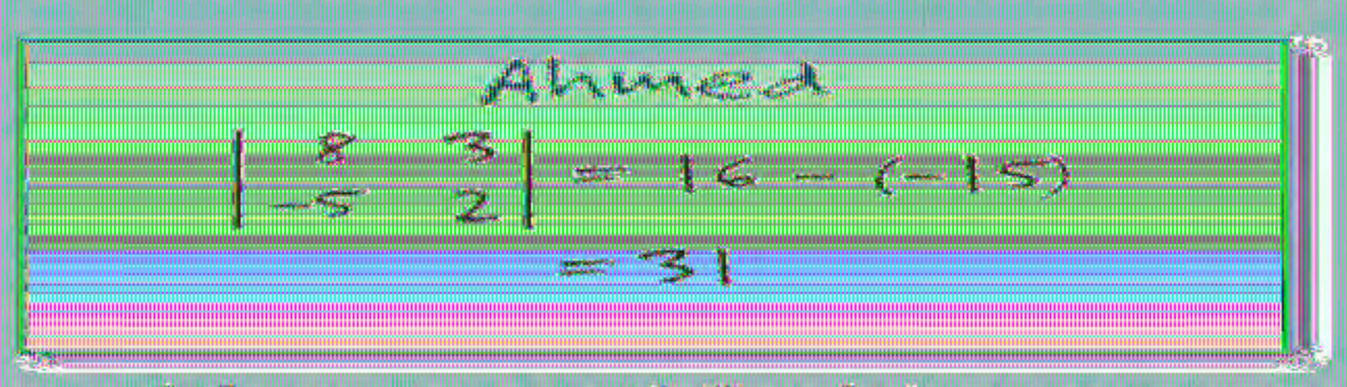
56. REASONING Some systems of equations cann

a. Find the value of $\begin{vmatrix} a & b \\ f & g \end{vmatrix}$. When is the va

b. Choose values for a, b, f, and g to make th
What type of system is formed?

57. REASONING What can you determine about
the determinant of the coefficients is 0?

58. CRITIQUE Ahmed and Hamdan are finding



Is either of them correct? Explain your reasor

59. CHALLENGE Find the determinant of a 3×3

$$a_{mn} = \begin{cases} 0 & \text{if } m + n \text{ is even} \\ m + n & \text{if } m + n \text{ is odd} \end{cases}$$

60. OPEN ENDED Write a 2×2 matrix with each
a. The determinant equals 0.
b. The determinant equals 25.
c. The elements are all negative numbers an

61. WRITING IN MATH Describe the possible grap
linear equations if the determinant of the ma

Standardized Test Practice

62. Tariq paid AED 25.25 to play three games of miniature golf and two rides on go-karts. Badr paid AED 25.75 for four games of miniature golf and one ride on the go-karts. How much does one game of miniature golf cost?

A AED 4.25
B AED 4.75
C AED 5.25
D AED 5.75

63. Use the table to determine the expression that best represents the number of faces of any prism having a base with n sides.

Base	Sides of Base	Faces of Prisms
triangle	3	5
quadrilateral	4	6
pentagon	5	7
hexagon	6	8
heptagon	7	9
octagon	8	10

F $2(n - 1)$
G $2(n + 1)$
H $n + 2$
J $2n$

Spiral Review

Determine whether each matrix product is defined. If so, product. (Lesson 1-6)

66. $A_{4 \times 2} \cdot B_{2 \times 6}$

67. $C_{5 \times 4} \cdot D_{5 \times 3}$

69. BUSINESS The table lists the prices at the Sandwich Shoppe. (Lesson 1-5)

a. List the prices in a 4×3 matrix.
b. The manager decides to cut the prices of every item by 20%. List this new set of data in a 4×3 matrix.
c. Subtract the second matrix from the first and deter the savings to the customer for each sandwich.

Graph each function.

70. $f(x) = 2|x - 3| - 4$

71. $f(x) = -3$ $|2x|$

Skills Review

Solve each system of equations.

73. $2x - 5y = -26$
$5x + 3y = -34$

74. $4y + 6x = 10$
$2x - 7y = 22$

Solving Systems of Inverse Matrices

Then | **Now** | **Why?**

- You solved systems of linear equations algebraically.

- 1 Find the inverse of a 2 × 2 matrix.
- 2 Write and solve matrix equations for a system of equations.

- Muna's Sandwich S… lunch options as sh…

To determine how … individual item cos… the following matri… which *w* represents … sandwich, *s* the co… *d* the cost of a drin…

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 2 \\ 4 & 3 & 4 \end{bmatrix} \begin{bmatrix} w \\ s \\ d \end{bmatrix}$$

New Vocabulary
identity matrix
square matrix
inverse matrix
matrix equation
variable matrix
constant matrix

Mathematical Practices
Use appropriate tools strategically.

1 Identity and Inverse Matrices

… multiplicative inverses if their pr… for matrices, the **identity matrix** is a… matrix, equals that same matrix. A s… rows and columns.

2 × 2 Identity Matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Key Concept: Identity Matrix f…

Words The identity matrix for n… of the main diagonal, fr… For any square matrix *A*…

Symbols If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then *A*…

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

Two $n \times n$ matrices are **inverses** of e… If matrix *A* has an inverse symbolize…

StudyTip

Structure
Since multiplication of matrices is not commutative, it is necessary to check the product in both orders.

Example 4 **Verify Inverse M**

Determine whether the ma

a. $A = \begin{bmatrix} -4 & 2 \\ -2 & 1 \end{bmatrix}$ and $B =$

If A and B are inverses, t

$$A \cdot B = \begin{bmatrix} -4 & 2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{4} & \\ \frac{1}{2} & \end{bmatrix}$$

$$= \begin{bmatrix} -1 + 1 & 2 - \\ -\frac{1}{2} + \frac{1}{2} & 1 - \end{bmatrix}$$

Since $A \cdot B \neq I$, they are

b. $F = \begin{bmatrix} 3 & -5 \\ -2 & 6 \end{bmatrix}$ and G

If F and G are inverses, t

$$F \cdot G = \begin{bmatrix} 3 & -5 \\ -2 & 6 \end{bmatrix} \cdot \begin{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{4} - \frac{5}{4} & \\ -\frac{6}{4} + \frac{6}{4} & -\frac{1}{2} \end{bmatrix}$$

$$G \cdot F = \begin{bmatrix} \frac{3}{4} & \frac{5}{8} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{4} - \frac{10}{8} & -\frac{15}{4} \\ \frac{3}{4} - \frac{6}{8} & -\frac{5}{4} \end{bmatrix}$$

Since $F \cdot G = G \cdot F = I$,

GuidedPractice

1. Determine whether $X =$

Some matrices do not have in
using the determinant.

KeyConcept **Inverse of a 2**

The inverse of matrix $A = \begin{bmatrix} a & \\ c & \end{bmatrix}$

Notice that $ad - bc$ is the value
is 0, the matrix cannot have an

Example 2 **Find the Inverse of a**

Find the inverse of each matrix,

a. $P = \begin{bmatrix} 7 & -5 \\ 2 & -1 \end{bmatrix}$

$$\begin{vmatrix} 7 & -5 \\ 2 & -1 \end{vmatrix} = -7 - (-10) \text{ or } 3$$

Since the determinant does n

$$P^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{7(-1) - (-5)(2)}\begin{bmatrix} -1 & -5 \\ -2 & 7 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} -1 & 5 \\ -2 & 7 \end{bmatrix} \text{ or } \begin{bmatrix} -\frac{1}{3} & \frac{5}{3} \\ -\frac{2}{3} & \frac{7}{3} \end{bmatrix}$$

CHECK **Find the product of the are inverses.**

$$\begin{bmatrix} 7 & -5 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{3} & \frac{5}{3} \\ -\frac{2}{3} & \frac{7}{3} \end{bmatrix}$$

b. $Q = \begin{bmatrix} -8 & -6 \\ 12 & 9 \end{bmatrix}$

$$\begin{vmatrix} -8 & -6 \\ 12 & 9 \end{vmatrix} = -72 - (-72) =$$

Since the determinant equals

Math History Link

Seki Kowa (1642–1708)
Known as The Arithmetical Sage, Seki Kowa was the first to develop the theory of determinants.

Guided Practice

2A. $\begin{bmatrix} 3 & 7 \\ 1 & -4 \end{bmatrix}$

2 Matrix Equations

Matrices ca

You can write a **matrix equatio**

$$x + 2y$$
$$3x - 6y$$

Write the left side of the matrix equa variable matrix. Write the right side

$$\begin{bmatrix} 1 & 2 \\ 3 & -6 \end{bmatrix} \cdot A$$

coefficient matrix

Study Tip

Inverse You can use this method to solve systems of equations only if A has an inverse. If A does not have an inverse, then the system either has no solution or infinitely many solutions.

Then solve the matrix equation other equation.

$$ax = b$$

$$\left(\frac{1}{a}\right)ax = \left(\frac{1}{a}\right)b$$

$$1x = \frac{b}{a}$$

$$x = \frac{b}{a}$$

Notice that the solution of the matrix and the constant matri



Real-World Link

Average gas prices increased fivefold from AED 0.68 per liter in 1977 to AED 0.92 per liter in 2007.

Source: U.S. Department of Energy

Real-World Examples

TRAVEL Halima stopped fo first station where she stop was AED 3.50 per liter. Ha 88.05. How much gasoline

A system of equations to re

$$x + y = 24.2$$
$$3.75x + 3.50y = 88.05$$

The matrix equation is $\begin{bmatrix} 3.7 \end{bmatrix}$

Step 1 Find the inverse o

$$A^{-1} = \frac{1}{3.50 - 3.75}$$

Step 2 Multiply each side

$$\frac{1}{0.25}\begin{bmatrix} 3.50 \\ -3.75 \end{bmatrix}$$

The solution is (13.4, 10.8), the first gas station, and y r

CHECK You can check your

Enter $\begin{bmatrix} 1 \\ 3.75 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 3.50 \end{bmatrix}$

Enter $\begin{bmatrix} 24.2 \\ 88.05 \end{bmatrix}$ as m

Multiply the inverse

Guided Practice

3. **COMIC BOOKS** Hamdan and used comic books. spent AED 15.75 on 10 same price, what is the

Check Your Understanding

Example 1 Determine whether the matrices in each pair are

1. $A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

2. $C =$

3. $F = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, $G = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$

4. H

Example 2 Find the inverse of each matrix, if it exists.

5. $\begin{bmatrix} 6 & -3 \\ -1 & 0 \end{bmatrix}$

6.

7. $\begin{bmatrix} -3 & 0 \\ 5 & 2 \end{bmatrix}$

8.

Example 3 Use a matrix equation to solve each system of eq

9. $-2x + y = 9$
$x + y = 3$

10. $4x - 2y = 22$
$6x + 9y = -$

12. **MONEY** Khalfan had 25 coins in total made u value of all the coins was AED 4. How many have?

Practice and Problem Solving

Example 1 Determine whether each pair of matrices are inv

13. $K = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$, $L = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$

14. $M =$

15. $P = \begin{bmatrix} 4 & 0 \\ 3 & 0 \end{bmatrix}$, $Q = \begin{bmatrix} -\frac{1}{2} & -1 \\ \frac{2}{3} & 5 \end{bmatrix}$

16. $R =$

Example 2 Find the inverse of each matrix, if it exists.

17. $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

18.

$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$

19.

20. $\begin{bmatrix} 1 & -1 \\ -6 & -1 \end{bmatrix}$

21.

$\begin{bmatrix} -5 & -4 \\ 4 & 2 \end{bmatrix}$

23. $\begin{bmatrix} 6 & -5 \\ 4 & 9 \end{bmatrix}$

24.

$\begin{bmatrix} -4 & -2 \\ 7 & 8 \end{bmatrix}$

Example 3

26. **BAKING** Badria is preparing a colored frostin she needs 25 milliliters of a 44% concentratio a 50% blue concentration of food coloring. H and red food coloring should be mixed to ma coloring?

PERSEVERANCE Use a matrix equation to solve e

27. $-x + y = 4$
$-x + y = -4$

28. $-x + y = 3$
$-2x + y = 6$

30. $3x + y = 3$
$5x + 3y = 6$

31. $y - x = 5$
$2y - 2x = 8$

33. $1.6y - 0.2x = 1$
$0.4y - 0.1x = 0.5$

34. $4y - x = -2$
$3y - x = 6$

36. **POPULATIONS** The diagram shows the
its suburbs.

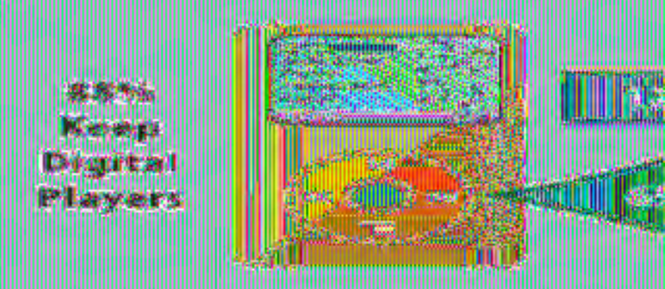


a. Write a matrix to represent the trans
population.

b. There are currently 16,275 people liv
suburbs. Assuming that the trends c
will live in the suburbs next year.

c. Use the inverse of the matrix from p
in the city last year.

37. **MUSIC** The diagram shows the trends i
ownership over the past five years for C
has either a digital audio player or a po
population of 25,000 people, of whom
portable CD players.



a. Write a matrix to represent the trans

b. Assume that the trends continue. Pr
audio players next year.

c. Use the inverse of the matrix from p
digital audio players last year.

H.O.T. Problems Use Higher-Order Thinking Skills

38. **CRITIQUE** Asma and Amna are setting
$5x + 7y = 19$ and $3y + 4x = 10$. Is eith

Asma

$$\begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 10 \end{bmatrix}$$

39. **CHALLENGE** Describe what a matrix eq

40. **REASONING** Determine whether the fol
Explain your reasoning.

A square matri

41. **OPEN ENDED** Write a matrix equation t

42. **WRITING IN MATH** When would yo
algebraic methods, and when would yo

Glencoe Algebra 2 Chapter 4 Matrix Algebra and Applications

Standardized Test Practice

43. The Yogurt Shoppe sells cones in three sizes: small, AED 0.89; medium, AED 1.19; and large, AED 1.39. One day Salem sold 52 cones. He sold seven more medium cones than small cones. If he sold AED 58.98 in cones, how many medium cones did he sell?

A 11 **B** 17 **C** 24 **D** 36

44. The chart shows an expression evaluated for different values of x.

x	$x^2 + x + 1$
1	3
2	7
3	13
4	21

A student concludes that for all values of x, $x^3 + x + 1$ produces a prime number. Which value of x serves as a counterexample to prove this conclusion false?

F −4 **G** −3 **H** 2 **J** 4

Spiral Review

Evaluate each determinant. (Lesson 1-7)

47. $\begin{vmatrix} 8 & -3 \\ 6 & -9 \end{vmatrix}$ **48.** $\begin{vmatrix} 9 & -7 \\ -5 & -3 \end{vmatrix}$

Find each product, if possible. (Lesson 1-6)

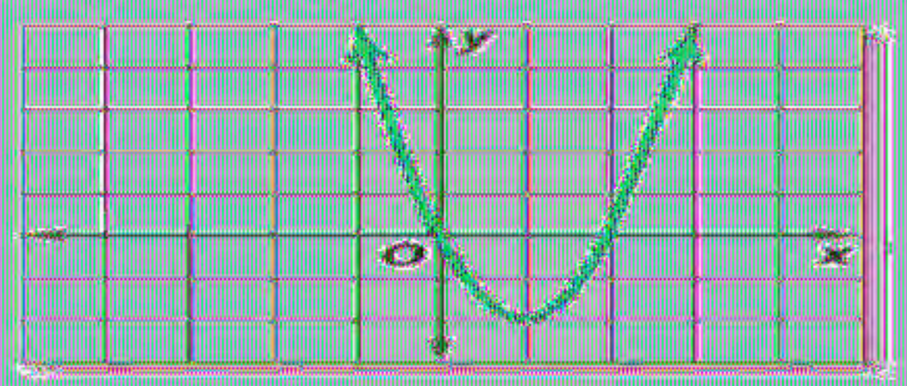
50. $\begin{bmatrix} 4 & 2 \\ -1 & -3 \end{bmatrix} \cdot \begin{bmatrix} 6 & 2 \\ 5 & 1 \end{bmatrix}$ **51.** $\begin{bmatrix} 8 & -2 \\ -4 & -5 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

53. MILK Manal's Family Dairy produces at most 200 liters of sk
each day for delivery to large bakeries and restaurants. Regu
least 15 liters of skim and 21 liters of whole milk each day. If
of skim milk is AED 0.82 and the profit on a liter of whole m
liters of each type of milk should the dairy produce each day
profits? (Lesson 1-4)

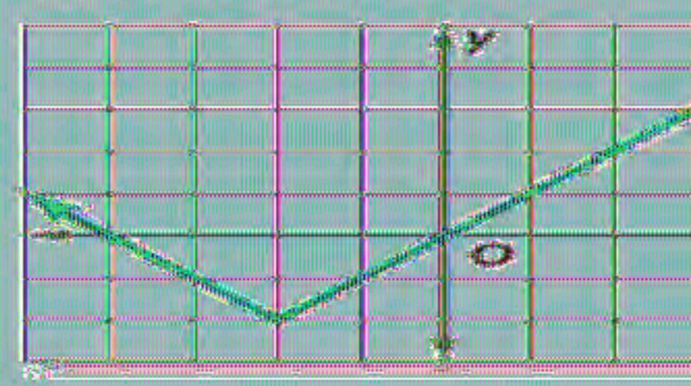
Skills Review

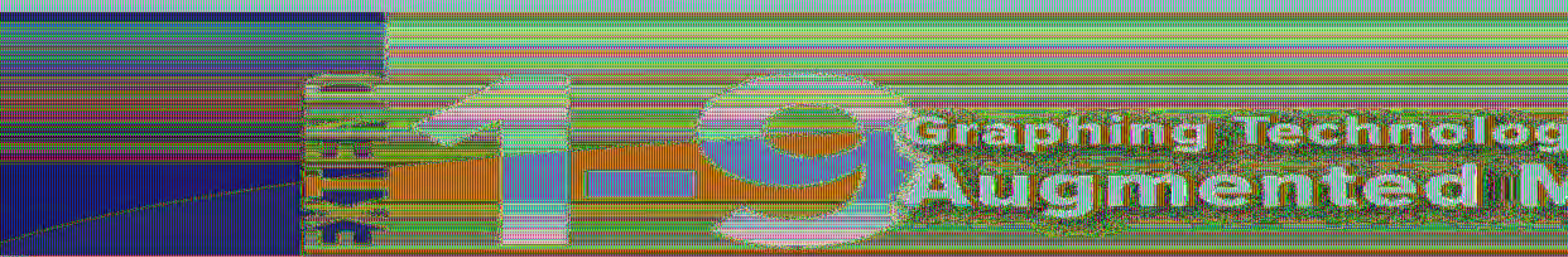
Identify the type of the function represented by each graph.

54.



55.





Using a graphing calculator, you can solve a system of linear equat
MATRIX function. An **augmented matrix** contains the coefficient ma
extra column containing the constant terms. You can use a graphin
reduce the augmented matrix so that the solution of the system of
easily determined.

Example

**Write an augmented matrix for the following system of
system by using a graphing calculator.**

$$2x + y + z = 1$$
$$3x + 2y + 3z = 12$$
$$4x + y + 2z = -1$$

Step 1 Write the augmented matrix and enter it into a

The augmented matrix $B = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 3 & 2 & 3 & 12 \\ 4 & 1 & 2 & -1 \end{bmatrix}$

Begin by entering the matrix.

KEYSTROKES: [2nd] [MATRIX] [ENTER] [E
1 [ENTER] [ENTER] [ENTER] [ENTE
4 [ENTER] [ENTER] [ENTER] [ENTE

Step 2 Find the reduced row echelon form (rref) using

KEYSTROKES: [2nd] [QUIT] [2nd] [MATRIX] [▶] [AL
[ENTER] [)] [ENTER]

Study the reduced echelon matrix. The first thr
the same as a 3 × 3 identity matrix. The first ro
$x = -4$, the second row represents $y = 3$, and t
represents $z = 6$. The solution is $(-4, 3, 6)$.

Exercises

**Write an augmented matrix for each system of equations.
a graphing calculator.**

1. $3x + 2y = -4$
   $4x + 7y = 13$

2. $2x + y = 6$

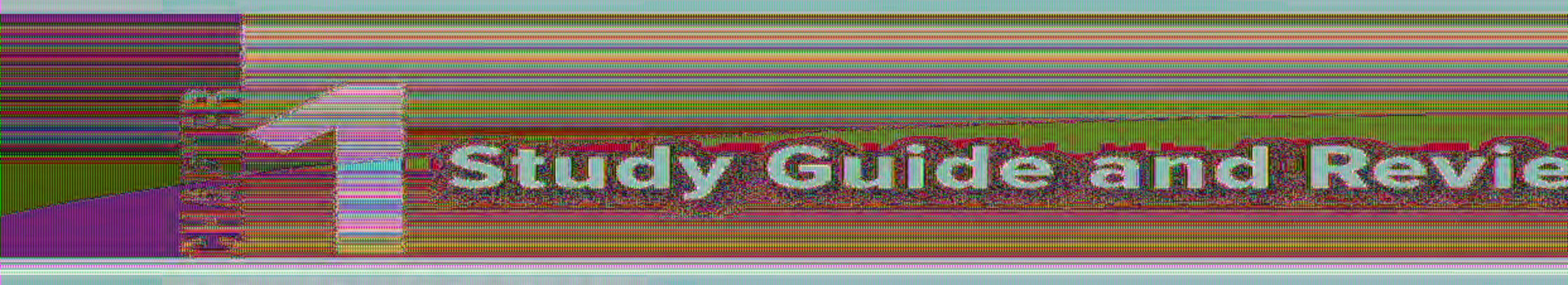
3. $2x + 2y = -4$
   $6x - 2y = 0$

4. $4x + 6y = 0$

5. $6x - 4y + 2z = -4$
   $8x - 2y = 7$

6. $5x - 5y + 5z$
   $2x - 2y + 6z$
   $2x + 2y + 2z$

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Study Guide

Key Concepts

Systems of Equations and Inequalities (Lessons 1-1 and 1-2)

- In the substitution method, one equation is solved for a variable and substituted to find the value of another variable. In the elimination method, one variable is eliminated by adding or subtracting the equations.
- The solution of a system of inequalities is found by graphing the inequalities and determining the intersection of the graphs.

Linear Programming (Lesson 1-3)

- Linear programming is a method for finding maximum or minimum values of a function over a given system of inequalities with each inequality representing a constraint.

Systems of Equations in Three Variables (Lesson 1-4)

- A system of equations in three variables can be solved algebraically by using the substitution method or the elimination method.

Operations with Matrices (Lessons 1-5 and 1-6)

- Matrices can be added or subtracted if they have the same dimensions. Add or subtract corresponding elements.
- Two matrices can be multiplied if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix.

Solving Systems Using Cramer's Rule (Lesson 1-7)

- If a determinant is nonzero, then the system has a unique solution. If the determinant is 0, then the system either has no solution or infinite solutions.

Solving Systems with Inverse Matrices (Lesson 1-8)

- An identity matrix is a square matrix with ones on the main diagonal and zeros in the other positions.
- Two matrices are inverses of each other if their product is the identity matrix.
- To solve a matrix equation, find the inverse of the coefficient matrix. Then multiply each side of the equation by the inverse of the coefficient matrix.

Foldables Study Organizer

Be sure the Key Concepts are noted in your Foldable.

Lesson-by-Lesson Review

Solving Systems of Equations

Solve each system of equations by graphing.

11. $3x + 4y = 8$
$x - 3y = -6$

12. $x + \frac{8}{3}y = 12$
$\frac{1}{2}x + \frac{4}{3}y = 6$

13. $y - 3x = 13$
$y = \frac{1}{3}x + 5$

14. $6x - 14y = 5$
$3x - 7y = 5$

15. **LAWN CARE** Khalid and Jasim each mow lawns. Khalid charges a AED 30 service fee and AED 10 per hour. Jasim charges a AED 10 service fee and AED 15 per hour. After how many hours will Khalid and Jasim charge the same amount?

Solve each system of equations by using either substitution or elimination.

16. $x + y = 6$
$3x - 2y = -2$

17. $5x - 2y = 4$
$-2y + x = 12$

18. $x + y = 3.5$
$x - y = 7$

19. $3y - 5x = 0$
$2y - 4x = -2$

20. **SCHOOL SUPPLIES** At an office supply store, Jamal bought 3 notebooks and 5 pens for AED 13.75. If a notebook costs AED 1.25 more than a pen, how much does a notebook cost? How much does a pen cost?

Solving Systems of Inequalities by Graphing

Solve each system of inequalities by graphing.

21. $y < 2x - 3$
$y \geq 4$

22. $|y| > 2$
$x > 3$

23. $y \geq x + 3$
$2y \leq x - 5$

24. $y > x + 1$
$x < -2$

25. **JEWELRY** Suha makes jewelry to sell at her mother's clothing store. She spends no more than 3 hours making jewelry on Saturdays. It takes her 15 minutes to set up her supplies and 25 minutes to make each bracelet. Draw a graph that represents this.

Optimization with Linear Programming

26. **FLOWERS** A florist can make a grand arrangement in 18 minutes or a simple arrangement in 10 minutes. The florist makes at least twice as many of the simple arrangements as the grand arrangements. The florist can work only 40 hours per week. The profit on the simple arrangements is AED 10 and the profit on the grand arrangements is AED 25. Find the number and type of arrangements that the florist should produce to maximize profit.

27. **MANUFACTURING** A shoe manufacturer makes outdoor and indoor soccer shoes. There is a two-step process for both kinds of shoes. Each pair of outdoor shoes requires 2 hours in step one and 1 hour in step two, and produces a profit of AED 20. Each pair of indoor shoes requires 1 hour in step one and 3 hours in step two, and produces a profit of AED 15. The company has 40 hours of labor available per day for step one and 60 hours available for step two. What is the manufacturer's maximum profit? What is the combination of shoes for this profit?

Systems of Equations in Three Variables

Solve each system of equations.

28. $a - 4b + c = 3$
$b - 3c = 10$
$3b - 8c = 24$

29. $2x - z = 14$
$3x - y + 5z = 0$
$4x + 2y + 3z = -2$

30. **AMUSEMENT PARKS** Harib, Ismail, and Humaid went to an amusement park. They purchased snacks from the same vendor. Their snacks and how much they paid are listed in the table. How much did each snack cost?

Name	Sandwiches	Popcorn	Sodas	Price
Harib	1	2	3	AED 15.25
Ismail	2	0	3	AED 14.00
Humaid	1	2	1	AED 10.25

Operations with Matrices

Perform the indicated operations. If the matrix does not exist, write *impossible*.

31. $3\left(\begin{bmatrix} -2 & 0 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ -3 & -4 \end{bmatrix}\right)$

32. $\begin{bmatrix} 2 \\ -6 \end{bmatrix} - \begin{bmatrix} -3 \\ 2 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix}$

33. RETAIL Current Fashions buys shirts, jeans, and shoes from a manufacturer, marks them up, and then sells them. The table shows the purchase price and the selling price.

Item	Purchase Price	Selling Price
shirts	AED 15	AED 35
jeans	AED 25	AED 55
shoes	AED 30	AED 85

a. Write a matrix for the purchase price.

b. Write a matrix for the selling price.

c. Use matrix operations to find the profit on 1 shirt, 1 pair of jeans, and 1 pair of shoes.

Multiplying Matrices

Find each product, if possible.

34. $\begin{bmatrix} 3 & -7 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

35. $\begin{bmatrix} -3 & 0 & 2 \\ 6 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 8 & -1 \\ -4 & 3 \\ 6 & 7 \end{bmatrix}$

36. $\begin{bmatrix} 2 & 11 \\ 0 & -3 \\ -6 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 8 & -5 \\ 12 & 0 & 9 \\ 4 & -6 & 7 \end{bmatrix}$

37. GROCERIES Mahmoud bought 1 liter of milk, 2 apples, 4 frozen dinners, and 1 box of cereal. The following matrix shows the prices for each item, respectively.

[AED 2.59 AED 0.49 AED 5.25 AED 3.99]

Use matrix multiplication to find the total amount of money Mahmoud spent at the grocery store.

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Solving Systems of Equations Using Cramer's Rule

Evaluate each determinant.

38. $\begin{vmatrix} 2 & 4 \\ 7 & -3 \end{vmatrix}$

39. $\begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{vmatrix}$

Use Cramer's Rule to solve each system of equations.

40. $3x - y = 0$
$5x + 2y = 22$

41. $5x + 2y = 4$
$3x + 4y + 2z = 6$
$7x + 3y + 4z = 29$

42. **JEWELRY** Amal paid AED 98.25 for 3 necklaces and 2 pairs of earrings. Wafa paid AED 133.50 for 2 necklaces and 4 pairs of earrings. Use Cramer's Rule to find out how much 1 necklace costs and how much 1 pair of earrings costs.

Solving Systems of Equations Using Inverse Matrices

Find the inverse of each matrix, if it exists.

43. $\begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$

44. $\begin{bmatrix} 2 & 5 \\ -5 & -13 \end{bmatrix}$

45. $\begin{bmatrix} 6 & -3 \\ -8 & 4 \end{bmatrix}$

Use a matrix equation to solve each system of equations.

46. $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

47. $\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

48. **HEALTH FOOD** Mahmoud sells nuts and raisins by the kilogram. Shaikha bought 2 kilograms of nuts and 2 kilograms of raisins for AED 23.50. Reham bought 3 kilograms of nuts and 1 kilogram of raisins for AED 22.25. What is the cost of 1 kilogram of nuts and 1 kilogram of raisins?

Solving Systems of Equations Using Cramer's Rule

Evaluate each determinant.

38. $\begin{vmatrix} 2 & 4 \\ 7 & -3 \end{vmatrix}$

39. $\begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{vmatrix}$

Use Cramer's Rule to solve each system of equations.

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Solve each system of equations by using either substitution or elimination.

1. $y = x + 4$
   $x + y = -12$

2. $3x + 5y = -7$
   $6x - 4y = 0$

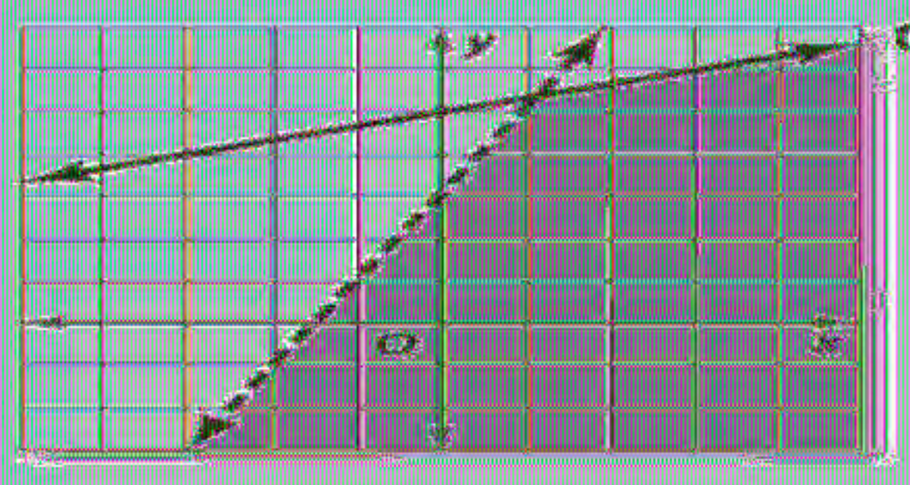
3. $5x + 2y = 4$
   $3y - 4x = -40$

4. $8x - 3y = -13$
   $-3x + 5y = 1$

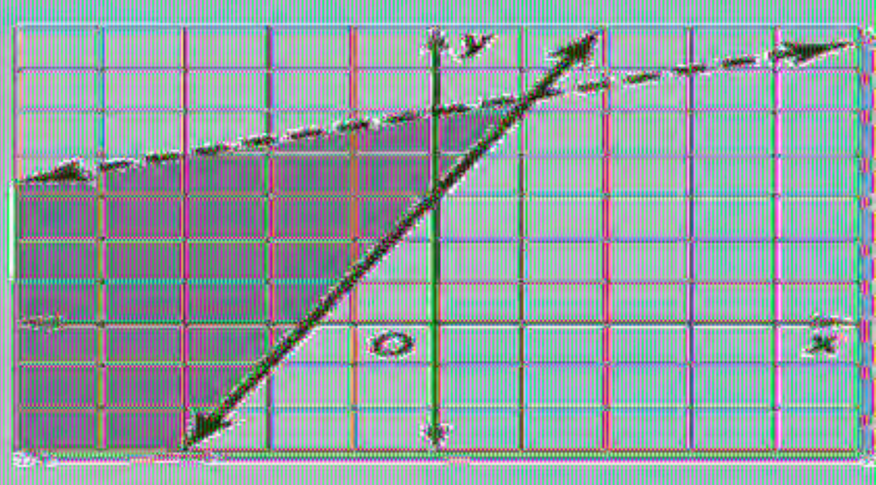
5. **MULTIPLE CHOICE** Which graph shows the solution of the system of inequalities?

$$y \leq 2x + 3$$
$$y < \frac{1}{3}x + 5$$

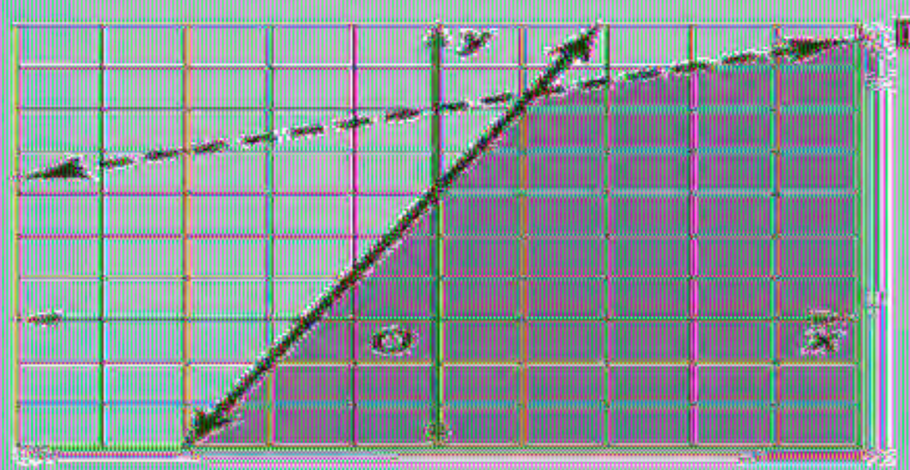
A



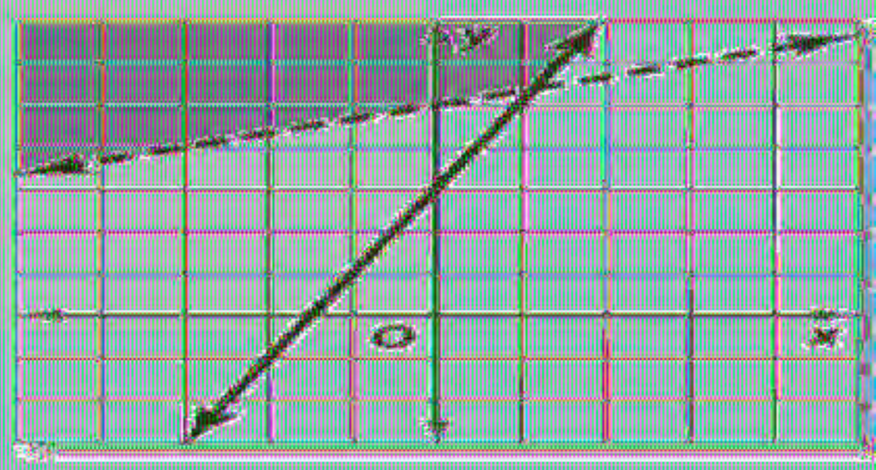
C



B



D



Solve each system of inequalities by graphing.

6. $x + y > 6$
   $x - y < 0$

7. $y \geq 2x - 5$
   $y \leq x + 4$

8. $3x + 4y \leq 12$
   $6x - 3y \geq 18$

9. $5y + 2x \leq 20$
   $4x + 3y > 12$

10. **SALONS** Sumayya is a nail technician. She allots 20 minutes for a manicure and 45 minutes for a pedicure in her 7-hour work day. No more than 5 pedicures can be scheduled each day. The prices are AED 18 for a manicure and AED 45 for a pedicure. How many manicures and pedicures should Ms. Sumayya schedule to maximize her daily income? What is her maximum daily income?

Short Answer Questions

Short answer questions require you to provide a solution to the problem, along with a method, explanation, and/or justification used to arrive at the solution.

Strategies for Solving Short Answer Questions

Short answer questions are typically graded using a **rubric**, or a scoring g
The following is an example of a short answer question scoring rubric.

Scoring Rubric	
Criteria	Score
Full Credit: The answer is correct and a full explanation is provided that shows each step.	2
Partial Credit: • The answer is correct but the explanation is incomplete. • The answer is incorrect but the explanation is correct.	1
No Credit: Either an answer is not provided or the answer does not make sense.	0

In solving short answer questions, remember to...

- explain your reasoning or state your approach to solving the probl
- show all of your work or steps.
- check your answer if time permits.

Standardized Test Example

Read the problem. Identify what you need to know. Then use problem to solve.

Company A charges a monthly fee of AED 14.50 plus AED 0. service. Company B charges AED 20.00 per month plus AED number of minutes would the total monthly charge be the sa

Read the problem carefully. You are given information about companies and their monthly charges. Since the situation inv a variable rate, you can set up and solve a system of equation

Example of a 2-point response:

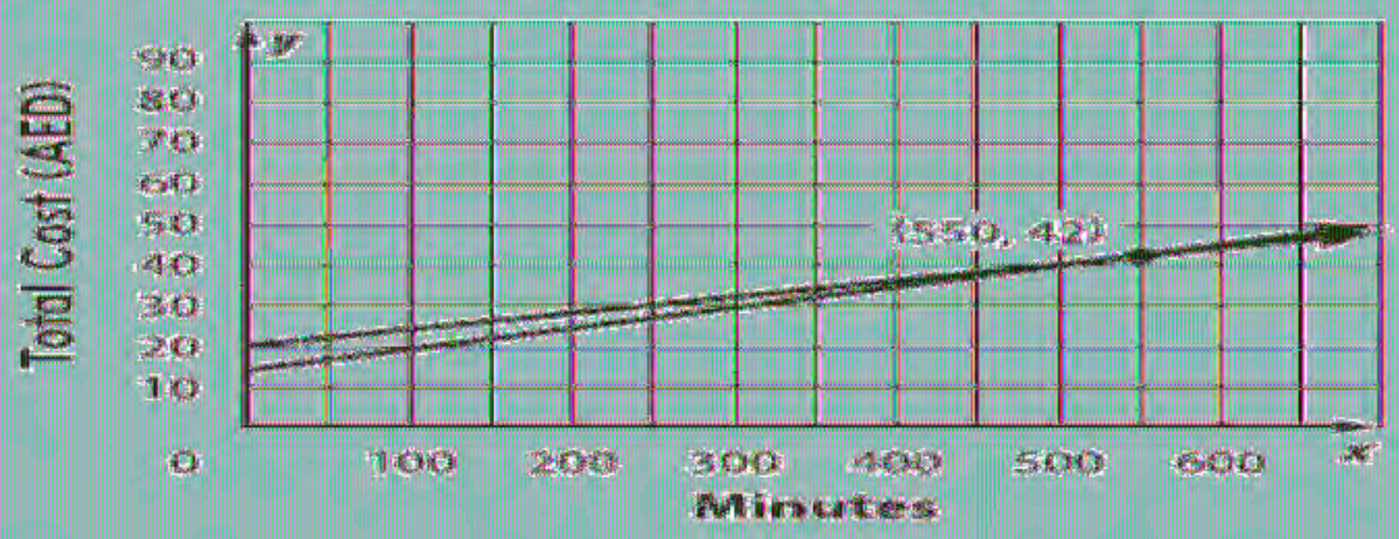
Set up and solve a system of equations.

flat fee + rate × minutes = total charges
y = total charges, x = minutes used

$y = 14.5 + 0.05x$ (Company A)

$y = 20 + 0.04x$ (Company B)

Solve the system by graphing.



The solution is (550, 42). So, with each company, if the the total monthly charge is AED 42.

The steps, calculations, and reasoning are clearly stated at the correct answer. So, this response is worth the ful

Exercises

Read each problem. Then use the information in the problem to solve.

1. Harib and Hassan borrowed AED 1400 to start a lawn mowing business. They charge their customers AED 45 per lawn, and with each lawn that they mow, they incur AED 10.50 in operating expenses. How many lawns must they mow in order to start earning a profit?

2. A circle of radius r is circumscribed about a square. What is the exact ratio of the area of the circle to the area of the square?

3. Mr. Ahmed can spend no more than AED 50 on art supplies. Packages of paint brushes cost AED 4.75 each, and boxes of colored pencils cost $6.50 each. He wants to buy at least 2 packages of each supply. Write a system of inequalities and plot the feasible region on a coordinate grid. Give three different solutions to the system.

4. Buthaina sells engraved necklaces over the Internet. She purchases 50 necklaces for AED 400, and it costs her an additional AED 3 for each personalized engraving. If she charges AED 20 for each necklace, how many will she need to sell in order to make a profit of at least AED 225?