Chapter 8: Systems of Particles and Extended Objects

Concept Checks

8.1. b **8.2.** a **8.3.** d **8.4.** b **8.5.** a

Multiple-Choice Questions

8.1. d **8.2.** b **8.3.** d **8.4.** b and d **8.5.** e **8.6.** a **8.7.** b **8.8.** d **8.9.** b **8.10.** e **8.11.** a **8.12.** c **8.13.** a **8.14.** b **8.15.** b **8.16.** b

Conceptual Questions

8.17. It is reasonable to assume the explosion is entirely an internal force. This means the momentum, and hence the velocity of the center of mass remains unchanged. Therefore, the motion of the center of mass remains the same.

8.18. The length of the side of the cube is given as d. If the cubes have a uniform mass distribution, then the center of mass of each cube is at its geometric center. Let m be the mass of a cube. The coordinates of the center of mass of the structure are given by:

$$X_{\text{cm}} = \frac{m\left(\frac{d}{2}+\frac{d}{2}+\frac{d}{2}+\frac{3d}{2}\right)}{4m} = \frac{3d}{4}, \quad Y_{\text{cm}} = \frac{m\left(\frac{d}{2}+\frac{d}{2}+\frac{d}{2}+\frac{3d}{2}\right)}{4m} = \frac{3d}{4} \quad \text{and} \quad Z_{\text{cm}} = \frac{m\left(\frac{d}{2}+\frac{d}{2}+\frac{d}{2}+\frac{3d}{2}\right)}{4m} = \frac{3d}{4}.$$

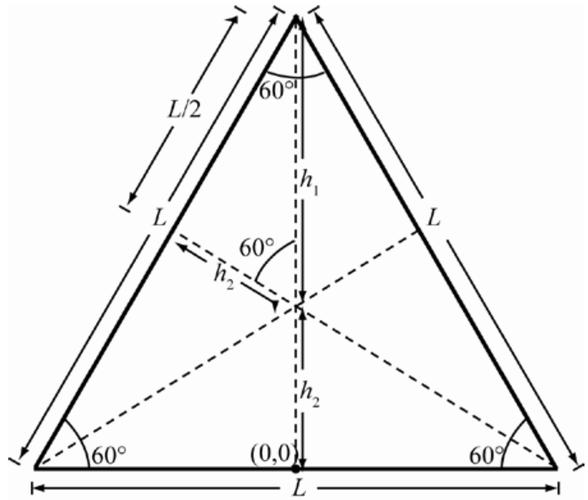
Therefore, the center of mass of the structure is located at $\vec{R} = \left(X_{\text{cm}},\ Y_{\text{cm}},\ Z_{\text{cm}}\right) = \left(\frac{3d}{4}, \frac{3d}{4}, \frac{3d}{4}\right)$.

8.19. After the explosion, the motion of the center of mass should remain unchanged. Since both masses are equal, they must be equidistant from the center of mass. If the first piece has x-coordinate x_1 and the second piece has x-coordinate x_2, then $\left|X_{\text{cm}} - x_1\right| = \left|X_{\text{cm}} - x_2\right|$. For example, since the position of the center of mass is still 100 m, one piece could be at 90 m and the other at 110 m: $\left|100-90\right| = \left|100-110\right|$.

8.20. Yes, the center of mass can be located outside the object. Take a donut for example. If the donut has a uniform mass density, then the center of mass is located at its geometric center, which would be the center of a circle. However, at the donut's center, there is no mass, there is a hole. This means the center of mass can lie outside the object.

8.21. It is possible if, for example, there are outside forces involved. The kinetic energy of an object is proportional to the momentum squared $\left(K \propto p^2\right)$. So if p increases, K increases.

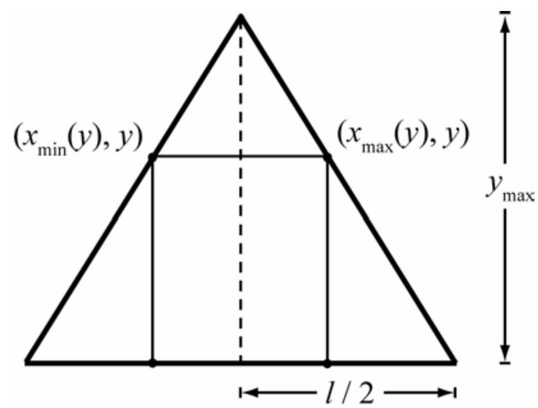
8.22. The intersection of the triangle's altitudes implies the triangle has a uniform mass density, meaning the center of mass is at the geometric center. To show this point by physical reasoning means using geometry to show where it is.



It can be seen that $h_1 \sin 60° = L/2$ and $h_1 \cos 60° = h_2$. Therefore,

$$h_1 = \frac{L}{2\sin 60°} = \frac{L}{2\left(\sqrt{3}/2\right)} = \frac{L}{\sqrt{3}} = \frac{L\sqrt{3}}{3} \Rightarrow h_2 = \frac{L\sqrt{3}}{3}\cos 60° = \frac{L\sqrt{3}}{3}\left(\frac{1}{2}\right) = \frac{L\sqrt{3}}{6}.$$

If the center of the bottom side of the triangle is (0, 0), then the center of mass is located at $\left(0, h_2\right) = \left(0, L\sqrt{3}/6\right)$. To calculate by direct measurement, note that due to symmetry by the choice of origin, the *x* coordinate of the center of mass is in the middle of the *x* axis. Therefore, $X_{\text{cm}} = 0$, which means only Y_{cm} must be determined.



Clearly, the *x* value of a point along the side of the triangle is dependent on the value of *y* for that point, meaning *x* is a function of *y*. When *y* is zero, *x* is *L*/2 and when *x* is zero, *y* is $y_{\max} = h_1 + h_2 = L\sqrt{3}/2$. The change in *x* should be linear with change in *y*, so $x = my + b$, where $m = \frac{\Delta x}{\Delta y} = \frac{\left(L/2\right) - 0}{0 - \left(L\sqrt{3}/2\right)} = -\frac{1}{\sqrt{3}}$.

Therefore, $\frac{L}{2} = -\frac{0}{\sqrt{3}} + b = 0 + b \Rightarrow b = \frac{L}{2}$ and $0 = -\frac{L\sqrt{3}}{2\sqrt{3}} + b = -\frac{L}{2} + b \Rightarrow b = \frac{L}{2}$. The equation for *x* is then given by $x\left(y\right) = -\frac{y}{\sqrt{3}} + \frac{L}{2}$. Since the mass density is uniform, the geometry of the triangle can be considered. $Y_{\text{cm}} = \frac{1}{A}\iint y\,dA$, where $A = \frac{L^2\sqrt{3}}{4}$ and $dA = dx\,dy$.

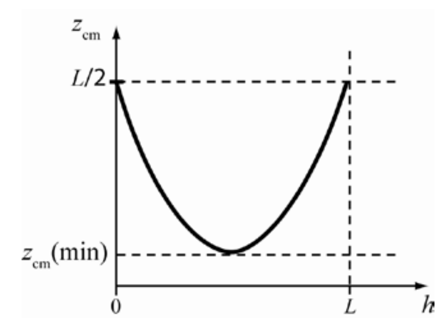
The integral then becomes:

$Y_{cm} = \frac{4}{L^2\sqrt{3}} \int_{y_{min}}^{y_{max}} y dy \int_{x_{min}(y)}^{x_{max}(y)} dx = \frac{4}{L^2\sqrt{3}} \int_0^{\frac{L\sqrt{3}}{2}} y\left(x_{max}(y) - x_{min}(y)\right) dy.$ Due to symmetry, $x_{max}(y) = -x_{min}(y)$ and $x_{max}(y) = x(y)$. Therefore,

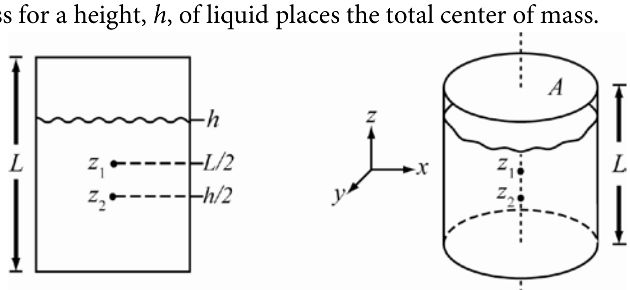
$$\begin{aligned}
Y_{cm} &= \frac{8}{L^2\sqrt{3}} \int_0^{\frac{L\sqrt{3}}{2}} yx(y) dy = \frac{8}{L^2\sqrt{3}} \int_0^{\frac{L\sqrt{3}}{2}} \left(\frac{-y^2}{\sqrt{3}} + \frac{yL}{2} \right) dy \\
&= \frac{8}{L^2\sqrt{3}} \left[\frac{y^2 L}{4} - \frac{y^3}{3\sqrt{3}} \right]_0^{\frac{L\sqrt{3}}{2}} = \frac{8}{L^2\sqrt{3}} \left[\frac{3L^3}{16} - \frac{L^3}{8} \right] = \frac{8}{L^2\sqrt{3}} \left[\frac{L^3}{16} \right] \\
&= \frac{L}{2\sqrt{3}} = \frac{L\sqrt{3}}{6}.
\end{aligned}$$

The center of mass is located at $R = (X_{cm}, Y_{cm}) = \left(0, \frac{L\sqrt{3}}{6}\right)$. This is consistent with reasoning by geometry.

8.23. (a) The empty can and the liquid should each have their centers of mass at their geometric centers, so initially the center of mass of both is at the center of the can (assuming the can is filled completely with soda). Assuming the liquid drains out uniformly, only the height changes and the cross sectional area remains constant, so the center of mass is initially at *L*/2 and changes only in height. As liquid drains, its mass *M* will drop by ΔM but the mass of the can, *m*, remains the same. As liquid drains, its center of mass will also fall such that if the liquid is at a height *h*, $0 < h < L$, its center of mass is at *h*/2. As long as $M - \Delta M > m$, the center of mass of both will also fall to some height h', $h/2 < h' < L$. Once $M - \Delta M < m$, the center of mass of both will begin to increase again until $M - \Delta M = 0$ and the center of mass is that of just the can at *L*/2. A sketch of the height of the center of mass of both as a function of liquid height is shown below.



(b) In order to determine the minimum value of the center of mass in terms of *L*, *M* and *m*, first consider where the center of mass for a height, *h*, of liquid places the total center of mass.



Z_1 is the center of mass of the can. Z_2 is the center of mass of the liquid. Notice the center of mass moves along the *z* axis only. *A* is the cross sectional area of the can in the *xy* plane. ρ_M is the density of the liquid. *h* is the height of the liquid.

The coordinate of the center of mass is given by

$$Z_{\text{cm}} = \frac{\frac{mL}{2} + \frac{Mh}{2}}{m+M}.$$

When $h = L$, $Z_{\text{cm}} = L/2$. When $h < L$, $h = \alpha L$, where $0 \le \alpha < 1$. In other words, the height of the liquid is a fraction, α, of the initial height, L. Initially the mass of the liquid is $M = \rho V = \rho AL$. When $h(\alpha) = \alpha L$, the mass of the liquid is $M(\alpha) = \rho A h(\alpha) = \alpha \rho AL = \alpha M$. This means the center of mass for some value of α is

$$Z_{\text{cm}}(\alpha) = \frac{\frac{mL}{2} + \frac{M(\alpha)h(\alpha)}{2}}{m + M(\alpha)} = \frac{\frac{mL}{2} + \frac{\alpha^2 ML}{2}}{m + \alpha M} = \frac{L}{2}\left(\frac{1+b\alpha^2}{1+b\alpha}\right).$$

where $b = M/m$ and M is the initial mass of the liquid. In order to determine the minimum value of Z_{cm}, $Z_{\text{cm}}(\alpha)$ must be minimized in terms of α to determine where α_{min} occurs and then determine $Z_{\text{cm}}(\alpha_{\text{min}})$.

$$\frac{dZ_{\text{cm}}(\alpha)}{d\alpha} = a\frac{d}{d\alpha}\left(\frac{1+b\alpha^2}{1+b\alpha}\right) = a\left[\frac{b^2\alpha^2 + 2b\alpha - b}{(1+b\alpha)^2}\right], \text{ where } a = L/2.$$

When $dZ_{\text{cm}}(\alpha)/d\alpha = 0 \Rightarrow b^2\alpha^2 + 2b\alpha - b = 0$. Using the quadratic equation, $\alpha = \frac{-1 \pm \sqrt{1+b}}{b}$. Since $b > 0$ and $\alpha > 0$, $\alpha_{\text{min}} = \frac{-1+\sqrt{1+b}}{b}$. Therefore, $Z_{\text{cm}}(\alpha_{\text{min}}) = a\left(\frac{1+b{\alpha_{\text{min}}}^2}{1+b\alpha_{\text{min}}}\right) = 2a\left(\frac{1+b-\sqrt{1+b}}{b\sqrt{1+b}}\right)$.

$$Z_{\text{cm}}(\alpha_{\text{min}}) = \frac{L\left(M + m - m\sqrt{1+\frac{M}{m}}\right)}{M\sqrt{1+\frac{M}{m}}}$$

If it is assumed that soda has a similar density to water and the can is made of aluminum, then the ratio of $M/m \approx 30$, giving a minimum Z_{cm} of about $L/6$.

8.24. (a) If the astronaut throws both at the same time, he gains their momentum of them moving at a velocity, v. If he throws one first at a velocity, v, he will recoil back at a velocity, v'. So when he throws the second item, he will gain its momentum at a velocity of $v - v'$, which is less than v. So he gains less momentum from throwing the second item after the first than if he throws both items at the same time. Therefore, he obtains maximum speed when he throws both at the same time.

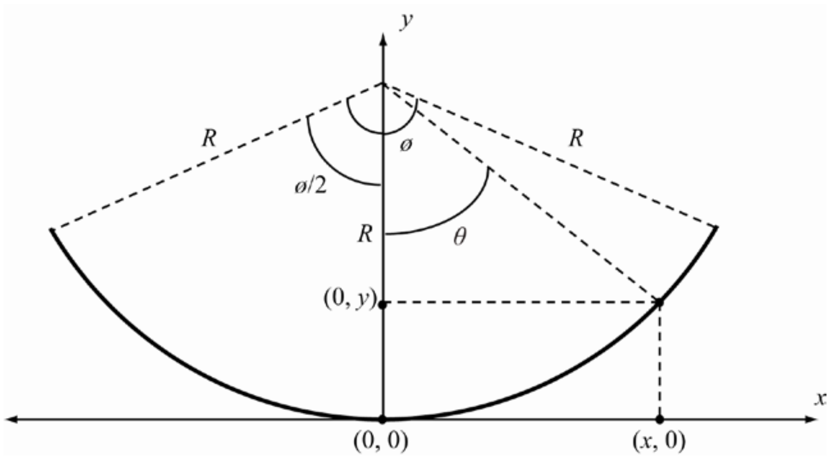
(b) If the astronaut throws the heavier object (tool box) first, it will give the astronaut a large velocity, v', so when he throws the lighter object (hammer), it will have a small velocity of $v - v'$. So its momentum contribution will be very small. However, if he throws the lighter item first, v' will be smaller in this scenario, so the momentum of the box will be dependent on $v - v'$, which is greater and contributes a large amount of momentum to the astronaut, giving him a larger velocity. Therefore, throwing the lighter object first will maximize his velocity.

(c) The absolute maximum velocity is when both items are thrown at the same time. Initially the momentum is zero and after the toss, the astronaut travels with velocity, v' and the box and hammer travel with velocity, v in the opposite direction.

$$\vec{p}_{\text{i}} = \vec{p}_{\text{f}} \Rightarrow 0 = Mv' - \left(\frac{M}{2} + \frac{M}{4}\right)v \Rightarrow v' = \frac{3}{4}v$$

Therefore, the maximum velocity is ¾ of the velocity at which he throws the two items.

8.25. Let the angle θ sweep through from $-\phi/2$ to $\phi/2$. Keeping R constant as θ increases, the length of the rod, $l = R\theta$, increases and in turn the mass, $m = \lambda l$, increases. Since the mass is uniformly distributed, the center of mass should be in the same location. So rather than bending a rod of constant length where θ and R change, keep R constant and change θ and l. Use Cartesian coordinates to determine the center of mass. Since the center of mass is a function of θ, it must be determined how the coordinates change with the angle θ.



$y = R - R\cos\theta,\ \ x = R\sin\theta,\ \ m = \lambda R\phi,\ \ dm = \lambda R d\theta$

$$X_{\text{cm}} = \frac{1}{m}\int x dm = \frac{1}{\lambda R\phi}\int_{-\phi/2}^{\phi/2} R\sin\theta \lambda R d\theta = \frac{R}{\phi}\int_{-\phi/2}^{\phi/2}\sin\theta d\theta = \left[-\frac{R}{\phi}\cos\theta\right]_{-\frac{\phi}{2}}^{\frac{\phi}{2}} = -\frac{R}{\phi}\left(\cos\frac{\phi}{2} - \cos\left(-\frac{\phi}{2}\right)\right) = 0$$

$$Y_{\text{cm}} = \frac{1}{m}\int y dm = \frac{1}{\lambda R\phi}\int_{-\phi/2}^{\phi/2}\left(R - R\cos\theta\right)\lambda R d\theta = \frac{R}{\phi}\int_{-\phi/2}^{\phi/2}\left(1-\cos\theta\right)d\theta = \left[\frac{R}{\phi}\left(\theta - \sin\theta\right)\right]_{-\frac{\phi}{2}}^{\frac{\phi}{2}}$$

$$= \frac{R}{\phi}\left(\frac{\phi}{2} - \left(-\frac{\phi}{2}\right)\right) - \frac{R}{\phi}\left(\sin\left(\frac{\phi}{2}\right) - \sin\left(-\frac{\phi}{2}\right)\right) = R - \frac{2R\sin\left(\frac{\phi}{2}\right)}{\phi}$$

$$\vec{R}_{\text{cm}} = \left(X_{\text{cm}}, Y_{\text{cm}}\right) = \left(0, R - \frac{2R\sin(\phi/2)}{\phi}\right)$$

8.26. As eggs A, B and/or C are removed, the center of mass will shift down and to the left. To determine the overall center of mass, use the center of the eggs as their center position, such that eggs A, B and C are located respectively at

$$\left(\frac{d}{2},\frac{d}{2}\right),\ \left(\frac{3d}{2},\frac{d}{2}\right),\ \left(\frac{5d}{2},\frac{d}{2}\right).$$

Since all of the eggs are of the same mass, *m*, and proportional to *d*, *m* and *d* can be factored out of the equations for X_{cm} and Y_{cm}.

(a) $X_{\text{cm}} = \frac{md}{11m}\left(2\left(-\frac{5}{2}\right) + 2\left(-\frac{3}{2}\right) + 2\left(-\frac{1}{2}\right) + \frac{1}{2} + 2\left(\frac{3}{2}\right) + 2\left(\frac{5}{2}\right)\right) = -\frac{d}{22}$, $Y_{\text{cm}} = \frac{md}{11m}\left(6\left(-\frac{1}{2}\right) + 5\left(\frac{1}{2}\right)\right) = -\frac{d}{22}$

$$\vec{R}_{\text{cm}} = \left(-\frac{d}{22}, -\frac{d}{22}\right)$$

(b) $X_{\text{cm}} = \frac{md}{11m}\left(2\left(-\frac{5}{2}\right)+2\left(-\frac{3}{2}\right)+2\left(-\frac{1}{2}\right)+2\left(\frac{1}{2}\right)+\frac{3}{2}+2\left(\frac{5}{2}\right)\right) = -\frac{3d}{22},\ Y_{\text{cm}} = \frac{md}{11m}\left(6\left(-\frac{1}{2}\right)+5\left(\frac{1}{2}\right)\right) = -\frac{d}{22}$

$\vec{R}_{\text{cm}} = \left(-\frac{3d}{22}, -\frac{d}{22}\right)$

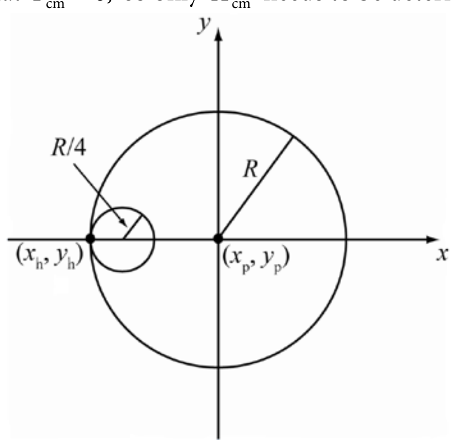
(c) $X_{\text{cm}} = \frac{md}{11m}\left(2\left(-\frac{5}{2}\right)+2\left(-\frac{3}{2}\right)+2\left(-\frac{1}{2}\right)+2\left(\frac{1}{2}\right)+2\left(\frac{3}{2}\right)+\frac{5}{2}\right) = -\frac{5d}{22},\ Y_{\text{cm}} = \frac{md}{11m}\left(6\left(-\frac{1}{2}\right)+5\left(\frac{1}{2}\right)\right) = -\frac{d}{22}$

$\vec{R}_{\text{cm}} = \left(-\frac{5d}{22}, -\frac{d}{22}\right)$

(d) $X_{\text{cm}} = \frac{md}{9m}\left(2\left(-\frac{5}{2}\right)+2\left(-\frac{3}{2}\right)+2\left(-\frac{1}{2}\right)+\frac{1}{2}+\frac{3}{2}+\frac{5}{2}\right) = -\frac{d}{2},\qquad Y_{\text{cm}} = \frac{md}{9m}\left(6\left(-\frac{1}{2}\right)+3\left(\frac{1}{2}\right)\right) = -\frac{d}{6}$

$\vec{R}_{\text{cm}} = \left(-\frac{d}{2}, -\frac{d}{6}\right)$

8.27. The center of the pizza is at $(0,0)$ and the center of the piece cut out is at $(-3R/4, 0)$. Assume the pizza and the hole have a uniform mass density (though the hole is considered to have a negative mass). Then the center of mass can be determined from geometry. Also, because of symmetry of the two circles and their *y* position, it can be said that $Y_{\text{cm}} = 0$, so only X_{cm} needs to be determined.



$A_{\text{p}} = \pi R^2,\ A_{\text{h}} = \pi\left(\frac{R}{4}\right)^2 = \pi\frac{R^2}{16},\ \left(x_{\text{p}}, y_{\text{p}}\right) = (0,0),\ \left(x_{\text{h}}, y_{\text{h}}\right) = \left(-\frac{3}{4}R, 0\right)$

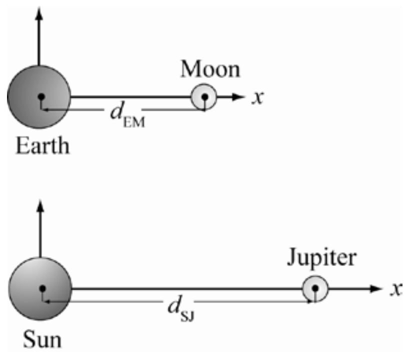
$$X_{\text{cm}} = \frac{x_{\text{p}}A_{\text{p}} - x_{\text{h}}A_{\text{h}}}{A_{\text{p}} - A_{\text{h}}} = \frac{0-\left(-\frac{3}{4}R\right)\left(\frac{\pi R^2}{16}\right)}{\pi R^2 - \frac{\pi R^2}{16}} = \frac{R}{20},\quad \vec{R}_{\text{cm}} = \left(\frac{R}{20}, 0\right)$$

8.28. Since the overall mass of the hourglass does not change and the center of mass must move from the top half to the bottom half, then the center of mass velocity, v_{cm}, must be non-zero and pointing down. As the sand flows from the top part of the hourglass to the lower part, v_{cm} changes with time. The magnitude of v_{cm} is larger when the sand has just started to flow than just before all the sand has flowed through. Thus $dv_{\text{cm}}/dt = a_{\text{cm}}$ must be in the opposite direction from v_{cm}, which is the upward direction. The scale must supply the force required to produce this upward acceleration, so the hourglass weighs more when the sand is flowing than when the sand is stationary. You can find a published solution to a similar version of this problem at the following reference: K.Y. Shen and Bruce L. Scott, American Journal of Physics, **53**, 787 (1985).

Exercises

8.29. **THINK:** Determine (a) the distance, d_1, from the center of mass of the Earth-Moon system to the geometric center of the Earth and (b) the distance, d_2, from the center of mass of the Sun-Jupiter system to the geometric center of the Sun. The mass of the Earth is approximately $m_{\text{E}} = 5.9742 \cdot 10^{24}$ kg and the mass of the Moon is approximately $m_{\text{M}} = 7.3477 \cdot 10^{22}$ kg. The distance between the center of the Earth to the center of the Moon is $d_{\text{EM}} = 384,400$ km. Also, the mass of the Sun is approximately $m_{\text{S}} = 1.98892 \cdot 10^{30}$ kg and the mass of Jupiter is approximately $m_{\text{J}} = 1.8986 \cdot 10^{27}$ kg. The distance between the center of the Sun and the center of Jupiter is $d_{\text{SJ}} = 778,300,000$ km.

SKETCH:



RESEARCH: Determine the center of mass of the two object system from $\vec{R} = \dfrac{\vec{r}_1 m_1 + \vec{r}_2 m_2}{m_1 + m_2}$. By considering the masses on the *x*-axis (as sketched), the one dimensional equation can be used for *x*. Assuming a uniform, spherically symmetric distribution of each planet's mass, they can be modeled as point particles. Finally, by placing the Earth (Sun) at the origin of the coordinate system, the center of mass will be determined with respect to the center of the Earth (Sun), i.e. $d_1(d_2) = x$.

SIMPLIFY:

(a) $d_1 = x = \dfrac{x_1 m_{\text{E}} + x_2 m_{\text{M}}}{m_{\text{E}} + m_{\text{M}}} = \dfrac{d_{\text{EM}} m_{\text{M}}}{m_{\text{E}} + m_{\text{M}}}$

(b) $d_2 = x = \dfrac{x_1 m_{\text{S}} + x_2 m_{\text{J}}}{m_{\text{S}} + m_{\text{J}}} = \dfrac{d_{\text{SJ}} m_{\text{J}}}{m_{\text{S}} + m_{\text{J}}}$

CALCULATE:

(a) $d_1 = \dfrac{(384,400 \text{ km})(7.3477 \cdot 10^{22} \text{ kg})}{(5.9742 \cdot 10^{24} \text{ kg}) + (7.3477 \cdot 10^{22} \text{ kg})} = \dfrac{2.8244559 \cdot 10^{28} \text{ km} \cdot \text{kg}}{6.047677 \cdot 10^{24} \text{ kg}} = 4670.3 \text{ km}$

(b) $d_2 = \dfrac{(7.783 \cdot 10^{8} \text{ km})(1.8986 \cdot 10^{27} \text{ kg})}{(1.98892 \cdot 10^{30} \text{ kg}) + (1.8986 \cdot 10^{27} \text{ kg})} = 742247.6 \text{ km}$

ROUND:

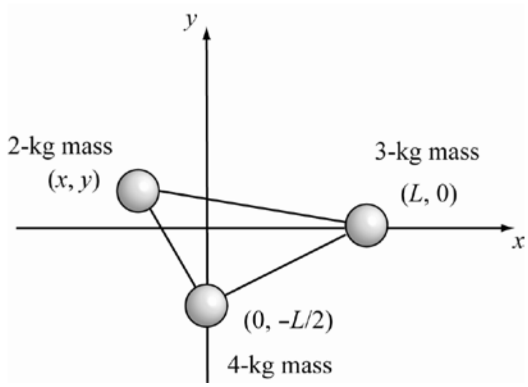
(a) d_{EM} has four significant figures, so $d_1 = 4670.$ km.

(b) d_{SJ} has four significant figures, so $d_2 = 742,200$ km.

DOUBLE-CHECK: In each part, the distance d_1 / d_2 is much less than half the separation distance $d_{\text{EM}} / d_{\text{SJ}}$. This makes sense as the center of mass should be closer to the more massive object in the two body system.

8.30. **THINK:** The center of mass coordinates for the system are $(L/4, -L/5)$. The masses are $m_1 = 2$ kg, $m_2 = 3$ kg and $m_3 = 4$ kg. The coordinates for m_2 are $(L, 0)$ and the coordinates for m_3 are $(0, -L/2)$. Determine the coordinates for m_1.

SKETCH:



RESEARCH: The *x* and *y* coordinates for m_1 can be determined from the equations for the center of mass in each dimension:

$$X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i \text{ and } Y = \frac{1}{M}\sum_{i=1}^{n} y_i m_i.$$

SIMPLIFY: $X = \dfrac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3} \Rightarrow x_1 = \dfrac{1}{m_1}\left(X\left(m_1 + m_2 + m_3\right) - x_2 m_2 - x_3 m_3\right)$

Similarly, $y_1 = \dfrac{1}{m_1}\left(Y\left(m_1 + m_2 + m_3\right) - y_2 m_2 - y_3 m_3\right)$.

CALCULATE: $x_1 = \left(\dfrac{1}{2 \text{ kg}}\right)\left(\dfrac{L}{4}\left(2 \text{ kg} + 3 \text{ kg} + 4 \text{ kg}\right) - L\left(3 \text{ kg}\right) - 0\left(4 \text{ kg}\right)\right) = -\dfrac{3}{8}L$

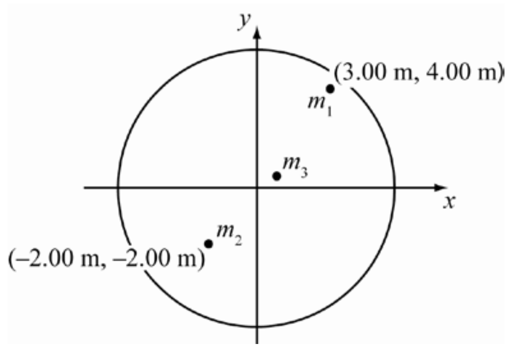
$$y_1 = \left(\frac{1}{2 \text{ kg}}\right)\left(-\frac{L}{5}\left(2 \text{ kg} + 3 \text{ kg} + 4 \text{ kg}\right) - 0\left(3 \text{ kg}\right) - \left(-\frac{L}{2}\right)\left(4 \text{ kg}\right)\right) = \frac{1}{10}L$$

ROUND: Rounding is not necessary since the initial values and the results are fractions, so m_1 is located at $(-3L/8, L/10)$.

DOUBLE-CHECK: The coordinates for m_1 are reasonable: since X_{cm} is positive and Y_{cm} is negative and both coordinates have comparatively small values (and thus the center of mass is close to the origin), it makes sense that *x* will be negative to balance the 3-kg mass and *y* will be positive to balance the 4-kg mass.

8.31. **THINK:** The mass and location of the first acrobat are known to be $m_1 = 30.0$ kg and $\vec{r}_1 = (3.00 \text{ m}, 4.00 \text{ m})$. The mass and location of the second acrobat are $m_2 = 40.0$ kg and $\vec{r}_2 = (-2.00 \text{ m}, -2.00 \text{ m})$. The mass of the third acrobat is $m_3 = 20.0$ kg. Determine the position of the third acrobat, $\vec{r}_3$, when the center of mass (com) is at the origin.

SKETCH:



RESEARCH: Let M be the sum of the three masses. The coordinates of m_3 can be determined from the center of mass equations for each dimension,

$$X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i \text{ and } Y = \frac{1}{M}\sum_{i=1}^{n} y_i m_i.$$

SIMPLIFY: Since $X = 0$, $X = \frac{1}{M}\left(x_1m_1 + x_2m_2 + x_3m_3\right) = 0 \Rightarrow x_3 = \frac{\left(-x_1m_1 - x_2m_2\right)}{m_3}$. Similarly, with $Y = 0$,

$$y_3 = \frac{\left(-y_1m_1 - y_2m_2\right)}{m_3}.$$

CALCULATE: $x_3 = \frac{\left(-(3.00\text{ m})(30.0\text{ kg}) - (-2.00\text{ m})(40.0\text{ kg})\right)}{20.0\text{ kg}} = -0.500\text{ m},$

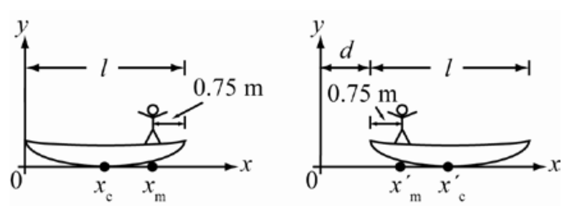
$$y_3 = \frac{\left(-(4.00\text{ m})(30.0\text{ kg}) - (-2.00\text{ m})(40.0\text{ kg})\right)}{20.0\text{ kg}} = -2.00\text{ m}$$

ROUND: $\vec{r}_3 = \left(-0.500\text{ m}, -2.00\text{ m}\right)$

DOUBLE-CHECK: The resulting location is similar to the locations of the other acrobats.

8.32. **THINK:** The man's mass is $m_\text{m} = 55$ kg and the canoe's mass is $m_\text{c} = 65$ kg. The canoe's length is $l = 4.0$ m. The man moves from 0.75 m from the back of the canoe to 0.75 m from the front of the canoe. Determine how far the canoe moves, d.

SKETCH:



RESEARCH: The center of mass position for the man and canoe system does not change in our external reference frame. To determine d, the center of mass location must be determined before the canoe moves. Then the new location for the canoe after the man moves can be determined given the man's new position and the center of mass position. Assume the canoe has a uniform density such that its center of mass location is at the center of the canoe, $x_\text{c} = 2.0$ m. The man's initial position is $x_\text{m} = l - 0.75\text{ m} = 3.25$ m. After moving, the canoe is located at x'_c and the man is located at $x'_\text{m} = x'_\text{c} + a$. a is the relative position of the man with respect to the canoe's center of mass and $a = -l/2 + 0.75\text{ m} = -1.25$ m. Then the distance the canoe moves is $d = x'_\text{c} - x_\text{c}$.

SIMPLIFY:

$$X=\frac{1}{M}\sum_{i=1}^{n}x_i m_i.$$

The center of mass is $X=\frac{1}{M}\left(x_{\text{m}}m_{\text{m}}+x_{\text{c}}m_{\text{c}}\right)$. After moving,

$X=\frac{1}{M}\left(x'_{\text{m}}m_{\text{m}}+x'_{\text{c}}m_{\text{c}}\right)=\frac{1}{M}\left(\left(x'_{\text{c}}+a\right)m_{\text{m}}+x'_{\text{c}}m_{\text{c}}\right)$. Since X does not change, the equations can be equated:

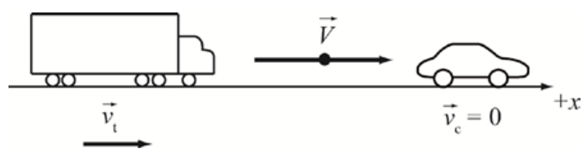
$$\frac{1}{M}\left(\left(x'_{\text{c}}+a\right)m_{\text{m}}+x'_{\text{c}}m_{\text{c}}\right)=\frac{1}{M}\left(x_{\text{m}}m_{\text{m}}+x_{\text{c}}m_{\text{c}}\right)$$

This implies $x_{\text{m}}m_{\text{m}}+x_{\text{c}}m_{\text{c}}=x'_{\text{c}}m_{\text{m}}+x'_{\text{c}}m_{\text{c}}+am_{\text{m}} \Rightarrow x'_{\text{c}}=\frac{x_{\text{m}}m_{\text{m}}+x_{\text{c}}m_{\text{c}}-am_{\text{m}}}{m_{\text{m}}+m_{\text{c}}}$.

CALCULATE: $x'_c=\frac{(3.25\text{ m})(55.0\text{ kg})+(2.00\text{ m})(65.0\text{ kg})-(-1.25\text{ m})(55.0\text{ kg})}{55.0\text{ kg}+65.0\text{ kg}}=3.1458\text{ m}$

Then $d=3.1458\text{ m}-2.00\text{ m}=1.1458\text{ m}$.
ROUND: As each given value has three significant figures, d = 1.15 m.
DOUBLE-CHECK: This distance is less than the distance traveled by the man (2.5 m), as it should be to preserve the center of mass location.

8.33. **THINK:** The mass of the car is $m_{\text{c}}=2.00\text{ kg}$ and its initial speed is $v_{\text{c}}=0$. The mass of the truck is $m_{\text{t}}=3.50\text{ kg}$ and its initial speed is $v_{\text{t}}=4.00\text{ m/s}$ toward the car. Determine (a) the velocity of the center of mass, $\vec{V}$, and (b) the velocities of the truck, $\vec{v}'_{\text{t}}$ and the car, $\vec{v}'_{\text{c}}$ with respect to the center of mass.
SKETCH:



RESEARCH:

(a) The velocity of the center of mass can be determined from $\vec{V}=\frac{1}{M}\sum_{i=1}^{n}m_i\vec{v}_i$.

Take $\vec{v}_{\text{t}}$ to be in the positive x-direction.
(b) Generally, the relative velocity, $\vec{v}'$, of an object with velocity, $\vec{v}$, in the lab frame is given by $\vec{v}'=\vec{v}-\vec{V}$, where $\vec{V}$ is the velocity of the relative reference frame. Note the speeds of the car and the truck relative to the center of mass do not change after their collision, but the relative velocities change direction; that is, $\vec{v}'_{\text{t}}(\text{before collision})=-\vec{v}'_{\text{t}}(\text{after collision})$ and similarly for the car's relative velocity.
SIMPLIFY:

(a) Substituting $\vec{v}_{\text{c}}=0$ and $M=m_{\text{c}}+m_{\text{t}}$, $\vec{V}=\frac{1}{M}\left(m_{\text{c}}\vec{v}_{\text{c}}+m_{\text{t}}\vec{v}_{\text{t}}\right)$ becomes $\vec{V}=\frac{\left(m_{\text{t}}\vec{v}_{\text{t}}\right)}{\left(m_{\text{c}}+m_{\text{t}}\right)}$.

(b) $\vec{v}'_{\text{t}}$ and $\vec{v}'_{\text{c}}$ before the collision are $\vec{v}'_{\text{t}}=\vec{v}_{\text{t}}-\vec{V}$ and $\vec{v}'_{\text{c}}=\vec{v}_{\text{c}}-\vec{V}=-\vec{V}$.
CALCULATE:

(a) $\vec{V}=\frac{(3.50\text{ kg})(4.00\hat{x}\text{ m/s})}{(3.50\text{ kg}+2.00\text{ kg})}=2.545\hat{x}\text{ m/s}$

(b) $\vec{v}'_{\text{t}}=(4.00\hat{x}\text{ m/s})-(2.545\hat{x}\text{ m/s})=1.4545\hat{x}\text{ m/s},\ \vec{v}'_{\text{c}}=-2.545\hat{x}\text{ m/s}$

ROUND: There are three significant figures for each given value, so the results should be rounded to the same number of significant figures.

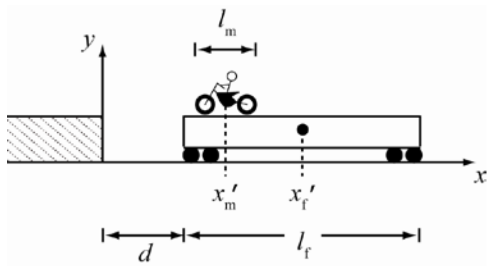
(a) $\vec{V}=2.55\hat{x}\text{ m/s}$

(b) Before the collision, $\vec{v}_{\text{t}}' = 1.45\hat{x}$ m/s and $\vec{v}_{\text{c}}' = -2.55\hat{x}$ m/s. This means that after the collision, the velocities with respect to the center of mass become $\vec{v}_{\text{t}}' = -1.45\hat{x}$ m/s and $\vec{v}_{\text{c}}' = 2.55\hat{x}$ m/s.

DOUBLE-CHECK: $\vec{V}$ is between the initial velocity of the truck and the initial velocity of the car, as it should be.

8.34. **THINK:** The motorcycle with rider has a mass of $m_{\text{m}} = 350.$ kg. The flatcar's mass is $m_{\text{f}} = 1500.$ kg. The length of the motorcycle is $l_{\text{m}} = 2.00$ m and the length of the flatcar is $l_{\text{f}} = 20.0$ m. The motorcycle starts at one of end of the flatcar. Determine the distance, *d*, that the flatcar will be from the platform when the motorcycle reaches the end of the flatcar.

SKETCH: After the motorcycle and rider drive down the platform:



RESEARCH: The flatcar-motorcycle center of mass stays in the same position while the motorcycle moves. First, the center of mass must be determined before the motorcycle moves. Then the new location of the flatcar's center of mass can be determined given the center of mass for the system and the motorcycle's final position. Then the distance, *d*, can be determined. Assume that the motorcycle and rider's center of mass and the flatcar's center of mass are located at their geometric centers. Take the initial center of mass position for the motorcycle to be $x_{\text{m}} = l_{\text{f}} - l_{\text{m}}/2$, and the initial center of mass for the flatcar to be $x_{\text{f}} = l_{\text{f}}/2$. The final position of the center of mass for the motorcycle will be $x_{\text{m}}' = d + l_{\text{m}}/2$, and the final position for the flatcar will be $x_{\text{f}}' = d + l_{\text{f}}/2$. Then *d* can be determined from

$$X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i .$$

SIMPLIFY: Originally, $X = \frac{1}{M}\left(x_{\text{m}}m_{\text{m}} + x_{\text{f}}m_{\text{f}}\right)$. After the motorcycle moves, $X = \frac{1}{M}\left(x_{\text{m}}'m_{\text{m}} + x_{\text{f}}'m_{\text{f}}\right)$.

As the center of mass remains constant, the two expressions can be equated:

$$\frac{1}{M}\left(x_{\text{m}}m_{\text{m}} + x_{\text{f}}m_{\text{f}}\right) = \frac{1}{M}\left(x_{\text{m}}'m_{\text{m}} + x_{\text{f}}'m_{\text{f}}\right)$$

$$x_{\text{m}}m_{\text{m}} + x_{\text{f}}m_{\text{f}} = \left(d + \frac{1}{2}l_{\text{m}}\right)m_{\text{m}} + \left(d + \frac{1}{2}l_{\text{f}}\right)m_{\text{f}}$$

$$x_{\text{m}}m_{\text{m}} + x_{\text{f}}m_{\text{f}} = d\left(m_{\text{m}} + m_{\text{f}}\right) + \frac{1}{2}l_{\text{m}}m_{\text{m}} + \frac{1}{2}l_{\text{f}}m_{\text{f}}$$

$$d = \frac{\left(x_{\text{m}} - \frac{1}{2}l_{\text{m}}\right)m_{\text{m}} + \left(x_{\text{f}} - \frac{1}{2}l_{\text{f}}\right)m_{\text{f}}}{m_{\text{m}} + m_{\text{f}}}$$

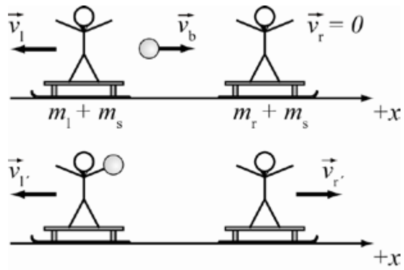
$x_{\text{m}} = l_{\text{f}} - \frac{l_{\text{m}}}{2}$ and $x_{\text{f}} = \frac{l_{\text{f}}}{2}$, therefore $d = \frac{\left(l_{\text{f}} - l_{\text{m}}\right)m_{\text{m}}}{m_{\text{m}} + m_{\text{f}}}$.

CALCULATE: $d = \frac{(20.0\text{ m} - 2.00\text{ m})(350.\text{ kg})}{350.\text{ kg} + 1500.\text{ kg}} = 3.4054\text{ m}$

ROUND: m_{m} has three significant figures, so the result should be rounded to d = 3.41 m.
DOUBLE-CHECK: It is reasonable that the distance moved is less than length of the flatcar.

8.35. **THINK:** The mass of the sled is $m_s = 10.0$ kg, the mass of the ball is $m_b = 5.00$ kg, and the mass of the student on the left is $m_l = 50.0$ kg. His relative ball-throwing speed is $v_{bl} = 10.0$ m/s. The mass of the student on the right is $m_r = 45.0$ kg and his relative ball-throwing speed is $v_{br} = 12.0$ m/s. Determine (a) the speed of the student on the left, v_l, after first throwing the ball, (b) the speed of the student on the right, v_r, after catching the ball, (c) the speed of the student on the left after catching the pass, v_l', and (d) the speed of the student on the right after throwing the pass, v_r'.

SKETCH:



RESEARCH: Momentum is conserved between each student and ball system. For each step, use $\vec{P}_i = \vec{P}_f$. In addition, the relative velocity of the ball is the difference between its velocity in the lab frame and the velocity of the student in the lab frame who has thrown it. That is, $\vec{v}_{bl} = \vec{v}_b - \vec{v}_l$ and $\vec{v}_{br} = \vec{v}_b - \vec{v}_r$. Recall each student begins at rest.

SIMPLIFY:

(a) Determine v_l after the ball is first thrown:

$$\vec{P}_i = \vec{P}_f \Rightarrow 0 = (m_s + m_l)\vec{v}_l + m_b\vec{v}_b \Rightarrow 0 = (m_s + m_l)\vec{v}_l + m_b(\vec{v}_{bl} + \vec{v}_l) \Rightarrow \vec{v}_l = -\frac{m_b\vec{v}_{bl}}{m_s + m_l + m_b}.$$

(b) Determine $\vec{v}_r$ after the student catches the ball. The velocity of the ball, $\vec{v}_b$, in the lab frame is needed. From part (a), $\vec{v}_l$ is known. Then $\vec{v}_b = \vec{v}_{bl} + \vec{v}_l$. So, $\vec{v}_b$ is known before it is caught. Now, for the student on the right catching the ball,

$$\vec{P}_i = \vec{P}_f \Rightarrow m_b\vec{v}_b = (m_b + m_r + m_s)\vec{v}_r \Rightarrow \vec{v}_r = \frac{m_b\vec{v}_b}{m_b + m_r + m_s}.$$

(c) Now the student on the right throws the ball and the student on the left catches it. To determine $\vec{v}_l'$, the velocity of the ball after it is thrown, $\vec{v}_b'$, is needed. It is known that $\vec{v}_{br} = \vec{v}_b - \vec{v}_r$. Then to determine $\vec{v}_b'$, consider the situation when the student on the right throws the ball. For the student on the right: $P_i = P_f \Rightarrow (m_s + m_r + m_b)\vec{v}_r = m_b\vec{v}_b' + (m_r + m_s)\vec{v}_r'$, where $\vec{v}_r$ is known from part (b) and $\vec{v}_{br} = \vec{v}_b' - \vec{v}_r' \Rightarrow \vec{v}_r' = \vec{v}_b' - \vec{v}_{br}$. Then, the fact that $(m_s + m_r + m_b)\vec{v}_r = m_b\vec{v}_b' + (m_r + m_s)(\vec{v}_b' - \vec{v}_{br})$ implies $\vec{v}_b' = \frac{(m_s + m_r + m_b)\vec{v}_r + (m_r + m_s)\vec{v}_{br}}{m_b + m_r + m_s}$. With $\vec{v}_b'$ known, consider the student on the left catching this ball: $P_i = P_f \Rightarrow m_b\vec{v}_b' + (m_l + m_s)\vec{v}_l = (m_b + m_l + m_s)\vec{v}_l'$. $\vec{v}_l$ is known from part (a) and $\vec{v}_b'$ has just been determined, so $\vec{v}_l' = \frac{m_b\vec{v}_b' + (m_l + m_s)\vec{v}_l}{m_b + m_l + m_s}$.

(d) $\vec{v}_{br} = \vec{v}_b' - \vec{v}_r' \Rightarrow \vec{v}_r' = \vec{v}_b' - \vec{v}_{br}$ and $\vec{v}_b'$ has been determined in part (c).

CALCULATE:

(a) $\vec{v}_l = -\frac{(5.00 \text{ kg})(10.0 \text{ m/s})}{10.0 \text{ kg} + 50.0 \text{ kg} + 5.00 \text{ kg}} = -0.76923 \text{ m/s}$

(b) $\vec{v}_b = 10.0 \text{ m/s} - 0.769 \text{ m/s} = 9.231 \text{ m/s}$, $\vec{v}_r = \frac{(5.00 \text{ kg})(9.23077 \text{ m/s})}{5.00 \text{ kg} + 45.0 \text{ kg} + 10.0 \text{ kg}} = 0.76923 \text{ m/s}$

(c) The ball is thrown to the left, or along the $-\hat{x}$ axis by the student on the right. That is, $\vec{v}_{\text{br}} = -12.0$ m/s.

$$\vec{v}_{\text{b}}' = \frac{(10.0 \text{ kg} + 45.0 \text{ kg} + 5.00 \text{ kg})(0.769 \text{ m/s}) + (45.0 \text{ kg} + 10.0 \text{ kg})(-12.0 \text{ m/s})}{5.00 \text{ kg} + 45.0 \text{ kg} + 10.0 \text{ kg}} = -10.23100 \text{ m/s}$$

$$\vec{v}_{\text{l}}' = \frac{(5.00 \text{ kg})(-10.2310 \text{ m/s}) + (50.0 \text{ kg} + 10.0 \text{ kg})(-0.769 \text{ m/s})}{5.00 \text{ kg} + 50.0 \text{ kg} + 10.0 \text{ kg}} = -1.49685 \text{ m/s}$$

(d) $\vec{v}_{\text{r}}' = (-10.231 \text{ m/s}) - (-12.0 \text{ m/s}) = 1.769 \text{ m/s}$

ROUND:

(a) $\vec{v}_{\text{l}} = -0.769$ m/s (to the left)

(b) $\vec{v}_{\text{r}} = 0.769$ m/s (to the right)

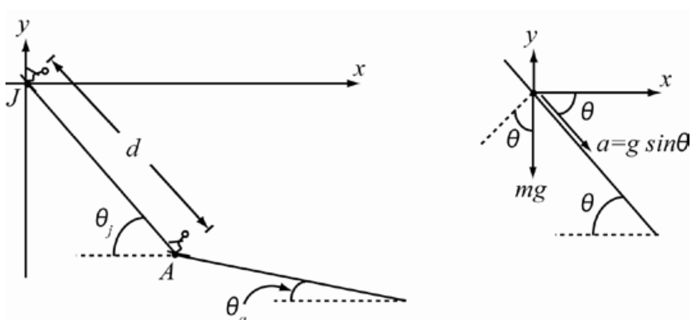
(c) $\vec{v}_{\text{l}}' = -1.50$ m/s (to the left)

(d) $\vec{v}_{\text{r}}' = 1.77$ m/s (to the right)

DOUBLE-CHECK: Before rounding, $|\vec{v}_{\text{l}}'| > |\vec{v}_{\text{l}}| > 0$ (where the initial speed was zero) and $|\vec{v}_{\text{r}}'| > |\vec{v}_{\text{r}}| > 0$, as expected.

8.36. **THINK:** Jack's mass is $m_{\text{J}} = 88.0$ kg. Jack's initial position is taken as $(0,0)$ and the angle of his slope is $\theta_{\text{J}} = 35.0°$. The distance of his slope is d = 100. m. Annie's mass is $m_{\text{A}} = 64.0$ kg. Her slope angle is $\theta_{\text{A}} = 20.0°$. Take her initial position to be $(d\cos\theta_{\text{J}}, -d\sin\theta_{\text{J}})$. Determine the acceleration, velocity and position vectors of their center of mass as functions of time, before Jack reaches the less steep section.

SKETCH:



RESEARCH: To determine the acceleration, velocity and position vectors for the center of mass, the vectors must be determined in each direction. Assuming a constant acceleration, the familiar constant acceleration equations can be used. In addition,

$$\vec{R} = \frac{1}{M}\sum_{i=1}^{n}\vec{r}_i m_i, \quad \vec{V} = \frac{d\vec{R}}{dt} = \frac{1}{M}\sum_{i=1}^{n}\vec{v}_i m_i, \quad \vec{A} = \frac{d\vec{V}}{dt} = \frac{1}{M}\sum_{i=1}^{n}\vec{a}_i m_i,$$

where each equation can be broken into its vector components.

SIMPLIFY: The magnitude of the net acceleration of each skier is $a = g\sin\theta$ down the incline of angle, θ. In the x-direction, $a_{\text{J}x} = (g\sin\theta_{\text{J}})\cos\theta_{\text{J}}$ and $a_{\text{A}x} = (g\sin\theta_{\text{A}})\cos\theta_{\text{A}}$. In the y-direction, $a_{\text{J}y} = -(g\sin\theta_{\text{J}})\sin\theta_{\text{J}} = -g\sin^2\theta_{\text{J}}$ and $a_{\text{A}y} = -(a\sin\theta_{\text{A}})\sin\theta_{\text{A}} = -g\sin^2\theta_{\text{A}}$. Then,

$$A_x = \frac{1}{M}(m_{\text{J}}a_{\text{J}x} + m_{\text{A}}a_{\text{A}x}) = \frac{g}{M}(m_{\text{J}}\sin\theta_{\text{J}}\cos\theta_{\text{J}} + m_{\text{A}}\sin\theta_{\text{A}}\cos\theta_{\text{A}}), \text{ where } M = m_{\text{J}} + m_{\text{A}} \text{ and}$$

$$A_y = \frac{1}{M}(m_{\text{J}}a_{\text{J}y} + m_{\text{A}}a_{\text{A}y}) = -\frac{g}{M}(m_{\text{J}}\sin^2\theta_{\text{J}} + m_{\text{A}}\sin^2\theta_{\text{A}}).$$

Each skier starts from rest. In the x-direction, $v_{\text{J}x} = a_{\text{J}x}t = g\sin\theta_{\text{J}}\cos\theta_{\text{J}}t$ and $v_{\text{A}x} = a_{\text{A}x}t = g\sin\theta_{\text{A}}\cos\theta_{\text{A}}t$. In the y-direction, $v_{\text{J}y} = a_{\text{J}y}t = -g\sin^2\theta_{\text{J}}t$ and $v_{\text{A}y} = a_{\text{A}y}t = -g\sin^2\theta_{\text{A}}t$.

Then,

$$V_x = \frac{1}{M}\left(m_{\text{J}}v_{\text{J}x} + m_{\text{A}}v_{\text{A}x}\right) = \frac{g}{M}\left(m_{\text{J}}\sin\theta_{\text{J}}\cos\theta_{\text{J}} + m_{\text{A}}\sin\theta_{\text{A}}\cos\theta_{\text{A}}\right)t = A_x t \text{ and}$$

$$V_y = \frac{1}{M}\left(m_{\text{J}}v_{\text{J}y} + m_{\text{A}}v_{\text{A}y}\right) = -\frac{g}{M}\left(m_{\text{J}}\sin^2\theta_{\text{J}} + m_{\text{A}}\sin^2\theta_{\text{A}}\right)t = A_y t.$$

The position in the x-direction is given by:

$$x_{\text{J}} = \frac{1}{2}a_{\text{J}x}t^2 + x_{\text{J}0} = \frac{1}{2}g\sin\theta_{\text{J}}\cos\theta_{\text{J}}t^2 \text{ and } x_{\text{A}} = \frac{1}{2}a_{\text{A}x}t^2 + x_{\text{A}0} = \frac{1}{2}g\sin\theta_{\text{A}}\cos\theta_{\text{A}}t^2 + d\cos\theta_{\text{J}}.$$

In the y-direction,

$$y_{\text{J}} = \frac{1}{2}a_{\text{J}y}t^2 + y_{\text{J}0} = -\frac{1}{2}g\sin^2\theta_{\text{J}}t^2 \text{ and } y_{\text{A}} = \frac{1}{2}a_{\text{A}y}t^2 + y_{\text{A}0} = -\frac{1}{2}g\sin^2\theta_{\text{A}}t^2 - d\sin\theta_{\text{J}}.$$

Then,

$$X = \frac{1}{M}\left(m_{\text{J}}x_{\text{J}} + m_{\text{A}}x_{\text{A}}\right) = \frac{1}{M}\left(\frac{1}{2}m_{\text{J}}g\sin\theta_{\text{J}}\cos\theta_{\text{J}}t^2 + \frac{1}{2}m_{\text{A}}g\sin\theta_{\text{A}}\cos\theta_{\text{A}}t^2 + m_{\text{A}}d\cos\theta_{\text{J}}\right) = \frac{1}{2}A_x t^2 + \frac{m_{\text{A}}}{M}d\cos\theta_{\text{J}}$$

$$Y = \frac{1}{M}\left(m_{\text{J}}y_{\text{J}} + m_{\text{A}}y_{\text{A}}\right) = -\frac{1}{M}\left(\frac{1}{2}m_{\text{J}}g\sin^2\theta_{\text{J}}t^2 + \frac{1}{2}m_{\text{A}}g\sin^2\theta_{\text{A}}t^2 + m_{\text{A}}d\sin\theta_{\text{J}}\right) = \frac{1}{2}A_y t^2 - \frac{m_{\text{A}}}{M}d\sin\theta_{\text{J}}.$$

CALCULATE:

$$A_x = \frac{\left(9.81 \text{ m/s}^2\right)}{88.0 \text{ kg} + 64.0 \text{ kg}}\left(\left(88.0 \text{ kg}\right)\sin 35.0°\cos 35.0° + \left(64.0 \text{ kg}\right)\sin 20.0°\cos 20.0°\right) = 3.996 \text{ m/s}^2$$

$$A_y = -\frac{\left(9.81 \text{ m/s}^2\right)}{88.0 \text{ kg} + 64.0 \text{ kg}}\left(\left(88.0 \text{ kg}\right)\sin^2\left(35.0°\right) + \left(64.0 \text{ kg}\right)\sin^2\left(20.0°\right)\right) = -2.352 \text{ m/s}^2$$

$$V_x = \left(3.996 \text{ m/s}^2\right)t, \quad V_y = \left(-2.352 \text{ m/s}^2\right)t$$

$$X = \frac{1}{2}\left(3.996 \text{ m/s}^2\right)t^2 + \frac{64.0 \text{ kg}}{\left(88.0 \text{ kg} + 64.0 \text{ kg}\right)}\left(100. \text{ m}\right)\cos\left(35.0°\right) = \left(1.998 \text{ m/s}^2\right)t^2 + 34.49 \text{ m}$$

$$Y = \frac{1}{2}\left(-2.352 \text{ m/s}^2\right)t^2 - \frac{64.0 \text{ kg}}{\left(88.0 \text{ kg} + 64.0 \text{ kg}\right)}\left(100. \text{ m}\right)\sin\left(35.0°\right) = \left(-1.176 \text{ m/s}^2\right)t^2 - 24.1506 \text{ m}$$

ROUND: Rounding to three significant figures, $A_x = 4.00 \text{ m/s}^2$, $A_y = -2.35 \text{ m/s}^2$, $V_x = \left(4.00 \text{ m/s}^2\right)t$ and $V_y = \left(-2.35 \text{ m/s}^2\right)t$, $X = \left(2.00 \text{ m/s}^2\right)t^2 + 34.5 \text{ m}$ and $Y = \left(-1.18 \text{ m/s}^2\right)t^2 - 24.2 \text{ m}$.

DOUBLE-CHECK: The acceleration of the center of mass is not time dependent.

8.37. **THINK:** The proton's mass is $m_{\text{p}} = 1.6726 \cdot 10^{-27}$ kg and its initial speed is $v_{\text{p}} = 0.700c$ (assumed to be in the lab frame). The mass of the tin nucleus is $m_{\text{sn}} = 1.9240 \cdot 10^{-25}$ kg (assumed to be at rest). Determine the speed of the center of mass, v, with respect to the lab frame.

SKETCH: A sketch is not necessary.

RESEARCH: The given speeds are in the lab frame. To determine the speed of the center of mass use $V = \frac{1}{M}\sum_{i=1}^{n} m_i v_i$.

SIMPLIFY: $V = \frac{1}{m_{\text{p}} + m_{\text{sn}}}\left(m_{\text{p}}v_{\text{p}} + m_{\text{sn}}v_{\text{sn}}\right) = \frac{m_{\text{p}}v_{\text{p}}}{m_{\text{p}} + m_{\text{sn}}}$

CALCULATE: $V = \frac{\left(1.6726 \cdot 10^{-27} \text{ kg}\right)\left(0.700c\right)}{\left(1.6726 \cdot 10^{-27} \text{ kg}\right) + \left(1.9240 \cdot 10^{-25} \text{ kg}\right)} = 0.0060329c$

ROUND: Since v_{p} has three significant figures, the result should be rounded to $V = 0.00603c$.

DOUBLE-CHECK: Since m_{sn} is at rest and $m_{\text{sn}} >> m_{\text{p}}$, it is expected that $V << v_{\text{p}}$.

8.38. **THINK:** Particle 1 has a mass of $m_1 = 2.0$ kg, a position of $\vec{r}_1 = (2.0 \text{ m}, 6.0 \text{ m})$ and a velocity of $\vec{v}_1 = (4.0 \text{ m/s}, 2.0 \text{ m/s})$. Particle 2 has a mass of $m_2 = 3.0$ kg, a position of $\vec{r}_2 = (4.0 \text{ m}, 1.0 \text{ m})$ and a velocity of $\vec{v}_2 = (0, 4.0 \text{ m/s})$. Determine (a) the position $\vec{R}$ and the velocity $\vec{V}$ for the system's center of mass and (b) a sketch of the position and velocity vectors for each particle and for the center of mass.

SKETCH: To be provided in the calculate step, part (b).

RESEARCH: To determine $\vec{R}$, use $X = \frac{1}{M}(x_1 m_1 + x_2 m_2)$ and $Y = \frac{1}{M}(y_1 m_1 + y_2 m_2)$. To determine $\vec{V}$, use $V_x = \frac{1}{M}(v_{1x} m_1 + v_{2x} m_2)$ and $V_y = \frac{1}{M}(v_{1y} m_1 + v_{2y} m_2)$.

SIMPLIFY: It is not necessary to simplify.

CALCULATE:

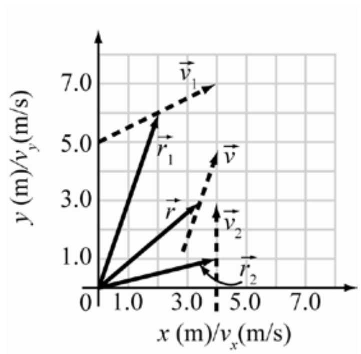
(a) $X = \frac{1}{2.00 \text{ kg} + 3.00 \text{ kg}}\left((2.00 \text{ m})(2.00 \text{ kg}) + (4.00 \text{ m})(3.00 \text{ kg})\right) = 3.20 \text{ m}$

$Y = \frac{1}{2.00 \text{ kg} + 3.00 \text{ kg}}\left((6.00 \text{ m})(2.00 \text{ kg}) + (1.00 \text{ m})(3.00 \text{ kg})\right) = 3.00 \text{ m}$

$V_x = \frac{1}{2.00 \text{ kg} + 3.00 \text{ kg}}\left((4.00 \text{ m/s})(2.00 \text{ kg}) + 0(3.00 \text{ kg})\right) = 1.60 \text{ m/s}$

$V_y = \frac{1}{2.00 \text{ kg} + 3.00 \text{ kg}}\left((2.00 \text{ m/s})(2.00 \text{ kg}) + (4.00 \text{ m/s})(3.00 \text{ kg})\right) = 3.20 \text{ m/s}$

(b)

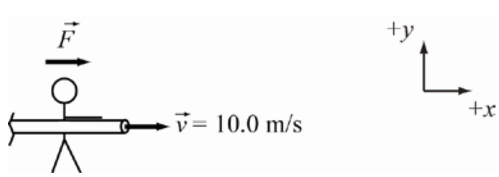


ROUND: Each given value has three significant figures, so the results should be rounded to X = 3.20 m, Y = 3.00 m, $V_x = 1.60$ m/s and $V_y = 3.20$ m/s.

DOUBLE-CHECK: $\vec{R}$ should point between $\vec{r}_1$ and $\vec{r}_2$, and $\vec{V}$ should point between $\vec{v}_1$ and $\vec{v}_2$.

8.39. **THINK:** The radius of the hose is r = 0.0200 m and the velocity of the spray is v = 10.0 m/s. Determine the horizontal force, $\vec{F}_f$, required of the fireman to hold the hose stationary.

SKETCH:



RESEARCH: By Newton's third law, the force exerted by the fireman is equal in magnitude to the force exerted by the hose. The thrust force of the hose can be determined from $\vec{F}_{\text{thrust}} = -\vec{v}_c dm/dt$. To determine dm/dt, consider the mass of water exiting the hose per unit time.

SIMPLIFY: The volume of water leaving the hose is this velocity times the area of the hose's end. That is,

$$\frac{dV_w}{dt} = Av = \pi r^2 v.$$

With $\rho_w = m / V_w$, $\frac{dm}{dt} = \rho_w \frac{dV_w}{dt} = \rho_w \pi r^2 v$. Now, by Newton's third law, $\vec{F}_f = -\vec{F}_{thrust}$, so

$\vec{F}_f = \vec{v}_c \frac{dm}{dt} = \vec{v}_c \rho_w \pi r^2 v$. Since v_c is in fact v, $F_f = \rho_w \pi r^2 v^2$.

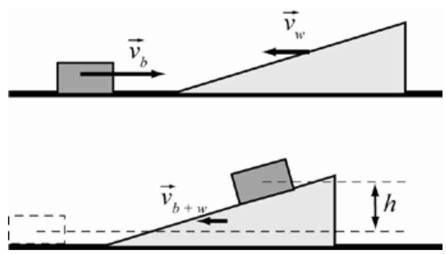
CALCULATE: $F_f = \pi\left(1000 \text{ kg/m}^3\right)\left(0.0200 \text{ m}\right)^2\left(10.0 \text{ m/s}\right)^2 = 125.7 \text{ N}$

ROUND: Since v has three significant figures, $\vec{F}_f = 126 \text{ N}$ in the direction of the water's velocity.

DOUBLE-CHECK: The result has units of force. Also, this is a reasonable force with which to hold a fire hose.

8.40. **THINK:** The block's mass is $m_b = 1.2$ kg. It has an initial velocity is $\vec{v}_b = 2.5$ m/s (with the positive x axis being the right direction). The wedge's mass is m_w and its initial velocity is $\vec{v}_w = -1.1$ m/s. Their final velocity when the wedge stops moving is $\vec{v}_{b+w}$. Determine (a) m_w, if the block's center of mass rises by $h = 0.37$ m and (b) $\vec{v}_{b+w}$.

SKETCH:



RESEARCH: Momentum is conserved. As this is an elastic collision, and there are only conservative forces, mechanical energy is also conserved. Use $P_i = P_f$, $\Delta K + \Delta U = 0$, $K = mv^2/2$ and $U = mgh$ to determine m_w and ultimately $\vec{v}_{b+w}$.

SIMPLIFY: It will be useful to determine an expression for $\vec{v}_{b+w}$ first:

$$\vec{P}_i = \vec{P}_f \Rightarrow m_b\vec{v}_b + m_w\vec{v}_w = \left(m_b + m_w\right)\vec{v}_{b+w} \Rightarrow \vec{v}_{b+w} = \frac{m_b\vec{v}_b + m_w\vec{v}_w}{m_b + m_w}.$$

(a) From the conservation of mechanical energy:

$$\Delta K + \Delta U = K_f - K_i + U_f - U_i = 0 \Rightarrow \frac{1}{2}\left(m_b + m_w\right)\vec{v}_{b+w}^{\ 2} - \frac{1}{2}m_b\vec{v}_b^{\ 2} - \frac{1}{2}m_w\vec{v}_w^{\ 2} + m_b gh = 0$$

$$\Rightarrow \frac{1}{2}\left(m_b + m_w\right)\frac{\left(m_b\vec{v}_b + m_w\vec{v}_w\right)^2}{\left(m_b + m_w\right)^2} - \frac{1}{2}m_b\vec{v}_b^{\ 2} - \frac{1}{2}m_w\vec{v}_w^{\ 2} + m_b gh = 0$$

$$\Rightarrow \frac{\left(m_b^{\ 2}\vec{v}_b^{\ 2} + 2m_b m_w\vec{v}_b\vec{v}_w + m_w^{\ 2}\vec{v}_w^{\ 2}\right)}{2\left(m_b + m_w\right)} - \frac{1}{2}m_b\vec{v}_b^{\ 2} - \frac{1}{2}m_w\vec{v}_w^{\ 2} + m_b gh = 0$$

Multiply the expression by $2\left(m_b + m_w\right)$:

$$m_b^{\ 2}\vec{v}_b^{\ 2} + 2m_b m_w\vec{v}_b\vec{v}_w + m_w^{\ 2}\vec{v}_w^{\ 2} - m_b\vec{v}_b^{\ 2}\left(m_b + m_w\right) - m_w\vec{v}_w^{\ 2}\left(m_b + m_w\right) + 2m_b gh\left(m_b + m_w\right) = 0$$

$$\Rightarrow m_b^{\ 2}\vec{v}_b^{\ 2} + 2m_b m_w\vec{v}_b\vec{v}_w + m_w^{\ 2}\vec{v}_w^{\ 2} - m_b^{\ 2}\vec{v}_b^{\ 2} - m_b m_w\vec{v}_b^{\ 2} - m_w m_b\vec{v}_w^{\ 2} + m_w^{\ 2}\vec{v}_w^{\ 2} + 2m_b^{\ 2}gh + 2m_b m_w gh = 0$$

$$\Rightarrow 2m_b m_w\vec{v}_b\vec{v}_w - m_b m_w\vec{v}_b^{\ 2} - m_b m_w\vec{v}_w^{\ 2} + 2m_b^{\ 2}gh + 2m_b m_w gh = 0$$

$$\Rightarrow m_w = -\frac{2m_b^{\ 2}gh}{2m_b\vec{v}_b\vec{v}_w - m_b\vec{v}_b^{\ 2} - m_b\vec{v}_w^{\ 2} + 2m_b gh} = \frac{2m_b gh}{\vec{v}_b^{\ 2} + \vec{v}_w^{\ 2} - 2\vec{v}_b\vec{v}_w - 2gh}.$$

(b) With m_{w} known, $\vec{v}_{\text{b+w}} = \dfrac{m_{\text{b}}\vec{v}_{\text{b}} + m_{\text{w}}\vec{v}_{\text{w}}}{m_{\text{b}} + m_{\text{w}}}$.

CALCULATE:

(a) $$m_{\text{w}} = \frac{2(1.20 \text{ kg})(9.81 \text{ m/s}^2)0.370 \text{ m}}{(2.5 \text{ m/s})^2 + (-1.10 \text{ m/s})^2 - 2(2.50 \text{ m/s})(-1.10 \text{ m/s}) - 2(9.81 \text{ m/s}^2)(0.370 \text{ m})}$$
$$= \frac{8.712 \text{ kg}\cdot\text{m}^2/\text{s}^2}{6.25 \text{ m}^2/\text{s}^2 + 1.21 \text{ m}^2/\text{s}^2 + 5.5 \text{ m}^2/\text{s}^2 - 7.2594 \text{ m}^2/\text{s}^2} = 1.528 \text{ kg}$$

(b) $$\vec{v}_{\text{b+w}} = \frac{(1.20 \text{ kg})(2.50 \text{ m/s}) + (1.528 \text{ kg})(-1.10 \text{ m/s})}{1.20 \text{ kg} + 1.528 \text{ kg}} = 0.4835 \text{ m/s}$$

ROUND: Each given value has three significant figures, so the results should be rounded to: $m_w = 1.53$ kg and $\vec{v}_{\text{b+w}} = 0.484$ m/s to the right.

DOUBLE-CHECK: These results are reasonable given the initial values.

8.41. **THINK:** For rocket engines, the specific impulse is $J_{\text{spec}} = \dfrac{J_{\text{tot}}}{W_{\text{expended fuel}}} = \dfrac{1}{W_{\text{expended fuel}}}\displaystyle\int_{t_0}^{t} F_{\text{thrust}}(t')dt'$.

(a) Determine J_{spec} with an exhaust nozzle speed of v.

(b) Evaluate and compare J_{spec} for a toy rocket with $v_{\text{toy}} = 800.$ m/s and a chemical rocket with $v_{\text{chem}} = 4.00$ km/s.

SKETCH: Not applicable.

RESEARCH: It is known that $\vec{F}_{\text{thrust}} = -v_{\text{c}}dm/dt$. Rewrite $W_{\text{expended fuel}}$ as $m_{\text{expended}}g$. With the given definition, J_{spec} can be determined for a general v, and for v_{toy} and v_{chem}.

SIMPLIFY: $J_{\text{spec}} = \dfrac{1}{m_{\text{expended}}g}\displaystyle\int_{m_0}^{m} -v dm = -\dfrac{v}{m_{\text{expended}}g}(m - m_0)$. Now, $m - m_0 = -m_{\text{expended}}$, so $J_{\text{spec}} = \dfrac{v}{g}$.

CALCULATE: $J_{\text{spec, toy}} = \dfrac{v_{\text{toy}}}{g} = \dfrac{800. \text{ m/s}}{(9.81 \text{ m/s}^2)} = 81.55 \text{ s}$, $J_{\text{spec, chem}} = \dfrac{v_{\text{chem}}}{g} = \dfrac{4.00\cdot 10^3 \text{ m/s}}{(9.81 \text{ m/s}^2)} = 407.75 \text{ s}$

$$\frac{J_{\text{spec, toy}}}{J_{\text{spec, chem}}} = \frac{v_{\text{toy}}}{v_{\text{chem}}} = \frac{800. \text{ m/s}}{4.00\cdot 10^3 \text{ m/s}} = 0.200$$

ROUND:

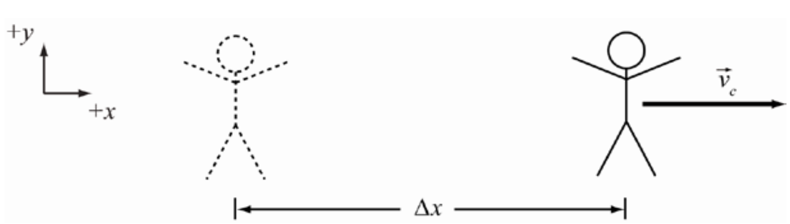
(a) $J_{\text{spec, toy}} = 81.6$ s

(b) $J_{\text{spec, chem}} = 408$ s and $J_{\text{spec, toy}} = 0.200 J_{\text{spec, chem}}$.

DOUBLE-CHECK: The units of the results are units of specific impulse. Also, as expected $J_{\text{spec, toy}} < J_{\text{spec, chem}}$.

8.42. **THINK:** The astronaut's total mass is m = 115 kg. The rate of gas ejection is $dm/dt = 7.00 \text{ g/s} = 0.00700 \text{ kg/s}$ and the leak speed is $v_{\text{c}} = 800.$ m/s. After $\Delta t = 6.00$ s, how far has the astronaut moved from her original position, Δx?

SKETCH:



RESEARCH: Assume that the astronaut starts from rest and the acceleration is constant. Δx can be determined from $\Delta x = (v_\text{i} + v_\text{f})\Delta t / 2$. To determine v_f, use the rocket-velocity equation $v_\text{f} - v_\text{i} = v_\text{c} \ln(m_\text{i} / m_\text{f})$. The loss of mass can be determined from $\Delta m = \frac{dm}{dt}\Delta t$.

SIMPLIFY: Since $v_\text{i} = 0$, $v_\text{f} = v_\text{c} \ln(m_\text{i} / m_\text{f})$, where $m_\text{i} = m$ and $m_\text{f} = m - \Delta m = m - \frac{dm}{dt}\Delta t$. Then,

$$v_\text{f} = v_\text{c} \ln\left(\frac{m}{m - \frac{dm}{dt}\Delta t}\right) \text{ and } \Delta x = \frac{1}{2} v_\text{f} \Delta t.$$

CALCULATE: $v_\text{f} = (800.\text{ m/s})\ln\left(\frac{115\text{ kg}}{115\text{ kg} - (0.00700\text{ kg/s})(6.00\text{ s})}\right) = 0.29223\text{ m/s}$

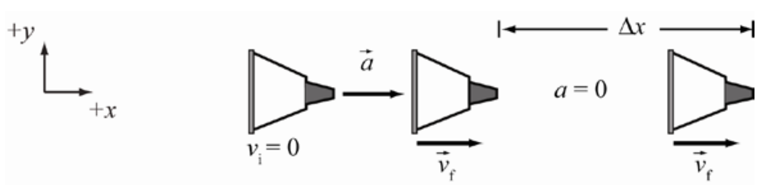
$$\Delta x = \frac{1}{2}(0.29223\text{ m/s})(6.00\text{ s}) = 0.87669\text{ m}$$

ROUND: The problem values have three significant figures, so the results should be rounded to $v_\text{f} = 0.292\text{ m/s}$ $\Delta x = 0.877\text{ m}$.

DOUBLE-CHECK: Considering how such a small amount of the total mass has escaped, this is a reasonable distance to have moved.

8.43. **THINK:** The mass of the payload is $m_\text{p} = 5190.0\text{ kg}$, and the fuel mass is $m_\text{f} = 1.551 \cdot 10^5\text{ kg}$. The fuel exhaust speed is $v_\text{c} = 5.600 \cdot 10^3\text{ m/s}$. How long will it take the rocket to travel a distance $\Delta x = 3.82 \cdot 10^8\text{ m}$ after achieving its final velocity, v_f? The rocket starts accelerating from rest.

SKETCH:



RESEARCH: The rocket's travel speed, v_f, can be determined from $v_\text{f} - v_\text{i} = v_\text{c} \ln(m_\text{i} / m_\text{f})$. Then Δt can be determined from $\Delta x = v\Delta t$.

SIMPLIFY: $v_\text{f} = v_\text{c} \ln\left(\frac{m_\text{p} + m_\text{f}}{m_\text{p}}\right)$, and $\Delta t = \Delta x / v_\text{f}$.

CALCULATE: $v_\text{f} = (5.600 \cdot 10^3\text{ m/s})\ln\left(\frac{5190.0\text{ kg} + 1.551 \cdot 10^5\text{ kg}}{5190.0\text{ kg}}\right) = 19209\text{ m/s}$,

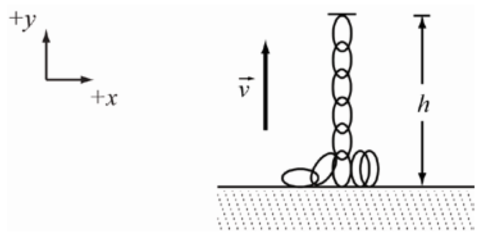
$$\Delta t = \frac{3.82 \cdot 10^8\text{ m}}{19209\text{ m/s}} = 19886\text{ s}$$

ROUND: Δx has three significant figures, so the result should be rounded to $\Delta t = 19{,}886\text{ s} = 5.52\text{ h}$.

DOUBLE-CHECK: This is a reasonable time for a rocket with such a large initial velocity to reach the Moon from the Earth.

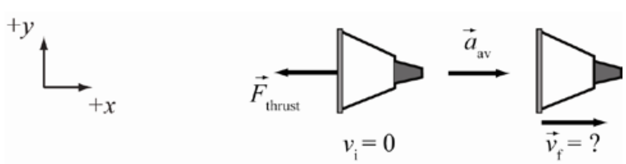
8.44. **THINK:** The linear density of the chain is $\lambda = 1.32\text{ kg/m}$, and the speed at which one end of the chain is lifted is v = 0.47 m/s. Determine (a) the net force acting on the chain, F_net and (b) the force, F, applied to the end of the chain when h = 0.15 m has been lifted off the table.

SKETCH:



RESEARCH:
(a) Since the chain is raised at a constant rate, *v*, the net force is the thrust force, $F_{\text{thrust}} = v_c dm/dt$. Since the chain's mass in the air is increasing, $F_{net} = v\,dm/dt$.
(b) The applied force can be determined by considering the forces acting on the chain and the net force determined in part (a): $F_{\text{net}} = \sum F_i$.
SIMPLIFY:
(a) $F_{\text{net}} = v\frac{dm}{dt} = v\lambda\frac{dh}{dt} = v\lambda v = v^2\lambda$
(b) $F_{\text{net}} = F_{\text{applied}} - mg \Rightarrow F_{\text{applied}} = F_{\text{net}} + mg = v^2\lambda + mg = v^2\lambda + \lambda hg$
CALCULATE:
(a) $F_{\text{net}} = (0.470 \text{ m/s})^2 (1.32 \text{ kg/m}) = 0.2916 \text{ N}$
(b) $F_{\text{applied}} = 0.2916 \text{ N} + (1.32 \text{ kg/m})(0.150 \text{ m})(9.81 \text{ m/s}^2) = 0.2916 \text{ N} + 1.942 \text{ N} = 2.234 \text{ N}$
ROUND: *v* and *h* each have three significant figures, so the results should be rounded to $F_{\text{net}} = 0.292$ N and $F_{\text{applied}} = 2.23$ N.
DOUBLE-CHECK: These forces are reasonable to determine for this system. Also, $F_{\text{net}} < F_{\text{applied}}$.

8.45. **THINK:** The thrust force is $\vec{F}_{\text{thrust}} = 53.2 \cdot 10^6$ N and the propellant velocity is $v = 4.78 \cdot 10^3$ m/s. Determine (a) *dm*/*dt*, (b) the final speed of the spacecraft, v_s, given $v_i = 0$, $m_i = 2.12 \cdot 10^6$ kg and $m_f = 7.04 \cdot 10^4$ kg and (c) the average acceleration, a_{av} until burnout.
SKETCH:



RESEARCH:
(a) To determine *dm*/*dt*, use $\vec{F}_{\text{thrust}} = -v_c dm/dt$.
(b) To determine v_f, use $v_f - v_i = v_c \ln(m_i / m_f)$.
(c) Δv is known from part (b). Δt can be determined from the equivalent ratios,
$\frac{dm}{dt} = \frac{\Delta m}{\Delta t}$, where $\Delta m = m_i - m_f$.
SIMPLIFY:
(a) Since $\vec{F}_{\text{thrust}}$ and $\vec{v}_c$ are in the same direction, the equation can be rewritten as:

$$F_{\text{thrust}} = v_c \frac{dm}{dt} \Rightarrow \frac{dm}{dt} = \frac{F_{\text{thrust}}}{v_c}.$$

(b) $v_{\text{i}}=0 \Rightarrow v_{\text{f}}=v_{\text{c}}\ln\left(\dfrac{m_{\text{i}}}{m_{\text{f}}}\right)$

(c) $\dfrac{dm}{dt}=\dfrac{\Delta m}{\Delta t} \Rightarrow \Delta t=\dfrac{\Delta m}{dm/dt},\ a_{\text{av}}=\dfrac{\Delta v}{\Delta t}=\dfrac{v_{\text{f}}}{\Delta m}\left(\dfrac{dm}{dt}\right)\quad (v_{\text{i}}=0)$

CALCULATE:

(a) $\dfrac{dm}{dt}=\dfrac{\left(53.2\cdot 10^6\text{ N}\right)}{\left(4.78\cdot 10^3\text{ m/s}\right)}=11129.7\text{ kg/s}$

(b) $v_{\text{f}}=\left(4.78\cdot 10^3\text{ m/s}\right)\ln\left(\dfrac{2.12\cdot 10^6\text{ kg}}{7.04\cdot 10^4\text{ kg}}\right)=1.6276\cdot 10^4\text{ m/s}$

(c) $a_{\text{av}}=\dfrac{\left(1.6276\cdot 10^4\text{ m/s}\right)}{\left(2.12\cdot 10^6\text{ kg}-7.04\cdot 10^4\text{ kg}\right)}\left(11129.7\text{ kg/s}\right)=88.38\text{ m/s}^2$

ROUND: Each given value has three significant figures, so the results should be rounded to $dm/dt=11100\text{ kg/s}$, $v_{\text{f}}=1.63\cdot 10^4\text{ m/s}$ and $a_{\text{av}}=88.4\text{ m/s}^2$.

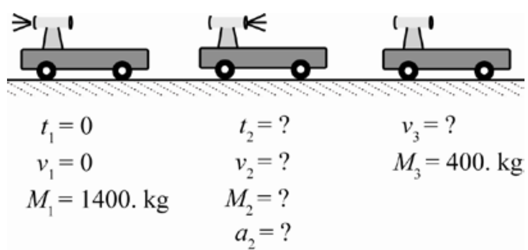
DOUBLE-CHECK: The results all have the correct units. Also, the results are reasonable for a spaceship with such a large thrust force.

8.46. **THINK:** The mass of the cart with an empty water tank is $m_{\text{c}}=400.\text{ kg}$. The volume of the water tank is $V=1.00\text{ m}^3$. The rate at which water is ejected in SI units is

$$dV/dt=\left(200.\ \frac{\text{L}}{\text{min}}\right)\left(\frac{1\text{ m}^3}{1000\text{ L}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)=0.003333\text{ m}^3/\text{s}.$$

The muzzle velocity is $v_{\text{c}}=25.0\text{ m/s}$. Determine (a) the time, t_2, to switch from backward to forward so the cart ends up at rest (it starts from rest), (b) the mass of the cart, M_2, and the velocity, v_2, at the time, t_2, (c) the thrust, F_{thrust}, of the rocket and (d) the acceleration, a_2, of the cart just before the valve is switched. Note the mass of the cart increases by 1000. kg when the water tank is full, as $m_{\text{w}}=\rho V=\left(1000.\text{ kg/m}^3\right)\left(1.00\text{ m}^3\right)$. That is, the initial mass is $M_1=1400.\text{ kg}$.

SKETCH:



RESEARCH:

(a) t_2 can be determined from the ratio, $\dfrac{M_1-M_2}{t_2-t_1}=\dfrac{dm}{dt}$, with $t_1=0$. Note that, $dm/dt=\rho dV/dt$. M_2 can be determined from $v_{\text{f}}-v_{\text{i}}=v_{\text{c}}\ln\left(m_{\text{i}}/m_{\text{f}}\right)$. When the cart stops moving, the water tank is empty and the total mass is $M_3=400\text{ kg}$.

(b) Using the mass determined in part (a), v_2 can be determined from $v_{\text{f}}-v_{\text{i}}=v_{\text{c}}\ln\left(m_{\text{i}}/m_{\text{f}}\right)$.

(c) Use $\vec{F}_{\text{thrust}}=-\vec{v}_{\text{c}}dm/dt$.

(d) Since $\vec{F}_{\text{thrust}}=M\vec{a}_{\text{net}}$, a_2 can be determined from this equation.

SIMPLIFY:

(a) Consider the first leg of the trip before the valve is switched:

$$v_2 - v_1 = v_c \ln\left(M_1 / M_2\right) \Rightarrow v_2 = v_c \ln\left(M_1 / M_2\right).$$

In the second leg, v_c changes direction, and the similar equation is

$$v_3 - v_2 = -v_c \ln\left(M_2 / M_3\right) \Rightarrow v_2 = v_c \ln\left(M_2 / M_3\right).$$

Then it must be that $\ln\left(M_2 / M_3\right) = \ln\left(M_2 / M_3\right)$, or $M_1 / M_2 = M_2 / M_3$. Then $M_2 = \sqrt{M_3 M_1}$. Now,

$$\frac{M_1 - M_2}{t_2} = \frac{dm}{dt} = \rho\frac{dV}{dt} \Rightarrow t_2 = \frac{M_1 - M_2}{\rho\frac{dV}{dt}} = \frac{M_1 - \sqrt{M_3 M_1}}{\rho\frac{dV}{dt}}.$$

(b) From above, $M_2 = \sqrt{M_3 M_1}$, $v_2 = v_c \ln\left(\frac{M_1}{M_2}\right)$.

(c) $\vec{F}_{\text{thrust}} = -\vec{v}_c \frac{dm}{dt} = -\vec{v}_c \rho \frac{dV}{dt}$

(d) $\vec{a}_2 = \frac{\vec{F}_{\text{thrust}}}{M_2}$

CALCULATE:

(a) $t_2 = \frac{1400.\text{ kg} - \sqrt{(400.\text{ kg})(1400.\text{ kg})}}{\left(1000.\text{ kg/m}^3\right)\left(0.003333\text{ m}^3/\text{s}\right)} = 195.5\text{ s}$

(b) $M_2 = \sqrt{(400.\text{ kg})(1400.\text{ kg})} = 748.33\text{ kg}$, $v_2 = (25.0\text{ m/s})\ln\left(\frac{(1400.\text{ kg})}{(748.33\text{ kg})}\right) = 15.66\text{ m/s}$

(c) Before the valve is switched, v_c is directed backward, i.e. $\vec{v}_c = -25.0$ m/s. Then $\vec{F}_{\text{thrust}} = -(-25.0\text{ m/s})\left(1000.\text{ kg/m}^3\right)\left(0.003333\text{ m}^3/\text{s}\right) = 83.33\text{ N}$ forward. After the valve is switched, $\vec{F}_{\text{thrust}}$ is directed backward, i.e. $\vec{F}_{\text{thrust}} = -83.33$ N.

(d) Before the valve is switched, $\vec{a}_2 = \frac{83.33\text{ N}}{748.33\text{ kg}} = 0.111355\text{ m/s}^2$.

ROUND:

Rounding to three significant figures:

(a) $t_2 = 196$ s

(b) $M_2 = 748$ kg and $v_2 = 15.7$ m/s.

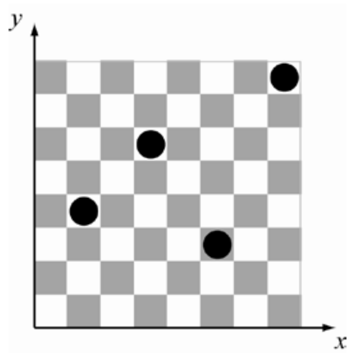
(c) $\vec{F}_{\text{thrust}} = -83.3$ N

(d) $\vec{a}_2 = 0.111\text{ m/s}^2$

DOUBLE-CHECK: All the units for the results are appropriate. Also, the results are of reasonable orders of magnitude.

8.47. **THINK:** The checkerboard has dimensions 32.0 cm by 32.0 cm. Its mass is $m_b = 100.$ g and the mass of each of the four checkers is $m_c = 20.0$ g. Determine the center of mass of the system. Note the checkerboard is 8 by 8 squares, thus the length of the side of each square is 32.0 cm/8 = 4.00 cm. From the figure provided, the following *x-y* coordinates can be associated with each checker's center of mass: $m_1 : (22.0\text{ cm}, 10.0\text{ cm})$, $m_2 : (6.00\text{ cm}, 14.0\text{ cm})$, $m_3 : (14.0\text{ cm}, 22.0\text{ cm})$, $m_4 : (30.0\text{ cm}, 30.0\text{ cm})$. Assuming a uniform density distribution, the checkerboard's center of mass is located at $(x_b, y_b) = (16.0\text{ cm}, 16.0\text{ cm})$.

SKETCH:



RESEARCH: To determine the system's center of mass, use the following equations: $X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i$ and $Y = \frac{1}{M}\sum_{i=1}^{n} y_i m_i$.

SIMPLIFY: $M = m_\text{b} + 4m_\text{c}$

$$X = \frac{1}{M}\left(x_\text{b}m_\text{b} + m_\text{c}\left(x_1 + x_2 + x_3 + x_4\right)\right),\ Y = \frac{1}{M}\left(y_\text{b}m_\text{b} + m_\text{c}\left(y_1 + y_2 + y_3 + y_4\right)\right)$$

CALCULATE: $M = 100.\text{ g} + 4\left(20.0\text{ g}\right) = 180.\text{ g}$

$$X = \frac{1}{180.\text{ g}}\left(16.0\text{ cm}\left(100.0\text{ g}\right) + 20.0\text{ g}\left(22.0\text{ cm} + 6.00\text{ cm} + 14.0\text{ cm} + 30.0\text{ cm}\right)\right) = 16.889\text{ cm}$$

$$Y = \frac{1}{180.\text{ g}}\left(16.0\text{ cm}\left(100.\text{ g}\right) + 20.0\text{ g}\left(10.0\text{ cm} + 14.0\text{ cm} + 22.0\text{ cm} + 30.0\text{ cm}\right)\right) = 17.33\text{ cm}$$

ROUND: $X = 16.9\text{ cm}$ and $Y = 17.3\text{ cm}$. The answer is (16.9 cm,17.3 cm).

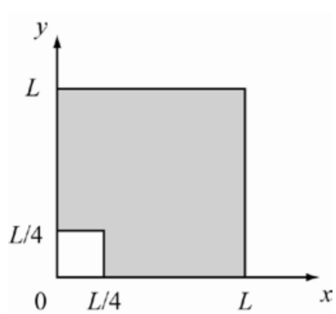
DOUBLE-CHECK: $m_\text{b} > m_\text{c}$, so it is reasonable to expect the system's center of mass to be near the board's center of mass.

8.48. **THINK:** The total mass of the plate is $M_\text{tot} = 0.205\text{ kg}$. The dimensions of the plate are L by L, $L = 5.70\text{ cm}$. The dimensions of the smaller removed plate are $L/4$ by $L/4$. The mass of the smaller removed plate is

$$\frac{M_\text{tot}}{A_\text{tot}} = \frac{m_\text{s}}{A_\text{s}} \Rightarrow m_\text{s} = A_\text{s}\frac{M_\text{tot}}{A_\text{tot}} = \left(\frac{L}{4}\right)^2 \frac{M_\text{tot}}{L^2} = \frac{1}{16}M_\text{tot}.$$

Determine the distance from the bottom left corner of the plate to the center of mass after the smaller plate is removed. Note the mass of the plate with the void is $m_\text{p} = M_\text{tot} - m_\text{s} = 15M_\text{tot}/16$.

SKETCH:



RESEARCH: The center of mass in each dimension is $X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i$ and $y = \frac{1}{M}\sum_{i=1}^{n} y_i m_i$. The center of mass of the plate with the void, (X_p, Y_p), can be determined by considering the center of mass of the total system as composed of the smaller plate of mass m_s and the plate with the void of mass m_p. Note the center of mass of the total system is at the total plate's geometric center, $(X,Y) = (L/2, L/2)$, assuming uniform density. Similarly, the center of mass of the smaller plate is at its center $(X_\text{s}, Y_\text{s}) = (L/8, L/8)$. The distance of the center of mass of the plate from the origin is then $d = \sqrt{X_\text{p}^2 + Y_\text{p}^2}$.

SIMPLIFY: $X = \frac{1}{M_\text{tot}}\left(X_\text{p} m_\text{p} + X_\text{s} m_\text{s}\right)$, and $X_\text{p} = \frac{\left(XM_\text{tot} - X_\text{s} m_\text{s}\right)}{M_\text{tot} - \frac{1}{16}M_\text{tot}} = \frac{L\left(\frac{1}{2}M_\text{tot} - \frac{1}{8}\left(\frac{1}{16}M_\text{tot}\right)\right)}{\frac{15}{16}M_\text{tot}} = \frac{21}{40}L.$

Similarly, $Y_\text{p} = \frac{\left(YM_\text{tot} - Y_\text{s} m_\text{s}\right)}{m_\text{p}} = \frac{L\left(\frac{1}{2}M_\text{tot} - \frac{1}{8}\left(\frac{1}{16}M_\text{tot}\right)\right)}{\frac{15}{16}M_\text{tot}} = \frac{21}{40}L.$

CALCULATE: $X_\text{p} = Y_\text{p} = \frac{21}{40}(5.70 \text{ cm}) = 2.9925 \text{ cm}$

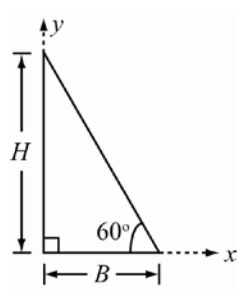
$d = \sqrt{(2.9925 \text{ cm})^2 + (2.9925 \text{ cm})^2} = 4.232 \text{ cm}$

ROUND: Since *L* has three significant figures, the result should be rounded to $d = 4.23$ cm.

DOUBLE-CHECK: It is expected that the center of mass for the plate with the void would be further from the origin than the center of mass for the total plate.

8.49. **THINK:** The height is *H* = 17.3 cm and the base is *B* = 10.0 cm for a flat triangular plate. Determine the *x* and *y*-coordinates of its center of mass. Since it is not stated otherwise, we assume that the mass density of this plate is constant.

SKETCH:



RESEARCH: Assuming the mass density is constant throughout the object, the center of mass is given by $\vec{R} = \frac{1}{A}\int_A \vec{r}\,dA$, where *A* is the area of the object. The center of mass can be determined in each dimension.

The *x* coordinate and the *y* coordinate of the center of mass are given by $X = \frac{1}{A}\int_A x\,dA$ and $Y = \frac{1}{A}\int_A y\,dA$, respectively. The area of the triangle is $A = HB/2$.

SIMPLIFY: The expression for the area of the triangle can be substituted into the formulae for the center of mass to get

$$X = \frac{2}{HB}\int_A x\,dA \text{ and } Y = \frac{2}{HB}\int_A y\,dA.$$

In the x-direction we have to solve the integral:

$$\int_A x dA = \int_0^B \int_0^{y_m(x)} xdydx = \int_0^B xdx \int_0^{y_m(x)} dy = \int_0^B xy_m(x)dx = \int_0^B xH(1-x/B)dx = H\int_0^B x-\left(x^2/B\right)dx$$
$$= H(\tfrac{1}{2}x^2 - \tfrac{1}{3}x^3/B)\Big|_0^B = \tfrac{1}{2}HB^2 - \tfrac{1}{3}HB^2 = \tfrac{1}{6}HB^2$$

Note that in this integration procedure the maximum for the y–integration depends on the value of x: $y_m(x) = H(1-x/B)$. Therefore we arrive at

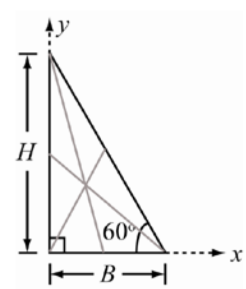
$$X = \frac{2}{HB}\int_A x\,dA = \frac{2}{HB}\cdot\frac{HB^2}{6} = \tfrac{1}{3}B$$

In the same way we can find that $Y = \tfrac{1}{3}H$.

CALCULATE: $X_{\text{com}} = \tfrac{1}{3}(10.0 \text{ cm}) = 3.33333 \text{ cm}$, $Y_{\text{com}} = \tfrac{1}{3}(17.3 \text{ cm}) = 5.76667 \text{ cm}$

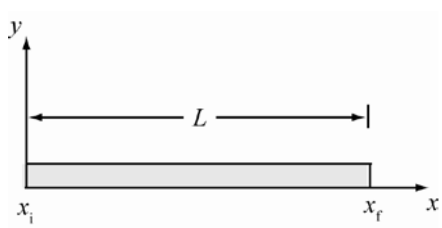
ROUND: Three significant figures were provided in the question, so the results should be written $X = 3.33$ cm and $Y = 5.77$ cm.

DOUBLE-CHECK: Units of length were calculated for both X and Y, which is dimensionally correct. We also find that the center of mass coordinates are inside the triangle, which always has to be true for simple geometrical shape without holes in it. Finally, we can determine the location of the center of mass for a triangle geometrically by connecting the center of each side to the opposite corner with a straight line (see drawing). The point at which these three lines intersect is the location of the center of mass. You can see from the graph that this point has to be very close to our calculated result of $(\tfrac{1}{3}B, \tfrac{1}{3}H)$.



8.50. **THINK:** The linear density function for a 1.00 m long rod is $\lambda(x) = 100.\text{ g/m} + 10.0x \text{ g/m}^2$. One end of the rod is at $x_{\text{i}} = 0$ m and the other end is situated at $x_{\text{f}} = 1.00$ m. The total mass, M of the rod and the center of mass coordinate are to be determined.

SKETCH:



RESEARCH:

(a) The linear density of the rod is given by $\lambda(x) = dm/dx$. This expression can be rearranged to get $\lambda(x)dx = dm$. An expression for $\lambda(x)$ was given so both sides can be integrated to solve for M.

(b) The center of mass coordinate is given by $X_{\text{com}} = \dfrac{1}{M}\int xdm$.

SIMPLIFY:

(a) Integrate both of sides of the linear density function to get:

$$\int_{x_{\text{i}}}^{x_{\text{f}}}\left(100.\text{ g/m} + 10.0x \text{ g/m}^2\right)dx = \int_0^M dm \Rightarrow \left[100.x \text{ g/m} + 5.0x^2 \text{ g/m}^2\right]_{x_{\text{i}}}^{x_{\text{f}}} = M.$$

(b) Substitute $dm = \lambda(x)dx$ into the expression for X_{com} to get

$$X_{\text{com}} = \frac{1}{M}\int_{x_{\text{i}}}^{x_{\text{f}}} x\lambda(x)dx.$$

The value calculated in part (a) for M can later be substituted. Substitute $\lambda(x)=100\text{ g/m}+10.0x\text{ g/m}^2$ into the expression for X_{com} to get

$$X_{\text{com}}=\frac{1}{M}\int_{x_i}^{x_f}\left(100.x\text{ g/m}+10.0x^2\text{ g/m}^2\right)dx\Rightarrow\left[\frac{1}{M}\left(50.0x^2\text{ g/m}+\frac{10.0}{3}x^3\text{ g/m}^2\right)\right]_{x_i}^{x_f}.$$

CALCULATE:

(a) $M=100.\text{ g/m}(1\text{ m})+5.0\text{ g}\dfrac{(1\text{ m})^2}{\text{m}^2}=105\text{ g}$

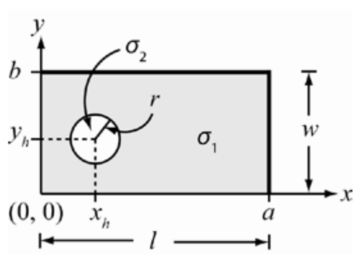
(b) $X_{\text{com}}=\dfrac{1}{105\text{ g}}\left(50.0(1\text{ m})^2\text{ g/m}+\dfrac{10.0}{3}(1\text{ m})^3\text{ g/m}^2\right)=0.50793651\text{ m}$

ROUND:
Rounding to three significant figures
(a) $M=105\text{ g}$
(b) $X_{\text{com}}=0.508\text{ m}$

DOUBLE-CHECK: The correct units were calculated for the mass and the center of mass so the results are dimensionally correct. Our result for the location of the center of mass of the rod, 50.8 cm, is just larger than the geometric center of the rod, 50.0 cm. This makes sense because the density of the rod increases slightly with increasing distance.

8.51. **THINK:** The area density for a thin, rectangular plate is given as $\sigma_1=1.05\text{ kg/m}^2$. Its length is a = 0.600 m and its width is b = 0.250 m. The lower left corner of the plate is at the origin. A circular hole of radius, r = 0.0480 m is cut out of the plate. The hole is centered at the coordinates $x_{\text{h}}=0.068\text{ m}$ and $y_{\text{h}}=0.068\text{ m}$. A round disk of radius, r is used to plug the hole. The disk, D, has a uniform area density of $\sigma_2=5.32\text{ kg/m}^2$. The distance from the origin to the modified plate's center of mass, R, is to be determined.

SKETCH:



RESEARCH: The center of mass, R, of an object can be defined mathematically as $R=\dfrac{1}{M}\sum_{i=1}^{n}\vec{r}_i m_i$ (1). In this equation, M is the total mass of the system. The vector $\vec{r}_i$ denotes the position of the i^{th} object's center of mass and m_{i} is the mass of that object. To solve this problem, the center of mass of the plate, R_{p}, and the center of mass of the disk, $R_{\text{D,}}$ must be determined. Then equation (1) can be used to determine the distance from the origin to the modified center of mass, R. First, consider the rectangular plate, P, which has the hole cut in it. The position of the center of mass, R_{p}, is not known. The mass of P can be denoted m_{p}. Consider the disk of material, d, that was removed (which has a uniform area density of σ_1), and denote its center of mass as R_{d} and its mass as m_{d}. Next, define S as the system of the rectangular plate, P, and the disc of removed material, d. The mass of S can be denoted $m_{\text{s}}=m_{\text{p}}+m_{\text{d}}$. The center of mass of S is $R_{\text{s}}=(a/2)\hat{x}+(b/2)\hat{y}$. m_{p} and m_{d} are not known but it is known that they have uniform area density of σ_1. The uniform area density is given by $\sigma=m/A$. Therefore, $m_{\text{p}}=\sigma_1 A_{\text{p}}$ and

$m_{\rm d} = \sigma_1 A_{\rm d}$, where $A_{\rm p}$ is the area of the plate minus the area of the hole and $A_{\rm d}$ is the area of the disk, *d*. The expressions for these areas are $A_{\rm p} = ab - \pi r^2$ and $A_{\rm d} = \pi r^2$. Substituting these area expressions into the expressions for $m_{\rm p}$ and $m_{\rm d}$ gives $m_{\rm p} = \sigma_1\left(ab - \pi r^2\right)$ and $m_{\rm d} = \sigma_1 \pi r^2$. So the center of mass of the system is given by:

$$\vec{R}_{\rm s} = \frac{\left(x_{\rm h}\hat{x} + y_{\rm h}\hat{y}\right)m_{\rm d} + \vec{R}_{\rm p}m_{\rm p}}{\sigma_1\left(ab - \pi r^2\right) + \sigma_1 \pi r^2} \quad (2).$$

Now, consider the disk, *D*, that is made of the material of uniform area density, σ_2. Define its center of mass as $\vec{R}_{\rm D} = x_{\rm h}\hat{x} + y_{\rm h}\hat{y}$. Also, define its mass as $m_{\rm D} = \sigma_2 \pi r^2$.

SIMPLIFY: Rearrange equation (2) to solve for $\vec{R}_{\rm p}$:

$$\vec{R}_{\rm p}m_{\rm p} = \vec{R}_{\rm s}\sigma_1 ab - \left(x_{\rm h}\hat{x} + y_{\rm h}\hat{y}\right)m_{\rm d} \Rightarrow \vec{R}_{\rm p} = \frac{\vec{R}_{\rm s}\sigma_1 ab - \left(x_{\rm h}\hat{x} + y_{\rm h}\hat{y}\right)m_{\rm d}}{m_{\rm p}}.$$

Now, substitute the values for $\vec{R}_{\rm s}$, $m_{\rm d}$ and $m_{\rm p}$ into the above equation to get:

$$\vec{R}_{\rm p} = \frac{\left(\frac{a}{2}\hat{x} + \frac{b}{2}\hat{y}\right)\sigma_1 ab - \left(x_{\rm h}\hat{x} + y_{\rm h}\hat{y}\right)\sigma_1 \pi r^2}{\sigma_1\left(ab - \pi r^2\right)}.$$

Once $\vec{R}_{\rm p}$ is solved, it can be substituted into the expression for $\vec{R}$ to get $\vec{R} = \dfrac{\vec{R}_{\rm p}m_{\rm p} + \vec{R}_{\rm D}m_{\rm D}}{m_{\rm p} + m_{\rm D}}$. Use the distance formula $R = \sqrt{R_x^{\ 2} + R_y^{\ 2}}$.

CALCULATE:

$$\vec{R}_{\rm p} = \frac{\left(\frac{0.600}{2}\hat{x} + \frac{0.250}{2}\hat{y}\right)\left(\left(1.05\text{ kg/m}^3\right)\left(0.600\text{ m}\right)\left(0.250\text{ m}\right)\right) - \left(0.068\hat{x} + 0.068\hat{y}\right)\left(1.05\text{ kg/m}^3\right)\pi\left(0.0480\text{ m}\right)^2}{\left(1.05\text{ kg/m}^3\right)\left(\left(0.600\text{ m}\right)\left(0.250\text{ m}\right) - \pi\left(0.0480\text{ m}\right)^2\right)}$$

$$= \left(0.31176\hat{x} + 0.12789\hat{y}\right)\text{ m}$$

$$\vec{R} = \frac{\left(0.31176\hat{x} + 0.12789\hat{y}\right)\text{ m}\left(0.1499\text{ kg}\right) + \left(0.068\hat{x} + 0.068\hat{y}\right)\text{ m}\left(0.038507\text{ kg}\right)}{0.1499\text{ kg} + 0.038507\text{ kg}}$$

$$= \left(0.26194\hat{x} + 0.11565\hat{y}\right)\text{ m}$$

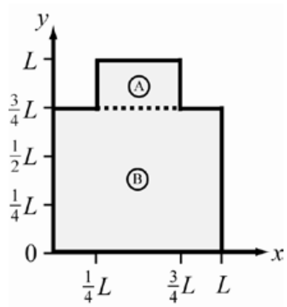
Then, the distance to the origin is given by $R = \sqrt{\left(0.26194\text{ m}\right)^2 + \left(0.11565\text{ m}\right)^2} = 0.28633\text{ m}$.

ROUND: Densities are given to three significant figures. For dimensions the subtraction rule applies, where all dimensions are known to three decimal places. The result should be rounded to $R = 0.286$ m.

DOUBLE-CHECK: The position of the center of mass for the modified system is shifted closer to the position of the disk, *D*, which has an area density of 5.32 kg/m^2. This is reasonable because the disk has a much higher area density than the rest of the plate. Also, the results are reasonable considering the given values.

8.52. **THINK:** The object of interest is a uniform square metal plate with sides of length, L = 5.70 cm and mass, m = 0.205 kg. The lower left corner of the plate sits at the origin. Two squares with side length, $L/4$ are removed from each side at the top of the square. Determine the *x*-coordinate and the *y*-coordinate of the center of mass, denoted $X_{\rm com}$ and $Y_{\rm com}$, respectively.

SKETCH:



RESEARCH: Because the square is uniform, the equations for X_{com} and Y_{com} can be expressed by

$$X_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \text{ and } Y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i.$$

M is the total mass of the system. In this problem it will be useful to treat the system as if it were made up of two uniform metal rectangles, R_A and R_B.

(a) The center of mass x-coordinate for rectangle A is $x_{\text{A}} = (L/2)\hat{x}$. The center of mass x-coordinate for rectangle B is $x_{\text{B}} = (L/2)\hat{x}$.

(b) The center of mass y-coordinate for rectangle A is $y_{\text{A}} = (7L/8)\hat{y}$. The center of mass y-coordinate for rectangle B is $y_{\text{B}} = (3L/8)\hat{y}$. Both rectangles have the same uniform area density, σ. The uniform area density is given by $\sigma = m_{\text{A}}/A_{\text{A}} = m_{\text{B}}/A_{\text{B}}$. Therefore, $m_{\text{A}} = m_{\text{B}}A_{\text{A}}/A_{\text{B}}$. The areas are given by the following expressions:

$$A_{\text{A}} = \left(\frac{L}{4}\right)\left(\frac{L}{2}\right) = \frac{L^2}{8} \text{ and } A_{\text{B}} = \left(\frac{3L}{4}\right)L = \frac{3L^2}{4}.$$

SIMPLIFY:

(a) $X_{\text{com}} = \dfrac{x_{\text{A}}m_{\text{A}} + x_{\text{B}}m_{\text{B}}}{m_{\text{A}} + m_{\text{B}}}$

Substitute the expression for m_{A} into the above equation to get:

$$X_{\text{com}} = \frac{x_{\text{A}}m_{\text{B}}\dfrac{A_{\text{A}}}{A_{\text{B}}} + x_{\text{B}}m_{\text{B}}}{m_{\text{B}}\dfrac{A_{\text{A}}}{A_{\text{B}}} + m_{\text{B}}} = \frac{x_{\text{A}}\left(\dfrac{A_{\text{A}}}{A_{\text{B}}}\right) + x_{\text{B}}}{\dfrac{A_{\text{A}}}{A_{\text{B}}} + 1}.$$

Then substitute the expressions for x_{A}, x_{B}, A_{A} and A_{B} to get:

$$X_{\text{com}} = \frac{\dfrac{L}{2}\left(\dfrac{L^2/8}{3L^2/4}\right) + \dfrac{L}{2}}{\dfrac{L^2/8}{3L^2/4} + 1} = \frac{\dfrac{L}{12} + \dfrac{L}{2}}{\dfrac{1}{6} + 1} = \frac{\dfrac{7L}{12}}{\dfrac{7}{6}} = \frac{1}{2}L.$$

(b) The same procedure can be used to solve for the y-coordinate of the center of mass:

$$Y_{\text{com}} = \frac{y_{\text{A}}\left(\dfrac{A_{\text{A}}}{A_{\text{B}}}\right) + y_{\text{B}}}{\dfrac{A_{\text{A}}}{A_{\text{B}}} + 1} = \frac{\dfrac{7L}{8}\left(\dfrac{1}{6}\right) + \dfrac{3L}{8}}{\dfrac{7}{6}} = \frac{\dfrac{7L}{48} + \dfrac{18L}{48}}{\dfrac{7}{6}} = \frac{25L}{48}\left(\frac{6}{7}\right) = \frac{25L}{56}.$$

CALCULATE:

(a) $X_{\text{com}} = \frac{1}{2}(5.70 \text{ cm}) = 2.85 \text{ cm}$

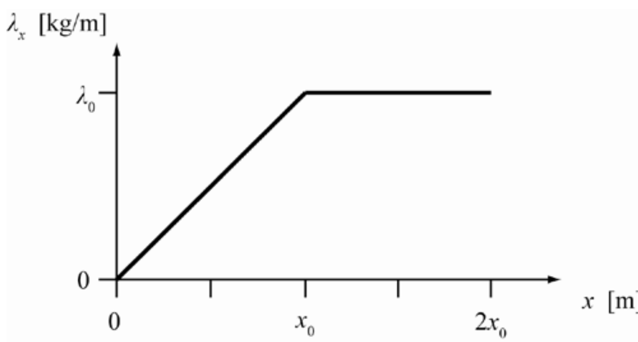
(b) $Y_{\text{com}} = \frac{25}{56}(5.70 \text{ cm}) = 2.5446 \text{ cm}$

ROUND: Three significant figures were provided in the question, so the results should be rounded to $X_{\text{com}} = 2.85 \text{ cm}$ and $Y_{\text{com}} = 2.54 \text{ cm}$.

DOUBLE-CHECK: Units of distance were calculated, which is expected when calculating the center of mass coordinates. The squares were removed uniformly at the top of the large square, so it makes sense that the *x*-coordinate of the center of mass stays at *L*/2 by symmetry and the *y*-coordinate of the center of mass is shifted slightly lower.

8.53. **THINK:** The linear mass density, $\lambda(x)$, is provided in the graph. Determine the location for the center of mass, X_{com}, of the object. From the graph, it can be seen that $\lambda(x) = \begin{Bmatrix} \frac{\lambda_0}{x_0}x, \; 0 \le x < x_0 \\ \lambda_0, \; x_0 \le x \le 2x_0 \end{Bmatrix}$.

SKETCH:



RESEARCH: The linear mass density, $\lambda(x)$, depends on *x*. To determine the center of mass, use the equation $X_{\text{com}} = \frac{1}{M}\int_L x\lambda(x)dx$. The mass of the system, M, can be determined using the equation $M = \int_L \lambda(x)dx$. In order to evaluate the center of mass of the system, two separate regions must be considered; the region from $x = 0$ to $x = x_0$ and the region from $x = x_0$ to $x = 2x_0$. The equation for X_{com} can be expanded to $X_{\text{com}} = \frac{1}{M}\int_0^{x_0} x\frac{\lambda_0}{x_0}xdx + \frac{1}{M}\int_{x_0}^{2x_0} \lambda_0 xdx$. The equation for M is $M = \int_0^{x_0} \frac{\lambda_0}{x_0}xdx + \int_{x_0}^{2x_0} \lambda_0 dx$.

SIMPLIFY: Simplify the expression for *M* first and then substitute it into the expression for X_{com}.

$$M = \int_0^{x_0} \frac{\lambda_0}{x_0}xdx + \int_{x_0}^{2x_0} \lambda_0 dx = \left[\frac{1}{2}\frac{\lambda_0}{x_0}x^2\right]_0^{x_0} + \left[x\lambda_0\right]_{x_0}^{2x_0} = \frac{1}{2}\lambda_0 x_0 + 2x_0\lambda_0 - x_0\lambda_0 = \frac{3}{2}x_0\lambda_0.$$

Substitute the above expression into the equation for X_{com} to get:

$$X_{\text{com}} = \frac{2}{3x_0\lambda_0}\left[\int_0^{x_0} x^2\frac{\lambda_0}{x_0}dx + \int_{x_0}^{2x_0} \lambda_0 xdx\right] = \frac{2}{3x_0\lambda_0}\left[\frac{1}{3}\lambda_0 x_0^2 + 2\lambda_0 x_0^2 - \frac{1}{2}\lambda_0 x_0^2\right] = \frac{2}{3x_0\lambda_0}\left[\lambda_0 x_0^2\left(\frac{2}{6} + \frac{12}{6} - \frac{3}{6}\right)\right]$$

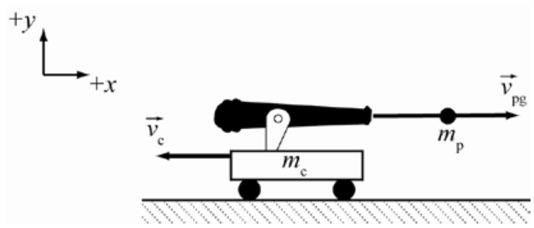
$$= \frac{2}{3x_0\lambda_0}\left(\frac{11}{6}\lambda_0 x_0^2\right) = \frac{11x_0}{9}.$$

CALCULATE: This step does not apply.

ROUND: This step does not apply.

DOUBLE-CHECK: The units for the result are units of length, so the answer is dimensionally correct. It is reasonable that the calculated value is closer to the denser end of the object.

8.54. **THINK:** The mass of the cannon is $m_c = 750$ kg and the mass of the projectile is $m_p = 15$ kg. The total mass of the cannon and projectile system is $M = m_c + m_p$. The speed of the projectile is $v_p = 250$ m/s with respect to the muzzle just after the cannon has fired. The cannon is on wheels and can recoil with negligible friction. Determine the speed of the projectile with respect to the ground, v_{pg}.
SKETCH:



RESEARCH: The problem can be solved by considering the conservation of linear momentum. The initial momentum is $\vec{P}_i = 0$ because the cannon and projectile are both initially at rest. The final momentum is $\vec{P}_f = m_c\vec{v}_c + m_p\vec{v}_{pg}$. The velocity of the recoiling cannon is v_c. The equation for the conservation of momentum is $\vec{P}_i = \vec{P}_f$. The velocity of the projectile with respect to the cannon's muzzle can be represented as $\vec{v}_p = \vec{v}_{pg} - \vec{v}_c$. Take $\vec{v}_{pg}$ to be in the positive *x*-direction.

SIMPLIFY: Rearrange the above equation so that it becomes $\vec{v}_c = \vec{v}_{pg} - \vec{v}_p$. Then substitute this expression into the conservation of momentum equation:

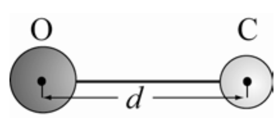
$$P_i = P_f \Rightarrow 0 = m_c v_c + m_p v_{pg} \Rightarrow 0 = m_c\left(v_{pg} - v_p\right) + m_p v_{pg} \Rightarrow v_{pg}\left(m_c + m_p\right) = m_c v_p \Rightarrow v_{pg} = \frac{m_c v_p}{\left(m_c + m_p\right)}.$$

CALCULATE: $v_{pg} = \dfrac{(750 \text{ kg})(250 \text{ m/s})}{(750 \text{ kg} + 15 \text{ kg})} = 245.098$ m/s

ROUND: The least number of significant figures provided in the question is three, so the result should be rounded to $v_{pg} = 245$ m/s.

DOUBLE-CHECK: The units of speed are correct for the result. The velocity calculated for the projectile with respect to the ground is slower than its velocity with respect to the cannon's muzzle, which is what is expected.

8.55. **THINK:** The mass of a carbon atom is $m_C = 12.0$ u and the mass of an oxygen atom is $m_O = 16.0$ u. The distance between the atoms in a CO molecule is $d = 1.13 \cdot 10^{-10}$ m. Determine how far the center of mass, X_{com}, is from the carbon atom. Denote the position of the carbon atoms as X_C and the position of the oxygen atom as X_O.
SKETCH:



RESEARCH: The center of mass of a system is given by $X_{com} = \dfrac{1}{M}\sum_{i=1}^{n} m_i x_i$.

The total mass of the system is $M = m_C + m_O$. It is convenient to assign the position of the oxygen atom to be at the origin, $X_O = 0$. Then the center of mass becomes

$$X_{com} = \frac{(0)m_O + m_C d}{m_O + m_C} = \frac{m_C d}{m_O + m_C}.$$

Once X_{com} is determined, then the distance from it to the carbon atom can be determined using the equation $X_{dC} = X_C - X_{com}$, where X_{dC} is the distance from the center of mass to the carbon atom.

SIMPLIFY: Substitute the expression $X_{com} = (m_C d)/(m_O + m_C)$ into the expression for X_{dC} to get $X_{dC} = X_C - \frac{m_C d}{m_O + m_C}$. Substitute $X_C = d$ to get $X_{dC} = d - \frac{m_C d}{m_O + m_C}$.

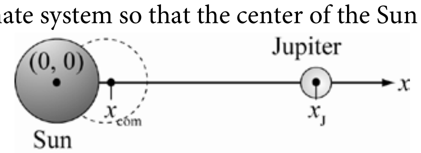
CALCULATE: $X_{dC} = (1.13 \cdot 10^{-10} \text{ m}) - \left(\frac{12.0 \text{ u}}{28.0 \text{ u}}\right)(1.13 \cdot 10^{-10} \text{ m}) = 6.4571 \cdot 10^{-11} \text{ m}$

ROUND: Three significant figures were provided in the problem so the answer should be rounded to $X_{dC} = 6.46 \cdot 10^{-11}$ m.

DOUBLE-CHECK: The center of mass of the system is closer to the more massive oxygen atom, as it should be.

8.56. **THINK:** The system to be considered consists of the Sun and Jupiter. Denote the position of the Sun's center of mass as X_S and the mass as m_S. Denote the position of Jupiter's center of mass as X_J and its mass as m_J. Determine the distance that the Sun wobbles due to its rotation about the center of mass. Also, determine how far the system's center of mass, X_{com}, is from the center of the Sun. The mass of the Sun is $m_S = 1.98892 \cdot 10^{30}$ kg. The mass of Jupiter is $m_J = 1.8986 \cdot 10^{27}$ kg. The distance from the center of the Sun to the center of Jupiter is $X_J = 7.78 \cdot 10^8$ km.

SKETCH: Construct the coordinate system so that the center of the Sun is positioned at the origin.



RESEARCH: The system's center of mass is given by $X_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$.

The total mass of the system is $M = m_S + m_J$. The dashed line in the sketch denotes the Sun's orbit about the system's center of mass. From the sketch it can be seen that the distance the sun wobbles is twice the distance from the Sun's center to the system's center of mass.

SIMPLIFY: $X_{com} = \frac{m_S X_S + m_J X_J}{m_S + m_J}$. The coordinate system was chosen in such a way that $X_S = 0$. The center of mass equation can be simplified to $X_{com} = \frac{m_J X_J}{m_S + m_J}$. Once X_{com} is determined, it can be doubled to get the Sun's wobble.

CALCULATE: $X_{com} = \frac{(1.8986 \cdot 10^{27} \text{ kg})(7.78 \cdot 10^8 \text{ km})}{1.98892 \cdot 10^{30} \text{ kg} + 1.8986 \cdot 10^{27} \text{ kg}} = 741961.5228 \text{ km}$

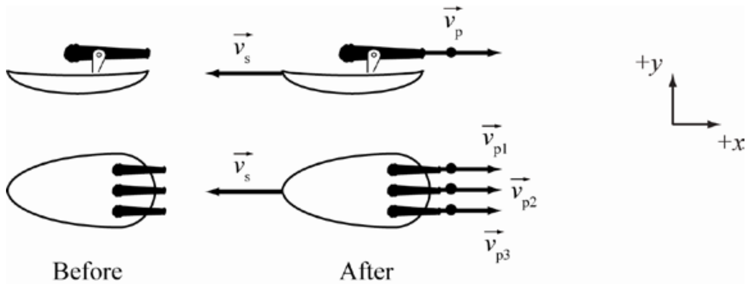
The Sun's wobble is $2(741961.5228 \text{ km}) = 1483923.046$ km.

ROUND: Rounding the results to three figures, $X_{com} = 7.42 \cdot 10^5$ km and the Sun's wobble is $1.49 \cdot 10^6$ km.

DOUBLE-CHECK: It is expected that the system's center of mass is much closer to the Sun than it is to Jupiter, and the results are consistent with this.

8.57. **THINK:** The mass of the battleship is $m_s = 136{,}634{,}000$ lbs. The ship has twelve 16-inch guns and each gun is capable of firing projectiles of mass, $m_p = 2700.$ lb, at a speed of $v_p = 2300.$ ft/s. Three of the guns fire projectiles in the same direction. Determine the recoil velocity, v_s, of the ship. Assume the ship is initially stationary.

SKETCH:



RESEARCH: The total mass of the ship and projectile system is $M = m_s + \sum_{i=1}^{n} m_{pi}$.

All of the projectiles have the same mass and same speed when they are shot from the guns. This problem can be solved considering the conservation of momentum. The equation for the conservation of momentum is $\vec{P}_i = \vec{P}_f$. $\vec{P}_i$ is the initial momentum of the system and $\vec{P}_f$ is the final momentum of the system. Assume the ship carries one projectile per gun. $\vec{P}_i = 0$ because the battleship is initially at rest and $\vec{P}_f = -\left(m_s + 9m_p\right)v_s + 3m_p v_p$.

SIMPLIFY: $\vec{P}_i = \vec{P}_f \Rightarrow 0 = -\left(m_s + 9m_p\right)v_s + 3m_p v_p \Rightarrow v_s = \dfrac{3m_p v_p}{\left(m_s + 9m_p\right)}$

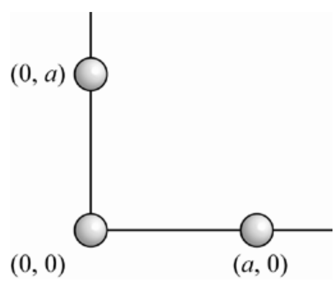
CALCULATE: $v_s = \dfrac{3(2700.\text{ lb})(2300.\text{ ft/s})}{\left(136{,}634{,}000\text{ lb} + 9(2700.\text{ lb})\right)} = 0.136325\text{ ft/s}$

ROUND: The values for the mass and speed of the projectile that are given in the question have four significant figures, so the result should be rounded to $v_s = 0.1363$ ft/s. The recoil velocity is in opposite direction than the cannons fire.

DOUBLE-CHECK: The mass of the ship is much greater than the masses of the projectiles, so it reasonable that the recoil velocity is small because momentum depends on mass and velocity.

8.58. **THINK:** The system has three identical balls of mass *m*. The *x* and *y* coordinates of the balls are $\vec{r}_1 = \left(0\hat{x}, 0\hat{y}\right)$, $\vec{r}_2 = \left(a\hat{x}, 0\hat{y}\right)$ and $\vec{r}_3 = \left(0\hat{x}, a\hat{y}\right)$. Determine the location of the system's center of mass, *R*.

SKETCH:



RESEARCH: The center of mass is a vector quantity, so the *x* and *y* components must be considered separately. The *x*- and *y*-components of the center of mass are given by

$$X_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \text{ and } Y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i.$$

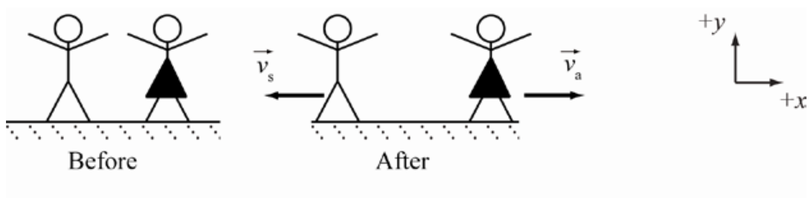
For this system, the equations can be rewritten as

$$X_{com} = \frac{m(0) + ma\hat{x} + m(0)}{3m} = \frac{a}{3}\hat{x} \text{ and } Y_{com} = \frac{m(0) + m(0) + ma\hat{y}}{3m} = \frac{a}{3}\hat{y}.$$

SIMPLIFY: The x and y components of the center of mass are known, so $\vec{R}_{com} = \frac{a}{3}\hat{x} + \frac{a}{3}\hat{y}.$

CALCULATE: This step is not necessary.
ROUND: This step is not necessary.
DOUBLE-CHECK: Considering the geometry of the system, the results are reasonable. In the *x*-direction we would expect the center of mass to be twice as far from the mass on the right as from the two on the left, and in the *y*-direction we would expect the center of mass to be twice as far from the upper mass as from the two lower ones.

8.59. **THINK:** Sam's mass is $m_S = 61.0$ kg and Alice's mass is $m_A = 44.0$ kg. They are standing on an ice rink with negligible friction. After Sam pushes Alice, she is moving away from him with a speed of $v_A = 1.20$ m/s with respect to the rink. Determine the speed of Sam's recoil, v_S. Also, determine the change in kinetic energy, ΔK, of the Sam-Alice system.
SKETCH:



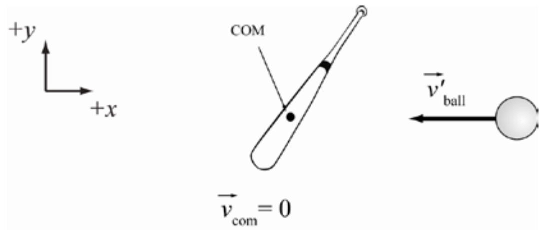
RESEARCH:
(a) To solve the problem, consider the conservation of momentum. The equation for conservation of momentum can be written $\vec{P}_i = \vec{P}_f$. $\vec{P}_i$ is the initial momentum of the system and $\vec{P}_f$ is the final momentum of the system. $\vec{P}_i = 0$ because Sam and Alice are initially stationary and $\vec{P}_f = -m_S\vec{v}_S + m_A\vec{v}_A$.
(b) The change in kinetic energy is $\Delta K = K_f - K_i = \left(m_S v_S^2\right)/2 + \left(m_A v_A^2\right)/2.$
SIMPLIFY:
(a) $\vec{P}_i = \vec{P}_f \Rightarrow 0 = -m_S\vec{v}_S + m_A\vec{v}_A \Rightarrow v_S = \frac{m_A\vec{v}_A}{m_S}$
(b) The expression determined for v_S in part (a) can be substituted into the equation for ΔK to get

$$\Delta K = \frac{1}{2}m_S\left(\frac{m_A\vec{v}_A}{m_S}\right)^2 + \frac{1}{2}m_A v_A^2.$$

CALCULATE:
(a) $v_S = \frac{(44.0 \text{ kg})(1.20 \text{ m/s})}{61.0 \text{ kg}} = 0.8656$ m/s
(b) $\Delta K = \frac{1}{2}(61.0 \text{ kg})\left(\frac{(44.0 \text{ kg})(1.20 \text{ m/s})}{61.0 \text{ kg}}\right)^2 + \frac{1}{2}(44.0 \text{ kg})(1.20 \text{ m/s})^2 = 54.53$ J
(c) Sam did work on Alice when he pushed her. The work that Sam did was the source of the kinetic energy. Sam was able to do this work by converting chemical energy that was stored in his body into mechanical energy. The energy stored in Sam's body was provided by food that he ate and his body processed.
ROUND: Three significant figures were provided in the problem so the results should be rounded accordingly to $v_S = 0.866$ m/s and $\Delta K = 55$ J.
DOUBLE-CHECK: Sam's mass is greater than Alice's so it reasonable that his recoil speed is slower than her sliding speed. The change in kinetic energy is reasonable considering the masses and velocities given.

8.60. **THINK:** The mass of the bat is m_{bat} and the mass of the ball is m_{ball}. Assume that the center of mass of the ball and bat system is essentially at the bat. The initial velocity of the ball is $\vec{v}_{\text{ball,i}} = -30.0$ m/s and the initial velocity of the bat is $\vec{v}_{\text{bat}} = 35.0$ m/s. The bat and ball undergo a one-dimensional elastic collision. Determine the speed of the ball after the collision.

SKETCH:



RESEARCH: In the center of mass frame, $\vec{v}_{\text{com}} = 0$. Since the collision is elastic, in the center of mass frame the final velocity of the ball, $\vec{v}_{\text{ball,f}}$, will be equal to the negative of the ball's initial velocity, $\vec{v}_{\text{ball,i}}$. This statement can be written mathematically as $\vec{v}_{\text{ball,i}} = -\vec{v}_{\text{ball,f}}$. Since the center of mass is in the bat, the $\vec{v}_{\text{com}}$ in the lab reference frame equals $\vec{v}_{\text{bat}}$. The following relationships can be written for this system: $\vec{v}'_{\text{ball,i}} = \vec{v}_{\text{ball,i}} - \vec{v}_{\text{com}}$ (1) and $\vec{v}'_{\text{ball,f}} = \vec{v}_{\text{ball,f}} - \vec{v}_{\text{com}}$ (2).

SIMPLIFY: Recall that $\vec{v}'_{\text{ball,i}} = -\vec{v}'_{\text{ball,f}}$. Therefore, the following equality can be written:

$\vec{v}_{\text{ball,i}} - \vec{v}_{\text{com}} = -\left(\vec{v}_{\text{ball,f}} - \vec{v}_{\text{com}}\right) \Rightarrow \vec{v}_{\text{ball,f}} = 2\vec{v}_{\text{com}} - \vec{v}_{\text{ball,i}}$.

Recall that $\vec{v}_{\text{com}}$ is equal to $\vec{v}_{\text{bat}}$, so the above expression can be rewritten as $\vec{v}_{\text{ball,f}} = 2\vec{v}_{\text{bat}} - \vec{v}_{\text{ball,i}}$.

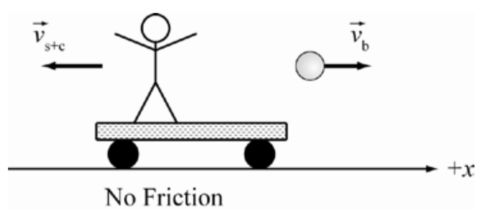
CALCULATE: $\vec{v}_{\text{ball,f}} = 2(35.0 \text{ m/s}) - (-30.0 \text{ m/s}) = 100.0 \text{ m/s}$

ROUND: Rounding to three significant figures: $\vec{v}_{\text{ball,f}} = 100.$ m/s

DOUBLE-CHECK: The initial velocities of the bat and ball are similar, but the bat is much more massive than the ball, so the speed of the ball after the collision is expected to be high.

8.61. **THINK:** The student's mass is $m_{\text{s}} = 40.0$ kg, the ball's mass is $m_{\text{b}} = 5.00$ kg and the cart's mass is $m_{\text{c}} = 10.0$ kg. The ball's relative speed is $v'_{\text{b}} = 10.0$ m/s and the student's initial speed is $v_{\text{si}} = 0$. Determine the ball's velocity with respect to the ground, $\vec{v}_{\text{b}}$, after it is thrown.

SKETCH:



RESEARCH: $\vec{v}_{\text{b}}$ can be determined by considering the conservation of momentum, $\vec{P}_{\text{i}} = \vec{P}_{\text{f}}$, where $p = mv$. Note the ball's relative speed is $\vec{v}'_{\text{b}} = \vec{v}_{\text{b}} - \vec{v}_{\text{s+c}}$, where $\vec{v}_{\text{b}}$ and $\vec{v}_{\text{s+c}}$ are measured relative to the ground.

SIMPLIFY: $\vec{P}_{\text{i}} = \vec{P}_{\text{f}} \Rightarrow 0 = (m_{\text{s}} + m_{\text{c}})\vec{v}_{\text{s+c}} + m_{\text{b}}\vec{v}_{\text{b}} \Rightarrow 0 = (m_{\text{s}} + m_{\text{c}})(\vec{v}_{\text{b}} - \vec{v}'_{\text{b}}) + m_{\text{b}}\vec{v}_{\text{b}} \Rightarrow \vec{v}_{\text{b}} = \dfrac{\vec{v}'_{\text{b}}(m_{\text{s}} + m_{\text{c}})}{m_{\text{s}} + m_{\text{c}} + m_{\text{b}}}$

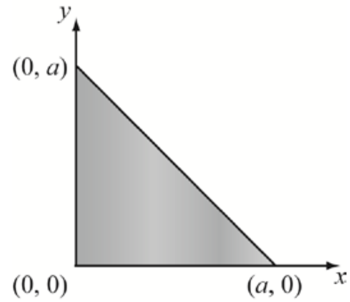
CALCULATE: $\vec{v}_{\text{b}} = \dfrac{(10.0 \text{ m/s})(40.0 \text{ kg} + 10.0 \text{ kg})}{(40.0 \text{ kg} + 10.0 \text{ kg} + 5.00 \text{ kg})} = 9.0909 \text{ m/s}$

ROUND: $\vec{v}_{\text{b}} = 9.09$ m/s in the direction of $\vec{v}'_{\text{b}}$ (horizontal)

DOUBLE-CHECK: It is expected that $v_{\text{b}} < v'_{\text{b}}$ since the student and cart move away from the ball when it is thrown.

8.62. **THINK:** Determine the center of mass of an isosceles triangle of constant density σ.

SKETCH:



RESEARCH: To determine the center of mass of a two-dimensional object of constant density σ, use $X = \frac{1}{A}\int_A \sigma x dA$ and $Y = \frac{1}{A}\int_A \sigma y dA.$

SIMPLIFY: Note the boundary condition on the hypotenuse of the triangle, $x + y = a.$ First, determine X. As x varies, take $dA = ydx$. Then the equation becomes $X = \frac{\sigma}{A}\int_0^a xydx.$ From the boundary condition, $y = a - x.$ Then the equation can be rewritten as $X = \frac{\sigma}{A}\int_0^a x(a-x)dx = \left[\frac{\sigma}{A}\left(\frac{1}{2}ax^2 - \frac{1}{3}x^3\right)\right]_0^a = \frac{a^3\sigma}{6A}.$ Similarly for Y, take $dA = xdy$ and $x = a - y$ to get $Y = \frac{\sigma}{A}\int_0^a y(a-y)dy = \left[\frac{\sigma}{A}\left(\frac{1}{2}ay^2 - \frac{1}{3}y^3\right)\right]_0^a = \frac{a^3\sigma}{6A},$ with $A = \int \sigma dA = \sigma \cdot \frac{bh}{2} = \frac{a^2\sigma}{2}$ we get $X = Y = \frac{2}{a^2\sigma}\cdot\frac{a^3\sigma}{6A} = \frac{a}{3}.$

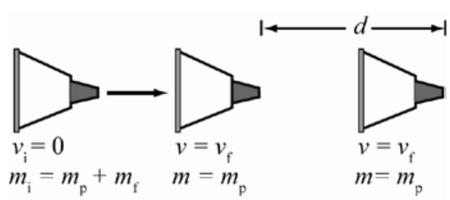
CALCULATE: This step is not applicable.

ROUND: This step is not applicable.

DOUBLE-CHECK: The center of mass coordinates that we obtained are contained within the isosceles triangle, as expected for a solid object.

8.63. **THINK:** The payload's mass is $m_{\text{p}} = 4390.0 \text{ kg}$ and the fuel mass is $m_{\text{f}} = 1.761 \cdot 10^5 \text{ kg}.$ The initial velocity is $v_{\text{i}} = 0.$ The distance traveled after achieving v_{f} is $d = 3.82 \cdot 10^8 \text{ m}.$ The trip time is $t = 7.00 \text{ h} = 2.52 \cdot 10^4 \text{ s}.$ Determine the propellant expulsion speed, v_{c}.

SKETCH:



RESEARCH: v_{c} can be determined from $v_{\text{f}} - v_{\text{i}} = v_{\text{c}} \ln(m_{\text{i}} / m_{\text{f}}).$ First, v_{f} must be determined from the relationship $v = d/t.$

SIMPLIFY: First, determine v_{f} from $v_{\text{f}} = d/t.$ Substitute this expression and $v_{\text{i}} = 0$ into the above equation to determine v_{c}:

$$v_{\text{c}} = \frac{v_{\text{f}}}{\ln\left(\frac{m_{\text{i}}}{m_{\text{f}}}\right)} = \frac{d}{t\ln\left(\frac{m_{\text{i}}}{m_{\text{f}}}\right)} = \frac{d}{t\ln\left(\frac{m_{\text{p}} + m_{\text{f}}}{m_{\text{p}}}\right)}.$$

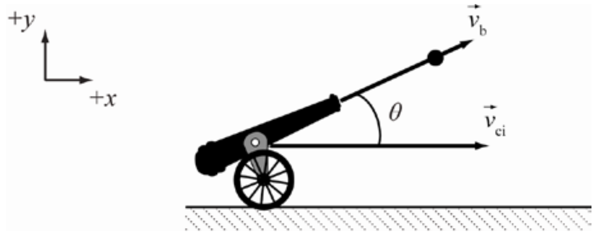
CALCULATE: $v_c = \dfrac{3.82\cdot 10^8 \text{ m}}{\left(2.52\cdot 10^4 \text{ s}\right)\ln\left(\dfrac{4390.0 \text{ kg}+1.761\cdot 10^5 \text{ kg}}{4390.0 \text{ kg}}\right)} = 4.079\cdot 10^3 \text{ m/s}$

ROUND: Since t has three significant figures, the result should be rounded to $v_c = 4.08$ km/s.

DOUBLE-CHECK: This expulsion velocity is reasonable.

8.64. **THINK:** The cannon's mass is M = 350 kg. The cannon's initial speed is $v_{ci} = 7.5$ m/s. The ball's mass is m = 15 kg and the launch angle is $\theta = 55°$. The cannon's final velocity after the shot is $v_{cf} = 0$. Determine the velocity of the ball relative to the cannon, $\vec{v}_b'$.

SKETCH:



RESEARCH: Use conservation of momentum, $\vec{P}_i = \vec{P}_f$, where $\vec{P} = m\vec{v}$. To determine the relative velocity, $\vec{v}_b'$, with respect to the cannon, use $\vec{v}_b' = \vec{v}_b - \vec{v}_c$, where $\vec{v}_b$ is the ball's velocity in the lab frame. Finally, since the cannon moves only in the horizontal (x) direction, consider only momentum conservation in this dimension. Take $\vec{v}_{ci}$ to be along the positive x-direction, that is $v_{ci} = +7.5$ m/s. With v_{bx} known, find v_b from the expression $v_{bx} = v_b \cos\theta$ and then v_b' can be determined.

SIMPLIFY: $P_{xi} = P_{xf} \Rightarrow \left(m_b + m_c\right)v_{ci} = m_c v_{cf} + m_b v_{bx}$. Note since v_{cf} is zero, $v_{bx} = v_{bx}'$, that is, the ball's speed relative to the cannon is the same as its speed in the lab frame since the cannon has stopped moving. Rearranging the above equation gives $v_{bx} = \dfrac{\left(m_b + m_c\right)v_{ci}}{m_b} \Rightarrow v_b = \dfrac{\left(m_b + m_c\right)v_{ci}}{m_b \cos\theta}$.

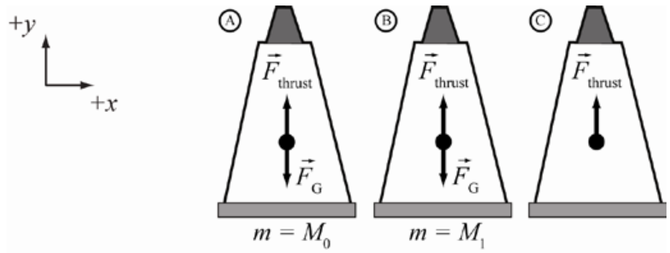
CALCULATE: $v_b = \dfrac{\left(15.0 \text{ kg} + 350 \text{ kg}\right)\left(7.50 \text{ m/s}\right)}{\left(15.0 \text{ kg}\right)\cos\left(55.0°\right)} = 318.2 \text{ m/s}$

ROUND: Each given value has three significant figures, so the result should be rounded to $v_b = 318$ m/s.

DOUBLE-CHECK: This is a reasonable speed at which to launch a cannonball. The component of the momentum of the cannon/cannon ball system in the x-direction before the ball is shot is $p_{x,\text{before}} = \left(350 \text{ kg} + 15 \text{ kg}\right)\left(7.5 \text{ m/s}\right) = 2737.5$ kg m/s. The component of the momentum of the cannon/cannon ball system in the x-direction after the ball is shot is $p_{x,\text{after}} = \left(15 \text{ kg}\right)\left(318.2 \text{ m/s}\right)\cos\left(55°\right) = 2737.68$ kg m/s. These components agree to within three significant figures.

8.65. **THINK:** The rocket's initial mass is $M_0 = 2.80\cdot 10^6$ kg. Its final mass is $M_1 = 8.00\cdot 10^5$ kg. The time to burn all the fuel is $\Delta t = 160.$ s. The exhaust speed is $v = v_c = 2700.$ m/s. Determine (a) the upward acceleration, a_0, of the rocket as it lifts off, (b) its upward acceleration, a_1, when all the fuel has burned and (c) the net change in speed, Δv in time Δt in the absence of a gravitational force.

SKETCH:



RESEARCH: To determine the upward acceleration, all the vertical forces on the rocket must be balanced. Use the following equations: $\vec{F}_{\text{thrust}} = -\vec{v}_{\text{c}} \dfrac{dm}{dt}$, $\vec{F}_{\text{g}} = m\vec{g}$, $\dfrac{dm}{dt} = \dfrac{\Delta m}{\Delta t}$. The mass of the fuel used is $\Delta m = M_0 - M_1$. To determine Δv in the absence of other forces (other than $\vec{F}_{\text{thrust}}$), use $v_{\text{f}} - v_{\text{i}} = v_{\text{c}} \ln(m_{\text{i}} / m_{\text{f}})$.

SIMPLIFY:

(a) $\dfrac{dm}{dt} = \dfrac{M_0 - M_1}{\Delta t}$

Balancing the vertical forces on the rocket gives

$$F_{\text{net}} = F_{\text{thrust}} - F_{\text{g}} = ma \Rightarrow M_0 a_0 = v_{\text{c}} \frac{dm}{dt} - M_0 g \Rightarrow a_0 = \frac{v_{\text{c}}}{M_0}\left(\frac{M_0 - M_1}{\Delta t}\right) - g \Rightarrow a_0 = \frac{v_{\text{c}}}{\Delta t}\left(1 - \frac{M_1}{M_0}\right) - g.$$

(b) Similarly to part (a):

$$F_{\text{net}} = F_{\text{thrust}} - F_{\text{g}} = ma \Rightarrow M_1 a_1 = v_{\text{c}} \frac{dm}{dt} - M_1 g \Rightarrow a_1 = \frac{v_{\text{c}}}{M_1}\left(\frac{M_0 - M_1}{\Delta t}\right) - g \Rightarrow a_1 = \frac{v_{\text{c}}}{\Delta t}\left(\frac{M_0}{M_1} - 1\right) - g.$$

(c) In the absence of gravity, $F_{\text{net}} = F_{\text{thrust}}$. The change in velocity due to this thrust force is $\Delta v = v_{\text{c}} \ln(M_0 / M_1)$.

CALCULATE:

(a) $a_0 = \left(\dfrac{2700.\text{ m/s}}{160\text{ s}}\right)\left(1 - \dfrac{8.00 \cdot 10^5\text{ kg}}{2.80 \cdot 10^6\text{ kg}}\right) - 9.81\text{ m/s}^2 = 2.244\text{ m/s}^2$

(b) $a_1 = \left(\dfrac{2700.\text{ m/s}}{160.\text{ s}}\right)\left(\dfrac{2.80 \cdot 10^6\text{ kg}}{8.00 \cdot 10^5\text{ kg}} - 1\right) - 9.81\text{ m/s}^2 = 32.38\text{ m/s}^2$

(c) $\Delta v = (2700.\text{ m/s})\ln\left(\dfrac{2.80 \cdot 10^6\text{ kg}}{8.00 \cdot 10^5\text{ kg}}\right) = 3382\text{ m/s}$

ROUND:

(a) $a_0 = 2.24\text{ m/s}^2$

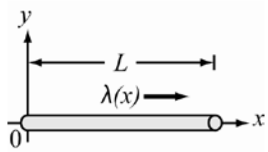
(b) $a_1 = 32.4\text{ m/s}^2$

(c) $\Delta v = 3380\text{ m/s}$

DOUBLE-CHECK: It can be seen that $a_1 > a_0$, as it should be since $M_1 < M_0$. It is not unusual for Δv to be greater than v_{c}.

8.66. **THINK:** The rod has a length of L and its linear density is $\lambda(x)=cx$, where c is a constant. Determine the rod's center of mass.

SKETCH:



RESEARCH: To determine the center of mass, take a differentially small element of mass: $dm=\lambda dx$ and use $X=\frac{1}{M}\int_L x\cdot dm=\frac{1}{M}\int_L x\lambda(x)dx$, where $M=\int_L dm=\int_L \lambda(x)dx$.

SIMPLIFY: First, determine M from $M=\int_0^L cxdx=\left[c\frac{1}{2}x^2\right]_0^L=\frac{1}{2}cL^2$. Then, the equation for the center of mass becomes:

$$X=\frac{1}{M}\int_0^L x(cx)dx=\frac{1}{M}\int_0^L cx^2dx=\frac{1}{M}c\left[\frac{1}{3}x^3\right]_0^L=\frac{1}{3M}cL^3.$$

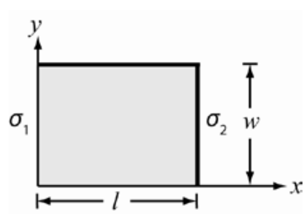
Substituting the expression for M into the above equation gives:

$$X=\frac{cL^3}{3\left(\frac{1}{2}cL^2\right)}=\frac{2}{3}L.$$

CALCULATE: This step is not applicable.
ROUND: This step is not applicable.
DOUBLE-CHECK: X is a function of L. Also, as expected, X is closer to the denser end of the rod.

8.67. **THINK:** The length and width of the plate are l = 20.0 cm and w = 10.0 cm, respectively. The mass density, σ, varies linearly along the length; at one end it is $\sigma_1=5.00\text{ g/cm}^2$ and at the other it is $\sigma_2=20.0\text{ g/cm}^2$. Determine the center of mass.

SKETCH:



RESEARCH: The mass density does not vary in width, i.e. along the y-axis. Therefore, the Y coordinate is simply $w/2$. To determine the X coordinate, use

$$X=\frac{1}{M}\int_A x\sigma(\vec{r})dA,\text{ where } M=\int_A \sigma(\vec{r})dA.$$

To obtain a functional form for $\sigma(\vec{r})$, consider that it varies linearly with x, and when the bottom left corner of the plate is at the origin of the coordinate system, σ must be σ_1 when x = 0 and σ_2 when $x = l$. Then, the conditions are satisfied by $\sigma(\vec{r})=\sigma(x)=\frac{(\sigma_2-\sigma_1)}{l}x+\sigma_1$.

SIMPLIFY: First determine M from $M=\int_A \sigma(\vec{r})dA=\int_0^l\int_0^w \sigma(x)dydx=\int_0^w dy\int_0^l\left(\frac{(\sigma_2-\sigma_1)}{l}x+\sigma_1\right)dx$. y is not dependent on x in this case, so

$$M = \left[y\right]_0^w \left[\frac{1}{2}\frac{(\sigma_2 - \sigma_1)}{l}x^2 + \sigma_1 x\right]_0^l = w\left(\frac{1}{2}\frac{(\sigma_2 - \sigma_1)}{l}l^2 + \sigma_1 l\right) = wl\left(\frac{1}{2}(\sigma_2 - \sigma_1) + \sigma_1\right) = \frac{wl}{2}(\sigma_2 + \sigma_1).$$

Now, reduce the equation for the center of mass:

$$X = \frac{1}{M}\int_A x\sigma(x)\,dA = \frac{1}{M}\int_0^l\int_0^w x\left(\frac{(\sigma_2 - \sigma_1)}{l}x + \sigma_1\right)dydx = \frac{1}{M}\int_0^w dy\int_0^l\left(\frac{(\sigma_2 - \sigma_1)}{l}x^2 + \sigma_1 x\right)dx$$

$$= \frac{1}{M}\left[y\right]_0^w\left[\frac{(\sigma_2 - \sigma_1)}{3l}x^3 + \frac{1}{2}\sigma_1 x^2\right]_0^l = \frac{1}{M}w\left(\frac{(\sigma_2 - \sigma_1)}{3l}l^3 + \frac{1}{2}\sigma_1 l^2\right) = \frac{1}{M}wl^2\left(\frac{1}{3}(\sigma_2 - \sigma_1) + \frac{1}{2}\sigma_1\right)$$

$$= \frac{1}{M}wl^2\left(\frac{1}{3}\sigma_2 + \frac{1}{6}\sigma_1\right)$$

Substitute the expression for M into the above equation to get

$$X = \frac{l\left(\frac{1}{3}\sigma_2 + \frac{1}{6}\sigma_1\right)}{\frac{1}{2}(\sigma_2 + \sigma_1)} = \frac{2l\left(\frac{1}{3}\sigma_2 + \frac{1}{6}\sigma_1\right)}{\sigma_2 + \sigma_1}.$$

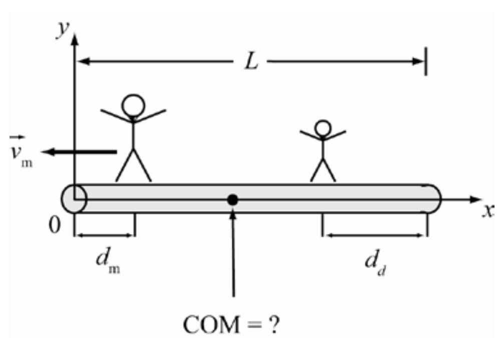
CALCULATE: $X = \dfrac{2(20.0\text{ cm})\left(\frac{1}{3}(20.0\text{ g/cm}^2) + \frac{1}{6}(5.00\text{ g/cm}^2)\right)}{20.0\text{ g/cm}^2 + 5.00\text{ g/cm}^2} = 12.00\text{ cm},\quad Y = \frac{1}{2}(10.0\text{ cm}) = 5.00\text{ cm}$

ROUND: The results should be written to three significant figures: $X = 12.0$ cm and $Y = 5.00$ cm. The center of mass is at (12.0 cm, 5.00 cm).

DOUBLE-CHECK: It is expected that the center of mass for the *x* coordinate is closer to the denser end of the rectangle (before rounding).

8.68. **THINK:** The log's length and mass are $L = 2.50$ m and $m_l = 91.0$ kg, respectively. The man's mass is $m_{\text{m}} = 72$ kg and his location is $d_{\text{m}} = 0.220$ m from one end of the log. His daughter's mass is $m_{\text{d}} = 20.0$ kg and her location is $d_{\text{d}} = 1.00$ m from the other end of the log. Determine (a) the system's center of mass and (b) the initial speed of the log and daughter, $v_{\text{l+d}}$, when the man jumps off the log at a speed of $v_{\text{m}} = 3.14$ m/s.

SKETCH:



RESEARCH: In one dimension, the center of mass location is given by $X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i$. Take the origin of the coordinate system to be at the end of log near the father. To determine the initial velocity of the log and girl system, consider the conservation of momentum, $\vec{p}_{\text{i}} = \vec{p}_{\text{f}}$, where $\vec{p} = m\vec{v}$. Note that the man's velocity is away from the daughter. Take this direction to be along the $-\hat{x}$ direction, so that $\vec{v}_{\text{m}} = -3.14\text{ m/s}\,\hat{x}$.

SIMPLIFY:

(a) $X = \frac{1}{M}\left(x_{\text{m}}m_{\text{m}} + x_{\text{d}}m_{\text{d}} + x_{\text{l}}m_{\text{l}}\right) = \frac{\left(d_{\text{m}}m_{\text{m}} + \left(L - d_{\text{d}}\right)m_{\text{d}} + \frac{1}{2}Lm_{\text{l}}\right)}{m_{\text{m}} + m_{\text{d}} + m_{\text{l}}}$

(b) $\vec{p}_{\text{i}} = \vec{p}_{\text{f}} \Rightarrow 0 = m_{\text{m}}\vec{v}_{\text{m}} + \left(m_{\text{d}} + m_{\text{l}}\right)\vec{v}_{\text{d+l}} \Rightarrow \vec{v}_{\text{d+l}} = -\frac{m_{\text{m}}\vec{v}_{\text{m}}}{\left(m_{\text{d}} + m_{\text{l}}\right)}$

CALCULATE:

(a) $X = \frac{\left(\left(0.220\text{ m}\right)\left(72.0\text{ kg}\right) + \left(2.50\text{ m} - 1.00\text{ m}\right)\left(20.0\text{ kg}\right) + \frac{1}{2}\left(2.50\text{ m}\right)\left(91.0\text{ kg}\right)\right)}{72.0\text{ kg} + 20.0\text{ kg} + 91.0\text{ kg}} = 0.8721\text{ m}$

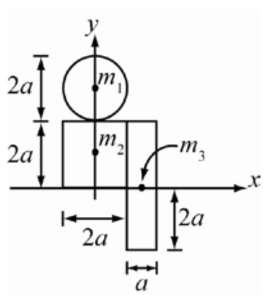
(b) $\vec{v}_{\text{d+l}} = -\frac{\left(72.0\text{ kg}\right)\left(-3.14\text{ m/s }\hat{x}\right)}{\left(20.0\text{ kg} + 91.0\text{ kg}\right)} = 2.0368\text{ m/s }\hat{x}$

ROUND: To three significant figures, the center of mass of the system is X = 0.872 m from the end of the log near the man, and the speed of the log and child is $v_{\text{d+l}} = 2.04$ m/s.

DOUBLE-CHECK: As it should be, the center of mass is between the man and his daughter, and $v_{\text{d+l}}$ is less than v_{m} (since the mass of the log and child is larger than the mass of the man).

8.69. **THINK:** Determine the center of mass of an object which consists of regularly shaped metal of uniform thickness and density. Assume that the density of the object is ρ.

SKETCH:



RESEARCH: First, as shown in the figure above, divide the object into three parts, m_1, m_2 and m_3. Determine the center of mass by using $\vec{R} = \frac{1}{M}\sum_{i=1}^{3} m_i\vec{r}_i$, or in component form $X = \frac{1}{M}\sum_{i=1}^{3} m_i x_i$ and $Y = \frac{1}{M}\sum_{i=1}^{3} m_i y_i$. Also, use $m = \rho At$ for the mass, where A is the area and t is the thickness.

SIMPLIFY: The center of mass components are given by:

$$X = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M} \text{ and } Y = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M}$$

The masses of the three parts are $m_1 = \rho\pi a^2 t$, $m_2 = \rho\left(2a\right)^2 t$ and $m_3 = \rho 4a^2 t$. The center of mass of the three parts are $x_1 = 0$, $y_1 = 3a$, $x_2 = 0$, $y_2 = a$, $x_3 = 3a/2$ and $y_3 = 0$. The total mass of the object is $M = m_1 + m_2 + m_3 = \rho\pi a^2 t + 4\rho a^2 t + 4\rho a^2 t = \rho a^2 t\left(8 + \pi\right)$.

CALCULATE: The center of mass of the object is given by the following equations:

$$X = \frac{0 + 0 + 4\rho a^2 t\left(3a/2\right)}{\rho a^2 t\left(8 + \pi\right)} = \left(\frac{6}{8 + \pi}\right)a;$$

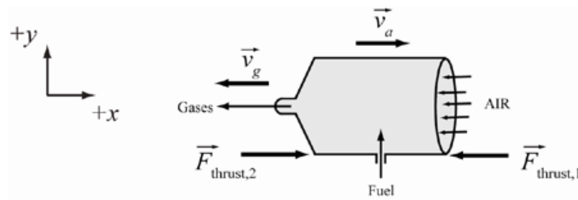
$$Y = \frac{\rho\pi a^2 t\left(3a\right) + 4\rho a^2 t\left(a\right) + 0}{\rho a^2 t\left(8 + \pi\right)} = \left(\frac{4 + 3\pi}{8 + \pi}\right)a.$$

ROUND: Rounding is not required.

DOUBLE-CHECK: The center of mass of the object is located in the area of m_2. By inspection of the figure this is reasonable.

8.70. **THINK:** A jet aircraft has a speed of 223 m/s. The rate of change of the mass of the aircraft is $(dM/dt)_{\text{air}} = 80.0$ kg/s (due to the engine taking in air) and $(dM/dt)_{\text{fuel}} = 3.00$ kg/s (due to the engine taking in and burning fuel). The speed of the exhaust gases is 600. m/s. Determine the thrust of the jet engine.

SKETCH:



RESEARCH: The thrust is calculated by using $\vec{F}_{\text{thrust}} = -\vec{v}\,dM/dt$, where $\vec{v}$ is the velocity of the gases or air, relative to the engine. There are two forces on the engine. The first force, $F_{\text{thrust,1}}$, is the thrust due to the engine taking in air and the second force, $F_{\text{thrust,2}}$, is the thrust due to the engine ejecting gases.

$$\vec{F}_{\text{thrust,1}} = -\vec{v}_{\text{a}}\left(\frac{dM}{dt}\right)_{\text{air}}, \quad \vec{F}_{\text{thrust,2}} = -\vec{v}_{\text{g}}\left[\left(\frac{dM}{dt}\right)_{\text{air}} + \left(\frac{dM}{dt}\right)_{\text{fuel}}\right]$$

The net thrust is given by $\vec{F}_{\text{thrust}} = \vec{F}_{\text{thrust,1}} + \vec{F}_{\text{thrust,2}}$.

SIMPLIFY: Simplification is not required.

CALCULATE: $\vec{F}_{\text{thrust,1}} = -(223 \text{ m/s } \hat{x})(80.0 \text{ kg/s}) = -17840 \text{ N } \hat{x}$,

$\vec{F}_{\text{thrust,2}} = -(600. \text{ m/s } (-\hat{x}))[80.0 \text{ kg/s} + 3.00 \text{ kg/s}] = 49800 \text{ N } \hat{x}$,

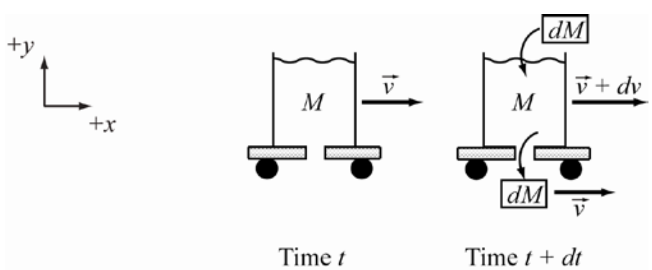
$\vec{F}_{\text{thrust}} = -17840 \text{ N } \hat{x} + 49800 \text{ N } \hat{x} = 31960 \text{ N } \hat{x}$

ROUND: To three significant figures, the thrust of the jet engine is $\vec{F}_{\text{thrust}} = 32.0 \text{ kN } \hat{x}$.

DOUBLE-CHECK: Since the $\hat{x}$ direction is in the forward direction of the aircraft, the plane moves forward, which it must. A jet engine is very powerful, so the large magnitude of the result is reasonable.

8.71. **THINK:** The solution to this problem is similar to a rocket system. Here the system consists of a bucket, a skateboard and water. The total mass of the system is M = 10.0 kg. The total mass of the bucket, skateboard and water remains constant at $\lambda = dM/dt = 0.100$ kg/s since rain water enters the top of the bucket at the same rate that it exits the bottom. Determine the time required for the bucket and the skateboard to reach a speed of half the initial speed.

SKETCH:



RESEARCH: To solve this problem, consider the conservation of momentum, $\vec{p}_{\text{i}} = \vec{p}_{\text{f}}$. The initial momentum of the system at time t is $p_{\text{i}} = Mv$. After time $t + dt$, the momentum of the system is $p_{\text{f}} = vdM + M(v + dv)$.

SIMPLIFY: $p_{\text{i}} = p_{\text{f}} \Rightarrow Mv = vdM + Mv + Mdv \Rightarrow Mdv = -vdM$

Dividing both sides by dt gives

$$M\frac{dv}{dt}=-v\frac{dM}{dt}=-v\lambda \text{ or } \frac{1}{v}\frac{dv}{dt}=-\frac{\lambda}{M} \Rightarrow \frac{1}{v}dv=-\frac{\lambda}{M}dt.$$

Integrate both sides to get

$$\int_{v=v_0}^{v}\frac{1}{v}dv=\int_{t=0}^{t}-\frac{\lambda}{M}dt \Rightarrow \ln v-\ln v_0=-\frac{\lambda}{M}t \Rightarrow \ln\left(\frac{v}{v_0}\right)=-\frac{\lambda}{M}t.$$

Determine the time such that $v=v_0/2$. Substituting $v=v_0/2$ into the above equation gives

$$\ln\left(\frac{v_0/2}{v_0}\right)=-\frac{\lambda}{M}t \Rightarrow t=-\frac{M}{\lambda}\ln\left(\frac{1}{2}\right)=\frac{M}{\lambda}\ln(2).$$

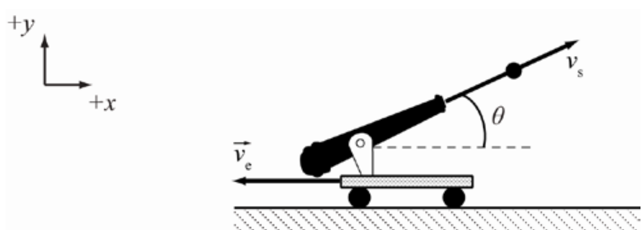
CALCULATE: $t=\dfrac{(10.0\text{ kg})\ln(2)}{0.100\text{ kg/s}}=69.3147\text{ s}$

ROUND: To three significant figures, the time for the system to reach half of its initial speed is t = 69.3 s.

DOUBLE-CHECK: It is reasonable that the time required to reduce the speed of the system to half its original value is near one minute.

8.72. **THINK:** The mass of a cannon is M = 1000. kg and the mass of a shell is m = 30.0 kg. The shell is shot at an angle of $\theta=25.0°$ above the horizontal with a speed of $v_s=500.$ m/s. Determine the recoil velocity of the cannon.

SKETCH:



RESEARCH: The momentum of the system is conserved, $p_i=p_f$, or in component form, $p_{xi}=p_{xf}$ and $p_{yi}=p_{yf}$. Use only the x component of the momentum.

SIMPLIFY: p_{xi} is equal to zero since both the cannon and the shell are initially at rest. Therefore,

$$p_{xi}=p_{xf} \Rightarrow mv_s\cos\theta+Mv_c=0 \Rightarrow v_c=-\frac{m}{M}v_s\cos\theta.$$

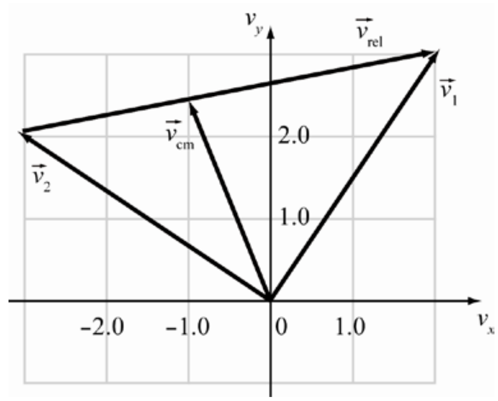
CALCULATE: $v_c=-\dfrac{(30.0\text{ kg})(500.\text{ m/s})\cos(25.0°)}{1000.\text{ kg}}=-13.595\text{ m/s}$

ROUND: To three significant figures: $v_c=-13.6$ m/s

DOUBLE-CHECK: The direction of the recoil is expected to be in the opposite direction to the horizontal component of the velocity of the shell. This is why the result is negative.

8.73. **THINK:** There are two masses, $m_1=2.0$ kg and $m_2=3.0$ kg. The velocity of their center of mass and the velocity of mass 1 relative to mass 2 are $\vec{v}_{cm}=(-1.00\hat{x}+2.40\hat{y})$ m/s and $\vec{v}_{rel}=(5.00\hat{x}+1.00\hat{y})$ m/s. Determine the total momentum of the system and the momenta of mass 1 and mass 2.

SKETCH:



RESEARCH: The total momentum of the system is $\vec{p}_{\text{cm}} = M\vec{v}_{\text{cm}} = m_1\vec{v}_1 + m_2\vec{v}_2$. The velocity of mass 1 relative to mass 2 is $\vec{v}_{\text{rel}} = \vec{v}_1 - \vec{v}_2$.

SIMPLIFY: The total mass M of the system is $M = m_1 + m_2$. The total momentum of the system is given by $\vec{p}_{\text{cm}} = M\vec{v}_{\text{cm}} = (m_1 + m_2)\vec{v}_{\text{cm}} = m_1\vec{v}_1 + m_2\vec{v}_2$. Substitute $\vec{v}_2 = \vec{v}_1 - \vec{v}_{\text{rel}}$ into the equation for the total momentum of the system to get $M\vec{v}_{\text{cm}} = m_1\vec{v}_1 + m_2(\vec{v}_1 - \vec{v}_{\text{rel}}) = (m_1 + m_2)\vec{v}_1 - m_2\vec{v}_{\text{rel}}$. Therefore, $\vec{v}_1 = \vec{v}_{\text{cm}} + \dfrac{m_2}{M}\vec{v}_{\text{rel}}$. Similarly, substitute $\vec{v}_1 = \vec{v}_2 + \vec{v}_{\text{rel}}$ into the equation for the total momentum of the system to get $M\vec{v}_{\text{cm}} = m_1\vec{v}_{\text{rel}} + (m_1 + m_2)\vec{v}_2$ or $\vec{v}_2 = \vec{v}_{\text{cm}} - \dfrac{m_1}{M}\vec{v}_{\text{rel}}$. Therefore, the momentums of mass 1 and mass 2 are $\vec{p}_1 = m_1\vec{v}_1 = m_1\vec{v}_{\text{cm}} + \dfrac{m_1 m_2}{M}\vec{v}_{\text{rel}}$ and $\vec{p}_2 = m_2\vec{v}_2 = m_2\vec{v}_{\text{cm}} - \dfrac{m_1 m_2}{M}\vec{v}_{\text{rel}}$.

CALCULATE:

(a)

$$\vec{p}_{\text{cm}} = (2.00\text{ kg} + 3.00\text{ kg})(-1.00\hat{x} + 2.40\hat{y}\ )\text{ m/s} = (-5.00\hat{x} + 12.0\hat{y})\text{ kg m/s}$$

$$\vec{p}_{\text{cm}} = (2.0\text{ kg} + 3.0\text{ kg})(-1.0\hat{x} + 2.4\hat{y}\ )\text{ m/s} = (-5.0\hat{x} + 12\hat{y})\text{ kg m/s}$$

(b) $$\vec{p}_1 = (2.00\text{ kg})(-1.00\hat{x} + 2.40\hat{y}\ )\text{ m/s} + \frac{(2.00\text{ kg})(3.00\text{ kg})}{2.00\text{ kg} + 3.00\text{ kg}}(5.00\hat{x} + 1.00\hat{y})\text{ m/s}$$
$$= (-2.00\hat{x} + 4.80\hat{y})\text{ kg m/s} + (6.00\hat{x} + 1.20\hat{y})\text{ kg m/s} = (4.00\hat{x} + 6.00\hat{y})\text{ kg m/s}$$

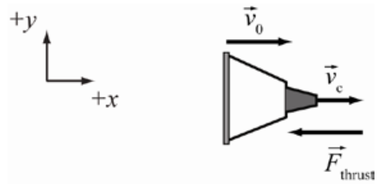
(c) $$\vec{p}_2 = (3.00\text{ kg})(-1.00\hat{x} + 2.40\hat{y}\ )\text{ m/s} - \frac{(2.00\text{ kg})(3.00\text{ kg})}{2.00\text{ kg} + 3.00\text{ kg}}(5.00\hat{x} + 1.00\hat{y})\text{ m/s}$$
$$= (-3.00\hat{x} + 7.20\hat{y})\text{ kg m/s} - (6.00\hat{x} + 1.20\hat{y})\text{ kg m/s} = (-9.00\hat{x} + 6.00\hat{y})\text{ kg m/s}$$

ROUND: The answers have already been rounded to three significant figures.

DOUBLE-CHECK: It is clear from the results of (a), (b) and (c) that $\vec{p}_{\text{cm}} = \vec{p}_1 + \vec{p}_2$.

8.74. **THINK:** A spacecraft with a total initial mass of $m_s = 1000.\text{ kg}$ and an initial speed of $v_0 = 1.00\text{ m/s}$ must be docked. The mass of the fuel decreases from 20.0 kg. Since the mass of the fuel is small compared to the mass of the spacecraft, we can ignore it. To reduce the speed of the spacecraft, a small retro-rocket is used which can burn fuel at a rate of $dM/dt = 1.00\text{ kg/s}$ and with an exhaust speed of $v_E = 100.\text{ m/s}$.

SKETCH:



RESEARCH:

(a) The thrust of the retro-rocket is determined using $F_{thrust} = v_c dM/dt$.

(b) In order to determine the amount of fuel needed, first determine the time to reach a speed of $v = 0.0200$ m/s. Use $v = v_0 - at$. By Newton's Second Law the thrust is also given by $\vec{F}_{thrust} = m_s\vec{a}$.

(c) The burn of the retro-rocket must be sustained for a time sufficient to reduce the speed to 0.0200 m/s, found in part (b).

(d) Use the conservation of momentum, $\vec{p}_i = \vec{p}_f$.

SIMPLIFY:

(a) $\vec{F}_{thrust} = -\vec{v}_c \dfrac{dM}{dt}$

(b) $t = \dfrac{v_0 - v}{a}$

The acceleration is given by $a = F_{thrust}/m_s$. Substitute this expression into the equation for t above to get $t = \dfrac{(v_0 - v)m_s}{F_{thrust}}$. Therefore, the mass of fuel needed is $m_F = \left(\dfrac{dM}{dt}\right)t = \left(\dfrac{dM}{dt}\right)\dfrac{(v_0 - v)m_s}{F_{thrust}}$.

(c) $t = \dfrac{(v_0 - v)m_s}{F_{thrust}}$

(d) $m_s\vec{v} = (M + m_s)\vec{v}_f \Rightarrow \vec{v}_f = \dfrac{m_s}{M + m_s}\vec{v}$, where M is the mass of the space station.

CALCULATE:

(a) The thrust is $\vec{F}_{thrust} = -(100.\text{ m/s})(1.00\text{ kg/s})\hat{v}_E = -100.0\text{ N }\hat{v}_c$, or 100.0 N in the opposite direction to the velocity of the spacecraft.

(b) $m_F = (1.00\text{ kg/s})\dfrac{(1.00\text{ m/s} - 0.0200\text{ m/s})1000.\text{ kg}}{100.0\text{ N}} = 9.800\text{ kg}$

(c) $t = \dfrac{(1.00\text{ m/s} - 0.0200\text{ m/s})1000.\text{ kg}}{100.0\text{ N}} = 9.800\text{ s}$

(d) $\vec{v}_f = \dfrac{1000.\text{ kg}(0.0200\text{ m/s})}{5.00\cdot 10^5\text{ kg} + 1000.\text{ kg}}\hat{v} = 3.992\cdot 10^{-5}\text{ m/s }\hat{v}$; that is, in the same direction as the spacecraft is moving.

ROUND: The answers should be expressed to three significant figures:

(a) $\vec{F}_{thrust} = -100.\text{ N }\hat{v}_c$

(b) $m_F = 9.80$ kg

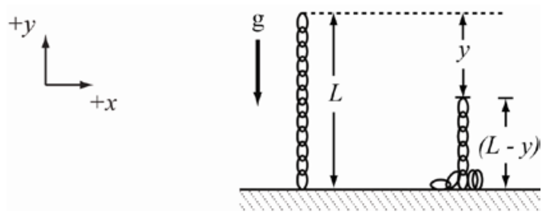
(c) $t = 9.80$ s

(d) $\vec{v}_f = 3.99\cdot 10^{-5}\text{ m/s }\hat{v}$

DOUBLE-CHECK: It is expected that the speed of the combined mass will be very small since its mass is very large.

8.75. **THINK:** A chain has a mass of 3.00 kg and a length of 5.00 m. Determine the force exerted by the chain on the floor. Assume that each link in the chain comes to rest when it reaches the floor.

SKETCH:



RESEARCH: Assume the mass per unit length of the chain is $\rho = M/L$. A small length of the chain, dy has a mass of dm, where $dm = Mdy/L$. At an interval of time dt, the small element of mass dm has reached the floor. The impulse caused by the chain is given by $J = F_J dt = \Delta p = vdm$. Therefore, the force F_J is given by $F_J = v\frac{dm}{dt} = v\frac{dm}{dy}\frac{dy}{dt}$.

SIMPLIFY: Using $dm/dy = M/L$ and $v = dy/dt$, the expression for force, F_J is

$$F_J = v^2\frac{M}{L}.$$

For a body in free fall motion, $v^2 = 2gy$. Thus, $F_J = 2Mgy/L$. There is another force which is due to gravity. The gravitational force exerted by the chain on the floor when the chain has fallen a distance y is given by $F_g = Mgy/L$ (the links of length y are on the floor). The total force is given by

$$F = F_J + F_g = \frac{2Mgy}{L} + \frac{Mgy}{L} = \frac{3Mgy}{L}.$$

When the last link of the chain lands on the floor, the force exerted by the chain is obtained by substituting $y = L$, that is, $F = \frac{3Mgy}{L} = 3Mg$.

CALCULATE: $F = 3(3.0 \text{ kg})(9.81 \text{ m/s}^2) = 88.29 \text{ N}$

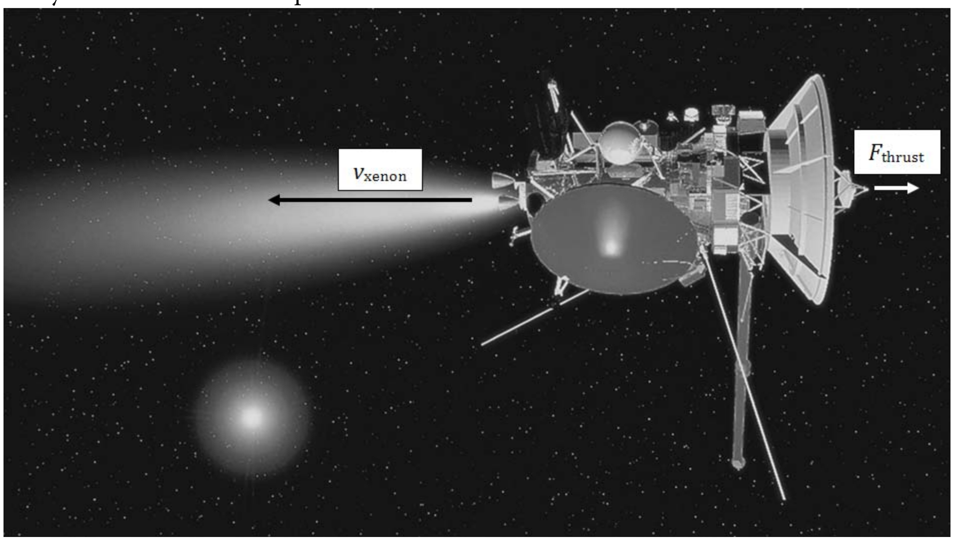
ROUND: To three significant figures, the force exerted by the chain on the floor as the last link of chain lands on the floor is $F = 88.3$ N.

DOUBLE-CHECK: F is expected to be larger than Mg due to the impulse caused by the chain as it falls.

Multi-Version Exercises

8.76. **THINK:** This question asks about the fuel consumption of a satellite. This is an example of rocket motion, where the mass of the satellite (including thruster) decreases as the fuel is ejected.

SKETCH: The direction in which the xenon ions are ejected is opposite to the direction of the thrust. The velocity of the xenon with respect to the satellite and the thrust force are shown.



RESEARCH: The equation of motion for a rocket in interstellar space is given by $\vec{F}_{\text{thrust}} = -\vec{v}_c \dfrac{dm}{dt}$. The velocity of the xenon ions with respect to the shuttle is given in km/s and the force is given in Newtons, or kg · m / s^2. The conversion factor for the velocity is given by $\dfrac{1000\text{ m/s}}{1\text{ km/s}}$.

SIMPLIFY: Since the thrust and velocity act along a single axis, it is possible to use the scalar form of the equation, $F_{\text{thrust}} = -v_c \dfrac{dm}{dt}$. The rate of fuel consumption equals the change in mass (the loss of mass is due to xenon ejected from the satellite), so solve for $\dfrac{dm}{dt}$ to get $\dfrac{dm}{dt} = -\dfrac{F_{\text{thrust}}}{v_c}$.

CALCULATE: The question states that the speed of the xenon ions with respect to the rocket is $v_c = v_{\text{xenon}}$ = 21.45 km/s. The thrust produced is $F_{\text{thrust}} = 1.187 \cdot 10^{-2}$ N. Thus the rate of fuel consumption is:

$$\begin{aligned}\frac{dm}{dt} &= -\frac{F_{\text{thrust}}}{v_c} \\ &= -\frac{1.187\cdot 10^{-2}\text{ N}}{21.45\text{ km/s}\cdot\dfrac{1000\text{ m/s}}{1\text{ km/s}}} \\ &= -5.533799534\cdot 10^{-7}\text{ kg/s} \\ &= -1.992167832\text{ g/hr}\end{aligned}$$

ROUND: The measured values are all given to four significant figures, and the final answer should also have four significant figures. The thruster consumes fuel at a rate of $5.534\cdot 10^{-7}$ kg/s or 1.992 g/hr.

DOUBLE-CHECK: Because of the cost of sending a satellite into space, the weight of the fuel consumed per hour should be pretty small; a fuel consumption rate of 1.992 g/hr is reasonable for a satellite launched from earth. Working backwards, if the rocket consumes fuel at a rate of $5.534\cdot 10^{-4}$ g/s, then the thrust is

$$-21.45\text{ km/s}\cdot\left(-5.534\cdot 10^{-4}\text{ g/s}\right) = 0.01187\text{ km}\cdot\text{g/s}^2 = 1.187\cdot 10^{-2}\text{ N}$$

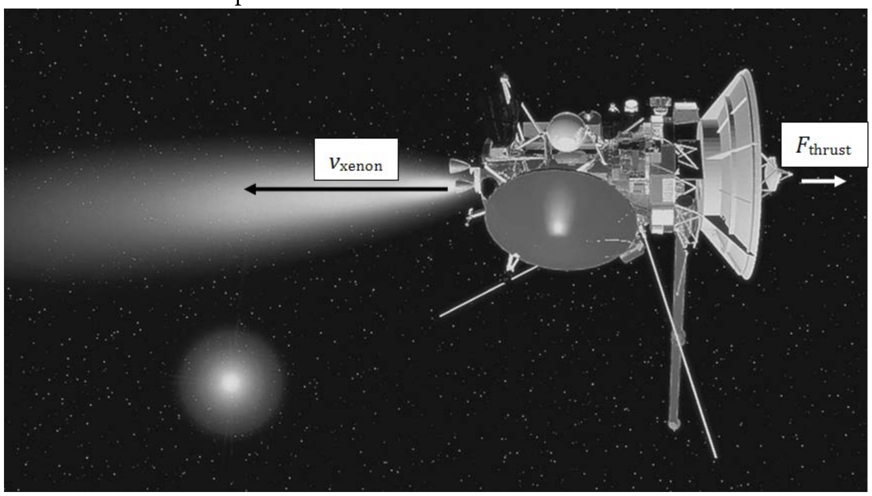
(the conversion factor is 1 km·g/s^2 = 1 kg·m/s^2). So, this agrees with the given thrust force of $1.187\cdot 10^{-2}$ N.

8.77. $F = v_c \dfrac{dm}{dt} = \left(23.75 \cdot 10^3 \text{ m/s}\right)\left(5.082 \cdot 10^{-7} \text{ kg/s}\right) = 1.207 \cdot 10^{-2} \text{ N}$

8.78. $v_c = \dfrac{F}{dm/dt} = \dfrac{1.299 \cdot 10^{-2} \text{ N}}{4.718 \cdot 10^{-7} \text{ kg/s}} = 26.05 \text{ km/s}$

8.79. **THINK:** This question asks about the speed of a satellite. This is an example of rocket motion, where the mass of the satellite (including thruster) decreases as the fuel is ejected.

SKETCH: The direction in which the xenon ions are ejected is opposite to the direction of the thrust. The velocity of the xenon with respect to the satellite and the thrust force are shown.



RESEARCH: Initially, the mass of the system is the total mass of the satellite, including the mass of the fuel: $m_{\text{i}} = m_{\text{satellite}}$ After all of the fuel is consumed, the mass of the system is equal to the mass of the satellite minus the mass of the fuel consumed: $m_{\text{f}} = m_{\text{satellite}} - m_{\text{fuel}}$. The change in speed of the satellite is given by the equation $v_{\text{f}} - v_{\text{i}} = v_c \ln\left(m_{\text{i}} / m_{\text{f}}\right)$, where v_c is the speed of the xenon with relative to the satellite.

SIMPLIFY: To make the problem easier, choose a reference frame where the initial speed of the satellite equals zero. Then $v_{\text{f}} - v_{\text{i}} = v_{\text{f}} - 0 = v_{\text{f}}$, so it is necessary to find $v_{\text{f}} = v_c \ln\left(m_{\text{i}} / m_{\text{f}}\right)$. Substituting in the masses of the satellite and fuel, this becomes $v_{\text{f}} = v_c \ln\left(m_{\text{satellite}} / \left[m_{\text{satellite}} - m_{\text{fuel}}\right]\right)$.

CALCULATE: The initial mass of the satellite (including fuel) is 2149 kg, and the mass of the fuel consumed is 23.37 kg. The speed of the ions with respect to the satellite is 28.33 km/s, so the final velocity of the satellite is:

$$\begin{aligned} v_{\text{f}} &= v_c \ln\left(m_{\text{satellite}} / \left[m_{\text{satellite}} - m_{\text{fuel}}\right]\right) \\ &= \left(28.33 \text{ km/s}\right)\ln\left(\frac{2149 \text{ kg}}{2149 \text{ kg} - 23.37 \text{ kg}}\right) \\ &= 3.0977123 \cdot 10^{-1} \text{ km/s} \end{aligned}$$

ROUND: The measured values are all given to four significant figures, and the weight of the satellite minus the weight of the fuel consumed also has four significant figures, so the final answer will have four figures. The change in the speed of the satellite is $3.098 \cdot 10^{-1}$ km/s or 309.8 m/s.

DOUBLE-CHECK: Alhough the satellite is moving quickly after burning all of its fuel, this is not an unreasonable speed for space travel. Working backwards, if the change in speed was $3.098 \cdot 10^{-1}$ km/s, then the velocity of the xenon particles was $v_c = \dfrac{\Delta v_{\text{satellite}}}{\ln(m_{\text{i}} / m_{\text{f}})}$, or

$$v_c = \frac{3.098 \cdot 10^{-1} \text{km /s}}{\ln(2149 \text{ kg} / \left[2149 \text{ kg} - 23.37 \text{ kg}\right])} = 28.33 \text{ km /s}.$$

This agrees with the number given in the question, confirming that the calculations are correct.

8.80. $\Delta v = v_c \ln\left(\frac{m_i}{m_f}\right)$

$$\frac{\Delta v}{v_c} = \ln\left(\frac{m_i}{m_f}\right)$$

$$e^{\frac{\Delta v}{v_c}} = \frac{m_i}{m_f}$$

$$m_f = m_i e^{-\frac{\Delta v}{v_c}}$$

$$m_{fuel} = m_i - m_f = m_i - m_i e^{-\frac{\Delta v}{v_c}} = m_i\left(1 - e^{-\frac{\Delta v}{v_c}}\right)$$

$$m_{fuel} = (2161\text{ kg})\left(1 - e^{-\frac{236.4\text{ m/s}}{20.61\cdot 10^3\text{ m/s}}}\right) = 24.65\text{ kg}$$

8.81. $\Delta v = v_c \ln\left(\frac{m_i}{m_f}\right)$

$$\frac{\Delta v}{v_c} = \ln\left(\frac{m_i}{m_f}\right)$$

$$e^{\frac{\Delta v}{v_c}} = \frac{m_i}{m_f}$$

$$m_i = m_f e^{\frac{\Delta v}{v_c}}$$

$$m_f = m_i - m_{fuel}$$

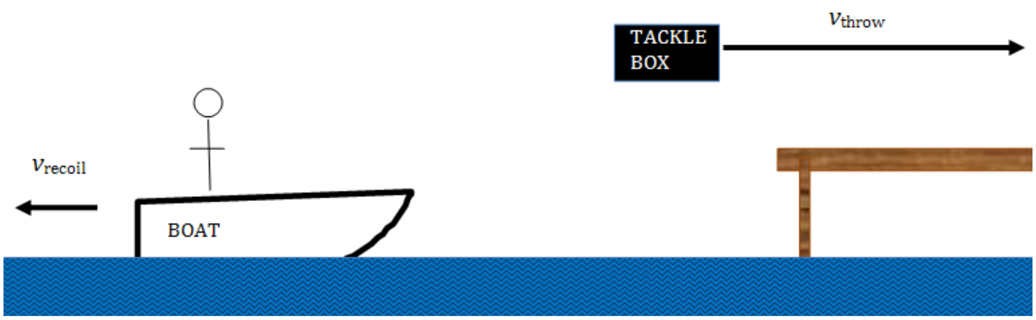
$$m_i = (m_i - m_{fuel})e^{\frac{\Delta v}{v_c}} = m_i e^{\frac{\Delta v}{v_c}} - m_{fuel} e^{\frac{\Delta v}{v_c}}$$

$$m_i e^{\frac{\Delta v}{v_c}} - m_i = m_{fuel} e^{\frac{\Delta v}{v_c}}$$

$$m_i = \frac{m_{fuel} e^{\frac{\Delta v}{v_c}}}{e^{\frac{\Delta v}{v_c}} - 1} = m_{fuel}\frac{1}{1 - e^{-\frac{\Delta v}{v_c}}} = (25.95\text{ kg})\frac{1}{1 - e^{-\frac{275.0\text{ m/s}}{22.91\cdot 10^3\text{ m/s}}}} = 2175\text{ kg}$$

8.82. **THINK:** The fisherman, boat, and tackle box are at rest at the beginning of this problem, so the total momentum of the fisherman, boat, and tackle box before and after the fisherman throws the tackle box must be zero. Using the principle of conservation of momentum and the fact that the momentum of the tackle box must cancel out the momentum of the fisherman and boat, it is possible to find the speed of the fisherman and boat after the tackle box has been thrown.

SKETCH: The sketch shows the motion of the tackle box, boat, and fisherman after the throw:



RESEARCH: The total initial momentum is zero, because there is no motion with respect to the dock. After the fisherman throws the tackle box, the momentum of the tackle box is $p_{\text{box}} = m_{\text{box}}v_{\text{box}} = m_{\text{box}}v_{\text{throw}}$ towards the dock. The total momentum after the throw must equal the total momentum before the throw, so the sum of the momentum of the box, the momentum of the boat, and the momentum of the fisherman must be zero: $p_{\text{box}} + p_{\text{fisherman}} + p_{\text{boat}} = 0.$ The fisherman and boat both have the same velocity, so $p_{\text{fisherman}} = m_{\text{fisherman}}v_{\text{fisherman}} = m_{\text{fisherman}}v_{\text{recoil}}$ away from the dock and $p_{\text{boat}} = m_{\text{boat}}v_{\text{boat}} = m_{\text{boat}}v_{\text{recoil}}$ away from the dock.

SIMPLIFY: The goal is to find the recoil velocity of the fisherman and boat. Using the equation for momentum after the tackle box has been thrown, $p_{\text{box}} + p_{\text{fisherman}} + p_{\text{boat}} = 0$, substitute in the formula for the momenta of the tackle box, boat, and fisherman: $0 = m_{\text{box}}v_{\text{throw}} + m_{\text{fisherman}}v_{\text{recoil}} + m_{\text{boat}}v_{\text{recoil}}$. Solve for the recoil velocity:

$$m_{\text{box}}v_{\text{throw}} + m_{\text{fisherman}}v_{\text{recoil}} + m_{\text{boat}}v_{\text{recoil}} = 0$$
$$m_{\text{fisherman}}v_{\text{recoil}} + m_{\text{boat}}v_{\text{recoil}} = -m_{\text{box}}v_{\text{throw}}$$
$$v_{\text{recoil}}\left(m_{\text{fisherman}} + m_{\text{boat}}\right) = -m_{\text{box}}v_{\text{throw}}$$
$$v_{\text{recoil}} = -\frac{m_{\text{box}}v_{\text{throw}}}{m_{\text{fisherman}} + m_{\text{boat}}}$$

CALCULATE: The mass of the tackle box, fisherman, and boat, as well as the velocity of the throw (with respect to the dock) are given in the question. Using these values gives:

$$\begin{aligned} v_{\text{recoil}} &= -\frac{m_{\text{box}}v_{\text{throw}}}{m_{\text{fisherman}} + m_{\text{boat}}} \\ &= -\frac{13.63 \text{ kg} \cdot 2.911 \text{ m/s}}{75.19 \text{ kg} + 28.09 \text{ kg}} \\ &= -0.3841685709 \text{ m/s} \end{aligned}$$

ROUND: The masses and velocity given in the question all have four significant figures, and the sum of the mass of the fisherman and the mass of the boat has five significant figures, so the final answer should have four significant figures. The final speed of the fisherman and boat is –0.3842 m/s towards the dock, or 0.3842 m/s away from the dock.

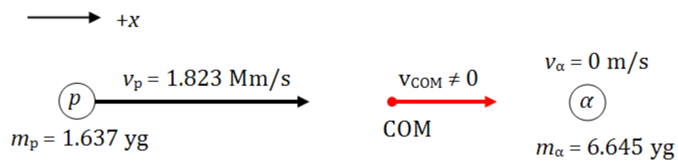
DOUBLE-CHECK: It makes intuitive sense that the much more massive boat and fisherman will have a lower speed than the less massive tackle box. Their momenta should be equal and opposite, so a quick way to check this problem is to see if the magnitude of the tackle box's momentum equals the magnitude of the man and boat. The tackle box has a momentum of magnitude 13.63 kg · 2.911 m/s = 39.68 kg·m/s after it is thrown. The boat and fisherman have a combined mass of 103.28 kg, so their final momentum has a magnitude of 103.28 kg · 0.3842 m/s = 39.68 kg·m/s. This confirms that the calculations were correct.

8.83. $$v_{\text{box}} = \frac{m_{\text{man}} + m_{\text{boat}}}{m_{\text{box}}}v_{\text{boat}} = \frac{77.49 \text{ kg} + 28.31 \text{ kg}}{14.27 \text{ kg}}\left(0.3516 \text{ m/s}\right) = 2.607 \text{ m/s}$$

8.84.
$$\left(m_{\text{man}} + m_{\text{boat}}\right)v_{\text{boat}} = m_{\text{box}}v_{\text{box}}$$
$$m_{\text{man}}v_{\text{boat}} + m_{\text{boat}}v_{\text{boat}} = m_{\text{box}}v_{\text{box}}$$
$$m_{\text{man}} = \frac{m_{\text{box}}v_{\text{box}} - m_{\text{boat}}v_{\text{boat}}}{v_{\text{boat}}} = m_{\text{box}}\frac{v_{\text{box}}}{v_{\text{boat}}} - m_{\text{boat}}$$
$$m_{\text{man}} = \left(14.91 \text{ kg}\right)\frac{3.303 \text{ m/s}}{0.4547 \text{ m/s}} - 28.51 \text{ kg} = 79.80 \text{ kg}$$

8.85. **THINK:** The masses and initial speeds of both particles are known, so the momentum of the center of mass can be calculated. The total mass of the system is known, so the momentum can be used to find the speed of the center of mass.

SKETCH: To simplify the problem, choose the location of the particle at rest to be the origin, with the proton moving in the $+x$ direction. All of the motion is along a single axis, with the center of mass (COM) between the proton and the alpha particle.



RESEARCH: The masses and velocities of the particles are given, so the momenta of the particles can be calculated as the product of the mass and the speed $p_\alpha = m_\alpha v_\alpha$ and $p_p = m_p v_p$ towards the alpha particle. The center-of-mass momentum can be calculated in two ways, either by taking the sum of the momenta of each particle ($P_{COM} = \sum_{i=0}^{n} p_i$) or as the product of the total mass of the system times the speed of the center of mass ($P_{COM} = M \cdot v_{COM}$).

SIMPLIFY: The masses of both particles are given in the problem, and the total mass of the system M is the sum of the masses of each particle, $M = m_p + m_\alpha$. The total momentum $P_{COM} = \sum_{i=0}^{n} p_i = p_\alpha + p_p$ and $P_{COM} = M \cdot v_{COM}$, so $M \cdot v_{COM} = p_\alpha + p_p$. Substitute for the momenta of the proton and alpha particle (since the alpha particle is not moving, it has zero momentum), substitute for the total mass, and solve for the velocity of the center of mass:

$$M \cdot v_{COM} = p_\alpha + p_p \Rightarrow$$
$$v_{COM} = \frac{p_\alpha + p_p}{M} = \frac{m_\alpha v_\alpha + m_p v_p}{m_\alpha + m_p} = \frac{m_\alpha \cdot 0 + m_p v_p}{m_\alpha + m_p} = \frac{m_p v_p}{m_\alpha + m_p}$$

CALCULATE: The problem states that the proton has a mass of $1.673 \cdot 10^{-27}$ kg and moves at a speed of $1.823 \cdot 10^{6}$ m/s towards the alpha particle, which is at rest and has a mass of $6.645 \cdot 10^{-27}$ kg. So the center of mass has a speed of

$$v_{COM} = \frac{m_p v_p}{m_{p\alpha} + m} = \frac{\left(1.823 \cdot 10^{6} \text{ m/s}\right)\left(1.673 \cdot 10^{-27} \text{ kg}\right)}{1.673 \cdot 10^{-27} \text{ kg} + 6.645 \cdot 10^{-27} \text{ kg}} = 3.666601346 \cdot 10^{5} \text{ m/s}$$

ROUND: The masses of the proton and alpha particle, as well as their sum, have four significant figures. The speed of the proton also has four significant figures. The alpha particle is at rest, so its speed is not a calculated value, and the zero speed does not change the number of figures in the answer. Thus, the speed of the center of mass is $3.667 \cdot 10^{5}$ m/s, and the center of mass is moving towards the alpha particle.

DOUBLE-CHECK: To double check, find the location of the center of mass as a function of time, and take the time derivative to find the velocity. The distance between the particles is not given in the problem, so call the distance between the particles at an arbitrary starting time $t = 0$ to be d_0. The positions of each particle can be described by their location along the axis of motion, $r_\alpha = 0$ and $r_p = d_0 + v_p t$.

Using this, the location of the center of mass is

$$R_{\text{COM}} = \frac{1}{m_{\text{p}} + m_{\alpha}}\left(r_{\text{p}} m_{\text{p}} + r_{\alpha} m_{\alpha}\right).$$

Take the time derivative to find the velocity:

$$\begin{aligned}
\frac{d}{dt} R_{\text{COM}} &= \frac{d}{dt}\left[\frac{1}{m_{\text{p}} + m_{\alpha}}\left(r_{\text{p}} m_{\text{p}} + r_{\alpha} m_{\alpha}\right)\right] \\
&= \frac{1}{m_{\text{p}} + m_{\alpha}} \frac{d}{dt}\left[\left(d_0 + v_{\text{p}} t\right) m_{\text{p}} + 0 \cdot m_{\alpha}\right] \\
&= \frac{1}{m_{\text{p}} + m_{\alpha}} \frac{d}{dt}\left(d_0 m_{\text{p}} + v_{\text{p}} m_{\text{p}} t + 0\right) \\
&= \frac{1}{m_{\text{p}} + m_{\alpha}} \frac{d}{dt}\left(d_0 m_{\text{p}} + v_{\text{p}} m_{\text{p}} t\right) \\
&= \frac{1}{m_{\text{p}} + m_{\alpha}}\left(0 + v_{\text{p}} m_{\text{p}}\right) \\
&= \frac{v_{\text{p}} m_{\text{p}}}{m_{\text{p}} + m_{\alpha}} \\
&= \frac{\left(1.823 \cdot 10^{6} \text{ m/s}\right)\left(1.673 \cdot 10^{-27} \text{ kg}\right)}{1.673 \cdot 10^{-27} \text{ kg} + 6.645 \cdot 10^{-27} \text{ kg}} \\
&= 3.666601346 \cdot 10^{5} \text{ m/s}
\end{aligned}$$

This agrees with the earlier result.

8.86. $(m_p + m_\alpha) v_{\text{cm}} = m_p v_p + m_\alpha v_\alpha$

Since $v_\alpha = 0$,

$$v_p = \frac{m_p + m_\alpha}{m_p} v_{\text{cm}} = \frac{1.673 \cdot 10^{-27} \text{ kg} + 6.645 \cdot 10^{-27} \text{ kg}}{\left(1.673 \cdot 10^{-27} \text{ kg}\right)}\left(5.509 \cdot 10^{5} \text{ m/s}\right) = 2.739 \cdot 10^{6} \text{ m/s}$$

Chapter 9: Circular Motion

Concept Checks

9.1. e **9.2.** b **9.3.** a **9.4.** d **9.5.** b **9.6.** d

Multiple-Choice Questions

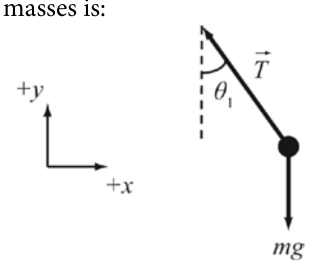
9.1. d **9.2.** c **9.3.** b **9.4.** d **9.5.** c **9.6.** a **9.7.** c **9.8.** a **9.9.** a **9.10.** c **9.11.** a **9.12.** a **9.13.** d **9.14.** d **9.15.** d **9.16.** a

Conceptual Questions

9.17. A ceiling fan is rotating in the clockwise direction, as viewed from below. This also means that the direction of angular velocity of the fan is in the clockwise direction. The angular velocity is decreasing or slowing down. This indicates that the angular acceleration is negative or in the opposite direction of the angular velocity. Therefore, the angular acceleration is in the counter-clockwise direction.

9.18. No, it will not. This is because when the actor swings across the stage there will be an additional tension on the rope needed to hold the action in circular motion. Note that the total mass of the rope and the actor is 3 lb + 147 lb = 150 lb. This is the maximum mass that can be supported by the hook. Therefore, the additional tension on the rope will break the hook.

9.19. The force body diagram for one of the masses is:



The force of tension in the *x*-axis is equal to the centripetal force, $T\sin\theta_1 = m\omega^2 r$. The force of the tension along the *y*-axis must be equal to the force of gravity, $T\cos\theta_1 = mg$. This means $\frac{T\sin\theta_1}{T\cos\theta_1} = \tan\theta_1 = \frac{\omega^2 r}{g}$; therefore, both θ_1 and θ_2 are the same, since they don't depend on the mass.

9.20. For the two points of interest, there are two forces acting on the person; the force of gravity and the normal force. These two forces combine to create the centripetal force. In case A: $F_{c,A} = F_{N,A} - F_g$ and case B: $-F_{c,A} = F_{N,B} - F_g$. This means that the normal force is $F_{N,A} = F_{c,A} + F_g = m\omega^2 r + mg$ and $F_{N,B} = F_g - F_{c,A} = mg - m\omega^2 r.$ Therefore, $F_{N,A}$ is greater than $F_{N,B}$.

9.21. The linear speed of the bicycle is given by $v = r\omega$. The smaller the diameter, *D*, the lower the linear speed for the same angular speed because $r = D/2$ so tires with a lower diameter than 25 cm will have a velocity too slow to be practical transportation.

9.22. Both the angular velocity and acceleration are independent of the radius. This means they are the same at the edge and halfway between the edge and center. The linear velocity and acceleration, however, do change with radius, *r*. At the edge $v_e = r\omega$ and $a_e = R\omega^2$. The halfway point gives $v_{1/2} = \frac{r\omega}{2}$ and $a_{1/2} = \frac{1}{2}R\omega^2$. Comparing the two points, it can be seen that $v_e = 2v_{1/2}$ and $a_e = 2a_{1/2}$.

9.23. For the car to stay on the road there must be a force in the direction of the centripetal force. In this case, the road is not banked, leaving the force of friction as the only possible choice. Since the car travels with constant speed, the force of friction holding the car on a circular path points in radial inward direction towards the center of the circle.

9.24. As the car makes the turn, both strings have a new angular position, θ. From the discussion of the conical pendulum on page 290 and 291, you can see that this angle is given by $\tan\theta = r\omega^2/g$, where r is the distance to the center of the circle. This means that the pendulum that is further away from the center has a larger angle. A larger angle means a larger sideways deflection of the pendulum, and thus the distance between the two pendula increase during the turn, both for a right turn and for a left turn. If the distance d between the two pendula is small compared to the turning radius r, however, this effect is hard to measure or see.

9.25. The kinetic energy when the point mass gets to the top of the loop is equal to the difference in potential energy between the height *h* and 2*R*. Rearranging $\frac{mv^2}{2} = mgh - 2mgR$ for v^2 gives $v^2 = 2g(h-2R)$. For the particle to stay connected to the loop, the centripetal force has to be greater than or equal to the force of gravity. This requirement means $v^2 = Rg$. Using these two equations, the height, *h,* can be determined:

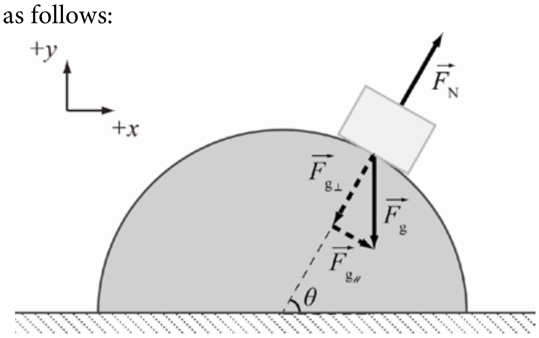
$$2g(h-2R) = Rg \Rightarrow h = 2R + \frac{1}{2}R = \frac{5}{2}R.$$

The height should be $5R/2$ or greater for the point to complete the loop.

9.26. The bob is moving in a horizontal circle at constant speed. This means that the bob experiences a net force equal to the centripetal force inwards. This force is equal to the horizontal component of the tension. The vertical component of the tension must be balanced by the force of gravity. The two forces acting on the bob are the tension and the force of gravity.

9.27. From our discussion of the conical pendulum on page 271, you can see that this angle is given by $\tan\theta = r\omega^2/g$. As the angular speed assumes larger and larger values, the angle *approaches* a value of $90°$, which is the condition that the string is parallel to the ground. However, the *exact* value of $90°$ cannot be reached, because it would correspond to an infinitely high value of the angular speed, which cannot be achieved.

9.28. A picture of the situation is as follows:



This picture tells us that the normal force can be related to the force of gravity by $F_N = F_{g\perp} = F_g \sin\theta = mg\sin\theta.$ In this situation, the normal force provides the centripetal force, so $F_c = mg\sin\theta$ and $a_c = g\sin\theta$. As θ decreases, $\sin\theta$ decreases, and therefore a_c decreases. The acceleration vector for circular motion has two components; the centripetal acceleration, a_c, and the tangential acceleration, $a_t = \cos\theta$, which increases as θ decreases to zero. This satisfies the requirement that $\vec{a} = a_t\hat{t} - a_c\hat{r}$.

9.29. The forces that you feel at the top and bottom of the loop are equal to the normal force. At the top and bottom of the loop, the normal force are along the vertical direction and are $N_{\text{T}} = mg - \frac{mv_{\text{T}}^2}{R}$ and $N_{\text{B}} = mg + \frac{mv_B^2}{R}$. If you experience weightlessness at the top, then $N_{\text{T}} = 0 \Rightarrow mg = mv_{\text{T}}^2 / R$. Energy conservation tells us that $\frac{1}{2}mv_{\text{B}}^2 = 2mgR + \frac{1}{2}mv_{\text{T}}^2$. Insert both of these results into the expression for the normal force at the bottom and find:

$$N_{\text{B}} = mg + mv_{\text{B}}^2 / R = mg + (4mgR + mv_{\text{T}}^2) / R = mg + 4mg + mg = 6mg$$

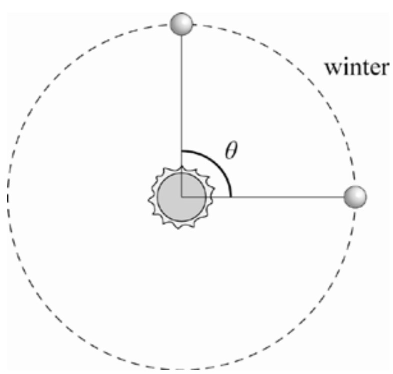
This means that the normal force exerted by the seat on you, your apparent weight, is indeed 6 times your weight at the top of the loop.

9.30. The combined weight of the five daredevils is W. To determine the strength of the rope needed, the tension at the bottom of the arc must be determined. At this point the centripetal force is equal to the difference between the tension and the force of gravity. The tension is equal to $T = \frac{mv^2}{R} + mg$ $= \frac{Wv^2}{Rg} + W$. The kinetic energy at the bottom of the arc is equal to the potential energy at the level of the bridge, $\frac{1}{2}mv^2 = mgR$ or $v^2 = 2gR$. Using this, $T = \frac{W2gR}{Rg} + W = 3W$. The rope must be able to withstand a tension equal to three times the combined weight of the daredevils.

Exercises

9.31. **THINK:** Determine the change in the angular position in radians. Winter lasts roughly a fourth of a year. There are 2π radians in a circle. Consider the orbit of Earth to be circular.

SKETCH:



RESEARCH: The angular velocity of the earth is $\omega = 2\pi / \text{yr}$. The angular position is given by $\theta = \theta_0 + \omega_0 t$.

SIMPLIFY: $\Delta\theta = \theta - \theta_0 = \omega_0 t$

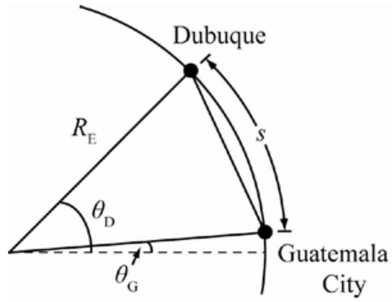
CALCULATE: $\Delta\theta = \frac{2\pi \text{ rad}}{\text{yr}}\left(\frac{1}{4}\text{ yr}\right) = \frac{\pi \text{ rad}}{2} = \frac{3.14 \text{ rad}}{2} = 1.57 \text{ rad}$

ROUND: Since π is used to three significant figures, the angle the Earth sweeps over winter is 1.57 rad. It would also be entirely reasonable to leave the answer as $\pi / 2$ radians.

DOUBLE-CHECK: This value makes sense, since there are four seasons of about equal length, so the angle should be a quarter of a circle.

9.32. **THINK:** Determine the arc length between Dubuque and Guatemala City. The angular positions of Dubuque and Guatemala City are $\theta_{\text{D}} = 42.50°$ and $\theta_{\text{G}} = 14.62°$, respectively. The radius of the Earth is $R_{\text{E}} = 6.37 \cdot 10^6$ m.

SKETCH:



RESEARCH: The length of an arc is given by $s = r\theta$, where r is the radius of the circle, and θ is the arc angle given in radians.

SIMPLIFY: $s = R_{\text{E}}(\theta_{\text{D}} - \theta_{\text{G}})\dfrac{2\pi}{360°}$, where the units of the angles are degrees.

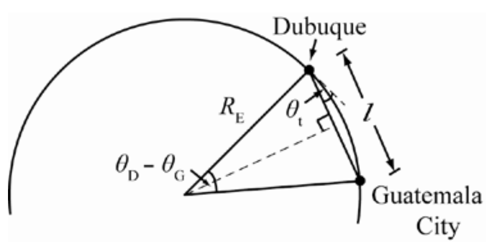
CALCULATE: $s = \left(6.37 \cdot 10^6 \text{ m}\right)\left(42.50° - 14.62°\right)\dfrac{2\pi}{360°} = 3.0996 \cdot 10^6 \text{ m}$

ROUND: The arc length's accuracy is given by the least accurate value used to determine it. In this case, the least accurate value is the radius of Earth, given to three significant figures, so the arc length is $3.10 \cdot 10^6$ m.

DOUBLE-CHECK: This is equal to 3100 km, a reasonable distance between the northern United States and Central America.

9.33. **THINK:** Determine the linear distance between Dubuque and Guatemala city. Also, determine the angle below the horizontal for a tunnel that connects the two. The angular positions of Dubuque and Guatemala City are $\theta_{\text{D}} = 42.50°$ and $\theta_{\text{G}} = 14.62°$, respectively. The radius of the Earth is $R_{\text{E}} = 6.37 \cdot 10^6$ m.

SKETCH:



RESEARCH: Use the triangle of the drawing to relate $\theta_{\text{D}} - \theta_{\text{G}}$, R_{E} and $l/2$. The right triangle gives rise to the equation $\sin\dfrac{\theta_{\text{D}} - \theta_{\text{G}}}{2} = \dfrac{l/2}{R_{\text{E}}}$. The angle of the tunnel is $\theta_{\text{t}} = \left(\dfrac{\theta_{\text{D}} - \theta_{\text{G}}}{2}\right)$.

SIMPLIFY: $l = 2R_{\text{E}} \sin\left(\dfrac{\theta_{\text{D}} - \theta_{\text{G}}}{2}\right)$

CALCULATE: $l = 2\left(6.37 \cdot 10^6 \text{ m}\right)\sin\left(\dfrac{42.50° - 14.62°}{2}\right) = 3.06914 \cdot 10^6 \text{ m}$ $\theta_{\text{t}} = \left(\dfrac{42.50° - 14.62°}{2}\right) = 13.94°$

ROUND: The length will have the same accuracy as the radius of Earth. The angle of the tunnel will be as accurate as the latitude of the cities. Therefore, the length of the tunnel is $3.07 \cdot 10^6$ m, with an angle of 13.94° below the surface of the Earth.

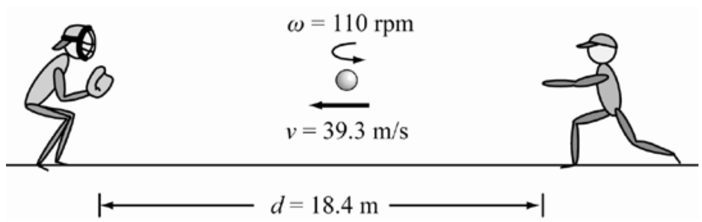
DOUBLE-CHECK: The length of the tunnel is a bit shorter than the arc length, which is expected. See the solution to Problem 9.32.

9.34. **THINK:** Determine the number of rotations the ball will make as it travels to the catcher's glove. The linear and angular speeds of the ball are $v = 88$ mph and $\omega = 110$ rpm . In SI units, these are

$$v = 88\text{ mph}\left(\frac{0.447\text{m / s}}{\text{mph}}\right) = 39.3\text{ m/s} \quad \text{and} \quad \omega = 110\text{ rpm}\left(\frac{2\pi\text{ rad}}{1\text{ rot}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 11.52\text{ rad/s.}$$

The ball travels a distance, $d = 60.5$ ft or 18.4 m.

SKETCH:



RESEARCH: The time it takes for the ball to reach the catcher is given by $t = v / d$. This time will then be used to calculate the number of rotations, given by $n = \omega t$. This number n will be in radians which will then have to be converted to rotations, where $1\text{ rad}\left(\frac{1\text{ rev}}{2\pi}\right) = 0.16$ revolution.

SIMPLIFY: $n = \omega\left(\frac{d}{v}\right)$

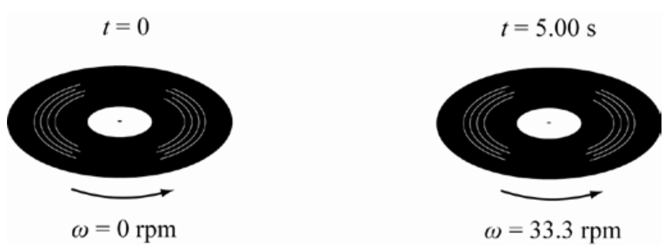
CALCULATE: $n = \frac{18.4\text{ m}(11.52\text{ rad/s})}{39.3\text{ m/s}} = 5.394\text{ rad}\left(\frac{1\text{ rev}}{2\pi}\right) = 0.8586$ rotations

ROUND: The linear speed of the ball, the distance traveled, and the angular speed of the ball are all given to three significant figures, so the number of rotations should be 0.859.

DOUBLE-CHECK: Dimensional analysis: $[n] = \text{rad/s} \cdot \frac{\text{m}}{\text{m/s}} \cdot \frac{\text{revolution}}{2\pi\text{ rad}}$. All units cancel giving a dimensionless quantity, as expected.

9.35. **THINK:** Determine the average angular acceleration of the record and its angular position after reaching full speed. The initial and final angular speeds are 0 rpm to 33.3 rpm. The time of acceleration is 5.00 s.

SKETCH:



RESEARCH: The equation for angular acceleration is $\alpha = \left(\omega_\text{f} - \omega_\text{i}\right) / \Delta t$. The angular position of an object under constant angular acceleration is given by $\theta = \frac{1}{2}\alpha t^2$.

SIMPLIFY: There is no need to simplify the equation.

CALCULATE: $\alpha = \frac{33.3\text{ rpm} - 0\text{ rpm}}{5.00\text{ s}(60\text{ s/min})} = 0.111\text{ rev/s}^2 \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} = 0.6974\text{ rad/s}^2$

$\theta = \frac{1}{2}(0.111\text{ rev/s})(5.00\text{ s})^2 = 1.3875\text{ rev} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} = 8.718\text{ rad}$

ROUND: To three significant figures, the angular acceleration and position are:

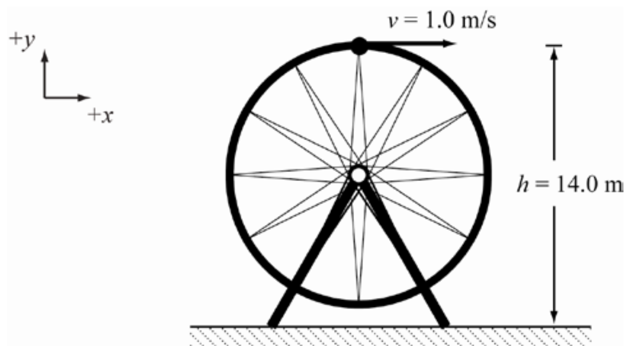
(a) $\alpha = 0.697 \text{ rad/s}^2$

(b) $\theta = 8.72 \text{ rad}$

DOUBLE-CHECK: The calculations yield the correct units of radians and rad/s^2.

9.36. **THINK:** Determine the horizontal distance the teddy bear travels during its fall. In order to do this, the height and the horizontal speed of the bear must be determined. The diameter of the wheel is 12.0 m, the bottom of which is 2.0 m above the ground. The rim of the wheel travels at a speed of $v = 1.0 \text{ m/s}$. The height of the bear is 14.0 m from the ground and is traveling at a speed of 1.0 m/s in the horizontal direction when it falls.

SKETCH:



RESEARCH: The horizontal distance is given by $x = vt$. The time is not yet known but can be determined from $h = \frac{1}{2}gt^2$.

SIMPLIFY: The time it takes the bear to fall is $t^2 = \frac{2h}{g}$ or $t = \sqrt{\frac{2h}{g}}$. The horizontal distance traveled is $x = vt = v\sqrt{\frac{2h}{g}}$.

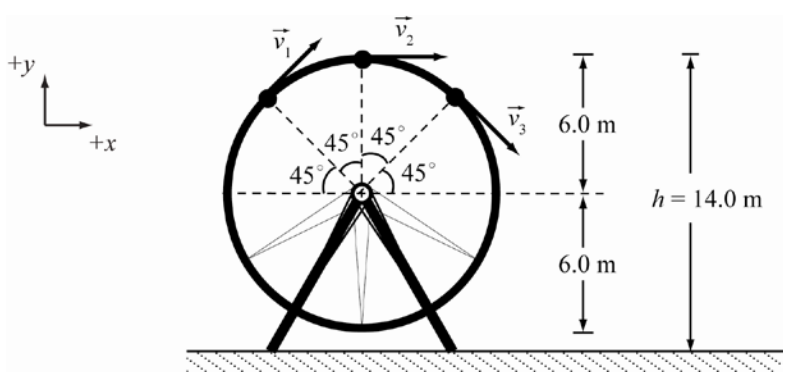
CALCULATE: $x = (1.00 \text{ m/s})\sqrt{\frac{2(14.0 \text{ m})}{9.81 \text{ m/s}^2}} = 1.6894 \text{ m}$

ROUND: The velocity is given to three significant figures, so round the distance to 1.69 m.

DOUBLE-CHECK: The bear lands a short distance from the base of the wheel, as one would expect given its small initial velocity.

9.37. **THINK:** Determine the distance between the three teddy bears. The bears will be traveling at 1.00 m/s but will have different directions and distances from the ground. The angle between adjacent bears is $45.0°$. The diameter of the wheel is 12.0 m and the bottom of the wheel is 2.00 m above the ground.

SKETCH:



RESEARCH: The height of bear 1 and 3 is the same and is $h_1 = (8.00 + 6.00\sin(45.0°))$ m. The second bear is $h_2 = 14.0$ m above the ground. The velocities of each bear in the horizontal and vertical directions are $v_x = v\cos\theta$ and $v_y = v\sin\theta$, where $\theta_1 = 45.0°$, $\theta_2 = 0°$ and $\theta_3 = -45.0°$. The distance between each bear before they are dropped is $\Delta d = 6.00\sin(45.0°)$ m. Use the regular equations for projectile motion: $\Delta x = v_x t$ and $\Delta y = v_y t - \frac{1}{2}gt^2$.

SIMPLIFY: For different initial heights, H, the time of the fall can be determined from $h(t) = v\sin(\theta)t - \frac{1}{2}gt^2 + H$. This is a quadratic equation with solution $t = \frac{v\sin\theta}{g} \pm \sqrt{\frac{v^2\sin^2\theta}{g^2} + \frac{2H}{g}}$.

Choose the positive root, that is, $t = \frac{v\sin\theta}{g} + \sqrt{\frac{v^2\sin^2\theta}{g^2} + \frac{2H}{g}}$. The change in distance is $\Delta x = v\cos(\theta)t = v\cos\theta\left(\frac{v\sin\theta}{g} + \sqrt{\frac{v^2\sin^2\theta}{g^2} + \frac{2H}{g}}\right)$, which means that the value of x is given by the equation $x = x_0 + v\cos\theta\left(\frac{v\sin\theta}{g} + \sqrt{\frac{v^2\sin^2\theta}{g^2} + \frac{2H}{g}}\right)$.

CALCULATE: For the first bear $x_0 = 0$, $H = h_1$, and $\theta_1 = 45.0°$. Recall $\cos 45.0° = \sin 45.0° = 1/\sqrt{2}$.

$$x_1 = 0 + \left((1.00\text{ m/s})\left(\frac{1}{\sqrt{2}}\right)\right)\left[\frac{1.00\text{ m/s}}{9.81\text{ m/s}^2}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{\frac{(1.00\text{ m/s})^2}{2(9.81\text{ m/s}^2)^2} + \frac{2(8.00 + 6.00/\sqrt{2})\text{ m}}{9.81\text{ m/s}^2}}\right] = 1.1693\text{ m}$$

The initial velocity of the second bear is horizontal, so the bear travels a horizontal distance of $\Delta x = 1.6894$ m (see solution to question 9.36). The second bear's position is $x_2 = x_0 + \Delta x = \Delta d + \Delta x = \frac{6.00\text{ m}}{\sqrt{2}} + 1.6894\text{ m} = 5.9320$ m from the origin. For the third bear, $x_0 = 2\Delta d$, $H = h_1$, and $\theta_2 = -45.0°$.

$$x_3 = 2\left(\frac{6.00\text{ m}}{\sqrt{2}}\right) + \frac{(1.00\text{ m/s})}{\sqrt{2}}\left[\frac{-1.00\text{ m/s}}{\sqrt{2}(9.81\text{ m/s}^2)} + \sqrt{\frac{(1.00\text{ m/s})^2}{2(9.81\text{ m/s}^2)^2} + \frac{2(8.00 + 6.00/\sqrt{2})\text{ m}}{9.81\text{ m/s}^2}}\right] = 9.5526\text{ m}$$

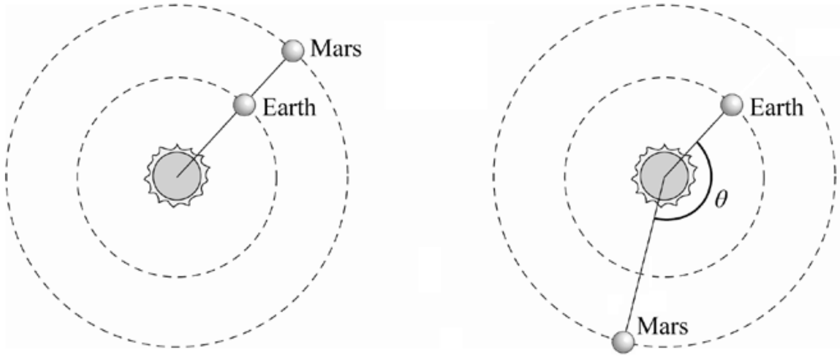
The distance between the first 2 bears is $\Delta d_{12} = 4.7627$ m. The distance between the last two bears is $\Delta d_{23} = 3.6206$ m.

ROUND: The velocity has three significant figures, so the results should also have three significant figures. The distances between the bears once they hit the ground are $\Delta d_{12} = 4.76$ m and $\Delta d_{23} = 3.62$ m.

DOUBLE-CHECK: The result is reasonable since $\Delta d_{12} > \Delta d_{23}$. This must be so since the third bear is in the air for a shorter time because the original horizontal velocity points towards the ground.

9.38. **THINK:** Determine (a) the angular distance between the two planets a year later, (b) the time it takes the two planets to align again and (c) the angular position the alignment occurs at. The radius and period of each planet's orbit are $r_M = 228\cdot 10^6$ km, $T_M = 687$ days, $r_E = 149.6\cdot 10^6$ km and $T_E = 365.26$ days.

SKETCH:



RESEARCH: The questions can be answered using $\theta = \omega t$ and $\omega = 2\pi / T$.

SIMPLIFY: The angular distance is

$$\Delta\theta = \theta_{\text{E}} - \theta_{\text{M}} = \omega_{\text{E}} T_{\text{E}} - \omega_{\text{M}} T_{\text{E}} = 2\pi\left(\frac{T_{\text{E}}}{T_{\text{E}}} - \frac{T_{\text{E}}}{T_{\text{M}}}\right) = 2\pi\left(1 - \frac{T_{\text{E}}}{T_{\text{M}}}\right).$$

The time it takes the planets to realign occurs when $\theta_{\text{E}} = \theta_{\text{M}} + 2\pi$ or $\omega_{\text{E}}\Delta t = \omega_{\text{M}}\Delta t + 2\pi$, so

$$\Delta t = \frac{2\pi}{\omega_{\text{E}} - \omega_{\text{M}}} = \frac{2\pi}{\frac{2\pi}{T_{\text{E}}} - \frac{2\pi}{T_{\text{M}}}} = \frac{T_{\text{E}} T_{\text{M}}}{T_{\text{M}} - T_{\text{E}}}.$$

The angular position is found by solving for the angle instead of the time. $\theta_{\text{M}} = \omega_{\text{M}}\Delta t \;\Rightarrow\; \Delta t = \theta_{\text{M}} / \omega_{\text{M}}$, so: $\theta_{\text{E}} = \omega_{\text{E}}\Delta t = \dfrac{\omega_{\text{E}}\theta_{\text{M}}}{\omega_{\text{M}}} = \theta_{\text{M}} + 2\pi \;\Rightarrow\; \theta_{\text{M}} = \dfrac{2\pi}{\dfrac{\omega_{\text{E}}}{\omega_{\text{M}}} - 1} - 2\pi = \dfrac{2\pi}{\dfrac{T_{\text{M}}}{T_{\text{E}}} - 1} - 2\pi = \dfrac{2\pi T_{\text{E}}}{T_{\text{M}} - T_{\text{E}}} - 2\pi.$ Subtract 2π from the answer, so that $\theta \le 2\pi$.

CALCULATE: $\Delta\theta = 2\pi\left(1 - \dfrac{365.26}{687}\right) = 2.9426 \text{ rad}$, $\Delta t = \dfrac{687(365.26)}{687 - 365.26} = 779.93 \text{ days}$,

$\theta = \dfrac{2\pi(365.26)}{687 - 365.26} - 2\pi = 0.84989 \text{ rad}$

ROUND: The periods of Mars and Earth have three significant figures, so the results should be rounded accordingly.

(a) $\Delta\theta = 2.94 \text{ rad}$

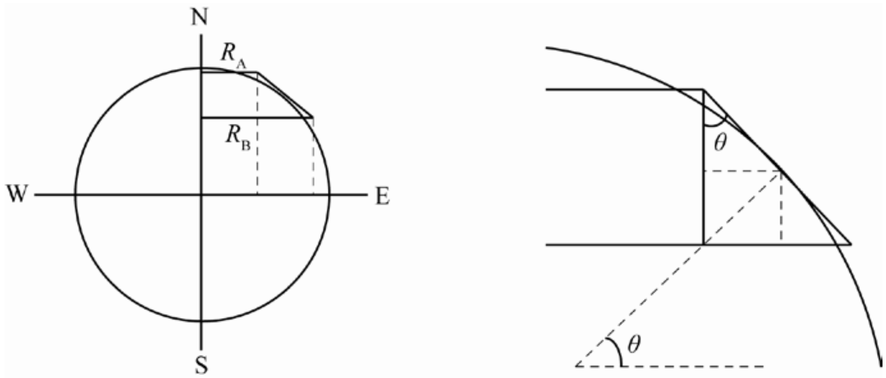
(b) $\Delta t = 780. \text{ days}$

(c) $\theta = 0.850 \text{ rad}$

DOUBLE-CHECK: The numbers are of the correct order for this solar system.

9.39. **THINK:** Determine (a) the magnitude and direction of the velocities of the pendulum at position *A* and *B*, (b) the angular speed of the pendulum motion, (c) the period of the rotation and (d) the effects of moving the pendulum to the equator. The latitude of the pendulum is $55.0°$ above the equator. The pendulum swings over a distance of d = 20.0 m. The period of the Earth's rotation is $T_{\text{E}} = 23 \text{ hr} + 56 \text{ min} = 86160 \text{ s}$ and the Earth's radius is $R_{\text{E}} = 6.37 \cdot 10^{6}$ m.

SKETCH:



RESEARCH: The following equations can be used: $\omega = \frac{2\pi}{T}$, $v = r\omega$, $R_A = R_E\cos\theta - \left(\frac{d}{2}\sin\theta\right)$ and $R_B = R_E\cos\theta + \left(\frac{d}{2}\sin\theta\right)$.

SIMPLIFY: The magnitudes of the velocities are:

$$v_A = R_A\omega_A = \frac{2\pi R_A}{T_E} = \frac{2\pi\left(R_E\cos\theta - \left(\frac{d}{2}\sin\theta\right)\right)}{T_E} \text{ and } v_B = \frac{2\pi\left(R_E\cos\theta + \left(\frac{d}{2}\sin\theta\right)\right)}{T_E}.$$

The angular speed of the rotation is related to the linear speed by $\Delta v = \omega_R d$. Rearranging gives:

$$\omega_R = \frac{\Delta v}{d} = \left(\frac{1}{d}\right)\frac{2\pi}{T_E}\left(\left(R_E\cos\theta + \left(\frac{d}{2}\sin\theta\right)\right) - \left(R_E\cos\theta - \left(\frac{d}{2}\sin\theta\right)\right)\right) = \frac{2\pi}{dT_E}d\sin\theta = \frac{2\pi}{T_E}\sin\theta.$$

The period is then $T_R = \frac{2\pi}{w_R} = \frac{2\pi}{\frac{2\pi}{T_E}\sin\theta} = \frac{T_E}{\sin\theta}$. At the equator, $\theta = 0°$.

CALCULATE:

(a) $v_A = 2\pi\left(\frac{\left(6.37\cdot 10^6 \text{ m}\right)\cos\left(55.0°\right) - \left(10.0 \text{ m}\right)\sin\left(55.0°\right)}{86{,}160 \text{ s}}\right) = 266.44277 \text{ m/s}$

$v_B = 2\pi\left(\frac{\left(6.37\cdot 10^6 \text{ m}\right)\cos\left(55.0°\right) + \left(10.0 \text{ m}\right)\sin\left(55.0°\right)}{86{,}160 \text{ s}}\right) = 266.44396 \text{ m/s}$

$\Delta v = v_B - v_A = 266.44396 \text{ m/s} - 266.44277 \text{ m/s} = 0.00119 \text{ m/s or } 1.19 \text{ mm/s}$

(b) $\omega_R = \frac{2\pi\sin\left(55.0°\right)}{86{,}160 \text{ s}} = 5.97\cdot 10^{-5} \text{ rad/s}$

(c) $T_R = \frac{86{,}160 \text{ s}}{\sin\left(55.0°\right)} = 105{,}182 \text{ s or about } 29.2 \text{ hours}$

(d) At the equator, $T_R = \lim_{\theta\to 0}\frac{T_E}{\sin\theta} = \infty$.

ROUND: The values given in the question have three significant figures, so the answers should also be rounded to three significant figures:

(a) The velocities are $v_A = 266.44277$ m/s and $v_B = 266.44396$ m/s, are in the direction of the Earth's rotation eastward. This means the difference between the velocities is $\Delta v = 1.19$ mm/s.

(b) The angular speed of rotation is $w_R = 1.19\cdot 10^{-4}$ rad/s.

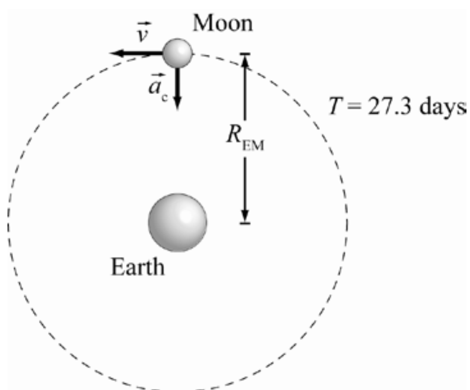
(c) The period of rotation is about 29.2 hours.

(d) At the equator there is no difference between the velocities at *A* and *B*, so the period is $T_\text{R} = \infty$. This means the pendulum does not rotate.

DOUBLE-CHECK: These are reasonable answers. If the difference in velocities was larger, these effects would be seen in everyday life but they are not. These are things pilots deal with when planning a flight path.

9.40. **THINK:** Determine the centripetal acceleration of the Moon around the Earth. The period of the orbit is $T = 27.3$ days and the orbit radius is $R = 3.85 \cdot 10^8$ m.

SKETCH:



RESEARCH: The centripetal acceleration is given by $a_\text{c} = \dfrac{v^2}{R}$. The radius, *R*, is known, so the speed, *v*, can be determined by making use of the period and noting that in this period the moon travels a distance equal to the circumference of a circle of radius *R*. Therefore,

$$v = \frac{\text{circumference}}{\text{period}} = \frac{2\pi R_{EM}}{T}.$$

SIMPLIFY: $a_\text{c} = \dfrac{v^2}{R} = \left(\dfrac{2\pi R_{EM}}{T}\right)^2 \left(\dfrac{1}{R_{EM}}\right) = \dfrac{4\pi^2 R_{EM}}{T^2}$

CALCULATE: Convert the period to seconds: 27.3 days = $2.3587 \cdot 10^6$ seconds. Therefore,

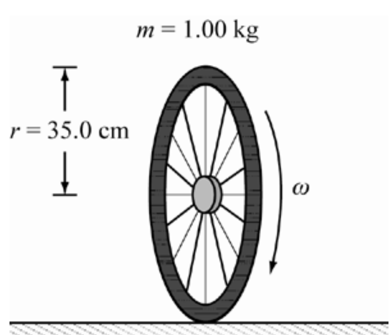
$$a_\text{c} = \frac{4\pi^2 \left(3.85 \cdot 10^8 \text{ m}\right)}{\left(23.587 \cdot 10^5 \text{ s}\right)^2} = 2.732 \cdot 10^{-3} \text{ m/s}^2.$$

ROUND: Since the values are given to three significant figures, $a_\text{c} = 2.73 \cdot 10^{-3}$ m/s^2.

DOUBLE-CHECK: This is reasonable for a body in uniform circular motion with the given values.

9.41. **THINK:** Determine the angular acceleration of a wheel given that it takes 1.20 seconds to stop when put in contact with the ground after rotating at 75.0 rpm. The wheel has a radius 35.0 cm and a mass of 1.00 kg.

SKETCH:



RESEARCH: Consider the angular speed of the wheel, and the necessary acceleration to bring that speed to zero in the given time. The angular acceleration is given by $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(\omega - \omega_0)}{t}$ and the rotational speed is given by:

$$\omega = (\text{rpm})\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right).$$

SIMPLIFY: Since the final rotational speed is zero, $\alpha = \frac{-\omega_0}{t} = -\frac{(\text{rpm})\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60 \text{ s}}\right)}{t}$.

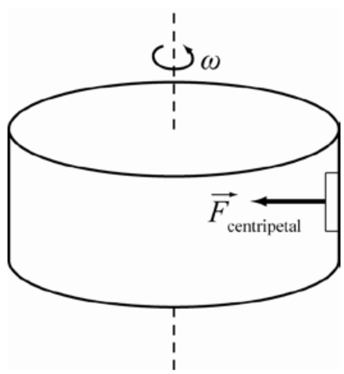
CALCULATE: $\alpha = -\frac{(75.0 \text{ rpm})(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s})}{(1.20 \text{ s})} = -6.54 \text{ rad/s}^2$

ROUND: Since the values are given to three significant figures, the result is $\alpha = -6.54 \text{ rad/s}^2$.

DOUBLE-CHECK: It is important that the acceleration is negative since it is slowing down the wheel. The magnitude seems reasonable based on the given values.

9.42. **THINK:** Determine the frequency of rotation required to produce an acceleration of $1.00 \cdot 10^5 g$. The radius is R = 10.0 cm.

SKETCH:



RESEARCH: Recall that the centripetal acceleration is given by $a_c = \omega^2 R$. Also, $\omega = 2\pi f$. Therefore, $a_c = (2\pi f)^2 R = 4\pi^2 f^2 R$.

SIMPLIFY: Solving for *f*, $f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$.

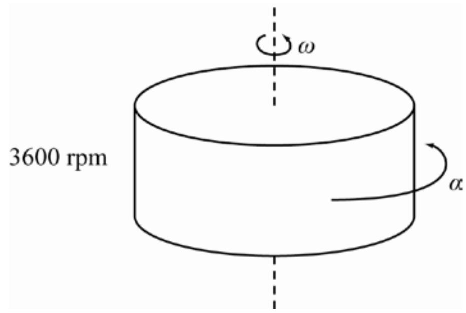
CALCULATE: $f = \frac{1}{2\pi}\sqrt{\frac{(1.00 \cdot 10^5)(9.81 \text{ m/s}^2)}{0.100 \text{ m}}} = 498.49 \text{ Hz}$

ROUND: Since all values are given to three significant figures, the result is $f = 498$ Hz.

DOUBLE-CHECK: A frequency of about 500 Hz seems reasonable to try to obtain an acceleration five orders of magnitude greater than *g*.

9.43. **THINK:** The initial angular speed is $\omega_0 = 3600. \text{ rpm} = 3600. \text{ rpm} \cdot \frac{2\pi \text{ rad}}{1 \text{ rotation}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = 120\pi \text{ rad/s}$. Calculate the time, t_1, it takes for the centrifuge to come to a stop $(\omega_1 = \omega(t_1) = 0)$ by using the average angular speed, $\bar{\omega}$, and the fact that that it completes $n = 60.0$ rotations. Use the time taken to stop to find the angular acceleration.

SKETCH:



RESEARCH: The average angular speed is given by $\bar{\omega} = \frac{1}{2}(\omega_f + \omega_0)$. Since the centrifuge completes 60.0 turns while decelerating, it turns through an angle of $\Delta\theta = 60.0 \text{ turns} \cdot \frac{2\pi \text{ rad}}{\text{turn}} = 120\pi \text{ rad}$. Use the two previous calculated values in the formula $\Delta\theta = \bar{\omega}t_1$ to obtain the time taken to come to a stop. Then, use the equation $\omega(t) = \omega_0 + \alpha t$ to compute the angular acceleration, α.

SIMPLIFY: The time to decelerate is given by, $t_1 = \frac{\Delta\theta}{\bar{\omega}} = \frac{\Delta\theta}{(\omega_1 + \omega_0)/2}$. Substituting this into the last equation given in the research step gives the equation, $\omega(t_1) = \omega_1 = \omega_0 + \alpha\frac{2\Delta\theta}{(\omega_1 + \omega_0)}$. Solving for α yields the equation: $\alpha = \frac{(\omega_1 - \omega_0)(\omega_1 + \omega_0)}{2\Delta\theta}$.

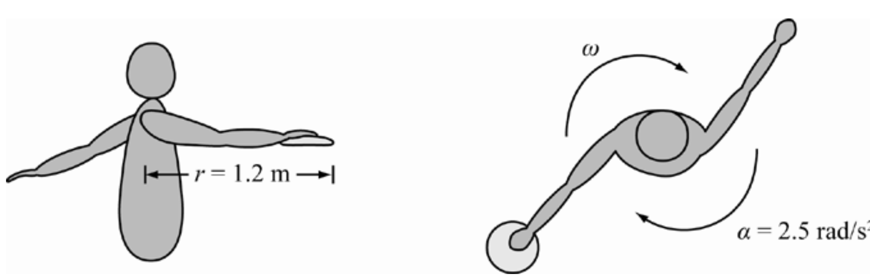
CALCULATE: $\alpha = \frac{(0 - 120\pi \text{ rad/s})(0 + 120\pi \text{ rad/s})}{2(120\pi \text{ rad})} = -60\pi \text{ rad/s}^2 = -188.496 \text{ rad/s}^2$

ROUND: Since the number of rotations is given to three significant figures, the final result should be also rounded to three significant figures: $\alpha = -188 \text{ rad/s}^2$.

DOUBLE-CHECK: The negative sign of α indicates deceleration, which is appropriate since the centrifuge is coming to a stop. The centrifuge decelerates from 120π rad/s to rest in $t_1 = \frac{2\Delta\theta}{(\omega_1 + \omega_0)} = \frac{2(120\pi \text{ rad})}{(0 + 120\pi \text{ rad/s})} = 2 \text{ s}$, and since the angular deceleration is constant, it must be the case that the deceleration is $60\pi \text{ rad/s}^2$. The answer is therefore reasonable.

9.44. **THINK:** A circular motion has a constant angular acceleration of $\alpha = 2.5 \text{ rad/s}^2$ and a radius of $r = 1.2$ m. Determine (a) the time required for the angular speed to reach 4.7 rad/s, (b) the number of revolutions to reach this angular speed of 4.7 rad/s, (c) the linear speed when the angular speed is 4.7 rad/s, (d) the linear acceleration when the angular speed is 4.7 rad/s, (e) the magnitude of the centripetal acceleration when the angular speed is 4.7 rad/s and (f) the magnitude of the discus' total acceleration.

SKETCH:



RESEARCH:

(a) Since the angular acceleration is constant, the time required to reach the final angular speed can be determined by means of the kinematic equation, $\omega = \omega_0 + \alpha t$, where $\omega_0 = 0.0$ rad/s.

(b) Once the time required to reach the angular speed, ω, is determined, the number of revolutions can be determined by setting $1\text{ rev} = 2\pi$ rad, where the number of radians is obtained from $d[\text{rad}] = \frac{1}{2}(\omega + \omega_0)t.$

(c) The linear speed, v, can be determined from the angular speed, ω, by the relation $v = \omega r$.

(d) The linear acceleration, a_t, can be obtained from the angular acceleration, α, by the relation $a_\text{t} = \alpha r$.

(e) The magnitude of the centripetal acceleration can be determined from the linear speed by the relation $a_\text{c} = \frac{v^2}{r}$.

(f) The total acceleration, a_T, can be found as the hypotenuse of a right angle triangle where the sides are the linear (tangential) acceleration, a_t, and the angular acceleration, α. The relationship is $a_\text{T} = \sqrt{a_\text{t}^2 + \alpha^2}$.

SIMPLIFY:

(a) $\omega = \omega_0 + \alpha t = \alpha t \Rightarrow t = \frac{\omega}{\alpha}$

(b) $d[\text{rad}] = \frac{1}{2}(\omega + \omega_0)t = \frac{1}{2}(\omega t)$, and to convert to the number of revolutions, $\text{rev} = \frac{d}{(2\pi)}$.

(c) $v = \omega r$

(d) $a_\text{t} = \alpha r$

(e) $a_\text{c} = \frac{v^2}{r}$

(f) $a_\text{T} = \sqrt{a_\text{t}^2 + \alpha^2}$

CALCULATE:

(a) $t = \frac{4.70\text{ rad/s}}{2.50\text{ rad/s}^2} = 1.88\text{ s}$

(b) $d[\text{rad}] = \frac{(4.70\text{ rad/s})(1.88\text{ s})}{2} = 4.42\text{ rad}$ or $4.42\text{ rad}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 0.70314\text{ rev}$

(c) $v = (4.70\text{ rad/s})(1.20\text{ m}) = 5.64\text{ m/s}$

(d) $a_\text{t} = (2.50\text{ rad/s}^2)(1.20\text{ m}) = 3.00\text{ m/s}^2$

(e) $a_\text{c} = \frac{(5.64\text{ m/s})^2}{1.20\text{ m}} = 26.5\text{ m/s}^2$

(f) $a_\text{T} = \sqrt{(2.88\text{ m/s}^2)^2 + (26.5\text{ m/s}^2)^2} = 26.656\text{ m/s}^2$

ROUND: Rounding to three significant figures:

(a) $t = 1.88$ s

(b) 0.703 revolutions

(c) $v = 5.64$ m/s

(d) $a_\text{t} = 3.00\text{ m/s}^2$

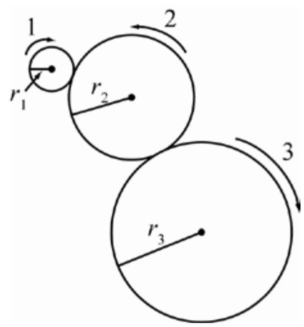
(e) $a_\text{c} = 26.5\text{ m/s}^2$

(f) $a_T = 26.7 \text{ m/s}^2$

DOUBLE-CHECK: Based on the given values, these results are reasonable.

9.45. **THINK:** Three coupled disks have radii $r_1 = 0.100$ m, $r_2 = 0.500$ m, $r_3 = 1.00$ m. The rotation rate of disk 3 is one revolution every 30.0 seconds. Determine (a) the angular speed of disk 3, (b) the tangential velocities of the three disks, (c) the angular speeds of disks 1 and 2 and (d) if the angular acceleration of disk 1 is $\alpha_1 = 0.100 \text{ rad/s}^2$, what are the angular accelerations of disks 2 and 3?

SKETCH:



RESEARCH:

(a) To obtain the angular speed of disk 3, use its rotation rate, $T = 30.0$ s, and the relationship between revolutions and radians, 2π rad/rev. Therefore, $\omega_3 = 2\pi / T$.

(b) Since the three disks are touching each other and there is no slipping, they all have the same tangential speed. Therefore, only one tangential speed must be determined. Since the angular speed of disk 3 is known, the tangential speed can be determined from $v = \omega_3 r_3$.

(c) Calculate the angular speed for disks 1 and 2 from the tangential speeds and the radii. That is, $\omega_1 = v / r_1$, and $\omega_2 = v / r_2$.

(d) Since the angular acceleration of disk 1 is known, its tangential acceleration can be determined. Since the disks are touching each other, and no slipping occurs, this tangential acceleration is common to all disks. The angular acceleration for disks 2 and 3 can be determined from this tangential acceleration and the radii. Therefore, $\alpha_1 = a_1 / r_1$, implies $a_1 = \alpha_1 r_1$. Since $a_1 = a_2 = a_3 = a$, $\alpha_2 = a / r_2$ and $\alpha_3 = a / r_3$.

SIMPLIFY:

(a) $\omega_3 = 2\pi / T$

(b) $v = \omega_3 r_3$

(c) $\omega_1 = v / r_1$, and $\omega_2 = v / r_2$.

(d) $\alpha_2 = a / r_2$ and $\alpha_3 = a / r_3$, where $a = \alpha_1 r_1$.

CALCULATE:

(a) $\omega_3 = \dfrac{(2\pi \text{ rad/rev})}{30.0 \text{ s}} = 0.209 \text{ rad/s}$

(b) $v = (0.209 \text{ rad/s})(1.00 \text{ m}) = 0.209 \text{ m/s}$ for all three disks.

(c) $\omega_1 = \dfrac{0.209 \text{ m/s}}{0.100 \text{ m}} = 2.09 \text{ rad/s}$ and $\omega_2 = \dfrac{0.209 \text{ m/s}}{0.500 \text{ m}} = 0.419 \text{ rad/s}$.

(d) $a = (0.100 \text{ rad/s}^2)(0.100 \text{ m}) = 1.00 \cdot 10^{-2} \text{ m/s}^2$.

Therefore, $\alpha_2 = \dfrac{1.00 \cdot 10^{-2} \text{ m/s}^2}{0.500 \text{ m}} = 2.00 \cdot 10^{-2} \text{rad/s}^2$ and $\alpha_3 = \dfrac{1.00 \cdot 10^{-2} \text{ m/s}^2}{1.00 \text{ m}} = 1.00 \cdot 10^{-2} \text{ rad/s}^2$.

ROUND: Keeping three significant figures, the results are:

(a) $\omega_3 = 0.209 \text{ rad/s}$

(b) $v = 0.209 \text{ m/s}$ for all three disks

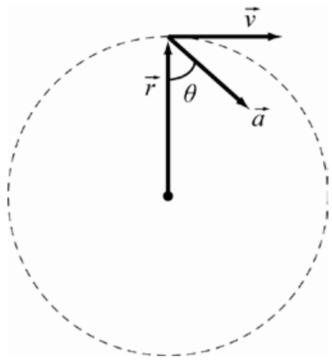
(c) $\omega_1 = 2.09$ rad/s and $\omega_2 = 0.419$ rad/s

(d) $\alpha_2 = 2.00 \cdot 10^{-2}$ rad/s^2 and $\alpha_3 = 1.00 \cdot 10^{-2}$ rad/s^2

DOUBLE-CHECK: Based on the given values, all the results are reasonable.

9.46. **THINK:** Determine the speed of a particle whose acceleration has a magnitude of $a = 25.0$ m/s^2 and makes an angle of $\theta = 50.0°$ with the radial vector.

SKETCH:



RESEARCH: To determine the tangential speed, v, recall that the centripetal acceleration is given by $a_c = \frac{v^2}{r}$. The centripetal acceleration is the projection of the total acceleration on the radial axis, i.e. $a_c = a_T \cos\theta$.

SIMPLIFY: Therefore, the tangential speed is given by $v = \sqrt{a_c r} = \sqrt{a_T r \cos\theta}$.

CALCULATE: $v = \sqrt{(25.0 \text{ m/s}^2)\cos(50.0°)(1.00 \text{ m})} = 4.01$ m/s

ROUND: The values are given to three significant figures, therefore the result is $v = 4.01$ m/s.

DOUBLE-CHECK: This result is reasonable based on the magnitudes of the given values.

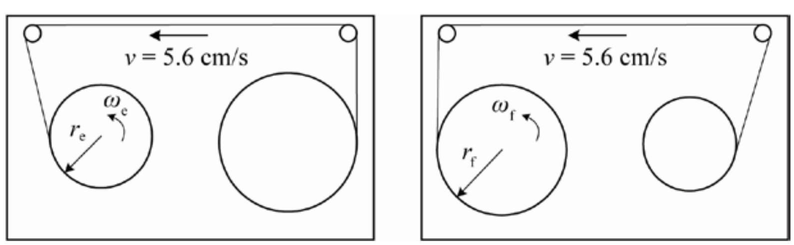
9.47. **THINK:** Determine the angular speed of the take-up spool in a tape recorder in the following cases:

(a) When the take-up spool is empty with radius, $r_e = 0.800$ cm.

(b) When the take-up spool is full with radius, $r_f = 2.20$ cm.

(c) Determine the average angular acceleration of the take-up spool if the length of the tape is $l = 100.80$ m. The magnetic tape has a constant linear speed of $v = 5.60$ cm/s.

SKETCH:



RESEARCH:

(a) & (b) To determine the angular speed, make use of the relationship $v = \omega r \Rightarrow \omega = \frac{v}{r}$.

(c) To determine an average angular acceleration, use the definition, $\alpha = \Delta w / \Delta t$, where the time is determined from $\Delta t = \frac{(\text{distance})}{(\text{speed})} = \frac{l}{v}$.

SIMPLIFY:

(a) $\omega_e = \frac{v}{r_e}$

(b) $\omega_f = \frac{v}{r_f}$

(c) $\alpha = \frac{\Delta\omega}{\Delta T} = \frac{\omega_f - \omega_e}{l/v} = \frac{v(\omega_f - \omega_e)}{l}$

CALCULATE:

(a) $\omega_e = \frac{5.60 \cdot 10^{-2}\text{ m/s}}{8.00 \cdot 10^{-3}\text{ m}} = 7.00\text{ rad/s}$

(b) $\omega_f = \frac{5.60 \cdot 10^{-2}\text{ m/s}}{2.20 \cdot 10^{-2}\text{ m}} = 2.54\text{ rad/s}$

(c) $\alpha = \frac{(5.60 \cdot 10^{-2}\text{ m/s})(2.54 - 7.00)}{100.80\text{ m}} = -2.48 \cdot 10^{-3}\text{ rad/s}^2$

ROUND: Keep three significant figures:

(a) $\omega_e = 7.00$ rad/s

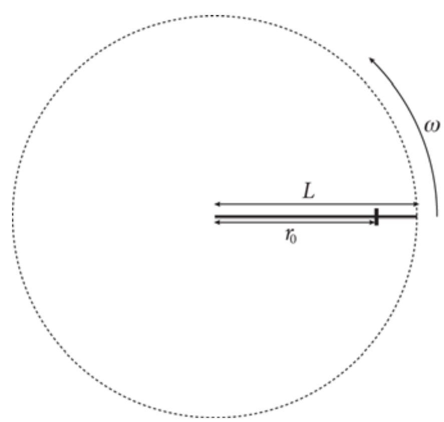
(b) $\omega_f = 2.54$ rad/s

(c) $\alpha = -2.48 \cdot 10^{-3}\text{ rad/s}^2$

DOUBLE-CHECK: It is reasonable that the angular speed of the spool when it's empty is greater than when it's full. Also, it is expected that the angular acceleration is negative since the angular speed is decreasing as the spool gets full.

9.48. **THINK:** Determine the radial velocity of a ring fitted around a rod as it reaches the end of the rod. The rod is spun in a horizontal circle at a constant angular velocity. The given values are the length of the rod, l = 0.50 m, the initial distance of the ring from the fixed end of the rod, $r_0 = 0.30$ m, and the constant angular velocity, $\omega = 4.0$ rad/s.

SKETCH:



RESEARCH: For the ring to move in a circular path at a fixed distance along the rod, it would require a centripetal acceleration of $a_c = \omega^2 r$ directed toward the center of the path. However, there is no force on the ring that will supply this acceleration, thus the inertia of the ring will tend to pull it outward along the rod. The resulting radial acceleration is equal to the missing centripetal acceleration, $a_c = \omega^2 r$. Since this radial acceleration depends on the radial position, the differential kinematic relations must be used:

$$\frac{dv_r}{dt} = \omega^2 r \Rightarrow \left(\frac{dv_r}{dr}\right)\left(\frac{dr}{dt}\right) = \omega^2 r,$$

where the second equation follows from using the chain rule of calculus.

SIMPLIFY: Since $\frac{dr}{dt} = v_{\text{r}}$, use separation of variables to set up the integral:

$$v_{\text{r}} dv_{\text{r}} = \omega^2 r dr \Rightarrow \int_0^v v_{\text{r}}' dv_{\text{r}}' = \omega^2 \int_{r_0}^l r' dr'$$

$$\left.\frac{(v_{\text{r}}')^2}{2}\right|_0^{v_{\text{r}}} = \omega^2 \left.\frac{(r')^2}{2}\right|_{r_0}^l$$

$$\frac{v_{\text{r}}^2}{2} = \frac{\omega^2\left(l^2 - r_0^2\right)}{2} \to v_{\text{r}} = \omega\sqrt{l^2 - r_0^2}.$$

CALCULATE: The speed is therefore, $v_{\text{r}} = (4.00 \text{ rad/s})\sqrt{(0.500 \text{ m})^2 - (0.300 \text{ m})^2} = 1.60 \text{ m/s}.$

ROUND: Since the angular velocity is given to three significant figures, the result remains $v_{\text{r}} = 1.60$ m/s.

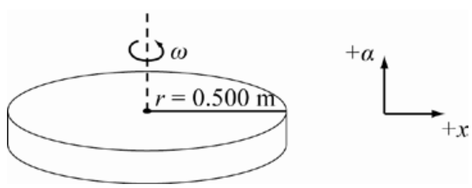
DOUBLE-CHECK: Based on the given values, the resulting radial velocity is reasonable.

9.49. **THINK:** A flywheel with a diameter of 1.00 m is initially at rest, and has an angular acceleration in terms of time as $\alpha(t) = 0.1t^2$, and has units of rad/s^2. Determine:

(a) The angular separation between the initial and final positions of a point on the rim 8.00 seconds after the rotations begin.

(b) Find the linear position, velocity, and acceleration of a point 8.00 seconds after the wheel starts rotating, where the starting position of the point is at $\theta = 0$. Use the known equations relating the position and velocity to the acceleration.

SKETCH:



RESEARCH:

(a) The angular separation can be determined by first considering the change in angular speed through the time period:

$$\Delta\omega = \int_{\text{i}}^{\text{f}} \alpha(t)\, dt.$$

Since the initial angular speed is zero, $\Delta\omega = \omega$. Then consider the change in the angle through the time period:

$$\Delta\theta = \int_{\text{i}}^{\text{f}} \omega\, dt.$$

(b) The angular acceleration and angular velocity are known and can be related to the tangential component of the linear acceleration and to the velocity through the equations $a_t = \alpha(t)r$ and $v = \omega r$. The radial component of the acceleration vector is the centripetal acceleration, $a_r = v^2/r$. The position will be on the circumference, given by $\vec{r} = r(\cos\theta)\hat{x} + r(\sin\theta)\hat{y}$ where the angle is known from (a). Note that in this case, the question indicates that $v_0 = 0$ and $\theta_0 = 0$. By convention, θ is measured counterclockwise from the positive x-axis.

SIMPLIFY:

(a) $\Delta\omega = \int_{\text{i}}^{\text{f}} \alpha\, dt$ and $\Delta\theta = \int_{\text{i}}^{\text{f}} \omega\, dt$.

(b) $a = \alpha(t)r$, $v = \omega(t)r$, $\vec{r} = r(\cos\theta)\hat{x} + r(\sin\theta)\hat{y}$ do not need simplifying.

CALCULATE:

(a) $\Delta\omega = \int_0^t (0.1)t^2 \; dt = \dfrac{0.1t^3}{3}$, $\Delta\theta = \int_{\text{i}}^{\text{f}} \omega \; dt = \int_0^8 \dfrac{0.1t^3}{3} \; dt = \left.\dfrac{0.1t^4}{12}\right|_0^8 = 34.13333 \text{ rad} = 5.43249 \text{ rev}$

This is 5 complete revolutions plus an additional 0.43249 of a revolution. Therefore the angular separation is given by $(0.43249)(2\pi) = 2.717$ rad.

(b) For the linear velocity, $v = \dfrac{0.1t^3 r}{3} = \dfrac{(0.1)(8.00 \text{ s})^3 (0.500 \text{ m})}{3} = 8.53333 \text{ m/s}$. The linear position,

$\vec{r} = (0.500 \text{ m})[\cos(2.717 \text{ rad})]\hat{x} + (0.500 \text{ m})[\sin(2.717 \text{ rad})]\hat{y} = -(0.4556 \text{ m})\hat{x} + (0.20597 \text{ m})\hat{y}$

For the tangential acceleration, $a_t = 0.1t^2 r = (0.1)(8.00 \text{ s})^2 (0.500 \text{ m}) = 3.200 \text{ m/s}^2$.

For the radial acceleration: $a_r = (8.53333 \text{ m/s})^2 / (0.500 \text{ m}) = 145.635 \text{ m/s}^2$.

Therefore the magnitude of the total acceleration is dominated by the centripetal acceleration:

$a = \sqrt{a_r^2 + a_t^2} = \sqrt{145.635^2 + 3.2^2} \text{ m/s}^2 = 145.671 \text{ m/s}^2$.

ROUND: The constant 0.1 in the function for α is treated as precise.

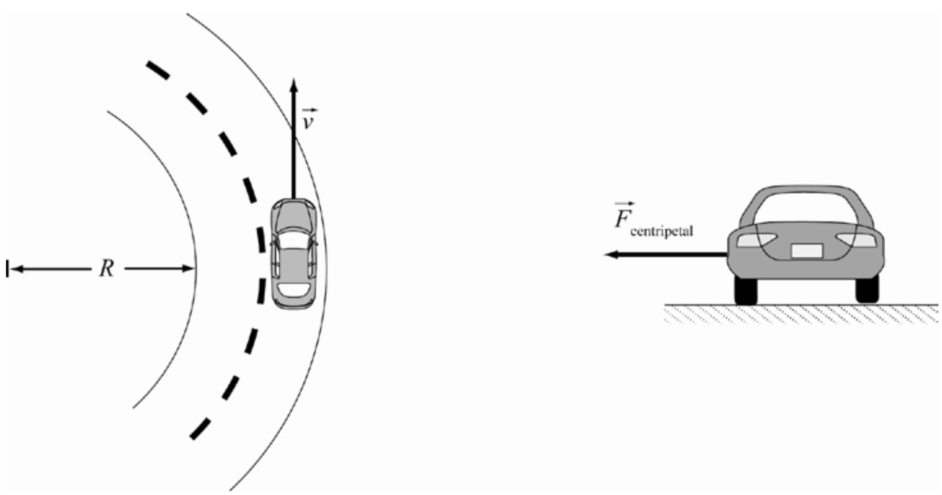
(a) Since all values are given to three significant figures, the result is $\Delta\theta = 2.72$ rad.

(b) To three significant figures, the results are $a_t = 3.20 \text{ m/s}^2$, $a_r = 146 \text{ m/s}^2$, and $v = 8.53$ m/s. The position of the point is $-0.456\hat{x} + 0.206\hat{y}$ (or 0.500 m from the center at an angle of $+2.72$ rad from its initial position).

DOUBLE-CHECK: Based on the given values, these results are reasonable. The magnitude of the linear position vector is $|\vec{r}| = \sqrt{(-0.4556 \text{ m})^2 + (0.20597 \text{ m})^2} = 0.500$ m, which is consistent with the requirement that the point is at the edge of the wheel.

9.50. **THINK:** Determine the force that plays the role of and has the value of the centripetal force on a vehicle of mass m = 1500. kg, with speed v = 15.0 m/s around a curve of radius R = 400. m.

SKETCH:



RESEARCH: The force that keeps the vehicle from slipping out of the curve is the force of static friction. The force can be calculated by recalling the form of the centripetal force,

$$F_c = m\frac{v^2}{R}.$$

SIMPLIFY: The equation is in its simplest form.

CALCULATE: $F_c = (1500.\text{ kg})\dfrac{(15.0\text{ m})^2}{400.\text{ m}} = 843.75\text{ N}$

ROUND: To three significant figures: $F_c = 844\text{ N}$

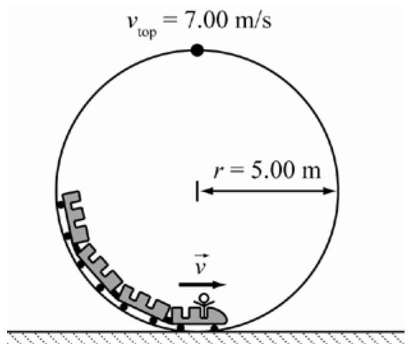
DOUBLE-CHECK: The coefficient of static friction can be determined from the equation $F_c = \mu_s mg$:

$$\Rightarrow \mu_s = \frac{F_c}{mg} = \frac{800\text{ N}}{1500\text{ kg}(9.81\text{ m/s}^2)} = 0.05.$$

This is within the expected values before slipping occurs. Therefore this is a reasonable force to obtain for the centripetal force.

9.51. **THINK:** The apparent weight of a rider on a roller coaster at the bottom of the loop is to be determined. From Solved Problem 9.1, the radius is r = 5.00 m, and the speed at the top of the loop is 7.00 m/s.

SKETCH:



RESEARCH: The apparent weight is the normal force from the seat acting on the rider. At the bottom of the loop the normal force is the force of gravity plus the centripetal force:

$$N = F_g + \frac{mv^2}{r} = mg + \frac{mv^2}{r}.$$

The velocity at the bottom of the loop can be determined by considering energy conservation between the configuration at the top and that at the bottom:

$$\frac{1}{2}mv^2 = mgh + \frac{1}{2}mv_t^2$$

where h = 2r. In Solved Problem 9.1 it as determined that the feeling of weightlessness at the top is achieved if $\dfrac{mv_t^2}{r} = mg$.

SIMPLIFY: Multiply the equation for energy conservation by a factor of $2/r$ and find:

$$\frac{mv^2}{r} = \frac{2mgh}{r} + \frac{mv_t^2}{r}.$$

Since h = 2r, this results in:

$$\frac{mv^2}{r} = 4mg + \frac{mv_t^2}{r}.$$

Insert this for the normal force and see

$$N = mg + \frac{mv^2}{r} = mg + 4mg + \frac{mv_t^2}{r} = mg + 4mg + mg = 6mg.$$

CALCULATE: Not needed.

ROUND: Not needed.

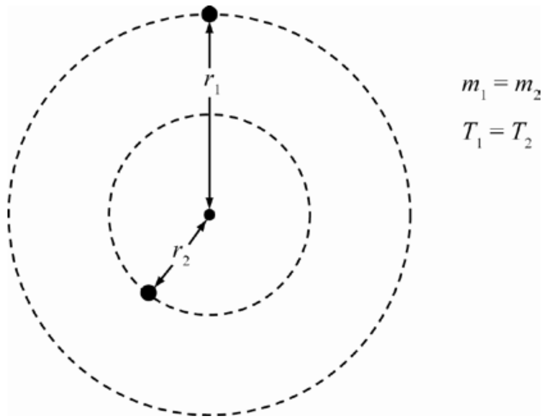
DOUBLE-CHECK: Our result means that you experience 6g of acceleration at the bottom of the loop, which seems like a large number, if you consider that the maximum acceleration during the launch of a Space Shuttle is kept to 3g. However, if you have ever had the opportunity to ride on such a roller coaster, then our result does not seem unreasonable.

9.52. **THINK:** Two skaters have equal masses and periods of rotation but the radius of one is half of the other. Determine:

(a) The ratio of their speeds.

(b) The ratio of the magnitudes of the forces on each skater.

SKETCH:



RESEARCH:

(a) The ratio of the speeds, v_1 / v_2, can be determined by considering the period of rotation, given by $T = 2\pi r / v$. Since the two skaters have the same period, $T = 2\pi r_1 / v_1 = 2\pi r_2 / v_2$.

(b) The force acting on each skater has only a centripetal component whose magnitude is mv^2 / r. Therefore the ratio of the magnitudes is simply $\frac{F_2}{F_1}$, and $\frac{F_2}{F_1} = \frac{m_2\left(v_2^2 / r_2\right)}{m_2\left(v_1^2 / r_1\right)}$.

SIMPLIFY:

(a) $\frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2} \Rightarrow \frac{r_1}{v_1} = \frac{r_2}{v_2} \Rightarrow \frac{r_1}{r_2} = \frac{v_1}{v_2}$

(b) $\frac{F_2}{F_1} = \frac{m_2\left(v_2^2 / r_2\right)}{m_2\left(v_1^2 / r_1\right)} = \frac{\left(v_2^2 / r_2\right)}{\left(v_1^2 / r_1\right)} = \frac{\left(r_2 / T_2^2\right)}{\left(r_1 / T_1^2\right)} = \frac{r_2}{r_1}$

CALCULATE:

(a) Since $r_2 = \frac{r_1}{2}$, $\frac{r_2}{r_1} = \frac{v_2}{v_1} = \frac{1}{2}$.

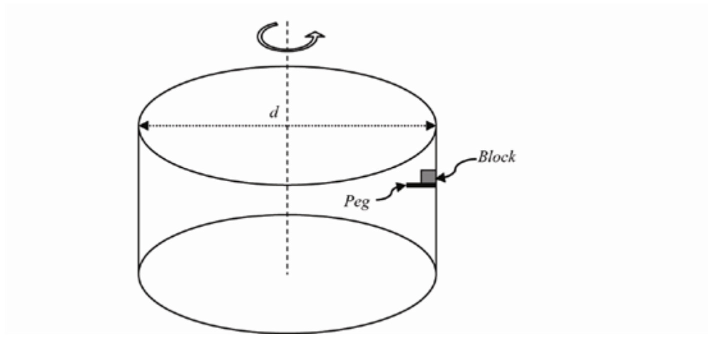
(b) $\frac{F_2}{F_1} = \frac{r_2}{r_1} = \frac{v_2}{v_1} = \frac{1}{2}$

ROUND: It is not necessary to round. The result for both parts (a) and (b) is a ratio of 1/2.

DOUBLE-CHECK: It is reasonable that by doubling the radius, both the speed of rotation and centripetal force also double.

9.53. **THINK:** Determine the minimum time required for a block held by a peg inside a cylinder to stay in place once the cylinder starts rotating with angular acceleration, α. The coefficient of static friction is given as μ. To avoid slipping in the vertical direction, balance the force due to gravity with the force due to friction between the block and the cylinder. For large values of the angular acceleration, we also obtain a significant force in tangential direction. However, we restrict our considerations to the case of small angular acceleration and neglect the tangential force.

SKETCH:



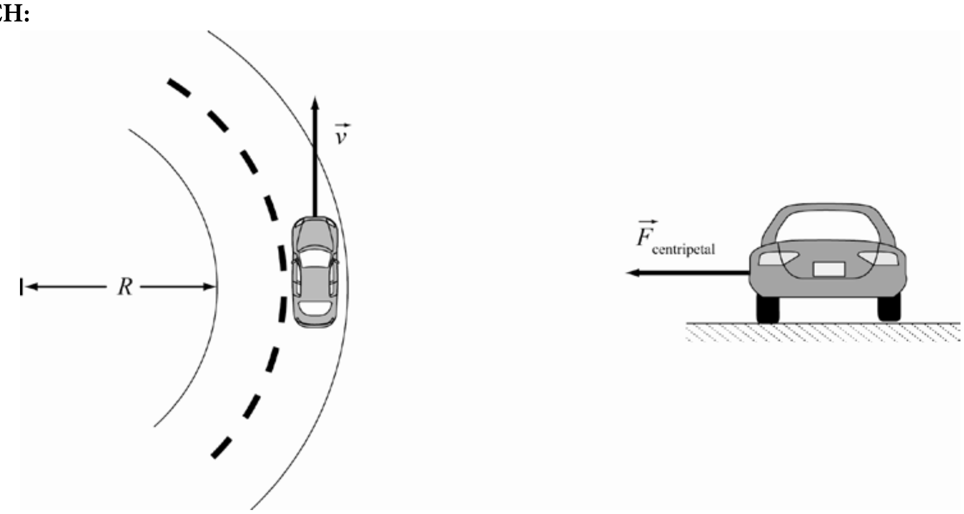
RESEARCH: The force due to friction is given by $f = \mu N$, and in this case N is simply the centripetal force, $F_c = mv^2 / (d/2)$. The time required to reach a suitable centripetal force can be determined by means of the angular speed, $\omega = v/r$, and the angular acceleration, $\alpha = \omega / t$.

SIMPLIFY: The centripetal force can be rewritten as $F_c = mv^2 / r = m(\omega r)^2 / (d/2) = m\omega^2 d/2$. Thus, the force of static friction is given by $f_f = \mu m(d/2)\omega^2$. Therefore, from the balancing of the vertical forces: $f = F_g$, or $\mu m\omega^2 d/2 = mg \Rightarrow \omega^2 = 2g/(\mu d)$. Since $t = \omega / \alpha$, the time interval is:

$$t = \sqrt{2g/\mu d}/\alpha = \sqrt{\frac{2g}{\mu d \alpha^2}}.$$

CALCULATE: There are no numbers to insert in this problem.
ROUND: There is nothing to round since there are no numerical values.
DOUBLE-CHECK: An easy check we can perform right away is to make sure that the units on the right-hand side of our formula indeed work out to be seconds.

9.54. **THINK:** The maximum velocity such that the car performs uniform circular motion without slipping must be determined. The coefficient of static friction is $\mu_s = 1.20$ and the radius of the circular path is r = 10.0 m.
SKETCH:



RESEARCH: Consider which force is providing the centripetal force. Since the car is not sliding, it is the force of static friction. Those two forces must be related to determine the maximum velocity. That is,

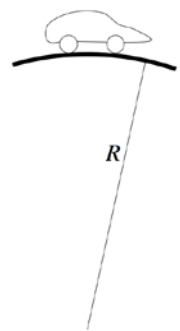
$$F_{\text{friction}} = F_{\text{centripetal}} \Rightarrow \mu_s mg = \frac{mv^2}{r}.$$

SIMPLIFY: $\mu_s mg = \dfrac{mv^2}{r} \Rightarrow v = \sqrt{\mu_s gr}$

CALCULATE: $v = \sqrt{\mu_s gr} = \sqrt{(1.20)(9.81 \text{ m/s}^2)(10.0 \text{ m})} = 10.84988 \text{ m/s}$

ROUND: Since the values given have three significant figures, the result is then v = 10.8 m/s.
DOUBLE-CHECK: This result may seem quite small for a racecar. But, consider that 10.8 m/s are ~24 mph, and that this is a very tight curve with a diameter of less than the length of a basketball court. It then seems reasonable that a car cannot go very fast through such a tight curve. Also, note that as expected, the maximum velocity is independent of the mass of the car.

9.55. **THINK:** Determine the maximum speed of a car as it goes over the top of a hill such that the car always touches the ground. The radius of curvature of the hill is 9.00 m. As the car travels over the top of the hill it undergoes circular motion in the vertical plane. The only force that can provide the centripetal force for this motion is gravity. Clearly, for small speeds the car remains in contact with the road due to gravity. But the car will lose contact if the centripetal acceleration exceeds gravity.
SKETCH:



RESEARCH: In the limiting case of the maximum speed we can set the centripetal acceleration equal to g: $g = v_{\text{max}}^2 / r$.

SIMPLIFY: Solve for the maximum speed and find $v_{\text{max}} = \sqrt{gr}$.

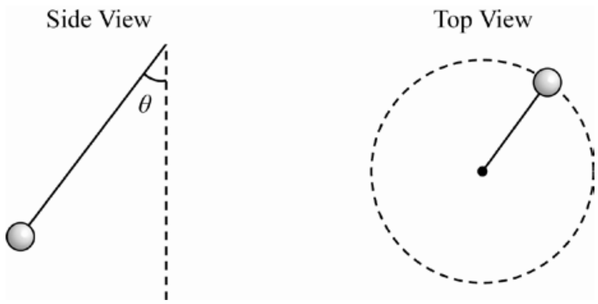
CALCULATE: $v_{\text{max}} = \sqrt{gr} = \sqrt{(9.81 \text{ m/s}^2)(9.00 \text{ m})} = 9.40 \text{ m/s}$

ROUND: Since the radius is given to three significant figures, the result is $v_{\text{max}} = 9.40$ m/s.

DOUBLE-CHECK: This speed of 9.40 m/s, which is approximately 21.0 mph, seems very small. But on the other hand, this is a very significant curvature at the top of the hill, equivalent to a good-sized speed bump. Going over this type of bump at more than 21 mph makes it likely that your car will lose contact with the road surface.

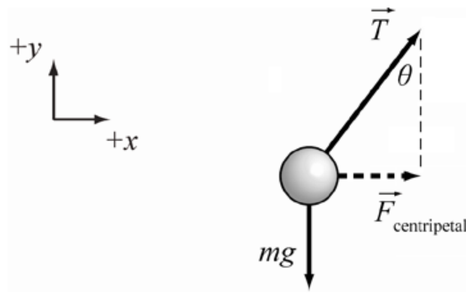
9.56. **THINK:** A ball attached to a string is in circular motion as described by the sketch. Determine:
(a) The free-body diagram for the ball.
(b) The force acting as the centripetal force.
(c) The required speed of the ball such that $\theta = 45.0°$.
(d) The tension on the string.

SKETCH:



RESEARCH:

(a)



(b) As shown in the sketch, the projection of the tension onto the horizontal plane provides the centripetal force. Therefore, $mv^2/r = T\sin\theta.$

(c) From the sketch, the force due to gravity is balanced by the projection of the tension on the vertical axis, i.e. $mg = T\cos\theta.$ From part (b), the centripetal force is given by $mv^2/r = T\sin\theta.$ By solving both equations for T and then equating them, the speed for the given angle can be determined.

(d) The tension on the string can most easily be found from $mg = T\cos\theta,$ for the given angle, $\theta.$

SIMPLIFY:

(a) Not applicable.

(b) Not applicable.

(c) $mg = T\cos\theta \Rightarrow T = \dfrac{mg}{\cos\theta}$, and $\dfrac{mv^2}{r} = T\sin\theta \Rightarrow T = \dfrac{mv^2}{r\sin\theta}.$

Equating the above equations gives $\dfrac{mg}{\cos\theta} = \dfrac{mv^2}{r\sin\theta} \Rightarrow v = \sqrt{gr\tan\theta},$ where $r = L\sin\theta.$

(d) $mg = T\cos\theta \Rightarrow T = \dfrac{mg}{\cos\theta}$

CALCULATE:

(a) Not applicable.

(b) Not applicable.

(c) $v = \sqrt{\left(9.81 \text{ m/s}^2\right)\left(\left(1.00 \text{ m}\right)\sin\left(45.0°\right)\right)\tan 45.0°} = 2.63376 \text{ m/s}$

(d) $T = \dfrac{\left(0.200 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right)}{\cos\left(45.0°\right)} = 2.7747 \text{ N}$

ROUND:

(a) Not applicable.

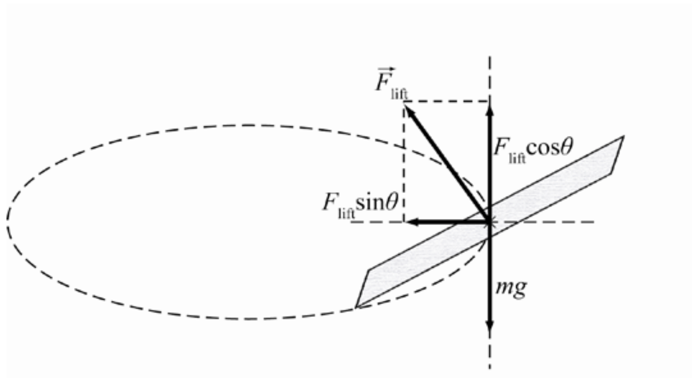
(b) Not applicable.

(c) Since values are given to three significant figures, the result is $v = 2.63$ m/s.

(d) Keeping three significant figures, $T = 2.77$ N.

DOUBLE-CHECK: All results are reasonable based on the given values. It is expected that the tension on the string will be greater than the weight of the ball.

9.57. **THINK:** Determine the banking angle for a plane performing uniform circular motion. The radius is 7.00 miles ($1.12654\cdot 10^4$ meters), the speed is 360. mph (160.93 m/s), the height is $2.00\cdot 10^4$ ft (6096 meters) and the plane length is 275 ft (83.82 meters).
SKETCH:



RESEARCH: Suppose *F* is the lift force, which makes an angle, θ, with the vertical as shown in the sketch. Also, suppose the weight of the plane is *mg*. Now, $F\cos\theta$ balances the weight of the plane when the plane is banked with the horizontal and $F\sin\theta$ provides the necessary centripetal force for the circular motion. Therefore,

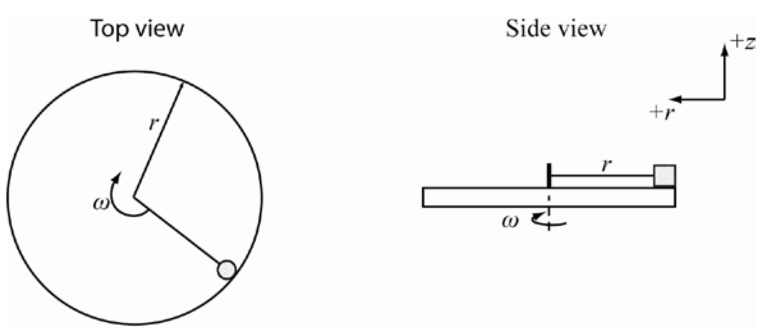
$$F\sin\theta = \frac{mv^2}{r} \text{ and } F\cos\theta = mg.$$

SIMPLIFY: Dividing the two equations gives $\tan\theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$.

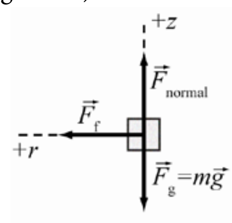
CALCULATE: $\theta = \tan^{-1}\left(\frac{(160.93 \text{ m/s})^2}{(1.12654\cdot 10^4 \text{ m})(9.81 \text{ m/s}^2)}\right) = 13.189°$

ROUND: Rounding to three significant figures, the result is an angle of approximately $13.2°$.
DOUBLE-CHECK: Based on the given values, the result is reasonable.

9.58. **THINK:** Determine the tension on the string attaching a cylinder (m = 20.0 g) to the center of a turntable as the angular velocity increases up to 60.0 rpm. The coefficient of static friction is $\mu_s = 0.800$ and the distance between the center of the turntable and the cylinder of l = 80.0 cm.
SKETCH:



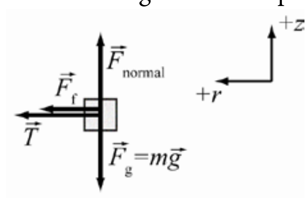
RESEARCH: As the turntable speeds up from the rest, the static friction force provides the centripetal force and no tension is built into the string for a while. The corresponding free body diagram for the cylinder under these conditions is presented. (Since the turntable speeds up very slowly, the tangential static friction force that acts on the cylinder from the turntable and keeps it moving with the turntable is important physically, but negligible in magnitude).



At a critical value, ω_1, of the angular velocity, the static friction force reaches its maximum value, so $F_\text{f} = \mu_\text{s} mg$ becomes

$$\frac{mv^2}{r} = m\omega_1^2 r = \mu_\text{s} mg \;\Rightarrow\; \omega_1 = \sqrt{\frac{\mu_\text{s} g}{r}}.$$

Once the angular velocity exceeds ω_1, static friction alone is not enough to provide the required centripetal force, and a tension is built into the string. The corresponding free body diagram is presented.



SIMPLIFY: The tension in the string when the angular velocity of the turntable is $\omega_2 = 60.0 \text{ rpm}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 2.00\pi \text{ rad/s}$ is calculated from the centripetal force at this velocity, $F_{\text{c}2} = m\omega_2^2 r,$ and the tension is given by $T = F_{\text{c}2} - F_\text{f} = m\omega_2^2 r - \mu_\text{s} mg = m\left(\omega_2^2 r - \mu_\text{s} g\right).$

CALCULATE: $T = (0.0200 \text{ kg})\left[(2.00\pi \text{ rad/s})^2 (0.800 \text{ m}) - (0.800)(9.81 \text{ m/s}^2)\right] = 0.475 \text{ N}$

ROUND: To three significant figures, $T = 0.475$ N.

DOUBLE-CHECK: This is a reasonable tension for the small system described.

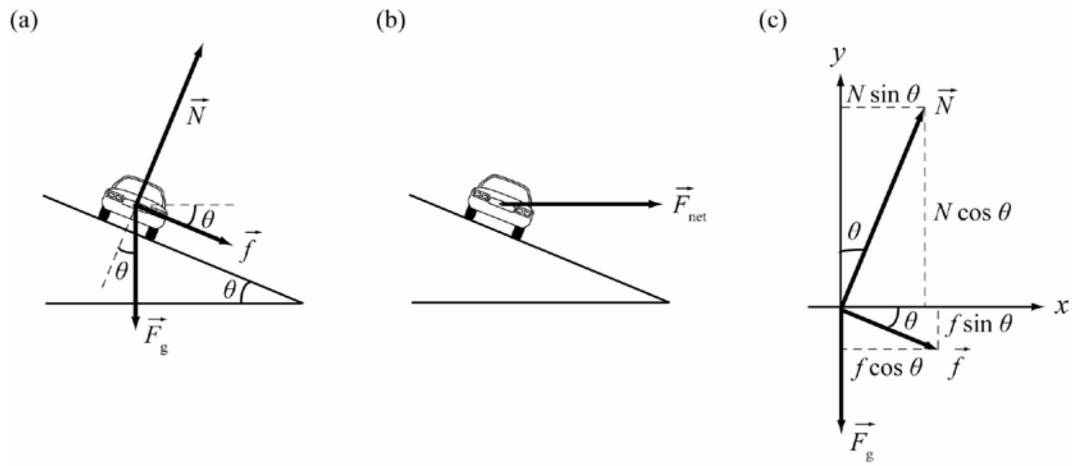
9.59. **THINK:** A speedway turn has a radius, *R*, and is banked at an angle of θ above the horizontal. This problem is a special case of Solved Problem 9.4, and the results of that solved problem will be used to obtain a solution to this problem. Determine:

(a) The optimal speed to take the turn when there is little friction present.

(b) The maximum and minimum speeds at which to take the turn if there is now a coefficient of static friction, μ_s.

(c) The value for parts (a) and (b) if $R = 400.$ m, $\theta = 45.0°$, and $\mu_s = 0.700$.

SKETCH:



RESEARCH: It was found in Solved Problem 9.4 that the maximum speed a car can go through the banked curve is given by

$$v_{max} = \sqrt{\frac{Rg(\sin\theta + \mu_s \cos\theta)}{\cos\theta - \mu_s \sin\theta}}.$$

SIMPLIFY:

(a) For the case of zero friction the case above approaches the limit of $v_{zero\ friction} = \sqrt{\dfrac{Rg\sin\theta}{\cos\theta}} = \sqrt{Rg\tan\theta}$.

(b) For the maximum speed we can use the formula already quoted above. The minimum speed that the car can travel through the curve is given by reversing the direction of the friction force. In this case the friction force points up the bank, because it needs to prevent the car from sliding down. Reversing the sign of the friction force leads to $v_{min} = \sqrt{\dfrac{Rg(\sin\theta - \mu_s \cos\theta)}{\cos\theta + \mu_s \sin\theta}}$.

CALCULATE:

(c) For the results from part (a):

$$v_{zero\ friction} = \sqrt{(400.\text{ m})(9.81\text{ m/s}^2)\tan 45.0^\circ} = 62.64184\text{ m/s}.$$

For the results from part (b), the minimum speed is:

$$v_{min} = \sqrt{\frac{(400.\text{ m})(9.81\text{ m/s}^2)(\sin 45.0^\circ - 0.700\cos 45.0^\circ)}{\cos 45.0^\circ + 0.700\sin 45.0^\circ}} = 26.31484\text{ m/s}.$$

and the maximum speed is:

$$v_{max} = \sqrt{\frac{(400.\text{ m})(9.81\text{ m/s}^2)(\sin 45.0^\circ + 0.700\cos 45.0^\circ)}{\cos 45.0^\circ - 0.700\sin 45.0^\circ}} = 149.1174\text{ m/s}.$$

ROUND:

(a) Not applicable.

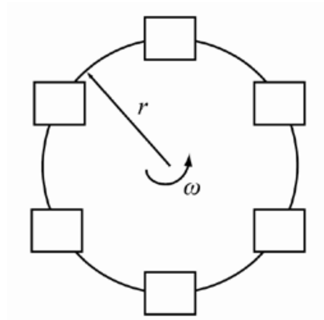
(b) Not applicable.

(c) $v_{zero\ friction} = 62.6$ m/s, $v_{min} = 26.3$ m/s and $v_{max} = 149$ m/s.

DOUBLE-CHECK: The results are reasonable considering that the friction-free speed should be within the minimum and maximum speed. The values for the given parameters are consistent with experiment.

9.60. **THINK:** A Ferris wheel has a radius of 9.00 m, and a period of revolution of T = 12.0 s. Let's start with part (a) and solve it all the way.

SKETCH:



RESEARCH: The constant speed of the riders can be determined by the equation for the speed, v = distance/time, where the distance is calculated from the circumference of the path.

SIMPLIFY: $v = \frac{d}{T} = \frac{2\pi r}{T}$

CALCULATE: $v = \frac{2\pi(9.00 \text{ m})}{12.0 \text{ s}} = 4.712 \text{ m/s}$

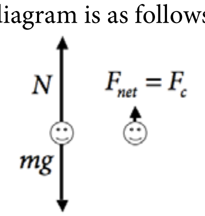
ROUND: Since the input values are given to two significant figures, the result for the linear speed is: $v = 4.7$ m/s.

DOUBLE-CHECK: For part (b) and part (c), realize that there is an essential difference between a Ferris wheel and a loop in a roller coaster: the speed of the Ferris wheel is gentle enough so that the riders do not get lifted out of their seats at the top. However, we need to check that the speed is actually sufficiently small so that this does not happen. In Solved Problem 9.1 we found that the minimum speed to experience weightlessness (i.e. zero normal force from the seat) at the top of the loop is $v_{N=0} = \sqrt{Rg}$. For the given value of R this speed works out to 9.4 m/s. Since our result is below this value, it is at least possible that a Ferris wheel could exist, which uses the values given here. Note that the centripetal acceleration from the speed use here is:

$$a_c = \frac{v^2}{R} = \frac{(4.71 \text{ m/s})^2}{9 \text{ m}} = 2.47 \text{ m/s}^2 = 0.25g.$$

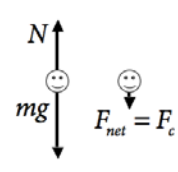
With the above information from our double-check we can solve parts (b) and (c):

(b) At the bottom of the ride the normal force has to balance gravity and in addition provide the centripetal force of 0.25 *mg*. The free-body diagram is as follows:



The normal force at the bottom of the path is thus: $N = mg + 0.25mg = 1.25mg$.

(c) At the top of the Ferris wheel gravity points in the direction of the centripetal force. The free-body diagram at the top is therefore:

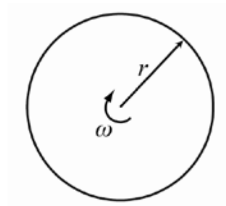


The normal force is in this case: $N = mg - 0.25mg = 0.75mg$.

Note the essential difference in parts (b) and (c): in part (b) the magnitude of the vector for the normal force is greater than that of the gravitational force, and in part (c) it is smaller.

9.61. **THINK:** The radius of the Ferris wheel is r = 9.00 m and its period is T= 12.0 s. Use these values to calculate ω. $\Delta\omega$ and $\Delta\theta$ are known when stopping at a uniform rate, which is sufficient to determine α. Also, the time it takes to stop, Δt, can be determined and with this, the tangential acceleration, a_t, can be determined.

SKETCH:



RESEARCH:

(a) $\omega = \dfrac{2\pi \text{ rad}}{T}$

(b) $\Delta\theta = \omega_i \Delta t + \dfrac{1}{2}\alpha\Delta t^2,\ \ \Delta\omega = \alpha\Delta t$

(c) $a_t = r\alpha$

SIMPLIFY:

(a) $\omega = \dfrac{2\pi}{T}$

(b) $\Delta\theta = \omega_i \Delta t + \dfrac{1}{2}\alpha\Delta t^2$ and $\Delta t = \dfrac{\Delta\omega}{\alpha} = \dfrac{\omega_f - \omega_i}{\alpha} = \dfrac{-\omega_i}{\alpha}$, since $\omega_f = 0$.

$$\Rightarrow \Delta\theta = \omega_i\left(\frac{-\omega_i}{\alpha}\right) + \frac{1}{2}\alpha\left(\frac{\omega_i^2}{\alpha^2}\right) = -\frac{\omega_i^2}{\alpha} + \frac{\omega_i^2}{2\alpha} = -\frac{\omega_i^2}{2\alpha} \Rightarrow \alpha = \frac{-\omega_i^2}{2\Delta\theta}$$

(c) $a_t = r\alpha$

CALCULATE:

(a) $\omega = \dfrac{2\pi \text{ rad}}{12.0 \text{ s}} = 0.5236 \text{ rad/s}$

(b) $\alpha = \dfrac{-(0.5236 \text{ rad/s})^2}{2(\pi/2) \text{ rad}} = -0.08727 \text{ rad/s}^2$

(c) $a_t = (-0.08727 \text{ rad/s}^2)(9.00 \text{ m}) = -0.785 \text{ m/s}^2$

ROUND: The given values have three significant figures, so the results should be rounded accordingly.

(a) $\omega = 0.524$ rad/s

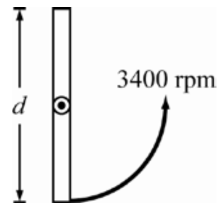
(b) $\alpha = -0.0873$ rad/s^2

(c) $a_t = -0.785$ m/s^2

DOUBLE-CHECK: These numbers are reasonable for a Ferris wheel. Note that the radius is only required for part (c). As expected, the value for the tangential acceleration is small compared to the gravitational acceleration g.

9.62. **THINK:** Determine the linear speed, given the blade's rotation speed and its diameter. To help determine the constant (negative) acceleration, it is given that it takes a time interval of 3.00 s for the blade to stop. The known values are ω = 3400. rpm, d = 53.0 cm.

SKETCH:



RESEARCH: $1 \text{ rpm} = \dfrac{2\pi \text{ rad}}{60 \text{ s}}$

(a) $v = \omega r = \dfrac{1}{2}\omega d$

(b) $\alpha = \dfrac{\Delta w}{\Delta t} = \dfrac{w_f - w_i}{\Delta t}$

SIMPLIFY: It is not necessary to simplify.

CALCULATE:

(a) $v = \left(3400.\left(\dfrac{2\pi}{60}\right)\right) \text{rad/s}\left(\dfrac{0.530 \text{ m}}{2}\right) = 94.35 \text{ m/s}$

(b) $\omega_f = 0,\ \omega_i = 3400.\left(\dfrac{2\pi}{60} \text{ rad/s}\right) = 356 \text{ rad/s and } \Delta t = 3 \text{ s, so } \alpha = \dfrac{-356 \text{ rad/s}}{3.00 \text{ s}} = -118.7 \text{ rad/s}^2.$

ROUND: The results should be rounded to three significant figures.

(a) $v = 94.3 \text{ m/s}$

(b) $\alpha = -119 \text{ rad/s}^2$

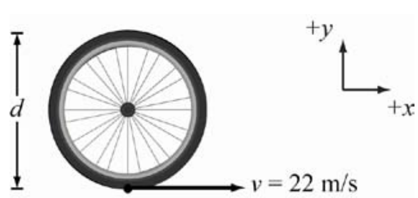
DOUBLE-CHECK: For lawn mower blades, these are reasonable values.

9.63. THINK:

(a) If the distance traveled can be determined, then the number of revolutions the tires made can be determined, since the diameter of the tires is known.

(b) The linear speed of the tires and the diameter of the tires are known, so the angular speed can be determined. The known variables are $v_i = 0,\ v_f = 22.0 \text{ m/s},\ \Delta t = 9.00 \text{ s},\ d = 58.0 \text{ cm}.$ Use $1 \dfrac{\text{rev}}{\text{s}} = 2\pi \dfrac{\text{rad}}{\text{s}} \Rightarrow 1 \dfrac{\text{rad}}{\text{s}} = \dfrac{1}{2\pi}\dfrac{\text{rev}}{\text{s}}.$

SKETCH:



RESEARCH: The circumference of a circle is given by $C = 2\pi r = \pi d.$ The displacement at constant acceleration is $\Delta x = v_i \Delta t + \dfrac{1}{2}a\Delta t^2$, where $v = \omega r.$

SIMPLIFY:

(a) $v_i = 0 \Rightarrow \Delta x = \dfrac{1}{2}a\Delta t^2,\ a = \dfrac{\Delta v}{\Delta t} \Rightarrow \Delta x = \dfrac{1}{2}\dfrac{\Delta v}{\Delta t}\Delta t^2 = \dfrac{1}{2}\Delta v \Delta t$

Let N = number of revolutions and the displacement is given by $\Delta x = \left(\dfrac{\text{displacement}}{\text{revolution}}\right)N.$ The displacement per revolution is simply the circumference, C, so

$$\Delta x = CN \Rightarrow N = \frac{\Delta x}{C} = \frac{1}{\pi d}\left(\frac{1}{2}\Delta v \Delta t\right) = \frac{\Delta v \Delta t}{2\pi d}.$$

(b) $\omega = \frac{v}{r} = \frac{v}{d/2} = \frac{2v}{d}$

CALCULATE:

(a) $N = \frac{(22.0 \text{ m/s})(9.00 \text{ s})}{2\pi(0.58 \text{ m})} = 54.33 \text{ revolutions}$

(b) $\omega = \frac{2(22.0 \text{ m/s})}{0.58 \text{ m}} = 75.86 \text{ rad/s} = \frac{75.86}{2\pi} \text{ rev/s} = 12.07 \text{ rev/s}$

ROUND: The results should be rounded to three significant figures.

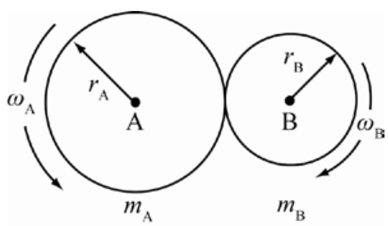
(a) $N = 54.3$ revolutions

(b) $\omega = 12.1$ rev/s

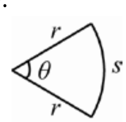
DOUBLE-CHECK: For the given values, these results are reasonable.

9.64. **THINK:** First, determine the number of revolutions gear A undergoes while slowing down. From this, determine the total arc-length of gear A. Gear B must have the same arc-length, from which the number of rotations undergone by gear B can be determined. The following values are given: $\omega_{\text{i,A}} = 120.$ rpm, $\omega_{\text{f,A}} = 60.0$ rpm, $\Delta t = 3.00$ s, $r_{\text{A}} = 55.0$ cm, $r_{\text{B}} = 30.0$ cm, $m_{\text{A}} = 1.00$ kg, $m_{\text{B}} = 0.500$ kg and $\Delta\omega_{\text{A}} = \omega_{\text{f,A}} - \omega_{\text{i,A}} = -60.0$ rpm. Use the conversion factor $1 \text{ rpm} = \frac{2\pi \text{ rad}}{60 \text{ s}}$.

SKETCH:



RESEARCH: The arc-length is given by $s = r\theta$.



The angular displacement is $\Delta\theta = \omega_{\text{i}}\Delta t + \frac{1}{2}\alpha t^2$ The angular acceleration is $\alpha = \frac{\Delta\omega}{\Delta t}$.

SIMPLIFY: The arc-length of gear A is given by $s_{\text{A}} = r_{\text{A}}\Delta\theta_{\text{A}} = r_{\text{A}}\left(\omega_{\text{i,A}}\Delta t + \frac{1}{2}\alpha_{\text{A}} t^2\right)$.

$$\alpha_{\text{A}} = \frac{\Delta\omega_{\text{A}}}{\Delta t} \Rightarrow s_{\text{A}} = r_{\text{A}}\left(\omega_{\text{i,A}}\Delta t + \frac{\Delta\omega_{\text{A}}\Delta t}{2}\right)$$

Gear B has the same arc-length, $s_{\text{A}} = s_{\text{B}}$. The angular displacement of gear B is $s_{\text{B}} = r_{\text{B}}\Delta\theta_{\text{B}}$, so $\Delta\theta_{\text{B}} = \frac{s_{\text{B}}}{r_{\text{B}}} = \frac{s_{\text{A}}}{r_{\text{B}}}$ (since $s_{\text{A}} = s_{\text{B}}$).

The number of rotations, *n*, of gear *B* is $n = \frac{\Delta\theta_{\text{B}}}{2\pi}$, so $n = \frac{s_{\text{A}}}{2\pi r_{\text{B}}} = \frac{1}{2\pi}\frac{r_{\text{A}}}{r_{\text{B}}}\left(\omega_{\text{i,A}}\Delta t + \frac{1}{2}\Delta\omega_{\text{A}}\Delta t\right)$

$$n = \frac{1}{2\pi}\frac{r_{\text{A}}}{r_{\text{B}}}\Delta t\left(\omega_{\text{i,A}} + \frac{1}{2}\Delta\omega_{\text{A}}\right)$$

CALCULATE: $n = \frac{1}{2\pi}\left(\frac{0.550 \text{ m}}{0.300 \text{ m}}\right)\left[\left(120.\left(\frac{2\pi}{60}\right)\right)\text{ s}^{-1} + \frac{1}{2}\left(-60.0\left(\frac{2\pi}{60}\right)\right)\text{ s}^{-1}\right](3.00 \text{ s})$

$$= \frac{1}{2\pi}\left(\frac{0.550 \text{ m}}{0.300 \text{ m}}\right)\left[4\pi - \pi\right](3.00) = \frac{9.00}{2}\left(\frac{0.550}{0.300}\right) = 8.250$$

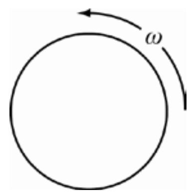
ROUND: Rounding the result to three significant figures gives n = 8.25 rotations.

DOUBLE-CHECK: There is an alternate solution. The average angular speed of A during the slowing down is $(120+60)/2 \text{ rpm} = 90 \text{ rpm}$. In 3 s, A undergoes $90(3/60) = 4.5$ rotations. Since B has a smaller radius, it undergoes a proportionally greater number of rotations. The proportionality is the ratio of the radii:

$$n = 4.5\left(\frac{0.55 \text{ m}}{0.30 \text{ m}}\right) = 8.25, \text{ as before.}$$

9.65. **THINK:** The angular acceleration is constant, so the uniform angular acceleration equations can be used directly. The known quantities are $\omega_\text{i} = 10.0$ rev/s, $\omega_\text{f} = 0$ and $\Delta t = 10.0$ min.

SKETCH:



RESEARCH: $\alpha = \frac{\Delta\omega}{\Delta t}, \quad \Delta\theta = \omega_\text{i}\Delta t + \frac{1}{2}\alpha t^2$

SIMPLIFY: Simplification is not necessary.

CALCULATE: $\alpha = \frac{-10.0 \text{ rev/s}(2\pi \text{ rad/rev})}{10.0 \text{ min}(60 \text{ s/min})} = -\frac{20\pi}{600} \text{ rad/s}^2 = -\frac{\pi}{30} \text{ rad/s}^2 = -0.1047 \text{ rad/s}^2$

$$\Delta\theta = (10.0 \text{ rev/s}(2\pi \text{ rad/rev}))(10.0 \text{ min}(60 \text{ s/min})) - \frac{1}{2}\left(\frac{\pi}{30}\text{rad/s}^2\right)(10.0 \text{ min}(60 \text{ s/min}))^2$$

$$= (20\pi \text{ rad/s})(600 \text{ s}) - \frac{\pi}{60} \text{ rad/s}^2 (600 \text{ s})^2 = 3.77 \cdot 10^4 \text{ rad} - 1.885 \cdot 10^4 \text{ rad} = 1.885 \cdot 10^4 \text{ rad}$$

ROUND: Rounding each result to three significant figures gives $\alpha = -0.105 \text{ rad/s}^2$ and $\Delta\theta = 1.88 \cdot 10^4$ rad.

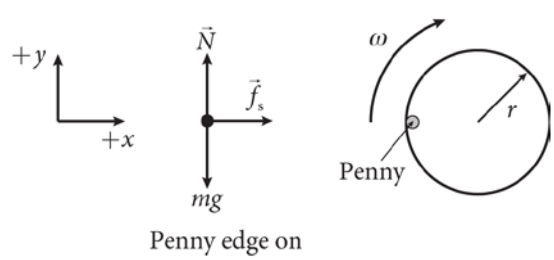
DOUBLE-CHECK: The average angular speed is

$$\frac{0 + 10.0 \text{ rev/s}}{2} = 5 \text{ rev/s} = 5(2\pi \text{ rad/s}).$$

The displacement during this time interval for the average speed is $\Delta\theta = \omega_\text{avg}\Delta t = (10\pi \text{ rad/s})(600 \text{ s}) = 1.885 \cdot 10^4 \text{ rad}$, as above. The results are consistent and reasonable.

9.66. **THINK:** The force of static friction between the penny and the phonograph disk provides the centripetal force to keep the penny moving in a circle.

SKETCH:



RESEARCH: The maximum force of static friction between the penny and the photograph disk is $f_s = \mu_s mg$. The centripetal force required to keep the penny moving in a circle is $F_c = mr\omega^2$. Frequency is related to angular frequency by $\omega = 2\pi f$.

SIMPLIFY: $mr\omega^2 = \mu_s mg \Rightarrow \mu_s = \dfrac{\omega^2 r}{g} \Rightarrow \mu_s = \dfrac{(2\pi f)^2 r}{g}$.

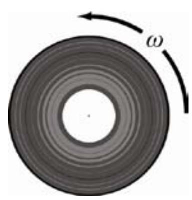
CALCULATE: $f = 33 \dfrac{\text{rev}}{\text{min}} \dfrac{\text{min}}{60 \text{ s}} = 0.5500 \text{ s}^{-1}$, $r = \dfrac{12 \text{ in}}{2} \dfrac{2.54 \text{ cm}}{\text{in}} \dfrac{1 \text{ m}}{100 \text{ cm}} = 0.1524 \text{ m}$,

$$\mu_s = \frac{\left[2\pi\left(0.5500 \text{ s}^{-1}\right)\right]^2 (0.1524 \text{ m})}{9.81 \text{ m/s}^2} = 0.1855.$$

ROUND: Rounding the result to two significant figures gives $\mu_s = 0.19$.

DOUBLE-CHECK: The results are reasonable for the given values.

9.67. **THINK:** The acceleration is uniform during the given time interval. The average angular speed during this time interval can be determined and from this, the angular displacement can be determined. **SKETCH:**



RESEARCH: $\omega_{avg} = \dfrac{\omega_f + \omega_i}{2}$, $\Delta\theta = \omega_{avg}\Delta t$

SIMPLIFY: Simplification is not necessary.

CALCULATE: $\omega_i = 33.33 \text{ rpm} = 33.33 \text{ rpm}\left(\dfrac{2\pi}{60 \text{ s}}\right) = 3.491 \text{ rad/s}$, $\omega_f = 0$

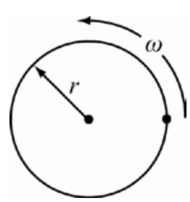
$$\Delta\theta = \left(\frac{3.491}{2} \text{ rad s}^{-1}\right)(15.0 \text{ s}) = 26.18 \text{ rad}$$

$$\text{number of rotations} = \frac{\Delta\theta}{2\pi} = 4.167$$

ROUND: Rounding the result to three significant figures gives the number of rotations to be 4.17 rotations.

DOUBLE-CHECK: These are reasonable results for a turntable.

9.68. **THINK:** Given the radius (2.0 cm) and rotation speed (250 rpm), the linear and angular speeds and acceleration can be determined.
SKETCH:



RESEARCH: $\omega = \text{rpm}\left(\dfrac{2\pi}{60} \text{ rad/s}\right)$, $v = \omega r$, $a = \omega^2 r$, $\alpha = 0$

SIMPLIFY: Simplification is not necessary.

CALCULATE: $\omega = 250.\left(\dfrac{2\pi}{60} \text{ rad/s}\right) = 26.18 \text{ rad/s}$, $v = \omega r = (26.18)(0.0200) \text{ m/s} = 0.5236 \text{ m/s}$

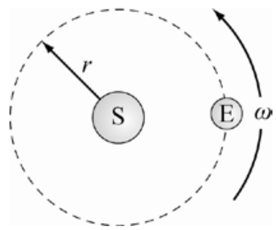
$a=\omega^2 r=(26.18)^2(0.020)\text{ m/s}^2=13.71\text{ m/s}^2,\ \alpha=0$

ROUND: Rounding the results to three significant figures gives $\omega=26.2$ rad/s, $v=0.524$ m/s, $a=14\text{ m/s}^2$, and $\alpha=0$.

DOUBLE-CHECK: The rotation speed is constant, so $\alpha=0$. The other values are likewise reasonable.

9.69. **THINK:** The angular acceleration of the Earth is zero. The linear acceleration is simply the centripetal acceleration. $r=1$ AU or $r=1.50\cdot 10^{11}$ m and $\omega=2\pi$ rad / year.

SKETCH:



RESEARCH: $a=\omega^2 r,\ 1\text{ yr}=3.16\cdot 10^7\text{ s}$

SIMPLIFY: Simplification is not necessary.

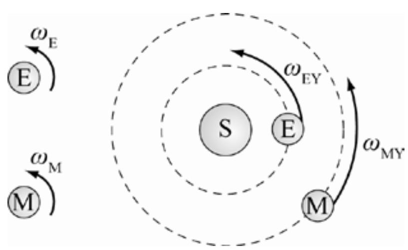
CALCULATE: $a=\left(\dfrac{2\pi}{3.16\cdot 10^7\text{ s}}\right)^2\left(1.50\cdot 10^{11}\text{ m}\right)=5.93\cdot 10^{-3}\text{ m/s}^2$

ROUND: Keeping three significant figures, $a=5.93\cdot 10^{-3}\text{ m/s}^2$.

DOUBLE-CHECK: The linear acceleration is rather small because the distance to the Sun is so great.

9.70. **THINK:** From the given data, the ratio of the angular accelerations of Mars and Earth can be determined.

SKETCH:



RESEARCH: $\omega_{\text{day}}=\dfrac{2\pi\text{ rad}}{1\text{ day}},\ \omega_{\text{yr}}=\dfrac{2\pi\text{ rad}}{1\text{ yr}}$

SIMPLIFY: $\omega_{\text{M}}=\dfrac{2\pi\text{ rad}}{24.6\text{ hr}},\ \omega_{\text{E}}=\dfrac{2\pi\text{ rad}}{24\text{ hr}},\ \omega_{\text{My}}=\dfrac{2\pi\text{ rad}}{687\text{ Earth-days}},\ \omega_{\text{Ey}}=\dfrac{2\pi\text{ rad}}{365\text{ Earth-days}}$

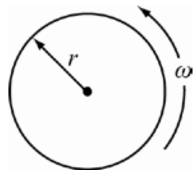
CALCULATE: $\dfrac{\omega_{\text{M}}}{\omega_{\text{E}}}=\dfrac{24.0\text{ hr}}{24.6}=0.9756,\ \dfrac{\omega_{\text{My}}}{\omega_{\text{Ey}}}=\dfrac{365}{687}=0.5319$

ROUND: Rounding the results to three significant figures gives $\dfrac{\omega_{\text{M}}}{\omega_{\text{E}}}=0.976$ and $\dfrac{\omega_{\text{My}}}{\omega_{\text{Ey}}}=0.532$.

DOUBLE-CHECK: The angular speed of Mars' orbit is 0.532 that of Earth. The latter is reasonable given that Mars is further from the Sun than Earth, as we will learn in Chapter 12.

9.71. **THINK:** Parts (a) and (b) can be solved using the constant angular acceleration equations. For part (c), calculate the angular displacement and, from this, compute the total arc-length, which is equal to the distance traveled.

SKETCH:



RESEARCH:

(a) $v = \omega r$

(b) $\alpha = \dfrac{\Delta\omega}{\Delta t}, \quad \Delta\theta = \omega_i \Delta t + \dfrac{1}{2}\alpha \Delta t^2$

(c) $s = r\Delta\theta, \quad \Delta\theta = 2\pi(\text{total revs.})$

SIMPLIFY:

(a) $\omega_{\text{i}} = \dfrac{v_{\text{i}}}{r}$

(b) $\Delta\omega = \omega_{\text{f}} - \omega_{\text{i}} = 0 - \dfrac{v_{\text{i}}}{r} = -\omega_{\text{i}}, \quad \Delta t = \dfrac{\Delta\omega}{\alpha} = -\dfrac{\omega_{\text{i}}}{\alpha}$

$$\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha\Delta t^2 = \omega_i\left(\frac{-\omega_i}{\alpha}\right) + \frac{\alpha}{2}\left(\frac{-\omega_i}{\alpha}\right)^2 = \frac{-\omega_i^2}{\alpha} + \frac{\omega_i^2}{2\alpha} = -\frac{\omega_i^2}{2\alpha} \Rightarrow \alpha = \frac{-\omega_i^2}{2\Delta\theta}$$

(c) $s = r\Delta\theta$

CALCULATE:

(a) $\omega_i = \dfrac{35.8 \text{ m/s}}{0.550 \text{ m}} = 65.09 \text{ s}^{-1}$

(b) $\alpha = \dfrac{-\left(65.09 \text{ s}^{-1}\right)^2}{2\left(2\pi(40.2)\right)} = -8.387 \text{ s}^{-2}$

(c) $s = (0.550 \text{ m})\left(2\pi(40.2)\right) = 138.92 \text{ m}$

ROUND: Rounding the results to three significant figures:

(a) $\omega_i = 65.1 \text{ s}^{-1}$

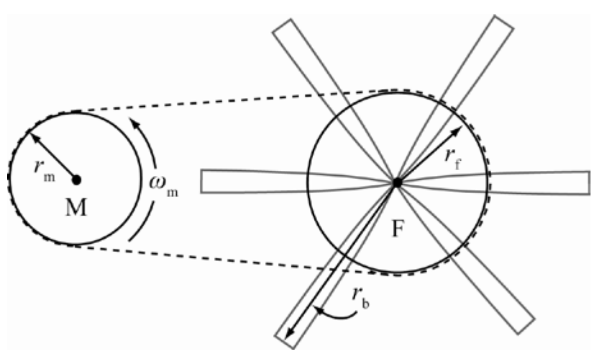
(b) $\alpha = -8.39 \text{ s}^{-2}$

(c) $s = 139 \text{ m}$

DOUBLE-CHECK: For the parameters given, these are reasonable results.

9.72. **THINK:** Everything in the problem rotates at constant angular speed. The two wheels have radii of $r_{\text{m}} = 2.00$ cm and $r_{\text{f}} = 3.00$ cm and rotate at the same linear speed.

SKETCH:



RESEARCH: $v = \omega r$

SIMPLIFY: $v_{\text{m}} = \omega_{\text{m}} r_{\text{m}}, \quad v_{\text{f}} = \omega_{\text{f}} r_{\text{f}}, \quad v_{\text{b}} = \omega_{\text{b}} r_{\text{b}}$

The wheels are attached by a belt, so $v_m = v_f \Rightarrow \omega_m r_m = \omega_f r_f \Rightarrow \omega_f = \frac{\omega_m r_m}{r_f}$. The blades are attached to wheel *F*, so $\omega_b = \omega_f \Rightarrow v_b = \omega_f r_b = \frac{\omega_m r_m}{r_f} r_b$.

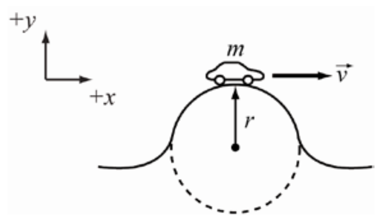
CALCULATE: $v_b = \frac{1}{0.03 \text{ m}}\left(1200\left(\frac{2\pi}{60 \text{ s}}\right)\right)(0.02 \text{ m})(0.15 \text{ m}) = 12.57 \text{ m/s}$

ROUND: Rounding the result to three significant figures gives $v_b = 12.6$ m/s.

DOUBLE-CHECK: From $v_b = \frac{\omega_m r_m r_b}{r_f}$, it can be seen that v_b grows with ω_m, r_m, r_b, and v_b decreases as r_f grows. All these relations are reasonable.

9.73. **THINK:** The net force due to gravity (down) and normal force from the hill (upward) equals the centripetal force determined by the car's speed and the path's radius of curvature. The force the car exerts on the hill is equal and opposite to the force of the hill on the car.

SKETCH:



RESEARCH: $F_g = mg$ acts downward, and let *N* be the upward force of the hill on the car. The net force, F_{net}, which is the centripetal force $F_c = m\frac{v^2}{r}$, acts downward.

SIMPLIFY: Taking upward force as positive and downward force as negative,

$$-F_{net} = N - F_g = N - mg = -F_c = -m\frac{v^2}{r} \Rightarrow N = m\left(g - \frac{v^2}{r}\right)$$

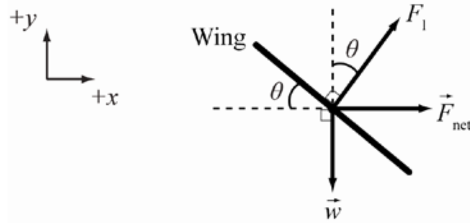
CALCULATE: $N = (1000. \text{ kg})\left(9.81 \text{ m/s}^2 - \frac{(60.0 \text{ m/s})^2}{370. \text{ m}} - \right) = 80.270 \text{ N}$

ROUND: To three significant figures, $N = 80.3$ N.

DOUBLE-CHECK: The equation confirms what we know from observation, namely that if v is large enough, then the normal force will go to zero and the car will lose contact with the ground.

9.74. **THINK:** A free body diagram will show all the forces acting on the plane. The net force is horizontal, directed towards the center of the radius of curvature. The speed is v = 4800 km/h and the turning radius is r = 290 km. The banking angle, θ, must be determined.

SKETCH:



RESEARCH: $\vec{F}_{\text{net}} = m\vec{a}, \ \vec{F}_{\text{net}} = \vec{w} + \vec{F}_1$

SIMPLIFY: $F_{\text{net}} = ma = m\dfrac{v^2}{r}, \ w = mg$

$\sum F_y = mg - F_1\cos\theta = 0 \ \Rightarrow \ mg = F_1\cos\theta \ \Rightarrow \ \dfrac{F_1}{m} = \dfrac{g}{\cos\theta}, \qquad \sum F_x = F_1\sin\theta = m\dfrac{v^2}{r} \ \Rightarrow \ \dfrac{F_1}{m} = \dfrac{v^2}{r\sin\theta}$

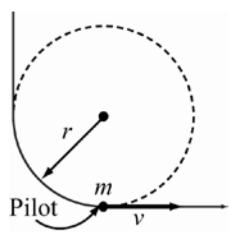
Equating the two above equations gives $\dfrac{g}{\cos\theta} = \dfrac{v^2}{r\sin\theta} \ \Rightarrow \ \tan\theta = \dfrac{v^2}{gr} \ \Rightarrow \ \theta = \tan^{-1}\left(\dfrac{v^2}{gr}\right).$

CALCULATE: $v = 4800 \text{ km/h} = 4800\left(\dfrac{10^3 \text{ m}}{3600 \text{ s}}\right) = 1.333\cdot 10^3 \text{ m/s}, \ \ r = 290 \text{ km} = 2.9\cdot 10^5 \text{ m}$

$$\theta = \tan^{-1}\frac{\left(1.333\cdot 10^3 \text{ m/s}\right)^2}{\left(9.81 \text{ m/s}^2\right)\left(2.9\cdot 10^5 \text{ m}\right)} = 32.00°$$

ROUND: The speed is given to three significant figures, so the result should be $\theta = 32.0°$.
DOUBLE-CHECK: A banking angle of $32°$ is reasonable for the SR-71.

9.75. **THINK:** From the linear speed and the radius, the centripetal acceleration can be determined. With the pilots' mass, the centripetal force can also be determined. The pilot's apparent weight is the combined effect of gravitational and centripetal accelerations.
SKETCH:



RESEARCH:

(a) $a_c = \dfrac{v^2}{r}, \ F_c = \dfrac{mv^2}{r}$

(b) $F_c = \dfrac{mv^2}{r}, \ F_g = mg, \ w = \dfrac{mv^2}{r} + mg$

SIMPLIFY: Simplification is not necessary.
CALCULATE:

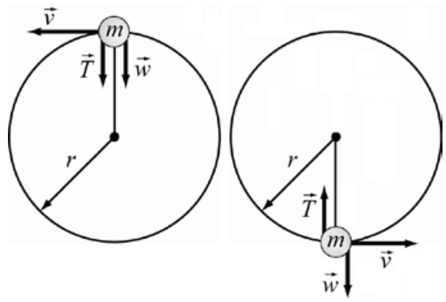
(a) $a_c = \dfrac{\left(500. \text{ m/s}\right)^2}{4000. \text{ m}} = 62.50 \text{ m/s}^2, \quad F_c = ma_c = \left(80.0 \text{ kg}\right)\left(62.50 \text{ m/s}^2\right) = 5.00\cdot 10^3 \text{ N}$

(b) $w = 5.00\cdot 10^3 \text{ N} + \left(80.0 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right) = 5784.8 \text{ N}$

ROUND: Round the results to three significant figures:
(a) $a_c = 62.5 \text{ m/s}^2$ and $F_c = 5.00\cdot 10^3 \text{ N}$
(b) $w = 5780 \text{ N}$
DOUBLE-CHECK: These are all reasonable values.

9.76. **THINK:** The net force on the ball is the centripetal force. Gravity and tension sum to produce this force. $m = 1.00$ kg, $r = 1.00$ m and $v = 10.0$ m/s. At the top of the circle, gravity and tension both point down. At the bottom of the circle, gravity still points down, but the tension points up.

SKETCH:



RESEARCH: $F_{\text{net}} = \dfrac{mv^2}{r}$

(a) $F_{\text{net}} = T + w$

(b) $F_{\text{net}} = T - w$

SIMPLIFY:

(a) $T = F_{\text{net}} - w = \dfrac{mv^2}{r} - mg$

(b) $T = F_{\text{net}} + w = \dfrac{mv^2}{r} + mg$

CALCULATE: $\dfrac{mv^2}{r} = \dfrac{(1.00\text{ kg})(10.0\text{ m/s})^2}{1.00\text{ m}} = 100.\text{ N},\;\; mg = (1.00\text{ kg})(9.81\text{ m/s}^2) = 9.81\text{ N}$

(a) $T = 100.\text{ N} - 9.81\text{ N} = 90.19\text{ N}$

(b) $T = 100.\text{ N} + 9.81\text{ N} = 109.8\text{ N}$

(c) The tension in the string is greatest at the bottom of the circle. As the ball moves away from the bottom, the tension decreases to its minimum value at the top of the circle. It then increases until the ball again reaches the bottom.

ROUND: Round the results to three significant figures.

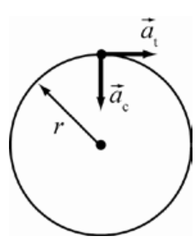
(a) $T = 90.2$ N

(b) $T = 110.$ N

DOUBLE-CHECK: If you are swinging the ball with a high speed like in this problem, the weight becomes almost negligible, and thus we should expect that the tensions at the bottom and top become almost identical. The tension is still highest at the bottom, as would be reasonably expected.

9.77. **THINK:** The car start slipping at the point where the magnitude of the total acceleration exceeds the maximum acceleration that can be provided by the friction force. The total acceleration of the car is composed of contributions from the centripetal and the tangential acceleration, which have to be added as vectors. Given here are $R = 36.0$ m, $a_{\text{t}} = 3.30\text{ m/s}^2$, $v_{\text{i}} = 0$ and $\mu = 0.950$.

SKETCH:



RESEARCH: The magnitude of the total acceleration is given by the tangential and radial acceleration, $a = \sqrt{a_{\text{t}}^2 + a_{\text{c}}^2}$. The centripetal acceleration is $a_{\text{c}} = v^2 / R$. Since the car accelerates at constant linear acceleration starting from rest, the speed as a function of time is $v = a_{\text{t}} t$. The maximum force of friction is given by $f = \mu mg$. So the maximum acceleration due to friction is $a_\text{f} = \mu g$. The distance traveled by then is $d = \frac{1}{2}a_\text{t}t^2$.

SIMPLIFY: Slippage occurs when $a_\text{f} = a$; so $a_\text{f} = \mu g = a = \sqrt{a_\text{t}^2 + a_\text{c}^2}$

$$\Rightarrow \mu^2 g^2 = a_\text{t}^2 + a_\text{c}^2 = a_\text{t}^2 + (v^2/R)^2 = a_\text{t}^2 + (a_\text{t}^2 t^2/R)^2 \quad \Rightarrow t^2 = R\sqrt{\mu^2 g^2 - a_\text{t}^2}\,/\,a_\text{t}^2$$

$$\Rightarrow d = \frac{1}{2}a_\text{t}t^2 = \frac{\frac{1}{2}R\sqrt{\mu^2 g^2 - a_\text{t}^2}}{a_\text{t}}$$

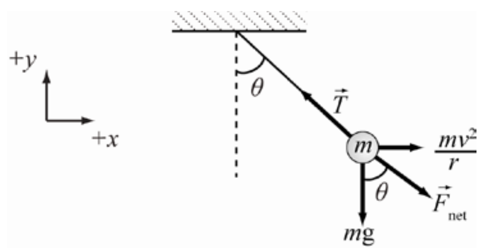
CALCULATE: $d = \dfrac{\frac{1}{2}(36.0 \text{ m})\sqrt{0.950^2(9.81 \text{ m/s}^2)^2 - (3.30 \text{ m/s}^2)^2}}{(3.30 \text{ m/s}^2)} = 47.5401 \text{ m}$

ROUND: Rounding to three significant figures gives $d = 47.5$ m.

DOUBLE-CHECK: d is proportional to R. This makes sense because a larger R implies less curvature and thus less centripetal force. d is also inversely proportional to a_t, which also makes sense since a smaller tangential acceleration implies a greater distance traveled before the maximum speed is attained.

9.78. **THINK:** The pendulum experiences a vertical force due to gravity and a horizontal centripetal force. These forces are balanced by the tension in the pendulum string. $r = 6.0$ m and $\omega = 0.020$ rev/s.

SKETCH:



RESEARCH: $F_\text{net}\sin\theta = \dfrac{mv^2}{r}$, $F_\text{net}\cos\theta = mg$, $v = \omega r$, $1 \text{ rev/s} = 2\pi$ rad/s

SIMPLIFY: $\dfrac{F_\text{net}}{m} = \dfrac{v^2}{r\sin\theta}$, $\dfrac{F_\text{net}}{m} = \dfrac{g}{\cos\theta}$

Equating the equations, $\dfrac{v^2}{r\sin\theta} = \dfrac{g}{\cos\theta} \Rightarrow \tan\theta = \dfrac{v^2}{rg} = \dfrac{\omega^2 r^2}{rg} = g\dfrac{\omega^2 r}{g} \Rightarrow \theta = \tan^{-1}\left(\dfrac{\omega^2 r}{g}\right)$.

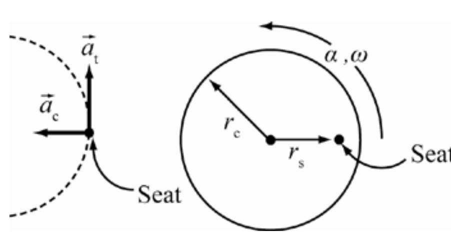
CALCULATE: $\theta = \tan^{-1}\left(\dfrac{\left(0.0200(2\pi)\text{s}^{-1}\right)^2(6.00 \text{ m})}{9.81 \text{ m/s}^2}\right) = 0.5533°$

ROUND: Rounding to three significant figures, $\theta = 0.553°$.

DOUBLE-CHECK: Such a small deviation is reasonable, given that the rotation is so slow.

9.79. **THINK:** Use the relationship between angular and centripetal acceleration. The given values are $r_\text{s} = 2.75$ m, $r_\text{c} = 6.00$ m, $\omega_\text{i} = 0$, $\omega_\text{f} = 0.600$ rev/s and $\Delta t = 8.00$ s.

SKETCH:



RESEARCH:

(a) $\alpha = \dfrac{\Delta\omega}{\Delta t}$

(b) $a_c = \dfrac{v_s^{\ 2}}{r_s}$, $v_s = \omega_s r_s$

(c) $\vec{a} = \vec{a}_c + \vec{a}_t$, $a_t = \alpha r_s$

SIMPLIFY:

(a) Simplification is not necessary.

(b) $a_c = \omega_s^{\ 2} r_s$

(c) $a = \sqrt{a_c^{\ 2} + a_t^{\ 2}}$, $\tan\theta = \left(\dfrac{a_t}{a_c}\right)$

CALCULATE:

(a) $\alpha = \dfrac{0.600(2\pi)\text{ rad/s}}{8.00\text{ s}} = 0.4712\text{ rad/s}^2$

(b) At 8.00 s, $\omega_s = 0.600$ rev/s, so $a_c = \left(0.600(2\pi)\text{ s}^{-1}\right)^2(2.75\text{ m}) = 39.08\text{ m/s}^2$ and $\alpha = 0.4712\text{ rad/s}^2$.

(c) $a = \sqrt{\left(39.08\text{ m/s}^2\right)^2 + \left(0.4712\text{ s}^{-2}\right)^2(2.75\text{ m})^2} = 39.10\text{ m/s}^2$ $\theta = \tan^{-1}\dfrac{\left(0.4712\text{ s}^{-2}\right)(2.75\text{ m})}{39.08\text{ m/s}^2} = 1.899°$

If the centripetal acceleration is along the positive x axis, then the direction of the total acceleration is $1.90°$ along the horizontal (rounded to three significant figures).

ROUND: Values are given to three significant figures, so the results should be rounded accordingly.

(a) $\alpha = 0.471\text{ rad/s}^2$

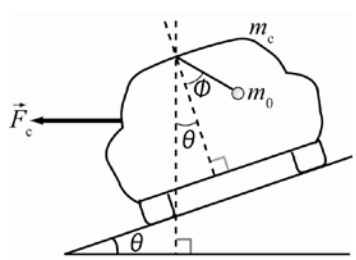
(b) $a_c = 39.1\text{ m/s}^2$ and $\alpha = 0.471\text{ rad/s}^2$.

(c) $a = 39.1\text{ m/s}^2$ at $\theta = 1.90°$.

DOUBLE-CHECK: The total acceleration is quite close to the centripetal acceleration, since the tangential acceleration and the angular acceleration are both quite small.

9.80. **THINK:** The forces acting on the ornament are the tension on the string and the force of gravity. The net force is the centripetal force acting towards the center of the track. The centripetal force is close to the car's friction with the ground. $m_c g = 10.0$ kN, $\theta = 20.0°$ and $\phi = 30.0°$. F_f is the frictional force acting on the car.

SKETCH:



RESEARCH: $T\cos(\theta+\phi) = m_0 g$, $T\sin(\theta+\phi) = m_0\dfrac{v^2}{r}$, $F_c = F_f = m_c\dfrac{v^2}{r}$

SIMPLIFY: $\dfrac{T}{m_0} = \dfrac{g}{\cos(\theta+\phi)}$

$\dfrac{T}{m_0} = \dfrac{v^2}{r\sin(\theta+\phi)}$

Equating the equations above gives $\frac{g}{\cos(\theta+\phi)} = \frac{v^2}{r\sin(\theta+\phi)} \Rightarrow \tan(\theta+\phi) = \frac{v^2}{rg} \Rightarrow \frac{v^2}{r} = g\tan(\theta+\phi).$

The force of friction then becomes $F_\text{f} = m_\text{c}\frac{v^2}{r} = m_\text{c} g\tan(\theta+\phi).$

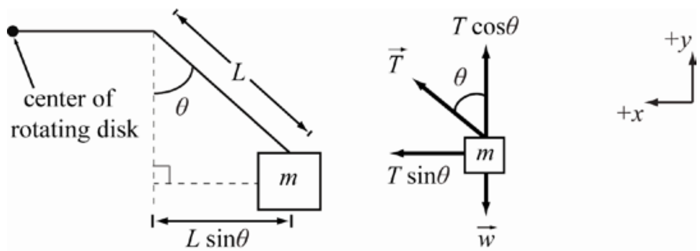
CALCULATE: $F_\text{f} = (10.0\cdot 10^3\text{ N})\tan(20.0° + 30.0°) = 1.192\cdot 10^4\text{ N}$

ROUND: Rounding to three significant figures gives $F_\text{f} = 1.19\cdot 10^4$ N.

DOUBLE-CHECK: This is a reasonable value for a car of this weight.

9.81. **THINK:** Both gravity and tension act on the passenger. The net force is the centripetal force acting towards the center. The given values are as follows: $\theta = 30.0°$, m = 65.0 kg, L = 3.20 m and $R_0 = 3.00$ m.

SKETCH:



RESEARCH: $T\cos\theta = w,\ w = mg,\ T\sin\theta = \frac{mv^2}{r},\quad r = R_0 + L\sin\theta$

SIMPLIFY: $v^2 = \frac{rT\sin\theta}{m},\quad T = \frac{w}{\cos\theta} = \frac{mg}{\cos\theta}$

(a) $v^2 = \frac{r\sin\theta}{m}\left(\frac{mg}{\cos\theta}\right) = rg\tan\theta \Rightarrow v = \sqrt{rg\tan\theta}$

(b) $T = \frac{mg}{\cos\theta}$ or $T = \frac{mv^2}{r\sin\theta}$.

CALCULATE:

(a) $v = \sqrt{(3.00\text{ m} + 3.20\sin 30.0°\text{ m})(9.81\text{ m/s}^2)(\tan 30.0°)} = 5.104\text{ m/s}$

(b) $T = \frac{(65.0\text{ kg})(9.81\text{ m/s}^2)}{\cos 30.0°} = 736.3\text{ N}$

ROUND: All values are given to three significant figures, so the results should be rounded accordingly.

(a) v = 5.10 m/s

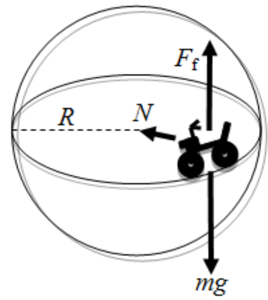
(b) T = 736 N

DOUBLE-CHECK: Note that the speed increases if the main disk, R_0, increases, or the length of the cable, L, increases, as it should.

Multi-Version Exercises

9.82. **THINK:** The only values given in this problem are the radius of the sphere and the coefficient of static friction between the motorcycle and the sphere. The motorcycle will stay on the surface as long as the vertical force exerted by the force of friction is at least as much as the weight of the motorcycle. The friction force is proportional to the normal force exerted by the wall of the dome, which is given by the centripetal force. Combine these to solve for the minimum velocity.

SKETCH:



RESEARCH: The centripetal force required to keep the motorcycle moving in a circle is $F_c = \frac{mv^2}{R}$. The friction force is $F_f = \mu_s N$, and it must support the weight of the motorcycle, s o $F_f \geq mg$.

SIMPLIFY: Since the normal force equals the centripetal force in this case, substitute F_c for N in the equation $F_f = \mu_s N$ to get $F_f = \mu_s F_c = \mu_s \frac{mv^2}{R}$. Combine this with the fact that the frictional force must be enough to support the weight of the motorcycle, so $mg \leq F_f = \mu_s \frac{mv^2}{R}$. Finally, solve the inequality for the velocity (keep in mind that the letters represent positive values):

$$\mu_s \frac{mv^2}{R} \geq mg \Rightarrow$$
$$\frac{R}{\mu_s m} \cdot \mu_s \frac{mv^2}{R} \geq \frac{R}{\mu_s m} \cdot mg \Rightarrow$$
$$v^2 \geq \frac{Rg}{\mu_s}$$
$$v \geq \sqrt{\frac{Rg}{\mu_s}}$$

CALCULATE: The radius of the sphere is 12.61 m, and the coefficient of static friction is 0.4601. The gravitational acceleration near the surface of the earth is about 9.81 m/s^2, so the speed must be:

$$v \geq \sqrt{\frac{Rg}{\mu_s}}$$
$$v \geq \sqrt{\frac{12.61 \text{ m} \cdot 9.81 \text{ m/s}^2}{0.4601}}$$
$$v \geq 16.3970579 \text{ m/s}$$

ROUND: Since the measured values are all given to four significant figures, the final answer will also have four figures. The minimum velocity is 16.40 m/s.

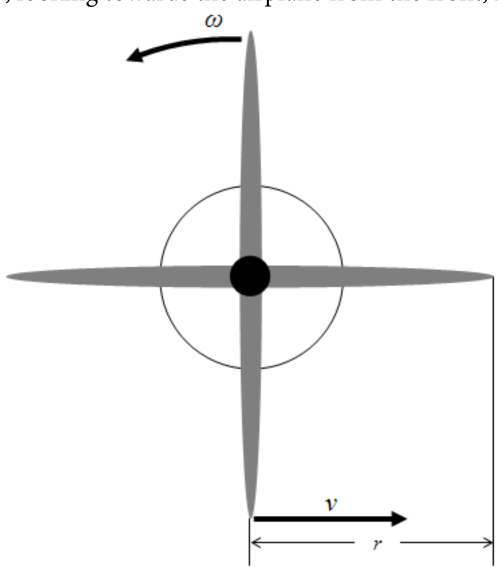
DOUBLE-CHECK: In this case, the motorcycle is traveling at 16.40 m/s, or about 59 kilometers per hour, which is a reasonable speed based on how fast motorcycles *can* go. It needs to travel 12.61(2π) = 79.23 meters to go all the way around the sphere, so it makes one revolution every 4.83 seconds, or between 12 and 13 revolutions per minute. These values all seem reasonable based on past experience with motorcycles.

9.83. $\mu_s = \frac{Rg}{v^2} = \frac{(13.75 \text{ m})(9.81 \text{ m/s}^2)}{(17.01 \text{ m/s})^2} = 0.4662$

9.84. $R = \frac{\mu_s v^2}{g} = \frac{(0.4741)(15.11)^2}{9.81 \text{ m/s}^2} = 11.03 \text{ m}$

9.85. **THINK:** The speed of a point on the tip of the propeller can be calculated from the angular speed and the length of the propeller blade. The angular speed of the propeller can be calculated from the frequency. Find the maximum length of the propeller blade such that the angular speed at the tip of the propeller blade is less than the indicated speed of sound.

SKETCH: A view, looking towards the airplane from the front, is shown.



RESEARCH: The linear velocity should be less than the speed of sound $v \le v_{\text{sound}}$. The magnitude of the linear velocity v is equal to the product of the radius of rotation r and the angular speed ω: $v = r\omega$. The angular speed is related to the rotation frequency by $\omega = 2\pi f$. The length of the propeller blade is twice the radius of the propeller ($d = 2r$). Finally, note that the rotation frequency is given in revolutions per minute and the speed of sound is given in meters per second, so a conversion factor of 60 seconds / minute will be needed.

SIMPLIFY: Use the equation for the linear speed ($v = r\omega$) and the equation for the rotation frequency to get $v = 2\pi f \cdot r$. Use this in the inequality $v \le v_{\text{sound}}$ to find that $2\pi f \cdot r \le v_{\text{sound}}$. Solve this for the length of the propeller blade r (note that ω is a positiv e number of revolutions per minute) to get $r \le \frac{v_{\text{sound}}}{2\pi f}$. The maximum length of the propeller blade is two times the largest possible value of $d = 2r = \frac{v_{\text{sound}}}{\pi f}$.

CALCULATE: The angular frequency f is given in the problem as 2403 rpm and the speed of sound is 343.0 m/s. The maximum length of the propeller blade is thus $d = \frac{343.0 \text{ m/s} \cdot 60 \text{ s/min}}{\pi \cdot 2403 \text{ rev/min}} = 2.726099649 \text{ m}$.

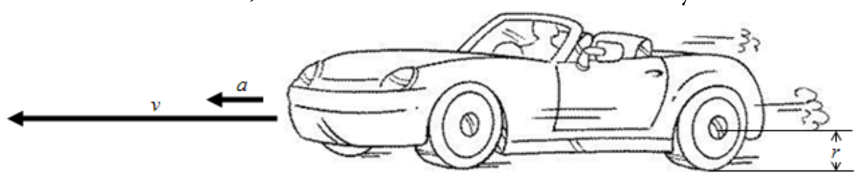
ROUND: The measured values from the problem (the angular frequency and speed of sound) are given to four significant figures, so the final answer should also have four significant figures. The maximum length of a propeller blade is 2.726 m.

DOUBLE-CHECK: For those familiar with propeller-driven aircraft, a total propeller length of about 2.7 m seems reasonable. Working backwards, if the propeller blade is 2.726 m and the linear speed at the tip of the propeller is 343.0 m/s, then the angular speed is $\omega = \frac{v}{r} = \frac{343.0 \text{ m/s}}{1.363 \text{ m}}$. The angular frequency is then $f = \frac{\omega}{2\pi} = \frac{343.0 \text{ m/s}}{2\pi \cdot 1.363 \text{ m}} = 40.05 \text{ rev/sec}$. Since there are 60 seconds in a minute, this agrees with the value of 2403 rev/min given in the problem, and the calculations were correct.

9.86. $f = \dfrac{v}{\pi d} = \dfrac{343.0 \text{ m/s}}{\pi(2.601 \text{ m})} = \left(41.98 \dfrac{1}{\text{s}}\right)\left(\dfrac{60 \text{ s}}{\text{min}}\right) = 2519 \text{ rpm}$

9.87. **THINK:** The linear acceleration can be computed from the change in the speed of the car and the time required to accelerate, both of which are given in the problem. The angular acceleration can be calculated from the linear acceleration and the radius of the tires. Since the car's acceleration is constant and it starts at rest, the motion of the car occurs in only one direction, which can be taken to be the +x direction, and the time that the car starts moving can be taken as time zero.

SKETCH: The car starts at rest, so the constant acceleration and velocity are in the same direction.



RESEARCH: The constant linear acceleration is the change in speed per unit time $a = \dfrac{\Delta v}{\Delta t}$. The relationship between linear acceleration a and angular acceleration α is given by $a = r\alpha$, where r is the radius of the rotating object.

SIMPLIFY: Since there are two expressions for the linear acceleration, $a = \dfrac{\Delta v}{\Delta t}$ and $a = r\alpha$, they must be equal to one another: $r\alpha = \dfrac{\Delta v}{\Delta t}$. Solve for the angular acceleration α to get $\alpha = \dfrac{\Delta v}{r\Delta t}$. The car starts at rest at time zero, the final velocity is equal to Δv and the total time is equal to Δt, giving $\alpha = \dfrac{v}{rt}$.

CALCULATE: After 3.945 seconds, the car's final speed is 29.13 m/s. The rear wheels have a radius of 46.65 cm, or $46.65 \cdot 10^{-2}$ m. The angular acceleration is then

$$\alpha = \frac{29.13 \text{ m/s}}{46.65 \cdot 10^{-2} \text{ m} \cdot 3.945 \text{ s}}$$
$$= 15.82857539 \text{ s}^{-2}$$

ROUND: The time in seconds, radius of the tires, and speed of the car are all given to four significant figures, so the final answer should also have four figures. The angular acceleration of the car is 15.83 s^{-2}.

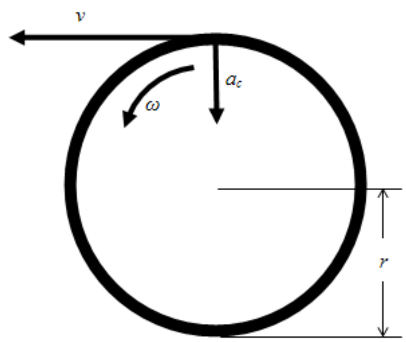
DOUBLE-CHECK: First note that the units (per second per second) are correct for angular acceleration. Working backwards, if the sports car accelerates with an angular acceleration of 15.83 s^{-2} for 3.945 seconds, it will have a final angular speed of (15.83·3.945) s^{-1}. With a tire radius of 46.65 cm, this means that the car's final speed will be (46.65·15.83·3.945) cm/s, or 29.13 m/s (when rounded to four significant figures), which agrees with the problem statement. This confirms that the first set of calculations was correct.

9.88. $v = r\alpha t = (0.4895 \text{ m})(14.99 \text{ s}^{-2})(3.997 \text{ s}) = 29.33 \text{ m/s}$

9.89. $r = \dfrac{v}{\alpha t} = \dfrac{29.53 \text{ m/s}}{(17.71 \text{ s}^{-2})(4.047 \text{ s})} = 0.4120 \text{ m} = 41.20 \text{ cm}$

9.90. **THINK:** The frequency and radius of the flywheel can be used to calculate the speed at the edge of the flywheel. The centripetal acceleration can be calculated from the linear speed and the radius of the flywheel.

SKETCH:



RESEARCH: The centripetal acceleration at the edge of the flywheel is $a_c = \frac{v^2}{r}$, where v is the linear speed at the edge of the flywheel and r is the flywheel's radius. The linear speed v is equal to the angular speed times the radius of the flywheel ($v = r\omega$), and the angular speed ω is related to the frequency f by the equation $\omega = 2\pi f$. The numbers are given in centimeters and revolutions per minute, so conversion factors of $\frac{1\text{ m}}{100\text{ cm}}$ and $\frac{1\text{ min}}{60\text{ sec}}$ may be needed.

SIMPLIFY: First, find the equation for the velocity in terms of the angular frequency to get $v = r\omega = r(2\pi f)$. Use this in the equation for centripetal acceleration to find

$$a_c = \frac{v^2}{r} = \frac{(2\pi rf)^2}{r} = 4r(\pi f)^2 .$$

CALCULATE: The radius is 27.01 cm, or 0.2701 m and the frequency of the flywheel is 4949 rpm. So the angular acceleration is $4 \cdot 27.01\text{ cm}\left(\pi \cdot 4949 \frac{\text{rev}}{\text{min}}\right)^2 = 2.611675581 \cdot 10^{10}\ \frac{\text{cm}}{\text{min}^2}$. Converting to more familiar units, this becomes

$$2.611675581 \cdot 10^{10}\ \frac{\text{cm}}{\text{min}^2} \cdot \frac{1\text{ m}}{100\text{ cm}} \cdot \left(\frac{1\text{min}}{60\text{sec}}\right)^2 = 7.254654393 \cdot 10^4\ \text{m/s}^2 .$$

ROUND: The radius and frequency of the flywheel both have four significant figures, so the final answer should also have four figures. The centripetal acceleration at a point on the edge of the flywheel is $7.255 \cdot 10^4$ m/s^2.

DOUBLE-CHECK: Work backwards to find the frequency from the centripetal acceleration and the radius of the flywheel. The linear velocity is $v = \sqrt{a_c r}$, the angular speed is $\omega = v / r = \frac{\sqrt{a_c r}}{r}$, and the frequency $f = \frac{\omega}{2\pi} = \frac{\sqrt{a_c r}}{2\pi r}$. The radius of the flywheel is 0.2701 m and the centripetal acceleration is $7.255 \cdot 10^4$ m/s^2, so the frequency is

$$\begin{aligned} f &= \frac{\sqrt{a_c r}}{2\pi r} \\ &= \frac{\sqrt{7.255 \cdot 10^4\ \text{m/s}^2 \cdot 0.2701\text{ m}}}{2\pi \cdot 0.2701\text{ m}} \\ &= 82.4853\ \text{s}^{-1} \cdot \frac{60\text{sec}}{1\text{min}} \\ &= 4949.117882\ \text{min}^{-1} \end{aligned}$$

After rounding to four significant figures, this agrees with the frequency given in the problem of 4949 rpm (revolutions per minute).

9.91. $a_c = r(2\pi f)^2$

$$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{r}} = \frac{1}{2\pi}\sqrt{\frac{8.629 \cdot 10^4\ \text{m/s}^2}{0.3159\text{ m}}} = \left(83.18\ \frac{1}{\text{s}}\right)\left(\frac{60\text{ s}}{\text{min}}\right) = 4991\text{ rpm}$$

Chapter 10: Rotation

Concept Checks

10.1. c **10.2.** c **10.3.** a **10.4.** f **10.5.** b **10.6.** c **10.7.** c **10.8.** b **10.9.** b **10.10** b

Multiple-Choice Questions

10.1. b **10.2.** c **10.3.** b **10.4.** d **10.5.** c **10.6.** c **10.7.** c **10.8.** d **10.9.** b **10.10.** e **10.11.** b **10.12.** a **10.13.** c **10.14.** b **10.15.** c **10.16.** b **10.17.** a **10.18.** b **10.19.** c **10.20.** b

Conceptual Questions

10.21. Rotational kinetic energy is given by $K_{\text{rot}} = \tfrac{1}{2}cMv^2$

The total kinetic energy for an object rolling without slipping is given by:

$$K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}Mv^2(1+c) \text{ with } c = 2/5 \text{ for a sphere} \Rightarrow$$

$$\frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{c}{1+c} = \frac{2/5}{1+2/5} = \frac{2}{7}$$

10.22. Assume negligible drag and no slipping. The object that reaches the bottom of the incline first will be the one with the lowest moment of inertia (that is, with the least resistance to rotation). The moments of inertia for the given objects are as follows: Thin ring: $I_{\text{r}} = MR^2$; Solid sphere: $I_{\text{ss}} = \dfrac{2}{5}MR^2$; Hollow sphere: $I_{\text{hs}} = \dfrac{2}{3}MR^2$; Homogeneous disk: $I_{\text{d}} = \dfrac{1}{2}MR^2$. Therefore, the order of the moments of inertia from smallest to greatest (assuming equal mass and radius) is I_{ss}, I_{d}, I_{hs} and I_{r}. Therefore, the order of finish of the objects in the race is: First: solid sphere; Second: homogeneous disk; Third: hollow sphere; Last: thin ring.

10.23. The net translational and rotational forces on both the solid sphere and the thin ring are, respectively, $F_{\text{net}} = ma = mg\sin\theta - f_{\text{static}}$ and $\tau = f_{\text{static}}r = I\alpha$, where the angular acceleration is $\alpha = a/r$. Since the moment of inertia for the solid sphere is $I = \left(2mr^2\right)/5$, the force of static friction is given by $f_{\text{static}} = 2ma/5$. Substitute this expression into the net force equation to solve for the acceleration of the solid sphere: $a_{\text{ss}} = 5g\left(\sin\theta\right)/7$. The moment of inertia for the thin ring is $I = mr^2$. Therefore, the force of static friction in this case is given by $f_{\text{static}} = ma$. Substitute this expression into the net force equation to solve for the acceleration of the thin ring: $a_{\text{r}} = g\left(\sin\theta\right)/2$. Therefore, the ratio of the acceleration is:

$$\frac{a_{\text{r}}}{a_{\text{ss}}} = \frac{\left(g\sin\theta\right)/2}{\left(5g\sin\theta\right)/7} = \frac{7}{10}.$$

10.24. The net translational and rotational forces on the solid sphere on the incline are, respectively, $F_{\text{net}} = ma = mg\sin\theta - f_{\text{static}}$ and $\tau = f_{\text{static}}r = I\alpha$, where the angular acceleration is $\alpha = a/r$. Since the moment of inertia for the solid sphere is $I = \left(2mr^2\right)/5$, the force of static friction is given by $f_{\text{static}} = 2ma/5$. Substituting this expression into the net force equation to solve for the acceleration gives $a = 5g\left(\sin\theta\right)/7$. Thus, $f_{\text{static}} = 2ma/5 = 2mg\left(\sin\theta\right)/7$. The limiting friction corresponding to a coefficient of static friction, μ_{s}, is $\max\left\{f_{\text{static}}\right\} = \mu_{\text{s}}F_{\text{normal}} = \mu_{\text{s}}mg\cos\theta$. For rolling without slipping to take place, it is required that

$$f_{\text{static}}\mu_s mg \max\{f_{\text{static}}\} \Rightarrow \frac{2mg\sin\theta}{7} \le {}_{s} \quad \cos\theta. \text{ Therefore, } \tan\theta \le \frac{7\mu_s}{2} \Rightarrow \theta \le \tan^{-1}\left(\frac{7\mu_s}{2}\right).$$

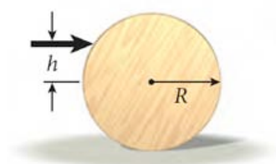
Thus, the maximum angle for which the sphere will roll without slipping is $\theta = \tan^{-1}\left(\frac{7\mu_s}{2}\right)$.

10.25. The "sharp horizontal blow" means that a force of magnitude F acts horizontally on the object along the arrow in the figure. With this force, we have to apply Newton's Second Law for linear motion ($F = Ma$) and Newton's Second Law for rotation ($\tau = I\alpha$). According to the problem text, the round object rolls without slipping. In Section 10.3 we have learned that this condition implies $v = R\omega$ and $a = R\alpha$. The force exerts a torque of magnitude $\tau = Fh$ (= magnitude of force times perpendicular distance) around the center of mass of the round object. Using all these relationships we can write

$$\tau = Fh = (Ma)h = M(R\alpha)h = MRh\alpha = I\alpha.$$

Simplifying and rearranging, we get

$$I = MRh \quad \Rightarrow \quad \frac{I}{MR^2} = \frac{h}{R}.$$



As a double-check, let's calculate h assuming a uniform solid sphere

$$h = \frac{RI}{MR^2} = \frac{R\left(\frac{2}{5}MR^2\right)}{MR^2} = \frac{2}{5}R.$$

So, for example, you might strike a cue ball with a horizontal cue a distance of $2R/5$ above the center of the cue ball to start it rolling without sliding.

10.26. (a) Since the path of the projectile is not a straight line about the origin (which would give an angular momentum of zero), the angular momentum can be determined by considering that the velocity of the projectile changes continuously along its path because of the change in the vertical component of velocity under the gravitational pull. If θ is the angle of projection, the horizontal component of velocity, $v_0\cos\theta_0$, remains unchanged throughout the path and at the maximum height, the vertical component of velocity is zero and it has only the horizontal $v_0\cos\theta_0$. The 'lever arm' for angular momentum at the maximum height is the maximum height itself, $\left(v_0^2\sin^2\theta_0\right)/2g$ so that the angular momentum is

$$L = \frac{\left(mv_0\cos\theta_0\right)\left(v_0^2\sin^2\theta_0\right)}{2g}.$$

Since the angular momentum is conserved in this case, the angular momentum above is the same throughout the path.

(b) Since the angular momentum does not change throughout the path, the rate of change is zero.

(c) The rate of change of this angular momentum is the net torque about the origin, which also equals zero, that is:

$$\tau = \frac{dL}{dt} = \frac{d(0)}{dt} = 0.$$

10.27. For each object we convert the initial potential energy into kinetic energy at the bottom of the ramp.

Sphere: $Mgh_0 = \frac{1}{2}Mv^2(1+c_{\text{sphere}}) \Rightarrow v^2 = 2gh_0 / (1+c_{\text{sphere}})$

Cylinder: $Mgh = \frac{1}{2}Mv^2(1+c_{\text{cylinder}}) \Rightarrow v^2 = 2gh / (1+c_{\text{cylinder}})$

If the speed is to be the same in both cases, this means:

$$2gh/(1+c_{\text{cylinder}}) = 2gh_0/(1+c_{\text{sphere}}) \Rightarrow$$

$$h = h_0 \frac{1+c_{\text{cylinder}}}{1+c_{\text{sphere}}} = h_0 \frac{1+1/2}{1+2/5} = h_0 \frac{3/2}{7/5} = h_0 \frac{15}{14}$$

10.28. To open a door (that is, to rotate a door about the hinges), a force must be applied so as to produce a torque about the hinges. Recall that torque is defined as $\vec{\tau} = \vec{r} \times \vec{F}$. The magnitude of this torque is then given by $\tau = rF\sin\theta$, where θ is the angle between the force applied at a point p, and the vector connecting the point p to the axis of rotation (to the axis of the hinges in this case). Therefore, torque is maximal when the applied force is perpendicular to the vector $\vec{r}$. That is, when the force is perpendicular to the plane of the door. Similarly, torque is minimal (i.e. zero) when the applied force is parallel to $\vec{r}$ (i.e. along the plane of the door).

10.29. Angular momentum is conserved, however, energy is not conserved; her muscles must provide an additional centripetal acceleration to her hands to pull them inwards. That force times the displacement is equal to the work that she does in pulling them in. Since she is doing work on the system, energy is not conserved.

10.30. Consider a particle of constant mass, m, which starts at position, r_0, moving with velocity, v, and having no forces acting on it. By Newton's first law of motion, the absence of forces acting on it means that it must continue to move in a straight line at the same speed, so its equation of motion is given by $r = r_0 + vt$. Its linear momentum is mv, so its angular momentum relative to the origin is given by $\vec{L} = \vec{r} \times m\vec{v} = (\vec{r}_0 + \vec{v}t) \times m\vec{v}$. The cross product is distributive over addition, so this can be rewritten as $\vec{L} = (\vec{r}_0 \times m\vec{v}) + (\vec{v}t \times m\vec{v})$. Clearly the vectors $\vec{v}t$ and $m\vec{v}$ are parallel, since they are both in the direction of $\vec{v}$, and the cross product of two parallel vectors is zero. So, the last term in the sum above comes to zero, and the expression can be rewritten as $\vec{L} = \vec{r}_0 \times m\vec{v}$. Now $\vec{r}_0$, m and $\vec{v}$ are all constants in this system, so it follows that $\vec{L}$ is also constant, as required by the law of conservation of angular momentum. Therefore, whether or not the particle has angular momentum is dependent on the r_0 vector, given non-zero velocity. If the path of the particle crosses the origin, $r_0 = 0$ and the particle has no angular momentum relative to the origin. In every other case, the particle will have constant, non-zero angular momentum relative to the origin.

10.31. Work is given by $W = Fd\cos\theta$ for linear motion, and by $W = \tau\theta$ for angular motion, where the torque, τ, is applied through a revolution of θ.

(a) Gravity points downward, therefore, $W_{\text{gravity}} = mg(s)\sin\theta$.

(b) The normal force acts perpendicular to the displacement. Therefore, $\cos 90 = 0 \Rightarrow W_{\text{normal}} = 0$.

(c) The frictional force considered in this problem is that of static friction since the cylinder is rolling without slipping. The direction of the static friction is opposite to that of the motion. Work done by the frictional force consists of two parts; one is the contribution by translational motion and the other is the contribution by rotational motion.

(i) Translational motion: $W_{\text{translation}} = (f_s)(s)\cos(180°) = -f_s s$

(ii) Rotational motion: $W_{\text{rotational}}\ \tau\ Rf\ \theta = (f\ s_s)\theta = {}_s$

Therefore, the total contribution to work done by the friction is zero.

10.32. Mechanical energy is conserved for the rolling motion without slipping. Setting the top as the vertical origin, the mechanical energy at the top is $E_t = K + U = 0 + 0$. The mechanical energy at the bottom is $E_b = K(\text{translational} + \text{rotational}) + U = \left[(1+c)mv^2/2\right] + (-mgh)$, where h is the vertical height. By conservation of energy, $E_t = E_b$ implies $v = \sqrt{2gh/(1+c)} = \sqrt{\frac{4gh}{3}}$, $\sin\theta = \frac{h}{s} \Rightarrow v = \sqrt{\frac{4}{3}sg\sin\theta}$, where c is 1/2 for the cylindrical object. Using $v_f^2 - v_i^2 = 2as$, the acceleration is

$$a = \frac{2gh/(1+c)}{2s} = \frac{2gs\sin\theta/(1+c)}{2S} = g\frac{\sin\theta}{(1+c)}.$$

For a cylinder, c = 1/2. Therefore, $a = \frac{2}{3}g\sin\theta$.

10.33. Prove that the pivot point about which the torque is calculated is arbitrary. First, consider the definition of torque, $\vec{\tau} = \vec{r} \times \vec{F}$. Therefore, for each of the applied forces, $\vec{F}_1$ and $\vec{F}_2 = -\vec{F}_1$, their contributions to the torque are given by $\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1$ and $\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2$, where $\vec{r}_1$ and $\vec{r}_2$ are the respective distances to the pivot point. The net torque is calculated from the algebraic sum of the torque contributions, that is, $\vec{\tau}_{\text{net}} = \vec{\tau}_1 + \vec{\tau}_2 = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = \vec{r}_1 \times \vec{F}_1 - \vec{r}_2 \times \vec{F}_1$. Since the cross product is distributive, $\vec{\tau}_{\text{net}} = (\vec{r}_1 - \vec{r}_2) \times \vec{F}_1 = \vec{d} \times \vec{F}_1$. Therefore, the net torque produced by a couple depends only on the distance between the forces, and is independent of the actual pivot point about which the contributing torques are calculated or the actual points where the two forces are applied.

10.34. It is actually the act of pulling in her arms that makes the figure skater increase her angular velocity. Since angular momentum is conserved $(I\omega_1 = I_2\omega_2 = \text{constant})$, by reducing her rotational inertia (by means of reducing the distance between her arms and hands to the axis of rotation), the figure skater increases her angular velocity.

10.35. By momentarily turning the handlebars to the left, the contact point of the motorcycle with the ground moves to the left of the center of gravity of the motorcycle so that the motorcycle leans to the right. Now the motorcycle can be turned to the right and the rider can lean to the right. The initial left turn creates a torque that is directed upwards, which deflects the angular momentum of the front tire upward and causes the motorcycle to lean to the right. In addition, as the motorcycle leans to the right, a forward pointing torque is induced that tends to straighten out the front wheel, preventing over-steering and oscillations. At low speeds, these effects are not noticeable, but at high speeds they must be considered.

10.36. The Earth-Moon system, to a good approximation, conserves its angular momentum (though the Sun also causes tides on the Earth). Thus, if the Earth loses angular momentum, the moon must gain it. If there were 400 days in a year in the Devonian period, the day was about 10% shorter, meaning the angular velocity of the Earth was about 10% greater. Since the rotational inertia of the Earth is virtually unchanged, this means that the rotational angular momentum of the Earth was then about 10% greater, and correspondingly the orbital angular momentum of the Moon was about 10% less.

10.37. In this problem, the key is that the monkey is trying to reach the bananas by climbing the rope. Since the monkey has the same mass as the bananas, if he didn't try to climb the rope, both the net torque and total angular momentum on the pulley would be zero. Take counterclockwise to be positive just for aesthetics.

(a) Consider the extra tension provided by the monkey on the rope by climbing (i.e. by pulling on the rope). Average out the force caused by the monkeys pulling with a constant force downwards, $\vec{T}$, on the monkey side. Therefore, the net torque on the pulley axis is provided by this extra force, $\vec{T}$, as

$$\vec{\tau}_{\text{net}} = \left[\left(\vec{F}_{\text{monkey}} + \vec{T}\right) - \vec{F}_{\text{banana}}\right]R = \vec{T}R.$$

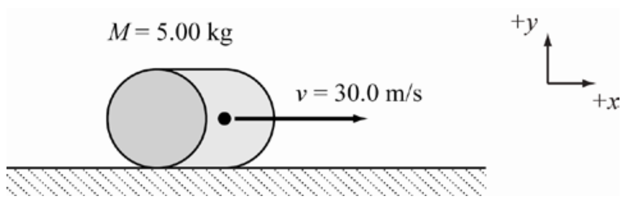
(b) Since there is now a non-zero net torque on the pulley, there is a non-zero total angular momentum given by

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} \Rightarrow \vec{L} = \int \tau dt.$$

Using the results of part (a), the above expression can be rewritten as $\vec{L} = \int \vec{T}Rdt = \vec{T}Rt.$ Recall that the extra "climbing" force was taken to be constant. In reality, the monkey's pulling will be time dependent and this will affect the final form of the time dependent angular momentum.

Exercises

10.38. **THINK:** Determine the energy of a solid cylinder as it rolls on a horizontal surface. The mass of the cylinder is $M = 5.00$ kg and the translational velocity of the cylinder's center of mass is $v = 30.0$ m/s.
SKETCH:



RESEARCH: Since the motion occurs on a horizontal surface, consider only the total kinetic energy of the cylinder, $K_{\text{T}} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. $I = cMR^2$ and, for a solid cylinder, $c = 1/2$ as in Table 10.1. For rolling without slipping, $v = \omega R$.

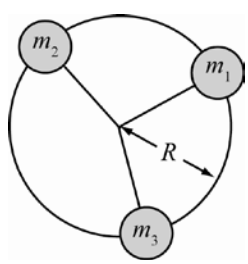
SIMPLIFY: $K_{\text{T}} = \frac{1}{2}Mv^2 + \frac{1}{2}\left(cMR^2\right)\left(\frac{v^2}{R^2}\right) = \frac{1}{2}Mv^2\left(1+c\right)$

CALCULATE: $K_{\text{T}} = \frac{1}{2}\left(5.00 \text{ kg}\right)\left(30.0 \text{ m/s}\right)^2\left(1+1/2\right) = 3375 \text{ J}$

ROUND: Both given values have three significant figures, so the result is rounded to $K_{\text{T}} = 3.38 \cdot 10^3$ J.

DOUBLE-CHECK: The calculated result has Joules as units, which are units of energy. This means that the calculated result is plausible.

10.39. **THINK:** The children can be treated as point particles on the edge of a circle and placed so they are all the same distance, R, from the center. Using the conversion, 1 kg = 2.205 lbs, the three masses are $m_1 = 27.2$ kg, $m_2 = 20.4$ kg and $m_3 = 36.3$ kg. Using the conversion 1 m = 3.281 ft, the distance is $R = 3.657$ m.
SKETCH:



RESEARCH: The moment of inertia for point particles is given by $I = \sum_i m_i r_i^2$.

SIMPLIFY: $I = \left(m_1 + m_2 + m_3\right)R^2$

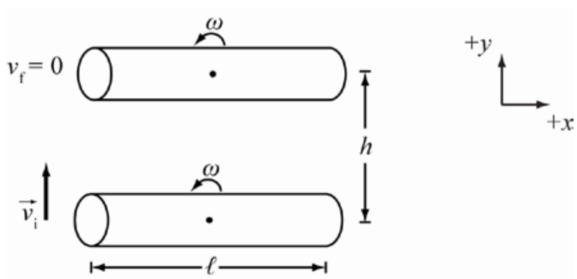
CALCULATE: $I = \left(27.2 \text{ kg} + 20.4 \text{ kg} + 36.3 \text{ kg}\right)\left(3.66 \text{ m}\right)^2 = 1123.9 \text{ kg m}^2$

ROUND: $I = 1.12 \cdot 10^3 \text{ kg m}^2$

DOUBLE-CHECK: Since the children are located on the edge of the merry-go-round, a large value for I is expected.

10.40. **THINK:** Since the pen with a length of l = 24 cm rotates at a constant rate, rotational kinetic energy remains constant so only the translational energy is converted to potential energy at a height of h = 1.2 m from release. Use kinematics to determine the time it takes the pen to reach the top and make 1.8 revolutions, in order to determine ω.

SKETCH:



RESEARCH: The pen has a translational kinetic energy of $K_T = mv^2/2,$ where v_i is the velocity at release. The potential energy at the top is given by $U = mgh$ and $mgh = mv_i^2/2.$ The rotational kinetic energy is given by $K_R = I\omega^2/2,$ where $I = ml^2/12.$ The initial velocity of the pen is determined from $v_f^2 = v_i^2 - 2gh$ and the time of flight is given by $t = -(v_f - v_i)/g.$ Angular velocity is given by $\omega = 2\pi(1.8 \text{ rev})/t.$

SIMPLIFY: The final velocity is zero, so the expression reduces to $0 = v_i^2 - 2gh \Rightarrow v_i = \sqrt{2gh}.$ The expression for the time of flight also reduces to

$$t = -\frac{(0 - v_i)}{g} = \sqrt{\frac{2h}{g}}.$$

The angular velocity is given by $\omega = \dfrac{3.6\pi}{\sqrt{2h/g}} = 3.6\pi\sqrt{\dfrac{g}{2h}}$ rad/s. The ratio is then:

$$\frac{K_R}{K_T} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2} = \frac{\frac{1}{2}\left(\frac{1}{12}ml^2\right)(3.6\pi)^2\frac{g}{2h}}{mgh} = \frac{12.96}{48}\pi^2\frac{l^2}{h^2}.$$

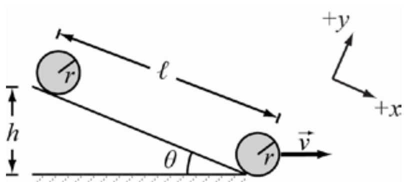
CALCULATE: $\dfrac{K_R}{K_T} = \dfrac{12.96}{48}\pi^2\left(\dfrac{0.24 \text{ m}}{1.2 \text{ m}}\right)^2 = 0.1066$

ROUND: To three significant figures: $\dfrac{K_R}{K_T} = 0.107.$

DOUBLE-CHECK: Remember that this is not a case of rolling without slipping. Here the translational motion is independent of the rotational motion, and so the ratio between translational and rotational kinetic energies could have almost any value. A simple double-check is thus not easily possible.

10.41. **THINK:** With no friction and no slipping, each object with mass, m = 1.00 kg, conserves energy. Since each object starts at the same height, they all have the same potential energy and hence kinetic energy after they travel a distance l = 3.00 m at an incline of $\theta = 35.0°$. Each ball has a radius of r = 0.100 m. Whichever object has the highest velocity at the bottom should reach the bottom first, and vice versa.

SKETCH:



RESEARCH: The constant *c* is related to the geometry of a figure. The values of *c* for different objects can be found in Table 10.1. The solid sphere has $c_1 = 2/5,$ the hollow sphere has $c_2 = 2/3$ and the ice cube has $c_3 = 0.$ Since energy is conserved, the velocity of each object at the bottom is $v = \sqrt{2gh/(1+c)}.$ The incline shows that $h = l\sin\theta.$

SIMPLIFY: $v_1 = \sqrt{\dfrac{2gl\sin\theta}{1+c_1}},\ v_2 = \sqrt{\dfrac{2gl\sin\theta}{1+c_2}},\ v_3 = \sqrt{\dfrac{2gl\sin\theta}{1+c_3}}$

CALCULATE: $v_1 = \sqrt{\dfrac{10}{7}}\sqrt{gl\sin\theta},\ v_2 = \sqrt{\dfrac{6}{5}}\sqrt{gl\sin\theta},\ v_3 = \sqrt{2}\sqrt{gl\sin\theta}$

(a) Since the velocity is inversely proportional to *c*, the object with a smaller *c* will have a larger velocity than that of one with a greater c, and will reach the end first. Since $c_1 < c_2,$ the solid sphere reaches the bottom first.

(b) Since $c_3 < c_1,$ and the velocity is inversely proportional to *c*, the ice cube travels faster than the solid ball at the base of the incline.

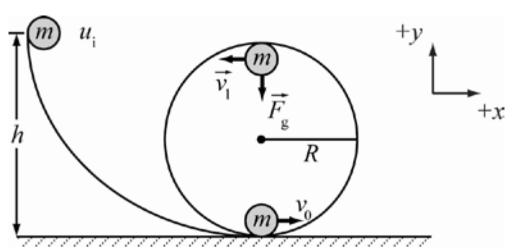
(c) $v_1 = \sqrt{\dfrac{10}{7}}\sqrt{(9.81\text{ m/s}^2)(3.00\text{ m})\sin(35.0°)} = 4.911\text{ m/s}$

ROUND: Parts (a) and (b) do not need to be rounded. (c) $v_1 = 4.91\text{ m/s}$

DOUBLE-CHECK: It is reasonable that the ice cube reaches the bottom first since it does not have to contribute any energy to rotational motion. As expected, the velocity of the sphere is less than it would be if it were in freefall $\left(v = \sqrt{2gl} = 8\text{ m/s}\right).$

10.42. **THINK:** With no friction and no slipping, the object of mass, *m*, and radius, *r*, will conserve energy. Therefore, the potential energy of the ball at height, *h*, should equal the potential energy at the top of the loop of radius, *R*, plus translational and rotational kinetic energy. For the ball to complete the loop, the minimum velocity required is the one where the normal force of the loop on the ball is 0 N, so that the centripetal force is solely the force of gravity on the ball.

SKETCH:



RESEARCH: The only force on the ball at the top of the loop is $F_g = mg = mv_1^2/R.$ The initial potential energy is given by $U_i = mgh$ and the final potential energy is given by $U_f = mg(2R).$ The kinetic energy at the top of the loop is $K = (1+c)mv_1^2/2,$ where the *c* value for a solid sphere is 2/5.

SIMPLIFY: The conservation of energy is given by

$$U_{\text{i}} = U_{\text{f}} + K \;\Rightarrow\; mgh = 2mgR + \frac{1}{2}(1+c)mv_1^{\,2}.$$

From the forces, $mg = \dfrac{mv_1^{\,2}}{R} \;\Rightarrow\; v_1^{\,2} = gR.$ Therefore, $mgh = 2mgR + \dfrac{1}{2}\left(\dfrac{7}{5}\right)mgR \Rightarrow h = R\left(2 + \dfrac{7}{10}\right) = \dfrac{27}{10}R.$

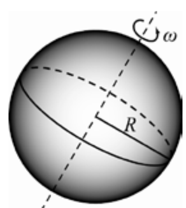
CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK: The height is greater than $2R$, which neglecting rotational energy would be the minimum energy needed, so the result is reasonable.

10.43. **THINK:** The change in energy should be solely that of the change in rotational kinetic energy. Assume the pulsar is a uniform solid sphere with $m \approx 2\cdot 10^{30}$ kg and $R = 12$ km. Initially, the pulsar rotates at $\omega = 60\pi$ rad/s and has a period, T which is increased by 10^{-5} s after 1 y. We calculate the power emitted by the pulsar by taking the time derivative of the rotational kinetic energy. The power output of the Sun is $P_{\text{Sun}} = 4\cdot 10^{26}$ W.

SKETCH:



RESEARCH: The kinetic energy is given by $K = I\omega^2/2,$ so

$$P_{\text{Crab}} = -\frac{dK}{dt} = -I\omega\frac{d\omega}{dt}.$$

The angular velocity is given by $\omega = 2\pi/T,$ so

$$\frac{d\omega}{dt} = -\frac{2\pi}{T^2}\frac{dT}{dt} = -\frac{2\pi}{\left(\frac{2\pi}{\omega}\right)^2}\frac{dT}{dt} = -\frac{\omega^2}{2\pi}\frac{dT}{dt}.$$

The moment of inertia of a sphere is

$$I = \frac{2}{5}mR^2.$$

SIMPLIFY: Combining our equations gives us

$$P_{\text{Crab}} = \frac{2}{5}mR^2\omega\frac{\omega^2}{2\pi}\frac{dT}{dt} = \frac{mR^2\omega^3}{5\pi}\frac{dT}{dt}.$$

CALCULATE: First we calculate the change in the period over one year,

$$\frac{dT}{dt} = \frac{10^{-5}\text{ s}}{(365\text{ days})(24\text{ hour/day})(3600\text{ s/hour})} = 3.17\cdot 10^{-13}.$$

The power emitted by the pulsar is

$$P_{\text{Crab}} = \frac{dK}{dt} = \frac{(2\cdot 10^{30}\text{ kg})(12\cdot 10^{3}\text{ m})^2(60\pi\,\text{rad/s})^3}{5\pi}(3.17\cdot 10^{-13}) = 3.89\cdot 10^{31}\text{ W}.$$

So the ratio of the power emitted by the pulsar to the power emitted by the Sun is

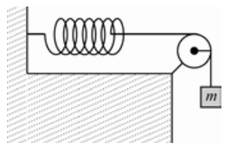
$$\frac{P_{\text{Crab}}}{P_{\text{Sun}}} = \frac{3.89\cdot 10^{31}\text{ W}}{4\cdot 10^{26}\text{ W}} = 9.73\cdot 10^{4}.$$

ROUND: $\dfrac{P_{\text{Crab}}}{P_{\text{Sun}}} = 1\cdot 10^{5}.$

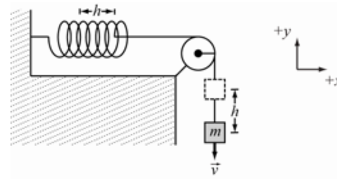
DOUBLE-CHECK: Our result for the ratio of the loss in rotational energy of the Crab Pulsar is close to the expected value of 100,000.

10.44. **THINK:** With no friction and no slipping, mechanical energy is conserved. This means that the potential energy of the block of mass m = 4.00 kg will be converted into the potential energy of the spring with a constant of k = 32.0 N/m, kinetic energy of the block and rotational energy of the pulley with a radius of R = 5.00 cm and mass M = 8.00 kg. If the block falls a distance h, then the spring is extended by a distance h as well. Consider the lower position of the block in parts (a) and (b) to be at zero potential. In part (a), the block falls a distance h = 1.00 m. In part (b), when the block comes to rest, the kinetic energy of the system is zero so that the block's potential energy is converted entirely into spring potential.

SKETCH: (a)



(b)



RESEARCH: The initial energy of the system is $E_\text{i} = U_\text{i} = mgh.$

(a) The final energy is $E_\text{f} = \frac{1}{2}k\left(x_0 - h\right)^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2.$

(b) The final energy is $E_\text{f} = k\left(x - h\right)^2/2.$ The moment of inertia of the wheel is $MR^2/2.$ With no slipping, $R\omega = v.$ Let $x_0 = 0$ for the spring equilibrium.

SIMPLIFY:

(a) $E_\text{i} = E_\text{f} \;\Rightarrow\; mgh = \frac{1}{2}kh^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2.$ Therefore,

$$mgh - \frac{1}{2}kh^2 = \left(\frac{1}{4}M + \frac{1}{2}m\right)v^2 \;\Rightarrow\; v = \sqrt{\frac{mgh - \frac{1}{2}kh^2}{\frac{1}{4}M + \frac{1}{2}m}}.$$

(b) $E_\text{i} = E_\text{f} \;\Rightarrow\; mgh = \frac{1}{2}kh^2 \;\Rightarrow\; h = \frac{2mg}{k}$

CALCULATE:

(a) $v = \sqrt{\dfrac{(4.00 \text{ kg})(9.81 \text{ m/s}^2)(1.00 \text{ m}) - \frac{1}{2}(32.0 \text{ N/m})(1.00 \text{ m})^2}{\frac{1}{4}(8.00 \text{ kg}) + \frac{1}{2}(4.00 \text{ kg})}} = 2.410 \text{ m/s}$

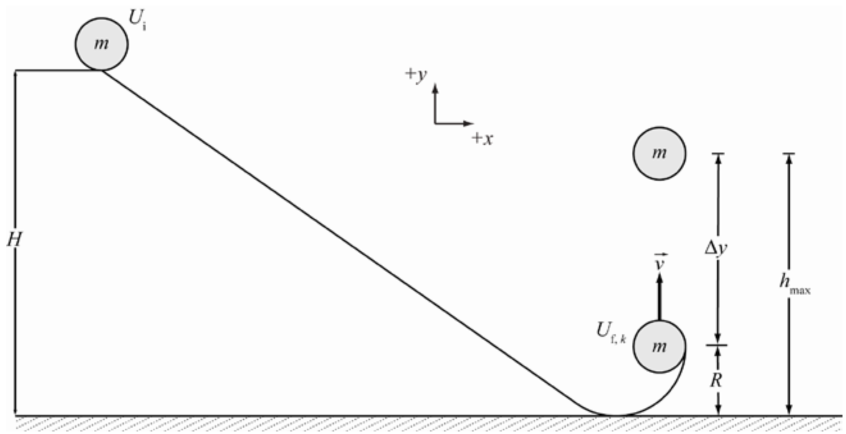
(b) $h = \dfrac{2(4.00 \text{ kg})(9.81 \text{ m/s}^2)}{32.0 \text{ N/m}} = 2.45 \text{ m}$

ROUND: Three significant figures:
(a) The block has a speed of v = 2.41 m/s after it has fallen 1.00 m.
(b) The maximum extension of the spring is h = 2.45 m.

DOUBLE-CHECK: For part (a), the speed should be less than it is for free fall $\left(v = \sqrt{2gh} = 4.4 \text{ m/s}\right)$, which it is. For part (b), the distance is reasonable.

10.45. **THINK:** With no friction and no slipping, the energy of the object of mass, m, and radius, r, is conserved. This means that the initial potential energy at height H = 6.00 m is equal to the potential energy at height R = 2.50 m, plus its rotational and translational energy. The object has a c value of 0.400. Using conservation of energy, the velocity of the object can be determined. Then, using kinematics, the maximum height the object achieves can be determined. Let the subscript i indicate the ball is at the top of the ramp, and the subscript f indicate the ball is at the end of the ramp, at the launch point.

SKETCH:



RESEARCH: The initial energy of the ball is $E_i = U_i = mgH$. The final energy of the ball is $E_f = U_f + K \Rightarrow E_f = mgR + \left[(1+c)mv^2/2\right]$, where c = 0.400. The kinematics equation for the velocity is $v_f^2 = v_i^2 + 2g\Delta y$. Since the ball is at rest at the top, the equation becomes $v^2 = 2g\Delta y \Rightarrow \Delta y = v^2/2g$. The maximum height achieved is $\Delta y + R = h_{max}$.

SIMPLIFY: $E_i = E_f \Rightarrow mgH = mgR + \frac{1}{2}(1+c)mv^2 \Rightarrow mg(H-R) = \frac{1}{2}(1+c)mv^2 \Rightarrow v^2 = \frac{2g(H-R)}{1+c}$

$$h_{max} = \Delta y + R = \frac{v^2}{2g} + R = \frac{(H-R)}{1+c} + R$$

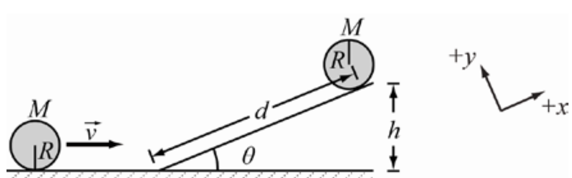
CALCULATE: $h_{max} = \frac{(6.00\text{ m} - 2.50\text{ m})}{1+0.400} + 2.50\text{ m} = 5.000\text{ m}$

ROUND: $h_{max} = 5.00\text{ m}$

DOUBLE-CHECK: If the object did not rotate, the mass is expected to reach its original height of 6 m. Since the object does rotate, the height it reaches should be less than the original height.

10.46. **THINK:** In both cases, energy should be conserved. In part (a), if the ball of mass, *M*, and radius, *R*, continues to spin at the same rate, then there is no change in rotational kinetic energy and only the translational energy is converted to potential energy. In part (b), there is slipping so both rotational and translational kinetic energy are converted to potential energy. The ball has an initial velocity of 3.00 m/s and travels a distance, *d*, up an incline with an angle of $\theta = 23.0°$.

SKETCH:



RESEARCH:

(a) The initial translational kinetic energy is given by $K_T = mv^2/2$, the initial rotational energy is given by $K_R = \frac{1}{2}I\omega^2$, and the final potential energy is given by $U_f = mgh$.

(b) The initial kinetic energy is $K = (1+c)mv^2/2$ and the final potential energy is $U_f = mgh$. The height of the ball is given by the expression $h = d\sin\theta$, and c = 2/5 for a solid sphere.

SIMPLIFY:

(a) $E_\text{i} = E_\text{f} \;\Rightarrow\; \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgd\sin\theta + \frac{1}{2}I\omega^2 \;\Rightarrow\; d = \frac{1}{2}\frac{v^2}{g\sin\theta}$

(b) $E_\text{i} = E_\text{f} \;\Rightarrow\; \frac{1}{2}(1+c)mv^2 = mgd\sin\theta \;\Rightarrow\; d = \frac{v^2(1+c)}{2g\sin\theta} = \frac{7}{10}\frac{v^2}{g\sin\theta}$

CALCULATE:

(a) $d = \dfrac{(3.00\text{ m/s})^2}{2(9.81\text{ m/s}^2)\sin(23.0°)} = 1.174\text{ m}$

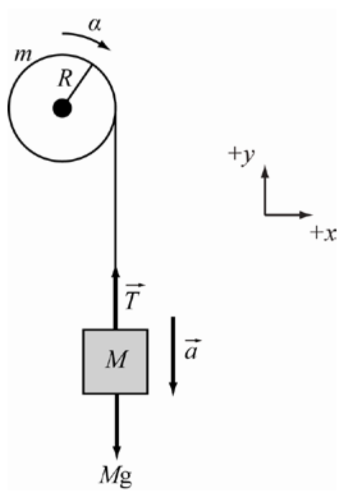
(b) $d = \dfrac{7(3.00\text{ m/s})^2}{10(9.81\text{ m/s}^2)\sin(23.0°)} = 1.644\text{ m}$

ROUND:
Rounding to three significant figures:
(a) d = 1.17 m
(b) d = 1.65 m

DOUBLE-CHECK: Since the ball in part (b) does contribute rotational energy to the potential, it is expected to go higher up the ramp and hence have a larger value for d.

10.47. **THINK:** The hanging block with a mass of M = 70.0 kg, will cause a tension, T, in the string that will in turn produce a torque, τ, in the wheel with a mass, m = 30.0 kg, and a radius, R = 40.0 cm. This torque will give the wheel an angular acceleration, α. If there is no slipping, then the angular acceleration of the wheel is directly related to the acceleration of the block.

SKETCH:



RESEARCH: The balance of forces is given by $T - Mg = -Ma$. The torque produced by the tension, T, is given by $\tau = TR = I\alpha$, where I of the wheel is $mR^2/2$. With no slipping, $R\alpha = a$.

SIMPLIFY: First, determine the tension, $T \Rightarrow T = M(g-a)$. This expression can be substituted into the torque equation to solve for a:

$$M(g-a)R = \frac{1}{2}mR^2\left(\frac{a}{R}\right) \;\Rightarrow\; MgR - MaR = \frac{1}{2}mRa \;\Rightarrow\; Mg = \left(\frac{1}{2}m + M\right)a \;\Rightarrow\; a = \frac{Mg}{\frac{1}{2}m + M}.$$

CALCULATE: $a = \dfrac{70.0\text{ kg}(9.81\text{ m/s}^2)}{\frac{1}{2}(30.0\text{ kg}) + 70.0\text{ kg}} = 8.079\text{ m/s}^2$

ROUND: $a = 8.08 \text{ m/s}^2$

DOUBLE-CHECK: Since there is tension acting opposite gravity, the overall acceleration of the hanging mass should be less than *g*.

10.48. **THINK:** The torque is simply the cross product of the vectors, $\vec{r} = (4\hat{x} + 4\hat{y} + 4\hat{z})$ m and $\vec{F} = (2\hat{x} + 3\hat{y})$ N.

SKETCH: Not applicable.

RESEARCH: $\vec{\tau} = \vec{r} \times \vec{F}$

SIMPLIFY: $\vec{\tau} = (4\hat{x} + 4\hat{y} + 4\hat{z}) \times (2\hat{x} + 3\hat{y})$ N m

$$= \left[8(\hat{x}\times\hat{x}) + 12(\hat{x}\times\hat{y}) + 8(\hat{y}\times\hat{x}) + 12(\hat{y}\times\hat{y}) + 8(\hat{z}\times\hat{x}) + 12(\hat{z}\times\hat{y})\right] \text{ N m}$$

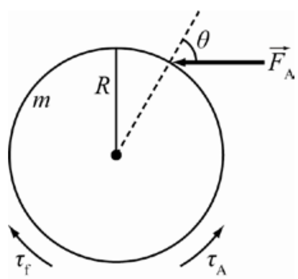
CALCULATE: $\vec{\tau} = \left[8(0) + 12(\hat{z}) + 8(-\hat{z}) + 12(0) + 8(\hat{y}) + 12(-\hat{x})\right] \text{ N m} = (-12\hat{x} + 8\hat{y} + 4\hat{z}) \text{ N m}$

ROUND: Not applicable.

DOUBLE-CHECK: The magnitude of the calculated torque is about 15. As required, this number is smaller than (or at most equal to) the product of the magnitudes of the force and the position vectors, which is about 25 in this case.

10.49. **THINK:** There are two forces, $F_1 = 70.0$ N (applied from 0 to 2.00 seconds) and $F_2 = 24.0$ N (applied after 2.00 seconds). These forces are applied at an angle of $\theta = 37.0°$ on the surface of a disk of mass, m = 14.0 kg, and diameter of d = 30.0 cm (radius, R = 15.0 cm). After 2.00 seconds, the disk moves at a constant angular speed, ω. This means that the sum of the torques is zero, so the torque produced by friction is equal and opposite the torque produced by the applied force. Assuming the frictional torque is constant, the angular acceleration, α, of the disk from 0 to 2.00 seconds can be calculated and ω can be determined.

SKETCH:



RESEARCH: The torque that F_A produces is $\tau_A = RF_A \sin\theta$. After t = 2.00 s, when ω is constant, $\sum \tau = 0 = \tau_A - \tau_f$, where τ_f is the frictional torque. For t = 0 to t = 2 s, $\sum \tau = \tau_A - \tau_f = \tau_{net}$ and $\tau_{net} = I\alpha$. Starting from rest, $\omega = \alpha t$. The rotational kinetic energy of the wheel after t = 2.00 s is then $K = I\omega^2/2$, where $I = mR^2/2$.

SIMPLIFY:

(a) $\sum \tau = \tau_A - \tau_f = 0 \Rightarrow \tau_A = \tau_f = RF_2 \sin\theta$

(b) $\tau_{net} = \tau_A - \tau_f = RF_1 \sin\theta - RF_2 \sin\theta = I\alpha \Rightarrow \alpha = \dfrac{R\sin\theta(F_1 - F_2)}{\frac{1}{2}mR^2} = \dfrac{2\sin\theta(F_1 - F_2)}{mR}$

$$\omega = \alpha t = \frac{2\sin\theta(F_1 - F_2)}{mR} t$$

(c) $K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{4}mR^2\omega^2$

CALCULATE:

(a) $\tau_{\text{f}} = (0.150 \text{ m})(24.0 \text{ N})\sin(37.0°) = 2.167 \text{ Nm}$

(b) $\omega = \dfrac{2\sin(37.0°)(70.0 \text{ N} - 24.0 \text{ N})(2.00 \text{ s})}{(14.0 \text{ kg})(0.150 \text{ m})} = 52.73 \text{ rad/s}$

(c) $K = \dfrac{1}{4}(14.0 \text{ kg})(0.150 \text{ m})^2 (52.73 \text{ rad/s})^2 = 218.96 \text{ J}$

ROUND:

(a) $\tau_{\text{f}} = 2.17 \text{ Nm}$

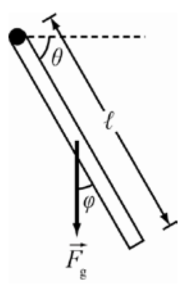
(b) $\omega = 52.7 \text{ rad/s}$

(c) $K = 219 \text{ J}$

DOUBLE-CHECK: Given the initial variables, these results are reasonable.

10.50. **THINK:** When the rod is at an angle of $\theta = 60.0°$ below the horizontal, the force of gravity acting at the center of mass of the rod, with mass, m = 2.00 kg, and length, l = 1.00 m, will produce a torque, τ, and hence an angular acceleration, α. If the rod has a uniform density, then the center of mass is at the geometric center of the rod.

SKETCH:



RESEARCH: From geometry it can be shown that $\theta + \varphi = 90°$. Therefore, $\varphi = 90° - \theta = 90° - 60° = 30°$. The torque that the force of gravity produces is $\tau = mg(l/2)\sin\varphi = I\alpha$, where $I = ml^2/3$.

SIMPLIFY: $mg\left(\dfrac{l}{2}\right)\sin\varphi = \dfrac{1}{3}ml^2\alpha \Rightarrow \alpha = \dfrac{3g\sin\varphi}{2l}$

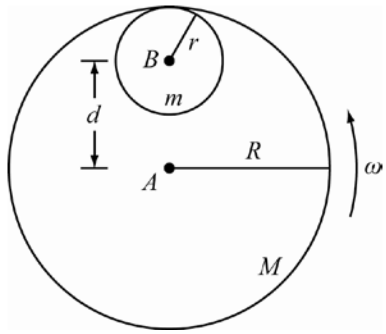
CALCULATE: $\alpha = \dfrac{3(9.81 \text{ m/s}^2)\sin(30.0°)}{2(1.00 \text{ m})} = 7.35 \text{ rad/s}^2$

ROUND: To three significant figures: $\alpha = 7.35 \text{ rad/s}^2$

DOUBLE-CHECK: The vertical component of the tangential acceleration at the end is given by $a_{\text{v}} = a_{\text{T}}\sin\theta = l\alpha\sin\theta \approx 6 \text{ m/s}^2$, which is less than g. This is expected since the pivot is causing the rod to swing and the vertical displacement of the end is slowing down and a smaller acceleration is expected.

10.51. **THINK:** Each object has its own moment of inertia, I_{A} and I_{B}. Disk *A* with a mass, M = 2.00 kg, and a radius, R = 25.0 cm, rotates about its center of mass while disk *B* with a mass, m = 0.200 kg and a radius, r = 2.50 cm, rotates a distance, $d = R - r$, away from the axis. This means the parallel axis theorem must be used to determine the overall moment of inertia of disk *B*, I_{B}'. The total moment of inertia is the sum of the two. If a torque, $\tau = 0.200 \text{ Nm}$, is applied then it will cause an angular acceleration, α. If the disk initially rotates at $\omega = -2\pi \text{ rad/s}$, then kinematics can be used to determine how long it takes to slow down.

SKETCH:



RESEARCH: The moment of inertia of disk A is $I_A = MR^2/2$. The moment of inertia of disk B is $I_B = mr^2/2$. Since disk B is displaced by $d = R - r$ from the axis of rotation, $I_B' = I_B + md^2$, by the parallel axis theorem. Therefore, the total moment of inertia is $I_{tot} = I_A + I_B'$. The torque that is applied produces $\tau = I_{tot}\alpha$, where $\alpha = (\omega_f - \omega_i)/\Delta t$.

SIMPLIFY:

(a) $I_{tot} = I_A + I_B' = I_A + I_B + m(R-r)^2$

$$= \frac{1}{2}MR^2 + \frac{1}{2}mr^2 + mR^2 - 2mRr + mr^2 = \left(\frac{1}{2}M + m\right)R^2 + \frac{3}{2}mr^2 - 2mRr$$

(b) $\tau = I_{tot}\alpha = \frac{I_{tot}(0-\omega_i)}{t} \Rightarrow t = -\frac{I_{tot}\omega_i}{\tau}$

CALCULATE:

(a) $I_{tot} = \left(\frac{1}{2}(2.00) + 0.200\right)(0.250)^2 + \frac{3}{2}(0.200)(0.0250)^2 - 2(0.200)(0.0250)(0.250) = 0.0726875 \text{ kg m}^2$

(b) $t = -\frac{(0.0726875 \text{ kg m}^2)(-2\pi \text{ rad/s})}{0.200 \text{ N m}} = 2.284 \text{ s}$

ROUND:

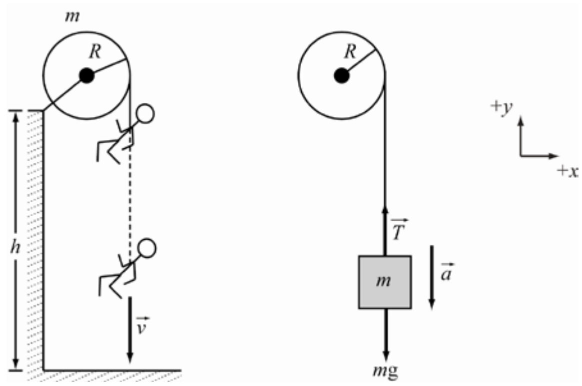
(a) $I_{tot} = 7.27 \cdot 10^{-2} \text{ kg m}^2$

(b) $t = 2.28$ s

DOUBLE-CHECK: Given the small masses and disk sizes, the moment of inertia should be small. Also, given the small torque and angular velocity, two seconds to come to a stop is reasonable.

10.52. **THINK:** The stuntman with a mass, $m = 50.0$ kg, will cause a tension, T, in the rope which produces a torque, τ, on the drum of mass, $M = 100.$ kg and radius, $R = 0.500$ m. This torque will cause the drum to have an angular acceleration, α, and if the rope does not slip, then it will be directly related to the stuntman's translational acceleration, a. If the stuntman starts from rest and needs to accelerate to $v = 4.00$ m/s after dropping a height, $h = 20.0$ m, then kinematics can be used to determine the acceleration.

SKETCH:



RESEARCH: The sum of the forces yields $T - mg = -ma$. The torque produced by the tension is given by $\tau = TR = I\alpha$. With no slipping, $R\alpha = a$. The velocity of the stuntman after falling a height, h, at an acceleration of a is given by $v_\text{f}^2 = v_\text{i}^2 + 2ah$, where $v_\text{i} = 0$. Also, if there is no slipping, $v = \omega R$. The angle the barrel makes is given by $\omega_\text{f}^2 = \omega_\text{i}^2 + 2\alpha\Delta\theta$.

SIMPLIFY:

(a) The tension is given by $T = m(g - a)$. Therefore, the torque is given by $\tau = TR = m(g - a)R = I_0\alpha$. This implies:

$$mgR - maR = I_0\frac{a}{R} \Rightarrow mgR^2 = maR^2 + I_0 a \Rightarrow a = \frac{mgR^2}{mR^2 + I_0}.$$

(b) $v^2 = 0 + 2ah \Rightarrow a = \dfrac{v^2}{2h}$; From part (a), $I_0 = \dfrac{mgR^2 - maR^2}{a} = mR^2\left(\dfrac{g}{a} - 1\right)$.

(c) $\alpha = \dfrac{a}{R}$

(d) $\Delta\theta = \dfrac{\omega_\text{f}^2}{2\alpha} = \dfrac{v^2}{2\alpha R^2}$, $\#\text{revolutions} = \dfrac{\Delta\theta}{2\pi} = \dfrac{v^2}{4\pi\alpha R^2}$

CALCULATE:

(a) No calculation is necessary.

(b) $a = \dfrac{(4.00 \text{ m/s})^2}{2(20.0 \text{ m})} = 0.400 \text{ m/s}^2$, $I = (50.0 \text{ kg})(0.500 \text{ m})^2\left(\dfrac{(9.81 \text{ m/s}^2)}{0.400 \text{ m/s}^2} - 1\right) = 294.0625 \text{ kg m}^2$

(c) $\alpha = \dfrac{0.400 \text{ m/s}^2}{0.500 \text{ m}} = 0.800 \text{ rad/s}^2$

(d) $\#\text{revolutions} = \dfrac{(4.00 \text{ m/s})^2}{4\pi(0.800 \text{ rad/s}^2)(0.500 \text{ m})^2} = 6.366$

ROUND:

Rounding to thre significant figures:

(a) Not applicable.

(b) $a = 0.400 \text{ m/s}^2$ and $I = 294 \text{ kg m}^2$.

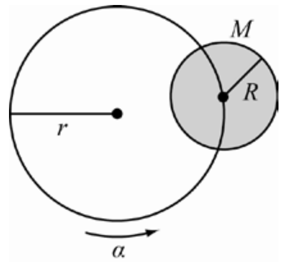
(c) $\alpha = 0.800 \text{ rad/s}^2$

(d) $\#\text{revolutions} = 6.37$

DOUBLE-CHECK: Given the large height and the small final velocity, the small accelerations and the few rotations of the drum are reasonable.

10.53. **THINK:** Since the center of mass of the tire with mass, $M = 23.5$ kg, is at a distance, $r = 1.10$ m, from the axis of rotation, the parallel axis theorem is used to determine the overall moment of inertia of the tire. Consider both cases where the tire is a thin hollow cylinder of radius, $R = 0.350$ m, and a thick hollow cylinder with radii, $R_1 = 0.300 \text{ m}$ and $R_2 = 0.400 \text{ m}$. The torque, $\tau = 20.0 \text{ N m}$, the athlete applies will cause an angular acceleration, α. Kinematics can be then be used to determine the linear speed after three rotations.

SKETCH:



RESEARCH: From the parallel axis theorem, the moment of inertia of a tire is $I = I_{cm} + Mr^2$. For a thin hollow cylinder, $I_{cm} = MR^2$ and for a thick hollow cylinder, $I_{cm} = M\left(R_1^2 + R_2^2\right)/2$. The torque is given by $\tau = I\alpha$. Then time can be determined from $\Delta\theta = \alpha t^2/2$, where $\Delta\theta$ is three rotations or 6π radians. Since the tire starts from rest, its final angular velocity is $\omega = \alpha t$ and its tangential velocity is $v = \omega r$.

SIMPLIFY:

(a) $\tau = I\alpha = \left(MR^2 + Mr^2\right)\alpha \Rightarrow \alpha = \dfrac{\tau}{M\left(R^2+r^2\right)}$, $\Delta\theta = \dfrac{1}{2}\alpha t_{\text{throw}}^{\;2} \Rightarrow t_{\text{throw}} = \sqrt{\dfrac{2\Delta\theta}{\alpha}} = \sqrt{\dfrac{12\pi\left(M\left(R^2+r^2\right)\right)}{\tau}}$

(b) $v = \omega r = \alpha t r = \sqrt{2\alpha\Delta\theta}\, r = r\sqrt{\dfrac{12\pi\tau}{M\left(R^2+r^2\right)}}$

(c) $t_{\text{throw}} = \sqrt{\dfrac{12\pi\left(M\left(r^2 + \dfrac{R_1^{\,2}+R_2^{\,2}}{2}\right)\right)}{\tau}}$, $\quad v = r\sqrt{\dfrac{12\pi\tau}{M\left(r^2 + \dfrac{R_1^{\,2}+R_2^{\,2}}{2}\right)}}$

CALCULATE:

(a) $t_{\text{throw}} = \sqrt{\dfrac{12\pi\left((23.5 \text{ kg})\left((0.350 \text{ m})^2 + (1.10 \text{ m})^2\right)\right)}{20.0 \text{ Nm}}} = 7.683 \text{ s}$

(b) $v = (1.10 \text{ m})\sqrt{\dfrac{12\pi(20.0 \text{ Nm})}{(23.5 \text{ kg})\left((0.350 \text{ m})^2 + (1.10 \text{ m})^2\right)}} = 5.39766 \text{ m/s}$

(c) $t_{\text{throw}} = \sqrt{\dfrac{12\pi\left((23.5 \text{ kg})\left((1.10 \text{ m})^2 + \dfrac{(0.300 \text{ m})^2 + (0.400 \text{ m})^2}{2}\right)\right)}{20.0 \text{ Nm}}} = 7.690 \text{ s}$

$$v = (1.1\text{ m})\sqrt{\frac{12\pi(20.0\text{ Nm})}{(23.5\text{ kg})\left((1.10\text{ m})^2 + \frac{(0.300\text{ m})^2 + (0.400\text{ m})^2}{2}\right)}} = 5.393\text{ m/s}$$

ROUND:

(a) $t_{\text{throw}} = 7.68$ s

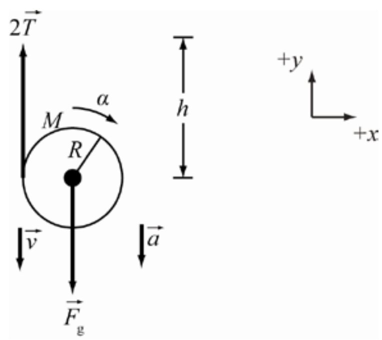
(b) $v = 5.40$ m/s

(c) $t_{\text{throw}} = 7.69$ s and $v = 5.39$ m/s

DOUBLE-CHECK: Given the small change in the two moments of inertia between the thin and thick cylinder, virtually identical values in time and velocity are reasonable.

10.54. **THINK:** Due to the symmetry of the barrel, assume the tension, T, in each rope is equal. The barrel with mass, M = 100. kg, and radius, R = 50.0 cm, will cause a tension, T, in the ropes that in turn produces a torque, τ, on the barrel and hence an angular acceleration, α. If the ropes do not slip, the angular acceleration will be directly related to the linear acceleration of the barrel. Once the linear acceleration is determined, kinematics can be used to determine the velocity of the barrel after it has fallen a distance, h = 10.0 m, assuming it starts from rest.

SKETCH:



RESEARCH: From the kinematic equations, $v_f^2 = v_i^2 + 2ah$, where the initial velocity is zero. The sum of the forces acting on the barrel is given by $2T - Mg = -Ma$. The tension in the ropes also cause a torque, $\tau = 2TR = I\alpha$, where $I = MR^2/2$.

SIMPLIFY: Summing the tensions in the ropes gives $2T = M(g-a)$. The torque this tension produces is

$$\tau = 2TR = MR(g-a) = \frac{1}{2}MR^2\left(\frac{a}{R}\right) \Rightarrow MgR = \frac{3}{2}MRa \Rightarrow a = \frac{2}{3}g.$$

The velocity is given by $v^2 = 2ah \Rightarrow v = \sqrt{\frac{4}{3}gh}$. The tension in one rope is $T = M(g-a)/2 = Mg/6$.

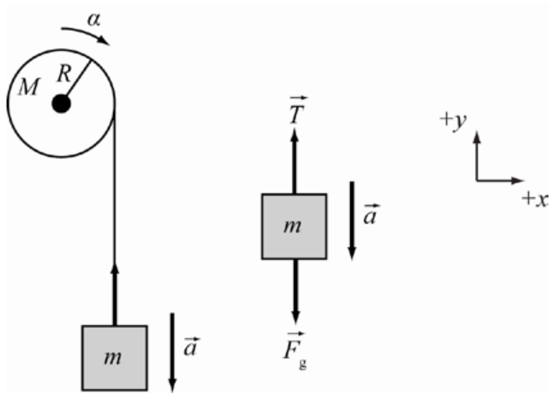
CALCULATE: $v = \sqrt{\frac{4}{3}(9.81\text{ m/s}^2)(10.0\text{ m})} = 11.437$ m/s, $T = \frac{1}{6}(100.\text{ kg})(9.81\text{ m/s}^2) = 163.5$ N

ROUND: Rounding to three significant figures, $v = 11.4$ m/s and T = 164 N.

DOUBLE-CHECK: If the barrel is in free fall, it would have a velocity of 14 m/s after falling 10 m, so a smaller velocity for this result is reasonable.

10.55. **THINK:** The hanging mass, m = 2.00 kg, will cause a tension, T, in the rope. This tension will then produce a torque, τ, on the wheel with a mass, M = 40.0 kg, a radius, R = 30.0 cm and a c value of 4/9. This torque will then give the wheel an angular acceleration, α. Assuming the rope does not slip, the angular acceleration of the wheel will be directly related to the linear acceleration of the hanging mass.

SKETCH:



RESEARCH: With no slipping, the linear acceleration is given by $a = \alpha R$. The tension can be determined by $T = m(g-a)$, which in turn produces a torque $\tau = TR = I\alpha$, where the moment of inertia of the wheel is $4MR^2/9$.

SIMPLIFY: To determine the angular acceleration:

$$TR = m(g-\alpha R)R = \frac{4}{9}MR^2\alpha \Rightarrow mgR = m\alpha R^2 + \frac{4}{9}MR^2\alpha \Rightarrow \alpha = \frac{mg}{\left(m+\frac{4}{9}M\right)R}.$$

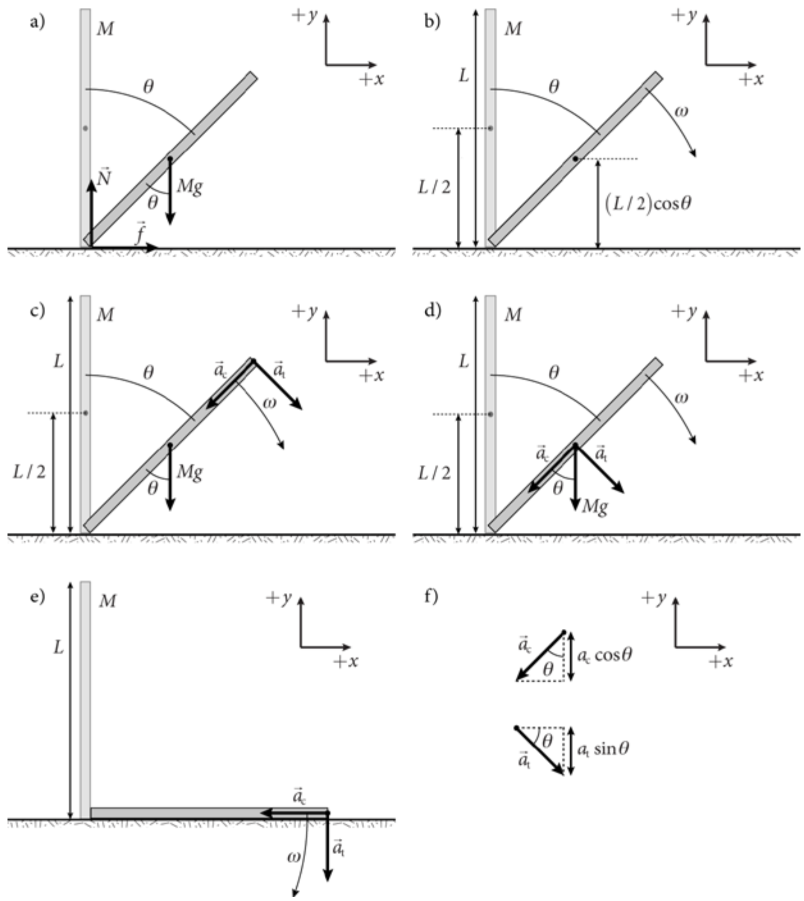
CALCULATE: $\alpha = \dfrac{2.00\text{ kg}\left(9.81\text{ m/s}^2\right)}{\left(2.00\text{ kg}+\frac{4}{9}(40.0\text{ kg})\right)(0.300\text{ m})} = 3.3067\text{ rad/s}^2$

ROUND: $\alpha = 3.31\text{ rad/s}^2$

DOUBLE-CHECK: Given the small hanging mass and the large mass of the wheel, this acceleration is reasonable.

10.56. **THINK:** As the rod with mass, M = 250.0 g and length, L = 50.0 cm, tips over, the torque, τ, caused by the force of gravity on the center of mass will change, which means the angular acceleration, α, of the rod will change with the angle it makes with the vertical. We can use energy conservation to calculate the angular velocity, ω, of the rod for any angle. The linear acceleration of any point on the rod is equal to the sum of the tangential acceleration plus the centripetal acceleration.

SKETCH:



RESEARCH: (a) In part a) of the sketch, we can see that the three forces acting on the rod are the normal force exerted by the table, the force of friction between the rod and the surface of the table, and the force of gravity.

(b1) To calculate the speed of the rod at $\theta = 45.0°$, we can use energy conservation. Conservation of mechanical energy gives us $K + U = K_0 + U_0$. The kinetic energy before the rod begins to fall is zero and at angle θ the kinetic energy is given by the kinetic energy of rotation $K = (1/2)I\omega^2$ where ω is the angular velocity and $I = (1/3)ML^2$. The potential energy before is $U_0 = mg(L/2)$ and the potential energy at angle θ is $U = mg(L/2)\cos\theta$ as illustrated in part b) of the sketch.

(b2) To calculate the vertical acceleration of the moving end of the rod, we need to calculate the tangential acceleration and the centripetal acceleration. The tangential acceleration can be calculated by realizing that that the force of gravity exerts a torque on the rod given by $\tau = mg(L/2)\sin\theta$ assuming the pivot on the table at the end of the rod. The angular acceleration is given by $\tau = I\alpha$ where $I = (1/3)ML^2$. The tangential acceleration can then be calculated from $a_t = L\alpha$. The centripetal acceleration is given by $a_c = L\omega^2$ where ω was obtained in part b1). As shown in part f) of the sketch, the vertical component of the tangential acceleration is $a_t \sin\theta$ and the vertical component of the centripetal acceleration is $a_c \cos\theta$.

(b3) To calculate the normal force exerted by the table on the rod, we need to calculate the vertical component of the tangential and centripetal acceleration of the center of mass of the rod. The angular acceleration is the same as calculated in part b2) so the tangential acceleration is $a_t = (L/2)\alpha$. The centripetal acceleration is $a_c = (L/2)\omega^2$. The vertical components of the tangential and centripetal acceleration are then $a_t \sin\theta$ and $a_c \cos\theta$ respectively. The normal force is then given by $N - Mg = Ma_v$, where a_v is the vertical acceleration of the center of mass of the rod.

(c) When the rod falls on the table, $\theta = 90.0°$ and we have

$$a_t = \frac{3}{2} g \sin 90.0° = \frac{3}{2} g.$$

The centripetal acceleration is given by

$a_c = L\omega^2$ where

$$\omega = \sqrt{\frac{3g}{L}\left(1 - \cos 90.0°\right)} = \sqrt{\frac{3g}{L}} \;\Rightarrow\; a_c = L\frac{3g}{L} = 3g.$$

SIMPLIFY:

(b1) We can combine the equations in b1) to obtain

$$\frac{1}{2}I\omega^2 + mg\frac{L}{2}\cos\theta = mg\frac{L}{2}.$$

We can rewrite the previous equation as

$$\frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 = mg\frac{L}{2}\left(1 - \cos\theta\right) \Rightarrow \omega = \sqrt{\frac{3g}{L}\left(1 - \cos\theta\right)}.$$

(b2) We can combine the equations in (b2) to get the tangential acceleration a_t

$$mg\frac{L}{2}\sin\theta = \frac{1}{3}mL^2\frac{a_t}{L} \to g\frac{1}{2}\sin\theta = \frac{1}{3}a_t \Rightarrow a_t = \frac{3}{2}g\sin\theta.$$

We can then write the vertical component of the acceleration as

$$a_v = a_t \sin\theta + a_c \cos\theta = -\frac{3}{2}g\sin^2\theta - L\omega^2\cos\theta.$$

(b3) We can combine the equations in (b3) to get

$$mg\frac{L}{2}\sin\theta = \frac{1}{3}mL^2\frac{a_t}{\left(L/2\right)} \to g\frac{1}{2}\sin\theta = \frac{2a_t}{3} \Rightarrow a_t = \frac{3}{4}g\sin\theta.$$

We can now write the vertical component of the acceleration as

$$a_v = -\frac{3}{4}g\sin^2\theta - \frac{L}{2}\omega^2\cos\theta.$$

The normal force is

$$N = m\left(g + a_v\right) = m\left(g - \frac{3}{4}g\sin^2\theta - \frac{L}{2}\omega^2\cos\theta\right).$$

(c) The linear acceleration at $\theta = 90.0°$ is

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(3g\right)^2 + \left(\frac{3}{2}g\right)^2} = 3g\sqrt{1 - \frac{1}{4}} = 3g\sqrt{\frac{3}{4}} = \frac{3\sqrt{3}}{2}g.$$

CALCULATE:

(a) Not necessary.

(b1) $\omega = \sqrt{\dfrac{3\left(9.81 \text{ m/s}^2\right)}{0.500 \text{ m}}\left(1 - \cos 45.0°\right)} = 4.1521 \text{ rad/s}.$

(b2) $a_v = -\dfrac{3}{2}\left(9.81 \text{ m/s}^2\right)\sin^2 45.0° - \left(0.500 \text{ m}\right)\left(4.1521 \text{ rad/s}\right)^2 \cos 45.0° = -13.453 \text{ m/s}^2.$

(b3) $N = \left(0.2500 \text{ kg}\right)\left(9.81 \text{ m/s}^2 - \dfrac{3}{4}\left(9.81 \text{ m/s}^2\right)\sin^2 45.0° - \dfrac{0.500 \text{ m}}{2}\left(4.1521 \text{ s}^{-1}\right)^2 \cos 45.0°\right) = 0.77091 \text{ N}.$

(c) $a = \dfrac{3\sqrt{3}}{2}\left(9.81 \text{ m/s}^2\right) = 25.487 \text{ m/s}^2.$

ROUND: Three significant figures:

(a) Not necessary

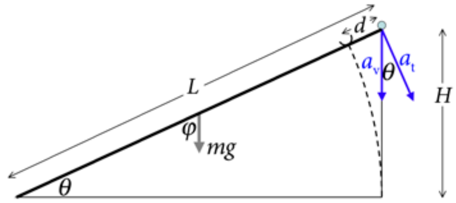
(b) $\omega = 4.15$ rad/s, $a_v = -13.5$ m/s^2 and $N = 0.771$ N.

(c) $a = 25.5$ m/s^2.

DOUBLE-CHECK: When θ 0, $\omega = 0$, i.e. the rod is perfectly upright and not rotating, looking at the equation for the normal force it can be seen that the normal force is equal to the force of gravity. The values for the accelerations may seem surprising because they are larger than g. However, we have to remember that the force of friction and the normal force must provide the centripetal force necessary to keep the rod rotating around one end. Note that the assumption that the friction force can provide the required centripetal force all the way to $\theta = 90.0°$ is unrealistic.

10.57. **THINK:** If we place the ball (blue dot) at the end of the board, it can only be caught by the cup (half circle), if the end of the board falls with a vertical component of the acceleration a_v, which is greater (or at least equal) to g. The cup is to be placed at a distance d away from the end so that it can be vertically under the ball and catch it when the board lands on the ground.

SKETCH:



RESEARCH: The board rotates about its lower end and has the same moment of inertia as a rod, $I = \frac{1}{3}mL^2$. The torque equation is $\tau = I\alpha$, where the torque is given by $\tau = Fr\sin\varphi = mg \cdot \frac{1}{2}L \cdot \sin\varphi$. The angular and tangential acceleration are related to each other via $a_t = \alpha L$. The vertical component of the tangential acceleration is then (see sketch) $a_v = a_t \cos\theta$.

Geometrical relations: Since $\theta = 90° - \varphi$, (see sketch), we find that $\sin\varphi = \cos\theta$. Also from the sketch, we see that the height of the vertical support stick is $H = L\sin\theta$. In addition (dashed circular segment in the sketch), we see that $d = L - L\cos\theta = L(1-\cos\theta)$.

SIMPLIFY:

$\tau = \frac{1}{2}mgL\sin\varphi = \frac{1}{2}mgL\cos\theta = \frac{1}{3}mL^2\alpha = \frac{1}{3}mLa_t \Rightarrow a_t = \frac{3}{2}g\cos\theta$

Inserting this result into $a_v = a_t\cos\theta$ from above, we find $a_v = \frac{3}{2}g\cos^2\theta$. If, as required, $a_v \geq g$, this means $\frac{3}{2}g\cos^2\theta \geq g$ or $\cos\theta \geq \sqrt{\frac{2}{3}}$. Since $\sin^2\theta + \cos^2\theta = 1$, this implies $\sin\theta \leq \sqrt{\frac{1}{3}}$. So, finally, from $H = L\sin\theta$ we see $H \leq \sqrt{\frac{1}{3}}L$.

CALCULATE:

(a) $H_{max} = \sqrt{\frac{1}{3}}L = \sqrt{\frac{1}{3}}(1.00 \text{ m}) = 0.57735$ m.

(b) $d = L(1-\cos\theta) = (1.00 \text{ m})(1-\sqrt{\frac{2}{3}}) = 0.1835$ m.

ROUND: Rounding to 3 digits leaves us with

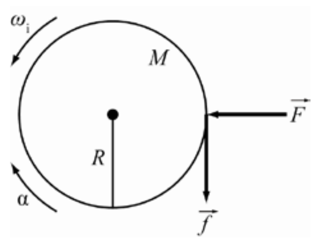
(a) $H_{max} = 0.577$ m.

(b) $d = 0.184$ m.

DOUBLE-CHECK: Clearly, this is a somewhat surprising result. However, it is also a standard lecture demonstration. When you see it you can convince yourself that these calculations for $\sin\theta$ are correct.

10.58. **THINK:** If the brakes applies an inward radial force, F = 100. N, and the contact has a coefficient of friction, $\mu_k = 0.200$, then this frictional force, f, will be perpendicular to F and cause a torque, τ, on the flywheel of mass, M = 120. kg, and radius, R = 80.0 cm. The torque can be used to determine the angular acceleration, α, of the wheel. Kinematics can then be used to determine the number of revolutions, n, the wheel will make and the time it will take for it to come to rest. The work done by torque should be the change in rotational energy, by conservation of energy. The flywheel has an initial angular speed of 500. rpm or $50\pi/3$ rad/s.

SKETCH:



RESEARCH: Similarly to the relation between the normal force and friction, $f = \mu_{\text{k}} F$, the friction causes a torque, $\tau = fR = I\alpha$, where the moment of inertia of the wheel is $I = MR^2/2$. Kinematics is used to determine the number of revolutions and the time it takes to come to an end, $\omega_{\text{f}}^2 = \omega_{\text{i}}^2 - 2\alpha\Delta\theta$ and $\omega_{\text{f}} - \omega_{\text{i}} = -\alpha t$. The work done by the friction is $W = \Delta K = -I\omega_{\text{i}}^2/2$.

SIMPLIFY: The angular acceleration is given by $\alpha = \frac{fR}{I} = \frac{\mu_{\text{k}} FR}{\frac{1}{2}MR^2} = \frac{2\mu_{\text{k}} F}{MR}$. Therefore, the total angular displacement is given by $\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta \Rightarrow |\Delta\theta| = \frac{\omega_0^2}{2\alpha}$. The number of revolutions, n, is given by $\frac{\Delta\theta}{2\pi} = \frac{\omega_0^2}{4\pi\alpha}$. The time to come to rest is $t = \omega_{\text{i}}/\alpha$. The work done is then $-MR^2\omega_{\text{i}}^2/4$.

CALCULATE:

$$\left(\frac{500 \text{ revolutions}}{1 \text{ min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ revolution}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = \frac{50\pi}{3} \text{ rad/sec}$$

$$\alpha = \frac{2(0.200)(100.\text{ N})}{(120.\text{ kg})(0.800\text{ m})} = 0.4167 \text{ rad/s}^2, \quad n = \frac{(50\pi/3 \text{ rad/s})^2}{4\pi 0.4167 \text{ rad/s}^2} = 523.60 \text{ revolutions}$$

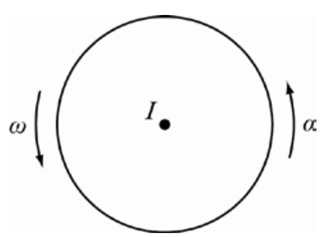
$$t = \frac{50\pi/3 \text{ rad/s}}{0.4167 \text{ rad/s}^2} = 125.66 \text{ s}, \quad W = -\frac{1}{4}(120.\text{ kg})(0.800\text{ m})^2\left(\frac{50\pi}{3}\text{ rad/s}\right)^2 = -52638 \text{ J}$$

ROUND: To three significant figures: $n = 524$ revolutions, $t = 126$ s and $W = -5.26 \cdot 10^4$ J.

DOUBLE-CHECK: Given the small friction, and hence the small torque, and the fast speed of the wheel, it would take a long time to stop, so the results are reasonable.

10.59. **THINK:** Assuming a constant angular acceleration, α, and $\Delta t = 25$ s, regular kinematics can be used to determine α and $\Delta\theta$. The total work done by torque, τ, should be converted entirely into rotational energy. $I = 25.0 \text{ kg m}^2$ and $\omega_{\text{f}} = 150.$ rad/s.

SKETCH:



RESEARCH: From kinematics, $\omega_\text{f} - \omega_\text{i} = \alpha\Delta t$ and $\Delta\theta = \alpha(\Delta t)^2/2.$ The torque on the wheel is $\tau = I\alpha.$ Since the torque is constant, the work done by it is $W = \tau\Delta\theta.$ The kinetic energy of the turbine is $K = I\omega^2/2.$

SIMPLIFY:

(a) $\omega_\text{f} = \alpha\Delta t \Rightarrow \alpha = \omega_\text{f}/\Delta t$

(b) $\tau = I\alpha$

(c) $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2$

(d) $W = \tau\Delta\theta$

(e) $K = \frac{1}{2}I\omega_\text{f}^2$

CALCULATE:

(a) $\alpha = \dfrac{150.\text{ rad/s}}{25.0\text{ s}} = 6.00\text{ rad/s}^2$

(b) $\tau = (25.0\text{ kg m}^2)(6.00\text{ rad/s}^2) = 150.\text{ N m}$

(c) $\Delta\theta = \frac{1}{2}(6.00\text{ rad/s}^2)(25.0\text{ s})^2 = 1875\text{ rad}$

(d) $W = (150.\text{ N m})(1875\text{ rad}) = 281{,}250\text{ J}$

(e) $K = \frac{1}{2}(25.0\text{ kg m}^2)(150.\text{ rad/s})^2 = 281{,}250\text{ J}$

ROUND:

(a) $\alpha = 6.00\text{ rad/s}^2$

(b) $\tau = 150.\text{ N m}$

(c) $\Delta\theta = 1880\text{ rad}$

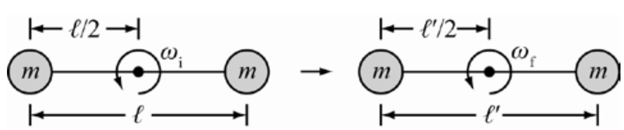
(d) $W = 281\text{ kJ}$

(e) $K = 281\text{ kJ}$

DOUBLE-CHECK: It is expected that the work and kinetic energy are equal. Since they were each determined independently and they are the same value, the procedure must have been correct.

10.60. **THINK:** Since the two masses have an equal mass of m = 6.00 kg, their center of mass will be at the geometric center, $l/2$, which is also the location of the axis of rotation. Initially, l = 1.00 m and then it extends to 1.40 m. When the length increases, the moment of inertia also increases. Since there are no external torques, conservation of angular momentum can be applied. The masses initially rotate at $\omega_\text{i} = 5.00\text{ rad/s}.$

SKETCH:



RESEARCH: The angular momentum before and after are $L_\text{i} = I_\text{i}\omega_\text{i}$ and $L_\text{f} = I_\text{f}\omega_\text{f}$. The moments of inertia for before and after are $I_\text{i} = 2m(l/2)^2$ and $I_\text{f} = 2m(l'/2)^2$. The conservation of angular momentum is represented by $L_\text{i} = L_\text{f}$.

SIMPLIFY: $L_i = L_f \Rightarrow 2m\left(\frac{l^2}{4}\right)\omega_i = 2m\left(\frac{l'^2}{4}\right)\omega_f \Rightarrow l^2\omega_i = l'^2\omega_f \Rightarrow \omega_f = \left(\frac{l}{l'}\right)^2\omega_i$

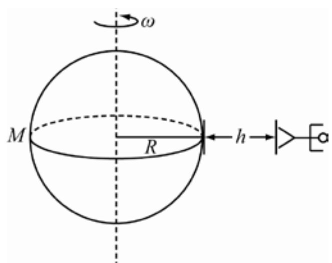
CALCULATE: $\omega_f = \left(\frac{1.00 \text{ m}}{1.40 \text{ m}}\right)^2 5.00 \text{ rad/s} = 2.551 \text{ rad/s}$

ROUND: To three significant figures: $\omega_f = 2.55 \text{ rad/s}$

DOUBLE-CHECK: Since the string length and hence the moment of inertia increases, a smaller rotational speed is expected, since angular momentum is conserved.

10.61. **THINK:** For the moment of inertia of the Earth, treat it as a solid sphere with mass, $M = 5.977 \cdot 10^{24}$ kg, and radius, R = 6371 km. The Chinese ($n = 1.30 \cdot 10^9$ people and m = 70.0 kg each) can be treated as a point mass of total mass, nm, standing on the surface of the Earth and then also at h = 1.00 m above the surface. Conservation of angular momentum relates the change in moment of inertia to the change in angular frequency, and hence period, of the Earth.

SKETCH:



RESEARCH: The Earth's (solid sphere) moment of inertia is $I_E = 2MR^2/5$, while the Chinese have a moment of inertia of $I_C = nmR^2$ on the surface of the Earth and $I'_C = nm(R+h)^2$ when standing on the chair. The angular momentum is $L = I\omega$. The period of the Earth's rotation is 1 day or 86,400 s, and is related to the angular velocity by $\omega = 2\pi/T$.

SIMPLIFY:

(a) The moment of inertia of Earth is $I_E = 2MR^2/5$.

(b) The moment of inertia of the Chinese people on Earth is $I_C = nmR^2$.

(c) The moment of inertia of the Chinese people on chairs is $I'_C = nm(R+1)^2$. The change in the moment of inertia for the Chinese people is $\Delta I_C = I'_C - I_C = nm(R^2 + 2R + 1 - R^2) = nm(2R+1)$.

(d) The conservation of angular momentum states, $(I_E + I_C)\omega_i = (\Delta I_C)\Delta\omega \Rightarrow (I_E + I_C)\frac{2\pi}{T_i} = (\Delta I_C)\frac{2\pi}{\Delta T}$.

Therefore, $\frac{\Delta T}{T} = \frac{\Delta I_C}{I_E + I_C}$ (fractional change) and $\Delta T = \frac{\Delta I_C}{I_E + I_C}T$ (total change).

CALCULATE:

(a) $I_E = \frac{2}{5}(5.977 \cdot 10^{24} \text{ kg})(6,371,000 \text{ m})^2 = 9.704 \cdot 10^{37} \text{ kg m}^2$

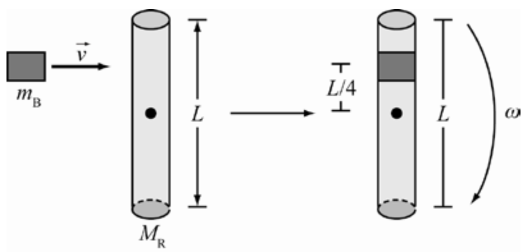
(b) $I_C = (1.30 \cdot 10^9)(70.0 \text{ kg})(6,371,000 \text{ m})^2 = 3.694 \cdot 10^{24} \text{ kg m}^2$

(c) $\Delta I_C = (1.30 \cdot 10^9)(70.0 \text{ kg})(2(6,371,000 \text{ m}) + 1.00 \text{ m}) = 1.1595 \cdot 10^{18} \text{ kg m}^2$

(d) $\frac{\Delta T}{T} = \frac{1.1595 \cdot 10^{18} \text{ kg m}^2}{9.704 \cdot 10^{37} \text{ kg m}^2 + 3.694 \cdot 10^{24} \text{ kg m}^2} = 1.1949 \cdot 10^{-20}$

ROUND:
(a) $I_{\text{E}} = 9.704 \cdot 10^{37}$ kg m^2
(b) $I_C = 3.69 \cdot 10^{24}$ kg m^2
(c) $\Delta I_C = 1.16 \cdot 10^{18}$ kg m^2
(d) $\dfrac{\Delta T}{T} = 1.19 \cdot 10^{-20}$
DOUBLE-CHECK: Despite the large number of Chinese people, the Earth is so massive that the rotation is hardly affected by their jump onto the surface.

10.62. **THINK:** The bullet with a mass, $m_{\text{B}} = 1.00 \cdot 10^{-2}$ kg, has a linear momentum. When it strikes the rod with length, L = 1.00 m, and mass, $m_{\text{R}} =$ 5.00 kg, the rod begins to rotate about its center and thus has an angular momentum. Conservation of momentum means the bullet's linear momentum is equal to the rod and bullet's angular momentum. Likewise, the bullet has a linear kinetic energy and the rod and bullet have a rotational kinetic energy so the change in kinetic energy is the difference between these two. The bullet can be treated as a point particle that is a distance $L/4$ from the axis of rotation.
SKETCH:



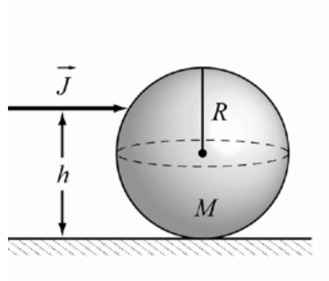
RESEARCH: The momentum of the bullet is given by $p = m_{\text{B}}v$. When it hits the rod at $L/4$ from the center, its linear momentum can be converted to angular momentum by $pL/4$. The moment of inertia of the rod is $m_{\text{R}}L^2/12$ and that of the bullet when in the rod is $m(L/4)^2$. The kinetic energy of the bullet is all translational, $K_{\text{T}} = m_{\text{B}}v^2/2$, while the kinetic energy of the bullet and rod together is all rotational, $K_{\text{R}} = I\omega^2/2$.
SIMPLIFY:
(a) The initial angular momentum is $L_{\text{i}} = m_{\text{B}}vL/4$. The final angular momentum is $L_{\text{f}} = I\omega = (m_{\text{R}}/12 + m_{\text{B}}/16)L^2\omega$. The angular velocity is then

$$L_{\text{i}} = L_{\text{f}} \Rightarrow \frac{m_{\text{B}}vL}{4} = \left(\frac{1}{12}m_{\text{R}} + \frac{1}{16}m_{\text{B}}\right)L^2\omega \Rightarrow \omega = \frac{m_{\text{B}}v}{\left(\frac{1}{3}m_{\text{R}} + \frac{1}{4}m_{\text{B}}\right)L}.$$

(b) $K_{\text{T}} = \frac{1}{2}m_{\text{B}}v^2$ and $K_{\text{R}} = \frac{1}{2}I\omega^2 = \left(\frac{1}{24}m_{\text{R}} + \frac{1}{32}m_{\text{B}}\right)L^2\omega^2$. Therefore, $\Delta K = K_{\text{R}} - K_{\text{T}}$.
CALCULATE:
(a) $$\omega = \frac{\left(1.00 \cdot 10^{-2}\text{ kg}\right)\left(100.\text{ m/s}\right)}{\left(\frac{5.00\text{ kg}}{3} + \frac{0.0100\text{ kg}}{4}\right)\left(1.00\text{ m}\right)} = 0.5991\text{ rad/s}$$
(b) $$\Delta K = \left(\frac{5.00\text{ kg}}{24} + \frac{1.00 \cdot 10^{-2}\text{ kg}}{32}\right)\left(1.00\text{ m}\right)^2\left(0.599\text{ rad/s}\right)^2 - \frac{1}{2}\left(1.00 \cdot 10^{-2}\text{ kg}\right)\left(100.\text{ m/s}\right)^2 = -49.925\text{ J}$$

ROUND:
To three significant figures,
(a) $\omega = 0.599$ rad/s
(b) $\Delta K = -49.9$ J
DOUBLE-CHECK: Given that the rod is five hundred times heavier than the bullet and that the bullet will lose energy from imbedding itself in the rod, a small ω and a negative value for ΔK is reasonable.

10.63. **THINK:** The sphere of mass, M, spins clockwise when a horizontal impulse J is exerted at a height h above the tabletop when $R < h < 2R$.
SKETCH:



RESEARCH: To calculate the linear speed after the impulse is applied, we use the fact that the impulse J can be written as $J = \Delta p = M\Delta v$. To get the angular velocity, we write the change in the angular momentum of the sphere as $\Delta L = \Delta p(h-R)$. To calculate the height where the impulse must be applied, we have to apply Newton's Second Law for linear motion, $F = Ma$, and Newton's Second Law for rotation, $\tau = I\alpha$. The torque is given by $\tau = F(h-R)$. The object rolls without slipping so from Section 10.3 we know that $v = R\omega$ and $a = R\alpha$. In addition, we can write the impulse as $J = F\Delta t$.
SIMPLIFY: a) Combining these relationships to get the linear velocity gives us

$$J = \Delta p = M\Delta v = Mv \quad \Rightarrow \quad v = \frac{J}{M}.$$

Combining these relationships to get the angular velocity gives us

$$\Delta L = \Delta p(h-R) = J(h-R)$$

$$\Delta L = I\Delta\omega = I\omega = \frac{2}{5}MR^2\omega$$

$$J(h-R) = \frac{2}{5}MR^2\omega$$

$$\omega = \frac{5J(h-R)}{2MR^2}.$$

b) Combining these relationships to get the height h_0 at which the impulse must be applied for the sphere to roll without slipping we get

$$F = Ma \quad \Rightarrow \quad \frac{J}{\Delta t} = MR\alpha$$

$$F(h_0 - R) = I\alpha \quad \Rightarrow \quad \frac{J}{\Delta t}(h_0 - R) = \frac{2}{5}MR^2\alpha.$$

Dividing these two equations gives us

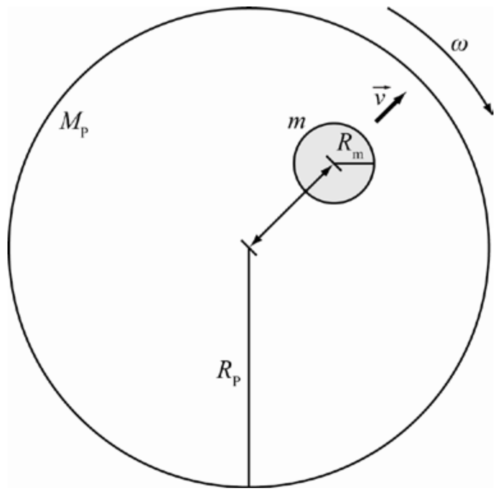
$$h_0 - R = \frac{2}{5}R \quad \Rightarrow \quad h_0 = \frac{7}{5}R.$$

CALCULATE: Not applicable.
ROUND: Not applicable.

DOUBLE-CHECK: The linear velocity will always be positive. However, the angular velocity can be positive or negative, depending on whether $h > R$ or $h < R$. The fact that $h_0 > R$ is consistent with the ball rolling to the right after the impulse is applied.

10.64. **THINK:** If the man, approximated by a cylinder of mass, m = 80.0 kg, and radius, $R_{\text{m}} = 0.200$ m, walks at a constant velocity, v = 0.500 m/s, then his distance, d, from the center of the platform of mass, $M_p = 400.$ kg, and radius, $R_{\text{p}} = 4.00$ m, will change linearly with time. The platform initially rotates at 6.00 rpm or 0.200π rad/s. Initially, the man and the platform have their center of mass on the axis of rotation, so their moments of inertia are summed. When the man is a distance, d, from the center, the parallel axis theorem is needed to determine his overall moment of inertia.

SKETCH:



RESEARCH: The distance, d, the man is from the center is $d = vt$. The moment of inertia of the platform is $I_{\text{p}} = M_{\text{p}}R_{\text{p}}^2/2$. The man has a moment of inertia of $I_{\text{m}} = mR_{\text{m}}^2/2$ and by the parallel axis theorem has a final moment of inertia of $I'_{\text{m}} = \left(mR_{\text{m}}^2/2\right) + md^2$. Conservation of angular momentum states $L_{\text{i}} = L_{\text{f}}$, where $L = \omega I$.

SIMPLIFY: The man's moment of inertia as a function of time is $I'_{\text{m}} = \left(mR_{\text{m}}^2/2\right) + mv^2t^2 = I_{\text{m}} + mv^2t^2$. The initial angular momentum of the system is $L_{\text{i}} = \omega_{\text{i}}\left(I_{\text{p}} + I_{\text{m}}\right)$. The angular momentum at time t is $L_{\text{f}} = \omega_{\text{f}}\left(I_{\text{p}} + I'_{\text{m}}\right)$. Therefore, $\left(I_{\text{p}} + I_{\text{m}}\right)\omega_{\text{i}} = \left(I_{\text{p}} + I_{\text{m}} + mv^2t^2\right)\omega_{\text{f}}$

$$\Rightarrow \omega_{\text{f}} = \frac{\left(I_{\text{p}} + I_{\text{m}}\right)\omega_{\text{i}}}{\left(I_{\text{p}} + I_{\text{m}} + mv^2t^2\right)} \Rightarrow \omega_f(t) = \omega_{\text{i}}\left(1 + \frac{2mv^2t^2}{M_{\text{p}}R_{\text{p}}^2 + mR_{\text{m}}^2}\right)^{-1}.$$

When the man reaches the end, $d = R_{\text{p}} \Rightarrow t = R_{\text{p}}/v$. Therefore, $\omega_{\text{f}} = \omega_{\text{i}}\left(1 + \dfrac{2mR_{\text{p}}^2}{M_{\text{p}}R_{\text{p}}^2 + mR_{\text{m}}^2}\right)^{-1}$.

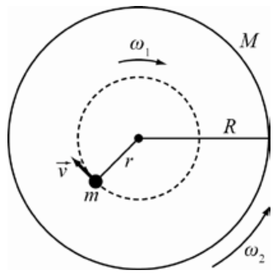
CALCULATE: $\omega_{\text{f}} = \left(0.200\pi \text{ rad/s}\right)\left(1 + \dfrac{2\left(80.0 \text{ kg}\right)\left(4.00 \text{ m}\right)^2}{\left(400. \text{ kg}\right)\left(4.00 \text{ m}\right)^2 + \left(80.0 \text{ kg}\right)\left(0.200 \text{ m}\right)^2}\right)^{-1} = 0.4489 \text{ rad/s}$

ROUND: To three significant figures: $\omega_{\text{f}} = 0.449$ rad/s.

DOUBLE-CHECK: As time increases (i.e. the man walks from the center), the overall moment of inertia increases, so a smaller angular velocity is expected.

10.65. **THINK:** Initially, the system has zero angular momentum. The boy with mass, m = 25.0 kg, can be treated as a point particle a distance, r = 2.00 m, from the center of the merry-go-round, which has a moment of inertia, $I_0 = 200.\text{ kg m}^2$. When the boy starts running with a velocity, v = 0.600 m/s, the merry-go-round will begin to rotate in the opposite direction in order to conserve angular momentum.

SKETCH:



RESEARCH: The initial angular momentum is $L_i = 0$. The angular velocity of the boy is $\omega_1 = v / r$. The moment of inertia of the boy is mr^2. The angular momentum is given by $L = I\omega$. The tangential velocity of the merry-go-round at r is $v_2 = \omega_2 r$. The boy's velocity relative to the merry-go-round (which is rotating in the opposite direction) is $v' = v + v_2$.

SIMPLIFY:

(a) $L_f = I_0\omega_2 - mr^2\omega_1 = 0 \Rightarrow \omega_2 = \dfrac{mr^2\omega_1}{I_0} \Rightarrow \omega_2 = \dfrac{mrv}{I_0}$

(b) $v' = v + r\omega_2$

CALCULATE:

(a) $\omega_2 = \dfrac{(25.0\text{ kg})(2.00\text{ m})(0.600\text{ m/s})}{200.\text{ kg m}^2} = 0.150\text{ rad/s}$

(b) $v' = 0.600\text{ m/s} + (2.00\text{ m})(0.150\text{ rad/s}) = 0.900\text{ m/s}$

ROUND:

(a) $\omega_2 = 0.150\text{ rad/s}$

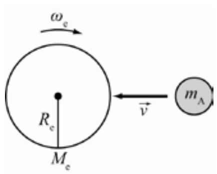
(b) $v' = 0.900\text{ m/s}$

DOUBLE-CHECK: Since the merry-go-round must move opposite to the boy, a relative velocity greater than the velocity compared to the ground makes sense. Also, since the boy and the merry-go-round have comparable moments of inertia, the comparable velocities are reasonable.

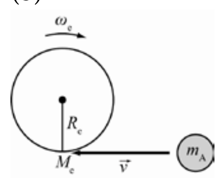
10.66. **THINK:** In every case, the momentum (angular and linear) must be conserved. If the asteroid with mass, $m_A = 1.00 \cdot 10^{22}\text{ kg}$, and velocity, $v = 1.40 \cdot 10^3\text{ m/s}$, hits the Earth, which has an angular speed of $\omega_E = 7.272 \cdot 10^{-5}\text{ rad/s}$, dead on (radially inward), then it will not contribute any of its linear momentum to the angular momentum of the planet, meaning the change in the Earth's rotation is solely a result of it gaining mass. If the asteroid hits the planet tangentially, then the full amount of the asteroid's linear momentum is contributed to the angular momentum. If the asteroid hits in the direction of Earth's rotation, it will add its momentum and the Earth will spin faster and vice versa for the opposite direction. The mass of the Earth is $m_E = 5.977 \cdot 10^{24}\text{ kg}$ and the radius is $R_E = 6371\text{ km}$. The Earth can be treated as a solid sphere.

SKETCH:

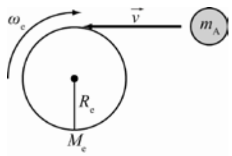
(a)



(b)



(c)



RESEARCH: The moment of inertia of Earth is $I_E = 2M_ER_E^2/5$. After the asteroid has collided, the moment of inertia of the system is then given by $I_T = I_E + m_AR_E^2$. The angular momentum is $L = I\omega$. Conservation of angular momentum applies in each case. The momentum the asteroid contributes is $p = m_Av$ and its linear momentum will be $\pm pR_E$, depending on which way it hits.

SIMPLIFY:

(a) $I_E\omega_E = I_T\omega_F \Rightarrow \omega_F = \dfrac{\frac{2}{5}M_ER_E^2}{\frac{2}{5}M_ER_E^2 + M_AR_E^2}\omega_E \Rightarrow \omega_F = \dfrac{2M_E}{2M_E + 5M_A}\omega_E$

(b) $I_E\omega_E + M_AvR_E = I_T\omega_F \Rightarrow \omega_F = \dfrac{\frac{2}{5}M_ER_E^2\omega_E + M_AvR_E}{\frac{2}{5}M_ER_E^2 + M_AR_E^2} \Rightarrow \omega_F = \dfrac{\frac{2}{5}M_ER_E\omega_E + M_Av_A}{\frac{2}{5}M_ER_E + M_AR_E}$

(c) $I_E\omega_E - M_AvR_E = I_T\omega_F \Rightarrow \omega_F = \dfrac{\frac{2}{5}M_ER_E\omega_E - M_Av_A}{\frac{2}{5}M_ER_E + M_AR_E}$

CALCULATE:

(a) $\omega_F = \dfrac{2\left(5.977\cdot 10^{24}\text{ kg}\right)\left(7.272\cdot 10^{-5}\text{ rad/s}\right)}{2\left(5.977\cdot 10^{24}\text{ kg}\right) + 5\left(1.00\cdot 10^{22}\text{ kg}\right)} = 7.2417\cdot 10^{-5}\text{ rad/s}$

(b)

$$\omega_F = \frac{\frac{2}{5}\left(5.977\cdot 10^{24}\text{ kg}\right)\left(6371000\text{ m}\right)\left(7.272\cdot 10^{-5}\text{ rad/s}\right) + \left(1.00\cdot 10^{22}\text{ kg}\right)\left(1.40\cdot 10^{3}\text{ m/s}\right)}{\frac{2}{5}\left(5.977\cdot 10^{24}\text{ kg}\right)\left(6371000\text{ m}\right) + \left(1.00\cdot 10^{22}\text{ kg}\right)\left(6371000\text{ m}\right)} = 7.333\cdot 10^{-5}\text{ rad/s}$$

(c)

$$\omega_F = \frac{\frac{2}{5}\left(5.977\cdot 10^{24}\text{ kg}\right)\left(6371000\text{ m}\right)\left(7.272\cdot 10^{-5}\text{ rad/s}\right) - \left(1.00\cdot 10^{22}\text{ kg}\right)\left(1.40\cdot 10^{3}\text{ m/s}\right)}{\left(\frac{2}{5}\left(5.977\cdot 10^{24}\text{ kg}\right) + \left(1.00\cdot 10^{22}\text{ kg}\right)\right)\left(6371000\text{ m}\right)} = 7.1502\cdot 10^{-5}\text{ rad/s}$$

ROUND:

To three significant figures:

(a) $\omega_F = 7.24\cdot 10^{-5}\text{ rad/s}$

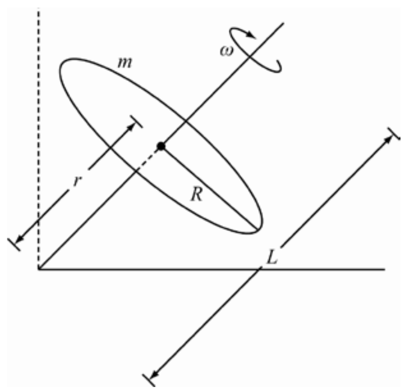
(b) $\omega_F = 7.33 \cdot 10^{-5}$ rad/s

(c) $\omega_F = 7.15 \cdot 10^{-5}$ rad/s

DOUBLE-CHECK: In part (a), it is expected that ω would be reduced very little, since the Earth gains a 0.4% mass on the surface and the moment of inertia is changed only slightly. In part (b), the asteroid would make the Earth spin faster, provided the velocity was great enough. In part (c), the asteroid would definitely make the Earth slow down its rotation.

10.67. **THINK:** If the disk with radius, R = 40.0 cm, is rotating at 30.0 rev/s, then the angular speed, ω, is 60.0π rad/s. The length of the gyroscope is L = 60.0 cm, so that the disk is located at $r = L/2$ from the pivot.

SKETCH:



RESEARCH: The precessional angular speed is given by $\omega_p = rmg / I\omega$. The moment of inertia of the disk is $I = mR^2/2$.

SIMPLIFY: $\omega_p = \dfrac{\frac{L}{2}mg}{\frac{1}{2}mR^2\omega} = \dfrac{Lg}{R^2\omega}$

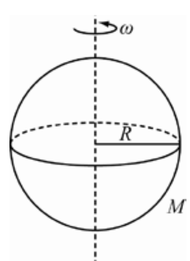
CALCULATE: $\omega_p = \dfrac{0.600 \text{ m}\left(9.81 \text{ m/s}^2\right)}{\left(0.400 \text{ m}\right)^2 60.0\pi \text{ rad/s}} = 0.19516$ rad/s

ROUND: $\omega_p = 0.195$ rad/s

DOUBLE-CHECK: The precession frequency is supposed to be much less than the frequency of the rotating disk. In this example, the disk frequency is about one thousand times the precession frequency, so it makes sense.

10.68. **THINK:** Assume the star with a mass, $M = 5.00 \cdot 10^{30}$ kg, is a solid sphere. After the star collapses, the total mass remains the same, only the radius of the star has changed. Initially, the star has radius, $R_i = 9.50 \cdot 10^8$ m, and period, $T_i = 30.0 \text{ days} = 2592000$ s, while after the collapse it has a radius, $R_f = 10.0$ km, and a period, T_f. To determine the final period, consider the conservation of angular momentum.

SKETCH:



RESEARCH: The moment of inertia of the star is $I = 2MR^2/5$, so initially it is $I_\text{i} = 2M\text{R}_\text{i}^2/5$ and afterwards it is $I_\text{f} = 2M\text{R}_\text{f}^2/5$. Angular momentum is conserved, so $L_\text{i} = L_\text{f}$, where $L = I\omega$. The period is related to the angular frequency by $T = 2\pi/\omega$ or $\omega = 2\pi/T$.

SIMPLIFY: $I_\text{i}\omega_\text{i} = I_\text{f}\omega_\text{f} \Rightarrow \dfrac{\frac{2}{5}M\text{R}_\text{i}^2(2\pi)}{T_\text{i}} = \dfrac{\frac{2}{5}M\text{R}_\text{f}^2(2\pi)}{T_\text{f}} \Rightarrow T_\text{f} = \dfrac{\text{R}_\text{f}^2}{\text{R}_\text{i}^2}T_\text{i}$

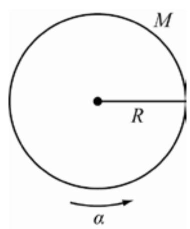
CALCULATE: $T_\text{f} = \dfrac{\left(10.0\cdot 10^3\text{ m}\right)^2\left(2{,}592{,}000\text{ s}\right)}{\left(9.50\cdot 10^8\text{ m}\right)^2} = 2.872\cdot 10^{-4}\text{ s}$

ROUND: $T_\text{f} = 2.87\cdot 10^{-4}\text{ s}$

DOUBLE-CHECK: Given the huge reduction in size, a large reduction in period, or increase in angular velocity is expected.

10.69. **THINK:** The flywheel with radius, R = 3.00 m, and $M = 1.18\cdot 10^6$ kg, rotates from rest to $\omega_\text{f} = 1.95$ rad/s in $\Delta t = 10.0\text{ min} = 600.\text{ s}$. The wheel can be treated as a solid cylinder. The angular acceleration α, can be determined using kinematics. The angular acceleration is then used to determine the average torque, τ.

SKETCH:



RESEARCH: The energy is all rotational kinetic energy, so $E = I\omega^2/2$. The moment of inertia of the wheel is $I = MR^2/2$. From kinematics, $\omega_\text{f} - \omega_\text{i} = \alpha\Delta t$. The torque is then given by $\tau = I\alpha$.

SIMPLIFY: The total energy is given by $E = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}MR^2\omega^2$. The angular acceleration is given by $\omega_\text{f} - 0 = \alpha\Delta t \Rightarrow \alpha = \omega_\text{f}/\Delta t$. The torque needed is

$$\tau = I\alpha = \left(\frac{1}{2}MR^2\right)\left(\frac{\omega_\text{f}}{\Delta t}\right) = \frac{MR^2\omega_\text{f}}{2\Delta t}.$$

CALCULATE: $E = \frac{1}{4}\left(1.18\cdot 10^6\text{ kg}\right)\left(3.00\text{ m}\right)^2\left(1.95\text{ rad/s}\right)^2 = 1.0096\cdot 10^7\text{ J}$

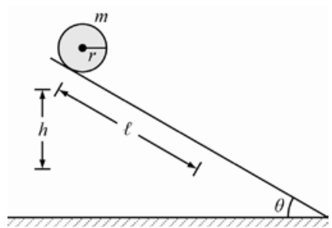
$$\tau = \frac{\left(1.18\cdot 10^6\text{ kg}\right)\left(3.00\text{ m}\right)^2\left(1.95\text{ rad/s}\right)}{2\left(600.\text{ s}\right)} = 17257.5\text{ N m}$$

ROUND: $E = 1.01\cdot 10^7$ J, $\tau = 17{,}300$ N m

DOUBLE-CHECK: The problem mentions that a huge amount of energy is needed for the experiment and the resulting energy is huge. It is reasonable that a huge torque would also be required.

10.70. **THINK:** With no friction and no slipping, energy is conserved. The potential energy of the hoop of mass, m = 2.00 kg, and radius, r = 50.0 cm, will be converted entirely into translational and rotational kinetic energy at l = 10.0 m down the incline with an angle of $\theta = 30.0°$. For a hoop, c = 1.

SKETCH:



RESEARCH: The change in height of the hoop is $h = l\sin\theta$. The initial potential energy of the hoop is *mgh*. The kinetic energy of the hoop is $K_f = (1+c)mv^2/2$. The *c* value for the hoop is 1.

SIMPLIFY: $U_i = K_f \Rightarrow mgl\sin\theta = (1+c)mv^2/2 \Rightarrow v = \sqrt{\dfrac{2gl\sin\theta}{1+c}}$

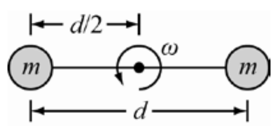
CALCULATE: $v = \sqrt{\dfrac{2(9.81\text{ m/s}^2)(10.0\text{ m})\sin(30.0°)}{1+1}} = 7.004\text{ m/s}$

ROUND: To three significant figures: $v = 7.00$ m/s

DOUBLE-CHECK: This value is less than the velocity the hoop would have going a distance, *h*, in free fall (v = 9.9 m/s), so it seems reasonable.

10.71. **THINK:** The oxygen atoms, $m = 2.66\cdot 10^{-26}$ kg, can be treated as point particles a distance, *d*/2 (where $d = 1.21\cdot 10^{-10}$ m) from the axis of rotation. The angular speed of the atoms is $\omega = 4.60\cdot 10^{12}$ rad/s.

SKETCH:



RESEARCH: Since the masses are equal point particles, the moment of inertia of the two is $I = 2m(d/2)^2$. The rotational kinetic energy is $K = I\omega^2/2$.

SIMPLIFY:

(a) $I = 2m\left(\dfrac{d^2}{4}\right) = \dfrac{1}{2}md^2$

(b) $K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{4}md^2\omega^2$

CALCULATE:

(a) $I = \dfrac{1}{2}(2.66\cdot 10^{-26}\text{ kg})(1.21\cdot 10^{-10}\text{ m})^2 = 1.9473\cdot 10^{-46}\text{ kg m}^2$

(b) $K = \dfrac{1}{4}(2.66\cdot 10^{-26}\text{ kg})(1.21\cdot 10^{-10}\text{ m})^2(4.60\cdot 10^{12}\text{ rad/s})^2 = 2.06\cdot 10^{-21}\text{ J}$

ROUND:

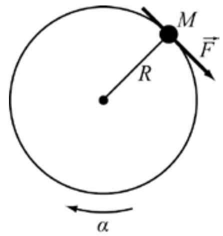
(a) $I = 1.95\cdot 10^{-46}\text{ kg m}^2$

(b) $K = 2.06\cdot 10^{-21}$ J

DOUBLE-CHECK: Since an oxygen molecule is so small, a very small moment of inertia and energy are expected.

10.72. **THINK:** If the force, *F*, is tangent to the circle's radius, then the angle between it and the radius, *R* = 0.40 m, is 90°. The bead with mass, *M* = 0.050 kg, can be treated as a point particle. The required angular acceleration, $\alpha = 6.0\text{ rad/s}^2$, is then found using the torque, τ.

SKETCH:



RESEARCH: The force produces a torque, $\tau = FR = I\alpha$. The moment of inertia of the bead is $I = MR^2$.

SIMPLIFY: $FR = I\alpha = MR^2\alpha \Rightarrow F = MR\alpha$.

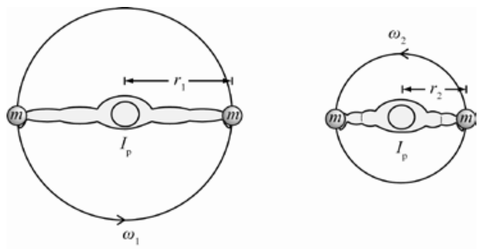
CALCULATE: $F = (0.0500 \text{ kg})(0.400 \text{ m})(6.00 \text{ rad/s}^2) = 0.120 \text{ N}$

ROUND: To three significant figures: $F = 0.120 \text{ N}$

DOUBLE-CHECK: For a small mass, a small force is reasonable.

10.73. **THINK:** Angular momentum will be conserved when the professor brings his arms and the two masses, because there is no external torque.

SKETCH:



RESEARCH: Conservation of angular momentum states $I_i\omega_i = I_f\omega_f$, and the moment of inertia at any point is $I = I_{\text{body}} + 2r^2m$. We assume that $I_{\text{body,i}} = I_{\text{body,f}}$.

SIMPLIFY: Substituting the moments of inertia into the conservation equation gives

$$\omega_f = \frac{I_i}{I_f}\omega_i = \frac{I_{\text{body,i}} + 2r_i^2 m}{I_{\text{body,f}} + 2r_f^2 m}\omega_i.$$

CALCULATE:

The initial angular speed is $\omega_i = 2\pi f = 2\pi(1.00 \text{ rev/min}) = 0.1047 \text{ rad/s}$.

So the final angular speed is

$$\omega_f = \frac{(2.80 \text{ kg m}^2) + 2(1.20 \text{ m})^2(5.00 \text{ kg})}{(2.80 \text{ kg m}^2) + 2(0.300 \text{ m})^2(5.00 \text{ kg})}(0.1047 \text{ rad/s}) = 0.4867135 \text{ rad/s}^{-1}.$$

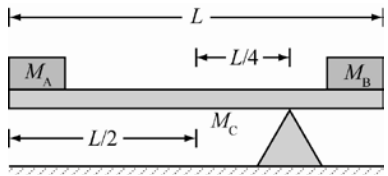
ROUND:

$\omega_f = 0.487 \text{ rad/s}$.

DOUBLE-CHECK: We find that the angular velocity increases from 0.105 rad/s to 0.487 rad/s. Does it make sense that the professor speeds up by pulling in the arms? If you have ever watched a figure skating competition, you know that the answer is yes, and that speeding up the rate of rotation by a factor of ~3 is very reasonable.

10.74. **THINK:** Determine the angular acceleration, which can be obtained by first determining the total torque. Make sure that the moments of inertia are calculated with respect to the pivot point. $M_A = 1.00$ kg, $M_B = 10.0$ kg, $M_C = 20.0$ kg and $L = 5.00$ m.

SKETCH:



RESEARCH: $\sum \tau = I\alpha$

For M_A: $I_A = M_A(3L/4)^2$. For M_B: $I_B = M_B(L/4)^2$. For the rod: $I_C = (1/12)M_C L^2 + M_C(L/4)^2$.
$I = I_A + I_B + I_C$, $\sum \tau = \tau_A + \tau_C - \tau_B$, $\tau_A = M_A g(3L/4)$, $\tau_B = M_B g(L/4)$ and $\tau_C = M_C g(L/4)$.

SIMPLIFY: $\sum \tau = gL\left(\frac{3}{4}M_A - \frac{1}{4}M_B + \frac{1}{4}M_C\right) = \frac{gL}{4}(3M_A - M_B + M_C)$

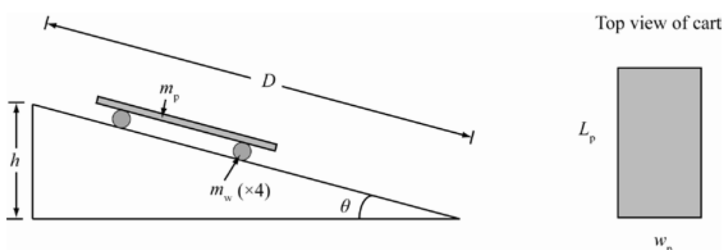
$$I = L^2\left(\frac{9}{16}M_A + \frac{1}{16}M_B + \left(\frac{1}{12} + \frac{1}{16}\right)M_C\right) = \frac{L^2}{16}\left(9M_A + M_B + \frac{7}{3}M_C\right), \quad \sum \tau = I\alpha \Rightarrow \alpha = \frac{\sum \tau}{I}$$

$$\alpha = \frac{gL}{4}\left(\frac{16}{L^2}\right)\left(\frac{3M_A - M_B + M_C}{9M_A + M_B + \frac{7}{3}M_C}\right) = \frac{4g}{L}\left(\frac{3M_A - M_B + M_C}{9M_A + M_B + \frac{7}{3}M_C}\right)$$

CALCULATE: $\alpha = \frac{4(9.81 \text{ m/s}^2)}{5.00 \text{ m}}\left(\frac{3(1.00 \text{ kg}) - 10.0 \text{ kg} + 20.0 \text{ kg}}{9(1.00 \text{ kg}) + 10.0 \text{ kg} + \frac{7}{3}20.0 \text{ kg}}\right) = 1.55 \text{ rad/s}^2$

ROUND: Using three significant figures, $\alpha = 1.55 \text{ rad/s}^2$. The positive sign indicates that the angular acceleration is counter clockwise.
DOUBLE-CHECK: Note that α decreases as *L* increases. This makes sense because *I* increases faster with *L* (*L* is squared) than does τ.

10.75. **THINK:** To determine the cart's final speed, use the conservation of energy. The initial gravitational potential energy is converted to kinetic energy. The total kinetic energy at the bottom is the sum of the translational and rotational kinetic energies. Use $m_p = 8.00$ kg, $m_w = 2.00$ kg, $L_p = 1.20$ m, $w_p = 60.0$ cm, $r = 10.0$ cm, $D = 30.0$ m and $\theta = 15.0°$.
SKETCH:



RESEARCH: The initial energy is $E_{tot} = U$ (potential energy). The final energy is $E_{tot} = K$ (kinetic energy). $U = M_{tot}gh$, $h = D\sin\theta$, $M_{tot} = m_p + 4m_w$, $K = M_{tot}v^2/2 + I\omega^2/2$, $\omega = v/r$ and $I = 4(m_w r^2/2)$.
SIMPLIFY:

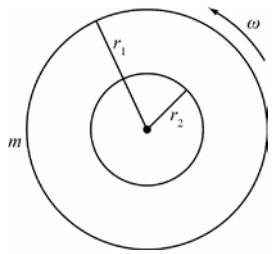
$$U = K \Rightarrow M_{tot}gh = \frac{1}{2}M_{tot}v^2 + \frac{1}{2}I\omega^2 \Rightarrow (m_p + 4m_w)gD\sin\theta = \frac{1}{2}(m_p + 4m_w)v^2 + \frac{1}{2}(4)\left(\frac{1}{2}m_w r^2\right)\left(\frac{v^2}{r^2}\right)$$

$$\Rightarrow \left(m_{\text{p}}+4m_{\text{w}}\right)gD\sin\theta=\left(\frac{1}{2}m_{\text{p}}+2m_{\text{w}}+m_{\text{w}}\right)v^2 \Rightarrow v=\sqrt{\frac{\left(m_{\text{p}}+4m_{\text{w}}\right)gD\sin\theta}{\frac{1}{2}m_{\text{p}}+3m_{\text{w}}}}$$

CALCULATE: $v=\sqrt{\dfrac{\left(8.00\text{ kg}+4\left(2.00\text{ kg}\right)\right)\left(9.81\text{ m/s}^2\right)\left(30.0\text{ m}\right)\sin 15.0^\circ}{\frac{1}{2}\left(8.00\text{ kg}\right)+3\left(2.00\text{ kg}\right)}}=11.04\text{ m/s}$

ROUND: The length of the incline is given to three significant figures, so the result should be rounded to $v = 11.0$ m/s.
DOUBLE-CHECK: This velocity is rather fast. In reality, the friction would slow the cart down. Note also that the radii of the wheels play no role.

10.76. **THINK:** Determining the moment of inertia is straightforward. To determine the torque, first determine the angular acceleration, α, and both $\Delta\omega$ and $\Delta\theta$ are known. Knowing α and *I*, the torque can be determined. $m = 15.0$ g, $r_1 = 1.5$ cm/2, $r_2 = 11.9$ cm/2, $\omega_{\text{i}} = 0$, $\omega_{\text{f}} = 4.3$ rev/s and $\Delta\theta = 0.25$ revs.
SKETCH:



RESEARCH: $\tau = I\alpha$, $I=\frac{1}{2}m\left(r_1^2+r_2^2\right)$, $\omega_{\text{f}}^2=\omega_{\text{i}}^2+2\alpha\Delta\theta$

SIMPLIFY:

(a) $I=\frac{1}{2}m\left(r_1^2+r_2^2\right)$

(b) $\alpha=\dfrac{\left(\omega_{\text{f}}^2-\omega_{\text{i}}^2\right)}{2\Delta\theta}$, $\tau=I\alpha=\dfrac{m}{4\Delta\theta}\left(r_1^2+r_2^2\right)\omega_{\text{f}}^2$ $\left(\omega_{\text{i}}=0\right)$

CALCULATE:

(a) $I=\frac{1}{2}\left(15.0\cdot 10^{-3}\text{ kg}\right)\left(\left(\frac{1.50\cdot 10^{-2}}{2}\text{ m}\right)^2+\left(\frac{11.9\cdot 10^{-2}\text{ m}}{2}\right)^2\right)=2.697\cdot 10^{-5}\text{ kg m}^2$

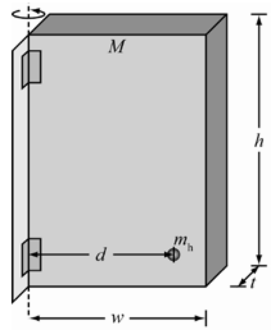
(b) $\Delta\theta=0.250\text{ revs}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)=1.571\text{ rad}$, $\omega_{\text{f}}=4.30\text{ rev/s}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)=27.02\text{ rad/s}$

$\alpha=\dfrac{\left(27.02\text{ rad/s}\right)^2}{2\left(1.571\text{ rad}\right)}=232.4\text{ rad/s}^2$, $\tau=2.697\cdot 10^{-5}\text{ kg m}^2\left(232.4\text{ rad/s}^2\right)=6.267\cdot 10^{-3}\text{ N m}$

ROUND: Rounding to three significant figures, (a) $I=2.70\cdot 10^{-5}\text{ kg m}^2$ and (b) $\tau=6.27\cdot 10^{-3}\text{ N m}$.
DOUBLE-CHECK: These results are reasonable for the given values.

10.77. **THINK:** Begin with the moment of inertia of the door about an axis passing through its center of mass, then use the parallel axis theorem to shift the axis to the edge of the door, and then add the contribution of the handle, which can be treated as a point particle. $\rho = 550.\text{ kg/m}^3$, w = 0.550 m, h = 0.790 m, t = 0.0130 m, d = 0.450 m and $m_{\text{h}} = 0.150$ kg.

SKETCH:



RESEARCH: $M=\rho V=\rho wht$, $I_{\text{center}}=\frac{1}{12}M\left(w^2+t^2\right)$, $I_{\text{edge}}=I_{\text{center}}+M\left(\frac{w}{2}\right)^2$, $I_{\text{handle}}=m_{\text{h}}d^2$

$I=I_{\text{edge}}+I_{\text{handle}}$

SIMPLIFY: $I=\frac{M}{12}\left(w^2+t^2\right)+\frac{M}{4}w^2+m_{\text{h}}d^2=\frac{M}{12}\left(4w^2+t^2\right)+m_{\text{h}}d^2$

CALCULATE: Substituting $M=\rho wht$ into the above equation yields:

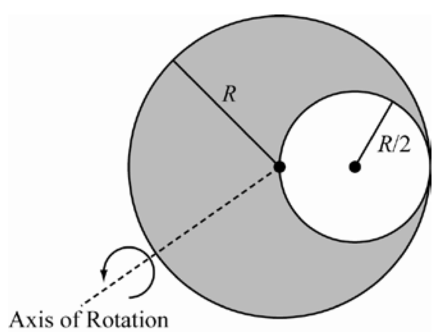
$$I=\frac{1}{12}\left(550.\text{ kg/m}^3\right)\left(0.550\text{ m}\right)\left(0.790\text{ m}\right)\left(0.0130\text{ m}\right)\left(4\left(0.550\text{ m}\right)^2+\left(0.0130\text{ m}\right)^2\right)+\left(0.150\text{ kg}\right)\left(0.450\text{ m}\right)^2$$
$$=0.3437\text{ kg m}^2.$$

ROUND: Rounding to three significant figures gives $I=0.344\text{ kg m}^2$.

DOUBLE-CHECK: This is a reasonable result for a door of this size. Note that the height of the door enters into only the calculation of the door's mass.

10.78. **THINK:** The moment of inertia of the machine part is the moment of inertia of the initial solid disk about its center, minus the moment of inertia of a solid disk of the amount of mass removed about its outside edge (which is at the center of the disk). M = mass of the disk without the hole cut out, and m = mass of the material cut out to make a hole.

SKETCH:



RESEARCH: $I_{\text{center}}=MR^2/2$ (disk spinning about its center). $I_{\text{edge}}=\frac{1}{2}m\left(R/2\right)^2+m\left(R/2\right)^2$ (disk spinning about its edge). The area of the hole is $\pi R^2/4$. The area of the disk without the hole is πR^2. The area of the disk with the hole is $\pi R^2-\pi R^2/4=3\pi R^2/4$. The area of the hole is 1/4 the area of the disk without the hole; therefore, because the disk has uniform density, $m=M/4$. The moment of inertia is

$$I=\frac{1}{2}MR^2-\left(\frac{1}{2}m\left(\frac{R}{2}\right)^2+m\left(\frac{R}{2}\right)^2\right).$$

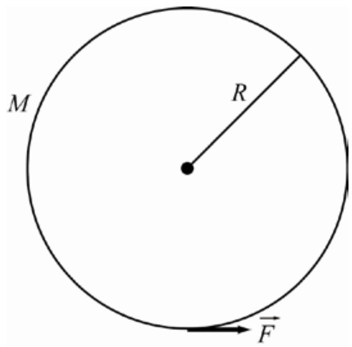
SIMPLIFY: Substitute $m = M/4$ into the above equation to get

$$I = \frac{1}{2}MR^2 - \left[\frac{M}{8}\left(\frac{R^2}{4}\right) + \frac{M}{4}\left(\frac{R^2}{4}\right)\right] = \frac{16}{32}MR^2 - \frac{1}{32}MR^2 - \frac{2}{32}MR^2 = \frac{13}{32}MR^2.$$

CALCULATE: This step is not necessary.
ROUND: This step is not necessary.
DOUBLE-CHECK: As expected, the moment of inertia decreases when the hole is cut out.

10.79. **THINK:** If the angular momentum and the torque are determined, the time can be determined by recalling that torque is the time rate of change of angular momentum. To determine the angular momentum, first determine the angular speed required to produce a centripetal acceleration equal to Earth's gravitational acceleration. From this, the angular momentum, *L*, of the space station can be determined. Finally, the torque can be determined from the given force and the radius of the space station. $R = 50.0$ m, $M = 2.40 \cdot 10^5$ kg and $F = 1.40 \cdot 10^2$ N.

SKETCH:



RESEARCH: $I = MR^2$, $L = \omega I$, $v = \omega R$, $\frac{v^2}{R} = g$, $\tau = FR$, $\tau = \frac{\Delta L}{\Delta t}$

SIMPLIFY: $\Delta t = \frac{\Delta L}{\tau} = \frac{\omega I - 0}{FR} = \frac{MR^2\omega}{FR} = \frac{MR\omega}{F}$, $\omega = \frac{v}{R} = \frac{1}{R}\sqrt{Rg} = \sqrt{\frac{g}{R}}$, $\Delta t = \frac{MR}{F}\sqrt{\frac{g}{R}} = \frac{M\sqrt{Rg}}{F}$

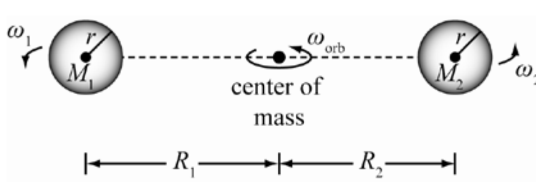
CALCULATE: $\Delta t = \frac{2.40 \cdot 10^5 \text{ kg}\sqrt{(50.0 \text{ m})(9.81 \text{ m/s}^2)}}{1.40 \cdot 10^2 \text{ N}} = 3.797 \cdot 10^4$ s

ROUND: The radius of the space station is given to three significant figures, so the result should be rounded to $\Delta t = 3.80 \cdot 10^4$ s.

DOUBLE-CHECK: The result is equal to about 10 hours. For such a relatively small thrust, this result is reasonable. As expected, this time interval increases if either the thrust decreases or the mass increases.

10.80. **THINK:** There is enough information given to determine the stars' rotational and translational kinetic energies directly and subsequently determine their ratio. Note that the orbital period is given as 2.4 hours. Use the values: $M_1 = 1.250M_{\text{Sun}}$, $M_2 = 1.337M_{\text{Sun}}$, $\omega_1 = 2\pi$ rad/2.8 s, $\omega_2 = 2\pi$ rad/0.023 s, r = 20.0 km, $R_1 = 4.54 \cdot 10^8$ m, $R_2 = 4.23 \cdot 10^8$ m and $\omega_{\text{orb}} = 2\pi$ rad/2.4 h.

SKETCH:



RESEARCH: $K_{\text{rot}} = \frac{1}{2}I\omega^2$, $I = \frac{2}{5}MR^2$, $K_{\text{orb}} = \frac{1}{2}Mv^2 = \frac{1}{2}MR^2\omega^2$

SIMPLIFY:

(a) $\frac{K_{1,\text{rot}}}{K_{2,\text{rot}}} = \frac{I\omega_1^2}{I\omega_2^2} = \frac{M_1 r_1^2 \omega_1^2}{M_2 r_2^2 \omega_2^2} = \frac{M_1 \omega_1^2}{M_2 \omega_2^2}$

(b) $\frac{K_{1,\text{rot}}}{K_{1,\text{orb}}} = \frac{\frac{1}{5}M_1 r^2 \omega_1^2}{\frac{1}{2}M_1 R_1^2 \omega_{\text{orb}}^2} = \frac{2r^2\omega_1^2}{5R_1^2\omega_{\text{orb}}^2}, \quad \frac{K_{2,\text{rot}}}{K_{2,\text{orb}}} = \frac{2r^2\omega_2^2}{5R_2^2\omega_{\text{orb}}^2}$

CALCULATE:

(a) $\frac{K_{1,\text{rot}}}{K_{2,\text{rot}}} = \frac{1.25/(2.8)^2}{1.337/(0.023)^2} = 6.308 \cdot 10^{-5}$

(b) $\frac{K_{1,\text{orb}}}{K_{1,\text{rot}}} = \frac{2(20 \cdot 10^3)^2/(2.8 \text{ s})^2}{5(4.54 \cdot 10^8)^2/(2.4 \text{ h} \cdot 3600 \text{ s/h})^2} = 0.00739128$

$\frac{K_{2,\text{rot}}}{K_{2,\text{orb}}} = \frac{2(20 \cdot 10^3 \text{ m})^2/(0.023 \text{ s})^2}{5(4.23 \cdot 10^8)^2/(2.4 \text{ h} \cdot 3600 \text{ s/h})^2} = 126.186$

ROUND: The rotation periods are given to at least three significant figures, so the results should be rounded to:

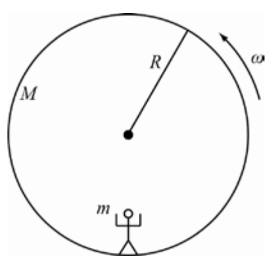
(a) $K_{1,\text{rot}}/K_{2,\text{rot}} = 6.31 \cdot 10^{-5}$

(b) $K_{1,\text{orb}}/K_{1,\text{rot}} = 7.39 \cdot 10^{-3}, \quad K_{2,\text{orb}}/K_{2,\text{rot}} = 126$

DOUBLE-CHECK: M_2 has a much faster rotational speed than M_1. The kinetic energy for M_1 is dominated by the orbit, while for M_2 it is dominated by rotational motion.

10.81. **THINK:** Conservation of angular momentum can be considered to determine the angular momentum of the merry-go-round. From this, the mass, M, of the merry-go-round can be determined. For parts (b) and (c), use the uniform acceleration equations to answer the problem. $R = 1.50$ m, $\omega = 1.30$ rad/s, $m = 52.0$ kg and $v = 6.80$ m/s (speed of the student just prior to jumping on).

SKETCH:



RESEARCH: $L_{\text{student}} = Rmv$, $L = I\omega$ (merry-go-round), $I = \frac{1}{2}MR^2 + mR^2$, $\Delta\theta = \frac{1}{2}\alpha t^2 + \omega_{\text{i}} t$, $\omega_{\text{f}} = \alpha t + \omega_{\text{i}}$, and $\tau = I\alpha$.

SIMPLIFY:

(a) $L_{\text{student}} = L \Rightarrow Rmv = I\omega = \left(\frac{1}{2}MR^2 + mR^2\right)\omega \Rightarrow Rmv - mR^2\omega = \frac{1}{2}MR^2\omega$

$\Rightarrow M = \frac{2}{\omega R^2}\left(Rmv - mR^2\omega\right) \Rightarrow M = \frac{2m}{R\omega}\left(v - R\omega\right) = 2m\left(\frac{v}{\omega R} - 1\right)$

(b) $\tau = I\alpha = \left(\frac{1}{2}MR^2 + mR^2\right)\alpha, \quad \alpha = \frac{\Delta\omega}{\Delta t} = -\frac{\omega}{t} \quad \left(\omega_{\text{i}} = \omega,\ \omega_{\text{f}} = 0,\ t_{\text{f}} = t,\ t_{\text{i}} = 0\right), \quad \tau = -\frac{\omega R^2}{t}\left(\frac{M}{2} + m\right)$

(c) $\Delta\theta = \frac{1}{2}\alpha t^2 + \omega_i t = \frac{1}{2}\left(-\frac{\omega}{t}\right)t^2 + \omega t = -\frac{1}{2}\omega t + \omega t = \frac{1}{2}\omega t$

CALCULATE:

(a) $M = 2(52.0\text{ kg})\left(\frac{6.80\text{ m/s}}{1.30\text{ rad s}^{-1}(1.50\text{ m})} - 1\right) = 258.7\text{ kg}$

(b) $\tau = -\frac{(1.30\text{ rad s}^{-1})(1.50\text{ m})^2}{35.0\text{ s}}\left(\frac{258.7\text{ kg}}{2} + 52.0\text{ kg}\right) = -15.15\text{ N m}$

(c) $\Delta\theta = \frac{1}{2}(1.30\text{ rad/s})(35.0\text{ s}) = 22.75\text{ rad} = 22.75\text{ rad}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 3.621\text{ rev}$

ROUND: Round the results to three significant figures.

(a) $M = 259\text{ kg}$

(b) $\tau = -15.2\text{ N m}$

(c) $\Delta\theta = 3.62$ revolutions

DOUBLE-CHECK: The results are all consistent with the given information.

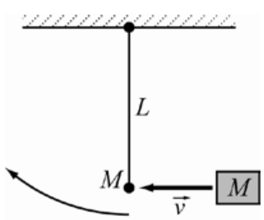
10.82. THINK:

(a) The speed of the pendulum just after the collision can be determined by considering the conservation of linear momentum. From the conservation of energy, the maximum height of the pendulum can be determined, since at this point, all of the initial kinetic energy will be stored as gravitational potential energy.

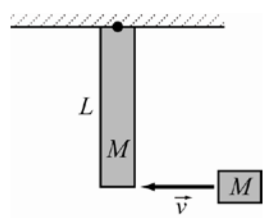
(b) From the conservation of angular momentum, the rotation speed of the pendulum just after collision can be determined. From the conservation of energy, the maximum height of the pendulum can be determined, since at this point, all of the initial rotational kinetic energy will be stored as gravitational potential energy. L = 0.48 m and v = 3.6 m/s.

SKETCH:

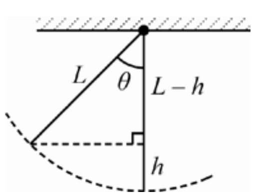
(a)



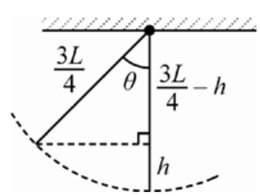
(b)



(c)



(d)



RESEARCH:

(a) $E = \text{constant} = K + U, \quad K = \frac{1}{2}mv^2, \quad U = mgh$

(b) $L = \text{constant} = I\omega, \quad I_{\text{rod}} = \frac{1}{3}ML^2, \quad I_{\text{proj}} = ML^2$

SIMPLIFY: v_0 is the speed of the projectile just prior to collision. v_p is the speed of the pendulum at the lower edge just after collision.

(a) $P_i = P_f \Rightarrow Mv_0 = (M+M)v_p \Rightarrow v_p = \frac{1}{2}v_0$; At the pendulum's maximum height,

$$K_f = (M+M)gh = \frac{1}{2}(M+M)v_p^{\,2}.$$

$$h = \frac{v_p^{\,2}}{2g} = \frac{v_0^{\,2}}{8g} \Rightarrow \cos\theta = 1 - \frac{h}{L} = 1 - \frac{v_0^{\,2}}{8gL} \Rightarrow \theta = \cos^{-1}\left(1 - \frac{v_0^{\,2}}{8gL}\right)$$

(b) $L = I\omega$, $\omega = v/L$, $L_i = I_i \frac{v_0}{L} = MLv_0$

$$L_f = (I_{rod} + ML^2)\omega_f = MLv_0 \Rightarrow \frac{4}{3}ML^2\omega_f = MLv_0 \Rightarrow \omega_f = \frac{3}{4L^2}Lv_0 = \frac{3v_0}{4L} \Rightarrow \frac{3v_0}{4L} = \frac{v_p}{L} \Rightarrow v_p = \frac{3}{4}v_0$$

At maximum height: $\frac{1}{2}I\omega_f^{\,2} = mgh \Rightarrow \frac{1}{2}\left(\frac{4}{3}ML^2\right)\left(\frac{3v_0}{4L}\right)^2 = 2Mgh \Rightarrow \frac{3}{8}v_0^{\,2} = 2gh \Rightarrow h = \frac{3v_0^{\,2}}{16g}.$

This is the height attained by the center of mass of the pendulum and projectile system. By symmetry, the center of mass of the system is located $3L/4$ from the top. So,

$$\cos\theta = 1 - \frac{h}{3L/4} = 1 - \frac{4h}{3L} = 1 - \frac{4}{3L}\cdot\frac{3v_0^{\,2}}{16g} = 1 - \frac{v_0^{\,2}}{4gL} \Rightarrow \theta = \cos^{-1}\left(1 - \frac{v_0^{\,2}}{4gL}\right).$$

CALCULATE:

(a) $\theta = \cos^{-1}\left(1 - \frac{(3.6 \text{ m/s})^2}{8(9.81 \text{ m/s}^2)(0.48 \text{ m})}\right) = 49.01°$

(b) $\theta = \cos^{-1}\left(1 - \frac{(3.6 \text{ m/s})^2}{4(9.81 \text{ m/s}^2)(0.48 \text{ m})}\right) = 71.82°$

ROUND: Round the results to three significant figures.

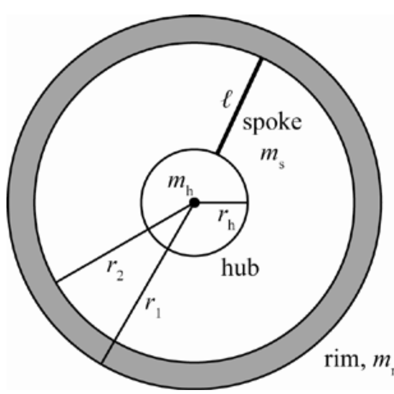
(a) $\theta = 49.0°$

(b) $\theta = 71.8°$

DOUBLE-CHECK: The rod swings higher. This is expected since the center of mass is higher than for the pendulum. The projectile exerts a greater torque on the rod.

10.83. **THINK:** The quantity of interest can be calculated directly from the given information. $m_r = 5.20$ kg, $m_h = 3.40$ kg, $m_s = 1.10$ kg, $r_1 = 0.900$ m, $r_2 = 0.860$ m, $r_h = 0.120$ m and $l = r_2 - r_h$.

SKETCH:



RESEARCH: $I_{\text{rim}}=\frac{1}{2}m_{\text{r}}\left(r_1^2+r_2^2\right)$, $I_{\text{spoke}}=\frac{1}{12}m_{\text{s}}l^2+m_{\text{s}}d^2$, (with $d=\frac{1}{2}l+r_{\text{h}}$), $I_{\text{hub}}=\frac{1}{2}m_{\text{h}}r_{\text{h}}^2$

SIMPLIFY: $I=I_{\text{rim}}+I_{\text{hub}}+12I_{\text{spoke}}$, $M=m_{\text{r}}+m_{\text{h}}+12m_{\text{s}}$, $R=r_1$

CALCULATE: $I_{\text{rim}}=\frac{1}{2}5.20\text{ kg}\left((0.900)^2+(0.860)^2\right)\text{ m}^2=4.029\text{ kg m}^2$

$$I_{\text{hub}}=\frac{1}{2}(3.40\text{ kg})(0.120\text{ m})^2=2.448\cdot 10^{-2}\text{ kg m}^2$$

$$I_{\text{spoke}}=\frac{1}{12}(1.10\text{ kg})(0.860\text{ m}-0.120\text{ m})^2+(1.10\text{ kg})(0.490\text{ m})^2=3.143\cdot 10^{-1}\text{ kg m}^2$$

$$I=I_{\text{rim}}+I_{\text{hub}}+12I_{\text{spoke}}=7.825\text{ kg m}^2,\ M=\left[5.20+3.40+12(1.10)\right]\text{ kg}=21.8\text{ kg}$$

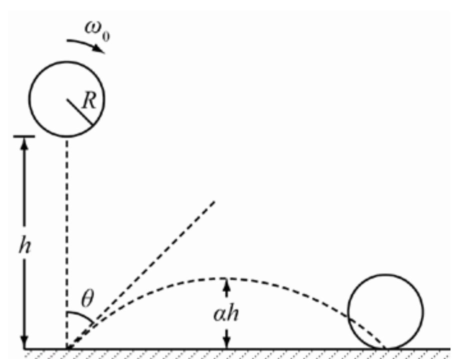
$$c=\frac{I}{MR^2}=\frac{7.825\text{ kgm}^2}{(21.8\text{ kg})(0.900\text{ m})^2}=0.4431$$

ROUND: Rounding to three significant figures, $c=0.443$.

DOUBLE-CHECK: It is reasonable that the moment of inertia is dominated by the rim and the spokes, and the hub is negligible.

10.84. **THINK:** To determine the angles in parts (a), the vertical and horizontal components of the velocity just after impact must be determined. To determine the vertical velocity, consider the conservation of energy. To determine the horizontal velocity, consider the linear and angular impulses experienced in either of the following two situations. Situation I: The ball slips on the floor during the entire impact time. Kinetic friction must be considered the entire time. Situation II: The ball stops slipping on the floor at some point during the impact. From this point for the duration of the impact, rolling motion is attained, and the usual equations relating angular and rotational speeds are applicable.

SKETCH:



RESEARCH: Energy conservation is given by $mgh=mv_0^2/2$. v_0 is the speed of the ball just prior to the impact for the first time. Also, from energy conservation: $mg(\alpha h)=\frac{1}{2}mv_{2y}^2$, where v_{2y} is the vertical velocity just after the impact of the ball with the ground. Linear impulse is given by

$$\int_{t_1}^{t_2}F(t)dt=p(t_2)-p(t_1).$$

Angular impulse is given by $\int_{t_1}^{t_2}\tau(t)dt=L(t_2)-L(t_1)=I(\omega_0-\omega_2)$.

SIMPLIFY: Just prior to impact: $mgh=mv_0^2/2 \Rightarrow v_0=\sqrt{2gh}$. Just after impact:

$$\alpha mgh=\frac{1}{2}mv_{2y}^2 \Rightarrow v_{2y}=+\sqrt{2\alpha gh}.$$

Situation I:

n is the normal force and μn is the frictional force. The impulses are as follows:

$$I_y = \int_{t_1}^{t_2} n(t)\,dt = m\sqrt{\alpha}v_0 - m(-v_0) = m\left(1+\sqrt{\alpha}\right)v_0, \quad I_x = \int_{t_1}^{t_2} \mu n\,dt = \mu I_y = \mu m\left(1+\sqrt{\alpha}\right)v_0 = mv_{2x}$$

$$I_\theta = \int_{t_1}^{t_2} \mu n R\,dt = R\mu I_y = R\mu m\left(1+\sqrt{\alpha}\right)v_0 = I(\omega_0 - \omega_2) \Rightarrow v_{2x} = \mu\left(1+\sqrt{\alpha}\right)v_0 = \mu\left(1+\sqrt{\alpha}\right)\sqrt{2gh}$$

$$\omega_2 = \omega_0 - \frac{1}{I}R\mu m\left(1+\sqrt{\alpha}\right)v_0 = \omega_0 - \frac{R}{I}\mu m\left(1+\sqrt{\alpha}\right)\sqrt{2gh} = \omega_0 - \frac{Rmv_{2x}}{I}$$

(a) $\tan\theta = \left(\dfrac{v_{2x}}{v_{2y}}\right)^{-1} = \left(\dfrac{\mu\left(1+\sqrt{\alpha}\right)\sqrt{2gh}}{\sqrt{2\alpha gh}}\right)^{-1} = \left(\dfrac{\mu\left(1+\sqrt{\alpha}\right)}{\sqrt{\alpha}}\right)^{-1}$

(b) The time, t, it takes for the ball to fall is $t = \dfrac{2v_{2y}}{g} = \dfrac{2}{g}\sqrt{2\alpha gh} = \sqrt{\dfrac{8\alpha h}{g}}$.

The distance traveled during this time is $d = v_{2x}t = \mu\left(1+\sqrt{\alpha}\right)\left(\sqrt{2gh}\right)\left(\sqrt{\dfrac{8\alpha h}{g}}\right) = 4\mu\left(1+\sqrt{\alpha}\right)h\left(\sqrt{\alpha}\right)$.

(c) $\omega_2 = \omega_0 - \dfrac{Rmv_{2x}}{I}$. The minimum ω_0 occurs when $R\omega_2 = v_{2x}$, where $R\omega_2$ is the velocity of the contact point. $R\omega_{0,\min} - \dfrac{R^2mv_{2x}}{I} = v_{2x}$, $\omega_{0,\min} = v_{2x}\left(\dfrac{1}{R}+\dfrac{Rm}{I}\right) = \dfrac{v_{2x}}{R}\left(1+\dfrac{mR^2}{I}\right) = \dfrac{\mu\left(1+\sqrt{\alpha}\right)\sqrt{2gh}}{R}\left(1+\dfrac{mR^2}{I}\right)$

Situation II:

After the ball stops slipping, there is a rolling motion and $\omega_2 R = v_{2x}$. The impulses are as follows.

$$I_x = \int_{t_1}^{t_2} \mu n\,dt = mv_{2x}, \quad I_\theta = \int_{t_1}^{t_2} \mu n R\,dt = I(\omega_0 - \omega_2) = RI_x$$

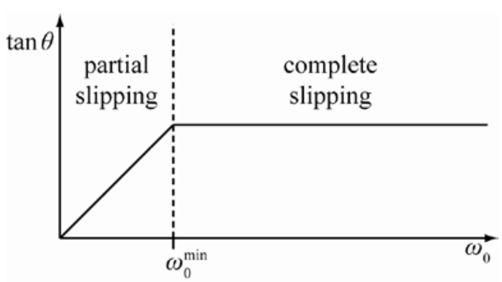
$$\Rightarrow I(\omega_0 - \omega_2) = Rmv_{2x} \Rightarrow I\left(\omega_0 - \frac{v_{2x}}{R}\right) = Rmv_{2x}.$$

Solve for v_{2x} by substituting $I = \left(2mR^2/5\right)$ into the above equation to get:

$$\frac{2}{5}mR^2\left(\omega_0 - \frac{v_{2x}}{R}\right) = Rmv_{2x} \Rightarrow \frac{2}{5}\left(\omega_0 R - v_{2x}\right) = v_{2x} \Rightarrow v_{2x}\left(1+\frac{2}{5}\right) = \frac{2}{5}\omega_0 R \Rightarrow v_{2x} = \frac{5}{7}\left(\frac{2}{5}\omega_0 R\right) = \frac{2}{7}\omega_0 R.$$

(d) $\tan\theta = \dfrac{v_{2x}}{v_{2y}} = \dfrac{\frac{2}{7}\omega_0 R}{\sqrt{2\alpha gh}}$

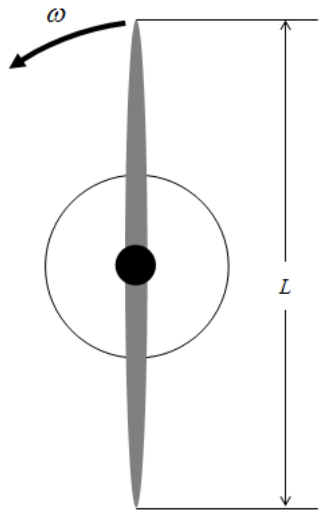
(e) $d = v_{2x}t = \dfrac{2}{7}\omega_0 R\sqrt{\dfrac{8\alpha h}{g}}$



CALCULATE: This step is not necessary.
ROUND: This step is not necessary.
DOUBLE-CHECK: When only partial slipping occurs, the horizontal distance traveled should and does depend on ω_0.

Multi-Version Exercises

10.85. **THINK:** The length and mass of the propeller, as well as the frequency with which it is rotating, are given. To find the kinetic energy of rotation, it is necessary to find the moment of inertia, which can be calculated from the mass and radius by approximating the propeller as a rod with constant mass density.
SKETCH: The propeller is shown as it would be seen looking directly at it from in front of the plane.



RESEARCH: The kinetic energy of rotation is related to the moment of inertia and angular speed by the equation $K_{\text{rot}} = \frac{1}{2}I\omega^2$. The angular speed $\omega = 2\pi f$ can be computed from the frequency of the propeller's rotation. Approximating the propeller as a rod with constant mass density means that the formula $I = \frac{1}{12}mL^2$ for a long, thin rod rotating about its center of mass can be used.
SIMPLIFY: Combine the equations for the moment of inertia and angular speed to get a single equation for the kinetic energy $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{12}mL^2\right)\cdot(2\pi f)^2$. Using algebra, this can be simplified to $K_{\text{rot}} = \frac{m}{6}(\pi L f)^2$. Since the angular speed is given in revolutions per minute, the conversion 1 minute = 60 seconds will also be needed.
CALCULATE: The propeller weighs m = 17.36 kg, it is L = 2.012 m long, and it rotates at a frequency of f = 3280. rpm. The rotational kinetic energy is

$$\begin{aligned} K_{\text{rot}} &= \frac{m}{6}(\pi L f)^2 \\ &= \frac{17.36\text{ kg}}{6}\left(\pi \cdot 2.012\text{ m} \cdot 3280.\text{ rpm} \cdot \frac{1\text{min}}{60\text{sec}}\right)^2 \\ &= 345{,}461.2621\text{ J} \end{aligned}$$

ROUND: The values in the problem are all given to four significant figures, so the final answer should have four figures. The propeller has a rotational kinetic energy of $3.455 \cdot 10^5$ J or 345.5 kJ.
DOUBLE-CHECK: Given the large amount of force needed to lift a plane, it seems reasonable that the energy in the propeller would be in the order of hundreds of kilojoules. Working backwards, if a propeller weighing 17.36 kg and having length 2.012 m has rotational kinetic energy 345.5 kJ, then it is turning at

$f=\frac{1}{\pi L}\sqrt{\frac{6K_{\text{rot}}}{m}}=\frac{1}{\pi\cdot 2.012\text{ m}}\sqrt{\frac{6\cdot 3.455\cdot 10^5\text{ J}}{17.36\text{ kg}}}$. This is 54.667 revolutions per second, which agrees with the given value of 54.667·60 = 3280 rpm. This confirms that the calculations were correct.

10.86.

$$K_{\text{rot}}=\frac{m}{6}(\pi Lf)^2$$

$$f=\frac{1}{\pi L}\sqrt{\frac{6K_{\text{rot}}}{m}}$$

$$=\frac{1}{\pi(2.092\text{ m})}\sqrt{\frac{6(422.8\cdot 10^3\text{ J})}{17.56\text{ kg}}}=57.8\text{ rev/s}=3470.\text{ rpm}$$

10.87.

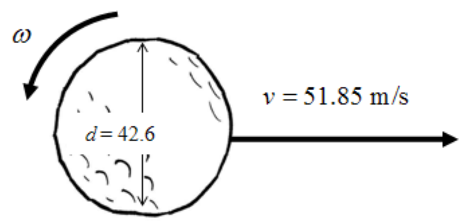
$$K_{\text{rot}}=\frac{m}{6}(\pi Lf)^2$$

$$m=\frac{6K_{\text{rot}}}{(\pi Lf)^2}$$

$$=\frac{6(124.3\cdot 10^3\text{ J})}{\left[\pi(1.812\text{ m})\left(2160.\text{ rpm}\cdot\frac{1}{60}\text{ s/min}\right)\right]^2}=17.76\text{ kg}$$

10.88. **THINK:** The total kinetic energy of the golf ball is the sum of the rotational kinetic energy and the translational kinetic energy. The translational kinetic energy can be calculated from the mass of the ball and the speed of the center of mass of the golf ball, both of which are given in the question. To find the rotational kinetic energy, it is necessary to find the moment of inertia of the golf ball. Though the golf ball is not a perfect sphere, it is close enough that the moment of inertia can be computed from the mass and diameter of the golf ball using the approximation for a sphere.

SKETCH: The golf ball has both rotational and translational motion.



RESEARCH: The total kinetic energy is equal to the translational kinetic energy plus the rotational kinetic energy $K=K_{\text{trans}}+K_{\text{rot}}$. The translational kinetic energy is computed from the speed and the mass of the golf ball using the equation $K_{\text{trans}}=\frac{1}{2}mv^2$. The rotational kinetic energy is computed from the moment of inertia and the angular speed by $K_{\text{rot}}=\frac{1}{2}I\omega^2$. It is necessary to compute the moment of inertia and the angular speed. The angular speed $\omega=2\pi f$ depends only on the frequency. To find the moment of inertia, first note that golf balls are roughly spherical. The moment of inertia of a sphere is given by $I=\frac{2}{5}mr^2$. The question gives the diameter d which is twice the radius (d / 2 = r). Since the frequency is given in revolutions per minute and the speed is given in meters per second, the conversion factor $\frac{1\text{ min}}{60\text{ sec}}$ will be necessary.

SIMPLIFY: First, find the moment of inertia of the golf ball in terms of the mass and diameter to get $I=\frac{1}{10}md^2$. Substituted for the angular speed and moment of inertia in the equation for rotational kinetic energy to get $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{10}md^2\right)(2\pi f)^2$. Finally, use the equations $K_{\text{trans}} = \frac{1}{2}mv^2$ and $K_{\text{rot}} = \frac{1}{2}\left(\frac{1}{10}md^2\right)(2\pi f)^2$ to find the total kinetic energy and simplify using algebra:

$$\begin{aligned} K &= K_{\text{trans}} + K_{\text{rot}} \\ &= \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{10}md^2\right)(2\pi f)^2 \\ &= \tfrac{1}{2}mv^2 + \tfrac{1}{5}m(\pi d f)^2 \end{aligned}$$

CALCULATE: The mass of the golf ball is 45.90 g = 0.04590 kg, its diameter is 42.60 mm = 0.04260 m, and its speed is 51.85 m/s. The golf ball rotates at a frequency of 2857 revolutions per minute. The total kinetic energy is

$$\begin{aligned} K &= \tfrac{1}{2}mv^2 + \tfrac{1}{5}m(\pi d f)^2 \\ &= \tfrac{1}{2}(0.04590\text{ kg})(51.85\text{ m/s})^2 + \tfrac{1}{5}(0.04590\text{ kg})\left(\pi \cdot 0.04260\text{ m} \cdot 2857\text{ rpm} \cdot \frac{1\text{ min}}{60\text{ sec}}\right)^2 \\ &= 62.07209955\text{ J} \end{aligned}$$

ROUND: The mass, speed, frequency, and diameter of the golf ball are all given to four significant figures, so the translational and rotational kinetic energies should both have four significant figures, as should their sum. The total energy of the golf ball is 62.07 J.

DOUBLE-CHECK: The golf ball's translational kinetic energy alone is equal to $\frac{1}{2}(0.04590\text{ kg})(51.85\text{ m/s})^2 = 61.7\text{ J}$, and it makes sense that a well-driven golf ball would have much more energy of translation than energy of rotation.

10.89. $K = K_{\text{trans}} + K_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{5}m(\pi d f)^2$

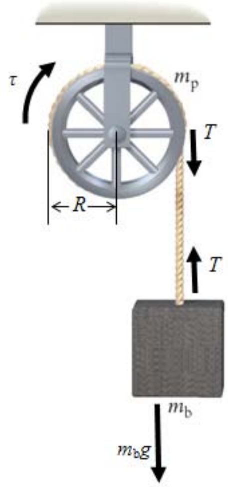
$$\begin{aligned} f &= \frac{1}{\pi d}\sqrt{5\left(\frac{K}{m} - \frac{1}{2}v^2\right)} \\ &= \frac{1}{\pi(0.04260\text{ m})}\sqrt{5\left(\frac{67.67\text{ J}}{0.04590\text{ kg}} - \frac{1}{2}(54.15\text{ m/s})^2\right)} = 47.79\text{ rev/s} = 2867\text{ rpm} \end{aligned}$$

10.90. $K = K_{\text{trans}} + K_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{5}m(\pi d f)^2$

$$\begin{aligned} v &= \sqrt{2\left(\frac{K}{m} - \frac{1}{5}(\pi d f)^2\right)} \\ &= \sqrt{2\left(\frac{73.51\text{ J}}{0.04590\text{ kg}} - \frac{1}{5}\left(\pi \cdot 0.04260\text{ m} \cdot 2875\text{ rpm} \cdot \frac{1}{60}\text{ min/s}\right)^2\right)} = 56.45\text{ m/s} \end{aligned}$$

10.91. **THINK:** The gravitational force on the block is transmitted through the rope, causing a torque on the pulley. The torque causes an angular acceleration, and the linear acceleration is calculated from the angular acceleration.

SKETCH: Use the figure from the text:



RESEARCH: The torque on the pulley is given by the tension on the rope times the radius of the pulley $\tau = TR$. This torque will cause an angular acceleration $\tau = I\alpha$, where the moment of inertia of the pulley is given by $I = m_{\text{p}}R^2$. The tension on the rope is given by $T - m_{\text{b}}g = -m_{\text{b}}a$ (the minus indicates that the block is accelerating downward). The linear acceleration of the block a is related to the angular acceleration of the pulley α by the equation $a = R\alpha$.

SIMPLIFY: First, substitute for the tension on the pulley $T = m_{\text{b}}g - m_{\text{b}}a$ in the equation for the torque τ to get $\tau = (m_{\text{b}}g - m_{\text{b}}a)R$. Then, substitute for the moment of inertia ($I = m_{\text{p}}R^2$) and angular acceleration ($\alpha = a/R$) in the equation $\tau = I\alpha$ to get $\tau = (m_{\text{p}}R^2)\left(\frac{a}{R}\right) = m_{\text{p}}Ra$. Combine these two expressions for the torque to get $(m_{\text{b}}g - m_{\text{b}}a)R = m_{\text{p}}Ra$. Finally, solve this expression for the linear acceleration a of the block:

$$(m_{\text{b}}g - m_{\text{b}}a)R = m_{\text{p}}aR$$
$$m_{\text{b}}gR - m_{\text{b}}aR + m_{\text{b}}aR = m_{\text{p}}aR + m_{\text{b}}aR$$
$$m_{\text{b}}gR = (m_{\text{p}}R + m_{\text{b}}R)a$$
$$\frac{m_{\text{b}}gR}{m_{\text{p}}R + m_{\text{b}}R} = a$$
$$a = \frac{m_{\text{b}}g}{m_{\text{p}} + m_{\text{b}}}$$

CALCULATE: The mass of the block is m_{b} = 4.243 kg and the mass of the pulley is m_{p} = 5.907 kg. The acceleration due to gravity is –9.81 m/s^2. So, the total (linear) acceleration of the block is

$$a = \frac{m_{\text{b}}g}{m_{\text{p}} + m_{\text{b}}} = \frac{-9.81 \text{ m/s}^2 \cdot 4.243 \text{ kg}}{5.907 \text{ kg} + 4.243 \text{ kg}} = -4.100869951 \text{ m/s}^2.$$

ROUND: The masses of the pulley and block are given to four significant figures, and the sum of their masses has five figures. On the other hand, the gravitational constant g is given only to three significant figures. So, the final answer should have three significant figures. The block accelerates downward at a rate of 4.10 m/s^2.

DOUBLE-CHECK: A block falling freely would accelerate (due to gravity near the surface of the Earth) at a rate of 9.81 m/s^2 towards the ground. The block attached to the pulley will still accelerate downward, but the rate of acceleration will be less (the potential energy lost when the block falls 1 meter will equal the kinetic energy of a block in free fall, but it will equal the kinetic energy of the block falling plus the rotational kinetic energy of the pulley in the problem). The mass of the pulley is close to, but a bit larger than, the mass of the block, so the acceleration of the block attached to the pulley should be a bit less than half of the acceleration of the block in free fall. This agrees with the final acceleration of 4.10 m/s^2, which is a bit less than half of the acceleration due to gravity.

10.92. $$a = \frac{m_\text{b} g}{m_\text{p} + m_\text{b}}$$

$$m_\text{p} = \frac{m_\text{b} g}{a} - m_\text{b} = m_\text{b}\left(\frac{g}{a} - 1\right) = (4.701\text{ kg})\left(\frac{9.81\text{ m/s}^2}{4.330\text{ m/s}^2} - 1\right) = 5.95\text{ kg}$$

10.93. $$a = \frac{m_\text{b} g}{m_\text{p} + m_\text{b}}$$

$$m_\text{p} = \frac{m_\text{b} g}{a} - m_\text{b} = m_\text{b}\left(\frac{g}{a} - 1\right)$$

$$\Rightarrow m_\text{b} = \frac{m_\text{p}}{\frac{g}{a} - 1} = \frac{5.991\text{ m}}{\frac{9.81\text{ m/s}^2}{4.539\text{ m/s}^2} - 1} = 5.16\text{ kg}$$