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8

McGraw-Hill Education

Integrated Math

United Arab Emirates Edition

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GRADE 8 • VOLUME 1

McGraw Hill Education

"Extensive knowledge and modern science must be acquired. The educational process we see today is in an ongoing and escalating challenge which requires hard work.
We succeeded in entering the third millennium, while we are more confident in ourselves."

H.H. Sheikh Khalifa Bin Zayed Al Nahyan
President of the United Arab Emirates

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Units organized by domain

This book is organized into units based on groups called domains.
MP Mathematical Practices are embedded throughout the course.

Mathematical Practices Handbook

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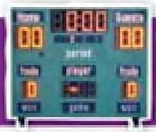

UNIT 5
Domain 8.SP

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MP Mathematical Practices
Mathematical Practices Handbook

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Web Design 101

Persevere with Problems

How do I make sense of a problem?

Making and using a step-by-step plan to solve a problem is like using directions to build a piece of furniture. If you follow the directions correctly, there is a good chance you will end up with a solid piece of furniture. Once you understand the meaning of the problem, you can decide what strategy will work best to solve it. You might try several strategies and then ask yourself, "Does this make sense?"

You have already used the four-step problem-solving plan in previous courses. Complete the graphic organizer that shows the four steps to solve the given problem.

> **Mathematical Practice 1**
>
> Make sense of problems and persevere in solving them.

Of the 480 students at Lincoln Middle School, one-third have traveled overseas. Of these, 15% have been to Australia. How many students have not been to Australia?

Step 1. Understand

What are the facts?

Sample answer: 480 total students; $\frac{1}{3}$ have traveled overseas; 15% of the $\frac{1}{3}$ have not been to Australia

Step 2. Plan

What strategy will you use to solve the problem above?

Sample answer: Solve a Simpler Problem

Step 3. Solve

Solve the problem. Show your steps below.

456 students have not been to Australia.

Step 4. Check

How do you know your answer is reasonable?

See students' work.

It's Your Turn!

Solve each problem by using the four-step problem-solving model.

1. About fifty percent of the population of Alaska lives within a 80.47 kilometers radius of Anchorage. If the total area of Alaska is 1,518,800 square kilometers, about what percent of the total land area is within 80.47 kilometers of Anchorage?

Understand What are you asked to find? Is there any information you will not use?

What percent of the total land area is within 80.47 kilometers of Anchorage?; yes; About 50% of the population lives within a 80.47 kilometers radius of Anchorage.

Plan How will you solve this problem?

Sample answer: Find the area of the 80.47 kilometers radius circle, then determine the percent of the total.

Solve Solve the problem. Show your steps below. What is the solution?

Sample answer: the area of the 80.47 kilometers radius circle is about 20341.767 square kilometers. This is about 1% of the total land area of Alaska.

Check Does your answer make sense?

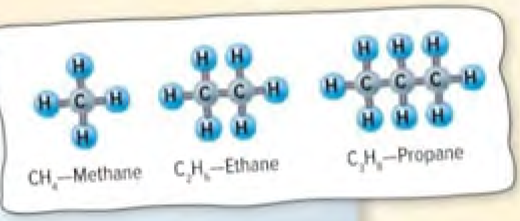
See students' work.

Check

Solve the problem using a different strategy to check your work.

2. The first three molecules for a certain family of hydrocarbons are shown. How many hydrogen atoms (H) are in a molecule containing 6 carbon atoms (C)?

14 hydrogen atoms



Find it in Your Book!

MP Persevere with Problems

Look at Chapter 1. Give an example of where Mathematical Practice 1 is used. Then explain why your example represents this practice.

See students' examples.

Reason Abstractly and Quantitatively

What does it mean to reason abstractly and quantitatively?

Mathematical Practice 2

Reason abstractly and quantitatively.

In math, we solve real-world problems where numbers and variables in an equation represent concrete objects. This involves thinking quantitatively.

Suppose you are given a AED 91.83 gift card to an online music store. Each song costs AED 7.16 to purchase and download. How many songs can you buy?

1. What values in the problem do we already know?
   AED 25 to spend, AED 1.95 per song

2. What are we trying to find?
   number of songs we can buy

3. What symbol can we use to represent the unknown value?
   Sample answer: *s*

Now that the problem is broken down into known and unknown values, we can manipulate the symbols in order to solve the problem. This is thinking abstractly.

4. Write an equation to solve the problem. Explain what each quantity or symbol represents.
   $25 = 1.95s$; 25 represents the total gift card amount, 1.95 represents how much it costs per song, and *s* represents the number of songs we can buy.

5. Use your equation to solve the problem and label your solution. Explain the meaning of the solution.
   12 songs; There is enough money on the gift card to buy 12 songs. There will be AED 1.60 left on the card, which is not enough to buy another song.

It's Your Turn!

Write and solve an equation for each of the following.

6. You are in the pit crew for a driver at a car race. The gas weighs 5.92 kilograms per liter. Your driver uses 0.25 liter per lap. With 42 laps to go, you put 60 liter of fuel in the tank of the car. Will your driver finish the race at the same rate without more gas?

   a. What values do we already know? What are we trying to find?

   5.92 pounds per gallon, 0.25 gallon per lap, 60 pounds of fuel;

   Sample answer: Is 60 pounds of fuel enough to finish the race?

   b. Write an equation to find the number of liter in 60 kilograms of fuel.

   Sample answer: $5.92g = 60$

   c. Use the equation to solve the problem and explain the meaning of the solution.

   No, the driver will not be able to finish the race; Sample answer: The car can go about 40.5 laps, which is less than the 42 laps he needs to finish.

7. A class trip is scheduled for an amusement [illegible] group admission prices are AED 113.86 per stud[illegible]ing is AED 66.11 pe[illegible]

   a. Complete the table to show the total cost of 10, 20, 30, and 40 students and two buses.

   b. Write an equation to show the total cost c if two buses transport s students to the park. **$c = 31s + 36$**

   c. There are a total of 78 students attending on two buses. What is the total cost? Label your solution and explain its meaning.

   AED 2,454; It will cost a total of AED 2,454 for 78 students to attend the class trip.

Number of Students, s	Total Cost, c (AED)
10	346
20	656
30	966
40	1,276

Find it in Your Book!

MP Reason Abstractly

Look at Chapter 2. Give an example of where Mathematical Practice 2 is used. Then explain why your example represents this practice.

See students' examples.

Construct an Argument

How do I construct a viable argument in math class?

Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

Suppose your friend told you that his rectangular flatscreen T.V. has congruent diagonals, simply because it was rectangular. How could you ask your friend to justify his argument? You could use inductive reasoning or deductive reasoning. *Inductive reasoning* uses examples to draw conclusions, while *deductive reasoning* uses definitions, rules, or facts.

1. How could you use *inductive reasoning* to justify why the following statement is true?

   All rectangles have diagonals that are congruent.

   Sample answer: Draw and measure the diagonals of several rectangles. By noticing that the measures of the diagonals are congruent, I can conclude that this statement is true.

2. How could you use *deductive reasoning* to justify why the following statement is false?

   Each angle of an equilateral triangle measures 90°.

   Sample answer: An equilateral triangle has 3 congruent angles. If each angle measures 90°, the sum of the angle measures would be 270°, but the sum of the angle measures in a triangle is 180°, not 270°.

3. Complete the graphic organizer to show that the statement below is *sometimes* true. **See students' work.**

The product of two integers is positive.

Example when it is true:	Example when it is false:
$-2(-3) = 6$	**$-2(3) = -6$**

It's Your Turn!

For each of the following statements, determine if the statement is *always*, *sometimes*, or *never* true. Justify your response using examples or counterexamples.

4. The sum of two rational numbers is a rational number.

always; Sample answer: When you add two fractions, decimals, or integers, you always get a fraction, decimal, or integer as an answer; For example, $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$ or 1.

5. The sum of two odd numbers is an odd number.

never; Sample answer: When you add two odd numbers, the result is always even; For example, $3 + 5 = 8$, $9 + 7 = 16$, $11 + 3 = 14$.

6. The volume of a pyramid is less than the volume of a prism with the same size base.

sometimes; Sample answer: If the height of the pyramid is more than three times the height of the prism, the volume of the pyramid will be greater.

Find it in Your Book!

MP Construct an Argument

Look at Chapter 1. Give an example of where Mathematical Practice 3 is used. Then explain why your example represents this practice.

See students' examples.

Model with Mathematics

How does math fit into your future?

No matter what career path you choose, you are sure to use math in your job or career. Graphic organizers arrange ideas so that you can make informed decisions. Using and understanding models such as graphs, tables, and diagrams helps you to simplify a complicated situation and to identify important quantities in a real-life situation.

Suppose you are a doctor or a nurse. A prescription directs a patient to take 2.5 cc (cubic centimeters) of a medicine per 22.7 kilograms of body weight.

MP Mathematical Practice 4

Model with mathematics.

1. What skill(s) would you use to see how much medicine you should give to a 56.7 kilogram person?

   writing and solving proportions

2. How much medicine would the 56.7 kilogram patient need?

   6.25 cc

3. What career path interests you? Research that career and complete the graphic organizer below. **See students' work.**

Education Required

____

____

____

Career: ____________________

____

____

____

How is math used in this career?

It's Your Turn!

Use the given tools to solve each problem.

4. You are saving money to buy a new game system. You received AED 50 as a graduation gift from your grandparents. You want to save AED 25 a week from mowing lawns.

a. **Tables** Complete the table to show the total amount saved after 1, 2, 3, 4, and 5 weeks.

Week, w	Total Saved, s (AED)
1	75
2	100
3	125
4	150
5	175

b. **Symbols** Write an equation to show the total amount saved s after w weeks. $s = 25w + 50$

c. **Algebra** Use the equation to determine the total amount saved after 17 weeks. AED 475

Use the table for Exercises 5 and 6.

Polly's Perfect Parties			
Cost of Food (per person)		Cost of Extras	
Appetizers	AED 33.79	Hall	AED 918.25
Buffet	AED 67.22	Linens	AED 55.09 per table
Sit-down Dinner	AED 94.58	Table and Chair Rental (seats 8)	AED 220.38 per table

5. Mrs. Fatma hired a party planner to plan Noha's dinner party. There will be 125 guests and she wants to offer appetizers and a buffet dinner. What is the cost, before tax, for the party?

AED 4,887.50

6. There is a $7\frac{1}{2}$% sales tax added to the party bill. Mrs. Fatma also wants to add an 18% tip for the servers. This will be figured before tax is added. What will be the total cost of the party?

AED 6,133.81

Find it in Your Book!

MP Model with Mathematics

Look at Chapter 1. Give an example of where Mathematical Practice 4 is used. Then explain why your example represents this practice.

See students' examples.

Use Math Tools

How do I use tools and strategies in math class?

Mathematical Practice 5

Use appropriate tools strategically.

Sometimes using math tools and strategies helps make solving problems easier if you know which tool to use in a given situation. Math tools are physical objects you use when solving problems. Paper and pencil, technology, or calculators are examples of tools.

1. List three other tools you can use to solve math problems.
   Sample answer: protractor, algebra tiles, and virtual manipulatives

Math strategies are more like skills or the ability to apply your math knowledge. Some math strategies are mental math, number sense, estimation, drawing a diagram, or solving a simpler problem.

2. List three other strategies you can use to solve math problems.
   Sample answer: acting it out, choosing the correct strategy, and choosing the correct operation

3. Complete the graphic organizer. **Sample answers are given.**

Problem	Tool	Strategy
You want to leave a 20% tip for your server.	not necessary	mental math
You want to determine how long it will take to drive from Abu Dhabi to Fujira	Internet, calculator	estimation
You are stuck while in the middle of solving an equation.	algebra tiles	number sense

It's Your Turn!

List the tools or strategies you would use to solve each problem. Then solve the problem.

4. A pre-election survey was taken in Ms. Noha's homeroom. The results for class president are shown in the table.

Class President	
Marwa	10
Karam	8
Asmaa	20
Sara	12

a. Based on the survey, if there are 850 students in the 8th grade, how many votes will Asmaa get?

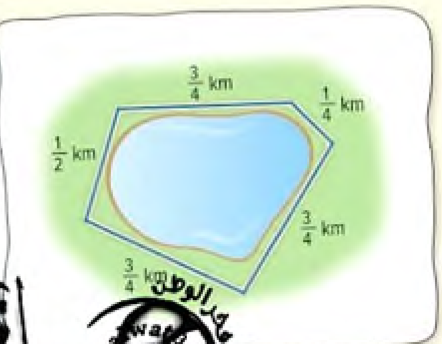
Sample answer: calculator; 340

b. A candidate needs to receive at least 51% of the votes to win the election. If every student votes, how many more votes would Asmaa need to win?

Sample answer: calculator, number sense; 94 votes

5. A walking path around a lake is in the shape of a pentagon like the one shown. If Natalie wants to walk $4\frac{1}{2}$ kilometers, how many times does she need to walk around the lake?

Sample answer: paper and pencil, number sense; $1\frac{1}{2}$ times



6. Write a word problem that requires the use of a protractor, a calculator, and one strategy, like mental math or estimation. Find the solution to your problem and explain how you used the tools to solve it.

See students' work.

Find it in Your Book!

MP Use Math Tools

Look at Chapter 1. Give an example of where Mathematical Practice 5 is used. Then explain why your example represents this practice.

See students' examples.

Attend to Precision

What does it mean to be precise?

Mathematical Practice 6

Attend to precision.

Communication is important to our daily life, whether it's in school, sports, at home, or hanging out with friends. If you can't clearly express your thoughts, no one will understand what you mean! Math also requires clear and precise communication by using labels, appropriate symbols, and clear definitions.

Suppose you and your brother want to paint two walls in your bedroom a new color. Your bedroom is 3.66 meters 12.7 centimeters long, 4.27 meters 20.32 centimeters wide, and has an 2.44 meters ceiling height.

1. What skill(s) would you use to see how much paint you need?
   Sample answer: multiplying to find the area

2. What information do you need to know in order to make your calculations?
   Sample answer: I need to know which 2 walls are being painted and how many square feet a gallon of paint covers.

You are painting two walls that are perpendicular to each other. They do not have doors or windows on them. A gallon of paint covers about 32.5 square meter.

3. What is the area of wall space you will be painting? Label your answer.
   20.15 m^2

4. How precise does the area need to be to determine how much paint you will need? Round the area and explain why you rounded to the place value you chose.
   Sample answer: Because the area that needs painted is less than the area a gallon of paint can cover, I chose to round the area to 20 square meters.

5. How many gallons of paint do you need? Round to an appropriate place value and label your answer. Explain your rounding.
   1 gallon; Sample answer: I cannot buy 0.63 gallon of paint, so I rounded up to the nearest whole number.

It's Your Turn!

6. Turn to page 7 in your text. Find the vocabulary term *rational number* and complete the graphic organizer for that term. **Sample answers are given.**

Rational Number

Definition	Types
numbers that can be expressed as the comparison of two integers	fractions, integers, whole numbers, decimals
Examples	**Non-Examples**
$\frac{1}{2}$, −3, 7, 2.25	π

7. Model trains come in different scales. The ratio for an HO scale train is 1:87, while the ratio for a Z scale train is 1:220. Suppose a Z scale model of a steam engine is 62 millimeters long. What is the length of the HO scale model of the same engine? To what place value should you round? Explain your reasoning.

Sample answer: about 157 mm; I rounded to the nearest millimeter to find the approximate length.

Find it in Your Book!

MP Attend to Precision

Look at Chapter 1. Give an example of where Mathematical Practice 6 is used. Then explain why your example represents this practice.

See students' examples.

Focus on Mathematical Practice 7

Make Use of Structure

What does it mean to use structure in math?

When you use structure in math, you might apply properties to solve equations or you might examine patterns in tables and graphs to describe relationships.

Mathematical Practice 7

Look for and make use of structure.

1. The table shows the diameters of several flying discs. Use the relationship between the radius and diameter of a circle to complete the table. Round to the nearest tenth.

Diameter (cm)	Radius (cm)	Circumference (cm)	Area (cm^2)
20	**10**	**62.8**	**314.2**
22	**11**	**69.1**	**380.1**
25	**12.5**	**78.5**	**490.9**

2. Describe the relationship between the diameter and radius of a circle. **The diameter is twice the radius.**

3. Describe the relationship between the circumference and diameter of a circle. **The ratio of the circumference of a circle to its diameter is equal to pi.**

4. Complete the graphic organizer by writing a formula in each box that shows the relationship between each term.
Sample answers are given.

Diameter and Radius
$d = 2r$

Radius and Circumference
$C = \pi(2r)$ or $C = 2\pi r$

Diameter and Area
$A = \pi\left(\frac{d}{2}\right)^2$ or $A = \frac{\pi d^2}{4}$

It's Your Turn!

Suppose you are training for a marathon. A marathon is 42.2 kilometers long. You can run 4.8 kilometers in 16 minutes.

5. At this rate, how many miles can you run in one hour?

11.25 kilometers

6. Complete the table and plot the points to make a line graph.

Time (h)	Distance (km)
1	**11.25**
2	**22.5**
3	**33.75**

7. Write an equation that shows the relationship between distance and time.

$d = 11.25t$

8. Estimate how long it will take to complete the marathon.

about 2 hours 20 minutes

Find it in Your Book!

MP Make Use of Structure

Look at Chapter 1. Give an example of where Mathematical Practice 7 is used. Then explain why your example represents this practice.

See students' examples.

Use Repeated Reasoning

What does it mean to look for repeated reasoning?

Mathematical Practice 8

Look for and express regularity in repeated reasoning.

Problems can often be solved by finding patterns or repeated processes. Sometimes you can even create shortcuts to solve a problem once you understand the pattern. For example, multiplication is a shortcut for repeating the same addition over and over.

Suppose you have a garden with a length of 1.8 meters and a width of 1.2 meters and you want to increase its size. Before making any changes, do some math!

1. What is the perimeter of the garden? **20 mtr**
   the area? **20 mtr^2**

2. If you double the dimensions of the garden, what is the new perimeter? **40 mtr** new area? **96 mtr^2**

3. What number can you multiply the original perimeter by to find the new perimeter? **2** What number can you multiply the original area by to find the new area? **4**

Oh no, the increased size of the garden is too big! Using the original dimensions of the garden, you increase the length to 2.7 meters and the width to 1.8 meters.

4. What is the new perimeter? **30 mtr** new area? **54 mtr^2**

5. What number can you multiply the original perimeter by to find the new perimeter? **1.5** What number can you multiply the original area by to find the new area? **2.25**

6. Try other changes in the dimensions of the garden to find the new perimeter and area of the garden.
   See students' work.

It's Your Turn!

7. Ahmed is mixing orange juice and apple juice in a ratio of 3 to 4 to make a fruit punch. He wants to make 35 cups of the punch. To determine how many cups of each juice he needs, he started making a table. Complete the table to find how many cups of each juice he will need. Then explain a shortcut you could use to solve the problem.

Orange Juice	Apple Juice	Total Cups
3	4	7
6	8	14
9	12	21
12	**16**	**28**
15	**20**	**35**

Sample answer: I know that 7 × 5 is 35, so I can also multiply 3 cups and 4 cups by 5 to find out how many cups of each juice he will need.

8. Amina's parents are going to pay her for doing chores 6 days a week and they offer her two payment plans.

Option A						
Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Total
AED 11.02	AED 22.04	AED 33.06	**AED 12**	**AED 15**	**AED 18**	**AED 63**

Option B						
Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Total
AED 2.75	AED 5.51	AED 11.02	**AED 6**	**AED 12**	**AED 24**	**AED 47.25**

Complete the table to determine which is the better option for Savannah to choose. Explain the pattern for each option.

Option A; Sample answer: The pattern in Option A is adding AED 3 each day. Amina will receive AED 18 on the 6th day and a total of AED 63 for the week. The pattern in Option B is multiplying by 2 each day. Amina will receive AED 24 on the 6th day and a total of AED 47.25 for the week.

Find it in Your Book!

MP Use Repeated Reasoning

Look at Chapter 1. Give an example of where Mathematical Practice 8 is used. Then explain why your example represents this practice.

See students' examples.

Review

Use the Mathematical Practices

Solve.

You are boxing and wrapping gifts for a club fundraiser. The charge to wrap a gift in the shape of a rectangular prism is shown in the table.

Total Surface Area	Cost
up to 88.9 cm^2	AED 18.37
91.44-137.16 cm^2	AED 29.38
over 139.7 cm^2	AED 44.08

a. Mariam wrapped three different boxes with measurements shown in the table. Complete the table with the cost per box and the cost per square centimeter. Which box has the least cost per square centimeter? **Box B**

Box	height cm.	width cm.	length cm.	Cost to Wrap	Cost per Square cm.
A	5.08	10.16	7.62	AED 8	AED 0.15
B	5.08	12.7	15.24	AED 12	AED 0.12
C	5.08	7.62	5.08	AED 5	AED 0.16

b. Which of those boxes has the least cost per cubic centimeter? Explain.

Box B; The volume of Box A is 393.3 cm^3, so the cost per cm^3 is AED 8 ÷ 393.3 or AED 0.02 per cm^3. The volume of Box B is 983.2 cm^3, so the cost per cm^3 is AED 12 ÷ 983.2 or AED 0.012 per cm^3. The volume of Box C is 196.65 cm^3, so the cost per cm^3 is AED 5 ÷ 196.65 or about AED 0.025 per cm^3.

Determine which mathematical practices you used to determine the solution. Shade the circles that apply.

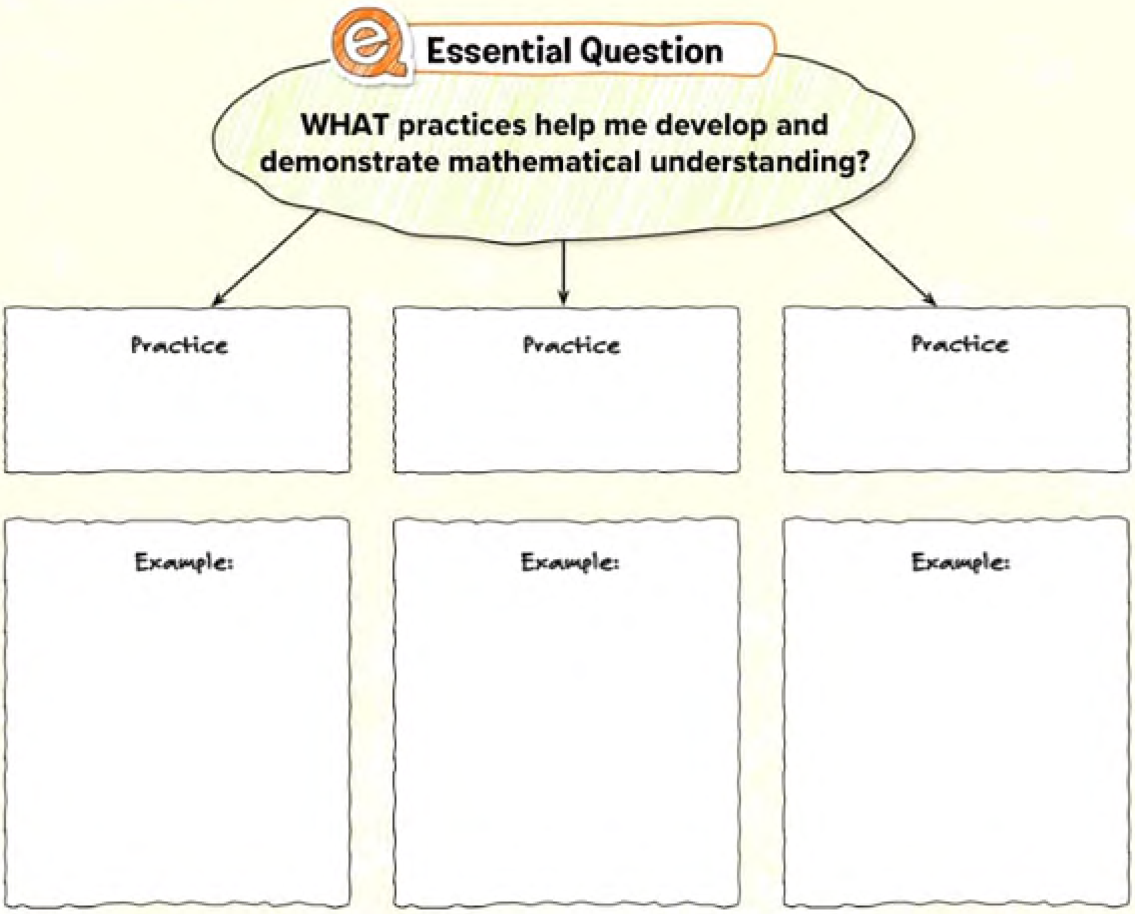
Which **Mathematical Practices** did you use?
Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Reflect

Answering the Essential Question

Use what you learned about the mathematical practices to complete the graphic organizer. List three practices that help you best demonstrate mathematical understanding. Then give an example for each practice.
See students' work.



Answer the Essential Question. WHAT practices help me develop and demonstrate mathematical understanding?

Sample answer: All of the mathematical practices help to develop and demonstrate mathematical understanding. For example, I can use math tools and look for structure to develop understanding. I can construct an argument and model with mathematics to demonstrate understanding.

The Number System

Essential Question

HOW can mathematical ideas be represented?

Chapter 1
Real Numbers

Rational numbers can be used to approximate the value of irrational numbers. In this chapter, you will perform operations on monomials and numbers written in scientific notation. You will then use rational approximations to estimate roots and to compare real numbers.

Essential Question

At the end of this unit, students should be able to answer "How can mathematical ideas be represented?"

Chapter 1 explores an essential question that assists students in answering the unit question. The lessons in this chapter include exercises that lead students to various aspects of the essential question. This unit focuses on the Number System (NS) domain. The chapter addresses the following:

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2).

Unit Project Preview

Have students write about both written music and musical instruments.

The Unit Project can be found on pages 103 –104.

Project 1 Preview

Collaborate

Music to My Ears Listening to music can be both fun and relaxing. But did you know that many interesting relationships exist between math and music? Even the ancient Greek mathematician Pythagoras observed and wrote about many of these relationships.

At the end of Chapter 1, you'll complete a project to find how math and music are connected. But for now, write about the connections between math and music in the space provided.

Math and Music

Chapter 1

Real Numbers

Essential Question

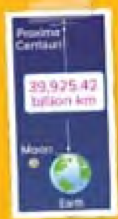
WHY is it helpful to write numbers in different ways?

Mathematical Practices
1, 3, 4, 5, 6, 7, 8

Math in the Real World

Space The average distance from Earth to the Moon is about 384,403 kilometers. The Sun is the closest star to Earth and is about 150 million kilometers away. The next closest star is Proxima Centauri which is about 4.22 light years away from Earth.

A light year is defined as 9.461 billion kilometers. Find and label the distance in kilometers from Earth to Proxima Centauri.



FOLDABLES Study Organizer

1. Cut out the Foldable in the back of this book.
2. Place your Foldable on page 100.
3. Use the Foldable throughout this chapter to help you learn about real numbers.

Focus narrowing the scope

This chapter focuses on content from the **Number System** (NS) domain.

Coherence connecting within and across grades

Previous	Now	Next
Students compared positive and negative fractions, mixed numbers, decimals, and percents and graphed them on a number line.	Students simplify real number expressions using integer exponents and laws of exponents.	Students will apply real numbers to the Pythagorean Theorem, finding area and volume and solving other real-world problems.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

Launch the Chapter

Math in the Real World

Space Remind students that to convert from light years to kilometers, students must multiply 4.22 by 9,461.

Throughout this text, refer to the following icons to find differentiated strategies to meet the needs of all learners.

- AL Approaching-Level Learners
- OL On-Level Learners
- BL Beyond-Level Learners
- LA Language Acquisition

What Tools Do You Need?

Vocabulary Activity

LA As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define: Scientific notation is when a number is written as the product of a single digit factor and an integer power of 10.

Example: $99{,}000 = 9.9 \times 10^4$

Ask:

- *What are some real-world examples of using scientific notation?* **Sample answers: The equatorial circumference of the Sun is about 4.36×10^6 kilometers. The mass of a proton is 1.673×10^{-24} grams.**

Use a Mnemonic Device

LA Have students read the *Use a Mnemonic Device* section. When working with the order of operations, remind students that when it comes to the last two rungs of the ladder (multiplying and dividing and adding and subtracting), they perform these operations in order from left to right.

What Tools Do You Need?

Vocabulary

base	perfect cube	repeating decimal
cube root	perfect square	scientific notation
exponent	power	square root
irrational number	radical sign	terminating decimal
monomial	rational number	

Use a Mnemonic Device

When a mathematical expression has a combination of operations, the order of operations tells you which operation to perform first. How can you remember the orders easily? A mnemonic device is a verse or phrase to help you remember something.

In this case, it is *Please Excuse My Dear Aunt Sally*. On each rung of the ladder, fill in the operation that the mnemonic device represents. Then evaluate the numerical expression step-by-step.

$3(5 - 15)^2 - 7 \cdot 3 + 24 \div 6$

Please	parentheses	$3(-10)^2 - 7 \cdot 3 + 24 \div 6$
Excuse	exponents	$3(100) - 7 \cdot 3 + 24 \div 6$
My Dear	multiply and divide	$300 - 21 + 4$
Aunt Sally	add and subtract	283

What Do You Already Know?

Place a checkmark below the face that expresses how much you know about each concept. Then scan the chapter to find a definition or example of it. See students' work.

(red face) I have no clue. (orange face) I've heard of it. (green face) I know it!

Integers				
Concept	(red face)	(orange face)	(green face)	Definition or Example
irrational numbers				
multiplying and dividing monomials				
negative exponents				
rational numbers				
scientific notation				
square roots				

When Will You Use This?

Here is an example of how real numbers are used in the real world.

Activity Use the Internet or another media source to find a description of the metric system. How do you convert one measurement to another using the metric system?

See students' work.

What Do You Already Know?

In this activity students assess their prior knowledge by choosing a face to represent their knowledge about concepts in the chapter.

After completing the chapter, have students return to this page and have them reevaluate their knowledge level about the content.

When Will You Use This?

Activity

Students may not realize how frequently they use real numb in everyday life. Most of the numbers they use every day (integers, fractions, decimals, etc.) are real numbers.

Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

Quick Check

If students have difficulty with the exercises, present an additional example to clarify any misconceptions.

Exercises 1–3

Find $4 \cdot 3 \cdot 3 \cdot 4 \cdot 4$. **576**

Exercises 4–9

Find the prime factorization of 72. **$2 \times 2 \times 2 \times 3 \times 3$**

Are You Ready?

Try the Quick Check below.

Quick Review

Example 1

Find $5 \cdot 4 \cdot 5 \cdot 4 \cdot 5$.

$$\begin{aligned} 5 \cdot 4 \cdot 5 \cdot 4 \cdot 5 &= 4 \cdot 4 \cdot 5 \cdot 5 \cdot 5 \\ &= (4 \cdot 4) \cdot (5 \cdot 5 \cdot 5) \\ &= 16 \cdot 125 \\ &= 2{,}000 \end{aligned}$$

Example 2

Find the prime factorization of 60.

60

6 × 10

2 × 3 × 2 × 5

The prime factorization of 60 is $2 \times 2 \times 3 \times 5$.

Quick Check

Simplify Expressions Find each product.

1. $2 \cdot 2 \cdot 4 \cdot 4 \cdot 4 =$ **256**
2. $(-8)(-8)(5)(5)(-8) =$ **−12,800**
3. The students at Hampton Middle School raised $8 \cdot 8 \cdot 2 \cdot 8 \cdot 2$ dirhams to help build a new community center. How much money did they raise?

AED 2,048

Prime Factorization Find the prime factorization of each number.

4. 36 **$2 \times 2 \times 3 \times 3$**
5. 24 **$2 \times 2 \times 2 \times 3$**
6. 18 **$2 \times 3 \times 3$**
7. 100 **$2 \times 2 \times 5 \times 5$**
8. 121 **11×11**
9. −42 **$-1 \times 2 \times 3 \times 7$**

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7 8 9

Lesson 1

Rational Numbers

Vocabulary Start-Up

Numbers that can be written as a comparison of two integers, expressed as a fraction, are called **rational numbers**.

Complete the graphic organizer. Sample answers are given.

Rational Number

Define in your own words: Rational numbers are numbers that can be written as a ratio of two integers.

Examples:
- Percent: $19\% = \frac{19}{100}$
- Decimal: $0.32 = \frac{8}{25}$
- Fraction: $\frac{3}{4} = 0.75$
- Mixed Numbers: $3\frac{1}{11} = \frac{34}{11}$

The root of the word *rational* is *ratio*. Describe the relationship between rational numbers and ratios. Sample answer: Rational numbers are written as ratios in the form $\frac{a}{b}$, where a and b are integers, and $b \neq 0$.

Essential Question

WHY is it helpful to write numbers in different ways?

Vocabulary

rational number
repeating decimal
terminating decimal

Mathematical Practices
1, 3, 4, 6, 7, 8

Real-World Link

During a recent regular season, a Texas Ranger baseball player had 126 hits and was at bat 399 times. Write a fraction in simplest form to represent the ratio of the number of hits to the number of at bats. $\frac{6}{19}$

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Write fractions as decimals and decimals a fractions.

Coherence connecting within and across grade

Previous Students converted between different forms of rational numbers.

Now Students write fractions as terminating and repeating decimals.

Next Students w and order numbers.

Rigor pursuing concepts, fluency, and applicati

See the Levels of Complexity chart on page 11.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole gr group, think-pair-share activity, or independent activi

LA Make up sets of index cards with di types of number sets, such as odd and ev fractions, and decimals (one number per card). Each cards should have 10–20 different numbers. Divide s into small groups and give each group a set of cards to classify each number in as many ways as they can

Alternate Strategy

AL LA Hang three large pieces of chart paper aro room. Label each one with a different term (Fraction, Terminating Decimal, and Repeating Decimal). Hand o notes with one number on each. Give students one m decide what type of number is on the sticky note, the them up to place sticky notes on the appropriate char MP 1, 2, 6

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Write a fraction as a decimal.

AL
- *What operation does the fraction bar represent?* **division**
- *What is the first step to write $\frac{5}{8}$ as a decimal?* **Divide 5 by 8.**

OL
- *How can you use estimation to determine if your answer is reasonable?* **Sample answer: $\frac{5}{8} \approx \frac{1}{2}$ and $0.625 \approx 0.5$**

BL
- *What are the fraction-decimal equivalents for any fraction with a denominator of 8?* **$\frac{1}{8} = 0.125$, $\frac{2}{8} = 0.25$, $\frac{3}{8} = 0.375$, $\frac{4}{8} = 0.5$, $\frac{5}{8} = 0.625$, $\frac{6}{8} = 0.75$, $\frac{7}{8} = 0.875$, $\frac{8}{8} = 1$**

Need Another Example?

Write $\frac{3}{16}$ as a decimal. **0.1875**

2. Write a mixed number as a decimal.

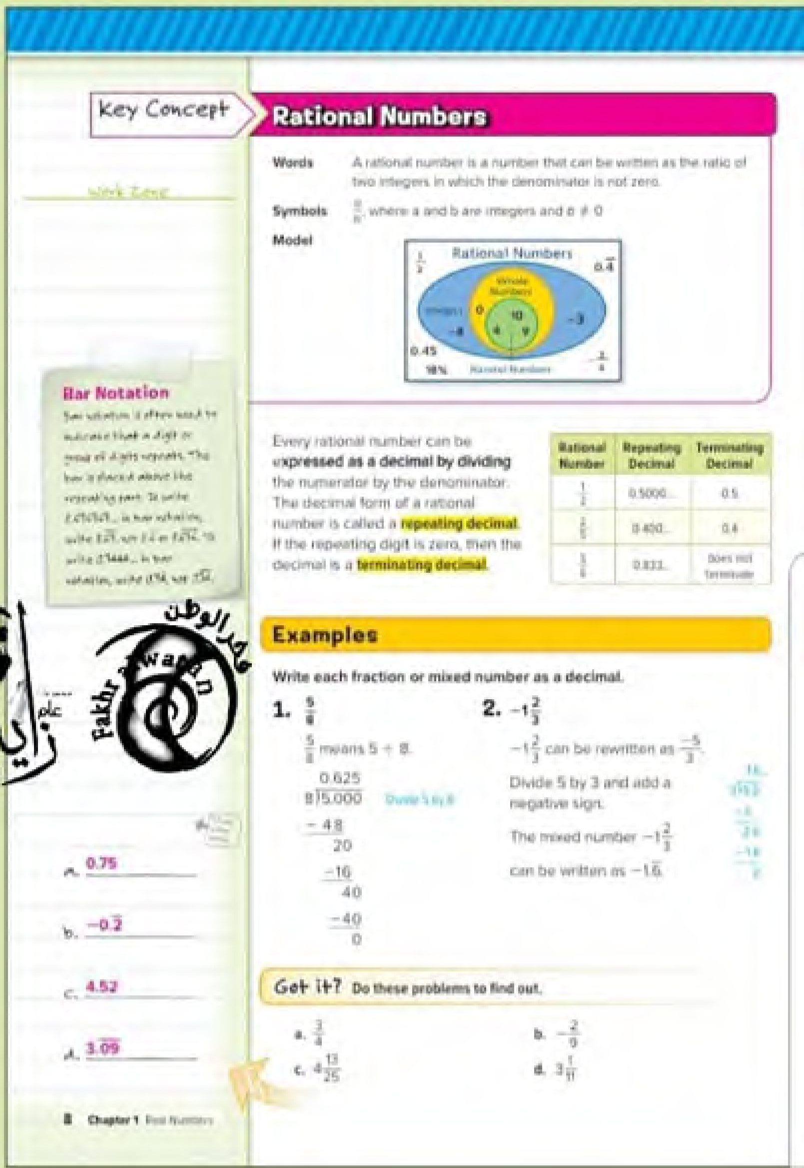
AL
- *What is the first step to write $-1\frac{2}{3}$ as a decimal?* **Rewrite $-1\frac{2}{3}$ as the improper fraction $-\frac{5}{3}$.**

OL
- *What does the bar over the 6 represent?* **The digit 6 repeats.**

BL
- *How do you determine whether or not a decimal is a terminating decimal?* **If the repeating digit is zero, then it is a terminating decimal.**
- *Explain another way you can write the mixed number as a decimal.* **Sample answer: Write $\frac{2}{3}$ as a decimal. Then insert -1 in front of the decimal point.**

Need Another Example?

Write $-3\frac{2}{11}$ as a decimal. **$3.\overline{18}$**


Key Concept **Rational Numbers**

Words A rational number is a number that can be written as the ratio of two integers in which the denominator is not zero.

Symbols $\frac{a}{b}$, where a and b are integers and $b \neq 0$

Model

Work Zone

Bar Notation

Every rational number can be **expressed as a decimal by dividing** the numerator by the denominator. The decimal form of a rational number is called a **repeating decimal**. If the repeating digit is zero, then the decimal is a **terminating decimal**.

Rational Number	Repeating Decimal	Terminating Decimal
$\frac{1}{2}$	0.5000...	0.5
$\frac{2}{5}$	0.400...	0.4
$\frac{5}{6}$	0.833...	Does not terminate

Examples

Write each fraction or mixed number as a decimal.

1. $\frac{5}{8}$

$\frac{5}{8}$ means $5 \div 8$.

0.625
8)5.000 Divide 5 by 8
− 48
20
− 16
40
− 40
0

2. $-1\frac{2}{3}$

$-1\frac{2}{3}$ can be rewritten as $\frac{-5}{3}$.

Divide 5 by 3 and add a negative sign.

The mixed number $-1\frac{2}{3}$ can be written as $-1.\overline{6}$.

Got It? Do these problems to find out.

a. $\frac{3}{4}$ b. $-\frac{2}{9}$
c. $4\frac{13}{25}$ d. $3\frac{1}{11}$

a. 0.75
b. $-0.\overline{2}$
c. 4.52
d. $3.\overline{09}$

8 Chapter 1 Real Numbers

Example

3. In a recent season, first baseman Marwan had 175 hits in 530 at bats. To the nearest thousandth, find his batting average.

To find his batting average, divide the number of hits, 175, by the number of at bats, 530.

175 [÷] 530 [ENTER] 0.3301886792

Look at the digit to the right of the thousandths place. Since $1 < 5$, round down.

Marwan's batting average was 0.330.

Got It? **Do this problem to find out.**

e. In a recent season, a race car driver won 6 of the 36 total races held. To the nearest thousandth, find the part of races he won.

e. 0.167

Examples

4. Write 0.45 as a fraction.

$0.45 = \frac{45}{100}$ 0.45 is 45 hundredths

$= \frac{9}{20}$ Simplify.

5. Write $0.\overline{5}$ as a fraction in simplest form.

Assign a variable to the value $0.\overline{5}$. Let $N = 0.555...$. Then perform operations on N to determine its fractional value.

$N = 0.555...$

$10(N) = 10(0.555...)$ Multiply each side by 10 because 1 digit repeats.

$10N = 5.555...$ Multiplying by 10 moves the decimal point 1 place to the right.

$-N = 0.555...$ Subtract N = 0.555... to eliminate the repeating part.

$9N = 5$ Simplify.

$N = \frac{5}{9}$ Divide each side by 9.

The decimal $0.\overline{5}$ can be written as $\frac{5}{9}$.

Examples

3. Solve a real-world example involving equivalent fractions and decimals.

- AL • *What fraction is used to find the batting average?* $\frac{175}{530}$
- OL • *What place value do you look at to round to the nearest thousandth?* ten-thousandths
- BL • *How many hits would you expect in 590 at bats?* 195

Need Another Example?
When Hadia went strawberry picking, 28 of the 54 strawberries she picked weighed less than 2 ounces. To the nearest thousandth, find the portion of the strawberries that weighed less than 2 ounces. 0.519

4. Write a decimal as a fraction.

- AL • *Write 0.45 in word form.* forty-five hundredths
- OL • *When writing 0.45 as a fraction, what number would you use in the denominator? Why?* 100; 0.45 is read as forty-five hundredths, so the fraction is 45 out of 100.
- BL • *How can you use estimation to determine if your answer is reasonable?* Sample answer: 9 out of 20 is a little less than 0.5 and 0.45 is a little less than 0.5.

Need Another Example?
Write 0.32 as a fraction in simplest form. $\frac{8}{25}$

5. Write a decimal as a fraction.

- AL • *What does the bar over the 5 represent?* The digit 5 repeats.
- OL • *Why do you need to multiply 0.555... by 10?* because only the first digit repeats
- BL • *Describe the pattern in the decimal equivalents for fractions with denominators of 9.* The numerator is the digit that repeats. For example, $\frac{1}{9} = 0.\overline{1}$.

Need Another Example?
Write $0.\overline{7}$ as a fraction in simplest form. $\frac{7}{9}$

Name ______________________ My Homework ______________________

Independent Practice

Write each fraction or mixed number as a decimal. [illegible]

1. $\frac{2}{5}$ = **0.4**

2. $2\frac{1}{8}$ = **2.125**

3. $\frac{33}{40}$ = **0.825**

4. $\frac{4}{33}$ = **$0.\overline{12}$**

5. $-\frac{6}{11}$ = **$-0.\overline{54}$**

6. $-7\frac{8}{45}$ = **$-7.1\overline{7}$**

7. **Identify Repeated Reasoning** The table shows statistics about the students at Carter Junior High. [illegible]

Number of Siblings	Fraction of Students
None	$\frac{1}{15}$
One	$\frac{1}{3}$
Two	$\frac{5}{12}$
Three	$\frac{1}{6}$
Four or more	$\frac{1}{60}$

a. Express the fraction of students with no siblings as a decimal. **$0.0\overline{6}$**

b. Find the decimal equivalent for the fraction of students with three siblings. **$0.1\overline{6}$**

c. Write the fraction of students with one sibling as a decimal. Round to the nearest thousandth. **0.333**

d. Write the fraction of students with two siblings as a decimal. Round to the nearest thousandth. **0.417**

Write each decimal as a fraction or mixed number in simplest form. [illegible]

8. -0.4 = **$-\frac{2}{5}$**

9. -7.32 = **$-7\frac{8}{25}$**

10. $0.\overline{2}$ = **$\frac{2}{9}$**

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–13, 20–31	14, 15, 32, 33	16–19
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–13, 15, 16, 18, 19, 32, 33
OL	On Level	1–13 odd, 14–16, 18, 19, 32, 33
BL	Beyond Level	14–19, 32, 33

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	17
3 Construct viable arguments and critique the reasoning of others.	18
4 Model with mathematics.	19
6 Attend to precision.	14, 15
7 Look for and make use of structure.	16
8 Look for and express regularity in repeated reasoning.	7, 22–24

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Ask students to write $\frac{5}{6}$ as a decimal. $0.8\overline{3}$

Watch Out!

Common Error In Exercises 14 and 15, students may not know what fractional part of an inch each tick mark on the ruler represents. Encourage them to count the number of spaces from 0 to 1. This is the denominator. The number of spaces the length of the item covers is the numerator.

Copy and Solve Write each decimal as a fraction or mixed number in simplest form. Show your work on a separate piece of paper.

11. $-0.\overline{45}$ $-\frac{5}{11}$

12. $2.\overline{7}$ $2\frac{7}{9}$

13. 5.55 $5\frac{11}{20}$

Be Precise Write the length of each insect as a fraction or mixed number and as a decimal.

14. 

$\frac{7}{8}$ cm; 0.875 cm

15. 

$1\frac{1}{16}$ cm; 1.0625 cm

H.O.T. Problems Higher Order Thinking

16. **Identify Structure** Give an example of a repeating decimal where two digits repeat. Explain why your number is a rational number.
Sample answer: $0.\overline{12}$; Since $0.\overline{12} = \frac{4}{33}$, it is a rational number.

17. **Persevere with Problems** Explain why any rational number is either a terminating or repeating decimal. Sample answer: When dividing, there are two possibilities for the remainder. If the remainder is 0, the decimal terminates. If the remainder is not 0, then the decimal begins to repeat at the point where the remainder repeats or equals the original dividend.

18. **Make a Conjecture** Compare 0.1 and $0.\overline{1}$, 0.13 and $0.\overline{13}$, and 0.157 and $0.\overline{157}$ when written as fractions. Make a conjecture about expressing repeating decimals like these as fractions.
Sample answer: When the digits repeat, the repeating digits are the numerator and 1 less than the decimal place value is the denominator.

19. **Model with Mathematics** Write two decimals, one repeating and one terminating, with values between 0 and 1. Then write an inequality that shows the relationship between your two decimals.
Sample answer: 0.5, 0.555...; 0.5 < 0.555...

Name ______________ My Homework ______________

Extra Practice

20. Write $\frac{5}{9}$ as a decimal. $0.\overline{5}$

$$\begin{array}{r} 0.55 \\ 9\overline{)5.00} \\ -45 \\ \hline 50 \\ -45 \\ \hline 5... \end{array}$$

Homework Help

21. Write $7.\overline{15}$ as a mixed number in simplest form. $7\frac{5}{33}$

$N = 7.151515...$

$100(N) = 100(7.151515...)$

$100N = 715.151515...$

$-N = 7.151515...$

$99N = 708$

$N = \frac{708}{99}$ or $7\frac{5}{33}$

Identify Repeated Reasoning Write each fraction or mixed number as a decimal.

22. $\frac{4}{5} =$ **0.8**

23. $5\frac{5}{16} =$ **5.3125**

24. $-6\frac{13}{15} =$ **$-6.8\overline{6}$**

Write each decimal as a fraction or mixed number in simplest form.

25. $-1.55 =$ **$-1\frac{11}{20}$**

26. $3.\overline{8} =$ **$3\frac{8}{9}$**

27. $-0.\overline{09} =$ **$-\frac{1}{11}$**

Write the rainfall amount for each day as a fraction or mixed number.

28. Friday **$\frac{2}{25}$ ml**

29. Saturday **$2\frac{2}{5}$ ml**

30. Sunday **$\frac{7}{200}$ ml**

Day	Rainfall (mL)
Friday	0.08
Saturday	2.4
Sunday	0.035

31. The table shows three popular flavors according to the results of a survey. What is the decimal value of those who liked vanilla, chocolate, or strawberry? Round to the nearest hundredth. **0.45**

Flavor	Fraction
Vanilla	$\frac{3}{10}$
Chocolate	$\frac{1}{11}$
Strawberry	$\frac{1}{18}$

Power Up! Test Practice

Exercises 32 and 33 prepare students for more rigorous thinking needed for the assessment.

32.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1, MP8
Scoring Rubric	
1 point	Students correctly answer each part of the question.

33.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

32. Determine if the number in each situation is rational.

a. The position of a submarine with respect to the surface of the water is −225.4 feet. ■ Rational □ Not Rational

b. A mechanic uses a wrench that is labeled $\frac{13}{16}$ inch. ■ Rational □ Not Rational

c. The circumference of a pizza in 16π or 50.2654824574... inches. □ Rational ■ Not Rational

d. Khaled earned a score of 86.7% on his science test. ■ Rational □ Not Rational

33. The table shows the number of free throws each player made during the last basketball season. Determine if each statement is true or false.

Player	Free Throws Made	Free Throws Attempted
Fatima	18	20
Maha	13	24
Yasmine	15	22
Gihan	10	14

a. Fatima made $\frac{9}{10}$ of her free throw attempts. ■ True □ False

b. Maha made $\frac{7}{12}$ of her free throw attempts. □ True ■ False

c. Yasmine made $\frac{15}{22}$ of her free throw attempts. ■ True □ False

d. Gihan made $\frac{4}{7}$ of her free throw attempts. □ True ■ False

Spiral Review

Fill in each ◯ with >, <, or = to make a true statement.

34. $2\frac{7}{8}$ (>) 2.75

35. $\frac{-1}{3}$ (>) $\frac{-7}{3}$

36. $\frac{5}{7}$ (<) $\frac{4}{5}$

37. $3\frac{6}{11}$ (=) $3.\overline{54}$

38. At the grocery store, Karimah was comparing the unit price for two different packages of laundry detergent. One package was AED 0.2692 per 28.35 grams. The other package was AED 13.37 for 1,474.18 grams. Which package had the lower unit price?

Explain. **The package that was AED 3.64 for 52 ounces had a unit price of AED 0.07 per ounce so it had the lower unit price.**

Lesson 2

Powers and Exponents

Real-World Link

Savings Yunus decided to start saving money by putting a fils in his cashbox, then doubling the amount he saves each week. Use the questions below to find how much money Yunus will save in 8 weeks.

1. Complete the table below to find the amount Yunus saved each week and the total amount in his cashbox.

Week	0	1	2	3	4	5	6
Weekly Savings	1 fils	2 fils	4fils	8fils	16fils	32fils	64fils
Total Savings	1 fils	3 fils	7fils	15fils	31fils	63fils	AED 127

2. How many 2s are multiplied to find his savings in Week 4? 4
   Week 5? 5

3. How much money will Yunus save in Week 8? AED2.56

4. Continue the table to find when he will have enough to buy a pair of shoes for AED 80. after week 12

Week	7	8	9	10	11	12
Weekly Savings	AED 1.28	AED 2.56	AED 5.12	AED 10.23	AED 20.47	AED 40.96
Total Savings	AED 2.55	AED 5.11	AED 10.23	AED 20.47	AED 40.95	AED 81.91

Essential Question

WHY is it helpful to write numbers in different ways?

Vocabulary

power
base
exponent

Mathematical Practices
1, 3, 4, 8

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Write and evaluate expressions involving powers and exponents.

Coherence connecting within and across grades

Previous
Students used the order of operations to evaluate expressions without exponents.

Now
Students write and evaluate expressions involving exponents.

Next
Students will use laws of exponents to simplify expressions involving exponents.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 19.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Roundrobin** Have students work in a group to make and complete a table similar to the one shown using a factor other than 3. MP 1, 4

Alternate Strategy

BL LA **Pairs Discussion** Have students work in pairs to answer the following question. MP 1, 2

Ask:

- *Without multiplying it out, how can you determine Younis's weekly savings in the 15th week?* **Sample answer: Find 2^{15}.**

Engage Explore **Explain** Elaborate Evaluate

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Write expressions using powers.

- AL • *How many times is -2 used as a factor?* **3 times**
  • *How many times is 3 used as a factor?* **4 times**
- OL • *What will be the exponent for -2?* **3**
  • *What will be the exponent for 3?* **4**
- BL • *What is the difference between the expressions $(-2)^3$ and -2^3? Are they equivalent?* **The first expression is the cube of -2. The second expression is the negative cube of 2. They are equivalent in this case because both expressions equal -8.**
  • *Are the expressions $(-2)^4$ and -2^4 equivalent? Explain.* **no; $(-2)^4 = 16$ while $-2^4 = -16$**

Need Another Example?
Write $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot 4 \cdot 4 \cdot 4$ as an expression using exponents. **$(-9)^5 \cdot 4^3$**

2. Write expressions using powers.

- AL • *How many a's are there? b's?* **2; 3**
- OL • *What will be the exponent for a? for b?* **2; 3**
  • *What property allows us to group the factors differently?* **Associative Property**
- BL • *Write an expression with 3 different bases, trade with a partner, and express the expression with exponents.* **See students' work.**

Need Another Example?
Write $x \cdot y \cdot x \cdot y \cdot x \cdot x$ as an expression using exponents.
$x^4 \cdot y^2$

Work Zone

Write and Evaluate Powers

A product of repeated factors can be expressed as a **power**, that is, using an exponent and a base.

4 factors
$2 \cdot 2 \cdot 2 \cdot 2 = 2^4$

The **base** is the common factor.
The **exponent** tells how many times the base is used as a factor.

Powers are read in a certain way.

Read and Write Powers

Power	Words	Factors
3^1	3 to the first power	3
3^2	3 to the second power or 3 squared	$3 \cdot 3$
3^3	3 to the third power or 3 cubed	$3 \cdot 3 \cdot 3$
3^4	3 to the fourth power or 3 to the fourth	$3 \cdot 3 \cdot 3 \cdot 3$
⋮	⋮	⋮
3^n	3 to the *n*th power or 3 to the *n*th	$\underbrace{3 \cdot 3 \cdot 3 \cdot \ldots \cdot 3}_{n \text{ factors}}$

Examples

Write each expression using exponents.

1. $(-2) \cdot (-2) \cdot (-2) \cdot 3 \cdot 3 \cdot 3 \cdot 3$

The base -2 is a factor 3 times, and the base 3 is a factor 4 times.

$(-2) \cdot (-2) \cdot (-2) \cdot 3 \cdot 3 \cdot 3 \cdot 3 = (-2)^3 \cdot 3^4$

2. $a \cdot b \cdot b \cdot a \cdot b$

Use the properties of operations to rewrite and group like bases together. The base a is a factor 2 times, and the base b is a factor 3 times.

$a \cdot b \cdot b \cdot a \cdot b = a \cdot a \cdot b \cdot b \cdot b$
$= a^2 \cdot b^3$

Got It? Do these problems to find out.

a. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$ b. $4 \cdot 4 \cdot 4 \cdot 5 \cdot 5$ c. $m \cdot m \cdot n \cdot n \cdot m$

a. $\left(\frac{1}{2}\right)^4$
b. $4^3 \cdot 5^2$
c. $m^3 \cdot n^2$

Example

3. Evaluate $\left(-\frac{2}{3}\right)^4$.

$\left(-\frac{2}{3}\right)^4 = \left(-\frac{2}{3}\right)\cdot\left(-\frac{2}{3}\right)\cdot\left(-\frac{2}{3}\right)\cdot\left(-\frac{2}{3}\right)$ Write the power as a product.

$= \frac{16}{81}$ Multiply.

Evaluate
Remember that to evaluate an expression means to find its value.

Got it? Do these problems to find out.

d. 4^4 e. $(-2)^6$ f. $\left(\frac{1}{5}\right)^3$

d. 256

e. 64

f. $\frac{1}{125}$

Example

4. The deck of a skateboard has an area of about $2^5 \cdot 7$ square inches. What is the area of the skateboard deck?

$2^5 \cdot 7 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7$ Write the power as a product.

$= (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \cdot 7$ Associative Property

$= 32 \cdot 7$ or 224 Multiply.

The area of the skateboard deck is about 224 square inches.

Got it? Do this problem to find out.

g. A school basketball court has an area of $2^3 \cdot 3 \cdot 5^2 \cdot 7$ square feet. What is the area of a school basketball court?

g. 4,200 ft^2

Examples

Evaluate each expression if $a = 3$ and $b = 5$.

5. $a^2 + b^4$

$a^2 + b^4 = 3^2 + 5^4$ Replace a with 3 and b with 5.

$= (3 \cdot 3) + (5 \cdot 5 \cdot 5 \cdot 5)$ Write the powers as products.

$= 9 + 625$ or 634 Add.

6. $(a - b)^2$

$(a - b)^2 = (3 - 5)^2$ Replace a with 3 and b with 5.

$= (-2)^2$ Perform operations in the parentheses first.

$= (-2) \cdot (-2)$ or 4 Write the powers as products. Then simplify.

Examples

3. Evaluate powers.

- AL • *What does "to the fourth power" mean?* **Use the base as a factor 4 times.**
- OL • *Why is the result positive?* **The exponent is even and the base is negative, so the product is always positive.**
- BL • *Compare the numerator of the problem with the numerator of the result and compare the denominator of the problem with the denominator of the result. What do you notice?* **$2^4 = 16$ and $3^4 = 81$**

Need Another Example?
Evaluate $\left(\frac{1}{4}\right)^3$. $\frac{1}{64}$

4. Evaluate powers.

- AL • *Which operation do we perform first?* **the power**
- OL • *What is the value of 2^5?* **32**
- BL • *What is the approximate area of the deck in square feet? Round to the nearest tenth.* **224 ÷ 144 or about 1.6 ft**

Need Another Example?
A racquetball court has an area of $2 \cdot 4^2 \cdot 5^2$ square feet. What is the area of the racquetball court? **800 ft^2**

5. Evaluate algebraic expressions.

- AL • *What number replaces a? b?* **3; 5**
- BL • *Do you calculate the powers first or add first? Explain.* **powers; Powers and expressions in parentheses should be performed first.**

Need Another Example?
Evaluate $x^3 + y^5$ if $x = 4$ and $y = 2$. **96**

6. Evaluate algebraic expressions.

- OL • *Look back at Example 5. Identify at least two ways in which Example 5 and Example 6 differ.* **Sample answer: Example 5 is addition, Example 6 is subtraction; In Example 5, the power is performed first. In Example 6, the subtraction is performed first.**

Need Another Example?
Evaluate $(x + y)^2$ if $x = 4$ and $y = 2$. **36**

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL BL LA Numbered Heads Together Assign students to 3- or 4-person learning teams, making sure to pair an Approaching Level student with a Beyond Level student. Each team completes Exercises 1–10 making sure that every member understands. Randomly select one member from each team to orally present their solution to the class. MP 1, 8

Watch Out!

Common Error Students can lose track of the negative sign when evaluating expressions that have a negative base. Remind them that when the base is negative an even exponent results in a positive product and an odd exponent results in a negative product.

h. 17

i. 125

j. 715

Got it? Do these problems to find out.

Evaluate each expression if $c = -4$ and $d = 9$.

h. $c^2 + d^2$

i. $(c + d)^3$

j. $d^3 - (c^2 - 2)$

Guided Practice

Write each expression using exponents. (Examples 1 and 2)

1. $(-11)(-11)(-11) =$ $(-11)^3$

2. $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 =$ $2^3 \cdot 3^3$

3. $r \cdot s \cdot r \cdot r \cdot s \cdot s \cdot r \cdot r =$ $r^5 \cdot s^3$

Evaluate each expression. (Example 3)

4. $2^6 =$ 64

5. $(-4)^4 =$ 256

6. $\left(\frac{1}{7}\right)^3 =$ $\frac{1}{343}$

7. The table shows the average weights of some endangered mammals. What is the weight of each animal? (Example 4)

black bear: 350 lb; key deer: 75 lb; panther: 120 lb

Animal	Weight (lb)
Black bear	$2 \cdot 5^2 \cdot 7$
Key deer	$3 \cdot 5^2$
Panther	$2^3 \cdot 3 \cdot 5$

Evaluate each expression if $x = 2$ and $y = 10$. (Examples 5 and 6)

8. $x^2 + y^4 =$ 10,004

9. $(x^2 + y)^3 =$ 2,744

10. **Building on the Essential Question** How can I write repeated multiplication using powers? The repeated factor is the base. The number of times it repeats is the exponent.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

Name ______________ My Homework ______________

Independent Practice

Write each expression using exponents. (Examples 1 and 2)

1. $(-5)(-5)(-5)(-5) =$ **$(-5)^4$**
2. $3 \cdot 3 \cdot 5 \cdot q \cdot q \cdot q =$ **$3^2 \cdot 5 \cdot q^3$**
3. $m \cdot m \cdot m \cdot m \cdot m =$ **m^5**

Evaluate each expression. (Example 3)

4. $(-9)^4 =$ **6,561**
5. $\left(\frac{1}{3}\right)^4 =$ **$\frac{1}{81}$**
6. $\left(\frac{5}{7}\right)^3 =$ **$\frac{125}{343}$**

7. In the United States, nearly $8 \cdot 10^9$ text messages are sent every month. About how many text messages is this?
(Example 4) **8,000,000,000 or 8 billion**

8. Interstate 70 stretches almost $2^3 \cdot 5^2 \cdot 11$ miles across the United States. About how many miles long is Interstate 70?
(Example 4) **2,200 mi**

Evaluate each expression. (Examples 5 and 6)

9. $g^5 - h^3$ if $g = 2$ and $h = 7$ **−311**
10. $c^2 + d^3$, if $c = 8$ and $d = -3$ **37**
11. $a^2 \cdot b^6$ if $a = \frac{1}{2}$ and $b = 2$ **16**
12. $(r - s)^3 + r^2$ if $r = -3$ and $s = -4$ **10**

Engage Explore **Explain Elaborate Evaluate**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–12, 17–23	13, 24–29	14–16
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–16, 28, 29
OL	On Level	1–11 odd, 13–16, 28, 29
BL	Beyond Level	13–16, 28, 29

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
2 Reason abstractly and quantitatively.	16
3 Construct viable arguments and critique the reasoning of others.	27
4 Model with mathematics.	13
7 Look for and make use of structure.	14
8 Look for and express regularity in repeated reasoning.	15

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write about how they think powers and exponents connect with the topic of the next lesson, which is about multiplying and dividing monomials. See students' work.

13. **Model with Mathematics**

The metric system is based on powers of 10. For example, one kilometer is equal to 1,000 meters or 10^3 meters. Write each measurement in meters as a power of 10.

a. hectometer (100 meters) 10^2

b. megameter (1,000,000 meters) 10^6

c. gigameter (1,000,000,000 meters) 10^9

d. petameter (1,000,000,000,000,000 meters) 10^{15}

H.O.T. Problems Higher Order Thinking

14. **Identify Structure** Write an expression with an exponent that has a value between 0 and 1. Sample answer: $\left(\frac{1}{2}\right)^2$

15. **Identify Repeated Reasoning** Describe the following pattern: $3^4 = 81, 3^3 = 27, 3^2 = 9, 3^1 = 3$. Then use a similar pattern to predict the value of 2^{-1}. Sample answer: As the exponent decreases by 1, the simplified answer is divided by 3; $\frac{1}{2}$

16. **Reason Abstractly** Simplify the expressions below to develop a rule for multiplying powers with the same base.

$2^2 \cdot 2^3 = 32$ or 2^{5} $\quad$ $3 \cdot 3^2 = 27$ or 3^{3}

$4^3 \cdot 4 = 256$ or 4^{4} $\quad$ $x^2 \cdot x^3 = x^{5}$

Sample answer: Keep the bases the same, and add the exponents.

Name ______________________ My Homework ______________________

Extra Practice

17. Write $3 \cdot p \cdot p \cdot p \cdot 3 \cdot 3$ using exponents.

$3^3 \cdot p^3$

$3 \cdot p \cdot p \cdot p \cdot 3 \cdot 3 = 3 \cdot 3 \cdot 3 \cdot p \cdot p \cdot p$

$= 3^3 \cdot p^3$

18. Evaluate $x^3 + y^4$ if $x = -3$ and $y = 4$.

229

$x^3 + y^4 = (-3)^3 + 4^4$

$= (-3) \cdot (-3) \cdot (-3) + 4 \cdot 4 \cdot 4 \cdot 4$

$= (-27) + 256$

$= 229$

Write each expression using exponents.

19. $\left(-\frac{5}{6}\right)\left(-\frac{5}{6}\right)\left(-\frac{5}{6}\right) = \left(-\frac{5}{6}\right)^3$

20. $s \cdot (7) \cdot s \cdot (7) \cdot (7) = 7^3 \cdot s^2$

21. $4 \cdot b \cdot b \cdot 4 \cdot b \cdot b = 4^2 \cdot b^4$

Evaluate each expression.

22. $k^4 \cdot m$, if $k = 3$ and $m = \frac{5}{6}$ **$67\frac{1}{2}$**

23. $(c^3 + d^4)^2 - (c + d)^3$, if $c = -1$ and $d = 2$ **224**

Fill in each ◯ with <, >, or = to make a true statement.

24. $(6 - 2)^2 + 3 \cdot 4$ (>) 5^2

25. $5 + 7^2 + 3^3$ (=) 3^4

26. $\left(\frac{1}{2}\right)^4$ (=) $\left(\frac{1}{4}\right)^2$

27. **Multiple Representations** A square has a side length of *s* inches. **a–c. See Answer Appendix**

a. **Tables** Copy and complete the table showing the side length, perimeter, and area of the square on a separate piece of paper.

b. **Graphs** On a separate piece of grid paper, graph the ordered pairs (side length, perimeter) and (side length, area) on the same coordinate plane. Then connect the points for each set.

c. **Words** On a separate sheet of paper, compare and contrast the graphs of the perimeter and area of the square. Which graph is a line?

Side Length (in)	Perimeter (in)	Area (in^2)
1	4	1
2		
3		
4		
5		
⋮		
10		

Power Up! Test Practice

Exercises 28 and 29 prepare students for more rigorous thinking needed for the assessment.

28.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

29.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1, MP8
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

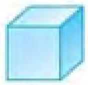
28. Hard drive storage capacity is measured in bytes using the metric system. The metric system is based on powers of 10. For example, 1 kilobyte is equal to 1,000 bytes or 103 bytes. The table shows some common units of storage capacity. Select the correct power of 10 to complete the table.

Unit	Number of Bytes
megabyte	1,000,000
terabyte	1,000,000,000,000
gigabyte	1,000,000,000

Unit	Power of 10
megabyte	10^6
terabyte	10^{12}
gigabyte	10^9

10^3 | 10^6 | 10^9 | 10^{12} | 10^{15}

29. A cube has the dimensions shown below.



What is the volume of the cube expressed as a power?

$6^3\ in^3$

Spiral Review

30. The table below shows the number of ants in an ant farm on different days. The number of ants doubles every ten days.

Day	51	61	71
Number of Ants	320	640	1,280

a. How many ants were in the farm on Day 1? 10

b. How many ants will be in the farm on Day 91? 5,120

Add.

31. $-12 + (-19) =$ -31

32. $-8 + (-11) =$ -19

33. $-5 + 6 =$ 1

Lesson 3

Multiply and Divide Monomials

Real-World Link

Arachnids Spiders in North America can range in size from 1 millimeter in length to 7.6 centimeters in length. Use the table to see how other metric measurements of length are related to the millimeter.

Unit of Length	Times Longer than a Millimeter	Written Using Powers
Millimeter	1	10^0
Centimeter	$1 \times 10 =$ **10**	10^1
Decimeter	$10 \times 10 =$ **100**	$10^1 \times 10^1 = 10^2$
Meter	$100 \times 10 = 1{,}000$	$10^2 \times 10^1 = 10^{\mathbf{3}}$
Dekameter	$1{,}000 \times 10 = 10{,}000$	$10^3 \times 10^1 = 10^{\mathbf{4}}$
Hectometer	$10{,}000 \times 10 =$ **100,000**	$10^4 \times 10^1 = 10^5$
Kilometer	$100{,}000 \times 10 =$ **1,000,000**	$10^5 \times 10^1 = 10^{\mathbf{6}}$

1. Look at the entries in the last column. What do you observe about the exponents of the factors and the exponent of the product for each entry? **Sample answer: The exponent of the product is the sum of the exponents of the factors.**

2. A *megameter* is 100,000,000 × 10 or 1,000,000,000 times longer than a millimeter. Extend the pattern to write this number using powers. **10^9**

Essential Question

WHY is it helpful to write numbers in different ways?

Vocabulary

monomial

Mathematical Practices

1, 3, 4, 7

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Simplify real number expressions by multiplying and dividing monomials.

Coherence connecting within and across grades

Previous Students evaluated expressions involving exponents.

Now Students use the Laws of Exponents to multiply and divide monomials.

Next Students will use the Laws of Exponents to find the power of a power.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 27.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

AL LA **Round Robin** Have students work in teams of three to extend the pattern shown in the table. For example, the first student completes the entry for centimeter, then passes it to the next student to complete the entry for decimeter, and so on until the table is completed. Then students discuss the answers for Exercises 1 and 2. MP 1, 7, 8

Alternate Strategy

BL LA Have students research the prefixes for the measurements and note which ones have Greek roots and which have Latin roots. Then have students compile a list of other words in the English language which use those roots. MP 1, 6

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1–2. Find the product of powers.

AL
- *In Example 1, what is the exponent of the first factor? the second factor?* **2; 1**
- *Are the bases the same?* **yes**

OL
- *In Example 2, are the bases the same?* **yes**
- *In Example 2, how will you find the exponent in the answer?* **add the exponents for each factor together**

BL
- *When you multiply c^3 and c^5, why isn't the coefficient of the product 2?* **When you multiply, you don't add the coefficients, you multiply them. The coefficients of c^3 and c^5 are 1; $1 \cdot 1 = 1$.**

Need Another Example?
Simplify each expression using the Laws of Exponents.
a. $r^4 \cdot r^6$ r^{10} b. $7^6 \cdot 7$ 7^7

3. Find the product of powers.

AL
- *What are the coefficients of each factor?* **−3 and 4**
- *What is the exponent of the first factor? the second factor?* **2; 5**

OL
- *When finding the product of the two terms, what do you do with the coefficients?* **multiply them**
- *When finding the product of the two terms, what do you do with the exponents if the bases are the same?* **add them**
- *What properties allow you to regroup the factors so that the variables are together?* **Commutative and Associative Properties of Multiplication**

BL
- *Simplify $3x^4y^2 \cdot 3x^6yz^4$.* $9x^{10}y^3z^4$

Need Another Example?
Simplify $-7x^2 \cdot 11x^4$ using the Laws of Exponents. $-77x^6$

Key Concept

Product of Powers

Words	To multiply powers with the same base, add their exponents.	
Examples	Numbers $2^4 \cdot 2^3 = 2^{4+3}$ or 2^7	Algebra $a^m \cdot a^n = a^{m+n}$

Work Zone

A **monomial** is a number, a variable, or a product of a number and one or more variables. You can use the Laws of Exponents to simplify monomials.

$$3^2 \cdot 3^4 = \underbrace{\underbrace{(3 \cdot 3)}_{\text{2 factors}} \cdot \underbrace{(3 \cdot 3 \cdot 3 \cdot 3)}_{\text{4 factors}}}_{\text{6 factors}} \text{ or } 3^6$$

Notice that the sum of the original exponents is the exponent in the final product.

Examples

Simplify using the Laws of Exponents.

1. $5^2 \cdot 5$

$5^2 \cdot 5 = 5^2 \cdot 5^1$ $5 = 5^1$
$= 5^{2+1}$ The common base is 5.
$= 5^3$ or 125   Add the exponents. Simplify.

Check $5^2 \cdot 5 = (5 \cdot 5) \cdot 5$
$= 5 \cdot 5 \cdot 5$
$= 5^3$ ✓

2. $c^3 \cdot c^5$

$c^3 \cdot c^5 = c^{3+5}$ The common base is c.
$= c^8$ Add the exponents.

3. $-3x^2 \cdot 4x^5$

$-3x^2 \cdot 4x^5 = (-3 \cdot 4)(x^2 \cdot x^5)$ Commutative and Associative Properties
$= (-12)(x^{2+5})$ The common base is x.
$= -12x^7$ Add the exponents.

Got it? Do these problems to find out.

a. $9^3 \cdot 9^2$ b. $a^3 \cdot a^2$ c. $-2m(-8m^5)$

a. 9^5 or 59,049
b. a^5
c. $16m^6$

Quotient of Powers

Key Concept

Words To divide powers with the same base, subtract their exponents.

Examples

Numbers: $\frac{3^7}{3^3} = 3^{7-3}$ or 3^4

Algebra: $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

STOP and Reflect

Explain below why the Quotient of Powers rule cannot be used to simplify the expression $\frac{x^7}{y^3}$.

The rule can only be used when the bases are the same. In this expression, one base is *x* and the other base is *y*.

There is also a Law of Exponents for dividing powers with the same base.

$$\frac{5^7}{5^4} = \frac{\overbrace{5 \cdot 5 \cdot 5 \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{5}}^{7 \text{ factors}}}{\underbrace{\cancel{5} \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{5}}_{4 \text{ factors}}} \text{ or } 5^3$$

Notice that the difference of the original exponents is the exponent in the final quotient.

Examples

Simplify using the Laws of Exponents.

4. $\frac{4^8}{4^2}$

$\frac{4^8}{4^2} = 4^{8-2}$ The common base is 4.

$= 4^6$ or 4,096 Simplify.

5. $\frac{n^9}{n^4}$

$\frac{n^9}{n^4} = n^{9-4}$ The common base is *n*.

$= n^5$ Simplify.

6. $\frac{2^5 \cdot 3^5 \cdot 5^2}{2^2 \cdot 3^4 \cdot 5}$

$\frac{2^5 \cdot 3^5 \cdot 5^2}{2^2 \cdot 3^4 \cdot 5} = \left(\frac{2^5}{2^2}\right)\left(\frac{3^5}{3^4}\right)\left(\frac{5^2}{5}\right)$ Group by common base.

$= 2^3 \cdot 3^1 \cdot 5^1$ Subtract the exponents.

$= 8 \cdot 3 \cdot 5$ $2^3 = 8$

$= 120$ Simplify.

Got it? Do these problems to find out.

d. $\frac{5^7}{5^4}$ e. $\frac{x^{10}}{x^3}$ f. $\frac{12w^5}{2w}$

g. $\frac{3^4 \cdot 5^2 \cdot 7^3}{3^2 \cdot 5 \cdot 7^3}$ h. $\frac{5^5 \cdot 7^4 \cdot 8^3}{5^3 \cdot 7^2 \cdot 8^2}$ i. $\frac{(-2)^5 \cdot 3^4 \cdot 5^7}{(-2)^2 \cdot 3 \cdot 5^4}$

d. 5^3 or 125

e. x^7

f. $6w^4$

g. $3^2 \cdot 5 \cdot 7^2$ or 2,205

h. $5^2 \cdot 7^2 \cdot 8$ or 9,800

i. $(-2)^3 \cdot 3^3 \cdot 5^3$ or $-27{,}000$

Examples

4–5. Find the quotient of powers.

AL
- *In Example 4, what is the exponent of the numerator?* 8
- *In Example 4, what is the exponent of the denominator?* 2
- *Are the bases the same?* yes
- *What do you do to find the exponent of the quotient?* subtract 2 from 8

OL
- *In Example 5, are the bases the same?* yes
- *In Example 5, how will you find the exponent in the answer?* subtract 4 from 9

BL
- *When dividing powers with the same base, what do you do with the exponents and the base?* subtract the exponents, keep the base the same

Need Another Example?

Simplify each expression using the Laws of Exponents.

a. $\frac{6^{12}}{6^2}$ 6^{10} b. $\frac{a^{14}}{a^8}$ a^6

6. Find the quotient of powers.

AL
- *What terms have the same base?* 2^5 and 2^2, 3^5 and 3^4, 5^2 and 5
- *Rewrite the problem so the terms are grouped by common bases.* $\frac{2^5}{2^2} \cdot \frac{3^5}{3^4} \cdot \frac{5^2}{5}$
- *What is* $\frac{2^5}{2^2}$? 2^3 or 8 $\frac{3^5}{3^4}$? 3^1 or 3 $\frac{5^2}{5}$? 5^1 or 5
- *What is* $8 \cdot 3 \cdot 5$? 120

OL
- *How can the expression be rewritten as the multiplication of 3 separate fractions?* $\frac{2^5}{2^2} \cdot \frac{3^5}{3^4} \cdot \frac{5^2}{5}$
- *What is each factor in simplified form?* 2^3; 3; 5

BL
- *Simplify* $\frac{5^7x^4y^8}{5^4xy^4}$. $5^3x^3y^4$ or $125x^3y^4$

Need Another Example?

Simplify $\frac{2^4 \cdot 5^3 \cdot 9^2}{2^3 \cdot 5 \cdot 9}$ 450

Example

7. Find the quotient of powers.

AL • *What do you need to find?* **how many times longer Hawaii's shoreline is than New Hampshire's**

• *Will you need to add, subtract, multiply, or divide?* **divide**

• *What is the division problem you need to solve?* $\frac{2^{10}}{2^7}$

OL • *What operation will you need to use to solve the problem?* **division**

• *What expression represents this situation?* $\frac{2^{10}}{2^7}$

BL • *The Mississippi River is a little over 3^7 miles in length, while the Kansas River is about 9^3 miles in length. Describe how to solve this problem without evaluating the numbers and dividing.* **See students' work.**

Need Another Example?
One centimeter is equal to 10 millimeters, and one kilometer is equal to 10^6 millimeters. How many centimeters are in one kilometer? **10^5 cm**

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Check** Have students work in pairs to solve Exercises 1–7. One partner will solve the problem while the other one coaches. The coach may choose to help by having the partner write out the expressions, (i.e. 2^4 as $2 \cdot 2 \cdot 2 \cdot 2$) and then perform the appropriate operation. Students switch roles for the each problem. After every two problems, the pairs check their answers with another pair and discuss any differences. MP 1, 3, 6, 7, 8

BL LA **Pairs Discussion** Have students work in pairs to complete the Enrich worksheet *Dividing Powers with Different Bases*. Have them trade their solutions with another pair of students and discuss any differences. MP 1, 3, 6, 7, 8

Example

7. Hawaii's total shoreline is about 2^{10} miles long. New Hampshire's shoreline is about 2^7 miles long. About how many times longer is Hawaii's shoreline than New Hampshire's?

To find how many times longer, divide 2^{10} by 2^7.

$\frac{2^{10}}{2^7} = 2^{10-7}$ or 2^3 Quotient of Powers

Hawaii's shoreline is about 2^3 or 8 times longer.

Guided Practice

Simplify using the Laws of Exponents. Examples 1–6

1. $4^5 \cdot 4^3 =$ **4^8 or 65,536**
2. $-2a(3a^4) =$ **$-6a^5$**
3. $\frac{y^8}{y^5} =$ **y^3**
4. $\frac{24k^7}{6k^4} =$ **$4k^3$**
5. $\frac{2^2 \cdot 3^3 \cdot 4^2}{2 \cdot 3 \cdot 4} =$ **$2 \cdot 3^2 \cdot 4$ or 72**
6. $\frac{(-3)^4 \cdot (-4)^3 \cdot 5^2}{(-3)^2 \cdot (-4) \cdot 5} =$ **$(-3)^2 \cdot (-4)^2 \cdot 5$ or 720**

7. The table shows the number of people worldwide that speak certain languages. How many times as many people speak French than Sicilian? Example 7 **2^4 or 16 times**

Language	Total (millions)
French	2^7
Sicilian	2^3

8. **Building on the Essential Question** How can I use the properties of integer exponents to simplify algebraic and numeric expressions? **Sample answer: If multiplication or division expressions contain powers with the same base, you can use the properties to simplify before you multiply or divide.**

Rate Yourself!
Are you ready to move on? Shade the section that applies.
YES ? NO

FOLDABLES Time to update your Foldable!

Independent Practice

Simplify using the Laws of Exponents.

1. $(-6)^2 \cdot (-6)^5 =$ $(-6)^7$ or $-279{,}936$

2. $-4a^5(6a^5) =$ $-24a^{10}$

3. $(-7a^4bc^3)(5ab^4c^2) =$ $-35a^5b^5c^5$

4. $\frac{8^{15}}{8^{13}} =$ 8^2 or 64

5. $\frac{16r^4}{8r} =$ $2r^3$

6. $\frac{x^5y^{14}}{x^3y^9} =$ x^2y^5

7. $\frac{3^4x^4}{3x^2} =$ 3^3x^2 or $27x^2$

8. $\frac{4^5 \cdot 5^3 \cdot 6^2}{4^4 \cdot 5^2 \cdot 6} =$ $4 \cdot 5 \cdot 6$ or 120

9. $\frac{6^3 \cdot 6^5 \cdot 6^4}{6^2 \cdot 6^3 \cdot 6^2} =$ 6^5 or 7,776

10. $\frac{(-2)^5 \cdot (-3)^4 \cdot (-5)^3}{(-2)^3 \cdot (-3) \cdot (-5)^2} =$ $(-2)^2 \cdot (-3)^3 \cdot (-5)$ or 540

11. The processing speed of a certain computer is 10^{11} instructions per second. Another computer has a processing speed that is 10^3 times as fast. How many instructions per second can the faster computer process?

10^{14} instructions

12. The table shows the seating capacity of two different facilities. About how many times as great is the capacity of Safa Park than a typical movie theater?

3^4 or 81 times

Place	Seating Capacity
Movie theater	3^5
Safa Park	3^9

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

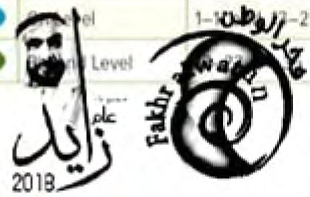
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–12, 24–32	13–19, 33–35	20–23
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–13, 15–19 odd, 20, 21, 23, 34, 35
OL	On Level	[illegible]–21, 23, 34, 35
BL	Beyond Level	[illegible]



Watch Out!

Common Error When multiplying and dividing monomials, students may try to add or subtract exponents with powers that do not have the same base. Remind students that adding and subtracting exponents only works if the bases are like bases.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	14–19, 22, 33
3 Construct viable arguments and critique the reasoning of others.	21, 23
7 Look for and make use of structure.	20

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students explain how to evaluate exponents when multiplying and dividing numbers with the same base in exponential form. See students' work.

13. Refer to the information in the table.

Power of Ten	U.S. Name
10^3	one thousand
10^6	one million
10^9	one billion
10^{12}	one trillion
10^{15}	one quadrillion
10^{18}	one quintillion

a. How many times as great is one quadrillion than one million?
10^9 times greater

b. One quintillion is one trillion times as great as what number?
10^6 or one million

Persevere with Problems Find each missing exponent.

14. $(6^{\square})(6^3) = 6^5$ 2

15. $3x^{\square} \cdot 4x^3 = 12x^{12}$ 9

16. $p^3 \cdot p^{\square} \cdot p^2 = p^9$ 4

17. $\frac{3^{\square}}{3^2} = 3^4$ 6

18. $\frac{5^9}{5^{\square}} = 5^4$ 5

19. $2x^{\square} \cdot \frac{3x^2}{x^6} = 6x^3$ 7

H.O.T. Problems Higher Order Thinking

20. **Identify Structure** Write a multiplication expression with a product of 5^{13}.
Sample answer: $5^{10} \cdot 5^3$

21. **Justify Conclusions** Is $\frac{3^{100}}{3^{99}}$ *greater than, less than, or equal to 3?* Explain your reasoning to a classmate. equal; Sample answer: Using the quotient of powers, $\frac{3^{100}}{3^{99}} = 3^{100-99}$ or 3^1, which is 3.

22. **Persevere with Problems** What is twice 2^{30}? Write using exponents. Explain your reasoning.
2^{31}; $2 \cdot 2^{30} = 2^{31}$

23. **Use a Counterexample** Determine whether the statement below is true or false. If *true*, explain your reasoning. If *false*, give a counterexample.

For any integer a, $(-a)^2 = -a^2$.

false; Sample answer: If $a = 3$, then $(-3)^2 = 9$, but $-3^2 = -9$.

Name ______ My Homework ______

Extra Practice

Simplify using the Laws of Exponents.

24. $(3x^8)(5x) =$ $15x^9$

$(3x^8)(5x) = 3 \cdot 5 \cdot x^8 \cdot x$

$= 15 \cdot x^{8+1}$

$= 15x^9$

25. $\frac{h^7}{h^6} =$ h^1 or h

$\frac{h^7}{h^6} = h^{7-6}$

$= h^1$ or h

26. $2g^2 \cdot 7g^6 =$ $14g^8$

27. $(8w^4)(-w^7) =$ $-8w^{11}$

28. $(-p)(-9p^2) =$ $9p^3$

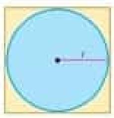
29. $\frac{2^9}{2} =$ 2^8 or 256

30. $\frac{36d^{10}}{6d^5} =$ $6d^5$

31. $\frac{5^3 \cdot 7^5 \cdot 10}{5 \cdot 7^4} =$ $5^2 \cdot 7^1 \cdot 10$ or 1,750

32. $\frac{(-3)^2 \cdot 4^3 \cdot (-1)^8}{4 \cdot (-1)^5} =$ $(-3)^2 \cdot 4^2 \cdot (-1)^3$ or -144

33. **Persevere with Problems** The figure at the right is composed of a circle and a square. The circle touches the square at the midpoints of the four sides.



a. What is the length of one side of the square? $2r$

b. The formula $A = \pi r^2$ is used to find the area of a circle. The formula $A = 4r^2$ can be used to find the area of the square. Write the ratio of the area of the circle to the area of the square in simplest form.
$\frac{\pi}{4}$

c. Complete the table.

Radius (units)	2	3	4	2r
Area of Circle (units^2)	$\pi(2)^2$ or 4π	9π	16π	$4\pi r^2$
Length of 1 Side of the Square	4	6	8	$4r$
Area of Square (units^2)	4^2 or 16	36	64	$16r^2$
Ratio $\left(\frac{\text{Area of circle}}{\text{Area of square}}\right)$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$

d. What can you conclude about the relationship between the areas of the circle and the square? The ratio is $\frac{\pi}{4}$.

Power Up! Test Practice

Exercises 34 and 35 prepare students for more rigorous thinking needed for the assessment.

34.

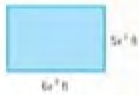
This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

35.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

34. Which expression(s) could be used to represent the area of the rectangle? Select all that apply.



- [ ] $\frac{6x^8}{5x^2}$ ft^2
- [x] $6x^8(5x^2)$ ft^2
- [x] $30x^{10}$ ft^2
- [ ] $\frac{6}{5}x^6$ ft^2

35. The table shows the approximate populations of four states.

State	Alabama	Idaho	Illinois	Wyoming
Approximate Population	3^{14}	3^{13}	3^{15}	3^{12}

Select the correct state to make each statement true.

Idaho / Alabama / Illinois / Wyoming

Statement 1: The population of **Idaho** is about $\frac{1}{3}$ the population of Alabama.

Statement 2: The population of **Alabama** is about 9 times greater than the population of Wyoming.

Statement 3: The population of **Illinois** is about 27 times greater than the population of **Wyoming**.

Spiral Review

Multiply or divide.

36. $14(-2) = -28$

37. $-20(-3) = 60$

38. $-5(7) = -35$

39. $-12 \div (-4) = 3$

40. $63 \div (-7) = -9$

41. $250 \div (-50) = -5$

42. Three-fourths of a pan of lasagna is to be divided equally among 6 people. What part of the lasagna will each person receive?

$\frac{1}{8}$

30 Chapter 1 Real Numbers

Lesson 4

Powers of Monomials

Real-World Link

Aquariums The Marine Club at Westview Middle School purchased an aquarium. The aquarium is in the shape of a cube with a side length of 2^4 inches. Use the questions to find the amount of water the aquarium will hold.

1. Write a multiplication expression to represent the volume of the aquarium. $2^4 \cdot 2^4 \cdot 2^4$
2. Simplify the expression. Write as a single power of 2. 2^{12}
3. Using 2^4 as the base, write the multiplication expression $2^4 \cdot 2^4 \cdot 2^4$ using an exponent. $(2^4)^3$
4. Explain why $(2^4)^3 = 2^{12}$. **Sample answer: Both expressions represent the volume of the same cube.**
5. Use a calculator to find the volume of the tank. **4,096** cubic inches
6. One gallon of water is equal to 231 cubic inches. Write an expression to find how many gallons of water the tank will hold if it is filled to the top. $\frac{4{,}096}{231}$
7. How many gallons of water will the aquarium hold? Round your answer to the nearest gallon. **18** gallons

Essential Question

WHY is it helpful to write numbers in different ways?

Mathematical Practices 1, 3, 4, 7

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Use the Laws of Exponents to find powers of monomials.

Coherence connecting within and across grades

Previous	Now	Next
Students used the Laws of Exponents to multiply and divide monomials.	Students use the Laws of Exponents to find the power of a power.	Students will use the Laws of Exponents to evaluate expressions with negative exponents.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 35.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Think-Write-Pair** Have students work in pairs. After they have completed the Real-World Link, give them a few minutes to think about how to simplify $(a^m)^n$. Then ask them to write a rule they can use to find the power to a power. MP 1, 2, 7, 8

Alternate Strategy

AL LA Remind students that the volume of a cube is found by multiplying the side length by itself three times. First have them write the side length three times. Then, have them rewrite the expression without exponents. Finally have them write the expression using only one base and one exponent. MP 1, 7, 8

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Find the power of a power.

AL
- *In the expression $(8^4)^3$ what does the exponent 3 mean?* **It means you use 8^4 as a factor three times.**
- *What expression represents that situation?* $8^4 \cdot 8^4 \cdot 8^4$
- *What is 4 + 4 + 4?* **12**
- *What is another way to write 4 + 4 + 4 using multiplication?* $4 \cdot 3$
- *What is $(8^4)^3$ simplified?* 8^{12}

OL
- *When finding the power of a power, do you add, subtract, multiply, or divide the exponents?* **multiply**
- *What numbers will you multiply?* **4 and 3**

BL
- *Describe how you would simplify the expression $[(7^2)^4]^5$. Then simplify.* **Multiply all of the exponents together; 7^{40}**

Need Another Example?
Simplify $(5^2)^8$ using the Laws of Exponents. 5^{16}

2. Find the power of a power.

AL
- *When finding the power of a power, do you add, subtract, multiply, or divide the exponents?* **multiply**
- *What is the base?* k
- *What are the exponents?* **7 and 5**

OL
- *What numbers are multiplied together?* **7 and 5**

BL
- *What do you think will happen when you raise $3m^2$ to the third power?* **The 3 and m^2 will both be raised to the third power.**

Need Another Example?
Simplify $(a^3)^7$ using the Laws of Exponents. a^{21}

Key Concept

Power of a Power

Words To find the power of a power, multiply the exponents.

Examples
Numbers: $(5^2)^3 = 5^{2 \cdot 3}$ or 5^6
Algebra: $(a^m)^n = a^{m \cdot n}$

Work Zone

You can use the rule for finding the *product of powers* to discover another Law of Exponents for finding the *power of a power*.

$$(6^4)^5 = \underbrace{(6^4)(6^4)(6^4)(6^4)(6^4)}_{\text{5 factors}}$$

$$= 6^{4+4+4+4+4} \quad \text{Apply the rule for the product of powers.}$$

$$= 6^{20}$$

Notice that the product of the original exponents, 4 and 5, is the final power 20.

Examples

Simplify using the Laws of Exponents.

1. $(8^4)^3$

$(8^4)^3 = 8^{4 \cdot 3}$ Power of a Power

$= 8^{12}$ Simplify

2. $(k^7)^5$

$(k^7)^5 = k^{7 \cdot 5}$ Power of a Power

$= k^{35}$ Simplify

Got it? Do these problems to find out.

a. $(2^5)^2$ b. $(w^4)^6$ c. $[(3^2)^3]^2$

a. 2^{10}
b. w^{24}
c. 3^{12}

Power of a Product

Key Concept

Words To find the power of a product, find the power of each factor and multiply.

Examples

Numbers: $(6x^2)^3 = (6)^3 \cdot (x^2)^3$ or $216x^6$

Algebra: $(ab)^m = a^m b^m$

Extend the power of a *power* rule to find the Laws of Exponents for the power of a product.

$$(3a^2)^5 = \underbrace{(3a^2)(3a^2)(3a^2)(3a^2)(3a^2)}_{\text{5 factors}}$$

$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot a^2 \cdot a^2 \cdot a^2 \cdot a^2 \cdot a^2$

$= 3^5 \cdot (a^2)^5$ Write using powers.

$= 243 \cdot a^{10}$ or $243a^{10}$ Power of a Power

Common Error
When finding the power of a power, do not add the exponents. $(8^4)^3 = 8^{12}$, not 8^7.

Examples

Simplify using the Laws of Exponents.

3. $(4p^3)^4$

$(4p^3)^4 = 4^4 \cdot p^{3 \cdot 4}$ Power of a Product

$= 256p^{12}$ Simplify

4. $(-2m^7n^6)^5$

$(-2m^7n^6)^5 = (-2)^5 m^{7 \cdot 5} n^{6 \cdot 5}$ Power of a Product

$= -32m^{35}n^{30}$ Simplify

Got it? Do these problems to find out.

d. $(8b^9)^2$ e. $(6x^5y^{11})^4$ f. $(-5w^2z^8)^3$

d. $64b^{18}$

e. $1{,}296x^{20}y^{44}$

f. $-125w^6z^{24}$

Examples

3. Find the power of a product.

AL
- *What is the base that is raised to the 4th power?* $4p^3$
- *When you raise $4p^3$ to the 4th power, what needs to happen?* 4 is raised to the 4th power and p^3 is raised to the 4th power
- *What is 4^4?* 256
- *What is p^3 to the 4th power?* p^{12}

OL
- *Does the exponent 4 need to be applied to both factors inside the parentheses?* yes
- *What exponents are multiplied?* 1 and 4; 3 and 4

BL
- *In your own words, explain the Power of a Product law.* See students' work.

Need Another Example?
Simplify $(3c^6)^3$ using the Laws of Exponents. $27c^{18}$

4. Find the power of a product.

AL
- *What is the base that is raised to the 5th power?* $-2m^7n^6$
- *What is $(-2)^5$?* -32
- *What is m^7 to the 5th power?* m^{35}
- *What is n^6 to the 5th power?* n^{30}

OL
- *How many factors are inside of the parentheses?* 3
- *Does the exponent 5 need to be applied to each factor inside the parentheses?* yes
- *What exponents are multiplied?* 1 and 5; 7 and 5; 6 and 5

BL
- *Simplify $[(2a^5b^3)^2]^3$.* $64a^{30}b^{18}$

Need Another Example?
Simplify $(-4p^5q)^2$. $16p^{10}q^2$

Example

5. Find the area.

AL
- *What do you need to find out?* the area of the logo
- *What shape is the logo?* square
- *How do you find the area of a square?* square one side
- *What is the length of one side of the logo?* $7a^4b$
- *How do you square $7a^4b$?* square each factor

OL
- *What is the length of one side of the logo?* $7a^4b$
- *What expression represents the area of the square?* $(7a^4b)^2$

BL
- *Suppose the smaller squares in the corners of the logo take up about 30% of the total area. What is the area of the logo that does not contain those squares?* $34.3a^8b^2$

Need Another Example?
Find the volume of a cube with side lengths of $6mn^7$.
$216m^3n^{21}$ cubic units

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Round Robin** Have students work in pairs to solve Exercises 1–7. Students take turns completing each step of the solution until the expression is simplified. The pairs check their answers with another pair and discuss any differences. MP 1, 2, 3, 6, 7

BL LA **Trade-a-Problem** Give students the following problem to solve: $(3a^{\square}b^{5})^{\square} = \square a^{16}b^{20}$. (4, 4, 81) Then have them create five of their own problems similar to that problem. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. MP 1, 2, 3, 7

STOP and Reflect
How do you know when an expression is in simplest form? Explain below.

Sample answer: Each base appears only once, there are no powers of powers, and all fractions are in simplest form.

Example

5. A magazine offers a special service to its subscribers. If they scan the square logo shown on a smartphone, they can receive special offers from the magazine. Find the area of the logo.

$A = s^2$ — Area of a square
$A = (7a^4b)^2$ — Replace s with $7a^4b$.
$A = 7^2(a^4)^2(b^1)^2$ — Power of a Product
$A = 49a^8b^2$ — Simplify.

The area of the logo is $49a^8b^2$ square units.

Guided Practice

Simplify using the Laws of Exponents. (Examples 1–4)

1. $(3^2)^5 =$ 3^{10} or 59,049
2. $(h^6)^4 =$ h^{24}
3. $[(2^3)^2]^3 =$ 2^{18} or 262,144
4. $(7w^7)^3 =$ $343w^{21}$
5. $(5g^8k^{12})^4 =$ $625g^{32}k^{48}$
6. $(-6r^5s^3)^2 =$ $36r^{10}s^{6}$

7. The floor of the commons room at King Middle School is in the shape of a square with side lengths of x^2y^3 feet. New tile is going to be put on the floor of the room. Find the area of the floor. (Example 5) x^4y^6 ft²

8. **Building on the Essential Question** How does the Product of Powers law apply to finding the power of a power?
Sample answer: You can write out the power of a power as a multiplication problem with factors having the same base. Then you can apply the Product of Powers law to simplify.

Rate Yourself!
How confident are you about powers of monomials? Check the box that applies.

FOLDABLES Time to update your Foldable!

Name ______________________ My Homework ______________________

Independent Practice

Simplify using the Laws of Exponents. (Examples 1–4)

1. $(4^2)^3 =$ **4^6**
2. $(5^3)^3 =$ **5^9**
3. $(a^7)^6 =$ **a^{42}**
4. $(h^4)^9 =$ **h^{36}**
5. $[(3^2)^2]^2 =$ **3^8**
6. $[(5^2)^2]^2 =$ **5^8**
7. $(5j^6)^4 =$ **$625j^{24}$**
8. $(11c^4)^3 =$ **$1{,}331c^{12}$**
9. $(6a^2b^6)^3 =$ **$216a^6b^{18}$**
10. $(2m^5n^{11})^6 =$ **$64m^{30}n^{66}$**
11. $(-3w^3z^8)^5 =$ **$-243w^{15}z^{40}$**
12. $(-5r^4s^{12})^4 =$ **$625r^{16}s^{48}$**

13. A shipping box is in the shape of a cube. Each side measures $3c^5d^2$ inches. Express the volume of the cube as a monomial. (Example 5)
$27c^{15}d^6$ in^3

14. Tahani is decorating her patio with a planter in the shape of a cube like the one shown. Find the volume of the planter. (Example 5)
$27w^{12}$ units^3

Copy and Solve **Simplify. Show your work on a separate sheet of paper.**

15. $[(3x^2y^3)^2]^3$ **$729x^{12}y^{18}$**
16. $\left(\frac{3}{5}a^6b^9\right)^2$ **$\frac{9}{25}a^{12}b^{18}$**
17. $(-2v^7)^3(-4v^2)^4$ **$-2{,}048v^{29}$**

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises: 1–14, 23–34	15–18, 35–40	19–22
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–15, 17, 19, 22, 39, 40
OL	On Level	1–13 od[illegible]9, 22, 39, 40
BL	Beyond Level	15–[illegible]

Watch Out!

Common Error Students often confuse when they add exponents with when they multiply exponents. Remind them to multiply exponents only when they raise a power to a power.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	20, 21, 38
3 Construct viable arguments and critique the reasoning of others.	19, 22
7 Look for and make use of structure.	18

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write a paragraph telling how yesterday's lesson on the Product of Powers law helped them with today's lesson. See students' work.

18. **Identify Structure** Draw a line connecting the Law(s) of Exponents you would use to simplify each of the expressions. Then simplify each one.

- Product of Powers
- Quotient of Powers
- Power of a Power
- Power of a Product

- $(a^9)^3 = a^{27}$
- $(m^8) \div (m^4) = m^4$
- $5x^2 \cdot (-7x^4) = -35x^6$
- $\frac{(xy^4)^3}{xy} = x^2y^{11}$
- $(n^6)^8 = n^{48}$

H.O.T. Problems Higher Order Thinking

19. **Reason Inductively** The table gives the area and volume of a square and cube, respectively, with side lengths shown.

Side Length (units)	x	$2x$	$3x$
Area of Square (units²)	x^2	$(2x)^2$ or $4x^2$	$(3x)^2$ or $9x^2$
Volume of Cube (units³)	x^3	$(2x)^3$ or $8x^3$	$(3x)^3$ or $27x^3$

a. Complete the table.

b. Describe how the area and volume are each affected if the side length is doubled. Then describe how they are each affected if the side length is tripled.
If the side length is doubled, the area is quadrupled and the volume is multiplied by 8. If the side length is tripled, the area is multiplied by 9 and the volume is multiplied by 27.

Persevere with Problems Solve each equation for x.

20. $(7^x)^3 = 7^{15}$ 5

21. $(-2m^3n^4)^x = -8m^9n^{12}$ 3

22. **Reason Inductively** Compare how you would correctly simplify the expressions $(2a^3)(4a^6)$ and $(2a^3)^6$.
Sample answer: To simplify $(2a^3)(4a^6)$, multiply 2 by 4. Then add the exponents 3 and 6 and write this sum as the final exponent on a. To simplify $(2a^3)^6$, evaluate 2^6. Then multiply the exponents 3 and 6 and write this product as the final exponent on a.

Name ______________________ My Homework ______________________

Extra Practice

Simplify using the Laws of Exponents.

23. $(2^2)^7 =$ 2^{14}

$(2^2)^7 = 2^{2 \cdot 7}$

$= 2^{14}$

24. $(8v^9)^5 =$ $32{,}768v^{45}$

$(8v^9)^5 = 8^5 \cdot v^{9 \cdot 5}$

$= 32{,}768v^{45}$

25. $(3^4)^2 =$ 3^8

26. $(m^8)^5 =$ m^{40}

27. $(z^{11})^5 =$ z^{55}

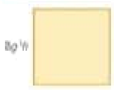
28. $[(4^3)^2]^2 =$ 4^{12}

29. $[(2^3)^3]^2 =$ 2^{18}

30. $(14y)^4 =$ $38{,}416y^4$

Express the area of each square as a monomial.

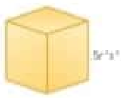
31. $64g^8h^2$ units^2



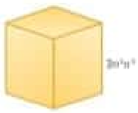
32. $144d^{12}e^{14}$ units^2

Express the volume of each cube as a monomial.

33. $125r^6s^9$ units^3



34. $343m^{18}n^{27}$ units^3



Simplify.

35. $(0.5k^5)^2 =$ $0.25k^{10}$

36. $(0.3p^7)^3 =$ $0.027p^{21}$

37. $\left(\frac{1}{4}w^5z^3\right)^2 =$ $\frac{1}{16}w^{10}z^6$

38. **Persevere with Problems** A ball is dropped from the top of a building. The expression $4.9x^2$ gives the distance in meters the ball has fallen after x seconds. Write and simplify an expression that gives the distance in meters the ball has fallen after x^2 seconds, after x^3 seconds.

$4.9(x^2)^2 = 4.9x^4$ meters; $4.9(x^3)^2 = 4.9x^6$ meters

Power Up! Test Practice

Exercises 39 and 40 prepare students for more rigorous thinking needed for the assessment.

39.

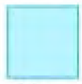
This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

40.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

39. Mouna has four pieces of carpet in the shape of a square like the one shown. He wants to use them together to carpet a portion of his basement. What is the area of the space he can cover with the carpet?



$16x^4$ yd^2

40. Select the correct expression to represent the volume of each cube.

$2a^{12}b^6$	$8a^{12}b^9$
$4a^6b^3$	$12a^6b^5$
$4a^9b^6$	$64a^6b^9$
$6a^6b^6$	$64a^9b^6$
$8a^{12}b^6$	$64a^9b^{12}$

$64a^9b^6$ | $8a^{12}b^9$ | $64a^6b^9$

Spiral Review

Simplify using the Laws of Exponents.

41. $6^4 \cdot 6^7 = 6^{11}$

42. $18^3 \cdot 18^5 = 18^8$

43. $(-3x^{[illegible]})(-6x^{[illegible]}) = 18x^{14}$

44. $(-9a^4)(2a^7) = -18a^{11}$

45. The table shows the heights of some United States waterfalls. What is the height of each waterfall?

Bridalveil: 620 ft; Fall Creek: 256 ft; Shoshone: 212 ft

Waterfall	Height (ft)
Bridalveil (California)	$2^2 \cdot 5 \cdot 31$
Fall Creek (Tennessee)	2^8
Shoshone (Idaho)	$2^2 \cdot 53$

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Problem-Solving Investigation

The Four-Step Plan

Mathematical Practices 1, 3, 4

Case #1 Texting Trail

Laila received a text about a concert. She forwarded the text to two of her friends. They each forwarded it to two more friends, and so on.

How many texts were sent at the 4th stage?

1 Understand *What are the facts?*

You know that each person at each stage sends a text to two people. You can use counters to represent the trail of texts sent.

2 Plan *What is your strategy to solve this problem?*

Use red counters to represent the texts in the first stage.
Use yellow counters to show the texts sent at the second stage. Continue the pattern. Draw the counters representing the number of texts sent in the 4th stage.

3 Solve *How can you apply the strategy?*

There are 16 counters in the 4th row. So, 16 texts were sent during the 4th stage.

4 Check *Does the answer make sense?*

The number of texts at each stage is a power of 2. So, find 2^4.
Since $2^4 = 16$, the answer is correct. ✓

Analyze the Strategy

Justify Conclusions At what stage would there be more than 1,000 texts sent? Explain.

stage 10; Sample answer: The ninth stage could be represented by 2^9 or 512, and the tenth stage could be represented by 2^{10} or 1,024. So, in the tenth stage, more than 1,000 texts would be sent.

Focus narrowing the scope

Objective Solve problems by using the four-step plan.
This lesson emphasizes **Mathematical Practice 4** Model with Mathematics.

Four-Step Plan Students can use the four-step plan to break down any word problem and find a method to solve it.

Coherence connecting within and across grades

Now
Students solve non-routine problems.

Next
Students will apply the four-step plan to compute with scientific notation.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 41.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

The problems on pages 39 and 40 are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance. The problem on page 39 walks students through the solution, while the problem on page 40 asks students to come up with their own solutions.

Case #1 Texting Trail

BL Have students extend the problem by having them answer the question below.

Ask:

- *If Laila sends the text to three of her friends, and they each forward the text to three more friends, how many texts will be sent at the 4th stage?* **Sample answer: The number of texts at each stage is a power of 3, so the 4th stage will have 3^4 or 81 texts.**

Case #2 Green Mileage

AL LA **Roundrobin** Have students work in pairs to share the steps of four-step plan and their strategy for each step. One student explains and shares how they completed the first step, Understand. The second student explains and shares how they completed the second step, Plan. Then the first student explains and shares how they completed the third step, Solve. The second student explains and shares how they completed the last step, Check. MP 1, 3, 5

BL LA **Think-Pair-Share** Have students work in pairs. Give students one minute to think about one way they could solve the problem. Have them share their responses with their partner. If they have the same answer, have them think of another way to solve and explain why each way works to solve the problem. MP 1, 2, 3

Need Another Example?

Abdulla paid for a AED 5 sandwich with a AED 20 bill. The cashier has AED 1, AED 5, and AED 10 bills in the register. How many different ways could Abdulla get his change? **6 different ways**

Case #2 Green Mileage

A test of a hybrid car resulted in 4,840 miles driven using 88 gallons of gas.

At this rate, how many gallons of gas will this vehicle need to travel 1,155 miles?

1 Understand

Read the problem. What are you being asked to find?

I need to find **how many gallons of gas the car will need for 1,155 miles**

Underline key words and values in the problem. What information do you know?

The hybrid car can travel **4,840** miles using **88** gallons of gas.

Is there any information that you do *not* need to know?

I do not need to know **the car was a hybrid**

2 Plan

How do the facts relate to one another?

Sample answer: You are comparing miles to gallons. You can write a ratio.

3 Solve

Write and solve a proportion comparing miles to gallons. Let *g* represent the amount of gas needed to travel 1,155 miles.

$$\frac{\text{miles}}{\text{gallons}} \rightarrow \frac{4{,}840}{88} = \frac{1{,}155}{g}$$

How many gallons of gas will the car use to travel 1,155 miles? **21**

4 Check

Use information from the problem to check your answer.

Sample answer: 4,840 miles ÷ 88 gallons = 55 miles per gallon;

1,155 miles ÷ 55 miles per gallon = 21 gallons. The answer is correct.

Work with a small group to solve the following cases. Show your work on a separate piece of paper.

Case #3 Class Trip

All of Mr. Khalifa's science classes are going to the Natural History Museum. A tour guide is needed for each group of eight students. His classes have 28 students, 35 students, 22 students, 33 students, and 22 students.

How many tour guides are needed?

18 tour guides

Case #4 Gardening

Mrs. Lubna is designing her garden in the shape of a rectangle. The area of her garden is 2 times greater than the area of the rectangle shown.

Write the area of Mrs. Lubna's garden in simplest form.

64[illegible] ft^2

Case #5 Toothpicks

Hani will make the figures at the right using toothpicks.

Write an expression that can be used to find the number of toothpicks needed to make any figure. Then find the number of toothpicks needed to make the 100th figure.

$2n + 1$; 201 toothpicks

Use any strategy!

Case #6 Number Sense

Study the following sequence:

$1 - \frac{1}{2}, 1 - \frac{1}{2}, 1 - \frac{1}{3}, 1 - \frac{1}{4}, \ldots, 1 - \frac{1}{48}, 1 - \frac{1}{49}$, and $1 - \frac{1}{50}$

What is the product of all of the terms?

$\frac{1}{100}$

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** EVALUATE

2 Collaborate

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 3, 4	Exercises 5	Exercises 6
Level 3			●
Level 2		●	
Level 1	●		

AL LA Numbered Heads Together Assign students to 3- or 4-person learning teams. Each team completes Cases 3–6, making sure that every member understands. Call on a specific member from one team to present the team's solution to the class. MP 1, 3, 5

BL LA Trade-a-Problem Have students work in pairs to write their own problem similar to Case #5 where they model a pattern. Then have them trade their problems with each other and solve. MP 1, 2, 4

Mid-Chapter Check

If students have trouble with Exercises 1–9, they may need help with the following concepts.

Concept	Exercise(s)
writing fractions as decimals (Lesson 1)	3
writing decimals and fractions (Lesson 1)	4
evaluating powers (Lesson 2)	1, 5
Product of Powers (Lesson 3)	2, 6
Quotient of Powers (Lesson 3)	7, 9
Power of a Power (Lesson 4)	8

Vocabulary Activity

LA **Think-Pair-Share** Have students work in pairs to complete Exercise 1. Give them about one minute to individually think through their response. Then have them share their responses with a partner. Call on one set of pairs to share their responses with the class. MP 1, 3

Alternate Strategies

AL LA Have students work with a partner. As one student verbally defines power, have the other student write the definition and read it back to the first student.

BL LA Have students verbally explain the difference between 2^5 and 5^2.

Mid-Chapter Check

Vocabulary Check

1. MP **Be Precise** Define *power* using the words *base* and *exponent*. Give an example of a power and label the base and exponent.
Sample answer: A power is when a number, called the base, is multiplied by itself a number of times. The number of times the base is a factor is the exponent. In 5^2, 5 is the base, 2 is the exponent.

2. Describe the Product of Powers rule. Give an example.
Sample answer: The Product of Powers rule allows you to multiply powers with the same base, for example, $3^4 \cdot 3^6 = 3^{10}$.

Skills Check and Problem Solving

3. Write $1\frac{7}{16}$ as a decimal. 1.4375

4. Write $0.\overline{15}$ as a fraction in simplest form. $\frac{5}{33}$

5. The mass of a baseball glove is 5 • 5 • 5 • 5 grams. Write the mass using exponents. Then find the value of the expression. 5^4; 625

Simplify using the Laws of Exponents.

6. $2^3a^7 \cdot 2a^3 =$ 2^4a^{10} or $16a^{10}$

7. $\frac{24y^4}{4y^2} =$ $6y^2$

8. $(2p^3r^2)^3 =$ $8p^9r^6$

9. MP **Persevere with Problems** Write two algebraic expressions, one with a quotient of x^5 and one with a product of x^5.
Sample answer: $\frac{x^8}{x^3}$ and $x^2(x^3)$

The Number System

Lesson 5

Negative Exponents

Real-World Link

Insects The table shows the approximate wing beats per minute for certain insects.

Insect	Wing Beats per Minute
house fly	10,000
small butterfly	100

1. Write a ratio in simplest form that compares the number of wing beats for a butterfly to a housefly. $\frac{1}{100}$

2. Write the ratio as a fraction with an exponent in the denominator and as a decimal. $\frac{1}{10^2}$; 0.01

3. Complete the 1st 4 rows of the table showing the exponential and standard forms of power of 10.

4. What operation is performed when you move down the table? division

5. What happens to the exponent? It decreases by one.

6. Extend the table to include the next three entries.

Exponential Form	Standard Form
10^3	1,000
10^2	100
10^1	10
10^0	1
10^{-1}	10^{-1} or 0.1
10^{-2}	$\frac{1}{100}$ or 0.01
10^{-3}	$\frac{1}{1,000}$ or 0.001

Essential Question

WHY is it helpful to write numbers in different ways?

Mathematical Practices 1, 3, 4, 7

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Simplify expressions involving negative exponents.

Coherence connecting within and across grades

Previous Students used the Laws of Exponents to simplify expressions involving exponents.

Now Students use the Laws of Exponents to write and simplify expressions involving negative exponents.

Next Students will use the Laws of Exponents to write and simplify expressions written in scientific notation.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 47.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA Think-Pair-Share Have students work in pairs to complete Exercises 1–6. MP 1, 2, 4, 6, 7, 8

Ask:

- *How is 100 written as a power of 10? 1,000?* 10^2; 10^3

Alternate Strategy

BL LA Roundrobin Have students work in pairs to complete Exercises 1–6. Then have them extend the pattern shown in the table. MP 1, 2, 4, 6, 7, 8

Ask:

- *What pattern do you see in the powers of 10?* Sample answer: As the exponent increases by 1, the standard decimal notation of the number increases 10 times or is multiplied by 10.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions to differentiate instruction.

Examples

1. Write expressions using positive exponents.

- AL • *What do you notice about the exponent?* **It is negative.**
- OL • *Explain how to write 6^{-3} using positive exponents.* **Write as a fraction with 1 in the numerator and 6^3 in the denominator.**

Need Another Example?
Write 4^{-4} using a positive exponent. $\frac{1}{4^4}$

2. Write expressions using positive exponents.

- OL • *If the base is a variable, does the process to write the expression using a positive exponent change compared to when the base is a numerical value?* **no, it is the same**
- BL • *Could "a" be equal to 0? Why or why not?* **no; There cannot be a 0 in the denominator of a fraction.**

Need Another Example?
Write c^{-7} using a positive exponent. $\frac{1}{c^7}$

3. Write expressions using negative exponents.

- AL • *What do you notice about the numerator?* **It is 1.**
- • *Where is the power located?* **in the denominator**
- BL • *Write $\frac{1}{5^{-2}}$ using a positive exponent.* **5^2**

Need Another Example?
Write $\frac{1}{f^5}$ using a negative exponent. f^{-5}

4. Write expressions using negative exponents.

- AL • *How can 36 be written as a power?* **6^2**
- BL • *Write your own examples, similar to Examples 1–4. Trade with a partner to complete each other's examples.* **See students' work.**

Need Another Example?
Write $\frac{1}{9}$ as an expression using a negative exponent other than −1. 3^{-2}

Key Concept

Zero and Negative Exponents

Words Any nonzero number to the zero power is 1. Any nonzero number to the negative *n* power is the multiplicative inverse of its *n*th power.

Examples

Numbers: $5^0 = 1$; $7^{-3} = \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7}$ or $\frac{1}{7^3}$

Algebra: $x^0 = 1, x \neq 0$; $x^{-n} = \frac{1}{x^n}, x \neq 0$

Work Zone

Negative Exponents
Remember that 6^{-2} is equal to $\frac{1}{6^2}$, not -2×6 or -12.

You can use exponents to represent very small numbers. Negative powers are the result of repeated division.

Examples

Write each expression using a positive exponent.

1. 6^{-2}

$6^{-2} = \frac{1}{6^2}$ Definition of negative exponent

2. a^{-5}

$a^{-5} = \frac{1}{a^5}$ Definition of negative exponent

Got It? Do these problems to find out.

a. 7^{-2} b. b^{-4}
c. 5^0 d. m^{-3}

a. $\frac{1}{7^2}$
b. $\frac{1}{b^4}$
c. 1
d. $\frac{1}{m^3}$

Examples

Write each fraction as an expression using a negative exponent other than −1.

3. $\frac{1}{5^2}$

$\frac{1}{5^2} = 5^{-2}$ Definition of negative exponent

4. $\frac{1}{36}$

$\frac{1}{36} = \frac{1}{6^2}$ Definition of exponent

$= 6^{-2}$ Definition of negative exponent

Got It? Do these problems to find out.

e. $\frac{1}{8^3}$ f. $\frac{1}{4}$
g. $\frac{1}{c^5}$ h. $\frac{1}{27}$

e. 8^{-3}
f. 2^{-2}
g. c^{-5}
h. 3^{-3}

Example

STEM One human hair is about 0.001 inch in diameter. Write the decimal as a power of 10.

$0.001 = \frac{1}{1{,}000}$

$= \frac{1}{10^3}$

$= 10^{-3}$

A human hair is 10^{-3} inch thick.

ot it? Do this problem to find out.

i. STEM A water molecule is about 0.0000000001 meter long. Write the decimal as a power of 10.

STOP and Reflect

Explain below the difference between the expressions $(-4)^2$ and 4^{-2}.

Sample answer: The expression $(-4)^2$ means −4 to the second power. $(-4)^2 = (-4)(-4)$ or 16. The expression 4^{-2} means 4 to the negative two power. $4^{-2} = \frac{1}{4^2}$ or $\frac{1}{16}$

i. 10^{-10}

ltiply and Divide with Negative Exponents

e Product of Powers and the Quotient of Powers rules can be used multiply and divide powers with negative exponents.

xamples

nplify each expression.

. $5^3 \cdot 5^{-5}$

$5^3 \cdot 5^{-5} = 5^{3+(-5)}$

$= 5^{-2}$

$= \frac{1}{5^2}$ or $\frac{1}{25}$

$\frac{w^{-1}}{w^{-4}}$

$\frac{w^{-1}}{w^{-4}} = w^{-1-(-4)}$

$= w^{(-1)+4}$ or w^3

ot it? Do these problems to find out.

j. $3^{-8} \cdot 3^2$

k. $\frac{11^2}{11^4}$

l. $n^9 \cdot n^{-4}$

m. $\frac{b^{-4}}{b^{-7}}$

j. $\frac{1}{729}$

k. $\frac{1}{121}$

l. n^5

m. b^3

Examples

5. Write expressions using negative exponents.

AL • *Write 0.001 in word form.* **one thousandth**

OL • *Explain how to write 0.001 as a fraction.* **Sample answer: Because 0.001 is read as "one thousandth," the fraction is $\frac{1}{1{,}000}$.**

• *How can 1,000 be written as a power?* 10^3

• *How can $\frac{1}{1{,}000}$ be written as a power?* $\frac{1}{10^3}$ **or** 10^{-3}

BL • *If you are writing the decimals 0.1, 0.01, 0.001, 0.0001, and so on, as decimals, what will the base of the power always be? Explain.* **10; Sample answer: The place-value decimal system is a base 10 system.**

Need Another Example?

A grain of salt has a mass of about 0.0001 gram. Write the decimal as a power of 10. 10^{-4}

6–7. Simplify expressions with negative exponents.

AL • *In Example 6, to multiply powers with the same base, do you add, subtract, multiply, or divide the exponents?* **add**

• *In Example 7, to divide powers with the same base, do you add, subtract, multiply, or divide the exponents?* **subtract**

OL • *What is 3 + (−5)?* **−2**

• *How would you write 5^{-2} using a positive exponent?* $\frac{1}{5^2}$ **or** $\frac{1}{25}$

• *What is −1 − (−4) rewritten as an addition expression?* **−1 + 4**

BL • *In Example 6, why do we not leave the answer as 5^{-2}?* **Sample answer: a simplified answer does not contain any negative exponents.**

Need Other Examples?

Simplify each expression.

a. $4^{-5} \cdot 4^{-3}$ $\frac{1}{4^8}$ **b.** $\frac{c^{-5}}{c^{-7}}$ c^2

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Rally Coach Students work in pairs to complete Exercises 1–13. Partner A works Exercise 1 while Partner B watches, listens, coaches, and praises. Then the partners trade roles for Exercise 2. Partners continue taking turns until all problems are completed. MP 1, 2, 6, 7, 8

BL LA Round Table For Exercise 13, assign students to a 4–5 person team. The first team member verbally explains what he or she has learned about the relationship between positive and negative exponents while the rest of the team listens attentively. Then the next team member states whether he or she agrees or disagrees and adds to the answer. Once every team member has contributed to the answer, the students work individually to write a summarized response. Team members then compare answers. MP 1, 2, 3, 6, 7, 8

Guided Practice

Write each expression using a positive exponent. (Examples 1 and 2)

1. $2^{-4} = \frac{1}{2^4}$
2. $4^{-3} = \frac{1}{4^3}$
3. $a^{-4} = \frac{1}{a^4}$
4. $g^{-7} = \frac{1}{g^7}$

Write each fraction as an expression using a negative exponent other than −1. (Examples 3 and 4)

5. $\frac{1}{3^4} = 3^{-4}$
6. $\frac{1}{m^5} = m^{-5}$
7. $\frac{1}{16} = 4^{-2}$ or 2^{-4}
8. $\frac{1}{49} = 7^{-2}$

9. An American green tree frog tadpole is about 0.00001 kilometer in length when it hatches. Write this decimal as a power of 10. (Example 5) 10^{-5}

Simplify. (Examples 6 and 7)

10. $3^{-3} \cdot 3^{-2} = \frac{1}{243}$
11. $r^{-7} \cdot r^{3} = \frac{1}{r^4}$
12. $\frac{p^{-2}}{p^{-12}} = p^{10}$

13. **Building on the Essential Question** How are negative exponents and positive exponents related?
Sample answer: Negative exponents are the result of repeated division and positive exponents are the result of repeated multiplication. You can rewrite an expression with a negative exponent to an expression with a positive exponent by using the multiplicative inverse.

Rate Yourself!

How well do you understand understand negative exponents? Circle the image that applies.

Name ________ My Homework ________

Independent Practice

Write each expression using a positive exponent. (Examples 1 and 2)

1. $7^{-10} = \frac{1}{7^{10}}$
2. $(-5)^{-4} = \frac{1}{(-5)^4}$
3. $g^{-7} = \frac{1}{g^7}$
4. $w^{-12} = \frac{1}{w^{12}}$

Write each fraction as an expression using a negative exponent other than −1. (Examples 3 and 4)

5. $\frac{1}{12^4} = 12^{-4}$
6. $\frac{1}{(-5)^7} = (-5)^{-7}$
7. $\frac{1}{125} = 5^{-3}$
8. $\frac{1}{1{,}024} = 2^{-10}$ or 4^{-5}

9. The table shows different metric measurements. Write each decimal as a power of 10. (Example 5) $10^{-1}, 10^{-2}, 10^{-3}, 10^{-6}$

Measurement	Value
Decimeter	0.1
Centimeter	0.01
Millimeter	0.001
Micrometer	0.000001

10. STEM An atom is a small unit of matter. A small atom measures about 0.0000000001 meter. Write the decimal as a power of 10. (Example 5)
10^{-10}

Simplify. (Examples 6 and 7)

11. $2^{-3} \cdot 2^{-4} = \frac{1}{128}$
12. $s^{-5} \cdot s^{-2} = \frac{1}{s^7}$
13. $y^{-1} \cdot y^4 = y^3$
14. $(3a)(a^{-3}) = \frac{3}{a^2}$
15. $\frac{3^{-1}}{3^{-5}} = 81$
16. $\frac{a^{-4}}{a^{-6}} = a^2$
17. $\frac{y^{-6}}{y^{-10}} = y^4$
18. $\frac{z^{-4}}{z^{-8}} = z^4$

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as homework assignment. The Extra Practice page can be us for additional reinforcement or as a second-day assignme

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Leve indicating the lowest level of complexity.

	Exercises		
	1–18, 25–38	19, 20, 39–43	21–24
Level 3			•
Level 2		•	
Level 1	•		

Suggested Assignments

You can use the table below that includes exercises of al complexity levels to select appropriate exercises for you students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–19, 21, 22, 24, 42, 43
OL	On Level	1–17 odd, 19–22, 24, 42, 43
BL	Beyond Level	19–24, 42, 43

عام زايد 2018

فخر الوطن Fakhr Alwatan

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	23, 39–41
2 Reason abstractly and quantitatively.	24
3 Construct viable arguments and critique the reasoning of others.	20
7 Look for and make use of structure.	21, 22

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write the following expressions using a positive exponent:

$\frac{1}{r^{-3}}$ r^3 t^{-5} $\frac{1}{t^5}$

19. **STEM** The mass of a molecule of penicillin is 10^{-18} kilogram and the mass of a molecule of insulin is 10^{-23} kilogram. How many times greater is the mass of a molecule of penicillin than the mass of a molecule of insulin?

10^5 or 100,000 times

20. **Justify Conclusions** A common flea that is 2^{-4} inch long can jump about 2^3 inches high. About how many times its body size can a flea jump? Explain your reasoning.

2^7 or 128 times; $2^3 \div 2^{-4} = 2^{3-(-4)}$ or 2^7

H.O.T. Problems Higher Order Thinking

21. **Identify Structure** Without evaluating, order 11^{-3}, 11^2, and 11^0 from least to greatest. Explain your reasoning.

11^{-3}, 11^0, 11^2; Sample answer: The exponents in order from least to greatest are −3, 0, 2.

22. **Identify Structure** Write an expression with a negative exponent that has a value between 0 and $\frac{1}{2}$.

Sample answer: 3^{-2}, $3^{-2} = \frac{1}{3^2}$ or $\frac{1}{9}$

23. **Persevere with Problems** Select several fractions between 0 and 1. Find the value of each fraction after it is raised to the −1 power. Explain the relationship between the −1 power and the original fraction.

Sample answer: $\left(\frac{1}{2}\right)^{-1} = 2$, $\left(\frac{34}{43}\right)^{-1} = \frac{43}{34}$, $\left(\frac{56}{65}\right)^{-1} = \left(\frac{65}{56}\right)$; When you raise a fraction to the −1 power, it is the same as finding the reciprocal of the fraction.

24. **Reason Abstractly** For each power, write an equivalent multiplication expression with two factors. The first factor should have a positive exponent and the second factor should have a negative exponent. **Sample answers are given.**

a. $10^4 =$ **$10^6 \cdot 10^{-2}$**

b. $8^2 =$ **$8^5 \cdot 8^{-3}$**

c. $x^7 =$ **$x^{12} \cdot x^{-5}$**

Name ____________________ My Homework ____________________

Extra Practice

25. Write 3^{-5} using positive exponents. $\frac{1}{3^5}$

Homework Help

$(3)^{-5} = \frac{1}{3^5}$

26. Simplify $(4^{-4})(4^2)$. $\frac{1}{16}$

$(4^{-4})(4^2) = 4^{-4+2}$

$= 4^{-2}$

$= \frac{1}{4^2}$ or $\frac{1}{16}$

Write each expression using a positive exponent.

27. $6^{-8} = \frac{1}{6^8}$

28. $(-3)^{-5} = \frac{1}{(-3)^5}$

29. $s^{-9} = \frac{1}{s^9}$

30. $t^{-11} = \frac{1}{t^{11}}$

Simplify.

31. $z^2 \cdot z^{-3} = \frac{1}{z}$

32. $n^{-1} \cdot n^3 = n^2$

33. $\frac{b^{-7}}{b^5} = \frac{1}{b^{12}}$

34. $\frac{x^4}{x^{-2}} = x^6$

35. $2^{-4} = \frac{1}{16}$

36. $(-5)^{-4} = \frac{1}{625}$

37. $(-10)^{-4} = \frac{1}{10{,}000}$

38. $(0.5)^{-4} = 16$

MP **Persevere with Problems** Find the missing exponent.

39. $\frac{17^{\blacksquare}}{17^4} = 17^8$ 12

40. $\frac{k^6}{k^{\blacksquare}} = k^2$ 4

41. $\frac{p^{-1}}{p^{\blacksquare}} = p^{10}$ −11

Power Up! Test Practice

Exercises 42 and 43 prepare students for more rigorous thinking needed for the assessment.

42.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

43.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1, MP7
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

42. The diameter of the average human cell is about 4^{-4} inch. Which of the following expressions are equivalent to this diameter? Select all that apply.

- [x] $\frac{1}{4^4}$ in.
- [ ] $-\frac{1}{4^4}$ in.
- [x] $\frac{1}{256}$ in.
- [x] 0.00390625 in.

43. The table shows the values of different measurements in the metric system.

Measurement	Value
micrometer	0.000001 m
millimeter	0.001 m
nanometer	0.000000001 m
picometer	0.000000000001 m

Select the correct answer to write each measurement as a power of 10.

Measurement	Power of 10
micrometer	10^{-6}
millimeter	10^{-3}
nanometer	10^{-9}
picometer	10^{-12}

10^{-12}	10^{-5}
10^{-9}	10^{-3}
10^{-6}	10^{-2}

Spiral Review

Evaluate.

44. 10^2 = 100

45. 10^3 = 1,000

46. 10^6 = 1,000,000

47. 10^5 = 100,000

Find each missing value.

48. 0.003 × 1,000 = 3

49. 0.079 × 100 = 7.9

50. 0.00041 × 10,000 = 4.1

51. 987 ÷ 100 = 9.87

52. 3,400 ÷ 1,000 = 3.4

53. 7,450 ÷ 10 = 745

Lesson 6

Scientific Notation

Real-World Link

Electronics A single sided, single layer DVD has a storage capacity of 4.7 gigabytes. One gigabyte is equal to 10^9 bytes.

1. Write a multiplication expression that represents how many bytes can be stored on the DVD. **4.7×10^9**
2. Complete the table below.

Expression	Product	Expression	Product
$4.7 \times 10^1 = 4.7 \times 10$	47	$4.7 \times 10^{-1} = 4.7 \times \frac{1}{10}$	0.47
$4.7 \times 10^2 = 4.7 \times 100$	**470**	$4.7 \times 10^{-2} = 4.7 \times \frac{1}{100}$	**0.047**
$4.7 \times 10^3 = 4.7 \times 1{,}000$	**4,700**	$4.7 \times 10^{-3} = 4.7 \times \frac{1}{1000}$	**0.0047**
$4.7 \times 10^4 = 4.7 \times$ **10,000**	**47,000**	$4.7 \times 10^{-4} = 4.7 \times \frac{1}{10{,}000}$	**0.00047**

3. If 4.7 is multiplied by a positive power of 10, what relationship exists between the decimal point's new position and the exponent? **Sample answer: When the power is positive, the number of the exponent gives the number of places the decimal point moves to the right in the product.**
4. When 4.7 is multiplied by a negative power of 10, how does the new position of the decimal point relate to the negative exponent? **Sample answer: When the power is negative, the number of the exponent gives the number of places the decimal point moves to the left in the product.**

Essential Question

WHY is it helpful to write numbers in different ways?

Vocabulary

scientific notation

Mathematical Practices

1, 3, 4, 7

Which **Mathematical Practices** did you use?
Shade the circle(s) that applies.

- ① Persevere with Problems
- ② Reason Abstractly
- ③ Construct an Argument
- ④ Model with Mathematics
- ⑤ Use Math Tools
- ⑥ Attend to Precision
- ⑦ Make Use of Structure
- ⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Use scientific notation to write large and small numbers.

Coherence connecting within and across grades

Previous Students wrote and simplified expressions involving exponents.

Now Students use scientific notation to write very large and very small numbers.

Next Students will compu with scientific notati

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 55.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, sm group, think-pair-share activity, or independent activity.

Prepare several pairs of index cards, enough so that each student in your class will have one. On one card, write a power of 10 such as 10^5 or 10^{-3}, and on another card, write the value of the number in standar decimal notation. Have students find the person who has t card that is equivalent to their card.

Alternate Strategy

BL LA **Roundrobin** In teams, have students answer t following question orally. Students listen to ideas and resp

MP 1, 2, 3

Ask:

- *List at least 5 examples of places where scientific nota might be used.* **Listen to student discussion.**

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1–2. Write in standard form.

AL • *In Example 1, what is 10^4?* **10,000**

• *In Example 1, by multiplying by 10,000, will that yield in a product greater than 5.34 or less?* **greater**

OL • *How are Examples 1 and 2 different?* **Sample answer: In Example 1, the exponent is positive. In Example 2, the exponent is negative.**

• *In Example 2, why is the product less than 3.27?* **By multiplying by 10^{-3}, you are really dividing by 10^3.**

BL • *In Example 1, why did we annex two zeros while the exponent was 4?* **5.34 already had two decimal places.**

Need Other Examples?

Write each number in standard form.

a. 9.62×10^5 **962,000** b. 2.85×10^{-5} **0.0000285**

3–4. Write in scientific notation.

AL • *In Example 3, between what two numbers should we place the decimal point? Example 4?* **between 3 and 7; between 3 and 1**

OL • *How are Examples 3 and 4 different?* **The number in Example 3 is greater than 1. The number in Example 4 is less than 1.**

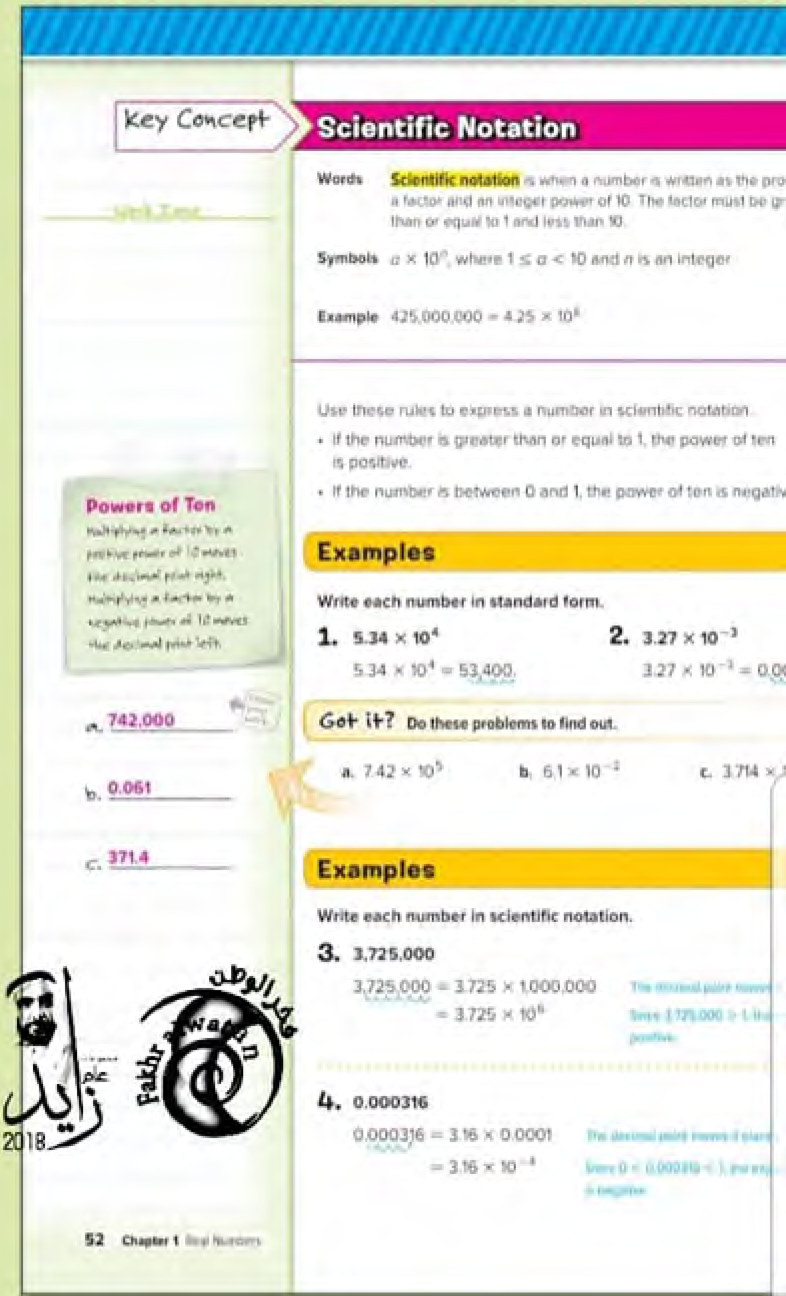
• *How do you know how many places to move the decimal point?* **The exponent on 10 indicates how many places to move the decimal point.**

BL • *Why do we not count only the zeros when determining the value of the exponent?* **Sample answer: We have to count how many places we moved the decimal, including digits other than 0.**

Need Other Examples?

Write each number in scientific notation.

a. 931,500,000 **9.315×10^8** b. 0.0044 **4.4×10^{-3}**



Key Concept **Scientific Notation**

Words **Scientific notation** is when a number is written as the pro… a factor and an integer power of 10. The factor must be gr… than or equal to 1 and less than 10.

Symbols $a \times 10^n$, where $1 \le a < 10$ and n is an integer

Example $425{,}000{,}000 = 4.25 \times 10^8$

Use these rules to express a number in scientific notation.

- If the number is greater than or equal to 1, the power of ten is positive.
- If the number is between 0 and 1, the power of ten is negativ…

Powers of Ten
Multiplying a factor by a positive power of 10 moves the decimal point right. Multiplying a factor by a negative power of 10 moves the decimal point left.

Examples

Write each number in standard form.

1. 5.34×10^4
$5.34 \times 10^4 = 53{,}400.$

2. 3.27×10^{-3}
$3.27 \times 10^{-3} = 0.00…$

Got it? Do these problems to find out.

a. 7.42×10^5 b. 6.1×10^{-2} c. $3.714 \times …$

a. 742,000
b. 0.061
c. 371.4

Examples

Write each number in scientific notation.

3. 3,725,000
$3{,}725{,}000 = 3.725 \times 1{,}000{,}000$
$= 3.725 \times 10^6$

4. 0.000316
$0.000316 = 3.16 \times 0.0001$
$= 3.16 \times 10^{-4}$

52 Chapter 1 Real Numbers

Got it? Do these problems to find out.

d. 14,140,000 e. 0.00876 f. 0.114

d. 1.414×10^{7}

e. 8.76×10^{-3}

f. 1.14×10^{-1}

Example

Refer to the table at the right. Order the countries according to the amount of money visitors spent in the UAE from greatest to least.

Dirhams Spent by International Visitors in the UAE	
Country	Dirhams Spent
Bahrain	1.03×10^{7}
Oman	1.83×10^{6}
Qatar	7.15×10^{6}
Saudi Arabia	1.06×10^{7}

Bahrain and Saudi Arabia — Qatar and Oman

Step 1 $\left\{\begin{matrix}1.06 \times 10^{7}\\ 1.03 \times 10^{7}\end{matrix}\right\} > \left\{\begin{matrix}7.15 \times 10^{6}\\ 1.83 \times 10^{6}\end{matrix}\right\}$ ← Group the numbers by their powers of 10.

Step 2 $1.06 > 1.03$ $7.15 > 1.83$ ← Order the decimals.

Saudi Arabia, Bahrain, Qatar, Oman

Got it? Do this problem to find out.

g. Some of the top U.S. cities visited by overseas travelers are shown in the table. Order the cities according to the number of visitors from least to greatest.

U.S. City	Number of Visitors
Boston	7.21×10^{5}
Las Vegas	1.3×10^{6}
Los Angeles	2.2×10^{6}
Metro D.C. area	9.01×10^{5}

g. Boston, Metro D.C. area, Las Vegas, Los Angeles

Example

STEM If you could walk at a rate of 2 meters per second, it would take you 1.92×10^{8} seconds to walk to the moon. Is it more appropriate to report this time as 1.92×10^{8} seconds or 6.09 years? Explain your reasoning.

The measure 6.09 years is more appropriate. The number 1.92×10^{8} seconds is very large so choosing a larger unit of measure is more meaningful.

Examples

5. Compare and order with scientific notation.

- AL • *What is the first thing you should look at when comparing numbers in scientific notation?* **Sample answer: look at the exponents, the greater the exponent, the greater the number**
- OL • *Why is Mexico not the country with the greatest number of dollars spent?* **Sample answer: While $7.15 > 1.03$, $7.15 > 1.83$, $7.15 > 1.06$, each of these numbers is multiplied by a power of 10. Multiplying by 10^{7} yields a larger number than multiplying by 10^{6}.**
- BL • *How could you write Canada's dollar amount using an exponent of 6?* **10.3×10^{6}**

Need Another Example?
The table lists the maximum frequency for the colors of the visible light spectrum. List the colors from greatest to least frequency. **violet, blue, green, orange, red**

Color	Maximum Frequency
Blue	6.7×10^{14}
Green	6.1×10^{14}
Orange	5.1×10^{14}
Red	3.8×10^{14}
Violet	7.5×10^{14}

6. Use scientific notation.

- AL • *Which is the larger unit of time: seconds or years?* **years**
- OL • *Explain why years is a more meaningful unit.* **Sample answer: It is more difficult to think about how long 1.92×10^{8} seconds is, but it is easier to think about how long 6.09 years is.**
- BL • *How could you convert seconds to years?* **Divide by 60 to convert seconds to minutes. Then divide by 60 to convert minutes to hours. Then divide by 24 to convert hours to days. Finally, divide by 365 $\left(\text{or } 365\frac{1}{4}\right)$ to convert days to years.**

Need Another Example?
One light year is about 9.46×10^{12} kilometers or 9.46×10^{18} millimeters. Is it more appropriate to report this length as 9.46×10^{12} kilometers or 9.46×10^{18} millimeters? Explain your reasoning. **Sample answer: 9.46×10^{12} kilometers; the number is very large, so choosing a larger unit of measure is more meaningful.**

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Teammates Consult** Teams discuss Exercise 1 with Student 1 leading the discussion. When everyone on the team has contributed to the discussion, team members silently write their own answers without further discussion. Repeat the process for Exercise 2 with Student 2 leading the discussion. MP 1, 3

BL LA **Trade-a-Problem** Have students research on the Internet for a large or small number that could be changed to scientific notation (for example, the salary of an NBA player or the number of chocolate candies produced each day). Have students create a problem using their research and trade problems with a partner. Students solve the other's problem and compare answers. MP 1, 2, 4, 5

Watch Out!

Common Error When learning to write numbers in scientific notation, students may think that a negative exponent indicates that a number is negative. Remind students of the meaning of a negative exponent.

h. 7.31 centimeters per year; the number is very small so choosing a smaller unit of measure is more meaningful.

Got it? Do this problem to find out.

h. STEM In an ocean, the sea floor moved 475 kilometers over 65 million years. Is it more appropriate to report this rate as 7.31×10^{-5} kilometer per year or 7.31 centimeters per year? Explain your reasoning.

Guided Practice

Write each number in standard form. (Examples 1 and 2)

1. $9.931 \times 10^5 =$ **993,100**

2. $6.02 \times 10^{-4} =$ **0.000602**

Write each number in scientific notation. (Examples 3 and 4)

3. $8{,}785{,}000{,}000 =$ **8.785×10^9**

4. $0.524 =$ **5.24×10^{-1}**

5. The table lists the total value of music shipments for four years. List the years from least to greatest dollar amount. (Example 5)

year 3, year 4, year 2, year 1

Year	Music Shipments (AED)
1	1.22×10^{10}
2	1.12×10^{10}
3	7.15×10^{8}
4	1.06×10^{9}

6. STEM A plant cell has a diameter of 1.3×10^{-8} kilometer. Is it more appropriate to report the diameter of a plant cell as 1.3×10^{-8} kilometer or 1.3×10^{-2} millimeter? Explain your reasoning. (Example 6)

1.3×10^{-2} millimeter; the number is very small so choosing a smaller unit of measure is more meaningful.

7. **Building on the Essential Question** How is scientific notation useful in the real world?

Sample answer: Scientific notation makes it easier for scientists to write the very large or very small numbers that they work with.

Rate Yourself!

I understand how to write numbers in scientific notation.

Great! You're ready to move on

I still have some questions about how to write numbers in scientific notation.

Name ______________________ My Homework ______________________

Independent Practice

Write each number in standard form. (Examples 1 and 2)

1. $3.16 \times 10^3 =$ **3,160**
2. $1.1 \times 10^{-4} =$ **0.00011**
3. $2.52 \times 10^{-5} =$ **0.0000252**

Write each number in scientific notation. (Examples 3 and 4)

4. $43{,}000 =$ **4.3×10^4**
5. $0.0072 =$ **7.2×10^{-3}**
6. $0.0000901 =$ **9.01×10^{-5}**

7. The areas of the world's oceans are listed in the table. Order the oceans according to their area from least to greatest. (Example 5)

Arctic, Southern, Indian, Atlantic, Pacific

World's Oceans

Ocean	Area (mi^2)
Atlantic	2.96×10^7
Arctic	5.43×10^6
Indian	2.65×10^7
Pacific	6×10^7
Southern	7.85×10^6

8. The space shuttle can travel about 8×10^5 centimeters per second. Is it more appropriate to report this rate as 8×10^5 centimeters per second or 8 kilometers per second? Explain. (Example 5)

8 kilometers per second; the number is very large so choosing a larger unit of measure is more meaningful.

9. The inside diameter of a certain size of ring is 1.732×10^{-2} meter. Is it more appropriate to report the ring diameter as 1.732×10^{-2} meter or 17.32 millimeters? Explain. (Example 6)

17.32 millimeters; the number is small so choosing a smaller unit of measure is more meaningful.

Fill in each ◯ with <, >, or = to make a true statement.

10. $678{,}000$ **<** 6.78×10^6
11. 6.25×10^3 **<** 6.3×10^3

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

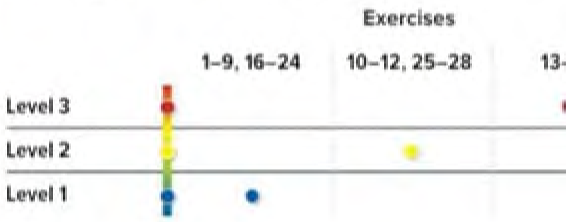
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used homework assignment. The Extra Practice page can be for additional reinforcement or as a second-day assign

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Le indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of complexity levels to select appropriate exercises for yo students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11, 13, 15, 27, 28
OL	On Level	1–9 odd, 10–13, 15, 27, 28
BL	Beyond Level	10–15, 27, 28

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	14
3 Construct viable arguments and critique the reasoning of others.	13
4 Model with mathematics.	12, 15
7 Look for and make use of structure.	25, 26

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Ask students to explain the steps to write a number between 0 and 1 in scientific notation. Tell them to use an example to illustrate their explanation. **See students' work.**

12. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–c.

a. Find Ahmed's and Khalid's heights in nanometers.
Ahmed: 1,700,000,000 nanometers, Khalid: 1,520,000,000 nanometers

b. Write each height using scientific notation.
Ahmed: 1.7×10^9 nanometers, Khalid: 1.52×10^9 nanometers

c. Give an example of something that would be appropriately measured by nanometers. **Sample answers: bacteria; switches inside of computers; fiber optic filament**

H.O.T. Problems Higher Order Thinking

13. **Justify Conclusions** Determine whether 1.2×10^5 or 1.2×10^6 is closer to one million. Explain. **1.2×10^6; 1.2×10^5 is only 120,000, but 1.2×10^6 is just over one million.**

14. **Persevere with Problems** Compute and express each value in scientific notation.

a. $\frac{(130{,}000)(0.0057)}{0.0004} =$ **1.8525×10^6**

b. $\frac{(90{,}000)(0.0016)}{(200{,}000)(30{,}000)(0.00012)} =$ **2×10^{-4}**

15. **Model with Mathematics** Write two numbers in scientific notation with values between 100 and 1,000. Then write an inequality that shows the relationship between your two numbers.

Sample answer: 3.01×10^2, 5.01×10^2; $3.01 \times 10^2 < 5.01 \times 10^2$

Name ______ My Homework ______

Extra Practice

16. Write 7.113×10^7 in standard form.

71,130,000

$7.113 \times 10^7 = 71130000$, The decimal point moves 7 places right.

Homework Help

17. Write 0.00000707 in scientific notation.

7.07×10^{-6}

$0.00000707 = 7.07 \times 0.000001$
$= 7.07 \times 10^{-6}$

The decimal point moves 6 places. Since $0 < 0.00000707 < 1$, the exponent is negative.

Write each number in standard form.

18. $2.08 \times 10^2 =$ **208**

19. $7.8 \times 10^{-3} =$ **0.0078**

20. $8.73 \times 10^{-4} =$ **0.000873**

Write each number in scientific notation.

21. 6,700 = $\mathbf{6.7 \times 10^3}$

22. 52,300,000 = $\mathbf{5.23 \times 10^7}$

23. 0.037 = $\mathbf{3.7 \times 10^{-2}}$

24. **STEM** The table shows the mass in grams of one atom of each of several elements. List the elements in order from the least mass to greatest mass per atom.

hydrogen, carbon, oxygen, silver, gold

Element	Mass per Atom
Carbon	1.995×10^{-23} g
Gold	3.272×10^{-22} g
Hydrogen	1.674×10^{-24} g
Oxygen	2.658×10^{-23} g
Silver	1.792×10^{-22} g

Identify Structure Arrange each set of numbers in increasing order.

25. 216,000,000, 2.2×10^3, 3.1×10^7, 310,000

$\mathbf{2.2 \times 10^3}$, **310,000**, $\mathbf{3.1 \times 10^7}$, **216,000,000**

26. 4.56×10^{-2}, 4.56×10^3, 4.56×10^2, 4.56×10^{-3}

$\mathbf{4.56 \times 10^{-3}}$, $\mathbf{4.56 \times 10^{-2}}$, $\mathbf{4.56 \times 10^2}$, $\mathbf{4.56 \times 10^3}$

Power Up! Test Practice

Exercises 27 and 28 prepare students for more rigorous thinking needed for the assessment.

27.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

28.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly order the four teams and their attendance AND answer the question.
1 point	Students correctly order the teams but fail to answer the question OR students order three teams and may or may not answer the question.

Power Up! Test Practice

27. The thermosphere layer of the atmosphere is between 90,000 and 110,000 meters above sea level. Which of the following elevations are in the thermosphere? Select yes or no.

a. 9.8×10^{-4} ☐ Yes ■ No

b. 1.04×10^5 ■ Yes ☐ No

c. 9.72×10^4 ■ Yes ☐ No

d. 1.45×10^5 ☐ Yes ■ No

28. The attendance for four Major League baseball teams for a recent year is shown below.

Team	Attendance
Los Angeles	3.06×10^6
Miami	22.2×10^5
Pittsburgh	20.9×10^5
St. Louis	3.26×10^6

Sort the teams from least to greatest attendance.

	Team	Attendance
Least	Pittsburgh	20.9×10^5
	Miami	22.2×10^5
	Los Angeles	3.06×10^6
Greatest	St. Louis	3.26×10^6

Which team had the greatest attendance? St. Louis

Spiral Review

Find each sum or difference.

29. $9.7 + 0.532 =$ 10.232

30. $4.39 - 0.035 =$ 4.355

31. $679 - 1.4 =$ 677.6

Simplify. Express using exponents.

32. $3a^4 \cdot 12a^2 =$ $36a^6$

33. $(5x)^2 \cdot 2x^5 =$ $50x^7$

34. $\frac{3^9}{3^2} =$ 3^7

Lesson 7

Compute with Scientific Notation

Real-World Link

E-mail Every day, nearly 130 billion spam E-mails are sent worldwide! Use the steps below to find out how many are sent each year. The numbers are too large even for your calculator.

1. Express 130 billion in scientific notation.
   1.3×10^{11}

2. Round 365 to the nearest hundred and express it in scientific notation.
   $400; 4 \times 10^2$

3. Write a multiplication expression using the number in Exercises 1 and 2 to represent the total number of spam E-mails sent each year.
   $(1.3 \times 10^{11})(4 \times 10^2)$

4. If you use the Commutative Property of Multiplication, you can rewrite the expression in Exercise 3 as $(1.3 \times 4)(10^{11} \times 10^2)$. Evaluate this expression to find the number of spam E-mails sent in a year. Express the result in both scientific notation and standard form.
   5.2×10^{13}; 52,000,000,000,000

Essential Question

WHY is it helpful to write numbers in different ways?

MP Mathematical Practices
1, 3, 4

Which **MP Mathematical Practices** did you use?
Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Compute with numbers written in scientific notati

Coherence connecting within and across grades

Previous
Students wrote very small and very large numbers using scientific notation.

Now
Students add, subtract, multiply, and divide numbers written in scientific notation.

Next
Students will use the calculator to work wit numbers in scientific notation

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 63.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, sm group, think-pair-share activity, or independent activity.

AL LA Team Consult Have students work in small gro Give each group a set of index cards with one of the follow expressions written on each card: 6.1×10^{14}, 7.4×10^{-5}, 2×10^{9}, 6.5×4^{10}. Have groups discuss which card is diffe from the others. Encourage groups to discuss why there mi be more than one possible answer. MP 1, 3

Alternate Strategy

BL Have students discuss how the Commutative Property Multiplication works when multiplying or dividing numbers written in scientific notation. MP 1, 7

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Multiply numbers written in scientific notation.

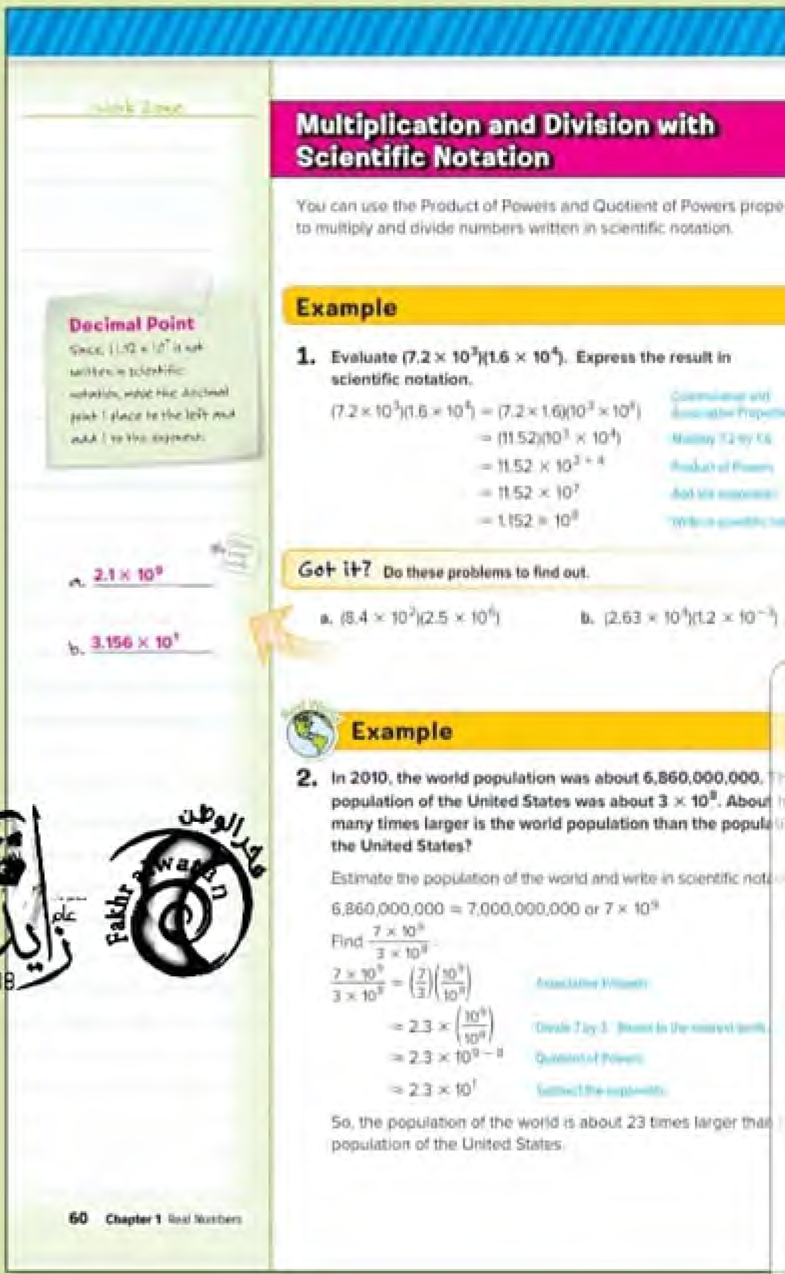
- AL • *If you want to keep the expression in scientific notation, what pairs of numbers would be easy to multiply?* **7.2 and 1.6; 10^3 and 10^4**
- OL • *When simplifying $10^3 \times 10^4$, do you add, subtract, multiply, or divide the exponents?* **add**
- BL • *Why do you need to rewrite 11.52×10^7 as 1.152×10^8?* **11.521×10^7 is not written in scientific notation; 11.52 is not between 1 and 10.**

Need Another Example?
Evaluate $(1.1 \times 10^{-3})(2.5 \times 10^9)$. Express the result in scientific notation. **2.75×10^6**

2. Divide numbers written in scientific notation.

- AL • *What operation will you need to perform to solve this problem?* **division**
  - • *What is 6,860,000,000 rounded to the nearest billion?* **7,000,000,000**
  - • *What is 7,000,000,000 written in scientific notation?* **7.0×10^9**
- OL • *What expression represents the situation?* **$\frac{7 \times 10^9}{3 \times 10^8}$**
- BL • *Why would you want to know how many times larger the world population is than the population of the U.S.?* **See students' work.**

Need Another Example?
The largest planet in our solar system is Jupiter with a diameter of about 143,000 kilometers. The smallest planet in our solar system is Mercury with a diameter of about 5×10^3 kilometers. About how many times greater is the diameter of Jupiter than the diameter of Mercury? **Sample answer: 3×10^1 or 30 times greater**



Work Zone

Multiplication and Division with Scientific Notation

You can use the Product of Powers and Quotient of Powers prope[rties] to multiply and divide numbers written in scientific notation.

Example

1. Evaluate $(7.2 \times 10^3)(1.6 \times 10^4)$. Express the result in scientific notation.

$(7.2 \times 10^3)(1.6 \times 10^4) = (7.2 \times 1.6)(10^3 \times 10^4)$
$= (11.52)(10^3 \times 10^4)$
$= 11.52 \times 10^{3+4}$
$= 11.52 \times 10^7$
$= 1.152 \times 10^8$

Decimal Point
Since 11.52×10^7 is not written in scientific notation, move the decimal point 1 place to the left and add 1 to the exponent.

Got it? Do these problems to find out.

a. $(8.4 \times 10^2)(2.5 \times 10^6)$ b. $(2.63 \times 10^4)(1.2 \times 10^{-3})$

a. 2.1×10^9
b. 3.156×10^1

Example

2. In 2010, the world population was about 6,860,000,000. The population of the United States was about 3×10^8. About how many times larger is the world population than the popula[tion of] the United States?

Estimate the population of the world and write in scientific not[ation].

6,860,000,000 ≈ 7,000,000,000 or 7×10^9

Find $\frac{7 \times 10^9}{3 \times 10^8}$.

$\frac{7 \times 10^9}{3 \times 10^8} = \left(\frac{7}{3}\right)\left(\frac{10^9}{10^8}\right)$
$\approx 2.3 \times \left(\frac{10^9}{10^8}\right)$
$= 2.3 \times 10^{9-8}$
$= 2.3 \times 10^1$

So, the population of the world is about 23 times larger tha[n the] population of the United States.

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Got it? Do this problem to find out.

c. The surface area of Lake Superior, the largest of the Great Lakes, is 8×10^4 square kilometers. The surface area of the smallest Great Lake, Ontario, is 18,160 square kilometers. About how many times as great is the area covered by Lake Superior than Lake Ontario?

c. 4 times

Addition and Subtraction with Scientific Notation

When adding or subtracting decimals in standard form, it is necessary to line up the place values. In scientific notation, the place value is represented by the exponent. Before adding or subtracting, both numbers must be expressed in the same form.

Examples

Evaluate each expression. Express the result in scientific notation.

3. $(6.89 \times 10^4) + (9.24 \times 10^5)$

$(6.89 \times 10^4) + (9.24 \times 10^5)$
$= (6.89 \times 10^4) + (92.4 \times 10^4)$ — Write 9.24×10^5 as 92.4×10^4.
$= (6.89 + 92.4) \times 10^4$ — Distributive Property
$= 99.29 \times 10^4$ — Add 6.89 and 92.4.
$= 9.929 \times 10^5$ — Rewrite in scientific notation.

4. $(7.83 \times 10^8) - 11{,}610{,}000$

$(7.83 \times 10^8) - (1.161 \times 10^7)$ — Rewrite 11,610,000 in scientific notation.

$(7.83 \times 10^8) - (1.161 \times 10^7)$
$= (78.3 \times 10^7) - (1.161 \times 10^7)$ — Write 7.83×10^8 as 78.3×10^7.
$= (78.3 - 1.161) \times 10^7$ — Distributive Property
$= 77.139 \times 10^7$ — Subtract 1.161 from 78.3.
$= 7.7139 \times 10^8$ — Rewrite in scientific notation.

STOP and Reflect

Explain below how to estimate the sum of (4.215×10^{-2}) and (7.2×10^{-4}). Then find the estimate.

Sample answer: convert both expressions to the same power of 10. Then round the digit terms and add. The approximate sum is 4×10^{-2}.

Examples

3. Add numbers written in scientific notation.

AL
- *Why do you need to rewrite 9.24×10^5 as 92.4×10^4?* You need to be able to line up the place values. To do that, the numbers must by multiplied by the same power of 10.
- *Why do you change the exponent for 10 in 9.24×10^5 to a 4 when you rewrite the expression?* When you move the decimal point one place to the right, you are multiplying by 10. You need to decrease the exponent on the 10 to reflect that.

OL
- *Why is 99.29×10^4 not the correct answer?* It is not written in scientific notation. In order for it to be in scientific notation, there can only be one place value to the left of the decimal point.
- *How do rewrite 99.29×10^4 in scientific notation?* Moving the decimal point one place to the left is the same as multiplying by another power of 10. Move the decimal one place to the left and add one to the exponent.

BL
- *Is there another way to solve this problem? Explain.* See students' work.

Need Another Example?
Evaluate $(2.85 \times 10^7) + (1.61 \times 10^9)$. Express the result in scientific notation. 1.6385×10^9

4. Subtract numbers written in scientific notation.

AL
- *Are the numbers written in the same form?* no
- *What is 11,610,000 written in scientific notation?* 1.161×10^7
- *Are the powers of 10 the same in both numbers?* no

OL
- *What do you need to do first to subtract the numbers?* Rewrite 11,610,000 in scientific notation.
- *Is 77.139×10^7 written in scientific notation?* No, it needs to be written as 7.7139×10^8.

BL
- *Is it easier to solve this problem using scientific notation or using standard form? Explain.* See students' work.

Need Another Example?
Evaluate $(8.23 \times 10^6) - 391{,}000$. Express the result in scientific notation. 7.839×10^6

Example

5. Add numbers written in scientific notation.

- AL • *Are the numbers written in the same form?* **no**
- • *What do you need to do first to subtract the numbers?* **Rewrite 593,000 in scientific notation.**
- • *What is 593,000 written in scientific notation?* **5.93×10^5**
- • *Why is 5.93×10^5 rewritten as 0.593×10^6?* **so the two expressions have the same power of 10**
- OL • *What is 5.93×10^5 rewritten so the exponent of 10 is 6?* **0.593×10^6**
- BL • *Evaluate $7{,}930{,}000 - (2.12 \times 10^3)(4.2 \times 10^2)$. Write the result in scientific notation.* **7.0396×10^6**

Need Another Example?
Evaluate $6{,}450{,}000{,}000 - (8.27 \times 10^7)$. Express the result in scientific notation. **6.3673×10^9**

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL Think-Pair-Share Have students work in pairs. Before students begin Exercises 1–7, have them make index cards with rules on multiplying, dividing, adding, and subtracting with scientific notation. They should take a minute before solving each Exercise to determine the steps they need to take to solve the problem. After they complete each Exercise, have them share their responses with their partner. When all of the work is completed, call on one student per Exercise to share their response within a small group or a large group discussion. MP 1, 3, 6, 7

BL Pairs Project Have students work in pairs. Pairs should summarize the Laws of Exponents, including calculations with scientific notation, through a poem, a rap, or a song. They can record the presentations using a video camera create a small book on the computer. MP 1, 2, 4, 6

5. $593{,}000 + (7.89 \times 10^6)$

$593{,}000 + (7.89 \times 10^6)$
$= (5.93 \times 10^5) + (7.89 \times 10^6)$
$= (0.593 \times 10^6) + (7.89 \times 10^6)$
$= (0.593 + 7.89) \times 10^6$
$= 8.483 \times 10^6$

Got it? Do these problems to find out.

d. $(8.41 \times 10^3) + (9.71 \times 10^4)$ — d. 1.0551×10^5
e. $(1.263 \times 10^9) - (1.525 \times 10^7)$ — e. 1.24775×10^9
f. $(6.3 \times 10^5) + 2{,}700{,}000$ — f. 3.33×10^6

Guided Practice

Evaluate each expression. Express the result in scientific notation.

1. $(2.6 \times 10^5)(1.9 \times 10^2) =$ 4.94×10^7
2. $\frac{8.37 \times 10^8}{2.7 \times 10^3} =$ 3.1×10^5
3. In 2005, 8.1×10^{10} text messages were sent in the United States. In 2010, the number of annual text messages had risen to 1,810,000,000,000. About how many times as great was the number of text messages in 2010 than 2005? 20

Evaluate each expression. Express the result in scientific notation.

4. $(8.9 \times 10^9) + (4.2 \times 10^5) =$ 8.9042×10^9
5. $(9.64 \times 10^8) - (5.29 \times 10^5) =$ 9.5871×10^8
6. $(1.35 \times 10^6) - (117{,}000) =$ 1.233×10^6
7. $5{,}400 + (6.8 \times 10^5) =$ 6.854×10^5
8. **Building on the Essential Question** How does scientific notation make it easier to perform computations with very large or very small numbers? **Sample answer: Computing with scientific notation is easier because you can work with more compact numbers and use properties of math with powers of 10.**

Rate Yourself!
Are you ready to move on? Shade the section that applies.
YES ? NO

62 Chapter 1 Real Numbers

Name ______________________ My Homework ______________________

Independent Practice

Evaluate each expression. Express the result in scientific notation. (Examples 1 and 2)

1. $(3.9 \times 10^2)(2.3 \times 10^6) =$ **8.97×10^8**

2. $(4.18 \times 10^{-4})(9 \times 10^{-4}) =$ **3.762×10^{-7}**

3. $(9.75 \times 10^3)(8.4 \times 10^{-6}) =$ **8.19×10^{-2}**

4. $\frac{9.45 \times 10^{10}}{1.5 \times 10^6} =$ **6.3×10^4**

5. $\frac{1.14 \times 10^6}{4.8 \times 10^{-6}} =$ **2.375×10^{11}**

6. $\frac{9 \times 10^{-11}}{2.4 \times 10^8} =$ **3.75×10^{-19}**

7. **STEM** Neurons are cells in the nervous system that process and transmit information. An average neuron is about 5×10^{-6} meter in diameter. A standard table tennis ball is 0.04 meter in diameter. About how many times as great is the diameter of a ball than a neuron? (Example 1)

8,000 times

Evaluate each expression. Express the result in scientific notation. (Examples 3–5)

8. $(9.5 \times 10^{11}) + (6.3 \times 10^9) =$ **9.563×10^{11}**

9. $(1.03 \times 10^9) - (4.7 \times 10^7) =$ **9.83×10^8**

10. $(1.357 \times 10^9) + 590{,}000 =$ **1.35759×10^9**

11. $87{,}100 - (6.34 \times 10^1) =$ **8.70366×10^4**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be usec
homework assignment. The Extra Practice page can b
for additional reinforcement or as a second-day assign

Levels of Complexity

The levels of the exercises progress from 1 to 3, with L
indicating the lowest level of complexity.

	Exercises 1–11, 17–24	12, 25, 26	13
Level 3			
Level 2		•	
Level 1	•		

Suggested Assignments

You can use the table below that includes exercises of
complexity levels to select appropriate exercises for y
students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–11, 13, 14, 16, 25, 26
OL	On Level	1–11 odd, 12–14, 16, 25, 26
BL	Beyond Level	12–16, 25, 26

Watch Out!

Common Error A student may multiply (8×10^6) by (3
get the answer 24×10^{10}. When this happens, remind
the product should be written in scientific notation whic
it must be in the form of $a \times 10^n$, where $1 \leq a < 10$.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	12, 15, 23
3 Construct viable arguments and critique the reasoning of others.	13, 14
4 Model with mathematics.	16

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write a short paragraph to describe how yesterday's lesson on scientific notation helped them with today's lesson. **See students' work.**

Watch Out!

Find the Error In Exercise 13, Enrique made a mistake when he was subtracting a negative integer from another negative integer. This is a common error with students. It may be helpful to review the basic skills of adding and subtracting integers.

12. **Persevere with Problems** Dubai Park and Resorts in Dubai is rectangular in shape and measures approximately 1.37×10^4 feet by 2.64×10^2 feet. If one acre is equal to 4.356×10^4 square feet, how many acres does Dubai Park and Resorts cover? Round to the nearest hundredth. **about 83.03 acres**

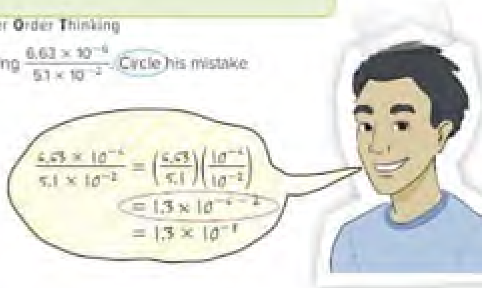
H.O.T. Problems Higher Order Thinking

13. **Find the Error** Enrique is finding $\frac{6.63 \times 10^{-6}}{5.1 \times 10^{-2}}$. Circle his mistake and correct it.

$\frac{6.63 \times 10^{-6}}{5.1 \times 10^{-2}} = \left(\frac{6.63}{5.1}\right)\left(\frac{10^{-6}}{10^{-2}}\right)$

$= 1.3 \times 10^{-6-(-2)}$

$= 1.3 \times 10^{-4}$



14. **Which One Doesn't Belong?** Identify the expression that does not belong with the other three. Explain your reasoning.

14.28×10^{9}	$(3.4 \times 10^{4})(4.2 \times 10^{5})$	1.4×10^{9}	$(3.4)(4.2) \times 10^{(4+5)}$

1.4×10^9; Sample answer: All of the other expressions are equivalent.

15. **Persevere with Problems** A *googol* is the number 1 followed by 100 zeros.

a. What is one googol written in scientific notation? **1×10^{100}**

b. How many times greater is a googol of meters than a nanometer? **10^{109} times**

c. There are about 2.5×10^{10} red blood cells in the average adult. About how many adults would it take to have a total of 1 googol red blood cells? **about 4×10^{89} adults**

16. **Model with Mathematics** Write an addition expression and a subtraction expression, each with a value of 2.4×10^{-3}.
Sample answers: $(2.15 \times 10^{-3}) + (2.5 \times 10^{-4})$; $(2.56 \times 10^{-3}) - (1.6 \times 10^{-4})$

Name ________________ My Homework ________________

Extra Practice

Evaluate each expression. Express the result in scientific notation.

17. $(3.7 \times 10^{-2})(1.2 \times 10^{3}) = 4.44 \times 10^{1}$

$(3.7 \times 10^{-2})(1.2 \times 10^{3}) = (3.7 \times 1.2) \times (10^{-2} \times 10^{3})$

$= 4.44 \times 10^{-2+3}$

$= 4.44 \times 10^{1}$

Homework Help

18. $\frac{4.64 \times 10^{-4}}{2.9 \times 10^{-6}} = 1.6 \times 10^{2}$

$\frac{4.64 \times 10^{-4}}{2.9 \times 10^{-6}} = \frac{4.64}{2.9} \times \frac{10^{-4}}{10^{-6}}$

$= 1.6 \times 10^{-4-(-6)}$

$= 1.6 \times 10^{2}$

19. $\frac{3.24 \times 10^{-4}}{8.1 \times 10^{-7}} = 4 \times 10^{2}$

20. $(7.3 \times 10^{5}) + 2,400,000 = 3.13 \times 10^{6}$

21. $(8.64 \times 10^{5}) + (1.334 \times 10^{10}) =$ 1.334864×10^{10}

22. $(1.21 \times 10^{5}) - 9,500 =$ 1.115×10^{5}

23. **Persevere with Problems** A circular swimming pool holds 1.22×10^{6} cubic inches of water. It is being filled at a rate of 1.5×10^{3} cubic inches per minute. How many hours will it take to fill the swimming pool? $13\frac{5}{9}$ h

24. **Financial Literacy** In 2010, the national debt of the United States was about 14 trillion dollars. In 2003 it was about 7×10^{12} dollars. About how many times larger was the national debt in 2010 than in 2003? 2 times

Hands-On Activity 2

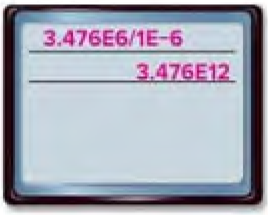
A human blood cell is about 1×10^{-6} meter in diameter. The Moon is about 3.476×10^{6} meters in diameter. How many times greater is the diameter of the Moon than the diameter of a blood cell?

Step 1 Press CLEAR to clear the home screen.

Step 2 Perform the following keystrokes:

3.476 2nd EE 6 ÷ 1 2nd EE −6 ENTER

Copy your calculator screen on the blank screen shown.



Step 3 Write the value in standard form.

3,476,000,000,000

So, the diameter of the Moon is 3,476,000,000,000 times greater than the diameter of a human blood cell.

Hands-On Activity 3

When in "Normal" mode, a calculator will show answers in scientific notation only if they are very large numbers or very small numbers. You can set your calculator to show scientific notation for all numbers by using the "Sci" mode.

Step 1 Press CLEAR to clear the home screen. Put your calculator in scientific mode by pressing MODE ▶ ENTER. Then press CLEAR to return to the home screen.

Step 2 Complete the table by entering the numbers in the first column into your calculator.

Enter	Calculator Notation	Standard Form
14 ÷ 100	1.4 E−1	.14
60 − 950	−8.9 E2	−8900
360 · 15	5.4 E3	5,400
1 + 1	2 E0	2

Hands-On Activity 2

AL Remind students what the E on the calculator screen represented in the first Hands-On Activity. Explain that they w now work backward and will need to manually enter the E symbol using the calculator button.

BL **LA** Before beginning the given steps, give students one minute to decide how they would set up and solve the problem if it was not in scientific notation. Have them expla what operation they would use and why. MP 1, 2, 7

Hands-On Activity 3

AL Give students one answer in each column to model w the answers should look like.

BL Have students predict what the answers will be for bo calculator notation and standard form before using the calculator. If predictions were incorrect, have them reflect o their mistake. MP 1, 6

Power Up! Test Practice

Exercises 25 and 26 prepare students for more rigorous thinking needed for the assessment.

25.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP6
Scoring Rubric	
1 point	Students correctly answer each part of the question.

26.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP7
Scoring Rubric	
2 points	Students order all four countries, state the densities, and determine the country with the greatest population density.
1 point	Students order all four countries, select the country with the greatest density, but fail to correctly identify density OR students order the countries and state the densities but fail to determine the country with the greatest density OR students correctly order 2–3 countries and state the densities and select the country with the greatest population.

Power Up! Test Practice

25. There are approximately 45 hundred species of mammals on Earth and 2.8×10^4 species of fish. Fill in each box to make a true statement.

There are more species of **fish** than species of **mammals** on Earth. The difference in the number of species is **23,500**.

26. *Population density* is a measure of how many people are living in a region. To calculate population density, divide the population of a region by the area in square miles. The table shows the approximate populations and areas of different countries.

Country	Population	Area (mi²)
China	1.332×10^9	3.7×10^6
Poland	3.84×10^7	1.2×10^5
Sweden	9.6×10^6	1.6×10^5
United States	3.15×10^8	3.5×10^6

Sort the countries from least to greatest population density.

	Country	Population Density (people per mi²)
Least	**Sweden**	**60**
	United States	**90**
	Poland	**320**
Greatest	**China**	**360**

Which country has the greatest population density? **China**

Spiral Review

27. A cube measures 6.6 inches on each side.

a. Find the area of one face of the cube. **43.56 in²**

b. Find the volume of the cube. **287.496 in³**

28. Complete the table shown.

x	x^2	x^3	x	x^2	x^3
1	**1**	**1**	7	**49**	**343**
2	**4**	**8**	8	**64**	**512**
3	**9**	**27**	9	**81**	**729**
4	**16**	**64**	10	**100**	**1,000**
5	**25**	**125**	11	**121**	**1,331**
6	**36**	**216**	12	**144**	**1,728**

Inquiry Lab

Graphing Technology: Scientific Notation Using Technology

Inquiry **WHAT are the similarities and differences between a number written in scientific notation and the calculator notation of the number shown on a screen?**

Mathematical Practices 1, 3, 5

The table shows the mass of some planets in our solar system. What is the mass of Earth written in scientific notation?

Planet	Mass (kg)
Earth	5,973,700,000,000,000,000,000,000
Mars	641,850,000,000,000,000,000,000
Saturn	568,510,000,000,000,000,000,000,000

What do you know? **the mass of three planets written in standard form**

What do you need to find? **the mass of Earth written in scientific notation**

Hands-On Activity 1

You will use a graphing calculator to explore how scientific notation is displayed using technology.

Step 1 Press CLEAR to clear the home screen.

Step 2 Enter the value in standard form for Earth's mass. Press ENTER.

Copy your calculator screen on the blank screen shown.

5973700000000000
000000000
5.9737E24

Step 3 Write the value for Earth's mass using scientific notation.
5.9737×10^{24}

Focus narrowing the scope

Objective Interpret scientific notation when using technology.

Coherence connecting within and across grades

Now
Students use technology to write numbers in scientific notation.

Next
Students will find and use square roots and cube roots.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 68.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activity 1 is intended to be used as a whole-group activity. Activity 1 is designed to provide more guidance to students than Activities 2 and 3.

Materials: graphing calculator

Hands-On Activity 1

AL LA **Think-Pair-Share** Have students individually think through their responses to the following question, then share their responses with a partner. MP 1, 6

Ask:

- *Why is it important to accurately count the number of zeros in Earth's mass and enter all of them in the calculator?* Sample answer: The number of zeros affects the value of the exponent.

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–5, 11	Exercises 6–10, 12	Exercises 13, 14
Level 3			●
Level 2		●	
Level 1	●		

Investigate

AL LA **Think-Pair-Share** Have students work individually to complete Exercises 1–5, then share their answers with a partner. MP 1, 3

Ask:

- *Refer to Exercise 3. Ask your partner to share their response. Ask them to clarify anything you do not understand.* See students' responses.

Analyze and Reflect

AL LA **Pairs Discussion** Have students work in pairs to answer the following extension question. MP 1, 7, 8

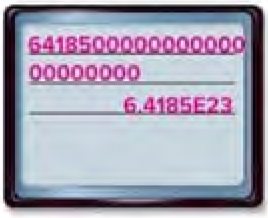
Ask:

- *Refer to Exercise 10. How do you know what the value of the exponent will be without using a calculator?* Sample answer: After placing the decimal point, count the number of digits to the right of the decimal point including the zeros.

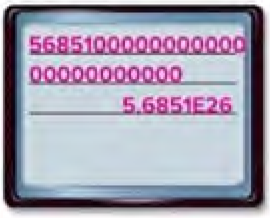
Investigate

MP **Use Math Tools** Work with a partner. Repeat Steps 1 and 2 of the activity on the previous page for each of the following.

1. mass of Mars



2. mass of Saturn



3. What does the E symbol represent on the calculator screen? The E symbol represents the "× 10" part of an expression written in scientific notation.

   What does the value after the E symbol represent? The value after the E symbol represents the exponent.

4. Based on your answer for Exercise 1, what is the mass of Mars in scientific notation? 6.4185×10^{23} kg

5. Based on your answer for Exercise 2, what is the mass of Saturn in scientific notation? 5.6851×10^{26} kg

Analyze and Reflect

Work with a partner to complete the table.

	Calculator Notation	Scientific Notation	Standard Form
6.	3.1E7	3.1×10^7	31,000,000
7.	6.39 E10	6.39×10^{10}	63,900,000,000
8.	2.357 E–2	2.357×10^{-2}	0.02357
9.	1.7E–11	1.7×10^{-11}	0.000000000017

10. MP **Reason Inductively** The Moon has a mass of about 73,600,000,000,000,000,000,000 kilograms. Without entering the value in your calculator, predict how the mass of the Moon will be displayed on the calculator screen. 7.36 E22

Investigate

BL LA **Paired Heads Together** Pair an Approaching
l student with a Beyond Level student to work together
ercise 11. Have each student individually think through
solution. Then have them share their solution with a
er. Each partner should practice speaking aloud the
ess they used to solve the problem. MP 1, 3, 6

Analyze and Reflect

In Exercise 12, show students how to set up the first
lem without a calculator (5,000 ÷ 0.000001). Give students
answer in each column to model what the answers should
like.

Have students use the Internet or another source
ok up lengths of objects in meters. Then have them
rmine how many micrometers are in each of their
urements. MP 1, 5

Create

Students should be able to answer "WHAT are the
arities and differences between a number written in
tific notation and the calculator notation of the number
n on a screen?" Check for student understanding and
de guidance, if needed.

Investigate

11. MP **Use Math Tools** Work with a partner. Write down the keystrokes and fill in the calculator screen to find $(6.2 \times 10^5)(2.3 \times 10^7)$ using a calculator in "Sci" mode. Write your final answer in standard form.

Keystrokes: 6.2 [2nd] [EE] 5 [×] 2.3 [2nd] [EE] 7 [ENTER]

Calculator screen:
6.2E5*2.3E7
1.426E13

Answer in standard form: 14,260,000,000,000

Analyze and Reflect

12. MP **Use Math Tools** A *micrometer* is 0.000001 meter. Use your calculator to determine how many micrometers are in each of the following. Write your answer in both calculator and scientific notation.

	Calculator Notation	Scientific Notation
5,000 meters	5E9	5×10^9
4.08E14 meters	4.08E20	4.08×10^{20}
2.9E-10 meter	2.9E-4	2.9×10^{-4}

Create

13. MP **Use Math Tools** Write a subtraction expression involving two numbers written in scientific notation. Then write the keystrokes and fill in the calculator screen to find the answer using a calculator in "Sci" mode. Write your final answer in standard form. **Sample answers are given.**

Expression: $(8.5 \times 10^{-3}) - (4.8 \times 10^{-5})$

Keystrokes: 8.5 [2nd] [EE] [(−)] 3 [−] 4.8 [2nd] [EE] [(−)] 5 [ENTER]

Calculator screen:
8.5E-3 - 4.8E-5
8.452E-3

Answer in standard form: 0.008452

14. Inquiry WHAT are the similarities and differences between a number written in scientific notation and the calculator notation of the number shown on a screen? **Sample answer: Both contain the same factor. In scientific notation, the power of ten is written with an exponent. In calculator notation, the exponent for the power of ten is written after the E.**

Lesson 8

Roots

Vocabulary Start-Up

A **square root** of a number is one of its two equal factors. Numbers such as 1, 4, 9, 16, and 25 are called **perfect squares** because they are squares of integers.

Complete the graphic organizer. **Sample answers are given.**

square root

I think this word means... **the opposite of squaring.**	How does this word fit with other words and concepts I know? **exponents**
Are there parts of the word that I recognize? **square**	What makes this an important word for me to know? **I can undo squaring a number.**

What is the relationship between squaring a number and finding the square root? **Sample answer: Squaring a number and finding a square root are inverse operations.**

Essential Question

WHY is it helpful to write numbers in different ways?

Vocabulary

square root
perfect square
radical sign
cube root
perfect cube

Mathematical Practices
1, 3, 4

Real-World Link

The square base of the Great Pyramid of Giza covers almost 562,500 square feet. How could you determine the length of each side of the base?

Sample answer: Find the square root of 562,500.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Find square roots and cube roots.

Coherence connecting within and across grades

Previous Students evaluated expressions involving rational numbers.

Now Students find square roots and cube roots.

Next Students will estimate square roots of non-perfect squares.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 75.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, sm group, think-pair-share activity, or independent activity.

LA Pairs Discussion Give each student pair 16 square math tiles. Have them make as many different rectangles as they can using all 16 tiles. Ask them t record their dimensions in a table like the one below. MP 1, 4

Ask:

- *How are these numbers related to the number 16?* **Sample answer: They are all factors of 16.**

Length	16				
Width	1				

Alternate Strategy

BL LA Ask students if it matters whether the factors are both positive or both negative when dealing with perfect squares. For example, is $-2(-2)$ the same as $2(2)$?

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1–4. Find square roots.

AL
- *Which examples indicate a positive square root only? negative only?* **Example 1; Example 3**

OL
- *What does the symbol in front of the radical sign in Example 2 indicate?* **We need to determine both the positive and negative square root.**
- *Why is there no solution to Example 4?* **There is no real number by which you can multiply itself to equal −16.**

BL
- *How can you use mental math to determine the square root in Example 2?* **Determine the square root of 121, which is 11. Then add the decimal point.**
- *Generate an example of a square root with no solution.* **Sample answer: $\sqrt{-25}$**

Need Other Examples?

Find each square root.

a. $\sqrt{225}$ **15**

b. $\pm\sqrt{1.44}$ **±1.2**

c. $-\sqrt{\frac{49}{64}}$ **$-\frac{7}{8}$**

d. $\sqrt{-81}$ **no real square root**

5. Solve an equation involving a square root.

AL
- *How many answers will there be? Explain.* **2; There will be a positive and a negative square root.**

OL
- *How can you check to see if your answers are reasonable?* **Substitute each answer into the equation.**

BL
- *Why do we take the square root of each side of the equation?* **The variable is squared. Squaring and taking a square root are inverse operations.**
- *Write your own equation that can be solved by taking the square root of both sides.* **Sample answer: $m^2 = 225$**

Need Another Example?

Solve $x^2 = 144$. Check your solution(s). **12, −12**

Key Concept

Square Root

Words A square root of a number is one of its two equal factors.

Symbols If $x^2 = y$, then x is a square root of y.

Example $5^2 = 25$ so 5 is a square root of 25.

Every positive number has *both* a positive and negative square root. In most real-world situations, only the positive or *principal* square root is considered. A **radical sign**, $\sqrt{\ }$, is used to indicate the principal square root. If $n^2 = a$, then $n = \pm\sqrt{a}$.

Examples

Find each square root.

1. $\sqrt{64}$

$\sqrt{64} = 8$ Find the positive square root of 64; $8^2 = 64$

2. $\pm\sqrt{1.21}$

$\pm\sqrt{1.21} = \pm1.1$ Find both square roots of 1.21; $1.1^2 = 1.21$

3. $-\sqrt{\frac{25}{36}}$

$-\sqrt{\frac{25}{36}} = -\frac{5}{6}$ Find the negative square root of $\frac{25}{36}$; $\left(\frac{5}{6}\right)^2 = \frac{25}{36}$

4. $\sqrt{-16}$

There is no real square root because no number times itself is equal to −16.

Got it? Do these problems to find out.

a. $\sqrt{\frac{9}{16}}$ b. $\pm\sqrt{0.81}$ c. $-\sqrt{49}$ d. $\sqrt{-100}$

a. $\frac{3}{4}$

b. ±0.9

c. −7

d. no real solution

Example

5. Solve $t^2 = 169$. Check your solution(s).

$t^2 = 169$ Write the equation.

$t = \pm\sqrt{169}$ Definition of square root

$t = 13$ and -13 **Check** $13 \cdot 13 = 169$ and $(-13)(-13) = 169$ ✓

Got it? Do these problems to find out.

e. $289 = a^2$ f. $m^2 = 0.09$ g. $y^2 = \frac{4}{25}$

e. ±17

f. ±0.3

g. $\pm\frac{2}{5}$

Roots

Key Concept

A **cube root** of a number is one of its three equal factors.

If $x^3 = y$, then x is the cube root of y.

...uch as 8, 27, and 64 are **perfect cubes** because they are ...of integers.

...2 or 2^3 $\quad 27 = 3 \cdot 3 \cdot 3$ or 3^3 $\quad 64 = 4 \cdot 4 \cdot 4$ or 4^3

...l $\sqrt[3]{\ }$ is used to indicate a cube root of a number.

...hen $n = \sqrt[3]{a}$. You can use this relationship to solve equations ...e cubes.

...les

...cube root.

... = 5 $\quad 5^3 = 5 \cdot 5 \cdot 5$ or 125

... = −3 $\quad (-3)^3 = (-3) \cdot (-3) \cdot (-3)$ or −27

Cube Roots

While $\sqrt{-16}$ is not a real number, $\sqrt[3]{-27}$ is a real number. $-3 \cdot -3 \cdot -3 = -27$

Do these problems to find out.

...9 $\quad$ i. $\sqrt[3]{-64}$ $\quad$ j. $\sqrt[3]{1{,}000}$

h. 9

i. −4

j. 10

...ample

...has a planter in the shape of a cube that holds 8 cubic ...f potting soil. Solve the equation $8 = s^3$ to find the side ...s of the container.

...$= s^3$ Write the equation

...$= s$ Take the cube root of each side.

...$= s$ Definition of cube root

...ch side of the container is 2 feet.

...$(2)^3 = 8$ ✓

Examples

6. Find cube roots.

AL • *What does the 3 represent in front of the radical sign?* cube root

• *What does it mean to find the cube root of 125?* Find the number that when used as a factor three times equals 125.

OL • *What number when used as a factor three times is equal to 125?* 5

• *How can you check your answer?* Find 5^3.

BL • *List the perfect cubes between 1 and 100.* 1, 8, 27, 64

Need Another Example?

Find $\sqrt[3]{27}$. 3

7. Find cube roots.

AL • *How is Example 6 different from Example 7?* Example 7 asks to determine the cube root of a negative number.

OL • *You cannot find the square root of a negative number. Why is it possible to find the cube root of a negative number?* Sample answer: You can use a negative number as a factor three times and the product is negative.

BL • *Do you think it is possible to find the fourth root of a negative number? Explain.* no; a positive number to the fourth power is positive and a negative number to the fourth power is also positive.

Need Another Example?

Find $\sqrt[3]{-1{,}000}$. −10

8. Use cube roots.

AL • *Why is −2 feet not a reasonable solution to the equation?* Sample answer: Since distance cannot be negative, we only use the principal square root.

OL • *How could we find the area of one side of the planter?* Because we found the side length of the planter, we can square the length to find the area of that side.

Need Another Example?

An box that is shaped like a cube has a volume of 125 cubic inches. What is the length of one side of the box? 5 in.

uided Practice

rmative Assessment Use these exercises to assess dents' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

L BL LA **Numbered Heads Together** Place students in ams. Assign each person in the team one exercise to solve. ve one minute to complete the assigned exercise. Have all the students who completed Exercise 1 compare answers, the students who completed Exercise 2 compare answers, d so on, and then report back to their own team. Each rson will verbally explain to the team how to solve their signed problem. Let students ask questions to team mbers if they are unsure how a problem was solved. MP 1, 3

/atch Out!

mmon Error Sometimes students are confused by oblems that include fractions. When students have to d the square root of a fraction such as $\sqrt{\frac{36}{49}}$, suggest that ey rewrite the fraction as $\frac{\sqrt{36}}{\sqrt{49}}$ so that they remember find the square root of both the numerator and nominator.

k. 1.5 feet

Got it? Do this problem to find out.

k. An aquarium in the shape of a cube that will hold 25 gallons of water has a volume of 3.375 cubic feet. Solve $s^3 = 3.375$ to find the length of one side of the aquarium.

Guided Practice

Find each square root. (Examples 1–4)

1. $-\sqrt{1.69} =$ **−1.3**
2. $\pm\sqrt{\frac{49}{144}} =$ **$\pm\frac{7}{12}$**
3. $\sqrt{-1.44} =$ **no real solution**

Solve each equation. Check your solution(s). (Example 5)

4. $p^2 = 36$ **6 or −6**
5. $t^2 = \frac{1}{9}$ **$\frac{1}{3}$ or $-\frac{1}{3}$**
6. $6.25 = r^2$ **2.5 or −2.5**

Find each cube root. (Examples 6 and 7)

7. $\sqrt[3]{216} =$ **6**
8. $\sqrt[3]{-125} =$ **−5**
9. $\sqrt[3]{-8} =$ **−2**

10. A cube-shaped packing box can hold 729 cubic inches of packing material. Solve $729 = s^3$ to find the length of one side of the box. (Example 8) **9 in.**

11. **Building on the Essential Question** Why would I need to use square roots and cube roots?
Sample answer: If I knew the area of a square or the volume of a cube, I could find the length of one side by finding the square root or the cube root.

Rate Yourself!

☐ I understand how to find square roots and cube roots.

Great! You're ready to move on!

☐ I still have some questions about finding square roots and cube roots.

Name ______ My Homework ______

Independent Practice

Find each square root. (Examples 1–4)

1. $\sqrt{16} =$ **4**
2. $-\sqrt{484} =$ **−22**
3. $\sqrt{-36} =$ **no real solution**
4. $\pm\sqrt{\frac{9}{49}} =$ **$\pm\frac{3}{7}$**
5. $-\sqrt{2.56} =$ **−1.6**
6. $\sqrt{-0.25} =$ **no real solution**

Solve each equation. Check your solution(s). (Example 5)

7. $v^2 = 81$ **±9**
8. $w^2 = \frac{36}{100}$ **$\pm\frac{3}{5}$**
9. $0.0169 = c^2$ **±0.13**

Find each cube root. (Examples 6 and 7)

10. $\sqrt[3]{1{,}728} =$ **12**
11. $\sqrt[3]{-0.125} =$ **−0.5**
12. $\sqrt[3]{\frac{27}{125}} =$ **$\frac{3}{5}$**

13. A group of 169 students needs to be seated in a square formation for a yearbook photo. Solve the equation $169 = s^2$ to find how many students should be in each row. (Example 8) **13 students**

14. Kamilia wants to build a storage container in the shape of a cube to hold 15.625 cubic meters of hay for her horse. Solve the equation $15.625 = s^3$ to find the length of one side of the container. (Example 8)
2.5 m

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used a homework assignment. The Extra Practice page can be for additional reinforcement or as a second-day assignm

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Lev indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of complexity levels to select appropriate exercises for yo students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–15, 17, 22–24, 43, 44
OL	On Level	1–13 odd, 15–17, 22–24, 43, 44
BL	Beyond Level	15–24, 43, 44

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	15–21, 42
2 Reason abstractly and quantitatively.	22
3 Construct viable arguments and critique the reasoning of others.	23, 24

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Tell students that the next lesson is about estimating square roots of non-perfect squares. Have them write how finding the square root of a perfect square might help them to estimate the square root of a number that is not a perfect square. See students' work.

Persevere with Problems Given the area of each square, find the perimeter.

15. Area = 121 square inches — 44 in.

16. Area = 25 square feet — 20 ft

17. Area = 36 square meters — 24 m

H.O.T. Problems Higher Order Thinking

Persevere with Problems Find each value.

18. $(\sqrt{36})^2 =$ 36

19. $\left(\sqrt{\frac{25}{81}}\right)^2 = \frac{25}{81}$

20. $(\sqrt{199})^2 =$ 199

21. $(\sqrt{x})^2 =$ x

22. **Reason Abstractly** Based on your solutions to Exercises 18–21, write a rule that could be used to simplify the square of any square root of a number.
Sample answer: The square of any square root of a number is the same as the original number.

23. **Reason Inductively** Explain why $\sqrt{-4}$ is not a real number, but $\sqrt[3]{-8}$ is.
Sample answer: There are no two equal numbers that have a product of -4, but $(-2)(-2)(-2) = -8$.

24. **Reason Inductively** Describe the difference between an exact value and an approximation when finding square roots of numbers that are not perfect squares. Give an example of each.
Sample answer: The exact value of a square root is given using the square root symbol, such as $\sqrt{13}$. An approximation is a decimal value, such as $\sqrt{13} \approx 3.6$.

Name ______ My Homework ______

Extra Practice

Find each square root.

25. $-\sqrt{81} = -9$

Homework Help: $9 \cdot 9 = 81$

So, $-\sqrt{81} = -9$.

26. $-\sqrt{\frac{64}{225}} = -\frac{8}{15}$

27. $-\sqrt{\frac{16}{25}} = -\frac{4}{5}$

28. $\pm\sqrt{1.44} = \pm 1.2$

Find each cube root.

29. $\sqrt[3]{-216} = -6$

30. $\sqrt[3]{-512} = -8$

31. $\sqrt[3]{-1{,}000} = -10$

32. $\sqrt[3]{-343} = -7$

Solve each equation. Check your solution(s).

33. $b^2 = 100$ ± 10

34. $\frac{9}{64} = c^2$ $\pm\frac{3}{8}$

35. $a^2 = 1.21$ ± 1.1

36. $\frac{1}{8} = z^3$ $\frac{1}{2}$

37. $1.331 = c^3$ 1.1

38. $m^3 = 8{,}000$ 20

39. $\sqrt{x} = 5$ 25

40. $\sqrt{y} = 20$ 400

41. $\sqrt{z} = 10.5$ 110.25

42. **Persevere with Problems** A concert crew needs to set up some chairs on the floor level. The chairs are to be placed in a square pattern consisting of four square sections. If one of the square sections holds 900 chairs, how many chairs will there be along each length of the larger square? 60 chairs

Power Up! Test Practice

xercises 43 and 44 prepare students for more rigorous inking needed for the assessment.

3.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

4.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK3
Mathematical Practice	MP1, MP8
Scoring Rubric	
2 points	Students correctly label each figure and answer the question, explaining their response.
1 point	Students correctly label each figure and answer the question but fail to explain their response OR answer the question and explain their response but fail to correctly label each figure.

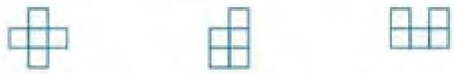
Power Up! Test Practice

43. Mr. Fahad has a square cornfield. Which of the following could be the area of the cornfield if the sides are measured in whole numbers? Select all that apply.

- ☐ 164,000 ft^2
- ☐ 156,816 ft^2 ■
- ☐ 174,724 ft^2 ■
- ☐ 215,908 ft^2

44. The area of each square in the figures below is 81 square units. Select the perimeter of each figure.

88 units	99 units
90 units	108 units
94 units	117 units



108 units — 90 units — 108 units

Do any of the figures have the same perimeter? If so, explain why.

Figures 1 and 3; Sample answer: The distance around both figures is the same.

Spiral Review

Evaluate each expression.

45. $13^3 =$ **2,197**

46. $25^2 =$ **625**

47. $15^3 =$ **3,375**

48. $34^2 =$ **1,156**

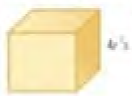
49. $5 \cdot \sqrt{121} =$ **55**

50. $-6 \cdot \sqrt{36} =$ **−36**

51. $10 \cdot \sqrt[3]{8} =$ **20**

52. $-4 \cdot \sqrt{144} =$ **−48**

Express the volume of each cube as a monomial.

53. 

$64r^6s^3$ units^3

54. 

$729m^6n^{12}$ units^3

Inquiry Lab

Roots of Non-Perfect Squares

Inquiry **HOW can you estimate the square root of a non-perfect square number?**

MP Mathematical Practices 1, 3, 4, 5

Mahdi is making a quilting piece from a square pattern as shown. Each of the dotted lines is 6 inches. What is the approximate length of one side of the square?

What do you know? **The figure is a square.**

What do you need to find? **the approximate length of one side**

Hands-On Activity

Step 1 The outline of the square on dot paper is shown. Draw dotted lines connecting opposite vertices.

When you draw the lines, four triangles that are the same shape and size are formed. What are the dimensions of the triangles?

base = **3** units height = **3** units

The area of one triangle is **4.5** square units.

The area of the square is **18** square units.

Step 2 Copy and cut out the square in Step 1 on another sheet of paper.

Step 3 Place one side of your square on the number line. Between what two consecutive whole numbers is $\sqrt{18}$, the side length of the square, located? **4 and 5**

Sample answers are given.

The side of the square is closer to which one of the two whole numbers? **4** Estimate $\sqrt{18}$. **4.2**

So, one side of the square is about **4.2** units long.

Focus narrowing the scope

Objective Estimate square roots of non-perfect squares.

Coherence connecting within and across grades

Now
Students approximate the square root of an irrational number using dot paper and number lines.

Next
Students will approximate the square root of an irrational number using p squares.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 80.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activit

Hands-On Activity

AL Give students dot paper and have them draw a square with side lengths of 4 units. Continue the activity using the given steps. Check that students understand the connectio between the area of the square and the length of the side Repeat the activity with the given square.

BL **LA** **Trade-a-Problem** Give students dot paper and have them draw their own squares of different sizes, simil the activity. Students trade papers, cut out the squares, ar use the number line to find the approximate length of the

MP 1, 4, 5

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–3	Exercises 4–7	Exercises 8, 9
Level 3			●
Level 2		●	
Level 1	●		

Analyze and Reflect

AL LA Have students draw and label a square with its area and side lengths next to a list of perfect square numbers. Have students verbally explain the connection between the area of the square and the length of the side. MP 1, 3, 4, 7

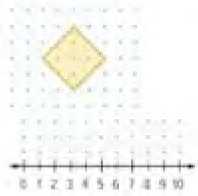
Create

Inquiry Students should be able to answer "HOW can you estimate the square root of a non-perfect square number?" Check for student understanding and provide guidance, if needed.

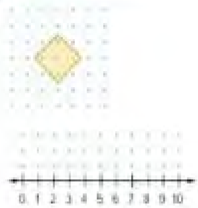
Investigate

MP **Use Math Tools** Work with a partner. Determine the two consecutive whole numbers the side length of each square is located between using the method shown in the Investigation.

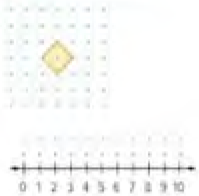
1. 2 and 3



2. 2 and 3



3. 1 and 2



Analyze and Reflect

MP **Use Math Tools** Estimate the side length of each square in Exercises 1–3. Verify your estimate by using a calculator. Sample answers are given.

4. Exercise 1
   Estimate 2.9
   Calculator 2.8

5. Exercise 2
   Estimate 2.0
   Calculator 2.1

6. Exercise 3
   Estimate 1.5
   Calculator 1.4

7. MP **Reason Inductively** How does using a square model help you find the square root of a non-perfect square? Sample answer: The side length of the square is the square root of the area of the square. Place the side on a number line and estimate the length.

Create

8. MP **Model with Mathematics** Write a real-world problem that involves estimating a square root. Use the method shown in the activity to solve your problem.
   Sample answer: The area of a square garden is 30 square feet. What is the length of a side of the garden; about 5.5 ft

9. Inquiry HOW can you estimate the square root of a non-perfect square?
   Sample answer: Determine between which two consecutive whole numbers the square root lies. Then determine which of the two numbers is closer to the square root.

Lesson 9

Estimate Roots

Real-World Link

Gravity Legend states that while sitting in his garden one day, Sir Isaac Newton was struck on the head by an apple. Suppose the apple was 25 feet above his head. Use the following steps to find how long it took the apple to fall.

1. What is the square root of 25? **5**

2. The formula $t = \frac{\sqrt{h}}{4}$ can be used to find the time t in seconds it will take an object to fall from a certain height h in feet. How long did it take the apple to fall?
   1.25 s

3. Suppose another apple was 13 feet above the ground. Use the formula to write an equation representing the time it would have taken for the apple to hit the ground.
   $t = \frac{\sqrt{13}}{4}$

4. Can you write $\frac{\sqrt{13}}{4}$ without a radical sign? Explain.
   no; Sample answer: $\sqrt{13}$ is not a perfect square so you cannot write it without the radical sign.

Essential Question

WHY is it helpful to write numbers in different ways?

Mathematical Practices
1, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Estimate square and cube roots.

Coherence connecting within and across grades

Previous Students estimated square roots of irrational numbers using dot paper.

Now Students will estimate square roots and cube roots of non-perfect square and cubes.

Next Students will compare and order real numbers.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 85.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA Team Game Play the following game to review how to find square roots of perfect squares of the numbers 1 through 10. Designate a leader to call out each of the following numbers one at a time: 100, 9, 25, 64, 4 4, 36, 81, 16. Ask students to hold up the square root of the number using their fingers. For example, if the leader calls ou 81, the students should hold up 9 fingers. MP 1, 4

Alternate Strategy

AL Have students make a perfect square chart to keep out on their desk. Tell students that they can use the square root of perfect squares to estimate the square roots of non-perfec squares, as will follow in this lesson.

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Estimate square roots.

AL
- *Identify the perfect squares that are close to 83.* Sample answer: 81 and 100
- *Between what two perfect squares is 83?* 81 and 100

OL
- *What perfect square is 83 closest to?* 81
- *What is the best integer estimate for $\sqrt{83}$? Explain.* 9; 83 is closer to 81 than to 100, so $\sqrt{83}$ is closer to $\sqrt{81}$, which is 9 than $\sqrt{100}$, which is 10.

BL
- *What is the best integer estimate for $\sqrt{84}$?* 9
  What is the best integer estimate for $\sqrt{85}$? 9
  At what point would the next successive integer be the best estimate? When the number inside the radical sign is 91 or greater, 10 would be the best estimate.

Need Another Example?
Estimate $\sqrt{54}$ to the nearest integer. 7

2. Estimate cube roots.

AL
- *Identify the perfect cubes that are close to 320.* Sample answer: 216 and 343
- *What is the smallest perfect cube greater than 320?* 343

OL
- *Between what two integers does $\sqrt[3]{320}$ lie?* 6 and 7
- *What is the best integer estimate for $\sqrt[3]{320}$? Explain.* 7; 320 is closer to 343 than to 216, so $\sqrt[3]{320}$ is closer to $\sqrt[3]{343}$ than $\sqrt[3]{216}$.

BL
- *What is the smallest perfect cube greater than 343?* $8^3 = 512$
- *What is the best integer estimate for $\sqrt[3]{510}$?* 8

Need Another Example?
Estimate $\sqrt[3]{100}$ to the nearest integer. 5

Work Zone

Estimate Square and Cube Roots

You know that $\sqrt{8}$ is not a whole number because 8 is not a perfect square.

The number line below shows that $\sqrt{8}$ is between 2 and 3. Since 8 is closer to 9 than 4, the best whole number estimate for $\sqrt{8}$ is 3.

Examples

1. Estimate $\sqrt{83}$ to the nearest integer.

- The largest perfect square less than 83 is 81. $\sqrt{81} = 9$
- The smallest perfect square greater than 83 is 100. $\sqrt{100} = 10$

Plot each square root on a number line. Then estimate $\sqrt{83}$.

$81 < 83 < 100$ Write an inequality.
$9^2 < 83 < 10^2$ $81 = 9^2$ and $100 = 10^2$
$\sqrt{9^2} < \sqrt{83} < \sqrt{10^2}$ Find the square root of each number.
$9 < \sqrt{83} < 10$ Simplify.

So, $\sqrt{83}$ is between 9 and 10. Since $\sqrt{83}$ is closer to $\sqrt{81}$ than $\sqrt{100}$, the best integer estimate for $\sqrt{83}$ is 9.

Inequalities
81 < 83 < 100 is read 81 is less than 83 which is less than 100, so 83 is between 81 and 100.

2. Estimate $\sqrt[3]{320}$ to the nearest integer.

- The largest perfect cube less than 320 is 216. $\sqrt[3]{216} = 6$
- The smallest perfect cube greater than 320 is 343. $\sqrt[3]{343} = 7$

$216 < 320 < 343$ Write an inequality.
$6^3 < 320 < 7^3$ $216 = 6^3$ and $343 = 7^3$
$\sqrt[3]{6^3} < \sqrt[3]{320} < \sqrt[3]{7^3}$ Find the cube root of each number.
$6 < \sqrt[3]{320} < 7$ Simplify.

So, $\sqrt[3]{320}$ is between 6 and 7. Since 320 is closer to 343 than 216, the best integer estimate for $\sqrt[3]{320}$ is 7.

Got it? Do these problems to find out.

a. $\sqrt{35}$ b. $\sqrt{170}$ c. $\sqrt{44.8}$

d. $\sqrt[3]{62}$ e. $\sqrt[3]{25}$ f. $\sqrt{129.6}$

a. 6

b. 13

c. 7

d. 4

e. 3

f. 5

Example

3. Marwan wants to fence in a square portion of the yard to make a play area for his new rabbit. The area covered is 2 square meters. How much fencing should Marwan buy?

$\sqrt{2}$ = 2 m² $\sqrt{2}$ =

Marwan will need $4 \cdot \sqrt{2}$ meters of fencing. The square root of 2 is between 1 and 2 so $4 \cdot \sqrt{2}$ is between 4 and 8. Is this the best approximation? You can truncate the decimal expansion of $\sqrt{2}$ to find better approximations.

Estimate $\sqrt{2}$ by truncating, or dropping, the digits after the first decimal place, then after the second decimal place, and so on until an appropriate approximation is reached.

$\sqrt{2} = 1.414213562$ Use a calculator.

$\sqrt{2} = \mathbf{1.4}14213562$ Truncate, or drop, the digits after the first decimal place. $\sqrt{2}$ is between 1.4 and 1.5.

$5.6 < 4\sqrt{2} < 6.0$ $4 \cdot 1.4 = 5.6$ and $4 \cdot 1.5 = 6.0$

To find a closer approximation, expand $\sqrt{2}$ then truncate the decimal expansion after the first two decimal places.

$\sqrt{2} = \mathbf{1.41}4213562$ $\sqrt{2}$ is between 1.41 and 1.42.

$5.64 < 4\sqrt{2} < 5.68$ $4 \cdot 1.41 = 5.64$ and $4 \cdot 1.42 = 5.68$

The approximations indicate that Wyatt should buy 6 meters of fencing.

STOP and Reflect

What is the difference between an exact value and an approximate value when finding square roots of numbers that are not perfect squares? Explain below.

Sample answer: The exact value of a nonperfect square is not rational so you cannot write the exact value in decimal notation. You use an approximation by rounding the decimal to a given place.

Got it? Do this problem to find out.

g. Marwa needs to put trim around a circular tablecloth with a diameter of 36 inches. Use the equation $C = \pi d$ to find three sets of approximations for the amount of trim she will need. Truncate the value of π to the ones, tenths, and hundredths place. Then determine how much trim she should buy.

g. **Sample answer: 108 in. and 144 in.; 111.6 in. and 115.2 in.; 113.04 in. and 113.40 in.; 114 in.**

Example

3. Estimate lengths with square roots.

AL • *Why is the side length of the play area $\sqrt{2}$ meters?* **Sample answer: To find the side length of the square, we have to take the square root of the area of the square. Because 2 is not a perfect square, we can leave the side length as $\sqrt{2}$.**

• *To find how much fencing Wyatt needs, do you need to find the side length, the area, or the perimeter?* **perimeter**

OL • *If the side length of the play area is $\sqrt{2}$ meters, what is the perimeter?* **$\sqrt{2} \times 4$ or $4\sqrt{2}$ meters**

• *What does it mean to truncate?* **Sample answer: drop digits after the first decimal place, the second decimal place, and so on until an appropriate approximation is reached.**

BL • *Why is it helpful to truncate the decimal approximation of $\sqrt{2}$?* **Sample answer: He does not want to buy too much or too little. So, truncating makes you get a closer estimate than by estimating to the nearest whole number.**

• *Do you need to find the exact amount of fencing needed to fence in the play area? Explain.* **Sample answer: no; Wyatt just needs an estimate of how much fencing he should buy.**

Need Another Example?

A square flower garden has an area of 250 square feet. A stone path runs along the outermost edge of the flower garden. Find three sets of approximations for the length of the path. Then determine the length of the path rounded to the nearest tenth. **Sample answer: 60 ft and 64 ft; 63.2 ft and 63.6 ft; 63.24 ft and 63.28 ft; 63.2 ft.**

Example

4. Estimate square roots.

AL • *How does the diagram on the student page help you to understand the expression given in the example?* **Sample answer: The shorter side is 2. The longer side is $1 + \sqrt{5}$. The expression is the longer side divided by the shorter side.**

OL • *What is an approximation for the numerator?* **about 3**

• *Explain the steps you would need to follow to estimate $\sqrt{5}$.* **Sample answer: First find the greatest perfect square less than 5 and the smallest perfect square greater than 5. You can use the square roots of the perfect squares to estimate $\sqrt{5}$ to the nearest whole number.**

BL • *Generate your own expression involving a square root and at least two other operations. Approximate the value of your expression.* **See students' work.**

Need Another Example?

To estimate the time in seconds it will take an object to fall h feet, you can use the expression $\frac{\sqrt{h}}{4}$. About how long will it take an object to fall from a height of 38 feet? **1.5 s**

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–9. Have them trade their solutions with another pair of students and discuss any differences. Students listen carefully and then respond verbally to clarify any misconceptions. MP 1, 3, 6

Example

4. The *golden rectangle* is found frequently in the nautilus shell. The length of the longer side divided by the length of the shorter side is equal to $\frac{1 + \sqrt{5}}{2}$. Estimate this value.

First estimate the value of $\sqrt{5}$.

$$4 < 5 < 9$$
$$2^2 < 5 < 3^2$$
$$\sqrt{2^2} < \sqrt{5} < \sqrt{3^2}$$
$$2 < \sqrt{5} < 3 \quad \text{Simplify.}$$

Since 5 is closer to 4 than 9, the best integer estimate for $\sqrt{5}$ is 2. Use this value to evaluate the expression.

$$\frac{1 + \sqrt{5}}{2} \approx \frac{1 + 2}{2} \text{ or } 1.5$$

Guided Practice

Estimate to the nearest integer.

1. $\sqrt{28} \approx$ **5**
2. $\sqrt{135} \approx$ **12**
3. $\sqrt{38.7} \approx$ **6**
4. $\sqrt{51} \approx$ **4**
5. $\sqrt{200} \approx$ **6**
6. $\sqrt{95} \approx$ **5**

7. **STEM** Humaid dropped a tennis ball from a height of 60 meters. The time in seconds it takes for the ball to fall 60 feet is $0.25(\sqrt{60})$. Find three sets of approximations for the amount of time it will take. Then determine how long it will take for the ball to hit the ground. **Sample answer: between 1.75 and 2 seconds, between 1.925 and 1.95 seconds, between 1.935 and 1.9375 seconds; about 2 seconds**

8. The number of swings back and forth of a pendulum of length L in inches each minute is $\frac{375}{\sqrt{L}}$. About how many swings will a 40-inch pendulum make each minute? **62.5 swings**

9. **Building on the Essential Question** How can I estimate the square root of a non-perfect square? **Sample answer: Find the perfect squares less than the number and the perfect square greater than the number. Determine which one the original number is closer to and take the square root of that number.**

Rate Yourself!

How confident are you about finding the square root of a nonperfect square? Mark an X in the section that applies.

Name ______________________ My Homework ______________________

Independent Practice

Estimate to the nearest integer. (Examples 1 and 2)

1. $\sqrt{23} \approx$ **5**
2. $\sqrt{197} \approx$ **14**
3. $\sqrt{15.6} \approx$ **4**
4. $\sqrt{85.1} \approx$ **9**
5. $\sqrt[3]{22} \approx$ **3**
6. $\sqrt[3]{34} \approx$ **3**
7. $\sqrt[3]{989} \approx$ **10**
8. $\sqrt[3]{250} \approx$ **6**

9. The area of Majidah's square garden is 345 square feet. One side of the garden is next to a shed. She wants to put a fence around the other three sides of the garden. Find three sets of approximations for the amount of fence it will take. Then determine how much fence she should buy.
(Example 3) **Sample answer: 54 ft and 57 ft; 55.5 ft and 55.8 feet; 55.71 ft and 55.74 feet; 56 feet**

10. In Little League, the bases are squares with sides of 14 inches. The expression $\sqrt{(s^2 + s^2)}$ represents the distance *diagonally across* a square of side length s. Estimate the diagonal distance across a base to the nearest inch. (Example 4) **20 in.**

11. **STEM** The formula $t = \frac{\sqrt{h}}{4}$ represents the time t in seconds that it takes an object to fall from a height of h feet. If a rock falls from a height of 125 feet, estimate how long it will take to reach the ground. (Example 4)
about 2.75 seconds

Order each set of numbers from least to greatest.

12. $\{7, 9, \sqrt{50}, \sqrt{85}\}$ **7, $\sqrt{50}$, 9, $\sqrt{85}$**

13. $\{\sqrt[3]{105}, 7, 5, \sqrt{38}\}$ **$\sqrt[3]{105}$, 5, $\sqrt{38}$, 7**

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

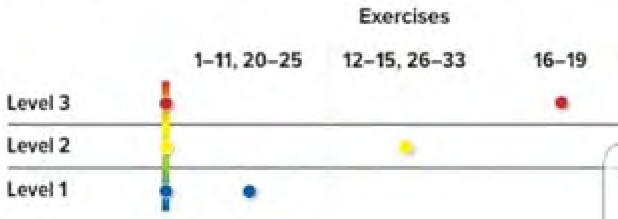
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–11, 13, 15, 17–19, 32, 33
OL	On Level	1–11 odd, 12, 13, 15, 17–19, 32, 33
BL	Beyond Level	12–19, 32, 33

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	14, 16
3 Construct viable arguments and critique the reasoning of others.	17–19
5 Use appropriate tools strategically.	31

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write about how finding the square or cube root of a perfect square or cube helped them to estimate the square or cube root of a number that is not a perfect square or cube. **See students' work.**

Watch Out!

Common Error In Exercise 17, Jasmine found the number that when multiplied by 2 equals 200. She should have recognized that $\sqrt{200}$ is between $\sqrt{196}$ and $\sqrt{225}$, so $\sqrt{200}$ is between 14 and 15.

14. **Persevere with Problems** Amal purchased a storage cube that has a volume of 4 cubic feet. She wants to put it on a bookshelf that is 12 inches tall. Will the cube fit? Explain. **No: 12 inches = 1 foot and the cube root of 4 > 1.**

15. Without a calculator, determine which is greater, $\sqrt{94}$ or 10. Explain your reasoning. **10; Since 94 is less than 100, $\sqrt{94}$ is less than 10.**

H.O.T. Problems Higher Order Thinking

16. **Persevere with Problems** Find two numbers that have square roots between 7 and 8. One number should have a square root closer to 7 and the other number should have a square root closer to 8. Justify your answer.
Sample answers: 50; 60. Since $49 < 50 < 64$ and 50 is closer to 49 than to 64, $\sqrt{50}$ is closer to 7 than to 8. Since $49 < 60 < 64$ and 60 is closer to 64 than to 49, $\sqrt{60}$ is closer to 8 than to 7.

17. **Find the Error** Yasmine is estimating $\sqrt{200}$. Find her mistake and correct it. **She incorrectly estimated. She found half of 200, not the square root. Since $196 < 200 < 225$, the square root of 200 is between 14 and 15. Since 200 is closer to 196, the square root of 200 is about 14.**

18. **Construct an Argument** If $x^4 = y$, then x is the fourth root of y. Explain how to estimate the fourth root of 30. Find the fourth root of 30 to the nearest whole number.
Since $16 < 30 < 81$, the fourth root is between 2 and 3. Since 30 is closer to 16 than 81, the fourth root of 30 is about 2.

19. **Reason Inductively** Suppose x is a number between 1 and 10 and y is a number between 10 and 20. Determine whether the statement below is always, sometimes or never true. Explain your reasoning.

$$\sqrt{x} > \sqrt[3]{y}$$

sometimes; Sample answer: $\sqrt{9}$ is greater than $\sqrt[3]{18}$, but $\sqrt{4}$ is less than $\sqrt[3]{18}$.

Name ______________________ My Homework ______________________

Extra Practice

Estimate to the nearest integer.

20. $\sqrt{44} \approx$ 7

Homework Help

$36 < 44 < 49$
$6^2 < 44 < 7^2$
$\sqrt{6^2} < \sqrt{44} < \sqrt{7^2}$
$\sqrt{44}$ is closer to $\sqrt{49}$ or 7.

21. $\sqrt[3]{199} \approx$ 6

$125 < 199 < 216$
$5^3 < \sqrt[3]{199} < 6^3$
$\sqrt{5^3} < \sqrt[3]{199} < \sqrt{6^3}$
$\sqrt[3]{199}$ is closer to $\sqrt{216}$ or 6.

22. $\sqrt{125} \approx$ **11**

23. $\sqrt{23.5} \approx$ **5**

24. $\sqrt[3]{59} \approx$ **4**

25. $\sqrt[3]{430} \approx$ **8**

Estimate the solution of each equation to the nearest integer.

26. $y^2 = 55$ **7 or −7**

27. $d^2 = 95$ **10 or −10**

28. $p^2 = 6.8$ **3 or −3**

The volume of each cube is given. Estimate the side length of the cube to the nearest integer. Use the formula $V = s^3$.

29. **6 in.**

30. **8 cm**

31. **MP Use Math Tools** Abdullah is buying a bag of grass seed. The two-pound bag will cover 1,000 square feet of lawn. Estimate the side length of the largest square Abdullah could seed if he purchases 5 bags.

70 feet on each side

Power Up! Test Practice

Exercises 32 and 33 prepare students for more rigorous thinking needed for the assessment.

32.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practice	MP1, MP8
Scoring Rubric	
3 points	Students correctly answer each part of the question.
2 points	Students correctly model and find the radius for two pools.
1 point	Students correctly model and find the radius of one pool OR students find the radius of all three pools but fail to correctly model.

33.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

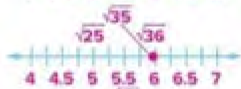
32. The radius of a circle with area A can be approximated using the formula $r = \sqrt{\frac{A}{3}}$. Use a number line to estimate the radius of each circular swimming pool to the nearest integer.

Pool 1: $A = 240$ ft^2

$r \approx$ 9 ft

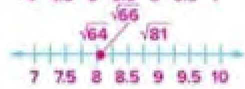
Pool 2: $A = 105$ ft^2

$r \approx$ 6 ft



Pool 3: $A = 198$ ft^2

$r \approx$ 8 ft



33. After an accident, officials use the formula $s = \sqrt{24m}$ to estimate the speed a car was traveling based on the length of the car's skid marks. In the formula, s represents the speed in miles per hour and m is the length of the skid marks in feet. If a car leaves a skid mark of 50 feet, what was its approximate speed?

35 mph

Spiral Review

Write each of the following as a fraction in simplest form.

34. $-36 = \frac{-36}{1}$ **35.** $1.7 = \frac{17}{10}$ **36.** $-0.048 = \frac{-6}{125}$ **37.** $98\% = \frac{49}{50}$

38. Of the 150 students in Mr. Majid's classes, 16% play soccer, $\frac{9}{25}$ play basketball, 3^3 play football and 14 do not play a sport at all. Write the number of students in order from least to greatest.

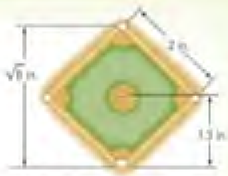
14, 24, 27, 54

Lesson 10

Compare Real Numbers

Real-World Link

Sports Major League baseball has rules for the dimensions of the baseball diamond. A model of the diamond is shown.



1. On the model, the distance from the pitching mound to home plate is 1.3 inches. Is 1.3 a rational number? Explain.
   Yes; it can be written as $\frac{13}{10}$.

2. On the model, the distance from first base to second base is 2 inches. Is 2 a rational number? Explain.
   Yes; it can be written as $\frac{2}{1}$.

3. The distance from home plate to second base is $\sqrt{8}$ inches. Using a calculator, find $\sqrt{8}$. Does it appear to terminate or repeat?
   2.828427125; Sample answer: It does not repeat. It appears to terminate.

4. To determine if the number terminates, on your calculator, multiply your answer to $\sqrt{8}$ by itself. Do not use the x^2 button.
   Is the answer 8? **no**

5. Based on your results, can you classify $\sqrt{8}$ as a rational number? Explain.
   No; it is not a repeating decimal.

Which MP **Mathematical Practices** did you use?
Shade the circle(s) that applies.

1 Persevere with Problems
2 Reason Abstractly
3 Construct an Argument
4 Model with Mathematics
5 Use Math Tools
6 Attend to Precision
7 Make Use of Structure
8 Use Repeated Reasoning

Essential Question

WHY is it helpful to write numbers in different ways?

Vocabulary

irrational number
real number

MP Mathematical Practices
1, 3, 4, 6

Focus narrowing the scope

Objective Compare mathematical expressions.

Coherence connecting within and across grades

Previous
Students estimated roots of non-perfect squares and cubes.

Now
Students compare and order real numbers.

Next
Students will identify and classify irrational and complex numbers.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 93.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Pairs Discussion** Have students work in pairs. First give each member of the pair two index cards and have them write the definition of rational numbers and give some examples on one card. Then ask them to write a definition of irrational numbers based on their definition of rational numbers. Have them share their ideas with their partner, and then verify their definitions by using their text.
MP 1, 2, 6

Alternate Strategy

AL Hang five pieces of chart paper around the room. Label each one with a kind of number (Natural, Whole, Integer, Rational, and Irrational). Hand out sticky notes with one number on each note. Have students place sticky notes on appropriate chart paper. MP 1, 5

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Classify real numbers.

- AL • *What kind of decimal number is this?* **repeating**
- • *Is it a rational number?* **yes**
- OL • *How can the number be written as a ratio between two integers?* $\frac{25}{99}$
- BL • *Are all repeating decimals rational numbers? Explain.* **Yes; a repeating decimal is any number that can be written as a ratio of two integers.**

Need Another Example?
Name all sets of numbers to which 0.0909... belongs. **rational**

2. Classify real numbers.

- AL • *What is the value of $\sqrt{36}$?* **6**
- • *Is it a rational number?* **yes**
- OL • *How can 6 be written as a ratio of two integers?* $\frac{6}{1}$
- BL • *Give an example of a rational number.* **See students' work.**

Need Another Example?
Name all sets of numbers to which $\sqrt{25}$ belongs. **natural, whole, integer, rational**

3. Classify real numbers.

- AL • *Use a calculator. What is the value of $-\sqrt{7}$?* **−2.645751311...**
- • *Does the decimal repeat or terminate?* **no**
- OL • *Can the number be written as a fraction? Is the number rational?* **no; no**
- BL • *Give another reason why this number is irrational.* **Sample answer: It cannot be written as a fraction.**

Need Another Example?
Name all sets of numbers to which $-\sqrt{12}$ belongs. **irrational**

Key Concept

Real Numbers

	Rational Number	Irrational Number
Words	A rational number is a number that can be expressed as the ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$.	An **irrational number** is a number that cannot be expressed as the ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$.
Examples	$-2, 5, 3.\overline{76}, -12\frac{7}{8}$	$\sqrt{2} = 1.414213562...$

Work Zone

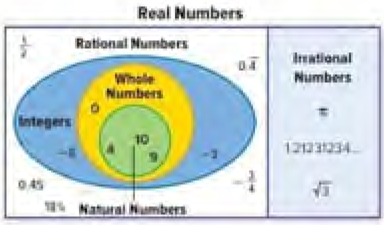
STOP and Reflect

Explain below how you know that $\sqrt{2}$ is an irrational number.

Sample answer: When you use a calculator, the decimal does not terminate nor repeat so $\sqrt{2}$ is not a rational number.

Numbers that are not rational are called irrational numbers. The square root of any number that is not a perfect square number is irrational. The set of rational numbers and the set of irrational numbers together make up the set of **real numbers**. Study the Venn diagram below.

Real Numbers



Examples

Name all sets of numbers to which each real number belongs.

1. 0.2525... The decimal ends in a repeating pattern. It is a rational number because it is equivalent to $\frac{25}{99}$.

2. $\sqrt{36}$ Since $\sqrt{36} = 6$, it is a natural number, a whole number, an integer, and a rational number.

3. $-\sqrt{7}$ $-\sqrt{7} \approx -2.64575131...$ The decimal does not terminate nor repeat, so it is an irrational number.

Got it? Do these problems to find out.

a. $\sqrt{10}$ b. $-2\frac{2}{5}$ c. $\sqrt{100}$

a. **irrational**

b. **rational**

c. **natural, whole, integer, rational**

The Number System

Compare and Order Real Numbers

You can compare and order real numbers by writing them in the same notation. Write the numbers in decimal notation before comparing or ordering them.

Examples

Fill in each ◯ with <, >, or = to make a true statement.

4. $\sqrt{7}$ ◯ $2\frac{2}{3}$

$\sqrt{7} = 2.64575131\ldots$

$2\frac{2}{3} = 2.666666666\ldots$

Since 2.64575131... is less than 2.66666666..., $\sqrt{7} < 2\frac{2}{3}$.

5. 15.7% ◯ $\sqrt{0.02}$

15.7% = 0.157

$\sqrt{0.02} \approx 0.141$

Since 0.157 is greater than 0.141, 15.7% > $\sqrt{0.02}$.

6. **Order the set $\{\sqrt{30}, 6, 5\frac{4}{5}, 5.\overline{36}\}$ from least to greatest. Verify your answer by graphing on a number line.**

Write each number as a decimal. Then order the decimals.

$\sqrt{30} \approx 5.48$

$6 = 6.00$

$5\frac{4}{5} = 5.80$

$5.\overline{36} \approx 5.37$

From least to greatest, the order is $5.\overline{36}$, $\sqrt{30}$, $5\frac{4}{5}$, and 6.

Got it? **Do these problems to find out.**

d. $\sqrt{11}$ (<) $3\frac{1}{3}$ e. $\sqrt{17}$ (>) 4.03 f. $\sqrt{6.25}$ (=) 250%

g. Order the set $\{-7, -\sqrt{60}, -7\frac{7}{10}, -\frac{66}{9}\}$ from least to greatest. Verify your answer by graphing on the number line below.

g. $-\sqrt{60}, -7\frac{7}{10}, -\frac{66}{9}, -7$

Examples

4. Compare real numbers.

AL • *In what form could both numbers be expressed to make it easier to compare?* **Sample answer: decimal**

OL • *How does using the number line help you compare numbers?* **You can plot the numbers and visualize them.**

BL • *After writing both numbers as decimals, how can you quickly determine which one is greater?* **The largest place value that has a different number is the hundredths place. 6 > 4, so 2.66 > 2.64.**

Need Another Example?

Is $\sqrt{15}$ <, >, or = $3\frac{7}{8}$? **<**

5. Compare real numbers.

AL • *What is 15.7% written as a decimal?* **0.157**

OL • *To what place value would you round $\sqrt{0.02}$? Explain.* **Thousandths; 15.7% written as a decimal stops at the thousandths place.**

BL • *Write your own comparison problem involving at least one irrational number.* **See students' work.**

Need Another Example?

Is 12.3% <, >, or = $\sqrt{0.01}$? **>**

6. Order real numbers.

AL • *In what form could we express each number?* **decimal**

• *To what place value do you need to round each of the numbers?* **hundredths**

OL • *How does the number line help order the numbers?* **It provides a visual reference that can verify the solution.**

BL • *How could you use mental math to order at least three of the four numbers?* **Sample answer: I know that $5\frac{4}{5}$ is 5.8 so I could order 6, $5\frac{4}{5}$, and $5.\overline{36}$.**

Need Another Example?

Order the set $\{\sqrt{15}, 3, 4\frac{1}{2}, 4.\overline{21}\}$ from least to greatest. Verify your answer by graphing on a number line. **3, $\sqrt{15}$, $4.\overline{21}$, $4\frac{1}{2}$**

Example

7. Use real numbers.

- AL • *To determine how much farther Kia can see what operation will be used?* subtraction
- OL • *What expression can be used to find the distance that Frida can see?* $1.23 \cdot \sqrt{1{,}250}$
  - • *What expression can be used to find the distance that Kia can see?* $1.23 \cdot \sqrt{1{,}362}$
- BL • *About how far could someone see to the horizon from the top of the Burj Khalifa skyscraper in Abu Dhabi, at a height of 2,723 feet?* 64.2 mi
  - • *About how far could someone see to the horizon if they were travelling in an airplane at an altitude of 35,000 feet?* 230.1 mi

Need Another Example?

The time in seconds that it takes an object to fall d feet is $\frac{\sqrt{d}}{4}$. About how many seconds would it take for a volleyball thrown 32 feet up in the air to fall from its highest point to the sand? 1.4 s

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Consult** Have students draw and color-code their own real numbers Venn diagram. Have them write at least 3 examples in each set. Then have them trade diagrams to check the accuracy of each other's work. MP 1, 4, 7

BL LA **Trade-a-Problem** Have students create their own real-world problem, similar to Example 7. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. MP 1, 3, 4

Example

7. **On a clear day, the number of miles a person can see to the horizon is about 1.23 times the square root of his or her distance from the ground in feet. Suppose Farida is at the Empire State Building observation deck at 1,250 feet and Fayza is at the Freedom Tower observation deck at 1,362 feet. How much farther can Fayza see than Farida?**

Use a calculator to approximate the distance each person can see.

Farida: $1.23 \cdot \sqrt{1{,}250} = 43.49$ Fayza: $1.23 \cdot \sqrt{1{,}362} = 45.39$

Fayza can see 45.39 − 43.49 or 1.90 miles farther than Farida.

Guided Practice

Name all sets of numbers to which each real number belongs.

1. 0.050505... rational
2. $-\sqrt{64}$ integer, rational
3. $\sqrt{17}$ irrational

Fill in each ◯ with <, >, or = to make a true statement.

4. $\sqrt{15}$ (>) 3.5
5. $\sqrt{2.25}$ (=) 150%
6. $\sqrt{6.2}$ (>) $2.\overline{4}$

7. Order the set $\{\sqrt{5}, 220\%, 2.25, 2.\overline{2}\}$ from least to greatest. Verify your answer by graphing on a number line.
$220\%, 2.\overline{2}, \sqrt{5}, 2.25$

8. The formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ can be used to find the area A of a triangle. The variables a, b, and c are the side measures and s is one half the perimeter. Use the formula to find the area of a triangle with side lengths of 7 centimeters, 9 centimeters, and 10 centimeters. about 30.6 cm^2

9. **Building on the Essential Question** How are real numbers different from irrational numbers?
Sample answer: Real numbers contain the sets of rational and irrational numbers. So all irrational numbers are real numbers but not all real numbers are irrational numbers.

Rate Yourself!

How well do you understand real numbers? Circle the image that applies.

Clear Somewhat Clear Not So Clear

Name ________________ My Homework ________________

Independent Practice

Name all sets of numbers to which each real number belongs. (Examples 1–3)

1. $\frac{2}{3}$ rational
2. $-\sqrt{20}$ irrational
3. $7.\overline{2}$ rational
4. $\frac{12}{4}$ natural, whole, integer, rational

Fill in each ◯ with <, >, or = to make a true statement. (Examples 4 and 5)

5. $\sqrt{10}$ (<) 3.2
6. $5\frac{1}{6}$ (=) $5.1\overline{6}$
7. $2.\overline{21}$ (<) $\sqrt{5.2}$

Order each set of numbers from least to greatest. Verify your answer by graphing on a number line. (Example 6)

8. $\{-415\%, -\sqrt{17}, -4.\overline{1}, -4.08\}$

−4.18 −4.16 −4.14 −4.12 −4.1 −4.08

$-415\%, -\sqrt{17}, -4.\overline{1}, -4.08$

9. $\{\sqrt{5}, \sqrt{6}, 2.5, 2.55, \frac{7}{3}\}$

2.1 2.2 2.3 2.4 2.5 2.6 2.7

$\sqrt{5}, \frac{7}{3}, \sqrt{6}, 2.5, 2.55$

10. The equation $s = \sqrt{30fd}$ can be used to find a car's speed s in miles per hour given the length d in feet of a skid mark and the friction factor f of the road. Police measured a skid mark of 90 feet on a dry concrete road. If the speed limit is 35 mph, was the car speeding? Explain. (Example 5)

Friction Factor		
Road	Concrete	Tar
Wet	0.4	0.5
Dry	0.8	1.0

Yes; $\sqrt{30 \times 0.8 \times 90} \approx 46$, so the car was speeding.

11. The surface area in square meters of the human body can be found using the expression $\sqrt{\frac{hm}{3{,}600}}$ where h is the height in centimeters and m is the mass in kilograms. Find the surface area of a 15-year-old boy with a height of 183 centimeters and a mass of 74 kilograms. (Example 7)

about 1.9 m^2

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–11, 21–29	12–15, 30–35	16–20
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–11, 13, 15, 16, 20, 34, 35
OL	On Level	1–11 odd, 12–16, 20, 34, 35
BL	Beyond Level	12–20, 34, 35

Watch Out!

Common Error If students have trouble comparing numbers, have them rewrite all the numbers as decimals before graphing on a number line.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	17–19, 30
3 Construct viable arguments and critique the reasoning of others.	16
4 Model with Mathematics.	20
6 Attend to precision.	12

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write one example of each of the following: a whole number, an integer that is not a whole number, a rational number that is not an integer, and an irrational number. **Sample answer: 8; −7; $\frac{1}{2}$; $\sqrt{75}$**

12. **Be Precise** Write a brief description and give an example of each type of number in the graphic organizer shown. **Sample answer:**

natural	whole	integer	rational	irrational
the counting numbers; 1, 2, 3	the counting numbers and 0; 0	the whole numbers and their opposites; -2	integers, all +/− fractions and repeating decimals; −1.2	decimals that do not repeat; $\sqrt{35}$

Use estimation to fill in each ◯ with <, >, or = to make a true statement.

13. 3π (>) $\sqrt{78}$

14. π^2 (<) $3 \cdot \sqrt{15}$

15. $\sqrt{980}$ (<) $4\pi^2$

H.O.T. Problems Higher Order Thinking

16. **Use a Counterexample** Give a counterexample for the statement *All square roots are irrational numbers.* Explain your reasoning.
Sample answer: $\sqrt{4}$; $\sqrt{4} = 2$ and 2 is a rational number

Persevere with Problems Tell whether the following statements are *always*, *sometimes*, or *never* true. If a statement is not always true, explain.

17. Integers are rational numbers. **always**

18. Rational numbers are integers. **sometimes; 3 or $\frac{3}{1}$ is a rational number and an integer, but $\frac{2}{3}$ is a rational number and not an integer.**

19. The product of a non-zero rational number and an irrational number is irrational. **always**

20. **Model with Mathematics** Identify two numbers, one rational number and one irrational number, that are between 1.4 and 1.6. Include the decimal approximation of the irrational number to the nearest hundredth.
Sample answer: 1.5; $\sqrt{2} \approx 1.41$

Name ______________ My Homework ______________

Extra Practice

21. Name all sets of real numbers to which $\sqrt{10}$ belongs. irrational

Homework Help

$\sqrt{10} \approx 3.16227766...$ Since the decimal does not terminate nor repeat, it is an irrational number.

22. Fill in ◯ with <, >, or = to make $5.1\overline{5}$ (>) $\sqrt{26}$ a true statement.

Write each number as a decimal.

$5.1\overline{5} = 5.155555...$

$\sqrt{26} \approx 5.099019...$

Since 5.155555... is greater than 5.099019..., $5.1\overline{5} > \sqrt{26}$.

Name all sets of numbers to which each real number belongs.

23. 14

natural, whole, integer, rational

24. $-\sqrt{16}$

integer, rational

25. $-\sqrt[3]{90}$

irrational

Fill in each ◯ with <, >, or = to make a true statement.

26. $\sqrt{12}$ (<) 3.5

27. $6\frac{1}{3}$ (>) $\sqrt[3]{240}$

28. 240% (=) $\sqrt{5.76}$

29. About how much greater is the perimeter of a square with area 250 square meters than a square with an area of 125 square meters?

18.52 m

30. MP **Persevere with Problems** In the sequence 4, 12, ■, 108, 324, the missing number can be found by simplifying $\sqrt{ab}$ where a and b are the numbers on either side of the missing number. Find the missing number.

36

Fill in each ◯ with <, >, or = to make a true statement.

31. $3 + \sqrt{7}$ (<) 6

32. $4 - \sqrt{10}$ (<) $\sqrt{2}$

33. 13 (>) $8 + \sqrt{20}$

Power Up! Test Practice

Exercises 34 and 35 prepare students for more rigorous thinking needed for the assessment.

34.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP7
Scoring Rubric	
2 points	Students correctly plot and label the four points and determine who earned the most points on the test.
1 point	Students correctly plot and label the four points but fail to indicate who earned the most points on the test OR students correctly plot and label three of the four points and may or may not determine who earned the most points on the test.

35.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

34. Mr. Jasim gave his students a test that was worth 100 points. Rashid earned an 84%, Jalal earned $\frac{5}{6}$ of the total points, Hasan earned $\sqrt{7225}$ points, and Saeed earned $\frac{83}{100}$ points. Plot points on the number line to represent each students' score.

Number of Points

Which student earned the most points?

Hasan

35. The diagonal of a rectangular room is $\sqrt{289}$ feet long. To which sets of numbers does $\sqrt{289}$ belong? Select all that apply.

- [x] real
- [x] rational
- [x] whole
- [x] integer
- [ ] irrational
- [x] natural

Spiral Review

36. Order the set $\{7, \sqrt{53}, \sqrt{32}, 6\}$ from least to greatest. $\sqrt{32}, 6, 7, \sqrt{53}$

Solve each equation.

37. $r^2 = 25$ 5 or −5

38. $y^2 = \frac{1}{49}$ $\frac{1}{7}$ or $-\frac{1}{7}$

39. $0.64 = a^2$ 0.8 or −0.8

Evaluate each expression. Express the result in scientific notation.

40. $(7.2 \times 10^4)(1.1 \times 10^{-6}) =$ 7.92×10^{-2}

41. $(3.6 \times 10^3) + (5.7 \times 10^5) =$ 5.736×10^5

42. The table shows the approximate population of several countries. Order the countries from the greatest population to the least population.

China, India, United States, Indonesia

Country	Population
China	1.3×10^9
India	1.2×10^9
Indonesia	2.3×10^8
United States	3.1×10^8

21ST CENTURY CAREER in Engineering

Robotics Engineer

Are you mechanically inclined? Do you like to find new ways to solve problems? If so, a career as a robotics engineer is something you should consider. Robotics engineers design and build robots to perform tasks that are difficult, dangerous, or tedious for humans. For example, a robotic insect was developed based on a real insect. Its purpose was to travel over water surfaces, take measurements, and monitor water quality.

Is This the Career for You?

Are you interested in a career as a robotics engineer? Take some of the following courses in high school.

- Calculus
- Electro-Mechanical Systems
- Fundamentals of Robotics
- Physics

Turn the page to find out how math relates to a career in Engineering.

Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace.

This lesson emphasizes **MP Mathematical Practice 4** Model with Mathematics.

Coherence connecting within and across grades

Previous
Students performed calculations using exponents and scientific notation.

Now
Students apply the content standard to solve problems in the workplace.

Rigor pursuing concepts, fluency, and applications

See the Career Project on page 98.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ask students to read the information on the student page about robotics engineers and answer the following questions.

Ask:

- *What does a robotics engineer do?* **designs and builds robots that perform tasks for humans**
- *What kinds of classes should you take if you want to be a robotics engineer?* **Calculus, Electro-Mechanical Systems, Fundamentals of Robotics, Physics**

2 Collaborate

AL LA **Think-Pair-Share** Have students work in pairs to complete Exercises 2–3 using the following scaffolded questions. Sample answers shown are for Exercise 2. MP 1, 6, 7

Ask:

- *What notation does the table use?* **standard form**
- *What power of 10 do you need to multiply the measurement by to write it in scientific notation? Explain.* **−2; Sample answer: In standard form, the length is less than one, so the power of ten is negative. The decimal needs to be moved two placed to the right, so the power is 10^{-2}.**

BL LA **Trade-a-Problem** Have students work in pairs to create their own real-world problem comparing an attribute of the robot insect to a living insect, similar to Exercise 5. Then have them trade their problems with each other to solve each other's problem. MP 1, 2, 4

Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

Career Facts

Cars are manufactured and surgeries are performed using robotics. Other industries that use robotics include aerospace, life sciences, and pharmaceuticals. Some robotic systems include robotic manipulators, robotic hands, mobile robots, walking robots, and aids for disabled persons.

MP Relying on Robots

Use the information in the table to solve each problem.

1. Write the mass of the robot in standard form. **0.00035 kg**
2. Write the length of the robot in scientific notation. **9×10^{-2} m**
3. Write the leg diameter of the robot in scientific notation. **2×10^{-1} mm**
4. What is the mass in milligrams? Write in standard form. **350 mg**
5. Real insects called water striders can travel 8.3 times faster than the robot. Write the speed of water striders in scientific notation. **1.494×10^{3} mm/s**

Robotic Insect Characteristics	
Mass	3.5×10^{-4} kg
Length	0.09 m
Leg Diameter	0.2 mm
Speed	180 mm/s

MP Career Project

It's time to update your career portfolio! Investigate the education and training requirements for a career in robotics engineering.

What skills would y[ou] need to improve to succeed in this car[eer]

-
-
-
-
-

Chapter Review

Vocabulary Check

Complete the crossword puzzle using the vocabulary list at the beginning of the chapter.

Across

5. a rational number whose cube root is a whole number
8. a number, a variable, or a product of a number and one or more variables
9. numbers that can be written as a comparison of two integers, expressed as a fraction

Down

1. the symbol used to indicate a positive square root
2. a product of repeated factors using a base and exponent
3. this tells how many times a number is used as a factor
4. a rational number whose square root is a whole number
6. in a power, the number that is the common factor
7. one of a number's three equal factors

Vocabulary Check

LA **Numbered Heads Together** Assign students to 3- or 4-person learning teams. Each member is assigned a number from 1 to 4. Each team completes the Vocabulary Check, making sure every team member understands the terms and their definitions. Call on a specific number from one team to present the group's solution to the class. MP 1, 6

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- base (Lesson 2)
- cube root (Lesson 8)
- exponent (Lesson 2)
- monomial (Lesson 3)
- perfect cube (Lesson 8)
- perfect square (Lesson 8)
- power (Lesson 2)
- radical sign (Lesson 8)
- rational number (Lesson 1)

Key Concept Check

FOLDABLES LA A completed Foldable for this chapter should include a review of exponents and scientific notation.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

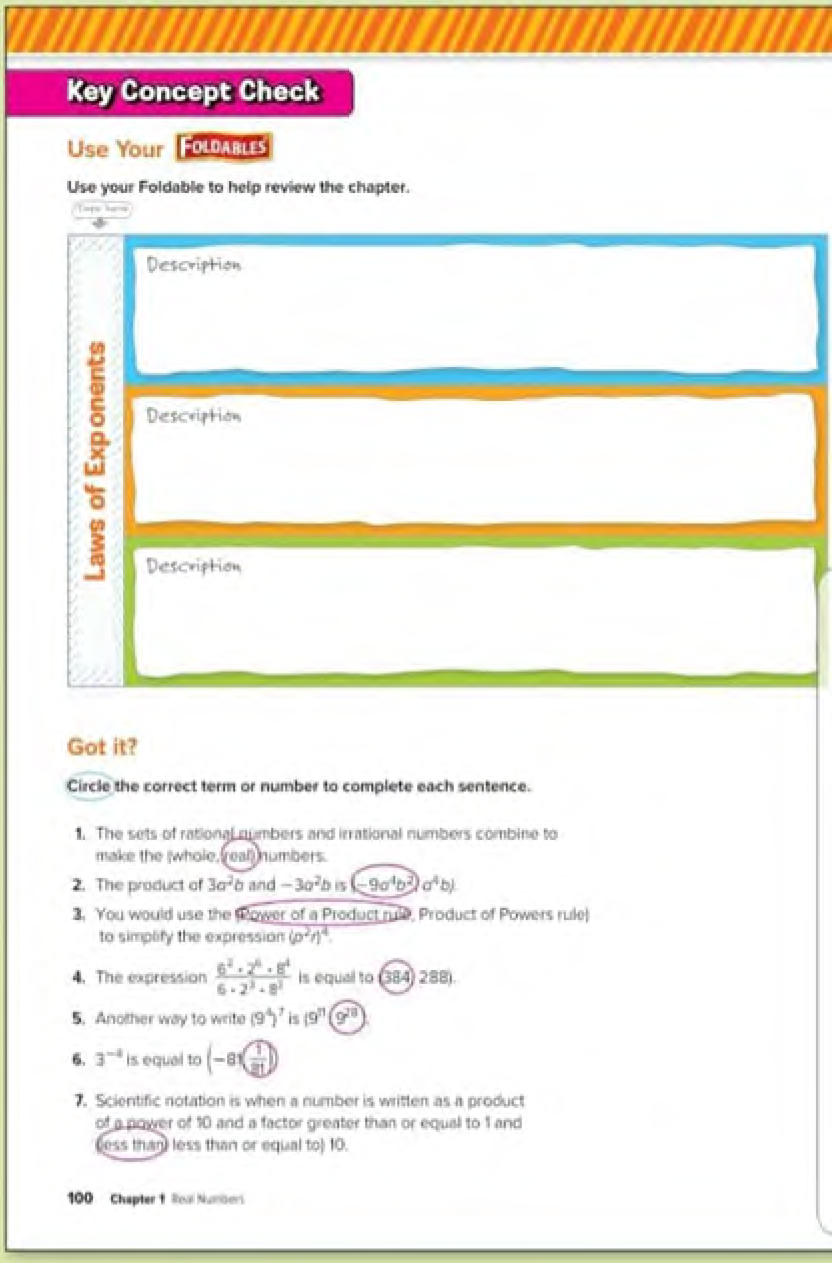
Ideas for Use

LA Have students work in pairs to discuss their Foldables. Have them practice speaking in a collaborative setting by sharing how they have completed their Foldable thus far and how they could finish it. Have each student complete their Foldable and trade with their partner to discuss any similarities and differences. MP 1, 3, 4, 5

Got It?

If students have trouble with Exercises 1–7, they may need help with the following concept(s).

Concept	Exercise(s)
real numbers (Lesson 10)	1
Product of Powers (Lesson 3)	2 , 4
Power of a Power (Lesson 4)	3, 5
negative exponents (Lesson 5)	6
scientific notation (Lesson 6)	7


Key Concept Check

Use Your FOLDABLES

Use your Foldable to help review the chapter.

Laws of Exponents

Description

Description

Description

Got it?

Circle the correct term or number to complete each sentence.

1. The sets of rational numbers and irrational numbers combine to make the (whole, real) numbers.
2. The product of $3a^2b$ and $-3a^2b$ is ($-9a^4b^2$, a^4b).
3. You would use the (Power of a Product rule, Product of Powers rule) to simplify the expression $(p^2r)^4$.
4. The expression $\frac{6^2 \cdot 2^6 \cdot 8^4}{6 \cdot 2^3 \cdot 8^3}$ is equal to (384, 288).
5. Another way to write $(9^4)^7$ is (9^{11}, 9^{28}).
6. 3^{-4} is equal to (-81, $\frac{1}{81}$).
7. Scientific notation is when a number is written as a product of a power of 10 and a factor greater than or equal to 1 and (less than, less than or equal to) 10.

100 Chapter 1 Real Numbers

Power Up! Performance Task

Planetary Play

The table shows some of the planets' approximate distances from the sun.

Planet	Approximate Distance from the Sun (km)
Mercury	5.8×10^7
Venus	1.1×10^8
Earth	1.5×10^8
Mars	2.3×10^8
Jupiter	7.8×10^8
Saturn	1.4×10^9
Neptune	4.5×10^9

Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A
How much farther is Earth from the Sun than it is from Venus? How many times further is Neptune from the Sun than Mercury is from the Sun? Explain.

Part B
It takes Mercury approximately 2.4×10^{-1} Earth years to orbit the Sun. Write this in standard form and in fraction form.

Part C
The volume of Saturn is 8.27×10^{14}. This is about 766 times the volume of Earth. What is the approximate volume of Earth? Show your work.

Part D
In her science class, Nadia needs to make a model showing the planets' relative distance from the Sun. She wants to use centimeter grid paper to create her model. If 1 centimeter on the model represents 100,000,000 kilometers, write a proportion Nadia could use to create her model on the grid paper. Use the proportion to find the distance for each planet on the model. Then use a ruler and centimeter grid paper to draw a model similar to Nadia's.

Power Up! Performance Task

This Performance-Based Assessment requires students to solve multi-step problems through abstract reasoning, precision, and perseverance. This practice scenario can be used to help students prepare for the thinking skills that will used on the Assessment.

A complete scoring rubric with answers to the Exercises can be found at the back of the book.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- How can you determine if a number is a rational number? (p. 10)
- How can I write repeated multiplication using powers? (p. 18)
- How can I use the properties of integer exponents to simplify algebraic and numeric expressions? (p. 26)
- How does the Product of Powers law apply to finding the power of a power? (p. 34)
- How are negative exponents and positive exponents related? (p. 46)
- How is scientific notation useful in the real world? (p. 54)
- How does scientific notation make it easier to perform computations with very large or very small numbers? (p. 62)
- Why would I need to use square roots and cube roots? (p. 74)
- How can I estimate the square root of a non-perfect square? (p. 84)
- How are real numbers different from irrational numbers? (p. 92)

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their classmate before they complete the graphic organizer.

MP 1, 2, 5

Reflect

Answering the Essential Question

Use what you learned about numbers to complete the graphic organizer. For each category, describe why you would use that form for the number $35{,}036\frac{1}{3}$. Then write the number in that form. If you would not use the number in that form, explain why. Sample answers are given.

WHY is it helpful to write numbers in different ways?

Decimal

If you were talking about money, you would want to write the number as a decimal and round to the nearest hundredth; AED 35,036.33

Power

This number cannot be written as a power.

Fraction

You would use the number as a mixed number in an addition or subtraction problem and as an improper fraction in a multiplication or division problem; $35{,}036\frac{1}{3}$; $\frac{105109}{3}$

Scientific Notation

You would use the number in scientific notation if it was part of a computation with a very large or very small number; 3.50363×10^4

Answer the Essential Question. WHY is it helpful to write numbers in different ways?

See students' work.

PROJECT 1

Music to My Ears When you listen to music, you may not be aware of the math used to create it. In this project you will:

- **Collaborate** with your classmates as you research the connections between math and music.
- **Share** the results of your research in a creative way.
- **Reflect** on how mathematical ideas can be represented.

By the end of this project, you just might be ready to write a hit song!

Collaborate

Go Online Work with your group to research and complete each activity. You will use your results in the Share section on the following page.

1. Choose a song on a CD or on your MP3 player. Listen to the song and describe the beats or rhythm using repeating numbers. For example, a song may have a rhythm that can be described by 1-2-3-1-2-3-... .

2. Research and [illegible]e the differ[illegible] types of musical notes. Make s[illegible] use rational numbers [illegible] include [illegible] of each note on [illegible] music.

3. Research Pythagoras' findings about music, notes and frequency, and harmony. Write a few paragraphs about what you found and create a list of the types of numbers you find in your research.

4. Describe the Fibonacci Sequence. Then give some examples of how Fibonacci numbers are found in music.

5. Find the digital music sales in a recent year. Write this number in both standard form and scientific notation. Then compare the digital music sales to CD music sales for the same year. Create a display to show what you find.

Launch the Project

Objective Research the connections between n
and music.

Music to My Ears

This project is designed to be completed by a g
of 4 or 5 students over several days or several
weeks. It utilizes concepts from the Number Sys
domain. You may choose to complete this proje
after completing the chapters within this domair

Collaborate

Have students work in teams to research inform
about math and music. Together, they should be
to gather the necessary information to complete
Exercises 1–5. Students should show their work
separate piece of paper.

Share

After each group gives their presentation, discuss similarities and differences in each group's findings.

21st Century Skills

You may want your students to connect their projects to a 21st century skill. Check out the suggestion below and on the student page.

connect with Language Arts

Civic Literacy Research a place or event in your community that centers on music. Write an essay that describes the event. Some points to include are:

- the type of entertainment offered and what music is played, and
- an explanation of how the music may affect those at the event.

Reflect

Students should work on their own to reflect on how the chapter from this unit and the objective of the project relate to the Essential Question.

Share

With your group, decide on a way to share what you have learned about math and music. Some suggestions are listed below, but you can also think of other creative ways to present your information. Remember to show how you used mathematics to complete each of the activities in this project!

- Create your own short piece of music based on your knowledge of notes and frequency. Make a recording of the music and explain how it is harmonious.
- Use presentation software to demonstrate some ways math and music are connected.

Use the note on the right to connect this project with other subjects.

connect with Health

Health Literacy Many studies have been done that show a positive connection between music and good health. Research the internet to find information about one such study.

Reflect

6. **Answer the Essential Question** HOW can mathematical ideas be represented? **See students' work.**

a. How did you use what you learned about real numbers in this chapter to represent mathematical ideas in this project?

b. In this project, you discovered how mathematical ideas are represented in music. Explain how mathematical ideas are represented in other parts of culture.

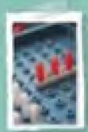
Expressions and Equations

Essential Question

HOW can you communicate mathematical ideas effectively?

Chapter 2
Equations in One Variable

Linear equations in one variable can have one solution, infinitely many solutions, or no solutions. In this chapter, you will write and solve two-step equations and solve equations with variables on both sides.



Chapter 3
Equations in Two Variables

In a proportional relationship, the unit rate is the slope of the graph. In this chapter, you will graph equations of the form $y = mx$ and $y = mx + b$. You will then solve systems of equations algebraically and by graphing.

Essential Question

At the end of this unit, students should be
answer "How can you communicate mathe
ideas effectively?"

Each chapter explores a different essentia
that assists students in answering the unit
The lessons in each chapter include exerc
lead students to various aspects of the es
question.

**This unit focuses on the Expressions a
Equations (EE) domain. The chapter ad
the following: Understand the connect
between proportional relationships, lin
linear equations.**

5. Graph proportional relationships, interp
unit rate as the slope of the graph. Co
different proportional relationships rep
different ways.
6. Use similar triangles to explain why th
the same between any two distinct po
non-vertical line in the coordinate plan
the equation y = mx for a line through
and the equation y = mx + b for a line
the vertical axis at b.

**Analyze and solve linear equations an
simultaneous linear equations.**

7. Solve linear equations in one variable

continue

2018

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

c. Solve real-world and mathematical problems leading to two linear equations in two variables.

Unit Project Preview

Collaborate

Ask students what they know about designing a Web page. Have students include drawings and descriptions of photos that they would include on their Web page.

The Unit Project can be found on pages 259 – 260.

Project 2 Preview

Collaborate

Web Design 101 A Web page is a useful way of presenting a summary of facts and statistics about a subject.

To design a Web page, you must first collect all the information that you want to include on the page. You will also need to decide how to balance the text and graphics. This will ensure that your Web page not only looks good but is functional too.

At the end of Chapter 3, you'll complete a project in which you will plan the design of a Web page about your favorite insect or other animal. But for now, it's time to do an activity in your book. Choose a subject that interests you. In the space provided, make an outline of all the items you would include if you were to design a Web page about that subject.

My Web page about ______

would include:

Chapter 2

Equations in One Variable

Essential Question

WHAT is equivalence?

Mathematical Practices
1, 2, 3, 4, 5, 7

Math in the Real World

Tips Maria and her family had salads and dinner at a local restaurant. Her mother wants to leave an 18% tip. The proportion $\frac{p}{AED\ 35.60} = \frac{18}{100}$ can be used to find the amount of tip she should leave.

Use the proportion to find the amount of tip Maria's mother should leave. Then find the total amount.

FOLDABLES Study Organizer

1 Cut out the Foldable in the back of this book.

2 Place your Foldable on page 164.

3 Use the Foldable throughout this chapter to help you learn about solving equations.

Focus narrowing the scope

This chapter focuses on content from the **Expressions an Equations** (EE) domain.

Coherence connecting within and across grades

Previous
Students wrote and solved one- and two-step linear equations.

Now
Students solve multi-step equations.

Next
Students will use tables, graphs, an equations to repre functions.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from concep understanding and procedural skills and fluency, to applic and critical thinking.

Launch the Chapter

Math in the Real World

Tips Ask students to describe the method used to solve proportion. They should give responses that include findi the cross products and then solving for the unknown.

What Tools Do You Need?

Vocabulary Activity

LA As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define: A two-step equation is an equation that contains two operations.

Example: $3x - 2 = 4$

Ask:

- *What are the two operations in this equation?* **multiplication and subtraction**

Writing Math

LA Have students read the Writing Math section to learn that to justify an answer means to give reasons why that answer is correct.

Ask:

- *In Step 1, why is it not enough to find the amount of the discount?* **Sample answer: Finding the amount of the discount does not answer the question of whether or not Mariah has enough money to pay for the two DVDs at-a-time plan.**
- *In Step 2, why does the answer to the problem have no numbers in it?* **Sample answer: The original question asks, "If she has AED 10.00 to spend, does she have enough?" So, the answer should be a sentence that tells whether or not she has enough money.**
- *In Step 3, why do you think that the justification is written in complete sentences?* **Sample answer: Complete sentences make the explanation of the justification clearer. It is easier to see if the question asked in the problem is answered completely.**

What Tools Do You Need?

Vocabulary

coefficient	multiplicative inverse	properties
identity	null set	two-step equation

Study Skill: Writing Math

Justify Your Answer When you justify your answer, you give reasons why your answer is correct.

The different plans an online movie rental company offers are shown. Amina wants to purchase the 2 DVDs at-a-time plan. This month, the plans are advertised at $\frac{1}{4}$ off. If she has AED 10.00 to spend, does she have enough? Justify your answer.

Online DVD Rental	
Plan	Monthly Price (AED)
3 DVDs at-a-time	14.50
2 DVDs at-a-time	12.00
1 DVD at-a-time	8.00

Step 1 **Solve the problem.**	Find the discount. $\frac{1}{4}$ of $12 = 3$ Find the discounted price in AED $12 - 3 = 9$
Step 2 **Answer the question.**	Amina does have enough money.
Step 3 **Justify your answer.** Always write complete sentences.	Amina has enough money because AED 9.00 is less than AED 10.00.

You can buy 3 used CDs at The Music Shoppe for AED 12.99, or you can buy 5 for AED 19.99 at Quality Sounds. Which is the better buy? Justify your answer.

Step 1 **Solve the problem.**	**Quality Sounds: $19.99 \div 5 \approx 4$** **Music Shoppe: $12.99 \div 3 \approx 4.33$**
Step 2 **Answer the question.**	**Quality Sounds**
Step 3 **Justify your answer.** Always write complete sentences.	**Since $4 < 4.33$, Quality Sounds is the better buy.**

What Do You Already Know?

Read each statement. Decide whether you agree (A) or disagree (D). Place a checkmark in the appropriate column and then justify your reasoning. See students' work.

Equations in One Variable			
Statement	**A**	**D**	**Why?**
To solve an equation with a fractional coefficient, multiply each side of the equation by the multiplicative inverse of the fraction.			
The first step to solve $-2x + 9 = 6$ is to divide both sides by -2.			
If an equation has variables on both sides of the equals sign, the first thing you should do is get the variables on the same side of the equation.			
Equations can have no solution, one solution, or infinitely many solutions.			
You would use the Associative Property first to solve the equation $6(x - 3) + 10 = 2(3x + 4)$.			
An equation that is true for every value of the variable is called an identity.			

When Will You Use This?

Here is an example how equations are used in the real world.

Activity Do you or your parents have a texting plan? If so, how much does it cost per text or per month? Ask your parents to help you research different texting plans. Then compare and contrast each plan.

See students' work.

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What Do You Already Know?

In this activity, students assess their prior knowledge by determining whether they agree or disagree with each statement about concepts in this chapter.

- You may want to add a third option of "I don't know" for those students who do not have any prior knowledge of the content of the statement.
- After completing the chapter, have students return to this page and see if any of their responses would change now that they have finished the chapter.

When Will You Use This?

Activity

Students may not realize how much they already know about equations. They have already solved one-step and two-step equations and applied them to real-world situations.

Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

Quick Check

If students have difficulty with the exercises, present an additional example to clarify any misconceptions.

Exercises 1–3
Solve $z - 6 = -10$. **−4**

Exercises 4–6
Solve $12 = \frac{a}{-7}$. **−84**

Are You Ready?

Try the Quick Check below.

Quick Review

Example 1

Solve $44 = k - 7$.

$44 = k - 7$ Write the equation.
$+7 = \quad +7$ Addition Property of Equality
$51 = k$

Example 2

Solve $18m = -360$.

$18m = -360$ Write the equation.
$\frac{18m}{18} = \frac{-360}{18}$ Division Property of Equality
$m = -20$ Simplify.

Quick Check

One-Step Equations Solve each equation. Check your solution.

1. $n + 8 = -9$ **−17**
2. $4 = p + 19$ **−15**
3. $-4 + a = 15$ **19**
4. $3c = -18$ **−6**
5. $-42 = -6b$ **7**
6. $\frac{w}{4} = -8$ **−32**
7. Sami has 18 more marbles than Samira. If Sami has 92 marbles, write and solve an equation to determine the number of marbles Samira has.
   $18 + h = 92$; 74 marbles

How Did You Do? Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

① ② ③ ④ ⑤ ⑥ ⑦

Lesson 1

Solve Equations with Rational Coefficients

Vocabulary Start-Up

Two numbers with a product of 1, such as $\frac{3}{4}$ and $\frac{4}{3}$, are called reciprocals or **multiplicative inverses**.

Complete the graphic organizer. Sample answers are given.

multiplicative inverses

Define it	Describe it
Two fractions that multiply to give 1.	The numerator and denominator of a fraction switch places.
List Some Examples	**List Some Nonexamples**
$\frac{2}{3}$ and $\frac{3}{2}$ $\frac{1}{2}$ and $\frac{2}{1}$	$2\frac{1}{3}$ and $\frac{7}{3}$ $\frac{1}{2}$ and $-\frac{1}{2}$

Describe how a multiplicative inverse is used in division of fractions.
When you divide fractions, multiply the dividend by the multiplicative inverse of the divisor.

Essential Question

WHAT is equivalence?

Vocabulary

multiplicative inverse
coefficient

Mathematical Practices
1, 3, 4, 7

Real-World Link

How can the action of the motorcyclist in the photo help you remember what the multiplicative inverse is? The motorcyclist is upside down. You switch the numerator and denominator of a fraction in the multiplicative inverse.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1 Persevere with Problems
2 Reason Abstractly
3 Construct an Argument
4 Model with Mathematics
5 Use Math Tools
6 Attend to Precision
7 Make Use of Structure
8 Use Repeated Reasoning

Focus narrowing the scope

Objective Solve equations with rational coefficients.

Coherence connecting within and across grades

Previous
Students solved one- and two-step equations with integer coefficients.

Now
Students solve one-step equations with rational coefficients.

Next
Students solve multi-step equations with rational coefficients.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 115.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Find the Fib** Have students work in teams. Each student should write down 2 facts about multiplicative inverses and 1 fib, or 2 pairs of numbers that are multiplicative inverses and 1 pair that is not. The job of the team is to identify the fib. Encourage students to use rational numbers, such as decimals, fractions, and mixed numbers. MP 1, 3

Alternate Strategy

AL Remind students that they used the multiplicative inverse, or reciprocal, when dividing fractions and mixed numbers. Then review how to divide fractions and mixed numbers before beginning the Vocabulary Start-Up.

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Solve an equation with a fractional coefficient.

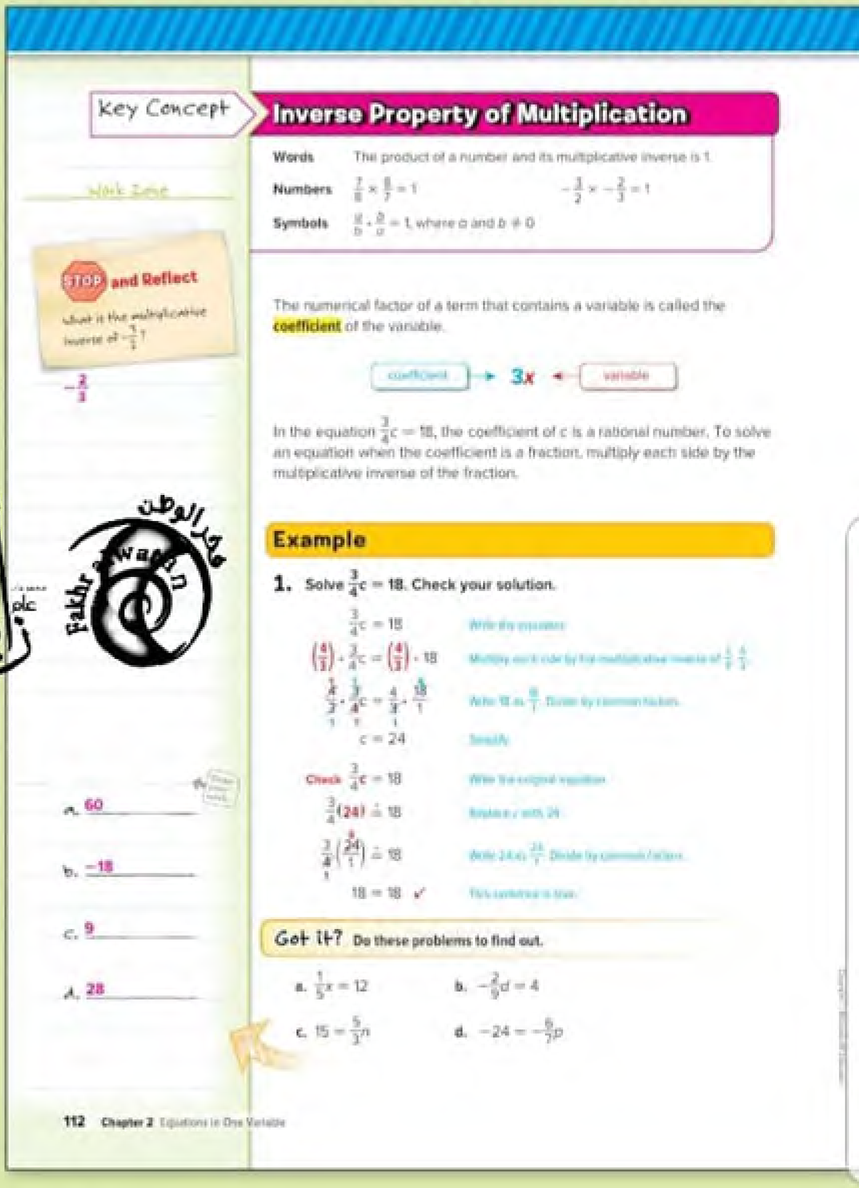
AL
- *What is a coefficient?* **the numerical factor of a term with a variable**
- *What is the coefficient in the equation?* $\frac{3}{4}$
- *If the coefficient was 3, what would be the first step in solving the equation?* **Divide each side by 3.**
- *Using that theory, what is the first step in solving* $\frac{3}{4}c = 18$*?* **Divide each side by $\frac{3}{4}$.**

OL
- *Dividing by $\frac{3}{4}$ is the same as multiplying by what number?* $\frac{4}{3}$
- *What is the multiplicative inverse of $\frac{3}{4}$?* $\frac{4}{3}$

BL
- *Would the solution change if you divided both sides by $\frac{3}{4}$? Explain.* **no, Sample answer: Dividing by a fraction is the same as multiplying by its reciprocal.**
- *Suppose your friend solved the equation by multiplying both sides by 4, and then dividing both sides by 3. Is this method correct? Explain.* **yes; Sample answer: Multiplying by 4 and dividing by 3 is the same as multiplying by $\frac{4}{3}$.**

Need Another Example?

Solve $\frac{2}{3}a = 12$. Check your solution. **18**



Key Concept **Inverse Property of Multiplication**

Words	The product of a number and its multiplicative inverse is 1.	
Numbers	$\frac{7}{8} \times \frac{8}{7} = 1$	$-\frac{3}{2} \times -\frac{2}{3} = 1$
Symbols	$\frac{a}{b} \cdot \frac{b}{a} = 1$, where a and $b \neq 0$	

The numerical factor of a term that contains a variable is called the **coefficient** of the variable.

coefficient → $3x$ ← variable

In the equation $\frac{3}{4}c = 18$, the coefficient of c is a rational number. To solve an equation when the coefficient is a fraction, multiply each side by the multiplicative inverse of the fraction.

Work Zone

STOP and Reflect: What is the multiplicative inverse of $-\frac{3}{2}$? $-\frac{2}{3}$

Example

1. Solve $\frac{3}{4}c = 18$. Check your solution.

$$\frac{3}{4}c = 18$$
$$\left(\frac{4}{3}\right) \cdot \frac{3}{4}c = \left(\frac{4}{3}\right) \cdot 18$$
$$\frac{4}{3} \cdot \frac{3}{4}c = \frac{4}{3} \cdot \frac{18}{1}$$
$$c = 24$$

Check
$$\frac{3}{4}c = 18$$
$$\frac{3}{4}(24) \stackrel{?}{=} 18$$
$$\frac{3}{4}\left(\frac{24}{1}\right) \stackrel{?}{=} 18$$
$$18 = 18 \checkmark$$

Got It? Do these problems to find out.

a. $\frac{1}{5}x = 12$ **60**

b. $-\frac{2}{9}d = 4$ **−18**

c. $15 = \frac{5}{3}n$ **9**

d. $-24 = -\frac{6}{7}p$ **28**

112 Chapter 2 Equations in One Variable

Example

2. Solve $1\frac{1}{2}s = 16\frac{1}{2}$. Check your solution.

$$1\frac{1}{2}s = 16\frac{1}{2}$$

$$\frac{3}{2}s = \frac{33}{2}$$

$$\left(\frac{2}{3}\right) \cdot \frac{3}{2}s = \left(\frac{2}{3}\right) \cdot \frac{33}{2}$$

$$\frac{2}{3} \cdot \frac{3}{2}s = \frac{2}{3} \cdot \frac{33}{2}$$

$$s = 11$$

Got it? Do these problems to find out.

d. $4\frac{1}{6} = 3\frac{1}{3}c$ e. $-9\frac{5}{8}w = 108$ f. $1\frac{7}{8}y = 4\frac{1}{2}$

d. $1\frac{1}{4}$

e. $-11\frac{17}{77}$

f. $2\frac{2}{5}$

Solve Equations with Decimal Coefficients

In the equation $3.15 = 0.45n$ the coefficient of n is a decimal. To solve an equation with a decimal coefficient, divide each side of the equation by the coefficient.

Quick Review

Example

3. Solve $3.15 = 0.45n$. Check your solution.

$$3.15 = 0.45n$$

$$\frac{3.15}{0.45} = \frac{0.45n}{0.45}$$

$$7 = n$$

Check $3.15 = 0.45n$

$$3.15 = 0.45(7)$$

$$3.15 = 3.15 \checkmark$$

Got it? Do these problems to find out.

g. $4.9 = 0.7t$ h. $-1.4m = 2.1$ i. $-5.6k = -12.88$

g. 7

h. −1.5

i. 2.3

Examples

2. Solve an equation with a fractional coefficient.

AL
- *What is the coefficient in the equation?* $1\frac{1}{2}$
- *What is an improper fraction?* a fraction in which the numerator is greater than or equal to the denominator
- *Why do you need to change the mixed numbers to improper fractions?* When multiplying, we multiply the numerators and multiply the denominators. A mixed number has a whole number part that needs to be accounted for.
- *How do you write $1\frac{1}{2}$ as an improper fraction?* The number 1 can be thought of as $\frac{2}{2}$. Then add $\frac{1}{2}$ to obtain $\frac{3}{2}$.

OL
- *After renaming the mixed number as an improper fraction, what is the next step to solve the equation?* Multiply each side of the equation by the multiplicative inverse of the improper fraction.
- *What is the multiplicative inverse of $1\frac{1}{2}$?* $\frac{2}{3}$

BL
- *Suppose your friend rewrote the equation as $1.5s = 16.5$ and then divided both sides by 1.5. Is this method correct? Explain.* yes; Sample answer: The decimals are equivalent forms of the fractions, so the answer would be equivalent.

Need Another Example?

Solve $2\frac{1}{4}b = 18$. Check your solution. 8

3. Solve an equation with a decimal coefficient.

AL
- *What is the coefficient in the equation?* 0.45
- *What operation is indicated by the coefficient?* multiplication

OL
- *How do you solve the equation?* Divide each side by 0.45.
- *Why is it important to check your solution?* Sample answer: I can verify that I did not make calculation errors.

BL
- *How can you use estimation to check your answer for reasonableness?* Sample answer: $0.45 \approx 0.5$ and $3.15 \approx 3$. If the equation was $3 = 0.5n$, half of 6 is 3, so n would equal 6. Since 6 is close to 7, the answer is reasonable.
- *Solve $0.3x = 2\frac{1}{4}$.* 7.5 or $7\frac{1}{2}$

Need Another Example?

Solve $10.8 = 0.9n$. Check your solution. 12

Example

4. Solve equations with rational coefficients.

- AL • *What is 75% written as a decimal?* 0.75
- OL • *What equation would you use to solve the problem?* $0.75n = 18$
  • *What is the first step in solving the equation?* Divide each side by 0.75.
- BL • *Is there another way to write the equation? Explain.* yes; Sample answer: You could write 75% as the fraction $\frac{3}{4}$.

Need Another Example?
Antonio has some fabric that he will use to make curtains. Forty-five percent, or 6 yards of the fabric is green. Define a variable. Then write and solve an equation to determine how many yards of fabric he has altogether. f = the total amount of fabric; $0.45f = 6$; $13\frac{1}{3}$

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Round Robin** Have students work in pairs to complete Exercises 1–4. One student should complete Exercise 1, then pass it to the other student who will check the answer. Students trade roles for the next exercise, and so on. MP 1

BL LA **Trade-a-Problem** Have students create their own word problem, similar to Exercise 4. Challenge students to use a fraction or mixed number, decimal, and a percent in their problem. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. MP 1, 3, 4

Quick Review
To write a percent as a decimal, move the decimal point two places to the left. Add zeros if necessary. For example, 18% = 0.18 and 75% = 0.75.

Example

4. Munira's softball team won 75%, or 18, of its games. Define a variable. Then write and solve an equation to determine the number of games the team played.

Munira's softball team won 18 games, which was 75% of the games played. Let n represent the number of games played. Write and solve an equation.

$0.75n = 18$ Write the equation. Write 75% as 0.75.

$\frac{0.75n}{0.75} = \frac{18}{0.75}$ Division Property of Equality

$n = 24$ Simplify.

Munira's softball team played 24 games.

Guided Practice

Solve each equation. Check your solution. (Examples 1–3)

1. $60 = \frac{3}{4}p$ **80**
2. $-\frac{27}{25}x = -\frac{9}{5}$ **$\frac{5}{3}$ or $1\frac{2}{3}$**
3. $-2.7t = 810$ **−300**

4. Huda has read 70% of the total pages in a book she is reading for English class. Huda has read 84 pages. Define a variable. Then write and solve an equation to determine how many pages are in the book. (Example 4)
p = total pages in book; $0.7p = 84$; 120 pages

5. **Building on the Essential Question** How is the multiplicative inverse used to solve an equation that has a rational coefficient?
To solve an equation with a coefficient that is a fraction, multiply each side of the equation by the multiplicative inverse of the fraction.

Rate Yourself!
Are you ready to move on? Shade the section that applies.
I have a few questions. | I'm ready to move on. | I have a lot of questions.

Name ______________ My Homework ______________

Independent Practice

Solve each equation. Check your solution. (Examples 1–3)

1. $6 = \frac{1}{12}v$ **72**
2. $-\frac{2}{3}w = 60$ **−90**
3. $-\frac{7}{8}k = -21$ **24**
4. $9.6 = 1.2b$ **8**
5. $0.75a = -9$ **−12**
6. $-413.4 = -15.9n$ **26**
7. $3\frac{1}{10}s = 6\frac{1}{5}$ **2**
8. $2\frac{2}{9} = -\frac{4}{5}m$ $-\frac{25}{9}$ **or** $-2\frac{7}{9}$
9. $-2\frac{4}{5} = -3\frac{1}{2}n$ $\frac{4}{5}$

Define a variable. Then write and solve an equation for each situation. (Example 4)

10. The Farouq family drove a total of 180 miles on their road trip. This distance is 1.5 times the distance they drove on the first day. How many miles did the Farouq family drive on the first day?
d = miles on first day; $1.5d = 180$; 120 mi

11. Ali correctly answered 80% of the questions on a language arts quiz. If he answered 16 questions correctly, how many questions were on the language arts quiz?
q = total questions; $0.8q = 16$; 20 questions

12. **Financial Literacy** Ismail deposited 60% of his paycheck into his savings account. What was the amount of his paycheck?
a = amount of paycheck; $0.60a = 41.67$; AED 6,945

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

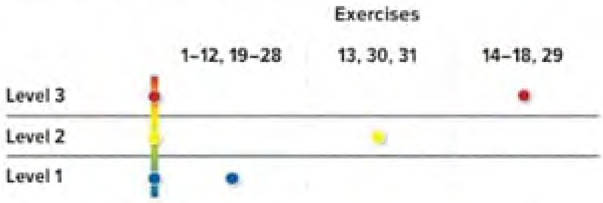
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–11, 13, 14, 17, 18, 30, 31
OL	On Level	1–11 odd, 12–14, 17, 18, 30, 31
BL	Beyond Level	12–18, 30, 31

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	15, 16
3 Construct viable arguments and critique the reasoning of others.	17, 18, 29
4 Model with mathematics.	14
7 Look for and make use of structure.	13

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students describe the first step to solve the equation $-\frac{1}{3}y = 2.5$. **Multiply both sides of the equation by −3.**

13. **Identify Structure** Suppose the numbers, $1\frac{1}{3}$, 0.2, −5, $-\frac{1}{2}$, are each coefficients in separate equations. Choose whether you would solve the equation by multiplying each side by the multiplicative inverse of the coefficient or by dividing each side by the coefficient. Write the number in the appropriate space.

Multiplicative Inverse	Division
$1\frac{1}{3}$ $-\frac{1}{2}$	0.2 −5

H.O.T. Problems Higher Order Thinking

14. **Model with Mathematics** Write a real-world problem that can be represented by the equation $\frac{3}{4}c = 21$. **Sample answer: Three fourths of the students in Abdul's homeroom study Spanish. Twenty-one students in his homeroom study Spanish. How many students are in Abdul's homeroom?**

Persevere with Problems Determine whether each statement is *true* or *false*. Explain your reasoning.

15. The product of a fraction and its multiplicative inverse is 1. **true; Sample answer: The product of $\frac{3}{4}$ and $\frac{4}{3}$ is $\frac{12}{12}$, which simplifies to 1.**

16. To solve an equation with a coefficient that is a fraction, divide each side of the equation by the reciprocal of the fraction. **false; Sample answer: You would multiply, not divide, by the reciprocal of the fraction. For example, to solve $\frac{2}{3}x = 20$, multiply each side by $\frac{3}{2}$.**

17. **Reason Inductively** Complete the statement: If $10 = \frac{1}{5}x$, then $x + 3 = \blacksquare$. Explain your reasoning. **53; Since $10 = \frac{1}{5}x$, then $x = 50$ and $x + 3 = 53$.**

18. **Justify Conclusions** Suppose your friend says he can solve $3x = 15$ by using the Multiplication Property of Equality. Is he correct? Justify your response. **Sample answer: Yes; he can multiply each side of the equation by $\frac{1}{3}$ instead of dividing by 3.**

Name ________________ My Homework ________________

Extra Practice

Solve each equation. Check your solution.

19. $\frac{1}{2} = \frac{2}{5}z$

Homework Help

$\frac{5}{2} \cdot \frac{1}{2} = \frac{5}{2} \cdot \frac{2}{5}z$

$\frac{5}{4} = 1z$

$1\frac{1}{4} = z$

20. $-\frac{3}{4}t = 5$ **$-6\frac{2}{3}$**

21. $-\frac{2}{9}g = -\frac{7}{9}$ **$3\frac{1}{2}$**

22. $0.6w = 0.48$ **0.8**

23. $-226.8 = 21.6y$ **-10.5**

24. $-30 = 1.25c$ **-24**

25. $1\frac{1}{2}x = 9\frac{9}{20}$ **$6\frac{3}{10}$**

26. $-12\frac{2}{3} = -1\frac{1}{9}y$ **$11\frac{2}{5}$**

27. $1\frac{5}{7} = 1\frac{13}{14}a$ **$\frac{8}{9}$**

28. One third of the bagels in a bakery are sesame bagels. There are 72 sesame bagels. Define a variable. Then write and solve an equation to find how many bagels there were in the bakery.

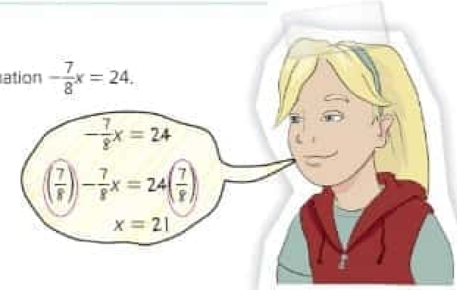
b = the total number of bagels; $\frac{1}{3}b = 72$; 216 bagels

29. MP **Find the Error** Nadia is solving the equation $-\frac{7}{8}x = 24$. Circle her mistake and correct it.

$-\frac{7}{8}x = 24$

$\left(-\frac{8}{7}\right)\left(-\frac{7}{8}\right)x = 24\left(-\frac{8}{7}\right)$

$x = -27\frac{3}{7}$



Watch Out!

Find the Error In Exercise 29, Nadia did not mu
correct multiplicative inverse. The multiplicative
is $-\frac{8}{7}$.

Up! Test Practice

d 31 prepare students for more rigorous
l for the assessment.

requires students to reason abstractly and when problem solving.	
wledge	DOK1
Practice	MP1, MP4
ric	
	Students correctly select the correct equations and correctly solve the equations.
	Students correctly select and solve one of the equations OR students select the correct equations but solve only one correctly.

requires students to explain and apply mathematical solve problems with precision, while making use of	
wledge	DOK1
Practice	MP1
ric	
	Students correctly answer the question.

Power Up! Test Practice

30. Select the correct equation for each situation. Then solve each problem.

$\frac{x}{5} = 240$	$5x = 240$
$\frac{x}{240} = 0.5$	$0.05x = 240$
$0.5x = 240$	$\frac{5}{x} = 240$

a. The Mahmoud family drove a total of 240 miles on their road trip. This distance is 5 times the distance they drove on the first day. How many miles did the family drive on the first day?

Equation: **$5x = 240$** Solution: **48 mi**

b. There are 240 students in Al Rihab's school. This is 5% of the total students in the school district. How many students are there in the school district?

Equation: **$0.05x = 240$** Solution: **4,800 students**

31. The table shows how many miles Fawzia has run this week.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Miles	6.5	2.9	4.2	5.5	3.1

The total distance this week is 1.5 times the distance that she ran last week. How many miles did Fawzia run last week? **14.8 mi**

Spiral Review

Solve each equation. Check your solution.

32. $w + 5 = -20$ **−25**

33. $x - 17 = -32$ **−15**

34. $t + 7.2 = 1.65$ **−5.55**

35. $-0.4 = g - 4.9$ **4.5**

36. $y - \frac{2}{5} = 1\frac{3}{5}$ **2**

37. $-5\frac{1}{6} = 2\frac{1}{3} + p$ **$-7\frac{1}{2}$**

38. Financial Literacy Shaima saved AED 65.35 more than her brother Ali and AED 37.50 less than her sister Alia. Alia saved AED 127.75. Write and solve equations to find how much money Shaima and Ali saved.

Shaima: $s + 37.50 = 127.75$; AED 90.25; Ali: $s - 65.35 = d$; AED 24.90

Inquiry Lab
Solve Two-Step Equations

Inquiry **HOW does a bar diagram help you solve a real-world problem involving a two-step equation?**

MP Mathematical Practices 1, 2, 3, 4

Manal bought two large postcards and four small postcards at a souvenir shop. Each small postcard costs AED 0.50. If Manal spent AED 5.00 on postcards, what is the cost of one large postcard?

What do you know? **Two large postcards and 4 small postcards cost AED 5.00. The small postcards are AED 0.50 each.**

What do you need to find? **the cost of one large postcard**

Hands-On Activity

Step 1 The bar diagram represents the total number of postcards and the total cost. Label the missing parts.

AED 5					
large	large	small	small	small	small
?	?	AED 0.50	AED 0.50	AED 0.50	AED 0.50

Step 2 Fill in the boxes to write an equation that represents the bar diagram. The cost of a large postcard is the unknown, so it is represented by the variable p.

$2p +$ **AED 2** $=$ **AED 5**

Step 3 Find the cost of the large postcards by working backward.

AED 3		AED 2			
large	large	small	small	small	small
AED 1.50	AED 1.50	AED 0.50	AED 0.50	AED 0.50	AED 0.50

The cost of one large postcard is **3 ÷ 2 or AED 1.50**

Focus narrowing the scope

Objective Use a bar diagram to write equations.

Coherence connecting within a

Now
Students use bar diagrams to write and solve two-step equations.

Ne
Stud
the a
and

Rigor pursuing concepts, fluency

See the Levels of Complexity chart

ENGAGE EXPLORE EXPLAIN ELA

1 Launch the Lab

The activity is intended to be used a

Hands-On Activity

AL LA **Rally Coach** Have stude
complete the Activity. Student 1 com
through their process aloud, while St
and coaches. Pairs alternate roles e
complete. Call on student pairs to p
solutions for the activity. MP 1, 3, 5

BL Have students alter the scenar
diagram and solution would change
choose to alter the scenario so that
number of large postcards or a diffe
postcards. They could also choose to
size of postcard and the total amount

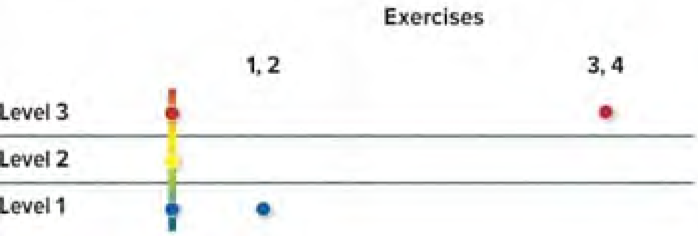
ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

Collaborate

The **Investigate** section is intended to be used as a small-group investigation. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Pairs Consult** Have students work in pairs to complete Exercises 1 and 2. Have Student 1 lead the discussion for Exercise 1, then have Student 2 lead the discussion for Exercise 2. Each person is responsible for asking questions and be sure that they and their partner understand how to use a bar diagram to write and solve an equation. Then call on one pair to present their results to the class. MP 1, 3, 5

Create

BL LA **Think-Pair-Share** Give students 1–2 minutes to brainstorm for Exercise 3. Have students work in pairs to complete Exercise 3. Choose one partner from each group to share their word problem with the class. MP 1, 3, 4, 5

Inquiry Students should be able to answer "HOW does a bar diagram help you solve a real-world problem involving a two-step equation?" Check for student understanding and provide guidance, if needed.

Investigate

MP **Reason Abstractly** Work with a partner. Use a bar diagram to write and solve an equation for each exercise.

1. Sameh and two friends went to the movies and spent a total of AED 42. The movie tickets were AED 5 each and they each bought a popcorn combo. What is the cost of one popcorn combo?

AED 15 + 3x = AED 42; x = AED 9

2. Four medium postcards and 4 small postcards cost AED 5. What is the cost of one medium postcard?

4x + AED 2 = AED 5; x = AED 0.75

Create

3. MP **Model with Mathematics** Write and solve a word problem that could be represented by the bar diagram shown.

Sample answer: I bought 2 shirts and 3 pairs of socks. The socks cost AED 6 each. If she spent a total of AED 72, and the shirts were the same price, how much did she pay for each shirt? AED 27

4. Inquiry HOW does a bar diagram help you solve a real-world problem involving a two-step equation?

Sample answer: You can visualize all of the parts of the problem using a bar diagram. Then you can use the work backward strategy to solve.

Exp

Lesson 2

Solve Two-Step Equations

Vocabulary Start-Up

Recall that in mathematics, **properties** are statements that are true for any number.

Complete the graphic organizer by matching the Property of Equality with the correct example.

Addition Property of Equality

Division Property of Equality

Multiplication Property of Equality

Subtraction Property of Equality

$\frac{1}{2}x = 10$
$2 \cdot \frac{1}{2}x = 10 \cdot 2$

$3x = 9$
$\frac{3x}{3} = \frac{9}{3}$

$x + 3 = 1$
$x + 3 - 3 = 1 - 3$

$x - 5 = 6$
$x - 5 + 5 = 6 + 5$

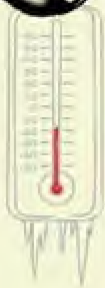
Essential Question

WHAT is equivalence?

Vocabulary

properties
two-step equation

Mathematical Practices
1, 2, 3, 4



Real-World Link

A property in science is a trait of matter that is always true under a given set of conditions. For example, pure water freezes at 0°C. How is the definition of *property* similar in science and math? **Sample answer:**

In science, a property is always true for members of a group. In math, a property is true for members of a group of numbers.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Solve two-step equations.

Coherence connecting within and acr

Previous
Students used bar diagrams to write and solve two-step equations.

Now
Students solve two-step equations.

Rigor pursuing concepts, fluency, and a

See the Levels of Complexity chart on page

ENGAGE EXPLORE EXPLAIN ELABORATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a
group, think-pair-share activity, or independ

LA Pairs Discussion Have st
pairs to complete the graphic or
a pair of students to share their responses
MP 1, 5

Alternate Strategies

AL Have students decode the operation
equation to help identify the property. MP 1,

BL LA Have students discuss with a pa
property means in science and what the te
Have them also discuss the meanings of th
the real world, such as owning property in r

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Solve two-step equations.

AL
- *How can we set up the model for this equation?* Place two x-tiles and three 1-tiles on one mat. Place seven 1-tiles on the other mat.
- *Why do we remove three 1-tiles from each mat?* so that the x-tiles are by themselves
- *Why do we separate the remaining tiles into 2 equal groups?* so that there is one x-tile in each group

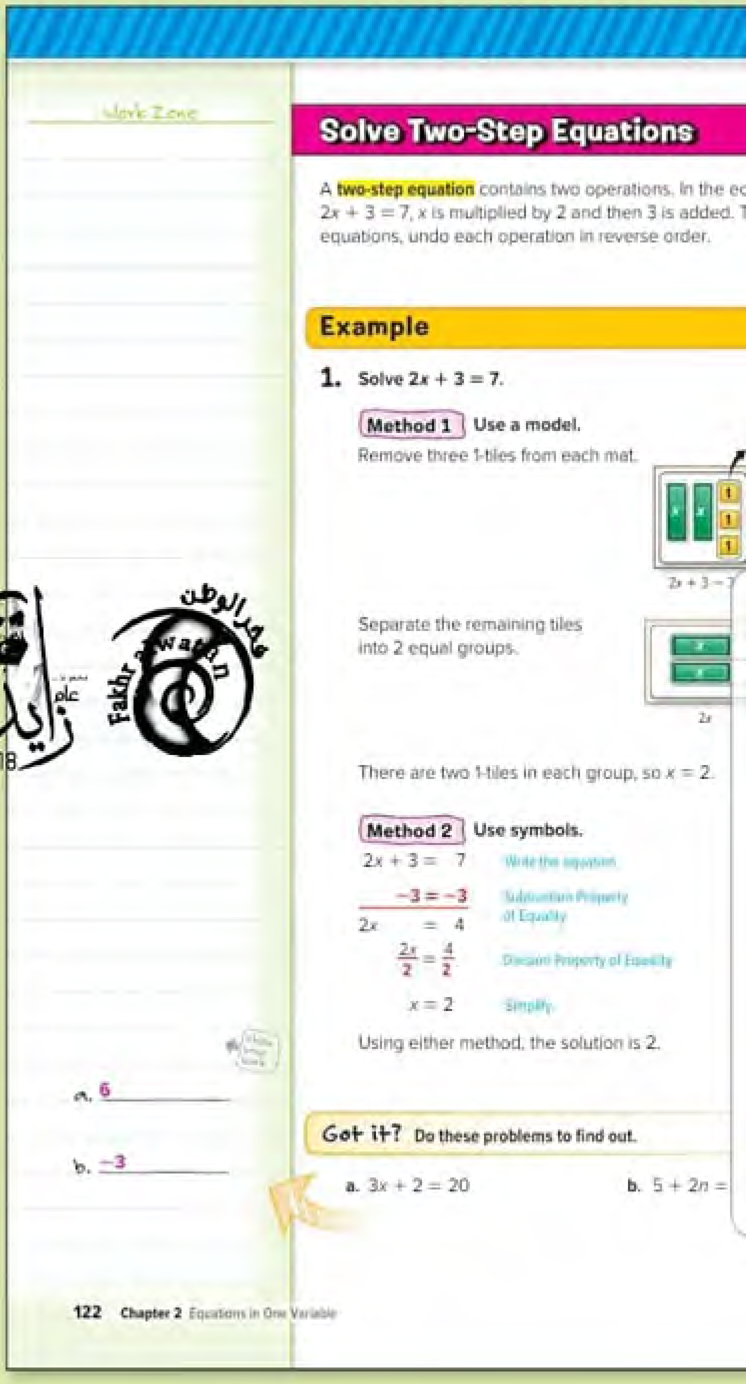
OL
- *In Method 2, what property allows us to subtract 3 from each side?* Subtraction Property of Equality
- *What step does subtracting 3 from each side in Method 2 correspond to in Method 1?* removing three 1-tiles from each mat
- *In Method 2, what property allows us to divide each side by 2?* Division Property of Equality
- *What step does dividing each side by 2 in Method 2 correspond to in Method 1?* separating the tiles into two groups

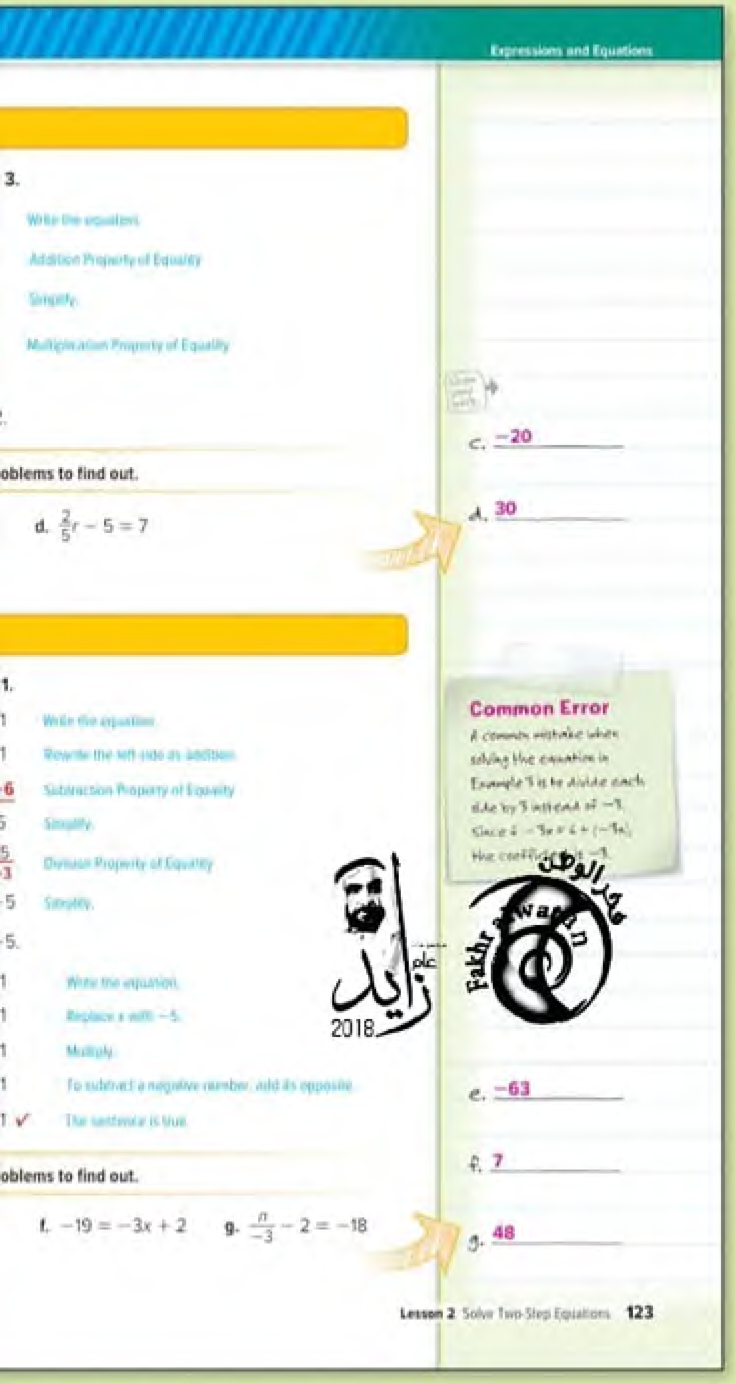
BL
- *Compare and contrast the methods. Which do you prefer?* Sample answer: Method 1 allows me to visualize the equation and its solution; Method 2 is often quicker; See students' preferences.

Need Another Example?
Solve $5y + 1 = 26$. 5

Watch Out!

Common Error In Example 1, students may want to add 3 tiles to each side because the equation has addition in it. Remind them that to solve for a variable, we have to "undo" each operation by doing the opposite.




Expressions and Equations

3.

Write the equation.

Addition Property of Equality

Simplify.

Multiplication Property of Equality

oblems to find out.

d. $\frac{2}{5}r - 5 = 7$

c. -20

d. 30

Common Error

A common mistake when solving the equation in Example 3 is to divide each side by 3 instead of −3. Since 6 − 3x = 6 + (−3x), the coefficient is −3.

1.

Write the equation.

Rewrite the left side as addition.

Subtraction Property of Equality

Simplify.

Division Property of Equality

Simplify.

Write the equation.

Replace x with −5.

Multiply.

To subtract a negative number, add its opposite.

The sentence is true.

oblems to find out.

f. $-19 = -3x + 2$ g. $\frac{n}{-3} - 2 = -18$

e. -63

f. 7

g. 48

Lesson 2 Solve Two-Step Equations 123

Examples

2. Solve two-step equations.

AL
- *What two operations are being performed on the variable?* **multiplication and subtraction**
- *In the order of operations, which operation is performed first?* **multiplication**

OL
- *In order to isolate the variable, what should we do first? Why?* **Add 3 to each side. We need to undo the operations by performing them in reverse order of the order of operations.**
- *After we add 3 to each side, what does the equation become?* $28 = \frac{1}{4}n$
- *Because the coefficient is a fraction, we have to divide by the fraction to undo the multiplication. How do you divide a fraction by a fraction?* **Multiply by the reciprocal.**

BL
- *How can you solve the equation $28 = \frac{1}{4}n$ mentally?* **Dividing by $\frac{1}{4}$ is the same as multiplying by 4, and $28 \times 4 = 112$.**

Need Another Example?

Solve $-4 = \frac{1}{3}z + 2$. -18

3. Solve two-step equations.

AL
- *How can we rewrite the subtraction equation as an addition equation?* $6 + (-3x) = 21$
- *What number is being multiplied by the variable?* -3
- *What number is being added to the variable?* 6

OL
- *What should we do first to isolate the variable?* **Subtract 6 from each side.**
- *After subtracting 6 from each side, what does the equation become?* $-3x = 15$
- *What is the next step?* **Divide each side by −3.**

BL
- *Why do we change the subtraction to addition before solving?* **Sample answer: to help remind us that the number being multiplied by the variable is negative and to show that the 6 is being added, not subtracted**

Need Another Example?

Solve $8 - 3x = 14$. -2

Example

4. Solve two-step equations.

- AL • *What two operations are being performed on the variable C?* multiplication and addition
- OL • *What is the first step in solving the equation $-27 = 1.8C + 32$?* Subtract 32 from each side.
  • *After subtracting 32 from each side, what does the equation become?* $-59 = 1.8C$
  • *What is the next step?* Divide each side by 1.8.
- BL • *Using the idea of inverse operations and the formula $F = 1.8C + 32$, how could you generate a formula that gives degrees Celsius in terms of degrees Fahrenheit?* Sample answer: Subtract 32 from each side. The formula becomes $F - 32 = 1.8C$. Then divide each side by 1.8. The formula becomes $\frac{F - 32}{1.8} = C$.

Need Another Example?
Manal will put trim molding around a rectangular table. The table is 45 inches long. She has 150 inches of trim. Solve the equation $150 = 2w + 90$ to find the width of the table. 30 in.

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Discussion** For Exercises 1–3, have students refer to the steps they used in Examples 1–3. For Exercises 2 and 3, remind them to rewrite each equation as an addition equation. Tell them to follow the steps in the same order for the similar problems. They may choose to make an order of operations chart to help remember how to work backward. MP 1, 5, 6

BL LA **Trade-a-Problem** Have students write a real-world problem that can be represented by a two-step equation. Have them trade problems with another student. Each student writes and solves the equation to solve the problem. MP 1, 4

Example

4. STEM Chicago's lowest recorded temperature in degrees Fahrenheit is −27°. Solve the equation $-27 = 1.8C + 32$ to convert to degrees Celsius.

$-27 = 1.8C + 32$ Write the equation.
$-32 = \quad -32$ Subtraction Property of Equality
$-59 = 1.8C$ Simplify.
$\frac{-59}{1.8} = \frac{1.8C}{1.8}$ Division Property of Equality
$-32.8 = C$ Simplify. Check the solution.

So, Chicago's lowest recorded temperature is about −32.8 degrees Celsius.

Guided Practice

Solve each equation. Check your solution. (Examples 1–3)

1. $6x + 5 = 29$ 4
2. $3 - 5y = -37$ 8
3. $\frac{2}{3}x - 5 = 7$ 18

4. Sara went to the movies with some of her friends. The tickets cost AED 6.50 each, and they spent AED 17.50 on snacks. The total amount paid was AED 63.00. Solve the equation $63 = 6.50p + 17.50$ to determine how many people went to the movies. (Example 4)
7 people

5. **Building on the Essential Question** How can you use the *work backward* problem-solving strategy to solve a two-step equation?
Sample answer: You identify the order in which operations would be performed on the variable, then you undo each operation using its inverse operation in reverse order.

Rate Yourself!
How confident are you about solving equations? Check the box square that applies.

Name ______________________ My Homework ______________________

Independent Practice

Solve each equation. Check your solution. (Examples 1–3)

1. $5 = 4a - 7$ **3**
2. $16 = 5x - 9$ **5**
3. $3 - 8c = 35$ **−4**
4. $-\frac{1}{2}x - 7 = -11$ **8**
5. $15 - \frac{w}{4} = 28$ **−52**
6. $-3 - 6x = 9$ **−2**

7. Suad received a AED 50 gift card to an online store. She wants to purchase some bracelets that cost AED 8 each. There will be a AED 10 overnight delivery fee. Solve $8n + 10 = 50$ to find the number of bracelets she can purchase. (Example 4) **5 bracelets**

8. Munira paid AED 75 to join a summer golf program. The course where she plays charges AED 30 per round. Since she is a student, she receives a AED 10 discount per round. If Munira spent AED 375, use the equation $375 = 20g + 75$ to find how many rounds of golf she played. (Example 4) **15 rounds**

Copy and Solve **Solve each equation. Show your work on a separate piece of paper.**

9. $\frac{a - 4}{5} = 12$ **64**
10. $\frac{n + 3}{8} = -4$ **−35**
11. $\frac{6 + z}{10} = -2$ **−26**

12. **Reason Abstractly** If Mr. Mohamed wants to put new carpeting in the room shown, how many square feet should he order? **140 ft²**

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as homework assignment. The Extra Practice page can be us for additional reinforcement or as a second-day assignme

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Leve indicating the lowest level of complexity.

	Exercises		
	1–8, 17–27	9–13, 28, 29	14–16
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of al complexity levels to select appropriate exercises for you students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–8, 9–13 odd, 15, 16, 28, 29
OL	On Level	1–7 odd, 9–13, 15, 16, 28, 29
BL	Beyond Level	9–16, 28, 29

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	14
2 Reason abstractly and quantitatively.	12
3 Construct viable arguments and critique the reasoning of others.	16
4 Model with mathematics.	13, 15
7 Look for and make use of structure.	27

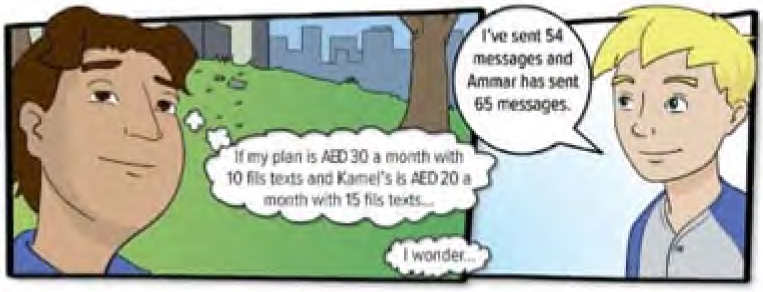
Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write a two-step equation and explain how to solve it. **See students' work.**

13. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.



a. The equation $50 = 28.10 + 0.15m$ represents the additonal number of messages Kamel can send with a budget of AED 50. Solve the equation to find the number of messages he has left to send. **146 messages**

b. The equation $50 = 36.50 + 0.10m$ represents the additional number of messages Ammar can send with a budget of AED 50. Solve the equation to find the number of messages he has left to send. **135 messages**

H.O.T. Problems Higher Order Thinking

14. **Persevere with Problems** Solve $(x + 5)(x + 5) = 49$.
(*Hint:* There are two solutions.)
−12 and 2

15. **Model with Mathematics** Write a real-world problem that could be solved by using the equation $3x - 25 = 125$. Then solve the equation.
Sample answer: I saved x dirhams each week for 3 weeks. I spent AED 25 and had AED 125 left. How much did I save each week?; AED 50

16. **Use a Counterexample** Determine if the following statement is *true* or *false*. If *false*, provide a counterexample.

An equation with an integer coefficient will always have an integer solution.

false; Sample answer: The coefficient of $-3x + 1 = 8$ is -3. However, its solution is $-\frac{7}{3}$, which is not an integer.

Name ______________________ My Homework ______________________

Extra Practice

Solve each equation. Check your solution.

17. $2h + 9 = 21$

Homework Help

$2h + 9 = 21$
$-9 = -9$
$\frac{2h}{2} = \frac{12}{2}$
$h = 6$

18. $12 - \frac{3}{5}p = -27$

$12 - \frac{3}{5}p = -27$
$-12 \quad = -12$
$-\frac{3}{5}p = -39$
$\left(-\frac{5}{3}\right)\left(-\frac{3}{5}p\right) = -39\left(-\frac{5}{3}\right)$
$p = 65$

19. $11 = 2b + 17$ **−3**

20. $-17 = 6p - 5$ **−2**

21. $2g - 3 = -19$ **−8**

22. $13 = \frac{g}{3} + 4$ **27**

23. $13 - 3d = -8$ **7**

24. $-\frac{2}{3}m - 4 = 10$ **−21**

25. $-5y - 25 = 25$ **−10**

26. Some friends decide to go to the aquarium together. Each person pays AED 7.50 to get in. They spend a total of AED 40 for the shark exhibit. The total cost is AED 70. Solve $7.5x + 40 = 70$ to find how many people went to the aquarium.

4 people

27. MP **Identify Structure** Shaker had AED 26 when he went to the fair. After playing 7 games, he had AED 15.50 left. Solve $15.50 = 26 - 7p$ to find the price for each game. Then list the Properties of Equality you used to solve the equation.

AED 1.50; Sample answer: Subtraction Property of Equality, Division Property of Equality

Power Up! Test Practice

Exercises 28 and 29 prepare students for more rigorous thinking needed for the assessment.

28.

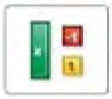
This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly model and solve the equation.
1 point	Students correctly model but fail to solve the equation OR students correctly solve the equation but make errors in modeling.

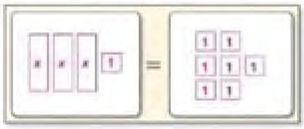
29.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

28. Use the algebra tiles to model the equation $3x + 1 = 7$ on the equation mat below. Then solve the equation.





$x =$ 2

29. Determine if the value of the variable is a solution of each equation. Select yes or no.

a. $5x - 4 = 31, x = 5.4$ ☐ yes ■ no

b. $\frac{3}{4}n + 4 = 10, n = 8$ ■ yes ☐ no

c. $-3 + 4y = 7, y = 2.5$ ■ yes ☐ no

Spiral Review

Solve each equation. Check your solution.

30. $t - 17 = 5$ **22**

31. $a - 5 = 14$ **19**

32. $9 = 5 + x$ **4**

Write and solve an equation for each of the following.

33. Sami is 9 years younger than his brother. His brother is 21. How old is Sami?
$s + 9 = 21$; 12 years

34. Ghada spent AED 45 more on boots than she did on a pair of jeans. She spent AED 79.50 on the boots. How much did she spend on the jeans?
$j + 45 = 79.50$; AED 34.50

35. The product of two integers is 72. If one integer is 18, what is the other integer? **$18x = 72$; 4**

Lesson 3

Write Two-Step Equations

Real-World Link

Robotics You want to attend a two-week robotic day camp that costs AED 700. Your parents will pay the deposit of AED 400 if you pay the rest in weekly payments of AED 15. Use the questions below to help you find the number of weeks you will need to make payments.

Essential Question

WHAT is equivalence?

Mathematical Practices
1, 2, 3, 4

1. Complete the table below. How much is paid after 2, 3, and 4 weeks?

Payments	Amount Paid
0	$400 + 15(0) = 400$
1	$400 + 15(1) = 415$
2	$400 + 15(2) = 430$
3	$400 + 15(3) = 445$
4	$400 + 15(4) = 460$

2. It will take a long time to solve the problem with a table. Instead, write and solve an equation to find the number of payments p you will need to make.

$400 + 15p = 700; p = 20$

3. How many payments will you make? 20

4. Suppose you received AED 75 in graduation money that you want to use towards the camp. Write and solve an equation to find the number of payments p you will need to make.

$400 + 75 + 15p = 700; 15$

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Write two-step equations that represent situatio

Coherence connecting within and across grades

Previous
Students solved two-step equations.

Now
Students write two-step equations that represent real-world situations.

Next
Students will mode solve equations wi variables on each s

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 133.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, s group, think-pair-share activity, or independent activity.

LA Team-Pair-Solo Have students work in s teams to complete Exercise 1, ensuring that ea student understands how to complete the table. Then ha them work in pairs to complete Exercise 2 and individual complete Exercise 3. Finally, have them rejoin their origin teams to discuss and compare solutions and complete Exercise 4. MP 1, 3, 5

Alternate Strategy

AL Help students write the equation in Exercise 2 by w the equation 400 + 15(__) = ___ on the board and havi them fill in the two blanks. MP 1, 5

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Translate sentences into equations.

AL
- *Which operation is indicated by the words "less than"?* **subtraction**
- *How would you represent "three times a number" using an algebraic expression?* $3n$

OL
- *If the variable is not given to you, how do you know what letter to use?* **Sample answer: You can choose any letter to represent "a number"**
- *How would you represent "eight less than three times a number" using an algebraic expression?* $3n - 8$

BL
- *Could we write the equation as* $8 - 3n = -23$*? Why or why not?* **no; Subtraction is not commutative.** $8 - 3n$ **represents eight less three times a number, not eight less *than* three times a number.**

Need Another Example?
Translate *three more than half a number is 15* into an equation. $\frac{1}{2}n + 3 = 15$

2. Translate sentences into equations.

AL
- *What operation do the words "more than" represent?* **addition**
- *How would you represent "one-fifth of a number" using an algebraic expression?* $\frac{1}{5}n$

OL
- *How would you represent "seven more than one-fifth of a number" using an algebraic expression?* $\frac{1}{5}n + 7$

BL
- *Could we write the equation as* $13 = 7 + \frac{1}{5}n$*? Explain.* **yes; Addition is commutative.**

Need Another Example?
Translate *nineteen is two more than five times a number* into an equation. $19 = 5n + 2$

Work Zone

Translate Sentences into Equations

There are three steps to writing a two-step equation.

Words	Describe the situation. Use only the most important words.
Variable	Define a variable to represent the unknown quantity.
Equation	Translate your verbal model into an algebraic equation.

You know how to write verbal sentences as one-step equations. Some verbal sentences translate into two-step equations.

Examples

Translate each sentence into an equation.

1. Eight less than three times a number is −23.

Words	Eight less than three times a number is −23.
Variable	Let n represent the number.
Equation	$3n - 8 = -23$

2. Thirteen is 7 more than one-fifth of a number.

Words	Thirteen is 7 more than one-fifth of a number.
Variable	Let n represent the number.
Equation	$13 = \frac{1}{5}n + 7$

STOP and Reflect

Name 3 words that indicate an addition statement.

Sample answer: more, total, sum

Got it? **Do these problems to find out.**

a. Fifteen equals three more than six times a number. $15 = 6n + 3$

b. Ten increased by the quotient of a number and 6 is 5. $10 + \frac{n}{6} = 5$

c. The difference between 12 and $\frac{2}{3}$ of a number is 18. $12 - \frac{2}{3}n = 18$

Expressions and Equations

Examples

3. **You buy 3 books that each cost the same amount and a magazine, all for AED 55.99. You know that the magazine costs AED 1.99. How much does each book cost?**

Words	Three books and a magazine cost AED 55.99.
Variable	Let b represent the cost of one book.
Equation	$3b + 1.99 = 55.99$

$3b + 1.99 = 55.99$ Write the equation.

$-1.99 = -1.99$ Subtraction Property of Equality

$3b = 54.00$ Simplify.

$\frac{3b}{3} = \frac{54.00}{3}$ Division Property of Equality

$b = 18$ Simplify.

So, the books each cost AED 18.

4. **A personal trainer buys a weight bench for AED 500 and w weights for AED 24.99 each. The total cost of the purchase is AED 849.86. How many weights were purchased?**

Words	Bench plus AED 24.99 per weight equals AED 849.86
Variable	Let w represent the number of weights.
Equation	$500 + 24.99 \cdot w = 849.86$

$500 + 24.99w = 849.86$ Write the equation.

$-500 \quad = -500$ Subtraction Property of Equality

$24.99w = 349.86$ Simplify.

$\frac{24.99w}{24.99} = \frac{349.86}{24.99}$ Division Property of Equality

$w = 14$ Simplify.

So, 14 weights were purchased.

Got it? **Do this problem to find out.**

d. The current temperature is 54°F. It is expected to rise 2.5°F each hour. In how many hours will the temperature be 84°F?

d. 12 h

Examples

3. Write and solve two-step equations.

AL
- *What variable can we use to represent the books?* Sample answer: b
- *What expression could be used to represent the total cost of the 3 books?* $3b$

OL
- *What expression could be used to represent the total cost of the books and the magazine?* $3b + 1.99$
- *What equation could be used to determine the cost of one book?* $3b + 1.99 = 55.99$
- *What are the two steps we can use to solve the equation?* First, subtract 1.99 from each side. Then divide each side by 3.

BL
- *Could we write the equation as $1.99 + 3b = 55.99$? Explain.* yes; Addition is commutative.

Need Another Example?

Fawzia paid AED 7 for her admission ticket to the fair and bought 12 ride tickets. She spent a total of AED 31 on admission and ride tickets. Write and solve an equation to determine the cost of one ride ticket. $7 + 12x = 31$; AED 2

4. Write and solve two-step equations.

AL
- *What expression could be used to represent the total cost of the weights?* $24.99w$

OL
- *What expression could be used to represent the total cost of the bench and the weights?* $500 + 24.99w$
- *What equation could be used to determine the cost of one weight?* $500 + 24.99w = 849.86$
- *What steps are used to solve the equation?* First, subtract 500 from each side. Then divide each side by 24.99.

BL
- *Could we write the equation as $24.99w + 500 = 849.86$? Explain.* yes; Addition is commutative.

Need Another Example?

Membership at the health club is AED 15 per month plus AED 10 for each exercise class taken. Fawzia paid AED 75 for the month of September. Write and solve an equation to determine the number of exercise classes Fawzia took in September.

$15 + 10c = 75$; 6 classes

Example

5. **Write and solve two-step equations.**

AL
- *What variable could be used to represent the cost of your friend's lunch?* **Sample answer: f**
- *What expression could be used to represent the cost of your lunch?* **$f + 3$**

OL
- *What expression could be used to represent the cost of both lunches altogether?* **$f + f + 3$ or $2f + 3$**
- *What equation could be used to determine the cost of both lunches?* **$f + f + 3 = 19$ or $2f + 3 = 19$**
- *Why can we write $f + f$ as $2f$?* **Sample answer: The variable f represents an unknown quantity. If we have two of the same unknown quantity, we have $2f$.**

BL
- *If $f = 8$, how much did you spend on your lunch?* **AED 11**

Need Another Example?
You and your friend spent a total of AED 33 for dinner. Your dinner cost AED 5 less than your friend's. Write and solve an equation to determine how much you spent for dinner.
$d + (d - 5) = 33$ or $2d - 5 = 33$; AED 14

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–5 and have them use yellow to highlight the word "is" for the equals sign. Have them use blue to highlight the variable and green to highlight its coefficient or divisor. Have them underline the number that is added or subtracted to the variable. MP 1, 5

BL LA **Trade-a-Problem** Have students write a sentence that can be translated into a two-step equation. Then have them trade their sentences with a partner. Each partner writes and solves the equation the other student wrote. MP 1, 4, 5

Defining the Variable
When the equation is solved, you can refer back to the definition of the variable to see if the question is answered or if additional steps are needed.

Example

5. **Your and your friend's lunch cost AED 69. Your lunch cost AED 3 more than your friend's. How much was your friend's lunch?**

Words	Your friend's lunch plus your lunch equals AED 69.
Variable	Let f represent the cost of your friend's lunch.
Equation	$f + f + 3 = 69$

$f + f + 3 = 69$ Write the equation.
$2f + 3 = 69$ $f + f = 2f$
$-3 = -3$ Subtraction Property of Equality
$2f = 66$ Simplify.
$\frac{2f}{2} = \frac{66}{2}$ Division Property of Equality
$f = 33$ Simplify.

Your friend spent AED 33.

Guided Practice

Translate each sentence into an equation. (Examples 1 and 2)

1. One more than three times a number is 7. **$3n + 1 = 7$**
2. Seven less than one-fourth of a number is -1. **$\frac{1}{4}n - 7 = -1$**
3. The quotient of a number and 5, less 10, is 3. **$\frac{n}{5} - 10 = 3$**
4. You already owe AED 4.32 in overdue rental fees and are returning a movie that is 4 days late. Now you owe AED 6.48. Define a variable. Then write and solve an equation to find the daily fine for an overdue movie. (Examples 3–5)
d = the daily fine; $4.32 + 4d = 6.48$; AED 0.54
5. **Building on the Essential Question** Why is it important to define a variable before writing an equation?
Sample answer: Assigning a variable to the unknown helps you when translating the verbal model into an algebraic equation.

Rate Yourself!
- ☐ I understand how to write two-step equations.
  Great! You're ready to move on!
- ☐ I still have some questions about writing two-step equations.

Name: ______________________ My Homework ______________________

Independent Practice

Translate each sentence into an equation. (Examples 1 and 2)

1. Four less than five times a number is equal to 11. $5n - 4 = 11$
2. Fifteen more than half a number is 9. $\frac{1}{2}n + 15 = 9$
3. Six less than seven times a number is equal to −20. $7n - 6 = -20$
4. Eight more than four times a number is −12. $4n + 8 = -12$

Define a variable. Then write and solve an equation to solve each problem. (Examples 3–5)

5. **Financial Literacy** The cost for a certain music plan is AED 9.99 per year plus AED 0.25 per song you download. If you paid AED 113.74 one year, find the number of songs you downloaded. s = the number of songs; $0.25s + 9.99 = 113.74$; 415 songs

6. Amira has saved AED 725 for a new guitar and lessons. Her guitar costs AED 475, and guitar lessons are AED 25 per hour. Determine how many hours of lessons she can afford. x = the number of hours; $475 + 25x = 725$; 10 hours

7. From ground level to the tip of the torch, the Statue of Liberty and its pedestal are 92.99 meters tall. The pedestal is 0.89 meter taller than the statue. How tall is the Statue of Liberty?
s = height of the Statue of Liberty; $s + (s + 0.89) = 92.99$; 46.05 m

8. **Reason Abstractly** Azza would like to take snowboarding lessons at Ski Dubai. She has saved AED 920 for lessons and a junior season pass. How many more semi-private lessons than private lessons can she take? 5 lessons

Ski Dubai Snowboarding Lessons	
Semi-Private	AED 140/lesson
Private	AED 300/lesson
Junior Season Pass	AED 220

ENGAGE EXPLORE **EXPLAIN ELABORATE EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–7, 14–19	8, 9, 20–22	10–13
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 10, 12, 13, 21, 22
OL	On Level	1–7 odd, 8–10, 12, 13, 21, 22
BL	Beyond Level	8–13, 21, 22

Watch Out!

Common Error Students may have difficulty translating word problems into expressions and equations. You may wish to hav students make a list of words that relate to the signs in an equation, such as "is" for "=" or "more than" for "+". Have the refer to this list when translating.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	11
2 Reason abstractly and quantitatively.	8
3 Construct viable arguments and critique the reasoning of others.	20
4 Model with mathematics.	10, 12, 13

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write and solve a two-step equation for the following situation: An appliance repairman charges AED 35 for a house call and AED 30 per hour. The total cost of a house call and repair job was AED 125. How many hours did the repair job take? $30h + 35 = 125$; $h = 3$

9. When diving, the peregrine falcon can reach speeds of up to 175 miles per hour. Write and solve equations to find each of the following.

a. The top speed of a peregrine falcon is 20 miles per hour less than three times the top speed of a cheetah. What is the cheetah's top speed? $175 = 3c - 20$; 65 mph

b. A sailfish can swim up to 1 mile per hour less than one fifth the top speed of a peregrine falcon. Find the top speed that a sailfish can swim. $s = \frac{1}{5} \cdot 175 - 1$; 34 mph

c. The peregrine falcon can reach speeds about 13 miles per hour more than 6 times the speed of the fastest human. What is the approximate top speed of the fastest human? $175 = 6h + 13$; 27 mph

H.O.T. Problems Higher Order Thinking

10. **Model with Mathematics** If 12 less than 4 times a number is 8, the number is 5. Write a different sentence where the unknown number is also 5. Sample answer: 6 times a number plus 5 equals 35.

11. **Persevere with Problems** The ages of three siblings combined is 27. The oldest is twice the age of the youngest. The middle child is 3 years older than the youngest. Write and solve an equation to find the ages of each sibling. $n + 2n + (n + 3) = 27$; 6, 9, 12

12. **Model with Mathematics** Write about a real-world situation that can be solved using a two-step equation. Then write the equation and solve the problem. Sample answer: Renting a locker at the gym costs AED 7 a week. You get a AED 4 discount when you return the key. If your total cost is AED 24 and you returned the key, how many weeks did you rent a locker? $7x - 4 = 24$; 4 weeks

13. **Model with Mathematics** Describe two real-world situations that can be represented by the same two-step equation. See students' work.

Situation 1: ______

Situation 2: ______

Name ______________________ My Homework ______________________

Extra Practice

Translate each sentence into an equation.

14. Twenty-two less than three times a number is −70. $3n - 22 = -70$

Words: Twenty-two less than three times a number is −70.

Variable: Let n represent the number.

Equation: $3n - 22 = -70$

15. The product of a number and 4 increased by 16 is −2. $4n + 16 = -2$

16. Twelve less than the one-fifth of a number is −7. $\frac{1}{5}n - 12 = -7$

17. Six more than nine times a number is 456. $6 + 9n = 456$

Define a variable. Then write and solve an equation to solve each problem.

18. It costs AED 13 for admission to an amusement park, plus AED 1.50 for each ride. If you have a total of AED 35.50 to spend, what is the greatest number of rides you can go on? r = the number of rides; $13 + 1.50r = 35.50$; 15 rides

19. Maher went to the batting cages to practice hitting. He rented a helmet for AED 4 and paid AED 0.75 for each group of 20 pitches. If he spent a total of AED 7 at the batting cages, how many groups of pitches did he pay for?

x = the number of groups of pitches; $4 + 0.75x = 7$; 4 groups

20. **Make a Conjecture** Majed and Basam are each trying to save AED 600 for a summer trip. Majed started with AED 150 and earns AED 7.50 per hour working at a grocery store. Basam has nothing saved, but he earns AED 12 per hour painting houses.

a. Make an argument about who will take longer to save enough money for the trip. Justify your reasoning. Sample answer: Majed; although he has money saved, he makes considerably less per hour than Basam. So, he will have to work longer.

b. Write and solve two equations to check your conjecture.

$7.50h + 150 = 600$; 60 h; $12h = 600$; 50 h

Power Up! Test Practice

Exercises 21 and 22 prepare students for more rigorous thinking needed for the assessment.

21.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

22.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly select the correct equations and correctly solve the equations.
1 point	Students correctly select and solve one of the equations OR students select the correct equations but solve only one correctly.

Power Up! Test Practice

21. Use the figure to fill in each blank to make a true statement.
The simplified expression for the perimeter of the rectangle is **6w + 6**.
An equation that can be used to find w is **6w + 6 = 36**. The width of the rectangle is **5 units**.

22. Model each situation below with an equation. Select the correct equation for each situation. Then solve each problem.

a. A company employs 72 workers. It plans to increase the number of employees by 6 per month until it has twice its current workforce. How many months will it take to double the number of employees?

$6m + 72 = 96$	$144 - 72m = 6$
$6m + 72 = 144$	$144 - 6m = 96$

Equation: **6m + 72 = 144** Solution: **12 months**

b. Fahed's fish tank holds 144 gallons of water. To clean the tank, he drains the water at a rate of 6 gallons per minute until the level is two-thirds its original level. How many minutes does it take to drain the tank for cleaning?

Equation: **144 − 6m = 96** Solution: **8 min**

Spiral Review

Solve each equation. Check your solution.

23. $\frac{y}{7} = 22$ **154**

24. $\frac{g}{6} = -108$ **−648**

25. $-6 = \frac{n}{8} + 1$ **−56**

26. $-15 = -4p + 9$ **6**

27. In a recent football game, the Dubai Ahli team scored 14 points less than the Baniyas. Write and solve an equation to find the total points the Baniyas scored. **p − 14 = 17; 31 points**

Preseason Week 4	
Team	Total Points
Ahli	17
Baniyas	p

Problem-Solving Investigation

Work Backward

Mathematical Practices 1, 4, 7

Case #1 Game Switcheroo!

Qasem and Adel traded video games. Adel gave Qasem one fourth of his video games in exchange for 6 video games. Then he sold 3 video games and gave 2 video games to his brother. Adel ended up with 16 video games.

How many video games did Adel have when he started?

1 Understand *What are the facts?*

- Adel now has 16 games.
- He gave some away, sold some, and traded some.

2 Plan *What is your strategy to solve this problem?*

Start with the ending number of video games, 16, and work backward.

- Start: Adel has 16 video games. → 16
- Add the games he gave to his brother. + 2 → 18
- Add the games he sold. + 3 → 21
- Subtract the games he traded with Qasem. − 6 → 15
- This number is $\frac{3}{4}$ of his games. × $\frac{4}{3}$ → 20 End

3 Solve *How can you apply the strategy?*

So, Adel had 20 video games at the beginning.

4 Check *Does the answer make sense?*

Start with 20. Perform operations in reverse order.

Analyze the Strategy

Identify Structure How is working backward similar to solving an equation?

When you solve an equation, you work backward through the order of operations.

Focus narrowing the scope

Objective Solve problems by working backward.
This lesson emphasizes **Mathematical Practice 7** Identify Structure.

Work Backward Working backward has a strong link to solving equations. In many equations, inverse operations may be used to solve for a variable. Addition and subtraction are inverse operations, as are multiplication and division.

Coherence connecting within and across grades

Now
Students solve non-routine problems.

Next
Students will apply the work backward strategy to solve equations with variables on each side.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 139.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

The problems on pages 137 and 138 are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance.

Case #1 Game Switcheroo!

BL Extend the problem by asking the questions below.

- *Write an equation that can be used to represent the problem described in Game Switcheroo! Define what the variable represents.* Sample answer: $g - \frac{1}{4}g + 6 - 3 - 3 = 16$ or $\frac{3}{4}g + 1 = 16$; g represents the number of video games Alex had when he started
- *What types of problems can best be solved by the work backward strategy?* Sample answer: when the problem gives the end results and you need to find the initial conditions

Case #2 Shoot the Rapids

AL LA **Popcorn Share** Ask the questions below and give students time to think. Then call, "Popcorn," and have students quickly and voluntarily pop up from their chairs one at a time to share an answer aloud to the class. Seated students should write answers and mark incorrect answers. MP 1, 3

Ask:

- *What information do you need to know to solve the problem?* **Sample answer: I need to know how much money was raised and what portion each person donated.**
- *What is the first operation you need to do to solve the problem?* **Divide the total by 4.**

BL LA **Value Line** Ask students how well they understand the *work backward* strategy. Then, have students place themselves on a pretend line where 10 represents that they understand completely and 1 represents that they do not understand. Pair the students, matching #1 with #10 and so on, for discussion and clarification. MP 1, 3, 5

Need Another Example?

On Monday morning, Manal spent $\frac{3}{5}$ of her money for an art class. She then bought paints and brushes for AED 85 and collected AED 35 for her pet walking business. At the end of the day she had AED 150. How much money did she have when she began the day? **AED 500**

Case #2 Shoot the Rapids

Sana raised money for a white water rafting trip. Manal made the first donation. Khalid's donation was twice Manal's donation. Zahra's mother tripled what Sana had raised so far. Now Sana has AED 120.

How much did Manal donate?

1 Understand

Read the problem. What are you being asked to find?

I need to find **the amount Manal gave**

Underline key words and values. What information do you know?

Manal donated **first**, Khalid **doubled Manal's donation** and Zahra's mother **tripled** the whole amount collected.

Is there any information that you do *not* need to know?

I do not need to know **that the money is for a rafting trip**.

2 Plan

Choose a problem-solving strategy.

I will use the **work backward** strategy.

3 Solve

Use your problem-solving strategy to solve the problem.

Sana raised a total of **AED 120**.

Go Back Divide that amount by 4. One part is Khalid's and Manal's donation and three parts is the amount donated by Zahra's mother. AED 120 ÷ 4 = **30**

Go Back Divide that amount by 3. One part is for Manal's donation and two parts is the amount that Khalid donated. **AED 30** ÷ 3 = **AED 10**

Manal was the first to donate. So, Manal donated **AED 10**.

4 Check

Use information from the problem to check your answer.

Begin with AED 10 and perform operations in reverse. **AED 10** × 2 = AED 20; AED 20 + AED 10 = **AED 30**, **AED 30** × 3 = AED 90; **AED 90** + **AED 30** = **AED 120**.

138 Chapter 2 Equations in One Variable

Work with a small group to solve the following cases.
Show your work on a separate piece of paper.

Case #3 Financial Literacy

Ruqaya has AED 75. She buys jeans that are discounted 30% and then uses an in-store coupon for AED 10 off the discounted price. After paying AED 3.48 in sales tax, she receives AED 17.34 in change.

What was the original price of the jeans?

AED 86.35

Case #4 Schedule

Amira needs to be at school at 7:45 A.M. It takes her 15 minutes to walk to school, $\frac{5}{12}$ hour to eat breakfast, 0.7 hour to get dressed, and 0.15 hour to shower.

At what time should Amira get up to be at school [illegible] minutes early?

6:09 A.M.

Case #5 Financial Literacy

At the end of the month, Mr. Rashed has AED 1,473.61 in his checking account. His checkbook showed the following transactions.

If he made an initial deposit of AED 75.00 that is not shown in his checkbook, what was his balance at the beginning of the month?

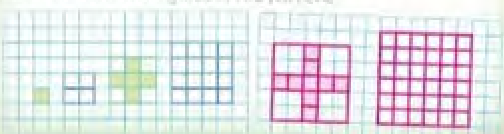
Chk No.	Date	Payment or Withdrawal	Deposit
		[illegible]	[illegible]
132		[illegible]	
		[illegible]	
133		[illegible]	
			[illegible]

AED 1,163.50

Case #6 Geometry

Study the pattern below.

Draw the next two figures in the pattern.



Use any strategy!

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** EVALUATE

2 Collaborate

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 3, 4	Exercises 5	Exercises 6
Level 3			•
Level 2		•	
Level 1	•		

AL LA Roundrobin Have students work in pairs to complete Case 6. Each member of the pair takes turns extending the pattern, speaking to their partner and explaining the step. Challenge them to draw the seventh and eighth figures. MP 1, 3, 8

BL LA Trade-a-Problem Have students create their own problem using the *work backward* strategy. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together, listening carefully, to determine the errors. MP 1, 3, 4

Mid-Chapter Check

If students have trouble with Exercises 1–9, they may need help with the following concepts.

Concept	Exercise(s)
solving equations with rational coefficients (Lesson 1)	1, 3, 4, 5
solving two-step equations (Lesson 2)	2, 6, 7, 8
writing two-step equations (Lesson 3)	9

Vocabulary Activity

LA **Think-Pair-Share** Have students work in pairs to complete Exercises 1 and 2. Give them about one minute to individually think through their response. Then have them share their responses with a partner. Call on one set of pairs to share their responses with the class. MP 1, 3, 6

Alternate Strategies

AL LA Provide students with several pairs of multiplicative inverses, each written on separate cards. Have students match each number with its multiplicative inverse. Then have students explain to a partner what it means for two numbers to be multiplicative inverses.

BL LA Have students think about what it means to say two numbers are multiplicative inverses. Then have them discuss whether or not all rational numbers have a multiplicative inverse.

Mid-Chapter Check

Vocabulary Check

1. **Be Precise** Define *multiplicative inverse*. Give an example of a number and its multiplicative inverse. (Lesson 1)
   Sample answer: The product of a number and its multiplicative inverse is 1; $\frac{2}{3} \cdot \frac{3}{2} = 1$

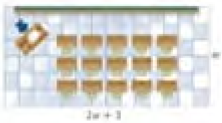
2. Fill in the blank in the sentence below with the correct term. (Lesson 2)
   The first step in solving the equation $3x + 4 = 20$ is to subtract 4 from each side. This is an example of the Subtraction Property of Equality.

Skills Check and Problem Solving

Solve each equation. Check your solution. (Lessons 1-2)

3. $\frac{2}{3}x = -8$ **−12**
4. $-4.5 = -0.15p$ **30**
5. $2\frac{1}{3}c = 2\frac{1}{10}$ $\frac{9}{10}$
6. $3m + 5 = 14$ **3**
7. $-2k + 7 = -3$ **5**
8. $11 = \frac{1}{3}u + 2$ **27**

9. **Persevere with Problems** A diagram of a room is shown. If the perimeter of the room is 78 feet, what is the area of the floor of the room? (Lesson 3)
   324 ft²



Inquiry Lab

Equations with Variables on Each Side

Inquiry **HOW do you use the Properties of Equality when solving an equation using algebra tiles?**

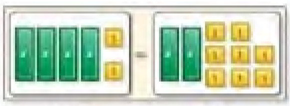
MP Mathematical Practices 1, 3, 5

Fatima bought 4 pens and a bottle of nail color. Her sister bought 2 of the same pens and 4 bottles of nail color, and spent the same amount as Fatima. The nail color cost AED 2. Use algebra tiles to find the cost of each pen.

Hands-On Activity 1

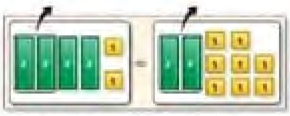
The equation $4x + 2 = 2x + 8$ represents the real-world situation above. Use algebra tiles to model and solve the equation.

Step 1 Model the equation.



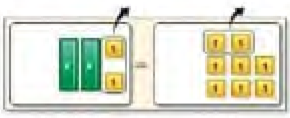
$4x + 2 = 2x + 8$

Step 2 Remove 2 x-tiles from each side of the mat until there are x-tiles on only one side.



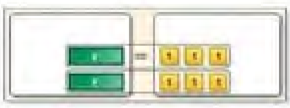
$4x - 2x + 2 = 2x - 2x + 8$

Step 3 Remove 2 1-tiles from each side of the mat until the x-tiles are by themselves on one side.



$2x + 2 - 2 = 8 - 2$

Step 4 Separate the tiles in 2 equal groups.



$2x = 6$

Check $4 \cdot 3 + 2 \stackrel{?}{=} 2 \cdot 3 + 8$

$14 = 14$ ✓

So, each pen costs AED 3.

Focus narrowing the scope

Objective Solve equations with variables on each side using algebra tiles.

Coherence connecting within and across grades

Now
Students model and solve equations with variables on each side of the equals sign.

Next
Students will solve equations with variables on each side of the equals sign.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 143.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

Materials: algebra tiles, equation mats

Hands-On Activity 1

AL LA **Cooperative Play** Have students go online to access virtual manipulative algebra tiles or provide them with concrete algebra tiles. Ask the students how algebra tiles might be useful in modeling and solving equations with variables on each side of the equal sign. Allow them time to play and experiment. MP 1, 5

BL Have students provide justification for each step. For example, in Step 2, have them justify why removing 2 x-tiles from each side assists in the process to solve the equation. MP 1, 3

Hands-On Activity 2

AL LA Mirror Mirror Have students work with a partner to complete Activity 2. One student reads each step aloud, while the other student models the action using virtual or concrete manipulatives. Then the first student draws the tiles in the blank mats given in the text. MP 1, 3, 5

Ask:

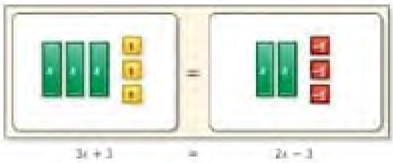
- *How would you model the equation $3x + 3 = 2x - 3$ using algebra tiles?* **Place three x-tiles and three 1-tiles on the left side of an equation mat, and two x-tiles and three -1-tiles on the right side.**
- *What do you need to do next so that the x-tiles are only on one side of the equation?* **Since there are three x-tiles one side and two x-tiles on the other side, remove two x-tiles from each side of the mat.**
- *What does removing the two x-tiles represent in the equation?* **subtracting $2x$ from each side of the equation**
- *What is left on the mat?* **one x-tile and three 1-tiles on the left and three -1-tiles on the right**
- *What do we need to do next?* **Remove the 1-tiles from the left side of the mat so that the x-tile is by itself.**
- *Can you remove three 1-tiles from each side of the mat? Explain.* **no; The 1-tiles are only on the left side of the mat. In order to remove them from each side, we need to add three -1-tiles to each side.**
- *What is left on the mat?* **one x-tile on the left and six -1-tiles on the right**

BL Have students rewrite the equation in the activity so that it is a one-step equation. Have them justify why the equation they rewrote represents the same equation in the activity. MP 1, 3

Hands-On Activity 2

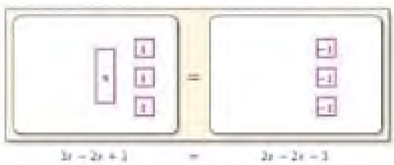
Use algebra tiles to model and solve $3x + 3 = 2x - 3$. Draw the tiles in the blank mats shown. The first step is done for you.

Step 1 Model the equation.

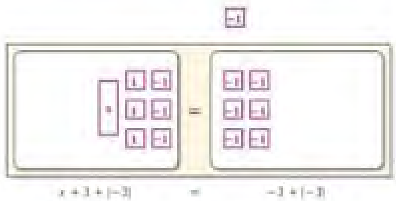


6

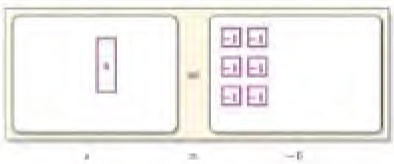
Step 2 Remove 2 x-tiles from each side of the mat in Step 1 so that there is an x-tile by itself on the left side. Draw the tiles that remain.



Step 3 To isolate the x-tile, it is not possible to remove the same number of 1-tiles from each side of the mat. Add three -1-tiles to each side of the mat. Draw the tiles.



Step 4 Remove the zero pairs from the left side. There are six -1-tiles on the right side of the mat. The x-tile is isolated on the left side of the mat. Draw the tiles that remain.



So, $x =$ **-6**.

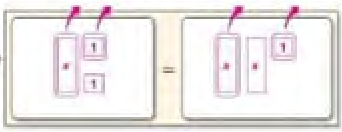
Check $3(-6) + 3 \stackrel{?}{=} 2(-6) - 3$

$-15 = -15$ ✓

Investigate

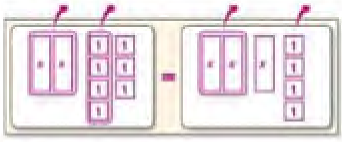
Use Math Tools Work with a partner. Model and solve each equation. Show your work using drawings. Write the solution below the mat.

1. $x + 2 = 2x + 1$



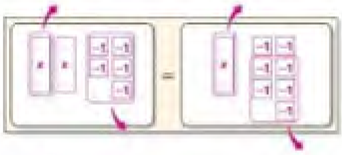
$x = 1$

2. $2x + 7 = 3x + 4$



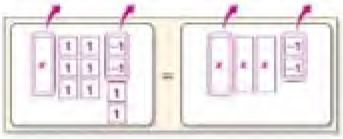
$x = 3$

3. $2x - 5 = x - 7$



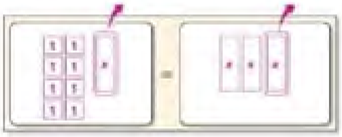
$x = -2$

4. $x + 6 = 3x - 2$



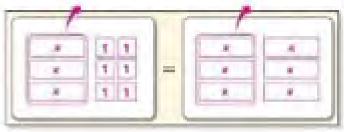
$x = 4$

5. $8 + x = 3x$



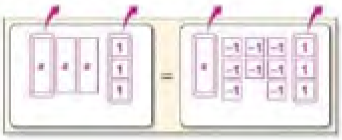
$x = 4$

6. $3x + 6 = 6x$



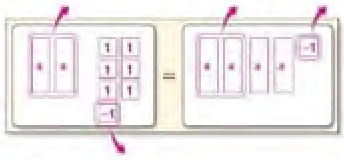
$x = 2$

7. $3x + 3 = x - 5$



$x = -4$

8. $2x + 5 = 4x - 1$



$x = 3$

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–8	Exercises 9–12	Exercises 13–15
Level 3			●
Level 2		●	
Level 1	●		

Investigate

AL LA Team-Pair-Solo Have students work in teams of four to complete Exercises 1–3, then work with a partner to complete Exercises 4–6. Have them complete Exercises 7 and 8 individually and then check their answers with their original partner or team. MP 1, 5

BL LA Pairs Discussion Have students complete Exercises 1–8 without the use of manipulatives. Ask them to say each step aloud, such as "subtract x from each side" and write what the new equation would be. Have them draw what the final step would look like if they did use manipulatives. MP

Analyze and Reflect

AL LA **Pairs Discussion** For Exercises 9 and 10, have students work with a partner. Following the directions, have one student solve the equation by removing the 1-tiles first, and the other partner solve by removing the *x*-tiles first. Compare answers. Then, have students write a paragraph explaining the steps they used to solve the equation and why both partners had the same solutions even though they followed different steps. MP 1, 3, 5

Create

BL LA **Trade-a-Problem** Have students trade the real-world problem they wrote for Exercise 13 with a partner and solve each other's problem. Have them respond to the questions below. MP 1, 3, 4

Ask:

- *Does the story problem make sense in the real world?* See students' work.
- *What does the variable x represent?* See students' work.

Inquiry Students should be able to answer "HOW do you use the Properties of Equality when solving an equation using algebra tiles?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

Work with a partner. One of you should solve the following equations by removing 1-tiles first. The other one should solve the equations by removing *x*-tiles first. Compare your answers. See students' work for solutions.

9. $x + 4 = 3x - 4$

$x =$ 4

10. $4x + 2 = x - 4$

$x =$ −2

11. MP **Reason Inductively** Does it matter whether you remove *x*-tiles or 1-tiles first? Is one way more convenient? Explain. You can remove either tile first. The order in which you add quantities to each side of an equation does not affect its solution, however, it may be more convenient to isolate the variable first.

12. MP **Use Math Tools** Explain why you can remove an *x*-tile from each side of the mat. The value of *x* is the same on each side of the mat.

Create

13. MP **Identify Structure** Write a real-world problem that could be represented by the equation $x + 4 = 3x - 4$. Then use algebra tiles to find a solution to your problem. Sample answer: Grace and Mateo are the same age. Grace is 4 years older than Axel. Mateo age is four years less than three times Axel's age. How old is Axel?; 4 years

14. MP **Identify Structure** Pizza Shack charges AED 8 per pizza with a AED 4 delivery fee. Pizza on the Plaza charges AED 10 per pizza, but does not charge a delivery fee. Write an equation that could be used to find the number of pizzas for which the cost, including delivery, will be the same. Then use algebra tiles to find the solution. $8x + 4 = 10x$; 2 pizzas

15. Inquiry HOW do you use the Properties of Equality to solve an equation using algebra tiles? Sample answer: The Properties of Equality allow you to add or remove tiles from each side and to divide tiles into groups.

Lesson 4

Solve Equations with Variables on Each Side

Real-World Link

Cell Phones A wireless company offers two cell phone plans. Plan A charges AED 24.95 per month plus AED 0.10 per minute for calls. Plan B charges AED 19.95 per month plus AED 0.20 per minute. Use the questions to find when the two plans cost the same.

1. Complete the table.

Minutes (m)	Plan A 24.95 + 0.10m	Plan B 19.95 + 0.20m
10	25.95	21.95
20	26.95	23.95
30	27.95	25.95
40	28.95	27.95
50	29.95	29.95
60	30.95	31.95
70	31.95	33.95

2. For what value(s) does Plan A cost less?
values greater than 50 min

3. For what value(s) does Plan B cost less?
values less than 50 min

4. For what value(s) do both Plans cost the same?
50 min

Essential Question

WHAT is equivalence?

Mathematical Practices 1, 3, 4

Which MP **Mathematical Practices** did you use?
Shade the circle(s) that applies.

1 Persevere with Problems
2 Reason Abstractly
3 Construct an Argument
4 Model with Mathematics
5 Use Math Tools
6 Attend to Precision
7 Make Use of Structure
8 Use Repeated Reasoning

Focus narrowing the scope

Objective Solve equations with variables on each side

Coherence connecting within and across grade

Previous
Students modeled and solved equations with variables on each side of the equals sign.

Now
Students solve equations with variables on each side of the equals sign.

Next
Students will ... step equation

Rigor pursuing concepts, fluency, and applicati

See the Levels of Complexity chart on page 149.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUAT

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole gro group, think-pair-share activity, or independent activi

LA Roundrobin Have students work in pa complete Exercises 1–4. For Exercise 1, have fill in the first row of the table, Student 2 fills in the se Student 1 fills in the third row, and so on. Have stude together to complete Exercises 2–4. Have them justi responses using their answers in the table. MP 1, 3, 5

Alternate Strategy

BL Have students write an equation that can be us determine when both plans will cost the same. MP 1,

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions to differentiate instruction.

Examples

1. Solve equations with variables on each side.

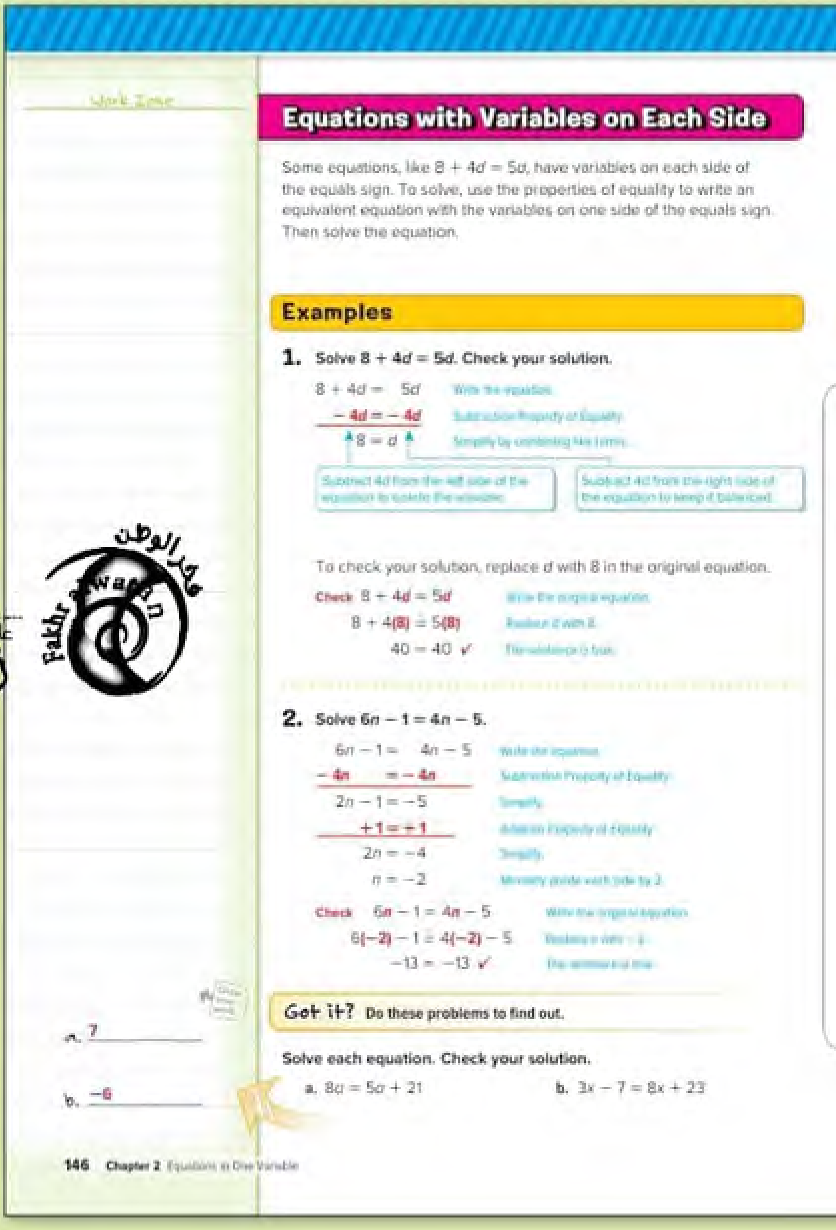
- AL • *Circle the expressions 4d and 5d. Explain their location with respect to the equals sign.* **They are on opposite sides of the equals sign.**
- OL • *Why is it important to isolate the variable?* **Sample answer: It is easier to solve the equation if the variables are all on one side of the equation.**
  • *How can you check to make sure your solution is correct?* **Plug the solution back in to the original equation to check that the equation is true.**
- BL • *Why do we not subtract 5d from each side?* **We would be left with 0 on the right side. We need to have the variables on one side and the constants on the other side.**

Need Another Example?
Solve $7x + 4 = 9x$. Check your solution. **2**

2. Solve equations with variables on each side.

- AL • *Identify the like terms and the constants in the equation $6n - 1 = 4n - 5$.* **like terms: 6n and 4n, −1 and −5; constants: −1, −5**
- OL • *What is the first step to solve the equation?* **Subtract 4n from each side.**
  • *What is the next step?* **Add 1 to each side.**
  • *What is the third step?* **Divide each side by 2.**
- BL • *Could we have started our first step by doing something differently? Explain.* **Sample answer: yes; We could have subtracted 6n from each side. This would result in the equation $-1 = -2n - 5$. Then we could add 5 to each side. Finally, we could divide each side by −2. The solution is the same, $n = -2$.**

Need Another Example?
Solve $3x - 2 = 8x + 13$. Check your solution. **−3**



Work Zone

Equations with Variables on Each Side

Some equations, like $8 + 4d = 5d$, have variables on each side of the equals sign. To solve, use the properties of equality to write an equivalent equation with the variables on one side of the equals sign. Then solve the equation.

Examples

1. Solve $8 + 4d = 5d$. Check your solution.

$8 + 4d = 5d$
$-4d = -4d$
$8 = d$

To check your solution, replace d with 8 in the original equation.

Check $8 + 4d = 5d$
$8 + 4(8) \stackrel{?}{=} 5(8)$
$40 = 40$ ✓

2. Solve $6n - 1 = 4n - 5$.

$6n - 1 = 4n - 5$
$-4n = -4n$
$2n - 1 = -5$
$+1 = +1$
$2n = -4$
$n = -2$

Check $6n - 1 = 4n - 5$
$6(-2) - 1 \stackrel{?}{=} 4(-2) - 5$
$-13 = -13$ ✓

Got it? Do these problems to find out.

Solve each equation. Check your solution.

a. $8a = 5a + 21$ b. $3x - 7 = 8x + 23$

a. 7
b. −6

Example

3. **Green's Gym charges a one time fee of AED 50 plus AED 30 per session for a personal trainer. A new fitness center charges a yearly fee of AED 250 plus AED 10 for each session with a trainer. For how many sessions is the cost of the two plans the same?**

Words	a fee of AED 50 plus AED 30 per session	is the same as	a fee of AED 250 plus AED 10 per session.
Variable	Let s represent the number of sessions.		
Equation	$50 + 30s = 250 + 10s$		

$50 + 30s = 250 + 10s$ — Write the equation.

$\underline{\quad -10s = \quad -10s}$ — Subtraction Property of Equality

$50 + 20s = 250$ — Simplify.

$\underline{-50 \quad = -50}$ — Addition Property of Equality

$20s = 200$ — Simplify.

$\frac{20s}{20} = \frac{200}{20}$ — Division Property of Equality

$s = 10$ — Simplify.

So, the cost is the same for 10 personal trainer sessions.

Check

Green's Gym: AED 50 plus 10 sessions at AED 30 per session

$50 + 10 \cdot 30 = 50 + 300$
$= \text{AED } 350$

new fitness center: AED 250 plus 10 sessions at AED 10 per session

$250 + 10 \cdot 10 = 250 + 100$
$= \text{AED } 350$ ✓

Got it? **Do this problem to find out.**

c. The length of a flag is 0.3 foot less than twice its width. If the perimeter is 14.4 feet longer than the width, find the dimensions of the flag.

c. $w = 3$ ft; $\ell = 5.7$ ft

Equations with Rational Coefficients

In some equations, the coefficients of the variables are rational numbers. Remember when working with fractions, you need to have a common denominator before you add or subtract.

Example

3. **Write an equation to represent a real-world problem.**

AL
- *What is the cost per session for a personal trainer at each gym?* **Green's Gym: AED 30 per session; new fitness center: AED 10 per session**
- *What variable can we use to represent the number of sessions?* **Sample answer: s**

OL
- *Write an expression to represent the total cost for Green's Gym.* **50 + 30s**
- *Write an expression to represent the total cost for the new fitness center.* **250 + 10s**
- *What equation can be used to find the number of sessions in which the costs will be the same?* **50 + 30s = 250 + 10s**
- *Why is the first step to solving the equation to subtract 10s from each side?* **to isolate the variable**

BL
- *For how many sessions is the cost the same? For what range of sessions does it cost less to go to Green's Gym?* **For 10 sessions, the cost is the same. For 0–9 sessions, it costs less to go to Green's Gym.**

Need Another Example?

The measure of an angle is 8 degrees more than its complement. If x represents the measure of the angle and $90 - x$ represents the measure of its complement, what is the measure of the angle? **49°**

Example

4. **Solve equations with rational coefficients.**

- AL • *Why do we write $\frac{2}{3}x$ as $\frac{4}{6}x$?* **to obtain a common denominator with $\frac{1}{6}x$**
- OL • *What is the first step to solve the equation?* **Add $\frac{1}{6}x$ to both sides.**
  - *What is the next step?* **Add 1 to each side.**
  - *What is the third step?* **Multiply both sides by $\frac{6}{5}$.**
- BL • *Could we have started our first step by doing something differently? Explain.* **Sample answer: yes; We could have subtracted $\frac{2}{3}x$ from both sides.**

Need Another Example?

Solve $\frac{3}{4}x + 2 = 7 + \frac{1}{3}x$. **12**

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL BL LA **Team-Pair-Solo** Have students complete Exercises 1 and 4 in a small team, making sure there is at least one Approaching Level student and at least one Beyond Level student in each team. Then have students work in pairs to complete Exercises 2 and 3. Have them answer Exercise 5 on their own. Then have them rejoin their original team to compare solutions to Exercises 2, 3, and 5. Have them discuss and resolve any differences. Call on one randomly selected student from each team to share their team's response to each exercise. MP 1, 3

Example

4. Solve $\frac{2}{3}x - 1 = 9 - \frac{1}{6}x$.

$\frac{4}{6}x - 1 = 9 - \frac{1}{6}x$ — The common denominator of the coefficients is 6. Rewrite the equation.

$+\frac{1}{6}x \quad = \quad +\frac{1}{6}x$ — Addition Property of Equality

$\frac{5}{6}x - 1 = 9$ — Simplify.

$+1 = +1$ — Addition Property of Equality

$\frac{5}{6}x = 10$ — Simplify.

$\left(\frac{6}{5}\right)\frac{5}{6}x = 10\left(\frac{6}{5}\right)$ — Multiplication Property of Equality

$x = 12$ — Simplify.

Got it? Do these problems to find out.

e. $\frac{1}{2}p + 7 = \frac{3}{4}p + 9$

f. $-\frac{5}{4}c - \frac{1}{2} = -\frac{3}{4} + \frac{5}{8}c$

e. **−8**

f. **$\frac{2}{15}$**

Guided Practice

Solve each equation. Check your solution.

1. $5n + 9 = 2n$ **−3**
2. $7y - 8 = 6y + 1$ **9**
3. $\frac{3}{5}x - 15 = \frac{6}{5}x + 12$ **−45**
4. A Car Rental charges AED 40 a day plus AED 0.25 per mile. Al Rashed Rent-A-Car charges AED 25 a day plus AED 0.45 per mile. What number of miles results in the same cost for one day? **75 mi**
5. **Building on the Essential Question** How is solving an equation with the variable on each side similar to solving a two-step equation? **Sample answer: Solving any equation uses the properties of equality. Solving an equation with the variable on each side usually adds an additional step to solving a two-step equation.**

Rate Yourself!

How well do you understand how to solve equations? Circle the figure that applies.

Clear — Somewhat Clear — Not So Clear

148 **Chapter 2** Equations in One Variable

Name ______________________ My Homework ______________________

Independent Practice

Solve each equation. Check your solution. (Examples 1, 2, 4)

1. $7a + 10 = 2a$ **−2**

2. $11x = 24 + 8x$ **8**

3. $8y - 3 = 6y + 17$ **10**

4. $5p + 2 = 4p - 1$ **−3**

5. $15 - \frac{1}{6}n = \frac{1}{6}n - 1$ **48**

6. $3 - \frac{2}{9}b = \frac{1}{3}b - 7$ **18**

7. Nine fewer than half a number is five more than four times the number. Define a variable, write an equation, and solve to find the number. (Example 3)
Let n = the number; $0.5n - 9 = 4n + 5$; −4

8. The table shows ticket prices for the local minor league baseball team for fan club members and non-members. For how many tickets is the cost the same for club members and non-members? (Example 3)
10 tickets

	Ticket Prices	
	Club Members	Non-Club Members
Membership Fee (one-time)	AED 30	none
Ticket Price	AED 3	AED 6

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

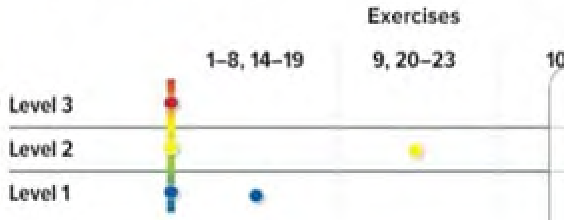
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be usec
homework assignment. The Extra Practice page can b
for additional reinforcement or as a second-day assign

Levels of Complexity

The levels of the exercises progress from 1 to 3, with L
indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises o
complexity levels to select appropriate exercises for y
students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–11, 13, 22, 23
OL	On Level	1–7 odd, 9–11, 13, 22, 23
BL	Beyond Level	9–13, 22, 23

Watch Out!

Common Error If students use the wrong operation w
eliminating a number from one side of an equation, re
them to use inverse operations.

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	12
2 Reason abstractly and quantitatively.	20, 21
3 Construct viable arguments and critique the reasoning of others.	9, 10
4 Model with mathematics.	11, 13

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

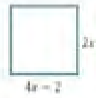
Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write the procedures they would use to solve an equation with variable terms on both sides such as $3x + 5 = 6x + 2$. See students' work.

9. **Multiple Representations** Refer to the square at the right.



a. **Words** Explain a method you could use to find the value of x.
Sample answer: Set the side lengths equal to each other and solve for x.

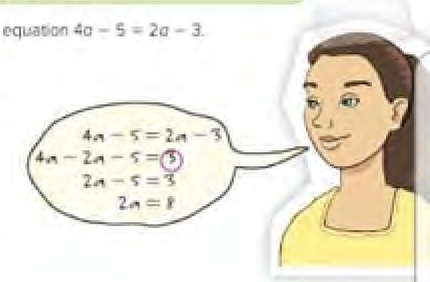
b. **Symbols** Write an equation to find the side length of the square.
$4x - 2 = 2x + 8$

c. **Algebra** What is the side length of the square?
18 units

H.O.T. Problems Higher Order Thinking

10. **Find the Error** Alma is solving the equation $4a - 5 = 2a - 3$. Circle her mistake and correct it.

$4a - 5 = 2a - 3$
$4a - 2a - 5 = 2a - 2a - 3$
$2a - 5 = -3$
$2a - 5 + 5 = -3 + 5$
$a = 1$



11. **Model with Mathematics** Write a real-world problem that can be solved using the equation $5x = 3x + 20$. Sample answer: You have 20 crafts made and continue to make crafts at the rate of 3 per hour. How many hours will it take you and your friend to make the same amount of crafts, if she makes crafts at a rate of 5 per hour.

12. **Persevere with Problems** Find the area of the rectangle at the right. 147 units^2

13. **Model with Mathematics** Write two equations so that each have variables on both sides and a solution of $\frac{1}{2}$. Sample answer: $3x + 6 = x + 7$, $1 - n = 3n - 1$

Name ______________________ My Homework ______________________

Extra Practice

Solve each equation. Check your solution.

14. $9g - 14 = 2g$

Homework Help

$9g - 14 = 2g$
$-9g \quad = -9g$
$\frac{-14}{-7} = \frac{-7g}{-7}$
$2 = g$

15. $-6f + 13 = 2f - 11$ **3**

16. $2.5h - 15 = 4h$ **−10**

17. $2z - 31 = -9z + 24$ **5**

18. Jamal averages 18 points a game and is the all-time scoring leader on his team with 483 points. Husam averages 21 points a game and is currently second on the all-time scorers list with 462 points. If both players continue to play at the same rate, how many more games will it take until Husam and Jamal have scored the same number of total points?

7 games

19. Eighteen less than three times a number is twice the number. Define a variable, write an equation, and solve to find the number.

Let n = the number; $3n - 18 = 2n$; 18

MP Reason Abstractly Write an equation to find the value of x so that each pair of polygons has the same perimeter. Then solve.

20.

$3x + 11 = 4x + 8$; 3

21.

$60x = 8x + 26$; 0.5

Power Up! Test Practice

Exercises 22 and 23 prepare students for more rigorous thinking needed for the assessment.

22.

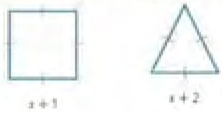
This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly model and solve the equation.
1 point	Students correctly model but fail to solve the equation OR students correctly solve the equation but make errors in modeling.

23.

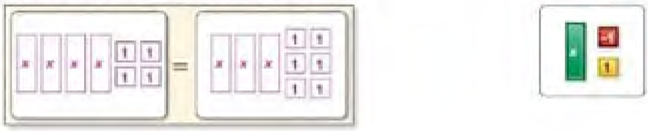
This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

22. The two regular polygons below have the same perimeter.



Use the algebra tiles to model an equation on the equation mat below that can be used to find x. Then solve the equation.



$x =$ **2**

23. Company A charges AED 28.50 plus AED 18 a room to clean carpet. Company B charges AED 16.50 plus AED 20 a room. Determine if each statement is true or false.

a. For 4 rooms, carpet cleaner B is cheaper. ■ True □ False

b. For 5 rooms, carpet cleaner A is cheaper. □ True ■ False

c. The equation $28.5 + 18x = 16.5 + 20x$ can be solved to find the number of rooms for which the total cost is the same. ■ True □ False

d. For 6 rooms, both carpet cleaners charge the same amount. ■ True □ False

Spiral Review

Use the Distributive Property to write each expression as an equivalent expression.

24. $6(x + 5) =$ **$6x + 30$**

25. $-8(y - 1) =$ **$-8y + 8$**

26. $-3(-5z + 12) =$ **$15z - 36$**

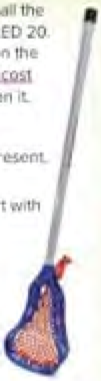
27. $\frac{1}{3}(6z + 10) =$ **$2z + \frac{10}{3}$**

Lesson 5

Solve Multi-Step Equations

Real-World Link

Lacrosse Coach Majeda wants to order uniform shirts for all the players (p) on her women's lacrosse team. Each shirt costs AED 20. There is an additional cost (d) for a player to put her name on the shirt. Use the steps below to write an equation for the total cost (c) if every player on the team orders a shirt with her name on it.



1. Circle the variables above and underline what they represent.

2. Write an expression that represents the cost of one shirt with a player's name on it.

   $20 + d$

   cost of shirt + cost of name

3. Use the expression to write an equation that can be used to find the total cost if every player on the team orders a shirt with her name on it.

   $p(20 + d) = c$

   number of players (cost of shirt + cost of name) = total cost

4. Suppose the total cost for 15 players to buy shirts is AED 420. Write an equation to show the total cost of the shirts if all of the players put their names on the shirts.

   $15(20 + d) = 420$

Essential Question

WHAT is equivalence?

Vocabulary

null set
identity

Math Symbols

∅ null set
{ } empty set

Mathematical Practices
1, 2, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Solve multi-step equations.

Coherence connecting within and across grades

Previous	Now	Next
Students solved equations with variables on each side of the equals sign.	Students solve multi-step equations.	Students will wr solve equations variables.

Rigor pursuing concepts, fluency, and application

See the Levels of Complexity chart on page 157.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole grou group, think-pair-share activity, or independent activity

LA Think-Pair-Share Have students wor pairs to complete Exercises 1–4. Give stude about one minute to think through their responses, the them share their responses with a partner. Then call o set of pairs to share their responses with the class. MP

Alternate Strategies

AL Ask students why the cost of one shirt with a pla name on it is represented by $20 + d$ instead of $20 \cdot d$

BL Ask students if the total cost represented by $p(20$ the same as $20p + pd$. Have them justify their response MP 1, 2, 3

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Solve multi-step equations.

AL
- *What operation is indicated by the parentheses?* multiplication
- *What operations are being performed on the variable d?* The variable d is being added to 20. Then that sum is multiplied by 15.

OL
- *What property allows you to rewrite $15(20 + d)$ as $300 + 15d$?* Distributive Property
- *After using the Distributive Property, what does the equation become?* $300 + 15d = 420$
- *What are the steps to solve this equation?* First, subtract 300 from each side. Then divide each side by 15.

BL
- *How can we check our work?* Replace the solution into the original equation for d and verify that the equation is true.
- *Is there a way that you could solve the equation without using the Distributive Property?* First, divide both sides by 15. Then subtract 20 from each side.

Need Another Example?
Solve $-3(4p - 6) = 54$. -3

Watch Out!

Common Error Students may forget to change the signs of both of the terms inside the grouping symbols when distributing a negative factor. For example, they may incorrectly simplify $-3(4p - 6)$ as $-12p - 18$ instead of $-12p + 18$. Remind them that the number and its sign are distributed over the sum or difference.

Work Zone

Solve Multi-Step Equations

Some equations contain expressions with grouping symbols. To sol these equations, first expand the expression using the Distributive Property. Then collect like terms if needed, and solve the equation using the Properties of Equality.

Example

1. Solve $15(20 + d) = 420$.

$15(20 + d) = 420$	Write the equation.
$300 + 15d = 420$	Distributive Property
$-300 \quad = -300$	Subtraction Property of Equality
$15d = 120$	Simplify.
$\frac{15d}{15} = \frac{120}{15}$	Division Property of Equality
$d = 8$	Simplify.

Got It? Do these problems to find out.

a. $-3(9 + x) = 33$ a. -20

b. $5(a - 7) = 25$ b. 12

Key Concept

Number of Solutions

	Null Set	One Solution	Identity
Words	no solution	one solution	infinitely many solu
Symbols	$a = b$	$x = a$	$a = a$
Example	$3x + 4 = 3x$ $4 = 0$ Since $4 \neq 0$, there is no solution.	$2x = 20$ $x = 10$	$4x + 2 = 4x + 2$ $2 = 2$ Since $2 = 2$, the so is all numbers.

Some equations have no solution. When this occurs, the solution the **null set** or empty set and is shown by the symbol Ø or { }. Oth equations may have every number as their solution. An equation is true for every value of the variable is called an **identity**.

...mples

...olve $6(x - 3) + 10 = 2(3x - 4)$.

$6(x - 3) + 10 = 2(3x - 4)$	Write the equation.
$6x - 18 + 10 = 6x - 8$	Distributive Property
$6x - 8 = 6x - 8$	Collect like terms.
$+8 = \ +8$	Addition Property of Equality
$6x = 6x$	Simplify.
$\frac{6x}{6} = \frac{6x}{6}$	Division Property of Equality
$x = x$	Simplify.

...he statement $x = x$ is *always* true. The equation is an identity and ...he solution set is all numbers.

Check		
	$6(x - 3) + 10 = 2(3x - 4)$	Write the original equation.
	$6(5 - 3) + 10 \stackrel{?}{=} 2[3(5) - 4]$	Substitute any value for x.
	$6(2) + 10 \stackrel{?}{=} 2(15 - 4)$	Simplify.
	$22 = 22$ ✓	

...olve $8(4 - 2x) = 4(3 - 5x) + 4x$.

$8(4 - 2x) = 4(3 - 5x) + 4x$	Write the equation.
$32 - 16x = 12 - 20x + 4x$	Distributive Property
$32 - 16x = 12 - 16x$	Collect like terms.
$+16x = \ +16x$	Addition Property of Equality
$32 = 12$	Simplify.

...he statement $32 = 12$ is *never* true. The equation has no ...olution and the solution set is $\varnothing$.

Check		
	$8(4 - 2x) = 4(3 - 5x) + 4x$	Write the equation.
	$8[4 - 2(2)] \stackrel{?}{=} 4[3 - 5(2)] + 4(2)$	Substitute any value for x.
	$8(0) \stackrel{?}{=} 4(-7) + 8$	Simplify.
	$0 \neq -20$ ✓	Since $0 \neq -20$, the equation has no solution.

STOP and Reflect

How do you know if the solution $5 = 0$ indicates no solution, one solution, or infinitely many solutions?

Since 5 never equals 0, there is no number that will make the equation true. So there is no solution.

...it? Do these problems to find out.

c. $3(6 - 4x) = -2(6x - 9)$ identity or all numbers

d. $2(3x + 5) = 5(2x - 4) - 4x$ null set or no solution

Examples

2. Solve a multi-step equation involving an identity.

AL
- *What operation is indicated by the parentheses?* multiplication
- *What property allows us to rewrite $6(x - 3)$ and $2(3x - 4)$ without parentheses?* Distributive Property

OL
- *After using the Distributive Property, what does the equation become?* $6x - 18 + 10 = 6x - 8$
- *What do you need to do next?* Simplify the left side of the equation.
- *After simplifying, what does the equation become?* $6x - 8 = 6x - 8$

BL
- *Do we need to continue to solve the equation to determine the set of possible solutions?* Sample answer: no; Because the expressions on both sides of the equals sign are the same, we know that this equation is an identity and the solution is the set of all numbers.

Need Another Example?
Solve $3(4x + 8) = 2(6x + 12)$. identity; all numbers

3. Solve a multi-step equation that has no solution.

AL
- *What operation is indicated by the parentheses?* multiplication
- *What is the first step to solve this equation?* Use the Distributive Property to eliminate the parentheses.

OL
- *After using the Distributive Property, what does the equation become?* $32 - 16x = 12 - 20x + 4x$
- *What happens if you add 16x to both sides?* You are left with $32 = 12$, which is not true.
- *What happens if you subtract 12 from each side?* You are left with $20 = 0$, which is not true.

BL
- *At what point could you see that there was not going to be a solution to this equation?* Sample answer: When we reached $32 - 16x = 12 - 16x$; both sides had 16x being subtracted from a number, but it was not the same number.

Need Another Example?
Solve $4(5x + 3) - 6x = 7(2x + 3)$. null set; no solution

Example

4. **Write and solve an equation involving a real-world problem.**

AL • *What variable could we use to represent the cost of a snack?* **Sample answer: s**

• *If a snack costs s dollars, what expression represents the cost of a ride ticket?* **$s - 1.5$**

OL • *What expression represents the cost of 3 snacks and 10 ride tickets?* **$3s + 10(s - 1.5)$**

• *What equation represents the cost of 3 snacks and 10 ride tickets being equal to AED 24?* **$3s + 10(s - 1.5) = 24$**

• *After solving the equation, what does $s = 3$ mean in the context of the problem?* **The cost of each snack was AED 3.**

BL • *How much is the cost of each ride ticket?* **AED 1.50**

Need Another Example?
The length of Majid's stride when walking is 4 inches greater than the length of Khalid's stride. If it takes Philip 5 steps and Khalid 6 steps to walk the same distance, what is the length of Khalid's stride? **20 in.**

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL BL LA **Pairs Discussion** Have students work in pairs to create a flowchart of the steps involved in solving a multi-step equation. Then have them use their flowchart to complete Exercises 1 and 2. MP 1, 5

BL LA **Pairs Discussion** Have students work in pairs to determine at which step they knew that Exercise 2 had no solution. Ask them if they could determine this information in a previous step. MP 1, 3

Example

4. **At the fair, Majed bought 2 snacks and 10 ride tickets. Each snack costs AED 1.50 less than a ride ticket. If he spent a total of AED 57.00, what was the cost of each ride?**

Write an equation to represent the problem.

$10s + 2(s - 1.5) = 57$ Write the equation.
$10s + 2s - 3 = 57$ Distributive Property
$12s - 3 = 57$ Collect like terms.
$+3 = +3$ Addition Property of Equality
$12s = 60$ Simplify.
$\frac{12s}{12} = \frac{60}{12}$ Division Property of Equality
$s = 5$ Simplify.

So, the cost of each snack was AED 5.

Guided Practice

Solve each equation. Check your solution. (Examples 1–3)

1. = 32 **2**

2. $8z - 22 = 3(3z + 11) - z$ **null set or no solution**

3. Mr. Sayed's class is holding a canned food drive for charity. Ahlam collected 10 more cans than Amira. Khaled collected twice as many cans as Ahlam. If they collected 130 cans altogether, how many cans did Ahlam collect? (Example 4) **35 cans**

4. **Building on the Essential Question** How many possible solutions are there to a linear equation in one variable? Describe each one.
Sample answer: There are 3 possible solutions to a linear equation in one variable: The null set where there are no solutions, one solution, or infinitely many solutions.

Rate Yourself!
Are you ready to move on? Shade the section that applies.

YES ? NO

FOLDABLES Time to update your Foldable

Name ______________________ My Homework ______________________

Independent Practice

Solve each equation. Check your solution. (Examples 1–3)

1. $-12(k + 4) = 60$ **-9**

2. $8(3a + 6) = 9(2a - 4)$ **-14**

3. $\frac{1}{3}h - 4\left(\frac{2}{3}h - 3\right) = \frac{2}{3}h - 6$ **6**

4. $8(c - 9) = 6(2c - 12) - 4c$ **identity or all real numbers**

Copy and Solve **Solve each equation. Show your work on a separate piece of paper.** (Examples 2 and 3)

5. $-10y + 18 = -3(5y - 7) + 5y$ **null set or no solution**

6. $8(t + 2) - 3(t - 4) = 6(t - 7) + 8$ **62**

7. $4(5 + 2x) - 5 = 3(3x + 7)$ **-6**

8. $6(2x - 8) + 3 = 15$ **5**

9. The school has budgeted AED 2,000 for an end-of-year party at the local park. The cost to rent the park shelter is AED 150. How much can the student council spend per student on food if each of the 225 students receives a AED 3.50 gift? (Example 4) **AED 4.72**

10. **Reason Abstractly** The table shows the number of students in each homeroom.

Teacher	Number of Students
Mr. Khalil	b
Mr. Sultan	$1.5(b + 2)$
Ms. Dana	15
Mrs. Amani	$2b - 9$

a. Write an equation to find the number of students in Mr. Khalil's homeroom if the total number of students is 90. **$90 = b + 1.5(b + 2) + 15 + (2b - 9)$ or $90 = 4.5b + 9$**

b. Solve the equation from part **a** to find the number of students in Mr. Khalil's homeroom. **18 students**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be
homework assignment. The Extra Practice page ca
for additional reinforcement or as a second-day as

Levels of Complexity

The levels of the exercises progress from 1 to 3, w
indicating the lowest level of complexity.

	Exercises	
	1–9, 15–22	10, 11, 23–25
Level 3		
Level 2		•
Level 1	•	

Suggested Assignments

You can use the table below that includes exerci
complexity levels to select appropriate exercises
students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11, 12, 14, 24, 25
OL	On Level	1–9 odd, 10–12, 14, 24, 25
BL	Beyond Level	10–14, 24, 25

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	13
2 Reason abstractly and quantitatively.	10
3 Construct viable arguments and critique the reasoning of others.	12
4 Model with mathematics.	11, 14
7 Look for and make use of structure.	23

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Ask students to write how the previous lesson on solving equations with variables on each side helped them with this lesson on solving multi-step equations. Use the writing prompt below. See students' work.

Ask:

- Learning how to solve an equation with variables on each side of the equals sign helped me to solve a multi-step equation because...

11. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.

a. Write an equation that can be used to determine the number of text messages Salah and Edhab can send for their plans to cost the same.
$20 + 0.15m = 30 + 0.1m$

b. Solve the equation from part **a** to find the number of text messages each person can send for their costs to be the same.
$m = 200$; 200 messages

H.O.T. Problems Higher Order Thinking

12. **Reason Inductively** Does a multi-step equation *always*, *sometimes*, or *never* have a solution? Explain your reasoning.
sometimes; Sample answer: An equation like $2x + 3 = 2x + 5$ has no solution.

13. **Persevere with Problems** The perimeter of a rectangle is $8(2x + 1)$ inches. The length of the sides of the rectangle are $3x + 4$ inches and $4x + 3$ inches. Write and solve an equation to find the length of each side of the rectangle.
$2(3x + 4) + 2(4x + 3) = 8(2x + 1)$; 13 in. and 15 in.

14. **Model with Mathematics** Write a real-world problem that can be solved using the Distributive Property. Then write and solve an equation for your real-world situation. Sample answer: My family spent AED 30 for lunch. They bought 5 sandwiches and 5 drinks. Each sandwich cost AED 3 more than each drink. How much did each sandwich cost?; $5x + 5(x + 3) = 30$; AED 4.50

Name ______________________ My Homework ______________________

Extra Practice

Solve each equation. Check your solution.

15. $9(j - 4) = 81$

$9j - 36 = 81$
$+36 = +36$
$9j = 117$
$\frac{9j}{9} = \frac{117}{9}$
$j = 13$

Homework Help

16. $8(4q - 5) - 7q = 5(5q - 8)$

$32q - 40 - 7q = 25q - 40$
$25q - 40 = 25q - 40$
$-25q \quad = -25q$
$-40 = -40$
The solution set is all numbers.

17. $\frac{1}{2}r + 2\left(\frac{3}{4}r - 1\right) = \frac{1}{4}r + 6$ **$4\frac{4}{7}$**

18. $-5(3m + 6) = -3(4m - 2)$ **-12**

19. $-7(k + 9) = 9(k - 5) - 14k$ **-9**

20. $10p - 2(3p - 6) = 4(3p - 6) - 8p$
null set or no solution

21. $12(x + 3) = 4(2x + 9) + 4x$
identity or all numbers

22. $0.2(x + 50) - 6 = 0.4(3x + 20)$ **-4**

23. MP **Identify Structure** Give an example of a multi-step equation for each of the following solutions.

a. all numbers **Sample answer: $3x + 5 = 3x - 2 + 7$**

b. null set **Sample answer: $2(x - 1) = 2x + 2$**

Up! Test Practice

nd 25 prepare students for more rigorous
d for the assessment.

n requires students to analyze and solve complex real-ms through the use of mathematical tools and models.	
owledge	DOK2
l Practice	MP1, MP4
oric	
	Students correctly model and solve the equation.
	Students correctly model but fail to solve the equation OR students correctly solve the equation but make errors in modeling.

n requires students to reason abstractly and y when problem solving.	
owledge	DOK1
l Practice	MP1
oric	
	Students correctly answer each part of the question.

Power Up! Test Practice

24. The table shows expressions to represent the number of students involved in different activities. The number of students involved in sports and student council is equal to the number of students involved in band and drama club. Model the situation with an equation. Select the correct expression for each box. Then solve the equation.

Activity	Number of Students
band	$3n - 2$
drama club	$2(2n + 1)$
sports	$5n + 7$
student council	n

$5n + 7$ + n = $3n - 2$ + $2(2n + 1)$

$5n + 7$	$3n - 2$
n	$2(2n + 1)$

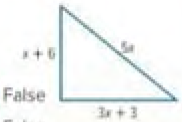
$n =$ 7

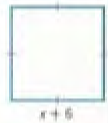
How many students participate in each activity?

sports: 42 students band: 19 students

student council: 7 students drama club: 30 students

25. The figures below have the same perimeter. Determine if each statement is true or false.





a. The value of x is 2. ☐ True ■ False

b. The perimeter of the triangle is 36 units. ■ True ☐ False

c. The perimeter of the square is 32 units. ☐ True ■ False

Spiral Review

Solve each inequality. Graph the solution set on a number line.

26. $a - 5 < 2$ $a < 7$

27. $x - 9 \geq -12$ $x \geq -3$

28. $5y \leq -30$ $y \leq -6$

29. $-\frac{n}{4} > -2$ $n < 8$

21ST CENTURY CAREER in Design

Skateboard Designer

If you love the sport of skateboarding, and you are creative and have strong math skills, you should think about a career designing skateboards. A skateboard designer applies engineering principles and artistic ability to design high-performance skateboards that are both strong and safe. To have a career in skateboard design, you should study physics and mathematics and have a good understanding of skateboarding.

Is This the Career for You?

Are you interested in becoming a skateboard designer? Take some of the following courses in high school.

- Digital Design
- Geometry
- Physics
- Trigonometry

Turn the page to find out how math relates to a career in Design.

Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace.

This lesson emphasizes **Mathematical Practice 4** Model with Mathematics.

Coherence connecting within and across grades

Previous	Now
Students solved multi-step equations.	Students apply the content standard to solve problems in the workplace.

Rigor pursuing concepts, fluency, and applications

See the Career Project on page 162.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ask students to read the information on the student page about skateboard design and answer the following questions.

Ask:

- *What kinds of classes should you take to be a skateboard designer?* **Digital Design, Geometry, Physics, Trigonometry**
- *What does a skateboard designer do?* **applies engineering principles and artistic ability to the design of skateboards**
- *What qualities are required for a skateboard?* **strength and safety**

Engage Explore **Explain** **Elaborate** Evaluate

Collaborate

L LA **Circle the Sage** Poll the class to see which udents have some knowledge about writing equations. hose students (the sages) spread out around the room. ssign the rest of the students to teams. Have the teams split with each team member going to a different sage, if ossible. Have the sages lead work for Exercises 1–4. When e exercises are complete, students return to their teams and mpare solutions. Students speak with their groups and scuss how the sages may have explained the steps fferently. MP 1, 3

L LA **Team Project** Have students work together in ams to discuss the requirements for three types of ateboards. Then have the students write a memorandum to skateboard designer detailing the requirements. MP 1, 3

Career Portfolio

hen students complete this page, have them add it to their areer Portfolio.

Career Facts

Tim Piumarta was one of the first skateboard designers to make the boards curve up at the edges, nose, and tail. These curves, called concave curves, strengthen the board and give riders more control.

It's Great to Skate

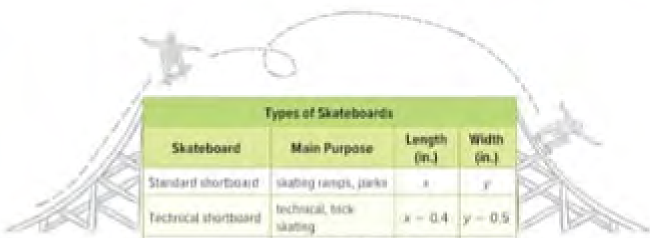
Use the information in the table to solve each problem.

1. The total width of two standard shortboards and a technical shortboard is 23.5 inches. Write an equation to represent the situation.
   $2y + (y - 0.5) = 23.5$

2. Solve the equation from Exercise 1 to find the width of a standard shortboard.
   8 in.

3. The total length of two longboards and a standard shortboard is 113.4 inches. Write and solve an equation to find the length of a longboard. $2(x + 14.7) + x = 113.4$; 42.7 in.

4. The total width of three technical shortboards is 4.5 inches more than the total width of two longboards. Write and solve an equation to find the width of a longboard.
   $3(y - 0.5) = 2(y + 1) + 4.5$; 9 in.



Types of Skateboards

Skateboard	Main Purpose	Length (in.)	Width (in.)
Standard shortboard	skating ramps, parks	x	y
Technical shortboard	technical, trick skating	$x - 0.4$	$y - 0.5$
Longboard	skating downhill, long rides	$x + 14.7$	$y + 1$

Career Project

It's time to update your career portfolio! Describe the skills that would be necessary for a skateboard designer to possess. Determine whether this type of career would be a good fit for you.

What problem-solving skill might you use as a skateboard designer?

Chapter Review

Vocabulary Check

Unscramble each of the clue words. After unscrambling all of the terms, use the numbered letters to find the name of a famous mathematician.

Clue	Answer	Numbered letters
OCIFCTIENEF	C O E F F I C I E N T	I = 2, T = 11
TECVALMITUPLI SEVNEIR	M U L T I P L I C A T I V E / I N V E R S E	A = 6; N = 13, R = 3
DEYTINTI	I D E N T I T Y	I = 4, N = 9
LLUNETS	N U L L S E T	
PERRITPEOS	P R O P E R T I E S	S = 1
REABIASVL	V A R I A B L E S	E = 10, S = 5
PEOCILRACR	R E C I P R O C A L	O = 12, C = 8, A = 7

S I R I S A A C N E W T O N
1 2 3 4 5 6 7 8 9 10 11 12 13

Complete each sentence using one of the unscrambled words above.

1. The **coefficient** is the numerical factor of a term that contains a variable.
2. Another name for the reciprocal is the **multiplicative inverse**.
3. In mathematics, statments that are true for all numbers are **properties**.
4. When writing an equation from a real-world problem, it is important to define the **variables**

Vocabulary Check

LA **Find the Fib** Have students work in and write down two facts and one fib usin words in the Vocabulary Check. For example, one fa be that properties are true for all numbers. One fib c that an identity equation is only true when the variab to 1. Each team member shares their facts and fib by aloud to the group. The responsibility of the group is carefully, discuss, and come to a consensus to identi

MP 1, 3, 5, 6

Alternate Strategy

AL LA To help students, you may wish to give th vocabulary list from which they can choose their ans A vocabulary list for this activity would include the fo terms.

- coefficient (Lesson 1)
- identity (Lesson 5)
- multiplicative inverse (Lesson 1)
- null set (Lesson 5)
- properties (Lesson 2)
- reciprocal (Lesson 1)
- variable (Lesson 1)

y Concept Check

LDABLES LA A completed Foldable for this chapter
uld include a review of solving multi-step equations.

ou choose not to use this Foldable, have students write a
ef review of the Key Concepts found throughout the chapter
l give an example of each.

eas for Use

Have students work in pairs to discuss their Foldables.
ve them practice speaking in a collaborative setting by
ring how they have completed their Foldable thus far and
v they could finish it. Have each student complete their
dable and trade with their partner to discuss any similarities
l differences. MP 1, 3, 5

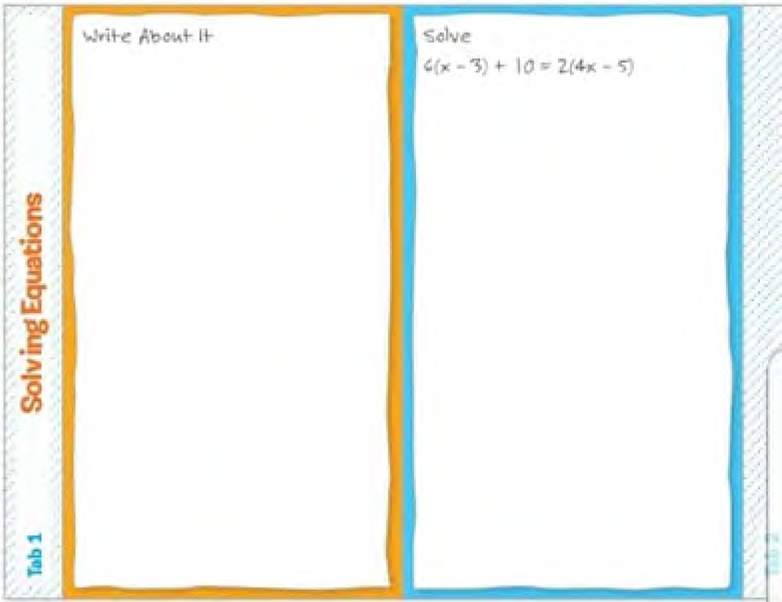
t It?

udents have trouble with Exercises 1–3, they may need
o with the following concept(s).

Concept	Exercise(s)
ving multi-step equations (Lesson 5)	1, 3
ving equations with variables on each side (Lesson 4)	2

Use Your FOLDABLES

Use your Foldable to help review the chapter.



Got it?

Number the steps in the order needed to solve each equation. Then solve the equation. Sample steps: 2 and 3

1. $5(x + 3) = 170$
   - 2 Subtract 15 from each side.
   - 1 Multiply x and 3 by 5.
   - 3 Divide each side by 5.
   - $x =$ 31

2. $2p - 9 = 6p + 7$
   - 3 Divide each side by 4.
   - 2 Subtract 7 from each side.
   - 1 Subtract $2p$ from each side.
   - $p =$ -4

3. $-\frac{2}{3}(a + 3) = \frac{5}{3}a - 19$
   - 2 Add $\frac{2}{3}a$ to each side.
   - 3 Add 19 to each side.
   - 1 Multiply a and 3 by $-\frac{2}{3}$.
   - 4 Multiply each side by $\frac{3}{7}$.
   - $a =$ $7\frac{2}{7}$

Power Up! Performance Task

Driving Motorbikes in the Desert

The Hazaa family goes to the desert once per month. They each drive their motorbikes same number of hours.

Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A

Five of the family members will ride motorbikes this month. The prices at Desert Zone 1 are listed in the table below. The total budget for the outing is AED 1300. The budget for food and drinks is AED 150. Write an equation to find how many hours each family member can ride a motor bike. Let *g* represent the number of hours ridden by each family member. Then solve your equation.

Desert Zone 1	
Cost per hour	AED 60
Insurance (one-time fee per visit)	AED 50

Part B

The next month, the family tries Desert Zone 2, which is on the other side of town. They set aside AED 170 for food and drink. Only four family members are riding this time. The total budget for this outing is AED 1450. The prices at Desert Zone 2 are listed in the table below. Write an equation to find how many hours each family member can ride. Let *g* represent the number of hours ridden by each family member. Then solve your equation.

Desert Zone 2	
Cost per hour	AED 50
Insurance (one-time fee per visit)	AED 70

Part C

The following month, Mr. Hazaa goes riding by himself. He debates on which desert zone to select based only on what it will cost to ride in each zone. How many hours would he need to ride to pay an equal amount at each desert zone?

Part D

Given the equation $6(8g + 2) + 45 = 153$, write a scenario similar to Parts A and B that could be represented by the equation. Solve for *g* and explain what each number in the equation represents, including the solution.

Power Up! Performance Tas

This Performance-Based Assessment requir
solve multi-step problems through abstract r
precision, and perseverance. This practice s
used to help students prepare for the thinki
used on the Assessment.

A complete scoring rubric with answers to t
be found on at the back of the book.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- How is the multiplicative inverse used to solve an equation that has a rational coefficient? (p. 114)
- How can you use the *work backward* problem-solving strategy to solve a two-step equation? (p. 124)
- Why is it important to define a variable before writing an equation? (p. 132)
- How is solving an equation with the variable on each side similar to solving a two-step equation? (p. 148)
- How many possible solutions are there to a linear equation in one variable? Describe each one. (p. 156)

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their classmate before they complete the graphic organizer.

MP 1, 3, 5

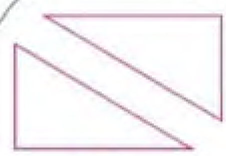
Reflect

Answering the Essential Question

Use what you learned about equivalence to complete the graphic organizer. Draw or write an example for each category.

Sample answers are given.

Geometric Figures



Fractions

$\frac{4}{8} = \frac{1}{2}$

Essential Question

WHAT is equivalence?

Algebraic Expressions

$5x + 3 - 2x = 3x + 3$

Numerical Expressions

$10 - 40 = -85 + 55$

Answer the Essential Question. WHAT is equivalence?

See students' work.

166 **Chapter 2** Equations in One Variable

Chapter 3

Equations in Two Variables

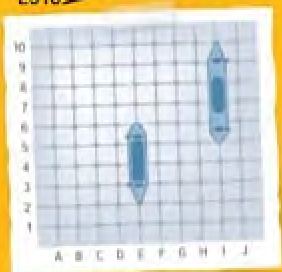
Essential Question

WHY are graphs helpful?

Mathematical Practices
1, 2, 3, 4, 5, 7

Math in the Real World

Games Zein is playing Attack of the Seas with Alia. Zein placed his ship between A5 and C5. To earn points, Alia guesses a point ... one of Zein's ship... Zein's ship on ... ate plane. If A... the locate ... ships?



FOLDABLES Study Organizer

1 Cut out the Foldable in the back of this book.

2 Place your Foldable on page 256.

3 Use the Foldable throughout this chapter to help you learn about equations in two variables.

Focus narrowing the scope

This chapter focuses on content from the **Expressio** **Equations** (EE) domain.

Coherence connecting within and across gra

Previous
Students graphed proportional relationships and identified the unit rate as the slope of the related linear function.

Now
Students use tables, graphs, and models to represent, analyze, and solve real-world problems related to linear equations.

Nex
Students compare

Rigor pursuing concepts, fluency, and applicat

The Levels of Complexity charts located throughou chapter indicate how the exercises progress from understanding and procedural skills and fluency, t and critical thinking.

Launch the Chapter

Math in the Real World

Games Tell students that the game board is simil coordinate plane. If they have trouble, have them point A5 as the ordered pair (A, 5).

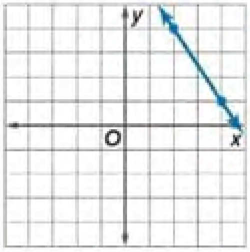
What Tools Do You Need?

Vocabulary Activity

LA As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define: Slope is the rate of change between any two points on a line. The ratio of the rise, or vertical change, to the run, or horizontal change.

Example:

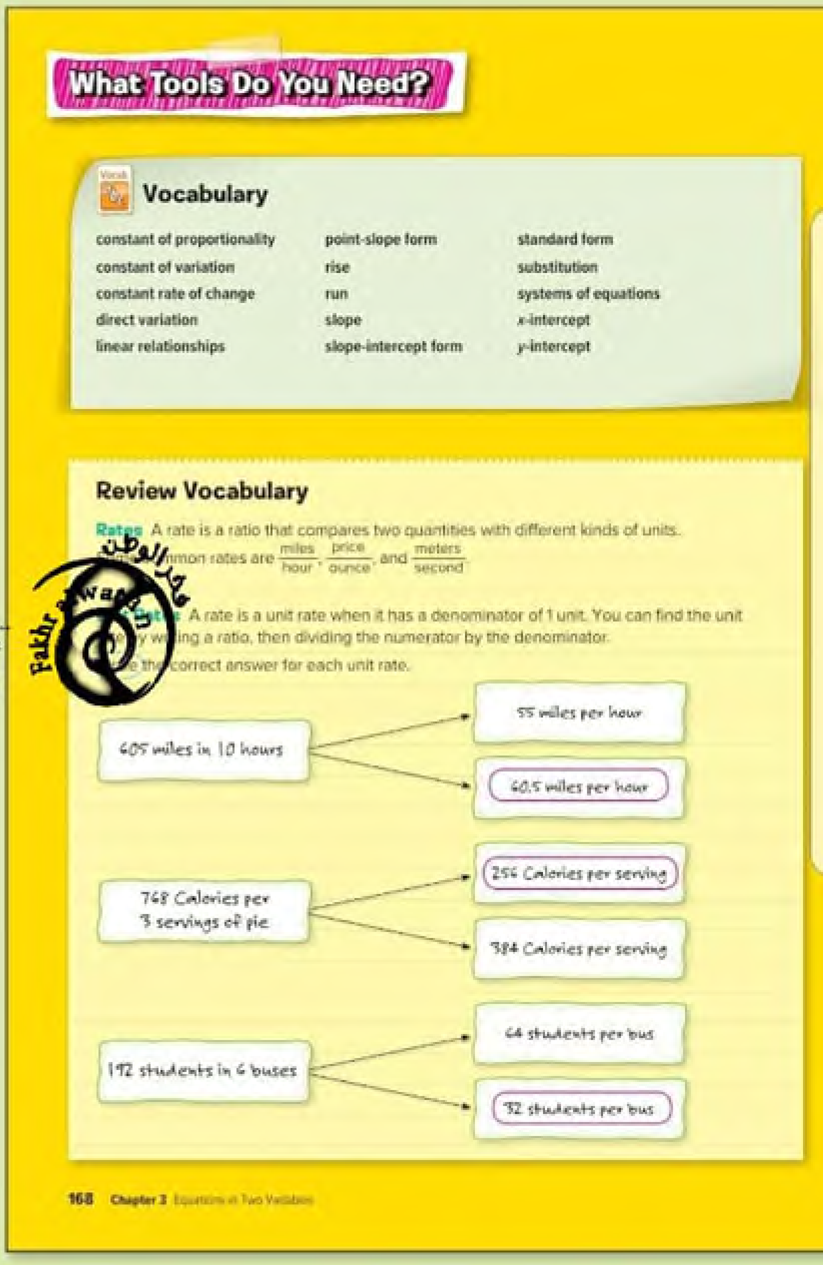


Ask:

- *What is the slope of this line?* $-\frac{3}{2}$

Review Vocabulary

LA To find each unit rate, have students first write the rate as a fraction. Then they can divide the numerator by the denominator to find the unit rate.



What Tools Do You Need?

Vocabulary

constant of proportionality	point-slope form	standard form
constant of variation	rise	substitution
constant rate of change	run	systems of equations
direct variation	slope	x-intercept
linear relationships	slope-intercept form	y-intercept

Review Vocabulary

Rates A rate is a ratio that compares two quantities with different kinds of units. [illegible]mmon rates are $\frac{\text{miles}}{\text{hour}}$, $\frac{\text{price}}{\text{ounce}}$, and $\frac{\text{meters}}{\text{second}}$.

[illegible]ates A rate is a unit rate when it has a denominator of 1 unit. You can find the unit [illegible] w[illegible]ing a ratio, then dividing the numerator by the denominator.

[illegible]e the correct answer for each unit rate.

What Do You Already Know?

List three things you already know about equations in two variables in the first section. Then list three things you would like to learn about equations in two variables in the second section. See students' work.

Equations in two variables

What I know	What I want to find out

When Will You Use This?

Here is an example of how systems of equations are used in the real world.

Activity Use the Internet to find the cost of ride tickets or a ride wrist band for a local fair. Which one do you think is a better deal?

See students' work.

What Do You Already Know?

In this activity students assess their prior knowledge by listi three things they already know and three things they would like to learn about concepts in the chapter.

- You may want to add a third option of "I don't know" for those students who do not have any prior knowledge of the topic.
- After completing the chapter, have students return to this page and have them add three new facts that they learn about the topic.

When Will You Use This?

Activity

Students compare different options to determine which one the better deal.

Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

Quick Check

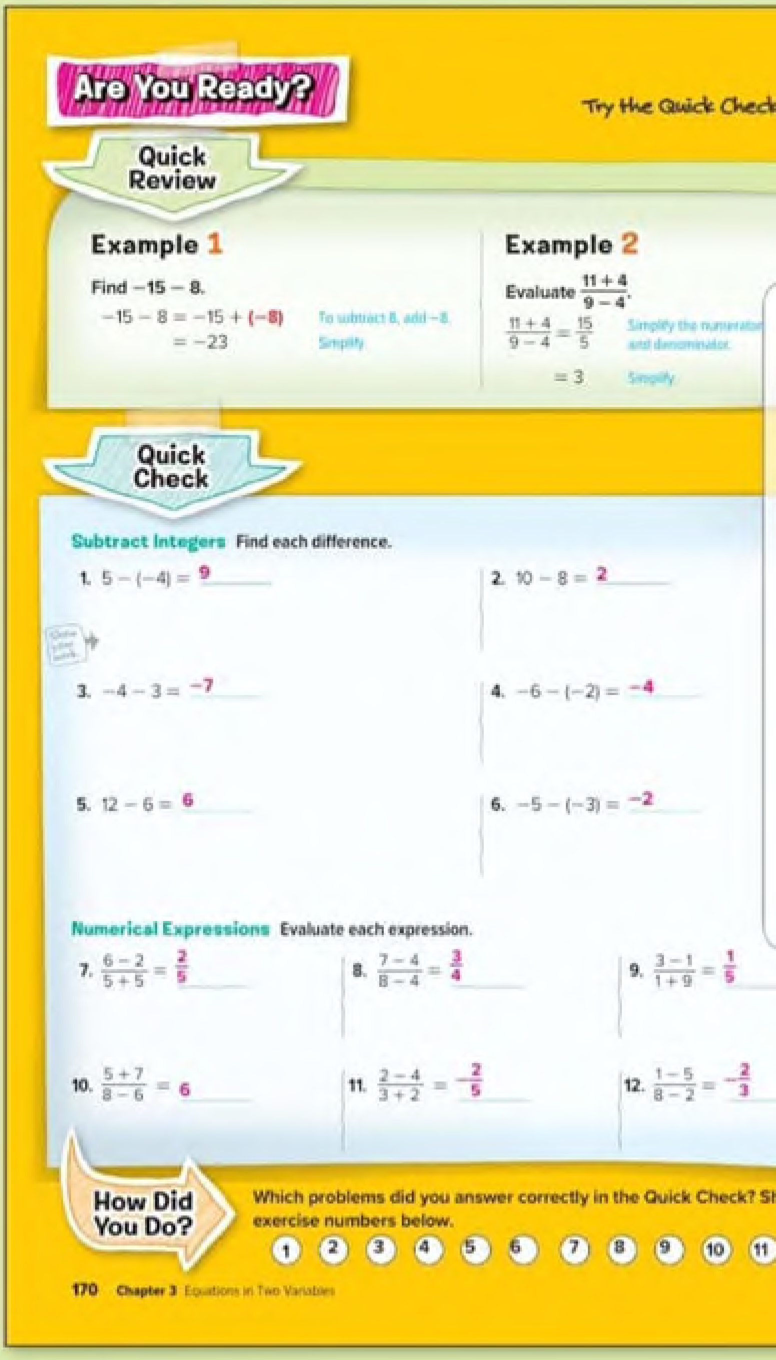
If students have difficulty with the exercises, present another example to clarify any misconceptions they may have.

Exercises 1–6

Find $-8 - 7$. -15

Exercises 7–12

Evaluate $\frac{4-8}{4-6}$. 2



Are You Ready?

Try the Quick Check

Quick Review

Example 1

Find $-15 - 8$.

$-15 - 8 = -15 + (-8)$ To subtract 8, add −8.

$= -23$ Simplify.

Example 2

Evaluate $\frac{11+4}{9-4}$.

$\frac{11+4}{9-4} = \frac{15}{5}$ Simplify the numerator and denominator.

$= 3$ Simplify.

Quick Check

Subtract Integers Find each difference.

1. $5 - (-4) = 9$
2. $10 - 8 = 2$
3. $-4 - 3 = -7$
4. $-6 - (-2) = -4$
5. $12 - 6 = 6$
6. $-5 - (-3) = -2$

Numerical Expressions Evaluate each expression.

7. $\frac{6-2}{5+5} = \frac{2}{5}$
8. $\frac{7-4}{8-4} = \frac{3}{4}$
9. $\frac{3-1}{1+9} = \frac{1}{5}$
10. $\frac{5+7}{8-6} = 6$
11. $\frac{2-4}{3+2} = -\frac{2}{5}$
12. $\frac{1-5}{8-2} = -\frac{2}{3}$

How Did You Do? Which problems did you answer correctly in the Quick Check? Sh exercise numbers below.

1 2 3 4 5 6 7 8 9 10 11

170 Chapter 3 Equations in Two Variables

Lesson 1

Constant Rate of Change

Real-World Link

Music Malek can download two Koranic verses from the Internet each minute. This is shown in the table below.

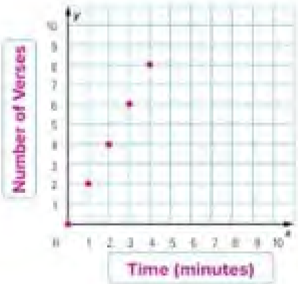
Time (minutes), x	0	1	2	3	4
Number of Verses, y	0	2	4	6	8

1. Compare the change in the number of verses y to the change in time x. What is the rate of change?

   Sample answer: The number of verses increases by 2, and the time increases by 1; 2 verses per minute

2. Graph the ordered pairs from the table on the graph shown. Label the axes. Then describe the pattern shown on the graph.

   Sample answer: The points appear to make a line.



Essential Question

WHY are graphs helpful?

Vocabulary

linear relationship
constant rate of change

Mathematical Practices
1, 3, 4, 5

Which **Mathematical Practices** did you use?
Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Identify proportional and nonproportional relationships by finding a constant rate of change.

Coherence connecting within and across gra

Previous
Students identified proportional and nonproportional relationships in tables and graphs.

Now
Students use tables and graphs to find the rate of change in a linear relationship.

Next
Students w
constant r
to the slop

Rigor pursuing concepts, fluency, and applicati

See the Levels of Complexity chart on page 175.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUA

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole g group, think-pair-share activity, or independent acti

LA Think-Pair-Solo Have students th their solution to Exercise 1. Then have th their responses with a partner. Have them work alo complete Exercise 2. Have them trade papers with and each partner checks the other's graph and ans have them discuss and resolve any differences. MP 1

Alternate Strategies

AL Have students explain the pattern shown in t
MP 1, 2, 3, 5, 7, 8

BL Have them extend the pattern in the table for 13, 14, and 15 minutes and confirm the rate of chang 7, 8

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Identify linear relationships.

AL
- *Is the balance increasing as the transactions increase, or decreasing?* **decreasing**
- *As the number of transactions increases by 3, what happens to the balance?* **It decreases by AED 30.**

OL
- *How do you determine if the relationship between two quantities is linear?* **if the rate of change is constant**
- *How do you find the constant rate of change, if it exists?* **Find the change in the balance per each transaction.**

BL
- *How could you find the balance after 5 transactons?* **Sample answer: Because the rate of change is −AED 10 per transaction, add AED 10 to the balance after 6 transactions. After 5 transactions, AED 140 + AED 10 = AED 150.**
- *Find the balance after 11 transactions.* **AED 90**
- *If you were to graph this relationship, what would the graph look like?* **Sample answer: a line sloping down from left to right**

Need Another Example?

The amount a babysitter charges is shown. Is the relationship between the number of hours and the amount charged linear? If so, find the constant rate of change. If not, explain your reasoning. **yes; The constant rate of change is $\frac{8}{1}$, or AED 8 per hour.**

Hours	Charge
1	AED 10
2	AED 18
3	AED 26
4	AED 34

Work Zone

Linear Relationships

Relationships that have straight-line graphs, like the one on the previous page, are called **linear relationships**. Notice that as the number of songs increases by 2, the time in minutes increases by 1.

Number of Verses, y	0	2	4	6	8
Time (minutes), x	0	1	2	3	4

+2 +2 +2 +2

+1 +1 +1 +1

Rate of Change

$\frac{2}{1}$ = 2 verses per minute

The rate of change between any two points in a linear relationship is the same or constant. A linear relationship has a **constant rate of change**.

Example

1. The balance in an account after several transactions is shown. Is the relationship between the balance and number of transactions linear? If so, find the constant rate of change. If not, explain your reasoning.

Number of Transactions	Balance (AED)
3	170
6	140
9	110
12	80

+3 +3 +3

−30 −30 −30

As the number of transactions increases by 3, the balance in the account decreases by AED 30.

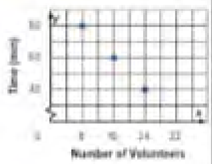
Since the rate of change is constant, this is a linear relationship. The constant rate of change is $\frac{-30}{3}$ or −AED 10 per transaction. This means that each transaction involved a AED 10 withdrawal.

Got it? **Do these problems to find out.**

a.

Cooling Water	
Time (min)	Temperature (°F)
5	95
10	90
15	86
20	82

b.



a. **No; the rate of change from 5 to 10 min, $\frac{90-95}{10-5}$ or −1°F per min, is not the same as the rate of change from 10 to 15 min, $\frac{86-90}{15-10}$ or −0.8°F per min, the relationship is not linear**

b. **Yes; the rate of change is −2.5 min per volunteer.**

Proportional Linear Relationships

Key Concept

Words Two quantities a and b have a proportional linear relationship if they have a constant ratio and a constant rate of change.

Symbols $\frac{b}{a}$ is constant and $\frac{\text{change in } b}{\text{change in } a}$ is constant.

To determine if two quantities are proportional, compare the ratio $\frac{b}{a}$ for several pairs of points to determine if there is a constant ratio.

Example

2. Use the table to determine if there is a proportional linear relationship between a temperature in degrees Fahrenheit and a temperature in degrees Celsius. Explain your reasoning.

Degrees Celsius	0	5	10	15	20
Degrees Fahrenheit	32	41	50	59	68

+5 +5 +5 +5

+9 +9 +9 +9

Constant Rate of Change

$\frac{\text{change in °F}}{\text{change in °C}} = \frac{9}{5}$

Since the rate of change is constant, this is a linear relationship.

To determine if the two scales are proportional, express the relationship between the degrees for several columns as a ratio.

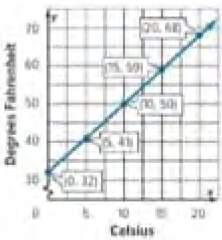
$\frac{\text{degrees Fahrenheit}}{\text{degrees Celsius}} \longrightarrow \frac{41}{5} = 8.2 \quad \frac{50}{10} = 5 \quad \frac{59}{15} = 3.9$

Since the ratios are not the same, the relationship between degrees Fahrenheit degrees Celsius is not proportional.

Check: Graph the points on the coordinate plane. Then connect them with a line.

The points appear to fall in a straight line so the relationship is linear. ✓

The line connecting the points does not pass through the origin so the relationship is not proportional. ✓



Proportional Relationships

Example

2. Identify proportional linear relationships.

AL
- *What is a linear relationship?* **a relationship that has a constant rate of change**
- *What is a proportional linear relationship?* **a relationship that has a constant rate of change and the ratios between the two pairs of quantities are the same**

OL
- *What are two methods you could use to determine if a proportional linear relationship exists?* **Make a table or make a graph.**
- *Is there a constant rate of change? Explain.* **yes; Sample answer: The degrees Fahrenheit increases by 9 for every 5 degree increase in degrees Celsius. The rate of change is $\frac{9}{5}$.**
- *Does this represent a linear relationship?* **yes**
- *Are the ratios of degrees Fahrenheit to degrees Celsius the same?* **no**
- *Does this represent a proportional relationship?* **no**
- *How does the graph illustrate that this relationship is linear, but not proportional?* **The graph is a straight line (linear), but it does not pass through the origin (not proportional).**

BL
- *Would you rather use a table or a graph to determine proportionality? Explain.* **Sample answer: Using a graph is more visual, but I need to use graph paper or a grid. Using a table is not as visual, but I don't need any materials to create a table.**

Need Another Example?

Use the table to determine if there is a proportional linear relationship between the speed (meters per second) and the time since a ball has been dropped. Explain your reasoning. **yes; The ratio of speed to time is a constant 9.8, so the relationship is proportional and linear.**

Speed (m/s)	0	9.8	19.6	29.4	39.2	49.0
Time (s)	0	1	2	3	4	5

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

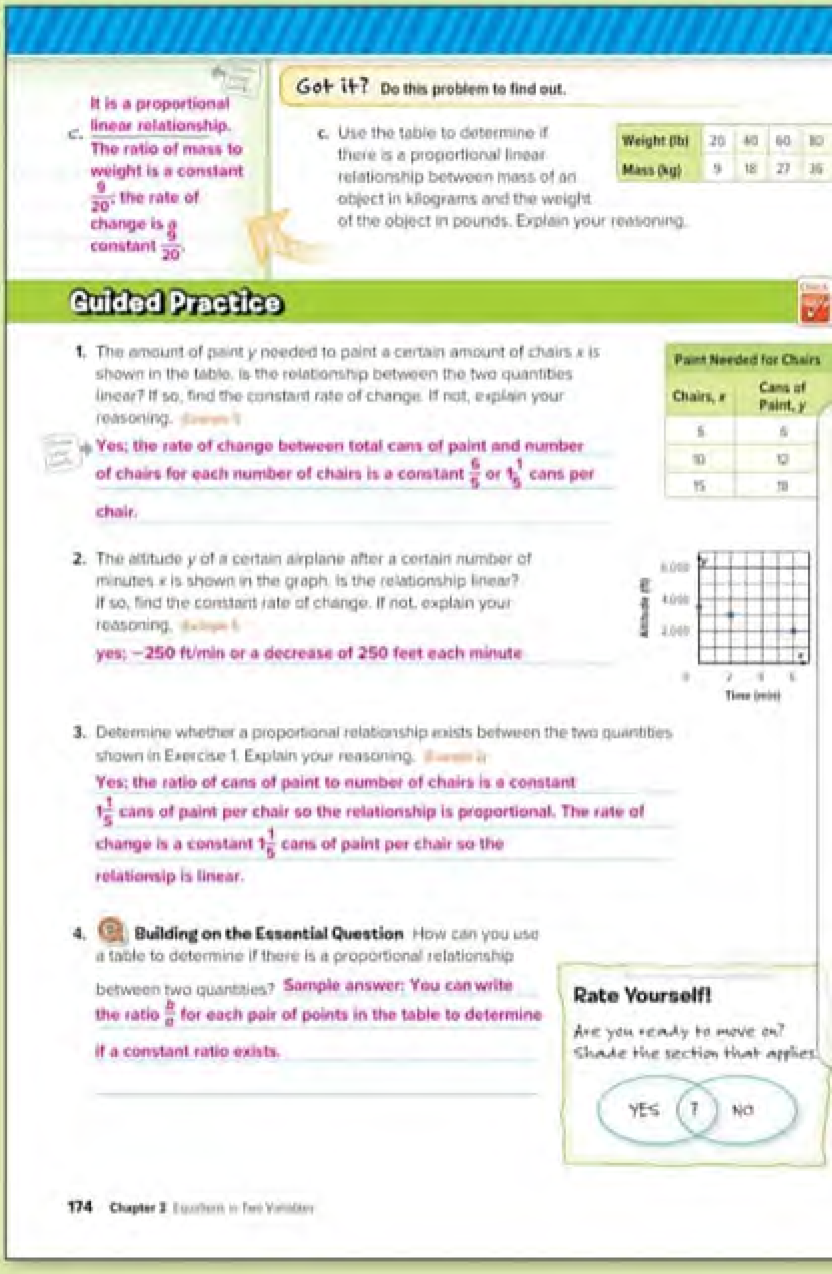
If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Teammates Consult** Have students work in small groups to discuss Exercise 1 with Student 1 leading the discussion. When everyone on the team has contributed and a solution is agreed upon, have all students record their answer in their textbooks. Repeat the process for Exercise 2 with Student 2 leading the discussion, and so on. MP 1, 2, 4, 5, 6, 7, 8

BL **LA** **Trade-a-Problem** Have students create their own real-world problem involving a constant rate of change. Have them trade their problems with a partner. Each partner generates a list of ordered pairs that represent the problem and graph them on a coordinate plane. Have them use the graph to verify whether the relationship demonstrates a constant rate of change. Then have them use the graph to determine whether the relationship is proportional. Have them justify their response. MP 1, 2, 4, 5, 6, 7, 8

Watch Out!

Common Error Students may have trouble determining if a relationship is proportional. Have students write the ratio in words to compare the quantities. They should make sure they compare the two quantities in the same order and that they do not invert any ratios.



c. It is a proportional linear relationship. The ratio of mass to weight is a constant $\frac{9}{20}$; the rate of change is a constant $\frac{9}{20}$.

Got it? Do this problem to find out.

c. Use the table to determine if there is a proportional linear relationship between mass of an object in kilograms and the weight of the object in pounds. Explain your reasoning.

Weight (lb)	20	40	60	80
Mass (kg)	9	18	27	36

Guided Practice

1. The amount of paint y needed to paint a certain amount of chairs x is shown in the table. Is the relationship between the two quantities linear? If so, find the constant rate of change. If not, explain your reasoning.
Yes; the rate of change between total cans of paint and number of chairs for each number of chairs is a constant $\frac{6}{5}$ or $1\frac{1}{5}$ cans per chair.

Paint Needed for Chairs	
Chairs, x	Cans of Paint, y
5	6
10	12
15	18

2. The altitude y of a certain airplane after a certain number of minutes x is shown in the graph. Is the relationship linear? If so, find the constant rate of change. If not, explain your reasoning.
yes; −250 ft/min or a decrease of 250 feet each minute

3. Determine whether a proportional relationship exists between the two quantities shown in Exercise 1. Explain your reasoning.
Yes; the ratio of cans of paint to number of chairs is a constant $1\frac{1}{5}$ cans of paint per chair so the relationship is proportional. The rate of change is a constant $1\frac{1}{5}$ cans of paint per chair so the relationsip is linear.

4. **Building on the Essential Question** How can you use a table to determine if there is a proportional relationship between two quantities? Sample answer: You can write the ratio $\frac{b}{a}$ for each pair of points in the table to determine if a constant ratio exists.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

YES ? NO

174 Chapter 3 Equations in Two Variables

Name ________________ My Homework ________________

Independent Practice

Determine whether the relationship between the two quantities shown in each table or graph is linear. If so, find the constant rate of change. If not, explain your reasoning. Example 1

1.

Cost of Electricity to Run Personal Computer	
Time (h)	Cost (AED)
5	15
8	24
12	36
24	72

Yes; the rate of change between cost and time for each hour is a constant AED 3 per hour.

2.

Distance Traveled by Falling Object				
Time (s)	1	2	3	4
Distance (m)	4.9	19.6	44.1	78.4

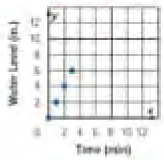
No; the rate of change from 1 to 2 seconds, 14.7 m/s, is not the same as the rate of change from 2 to 3 seconds, 24.5 m/s, so the rate of change is not constant.

3.

Italian Dressing Recipe				
Oil (c)	2	4	6	8
Vinegar (c)	$\frac{3}{4}$	$1\frac{1}{2}$	$2\frac{1}{4}$	3

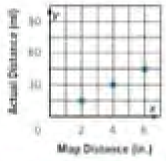
Yes; the rate of change between vinegar and oil for each cup of oil is a constant $\frac{3}{8}$ cup vinegar per cup of oil.

4.



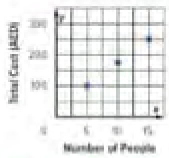
Yes; the rate of change between water level and time for each minute is a constant 2 in./min.

5.



Yes; the rate of change between the actual distance and the map distance for each inch on the map is a constant 7.5 mi/in.

6.



Yes; the rate of change between the total cost and the number of people is a constant AED 15/person.

Determine whether a proportional relationship exists between the two quantities shown in the following Exercises. Explain your reasoning. Example 2

7. Exercise 1

Yes; the ratio of the cost to time is a constant AED 3 per hour, so the relationship is proportional.

8. Exercise 3

Yes; the ratio of vinegar to oil is a constant $\frac{3}{8}$ cup vinegar per cup of oil, so the relationship is proportional.

9. Exercise 5

Yes; the ratio of actual distance to map distance is a constant $\frac{15}{2}$ miles per inch, so the relationship is proportional.

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as t homework assignment. The Extra Practice page can be use for additional reinforcement or as a second-day assignmen

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level indicating the lowest level of complexity.

	Exercises		
	1–9, 14–16	10, 17–21	11–13
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 12, 13, 20, 21
OL	On Level	1–9 odd, 10, 12, 13, 20, 21
BL	Beyond Level	10–13, 20, 21

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	11
2 Reason abstractly and quantitatively.	17, 18, 19
3 Construct viable arguments and critique the reasoning of others.	13
4 Model with mathematics.	12
5 Use appropriate tools strategically.	10

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and to apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students give a real-world example of a relationship that has a constant rate of change. **Sample answer: the distance traveled at a constant speed**

10. **Use Math Tools** Match each table with its rate of change.

2.4 ft/min

1.0 ft/min

−0.8 ft/min

0.25 ft/min

Time (minutes)	20	30	40
Altitude (feet)	170	162	154

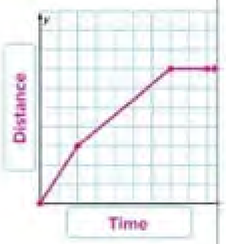
Time (minutes)	1	2	3
Distance (feet)	20	30	40

Time (minutes)	4	6	8
Height (feet)	1	1.5	2

Time (minutes)	5	10	15
Depth (feet)	12	24	36

H.O.T. Problems Higher Order Thinking

11. **Persevere with Problems** A cat starts walking, slows down, and then sits down to rest. Sketch a graph of the situation to represent the different rates of change. Label the *x*-axis "Time" and the *y*-axis "Distance". **Sample answer:**



12. **Model with Mathematics** Describe a situation with two quantities that have a proportional linear relationship.
Sample answer: Ahmed can read at a constant rate of 1.5 pages per minute. The number of pages read and the amount of time in minutes is proportional.

13. **Justify Conclusions** Each table shows a relationship with a constant rate of change. Is each relationship proportional? Justify your reasoning.

a.

Cost of Play Tickets (AED)				
t	1	2	3	4
c	3.50	4.00	4.50	5.00

no; Sample answer: $\frac{3.50}{1} \neq \frac{4.00}{2}$

b.

Cost of Play Tickets (AED)				
t	1	2	3	4
c	2.50	5.00	7.50	10.00

yes; Sample answer: $\frac{2.50}{1} = \frac{5.00}{2} = \frac{7.50}{3} = \frac{10.00}{4}$

Name ______________ My Homework ______________

Extra Practice

Determine whether the relationship between the two quantities shown in each table is linear. If so, find the constant rate of change. If not, explain your reasoning.

14.

Sale Price Comparison	
Retail Price (AED)	Sale Price (AED)
0	0
10	5
20	10
30	15
40	20
50	25
60	30

+10, +10, +10, +10, +10, +10 | +5, +5, +5, +5, +5, +5

Yes; the rate of change between the sale price and retail price is a constant value of $\frac{1}{2}$.

15.

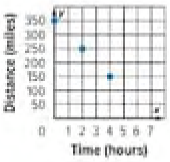
Total Number of Customers Helped at Jewelry Store	
Time (h)	Total Helped
1	12
2	24
3	36
4	60

No; the rate of change from 1 to 2 hours, $\frac{24-12}{2-1}$ or 12 per hour, is not the same as the rate of change from 3 to 4 hours, $\frac{60-36}{4-3}$ or 24 per hour, so the rate of change is not constant.

16. Determine whether a proportional relationship exists between the two quantities in Exercise 14. Explain your reasoning. Yes; the ratio of the sale price to the retail price is a constant $\frac{1}{2}$, so the relationship is proportional.

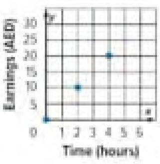
Reason Abstractly Find the constant rate of change for each graph and interpret its meaning.

17.



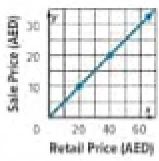
−50 mph; the distance decreased by 50 miles every hour.

18.



$5/h; earnings were $5 per hour.

19.



0.5; $\frac{1}{2}$ of retail price.

Power Up! Test Practice

Exercises 20 and 21 prepare students for more rigorous thinking needed for the assessment.

20.

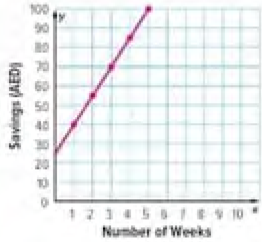
This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly graph the points, connect them, and state the constant rate of change.
1 point	Students correctly graph the points and connect them, but fail to state the constant rate of change OR students correctly plot the points and state the constant rate of change but fail to connect the points with a line OR students correctly plot 3–4 points and state the constant rate of change.

21.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part.

Power Up! Test Practice

20. The table shows the amount of money in Suhaib's savings account. Graph the points on the coordinate plane and connect them with a straight line.



Week	Savings (AED)
1	40
2	55
3	70
4	85
5	100

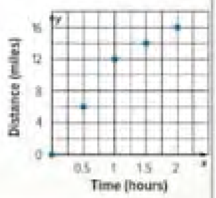
What is the constant rate of change? **AED 15 per week**

21. The graph shows the distance Mashael traveled on her 2-hour bike ride. Determine if each statement is true or false.

a. She traveled at a constant speed of 12 miles per hour for the entire ride. ☐ True ■ False

b. She traveled at a constant speed of 12 miles per hour for the first hour. ■ True ☐ False

c. She traveled at a constant speed of 4 miles per hour for the last hour. ■ True ☐ False



Spiral Review

Find the unit rate. Round to the nearest hundredth if necessary.

22. 60 miles on 2.5 gallons **24 mi/gal**

23. 4,500 kilobytes in 6 minutes **750 kB/min**

24. 10 red peppers for AED 5.50 **AED 0.55/red pepper**

25. 72.6 meters in 11 seconds **6.6 m/s**

Inquiry Lab

Graphing Technology: Rate of Change

Inquiry **HOW can you use a graphing calculator to determine the rate of change?**

Mathematical Practices 1, 3

t the school store, tickets to the football game re sold for AED 5 each. The equation $y = 5x$ can be used o find the total cost y of any number of tickets x. Find he rate of change.

Vhat do you know? **Tickets cost AED 5.**

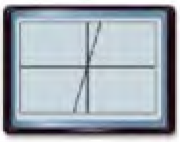
Vhat do you need to find? **the rate of change**

Hands-On Activity

ecall that a rate of change is a rate that describes how one quantity changes in elation to another.

Step 1 Enter the equation. Press [Y=] 5 [X,T,θ,n].

Step 2 Graph the equation in the standard viewing window. Press [Zoom] 6.



Step 3 Press [2nd] [TblSet] [▼] [▼] [ENTER] [▼] [ENTER] to generate the table automatically. Press [2nd] [Table] to access the table. Choose any two points on the line and find the rate of change. **Sample answer:**

$$\frac{\text{change in total cost}}{\text{change in number of tickets}} = \frac{\text{AED }(5 - 0)}{(1 - 0)\text{ tickets}}$$

$$= \frac{\text{AED 5}}{1\text{ ticket}}$$

o, the rate of change, or unit rate, is **AED 5 per ticket sold**.

Focus narrowing the scope

Objective Use a graphing calculator to find rate of change.

Coherence connecting within and across grades

Now
Students use technology to find the rate of change or slope.

Next
Students will find the rate of change or slope from tables, graphs, and equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 180.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Materials: graphing calculator

Hands-On Activity

AL LA **Pairs Interview** Have students work in pairs to complete the steps in the Activity. As they complete each step, have them interview each other by asking the questions below. MP 1, 2, 4, 5, 7, 8

- *What is the standard viewing window?* **The scale on each axis is −10 to 10, with an interval of 1.**
- *How do you access the table?* **Press** [2nd] [TABLE]
- *What does the rate of change mean in the context of the problem?* **Each ticket costs AED 5.**

Engage Explore Explain Elaborate Evaluate

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1, 2	Exercises 3	Exercises 4, 5
Level 3			●
Level 2		●	
Level 1	●		

Analyze and Reflect

AL BL LA **Numbered Heads Together** Assign students to 3- or 4-person learning teams, working together to write a response for Exercise 3. Make sure each member of the team verbally contributes and carefully listens to others. Then call on a specific person from each group to read their response out loud. Discuss any differences in responses. MP 1, 2, 4, 5, 7, 8

Create

BL LA **Trade-a-Problem** Have students trade their equations they wrote for Exercise 4 with a partner. Each partner should verify that the equation the other student wrote represents a line steeper than the equation given in Exercise 4. MP 1, 2, 7, 8

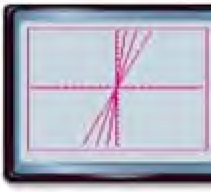
Inquiry Students should be able to answer "HOW can you use a graphing calculator to determine the rate of change?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner. School T-shirts are sold for AED 10 each and packages of markers are sold for AED 2.50 each.

1. For each item, write an equation that can be used to find the total cost y of x items. $y = 10x$; $y = 2.5x$

2. Graph the equations in the same window as the equation from the Activity. Copy your calculator screen on the blank screen shown.



Analyze and Reflect

3. Refer to Exercises 1 and 2. Find each rate of change. Is there a relationship between the steepness of the lines on the graph and the rates of change? Explain. $\frac{\text{AED } 10}{1 \text{ Tshirt}}$ or 10; $\frac{\text{AED } 2.50}{1 \text{ package}}$ or 2.5; yes; Sample answer: Lines with a greater rate of change are steeper than lines with a lesser rate of change.

Create

4. MP **Reason Inductively** Without graphing, write the equation of a line that is steeper than $y = \frac{1}{3}x$. Explain your reasoning.
$y = 3x$; Sample answer: $y = 3x$ has a rate of change of 3 and $y = \frac{1}{3}x$ has a rate of change of $\frac{1}{3}$. Since $3 > \frac{1}{3}$, $y = 3x$ has a steeper graph.

5. Inquiry HOW can you use a graphing calculator to determine the rate of change?
Sample answer: You can use the TABLE feature on a graphing calculator to find two points on the line. Then use the two points to find the rate of change.

Lesson 2

Slope

Vocabulary Start-Up

The term slope is used to describe the steepness of a straight line. **Slope** is the ratio of the **rise**, or vertical change, to the **run** or horizontal change.

Complete the graphic organizer. Sample answers are given.

slope

I think this word means...	How is this concept related to other math concepts?
downward or slant	It is a ratio.
Where have I heard this word in my life?	**What makes this an important word for me to know?**
to describe a roof or hillside	so I can describe the steepness of a straight line

Essential Question

WHY are graphs helpful?

Vocabulary

slope
rise
run

Mathematical Practices
1, 3, 4

Real-World Link

A ride at an amusement park rises 8 feet every horizontal change of 2 feet. How could you determine the slope of the ride?

Sample answer: Write the ratio of rise to run or 8 feet to 2 feet and simplify to 4.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Use tables and graphs to find the slope of a line.

Coherence connecting within and across grades

Previous Students used tables and graphs to find the constant rate of change in a linear relationship.

Now Students use tables and graphs to find the slope of a line.

Next Students will compare slopes of linear function

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 185.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, sm group, think-pair-share activity, or independent activity.

LA **Pairs Share** Have students read the definitio of *slope*. Instruct students to complete the graphic organizer individually, and then compare with a partner. MP 1, 5, 6

Alternate Strategies

AL **LA** Have students prepare a chart of words or phras that have similar meanings as slope. Display this chart in the class. MP 1, 5, 6

BL **LA** Have students research other real-world situatio that involve slope. Have them create drawings or print pictures from the Internet that display the slope of an object in the real world. MP 1, 4, 5

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

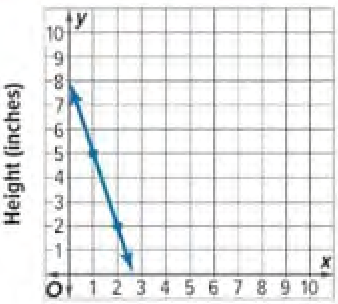
1. Find slope.

- AL • *How can you use a real-world scenario to remember $\frac{rise}{run}$?* **Sample answer: A helicopter rises before it moves forward.**
- OL • *If your friend wrote the slope as $\frac{48}{10}$, how could you explain to your friend their error?* **The slope is the rise over the run, not the run over the rise.**
- BL • *How could you make the treadmill more difficult to run on? easier?* **Sample answer: Increasing the rise of the treadmill will make it more difficult to run. Decreasing the rise of the treadmill will make it easier.**

2. Find slope using a graph.

- AL • *What is the rise? the run?* **2; 1**
- OL • *What does the slope represent in the context of the problem?* **The unit cost is AED 2 per muffin.**
- • *How does the slope compare to the constant rate of change?* **They are equivalent.**
- BL • *Why is this relationship a proportional linear relationship?* **The graph is a straight line that passes through the origin.**

Need Another Example?
The graph shows the position of a portrait leaning against a wall. Find the slope of the line.
$-\frac{3}{1}$ or -3



Work Zone

Find Slope Using a Graph or Table

Slope is a rate of change. It can be positive (slanting upward) or negative (slanting downward).

slope = $\frac{\text{rise}}{\text{run}}$ ← vertical change between any two points ← horizontal change between the same two points

Example

1. Find the slope of the treadmill.

slope = $\frac{rise}{run}$ Definition of slope

= $\frac{10 \text{ in.}}{48 \text{ in.}}$ rise = 10 in., run = 48 in.

= $\frac{5}{24}$ Simplify.

The slope of the treadmill is $\frac{5}{24}$.

Got it? Do this problem to find out.

a. A hiking trail rises 6 feet for every horizontal change of 100 feet. What is the slope of the hiking trail?

a. $\frac{3}{50}$

Translating Rise and Run

up → positive
down → negative
right → positive
left → negative

Examples

2. The graph shows the cost of muffins at a bake sale. Find the slope of the line.

Choose two points on the line. The vertical change is 2 units and the horizontal change is 1 unit.

slope = $\frac{rise}{run}$ Definition of slope

= $\frac{2}{1}$ rise = 2, run = 1

The slope of the line is $\frac{2}{1}$ or 2.

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3. **The table shows the number of pages Garrett has left to read after a certain number of minutes. The points lie on a line. Find the slope of the line.**

Time (min), x	Pages left, y
1	12
3	9
5	6
7	3

Choose any two points from the table to find the changes in the x- and y-values.

$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$ Definition of slope

$= \frac{9 - 12}{3 - 1}$ Use the points (1, 12) and (3, 9).

$= \frac{-3}{2}$ or $-\frac{3}{2}$ Simplify.

To check, choose two different points from the table and find the slope.

Check $\text{slope} = \frac{\text{change in } y}{\text{change in } x}$

$= \frac{3 - 6}{7 - 5}$

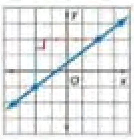
$= \frac{-3}{2}$ or $-\frac{3}{2}$ ✓

Slope
In linear relationships, no matter which two points you choose, the slope, or rate of change, of the line is always constant.

Got it? Do these problems to find out.

Find the slope of each line.

b.



c.

x	−6	−2	2	6
y	−2	−1	0	1

b. $\frac{3}{4}$

c. $\frac{1}{4}$

Slope Formula

Key Concept

Words The slope m of a line passing through points (x_1, y_1) and (x_2, y_2) is the ratio of the difference in the y-coordinates to the corresponding difference in the x-coordinates.

Symbols $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_2 \neq x_1$

Model

It does not matter which point you define as (x_1, y_1) and (x_2, y_2). However the coordinates of both points must be used in the same order.

Example

3. **Find slope using a table.**

AL
- *Does "change in y" refer to the rise or the run?* **rise**
- *Does "change in x" refer to the rise or the run?* **run**

OL
- *When you find $\frac{\text{change in } y}{\text{change in } x}$, what are you finding?* **the slope of the line through two given points**
- *What would the graph of the line look like?* **Sample answer: a line sloping downward**

BL
- *What will be the next point in the table?* **(9, 0)**
- *Why is a negative slope reasonable for this problem?* **Sample answer: The graph shows how many pages he has left to read, so as the minutes pass, he has fewer and fewer pages to read.**

Need Another Example?
The table shows the number of gallons of paint Mr. Omer used to paint the rooms in her house. Find the slope of the line. $\frac{3}{2}$

Gallons of Paint, x	2	4	6	8
Rooms Painted, y	3	6	9	12

Watch Out!

Common Error Students may have trouble with the slope formula because they use run over rise. Have students write down the formula and highlight the x and y values with different colors. Use the same colors to highlight the x- and y-coordinates for the points.

Example

4. Find slope using coordinates.

AL • *What is the x-coordinate of point R?* 1

• *What is the y-coordinate of point S?* 3

OL • *What is the "change in y"?* 3 − 2 or 1

• *What is the "change in x"?* −4 − 1 or −5

BL • *How can you predict, by looking at the graph, that the slope will be negative?* The line slopes downward from left to right.

• *If the slope was −5, how would that line compare to the line in this example?* The line with a slope of −5 would be steeper.

Need Another Example?

Find the slope of the line that passes through *A*(3, 3) and *B*(2, 0). 3

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Think-Pair-Solo** Have students think about their answers to Exercises 1, 2, and 4. Then have them pair with a partner to share responses, ensuring that both partners understand. Then have them work individually to complete Exercises 3 and 5. MP 1, 2, 5, 7, 8

BL LA **Trade-a-Problem** Have students create their own linear relationships. One relationship should be expressed as a graph, the other as a table, and the third as a pair of points. Have students trade with each other to find the slope of each relationship. Have them check each other's work. MP 1, 2, 5, 7, 8

Using the Slope Formula

To check Example 4, let $(x_1, y_1) = (-4, 3)$ and $(x_2, y_2) = (1, 2)$. Then find the slope.

a. $\frac{1}{3}$

c. $\frac{1}{2}$

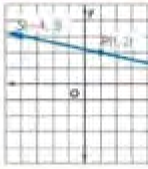
Example

4. Find the slope of the line that passes through *R*(1, 2), *S*(−4, 3).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \frac{3 - 2}{-4 - 1}$ $(x_1, y_1) = (1, 2)$, $(x_2, y_2) = (-4, 3)$

$m = \frac{1}{-5}$ or $-\frac{1}{5}$ Simplify.



Got it? Do these problems to find out.

d. *A*(2, 2), *B*(5, 3)

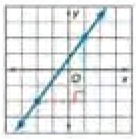
e. *J*(−7, −4), *K*(−3, −2)

Guided Practice

1. Find the slope of the storage shed's roof. (Example 1)
$\frac{1}{5}$ or $-\frac{1}{5}$

Find the slope of each line. (Examples 2 and 3)

2. $\frac{4}{3}$



3. 2

x	0	1	2	3
y	1	3	5	7

Find the slope of the line that passes through each pair of points. (Example 4)

4. *A*(−3, −2), *B*(5, 4) $\frac{3}{4}$

5. *E*(−6, 5), *F*(3, −3) $-\frac{8}{9}$

6. **Building on the Essential Question** In any linear relationship, explain why the slope is always the same.
Sample answer: The slope is the same as the rate of change. In a linear relationship, the rate of change is constant.

Rate Yourself!

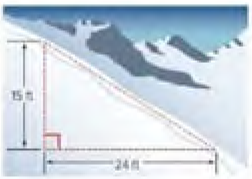
How well do you understand slope? Circle the image that applies.

Clear | Somewhat Clear | Not So Clear

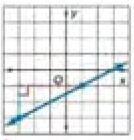
Name ______________ My Homework ______________

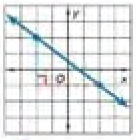
Independent Practice

1. Find the slope of a ski run that descends 15 feet for every horizontal change of 24 feet. (Example 1)
$-\frac{5}{8}$



Find the slope of each line. (Example 2)

2.

$\frac{1}{2}$

3.

$-\frac{3}{4}$

The points given in the table lie on a line. Find the slope of each line. (Example 3)

4. $-\frac{5}{2}$

x	0	2	4	6
y	9	4	−1	−6

5. 2

x	0	1	2	3
y	3	5	7	9

Find the slope of the line that passes through each pair of points. (Example 4)

6. $A(0, 1)$, $B(2, 7)$ 3

7. $C(2, 5)$, $D(3, 1)$ −4

8. $E(1, 2)$, $F(4, 7)$ $\frac{5}{3}$

9. **Justify Conclusions** Wheelchair ramps for access to public buildings are allowed a maximum of one inch of vertical increase for every one foot of horizontal distance. Would a ramp that is 10 feet long and 8 inches tall meet this guideline? Explain your reasoning to a classmate.
yes; $\frac{1}{15} < \frac{1}{12}$

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–8, 14–22	9, 10, 23, 24	11–13
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11, 13, 23, 24
OL	On Level	1–9 odd, 10, 11, 13, 23, 24
BL	Beyond Level	10–13, 23, 24

MATHEMATICAL PRACTICES

	Emphasis On	Exercise(s)
1	Make sense of problems and persevere in solving them.	12
3	Construct viable arguments and critique the reasoning of others.	9, 10, 11
4	Model with mathematics.	13
5	Use appropriate tools strategically.	18, 19

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

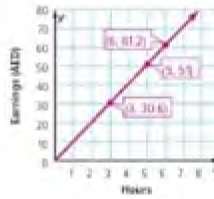
Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write an explanation of how to find the slope of a line given the points $A(1, 5)$ and $B(-7, 8)$. $-\frac{3}{8}$; **See students' work for explanations.**

10. **Multiple Representations** For working 3 hours, Dana earns AED 30.60. For working 5 hours, she earns AED 51. For working 6 hours, she earns AED 61.20.



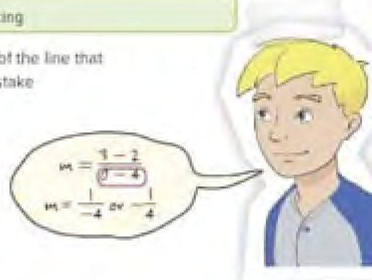
a. **Graphs** Graph the information with hours on the horizontal axis and money earned on the vertical axis. Draw a line through the points.

b. **Numbers** What is the slope of the line?
10.2

c. **Words** What does the slope of the line represent?
How does the slope relate to the unit rate? **the amount she made per hour, AED 10.20; the slope and the unit rate are the same**

H.O.T. Problems Higher Order Thinking

11. **Find the Error** Ammar is finding the slope of the line that passes through $X(0, 2)$ and $Y(4, 3)$. Circle his mistake and correct it.

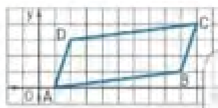


Ammar did not use the x-coordinates in the same order as the y-coordinates.

$m = \frac{3-2}{4-0}$

$m = \frac{1}{4}$

12. **Persevere with Problems** Two lines that are parallel have the same slope. Determine whether quadrilateral *ABCD* is a parallelogram. Justify your reasoning.



Slope of $\overline{AB}$: $m = \frac{1-0}{9-1}$ or $\frac{1}{8}$

Slope of $\overline{BC}$: $m = \frac{4-1}{10-9}$ or 3

Slope of $\overline{CD}$: $m = \frac{3-4}{2-10}$ or $\frac{1}{8}$

Slope of $\overline{DA}$: $m = \frac{0-3}{1-2}$ or 3

Since $\overline{AB}$ and $\overline{CD}$ are parallel, and $\overline{BC}$ and $\overline{DA}$ are parallel, quadrilateral *ABCD* is a parallelogram.

13. **Model with Mathematics** Give three points that lie on a line with each of the following slopes. **Sample answers are given.**

a. 5 **(1, 1), (2, 6), (3, 11)**

b. $\frac{1}{5}$ **(1, 1), (6, 2), (11, 3)**

c. −5 **(1, 1), (0, 6), (−1, 11)**

Name ______________________ My Homework ______________________

Extra Practice

14. Find the slope of a road that rises 12 feet for every horizontal change of 100 feet. $\frac{3}{25}$

slope $= \frac{\text{rise}}{\text{run}}$ Definition of slope

Homework Help $= \frac{12 \text{ ft}}{100 \text{ ft}}$ rise = 12 ft, run = 100 ft

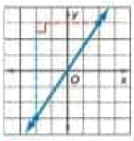
$= \frac{3}{25}$ Simplify.

15. Hammad is flying a kite in the park. The kite is a horizontal distance of 24 feet from Hammad's position and a vertical distance of 72 feet.

Find the slope of the kite string. 3

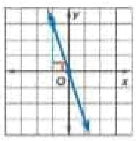
Find the slope of each line.

16.



$\frac{3}{2}$

17.



−3

Use Math Tools The points given in the table lie on a line. Find the slope of each line.

18. $\frac{2}{3}$

x	−3	3	9	15
y	−3	1	5	9

19. 2

x	−2	−1	1	2
y	−4	−2	2	4

Find the slope of the line that passes through each pair of points.

20. $M(-2, 3)$, $N(7, -4)$ $-\frac{7}{9}$

21. $G(-6, -1)$, $H(4, 1)$ $\frac{1}{5}$

22. $J(-9, 3)$, $K(2, 1)$ $-\frac{2}{11}$

Power Up! Test Practice

Exercises 23 and 24 prepare students for more rigorous thinking needed for the assessment.

23.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

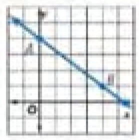
24.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Kowledge	DOK3
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students draw a line that passes through the points, determine the slope, and explain the meaning of the slope.
1 point	Students draw a line that passes through the points and determine the slope but fail to explain the meaning of slope OR students determine the slope and explain the meaning, but fail to draw the appropriate line.

Power Up! Test Practice

23. Line *AB* represents a steep hill.

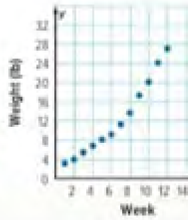
The coordinates of point *A* are **(0, 4)** and the coordinates of point *B* are **(4, 1)**. The slope of the hill is $-\frac{3}{4}$.



24. Fahed charted the growth of his puppy for several weeks and plotted the values on a graph. Draw a line that passes through the points (2, 4) and (10, 20).

What is the slope of the line? What does this slope represent?

$\frac{2}{1}$ **or 2; Sample answer: This means that the puppy is gaining about 2 pounds per week.**



Spiral Review

25. The wait time to ride the Thunder boats is 30 minutes when 180 people are in line. Write and solve a proportion to find the wait time when 240 people are in line.

$\frac{30}{180} = \frac{x}{240}$; **40 minutes**

Solve each proportion.

26. $\frac{5}{7} = \frac{a}{35}$ **25**

27. $\frac{12}{p} = \frac{36}{45}$ **15**

28. $\frac{3}{9} = \frac{21}{k}$ **63**

29. $\frac{n}{15} = \frac{17}{34}$ **7.5**

30. $\frac{-7}{10} = \frac{3.5}{j}$ **−5**

31. $\frac{12}{18} = \frac{-40}{x}$ **−60**

Lesson 3

Equations in $y = mx$ Form

Real-World Link

Charity The amount of money Essam can raise for the Wish Upon A Rainbow Bike-a-thon is shown in the table.

Biking Time (h), x	Money Raised (AED), y
2	20
4	40
6	60

Essential Question

WHY are graphs helpful?

Vocabulary

direct variation
constant of variation
constant of proportionality

Mathematical Practices
1, 3, 4

Recall that when the ratio of two variable quantities is constant, a proportional relationship exists. This relationship is called a **direct variation**. The constant ratio is called the **constant of variation** or **constant of proportionality**.

Complete the steps below to derive the equation for a direct variation.

$\frac{y_2 - y_1}{x_2 - x_1} = m$ Slope formula

$\frac{y - 0}{x - 0} = m$ $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (x, y)$

$\frac{y}{x} = m$ Simplify.

$y = m x$ Multiplication Property of Equality

1. Use the table to find the rate of change. Then write an equation in $y = mx$ form to represent the situation.
AED 10 per hour; $y = 10x$

Which Mathematical Practices did you use?
Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Use direct variation to solve problem

Coherence connecting within and across

Previous
Students found the constant rate of change or slope using tables and graphs.

Now
Students apply the concept of constant rate of change to direct variation.

Ne
Stud
equa
slope

Rigor pursuing concepts, fluency, and appli

See the Levels of Complexity chart on page 195.

ENGAGE EXPLORE EXPLAIN ELABORATE EVAL

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, think-pair-share activity, or independent a

AL LA Teammates Consult Hav discuss in groups the definitions of s proportional relationship, and how to find slope students work together to complete the Real-W choose one student from each group to present responses to the class. MP 1, 6, 7, 8

Alternate Strategy

BL Have students work in groups of three to how many letters they can write in 1 minute, 2 m minutes, and so on. One student writes, one st time, and the third student records the data in them analyze whether there is a constant ratio letters to number of minutes. MP 1, 7, 8

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Find the constant of variation.

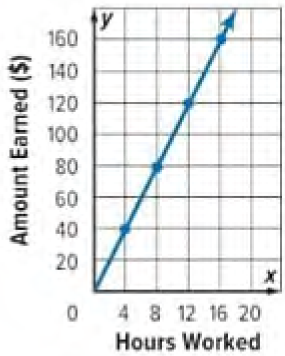
AL
- *Name a point that lies on the line.* **Sample answer: (2, 15)**
- *What does that point represent in the context of the problem?* **Robin earns AED 15 for 2 hours of babysitting.**
- *Name a point that does not lie on the line.* **Sample answer: (4, 4)**

OL
- *How would you find the constant of variation?* **Find $\frac{\text{amount earned}}{\text{time}}$ for points on the line.**
- *In the context of the problem, what does the ratio $\frac{7.5}{1}$ mean?* **Robin makes AED 7.50 for each hour she babysits.**

BL
- *Suppose Radhwa told you she made AED 30 for 4 hours of babysitting. How could you verify if that statement was correct?* **Sample answer: You could find the unit rate, or constant of variation, $\frac{30}{4}$, to see if it was AED 7.50.**

Need Another Example?

The amount of money Samira earns at her job is shown on the graph. Determine the amount Serena earns per hour.
AED 10 per hour



Key Concept **Direct Variation**

Words A linear relationship is a direct variation when the ratio of y to x is a constant, m. We say y varies directly with x.

Symbols $m = \frac{y}{x}$ or $y = mx$, where m is the constant of variation and $m \neq 0$

Example $y = 3x$

Graph

Work Zone

$y = mx$
In a direct variation equation $y = mx$, m represents the constant of variation, the constant of proportionality, the slope, and the unit rate.

The slope of the graph of $y = mx$ is m. Since (0, 0) is o
$y = mx$, the graph of a direct variation always passes t
origin.

Example

1. The amount of money Radwa earns while babysitting varies directly with the time as shown in the graph. Determine the amount that Radwa earns per hour.

To determine the amount Radwa earns per hour, or the unit rate, find the constant of variation.

Use the points (2, 150), (3, 225), and (4, 300).

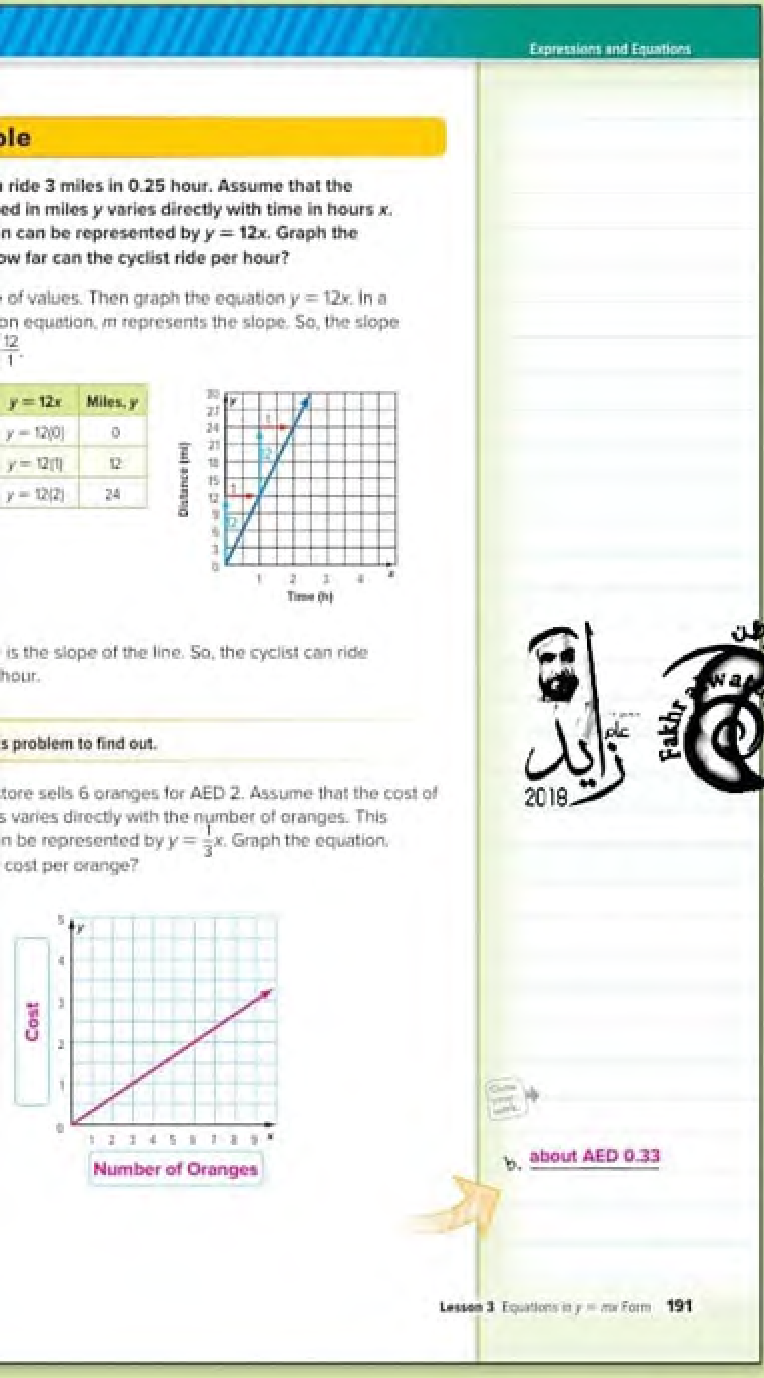
$\frac{\text{amount earned}}{\text{time}} \rightarrow \frac{150}{2}$ or $\frac{75}{1}$ $\frac{225}{3}$ or $\frac{75}{1}$ 3

So, Radwa earned AED 7.50 for each hour she

Got it? Do this problem to find out.

a. Two minutes after a skydiver opens his parach
descended 1,900 feet. After 5 minutes, he des
4,750 feet. If the distance varies directly with t
at what rate is the skydiver moving?

a. **−950 ft/min**


Expressions and Equations

ple

ride 3 miles in 0.25 hour. Assume that the
ed in miles y varies directly with time in hours x.
n can be represented by y = 12x. Graph the
ow far can the cyclist ride per hour?

of values. Then graph the equation y = 12x. In a
on equation, m represents the slope. So, the slope
$\frac{12}{1}$.

y = 12x	Miles, y
y = 12(0)	0
y = 12(1)	12
y = 12(2)	24

is the slope of the line. So, the cyclist can ride
hour.

s problem to find out.

tore sells 6 oranges for AED 2. Assume that the cost of
s varies directly with the number of oranges. This
n be represented by $y = \frac{1}{3}x$. Graph the equation.
cost per orange?

b. about AED 0.33

Lesson 3 Equations in y = mx Form 191

Example

2. Graph a direct variation equation.

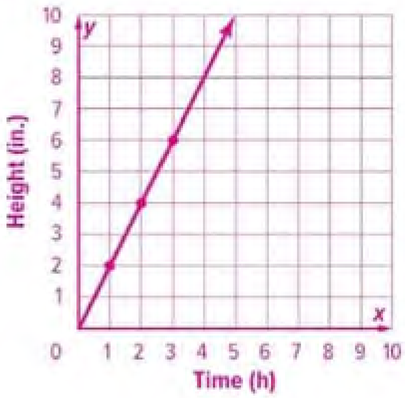
AL
- *Describe a method you could use to graph the equation y = 12x.* **Make a function table using the input values 0, 1, and 2.**
- *How far did the cyclist ride in 0 hours? 1 hour? 2 hours?* **0 mi; 12 mi; 24 mi**
- *How would you write this as ordered pairs?* **(0, 0), (1, 12), and (2, 24)**

OL
- *What is the slope of the line?* $\frac{12}{1}$ **or 12**
- *What does the slope mean in the context of the problem?* **The cyclist rides 12 miles in 1 hour.**
- *What are some synonyms for slope?* **constant rate of change, unit rate**

BL
- *How do you know the relationship is a proportional linear relationship?* **The line that represents the relationship is a straight line that passes through the origin.**
- *What is the unit rate if the cyclist rode 3 miles in 0.25 hour?* $\frac{12 \text{ mi}}{1 \text{ h}}$

Need Another Example?

Some types of bamboo can grow 7 inches in 3.5 hours. Assume that the height y varies directly with the time x. This situation can be represented by the equation y = 2x. Graph the equation. How fast can the bamboo grow per hour? **2 inches per hour**



Example

3. Compare proportional relationships.

AL • *What do you need to find?* which animal is faster

• *What do you need to compare to determine which animal is faster?* the speeds of the grizzly bear and the rabbit

OL • *Describe how to find the speed of the rabbit. Then find the speed.* The speed is also the unit rate. Since the slope and the unit rate are the same, the unit rate for the rabbit is $\frac{35 \text{ mi}}{1 \text{ h}}$ or 35 mph.

• *Describe how to find the speed of the grizzly bear. Then find the speed.* Find the slope of the line on the graph. The speed of the grizzly bear is $\frac{30}{1}$ or 30 mph.

• *How do the unit rates compare?* The rate for the rabbit is greater than the rate for the grizzly bear.

BL • *The distance traveled by a giraffe is shown in the table.*

Time (min)	5	10	15	20
Distance (mi)	$2\frac{2}{3}$	$5\frac{1}{3}$	8	$10\frac{2}{3}$

• *How does the giraffe's speed compare to the rabbit and grizzly bear?* The speed of the giraffe is 32 miles per hour, so it is in between the rabbit and grizzly bear.

Need Another Example?

Khalid spent the amounts shown in the table on tokens at Playtime Games.

Number of Tokens	15	54	25	36
Total Cost (AED)	3	10.80	5	7.20

Tokens at Game Time are AED 0.25 per token. Which arcade has the better price for tokens? Explain. Playtime Games; Sample answer: the unit rate for Playtime Games is AED 0.20 per token and the unit rate for Game Time is AED 0.25 per token; AED 0.20 < AED 0.25.

Key Concept **Compare Direct Variations**

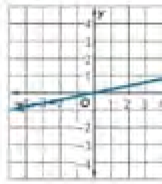
You can use tables, graphs, words, or equations to represent a
proportional relationships.

Work Zone

Table

x	15	20	25	30
y	3	4	5	6

Graph



Words y varies directly with x

Equation $y = \frac{1}{5}x$

STOP and Reflect In a proportional relationship, how is the unit rate represented on a graph? Explain below.

Sample answer: It is the slope of the line or $\frac{\text{rise}}{\text{run}}$.

When the x-value changes by an amount A, the y-value v
the corresponding amount mA.

Example

3. The distance y in miles covered by a rabbit in x hours can be represented by the equation $y = 35x$. The distance covered by a grizzly bear is shown on the graph. Which animal is faster? Explain.

Rabbit $y = 35x$

The slope or unit rate is 35 mph.

Grizzly Bear Find the slope of the graph.

$\frac{\text{rise}}{\text{run}} = \frac{30}{1}$ or 30

Since 35 > 30, the rabbit is the faster animal.

Got it? Do this problem to find out.

c. **Financial Literacy** Zaher's earnings for four we
part time job are shown in the table. Assume that
vary directly with the number of hours worked.

Time Worked (h)	15	12	22
Total Pay (AED)	1125.0	900.0	1650.0

He can take a job that will pay him AED 73 per hou
Which job has the better pay? Explain.

c. his current job; Sample answer: The unit rate of his current job is AED 75 per hour which is greater than AED 73 per hour.

Expressions and Equations

le

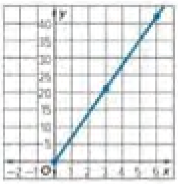
ervice dog is often considered to be 21 in human
e that the equivalent age in human years y varies
ts age as a dog x. Write and solve a direct variation
nd the human-year age of a dog that is 6 years old.

nt the dog's actual age and let y represent the
alent age.

now the human-year age or y-value when the dog is

og is 6 years old, the equivalent age in human

uation $y = 7x$.
hen $x = 6$ is 42. ✓



se problems to find out.

s travels 210 miles in $3\frac{1}{2}$ hours. Assume the
eled is directly proportional to the time traveled.
lve a direct variation equation to find how far the
el in 6 hours.

utterfly can fly 93 miles in 15 hours. Assume the
eled is directly proportional to the time traveled.
lve a direct variation equation to find how far the
terfly will travel in 24 hours.

d. $y = 60x$; 360 mi

e. $y = 6.2x$; 148.8 mi

Lesson 3 Equations in $y = mx$ Form 193

Example

4. Compare proportional relationships.

AL
- *How do you know that this situation is a direct variation?* **The problem indicates that the equivalent age in human years varies directly with its age as a dog.**
- *What is the form for a direct variation equation?* $y = mx$

OL
- *What value do you need to substitute for y in the direct variation equation? for x?* **21; 3**
- *What is the direct variation equation?* $y = 7x$
- *The equation is written, now what do you need to find?* **the age of the dog in human years, when the actual age of the dog is 6 years**
- *When a dog is 6 years old, what is its equivalent age in human years?* **42**

BL
- *What does the point (3, 21) represent?* **When a dog is 3 years old, the equivalent human age is 21.**
- *How would you find the human-year age of a dog that is 14 years old?* **Substitute 14 for x in the equation $y = 7x$ and simplify.**

Need Another Example?

At a certain store, four cans of soup cost AED 5. Assume the total cost varies directly with the number of cans purchased. Write and solve a direct variation equation to find how much it would cost to buy 10 cans of soup. **$y = 1.25x$; AED 12.50**

ractice

sessment Use these exercises to assess
erstanding of the concepts in this lesson.

ne of your students are not ready for
nments, use the differentiated activities below.

rs Discussion Have students work in pairs to
nilarities and differences of the following terms:
e, constant rate of change, and constant of
3, 6

d the Fib Students work in pairs to write two
fib for Exercise 3. For example, one fact could be
ob pays AED 7.25 per hour. One fib could be that
o pays AED 6.75 per hour. Then they trade their
ith another pair of students, not mentioning
e the facts and which one is the fib. Each pair of
ifies the facts and fibs. Then the two pairs of
e together as a team to resolve any differences.

Guided Practice

1. A color printer can print 36 pages in 3 minutes and 108 pages in 9 minutes. If the number of pages varies directly with the time, at what rate is the color printer printing? (Example 1)

   12 pages per minute

2. A new compact car can travel 288 miles on nine gallons of gas. The distance driven in miles y varies directly with the number of gallons of gas x. This situation can be represented by the equation $y = 32x$. (Examples 2 and 3)

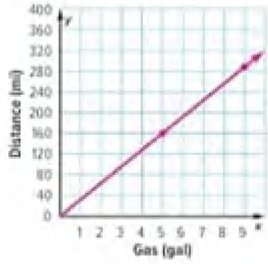
   a. Graph the equation on the coordinate plane shown.

   b. How many miles per gallon does the car get?

   32 miles per gallon

   c. The distance y traveled by a hybrid car using x gallons of gas can be represented by $y = 42x$. Which car gets better gas mileage? Explain.

   hybrid; Sample answer: The unit rate for the hybrid is 42 mpg. The unit rate for the new car is 32 mpg. 42 > 32



3. **Financial Literacy** Mai's current earnings are shown in the table. She was offered a new job that will pay AED 72 per hour. Assume that her earnings vary directly with the number of hours worked. Which job pays more an hour? (Example 3) **new job offer; Sample answer: The unit rate of her current job is AED 65.00 per hour which is lower than AED 72 per hour.**

Hours, x	Money Earned (AED), y
2	130.00
3	195.00
4	260.00
5	325.00

4. The height of a wide-screen television screen varies directly with its width. A television screen that is 60 centimeters wide and 33.75 centimeters high. Write and solve a direct variation equation to find the height of a television screen that is 90 centimeters wide.

   (Example 4) **$y = 0.5625x$; 50.625 cm**

5. **Building on the Essential Question** What is the relationship among the unit rate, slope, and constant rate of change of a proportional linear relationship?

   Sample answer: They all represent the same thing.

Rate Yourself!

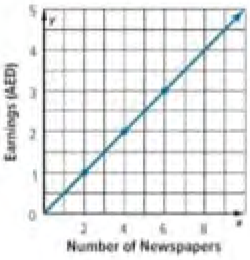
How well do you understand direct variation? Circle the image that applies.

Name ________________ My Homework ________________

Independent Practice

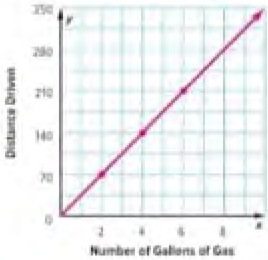
1. Tamer's earnings vary directly with the number of papers he delivers. The relationship is shown in the graph below. Determine the amount that Tamer earns for each paper he delivers. (Example 1)

AED 0.50 per paper



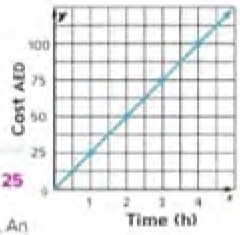
2. Hussein is buying a car that can travel 70 miles on two gallons of gas. Assume that the distance traveled in miles y varies directly with the amount of gas used x. This can be represented by $y = 35x$. Graph the equation on the coordinate plane. How many miles does the car get per gallon of gas? (Example 2)

35 miles per gallon



3. Anas was comparing computer repair companies. The cost y for Computer Access for x hours is shown in the graph. The cost for Computers R Us can be represented by the equation $y = 23.5x$. Which company's repair price is lower? Explain. (Example 3)

Computers R Us; Sample answer: The unit cost for Computer Access is AED 25 per hour. The unit cost for Computers R Us is AED 23.50. 23.5 < 25



4. The weight of an object on Mars varies directly with its weight on Earth. An object that weighs 50 pounds on Mars weighs 150 pounds on Earth. If an object weighs 120 pounds on Earth, write and solve a direct variation equation to find how much an object would weigh on Mars. (Example 4)

$y = \frac{1}{3}x$; 40 lb

Determine whether each linear function is a direct variation. If so, state the constant of variation. If not, explain why not.

5.

Pictures, x	5	6	7	8
Profit, y	20	24	28	32

yes; 4

6.

Age, x	10	11	12	13
Grade, y	5	6	7	8

no; Sample answer: The ratio of age to grade is not constant.

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVA**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to b
homework assignment. The Extra Practice page
for additional reinforcement or as a second-day

Levels of Complexity

The levels of the exercises progress from 1 to 3
indicating the lowest level of complexity.

	Exercises	
	1–4, 14–17	5–10, 18, 19
Level 3		
Level 2		•
Level 1	•	

Suggested Assignments

You can use the table below that includes exe
complexity levels to select appropriate exercis
students' needs.

Differentiated Homework Option		
AL	Approaching Level	1–5, 7, 9, 11, 13, 18, 19
OL	On Level	1, 3, 5–11, 13, 18, 19
BL	Beyond Level	5–13, 18, 19

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	8, 9, 10, 12
3 Construct viable arguments and critique the reasoning of others.	13, 17
4 Model with mathematics.	11

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students explain what a constant of variation is in a direct variation. **Sample answer: the ratio of any output value to its corresponding input value.**

Watch Out!

Common Error Students may have trouble with Exercises 8–10 because they must first find the constant of variation and then find the missing value. Stress to students that they should write the equation $y = mx$ and then substitute the known values into the equation to determine whether they should multiply or divide to solve for the missing value.

7. The number of centimeters varies directly with the number of inches. Find the measure of an object in centimeters if it is 50 inches long. **127 cm**

Inches, x	6	9	12	15
Centimeters, y	15.24	22.86	30.48	38.10

Persevere with Problems **If y varies directly with x, write an equation for the direct variation. Then find each value.**

8. If $y = -12$ when $x = 9$, find y when $x = -4$. $y = -\frac{4}{3}x$; $5\frac{1}{3}$

9. Find y when $x = 10$ if $y = 8$ when $x = 20$. $y = \frac{2}{5}x$; 4

10. If $y = -6$ when $x = -14$, find x when $y = -4$. $y = \frac{3}{7}x$; $-9\frac{1}{3}$

H.O.T. Problems Higher Order Thinking

11. **Model with Mathematics** Write three ordered pairs for a direct variation relationship where $y = 12$ when $x = 16$.
Sample answer: (4, 3), (8, 6), (0, 0)

12. **Persevere with Problems** The amount of stain needed to cover a wood surface is directly proportional to the area of the surface. If 3 pints are required to cover a square deck with a side of 7 feet, how many pints of stain are needed to paint a square deck with a side of 10 feet?
$6\frac{6}{49}$ **pt**

13. **Reason Inductively** Describe two real-world quantities that have a proportional linear relationship. Explain how you could change the situation to make the relationship nonproportional.
Sample answer: The total cost y of buying x boxes of popcorn is a proportional linear relationship. If you buy x boxes of popcorn and a drink for AED 1, the relationship between the total cost and the boxes of popcorn becomes nonproportional.

Name ________________ My Homework ________________

Extra Practice

Write and graph the direct variation equation that represents each situation.

14. Sameh used 3 gallons of paint to cover 1,050 square feet and 5 gallons to paint an additional 1,750 square feet. The area covered varies directly with the amount of paint used. How many square feet will one can of paint cover?

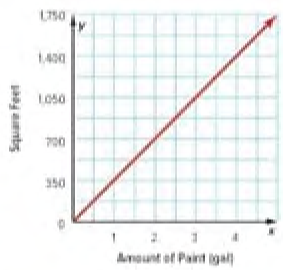
$y = 350x$; 350 square feet per gallon

$y = mx$

$1{,}050 = m(3)$

$350 = m$

$y = 350x$



15. Nahla purchased 2.5 pounds of cheese for AED 10.50. Her mother purchased 3 pounds of the same cheese for AED 12.60. The cost of cheese varies directly with the number of pounds purchased. How much does one pound of cheese cost?

$y = 4.2x$; AED 4.20 per pound

16. **STEM** When a 49 pound weight is attached to a spring, the spring stretches 7 inches. Assume that the length of the spring y varies directly with the weight attached x. Write and solve a direct variation equation to find the length of the spring when a 63 pound weight is attached.

$y = \frac{1}{7}x$; 9 in.

17. **Justify Conclusions** The money raised by the Drama Club selling raffle tickets is shown in the table. They can also raise money by selling tickets to the play for AED 6.25 per ticket. Assume that the money raised varies directly with the number of tickets sold. Which fundraiser has the potential to raise more money? Explain your reasoning to a classmate.

Raffle Tickets Sold	25	50	75	100
Money Raised (AED)	125	250	375	500

tickets to the play; Sample answer: The unit rate per raffle ticket is AED 5 and the unit rate per play ticket is AED 6.25. 6.25 > 5

ver Up! Test Practice

es 18 and 19 prepare students for more rigorous needed for the assessment.

test item requires students to explain and apply mathematical cepts and solve problems with precision, while making use of cture.	
th of Knowledge	DOK2
hematical Practices	MP1, MP4
ring Rubric	
oints	Students correctly model the slope and write the appropriate equation.
int	Students correctly model the slope OR write the appropriate equation.

test item requires students to reason abstractly and ntitatively when problem solving.	
th of Knowledge	DOK1
hematical Practice	MP1
ring Rubric	
int	Students correctly answer each part of the question.

Power Up! Test Practice

18. The table shows the amount of time a delivery truck has been driving and the distance traveled. The total distance traveled is a direct variation of the number of hours. Use the model below to find the slope. **Sample answer:**

Hours, x	2	5	7
Distance (mi), y	110	275	385

slope: $\dfrac{275 - 110}{5 - 2} = \dfrac{55}{1}$

Write an equation in $y = mx$ form to represent the situation. $y = 55x$

19. Students in a science class recorded lengths of a stretched spring, as shown in the table. Determine if each statement is true or false.

a. The relationship represents a constant rate of change. ■ True ☐ False

b. The slope of the relationship is 6 grams per centimeter. ■ True ☐ False

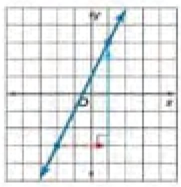
c. The equation that represents the relationship is $y = x + 10$. ☐ True ■ False

Length of Stretched Sprin	
Distance Stretched, x (centimeters)	Mass (gram
0	0
2	12
5	30
9	54
12	72

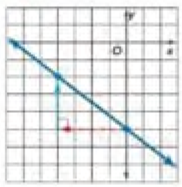
Spiral Review

Find the slope of each line.

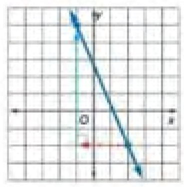
20. $\frac{2}{1}$ or 2



21. $-\frac{3}{4}$



22. $-\frac{7}{3}$



Find the slope of the line that passes through each pair of points.

23. (−1, 7) and (5, 7) 0

24. (1, 3) and (1, 0) undefined

25. (1, 2) and (5, 0) $-\frac{1}{2}$

Lesson 4

Slope-Intercept Form

Real-World Link

Football An interception in football is when a defensive player catches a pass made by an offensive player.

In a nonproportional linear relationship, the graph passes through the point (0, *b*) or the *y*-intercept. The **y-intercept** of a line is the *y*-coordinate of the point where the line crosses the *y*-axis.

Complete the steps to derive the equation for a nonproportional linear relationship by using the slope formula.

$\frac{y_2 - y_1}{x_2 - x_1} = m$ Slope formula

$(x_1, y_1) = (0, b)$

$\frac{y - b}{x - 0} = m$ $(x_2, y_2) = (x, y)$

$\frac{y - b}{x} = m$ Simplify.

$y - b = m \cdot x$ Multiplication Property of Equality

$y = mx + b$ Addition Property of Equality

slope → $y = mx + b$ ← y-intercept

How can knowing about an interception in football help you remember the definition of *y*-intercept?

Sample answer: In math, a line intersects an axis and a defensive football player intercepts the ball from the opposing team.

Essential Question

WHY are graphs helpful?

Vocabulary

y-intercept
slope-intercept form

Mathematical Practices
1, 3, 4

Which **MP Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Graph linear equations using the slope and *y*-intercept.

Coherence connecting within and across g

Previous Students graphed linear equations using ordered pairs.

Now Students write and graph linear equations in the form $y = mx + b$.

Ne Studen equati and y-i

Rigor pursuing concepts, fluency, and applic

See the Levels of Complexity chart on page 203.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALU

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whol group, think-pair-share activity, or independent a

LA Talking Chips Place students i discuss the steps to derive the equati nonproportional linear relationship. Give each st Students place a chip in the center of the table talk. All chips must be used, and students may n they have used all of their chips. At the complet activity, have a volunteer demonstrate the steps

Alternate Strategy

BL Ask students to use what they know abou proportional and nonproportional relationships t the graph of $y = 4x$ will change if it becomes y = **MP 1, 2, 3, 5, 7**

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Find the slope and *y*-intercept of a line.

- AL • *In slope-intercept form, which variable represents the slope?* **m** *the y-intercept?* **b**
- OL • *How would you write $y = \frac{2}{3}x - 4$ in slope-intercept form?* $y = \frac{2}{3}x + (-4)$

Need Another Example?
State the slope and *y*-intercept of the graph of the equation $y = \frac{3}{4}x - 5$. $\frac{3}{4}$; -5

2. Write the equation of a line.

- AL • *In slope-intercept form, which variable represents the slope?* **m** *the y-intercept?* **b**
- OL • *What is the equation in slope-intercept form?* $y = -3x - 4$
- BL • *Are the equations $y = 3x + (-4)$ and $y = -3x - 4$ equivalent? Explain.* **yes; Adding a negative is the same as subtracting its opposite.**

Need Another Example?
Write an equation of a line in slope-intercept form with a slope of -3 and a *y*-intercept -8. $y = -3x - 8$

3. Write an equation in slope-intercept form from a graph.

- AL • *What is the y-intercept?* **4**
- OL • *How could you find the slope of the line?* **From the *y*-intercept, count the number of units you need to move up/down and left/right to find the next point on the line.**
- BL • *How does the graph show that the slope is negative?* **The line slopes down from left to right, so the slope is negative.**

continued on page 201

Work Zone

Slope-Intercept Form of a Line

Nonproportional linear relationships can be written in th[e form]
$y = mx + b$. This is called the **slope-intercept form**. When [an equation]
is written in this form, *m* is the slope and *b* is the *y*-interc[ept].

Examples

1. State the slope and the *y*-intercept of the graph o[f the] equation $y = \frac{2}{3}x - 4$.

$y = \frac{2}{3}x + (-4)$ Write the equation in the form $y = mx + b$

$y = mx + b$ $m = \frac{2}{3}, b = -4$

The slope of the graph is $\frac{2}{3}$, and the *y*-intercept is [-4].

Got it? Do these problems to find out.

a. $y = -5x + 3$ b. $y = \frac{1}{4}x - 6$ c.

a. -5; 3

b. $\frac{1}{4}$; -6

c. -1; 5

Examples

2. Write an equation of a line in slope-intercept fo[rm with a slope of] -3 and a *y*-intercept of -4.

$y = mx + b$ Slope-intercept form

$y = -3x + (-4)$ Replace *m* with -3 and *b* with -4

$y = -3x - 4$ Simplify

3. Write an equation in slope-intercept form for the graph shown.

The *y*-intercept is 4. From (0, 4), you move down 1 unit and right 2 units to another point on the line.

So, the slope is $-\frac{1}{2}$.

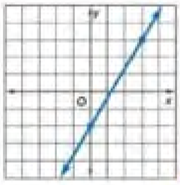
$y = mx + b$ Slope-intercept form

$y = -\frac{1}{2}x + 4$ Replace *m* with $-\frac{1}{2}$ and *b* with 4

...se problems to find out.

...uation in slope-intercept ... graph shown.

...uation of a line in slope-...rm with a slope of $\frac{3}{4}$ and ...t of −3.



a. $y = \frac{5}{3}x - 2$

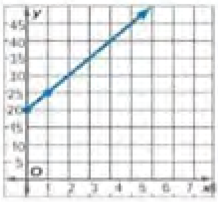
c. $y = \frac{3}{4}x - 3$

...the y-Intercept

...in slope-intercept form applies to a real-world ...e represents the rate of change and the y-intercept ...ial value.

...les

...ncil is selling T-shirts during UAE National Day. It ...0 for the design and AED 5 to print each shirt. The ...t x shirts is given by $y = 5x + 20$. Graph $y = 5x + 20$ ...pe and y-intercept.



...nd the slope and ...intercept.

$... = 5x + 20$ slope = 5, y-intercept = 20

...aph the y-intercept ... 20).

...ite the slope 5 as $\frac{5}{1}$. ...e it to locate a second ...int on the line. Go up 5 units ...d right 1 unit. Then draw a line ...ough the points.

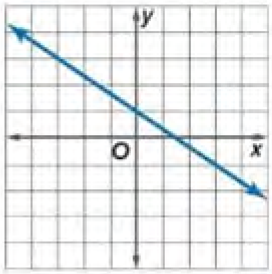
...slope and the y-intercept.

...epresents the cost in dirhams per T-shirt. The ...0 is the one-time charge in dirhams for the design.

Need Another Example?

Write an equation in slope-intercept form for the graph shown.

$y = -\frac{2}{3}x + 1$



Examples

4. Graph a line using slope-intercept form.

- AL • *What are the slope and y-intercept of the line?* 5 or $\frac{5}{1}$; 20
- OL • *How can you use the slope and y-intercept to graph the line?* Graph (0, 20) on a coordinate plane. Since the slope is positive, move up 5 units and right 1 unit to the next point.
- BL • *How much would it cost to print 9 shirts?* AED 65 *Describe two different ways you can find your answer.* Sample answer: I can extend the graph or I can substitute 9 for x in the equation $y = 5x + 20$ and find the corresponding value of y.

5. Interpret the slope and y-intercept.

- AL • *What are the slope and y-intercept?* 5; 20
- OL • *The slope of the line is $\frac{5}{1}$. What does this represent?* Since the slope is the same as a unit rate, this means that it costs AED 5 per shirt.
  - *What does the y-intercept represent?* the initial cost of the design
- BL • *If the equation of the line was $y = 5x + 10$, how would you interpret the y-intercept?* It would still represent the initial cost of the design, but the initial cost would now be AED 10, not AED 20.

Need Another Example?

A kayak rental pavilion charges AED 15.00 per hour and AED 2.50 for a brief lesson on kayak safety. The total cost y to rent the kayak for x hours is given by $y = 15x + 2.5$. Graph the equation using the slope and y-intercept. Then interpret the slope and y-intercept. See Answer Appendix.

d Practice

e **Assessment** Use these exercises to assess understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

Teammates Consult Have students work in teams dents. Have them discuss Exercise 1 with Student 1 e discussion. After everyone agrees on the solution, ent individually records their answer. Repeat the r Exercise 2 with Student 2 leading the discussion. rotating the leader role until all of the exercises have pleted. MP 1, 2, 4, 5, 6, 7, 8

Pairs Consult Have students work in pairs to alter rio in Exercise 1 so that it would be a proportional tionship. Then have them graph that relationship and t to the graph of the relationship given in Exercise 1 any differences in the slope and y-intercept. MP 1, 2, 4,

Out!

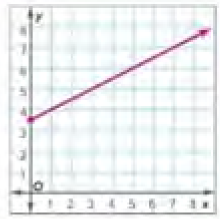
Error Students may have trouble graphing in slope-intercept form because they locate by reversing x and y and using $\frac{\text{difference in } x}{\text{difference in } y}$ ents write the slope as a fraction. Next to the , write "up" or "down" and next to the tor, write "right" or "left". Have students e y-intercept and then move up or down then t for the location to the next point.

Got it? Do these problems to find out.

A taxi fare y can be determined by the equation $y = 0.50x + 3.50$, where x is the number of miles traveled.

f. Graph the equation.

g. Interpret the slope and the y-intercept.



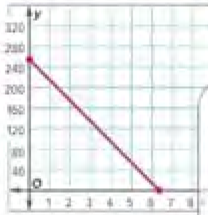
g. The slope represents the charge per mile, AED 0.50, and the y-intercept is a flat fee, AED 3.50.

Guided Practice

1. Ahamed is reading a 254-page book for school. He can read 40 pages in one hour. The equation for the number of pages he has left to read is $y = 254 - 40x$, where x is the number of hours he reads. (Examples 1, 4, and 5)

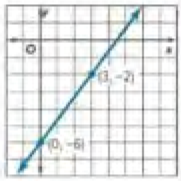
   a. State the slope and the y-intercept of the graph of the equation. -40; 254

   b. Graph the equation.

   c. Interpret what the slope and the y-intercept represent. the number of pages yet to be read decreases by 40 pages per hour; the total number of pages to read

2. Write an equation in slope intercept form for the graph shown. (Examples 2 and 3) $y = \frac{4}{3}x - 6$



3. **Building on the Essential Question** How does the y-intercept appear in these three representations: table, equation, and graph? Sample answer: In a table, the y-intercept is the y-value when the x-value is 0. In an equation written in slope-intercept form, the y-intercept is the constant. On a graph, the y-intercept is the point where the line crosses the y-axis.

Rate Yourself!

How confident are you about equations in slope-intercept form? Check the box that applies.

Name ____________ My Homework ____________

Independent Practice

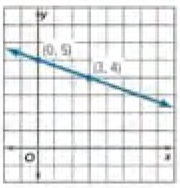
State the slope and the *y*-intercept for the graph of each equation. (Example 1)

1. $y = 3x + 4$ **3; 4**
2. $y = -\frac{3}{7}x - \frac{1}{7}$ **$-\frac{3}{7}$; $-\frac{1}{7}$**
3. $3x + y = -4$ **−3; −4**

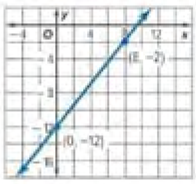
Write an equation of a line in slope-intercept form with the given slope and *y*-intercept. (Example 2)

4. slope: $-\frac{3}{4}$, *y*-intercept: −2 **$y = -\frac{3}{4}x - 2$**
5. slope: $\frac{5}{6}$, *y*-intercept: 8 **$y = \frac{5}{6}x + 8$**

Write an equation in slope-intercept form for each graph shown. (Example 3)

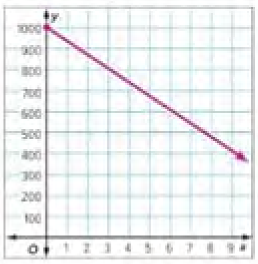
6. 

$y = -\frac{1}{3}x + 5$

7. 

$y = \frac{5}{4}x - 12$

8. A family is traveling to one of the Gulf countries for vacation. The equation $y = 1{,}000 - 65x$ represents the distance in miles remaining in their trip after *x* hours. (Examples 4 and 5)
   a. Graph the equation.
   b. Interpret the slope and the *y*-intercept. **The driving rate, 65 mph; the distance from which they began their trip, 1,000 miles.**



Copy and Solve. Graph each equation on a separate piece of grid paper. **9–11. See Answer Appendix.**

9. $y = \frac{1}{3}x - 5$
10. $y = -x + \frac{3}{2}$
11. $y = -\frac{4}{3}x + 1$

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVAL**

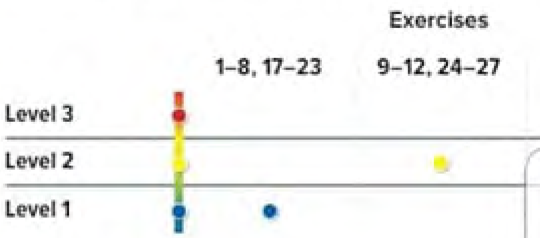
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be homework assignment. The Extra Practice page for additional reinforcement or as a second-day a

Levels of Complexity

The levels of the exercises progress from 1 to 3, indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exerc complexity levels to select appropriate exercise students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11, 12, 14–16, 26, 27
OL	On Level	1–7 odd, 9–12, 14–16, 26,
BL	Beyond Level	9–16, 26, 27

ATHEMATICAL PRACTICES	
Emphasis On	Exercise(s)
ake sense of problems and persevere in lving them.	13, 24
onstruct viable arguments and critique the asoning of others.	14, 15, 16
odel with mathematics.	12

ematical Practices 1, 3, and 4 are aspects of mathematical ing that are emphasized in every lesson. Students are n opportunities to be persistent in their problem solving, to ess their reasoning, and apply mathematics to real-world tions.

ative Assessment

this activity as a closing formative assessment before issing students from your class.

TICKET ut the Door

ve students explain how they would graph $y = -\frac{1}{4}x + 3$ ng the slope and y-intercept. **See students' work.**

12. **Model with Mathematics** At a fair, if you want to go on rides, you either buy an all-you-can-ride wristband for AED 25 or 7 tickets for AED 5.

    a. Write an equation in slope-intercept form for the total cost of any number of tickets at 7 tickets for AED 5. **y = AED 0.71x**

    b. Write an equation in slope-intercept form for the total cost of a wristband for all you can ride. **y = AED 25**

H.O.T. Problems Higher Order Thinking

13. **Persevere with Problems** The x-intercept is the x-coordinate of the point where a graph crosses the x-axis. What is the slope of a line that has a y-intercept but no x-intercept? Explain. **0; Sample answer: A line that has a y-intercept but no x-intercept is a horizontal line.**

14. **Reason Abstractly** Write an equation of a line that does not have a y-intercept. **Sample answer: $x = 4$**

15. **Justify Conclusions** Suppose the graph of a line has a negative slope and a positive y-intercept. Through which quadrants does the line pass? Justify your reasoning. **Quadrants I, II, and IV; If a y-intercept is graphed at (0, b), where b is positive, and then a line is drawn through the point so that it has a negative slope, the line will pass through Quadrants I, II, and IV.**

16. **Make a Conjecture** Describe what happens to the graph of $y = 3x + 4$ when the slope is changed to $\frac{1}{3}$. **Sample answer: The graph becomes less steep.**

Name ______________________ My Homework ______________________

Extra Practice

State the slope and the *y*-intercept for the graph of each equation.

17. $y = -5x + 2$ $-5; 2$

In the equation, $m = -5$ and $b = 2$ so the slope is -5 and the y-intercept is 2.

Homework Help

18. $y = \frac{1}{2}x - 6$ $\frac{1}{2}; -6$

19. $y - 2x = 8$ **2; 8**

Write an equation of a line in slope-intercept form with the given slope and *y*-intercept.

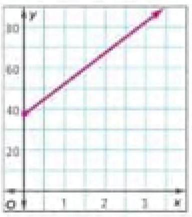
20. slope: $\frac{1}{2}$; *y*-intercept: 6
$y = \frac{1}{2}x + 6$

21. slope: -2; *y*-intercept: 3
$y = -2x + 3$

22. slope: $-\frac{3}{5}$; *y*-intercept: $-\frac{1}{5}$
$y = -\frac{3}{5}x - \frac{1}{5}$

23. **Persevere with Problems** The equation $y = 15x + 37$ can be used to approximate the temperature *y* in degrees Fahrenheit based on the number of chirps *x* a cricket makes in 15 seconds. Graph the equation to estimate the number of chirps a cricket will make in 15 seconds if the temperature is 80°F.

about 3 chirps

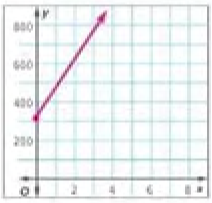


24. The Lakeside Marina charges a AED 350 rental fee for a boat in addition to charging AED 150 an hour for usage. The total cost *y* of renting a boat for *x* hours can be represented by the equation $y = 150x + 350$.

a. Graph the equation.

b. Interpret the slope and the *y*-intercept.

the hourly rental charge, AED 150, and the base rental fee, AED 350



25. Write an equation in slope-intercept form for the table shown.

Number of Pizzas	0	1	2	3	4
Cost (AED)	5	13	21	29	37

$y = 8x + 5$

...wer Up! Test Practice

...es 26 and 27 prepare students for more rigorous ...g needed for the assessment.

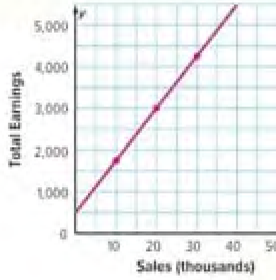
...s test item requires students to analyze and solve complex real-...rld problems through the use of mathematical tools and models.	
...th of Knowledge	DOK3
...thematical Practices	MP1, MP4
...oring Rubric	
...oints	Students correctly graph the line and give the correct equation.
...int	Students correctly graph the line OR give the correct equation.

...s test item requires students to explain and apply mathematical ...cepts and solve problems with precision, while making use of ...ucture.	
...th of Knowledge	DOK1
...thematical Practice	MP1
...oring Rubric	
...int	Students correctly answer each part of the question.

Power Up! Test Practice

26. The table shows Mr. Mubarak's total earnings as a car salesman for different sale amounts.

Sales (thousands), x	AED 10	AED 20	AED 30
Total Earnings, y	AED 1,750	AED 3,000	AED 4,250



Graph the points on the coordinate plane and connect them with a straight line.

Write an equation in slope-intercept form to represent the relationship. $y = 125x + 500$

27. Maha has 20 postcards in her collection. Each time she goes on vacation she buys 8 postcards to add to the collection. The total number of postcards y can be represented by the equation $y = 8x + 20$. Complete the following statements regarding the line.

The slope of the line is **8** and the y-intercept is **20**.

The **y-intercept** represents the number of postcards when she began collecting and the **slope** represents the number of postcards added each vacation.

Spiral Review

Solve each equation for d when $c = 0$.

28. $10c + 4d = 40$ **10**

29. $-5d = 2c + 10$ **−2**

30. $-4c - 6d = 24$ **−4**

Determine whether each linear relationship is proportional. If so, state the constant of proportionality.

31. yes; $\frac{4}{1}$ or 4

Pictures, x	5	6	7	8
Profit, y	20	24	28	32

32. yes; 0.07

Price, x	10	15	20	25
Tax, y	0.70	1.05	1.40	1.75

Expressions and Equations

Expressions and Equations

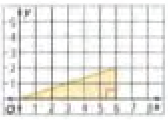
Inquiry Lab

Slope Triangles

Inquiry **HOW does graphing slope triangles on the coordinate plane help you analyze them?**

Mathematical Practices 1, 3, 5

onte ordered the plans shown to build a skateboard
mp. Each unit represents one foot. He wants to keep
e same slope of the ramp and extend the base of the
iangle three feet. How tall will the ramp be?

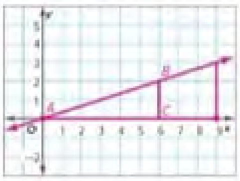


Hands-On Activity

efer to the graph shown above. Triangle ABC is formed by the rise, run, and
ection of the line $y = \frac{1}{3}x$ between points A and B.

Step 1 Graph $y = \frac{1}{3}x$ on the grid paper.
Draw a right triangle using the points $A(0, 0)$ and $B(6, 2)$.
Label the third point C.

What is the slope of $\overline{AB}$? $\frac{1}{3}$



Step 2 Select any two different points on the line. Label them D and E. Draw another triangle from these two points.

Is the slope of $\overline{DE}$ the same as the slope of $\overline{AB}$? Explain.
Sample answer: The slope is the same because all the triangles are made from points on the same line.

Step 3 Akram wants to expand the base of the ramp 3 feet. Graph and give the coordinates of the point that will represent the extended base of the ramp. **(9, 0)**

Create a right triangle using the line and that point. What will be the height of the new ramp? **3 feet**

Focus narrowing the scope

Objective Graph and analyze slope triangles.

Coherence connecting within and across grades

Now
Students use slope triangles to graph equations.

Next
Students will use x- and y-intercepts to graph a line.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 208.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The Activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA **Pairs Discussion** Have students work in pairs to complete the Activity. After they have completed the Activity have them answer and discuss the following question.
MP 1, 2, 4, 5, 6, 7, 8

Ask:

- *What is one method you could use to graph $y = \frac{1}{3}x$?* **Sample answer: The y-intercept is 0. You can use the y-intercept and the slope of the line to graph the next point on the line.**

BL Omit the Activity and proceed directly to the Investigate section.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Investigate

AL LA **Teammates Consult** Have students work in pairs. Teammate #1 leads the discussion for the first question. Each teammate contributes to the discussion, but they do not have to agree. Students then record their answers on their paper. Repeat steps for Exercise 2 with a new lead. MP 3, 5, 7

Analyze and Reflect

BL LA **Round Table Consensus** Have students work in teams of four. Students choose a leader to record the answers. Teammates must show agreement or disagreement with a thumbs up or down. If there is a disagreement, the team discusses the answer until there is consensus. MP 1, 3, 5, 7

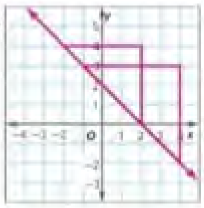
Create

inquiry Students should be able to answer "HOW does graphing slope triangles on the coordinate plane help you analyze them?" Check for student understanding and provide guidance, if needed.

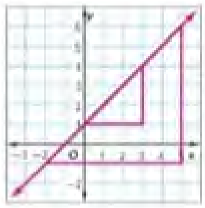
Investigate

Work with a partner. Draw two right triangles for each exercise using the rise, run, and portions of the line. Sample triangles are given.

1. $y = -x + 2$



2. $y = x + 1$



Analyze and Reflect

3. MP **Make a Conjecture** What do you notice about the shape and size of the pair of triangles in Exercises 1 and 2? Sample answer: The two triangles are the same shape but different sizes.

Create

Sample answers are given.

4. MP **Use Math Tools** The triangles in the activity are called *slope triangles*. Complete the graphic organizer by writing three observations about slope triangles.

equal angle measures in the triangles		Definition: Right triangles that have the hypotenuse on the same line.
	Slope Triangles	
same shape		may be different sizes

5. inquiry HOW does graphing slope triangles on the coordinate plane help you analyze them?
Sample answer: By graphing slope triangles on the coordinate plane you can tell that they have the same shape but they are different sizes.

Lesson 5

Graph a Line Using Intercepts

Real-World Link

Movies Mr. Zuhair spent AED 80 on movie tickets and drinks for his son and friends. The total cost of x movie tickets and y drinks is represented by the equation $8x + 4y = 80$.

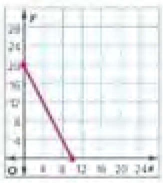
Item	Cost
ticket	AED 8
drink	AED 4

1. Complete the steps below to write the equation in slope-intercept form.

$$8x + 4y = 80$$
$$\boxed{-8x} \quad = \boxed{-8x}$$
$$\frac{4y}{\boxed{4}} = \frac{80 - 8x}{\boxed{4}}$$
$$y = 20 - 2x$$
$$y = \boxed{-2}\,x + \boxed{20}$$

slope — y-intercept

2. Graph the equation.



3. What does the point (0, 20) represent?

Mr. Zuhair could buy 20 drinks and no tickets.

Which Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

WHY are graphs helpful?

Vocabulary

x-intercept
standard form

Mathematical Practices
1, 3, 4

Focus narrowing the scope

Objective Graph an equation using the x- and y-intercept

Coherence connecting within and across grades

Previous Students wrote and graphed equations written in $y = mx + b$ form.

Now Students use the x- and y-intercepts to graph linear equations.

Next Students will write equations to repre tables and graphs.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 213.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, s group, think-pair-share activity, or independent activity.

Work together as a class to write the equatio slope-intercept form. Ask students why 8x represents the total cost of the tickets and 4y represent total cost of the drinks. Then ask if (7.5, 11) would make s as an ordered pair for this situation. MP 1, 7, 8

Alternate Strategy

AL LA **Act It Out** Have students work in pairs and a all of the possible solutions to the scenario, starting with replacing 0 for x, which represents 0 tickets. Have stude make a table with their findings. Then have them graph t ordered pairs and write an equation in slope-intercept for the graph. MP 1, 2, 4, 5, 7, 8

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Graph a line using intercepts.

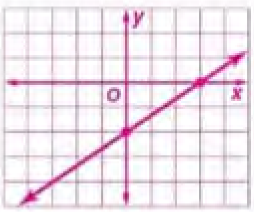
AL
- *In what form is the equation $y = 1.5x - 9$ written?* slope-intercept form
- *What is the slope?* 1.5 *What is the y-intercept?* -9

OL
- *Why is the y-intercept negative?* Slope-intercept form is $y = mx + b$, not $y = mx - b$.
- *Why do you replace y with 0 to find the x-intercept?* Since the x-intercept is on the x-axis, the value of y in the ordered pair is always 0.
- *What is the x-intercept?* 6
- *You now know two points on the line. What are they?* (0, −9) and (6, 0)

BL
- *Find two other points on the line.* Sample answer: (2, −6) and (4, −3)

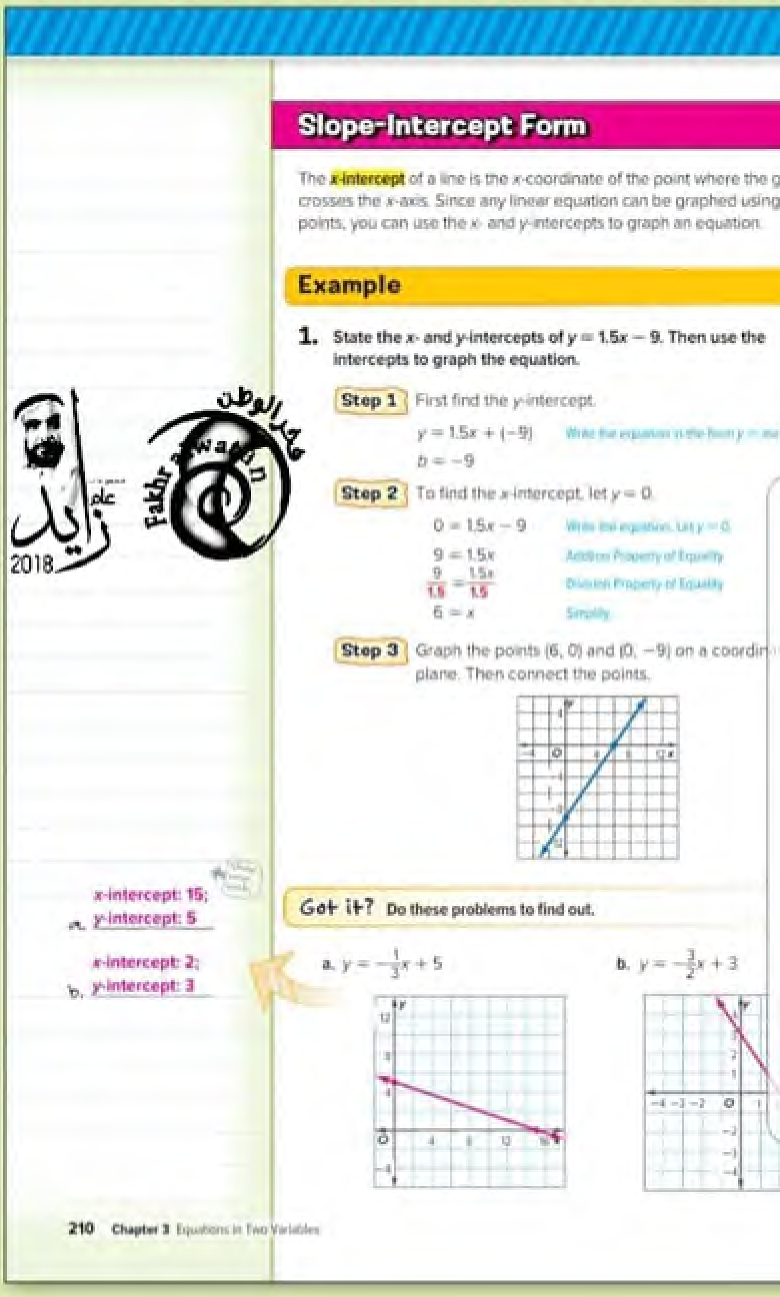
Need Another Example?

State the x- and y-intercepts of $y = \frac{2}{3}x - 2$. Then use the intercepts to graph the equation. x-intercept: 3; y-intercept = −2



Watch Out!

Common Error Students may have trouble graphing functions using the intercepts because they think that 0 is substituted for x to find the x-intercept, and 0 is substituted for y to find the y-intercept. Suggest that students solve the equations for y and check their graphs using the y-intercept and slope.



Slope-Intercept Form

The **x-intercept** of a line is the x-coordinate of the point where the g… crosses the x-axis. Since any linear equation can be graphed using… points, you can use the x- and y-intercepts to graph an equation.

Example

1. **State the x- and y-intercepts of $y = 1.5x - 9$. Then use the intercepts to graph the equation.**

Step 1 First find the y-intercept.

$y = 1.5x + (-9)$

$b = -9$

Step 2 To find the x-intercept, let $y = 0$.

$0 = 1.5x - 9$

$9 = 1.5x$

$\frac{9}{1.5} = \frac{1.5x}{1.5}$ Division Property of Equality

$6 = x$ Simplify

Step 3 Graph the points (6, 0) and (0, −9) on a coordin… plane. Then connect the points.

Got it? Do these problems to find out.

a. $y = -\frac{1}{3}x + 5$

b. $y = -\frac{3}{2}x + 3$

a. x-intercept: 15; y-intercept: 5

b. x-intercept: 2; y-intercept: 3

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Work Zone

Standard Form

When an equation is written in the form $Ax + By = C$, where $A \geq 0$, and A, B, and C are integers, it is written in **standard form**.

Examples

Al Shuroq Middle School wants to make AED 4,740 from yearbooks. Print yearbooks x cost AED 60 and digital yearbooks y cost AED 15. This can be represented by the equation $60x + 15y = 4{,}740$.

2. **Use the x- and y-intercepts to graph the equation.**

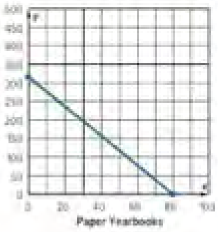
To find the x-intercept, let $y = 0$. To find the y-intercept, let $x = 0$.

$60x + 15y = 4{,}740$	$60x + 15y = 4{,}740$
$60x + 15(0) = 4{,}740$	$60(0) + 15y = 4{,}740$
$60x = 4{,}740$	$15y = 4{,}740$
$x = 79$	$y = 316$

3. **Interpret the x- and y-intercepts.**

The x-intercept is at the point (79, 0). This means they can sell 79 print yearbooks and 0 digital yearbooks to earn AED 4,740.

The y-intercept is at the point (0, 316). This means they can sell 0 print yearbooks and 316 digital yearbooks to earn AED 4,740.



STOP and Reflect

Describe below two different methods for graphing a line.

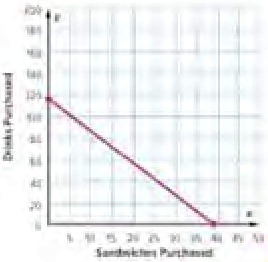
Sample answer: You can graph a line by using a table of values and plotting ordered pairs. You can also find the x- and y-intercepts of the line and graph those points.

y-intercept

When an equation is written in slope-intercept form, $y = mx + b$, the y-intercept is equal to b.

Got it? Do this problem to find out.

c. Mr. Mohsen spent AED 230 on lunch for his class. Sandwiches x cost AED 6 and drinks y cost AED 2. This can be represented by the equation $6x + 2y = 230$. Use the x- and y-intercepts to graph the equation. Then interpret the intercepts.



c. The x-intercept of 38.33 means that if 38.33 sandwiches and 0 drinks were purchased, the total cost would be AED 230. The y-intercept of 115 means that if 0 sandwiches and 115 drinks were purchased, the total cost would be AED 230.

Examples

2. **Use x- and y-intercepts to graph an equation.**

AL
- *Why is 60 the coefficient of x?* x represents the print yearbooks
- *Why is 15 the coefficient of y?* y represents the digital yearbooks

OL
- *How do you find the x-intercept? What is the x-intercept?* Replace y with 0 and solve for x; 79.
- *How do you find the y-intercept? What is the y-intercept?* Replace x with 0 and solve for y; 316.

BL
- *If the cost for print yearbooks were AED 75 and the total profit remained the same, what would the new equation be?* $75x + 15y = 4{,}740$

3. **Interpret x- and y-intercepts.**

AL
- *What is the ordered pair for the x-intercept?* (79, 0)
- *What is the ordered pair for the y-intercept?* (0, 316)

OL
- *What does the ordered pair (79, 0) represent?* They can sell 79 print yearbooks and 0 digital yearbooks to earn a total of AED 4,740.
- *What does the ordered pair (0, 316) represent?* They can sell 0 print yearbooks and 316 digital yearbooks to earn a total of AED 4,740.

BL
- *What does the point (30, 196) represent?* They sold 30 print yearbooks and 196 digital yearbooks.

Need Another Example?

The drama department sold AED 1,260 worth of tickets to a play. Student tickets x cost AED 5 and adult tickets y cost AED 9. This can be represented by the equation $5x + 9y = 1{,}260$. Use the x- and y-intercepts to graph the equation. Then interpret the intercepts. See Answer Appendix.

uided Practice

rmative Assessment Use these exercises to assess dents' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

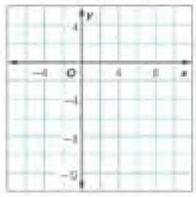
LA **Rally Coach** Have students work in pairs to mplete Exercises 1–4. Have Student 1 work through the ps to complete Exercise 1 while Student 2 watches, listens, aches, and praises. Have the students trade roles for ercise 2. Then have the students work individually for ercises 3 and 4 and then compare answers. MP 1, 2, 3, 4, 7, 8

LA **Trade-a-Problem** Have students generate their n real-world problem involving an equation expressed in ndard form. Have students trade their problem with a rtner, solve each other's problem and graph each other's uation, and discuss any differences in solutions. MP 1, 2, 4, 7, 8

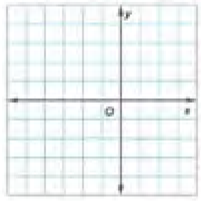
Guided Practice

State the *x*- and *y*-intercepts of each equation. Then use the intercepts to graph the equation.

1. $y = 3x - 9$ **x-intercept: 3; y-intercept: −9**



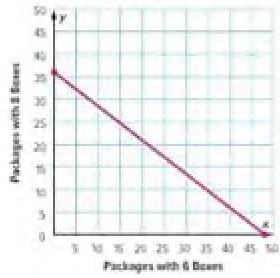
2. $y = \frac{1}{2}x + 2$ **x-intercept: −4; y-intercept: 2**



3. A store sells juice boxes in packages of 6 boxes and 8 boxes. They have 288 total juice boxes. This is represented by the function $6x + 8y = 288$. Use the x- and y-intercepts to graph the equation. Then interpret the x- and y-intercepts.

The x-intercept of 48 means that the store has 48 packages of 6 boxes and 0 packages of 8 boxes.

The y-intercept of 36 means that the store has 36 packages of 8 boxes and 0 packages of 6 boxes.



4. **Building on the Essential Question** How can the x-intercept and y-intercept be used to graph a linear equation? **Sample answer: You can graph a linear equation by finding the x- and y-intercepts and plotting the ordered pairs.**

Rate Yourself!

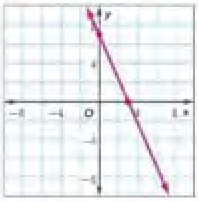
Are you ready to move on? Shade the section that applies.

Name ______________________ My Homework ______________________

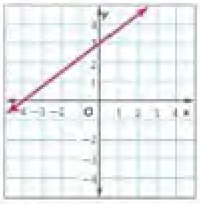
Independent Practice

State the x- and y-intercepts of each equation. Then use the intercepts to graph the equation. (Example 1)

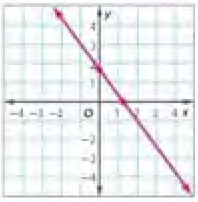
1. $y = -2x + 7$
   x-intercept: 3.5; y-intercept: 7



2. $y = \frac{3}{4}x + 3$
   x-intercept: −4; y-intercept: 3

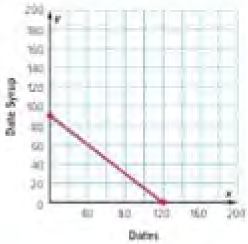


3. $12x + 9y = 15$
   x-intercept: $1\frac{1}{4}$; y-intercept: $1\frac{2}{3}$



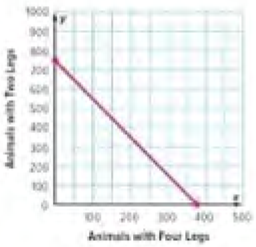
4. The table shows the cost for a Date Shop to buy bags of dates and cans of date syrup. The total cost forSaturday's shipment, AED 1,800, is represented by the equation $15x + 20y = 1,800$. Use the x- and y-intercepts to graph the equation. Then interpret the x- and y-intercepts. (Examples 2 and 3)

	Dates	Date Syrup
Cost per Unit (AED)	15	20
Amount Shipped	x	y



The x-intercept of 120 means that if the store purchased only dates, they would have 120 bags. The y-intercept of 90 means that if the store purchased only date syrup, they would have 90 cans.

5. The total number of legs, 1,500, on four-legged and two-legged animals in a zoo can be represented by the equation $4x + 2y = 1,500$. Use the x- and y-intercepts to graph the equation. Then interpret the x- and y-intercepts. (Examples 2 and 3)



The x-intercept of 375 means that if the zoo had only four-legged animals, there would be 375 of them. The y-intercept of 750 means that if the zoo had only two-legged animals, there would be 750 of them.

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–5, 10–15	6, 16, 17	7–9
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 9, 16, 17
OL	On Level	1–5 odd, 6, 7, 9, 16, 17
BL	Beyond Level	6–9, 16, 17

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	8
3 Construct viable arguments and critique the reasoning of others.	6, 7
4 Model with mathematics.	9

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students find the x- and y-intercepts of the equation $-x + 3y = 12$ and use the intercepts to graph the equation. **-12; 4; See students' graphs.**

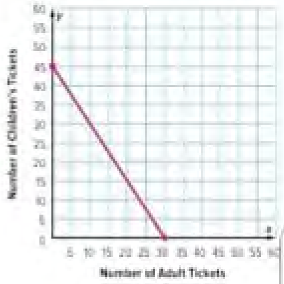
6. **Multiple Representations** The table shows the group rate for admission tickets for adults and children to an amusement park.

	Adult	Children
Ticket Price (AED)	45	30
Tickets Purchased	x	y

   a. **Symbols** The total cost of a group's tickets is AED 1,350. Write an equation to represent the number of adults' and children's tickets purchased. **$45x + 30y = 1{,}350$**

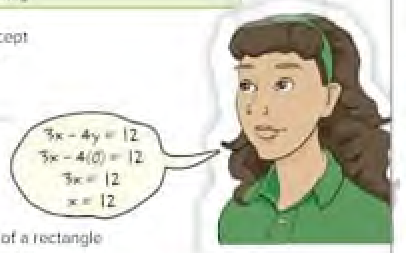
   b. **Words** What are the x- and y-intercepts and what do they represent? **The x-intercept of 30 means that if only adults bought tickets, 30 tickets would be sold. The y-intercept of 45 means that if only children's tickets were purchased, 45 tickets would be purchased.**

   c. **Graphs** Use the x- and y-intercepts to graph the equation. Use the graph to find the number of children's tickets purchased if 20 adult tickets were purchased. **15 children's tickets**



H.O.T. Problems Higher Order Thinking

7. **Find the Error** Ibtihal is finding the x-intercept of the equation $3x - 4y = 12$. Find her mistake and correct it. **After $3x = 12$, Ibtihal didn't divide both sides by 3 to get the x-intercept of 4.**



8. **Persevere with Problems** The perimeter of a rectangle that is x units wide and y units long is 24 centimeters.

   a. Write an equation in standard form for the perimeter. **$2x + 2y = 24$**

   b. Find the x- and y-intercepts. Does either intercept make sense as a solution for this situation? Explain. **Sample answer: The x-intercept is at the point (12, 0) and the y-intercept is at the point (0, 12). These points are not solutions in this situation because the length or width of the rectangle cannot be 0.**

9. **Model with Mathematics** Write two equations, one with an x-intercept but no y-intercept, and one with a y-intercept but no x-intercept. **Sample answers are given.**

   x-intercept equation: **$x = 2$**

   y-intercept equation: **$y = 2$**

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Name ______________________ My Homework ______________________

Extra Practice

10. State the *x*- and *y*-intercepts of the equation $y = \frac{2}{3}x - \frac{1}{3}$. Then use the intercepts to graph the equation.

Find the y-intercept.

$y = \frac{2}{3}x + (-\frac{1}{3})$

$b = -\frac{1}{3}$

Find the x-intercept.

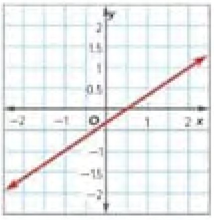
$y = \frac{2}{3}x + (-\frac{1}{3})$

$0 = \frac{2}{3}x + (-\frac{1}{3})$

$\frac{1}{3} = \frac{2}{3}x$

$(\frac{3}{2})\frac{1}{3} = (\frac{3}{2})\frac{2}{3}x$

$\frac{1}{2} = x$



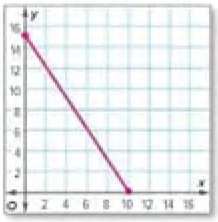
Copy and Solve **State the *x*- and *y*-intercepts of each equation. Then use the intercepts to graph each equation on a separate sheet of grid paper.** **11–13. See Answer Appendix.**

11. $2x + 3y = 24$

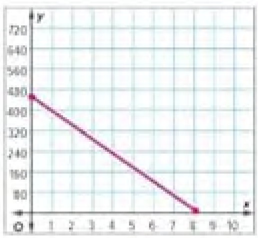
12. $y = -\frac{8}{9}x - 16$

13. $5x + 3y = 30$

14. Hana has 15 teaspoons of chocolate chips. She uses $1\frac{1}{2}$ teaspoons for each muffin. The total number of teaspoons of chocolate chips that she has left *y* after making *x* muffins can be given by $y = -\frac{3}{2}x + 15$. Graph the equation. Then interpret the *x*- and *y*-intercepts. **The *x*-intercept, 10, represents the number of muffins baked by using all of the chocolate chips. The *y*-intercept, 15, represents the number of teaspoons of chips before she baked any muffins.**



15. **Use Math Tools** Asmaa has AED 440 to pay a painter to paint her basement. The painter charges AED 55 per hour. The equation $y = 440 - 55x$ represents the amount of money *y* she has after *x* number of hours worked by the painter. Graph the equation. Then interpret the *x*- and *y*-intercepts. **The *x*-intercept, 8, represents the number of hours the painter worked to finish the basement. The *y*-intercept, 440, represents the total amount of money she has to pay the painter.**



...wer Up! Test Practice

...cises 16 and 17 prepare students for more rigorous ...king needed for the assessment.

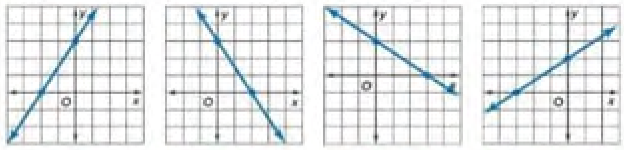
This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students place the correct equation with all four graphs.
1 point	Students place the correct equation with 3 of the 4 graphs.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

16. Match each equation to the appropriate graph below.

$2x - 3y =$ ...
$3x - 2y =$ ...
$3x + 2y$ ...
$2x + 3y$ ...



$3x - 2y = -6$ | $3x + 2y = 6$ | $2x + 3y = 6$ | $2x - 3y = -6$

17. The equation $12x - 10y = 600$ represents the total amount Student Council spent on supplies for a school fundraiser. Fill in the boxes to make a true statement.

The x-intercept of the line is **50** and the y-intercept is **−60**.

Spiral Review

Simplify each expression.

18. $-3(x + 6) =$ $-3x - 18$

19. $\frac{2}{3}(3x + 6) - 3 =$ $2x + 1$

20. $4t + 10 - 5 - 3t =$ $t + 5$

21. $5x + 6 - x =$ $4x + 6$

22. $-\frac{1}{4}(4x - 8) + 18 =$ $-x + 20$

23. $2a + 4 - 8a - 10 =$ $-6a - 6$

Problem-Solving Investigation

Guess, Check, and Revise

Mathematical Practices 1, 3, 4

Case #1 Polar Plunge

Mahdi's class is going to the zoo to see a polar bear exhibit. Student admission is AED 10 and adult admission is AED 25. They spent AED 345 on 30 tickets.

How many students and adults are going to the zoo?

1 Understand *What are the facts?*

The student cost is AED 10 and the adult cost is AED 25. There are 30 people on the trip.

2 Plan *What is your strategy to solve this problem?*

Make a guess and check to see if your guess is correct.

3 Solve *How can you apply the strategy?*

Make a table.

Students → s, Adults → a

s	a	10s + 25a	Check
26	4	10(26) + 25(4) = 360	too high
29	1	10(29) + 25(1) = 315	too low
28	2	10(28) + 25(2) = 330	still too low
27	3	10(27) + 25(3) = 345	correct

So, 27 students and 3 adults are going to the zoo.

4 Check *Does the answer make sense?*

27 + 3 = 30 and 10(27) + 25(3) = 345; the guess is correct. ✓

Analyze the Strategy

Justify Conclusions Twenty-three students and 5 adults would also spend AED 345 to get into the zoo. Explain why this cannot be the correct solution.

There are 30 people going to the zoo, not 23 + 5 or 28 people.

Focus narrowing the scope

Objective Solve problems by using the guess, check, and revise strategy. This lesson emphasizes **Mathematical Practice 3** Construct an Argument.

Guess, Check, and Revise This strategy is helpful in solving problems involving two different variables that are related. Guesses should be systematic and should be recorded so students can see which guesses are closer to the solution.

Coherence connecting within and across grades

Now

Students apply the content standard to solve non-routine problems.

Next

Students will apply the guess, check, and revise strategy to write and solve linear equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 219.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

The problems on pages 217 and 218 are intended for whole-group discussion on how to solve non-routine problems and provide scaffolded guidance. The problem on page 217 walks students through the solution, while the problem on page 218 asks students to come up with their own solutions.

Case #1 Polar Plunge

BL Have students extend the problem by having them answer the question below. MP 1, 2, 3, 4, 5, 6, 7, 8

Ask:

- *Would any combination of 30 student and adult tickets make a reasonable initial guess? Explain.* **No; Sample answer: An initial guess of 5 student tickets and 25 adult tickets would not be a reasonable initial guess since the cost would be AED 4(25) or AED 100 for only the adult tickets, and the class spent AED 66.**

Case #2 Coins

AL LA **Rally Coach** Have students work in pairs to solve the problem. Have Partner A work the first step, speaking out loud, while Partner B listens carefully, coaches, and praises. Next have Partner B work the second step while Partner A listens carefully, coaches, and praises. Partners take turns until they have solved the problem. MP 1, 2, 3, 4, 5

BL LA **Pairs Discussion** Have students work in pairs to answer the following question. MP 1, 3

Ask:

- *Why would you use the guess, check, and revise strategy for this problem instead of a different strategy?* **Sample answer: There are several unknown variables, so guess, check, and revise is a convenient way to solve to find a solution that fits all three variables.**

Need Another Example?

There are 58 passengers on an airline flight to Orlando. Passengers with first-class tickets paid AED 808 for their seats. Passengers with coach tickets paid AED 208 for their seats. The total amount paid for tickets on this flight was AED 15,664. How many tickets of each type were sold? **6 first-class tickets and 52 coach tickets**

Case #2 Coins

Zafer has AED 2.50 in 25 fils coins, 10 fils coins, and 5 fils coins. If he has 18 coins, how many of each coin does he have?

1 Understand

Read the problem. What are you being asked to find?

I need to find **18 coins that combine to form AED 2.50**

Underline key words and values in the problem. What information do you know?

There are **18** coins that have a sum of **AED 2.50**.

The coins are a combination of **25 fils, 10 fils, and 5 fils coins**

Is there any information that you do *not* need to know?

There is no unnecessary information.

2 Plan

Choose a problem-solving strategy.

I will use the **guess, check, and revise** strategy.

3 Solve

Use your problem-solving strategy to solve the problem.

25 fils	10 fils	5 fils	Sum	Number of Coins	Check
10	6	2	AED 3.20	18	too high
5	10	3	AED 2.40	18	too low
6	9	3	AED 2.55	18	too high
6	8	4	AED 2.50	18	correct

So, **one combination is six 25 fils, eight 10 fils, and four 5 fils coins.**

4 Check

Use information from the problem to check your answer.

(**6** × 0.25) + (**8** × 0.10) + (**4** × 0.05) = AED 2.50; the answer is correct.

Work with a small group to solve the following cases. Show your work on a separate piece of paper.

Case #3 Wrap it Up

Ahmed works part-time at a gift-wrapping store. The store sells wrapping paper rolls and square packages of wrapping paper. There are a total of 125 rolls and packages. Each roll costs AED 3.50 and each package costs AED 2.25. The total cost of all of the rolls and packages is AED 347.50.

How many rolls of wrapping paper are there?

53 rolls

Case #4 Family

Five siblings have a combined age of 195 years. The oldest is 13 years older than the youngest. The middle child, Jamila, is five years younger than Amena. The other two siblings are 6 years apart.

If the second oldest child is 42, what are the ages of the siblings?

32, 36, 40, 42, 45

Case #5 Sport Trading Cards

Baseball cards come in packages of 8 and 12. Kassem bought some of each type for a total of 72 baseball cards.

How many of each package did he buy?

Sample answer: 3 packages of 8 cards and 4 packages of 12 cards

Use any strategy!

Case #6 Future Careers

One hundred fifteen students could sign up to hear three different speakers for career day. Seventy students heard the nurse speak, 52 heard the firefighter, and 78 heard the Webmaster. Some students heard more than one speaker. The results are shown in the table above.

Number of Students	Speaker
15	all three
20	nurse and firefighter
30	Webmaster and nurse
12	firefighter only

How many students signed up only for Webmaster?

28 students

Engage Explore **Explain** **Elaborate** Evaluate

2 Collaborate

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 3, 4	5	6
Level 3			●
Level 2		●	
Level 1	●		

AL **LA** **Think-Pair-Share** Have students work in pairs to complete Case #4. One student should speak aloud their response. The other student should listen carefully to the response and ask for any clarification, if needed. Then call on one student to share their response with the class. MP 1, 2, 3, 4, 5

BL **LA** **Team Project** Have students work in teams to develop and administer a survey. Then have the teams write a problem like Case #6. Have each team present their problem to the class. Other teams should listen carefully and ask questions for clarification during the presentation. MP 1, 2, 3, 4, 5, 6

Mid-Chapter Check

If students have trouble with Exercises 1–8, they may need help with the following concepts.

Concept	Exercise(s)
linear relationships (Lesson 1)	1
slope (Lesson 2)	2–5
direct variation (Lesson 3)	6
interpreting slope (Lesson 4)	7, 8

Vocabulary Activity

LA **Think-Pair-Share** Have students work in pairs to complete Exercise 1. Give them about one minute to individually think through their response. Then have them share their responses with a partner. Call on one set of pairs to share their responses with the class. MP 1, 6

Alternate Strategies

AL Students may not understand the concept of *linear*. Have students identify objects in the classroom that have straight edges, such as a ruler or an eraser.

BL Have students research nonlinear relationships, such as exponential and quadratic relationships, and describe real-world relationships that are nonlinear.

Mid-Chapter Check

Vocabulary Check

1. MP **Be Precise** Define *linear relationship*. Give an example of a linear relationship. (Lesson 1)
   A relationship that has a straight-line graph. See students' work.

Skills Check and Problem Solving

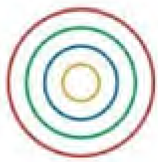
Find the slope of the line that passes through each pair of points. (Lesson 2)

2. $A(2, 5)$, $B(3, 1)$	3. $C(-1, 2)$, $D(-5, 2)$	4. $E(5, 2)$, $F(2, -3)$	5. $G(4, 3)$, $H(-2, -6)$
-4	0	$\frac{5}{3}$	$\frac{3}{2}$

6. Hamdan baked 3 cakes in $2\frac{1}{2}$ hours. Assume that the number of cakes baked varies directly with the number of hours. Write and solve a direct variation equation to find how many cakes can he bake in $7\frac{1}{2}$ hours. (Lesson 3)
   $y = 1\frac{1}{5}x$; **9 cakes**

7. The total money y Mohammed earned mowing x lawns is shown by the equation $y = 15x + 25$. What does the slope represent? (Lesson 4)
   AED 15 earned per lawn

8. MP **Persevere with Problems** A company logo has four concentric circles Suppose you graph the points (diameter, circumference) and connect them with a line. In terms of pi (π), what is the slope of the resulting line? (Lesson 4)
   π



Lesson 6

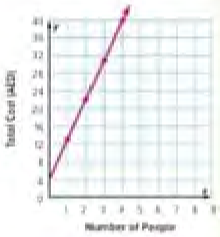
Write Linear Equations

Real-World Link

Zoo The cost for 1, 2, 3, and 4 people to go the zoo is shown in the table.

Number of People, x	1	2	3	4
Total Cost, y	AED 52	AED 88	AED 124	AED 160

1. Is the relationship linear? Explain.
   yes; Sample answer: The rate of change is constant for every pair of points in the table.
2. What is the slope of the related graph? 36
3. Choose an ordered pair. (2 , 88) Then substitute the values in the equation below. Sample answer is given.
   $y = m \cdot x + b$
   $88 = 36 \cdot 2 + b$
4. Solve for b to find the y-intercept.
   $b = 16$
5. Write an equation of the line in slope-intercept form.
   $y = 36x + 16$
6. Graph the data from the table on the coordinate plane.



Essential Question

WHY are graphs helpful?

Vocabulary

point-slope form

Mathematical Practices

1, 2, 3, 4, 5, 7

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Write an equation of a line.

Coherence connecting within and across grades

Previous Students wrote and graphed linear equations written in slope-intercept form.

Now Students will write linear equations in point-slope form and slope-intercept form.

Next Students will write and solve systems of linear equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 225.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Think-Pair-Solo** Give them about two minutes of "think time". Then have them work with a partner to complete Exercises 1–2 and discuss their responses. Then have them work individually to complete Exercises 3–6. Upon completion, have them discuss their solutions with their partner and resolve any differences. MP 1, 2, 4, 5, 7, 8

Alternate Strategy

BL Ask students to determine the equation of the line using the slope and y-intercept. Then have them alter the scenario so that the slope is greater and have them explain how this would affect the equation and the graph. MP 1, 2, 3, 4, 5, 7, 8

Engage Explore **Explain** Elaborate Evaluate

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Write an equation in point-slope form.

AL
- *What is the point-slope form of a linear equation?* $(y - y_1) = m(x - x_1)$
- *What ordered pair will we substitute for (x_1, y_1)?* **(−2, 3)**

OL
- *Can $x - (-2)$ be simplified?* **yes; It simplifies to $x + 2$.**
- *What is the equation in point-slope form?* $y - 3 = 4(x + 2)$

BL
- *If the equation of a line in point-slope form is $y + 1 = 5(x + 7)$, what is the slope and what is an ordered pair that the line passes through?* **The slope is 5 and an ordered pair is (−7, −1).**

Need Another Example?

Write the point-slope form of an equation for a line that passes through (2, 4) with slope $-\frac{3}{2}$. $y - 4 = -\frac{3}{2}(x - 2)$

2. Write an equation in slope-intercept form.

AL
- *What is the slope-intercept form of a linear equation?* $y = mx + b$
- *What operation do the parentheses in the point-slope form indicate?* **multiplication**

OL
- *What is the first step in rewriting the equation in slope-intercept form?* **Use the Distributive Property to rewrite $4(x + 2)$ as $4x + 8$.**
- *What is the next step?* **Add 3 to each side.**

BL
- *How can we check our answer?* **Substitute the coordinates of the point (−2, 3) into the equation to see if the equation is true.**

Need Another Example?

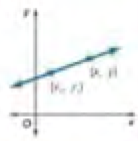
Write the equation $y - 4 = -\frac{3}{2}(x - 2)$ in slope-intercept form.
$y = -\frac{3}{2}x + 7$

Key Concept **Point-Slope Form of a Linear Equation**

Words The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.

Symbols $y - y_1 = m(x - x_1)$

Graph



Work Zone

Slope
The point-slope form of a linear equation is tied directly to the definition of slope.

You can write an equation of a line in slope-intercept form when you know the slope and the y-intercept. You can write an equation of a line in **point-slope form** when you are given the slope and the coordinates of a point on the line that is not the y-intercept.

Examples

1. Write an equation in point-slope form for the line that passes through (−2, 3) with a slope of 4.

$y - y_1 = m(x - x_1)$ Point-slope form
$y - 3 = 4[x - (-2)]$ $(x_1, y_1) = (-2, 3), m = 4$
$y - 3 = 4(x + 2)$ Simplify.

2. Write the slope-intercept form of the equation from Example 1.

$y - 3 = 4(x + 2)$ Write the equation.
$y - 3 = 4x + 8$ Distributive Property
$+3 = +3$ Addition Property of Equality
$y = 4x + 11$ Simplify.

Check Substitute the coordinates of the given point in the equation.

$y = 4x + 11$
$3 \stackrel{?}{=} 4(-2) + 11$
$3 = 3$ ✓

Got It? **Do this problem to find out.**

a. Write an equation in point-slope form and slope-intercept form for the line that passes through (−1, 2) and has a slope of $-\frac{1}{2}$.

a. $y - 2 = -\frac{1}{2}(x + 1)$;
$y = -\frac{1}{2}x + \frac{3}{2}$

Write a Linear Equation

Key Concept

From Slope and a Point	• Substitute the slope m and the coordinates of the point in $y - y_1 = m(x - x_1)$.
From Slope and y-intercept	• Substitute the slope m and y-intercept b in $y = mx + b$.
From a Graph	• Find the y-intercept b and the slope m from the graph, then substitute the slope and y-intercept in $y = mx + b$.
From Two Points	• Use the coordinates of the points to find the slope. Substitute the slope and coordinates of one of the points in $y - y_1 = m(x - x_1)$.
From a Table	• Use the coordinates of the two points to find the slope, then substitute the slope and coordinates of one of the points in $y - y_1 = m(x - x_1)$.

The form you use to write a linear equation is based on the information you are given.

Example

3. **Write an equation in point-slope form and slope-intercept form for the line that passes through (8, 1) and (−2, 9).**

Step 1 Find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \frac{9 - 1}{-2 - 8}$ $(x_1, y_1) = (8, 1), (x_2, y_2) = (-2, 9)$

$m = -\frac{8}{10}$ or $-\frac{4}{5}$ Simplify.

Step 2 Use the slope and the coordinates of either point to write the equation in point-slope form.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 1 = -\frac{4}{5}(x - 8)$ $(x_1, y_1) = (8, 1), m = -\frac{4}{5}$

So, the point-slope form of the equation is $y - 1 = -\frac{4}{5}(x - 8)$. In slope-intercept form, this is $y = -\frac{4}{5}x + \frac{37}{5}$.

Got it? Do these problems to find out.

c. (3, 0) and (6, −3)

d. (−1, 2) and (5, −10)

Sample answers are given.

c. $y + 3 = -1(x - 6)$; $y = -x + 3$

d. $y - 2 = -2(x + 1)$; $y = -2x$

Example

3. Write an equation given two points.

AL
- *What is the first value you will find?* the slope
- *What two points will we use?* (8, 1) and (−2, 9)

OL
- *How can you find the slope of the line?* Use the slope formula.
- *When finding the slope, does it matter which point is (x_1, y_1) and which point is (x_2, y_2)? Explain.* no; As long as we are consistent between numerators and denominators, the result will be the same.

BL
- *Can you write the equation of the line in point-slope form using either point? Why?* yes; Sample answer: No matter which point you use, the line still goes through both of them.
- *How would you write the equation in slope-intercept form?* First use the Distributive Property to rewrite $-\frac{4}{5}(x - 8)$ as $-\frac{4}{5}x + \frac{32}{5}$. Then add 1 to each side.

Need Another Example?
Write an equation in point-slope form and slope-intercept form for the line that passes through (3, 6) and (4, −2). Sample answer: $y - 6 = -8(x - 3)$; $y = -8x + 30$

Watch Out!

Common Error Students may be confused between point-slope form and slope-intercept form. Have students make an index card with the name and formula for each form that they can keep on their desks or tape to their books for easy reference.

Example

4. Write an equation given two points in a real-world context.

AL • *What are the points in the table written as ordered pairs?* (5, 165) and (10, 290)

• *What is the slope of the line?* 25

OL • *Could we use either point to substitute into the equation?* yes; The results are the same.

• *What is the point-slope form of the equation?* $y - 165 = 25(x - 5)$

BL • *What is the cost for attending 9 dog training sessions?* AED 265

Need Another Example?

The cost of different numbers of paper plates at a party supply store is shown in the table. Write an equation in point-slope form to represent the cost y of buying x paper plates.

$y - 5 = \frac{1}{4}(x - 20)$

Number of Plates	Total Cost (AED)
20	5
40	10

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Think-Pair-Share Give students several minutes to think through the answer for Exercise 1. Have them share with a partner. Then have them share their response in a small group. Compare answers. Continue for Exercises 2–4. MP 1, 2, 4, 7, 8

BL Have students research the Internet to find an example of a real-world linear relationship and write the equation of the line in point-slope form and slope-intercept form. MP 1, 4, 5, 7, 8

Example

4. The cost of assistance dog training sessions is shown in the table. Write an equation in point-slope form to represent the cost y of attending x dog training sessions.

Number of Sessions	Cost (AED)
5	165
10	290

Find the slope of the line. Then use the slope and one of the points to write the equation of the line.

$m = \frac{290 - 165}{10 - 5}$ [illegible]

$m = \frac{125}{5}$ or 25 Simplify.

$y - 165 = 25(x - 5)$ [illegible]

So, the equation of the line is $y - 165 = 25(x - 5)$.

Got It? Do this problem to find out.

e. The cost for making spirit buttons is show in the table. Write an equation in point-slope form to represent the cost y of making x buttons.

Number of Buttons	Cost (AED)
100	25
150	35

Sample answer:

e. $y - 25 = \frac{1}{5}(x - 100)$

Guided Practice

Write an equation in point-slope form and slope-intercept form for each line. (Examples 1–3)

1. passes through (2, 5), slope = 4

$y - 5 = 4(x - 2)$; $y = 4x - 3$

2. passes through (−3, 1) and (−2, −1)

Sample answer: $y - 1 = -2(x + 3)$; $y = -2x - 5$

3. Ruqaya is planning a party. The cost for 20 people is AED 290. The cost for 45 people is AED 590. Write an equation in point-slope form to represent the cost y of having a party for x people. (Example 4)

Sample answer: $y - 290 = 12(x - 20)$

4. **Building on the Essential Question** How does using the point-slope form of a linear equation make it easier to write the equation of a line? Sample answer: You can substitute the slope and a point into the equation. You do not need to find the y-intercept of the line.

Rate Yourself!

How confident are you about writing linear equations? Check the box that applies.

Name ______________________ My Homework ______________________

Independent Practice

Write an equation in point-slope form and slope-intercept form for each line. (Examples 1–3)

1. passes through (1, 9), slope = 2
   $y - 9 = 2(x - 1)$; $y = 2x + 7$

2. passes through (4, −1), slope = −3
   $y + 1 = -3(x - 4)$; $y = -3x + 11$

3. passes through (−4, −5), slope = $\frac{3}{4}$
   $y + 5 = \frac{3}{4}(x + 4)$; $y = \frac{3}{4}x - 2$

4. passes through (3, −6) and (−1, 2)
   Sample answer: $y + 6 = -2(x - 3)$; $y = -2x$

5. passes through (4, −4) and (6, −10)
   Sample answer: $y + 4 = -\frac{3}{2}(x - 4)$; $y = -\frac{3}{2}x + 2$

6. passes through (3, 4) and (5, −4)
   Sample answer: $y + 4 = -4(x - 5)$; $y = -4x + 16$

7. **STEM** For a science experiment, Mala measured the height of a plant every week. She recorded the information in the table. Assuming the growth is linear, write an equation in point-slope form to represent the height y of the plant after x weeks. (Example 4)

Weeks	Height (in.)
5	13
10	14

   Sample answer: $y - 14 = \frac{1}{5}(x - 10)$

8. After 2 seconds on a penalty kick in soccer, the ball travels 160 feet. After 2.75 seconds on the same kick, the ball travels 220 feet. Write an equation in point-slope form to represent the distance y of the ball after x seconds. (Example 4)
   Sample answer: $y - 160 = 80(x - 2)$

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–8, 15–18	Exercises 9–11, 19–24	Exercises 12–14
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11, 12, 23, 24
OL	On Level	1–7 odd, 9–12, 23, 24
BL	Beyond Level	9–14, 23, 24

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	13, 14
2 Reason abstractly and quantitatively.	12
5 Use appropriate tools strategically.	21, 22
7 Look for and make use of structure.	11

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

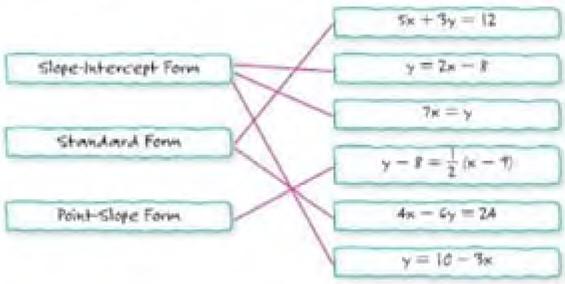
What is the equation in point-slope form of the line that passes through (−2, 4) and (3, −1)? $y - 4 = -1(x + 2)$

Write each equation in standard form.

9. $y - 4 = -3(x - 3)$
$3x + y = 13$

10. $y + 9 = 2(x + 5)$
$2x - y = -1$

11. **Identify Structure** Draw a line connecting the form of the equation to the correct equations.



H.O.T. Problems Higher Order Thinking

12. **Reason Abstractly** Write a linear equation that is in point-slope form. Identify the slope and name a point on the line.
Sample answer: $y - 3 = 4(x + 2)$; slope: 4, point on line: (−2, 3)

13. **Persevere with Problems** The equation of a line is $y = -\frac{1}{2}x + 6$. Write an equation in point-slope form for the same line. Explain the steps that you used.
Sample answer: $y - 5 = -\frac{1}{2}(x - 2)$; First, use the equation to find the slope and the coordinates of any point on the line. Then use the slope and coordinates to write an equation in point-slope form.

14. **Persevere with Problems** Order the steps to write a linear equation in slope-intercept form if you know the slope of the line and a point on the line.

4 Simplify the equation.

2 Use the Distributive Property to multiply the slope by x and x_1.

1 Substitute the slope m and the coordinates of the point (x_1, y_1) into the point-slope formula.

3 Use the Addition Property of Equality.

Name ______________________ My Homework ______________________

Extra Practice

Write an equation in point-slope form and slope-intercept form for each line.

15. passes through (−7, 10), slope = −4

$y - 10 = -4(x + 7)$; $y = -4x - 18$

$y - y_1 = m(x - x_1)$
$y - 10 = -4(x + 7)$
$y - 10 = -4x - 28$
$+10 = \quad +10$
$y = -4x - 18$

Homework Help

16. passes through (1, 2) and (3, 4)

$y - 4 = 1(x - 3)$; $y = x + 1$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{3-1} = \frac{2}{2}$ or 1

$y - y_1 = m(x - x_1)$
$y - 4 = 1(x - 3)$
$y - 4 = x - 3$
$+4 = +4$
$y = x + 1$

17. passes through (6, 2), slope = $\frac{2}{3}$

$y - 2 = \frac{2}{3}(x - 6)$; $y = \frac{2}{3}x - 2$

18. passes through (2, −2) and (4, −1)

Sample answer: $y + 1 = \frac{1}{2}(x - 4)$;
$y = \frac{1}{2}x - 3$

Write each equation in standard form.

19. $y + 1 = \frac{4}{5}(x - 3)$

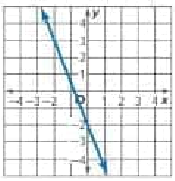
$4x - 5y = 17$

20. $y - 8 = -\frac{1}{2}(x + 4)$

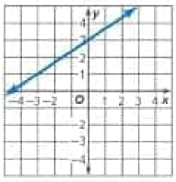
$x + 2y = 12$

Use Math Tools Write the point-slope form of an equation for each line graphed.

21. **Sample answer:** $y - 3 = -\frac{5}{2}(x + 2)$



22. **Sample answer:** $y - 1 = \frac{2}{3}(x + 3)$



Power Up! Test Practice

Exercises 23 and 24 prepare students for more rigorous thinking needed for the assessment.

23.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

24.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly model the slope and correctly represent the point-slope form of the line.
1 point	Students correctly model the slope OR correctly represent the point-slope form of the line.

Power Up! Test Practice

23. The table shows some ordered pairs that lie on a line. Which equations could represent the line? Select all that apply.

x	−1	0	1	2
y	−6	−2	2	6

■ $y = 4x - 2$ ☐ $y = -4x + 1$ ■ $y - 2 = 4(x - 1)$ ☐ $y - 2 =$

24. After 4 hours of driving, Zahra is 248 miles away from home. After 6 hours of driving, she is 372 miles from home. Select the correct values to complete the model below. **Sample answer:**

1
4
6

slope: $\dfrac{372 - 248}{6 - 4} = \dfrac{62}{1}$

What is the point-slope form of the line that represents this situation?

$y - 248 = 62(x - 4)$

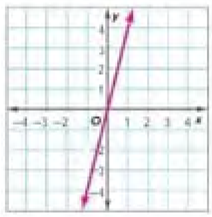
Spiral Review

25. Use the information in the table to find the constant rate of change in dirhams per hour.

Time (h)	0
Wage (AED)	0

AED 9 per hour

26. Graph $y = 4x$.



27. A train traveled 150 miles in $1\frac{1}{4}$ hours. At this rate, how far will the train travel after 5 hours? Assume that the distance traveled varies directly with the time traveled. Write an equation to represent the situation.

$d = 120t$; 600 mi

Inquiry Lab

Graphing Technology: Model Linear Behavior

Inquiry **HOW does using technology help you to determine if situations display linear behavior?**

Mathematical Practices 1, 3, 5

Simone and Lee walked to school at about 3 miles per hour. Use the Investigation to see if the relationship between time and distance is a linear relationship.

Hands-On Activity

Step 1 Connect a motion detector to your calculator. Start the data collection program by pressing APPS (CBL/CBR), ENTER, and then select Ranger, Applications, Meters, Dist Match.

Step 2 Place the detector on a desk or table so that it can read the motion of a walker.

Step 3 Mark the floor at a distance of 1 and 6 meters from the detector. Have a partner stand at the 1-meter mark.

Step 4 When you press the button to begin collecting data, have your partner begin to walk away from the detector at a slow but steady pace.

Step 5 Stop collecting data when your partner passes the 6-meter mark.

Step 6 Press ENTER to display a graph of the data. The *x*-values represent equal intervals of time in seconds. The *y*-values represent the distances from the detector in meters.

Describe the DISTANCE graph of the data. Does the relationship between time and distance appear to be linear? Explain.

Sample answer: The data appear to form a straight line; therefore, the relationship appears to be linear.

Focus narrowing the scope

Objective Use technology to model linear behavior.

Coherence connecting within and across grade

Now Students use technology to investigate linear behavior.

Next Students will graph families o equations.

Rigor pursuing concepts, fluency, and applicatio

See the Levels of Complexity chart on page 230.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUAT

1 Launch the Lab

The activity is intended to be used as a whole-group

Materials: graphing calculator, motion detector

Hands-On Activity

AL Make sure students understand how to use the detector. Have them perform the Activity several time they get used to working with it and have results that are representative of a steady walking pace without sudden acceleration or deceleration. If students do th several times, or if pairs of students are sharing the m detector, remind them to clear the memory after each performance of the Activity. MP 1, 5

Collaborate

he **Investigate** and **Analyze and Reflect** sections are
ntended to be used as small-group investigations. The **Create**
ection is intended to be used as independent exercises.

evels of Complexity

he levels of the exercises progress from 1 to 3, with Level 1
ndicating the lowest level of complexity.

	Exercises 1–3	4	5, 6
evel 3			●
evel 2		●	
evel 1	●		

Investigate

AL BL LA **Pairs Consult** Pair a Beyond Level student
ith an Approaching Level student to complete Exercises 1
oth students are responsible for ensuring that each one
nderstands and can explain their reasoning to each o
P 1, 3, 4, 5, 7, 8

Analyze and Reflect

BL Give students time to complete Exercise 4 on their own.
hen call on specific students to lead a whole class discussion
ased on their answers. Have the selected students answer
ny clarifying questions and discuss differences in answers.
P 1, 3, 4

Create

Inquiry Students should be able to answer "HOW does
raphing slope triangles on the coordinate plane help you
nalyze them?" Check for student understanding and provide
uidance, if needed.

Investigate

Use Math Tools Refer to the Activity. Work with a partner.

1. Use the TRACE feature on your calculator to find the y-intercept on the graph. Interpret its meaning. **Sample answer: The y-intercept is 0.91 and represents the starting distance from the detector to the person walking.**

2. Press STAT 1. The time data is in L1 and the distance data is in L2. Use these data to calculate the rate of change $\frac{\text{distance}}{\text{time}}$ for three pairs of points. **Sample answers are given.**

Point 1 (time, distance)	Point 2 (time, distance)	$\frac{\text{distance}_2 - \text{distance}_1}{\text{time}_2 - \text{time}_1}$	rate of change
(0, 0.91)	(2, 1.32)	$\frac{1.32 - 0.91}{2 - 0}$	0.205
(2, 1.32)	(4, 1.76)	$\frac{1.76 - 1.32}{4 - 2}$	0.220
(4, 1.76)	(6, 2.20)	$\frac{2.20 - 1.76}{6 - 4}$	0.220

3. **Justify Conclusions** Does the table in Exercise 2 support your conclusion about the graph in the Activity? Explain **yes; Sample answer: Since the rates of change between any two points are about the same, the graph does approximate a line and the relationship between time and distance is approximately linear.**

Analyze and Reflect

4. Predict how the graph and answers to Exercise 2 would change if the person in the activity were to:
   a. move at a steady but *quicker* pace *away* from the detector.
   The line would be steeper.
   b. move at a steady pace *toward* the detector.
   The slope of the line would be negative.

Create

5. **Reason Inductively** How could you change the situation to be one that does not display linear behavior? **Sample answer: The person could change their pace while walking.**

6. Inquiry HOW does using technology help you to determine if a situation displays linear behavior?
   Sample answer: You can easily graph a situation to determine if it is linear.

Inquiry Lab

Graphing Technology: Systems of Equations

Inquiry **HOW can I use a graphing calculator to find one solution for a set of two equations?**

MP Mathematical Practices
1, 3, 5, 7

Web site A charges AED 30 plus AED 10 per pound to ship an item. Web site B charges AED 10 plus AED 20 per pound to ship the same item. For an object that weighs x pounds, the charges for Web site A are represented by $y = 10x + 30$. The charges for Web site B are represented by $y = 20x + 10$. At what point are the charges the same?

What do you know? **Web site A charges AED 3 plus AED 1 per pound and Web site B charges AED 1 plus AED 2 per pound to ship an item.**

What do you need to know? **the point at which the charges are the same**

Hands-On Activity

Use a graphing calculator to generate a table of values for $y = 10x + 30$ and $y = 20x + 10$. Then use the table to find the total cost to ship objects that weigh 0, 1, 2, or 3 pounds.

Step 1 Press [Y=]. Then enter each equation.

Step 2 Set up the table. Press [2nd] [TblSet] to display the table setup screen. Press [▼] [▼] [▶] [ENTER] to highlight Indpnt: Ask. Then press [▼] [ENTER] to highlight Depend: Auto.

Step 3 Access the table by pressing [2nd] [Table]. Now key in your input values, pressing [ENTER] after each one. Fill in the table. The first one is done for you.

So, the point when the charges are the same is **(2, 50)**

Focus narrowing the scope

Objective Use graphing technology to find one solu
set of two equations.

Coherence connecting within and across gra

Now
Students use a graphing calculator to identify the solution of a system of linear equations.

Next
Students will identify and
solution of a system of lin

Rigor pursuing concepts, fluency, and applica

See the Levels of Complexity chart on page 232.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALU

1 Launch the Lab

The activity is intended to be used as a whole-grou

Materials: graphing calculator

Hands-On Activity

AL LA **Pairs Discussion** Have students work w
partner to understand that the input values in Step
shipping weights of 0, 1, 2, or 3 pounds. The values
Y_1 and Y_2 are the shipping charges for Web sites A
respectively. MP 1, 5, 7, 8

BL LA **Pairs Discussion** Provide students with a
site, Web site C, that charges AED 2 plus AED 1 per
ship the same item. Have students generate an equ
determine the number of pounds for a shipped item
have the same cost as for Web site A. Then have the
why the cost will never be the same for Web site C a
site B. MP 1, 2, 3, 5, 7, 8

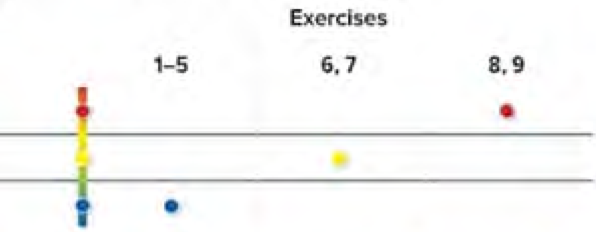
SE EXPLORE EXPLAIN ELABORATE EVALUATE

llaborate

estigate and **Analyze and Reflect** sections are d to be used as small-group investigations. The **Create** s intended to be used as independent exercises.

of Complexity

els of the exercises progress from 1 to 3, with Level 1 g the lowest level of complexity.



Investigate

LA **Value Line** After completing Exercise 4, s place themselves on a pretend number line fr number 10 representing that they understand the g of intersection of the lines in Exercise 4. Then, pair ents from both sides of the line to discuss Exercises MP 1, 5, 7, 8

Create

Rally Coach Have students work in pairs to e Exercise 7. Have Student 1 speak aloud about what t of intersection represents, while Student 2 listens, , and encourages. After both students have spoken, dents work independently to write their responses to 7. MP 1, 7

Students should be able to answer "HOW can I use ng calculator to find one solution for a set of two ns?" Check for student understanding and provide e, if needed.

Investigate

MP **Use Math Tools** Work with a partner. Refer to the Activity.

1. For what number of pounds are the charges for Web site A less than those for Web site B? **3 lb**

2. For what number of pounds are the charges for Web site A greater than the ones for Web site B? **0 or 1 lb**

3. Press GRAPH. Copy your calculator screen on the blank screen shown.

4. At what point do the two lines intersect? What does this ordered pair represent? **(2, 5); The x-coordinate, 2, is the number of pounds where the shipping charges are the same. The y-coordinate, 5, is the shipping charge.**

5. How does the point of intersection of the two lines compare to the answer to the Activity? **They are the same. The ordered pair is the solution to the equations $y = x + 3$ and $y = 2x + 1$.**

Analyze and Reflect

6. MP **Use Math Tools** Use a graphing calculator to graph each set of equations in the table. Find the point of intersection of the two lines.

Set of Equations	Point of Intersection
$y = 2x + 2$ $y = x + 2$	(0, 2)
$y = 3x + 5$ $y = -x - 3$	(−2, −1)

7. MP **Identify Structure** Explain what the point of intersection represents. **Sample answer: The point of intersection is the solution of the set of two equations.**

Create

8. MP **Model with Mathematics** Describe a real-world situation that would involve finding the point of intersection of two lines. **Sample answer: A health club offers a memberships that costs AED 20 per month with a AED 10 enrollment fee. Another club charges AED 25 per month with no enrollment fee. After how many months will the costs be the same?**

9. Inquiry HOW can I use a graphing calculator to find one solution for a set of two equations? **Sample answer: I can use a graphing calculator to graph the set of two equations. The graph will show the point of intersection which is the solution of the two equations.**

Lesson 7

Solve Systems of Equations by Graphing

Real-World Link

Activities A campground offers tubing, kayak, and bicycle rentals as shown.

	Deposit (AED)	Cost per Hour (AED)
Tube	50	20
Kayak	400	250
Water bike	500	350

1. Write an equation to represent the total cost y of renting a tube for any number of hours x. $y = 50 + 20x$
2. Write an equation to represent the total cost y of renting a kayak for any number of hours x. $y = 400 + 250x$
3. Write an equation to represent the total cost y of renting a water bike for any number of hours x. $y = 500 + 350x$
4. Find the cost to rent each item for 1, 2, 3, 4, and 5 hours.

Hours	Cost of Tube (AED)	Cost of Kayak (AED)	Cost of Water Bike (AED)
1	70	650	850
2	90	900	1200
3	110	1150	1550
4	130	1400	1900
5	150	1650	2250

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Essential Question

WHY are graphs helpful?

Vocabulary

systems of equations

Mathematical Practices

1, 3, 4, 7

2018

Focus narrowing the scope

Objective Solve systems of linear equations by gra

Coherence connecting within and across gra

Previous Students graphed and interpreted linear equations.

Now Students will use a graph to identify solutions of a system of two linear equations.

Next Students solving sy equation situatio

Rigor pursuing concepts, fluency, and applica

See the Levels of Complexity chart on page 239.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALU

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole g group, think-pair-share activity, or independent ac

LA Team-Pair-Solo Have students w teams to complete Exercise 1, in pairs t Exercise 2, and independently to complete Exerci have the original teams discuss their responses to and work together to complete the table in Exerci 4, 5, 7, 8

Alternate Strategy

AL Help students write the equations in Exercise providing them with the template, total cost = depo per hour • number of hours. MP 1, 2, 4, 5, 7, 8

NGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

Teach the Concept

the scaffolded questions for each example to differentiate uction.

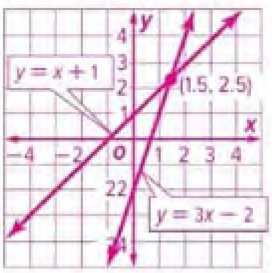
ample

Solve a system of equations by graphing.

- *Describe a method you could use to graph $y = -2x - 3$.* **Graph the y-intercept, −3. Since the slope is −2, move up 2 units and left 1 unit to graph the next point on the line. Then connect the points.**
- *Describe a method you could use to graph $y = 2x + 5$.* **Graph the y-intercept, 5. Since the slope is 2, move down 2 units and left 1 unit to graph the next point on the line. Then connect the points.**
- *Do the lines appear to intersect?* **yes**
- *At which point do they intersect?* **(−2, 1)**
- *What does it mean if the two lines intersect?* **The point of intersection is a solution of both equations.**

d Another Example?

e the system $y = 3x - 2$ and $y = x + 1$ by graphing.
2.5)



Work Zone

Systems of Equations

Two or more equations with the same set of variables are called a **system of equations**. For example, $y = 4x$ and $y = 4x + 2$ together are a system of equations.

You can estimate the solution of a system of equations by graphing the equations on the same coordinate plane. The ordered pair for th point of intersection of the graphs is the solution of the system because the point of intersection simultaneously satisfies both equations.

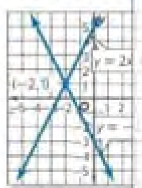
Example

1. **Solve the system $y = -2x - 3$ and $y = 2x + 5$ by graphing.**

Graph each equation on the same coordinate plane.

The graphs appear to intersect at (−2, 1).

Check this estimate by replacing x with −2 and y with 1.



Check $y = -2x - 3$ | $y = 2x + 5$

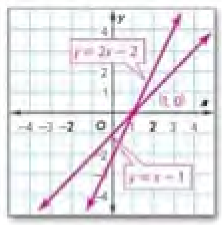
$1 \stackrel{?}{=} -2(-2) - 3$ | $1 \stackrel{?}{=} 2(-2) + 5$

$1 = 1$ ✓ | $1 = 1$ ✓

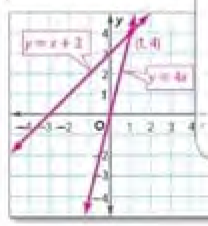
The solution of the system is (−2, 1).

Got it? Do these problems to find out.

a. $y = x - 1$
$y = 2x - 2$



b. $y = 4x$
$y = x + 3$



a. (1, 0)

b. (1, 4)

Examples

Jamal's Motorsports has motorcycles (two wheels) and ATVs (four wheels) in stock. The store has a total of 45 vehicles, that, together, have 130 wheels.

2. Write a system of equations that represents the situation.

Let y represent the motorcycles and x represent the ATVs.

$y + x = 45$

$2y + 4x = 130$

3. Solve the system of equations. Interpret the solution.

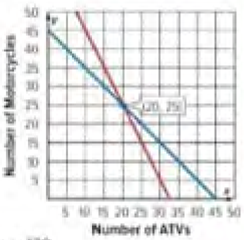
Write each equation in slope-intercept form.

$x + y = 45$ → $y = -x + 45$

$2y + 4x = 130$ → $2y = -4x + 130$ → $y = -2x + 65$

Graph both equations on the same coordinate plane. The equations intersect at (20, 25).

The solution is (20, 25). This means that the store has 20 ATVs and 25 motorcycles.



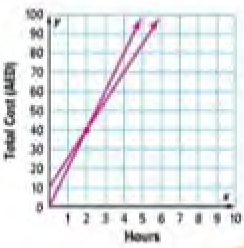
Check

$x + y = 45$ → $20 + 25 \stackrel{?}{=} 45$ → $45 = 45$ ✓

$2y + 4x = 130$ → $2(25) + 4(20) \stackrel{?}{=} 130$ → $130 = 130$ ✓

Got it? Do this problem to find out.

c. Creative Crafts gives scrapbooking lessons for AED 15 per hour plus a AED 10 supply charge. Scrapbooks Incorporated gives lessons for AED 20 per hour with no additional charges. Write and solve a system of equations that represents the situation. Interpret the solution.



c. $15x + 10 = y$; $20x = y$
(2, 40); If you take lessons for 2 hours, the costs at both stores are equal, at AED 40.

Examples

2. Write a system of equations to represent a real-world situation.

AL • *What could we let x represent? y?* Let x represent the ATVs and y represent the motorcycles.

OL • *If the number of motorcycles and ATVs total 45, what equation could represent this?* $y + x = 45$

• *How many wheels are on an ATV? a motorcycle?* 4; 2

• *If the total number of wheels is 130, what equation could represent this?* $2y + 4x = 130$

BL • *Is it possible to write different equations? Explain.* Yes; if the variables are reversed, the equations would be slightly different.

3. Solve a system of equations that represents a real-world situation.

AL • *How would you rewrite $2y + 4x = 130$ in slope-intercept form?* Subtract $4x$ from each side, and then divide both sides by 2.

OL • *Once the equations are graphed, what will the point of intersection represent?* The x-coordinate will be the number of ATVs, and the y-coordinate will be the number of motorcycles.

• *What is the point of intersection?* (20, 25)

BL • *What does the point (30, 15) represent in this situation?* There are 30 ATVs and 15 motorcycles. The total number of wheels would be 150 in this case.

Need Another Example?

Ms. Budour bought 14 packages of red and green pens for a total of 72 pens. The red pens come in packages of 6 and the green pens come in packages of 4. Write and solve a system of equations that represents the situation. Interpret the solution. See Answer Appendix.

Examples

4. Find the number of solutions for a system of equations.

- AL • *Graph each equation on a coordinate plane. What do you notice about the lines?* **They appear to be parallel.**
- • *If the lines are parallel, will they ever intersect?* **no**
- OL • *Since the lines do not intersect, does this system have a solution?* **no**
- BL • *Give one way you can tell there is no solution to the system of equations without graphing.* **Sample answer: The equations have the same slope, but different y-intercepts; therefore, they are parallel and will never intersect.**

Need Another Example?

Solve the system $y = \frac{1}{4}x - 1$ and $y = \frac{1}{4}x$ by graphing.

See Answer Appendix.

5. Find the number of solutions for a system of equations.

- AL • *What should you do first in order to graph the equations?* **Write $y - 3 = 2x - 2$ in slope-intercept form.**
- • *What is $y - 3 = 2x - 2$ written in slope-intercept form?* **$y = 2x + 1$**
- OL • *How does this equation compare to the other equation in the system?* **They are the same equation.**
- • *If the equations are the same, what is the solution of the system?* **Every point on the line is a solution so there are an infinite number of solutions.**
- BL • *How can you tell, without graphing, that there are an infinite number of solutions?* **Sample answer: After writing $y - 3 = 2x - 2$ in slope-intercept form, the new equation is exactly the same as the other equation in the system. Because they are the same equation, they will "intersect" everywhere.**

Need Another Example?

Solve the system $y = 3x - 2$ and $y - 2x = x - 2$ by graphing.

See Answer Appendix.

Work Zone

Number of Solutions

The graph of a system of equations indicates the number of solutions.

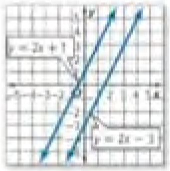
- If the lines intersect, there is one solution.
- If the lines are parallel, there is no solution.
- If the lines are the same, there are an infinite number of solutions.

Examples

Solve each system of equations by graphing.

4. $y = 2x + 1$
$y = 2x - 3$

Graph each equation on the same coordinate plane.



The graphs appear to be parallel lines. Since there is no coordinate point that is a solution of both equations, there is no solution for this system of equations.

Check Analyze the equations. Write them in standard form.

$y = 2x + 1$	$y = 2x - 3$
$y - 2x = 2x - 2x + 1$	$y - 2x = 2x - 2x - 3$
$y - 2x = 1$	$y - 2x = -3$

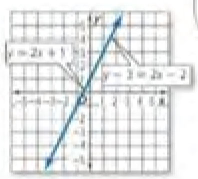
Since $y - 2x$ cannot simultaneously be 1 and -3, there is no solution. ✓

5. $y = 2x + 1$
$y - 3 = 2x - 2$

Write $y - 3 = 2x - 2$ in slope-intercept form.

$y - 3 = 2x - 2$ Write the equation.
$y - 3 + 3 = 2x - 2 + 3$ Add 3 to each side.
$y = 2x + 1$ Simplify.

Both equations are the same. Graph the line.



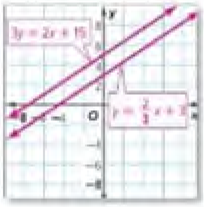
Any ordered pair on the graph will satisfy both equations. So, there are an infinite number of solutions of the system.

Expressions and Equations

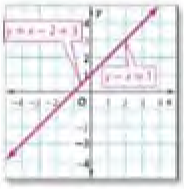
Got it? Do these problems to find out.

Solve each system of equations by graphing.

d. $y = \frac{2}{3}x + 3$
$3y = 2x + 15$



e. $y - x = 1$
$y = x - 2 + 3$



d. no solution

e. an infinite number of solutions

Slopes and Intercepts

When a linear system of equations has:

- different slopes and y-intercepts, there is one and only one solution.
- the same slope and different y-intercepts, there is no solution.
- the same slope and the same y-intercept, there is an infinite number of solutions.

Example

6. **A system of equations consists of two lines. One line passes through (2, 3) and (0, 5). The other line passes through (1, 1) and (0, −1). Determine if the system has *no solution, one solution, or an infinite number of solutions.***

To compare the two lines, write the equation of each line in slope-intercept form.

Find the slope of each line.

(2, 3) and (0, 5)

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{0 - 2}$ or -1

(1, 1) and (0, −1)

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 1}{0 - 1}$ or 2

Find the y-intercept for each line. Then write the equation.

Use the point (0, 5).
The y-intercept is 5.

$y = mx + b$
$y = -1x + 5$

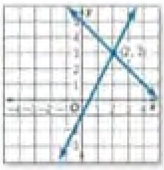
Use the point (0, −1).
The y-intercept is −1.

$y = mx + b$
$y = 2x - 1$

Since the lines have different slopes and different y-intercepts, they intersect in exactly one point.

Check Graph each line on a coordinate plane.

The lines intersect at (2, 3) so there is exactly one solution. ✓



Example

6. Analyze systems of equations.

AL
- *What information are we given?* **the points that each line passes through**
- *What do we need to determine?* **whether the system has no solution, one solution, or an infinite number of solutions**

OL
- *What is the slope of the line through (2, 3) and (0, 5)?* **−1** *the y-intercept?* **5**
- *What is the slope of the line through (1, 1) and (0, −1)?* **2** *the y-intercept?* **−1**
- *What is the equation, in slope-intercept form, of each line?* **$y = -1x + 5$ and $y = 2x - 1$**
- *Since the lines do not have the same slope, what do you know about the pair of equations?* **The lines are not parallel and they are not the same line, so the lines intersect in exactly one point.**

BL
- *How can you check to make sure the pair of equations has exactly one solution?* **Graph the lines and check where they intersect.**

Need Another Example?
A system of equations consists of two lines. One line passes through (−3, 9) and (2, 6). The other line passes through (−5, 7) and (2, 14). Determine if the system has no solution, one solution, or an infinite number of solutions. **one solution**

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Team-Pair-Solo** Have students work in a four-person team to complete Exercises 1 and 3. Then have them work in pairs to complete Exercises 2 and 4. Have them trade solutions with the other pair from their four-person team and discuss differences in solutions. Then have them work individually to complete Exercise 5. Upon completion, have them rejoin their original team to discuss solutions. Call on students to present their responses to the class. MP 1, 2, 4, 7, 8

BL LA **Trade-a-Problem** Have students work in pairs to write equations for three different sets of simultaneous linear equations. The first set should have exactly one solution. The second set should have no solution. The third set should have infinitely many solutions. Then have them trade sets of equations with another pair of students. Each pair graphs the first pair's sets of equations to determine if the equations were written correctly. MP 1, 2, 7, 8

Watch Out!

Common Error Students may have difficulty solving systems of equations by graphing because the lines of the equations they draw are not straight. Suggest that students use a straightedge to draw the lines. Remind them to check the point of intersection by substituting the values into both equations.

Got it? Do this problem to find out.

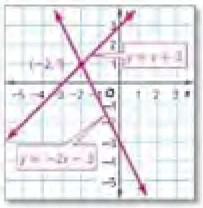
f. no solution

f. (0, 2), (1, 4) and (0, −1), (1, 1)

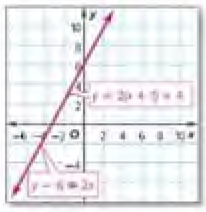
Guided Practice

Solve each system of equations by graphing. (Examples 1, 4, and 5)

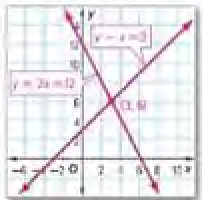
1. $y = x + 3$
   $y = -2x - 3$ **(−2, 1)**



2. $y - 6 = 2x$
   $y = 2(x + 1) + 4$ **an infinite number of solutions**



3. The sum of Salwa's age plus twice Hatems' age is 12. The difference of Salwa's age and Hatem's age is 3. Write and solve a system of equations to find their ages. Interpret the solution. (Examples 2 and 3)
   Sample answer: Let x Hatem's age and y = Salwa's age; y + 2x = 12, y − x = 3; Salwa is 6 years old and Hatem is 3 years old.



4. A system of equations consists of two lines. One line passes through (−1, 3) and (0, 1). The other line passes through (1, 4) and (0, 2). Determine if the system has *no solution*, *one solution*, or *an infinite number of solutions*. (Example 3) **one solution**

5. **Building on the Essential Question** How can you use a graph to solve a system of equations?
   Sample answer: Graphing the two equations will show whether or not the two equations intersect. If they intersect, the point of intersection is the solution.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

YES ? NO

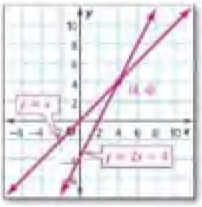
FOLDABLES Time to update your Foldable!

Name ______________________ My Homework ______________________

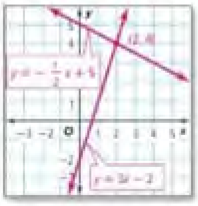
Independent Practice

Solve each system of equations by graphing. (Examples 1, 4, and 5)

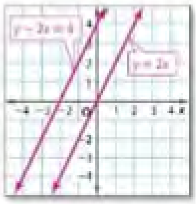
1. $y = x$
   $y = 2x - 4$ **(4, 4)**



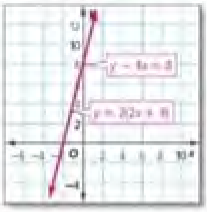
2. $y = -\frac{1}{2}x + 5$
   $y = 3x - 2$ **(2, 4)**



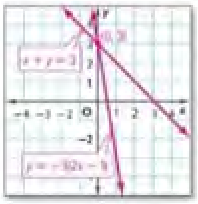
3. $y - 2x = 4$
   $y = 2x$ **no solution**



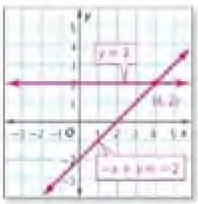
4. $y - 4x = 8$
   $y = 2(2x + 4)$ **an infinite number of solutions**



5. $x + y = 3$
   $y = -3(2x - 1)$ **(0, 3)**



6. $-x + y = -2$
   $y = 2$ **(4, 2)**



7. **Copy and Solve** A pet store currently has a total of 45 cats and birds. There are 7 more cats than birds. Find the number of cats and birds in the store. On a separate sheet of paper, write and solve a system of equations that represents the situation. Interpret the solution. (Examples 2 and 3)
   See Answer Appendix.

Copy and Solve A line passes through each pair of points. Determine if the system has *no solution*, *one solution*, or *an infinite number of solutions*. Show your work on a separate piece of paper. (Example 4)

8. (0, 3) and (−2, 5); (5, −2) and (0, 3) **an infinite number of solutions**

9. (4, 1) and (0, 1); (0, −4) and (4, 4) **one solution**

10. (−2, −2) and (0, 2); (1, 1) and (0, −1) **no solution**

ENGAGE EXPLORE **EXPLAIN ELABORATE EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–10, 14–17	Exercises 11, 18–22	Exercises 12, 13
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–11, 21, 22
OL	On Level	1–9 odd, 11, 21, 22
BL	Beyond Level	13, 21, 22

Watch Out!

Common Error For Exercise 7, students may have trouble interpreting the solution from the graph if they choose larger increments on the axes. Express to students that graphing does not always display solutions that can easily be seen and that smaller increments on the axes may help to find the solution.

EMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
ense of problems and persevere in them.	12, 13
with mathematics.	11
r and make use of structure.	18–20

tical Practices 1, 3, and 4 are aspects of mathematical hat are emphasized in every lesson. Students are portunities to be persistent in their problem solving, to heir reasoning, and apply mathematics to real-world .

e Assessment

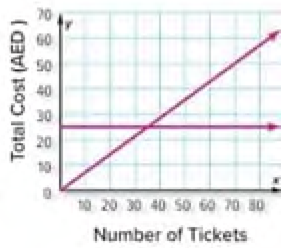
activity as a closing formative assessment before g students from your class.

KET
e Door

dents to solve the system $y = 2x + 6$ and $x - 6$ by graphing. **(−2, 2); See students' graphs.**

11. **Model with Mathematics** Ahmed decides to get an all-you-can-ride wristband for AED 25. Mohammed wants to use tickets instead. "Remember," Ahmed says to his friend, "each ride uses 2 tickets."

a. The equation $y = 0.71x$ represents the total cost y of x tickets at the rate of 7 tickets for AED 5. The equation $y = 25$ represents the cost of a wristband. Graph each equation on the same coordinate plane.



b. How many rides must each person ride for the total costs to be about the same? **18 rides**

H.O.T. Problems Higher Order Thinking

12. **Persevere with Problems** One equation in a system of equations is $y = 2x + 1$.

a. Write a second equation so that the system has (1, 3) as its only solution. **Sample answer: $y = -x + 4$**

b. Write an equation so that the system has no solution.
Sample answer: $y = 2x - 1$

c. Write an equation so that the system has infinitely many solutions.
Sample answer: $y - 2x = 1$

13. **Persevere with Problems** Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

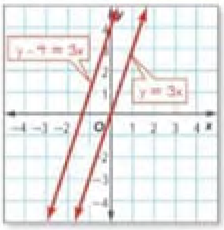
If the system $y = ax + b$ and $y = cx + d$ has exactly one solution, then $b = d$.

sometimes; Sample answer: $y = 2x + 1$ and $y = 5x + 1$ intersect at (0, 1) and $b = d$. However, $y = 2x + 1$ and $y = x + 2$ intersect at (1, 3), but $b \neq d$.

Extra Practice

Solve each system of equations by graphing.

14. $y = 3x$
$y - 4 = 3x$ no solution



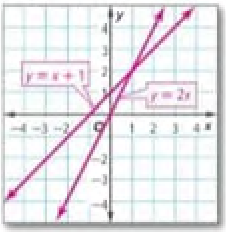
Write $y - 4 = 3x$ in slope-intercept form.

$y - 4 = 3x$
$y - 4 + 4 = 3x + 4$
$y = 3x + 4$

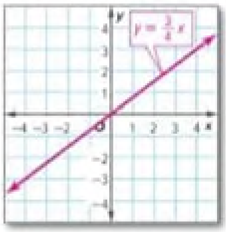
Graph the equations $y = 3x$ and $y = 3x + 4$ on the same coordinate plane.

The lines appear to be parallel, so there is no solution for this system of equations.

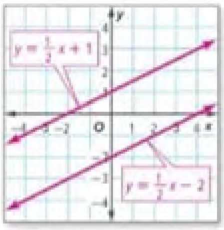
15. $y = 2x$
$y = x + 1$ (1, 2)



16. $y = \frac{3}{4}x$
$3x - 4y = 0$ an infinite number of solutions



17. $y = \frac{1}{2}x + 1$
$y = \frac{1}{2}x - 2$ no solution



Identify Structure Determine if each of the following systems of equations has *no solution*, *one solution*, or *an infinite number of solutions*. If there is a solution, find the solution. If not, explain why not.

18. $2x + 3y = 6$
$2x + 3y = 7$ no solution

Sample answer: Since $2x + 3y$ cannot simultaneously be 6 and 7, there is no solution.

19. $x + y = -2$
$y = x + 2$ (−2, 0)

one solution

20. $x + y = -3$
$2x + y = 1$ (4, −7)

one solution

Lesson 7 Solve Systems of Eq

r Up! Test Practice

and 22 prepare students for more rigorous
eded for the assessment.

item requires students to reason abstractly and ively when problem solving.	
Knowledge	DOK2
atical Practices	MP1, MP6
Rubric	
	Students correctly answer each part of the question.

item requires students to support their reasoning or the reasoning of others by justifying their response and ting arguments.	
Knowledge	DOK3
atical Practices	MP1, MP4, MP4
Rubric	
	Students correctly graph the system of equations, find the solution, and explain what the solution represents.
	Students correctly graph the system of equations OR students find the correct solution and explain what the solution represents.

Power Up! Test Practice

21. Determine if each system of equations has no solution, one solution, or infinitely many solutions. Select the correct answer.

a. $y = 3x - 1$
$y = -2x + 4$
☐ no solution ■ one solution ☐ infinitely many solutions

b. $y = 4x - 2$
$y = 4x + 5$
■ no solution ☐ one solution ☐ infinitely many solutions

c. $y = -x - 3$
$y = x$
☐ no solution ■ one solution ☐ infinitely many solutions

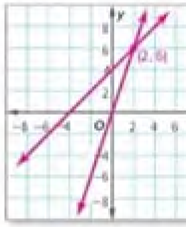
22. Haitham scored 3 times as many goals as Suleiman. Suleiman scored 4 fewer goals than Haitham. The number of goals scored by each person can be represented by the system of equations below.

$y = 3x$

$y = x + 4$

Graph these equations on the coordinate plane.

What is the solution to the system of equations? What does this situation represent?



(2, 6); Xander scored 2 goals, and Yolanda scored 6 goals.

Spiral Review

Solve.

23. $5x + 3y = 15$ for y when $x = 0$.
$y = 5$

24. $6x - 2y = 10$ for y when $x = 2$.
$y = 1$

25. $\frac{1}{2}x + 3y = 4$ for x when $y = 6$.
$x = -28$

26. $\frac{3}{4}x + 3y = 12$ for x when $y = 5$.
$x = -4$

27. $7x - 4y = 20$ for y when $x = 3$.
$y = \frac{1}{4}$

28. $7x - 4y = 20$ for y when $x = 5$.
$y = \frac{15}{4}$

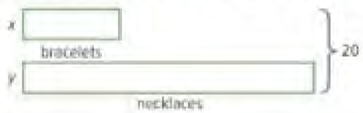
Lesson 8

Solve Systems of Equations Algebraically

Real-World Link

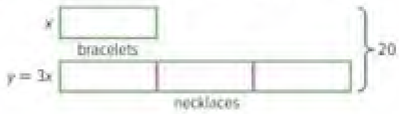
Jewelry Marwa sold 20 necklaces and bracelets at the craft fair. She sold 3 times as many necklaces as bracelets.

Step 1 The bar diagram below represents the situation.



An equation to represent the bar diagram is $x + y = 20$.

Step 2 Marwa sold 3 times as many necklaces as bracelets. Divide the necklace bar into sections to represent this.



Write an equation using only x to represent the total number of necklaces and bracelets.

$x + 3x = 20$ or $4x = 20$

Step 3 Solve the equation from Step 2. What does the solution represent? $x = 5$; Marwa sold 5 bracelets.

1. How many bracelets and necklaces did Marwa sell?
5 bracelets and 15 necklaces

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Essential Question

WHY are graphs helpful?

Vocabulary

substitution

MP **Mathematical Practices**

1, 3, 4, 7

Focus narrowing the scope

Objective Solve systems of equations algebr

Coherence connecting within and acro

Previous
Students found the solution to a system of equations by graphing.

Now
Students solve systems of equations using substitution.

Rigor pursuing concepts, fluency, and ap

See the Levels of Complexity chart on page 2

ENGAGE EXPLORE EXPLAIN ELABORATE E

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a w group, think-pair-share activity, or independe

AL LA **Teammates Consult** Hav in pairs. Before they begin the Real them complete the following activity. Write t $5x = x - 6$ on the board. Provide the followi

- Divide both sides by 5.
- Add 6 to both sides.
- Subtract x from each side.
- Divide both sides by 4.

Have students place the steps in the proper solve the equation. Be sure to tell students t steps below are correct. Then have pairs so Choose one student from each group to pre responses to the class. MP 1, 2, 4, 5, 7, 8

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

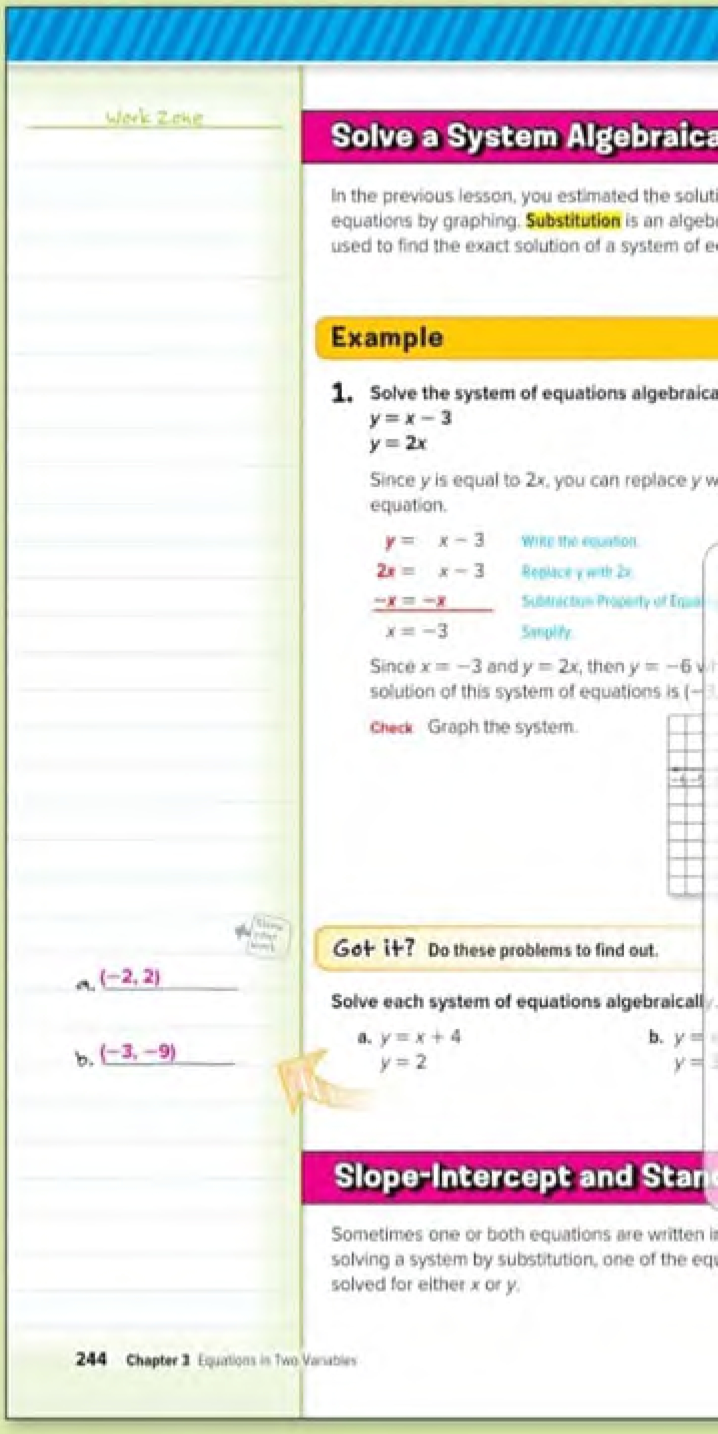
1. Solve a system of equations algebraically.

AL
- *What does the word "substitute" mean?* Sample answer: replace one thing with something equal.
- *What two expressions are equal to y?* $x - 3$ and $2x$

OL
- *Since y is equal to both $x - 3$ and $2x$, what can you conclude about the relationship between $x - 3$ and $2x$?* They are equal.
- *How can you use this fact to solve the system?* Sample answer: Replace y with $2x$ in the first equation or replace y with $x - 3$ in the second equation.

BL
- *When you solve for x, is this the solution of the system?* No; this is only the x-coordinate of the solution.
- *How would you find the y-coordinate of the solution?* Replace x with -3 in one of the equations and solve for y.

Need Another Example?
Solve the system $y = x + 15$ and $y = 4x$ algebraically. (5, 20)


Work Zone

Solve a System Algebraica

In the previous lesson, you estimated the soluti
equations by graphing. **Substitution** is an algeb
used to find the exact solution of a system of e

Example

1. Solve the system of equations algebraica
$y = x - 3$
$y = 2x$

Since y is equal to $2x$, you can replace y w
equation.

$y = x - 3$ Write the equation.
$2x = x - 3$ Replace y with 2x.
$-x = -x$ Subtraction Property of Equa
$x = -3$ Simplify.

Since $x = -3$ and $y = 2x$, then $y = -6$ w
solution of this system of equations is (−

Check Graph the system.

Got it? Do these problems to find out.

Solve each system of equations algebraicall

a. $y = x + 4$ b. y =
$y = 2$ y =

a. (−2, 2)
b. (−3, −9)

Slope-Intercept and Stan

Sometimes one or both equations are written i
solving a system by substitution, one of the equ
solved for either x or y.

244 Chapter 3 Equations in Two Variables

ystem of equations algebraically.

12

$8x + 4y =$	12	Write the equation.
$4(3x + 8) =$	12	Replace y with 3x + 8.
$x + 4 \cdot 8 =$	12	Distributive Property
$12x + 32 =$	12	Simplify.
$20x + 32 =$	12	Collect like terms.
$20x + 32 =$	12	
$-32 = -32$		Subtraction Property of Equality
$20x = -20$		Simplify.
$\frac{20x}{20} = \frac{-20}{20}$		Division Property of Equality
$x = -1$		Simplify.

-1, replace x with −1 in the equation $y = 3x + 8$ to ue of y.

8 or 5

n of this system is (−1, 5).

Substitution

When you replace a variable with an expression, write the expression inside parentheses. This will help you apply the Distributive Property correctly.

hese problems to find out.

26

d. $2x + 5y = 44$
$y = 6x - 4$

c. (2, 5)

d. (2, 8)

ples

okies and cakes were donated for a bake sale to the football team. There were four times as many d as cakes.

tem of equations to represent this situation.

diagram. Then write the system.

cakes				
ookies	cookies	cookies	cookies	} 75

There were 4 times as many cookies donated as cakes.

The total number of cakes and cookies is 75.

Examples

2. Solve a system of equations.

AL • *What does y equal in the first equation?* $3x + 8$

• *Can you replace y with 3x + 8 in the second equation?* yes

OL • *After substitution, what is the new equation?* $8x + 4(3x + 8) = 12$

• *What is the first step in solving this equation?* Use the Distributive Property to rewrite $4(3x + 8)$ as $12x + 32$.

• *Once you solve for x, what is the next step?* Find the value for y by replacing x with −1 in one of the equations.

BL • *After 3x + 8 is substituted for y in the second equation, why are parentheses used?* In the second equation y is multiplied by 4. When you substitute $3x + 8$ for y, you need to use parentheses so the expression is multiplied by 4.

Need Another Example?

Solve the system $y = 4x - 3$ and $3x + 2y = 38$ algebraically. (4, 13)

3. Write a system of equations.

AL • *What do you need to do?* Write two different equations that will represent the situation.

• *What does x represent?* the number of cakes donated

• *What does y represent?* the number of cookies donated

OL • *How many total items were donated?* 75

• *What equation can be used to represent this?* Let $x =$ the number of cakes and $y =$ the number of cookies; $x + y = 75$

• *What equation can be used to represent the relationship between the number of cookies and the number of cakes?* Let $x =$ the number of cakes and $y =$ the number of cookies; $y = 4x$

BL • *Suppose there was an equal number of cakes and pies donated, and four times as many cookies donated as cakes. Write a system of three equations if a total of 90 items were donated.* Let $x =$ the number of cakes, $y =$ the number of cookies, $z =$ the number of pies; $y = 4x$; $x + y + z = 90$; $z = x$

Example

4. Solve a system of equations algebraically.

AL
- *What is the system of equations you need to solve?* $y = 4x, x + y = 75$
- *What can you replace y with in the second equation?* $4x$

OL
- *After substitution, what is the new equation?* $x + 4x = 75$
- *What is the solution of the system?* (15, 60)
- *What does the solution mean?* There were 15 cakes and 60 cookies donated to the bake sale.

BL
- *Suppose there was an equal number of cakes and pies donated, and four times as many cookies donated as cakes. Solve a system to find how many of each item was donated if 90 total items were donated.* 15 cakes, 15 pies, 60 cookies

Need Another Example?
A store sold 84 black and gray T-shirts one weekend. They sold 5 times as many black T-shirts as gray T-shirts. Write and solve a system of equations to represent this situation. Then interpret the solution. Sample answer: $g + b = 84$, $b = 5g$; (70, 14); The store sold 70 black and 14 gray T-shirts.

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Consult** Have student work in pairs to solve Exercises 1–4. One student should solve Exercise 1 algebraically while the partner solves using graphing. Then they should compare their answers and discuss any differences. Students switch roles for Exercise 2 and so on. MP 1, 2, 7, 8

BL LA **Pairs Discussion** Have student work in pairs to compare and contrast solving a system of equations graphically and solving a system of equations algebraically. Then they should compile a list of pros and cons for each method. MP 1, 3

STOP and Reflect

Write a system of equations that has the solution (1, 7).

Sample answer: $y = x + 6$ and $y = 8x - 1$

4. **Solve the system in Example 3 algebraically. Interpret the solution.**

Since y is equal to $4x$, you can replace y with $4x$.

$x + y = 75$ Write the equations

$x + 4x = 75$ Replace y with 4x

$5x = 75$ Simplify

$\frac{5x}{5} = \frac{75}{5}$ Division Property of Equality

$x = 15$ Simplify

Since $x = 15$ and $y = 4x$, then $y = 60$ when $x = 15$. The solution is (15, 60). This means that 15 cakes and 60 cookies were donated.

Got It? **Do these problems to find out.**

Mr. Sami cooked 45 beefburgers and cheeseburgers at a cookout. He cooked twice as many cheeseburgers as beefburgers.

e. Write a system of equations to represent this situation.

f. Solve the system algebraically. Interpret the solution.

e. Sample answer: $y = 2x$; $x + y = 45$

f. Sample answer: (15, 30); This means that he cooked 15 beefburgers and 30 cheeseburgers.

Guided Practice

Solve each system of equations algebraically. (Examples 1 and 4)

1. $y = x + 7$, $y = 4$ (−3, 4)
2. $y = x + 5$, $y = 3x$ (2.5, 7.5)
3. $y = x - 9$, $y = -4x$ (1.8, −7.2)
4. $x + 3y = 1$, $y = 2x + 5$ (−2, 1)

5. Seven people went to the movies. The number of adults was one more than the number of children. Write a system of equations that represents the number of adults and children. Solve the system algebraically. Interpret the solution. (Examples 3 and 4)
Sample answer: $y = x + 1$; $y + x = 7$; (3, 4); Three children and four adults went to the movies.

6. **Building on the Essential Question** How can you solve a system of equations? Sample answer: I can solve system of equations algebraically and by graphing the equations on the same coordinate plane.

Rate Yourself!

How confident are you about solving systems of equations algebraically? Check the box that applies.

FOLDABLES Time to update your Foldable!

Name ______ My Homework ______

Independent Practice

Solve each system of equations algebraically. (Examples 1 and 2)

1. $y = x + 5$
   $y = 6$
   (1, 6)

2. $y = x + 12$
   $y = -18$
   (−30, −18)

3. $y = x - 10$
   $y = -12$
   (−2, −12)

4. $y = x + 15$
   $y = 2x$
   (15, 30)

5. $y = 2x - 3$
   $x + y = 18$
   (7, 11)

6. $y = \frac{1}{4}x$
   $x + 4y = 8$
   (4, 1)

7. $y = x + 12$
   $4x + 2y = 27$
   $\left(\frac{1}{2}, 12\frac{1}{2}\right)$ or (0.5, 12.5)

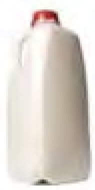
8. $10x + 3y = 19$
   $y = 2x + 5$
   $\left(\frac{1}{4}, 5\frac{1}{2}\right)$ or (0.25, 5.5)

Write and solve a system of equations that represents each situation. Use a bar diagram if needed. Interpret the solution. (Examples 3 and 4)

9. Youmna bought a total of 15 books and pens. She bought 7 more books than pens. How many of each did she buy?
   Sample answer: $s + p = 15$; $p + 7 = s$; (4, 11); She bought 11 books and 4 pens.

10. Together, Bilal and Hilal have 49 video games. Hilal has 11 more games than Bilal. How many games does each person have?
    Sample answer: $p + h = 49$; $h = p + 11$; (19, 30); Bilal has 19 games and Hilal has 30 games.

11. The cost of 8 muffins and 2 liters of milk is AED 18. The cost of 3 muffins and 1 liter of milk is AED 7.50. How much does 1 muffin and 1 liter of milk cost?
    Sample answer: $8x + 2y = 18$; $3x + y = 7.50$; (1.5, 3); A muffin costs AED 1.50 and 1 liter of milk costs AED 3.



ENGAGE EXPLORE EXPLAIN ELABORATE E

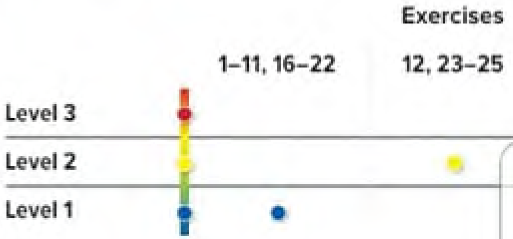
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant t homework assignment. The Extra Practice pa for additional reinforcement or as a second-d

Levels of Complexity

The levels of the exercises progress from 1 to indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes e complexity levels to select appropriate exer students' needs.

Differentiated Homework Opt		
AL	Approaching Level	1–11, 14, 15, 24, 25
OL	On Level	1–11 odd, 12, 14, 15, 2
BL	Beyond Level	12–15, 24, 25

CAL PRACTICES	
Emphasis On	Exercise(s)
problems and persevere in	13
e arguments and critique the hers.	12, 15
hematics.	23
ake use of structure.	14

actices 1, 3, and 4 are aspects of mathematical emphasized in every lesson. Students are ties to be persistent in their problem solving, to asoning, and apply mathematics to real-world

ssment
as a closing formative assessment before ents from your class.

solve the system of equations $x + \frac{1}{2}y = 15$ algebraically. **(8, 14)**

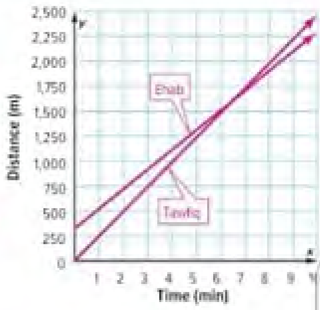
12. **Multiple Representations** The table shows the rates at which Ehab and Tawfiq are biking along the same trail.

Person	Rate (m/min)
Ehab	200
Tawfiq	250

a. **Algebra** Suppose Ehab began the trail 325 meters ahead of Tawfiq. Write a system of equations to represent the distance y each person will travel after any number of minutes x. **Ehab: $y = 200x + 325$; Tawfiq: $y = 250x$**

b. **Words** Which person was farther along the trail after 5 minutes? **Ehab**

c. **Graphs** Graph the system. Use the graph to determine when Tawfiq will catch up to Ehab. **Tawfiq catches up with Ehab after biking between 6 and 7 minutes and traveling about 1,600 meters.**



d. **Algebra** Solve the system of equations algebraically. Interpret your solution. How does your solution compare to your estimate in part **c**? **(6.5, 1,625); Tawfiq catches up with Ehab after biking 6.5 minutes and traveling 1,625 meters. Sample answer: The solution to part c was an estimate which is close to the exact solution found in part d.**

H.O.T. Problems Higher Order Thinking

13. **Persevere with Problems** What is the solution to the system $5x + y = 2$ and $y = -5x + 8$? Explain. **Ø; Sample answer: Adding $5x$ to each side of $y = -5x + 8$ results in the equation $5x + y = 8$. Since $5x + y$ cannot equal both 8 and 2, there are no values for x and y that make this system of equations true.**

14. **Identify Structure** Describe when it is better to use substitution to solve a system of equations rather than graphing. **Sample answer: When the equations are complex and cannot be easily graphed, or when the solution involves numbers that are not integers.**

15. **Which One Doesn't Belong?** Circle the system of equations that does not belong with the other three. Explain your reasoning.

$y = 3x - 5$ $y = -2x$	$y = 5x - 7$ $y = 2(2x - 3)$	$y = x + 3$ $y = -2x - 3$	$y = -2x$ $y = -2(3x - 2)$

Sample answer: The solution of $y = x + 3$ and $y = -2x - 3$ is $(-2, 1)$. The solution of the other three systems is $(1, -2)$.

Name ______________________ My Homework ______________________

Extra Practice

Solve each system of equations algebraically.

16. $y = 2x$

$y = x + 1$ (1, 2)

$y = x + 1$
$2x = x + 1$
$-x = -x$
$x = 1$
Since $x = 1$ and $y = 2x$, $y = 2$.

17. $y = 4x + 45$

$x = 4y$ **(−12, −3)**

18. $y = -2x$

$x = 0$ **(0, 0)**

19. $x + y = -3$

$y = x + 3$ **(−3, 0)**

20. $y = x + 4$

$y = 0$ **(−4, 0)**

21. $x - y = 6$

$y = -1$ **(5, −1)**

22. The length of the rectangle is 3 meters more than the width. The perimeter is 26 meters. Write and solve a system of equations that represents this situation. What are the dimensions of the rectangle?

Sample answer: $\ell = 3 + w$; $2\ell + 2w = 26$; (8, 5); The width is 5 meters and the length is 8 meters.

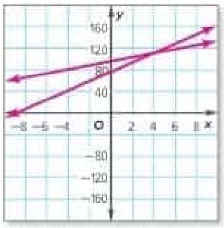
23. MP **Model with Mathematics** Ms. Kawthar wants to take her class on a trip to either the nature center or the zoo. The nature center charges AED 4 per student plus AED 95 for a 1-hour naturalist program. The zoo charges AED 9 per student plus AED 75 for a 1-hour guided tour.

a. Write a system of equations to represent this situation.

$y = 4x + 95$ and $y = 9x + 75$

b. Solve the system of equations algebraically and by graphing. Interpret the solution.

(4, 111); the costs, AED 111, are the same if 4 students attend either.



c. Ms. Kuaser has 22 students in her class. Determine which would cost less, the nature center or the zoo.

nature center

ower Up! Test Practice

cises 24 and 25 prepare students for more rigorous
ing needed for the assessment.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly model the situation, find the system of equations, and solve.
1 point	Students correctly model the situation OR find the system of equations and solve.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

24. A veterinarian examined 4 times as many dogs as cats today. In all, he examined 15 dogs and cats. Let *d* represent the number of dogs and *c* represent the number of cats that the veterinarian examined.

Use the labels to complete the bar diagram that models the situation.

c	cats				15
d	dogs	dogs	dogs	dogs	

Use the bar diagram to set up and solve a system of equations. How many dogs and cats did the veterinarian examine today?

$d = 4c$, $d + c = 15$; 3 cats and 12 dogs

25. In one volleyball game, Isam had 3 times as many spikes as Ismail. Together, they had 20 spikes. How many spikes did each player have? Write and solve a system of equations.

$n = 3v$, $n + v = 20$; Isam: 15 spikes, Ismail: 5 spikes

Spiral Review

Solve.

26. $p - 12 = 20$ **32**

27. $31 = r - 36$ **67**

28. $m + 1\frac{3}{8} = 5$ **$3\frac{5}{8}$**

29. $56.9 = 34 + p$ **22.9**

30. $0.97 + a = 2.6$ **1.63**

31. $x - 24 = 73$ **97**

32. $t + 5 = 30$ **25**

33. $r - 15 = 63$ **78**

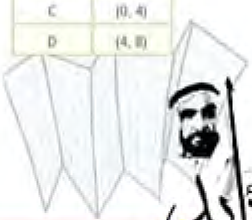
Inquiry Lab
Analyze Systems of Equations

Inquiry **HOW can you solve real-world mathematical problems using two linear equations in two variables?**

MP Mathematical Practices 1, 3, 5

A map uses a coordinate grid to show the locations of cities and towns. The map locations for four towns are shown in the table. Suppose Sami travels from Town A to Town B and Maryam travels from Town C to Town D. Do Sami's and Maryam's routes pass through a common location?

Town	Location
A	(0, 6)
B	(5, 1)
C	(0, 4)
D	(4, 8)

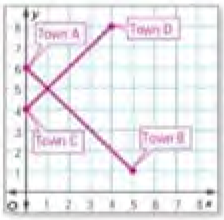


What do you know? **Sami's and Maryam's routes**

What do you need to know? **If Sami's and Maryam's routes pass through a common location.**

Hands-On Activity

Step 1 Plot and label the points of each town on the coordinate plane shown.



Step 2 Draw a red line segment to represent Sami's route and draw a blue line segment to represent Maryam's route.

Step 3 Find the slope of the lines that represent Sami's route and Maryam's route.

Sami's route: $m = -1$; Maryam's route: $m = 1$

What do the slopes tell you about the lines? Explain.

The lines will intersect in exactly one point; Sample answer: Since the slopes are different, the lines are not parallel or the same line so they will intersect in one point.

Step 4 Where do the two lines intersect? **(1, 5)**

So, Sami's and Maryam's routes pass through the common location (1, 5).

Focus narrowing the scope

Objective Solve real-world mathematical probl
systems of linear equations.

Coherence connecting within and across

Now
Students use technology to analyze systems of linear equations.

Next
Students will apply sytems of linear eq linear inequalities.

Rigor pursuing concepts, fluency, and app

See the Levels of Complexity chart on page 25

ENGAGE EXPLORE EXPLAIN ELABORATE E

1 Launch the Lab

The activity is intended to be used as a whole

Hands-On Activity

AL LA **Roundrobin** Have students work in
to complete the activity. Have each student c
of Steps 1–4. Students show agreement (thum
disagreement (thumbs down) after each stude
a step. If there is any disagreement, students
resolve it. MP 1, 4, 5, 6, 7, 8

BL LA **Pairs Discussion** Have students e
by placing Town E at the point (0, 7) and Town
(4, 5). Tell them that a third person travels from
Town F. Ask them to determine if that route will
a common location with either of Brent's or Ma
MP 1, 4, 5, 6, 7, 8

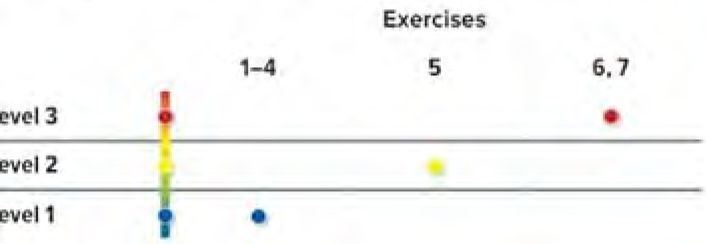
ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

Collaborate

he **Investigate** and **Analyze and Reflect** sections are ntended to be used as small-group investigations. The **Create** ection is intended to be used as independent exercises.

evels of Complexity

he levels of the exercises progress from 1 to 3, with Level 1 ndicating the lowest level of complexity.



Investigate

AL BL LA **Pair-Solo** Have students work with partners o write the equations for Exercise 1 and verify that the solution (1, 5) is a solution to both equations. Then have them individually omplete Exercises 3 and 4. Upon completion, have them iscuss their solutions with their partner. Have one pair of tudents volunteer to share their responses to Exercises 1–4 vith the class. MP 1, 2, 5, 7, 8

Create

BL LA **Pairs Discussion** For Exercise 6, have students vork in pairs to generate three different pairs of simultaneous near equations and their graphs that intersect at right angles o justify their conjecture. MP 1, 3, 4, 7, 8

Inquiry Students should be able to answer "HOW can you olve real-world mathematical problems using two linear quations in two variables?" Check for student understanding nd provide guidance, if needed.

Investigate

Refer to the Activity. Work with a partner.

1. Write an equation for the lines that represent Sami's routes and Maryam's routes.

   Sami's route: $y = -x + 6$ Maryam's route: $y = x + 4$

2. Solve the system of equations from Exercise 1 algebraically. (1, 5)

MP **Use Math Tools Write an equation for the line that passes through each pair of points. Use a graphing calculator to solve the system. Then describe the slope of each pair of lines.**

3. (0, −1) and (4, 3); (2, 1) and (0, 3)

   Equations: $y = x - 1$; $y = -x + 3$

   Solution: (2, 1); The slope of one line goes up from left to right and the slope of the other line goes down from left to right.

4. (0, 3) and (3, 9); (0, 2) and (3, 8)

   Equations: $y = 2x + 3$; $2x + 2$

   Solution: no solution; The slopes are the same so the lines are parallel and will not intersect.

Analyze and Reflect

5. MP **Reason Inductively** How can you determine if two lines will intersect using the slope? Sample answer: If the slopes of the lines are not the same, then the lines intersect.

Create

6. MP **Model with Mathematics** When two lines intersect to form a right angle, the slope of one is the opposite reciprocal of the slope of the other. Write a system of equations that form a right angle. Sample answer: $y = 2x$, $y = -\frac{1}{2}x$

7. Inquiry HOW can you solve real-world mathematical problems using two linear equations in two variables? Sample answer: You can graph the two equations to determine the point of intersection, or you can write the equations as a system and solve it algebraically.

21ST CENTURY CAREER in Music

Mastering Engineer

Do you love listening to music? Are you interested in the technical aspects of music-making? If so, a career creating digital masters might be something to think about! A mastering engineer produces digital masters and is responsible for making songs sound better, having the proper spacing between songs, removing extra noises, and assuring all the songs have consistent levels of tone and balance. Having a great-sounding master helps increase radio airplay and sales for recording artists.

Is This the Career for You?

Are you interested in a career as a mastering engineer? Take some of the following courses in high school.

- Algebra
- Music Appreciation
- Recording Techniques
- Sound Engineering

Turn the page to find out how math relates to a career in Music.

Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace.

This lesson emphasizes **MP Mathematical Practice 4** Model with Mathematics.

Coherence connecting within and across grades

Previous
Students determined if a relationship was linear and proportional.

Now
Students apply the content standard to solve problems in the workplace.

Rigor pursuing concepts, fluency, and applications

See the Career Project on page 254.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ask students to read the information on the student page about mastering engineers and answer the following questions.

Ask:

- *What do mastering engineers do?* **Sample answer: produce digital masters and makes songs sound better by removing extra noises**
- *What kinds of classes should you take if you want to be a mastering engineer?* **Algebra, Music Appreciation, Recording Techniques and Sound Engineering**

عام زايد 2018

Fakhr Alwatan فخر الوطن

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** EVALUATE

2 Collaborate

AL LA Numbered Heads Together Assign students to 3- or 4-person learning teams. Have each team answer the following questions, making sure that every member understands. Call on a specific number from each team to speak to the class about the team's solution. MP 1, 2, 3, 4, 7, 8

Ask:

- *How do you know if a relationship is linear?* **Sample answer: If the rate of change is constant, the relationship is linear.**
- *At a different studio, the cost of mastering 3 tracks is AED 250.50 and the cost of mastering 8 tracks is AED 668.00. What is the slope of the line containing the points (number of tracks, cost)? What does the slope represent?* **83.5; The cost per song is AED 83.50.**

BL LA Trade-a-Problem Have students work in pairs and write a real-world problem that could be represented by a system of equations. Use the information that a song produced at Dynamic Master costs AED 60 and a song at Mastering Mix costs AED 75. Then, have students trade their problems and solve.
MP 1, 2, 3, 4, 7, 8

Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

Career Facts

A current trend is to make CDs increasingly louder. To be competitive, engineers must artificially increase the sound levels throughout songs. The result is that songs do not have the same variations in loudness as before. Many artists are trying to reverse this trend to preserve the integrity of their music.

Mastering the Music

Use the information in the tables to solve each problem.

1. At Engineering Hits, is the relationship between the number of songs and the cost linear? Explain your reasoning. **No; the rate of change from 1 to 2 songs, $\frac{160-100}{2-1}$ or AED 60/song, is not the same as the rate of change from 2 to 3 songs, or $\frac{210-160}{3-2}$ or AED 50/song.**
2. Is there a proportional linear relationship between number of songs and cost at Dynamic Mastering? Explain your reasoning. **Yes; as the number of songs increases by 2, the cost increases by AED 120. The rate of change is constant, $\frac{60}{1}$, so this is a linear relationship. The ratios $\frac{\text{cost}}{\text{number of songs}}$ are all $\frac{60}{1}$, so the cost is proportional to the number of songs.**
3. Find the slope of the line represented in the Mastering Mix table. What does the slope represent? **75; As the number of songs increases by 1, the cost increases by AED 75.**
4. Is the linear relationship represented in the Mastering Mix table a direct variation? Explain. **No; the ratios $\frac{\text{cost}}{\text{number of songs}}$ are not the same for each pair of quantities.**
5. Write a direct variation equation to represent number of songs x and cost y at Dynamic Mastering. How much does it cost to master 11 songs? **$y = 60x$; AED 660**
6. For 4 or more songs at Engineering Hits, the cost varies directly as the number of songs. How much does it cost to master 6 songs? **AED 375**

Engineering Hits	
Number of Songs	Cost (AED)
1	100
2	160
3	210
4	250

Dynamic Mastering	
Number of Songs	Cost (AED)
2	120
4	240
6	360
8	480

Mastering Mix	
Number of Songs	Cost (AED)
1	125
3	275
5	425
7	575

Career Project

It's time to update your career portfolio! Find the name of the mastering engineer on one of your CDs. Use the Internet or another source to write a short biography of this engineer. Include a list of other artists whose songs he or she has mastered.

Do you think you would enjoy a career as a mastering engineer? Why or why not?

Chapter Review

Vocabulary Check

Complete the crossword puzzle using the vocabulary list at the beginning of the chapter.

Across

4. a value to describe the steepness of a straight line
5. a relationship in which the ratio of two variable quantities is constant
6. the horizontal change between the same two points
7. the *x*-coordinate of the point where the graph crosses the *x*-axis

Down

1. an algebraic model used to find the exact solution of a system of equations
2. the vertical change between any two points
3. the *y*-coordinate of the point where the line crosses the *y*-axis
4. when an equation is written in the form $Ax + By = C$

Vocabulary Check

LA **Round Table Consensus** Place students in groups of 3 or 4. In turns, each student orally gives the answer to one of the crossword puzzle clues. Team members show agreement or disagreement by giving the thumbs up or thumbs down sign. If there is any disagreement, students discuss and resolve until there is a consensus. MP 1, 3, 5, 6

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- direct variation (Lesson 3)
- rise (Lesson 2)
- run (Lesson 2)
- slope (Lesson 2)
- standard form (Lesson 5)
- substitution (Lesson 8)
- *x*-intercept (Lesson 5)
- *y*-intercept (Lesson 4)

Key Concept Check

FOLDABLES **LA** A completed Foldable for this chapter should include a review of the solutions of a system of linear equations.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

Ideas for Use

LA **Team Presentation** Have students works in pairs to discuss their Foldables. Have them practice speaking in a collaborative setting by sharing how they have completed their Foldable thus far and how they could finish it. Have each student complete their Foldable and trade with their partner to discuss any similarities and differences. MP 4, 5

Got It?

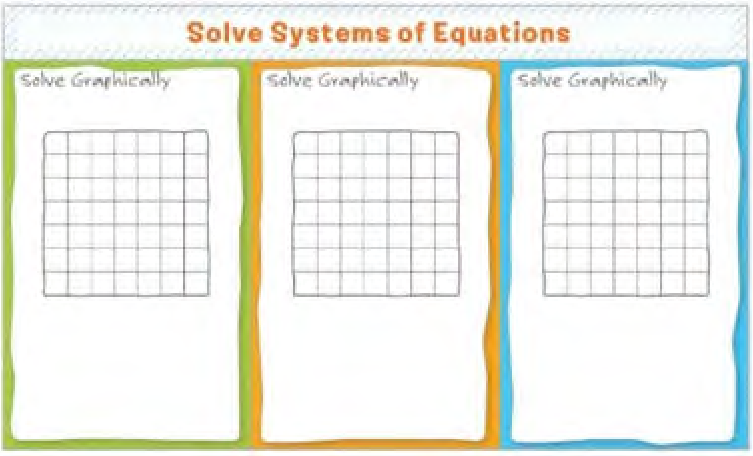
If students have trouble with Exercises 1–5, they may need help with the following concept(s).

Concept	Exercise(s)
write an equation from intercepts (Lesson 6)	1
write an equation from slope and y-intercept (Lesson 4)	2, 5
write an equation from two points (Lesson 6)	3
write an equation from slope and one point (Lesson 6)	4

Key Concept Check

Use Your FOLDABLES

Use your Foldable to help review the chapter.



Got it?

Match each set of information with the correct linear equation.

1. line that passes through (2, 0) and (0, 1)
2. line with a slope of 0.5 and a y-intercept of 1
3. line that passes through (4, 2) and the origin
4. line that has a slope of 0 and passes through (5, 4)
5. line that has an undefined slope and passes through (5, 4)

a. $y = 0.5x$
b. $x = 5$
c. $y = 0.5x + 1$
d. $y = -0.5x + 1$
e. $y = 4$

Expressions and Equations

Power Up! Performance Task

Play for a Prize

Ahmed likes to go to the arcade. Tickets are awarded in different ways for his favorite games.

Game #1	Game #2
two tickets per game PLUS one ticket for every sixty points scored	one ticket for every forty points scored

- The ticket dispenser awards the customer tickets based on the point total.
- For each game, x represents the number of points scored and y represents the number of tickets awarded.

Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A
Write an equation to model the number of tickets earned for each game. Interpret the slope and y-intercept for each situation.

Part B
Is either relationship proportional? Explain your reasoning.

Part C
Solve the system of equations algebraically. Interpret the solution.

Part D
Suppose Ahmed needs 10 more tickets to win a prize. Which game should he play? Explain your reasoning.

Power Up! Performance Task

This Performance-Based Assessment requires students to solve multi-step problems through abstract reasoning, precision, and perseverance. This practice scenario can be used to help students prepare for the thinking skills that will be used on the Assessment.

A complete scoring rubric with answers to the Exercises can be found at the back of the book.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- How can you use a table to determine if there is a proportional relationship between two quantities? (p. 174)
- In any linear relationship, explain why the slope is always the same. (p. 184)
- What is the relationship among the unit rate, slope, and constant rate of change of a proportional linear relationship? (p. 194)
- How does the *y*-intercept appear in these three representations: table, equation, and graph? (p. 202)
- How can the *x*-intercept and *y*-intercept be used to graph a linear equation? (p. 212)
- How does using the point-slope form of a linear equation make it easier to write the equation of a line? (p. 224)
- How can you use a graph to solve a system of equations? (p. 238)
- How can you solve a system of equations? (p. 246)

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their classmate before they complete the graphic organizer.

MP 1, 5

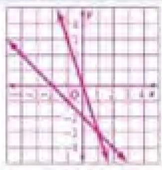
Reflect

Answering the Essential Question

Use what you learned about graphs to complete the graphic organizer. List three ways in which graphs are helpful. Then give an example for each way.
Sample answers are given.

Essential Question

WHY are graphs helpful?

You can find the slope of a line by choosing any two points on the line and applying the slope formula.	You can find the *y*-intercept by determining where the line crosses the *y*-axis.	You can solve a system of equations by graphing.
Example: Find the slope of the line that passes through the points (1, 5) and (3, −3); $-\frac{8}{2}$ or −4	Example: Find the *y*-intercept of the graph of the line that contains (−2, −1) and (2, 7); (0, 3)	Example: $y = -x - 2$ $y = -3x$ 

Answer the Essential Question. WHY are graphs helpful?

See students' work.

PROJECT 2

Web Design 101 When designing a good Web page, there are many details to consider in order to make your Web page stand out. In this project, you will:

- **Collaborate** with your classmates as you research an animal and design a Web page.
- **Share** the results of your research in a creative way.
- **Reflect** on how you communicate mathematical ideas effectively.

By the end of this Project, you will be ready to design a live Web page about your favorite animal!

Collaborate

Go Online Work with your group to research and complete each activity. You will use your results in the Share section on the following page.

1. Choose your favorite animal. Research information about that animal, such as the population over the past 10 years, the kinds of food it eats, sleeping habits, its average lifespan, average size, and average speed. Present this information using tables and graphs.
2. Use the distance formula, distance = rate × time, to write an equation that represents the distance your animal can travel at its average speed. Find the average speed of two other animals and write equations using the distance formula. Graph all three equations on the same coordinate plane. Then describe the graphs.
3. Research the elements needed to make a good Web page. Then make a sketch of your own Web page about the favorite animal that you selected in Exercise 1. Be sure to include tables, equations, graphs, and photos.
4. Find another animal in the same animal kingdom as your favorite animal. On the sketch of your Web page, include a link to this other animal and an equation that describes one of its characteristics.
5. Research the cost of taking a Web design class. Write an equation that represents the time it will take you to save enough money for the class. Share this equation as you write a few paragraphs that explain your plan on how to save enough money.

Launch the Project

Objective Research an animal and design a Web page about the animal.

Web Design 101

This project is designed to be completed by a grou of 4 or 5 students over several days or several weeks. It utilizes concepts from the Expressions an Equations domain. You may choose to complete th project after completing the chapters within this domain.

Collaborate

Have students work in teams to research informati about their favorite animal and Web design. Together, they should be able to gather the necessary information to complete Exercises 1–5. Students should show their work on a separate piece of paper.

Share

Collaborate

After each group gives their presentation, have students write one fact that they learned about each of the animals presented.

21st Century Skills

You may want your students to connect their projects to a 21st century skill. Check out the suggestion below and on the student page.

connect with Language Arts

Global Literacy Write a brochure describing the habitat of your favorite animal. Some questions to consider are:

- In which country or countries is the animal found?
- What types of terrain and climate does the animal's habitat have?

Reflect

On Your Own

Students should work on their own to reflect on how the chapters from this unit and the objective of the project relate to the Essential Question.

Share

Collaborate

With your group, decide on a way to share what you have learned about your animal and Web pages. Some suggestions are listed below, but you can also think of other creative ways to present your information. Remember to show how you used mathematics to complete each of the activities in this project!

- If possible, use Web page creation software to turn your design into a live Web page.
- Imagine you are going to be interviewed by a reporter about your work on this project. Write down what will be discussed in the interview. You may wish to actually record an interview.

Check out the note on the right to connect this project with other subjects.

connect with Economics

Business Literacy Research Web design jobs in your area. Find out the following:

- What type of education is required?
- What skills should a Web designer possess?

Reflect

On Your Own

6. **Answer the Essential Question** HOW can you communicate mathematical ideas effectively? **See students' work.**

   a. How did you use what you learned in the Equations in One Variable chapter to communicate mathematical ideas effectively in this project?

   b. How did you use what you learned in the Equations in Two Variables chapter to communicate mathematical ideas effectively in this project?