UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

عـام المجتـمع
YEAR OF COMMUNITY

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2031

2025-2026

Inspire Physics

UAE Edition
Grade 10 Advanced
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Physics

United Arab Emirates Edition

Grade 10 Advanced

McGraw Hill

MINISTRY OF EDUCATION

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Physics
Author names: Paul W. Zitzewitz, PhD, David G. Haase, PhD, Kathleen A. Harper, PhD

Source ISBN-13 9780021353163
Source ISBN-10 0021353166

Published by McGraw-Hill Education (UK) Limited
Unit 4, Foundation Park,
Roxborough Way,
Maidenhead SL6 3UD
T: +44 (0) 1628 502500
Website: www.mheducation.co.uk

Published by arrangement with McGraw Hill LLC

ISBN 978-1-39-896265-1
ISBN 1-39-896265-1

Contents


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MODULE 13
VIBRATIONS AND WAVES

ENCOUNTER THE PHENOMENON

How can this pendulum save a building from earthquake damage?

GO ONLINE to play a video about how dampers are used to prevent earthquake damage.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

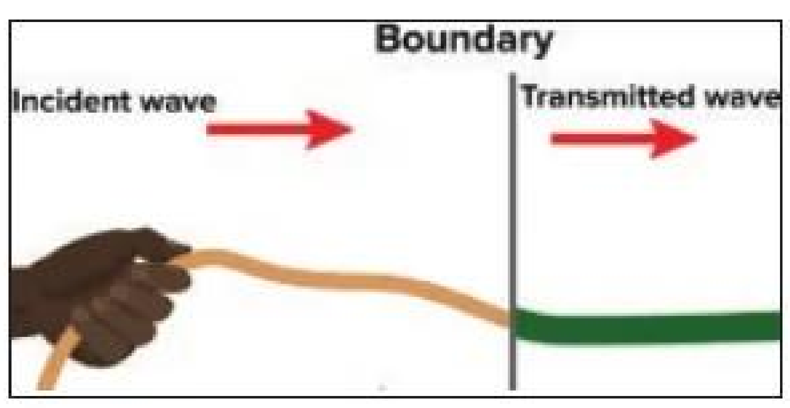
Make Your Claim Use your CER chart to make a claim about how a pendulum could save a building from earth-quake damage. Explain your reasoning.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.

LESSON 1: Explore & Explain: Pendulums



LESSON 2: Explore & Explain: Waves at Boundaries

Smithsonian

Additional Resources

LESSON 1
PERIODIC MOTION

FOCUS QUESTION
What are some types of repetitive motion?

Mass on a Spring

The bobbing of a mass on a spring or the swaying of a pendulum are examples of **periodic motion.** In each example, at one position the net force on the object is zero and the object is in equilibrium. When the object moves away from its equilibrium position, the net force on the system becomes nonzero. This net force acts to bring the object back toward equilibrium. The **period** (T) is the time needed for one full cycle of the motion. The **amplitude** of the motion is the maximum distance the object moves from the equilibrium position.

Simple harmonic motion In **Figure 1,** the force exerted by the spring is directly proportional to the distance it is stretched. When pulled down and released, the mass bobs up and down through equilibrium. Any system where the force acting to restore an object to its equilibrium position is directly proportional to the object's displacement shows **simple harmonic motion.**

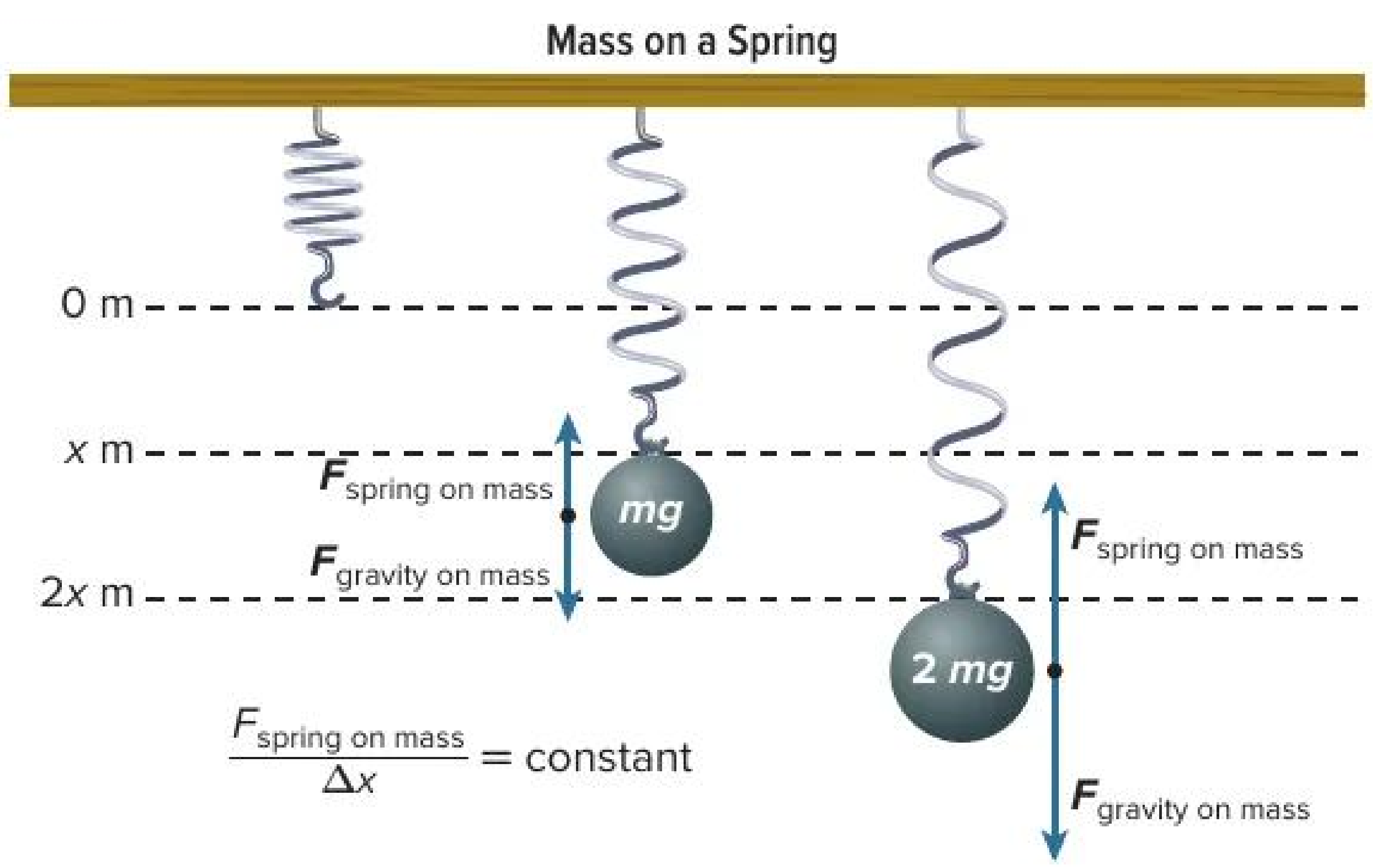


Figure 1 The force exerted on the mass by the spring is directly proportional to the mass's displacement.

Determine *the displacement if the mass is 0.5 mg.*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE
Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE
GO ONLINE to find these activities and more resources.

PhysicsLAB: Pendulum Vibrations
Plan and carry out an investigation to determine what variables affect a pendulum's period and use the resulting data to calculate the magnitude of the gravitation field.

Revisit the Encounter the Phenomenon Question
What information from this lesson can help you answer the Unit and Module questions?

Hooke's Law

Table 1 Force Magnitude-Stretch Distance in a Spring

Stretch Distance (m)	Magnitude of Force Exerted by Spring (N)
0.0	0.0
0.030	1.9
0.060	3.7
0.090	6.3
0.12	7.8

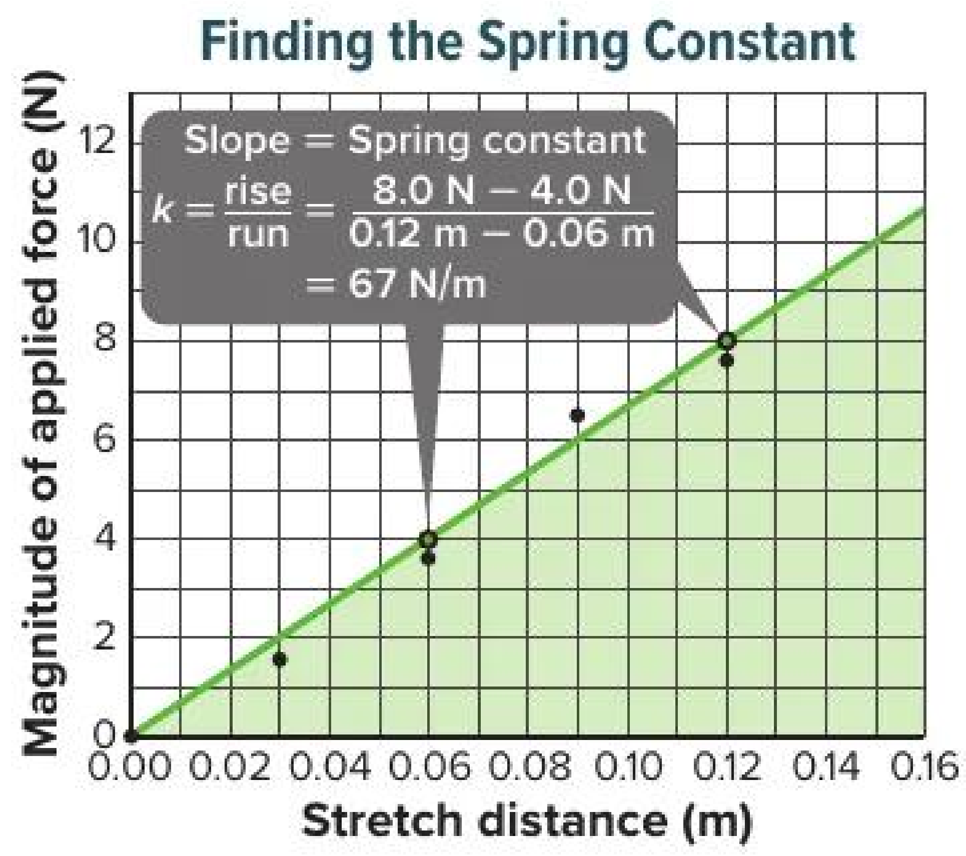


Figure 2 The spring constant can be determined from the slope of the force magnitude-stretch distance graph. The area under the curve is equal to the potential energy stored in the spring.

Hooke's Law

Table 1 shows the relationship between the magnitude of the force exerted by a spring and the distance the spring stretches. **Figure 2** is a graph of the data with the line of best fit. The linear relationship indicates that the magnitude of the force exerted by the spring is directly proportional to the amount the spring is stretched. A spring that exerts a force directly proportional to the distance stretched obeys **Hooke's law.**

Hooke's Law

The magnitude of the force exerted by a spring is equal to the spring constant times the distance the spring is stretched or compressed from its equilibrium position.

$$F = -kx$$

In this equation, k is the spring constant, which depends on the stiffness and other properties of the spring, and x is the distance the spring is stretched from its equilibrium position. Notice that k is the slope of the line in the magnitude of the force v. stretch distance graph. A steeper slope—a larger k—indicates that the spring is harder to stretch. The constant k has the same units as the slope, newtons/meter (N/m). The negative sign in Hooke's law indicates that the force is in the direction opposite the stretch or compression direction. The force exerted by the spring on the mass is always directed toward the spring's equilibrium position.

Hooke's law and real springs Not all springs obey Hooke's law. For example, rubber bands do not. Those that do obey Hooke's law are called elastic springs. Even for elastic springs, Hooke's law only applies over a limited range of distances. If a spring is stretched too far, it can become so deformed that the force is no longer proportional to the displacement.

Potential energy When you stretch a spring you transfer energy to the spring, giving it elastic potential energy. The work done by an applied force is equal to the area under a force v. distance graph like the one shown in **Figure 2.** This work is equal to the elastic potential energy stored in the spring. To calculate this stored energy, find the area of the triangle by multiplying one-half the base of the triangle, which is x, by the height of the triangle. According to Hooke's law, the height of the triangle—the magnitude of the force—is equal to kx.

Potential Energy in a Spring

The potential energy in a spring is equal to one-half times the product of the spring constant and the square of the displacement.

$$PE_{spring} = -\frac{1}{2}kx^2$$

This mathematical expression, which quantifies how the stored energy in a spring depends on its configuration, together with kinetic energy calculated from mass and speed data, allows the concept of the conservation of energy to be used to predict and describe system behavior. As shown in **Figure 3** on the next page, during horizontal simple harmonic motion the spring's elastic potential energy is converted to kinetic energy and then back to potential energy.

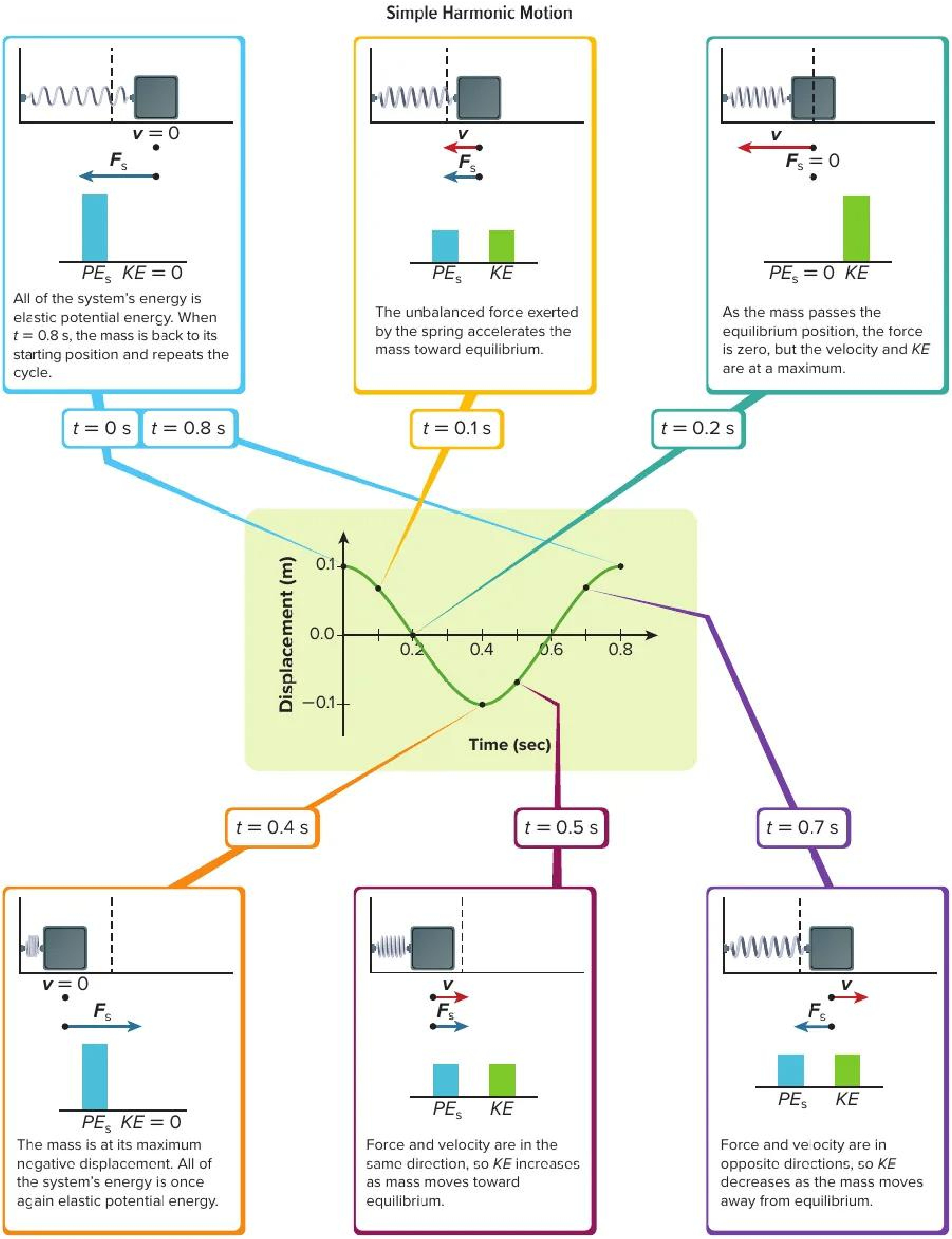


Figure 3 The total mechanical energy of the system is constant throughout the oscillation.

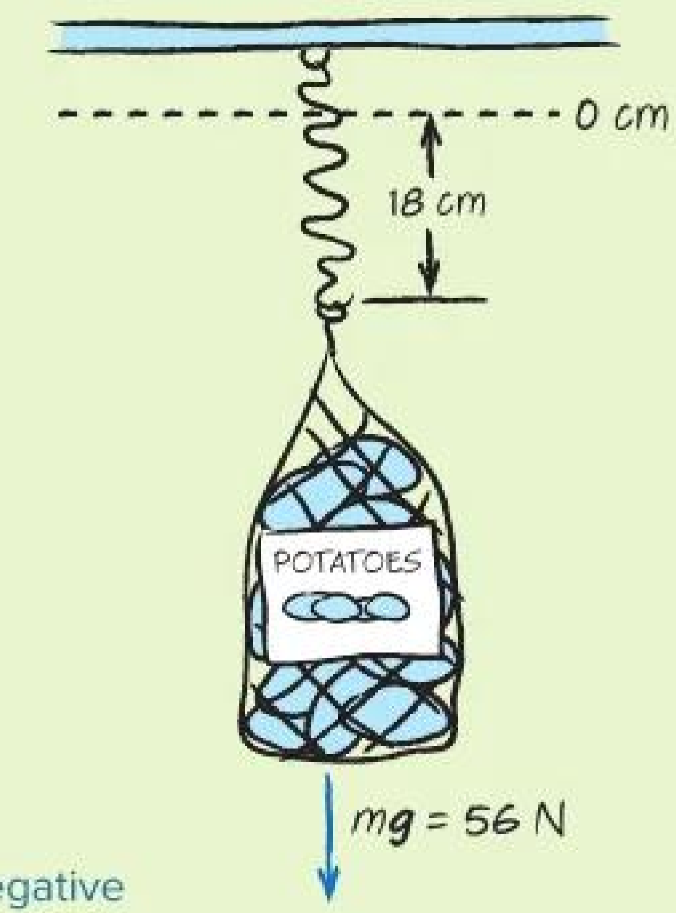
Explain *How can Hooke's law and the concept of conservation of energy be used to predict and describe the behavior of a system such as this?*

EXAMPLE Problem 1

THE SPRING CONSTANT AND THE ENERGY OF A SPRING A spring stretches by 18 cm when a bag of potatoes weighing 56 N is suspended from its end.

a. Determine the spring constant.

b. How much elastic potential energy does the stretched spring have?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Show and label the distance the spring has stretched and its equilibrium position.

Known	Unknown
$x = 18$ cm	$k = ?$
$F = -56$ N	$PE_s = ?$

2 SOLVE FOR THE UNKNOWN

a. Use Hooke's law and isolate k.

$$k = -\frac{F}{x}$$

$$= -\frac{-56\text{ N}}{0.18\text{ m}}$$ Substitute $F = -56$ N, $x = 0.18$ m. The force is negative because it is in the opposite direction of x.

$$= 310\text{ N/m}$$

b. $PE_s = \frac{1}{2}kx^2$

$$= \frac{1}{2}(310\text{ N/m})(0.18\text{ m})^2$$ Substitute $k = 310$ N/m, $x = 0.18$ m.

$$= 5.0\text{ J}$$

3 EVALUATE THE ANSWER

- **Are the magnitudes realistic?** N/m is the correct unit for the spring constant. (N/m)(m^2) = N•m = J, which is the correct unit for energy.
- **Is the magnitude realistic?** The average magnitude of the force the spring exerts is the average of 0 and 56 N. The work done is $W = Fx = (28\text{ N})(0.18\text{ m}) = 5.0$ J.

PRACTICE Problems

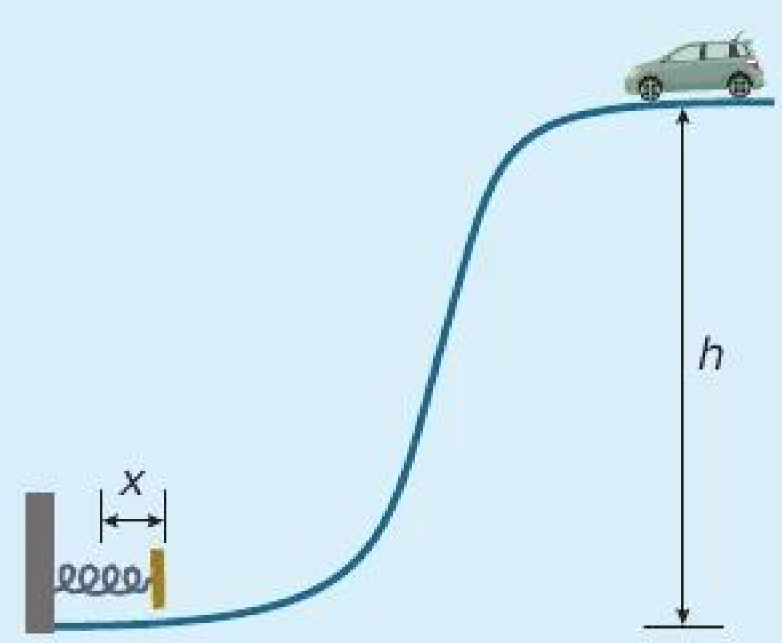
ADDITIONAL PRACTICE

1. What is the spring constant of a spring that stretches 12 cm when an object weighing 24 N is hung from it?
2. What is the elastic potential energy of a spring with $k = 144$ N/m that is compressed by 16.5 cm?
3. A spring has a spring constant of 56 N/m. How far will a block weighing 18 N stretch it?
4. **CHALLENGE** A spring has a spring constant of 256 N/m. How far must it be stretched to give it an elastic potential energy of 48 J?

PHYSICS Challenge

A car of mass m rests at the top of a hill of height h before rolling without friction into a crash barrier located at the bottom of the hill. The crash barrier contains a spring with a spring constant k, which is designed to bring the car to rest with minimum damage.

1. Determine, in terms of m, h, k, and g, the maximum distance (x) the spring will be compressed when the car hits it.
2. If the car rolls down a hill that is twice as high, by what factor will the spring compression increase?
3. What will happen after the car has been brought to rest?



Pendulums

Simple harmonic motion also occurs in the swing of a pendulum. A **simple pendulum** consists of a massive object, called the bob, suspended by a string or a light rod of length ℓ. The bob swings back and forth, as shown in **Figure 4.** The string or rod exerts a tension force (F_T), and gravity exerts a force (F_g) on the bob. Throughout the pendulum's path, the component of the gravitational force in the direction of the pendulum's circular path is a restoring force. At the left and right positions, the restoring force is at a maximum and the velocity is zero. At the equilibrium position, the restoring force is zero and the velocity is maximum.

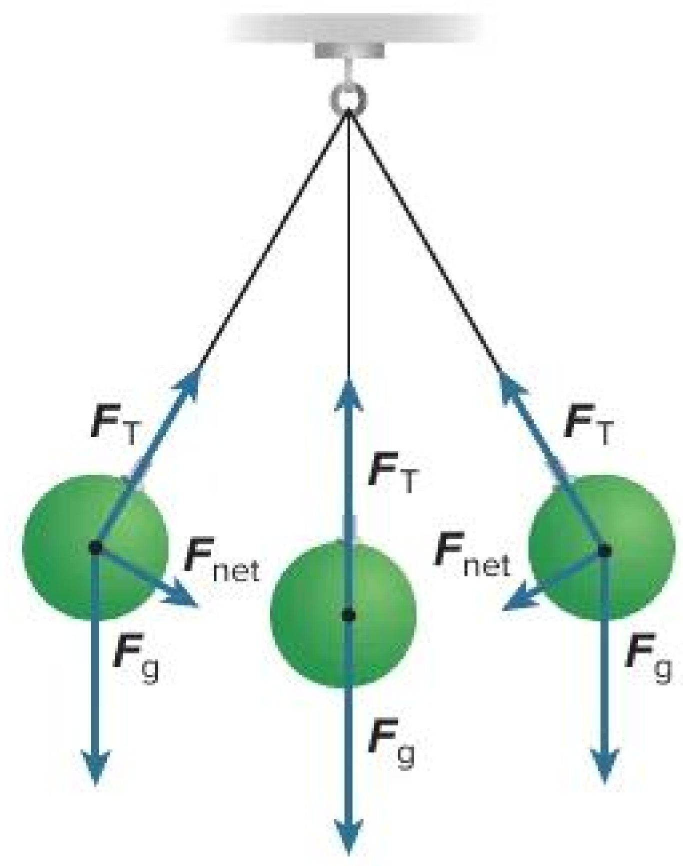


Figure 4 The pendulum's motion is an example of simple harmonic motion because the restoring force is directly proportional to the displacement from equilibrium.

For small angles (less than about 15°), the restoring force is proportional to the displacement from equilibrium. Similar to the motion of the mass on a spring discussed earlier, the motion of the pendulum is simple harmonic motion. The period of a pendulum is given by the following equation.

Period of a Pendulum

The period of a pendulum is equal to 2π times the square root of the length of the pendulum divided by the gravitational field.

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Notice that the period depends only on the length of the pendulum and the gravitational field, not on the mass of the bob or the amplitude of oscillation. One practical use of the pendulum is to measure g, which can vary slightly at different locations on Earth.

Get It?

Compare the period of a very massive pendulum, like the one shown at the beginning of the module, with the period of a pendulum with the same length but a tiny mass.

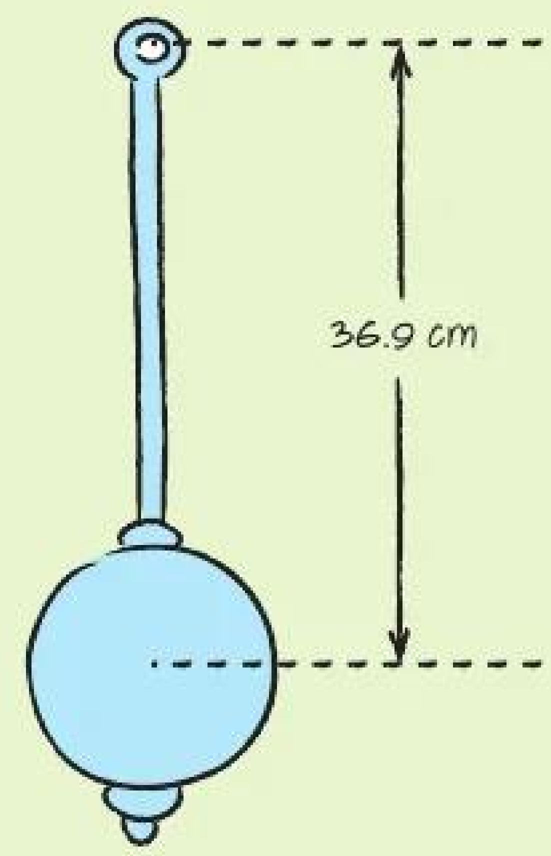
Resonance

To get a playground swing going, you can "pump" it by leaning back and pulling the chains at the same point in each swing. Another option is to have a friend give you repeated pushes at just the right times. **Resonance** occurs when forces are applied to a vibrating or oscillating object at time intervals equal to the period of oscillation. As a result, the amplitude of the vibration increases. Other familiar examples of resonance include rocking a car to free it from a snow bank and jumping rhythmically on a trampoline or a diving board to go higher.

Resonance in simple harmonic motion systems causes a larger and larger displacement as energy is added in small increments. As a child you may have been told to hold a seashell such as a conch up to your ear to "hear the sound of the ocean." The sound you hear when you hold a seashell or other similar-shaped object up to your ear actually comes from resonance. Sound waves resulting from background noise in the room interact with the seashell. Sounds with frequencies matching one of the natural frequencies at which the seashell vibrates result in resonance, and the sound becomes amplified and loud enough to hear. The large amplitude oscillations caused by resonance can also produce useful results. Resonance is used in musical instruments to amplify sounds and in clocks to increase accuracy.

EXAMPLE Problem 2

FINDING *g* USING A PENDULUM A pendulum with a length of 36.9 cm has a period of 1.22 s. What is the gravitational field at the pendulum's location?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Label the length of the pendulum.

Known	Unknown
$\ell = 36.9$ cm	$g = ?$
$T = 1.22$ s	

2 SOLVE FOR THE UNKNOWN

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$ Solve for g.

$$g = (2\pi)^2\frac{\ell}{T^2}$$

$$= \frac{4\pi^2(0.369\text{ m})}{(1.22\text{ s})^2}$$ Substitute $l = 0.369$ m, $T = 1.22$ s.

$$= 9.78\text{ m/s}^2 = 9.78\text{ N/kg}$$

3 EVALUATE THE ANSWER

- **Are the magnitudes realistic?** N/kg is the correct unit for gravitational field.
- **Is the magnitude realistic?** The calculated value of g is quite close to the accepted value of g, 9.8 N/kg. This pendulum could be at a higher elevation above sea level.

PRACTICE Problems ADDITIONAL PRACTICE

5. What is the period on Earth of a pendulum with a length of 1.0 m?

6. How long must a pendulum be on the Moon, where $g = 1.6$ N/kg, to have a period of 2.0 s?

7. CHALLENGE On a certain planet, the period of a 0.75-m-long pendulum is 1.8 s. What is g for this planet?

Check Your Progress

8. **Periodic Motion** Explain why a pendulum is an example of periodic motion.

9. **Energy of a Spring** The springs shown in **Figure 5** are identical. Contrast the potential energies of the bottom two springs.

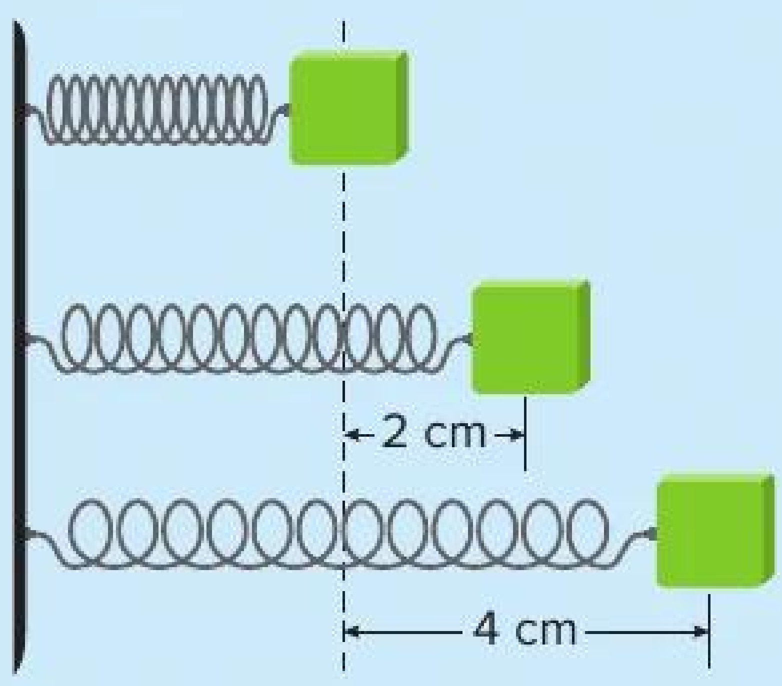


Figure 5

10. **Hooke's Law** Objects of various weights are hung from a rubber band that is suspended from a hook. The weights of the objects are plotted on a graph against the stretch of the rubber band. How can you tell from the graph whether the rubber band obeys Hooke's law?

11. **Pendulum** How must the length of a pendulum be changed to double its period? How must the length be changed to halve the period?

12. **Resonance** If a car's wheel is out of balance, the car will shake strongly at a specific speed but not at a higher or lower speed. Explain.

13. **Critical Thinking** How is uniform circular motion similar to simple harmonic motion? How are they different?

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LESSON 2
WAVE PROPERTIES

FOCUS QUESTION

What are some common types of waves?

Mechanical Waves

A **wave** is a disturbance that carries energy through matter or space without transferring matter. You have learned how Newton's laws of motion and the law of conservation of energy govern the behavior of particles. These laws also govern the behavior of waves. Water waves, sound waves, and the waves that travel along a rope or a spring are mechanical waves. Mechanical waves pass through a physical medium, such as water, air, or a rope.

Transverse waves A **wave pulse** is a single bump or disturbance that passes through a medium. In the left panel of **Figure 6,** the wave pulse disturbs the rope in the vertical direction, but the pulse travels horizontally. A wave that disturbs the particles in the medium perpendicular to the direction of the wave's travel is called a **transverse wave.** If the disturbances continue at a constant rate, a **periodic wave** is generated.

Longitudinal waves In a coiled spring toy, you can produce another type of wave. If you squeeze together several turns of the coiled spring toy and then suddenly release them, pulses will move away in both directions. The result is called a **longitudinal wave** because the disturbance is parallel to the direction of the wave's travel. Sound waves are longitudinal waves in which the molecules are alternately compressed or decompressed along the path of the wave.

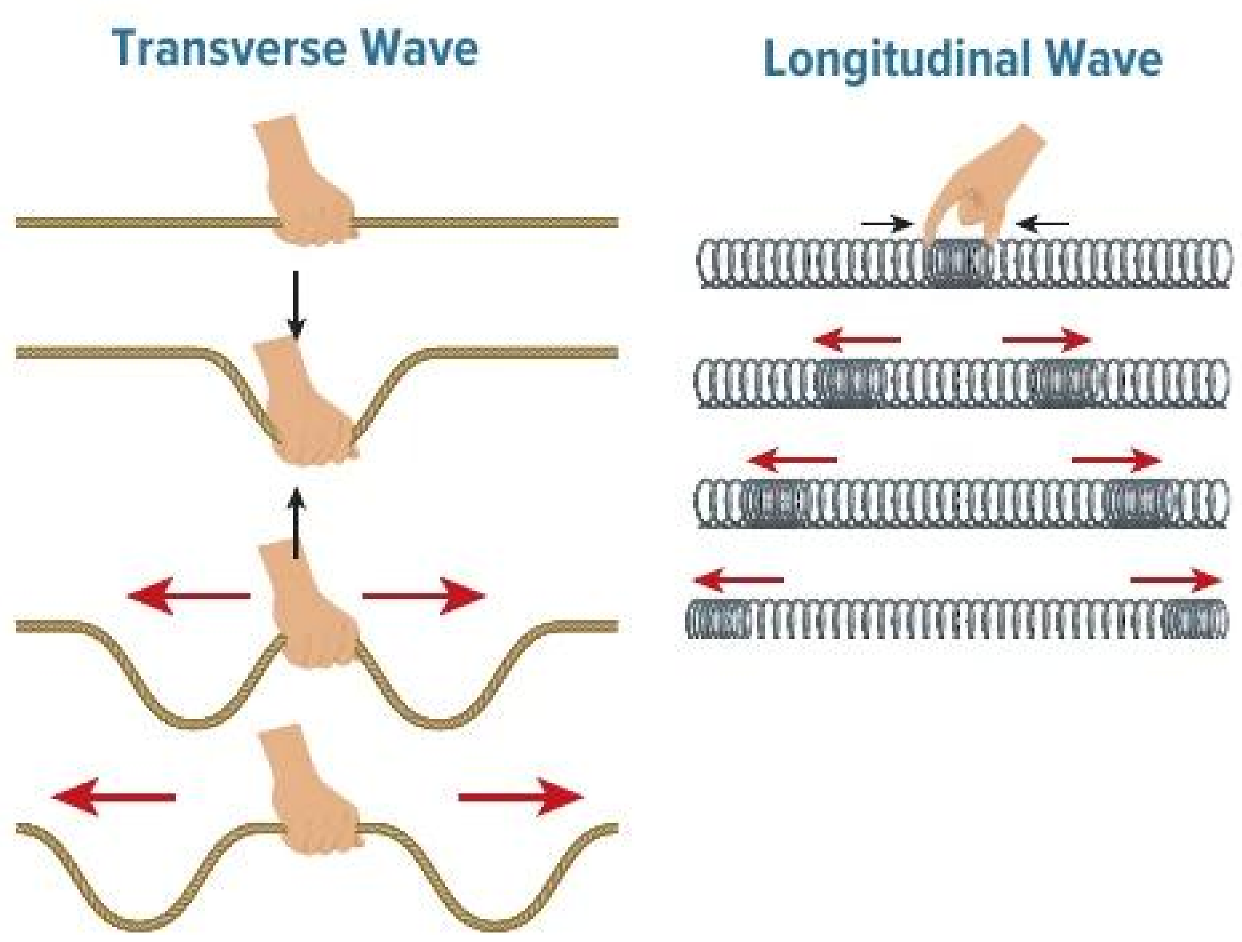


Figure 6 Shaking a rope up and down produces transverse wave pulses traveling in both directions. Squeezing and releasing the coils of a spring produces longitudinal wave pulses in both directions.

Explain *the difference between transverse and longitudinal waves.*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Applying Practices: Wave Characteristics

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

CCC **Identify Crosscutting Concepts**

Create a table of the crosscutting concepts and fill in examples you find as you read.

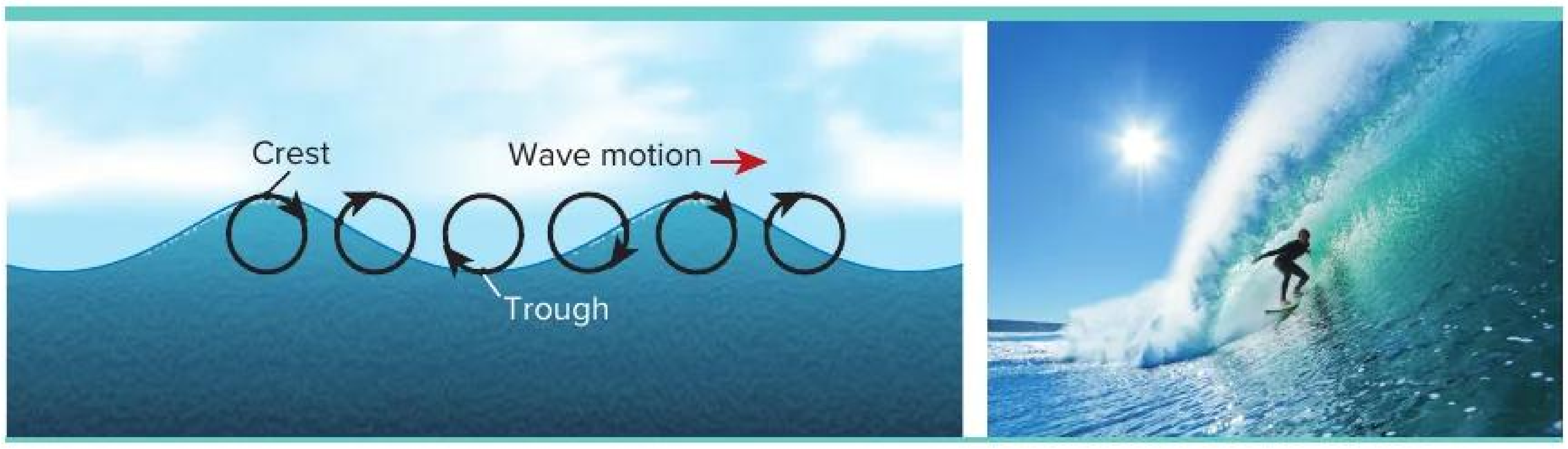


Figure 7 Surface waves in water cause movement both parallel and perpendicular to the direction of wave motion. When these waves interact with the shore, the regular, circular motion is disrupted and the waves break on the beach.

Surface waves Waves that are deep in a lake or an ocean are longitudinal. In a **surface wave,** however, the medium's particles follow a circular path that is at times parallel to the direction of travel and at other times perpendicular to the direction of wave travel, as shown in **Figure 7.** Surface waves set particles in the medium, in this case water, moving in a circular pattern. At the top and bottom of the circular path, particles are moving parallel to the direction of the wave's travel. This is similar to a longitudinal wave. At the left and right sides of each circle, particles are moving up or down. This up-and-down motion is perpendicular to the wave's direction, similar to a transverse wave.

Real-World Physics

Tsunamis

On March 11, 2011, a wall of water estimated to be ten meters high hit areas on the east coast of Japan—tsunami! A tsunami is a series of ocean waves that can have wavelengths over 100 km, periods of one hour, and wave speeds of 500–1000 km/h.

Wave Properties

Many types of waves share a common set of wave properties. Some wave properties depend on how the wave is produced, whereas others depend on the medium through which the wave is passing.

Amplitude How does the pulse generated by gently shaking a rope differ from the pulse produced by a violent shake? The difference is similar to the difference between a ripple in a pond and an ocean breaker—they have different amplitudes. You read earlier that the amplitude of periodic motion is the greatest distance from equilibrium. Similarly, as shown in **Figure 8,** a transverse wave's amplitude is the maximum distance of the wave from equilibrium. Since amplitude is a distance, it is always positive. You will learn more about measuring the amplitude of longitudinal waves when you study sound.

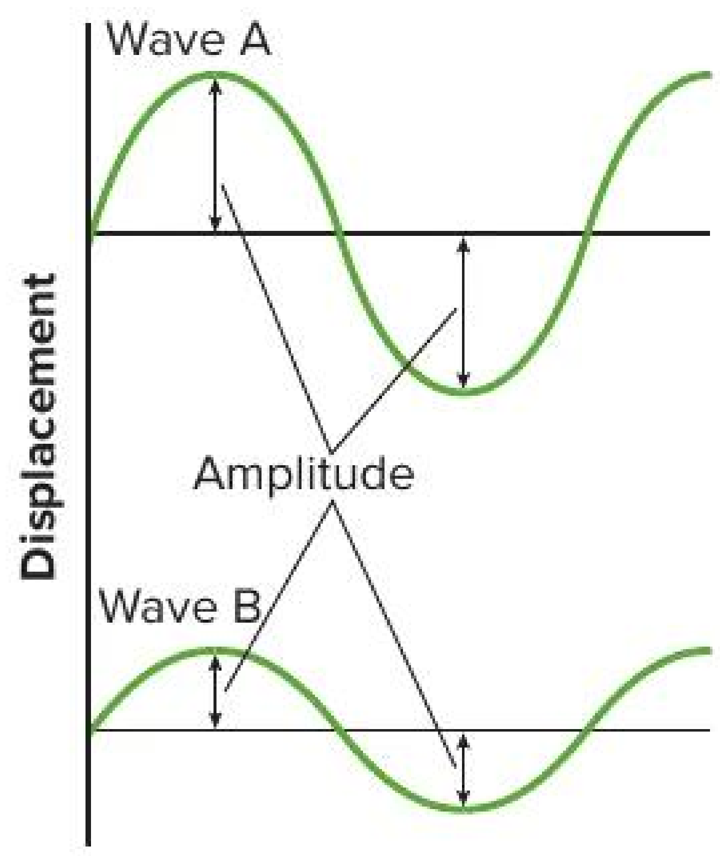


Figure 8 A wave's amplitude is measured from the equilibrium position to the highest or lowest point on the wave.

Energy of a wave Waves, including water waves, are examples of the many ways that energy manifests at the macroscopic scale. The energy of a wave is related to its amplitude, and a wave's amplitude depends on how the wave is generated. More energy must be added to the system to generate a wave with a greater amplitude. For example, strong winds produce larger water waves than those formed by gentle breezes. Waves with greater amplitudes transfer more energy. Whereas a wave with a small amplitude might move sand on a beach a few centimeters, a giant wave can uproot and move a tree.

For waves that move at the same speed, the rate at which energy is transferred is proportional to the square of the amplitude. Thus, doubling the amplitude of a wave increases the amount of energy that wave transfers each second by a factor of four.

Wavelength Rather than focusing on one point on a wave, imagine taking a snapshot of the wave so you can see the whole wave at one instant in time. The top image in **Figure 9** shows each low point on a transverse wave, called a **trough,** and each high point on a transverse wave, called a **crest,** of a wave. The shortest distance between points where the wave pattern repeats itself is called the **wavelength.** Crests are spaced by one wavelength. Each trough also is one wavelength from the next. The Greek letter lambda (λ) represents wavelength.

Speed and velocity How fast does a wave travel? The speed of a wave pulse can be found in the same way the speed of a moving car is determined. First, measure the displacement of one of the wave's crests or compressions (Δd), then divide this by the time interval (Δt) to find the speed.

$$v = \frac{\Delta d}{\Delta t}$$

For most mechanical waves, transverse and longitudinal, except water surface waves, wave speed does not depend on amplitude, frequency, or wavelength. Speed depends only on the medium through which the waves are passing and on the type of wave. For example, sound waves have characteristic speeds in water and in air. The temperature of the medium also makes a difference. Sound waves travel faster in warm, dry air than in cool, dry air.

Get It?
Summarize how changing a wave's amplitude, frequency, or wavelength affects the wave's speed.

Phase Any two points on a wave that are one or more whole wavelengths apart are said to be in phase. Particles in the medium are in phase with one another when they have the same displacement from the equilibrium position and the same velocity. Particles with opposite displacements from the equilibrium position and opposite velocities are 180° out of phase. A crest and a trough, for example, are 180° out of phase with each other. Two particles in a wave medium can be anywhere from 0° to 360° out of phase with each other.

Period Although wave speed and amplitude can describe both wave pulses and periodic waves, period (T) applies only to periodic waves. You have learned that the period of simple harmonic motion, such as the motion of a simple pendulum, is the time it takes for the motion to complete one cycle. Such motion is usually the source, or cause, of a periodic wave. The period of a wave is equal to the period of the source. In **Figure 9** the period (T) equals 0.04 s, which is the time it takes the source to complete one cycle. The same time is taken by P, a point on the rope, to return to its initial position and velocity.

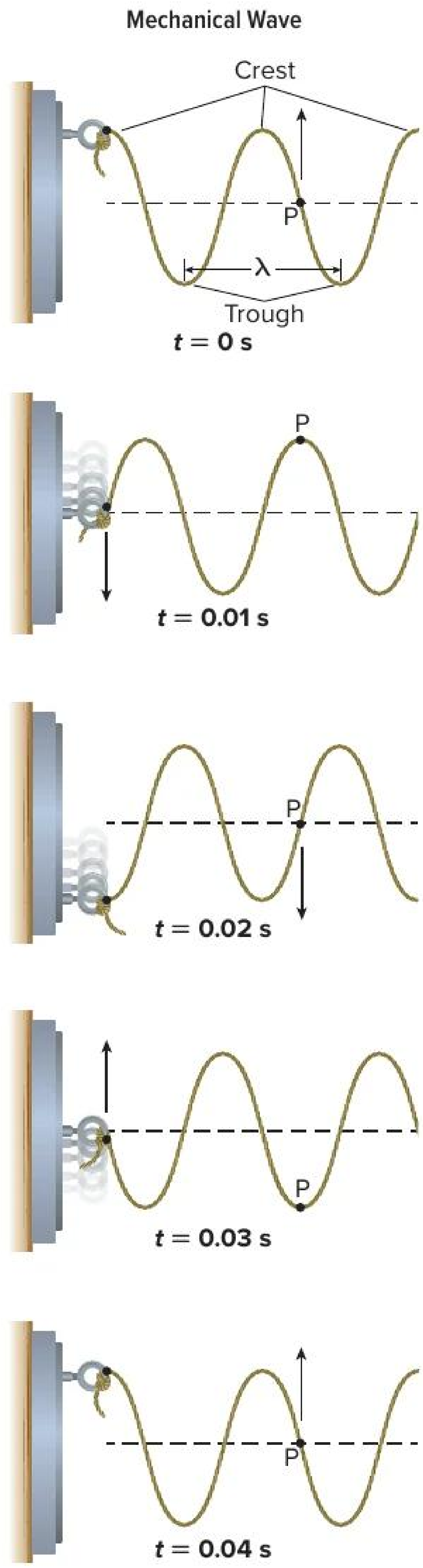


Figure 9 A mechanical oscillator moves the left end of the rope up and down, completing the cycle in 0.04 s.

Calculating frequency The **frequency** of a wave (f) is the number of complete oscillations a point on that wave makes each second. Frequency is measured in hertz (Hz). One hertz is one oscillation per second and is equal to 1/s or s^{-1}. The frequency and the period of a wave are related by the following equation.

Frequency of a Wave

The frequency of a wave is equal to the reciprocal of the period.

$$f = \frac{1}{T}$$

Both the period and the frequency of a wave depend only on the wave's source. They do not depend on the wave's speed or the medium.

Calculating wavelength You can directly measure a wave's wavelength by measuring the distance between adjacent crests or troughs. You can also calculate it because the wavelength and frequency of a wave are related to one another by the speed of travel of the wave, which depends on the type of wave and the medium through which it is passing. In the time interval of one period, a wave moves one wavelength. Therefore, the wavelength of a wave is the speed multiplied by the period, $\lambda = vT$. Using the relation that $f = \frac{1}{T}$, the wavelength equation is very often written in the following way.

Wavelength

The wavelength of a wave is equal to the velocity divided by the frequency.

$$\lambda = \frac{v}{f}$$

Graphing waves If you took a snapshot of a transverse wave on a coiled spring toy, it might look like one of the waves shown in **Figure 9.** This snapshot could be placed on a graph grid to show more information about the wave, as in the left panel of **Figure 10.** Measuring from peak to peak or trough to trough on such a snapshot provides the wavelength. Now consider recording the motion of a single particle, such as point P in **Figure 9.** That motion can be plotted on a displacement v. time graph, as in the graph on the right in **Figure 10.** Measuring from peak to peak or trough to trough in this graph provides the wave's period.

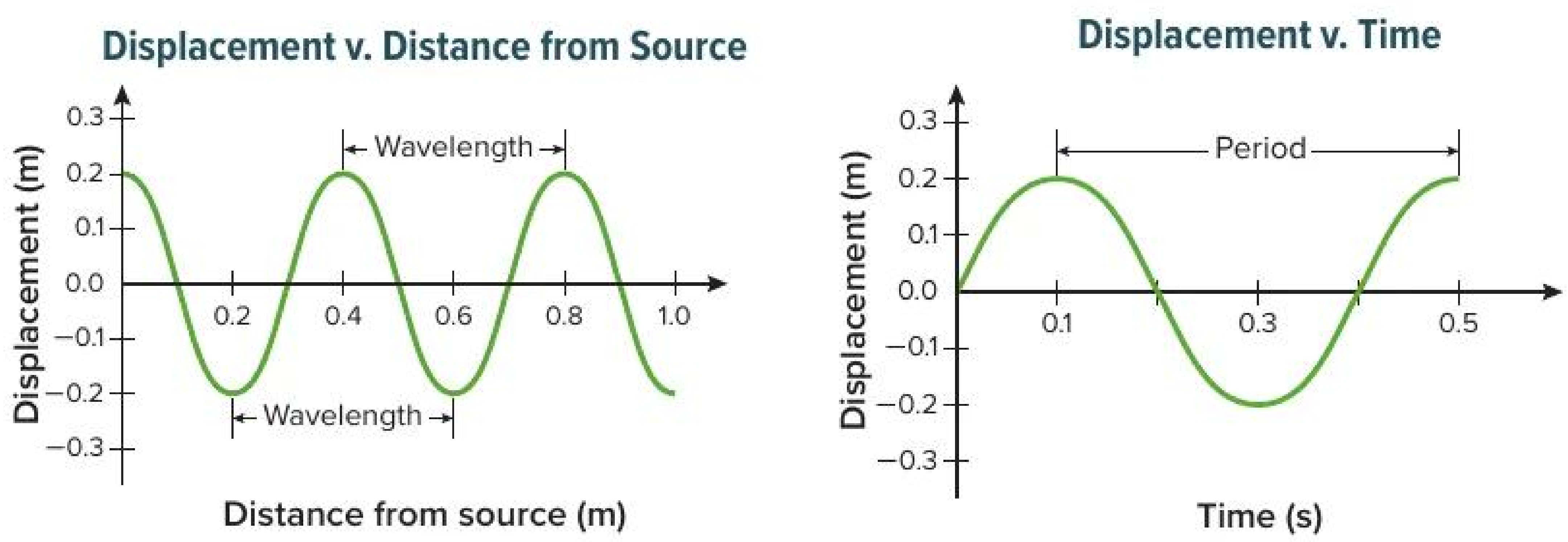


Figure 10 Graphing waves on different axes provides different kinds of information.

Determine *the period of the wave shown in the displacement v. time graph.*

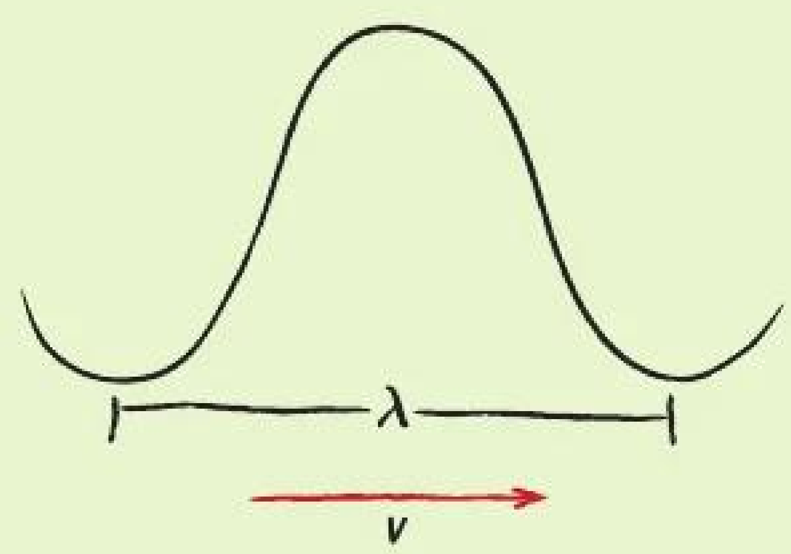
EXAMPLE Problem 3

CHARACTERISTICS OF A WAVE A sound wave has a frequency of 192 Hz and travels the length of a football field, 91.4 m, in 0.271 s.

a. What is the speed of the wave?
b. What is the wavelength of the wave?
c. What is the period of the wave?
d. If the frequency were changed to 442 Hz, what would be the new wavelength and period?

1 ANALYZE AND SKETCH THE PROBLEM

- Draw a diagram of the wave.
- Draw a velocity vector for the wave.



Known	Unknown
$f = 192$ Hz	$v = ?$
$\Delta d = 91.4$ m	$\lambda = ?$
$\Delta t = 0.271$ s	$T = ?$

2 SOLVE FOR THE UNKNOWN

a. Use the definition of velocity.

$$v = \frac{\Delta d}{\Delta t}$$
$$= \frac{91.4\text{ m}}{0.271\text{ s}}$$ Substitute $\Delta d = 91.4$ m, $\Delta t = 0.271$ s
$$= 337\text{ m/s}$$

b. Use the relationship between wave velocity, wavelength, and frequency.

$$\lambda = \frac{v}{f}$$
$$= \frac{337\text{ m/s}}{192\text{ Hz}}$$ Substitute $v = 337$ m/s, $f = 192$ Hz.
$$= 1.76\text{ m}$$

c. Use the relationship between period and frequency.

$$T = \frac{1}{f}$$
$$= \frac{1}{192}\text{ Hz}$$ Substitute $f = 192$ Hz.
$$= 0.00521\text{ s}$$

d. $\lambda = \frac{v}{f}$
$$= \frac{337\text{ m/s}}{442\text{ Hz}}$$ Substitute $v = 337$ m/s, $f = 442$ Hz.
$$= 0.762\text{ m}$$

$$T = \frac{1}{f}$$
$$= \frac{1}{442}\text{ Hz}$$ Substitute $f = 442$ Hz.
$$= 0.00226\text{ s}$$

3 EVALUATE THE ANSWER

- **Are the magnitudes realistic?** Hz has the unit s^{-1}, so $\left(\frac{m}{s}\right)/\text{Hz} = \left(\frac{m}{s}\right)\cdot s = m$, which is correct for λ.
- **Are the magnitudes realistic?** A typical sound wave travels at approximately 340 m/s in air, so 337 m/s is reasonable. The frequencies and periods are reasonable for sound waves. The frequency of 442 Hz is close to a 440-Hz A, which is A above middle-C on a piano.

PRACTICE Problems

ADDITIONAL PRACTICE

14. A sound wave produced by a clock chime is heard 515 m away 1.50 s later.

a. Based on these measurements, what is the speed of sound in air?

b. The sound wave has a frequency of 436 Hz. What is the period of the wave?

c. What is its wavelength?

15. How are the wavelength, frequency, and speed of a wave related? How do they depend on the medium through which the wave is passing and the type of wave?

16. What is the speed of a periodic wave disturbance that has a frequency of 3.50 Hz and a wavelength of 0.700 m?

17. How does increasing the wavelength by 50 percent affect the frequency of a wave on a rope?

18. The speed of a transverse wave in a string is 15.0 m/s. If a source produces a disturbance that has a frequency of 6.00 Hz, what is its wavelength?

19. Five wavelengths are generated every 0.100 s in a tank of water. What is the speed of the wave if the wavelength of the surface wave is 1.20 cm?

20. A periodic longitudinal wave that has a frequency of 20.0 Hz travels along a coiled spring toy. If the distance between successive compressions is 0.600 m, what is the speed of the wave?

21. How does the frequency of a wave change when the period of the wave is doubled?

22. Describe the change in the wavelength of a wave when the period is reduced by one-half.

23. If the speed of a wave increases to 1.5 times its original speed while the frequency remains constant, how does the wavelength change?

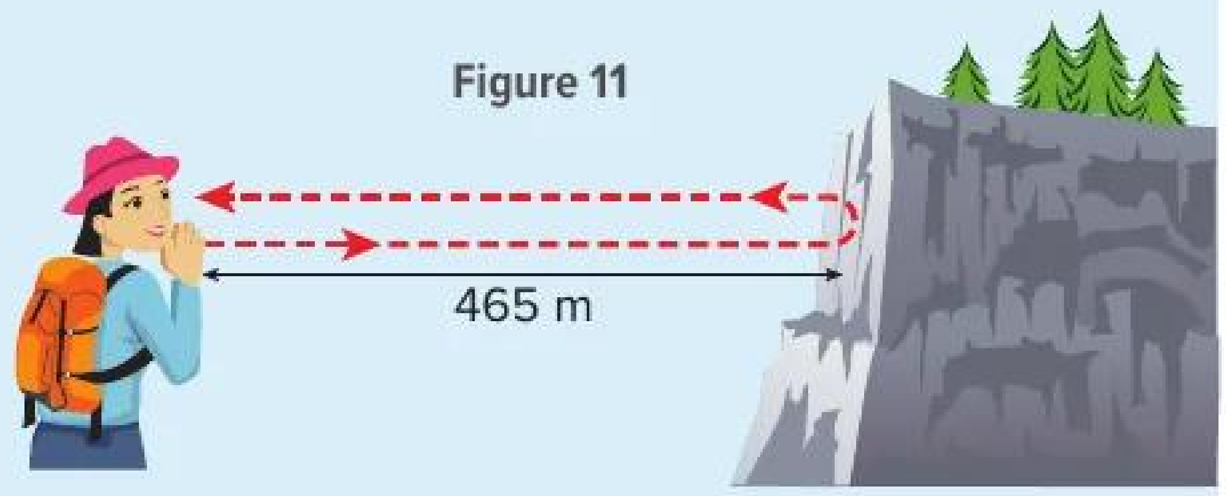
24. CHALLENGE A hiker shouts toward a vertical cliff as shown in **Figure 11.** The echo is heard 2.75 s later.

a. What is the speed of sound of the hiker's voice in air?

b. The wavelength of the sound is 0.750 m. What is its frequency?

c. What is the period of the wave?

Figure 11



Check Your Progress

25. **Transverse Waves** Suppose you and your lab partner are asked to demonstrate that a transverse wave transports energy without transferring matter. How could you do it?

26. **Wave Characteristics** You are creating transverse waves on a rope by shaking your hand from side to side. Without changing the distance your hand moves, you begin to shake it faster and faster. What happens to the amplitude, wavelength, frequency, period, and velocity of the wave?

27. **Longitudinal Waves** Describe longitudinal waves. What types of mediums transmit longitudinal waves?

28. **Speeds in Different Mediums** If you pull on one end of a coiled spring toy, does the pulse reach the other end instantaneously? What happens if you pull on a rope? What happens if you hit the end of a metal rod? Compare the pulses that are traveling through each of these three materials.

29. **Critical Thinking** If a raindrop falls into a pool, it produces waves with small amplitudes. If a swimmer jumps into a pool, he or she produces waves with large amplitudes. Why doesn't the heavy rain in a thunderstorm produce large waves?

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LESSON 3
WAVE BEHAVIOR

FOCUS QUESTION

What happens when two waves meet?

Waves at Boundaries

Recall from Lesson 2 that the speed of a mechanical wave depends only on the properties of the medium it passes through, not on the wave's amplitude or frequency. For example, for waves on a spring, the speed depends on the spring's tension and mass per unit length. Examine what happens when a wave travels from one medium to another. **Figure 12** shows a wave pulse traveling from a larger spring into a smaller one. The pulse that strikes the boundary is called the **incident wave.** One pulse from the larger spring continues in the smaller spring, but the speed is different in the smaller spring. Note that this transmitted wave pulse remains upward.

Some of the energy of the incident wave's pulse is reflected backward into the larger spring. This returning wave is called the reflected wave. Whether the **reflected wave** is upright or inverted depends on the characteristics of the two springs. For example, if the waves in the smaller spring have a greater speed because the spring is stiffer, then the reflected wave will be inverted.

Wave at a Boundary

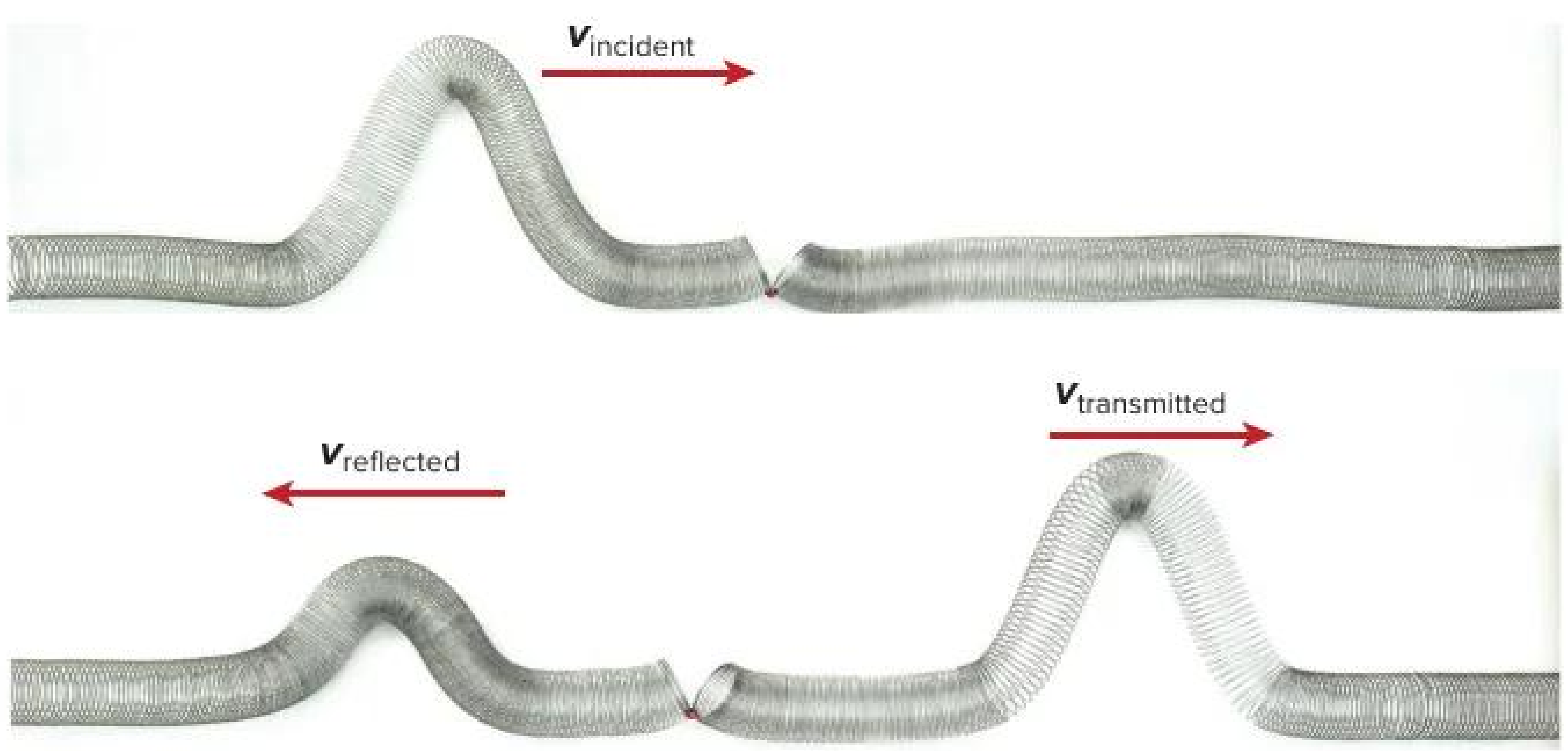


Figure 12 When the wave pulse meets the boundary between the two springs, a transmitted wave pulse and a reflected wave form.

Compare *the energy of the incident wave to the energy of the reflected wave.*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Reflection and Refraction

Analyze data from wave patterns in a ripple tank to predict the behavior of waves in water.

Review the News

Obtain information from a current news story about mechanical waves, such as seismic or water waves. Evaluate your source and communicate your findings to your class.

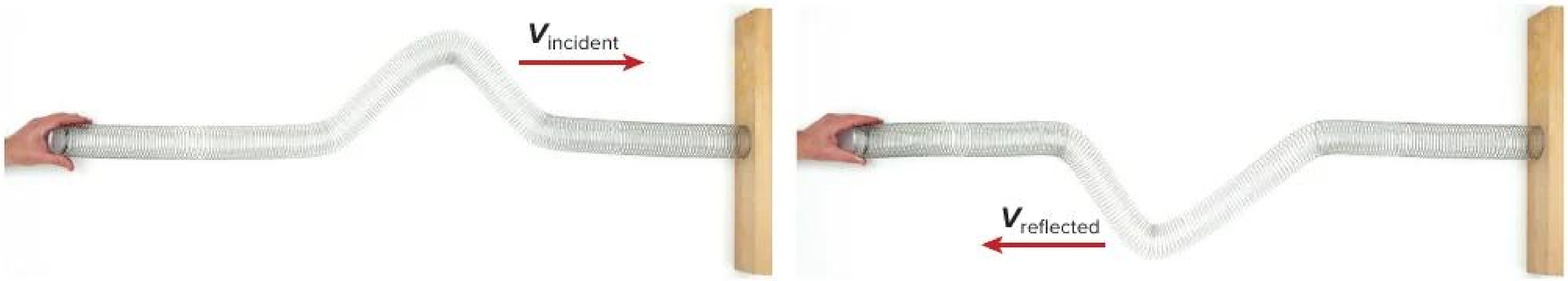


Figure 13 When a wave encounters a rigid boundary, the reflected wave is inverted. Note that the amplitude is not affected by the rigid boundary.

Rigid boundaries When a wave pulse hits a rigid boundary, the energy is reflected back, as shown in **Figure 13.** The wall is the boundary of a new medium through which the wave attempts to pass. Instead of passing through, the pulse is reflected from the wall with almost exactly the same amplitude as the pulse of the incident wave. Thus, almost all the wave's energy is reflected back. Very little energy is transmitted into the wall. Also note that the pulse is inverted.

Superposition of Waves

Suppose a pulse traveling along a spring meets a reflected pulse that is coming back from a boundary, as shown in **Figure 14.** In this case, two waves exist in the same place in the medium at the same time. Each wave affects the medium independently. The **principle of superposition** states that the displacement of a medium caused by two or more waves is the algebraic sum of the displacements caused by the individual waves. In other words, two or more waves can combine to form a new wave. If the waves move in the same medium, they can cancel or add or subtract to form a new wave of lesser or greater amplitude. They emerge unaffected by each other. The result of the superposition of two or more waves is called **interference.**

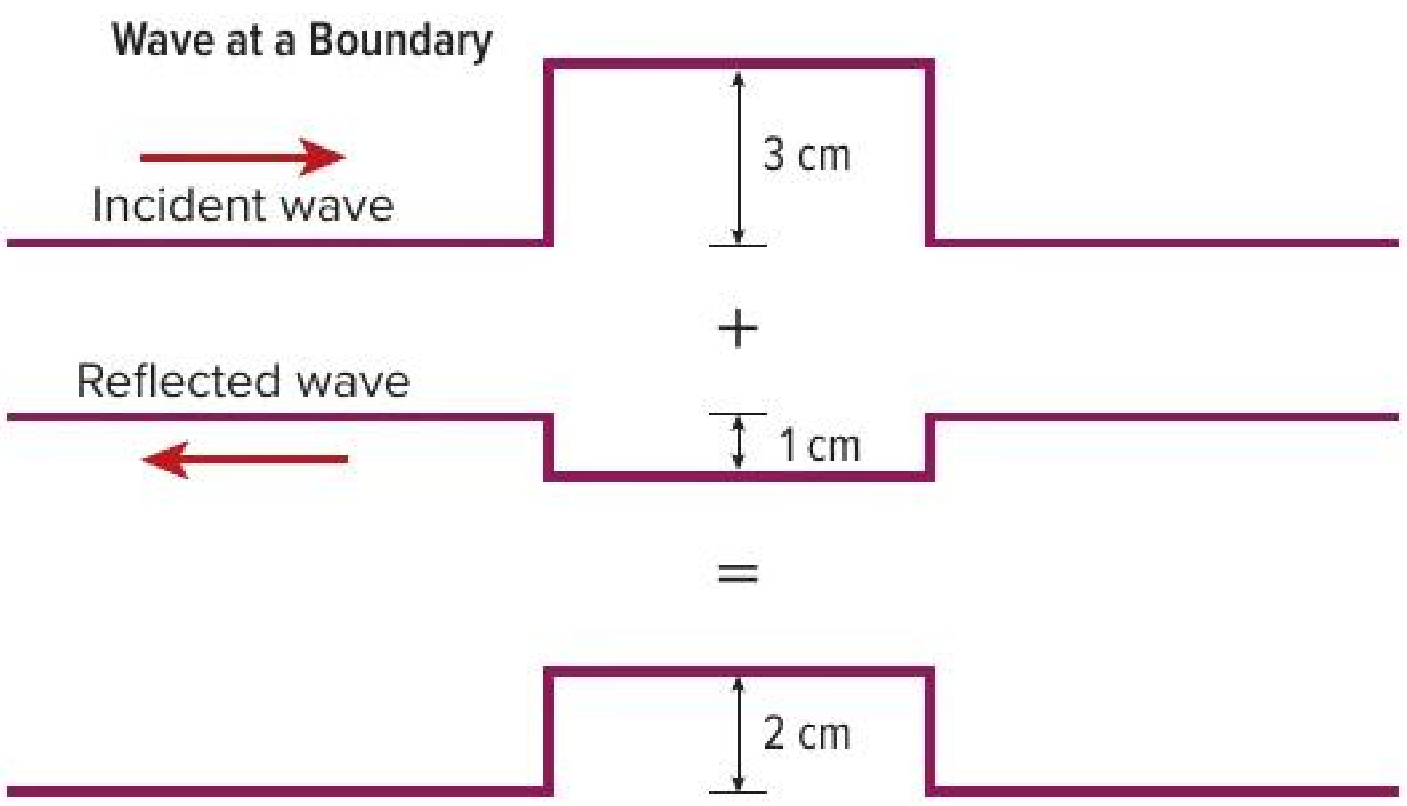


Figure 14 Waves add algebraically during superposition.

SCIENCE USAGE v. COMMON USAGE

Interference

Science usage: the result of superposition of two or more waves

The amplitude of the interference of several waves was much larger than the amplitude of the individual waves.

Common usage: the act of coming between in a way that hinders or impedes

Ehud was ejected from the game for an interference foul.

STEM CAREER Connection

Boat Designer

If you are drawn to the water, the career of boat designer might be for you. Whether designing ocean liners, freighters, yachts, or wakeboard boats, a boat designer must understand how a vessel will perform in various wave conditions.

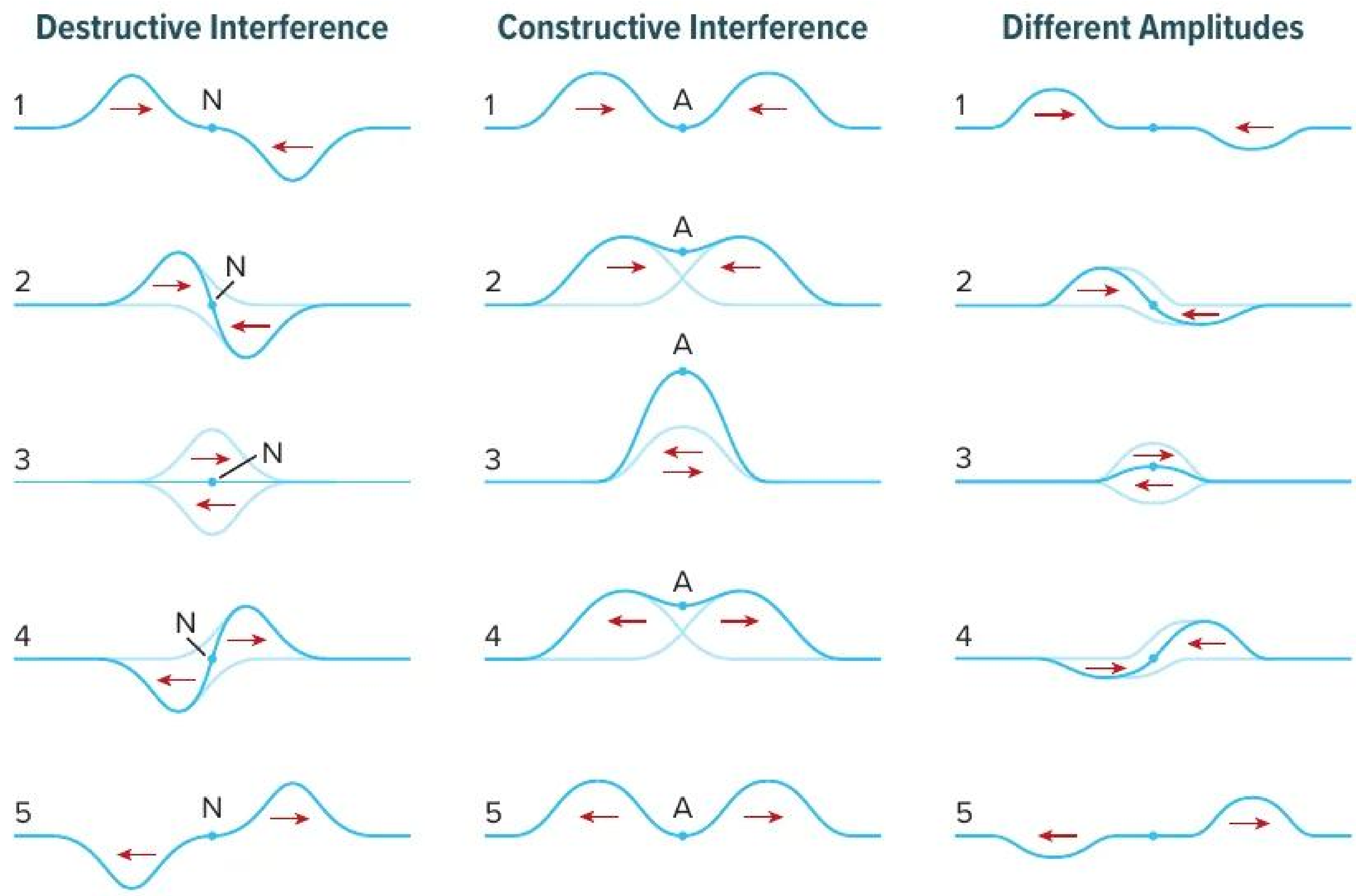


Figure 15 When waves add algebraically, the resulting combined waves can be quite different from the individual waves. **Summarize** *how waves behave during and after superposition.*

Wave interference Wave interference can be either constructive or destructive. The first panel in **Figure 15** shows the superposition of waves with equal but opposite displacements, causing destructive interference. When the pulses meet and are in the same location, the displacement is zero. Point N, which does not move at all, is called a **node.** The pulses travel horizontally and eventually emerge unaffected by each other.

Constructive interference occurs when wave displacements are in the same direction. The result is a wave that has an amplitude greater than those of the individual waves. A larger pulse appears at point A when the two waves meet. Point A has the largest displacement and is called the **antinode.** The two pulses pass through each other without changing their shapes or sizes. Even if the pulses have unequal amplitudes, the resultant pulse at the overlap is still the algebraic sum of the two pulses, as shown in the final panel of **Figure 15.**

Get It?
Compare the wave medium's displacement at a node and at an antinode.

Two reflections You can apply the concept of superimposed waves to the control of large-amplitude waves. Imagine attaching one end of a rope to a fixed point, such as a doorknob, a distance *L* away. When you vibrate the free end, the wave leaves your hand, travels along the rope toward the fixed end, is reflected and inverted at the fixed end, and returns to your hand. When it reaches your hand, the reflected wave is inverted and travels back down the rope. Thus, when the wave leaves your hand the second time, its displacement is in the same direction as it was when it left your hand the first time.

Standing waves Suppose you adjust the motion of your hand so that the period of vibration equals the time needed for the wave to make one round-trip from your hand to the door and back. Then, the displacement given by your hand to the rope each time will add to the displacement of the reflected wave. As a result, the amplitude of oscillation of the rope will be much greater than the motion of your hand. This large-amplitude oscillation is an example of mechanical resonance.

The ends of the rope are nodes and an antinode is in the middle, as shown in the top photo in **Figure 16.** Thus, the wave appears to be standing still and is called a **standing wave.** Note, however, that the standing wave is the interference of waves traveling in opposite directions. If you double the frequency of vibration, you can produce one more node and one more antinode on the rope. Then it appears to vibrate in two segments. When you further increase the vibration frequency, it produces even more nodes and antinodes, as shown in the bottom photo in **Figure 16.**

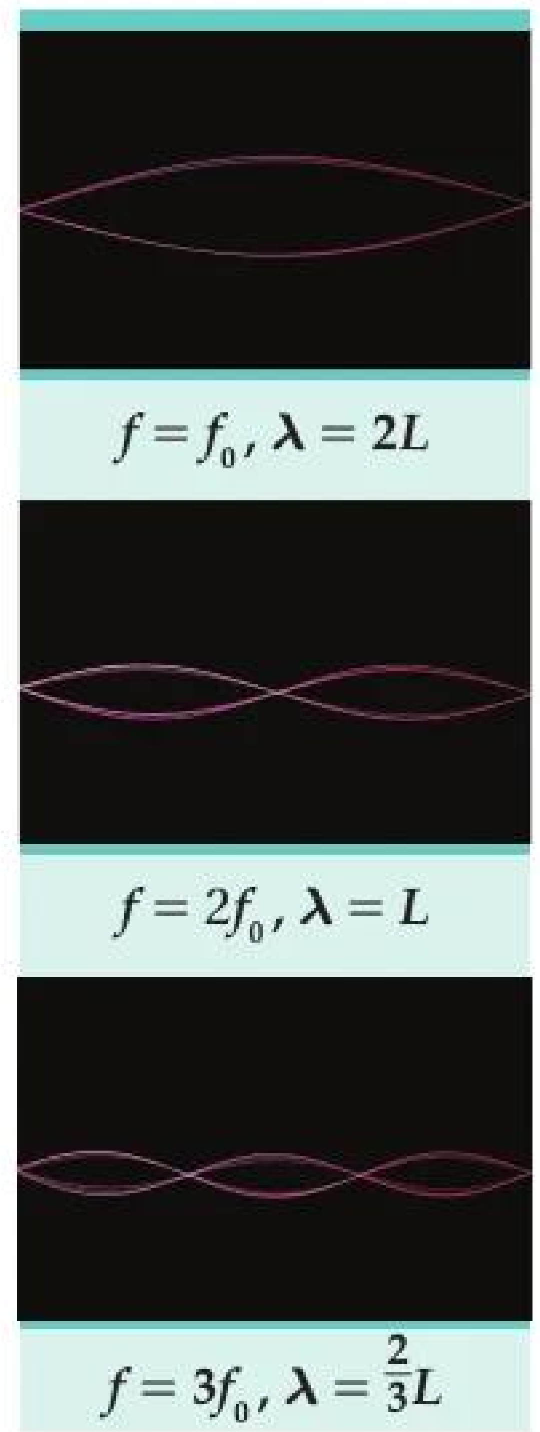


Figure 16 Interference produces standing waves only at certain frequencies.

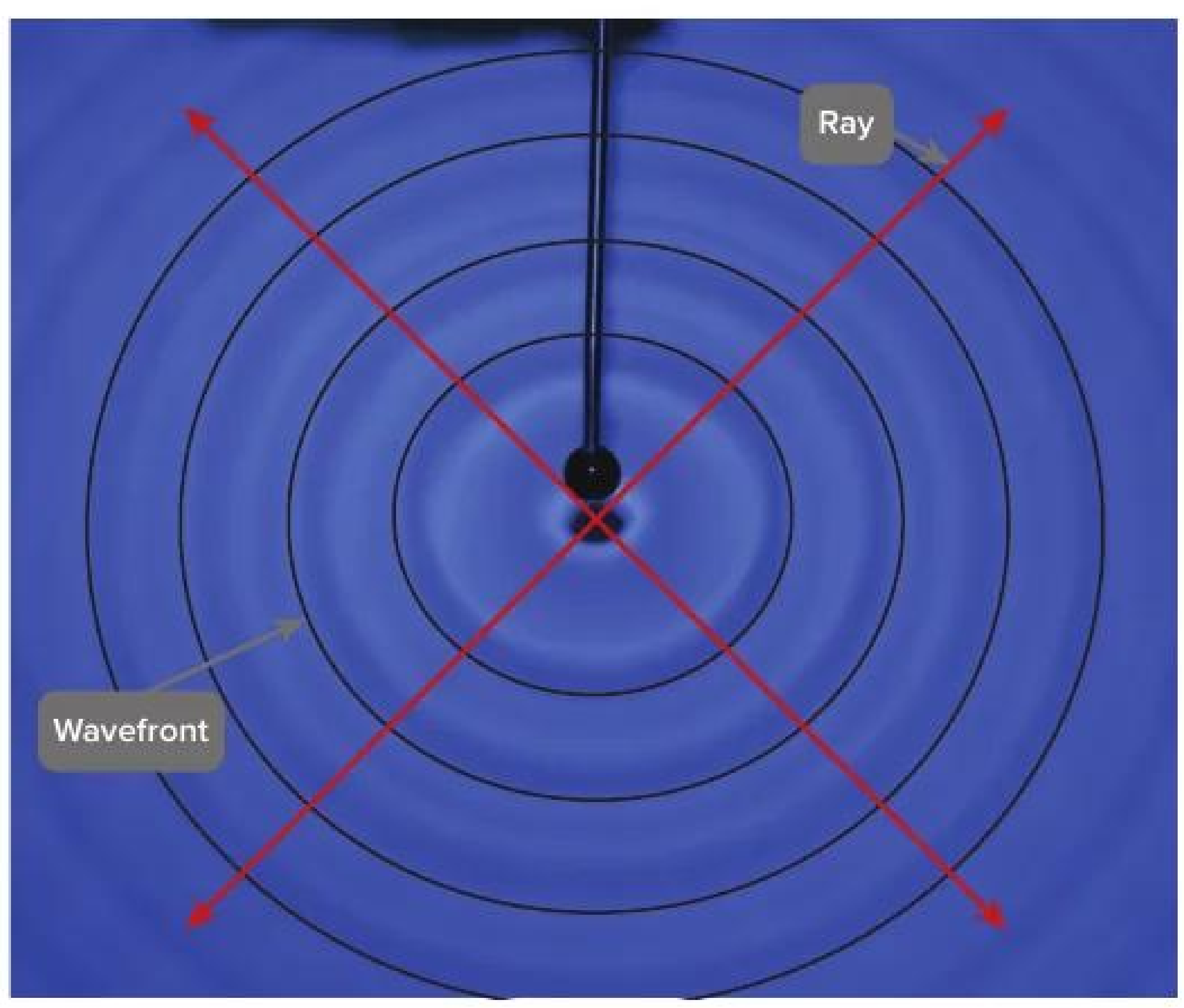
Predict *the wavelength if the frequency is four times the lowest frequency.*

Waves in Two Dimensions

You have studied waves on a rope and on a spring reflecting from rigid supports. During some of these interactions, the amplitude of the waves is forced to be zero by destructive interference. These mechanical waves travel in only one dimension. Waves on the surface of water, however, travel in two dimensions, and sound waves and electromagnetic waves will later be shown to travel in three dimensions. How can two-dimensional waves be represented?

Picturing waves in two dimensions When you throw a small stone into a calm pool of water, you see the circular crests and troughs of the resulting waves spreading out in all directions. You can sketch those waves by drawing circles to represent the wave crests. If you repeatedly dip your finger into water with a constant frequency, the resulting sketch would be a series of concentric circles, called wavefronts, centered on your finger. A **wavefront** is a line that represents the crest of a wave in two dimensions. Wavefronts can be used to show two-dimensional waves of any shape, including circular waves. The photo in **Figure 17** shows circular waves in water. The circles drawn on the diagram show the wavefronts that represent those water waves.

Figure 17 Waves spread out in a circular pattern from the oscillating source.



Whatever their shape, two-dimensional waves always travel in a direction that is perpendicular to their wavefronts. That direction can be represented by a **ray,** which is a line drawn at a right angle to the wavefront. When all you want to show is the direction in which a wave is traveling, it is convenient to draw rays instead of wavefronts. The red arrows in **Figure 17** are rays that show the water waves' direction of motion. One advantage of drawing wavefronts is when wavefronts are drawn to scale, they show the wave's wavelengths. In **Figure 17,** the wavelength equals the distance from one circle to the next.

Creating two-dimensional waves A ripple tank is a piece of laboratory equipment that is used to investigate the properties of two-dimensional waves. The main portion of the ripple tank shown in **Figure 18** is a shallow tank that contains a thin layer of water. A board attached to a mechanical oscillator produces waves with long, straight wavefronts. A lamp above the tank produces shadows below the tank that show the locations of the crests of the waves. The top photo in **Figure 18** shows a wave traveling through the ripple tank. The direction the wave travels is modeled by a ray diagram. For clarity, the wavefronts are not extended the entire length of the wave.

Reflection of two-dimensional waves The bottom row of pictures in **Figure 18** shows an incident ray encountering a rigid barrier placed at an angle to the ray's path. The orientation of the barrier is shown by a line, called the **normal,** drawn perpendicular to the barrier. The angle between the incident ray and the normal is called the angle of incidence and is labeled θ_i in the diagram. The angle between the normal and the reflected ray is called the angle of reflection and is labeled θ_r. The **law of reflection** states that the angle of incidence is equal to the angle of reflection. The law of reflection applies to many different kinds of waves, not just the waves in a ripple tank.

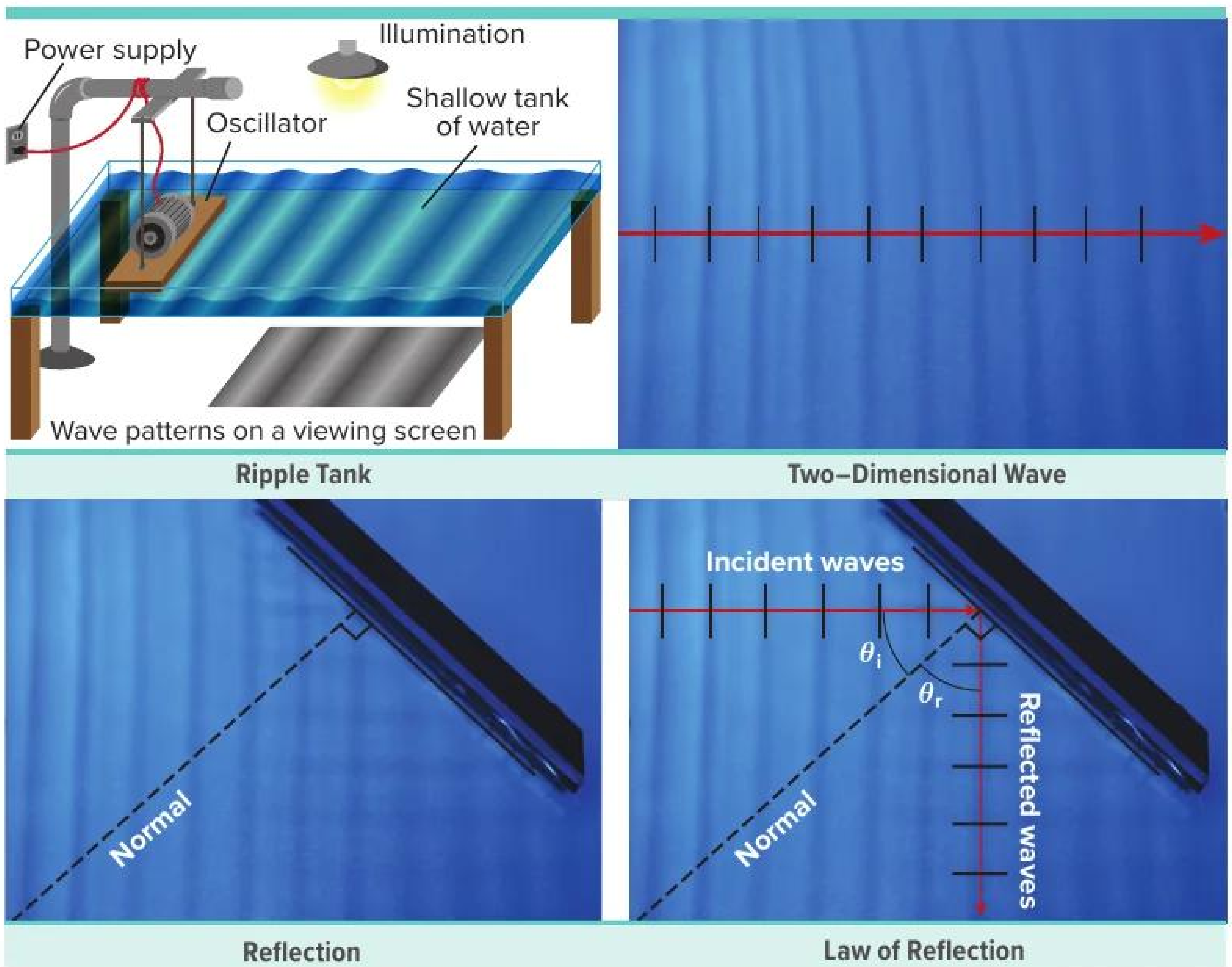


Figure 18 The ripple tank produces uniform waves that are useful for modeling wave behavior.

SCIENCE USAGE v. COMMON USAGE

Normal

Science usage: a line in a diagram that is drawn perpendicular to a surface
As measured from the normal, angles of incidence and reflection are equal.

Common usage: conforming to a type, standard, or regular pattern
Such cold temperatures in July are not normal.

CCC CROSSCUTTING CONCEPTS

Cause and Effect Write a procedure describing how you could use a ripple tank to gather empirical evidence to support the cause-and-effect relationship between the angle of a barrier and the angle of a reflected wave.

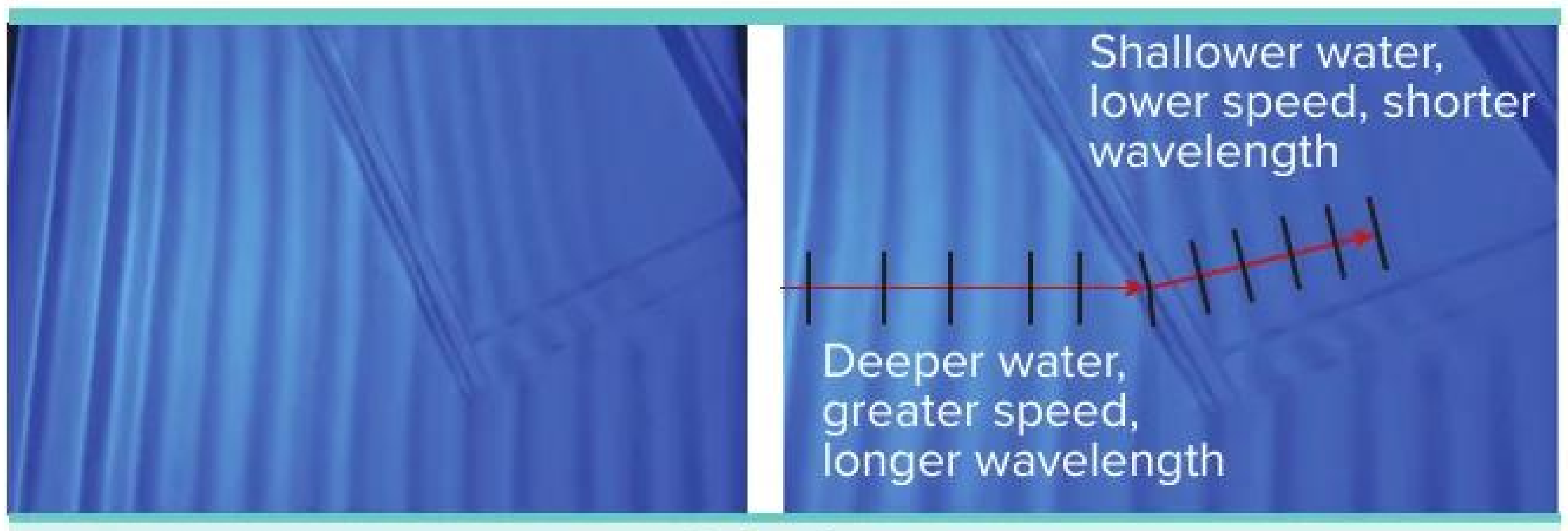


Figure 19 Waves in the ripple tank change direction as they enter shallower water.

Describe *how the wavelength changes as the wave travels into the shallow water.*

Refraction of waves in two dimensions A ripple tank can also model the behavior of waves as they travel from one medium into another. **Figure 19** shows a glass plate placed under the water in a ripple tank. The water above the plate is shallower than the water in the rest of the tank. As the waves travel from deep to shallow water, their speeds decrease and the direction of the waves changes. Such changes in speed are common when waves pass from one medium to another.

The waves in the shallow water are connected to the waves in the deep water. As a result, the frequency of the waves in the two mediums is the same. Based on the equation $\lambda = \frac{v}{f}$, the decrease in the speed of the waves means the wavelength is shorter in the shallower water. The change in the direction of waves at the boundary between two different mediums is known as **refraction.** **Figure 19** shows a wave front and ray model of refraction. Part of the wave will refract through the boundary, and part will be reflected from the boundary. Reflection and refraction occur for many different types of waves. Echoes are an example of reflection of sound waves by hard surfaces, such as the walls of a large gymnasium. Rainbows are the result of the reflection and refraction of light. As sunlight passes through a raindrop, reflection and refraction separate the light into its individual colors, producing a rainbow.

Check Your Progress

30. **Wave Characteristics** Which characteristics remain unchanged when a wave crosses a boundary into a different medium: frequency, amplitude, wavelength, velocity, direction?
31. **Superposition of Waves** Sketch two wave pulses whose interference produces a pulse with an amplitude greater than either of the individual waves.
32. **Refraction of Waves** In **Figure 19,** the wave changes direction as it passes from one medium to another. Can two-dimensional waves cross a boundary between two mediums without changing direction? Explain.
33. **Standing Waves** In a standing wave on a string fixed at both ends, how is the number of nodes related to the number of antinodes?
34. **Critical Thinking** As another way to understand wave reflection, cover the right-hand side of each drawing in the left panel in **Figure 15** with a piece of paper. The edge of the paper should be at point N, the node. Now, concentrate on the resultant wave, shown in darker blue. Note that it acts as a wave reflected from a boundary. Is the boundary a rigid wall? Repeat this exercise for the middle panel in **Figure 15.**

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ENGINEERING & TECHNOLOGY

Harnessing the Motion of the Ocean

Ocean waves represent a huge, almost untapped potential source of energy. Estimates suggest that in the United States, wave power could produce one-quarter to one-third of the electricity used each year. Currently wave power technology lags behind wind and solar power, but around the world, engineers are working to change this and harness the power of ocean waves.

Wave power generators convert mechanical energy from ocean waves to electrical energy.

Wave Power Technologies

One wave power technology, sometimes called a point absorber, looks like a large floating buoy. The buoy has parts that are attached but are able to move independently. As waves rise and fall, the buoy's different parts move relative to each other. This motion drives an energy converter device such as a hydraulic pump, which transforms the mechanical energy of motion into electrical energy. Two different versions of these buoys are being tested off the coast of Hawaii.

Another technology that is designed for offshore use looks like a giant metal sea snake. Its segments can move independently but are attached to one another as it floats on the surface of the water. The whole device is anchored to the ocean floor. The motion of ocean waves causes the segments to move relative to one another. The mechanical energy of the segments is converted by a hydraulic pump or other device to electrical energy. The resulting electrical current is transported to shore via a cable.

A different style of generator, called an oscillating water column or a terminator, is usually designed for onshore use. Waves push water into a column-shaped chamber through an underwater opening. Air is trapped above. The captured water column moves up and down like a piston. Air is forced in and out of an opening at the top of the column. The movement of the air turns a turbine and this mechanical motion is converted to electricity.

Successful wave power devices need to do more than just convert mechanical energy from waves into electricity. They must also be extremely durable to last in harsh ocean conditions. Engineers must design devices that will withstand storms, constant impacts from waves, and corrosive seawater.

So far none of these technologies have been adopted for large-scale use. Engineers and scientists will continue to develop and test prototypes, working towards a goal of wave power devices that are effective, reliable, durable, and affordable.

DEVELOP AND USE MODEL TO ILLUSTRATE

Choose one type of wave power technology. Find out how it converts mechanical energy from waves to electricity. Develop and use a model to illustrate how the technology works.

MODULE 13
STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 PERIODIC MOTION

- Simple harmonic motion results when the restoring force on an object is directly proportional to the object's displacement from equilibrium.
- The elastic potential energy of a spring that obeys Hooke's law is expressed by the following equation:

$$PE_s = \frac{1}{2}kx^2$$

- The period of a pendulum depends on the pendulum's length and the gravitational field strength at the pendulum's location. The period can be found using the following equation

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

- periodic motion
- period
- amplitude
- simple harmonic motion
- Hooke's law
- simple pendulum
- resonance

Lesson 2 PHOTOSYNTHESIS

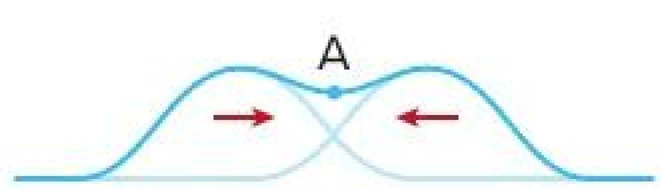
- Waves are disturbances that transfer energy without transferring matter.
- In transverse waves, the displacement of the medium is perpendicular to the direction the wave travels. In longitudinal waves, the displacement is parallel to the direction the wave travels.
- The velocity of a continuous wave is equal to the wave's frequency times its wavelength.

$$v = f\lambda$$

- wave
- wave pulse
- transverse wave
- periodic wave
- longitudinal wave
- surface wave
- trough
- crest
- wavelength
- frequency

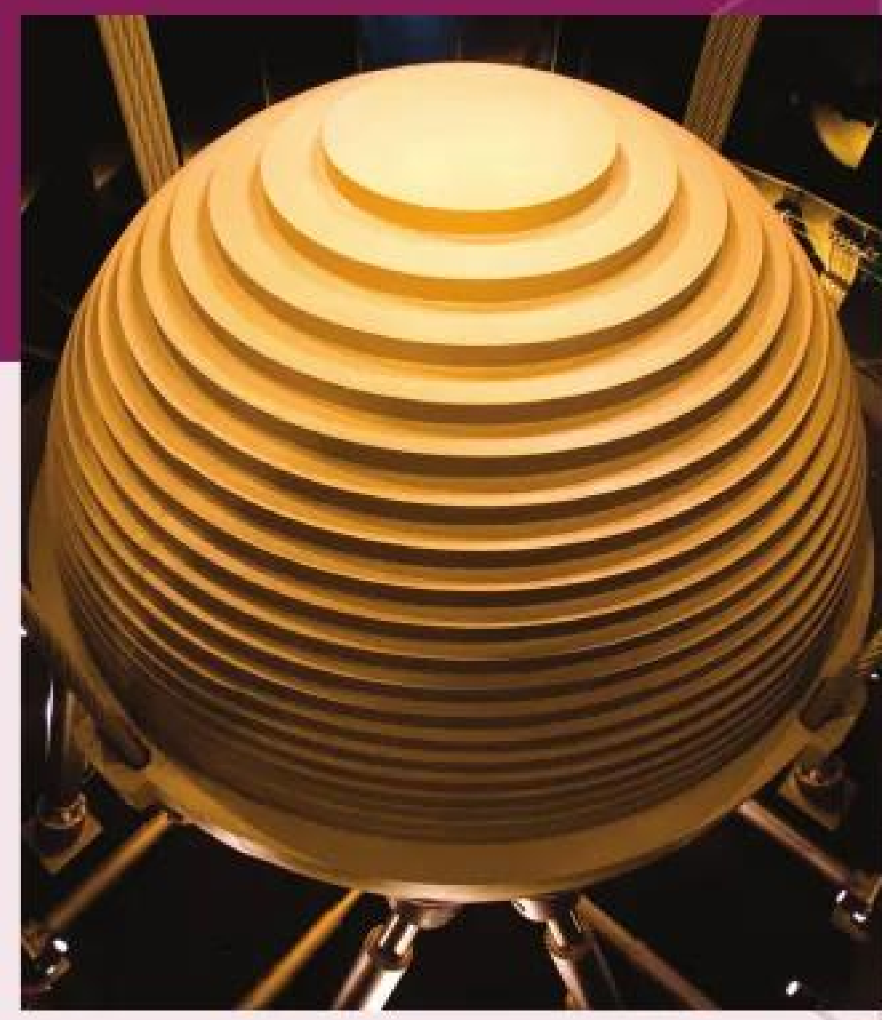
Lesson 3 WAVE BEHAVIOR

- When two-dimensional waves are reflected from boundaries, the angles of incidence and reflection are equal. The change in direction of waves at the boundary between two different mediums is called refraction.
- Interference occurs when two or more waves travel through the same medium at the same time. The principle of superposition states that the displacement of a medium resulting from two or more waves is the algebraic sum of the displacements of the individual waves.



- incident wave
- reflected wave
- principle of superposition
- interference
- node
- antinode
- standing wave
- wavefront
- ray
- normal
- law of reflection
- refraction

THREE-DIMENSIONAL THINKING
Module Wrap-Up



REVISIT THE PHENOMENON

How can this pendulum save a building from earthquake damage?

CER Claim, Evidence, Reasoning

Explain your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

How does the strength of gravity affect periodic motion?

An astronaut lands on an unknown planet and must determine the value of the gravitational acceleration g. He has the following instruments on hand.

Instrument A rests on the ground and shoots a ball vertically with a known speed. The astronaut can measure the time the ball takes to rise from its launch position and the time it takes to fall back to the launch position.

Instrument B is a simple unmarked weight suspended from a spring with constant $k = 22.5$ N/m.

Instrument C is a simple pendulum at the end of an arm that is 0.500 m long. The astronaut counts exactly 47 full swings in 1.00 min.

CER Analyze and Interpret Data

1. **Claim** Which instrument(s) can be used to determine g?
2. **Evidence and Reasoning** Explain how you made your decision.

MODULE 14
SOUND

Mike Long/Alamy Stock Photo

MODULE 14
SOUND

ENCOUNTER THE PHENOMENON

Why does a fire truck's siren change pitch as it passes you?

GO ONLINE to play a video how pitch changes when a source of sound moves past an observer.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

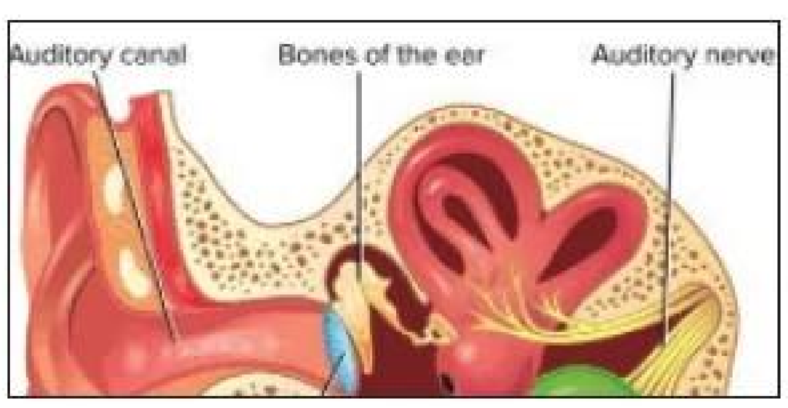
CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about why a fire truck's siren changes pitch as it passes you.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.



LESSON 1: Explore & Explain: Detecting Sound Waves

LESSON 2: Explore & Explain: Sources of Sound

Smithsonian

Additional Resources

LESSON 1

PROPERTIES AND DETECTION OF SOUND

FOCUS QUESTION

What factors affect the pitch of a sound?

Sound Waves

You already are familiar with several of the characteristics of sound, including volume, tone, and pitch, from your everyday experiences. Without thinking about it, you can use these, and other characteristics, to categorize many of the sounds that you hear.

Pressure variations Put your fingers against your throat as you hum or speak. Can you feel the vibrations? **Figure 1** shows a vibrating bell that can represent your vocal cords, a loudspeaker, or any other sound source. As it moves back and forth, the edge of the bell strikes the particles in the air. When the edge moves forward, air particles bounce off the bell with a greater velocity. When the edge moves backward, air particles bounce off the bell with a lower velocity.

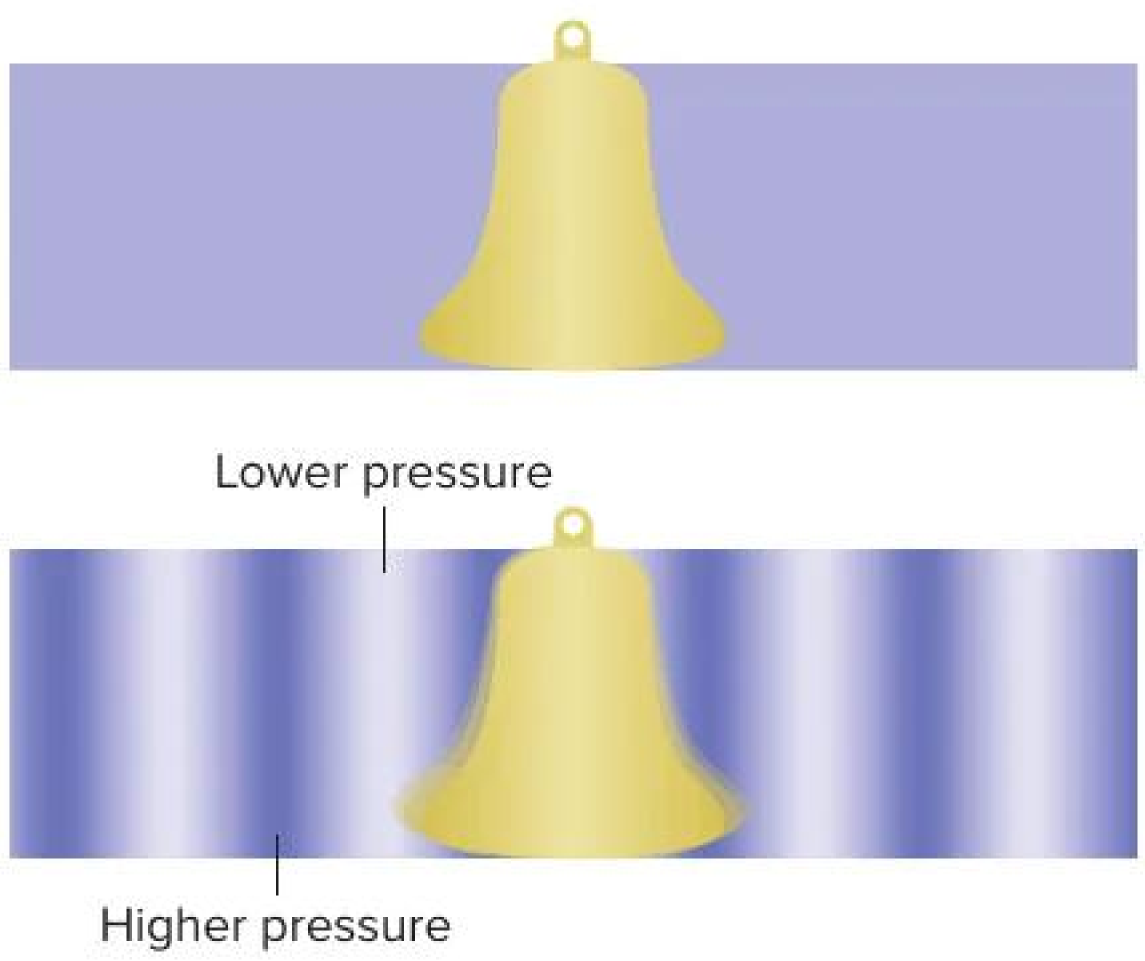


Figure 1 When the bell is at rest (top), the surrounding air is at average pressure. When the bell is struck (bottom), the vibrating edge creates regions of high and low pressure. Although the diagram shows the pressure regions moving in one direction, the waves move out in all directions.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Virtual Investigation: Doppler Effect

Analyze data to determine how a moving source affects the frequency of the detected sound wave.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

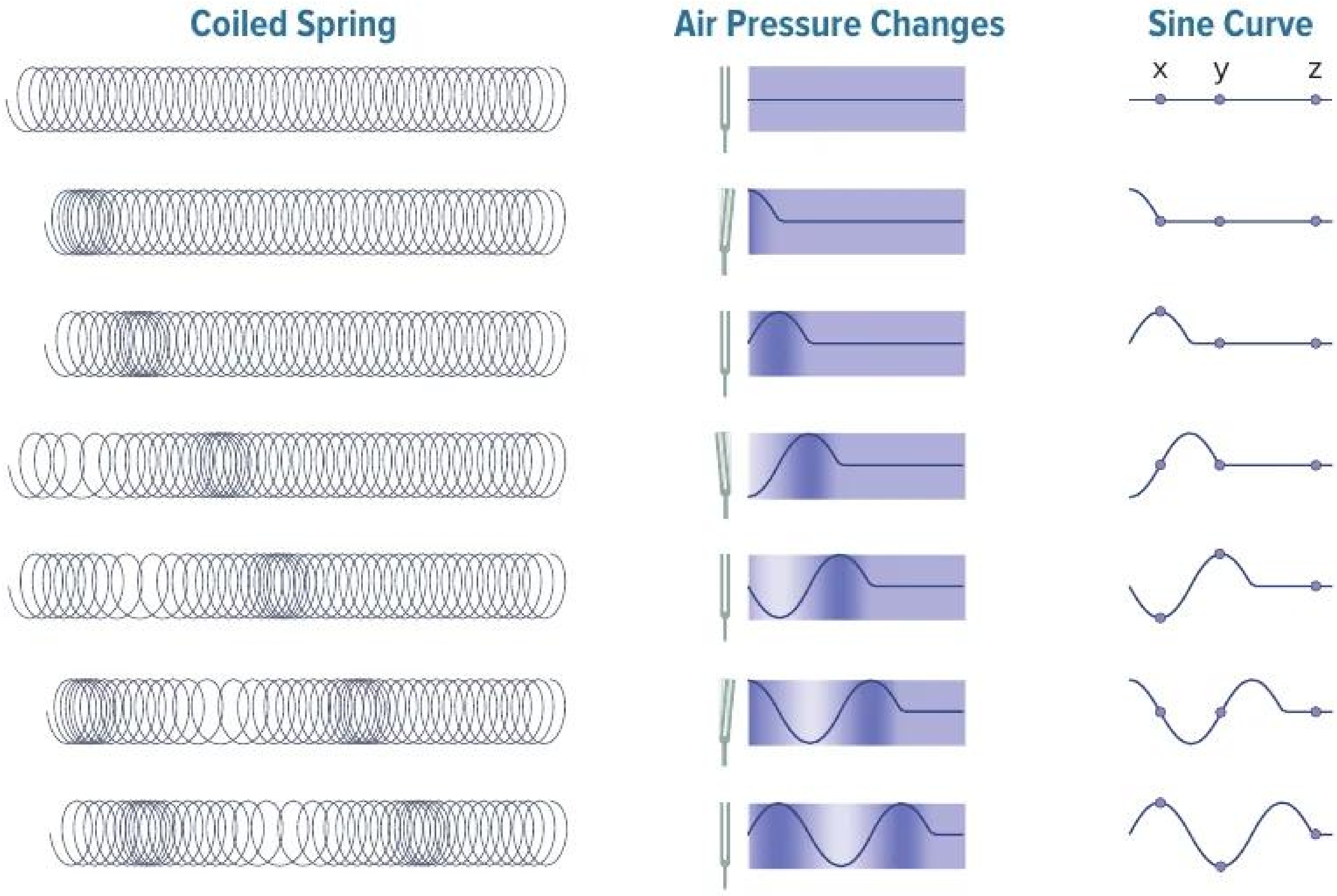


Figure 2 A coiled spring models the oscillations created by a sound wave. As the sound wave travels through the air, the air pressure rises and falls. The changes in the sine curves correspond to the changes in air pressure. Note that the positions of *x*, *y*, and *z* show that the wave, not the matter, travels forward.

The result of these velocity changes is that the forward motion of the bell produces a region where the air pressure is slightly higher than average, as shown in **Figure 1.** The backward motion produces slightly below-average pressure. Collisions of the air particles cause the pressure variations to move away from the bell in all directions. If you were to focus on one spot, you would see the value of the air pressure rise and fall. In this way, the pressure variations are transmitted through matter.

Describing sound

A pressure oscillation that is transmitted through matter is a **sound wave.** Sound waves travel through air because a vibrating source produces regular variations, or oscillations, in air pressure. The air particles collide, transmitting the pressure variations away from the source of the sound. The pressure of the air oscillates about the mean air pressure, as shown in **Figure 2.** The frequency of the wave is the number of oscillations in pressure each second. The wavelength is the distance between successive regions of high or low pressure. Because the motion of the particles in air is parallel to the direction of the wave's motion, sound is a longitudinal wave.

The wavelength and frequency of a wave are related to one another by the speed of travel of the wave. Just like any other wave, the speed of sound depends on the medium through which the sound passes. In air, the speed depends on the temperature, increasing by about 0.6 m/s for each 1°C increase in air temperature. At room temperature (20°C) and sea level, the speed of sound is 343 m/s. For the problems in this book, you may assume these conditions unless otherwise stated.

Get It?

Estimate the speed of sound through air at sea level if the temperature is 25°C.

In general, the speed of sound is greater in solids and liquids than in gases. **Table 1** lists the speeds of sound waves in various media. Sound cannot travel in a vacuum because there are no particles to collide.

Sound waves share the general properties of other waves. For example, they reflect off hard objects, such as the walls of a room. Reflected sound waves are called echoes. The time required for an echo to return to the source of the sound can be used to find the distance between the source and the reflective object. This principle is used by bats, by some cameras, and by ships that employ sonar. Two sound waves can interfere, causing dead spots at nodes where little sound can be heard. Recall that the frequency and wavelength of a wave are related to the speed of the wave by the equation $\lambda = \frac{v}{f}$.

Table 1 Speed of Sound in Various Media

Medium	Speed (m/s)
Air (0°C)	331
Air (20°C)	343
Helium (0°C)	965
Water (25°C)	1497
Seawater (25°C)	1535
Copper (20°C)	4760
Iron (20°C)	4994

Detection of Pressure Waves

Sound detectors transform sound energy—the kinetic energy of the vibrating particles of the transmitting medium—into another form of energy. A common detector is a microphone, which transforms sound energy into electrical energy. A microphone consists of a thin disk that vibrates in response to sound waves and produces an electrical signal.

The human ear

As shown in **Figure 3,** the human ear is a sound detector that receives pressure waves and converts them to electrical impulses. The tympanic membrane, also called the eardrum, vibrates when sound waves enter the auditory canal. Three tiny bones in the middle ear then transfer these vibrations to fluid in the cochlea. Tiny hairs lining the spiral-shaped cochlea detect certain frequencies in the vibrating fluid. These hairs stimulate nerve cells, which send impulses to the brain and produce the sensation of sound. The ear detects sound waves over a wide range of frequencies and is sensitive to an enormous range of amplitudes. In addition, human hearing can distinguish many different qualities of sound. Knowledge of both physics and biology is required to understand the complexities of the ear. The interpretation of sounds by the brain is even more complex, and is not totally understood.

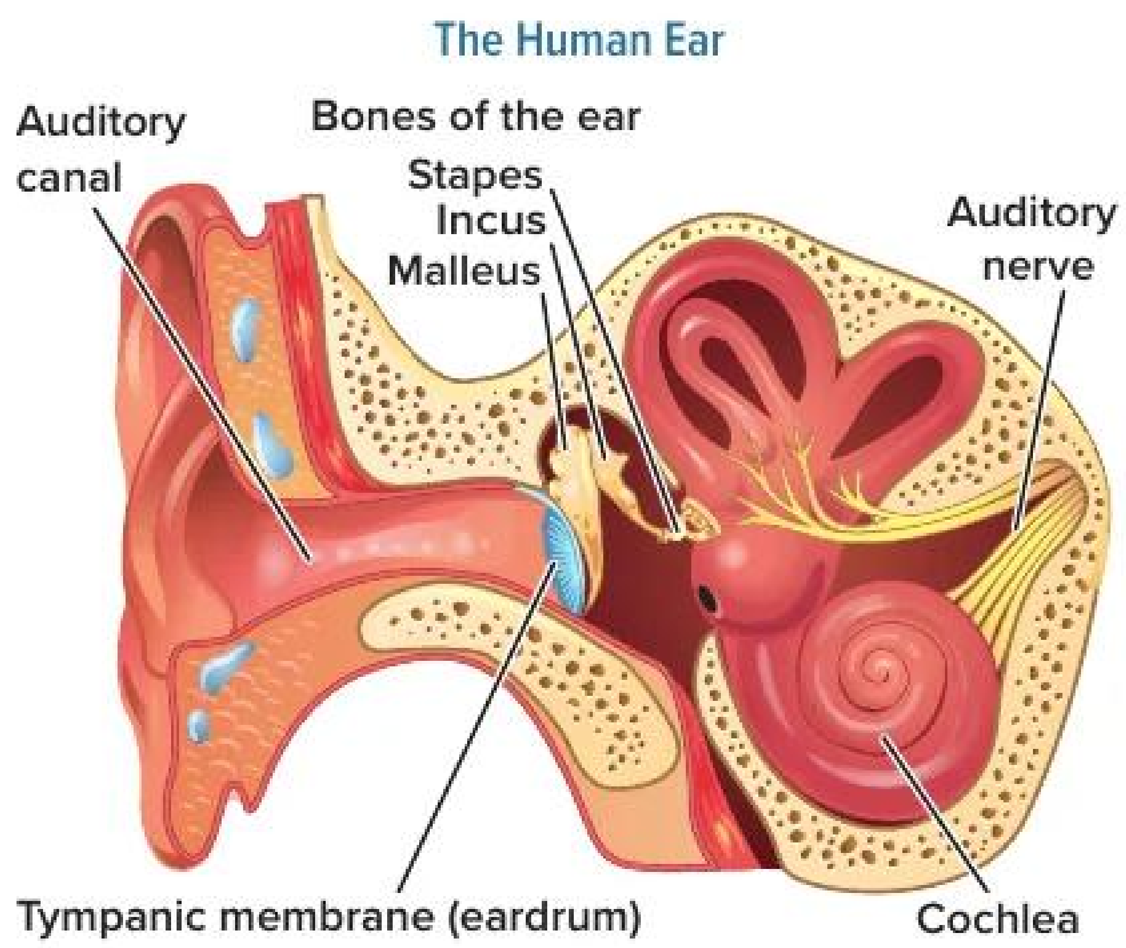


Figure 3 The human ear is a sense organ that translates sound vibrations from the external environment into nerve impulses that are sent to the brain for interpretation. The eardrum vibrates when sound waves enter the auditory canal. The bones in the middle ear—the malleus, the incus, and the stapes—move as a result of the vibrations. The vibrations are then transmitted to the inner ear, where they trigger nerve impulses to the brain. (Note: Diagram is not to scale.)

STEM CAREER Connection

Hearing Aid Specialist

Would you enjoy helping people improve their quality of life? A hearing aid specialist administers hearing tests and helps select and fit hearing aids for people.

Perceiving Sound

How humans perceive sound depends partly on the physical characteristics of sound waves, such as frequency and amplitude.

Pitch
Marin Mersenne and Galileo first determined that the pitch we hear depends on the frequency of vibration. **Pitch** is the highness or lowness of a sound, and it can be given a name on the musical scale. For instance, the note known as middle C has a frequency of 262 Hz. The highest note on a piano has a frequency of 4186 Hz. The human ear is not equally sensitive to all frequencies. Most people cannot hear sounds with frequencies below 20 Hz or above 16,000 Hz. Many animals, such as dogs, cats, elephants, and bats, are capable of hearing frequencies that humans cannot hear.

Get It?
Identify What characteristic of waves is pitch most closely linked to?

Loudness
Frequency and wavelength are two physical characteristics of sound waves. Another physical characteristic of sound waves is amplitude. Amplitude is the measure of the variation in pressure in a wave. The **loudness** of a sound is the intensity of the sound as perceived by the ear and interpreted by the brain. This intensity depends primarily on the amplitude of the pressure wave.

The human ear is extremely sensitive to variations in the intensity of sound waves. Recall that 1 atmosphere of pressure equals 1.01×10^5 Pa. The ear can detect pressure-wave amplitudes of less than one-billionth of an atmosphere, or 2×10^{-5} Pa. At the other end of the audible range, pressure variations of approximately 20 Pa or greater cause pain. It is important to remember that the ear detects pressure variations only at certain frequencies. Driving over a mountain pass changes the pressure on your ears by thousands of pascals, but this change does not take place at audible frequencies.

Because humans can detect a wide range of intensities, it is convenient to measure these intensities on a logarithmic scale called the **sound level.** The most common unit of measurement for sound level is the **decibel** (dB). The sound level depends on the ratio of the intensity of a given sound wave to that of the most faintly heard sound. This faintest sound is measured at 0 dB. A sound that is ten times more intense registers 20 dB. A sound that is another ten times more intense is 40 dB.

Get It?
Compare How much more intense is a sound that registers 80 dB than one of 40 dB?

SCIENCE USAGE v. COMMON USAGE

Pitch

Science usage: the highness or lowness of a sound, which depends on the frequency of vibration

The flute and the tuba produce very different pitches.

Common usage: the delivery of a ball by a pitcher to a batter

Sharon hit the pitch over the fence for a home run.

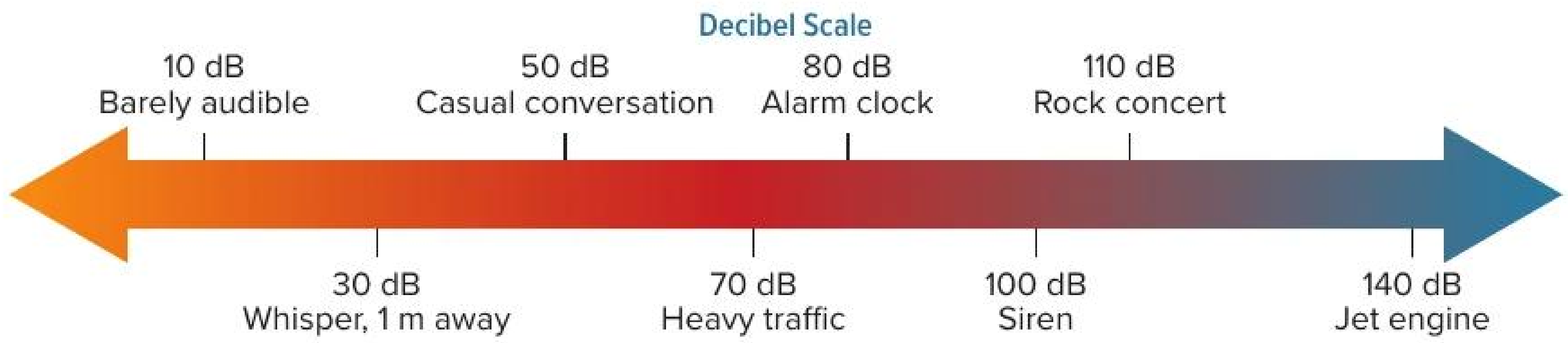


Figure 4 This decibel scale shows the sound level for a variety of sounds.

Infer *About how many times louder does an alarm clock sound than heavy traffic?*

Most people perceive a 10-dB increase in sound level as about twice as loud as the original level. **Figure 4** shows the sound level for a variety of sounds. In addition to intensity, pressure variations and the power of sound waves can be described by decibel scales.

The ear can lose its sensitivity, especially to high frequencies, after exposure to loud sounds in the form of noise or music. The longer a person is exposed to loud sounds, the greater the effect. A person can recover from short-term exposure in a period of hours, but the effects of long-term exposure can last for days or weeks. Long exposure to 100-dB or greater sound levels can produce permanent damage. Hearing loss also can result from loud music being transmitted to stereo headphones from personal music devices. In some cases, the listeners are unaware of just how high the sound levels really are. Cotton earplugs reduce the sound level only by about 10 dB. Special ear inserts can provide a 25-dB reduction. Specifically designed earmuffs and inserts, as shown in **Figure 5,** can reduce the sound level by up to 45 dB.

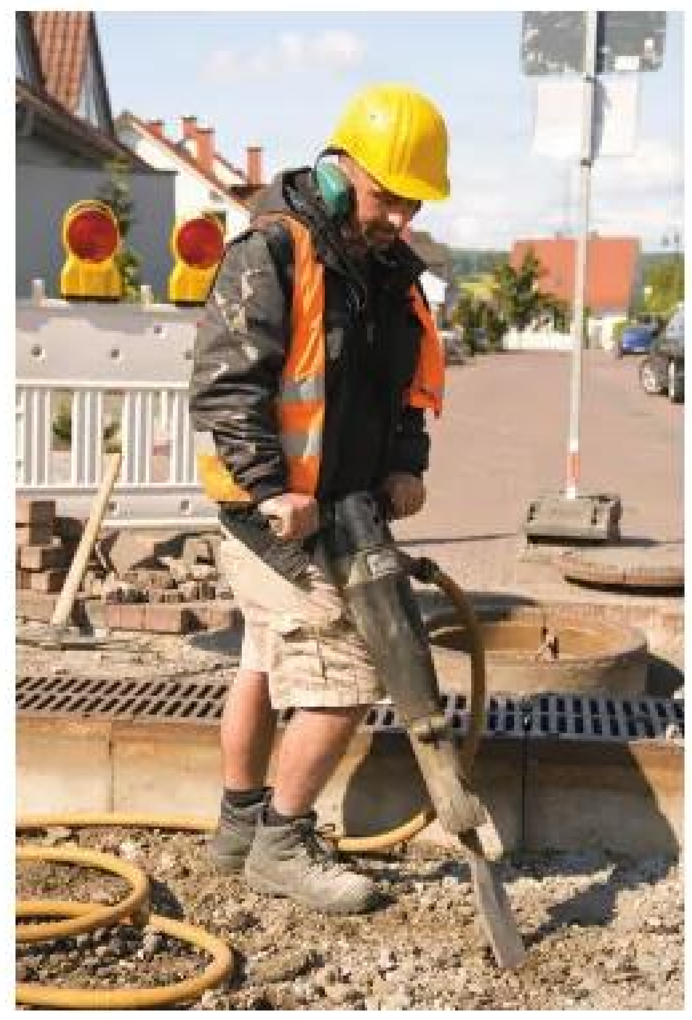


Figure 5 Hearing loss can occur with continuous exposure to loud sounds. Workers in many occupations, such as construction, wear ear protection. The jackhammer this worker is operating has a sound level of 130 dB.

The Doppler Effect

Have you ever noticed that the pitch of a fast car changed as the vehicle sped past you? The pitch was higher when the vehicle was moving toward you, then it dropped to a lower pitch as the vehicle moved away. The change in frequency of sound caused by the movement of either the source, the detector, or both is called the **Doppler effect.** The Doppler effect is illustrated in **Figure 6.**

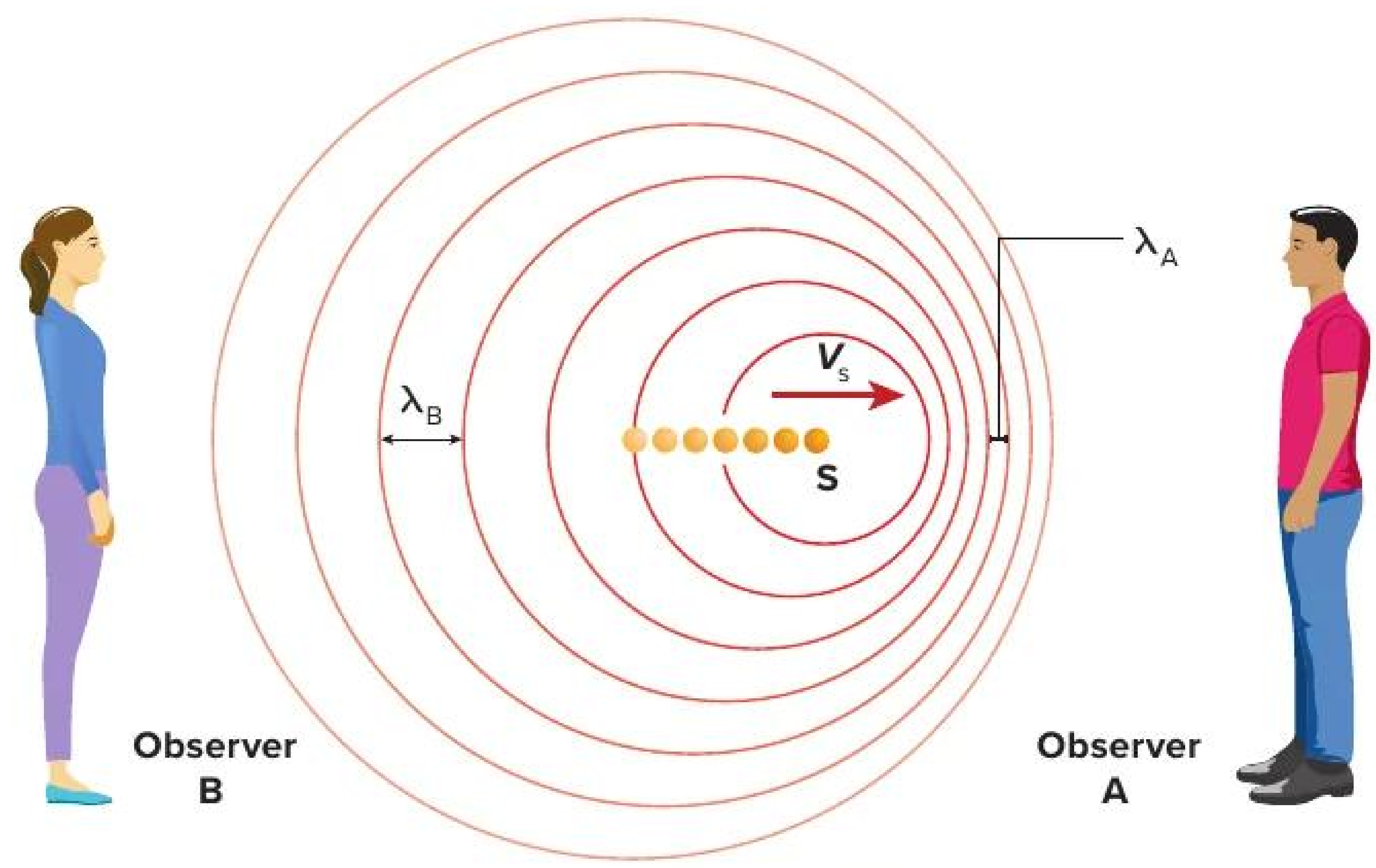


Figure 6 As a sound producing source moves toward observer A, the wavelength is shortened to λ_A. As the source moves away from observer B, the wavelength is lengthened to λ_B.

Describe *What is the relative difference in the frequency of the detected sound for each observer?*

The sound source (*S*) is moving to the right with a speed of v_s. The waves that the source emits spread in circles centered on the source at the time it produced the waves. As the source moves toward the sound detector, observer A in **Figure 6,** more waves are crowded into the space between them. The wavelength is shortened to λ_A. Because the speed of sound is not changed, more crests reach the ear each second, which means that the frequency of the detected sound increases. When the source is moving away from the detector, observer B in **Figure 6,** the wavelength is lengthened to λ_B, fewer crests reach the ear each second, and the detected frequency is lower.

A Doppler shift also occurs if the detector is moving and the source is stationary. As the detector approaches a stationary source, it encounters more wave crests each second than if it were still, and a higher frequency is detected. If the detector recedes from the source, fewer crests reach it each second, resulting in a lower detected frequency.

Get It?

Compare the wavelength and frequency heard by an observer in front of the moving fire engine at the beginning of the module with the wavelength and frequency heard by an observer behind the fire engine.

For any combination of moving source and moving observer, the frequency that the observer hears can be found using the relationship below.

Doppler Effect

The frequency perceived by a detector is equal to the velocity of the detector relative to the velocity of the wave, divided by the velocity of the source relative to the velocity of the wave, multiplied by the wave's frequency.

$$f_d = f_s \frac{v - v_d}{v - v_s}$$

In the Doppler effect equation, v is the velocity of the sound wave, v_s is the velocity of the sound's source, and v_d is the velocity of the observer of interest, who is detecting the sound. The subscript *d* is used instead of the letter *o* to avoid confusion with the number zero. The same subscripts are used to denote the corresponding frequencies.

Defining the coordinate system

As you solve problems using the above equation, be sure to define the coordinate system so that the positive direction is from the source to the detector. The sound waves will be approaching the detector from the source, so the velocity of sound is always positive.

Try drawing diagrams to confirm that the term $\frac{v - v_d}{v - v_s}$ behaves as you would predict based on what you have learned about the Doppler effect. Notice that for a source moving toward the detector (positive direction, which results in a smaller denominator compared to a stationary source) and for a detector moving toward the source (negative direction and increased numerator compared to a stationary detector), the detected frequency (f_d) increases.

Similarly, if the source moves away from the detector or if the detector moves away from the source, then f_d decreases. Read the Connecting Math to Physics feature below to see how the Doppler effect equation reduces when the source or observer is stationary.

CONNECTING MATH to Physics

Reducing Equations When an element in an equation is equal to zero, the equation might reduce to a form that is easier to use.

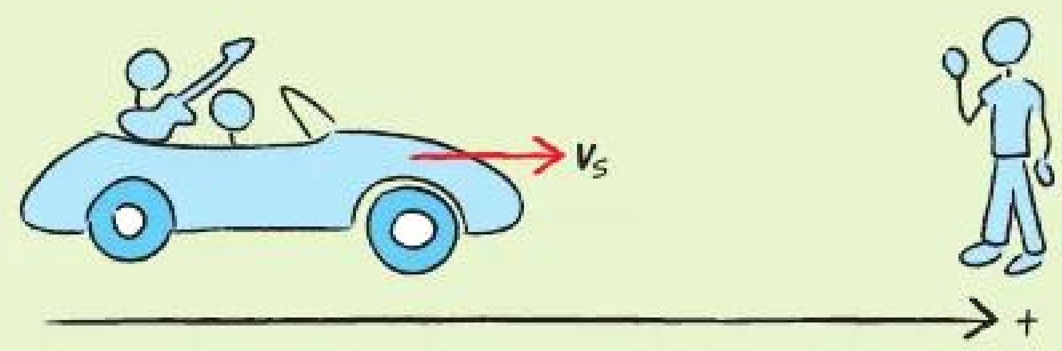
Stationary detector, source in motion: $v_d = 0$	Stationary source, detector in motion: $v_s = 0$
$f_d = f_s \frac{v - v_d}{v - v_s}$	$f_d = f_s \frac{v - v_d}{v - v_s}$
$= f_s \frac{v}{v - v_s}$	$= f_s \frac{v - v_d}{v}$
$= f_s \frac{\frac{v}{v}}{\frac{v}{v} - \frac{v_s}{v}}$	$= f_s\left(\frac{v}{v} - \frac{v_d}{v}\right)$
$= f_s \frac{1}{1 - \frac{v_s}{v}}$	$= f_s\left(1 - \frac{v_d}{v}\right)$

EXAMPLE Problem 1

THE DOPPLER EFFECT A guitar player sounds C above middle C (523 Hz) while traveling in a convertible at 24.6 m/s. If the car is coming toward you, what frequency would you hear? Assume that the temperature is 20°C.

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Establish a coordinate axis. Make sure that the positive direction is from the source to the detector.
- Show the velocities of the source and detector.



Known	Unknown
$v = +343$ m/s	$f_d = ?$
$v_s = +24.6$ m/s	
$v_d = 0$ m/s	
$f_s = 523$ Hz	

2 SOLVE FOR THE UNKNOWN

Use $f_d = f_s \frac{v - v_d}{v - v_s}$ with $v_d = 0$ m/s.

$$f_d = f_s \frac{1}{1 - \frac{v_s}{v}}$$

$$= 523 \text{ Hz} \left(\frac{1}{1 - \frac{24.6 \text{ m/s}}{343 \text{ m/s}}}\right)$$

Substitute $v = +343$ m/s, $v_s = +24.6$ m/s, and $f_s = 523$ Hz.

$$= 564 \text{ Hz}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** Frequency is measured in hertz.
- **Is the magnitude realistic?** The source is moving toward you, so the frequency should be increased.

PRACTICE Problems

ADDITIONAL PRACTICE

1. Repeat Example Problem 1, but with the car moving away from you. What frequency would you hear?
2. You are in an automobile, like the one in **Figure 7,** traveling toward a pole-mounted warning siren. If the siren's frequency is 365 Hz, what frequency do you hear? Use 343 m/s as the speed of sound.

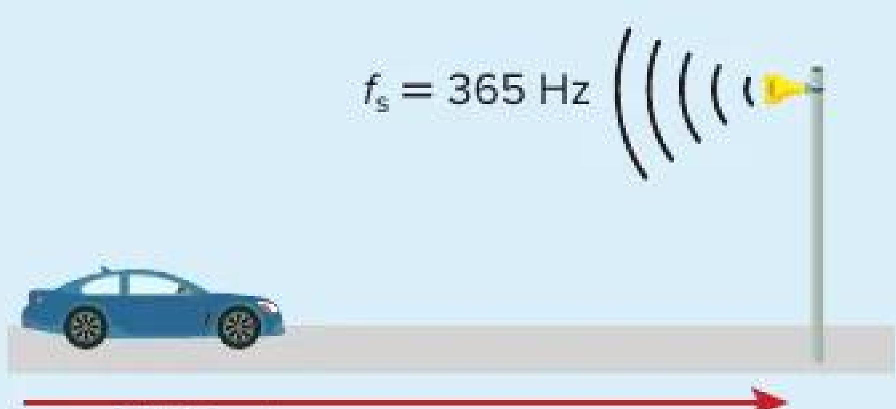


Figure 7

3. You are in an automobile traveling at 55 mph (24.6 m/s). A second automobile is moving toward you at the same speed. Its horn is sounding at 475 Hz. What frequency do you hear? Use 343 m/s as the speed of sound.
4. A submarine is moving toward another submarine at 9.20 m/s. It emits a 3.50-MHz ultrasound. What frequency would the second sub, at rest, detect? The speed of sound in water at the depth the submarines are moving is 1482 m/s.
5. **CHALLENGE** A trumpet plays middle C (262 Hz). How fast would it have to be moving to raise the pitch to C sharp (277 Hz)? Use 343 m/s as the speed of sound.

Applications of the Doppler effect The Doppler effect occurs in all wave motion, both mechanical and electromagnetic. It has many applications. Radar detectors use the Doppler effect to measure the speed of baseballs and automobiles. Astronomers observe light from distant galaxies and use the Doppler effect to measure their speeds. Physicians can detect the speed of the moving heart wall in a fetus by means of the Doppler effect in ultrasound.

BIOLOGY Connection Bats use the Doppler effect to detect and catch flying insects. When an insect is flying faster than a bat, the reflected frequency is lower, but when the bat is catching up to the insect, as in **Figure 8,** the reflected frequency is higher. Not only do bats use sound waves to navigate and locate their prey, but they often must do so in the presence of other bats. This means they must discriminate their own calls and reflections against a background of many other sounds of many frequencies. Scientists continue to study bats and their amazing abilities to use sound waves.

Figure 8 Bats use the Doppler effect to locate and catch flying insects, such as the moth shown here. As the bat catches up to the moth, the frequency of reflected sound waves increases.

Check Your Progress

6. **Wave Characteristics** What physical characteristic of a sound wave should be changed to alter the pitch? The loudness?
7. **Graph** The eardrum moves back and forth in response to the pressure variations of a sound wave. Sketch a graph of the displacement of the eardrum versus time for two cycles of a 1.0-kHz tone and of a 2.0-kHz tone.
8. **Effect of Medium** List two characteristics of sound that are affected by the medium through which the sound passes and two characteristics that are not affected.
9. **Decibel Scale** How many times greater is the sound pressure level of a typical rock concert (110 dB) than a normal conversation (50 dB)?
10. **Early Detection** In the nineteenth century, people put their ears to a railroad track to get an early warning of an approaching train. Why did this work?
11. **Bats** A bat emits short pulses of high-frequency sound and detects the echoes.
    a. In what way would the echoes from large and small insects compare if they were the same distance from the bat?
    b. In what way would the echo from an insect flying toward the bat differ from that of an insect flying away from the bat?
12. **Critical Thinking** Can a trooper using a radar detector at the side of the road determine the speed of a car at the instant the car passes the trooper? Explain.

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LESSON 2
THE PHYSICS OF MUSIC

FOCUS QUESTION
How is pitch controlled in a musical instrument?

Sources of Sound

In the middle of the nineteenth century, German physicist Hermann Helmholtz studied sound production in musical instruments and the human voice. In the twentieth century, scientists and engineers developed electronic equipment that permits a detailed study of sound and the creation of electronic instruments and recording devices so we can listen to music anywhere.

Recall that sound is produced by a vibrating object. The vibrations of the object create particle motions that cause pressure oscillations in the air. In brass instruments, such as the trumpet, the tuba, and the bugle, the lips of the performer vibrate, as shown in **Figure 9.** Reed instruments, such as the clarinet and the saxophone, have a thin wooden strip called a reed that vibrates as a result of air blown across it, as shown in **Figure 9.** In flutes and organ pipes, air is forced across an opening in a pipe. Air moving past the opening sets the column of air in the instrument into vibration.

Figure 9 The sound produced by an instrument is partly determined by the structure of the mouthpiece.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE
Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE
GO ONLINE to find these activities and more resources.

PhysicsLAB: Speed of Sound
Calculate the speed of sound using the relationship among frequency, wavelength, and speed.

Review the News
Obtain information from a current news story about sound waves and their applications. Evaluate your source and communicate your findings to your class.

Stringed instruments, such as the piano, the guitar, and the violin, work by setting wires or strings into vibration. In the piano, the wires are struck; for the guitar, they are plucked; and for the violin, the friction of the bow causes the strings to vibrate. The strings are attached to a sounding board that vibrates with the strings. The vibrations of the sounding board cause the pressure oscillations in the air that we hear as sound. Electric guitars use electronic devices to detect and amplify the vibrations of the guitar strings.

A loudspeaker has a cone that is made to vibrate by electrical currents. The surface of the cone creates the sound waves that travel to your ear and allow you to hear music. Musical instruments such as gongs, cymbals, and drums are other examples of vibrating surfaces that are sources of sound.

The human voice is produced by vibrations of the vocal cords, which are two membranes located in the throat. Air from the lungs rushing through the throat starts the vocal cords vibrating. The frequency of vibration is controlled by the muscular tension placed on the vocal cords. The more tension on the vocal cords, the more rapidly they vibrate, resulting in a higher pitch sound. If the vocal cords are more relaxed, they vibrate more slowly and produce lower-pitched sounds.

Get It?

Describe How does a vocalist sing higher pitched notes?

Resonance in Air Columns

If you have ever used just the mouthpiece of a brass or wind instrument, you know that while the vibration of your lips or the reed alone makes a sound, it is difficult to control the pitch. The long tube that makes up the instrument must be attached if music is to result. When the instrument is played, the air within this tube vibrates at the same frequency, or in resonance, with a particular vibration of the lips or reed. Remember that resonance increases the amplitude of a vibration by repeatedly applying a small external force to the vibrating air particles at the natural frequency of the air column. The length of the air column determines the frequencies of the vibrating air that will resonate. For wind and brass instruments, such as flutes, trumpets, and trombones, changing the length of the column of vibrating air varies the pitch of the instrument. The mouthpiece simply creates a mixture of different frequencies, and the resonating air column acts on a particular set of frequencies to amplify a single note, turning noise into music. A tuning fork above a hollow tube can provide resonance in an air column, as shown in **Figure 10.**

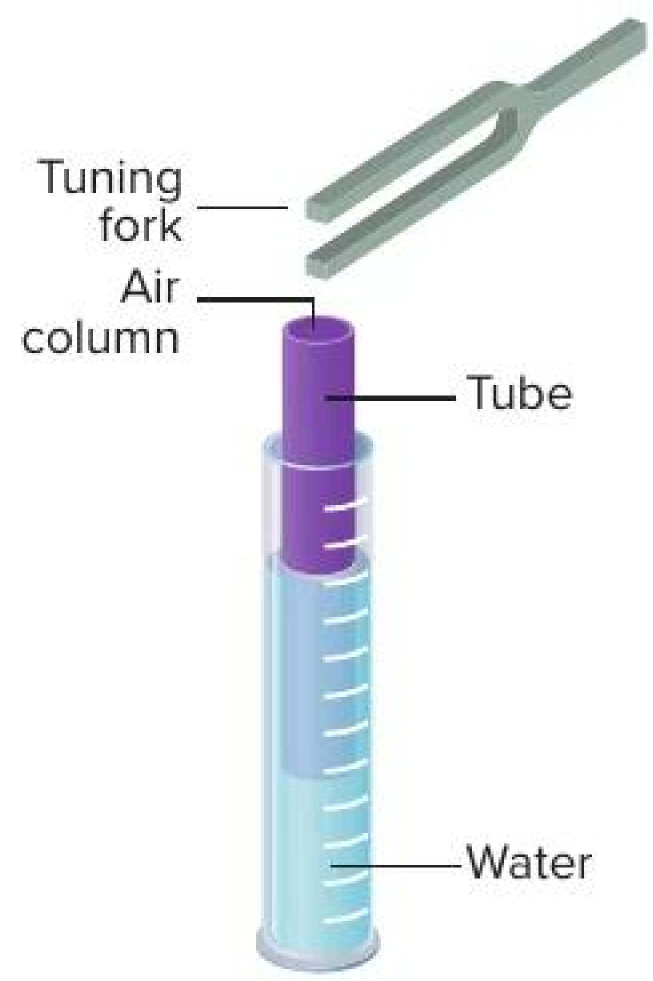


Figure 10 As the tube is raised or lowered, the length of the air column changes, which causes the sound's volume to change.

CCC CROSSCUTTING CONCEPTS

Cause and Effect Moving the tube in **Figure 10** up and down causes the volume of the sound to change. Develop a mathematical representation to relate the frequency, wavelength, speed of sound, and length of air column in the tube to explain this effect.

The tube is placed in water so that the bottom end of the tube is below the water surface. A resonating tube with one end closed to air is called a **closed-pipe resonator.** The length of the air column is changed by adjusting the height of the top of the tube above the water. If the tuning fork is struck with a rubber hammer, the sound alternately becomes louder and softer as the length of the air column is varied by moving the tube up and down in the water. The sound is loud when the air column is in resonance with the tuning fork because the resonating air column intensifies the sound of the tuning fork.

Standing pressure wave How does resonance occur? The vibrating tuning fork produces a sound wave. This wave of alternate high- and low-pressure variations moves down the air column. When the wave hits the water surface, it is reflected back up to the tuning fork, as shown in **Figure 11.** If the reflected high-pressure wave reaches the tuning fork at the same moment that the fork produces another high-pressure wave, then the emitted and returning waves reinforce each other. This reinforcement of waves creates a standing wave, and resonance occurs.

An **open-pipe resonator** is a resonating tube with both ends open that will resonate with a sound source. In this case, the sound wave does not reflect off a closed end, but rather off an open end. If the high-pressure part of the wave strikes the open end, the rebounding wave will be low-pressure at that point, as shown in **Figure 11.**

Resonance lengths **Figure 12** shows a standing sound wave in a pipe represented by a sine wave. Sine waves can represent either the air pressure or the displacement of the air particles. Recall that standing waves have nodes and antinodes. A node is the stationary point where two equal wave pulses meet and are in the same location. An antinode is the place of largest displacement when two wave pulses meet.

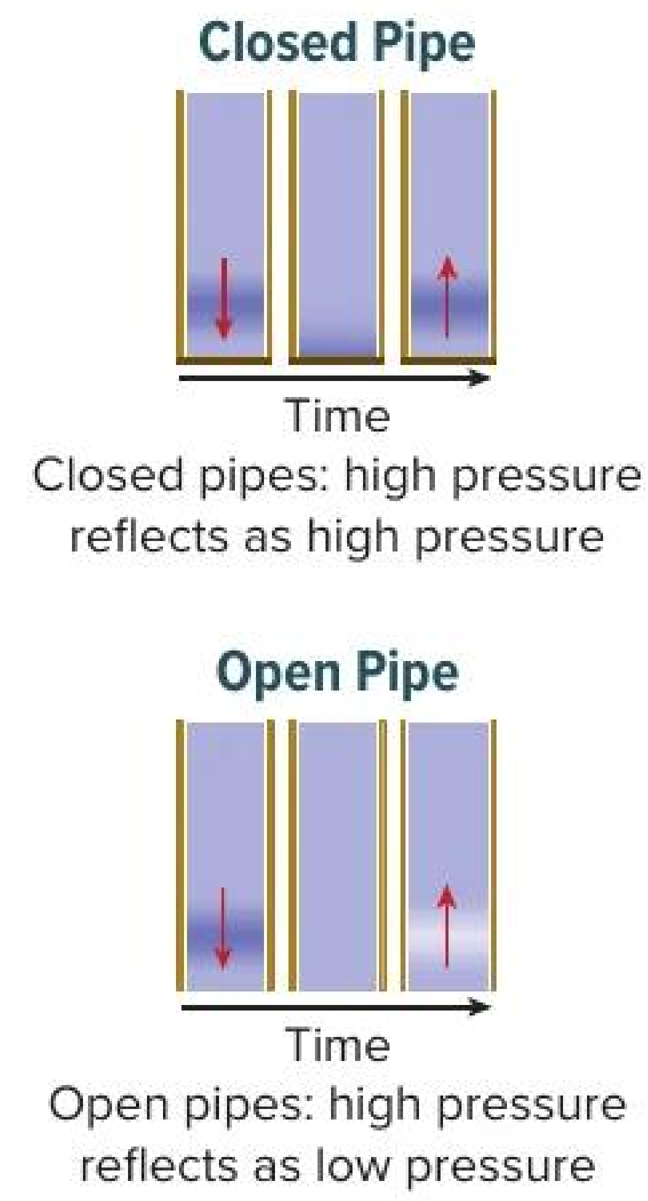


Figure 11 In closed pipes, the sound wave reflects off the closed end. High-pressure waves reflect as high pressure. In open pipes, the sound wave reflects off an open end. High-pressure waves are reflected as low pressure.

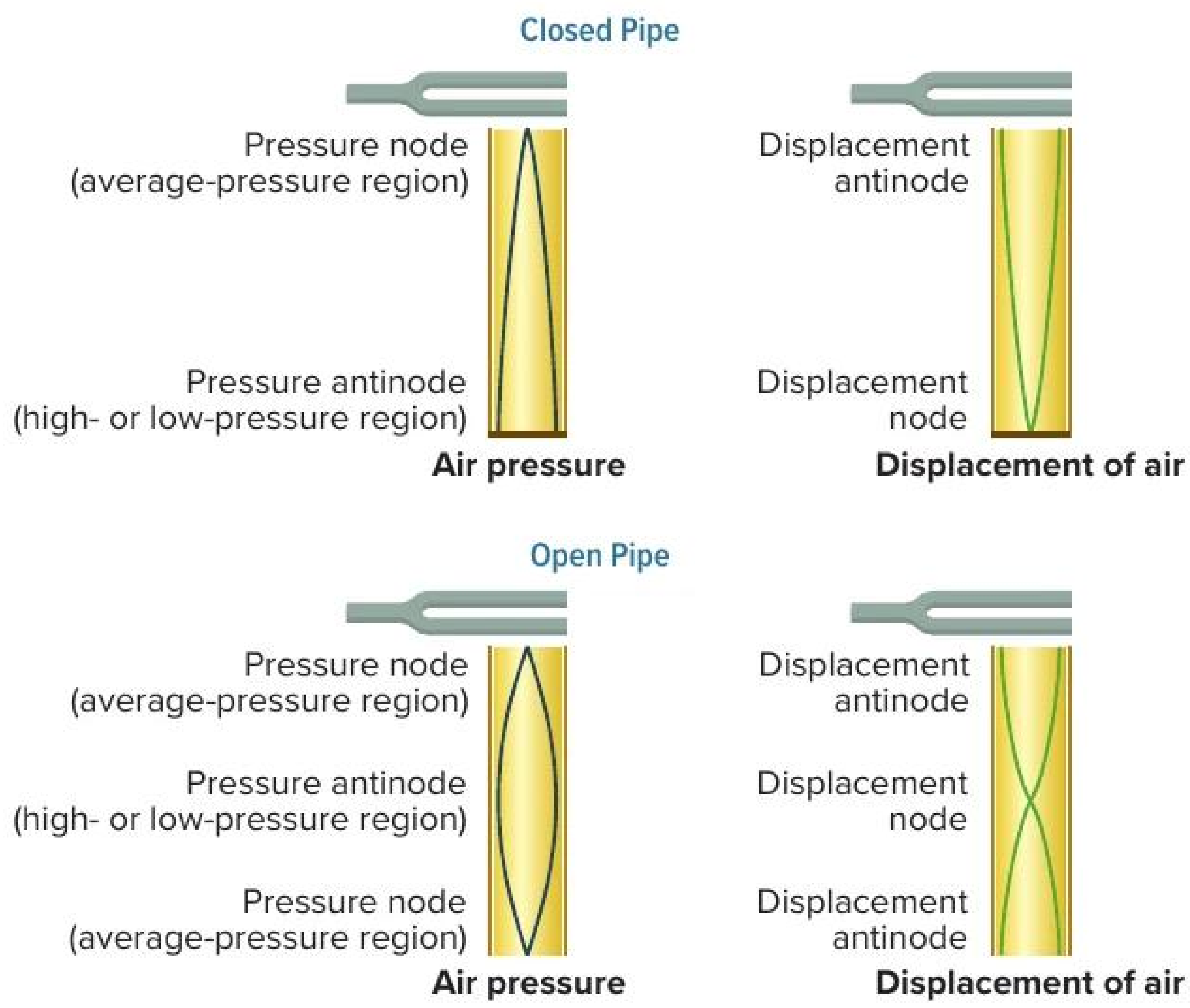


Figure 12 Standing waves in pipes can be represented by sine waves.

Identify *Which are the areas of mean atmospheric pressure in the air pressure graphs?*

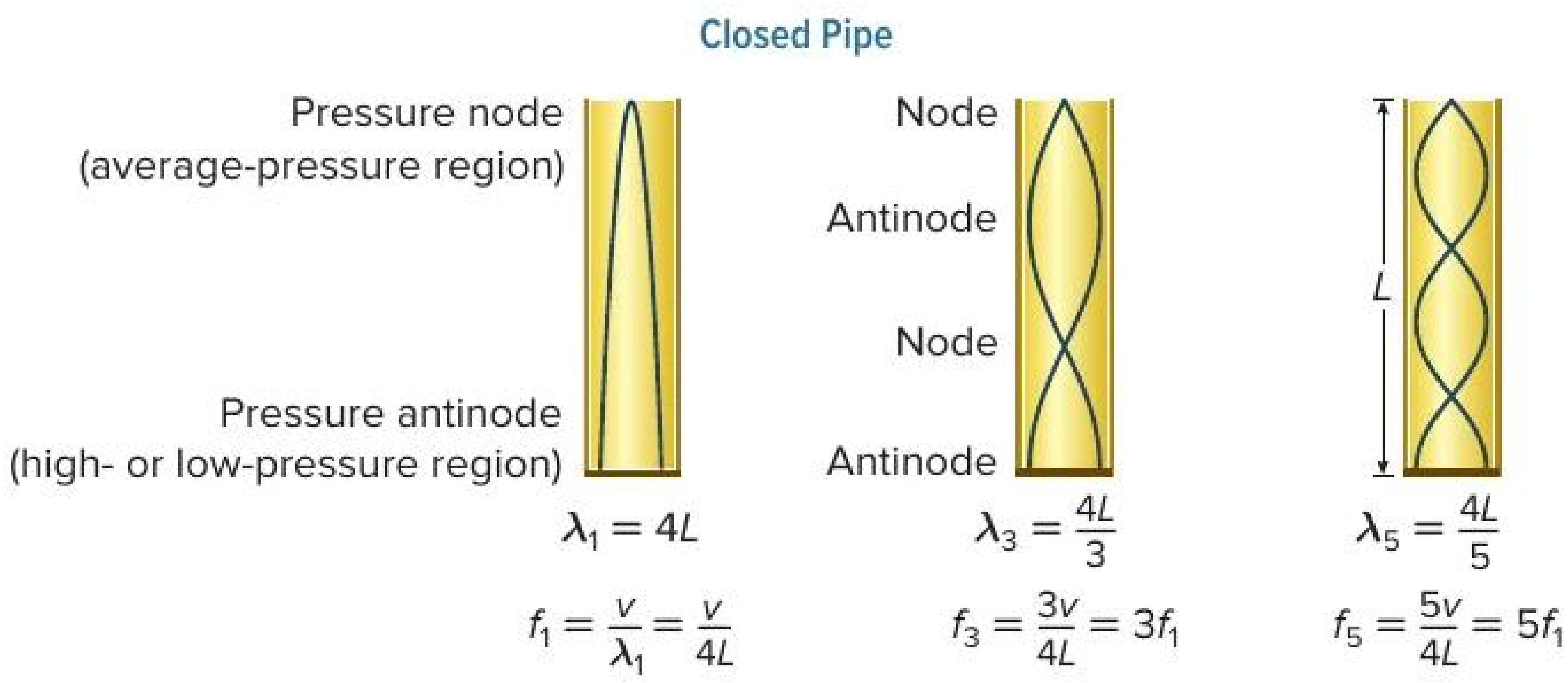


Figure 13 A closed pipe resonates when its length is an odd number of quarter wavelengths.

In the pressure graphs, the nodes are regions of mean atmospheric pressure. At the antinodes, the pressure oscillates between its maximum and minimum values. In the case of the displacement graph, the antinodes are regions of high displacement and the nodes are regions of low displacement. In both cases, two adjacent antinodes (or two nodes) are separated by one-half wavelength.

Get It?

Explain the difference between a node and an antinode on a displacement graph.

Resonance frequencies in a closed pipe If a closed end must act as a node, and an open end must act as an antinode, what is the shortest column of air that will resonate in a closed pipe? **Figure 13** shows that it must be one-fourth of a wavelength. As the frequency is increased, additional resonance lengths are found at half-wavelength intervals. Thus, columns of length $\frac{\lambda}{4}$, $\frac{3\lambda}{4}$, $\frac{5\lambda}{4}$, $\frac{7\lambda}{4}$, and so on will all be in resonance with a tuning fork that produces sound of wavelength λ.

In practice, the first resonance length is slightly longer than one-fourth of a wavelength. This is because the pressure variations do not drop to zero exactly at the open end of the pipe. Actually, the node is approximately 0.4 pipe diameters beyond the end. Additional resonance lengths, however, are spaced by exactly one-half of a wavelength. Measurements of the spacing between resonances can be used to find the velocity of sound in air, as in Example Problem 2.

Resonance frequencies in an open pipe The shortest column of air that can have nodes at both ends is one-half of a wavelength long, as shown in **Figure 14.** As the frequency is increased, additional resonance lengths are found at half-wavelength intervals. Thus, columns of length $\frac{\lambda}{2}$, λ, $\frac{3\lambda}{2}$, 2λ, and so on will be in resonance with a tuning fork.

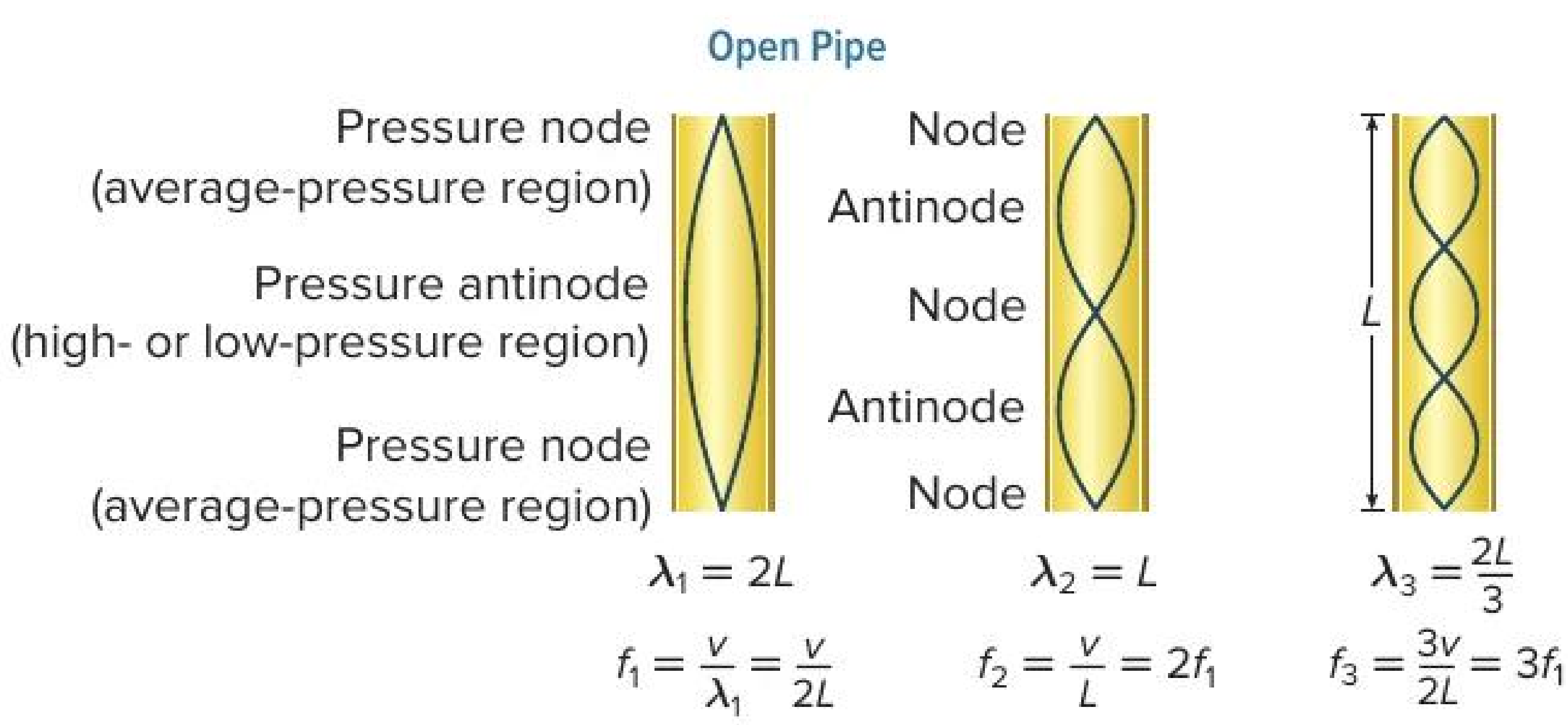


Figure 14 An open pipe resonates when its length is an even number of quarter wavelengths.

Explain *How does the length at which an open pipe resonates differ from the length at which a closed pipe resonates?*

Figure 15 A flute is an example of an open-pipe resonator. The hanging pipes of a marimba and seashells are examples of closed-pipe resonators.

If open and closed pipes of the same length are used as resonators, the wavelength of the resonant sound for the open pipe will be half as long as that for the closed pipe. Therefore, the frequency will be twice as high for the open pipe as for the closed pipe. For both pipes, resonance lengths are spaced by half-wavelength intervals.

Get It?

Predict A tuning fork plays a sound that has a wavelength of 0.78 m. A pipe that is 0.39 m long resonates with the tuning fork. Is the pipe open or closed? Explain your reasoning.

Hearing resonance Musical instruments use resonance to increase the loudness of particular notes. Open-pipe resonators include flutes, shown in **Figure 15.** Clarinets and the hanging pipes under marimbas and xylophones are examples of closed-pipe resonators. If you shout into a long tunnel, the booming sound you hear is the tunnel acting as a resonator. The seashell in **Figure 15** also acts as a closed-pipe resonator.

Real-World Physics

HEARING AND FREQUENCY The human auditory canal acts as a closed-pipe resonator that increases the ear's sensitivity for frequencies between 2000 and 5000 Hz, but the full range of frequencies that people hear extends from 20 to 20,000 Hz. A dog's hearing extends to frequencies as high as 45,000 Hz, and a cat's extends to frequencies as high as 100,000 Hz.

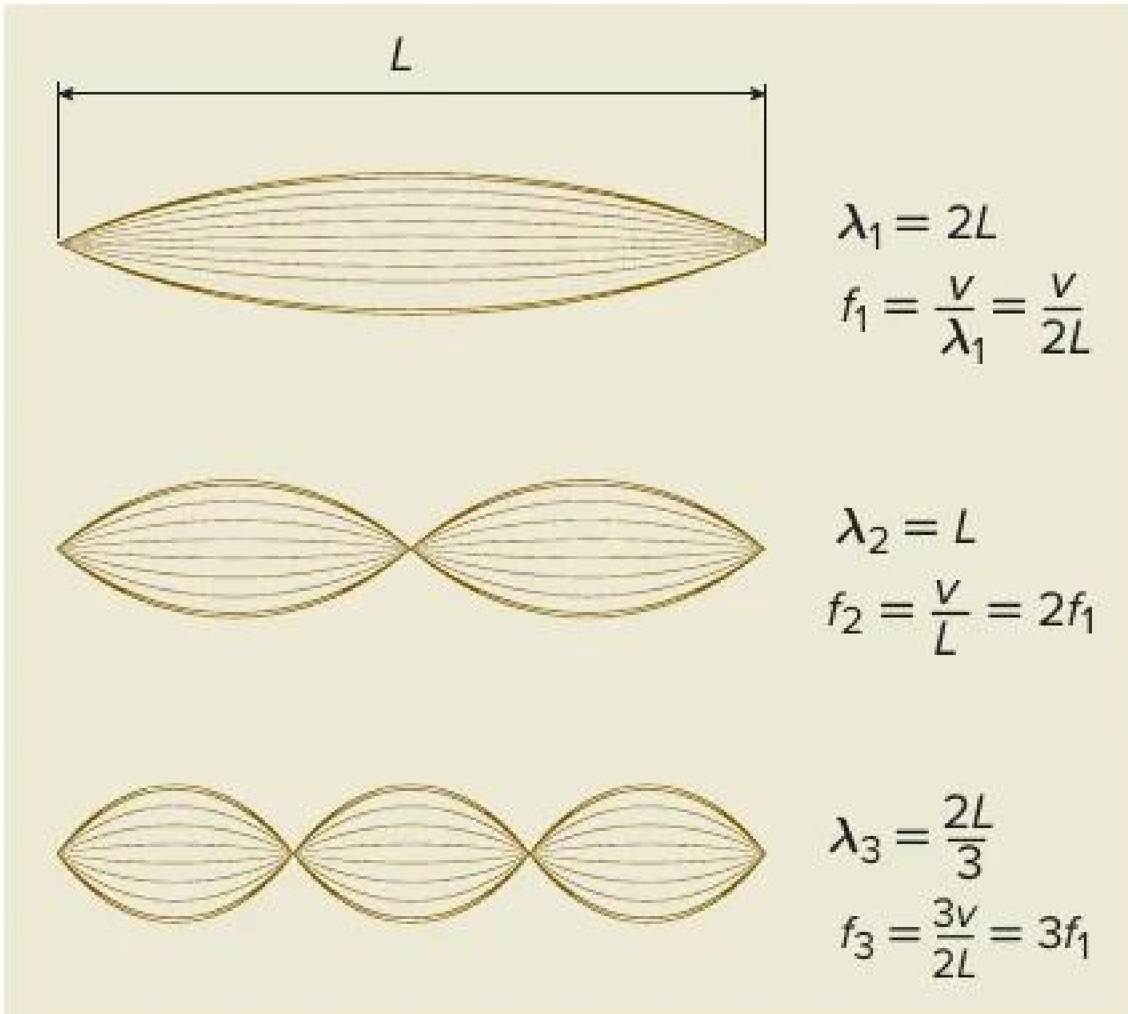


Figure 16 A string resonates with standing waves when its length is a whole number of half wavelengths.

Resonance on Strings

Although plucking, bowing, or striking strings produces variation in waveforms, waveforms on vibrating strings have many characteristics in common with standing waves on springs and ropes. A string on an instrument is clamped at both ends, and therefore, the string must have a node at each end when it vibrates. In **Figure 16,** you can see that the first mode of vibration has an antinode at the center and is one-half a wavelength long. The next resonance occurs when one wavelength fits on the string, and additional standing waves arise when the string length is $\frac{3\lambda}{2}$, 2λ, $\frac{5\lambda}{2}$, and so on. As with an open pipe, the resonant frequencies are whole-number multiples of the lowest frequency.

Recall that the speed of travel of a wave depends on the type of wave and the medium through which it is passing. For a string, the speed depends on the tension of the string, as well as its mass per unit length. The tighter the string, the faster the wave moves along it, and therefore, the higher the frequency of its standing waves. This makes it possible to tune a stringed instrument by changing the tension of its strings. Because strings are so small in cross-sectional area, they move very little air when they vibrate. This makes it necessary to attach them to a sounding board, which transfers their vibrations to the air and produces a stronger sound wave. Unlike the strings themselves, the sounding board should not resonate at any single frequency. Its purpose is to convey the vibrations of all the strings to the air, and therefore it should vibrate well at all frequencies produced by the instrument. Because of the complicated interactions among the strings, the sounding board, and the air, the design and construction of stringed instruments are complex processes, considered by many to be as much an art as a science.

Get It?

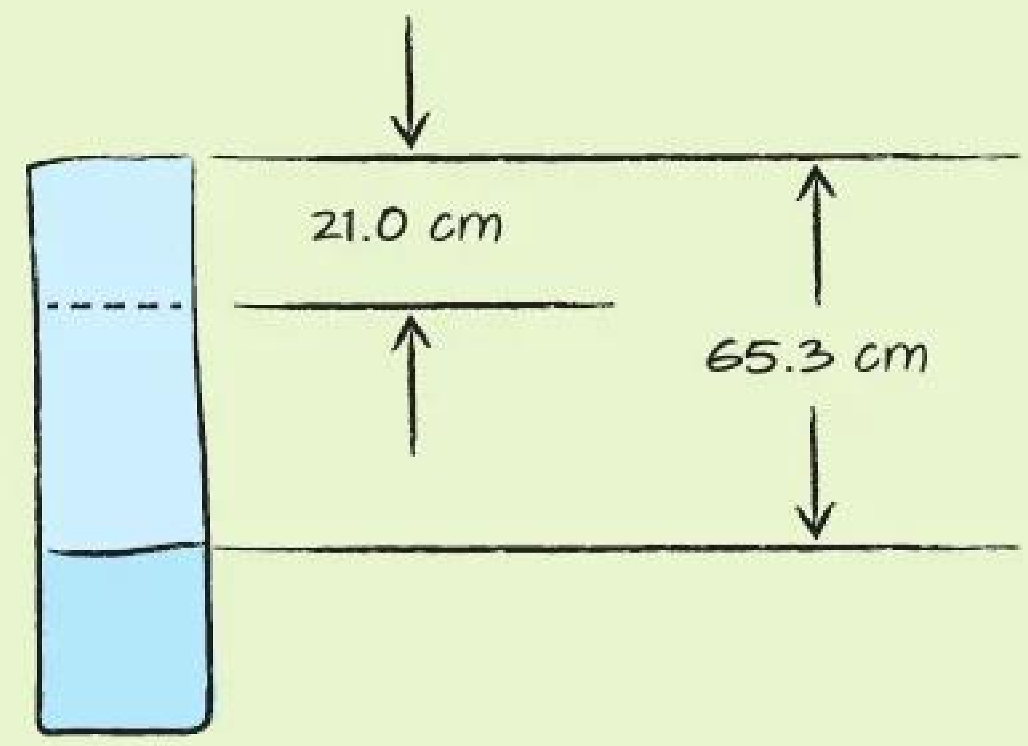
Describe the relationship between the tension of a string and the speed of a wave as it travels along the string.

EXAMPLE Problem 2

FINDING THE SPEED OF SOUND USING RESONANCE When a tuning fork with a frequency of 392 Hz is used with a closed-pipe resonator, the loudest sound is heard when the column is 21.0 cm and 65.3 cm long. What is the speed of sound in this case? Is the temperature warmer or cooler than normal room temperature, which is 20°C? Explain your answer.

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the closed-pipe resonator.
- Mark the resonance lengths.



Known	Unknown
$f = 392$ Hz	$v = ?$
$L_A = 21.0$ cm	
$L_B = 65.3$ cm	

2 SOLVE FOR THE UNKNOWN

Solve for the length of the wave using the length-wavelength relationship for a closed pipe.

$L_B - L_A = \frac{1}{2}\lambda$

$\lambda = 2(L_B - L_A)$ — Rearrange the equation for λ.

$= 2(0.653 \text{ m} - 0.210 \text{ m})$ — Substitute $L_B = 0.653$ m, $L_A = 0.210$ m.

$= 0.886$ m

Use $\lambda = \frac{v}{f}$

$v = f\lambda$ — Rearrange the equation for v.

$= (392 \text{ Hz})(0.886 \text{ m})$ — Substitute $f = 392$ Hz, $\lambda = 0.886$ m.

$= 347$ m/s

The speed is slightly greater than the speed of sound at 20°C, indicating that the temperature is slightly higher than normal room temperature.

3 EVALUATE THE ANSWER

- **Are the units correct?** $(\text{Hz})(\text{m}) = \left(\frac{1}{\text{s}}\right)(\text{m}) = \text{m/s}$. The answer's units are correct.
- **Is the magnitude realistic?** The speed is slightly greater than 343 m/s, which is the speed of sound at 20°C.

PRACTICE Problems

ADDITIONAL PRACTICE

13. A 440-Hz tuning fork is used with a resonating column to determine the velocity of sound in helium gas. If the spacing between resonances is 110 cm, what is the velocity of sound in helium gas?
14. The frequency of a tuning fork is unknown. A student uses an air column at 27°C and finds resonances spaced by 20.2 cm. What is the frequency of the tuning fork? Use the speed calculated in Example Problem 2 for the speed of sound in air at 27°C.
15. A 440-Hz tuning fork is held above a closed pipe. Find the spacing between the resonances when the air temperature is 20°C.
16. **CHALLENGE** A bugle can be thought of as an open pipe. If a bugle were straightened out, it would be 2.65-m long.
    a. If the speed of sound is 343 m/s, find the lowest frequency that is resonant for a bugle (ignoring end corrections).
    b. Find the next two resonant frequencies for the bugle.

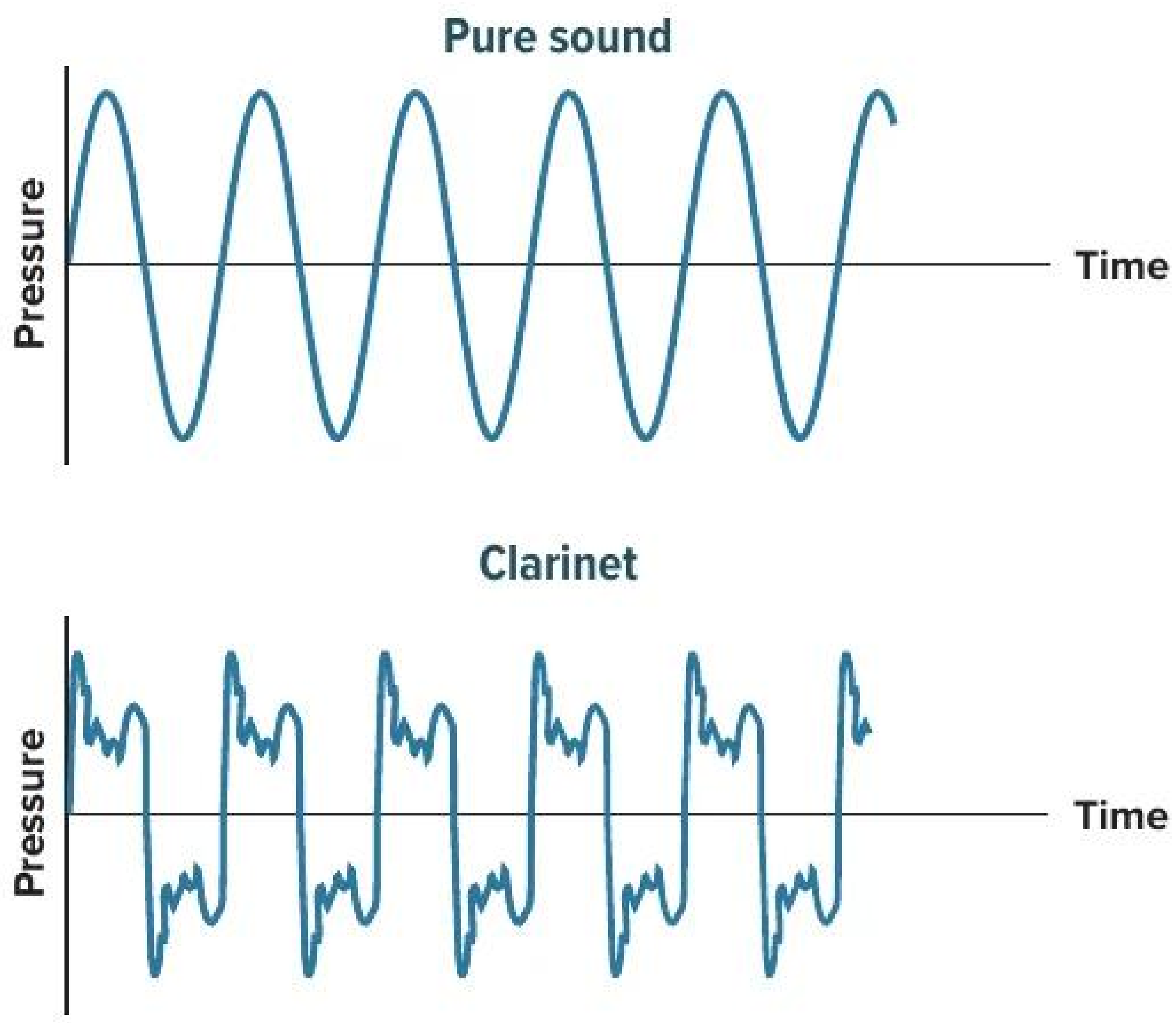


Figure 17 The pure sound produced by a tuning fork is represented by a simple sine wave. The more complex sound produced by a clarinet is represented in the bottom graph.

Sound Quality

A tuning fork produces a soft and uniform sound. This is because its tines vibrate like simple harmonic oscillators and produce the simple sine wave shown in the top graph in **Figure 17.** Sounds made by the human voice and musical instruments are much more complex, like the wave in the bottom graph in **Figure 17.** Both waves have the same frequency, or pitch, but they sound very different.

The complex sound wave is actually a blend of several different frequencies. The shape of the wave depends on the relative amplitudes of these frequencies. Different sources provide different combinations of frequencies. In musical terms, the difference between the waves from different instruments is called timbre (TAM bur), tone quality, or color.

The sound spectrum: fundamental and harmonics

The complex sound wave in **Figure 17** was made by a clarinet. Why does the clarinet produce such a sound wave? The air column in a clarinet acts as a closed pipe. Look back at **Figure 13,** which shows three resonant frequencies for a closed pipe. The clarinet acts as a closed pipe, so for a clarinet of length L the lowest frequency (f_1) that will be resonant is $\frac{v}{4L}$.

For a musical instrument, the lowest frequency of sound that resonates is called the **fundamental.** A closed pipe also will resonate at $3f_1$, $5f_1$, and so on. These higher frequencies, which are whole-number multiples of the fundamental frequency, are called **harmonics.** It is the addition of these harmonics that gives a clarinet its distinctive timbre.

Get It?

Explain the relationship between the fundamental and the harmonics of a musical instrument.

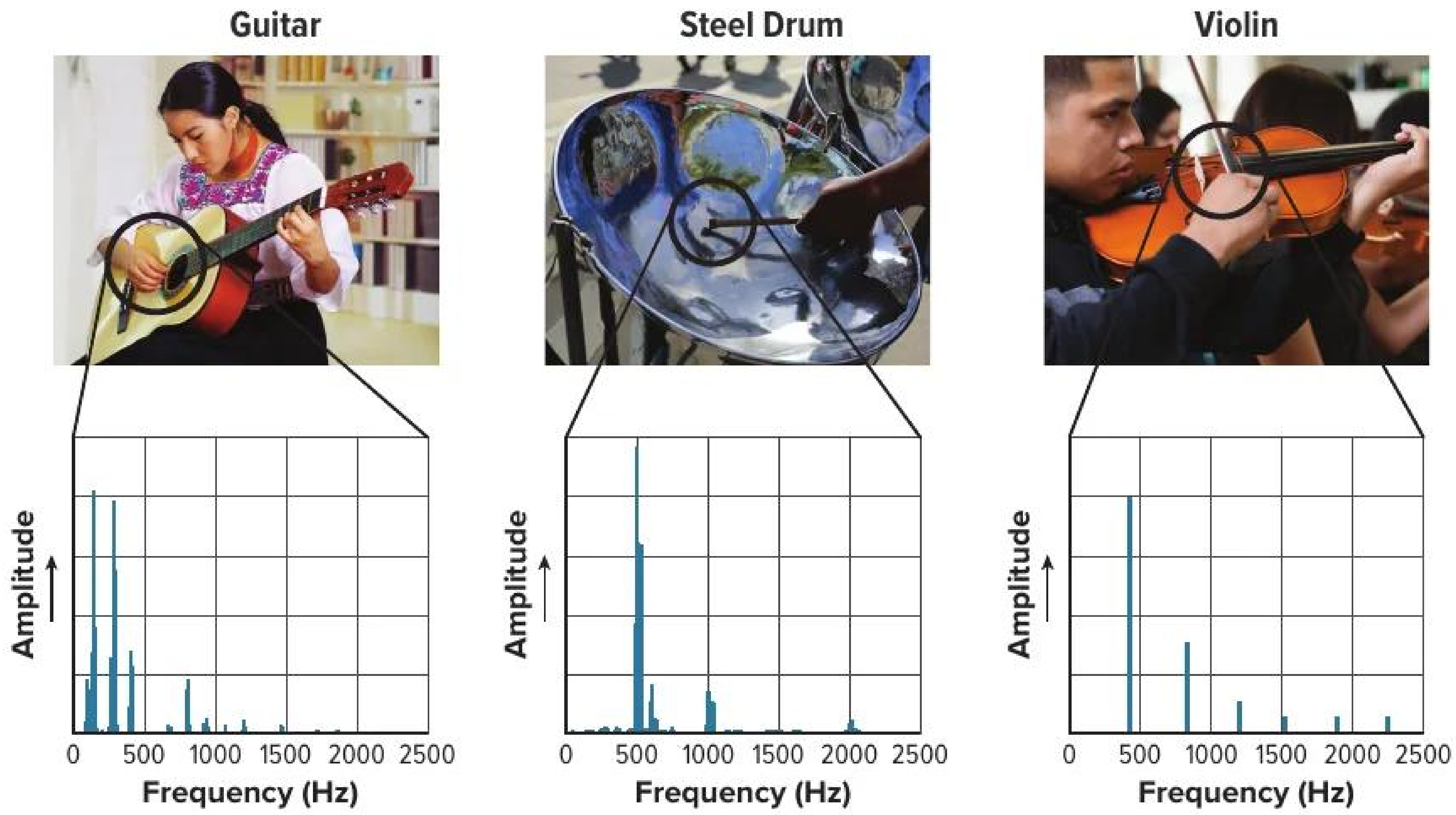


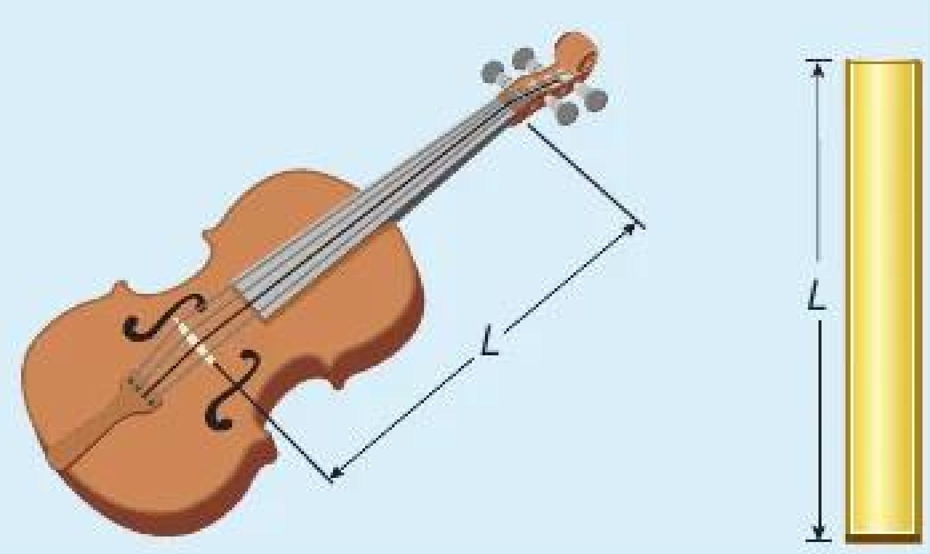
Figure 18 A guitar, a steel drum, and a violin produce characteristic sound spectra. Each spectrum is unique, as is the timbre of the instrument.

Some instruments, such as a flute, act as open-pipe resonators. Their fundamental frequency, which is also the first harmonic, is $f_1 = \frac{v}{2L}$ with subsequent harmonics at $2f_1$, $3f_1$, $4f_1$, and so on. Different combinations of these harmonics give each instrument its own unique timbre. Each harmonic on the instrument can have a different amplitude as well. A graph of the amplitude of a wave versus its frequency is called a sound spectrum. The spectra of three instruments are shown in **Figure 18.**

Consonance and dissonance When two different pitches are played at the same time, the resulting sound can be either pleasant or jarring. In musical terms, several pitches played together are called a chord. An unpleasant set of pitches is called **dissonance.** If the combination of pitches is pleasant, the sounds are said to be in **consonance.** What sounds pleasing varies between cultures, but most Western music is based upon the observations of Pythagoras of ancient Greece. He noted that pleasing sounds resulted when strings had lengths in small, whole-number ratios, such as 1:2, 2:3, or 3:4. This means their pitches (frequencies) will also have small, whole-number ratios.

PHYSICS Challenge

1. Determine the tension, F_T, in a violin string of mass *m* and length *L* that will play the fundamental note at the same frequency as a closed pipe also of length *L*. Express your answer in terms of *m*, *L*, and the speed of sound in air, *v*. The equation for the speed of a wave on a string is $v_{string} = \sqrt{\frac{F_T}{\mu}}$, where F_T is the tension in the string and μ is the mass per unit length of the string.
2. What is the tension in a string of mass 1.0 g and 40.0 cm long that plays the same note as a closed pipe of the same length?



Musical intervals Two notes with frequencies related by the ratio 1:2 are said to differ by an octave. For example, if a note has a frequency of 440 Hz, a note that is one octave higher has a frequency of 880 Hz. The fundamental and its harmonics are related by octaves; the first harmonic is one octave higher than the fundamental, the second is two octaves higher, and so on. It is the ratio of two frequencies, not the size of the interval between them, that determines the musical interval.

In other musical intervals, two pitches may be close together. For example, the ratio of frequencies for a "major third" is 4:5. An example is the notes C and E. The note C has a frequency of 262 Hz, so E has a $\left(\frac{5}{4}\right)(262\text{ Hz}) = 327\text{ Hz}$. In the same way, notes in a "fourth" (C and F) have a frequency ratio of 3:4, and those in a "fifth" (C and G) have a ratio of 2:3. More than two notes sounded together also can produce consonance. The three notes called do, mi, and sol make a major chord. For at least 2500 years, western music has recognized this as the sweetest of the three-note chords; it has the frequency ratio of 4:5:6.

Beats

You have seen that consonance is defined in terms of the ratio of frequencies. When the ratio becomes nearly 1:1, the frequencies become very close. Two frequencies that are nearly identical interfere to produce oscillating high and low sound levels called a **beat.** This phenomenon is illustrated in **Figure 19.** The frequency of a beat is the magnitude of difference between the frequencies of the two waves, $f_{\text{beat}} = |f_A - f_B|$. When the difference is less than 7 Hz, the ear detects this as a pulsation of loudness. Musical instruments often are tuned by sounding one against another and adjusting the frequency of one until the beat disappears.

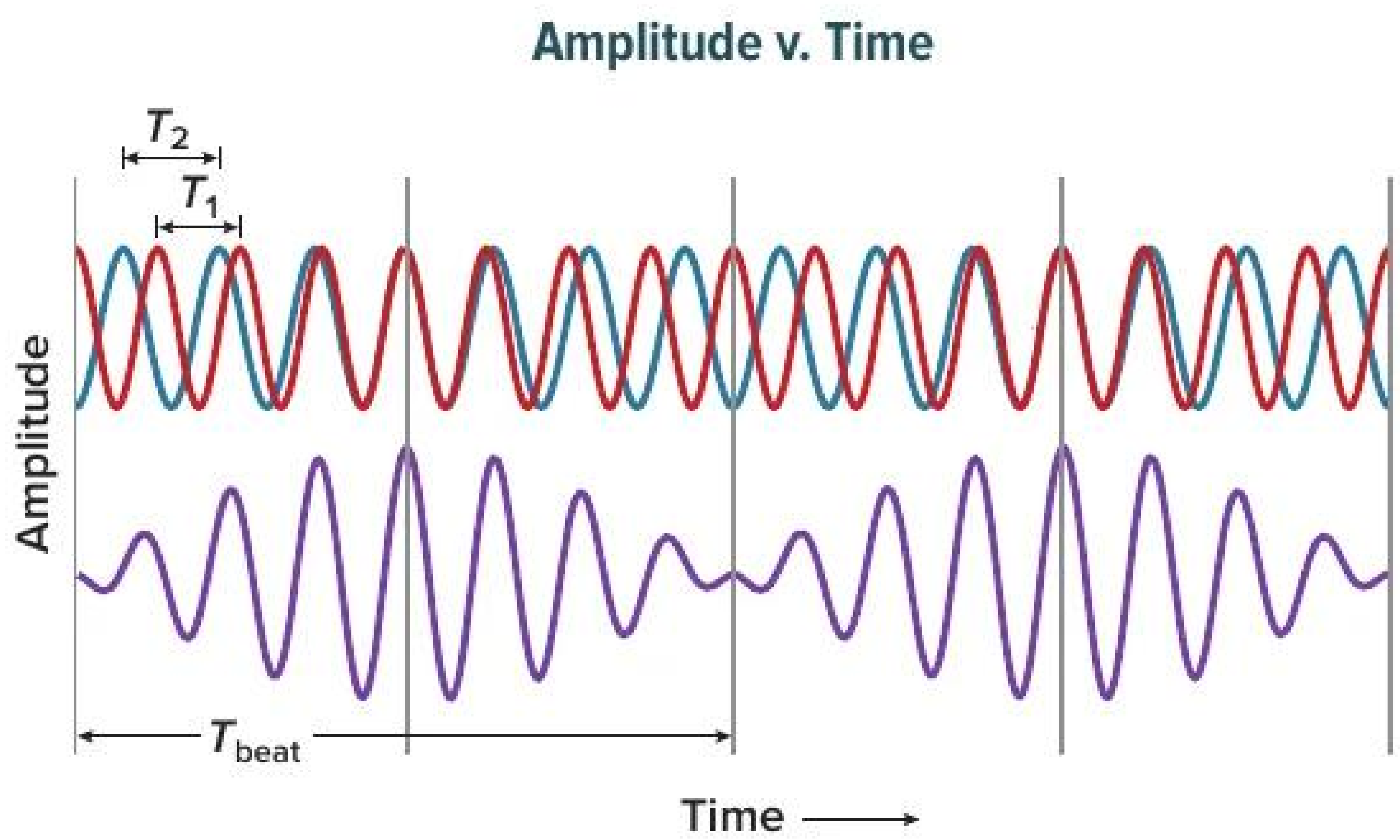


Figure 19 Beats occur as a result of the superposition of two sound waves of slightly different frequencies.

SCIENCE USAGE v. COMMON USAGE

Beat

Science usage: oscillation of wave amplitude that results from the superposition of two sound waves with almost identical frequencies

When the piano tuner no longer heard beats, she knew the piano was tuned properly.

Common usage: to strike repeatedly

Richard beat the drums while John played the guitar.

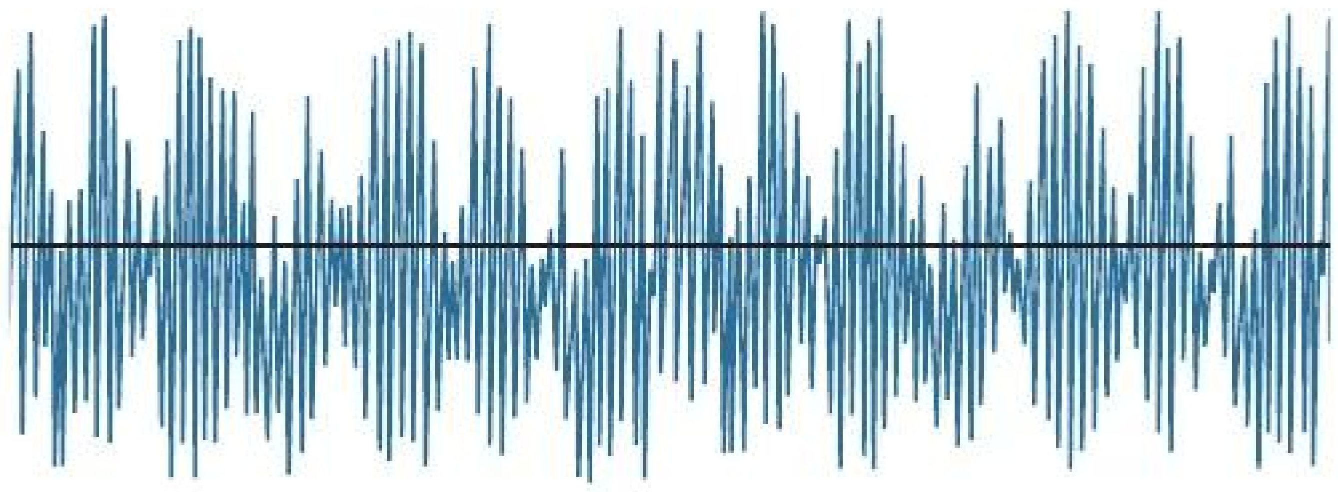


Figure 20 A noise wave consists of many different frequencies, all with about the same amplitude.

Sound Reproduction and Noise

When you listen to a live band or hear your school band practicing, you are hearing music produced directly by a human voice or musical instruments. You may want to hear live music every time you choose to listen to music. Most of the time, however, you likely listen to music that has been recorded and is played via electronic systems. To reproduce the sound faithfully, the system must accommodate all frequencies equally. A good stereo system keeps the amplitudes of all frequencies between 20 and 20,000 Hz to within a range of 3 dB.

A telephone system, on the other hand, needs only to transmit the information in spoken language. Frequencies between 300 and 3000 Hz are sufficient. Reducing the number of frequencies present helps reduce the noise. A noise wave is shown in **Figure 20.** Many frequencies are present with approximately the same amplitude. While noise is not helpful in a telephone system, some people claim that listening to white noise has a calming effect. For this reason, some dentists use noise to help their patients relax.

Check Your Progress

17. **Sound Sources** What is the vibrating object that produces sounds in each of the following?
    a. a human voice c. a tuba
    b. a clarinet d. a violin
18. **Resonance in Air Columns** Why is the tube from which a tuba is made much longer than that of a cornet?
19. **Resonance in Open Tubes** How must the length of an open tube compare to the wavelength of the sound to produce the strongest resonance?
20. **Resonance on Strings** A violin sounds a note of F sharp, with a pitch of 370 Hz. What are the frequencies of the next three harmonics produced with this note?
21. **Resonance in Closed Pipes** One closed organ pipe has a length of 2.40 m.
    a. What is the frequency of the note played?
    b. When a second pipe is played at the same time, a 1.40-Hz beat note is heard. By how much is the second pipe too long?
22. **Timbre** Why do various instruments sound different even when they play the same note?
23. **Beats** A tuning fork produces three beats per second with a second, 392-Hz tuning fork. What is the frequency of the first tuning fork?
24. **Critical Thinking** Strike a tuning fork with a rubber hammer and hold it at arm's length. Then press its handle against a desk, a door, a filing cabinet, and other objects. What do you hear? Why?

LEARNSMART® Go online to follow your personalized learning path to review, practice, and reinforce your understanding.

SCIENCE & SOCIETY

Out of Sight—Sonar Detection

Animals such as bats, whales, dolphins, and some birds use echolocation, also called bio-sonar, to detect objects when it is difficult for them to see (for example, at night or in cloudy waters). The animals emit high-frequency clicking noises. The sound waves from these noises bounce off nearby objects as echoes. The animals detect the echoes with their ears and process the information to determine the spatial imagery of their surroundings, including the location of prey.

Daniel Kish is blind. He uses echolocation to navigate around in his environment.

Human Echolocation

Some people who are blind also use echolocation to orient to their environment and move around safely without the help of a guide dog or cane. These people produce rapid clicking noises with their mouths and listen for the echoes as they reflect back from different surfaces and objects nearby. This acoustic information received from their ears is interpreted in the same part of the brain where people who have sight process visual information about their surroundings—in essence, they "see" by sound.

Californian Daniel Kish developed retinal cancer as an infant and soon became blind. Without the guidance of anyone, he instinctively used clicking noises at a young age as a compass to navigate in his environment—before the concept of human echolocation existed. Daniel is known as the "real-life batman" and can even mountain bike and sketch his surroundings using echolocation.

Daniel received his Master's degree in developmental psychology and developed the first step-by-step methods for teaching echolocation. He now trains visually impaired people to use echolocation as a valuable skill to move around in day-to-day life. Scientists have also found that it is possible to train people who are not blind to use echolocation.

Developing Echolocation Technologies

New assistive devices that are based on echolocation are being developed for people who are visually impaired. These include vibrating clothing and a wrist band that detects echoes bouncing off objects up to 14 feet away. Echolocation research may also be useful in the development of technology that uses artificial sonar, such as self-driving cars.

COMMUNICATE TECHNICAL INFORMATION

Research and choose a technology that uses echolocation. Summarize how the technology works in a short paragraph.

MODULE 14

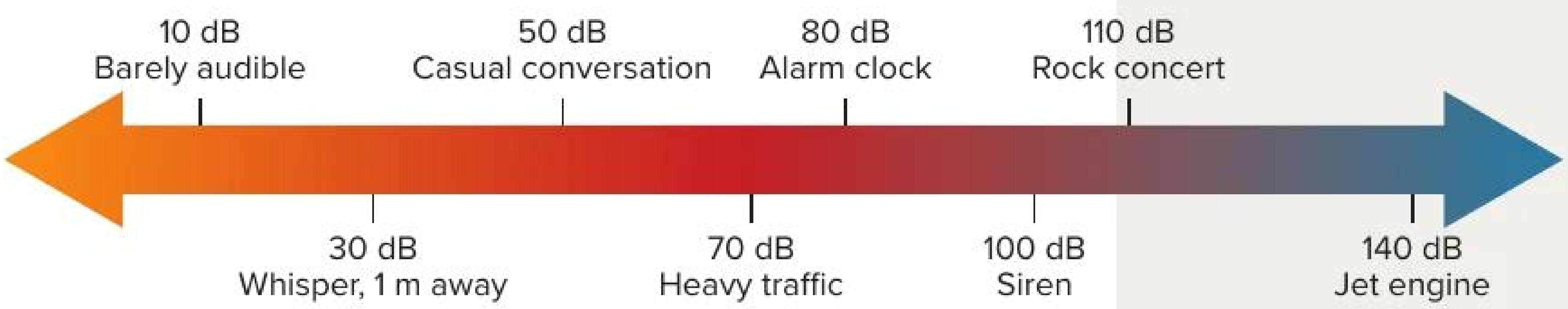
STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 PROPERTIES AND DETECTION OF SOUND

- sound wave
- pitch
- loudness
- sound level
- decibel
- Doppler effect

- Sound is a pressure variation transmitted through matter as a longitudinal wave. A sound wave has frequency, wavelength, speed, and amplitude. Sound waves reflect and interfere.
- Sound detectors convert the energy carried by a sound wave into another form of energy. The human ear is a highly efficient and sensitive detector of sound waves. The frequency of a sound wave is heard as its pitch. The loudness of sound as perceived by the ear and brain depends mainly on its amplitude. The pressure amplitude of a sound wave can be measured in decibels (dB).



- The Doppler effect is the change in frequency of sound caused by the motion of either the source or the detector.
- The Doppler effect is used in radar detectors, in medical ultrasound machines, and by astronomers. Bats also use the Doppler effect to detect and catch flying insects.

Lesson 2 THE PHYSICS OF MUSIC

- closed-pipe resonator
- open-pipe resonator
- fundamental
- harmonics
- dissonance
- consonance
- beat

- Sound is produced by a vibrating object in a medium.
- An air column can resonate with a sound source, thereby increasing the amplitude of its resonant frequency. A closed pipe resonates when its length is $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$, and so on. Its resonant frequencies are odd-numbered multiples of the fundamental. An open pipe resonates when its length is $\frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}$, and so on. Its resonant frequencies are whole-number multiples of the fundamental.
- A clamped string has a node at each end and resonates when its length is $\frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}$, and so on, just as with an open pipe. The string's resonant frequencies are also whole-number multiples of the fundamental.
- The frequencies and intensities of the complex waves produced by a musical instrument determine the timbre that is characteristic of that instrument.
- Two waves with almost the same frequency interfere to produce beats.

THREE-DIMENSIONAL THINKING

Module Wrap-Up

REVISIT THE PHENOMENON

Why does a fire truck's siren change pitch as it passes you?

CER Claim, Evidence, Reasoning

Explain your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

Can you test the Doppler effect?

In 1845, Dutch astronomer Christoph Buys Ballot tested the Doppler effect. A trumpet player sounded an A (440 Hz) while riding on a flatcar pulled by a train. Your project team wants to repeat the experiment. Rather than using a train and listening for beats, you plan to have a trumpet played in a rapidly moving car and have the car move fast enough so that the moving trumpet sounds one major third above a stationary trumpet.

CER Analyze and Interpret Data

1. **Claim** Should you try your experiment?
2. **Evidence and Reasoning** Justify your claim.

MODULE 15

FUNDAMENTALS OF LIGHT

MODULE 15

FUNDAMENTALS OF LIGHT

ENCOUNTER THE PHENOMENON

What does the light from a distant star or supernova tell us about it?

GO ONLINE to play a video about measuring the speed of light.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

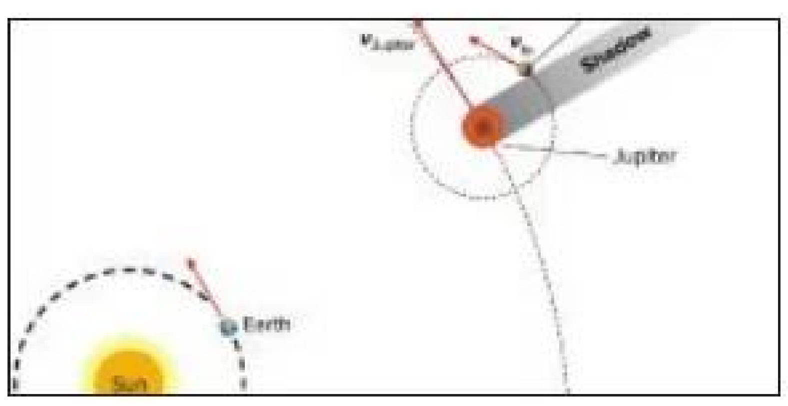
Make Your Claim Use your CER chart to make a claim about what light from a distant star or supernova tells us about it. Explain your reasoning.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.

LESSON 1: Explore & Explain: How Light Travels



LESSON 2: Explore & Explain: Speed of Light

Smithsonian

Additional Resources

LESSON 1

ILLUMINATION

FOCUS QUESTION

How does distance affect how bright a star appears?

Light

Light is electromagnetic radiation, like radio and microwaves. It can be modeled as a wave of changing electric and magnetic fields or as particles called photons. The wave model is useful for explaining many features of electromagnetic radiation, and the particle model explains other features. In this module, you will learn about the wave properties of light. You will learn about photons in a later module.

Light's path How does your body receive information? Many people respond to this question with the five senses, starting with sight and hearing. The sense of sight depends on light from your surroundings reaching your eyes. Did you ever wonder how light travels? Think of how a narrow beam of light, such as that of a flashlight or sunlight streaming through a small window, is made visible by dust particles in the air. You see the path of the light as a straight line. When your body blocks sunlight, you see your outline in a shadow, a result of light's straight path. **Figure 1** depicts light's straight path.

Figure 1 Light rays traveling in straight lines are evident in many situations.
Explain *how this photo demonstrates the wave properties of light.*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Quick Investigation: The Speed of Light
Use the relationship among the frequency, wavelength, and speed of a wave to calculate the speed of light.

Probeware Lab: Light Intensity and Distance
Analyze data to determine how distance from the source affects the intensity of light.

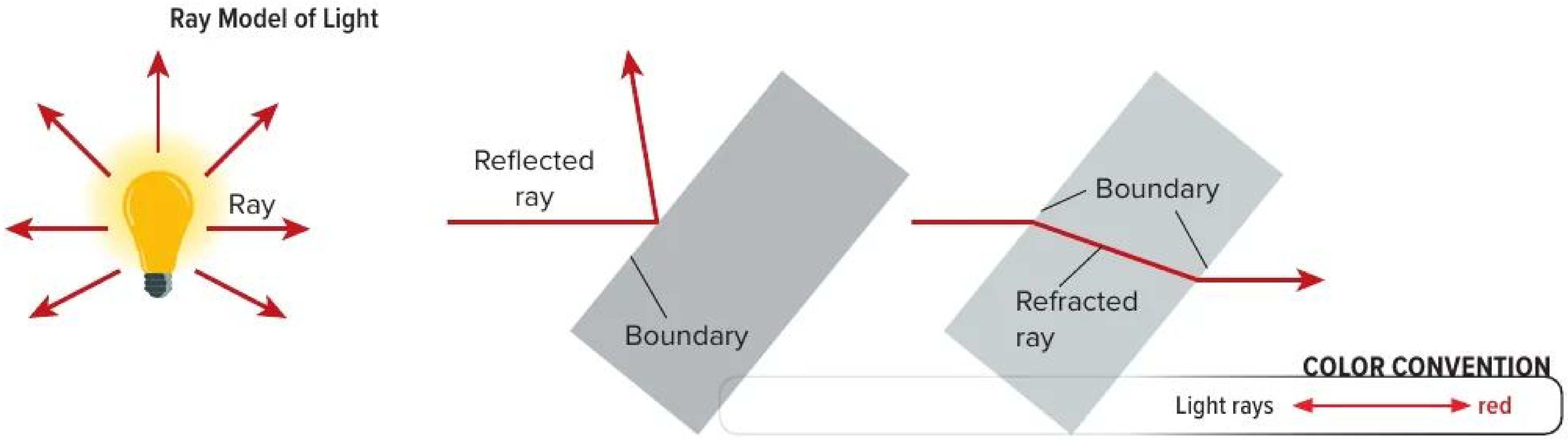


Figure 2 Light's straight-line path is demonstrated by the ray model. Light rays change direction when they are reflected or refracted by matter. In either case, light rays continue in a straight path.

Ray Model of Light

Imagine being in an empty, dark room when a small lightbulb is turned on in the center of the room. You can see around the room, and you can look at the bulb and see it. This must mean the bulb sends light in all directions. You could visualize the light coming from the bulb as an infinite number of arrows traveling straight away from the bulb in all directions. Each arrow represents a ray of light, which travels in a straight path until it reaches a boundary, as shown in **Figure 2.**

After interacting at a boundary, the ray still moves in a straight line, but its direction is changed. These basic principles—that light travels in straight lines and that its direction can be changed by encountering a boundary—constitute the **ray model of light.** The study of light interacting with matter is called ray optics or geometric optics.

Sources of light

What is the difference between sunlight and moonlight? For one, sunlight is much brighter. Another important fundamental difference is that the Sun produces and emits its own light, while the Moon is only visible because it reflects the Sun's light. Everything you see fits into one of these two categories. Objects such as the Sun and other stars that emit their own light are **luminous sources,** while those that you see due to light reflecting from them are illuminated sources, like the Moon and planets.

Luminous sources include natural sources such as flames and fireflies and human-made devices such as television screens, computer monitors, lasers, and tiny, light-emitting diodes. In **Figure 3,** the luminous source is fluorescent bulbs. They produce light from electrical energy. The other objects in the room are illuminated when the light from the bulbs is reflected off of them. In a room with no light, it would be impossible to see anything, because there is no light reflecting off of objects into your eyes.

Figure 3 Objects in the room are visible because of reflected light.

Recognize *What, if anything, would be visible if the room had no luminous source? Explain.*

Figure 4 Light is transmitted through the transparent and translucent candle holders. The candle cannot be seen clearly in the opaque holder because the light is absorbed.

Light and matter Objects can absorb, reflect, or transmit light. Objects that reflect and absorb light but do not transmit it are **opaque.** Many common objects—such as books, people, and backpacks—are opaque. Mediums that transmit and reflect light but do not allow objects to be seen clearly through them are **translucent** mediums. The frosted glass in **Figure 4** is a translucent medium. A **transparent** medium, such as air or glass, transmits most of the light that reaches it. **Figure 4** illustrates objects that are opaque, translucent, and transparent. Transparent mediums transmit light, but they also often reflect some light. For example, you can see the glass of the transparent candle holder. This is possible because light is reflected off the glass.

All three types of objects also absorb some light. There are various factors that determine how much light will be absorbed, but opaque objects usually absorb a greater portion of light than translucent or transparent objects.

Get It?
Explain why it is possible to see a fish through a glass fishbowl and also see the glass of the bowl.

Quantity of Light

If you were to have a flashlight shone at you from across the room, what factors would determine how bright that light would appear to you? Three main factors determine the brightness: the quantity of light the flashlight produces, the distance between the lightbulb and your eye, and the angle at which the light rays hit your eye. In this lesson, you will read about the first two of these factors.

Luminous flux With the ray model of light, a source that is brighter produces more light rays than a less bright source. Imagine again a single lightbulb sending rays in nearly all directions. How could you capture all the light it emits? You would need to construct a surface that completely encloses the bulb, as in **Figure 5.** The rate at which the bulb, a luminous source, produces light energy is called the **luminous flux** (P) and is measured in lumens (lm). The total amount of light that strikes the surface in a given unit of time depends only on the luminous flux of the source.

Luminous flux

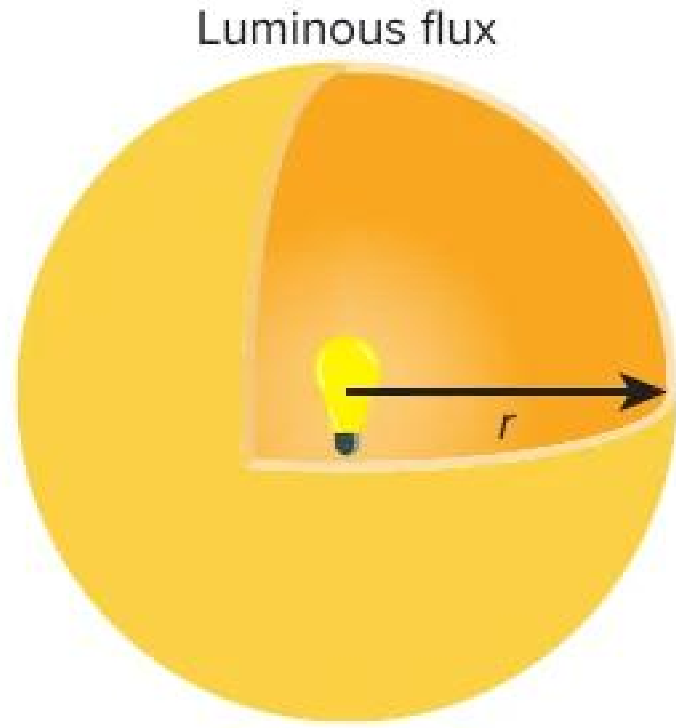


Figure 5 Luminous flux is the rate at which light rays are emitted from a luminous source.

lluminance Once you know the quantity of light being emitted by a luminous source, you can determine the amount of illumination the source provides to an object, such as a book. The luminous flux falling on a given surface area at any instant is called **illuminance** (*E*). It is measured in lux (lx), which is equivalent to lumens per square meter (lm/m^2). In this module, we assume, for simplification, that all light sources are point sources.

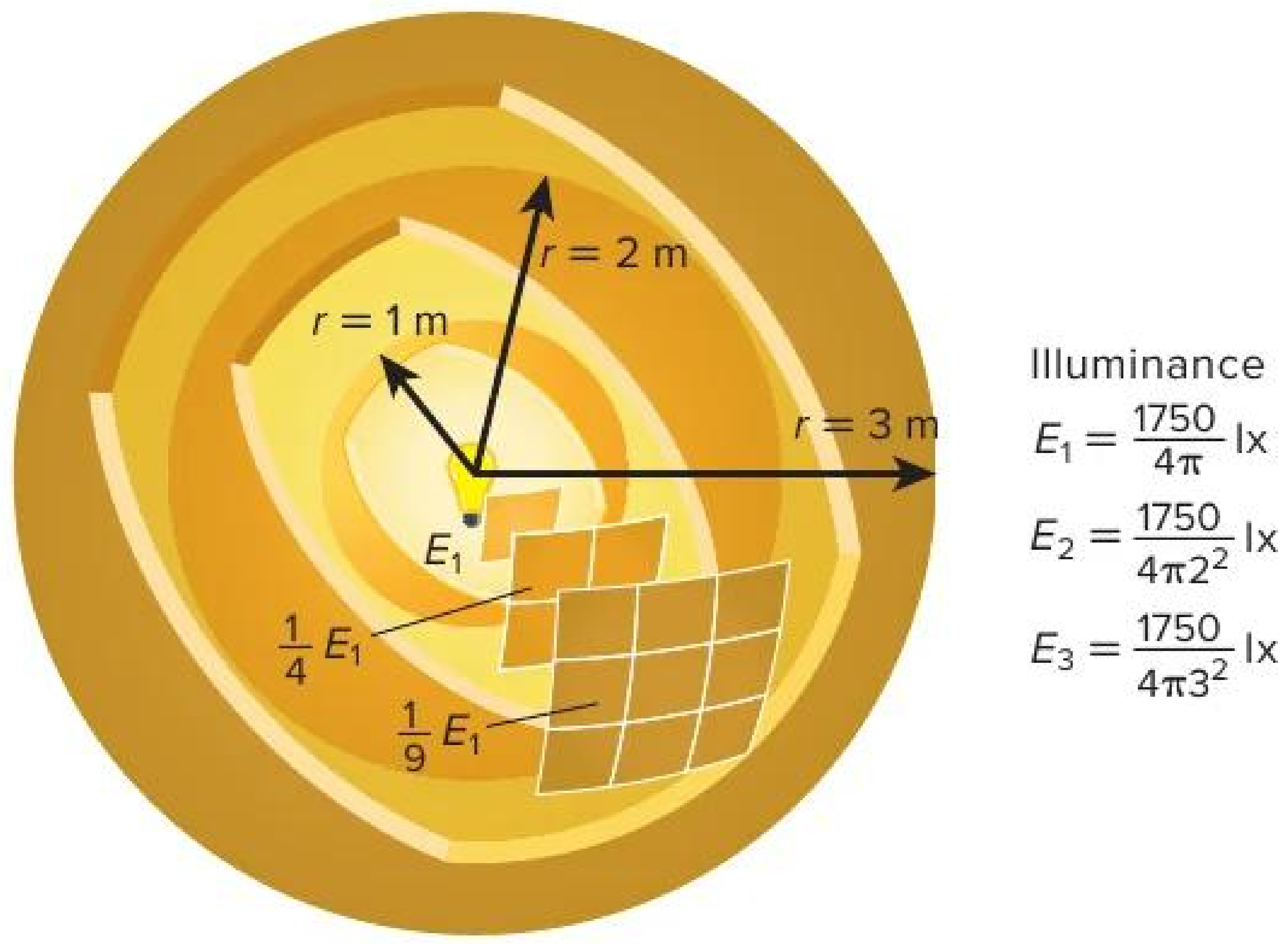


Figure 6 Illuminance (*E*) is the quantity of light that strikes a surface. As the distance from the luminous source (*r*) increases, *E* decreases. *E* depends on the inverse of *r* squared.

Consider the setup shown in **Figure 6.** The luminous flux of the source is 1750 lm (typical of a 26-W compact fluorescent bulb). What is the illuminance of the sphere's inside surface at $r = 1$ m? Because all the bulb's luminous flux strikes the surface, divide the luminous flux by the surface area of the sphere, $4\pi r^2$. The surface area is $4\pi(1.00\text{ m})^2 = 4\pi\text{ m}^2$, so the illuminance is $\frac{1750\text{ lm}}{4\pi\text{ m}^2} = 139$ lx. This mathematical relationship means that at a distance of 1.00 m from the bulb, 139 lm strikes each square meter.

Get It?
Define *illuminance*, and state the units that are used for illuminance.

Inverse-square relationship What if the sphere surrounding the lamp were larger? If the sphere's radius were 2.00 m, the luminous flux still would total 1750 lm because it only depends on the bulb. With a radius of 2.00 m, however, the area of the sphere would now be equal to $4\pi(2.00\text{ m})^2 = 16.0\pi\text{ m}^2$. The new area is four times larger than that of the 1.00-m sphere, as shown in **Figure 6.** The illuminance of the inside of the 2.00-m sphere is $\frac{1750\text{ lm}}{(160\pi\text{ m}^2)} = 34.8$ lx, so 34.8 lm strikes each square meter.

The illuminance on the inside surface of the 2.00-m sphere (E_2) is one-fourth the illuminance on the inside of the 1.00-m sphere. In the same way, the inside of a sphere with a 3.00-m radius has an illuminance only one-ninth $\left[\left(\frac{1}{3}\right)^2\right]$ as large as that of the 1.00-m sphere. **Figure 6** shows that the illuminance produced by a point source is proportional to $\frac{1}{r^2}$: an inverse-square relationship. In the case of the 3.00-m radius, only 15.5 lm strike each square meter inside the sphere. As the light rays spread out in straight lines in all directions from a point source, the number of light rays that illuminate a unit of area decreases as the square of the distance from the point source.

CCC CROSSCUTTING CONCEPTS

Systems and System Models Write a paragraph explaining how mathematical models were used in the study of light. Include information about the system and system boundaries that scientists used.

CONNECTING MATH to Physics

Direct and Inverse Relationships The illuminance provided by a source of light has both a direct and an inverse relationship.

Math	Physics
$y = \frac{x}{az^2}$	$E = \frac{P}{4\pi r^2}$
If z is constant, then y is directly proportional to x. • When x increases, y increases. • When x decreases, y decreases.	If r is constant, then E is directly proportional to P. • When P increases, E increases. • When P decreases, E decreases.
If x is constant, then y is inversely proportional to z^2. • When z^2 increases, y decreases. • When z^2 decreases, y increases.	If P is constant, then E is inversely proportional to r^2. • When r^2 increases, E increases. • When r^2 decreases, E decreases.

Luminous intensity

Some luminous sources are specified in candelas (cd). A candela is not a measure of luminous flux but of luminous intensity. The luminous intensity of a point source is the luminous flux that falls on 1 m^2 of the inside of a 1-m-radius sphere, so luminous intensity is luminous flux divided by 4π. A bulb with 1750 lm of flux has an intensity of $\frac{1750\ \text{lm}}{4\pi} = 139$ cd.

In **Figure 7,** the lightbulb is twice as far away from the screen as the candle. For the bulb to provide the same illuminance on its side of the screen as the candle does on the candle side of the screen, the bulb would have to be four times brighter than the candle. The lightbulb's luminous intensity, therefore, would have to be four times the candle's luminous intensity. If both sources in **Figure 7** had the same luminous intensity, the source at $2r$ would only provide one-quarter the illuminance to the screen. This is consistent with the inverse-square relationship we just developed.

Get It?

Describe what luminous intensity is a measure of and what its relationship is to illuminance.

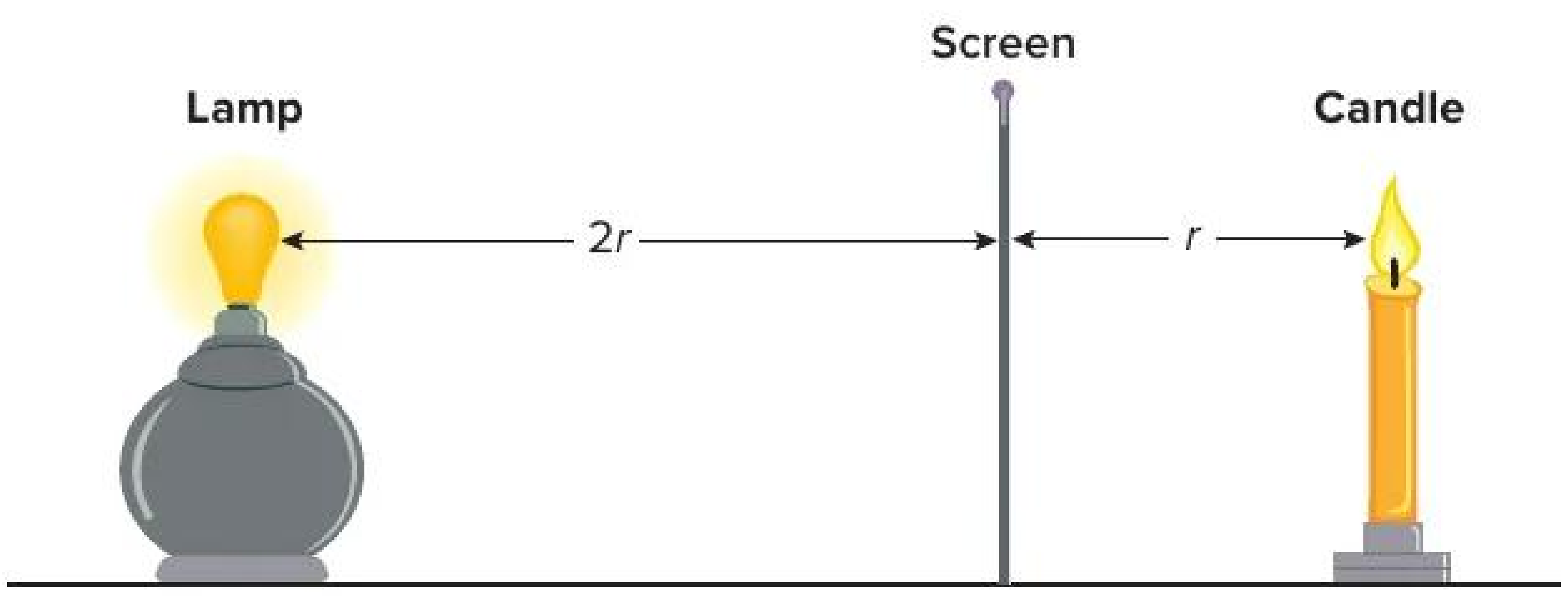


Figure 7 For the lightbulb and the candle to provide the same illuminance to the screen, the luminous intensity of the lightbulb is four times that of the candle.

Surface Illumination

Think again about the scenario in which a flashlight is shining at you from across the room. If the bulb has a small luminous intensity, the light will not be very bright. To increase the brightness, you could use a brighter bulb, thereby increasing the luminous flux, or you could move so that your eyes are closer to the light, decreasing the distance between the light source and your eyes. Following the simplification that we are treating all light sources as point sources, the illuminance and distance will follow the inverse-square relationship. In this case, and in all the cases we will deal with in this book, the illuminance caused by a point light source is represented by the following equation.

Real-World Physics

Illuminated Minds

When deciding how to achieve the correct illuminance on students' desktops in a classroom, architects must consider the luminous flux of the lights as well as the distance of the lights above the desktops. In addition, the efficiencies of the light sources are an important economic factor.

Point-Source Illuminance

If an object is illuminated by a point source of light, then the illuminance at the object is equal to the luminous flux of the light source divided by the surface area of the sphere whose radius is equal to the distance the object is from the light source.

$$E = \frac{P}{4\pi r^2}$$

Remember that the luminous flux of the light source is spreading out in all directions, so only some fraction of the luminous flux is available to illuminate the object. Use of this equation is valid only if the light from the luminous source strikes perpendicular to the surface it is illuminating. It is also only valid if the luminous source is small enough or far enough away to be considered a point source. Thus, the equation does not give accurate values of illuminance for long fluorescent lamps or lightbulbs that are close to the surfaces they illuminate.

Engineers who design lighting systems must understand how the light will be used. If an even illumination is needed to prevent dark areas, the common practice is to evenly space normal lights over the area to be illuminated, as was most likely done with the lights in your classroom. Because such light sources do not produce truly uniform light, however, engineers also design special light sources that control the spread of the light, such that they produce even illuminations over large surface areas. For safety reasons, this is extremely important for automobile headlights, as in **Figure 8.** Automobile engineers must consider these factors when designing headlights.

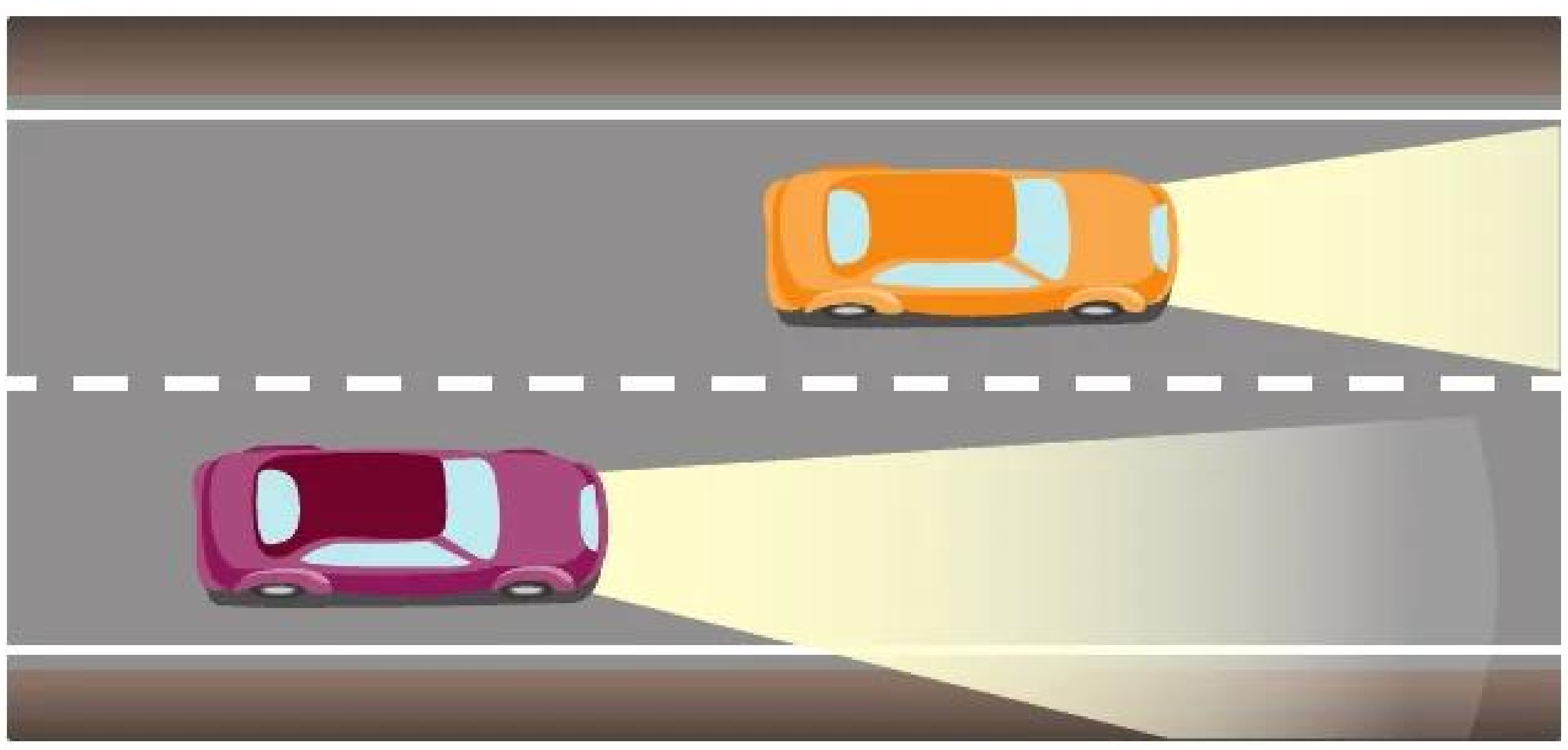


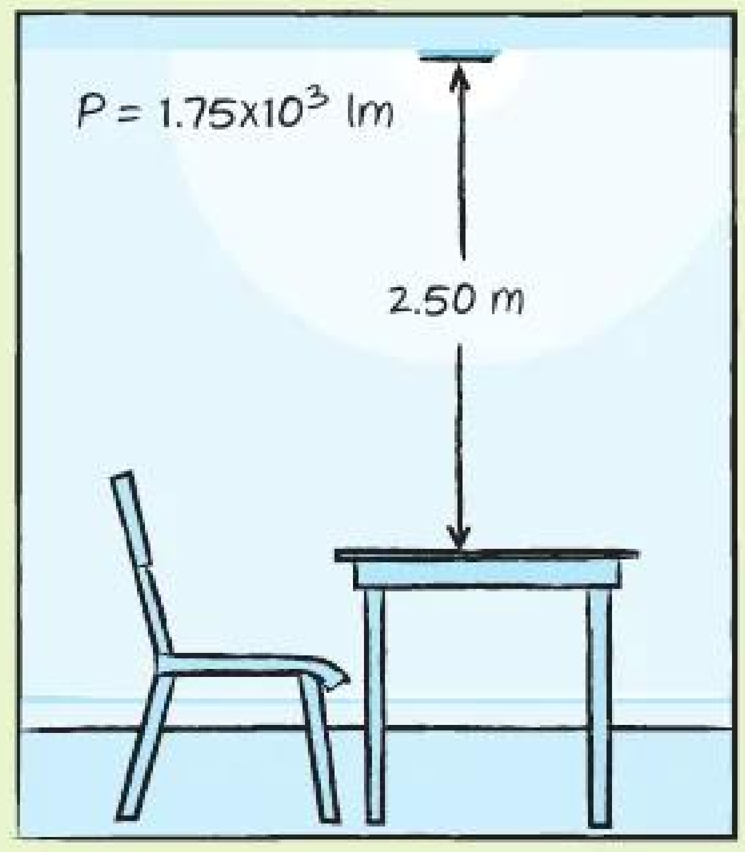
Figure 8 Dark areas can occur if headlights on cars are not set at the correct angles to adequately illuminate the road.

EXAMPLE Problem 1

ILLUMINATION OF A SURFACE What is the illuminance on your desktop if it is lit by a 1750-lm lamp that is 2.50 m above your desk?

1 ANALYZE AND SKETCH THE PROBLEM

- Assume the lightbulb is the point source.
- Diagram the position of the bulb and the desktop. Label P and r.



Known	Unknown
$P = 1.75\times10^3$ lm	$E = ?$
$r = 2.50$ m	

2 SOLVE FOR THE UNKNOWN

The surface is perpendicular to the direction in which the light ray is traveling, so you can use the point-source illuminance equation.

$$E = \frac{P}{4\pi r^2}$$

$$= \frac{1.75\times10^3 \text{ lm}}{4\pi(2.50 \text{ m})^2}$$ Substitute $P = 1.75\times10^3$ lm, $r = 2.50$ m

$$= 22.3 \text{ lm/m}^2$$

$$= 22.3 \text{ lx}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** The units of luminance are lm/m^2 = lx, which the answer agrees with.
- **Do the signs make sense?** All quantities are positive, as they should be.
- **Is the magnitude realistic?** Illuminance from an 1800-lm lamp at a distance of 2 m is about 20 lx.

PRACTICE Problems

ADDITIONAL PRACTICE

1. A lamp is moved from 30 cm to 90 cm above the pages of a book. Compare the illumination on the book before and after the lamp is moved.
2. Draw a graph of the illuminance produced by a lamp with a luminous flux of 2275 lm at distances from 0.50 m and 5.0 m.
3. A 64-cd point source of light is 3.0 m away from a painting. What is the illumination on the painting in lux?
4. A screen is placed between two lamps so that they illuminate the screen equally, as shown in **Figure 9.** The first lamp emits a luminous flux of 1445 lm and is 2.5 m from the screen. What is the distance of the second lamp from the screen if the luminous flux is 2375 lm?

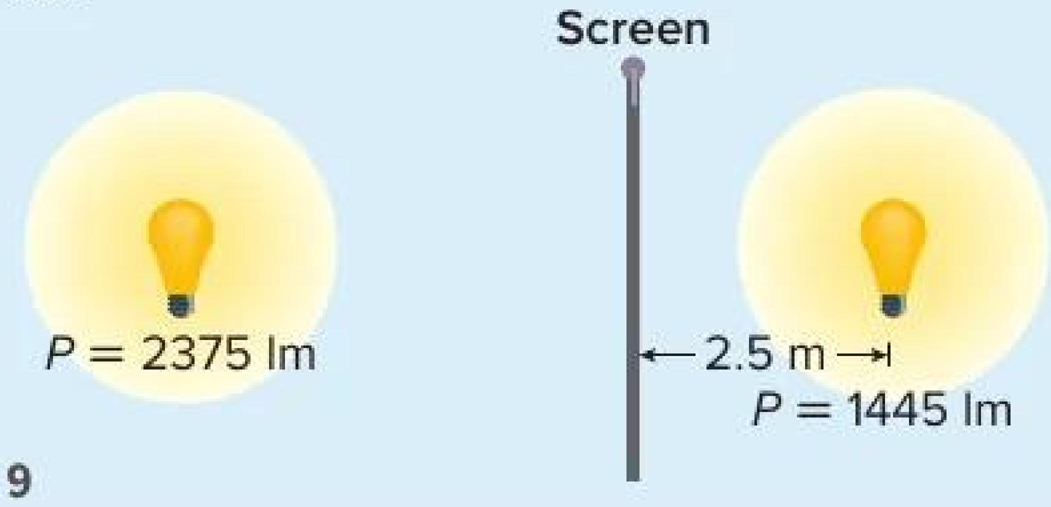


Figure 9

5. What is the illumination on a surface that is 3.0 m below a 150-W incandescent lamp that emits a luminous flux of 2275 lm?
6. A public school law requires a minimum illuminance of 160 lx at the surface of each student's desk. An architect's specifications call for classroom lights to be located 2.0 m above the desks. What is the minimum luminous flux that the lights must produce?
7. **CHALLENGE** Your local public library is planning to remodel the computer lab. The contractors have purchased fluorescent lamps with a rated luminous flux of 1750 lm. The desired illumination on the keyboard surfaces is 175 lx. Assume a single lamp illuminates each keyboard. What distance above the surface should the lights be placed to achieve the desired illumination? If the contractors had also already purchased fixtures to hold the lights that when installed would be 1.5 m above the keyboard surface, would the desired illuminance be achieved? If not, would the illuminance be greater or less than desired? What change in the lamp's luminous flux would be required to achieve the desired illuminance?

The Speed of Light

Arguments that light must travel at a finite speed have existed for more than 2400 years. By the seventeenth century, several scientists had performed experiments that supported the view that light travels at a finite speed, but that this speed is much faster than the speed of sound.

Actually measuring the speed of light was not an easy task in the seventeenth century. As you know from studying motion, if you can measure the time light takes to travel a certain distance, you can calculate the speed of light. However, the time that it takes light to travel between objects on Earth is much shorter than a human's reaction time. How could a seventeenth-century scientist solve this problem?

Clues from Io

Danish astronomer Ole Roemer was the first to measure the time it took for light to travel between two points with any success. Between 1668 and 1674, Roemer made 70 measurements of the 1.8-day orbital period of Io, one of Jupiter's moons. He recorded the times when Io emerged from Jupiter's shadow, as shown in **Figure 10.** He made his measurements as part of a project to improve maps by calculating the longitude of locations on Earth. This is an early example of the needs of technology driving scientific advances.

After making many measurements, Roemer was able to predict when the next eclipse of Io would occur. He compared his predictions with the actual measured times and found that Io's observed orbital period increased on average by about 13 s per orbit when Earth was moving away from Jupiter and decreased on average by about 13 s per orbit when Earth was approaching Jupiter. Roemer believed that Jupiter's moons were just as regular in their orbits as Earth's moon; thus, he wondered what might cause this discrepancy in the measurement of Io's orbital period. He considered another variable within the system, the movement and position of Earth relative to Jupiter.

The Eclipse of Io

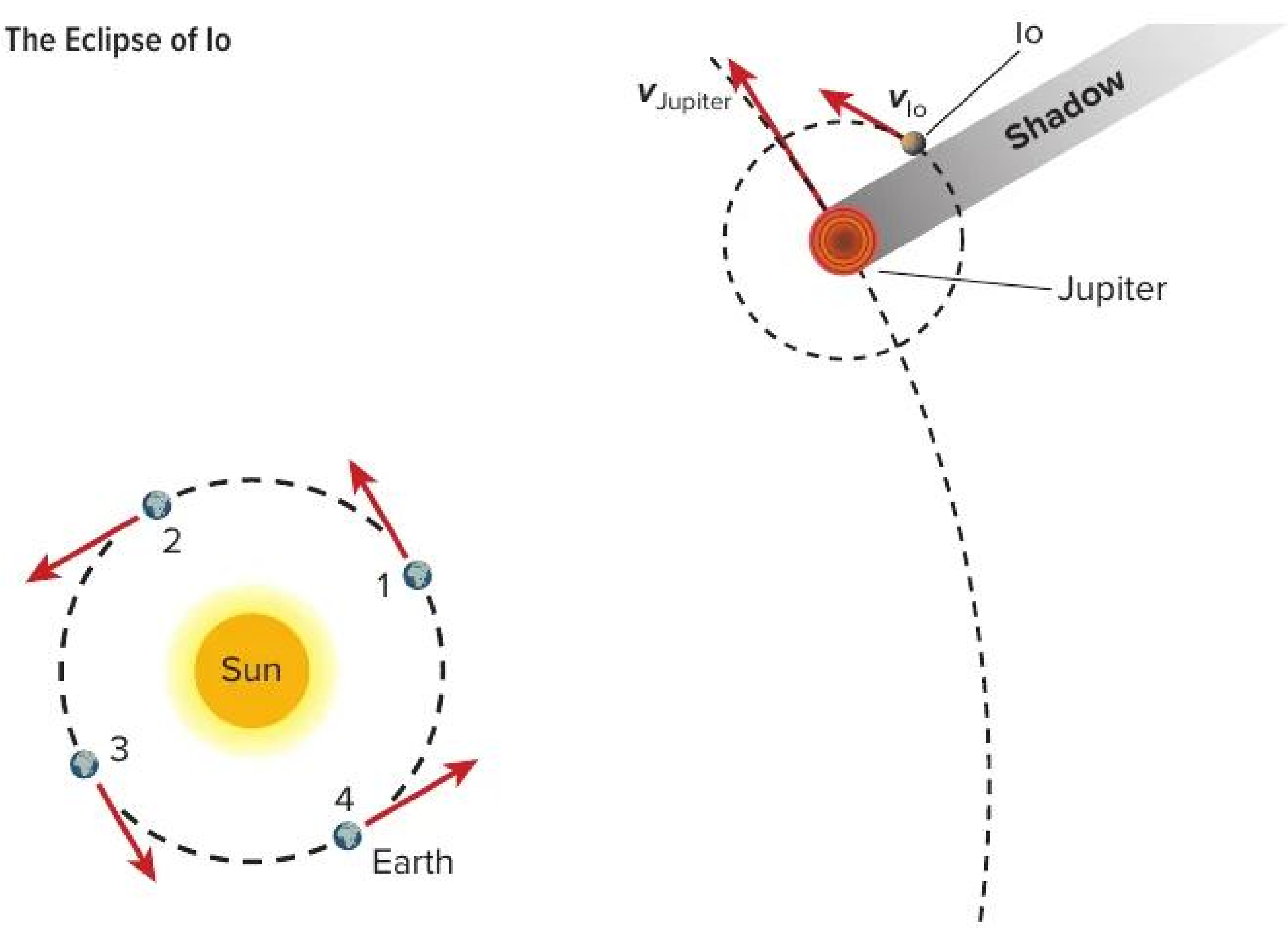


Figure 10 As Earth approaches Jupiter, the light reflected from Io takes less time to reach Earth than when Earth moves away from Jupiter. (Illustration is not to scale.)

Measuring the speed of light Roemer concluded that as Earth moved away from Jupiter, the light from each new appearance of Io took longer to reach Earth because it traveled farther. As Earth moved toward Jupiter, Io's orbital period seemed to decrease. During the 182.5 days it took for Earth to travel from position 1 to position 3, shown in **Figure 10,** there were (182.5 days)(1 Io eclipse/1.8 days) = 1.0×10^2 Io eclipses. Thus, for light to travel the diameter of Earth's orbit, he calculated that it takes (1.0×10^2 eclipses)(13 s/eclipse) = 1.3×10^3 s, or 22 min.

Using the presently known value of the diameter of Earth's orbit (2.93×10^{11} m), Roemer's value of 22 min gives a value for the speed of light of 2.9×10^{11} m/((22 min)(60 s/min)) = 2.2×10^8 m/s. Today, the speed of light is known to be closer to 3.0×10^8 m/s, so light takes 16.5 min, not 22 min, to cross Earth's orbit. Nevertheless, Roemer had successfully proved that light travels at a finite speed.

Michelson's measurements Although many measurements of the speed of light have been made, the most notable were performed by American physicist Albert A. Michelson. Between 1880 and the 1920s, he developed Earth-based techniques to measure the speed of light. In 1926 Michelson measured the time required for light to make a round trip between two California mountains 35 km apart. Michelson's best result was $(2.99796 \pm 0.00004)\times10^8$ m/s. For this work, he became the first American to receive a Nobel Prize in science.

The speed of light in a vacuum has its own special symbol, c. The International Committee on Weights and Measurements has measured and defined the speed of light in a vacuum to be c = 299,792,458 m/s. For many calculations, the value c = 3.00×10^8 m/s is precise enough. At this speed, light travels 9.46×10^{12} km in a year. This distance is called a light-year.

Check Your Progress

8. **Light** What evidence have you observed that light travels in a straight line?
9. **Light Properties** Why might you choose a window shade that is translucent? Opaque?
10. **Illuminance** Does one lightbulb provide more or less illuminance than two identical light-bulbs at twice the distance? Explain.
11. **Luminous Intensity** Two lamps illuminate a screen equally from distances shown in **Figure 11.** If Lamp A is rated 75 cd, what is Lamp B rated?

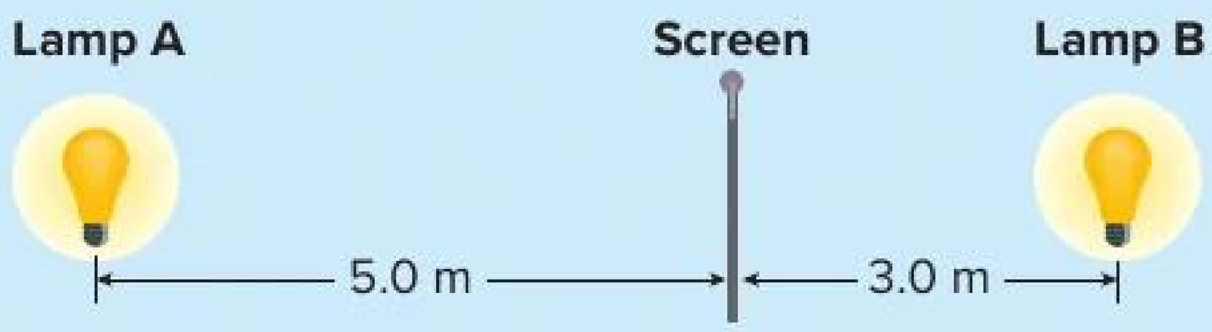


Figure 11

12. **Distance of a Light Source** A lightbulb illuminating your computer keyboard provides only half the illuminance that it should. If it is currently 1.0 m away, how far should it be to provide the correct illuminance?
13. **Light and Sound Travel** How far does light travel in the time it takes sound to travel 1 cm in air at 20°C?
14. **Distance of Light Travel** The distance to the Moon can be found with the help of mirrors left on the Moon by astronauts. A pulse of light is sent to the Moon and returns to Earth in 2.562 s. Using the defined value for the speed of light to the same precision, calculate the distance from Earth to the Moon.
15. **Critical Thinking** The correct time taken for light to cross Earth's orbit is 16.5 min, and the diameter of Earth's orbit is 2.98×10^{11} m. Calculate the speed of light using Roemer's method. Does this method appear to be accurate? Why or why not?

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LESSON 2
THE WAVE NATURE OF LIGHT

FOCUS QUESTION

How do scientists use the Doppler shift to determine how stars and galaxies are moving?

Diffraction and the Wave Model

In 1665 Italian scientist Francesco Maria Grimaldi observed that the edges of shadows are not perfectly sharp. He introduced a narrow beam of light into a dark room and held a rod in front of the light such that it cast a shadow on a white surface. The shadow cast by the rod was wider than the shadow should have been if light traveled in a straight line past the edges of the rod. Grimaldi also noted that the shadow was bordered by colored bands. He determined that both of these observations could be explained if light bent slightly. He called the bending of light as it passes the edge of a barrier **diffraction.**

Huygens' principle In 1678 Dutch scientist Christiaan Huygens used a wave model to explain diffraction. According to Huygens' principle, all the points of a wavefront of light can be thought of as new sources of smaller waves. These smaller waves, or wavelets, expand in every direction and are in step with one another. A flat, or plane, wavefront of light consists of an infinite number of point sources in a line. **Figure 12** illustrates Huygen's principle.

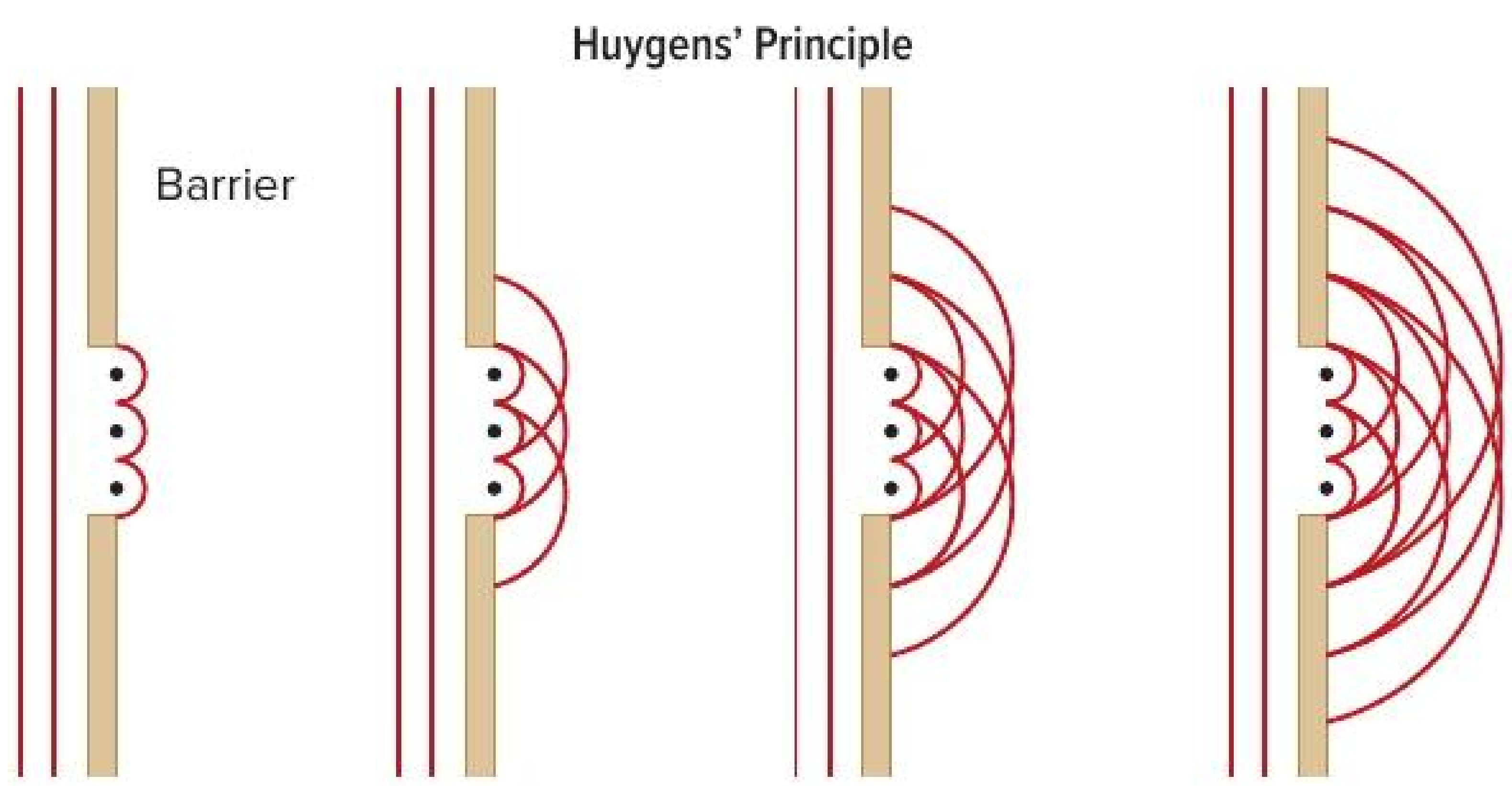


Figure 12 Huygens' wavelets combine to form a straight wavefront, except at the edges of the wave. The wavelets spread out in a circular manner when a barrier creates an edge.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Polarization

Carry out an investigation to determine the effect of polarizing filters on light.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

As a wavefront passes by an edge, the edge cuts the wavefront such that each circular wavelet generated by each Huygens' point will propagate as a circular wave in the region where the original wavefront was bent. This wave model explained the diffraction Grimaldi.

Color

In 1666 Newton performed experiments on the colors produced when a narrow beam of sunlight passed through a glass prism, as shown in **Figure 13.** Newton called the ordered arrangement of colors a spectrum. Using his later-disproved corpuscle (or particle) model of light, he thought that particles of light interacted with some unevenness in the glass to produce the spectrum.

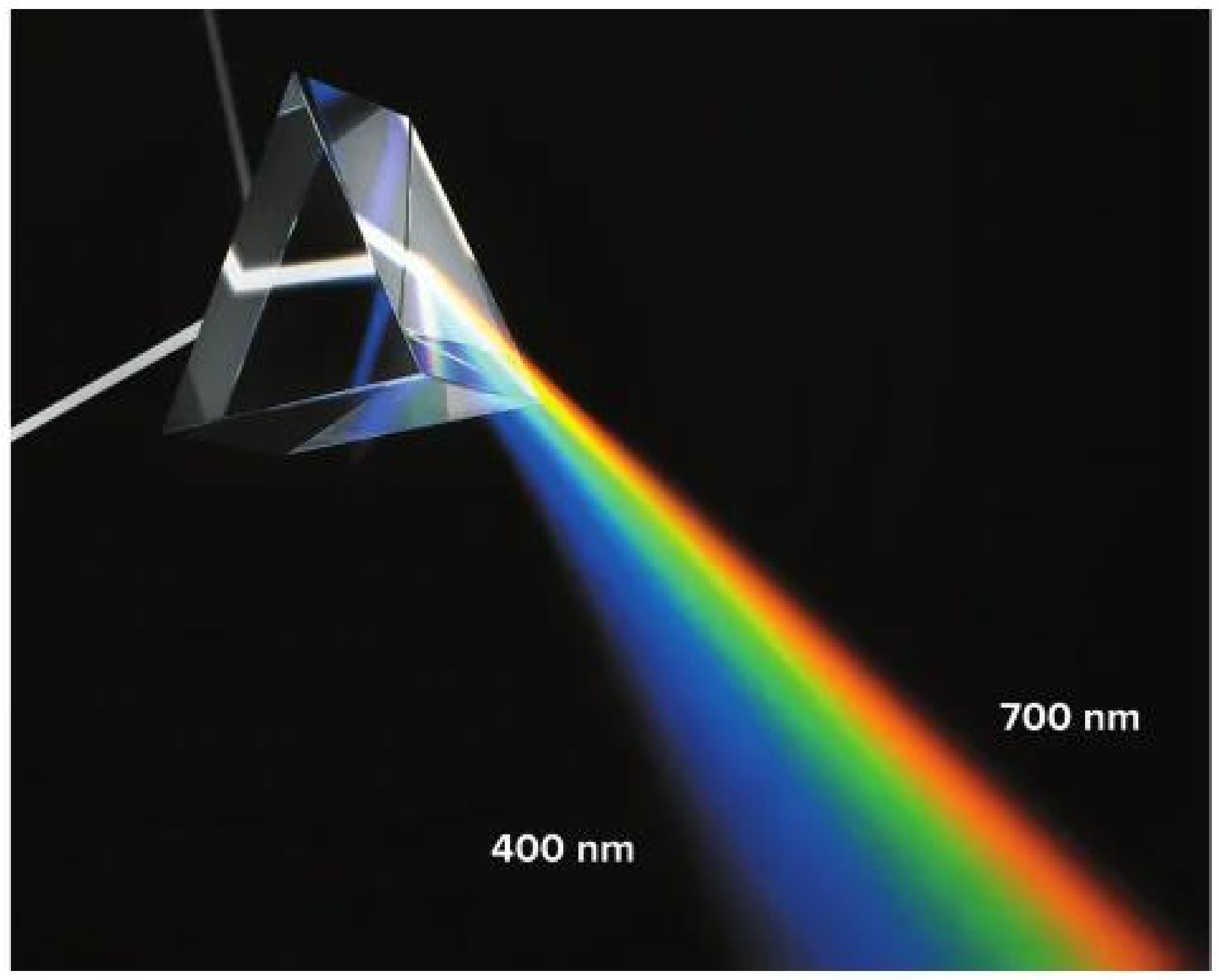


Figure 13 White light is separated into bands of color by a prism. Each color has a different wavelength.

To test this assumption, Newton allowed the spectrum from one prism to fall on a second prism. If the spectrum was caused by irregularities in the glass, he reasoned that the second prism would increase the spread in colors. Instead, the second prism reversed the spreading of colors and recombined them to form white light. After more experiments, Newton concluded that white light is composed of colors and that a property of the glass other than unevenness caused the light to separate into colors.

Different wavelengths

Can the wave model of light explain Newton's observations? For light to be a wave, it must have wavelength and frequency. The work of Grimaldi, Huygens, Newton, and others suggested that the color of light is related to wavelength. Visible light falls within the range of wavelengths from about 400 nm (4.00×10^{-7} m) to 700 nm (7.00×10^{-7} m), as shown in **Figure 13.** The longest visible wavelengths are seen as red light and the shortest as violet.

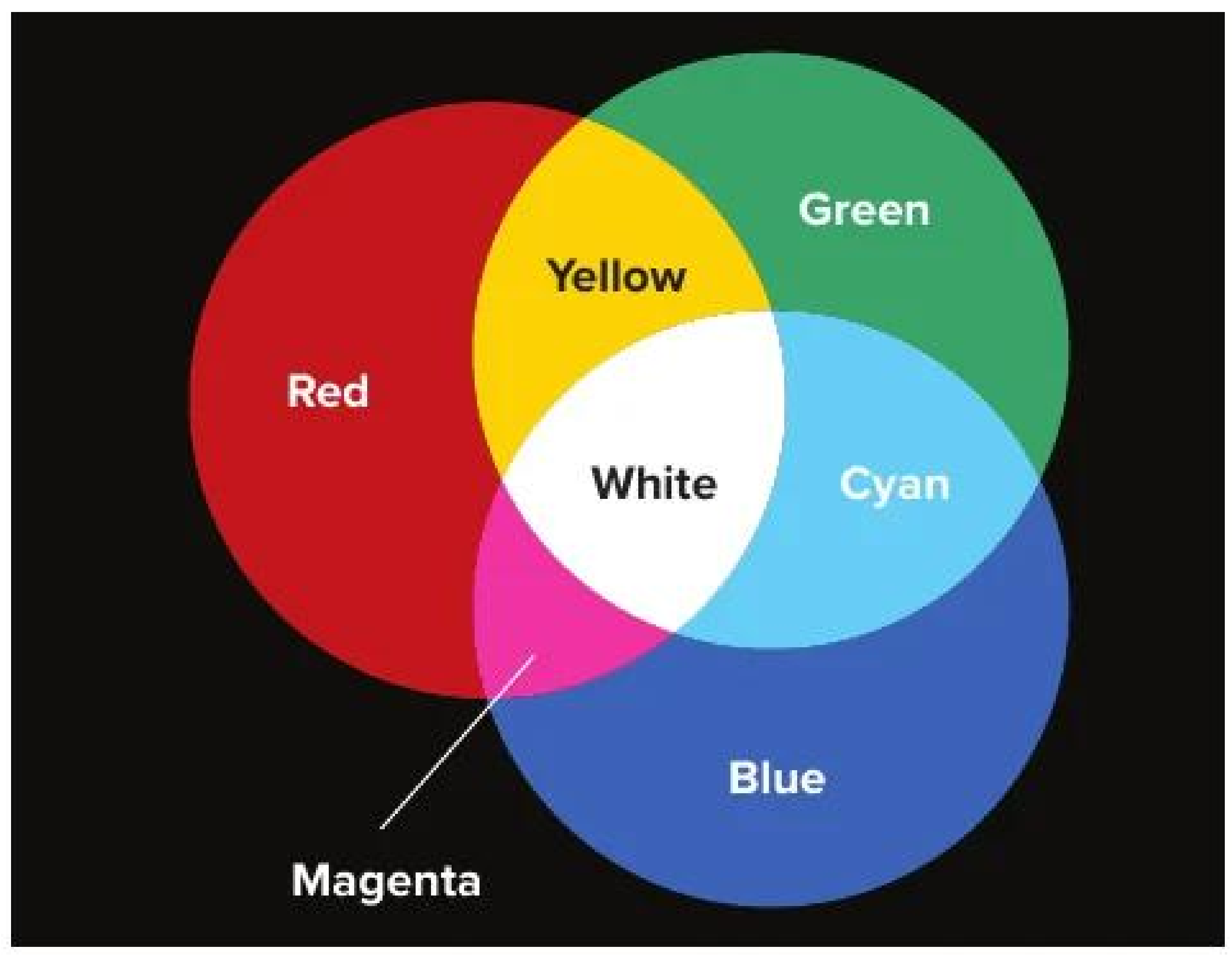


Figure 14 Red, green, and blue light, the primary colors, combine in pairs to produce yellow, cyan, or magenta light. The region where these three colors overlap on the screen appears white.

As white light crosses the boundary from air into glass and back into air in **Figure 13,** its wave nature causes each different color of light to be bent at a different angle. The shorter the wavelength, the more the light is bent. This unequal bending of the different colors causes the white light to be spread into a spectrum.

Color by addition of light

White light can be formed from colored light in a variety of ways. For example, when the correct intensities of red, green, and blue light are projected onto a white screen, as in **Figure 14,** white light is formed. This is called the additive color process, which is used in many television screens. A television screen uses three colors—red, green, and blue. Combinations of these produce the colors you see.

The correct intensities of red, green, and blue light appear on a screen as white light when combined. Because of this phenomenon, they are called the **primary colors** of light. The primary colors can be mixed in pairs to form three additional colors, as shown in **Figure 14.** Red and green light together produce yellow light, blue and green light produce cyan, and red and blue light produce magenta. The colors yellow, cyan, and magenta are called secondary colors. A **secondary color** is a combination of two primary colors. Note that these are slightly different from the primary and secondary colors you might have learned in art class; the reasons for this will be explained later in this module.

As shown in **Figure 14,** yellow light can be made from red light and green light. If yellow light and blue light are projected onto a white screen with the correct intensities, the surface will appear to be white. **Complementary colors** are two colors of light that can be combined to produce white light. Thus, yellow is a complementary color of blue, and vice versa, because the two colors of light combine to make white light. In the same way, cyan and red are complementary colors. Magenta and green are the other pair of complementary colors. A practical application of this is that yellowish laundry can be whitened with a bluing agent added to detergent.

Figure 15 The colors of objects we see are determined by which wavelengths of light are absorbed and which are reflected.

Explain *why the die that is yellow in the white light appears red in red light.*

Color by subtraction of light As you learned in the first lesson of this module, objects can reflect and transmit light. They also can absorb light. The color of an object depends on the wavelengths present in the light that illuminates the object. The color also depends on which wavelengths are absorbed by the object and which wavelengths are reflected. The natural existence or artificial placement of dyes in the material of an object, or pigments on its surface, gives the object color.

Dyes You are probably familiar with dyes that are used to color cloth. Dyes can be made from plant or insect extracts. For example, purple dye can be extracted from the berries of a black mulberry tree. The saffron crocus is a source of yellow dye. One type of red dye is extracted from an insect called a cochineal. A dye is a molecule that absorbs certain wavelengths of light and transmits or reflects others. When light is absorbed, its energy is transferred to the object that it strikes and is transformed into other forms of energy. A red shirt is red because the dyes in it reflect mostly red light to our eyes. When white light falls on the red object shown in **Figure 15,** the dye molecules in the object absorb most of the blue and green light and reflect mostly red light. When only blue light falls on the red object, very little light is reflected and the object appears to be almost black.

Get It?
Distinguish the difference between color by subtraction and color by addition.

Pigments The difference between a dye and a pigment is that pigments usually are made of crushed minerals rather than plant or insect extracts. For example, hematite produces a red pigment, and blue pigment can be obtained from azurite. Pigment particles can be seen with a microscope. A pigment that absorbs only one primary color and reflects two from white light is called a **primary pigment.** Yellow pigment absorbs blue light and reflects red and green light. Yellow, cyan, and magenta are the colors of primary pigments.

A pigment that absorbs two primary colors and reflects one color is called a **secondary pigment.** The colors of secondary pigments are red (which absorbs green and blue light), green (which absorbs red and blue light), and blue (which absorbs red and green light). Note that the primary pigment colors are these secondary colors of light. In the same way, the secondary pigment colors are light's primary colors.

The primary and secondary pigments are shown in **Figure 16.** When the primary pigments yellow and cyan are mixed, the yellow absorbs blue light and the cyan absorbs red light. **Figure 16** shows yellow and cyan combining to make green pigment. When yellow pigment is mixed with the secondary pigment blue, which absorbs green and red light, all the primary colors are absorbed, and the result is black. Yellow and blue are complementary pigments. Cyan and red, as well as magenta and green, are also complementary pigments.

CHEMISTRY Connection A color printer uses yellow, magenta, and cyan dots of pigment to make a color image on paper. Often, pigments that are used are finely ground compounds, such as titanium(IV) oxide (white), chromium(III) oxide (green), and cadmium sulfide (yellow). Pigments mix to form suspensions rather than solutions. Their chemical form is not changed in a mixture, so they still absorb and reflect the same wavelengths.

BIOLOGY Connection You can now begin to understand the colors that you see in **Figure 17.** The plants on the mountain look green because of the chlorophyll in them. One type of chlorophyll absorbs mostly red light and the other absorbs mostly blue light, but they both reflect green light. The energy in the red and blue light that is absorbed is used by the plants during photosynthesis to make food.

Figure 16 Magenta, cyan, and yellow are the primary pigments. Secondary pigments, red, green, and blue, are produced from mixing the primary pigments in pairs.

Figure 17 Chlorophyll in green leaves reflects mostly green light, giving the leaves their color.

Explain *why the plants are various shades of green.*

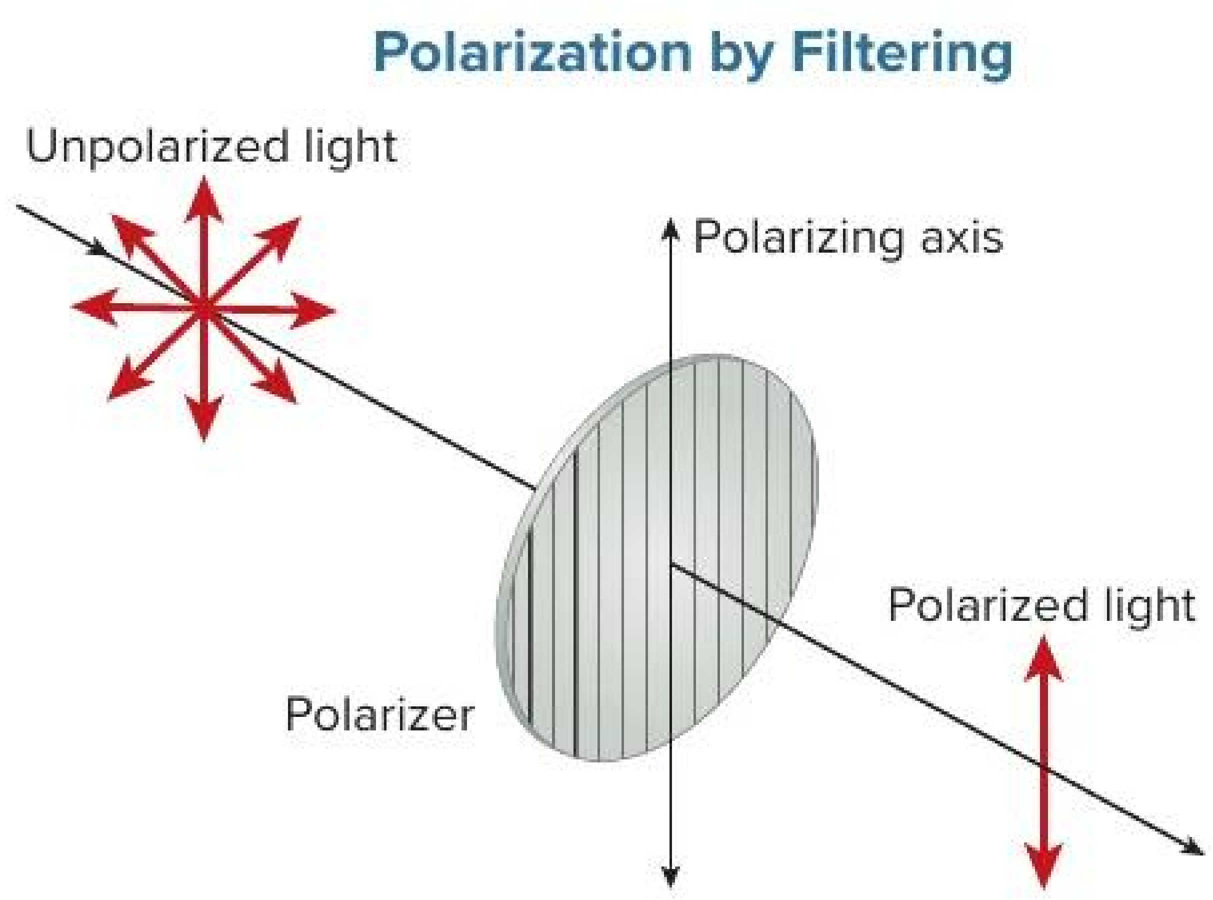


Figure 18 Nonpolarized light rays vibrate randomly in every direction perpendicular to the direction they travel. A polarizing medium blocks light that is not parallel to the polarizing axis.

Polarization of Light

Have you ever looked at light reflected off a road through polarizing sunglasses? If you rotate the glasses, the road first appears to be dark, then light, and then dark again. Light from a lamp, however, changes very little as the glasses are rotated. Why is there a difference? Normal lamplight is not polarized. However, the light that is coming from the road is reflected and has become polarized. **Polarization** is the production of light with a specific pattern of oscillation.

Recall that light behaves as a transverse wave. For waves on a rope, the oscillating medium is the rope. For light waves, the oscillating medium is the electric field. When this electric field oscillates in random directions, the light is nonpolarized. How can you filter nonpolarized light so that whatever passes through the filter is polarized light?

Get It?
Draw a diagram, with text explanations, showing nonpolarized light.

Polarization by filtering

The lines in the polarizer in **Figure 18** represent a polarizing axis. The light with the portion of the electric field that oscillates parallel to these lines passes through. The light with the portion of the electric field that oscillates perpendicular to these lines is absorbed. If a polarizer is placed in a beam of nonpolarized light, only the components of the waves in the same direction as the polarizing axis can pass through. As a result, half of the total light passes through, reducing the intensity of the light by half.

Get It?
Draw a second diagram, with text explanations, showing nonpolarized light passing through a filter as polarized light.

CHEMISTRY Connection Polarizing mediums contain long molecules in which electrons can oscillate, or move back and forth, all in the same direction. As light travels past the molecules, the electrons absorb light waves that oscillate in the same direction as the electrons. This allows light waves vibrating in one direction to pass, while the waves vibrating in the other direction are absorbed. The direction of a polarizing medium perpendicular to the long molecules is called the polarizing axis. Only waves oscillating parallel to that axis can pass through.

Polarization by reflection When you look through a polarizing filter at the light reflected by a sheet of glass and rotate the filter, you will see the light brighten and dim. The light is partially polarized parallel to the plane of the glass when it is reflected. Polarized reflected light causes glare. Polarizing sunglasses reduce glare from the polarized light reflected off roads. Photographers can use polarizing filters over camera lenses to block reflected light. This result is shown in **Figure 19.**

Figure 19 Glare is light that has been polarized by reflection. Photographers use polarizing filters to reduce glare.

Malus's law Suppose you produce polarized light with a polarizing filter. What would happen if you place a second polarizing filter in the path of the polarized light? If the polarizing axis of the second filter is parallel to that of the first, the light will pass through. If the polarizing axis of the second filter is perpendicular to that of the first, no light will pass through, as shown in **Figure 20.**

If the light intensity after the first polarizing filter is I_1 and the intensity after the second filter is I_2, how can you control I_2? I_2 depends only on I_1 and the angle between the axes of the filters, θ. If θ is 0, I_2 equals I_1; if θ is 90°, all of the light is blocked, resulting in I_2 being 0. This indicates that the intensity might depend on the cosine of θ. The actual relationship is a that of a cosine squared. The law that explains the reduction of light intensity as light passes through a second polarizing filter is **Malus's law.**

Malus's Law

The intensity of light coming out of a second polarizing filter is equal to the intensity of polarized light coming out of a first polarizing filter multiplied by the cosine, squared, of the angle between the polarizing axes of the two filters.

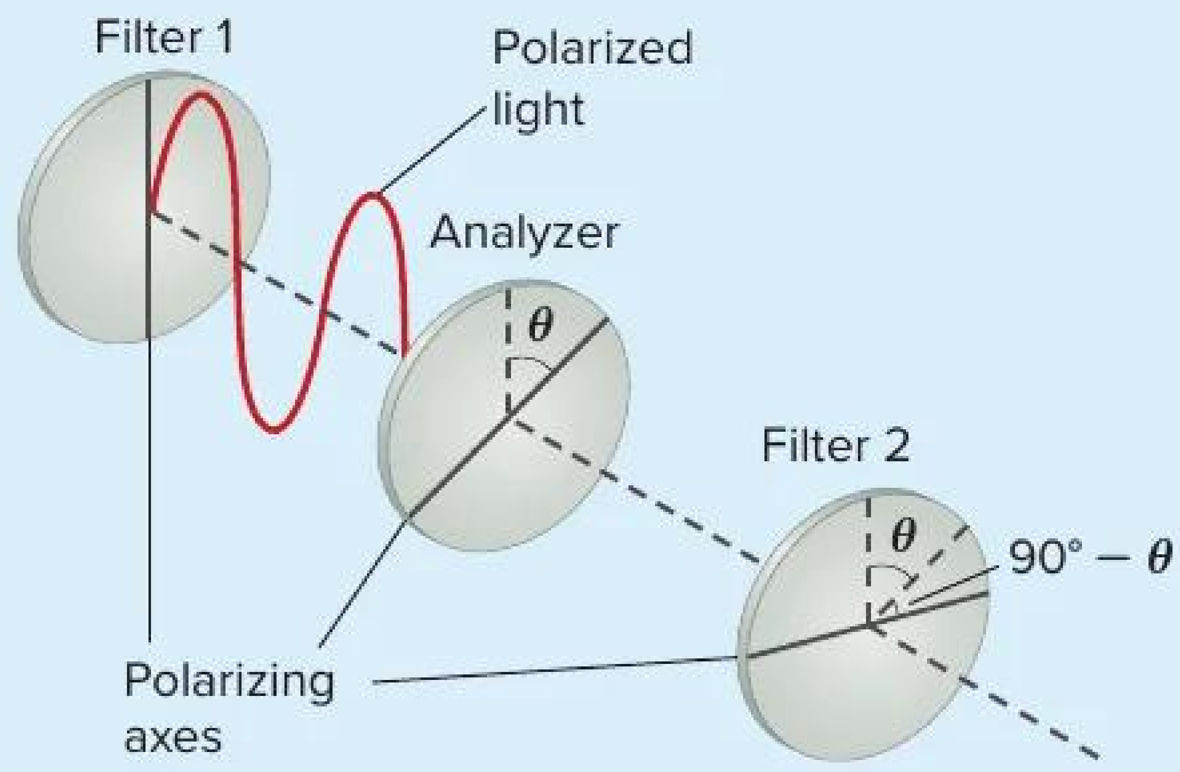
$$I_2 = I_1 \cos^2 \theta$$

Figure 20 Polarizing filters with their axes parallel will allow the light with the same orientation to pass. With the polarizing axes at a 45° angle, the filters allow some light to pass. If the axes of the filters are perpendicular, the second filter will block the light that has passed through the first filter.

PHYSICS Challenge

You place an analyzer filter between the two cross-polarized filters, such that its polarizing axis is not parallel to either of the two filters, as shown in the figure to the right.

1. You observe that some light passes through filter 2, though no light passed through filter 2 before you inserted the analyzer filter. Why does this happen?
2. The analyzer filter is placed at an angle of θ relative to the polarizing axis of filter 1. Derive an equation for the intensity of light coming out of filter 2 compared to the intensity of light coming out of filter 1.



Speed, Wavelength, and Frequency of Light

As you have learned, the source of a wave determines that wave's frequency (f), and the medium and the frequency together determine the wavelength (λ) of a wave. Because light has wave properties, the same mathematical models used to describe waves in general can be used to describe light. For light of a given frequency traveling through a vacuum, wavelength is a function of the speed of light (c), which can be written as $\lambda_o = c/f$. The development of the laser in the 1960s provided new ways to measure the speed of light. The frequency of light can be measured with extreme precision using lasers and the time standard of atomic clocks. Measurements of wavelengths of light, however, are much less precise.

All colors of light travel at c in a vacuum, though the wavelengths are different. Since $\lambda_o = c/f$, once the frequency of a light wave in a vacuum is measured, the wavelength can be determined.

Relative motion and light What happens if a light source travels toward you or you move toward the source? You have learned that the frequency of a sound heard by a listener changes if either the source or the listener of the sound is moving. The same is true for light. However, when you consider the velocities of a sound source and the observer, you are really considering each one's velocity relative to the medium through which the sound travels. This is not the case for light.

Get It?
Explain how the wavelength, frequency, and speed of light are mathematically related.

The Doppler effect The nature of light waves is such that they are not vibrations of the particles of a medium. The Doppler effect for light can involve only the relative velocity between the source and the observer. Remember that the only factors in the Doppler effect are the velocity components along the axis between the source and the observer, as shown in **Figure 21** on the next page.

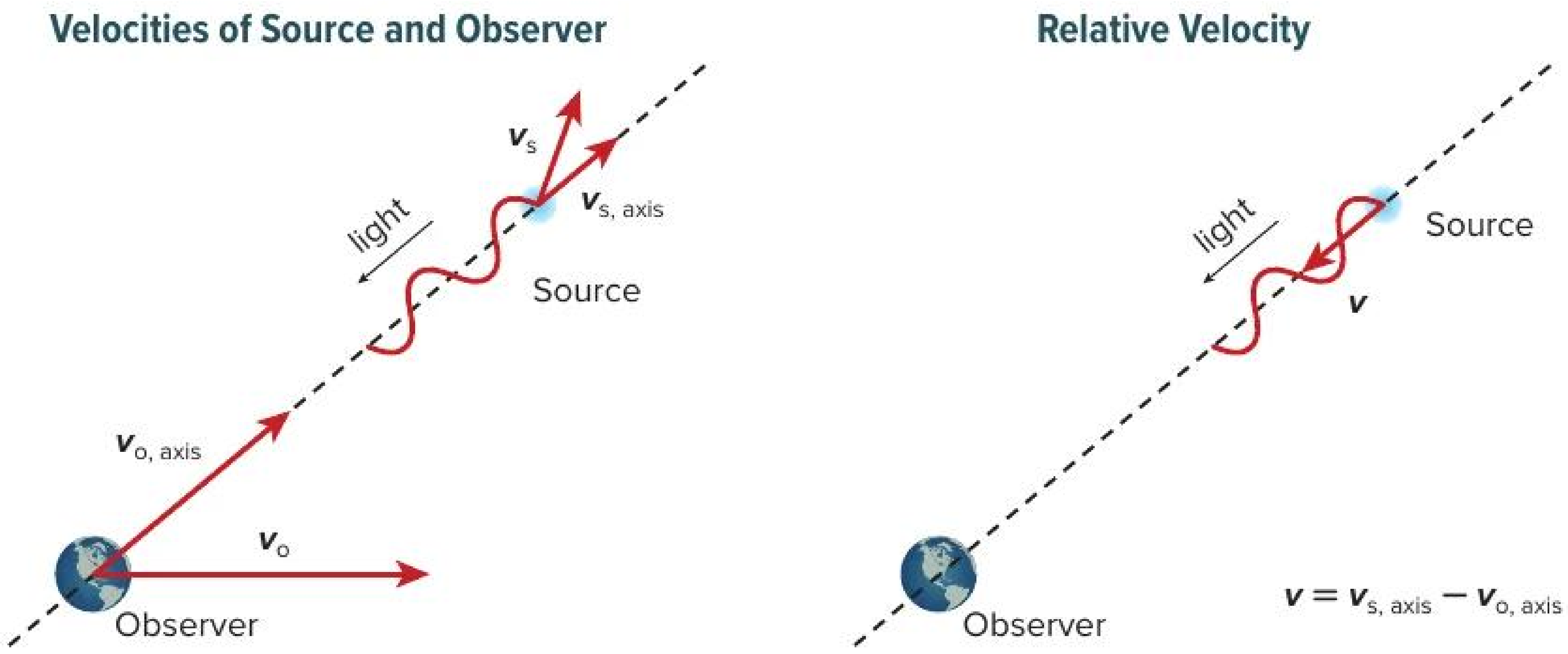


Figure 21 The Doppler effect describes how light frequency changes if an observer and a light source are moving toward or away from each other.

Doppler effect for light problems can be simplified by considering axial relative speeds that are much less than the speed of light ($v << c$). This simplification is used to develop the equation for the observed light frequency (f_{obs}), shown below.

Observed Light Frequency

The observed frequency of light from a source is equal to the actual frequency of the light generated by the source, times the quantity 1 plus the relative speed along the axis, divided by the speed of light, between the source and the observer if they are moving toward each other, or 1 minus the relative speed, divided by the speed of light, if they are moving away from each other.

$$F_{obs} = f\left(1 \pm \frac{v}{c}\right)$$

Applications Most applications of the Doppler effect for light are in astronomy, where phenomena are discussed more in terms of wavelength. Using the relationship $\lambda = c/f$ and the $v << c$ simplification, the following equation describes the Doppler shift ($\Delta\lambda$), the difference between the observed and the actual wavelengths.

Doppler Shift

The difference between the observed wavelength of light and the actual wavelength of light generated by a source is equal to the actual wavelength of light generated by the source, times the relative speed of the source and observer, divided by the speed of light.

$$(\lambda_{obs} - \lambda) = \Delta\lambda = \pm\left(\frac{v}{c}\right)\lambda$$

A positive change in wavelength occurs when the relative velocity of the source is away from the observer. In this case, the observed wavelength is longer than the original wavelength. The light appears closer to the red end of the spectrum than it normally would. We say this light is red shifted. A negative change in wavelength occurs when the relative velocity of the source is in a direction toward the observer. In this case, the observed wavelength is shorter than the original wavelength. This is known as a blue shift.

Because the speed of light is constant, when the wavelength is red shifted, the observed frequency is lower than the original due to the inverse relationship between the two variables. When light is blue shifted, the observed frequency is higher.

ASTRONOMY Connection Astronomers can determine how objects, such as galaxies, are moving relative to Earth by observing the Doppler shift of their light. This is done by observing the spectrum of light coming from stars in the galaxy using a spectrometer, as shown in **Figure 22.** The same elements that are present in the stars of galaxies emit light of specific wavelengths in labs on Earth. By comparing wavelength, astronomers can learn the velocities of objects toward or away from Earth.

Figure 22 The bottom hydrogen emission spectrum is red shifted compared to the laboratory spectrum, indicating the light source is moving away from Earth.

PRACTICE Problems

ADDITIONAL PRACTICE

16. Oxygen can be made to produce light with a wavelength of 513 nm. What is the frequency of this light?

17. A hydrogen atom in a galaxy moving with a speed of 6.55×10^6 m/s away from Earth emits light with a frequency of 6.16×10^{14} Hz. What frequency of light from that hydrogen atom would be observed by an astronomer on Earth?

18. A hydrogen atom in a galaxy moving with a speed of 6.55×10^6 m/s away from Earth emits light with a wavelength of 486 nm. What wavelength would be observed on Earth from that hydrogen atom?

19. CHALLENGE An astronomer is looking at the spectrum of a galaxy and finds that it has an oxygen spectral line of 525 nm, while the laboratory value is measured at 513 nm. Calculate how fast the galaxy would be moving toward or away from Earth and how you know.

STEM CAREER Connection

Photographer

Do you like capturing important events with a camera? Photographers must use light to compose a perfect picture or to achieve a desired effect. Understanding the fundamentals of light is an important part of being a good photographer.

In 1929, Edwin Hubble analyzed the light from many galaxies like the one shown in **Figure 23.** He observed that the light produced by familiar elements were at longer wavelengths than he had expected them to be. The light was shifted toward the red end of the spectrum. No matter what area of the sky he observed, almost all the galaxies were sending red shifted light to Earth. What do you think caused the spectral lines to be red shifted?

Hubble concluded that galaxies are moving away from Earth and suggested that the universe is expanding. Additional studies since then have supported this conclusion. As galaxies move, they sometimes collide and merge. The results of the collision depend on the size and speed of the galaxies.

Figure 23 Edwin Hubble observed that galaxies were sending red shifted light to Earth, indicating that they were moving away from Earth and the universe was expanding.

You have learned that some characteristics of light can be explained with a simple ray model of light, whereas others require a wave model of light. You can use both of these models to study how light interacts with mirrors and lenses. There are some aspects of light that can be understood only through the use of the wave model of light.

Check Your Progress

20. **Doppler Effect** Describe the relative motions of objects when light is red shifted and when light is blue shifted. Answer using the term *Doppler effect.*
21. **Addition of Light Colors** What color of light must be combined with blue light to obtain white light?
22. **Light and Pigment Interaction** What color will a yellow banana appear to be when illuminated by each of the following?
    a. white light
    b. green and red light
    c. blue light
23. **Pigment Colors** What are the secondary pigment colors, and why do they give objects the appearance of those colors?
24. **Combination of Pigments** What primary pigment colors must be mixed to produce red? Explain your answer in terms of color subtraction for pigment colors.
25. **Polarization** Describe a simple experiment you could do to determine whether sunglasses in a store are polarizing.
26. **Polarizing Sunglasses** Use **Figure 24** to determine the direction the polarizing axis of polarizing sunglasses should be oriented to reduce glare from the surface of a road: vertically or horizontally? Explain.

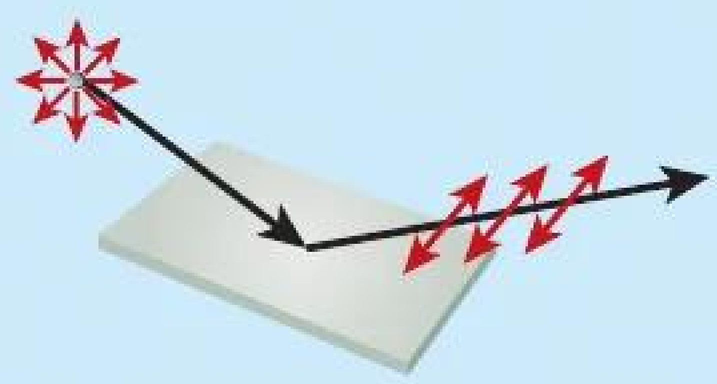


Figure 24

27. **Red Light** The speed of red light is slower in air and water than in a vacuum. The frequency, however, does not change when red light enters water. Does the wavelength change? If so, how?
28. **Critical Thinking** Astronomers have determined that our galaxy, the Milky Way, is moving toward Andromeda, a neighboring galaxy. Explain how they determined this. Can you think of a possible reason why the Milky Way is moving toward Andromeda?

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SCIENTIFIC BREAKTHROUGHS

Super-Efficient Solar Cells

Solar panels on homes and businesses are an increasingly popular alternative source of energy in today's society. This technology converts energy from the Sun into electricity people can use for the needs of everyday life. Solar energy is an inexhaustible source of clean energy. However, the current technology used to convert solar energy to electricity has limitations. Scientists working to optimize this technology have recently made a breakthrough that could greatly increase its efficiency.

Solar panels convert light waves into electricity.

What is a photovoltaic cell?

A solar panel is made up of many photovoltaic (PV) cells, which convert light waves into electrical current. Incoming light waves strike the semiconductor, usually silicon, and transfer energy to electrons. The electrons flow in one direction through the material, creating an electric current.

Limitations of Current Technology

Energy emitted by the Sun encompasses the entire electromagnetic spectrum. The silicon used in PV cells can absorb only a portion of these waves—from red to violet on the visible light spectrum. The remainder of the Sun's energy cannot be utilized by existing PV cells. As a result, PV cells are very inefficient. For example, most residential solar panels convert about 10 to 20 percent of the incoming energy to electricity, so solar panels need to be very large, especially in areas with cold and cloudy climates.

Hot Solar Cells

Scientists have developed a new solar thermo-photovoltaic (STPV) cell for converting solar energy to electricity. These STPV cells, called "hot solar cells," couple conventional PV cells with a light concentrator to utilize a greater percentage of the incoming energy from the Sun. The absorbing layer of carbon nanotubes turns energy from the Sun into thermal energy—with temperatures up to 1000°C! The emitting layer of nanophotonic crystals converts this thermal energy back into light that is specifically in the range of wavelengths that can be absorbed by the traditional PV cell. Initial results show that STPV cells create up to twice the amount of electrical energy compared to basic PV cells. This increased energy production can reduce the dependence on nonrenewable energy sources such as fossil fuels.

EVALUATE DESIGN SOLUTIONS

Using the Internet, gather information about the new STPV cell technology and evaluate this design solution. With a partner, discuss how the new technology might impact the cost, safety, reliability, or aesthetics of using solar energy to produce electricity. Summarize your discussion in a written paragraph.

MODULE 15

STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 ILLUMINATION

- Light can be modeled as a ray that travels in a straight path until it encounters a boundary. Mediums can be characterized as being transparent, translucent, or opaque, depending on how light interacts with them.
- The luminous flux of a light source is the rate at which light is emitted. It is measured in lumens (lm). Illuminance is the luminous flux per unit area. Illuminance is measured in lux (lx), or lumens per square meter (lm/m^2). For a point source, illuminance follows an inverse-square relationship with distance and a direct relationship with luminous flux.

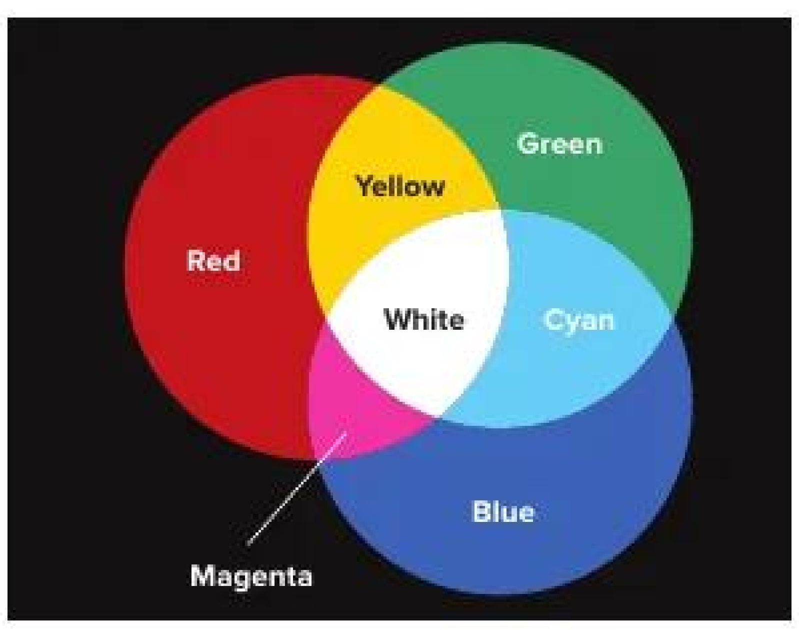
$$E = \frac{P}{4\pi r^2}$$

- Early measurements of the speed of light involved measurement of the time it takes for light to reach Earth from Jupiter's moon Io. Michelson used a land-based technique that involved the distance between two mountains and a set of rotating mirrors. In a vacuum, light has a constant speed of $c = 3.00\times10^8$ m/s.

- ray model of light
- luminous sources
- opaque
- translucent
- transparent
- luminous flux
- illuminance

Lesson 2 THE WAVE NATURE OF LIGHT

- In the wave model of light, all the points in a wavefront can be thought of as sources of smaller waves. As light travels past an edge, the wavefront is cut and each new wavelet generates a new circular wave.
- Visible light can have wavelengths between 400 and 700 nm. White light is a combination of the spectrum of colors, each color having a different wavelength. Combining the primary colors—red, blue, and green—forms white light. Combinations of two primary colors form the secondary colors, yellow, cyan, and magenta.



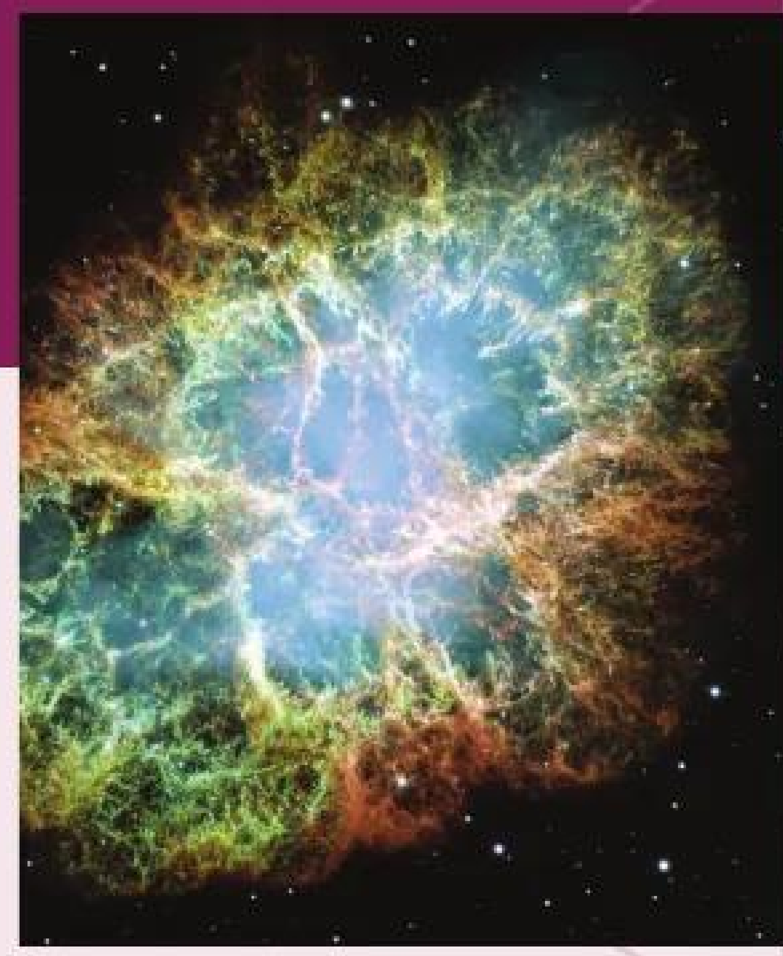
The primary pigments, cyan, magenta, and yellow, are used in combinations of two to produce the secondary pigments, red, blue, and green.

- Polarized light consists of waves whose electric fields oscillate with a specific pattern. Often, the oscillation is in a single plane. Light can be polarized with a polarizing filter or by reflection. Light waves traveling through a vacuum can be characterized in terms of frequency, wavelength, and the speed of light. Light waves are Doppler shifted based on the relative speed of the observer and light source along the axis of the observer and the light source.

- diffraction
- primary colors
- secondary color
- complementary colors
- primary pigment
- secondary pigment
- polarization
- Malus's law

THREE-DIMENSIONAL THINKING

Module Wrap-Up



REVISIT THE PHENOMENON

What does the light from a distant star or supernova tell us about it?

CER Claim, Evidence, Reasoning

Explain your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

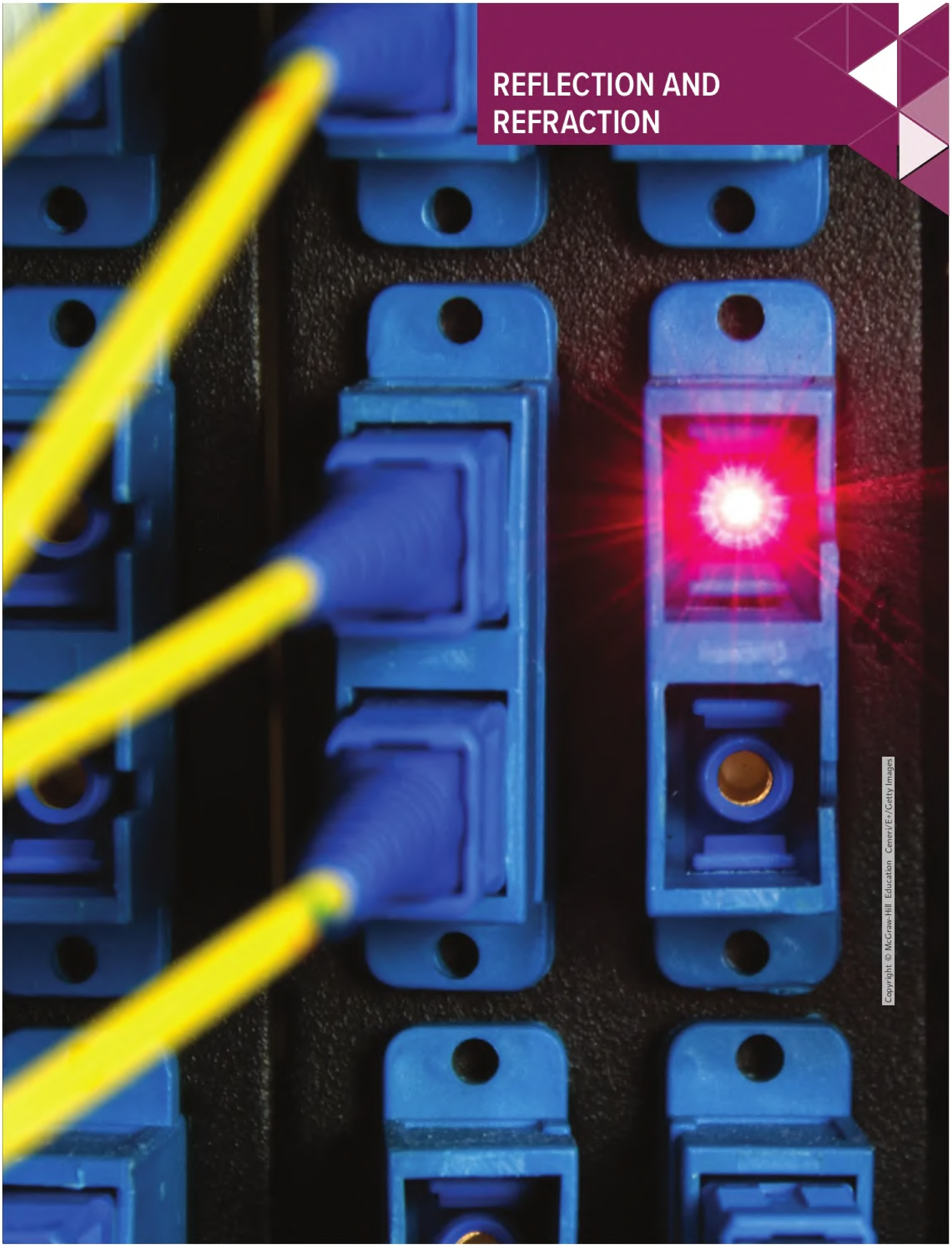
Can a traffic light appear to change color?

Suppose you are a traffic officer and you stop a driver for going through a red light. Further suppose the driver draws you a picture and explains that the light looked green because of the Doppler effect when he drove through it. The wavelength for red light is 645 nm and it is 545 nm for a green light.

CER Analyze and Interpret Data

1. **Claim** Would you have given the driver a ticket for running the red light or accepted his explanation?
2. **Evidence and Reasoning** How would you explain your decision to the driver?

REFLECTION AND REFRACTION



REFLECTION AND REFRACTION

ENCOUNTER THE PHENOMENON

How does light transmit information through a communication network?

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

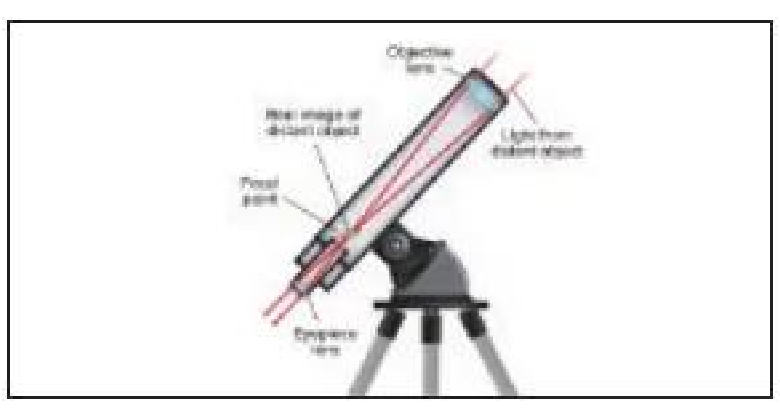
Make Your Claim Use your CER chart to make a claim about how light is used to transmit information in a communication network. Explain your reasoning.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.

LESSON 2: Explore & Explain: Images in Curved Mirrors



LESSON 4: Explore & Explain: Applications of Lenses

LESSON 1

REFLECTION OF LIGHT

FOCUS QUESTION

Why does light reflected from a mirror make an image while light reflected from a piece of paper does not?

Reflected Images

When you look at the surface of a body of water, you don't always see a clear reflection as in **Figure 1.** Sometimes, the wind causes ripples in the water, or passing boats produce waves. Disturbances on the water's surface prevent the light from reflecting in a manner such that a clear image is visible.

Almost 4000 years ago, Egyptians understood that the type of reflection you see from a still pond requires a smooth surface. They used polished metal mirrors to view their images. Artisans in sixteenth-century Venice created mirrors by coating the back of a flat piece of glass with a thin sheet of metal. Sharp, well-defined, reflected images were not possible until 1857, however, when Jean Foucault, a French scientist, developed a method of coating glass with silver.

Today, we don't only use mirrors to view our reflections. Mirrors are key components in lasers, telescopes, and other precise optical systems. We use ever-increasing precision to make modern mirrors. Today's mirrors are made by evaporating aluminum or silver onto highly polished glass. The same basic physics principles, however, govern reflection from the smooth surface of a pond, from a sixteenth-century mirror, and from a tiny mirror inside a laser.

Figure 1 Disturbances on the surface of a pond or lake produce a distorted reflected image.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Forensics Lab: A Little Time to Reflect
Carry out an investigation into the relationship between the angles of incident and reflection of light on a plane mirror.

CCC **Identify Crosscutting Concepts**
Create a table of the crosscutting concepts and fill in examples you find as you read.

The Law of Reflection

Figure 2 shows a light ray striking a reflecting surface. The normal is an imaginary line that is perpendicular to a surface at the location where light strikes the surface. The incident ray, the reflected ray, and the normal are in the same plane, which is perpendicular to the surface. Although light travels in three dimensions, the reflection of light is planar (two-dimensional). Planar relationships are known as the law of reflection.

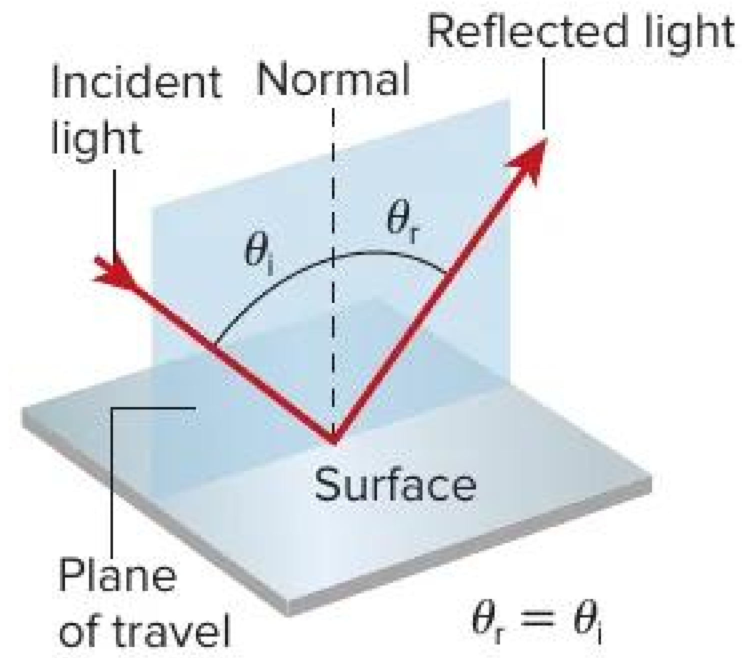


Figure 2 Reflecting light waves have an angle of reflection in the same plane as and equal to the angle of incidence.

COLOR CONVENTION

light ray ⟷ red

Law of Reflection

The angle that a reflected ray makes as measured from the normal to a reflective surface equals the angle that the incident ray makes as measured from the same normal.

$$\theta_r = \theta_i$$

Wave model We can describe this law in terms of the wave model of light. **Figure 3** shows a wavefront of light approaching a surface. The wavefront is perpendicular to the light ray. As each point along the wavefront reaches the surface, it reflects off that surface. Because all points are traveling at the same speed, they all travel the same total distance in the same time. Thus, the wavefront as a whole leaves the surface at an angle equal to its incident angle. Note that the wavelength of the light does not affect this process. The surface reflects red, green, and blue light all in the same direction.

Smooth surfaces Consider the light rays shown on the left in **Figure 4** on the next page. All the rays reflect off the surface parallel to one another. This occurs only if the reflecting surface is not rough on the scale of the wavelength of the light. We consider such a surface to be smooth. A smooth surface, such as a mirror, produces **specular reflection,** in which parallel light rays reflect in parallel.

Rough surfaces What happens when light strikes a sheet of paper? A sheet of paper might appear smooth, but on the scale of the wavelength of light, paper is actually quite rough. The right side of **Figure 4** shows light rays reflecting off a sheet of paper. All of the light rays are parallel before they strike the surface, but the reflected rays are not parallel, as shown. This scattering of light off a rough surface is called **diffuse reflection.**

The law of reflection applies to smooth and rough surfaces. For a rough surface, the angle that each incident ray makes with the normal equals the angle that its reflected ray makes with the normal. On a microscopic scale, however, the normals to the surface locations are not parallel.

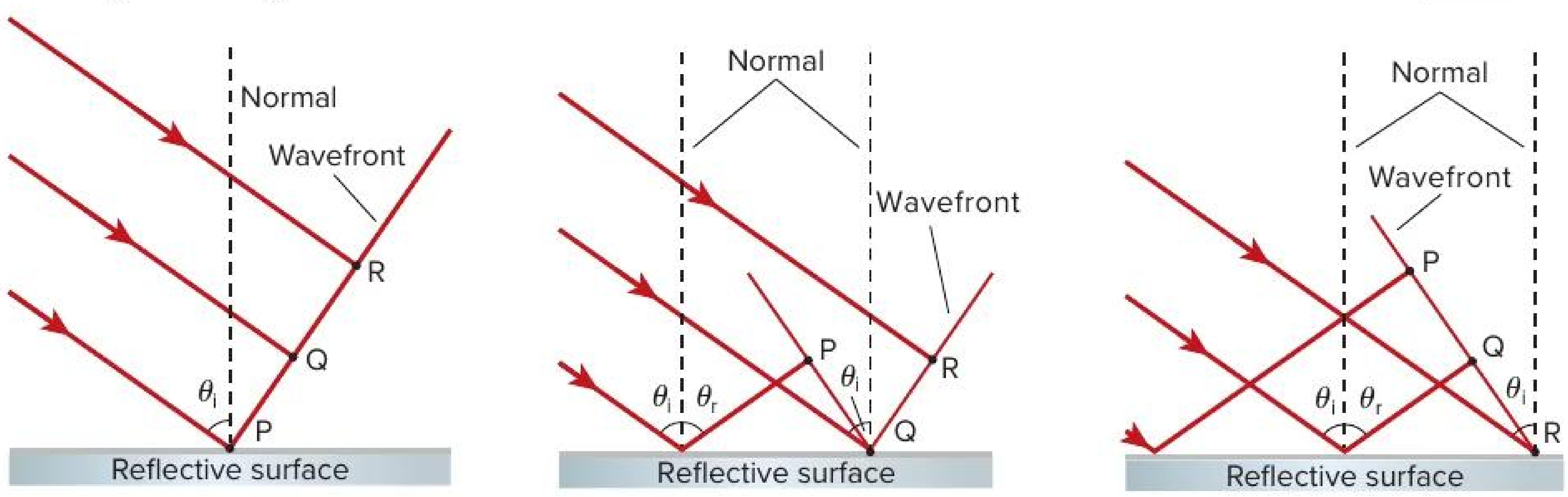


Figure 3 All parts of a wavefront reflect from a surface at the same angle. The angle of incidence equals the angle of reflection.

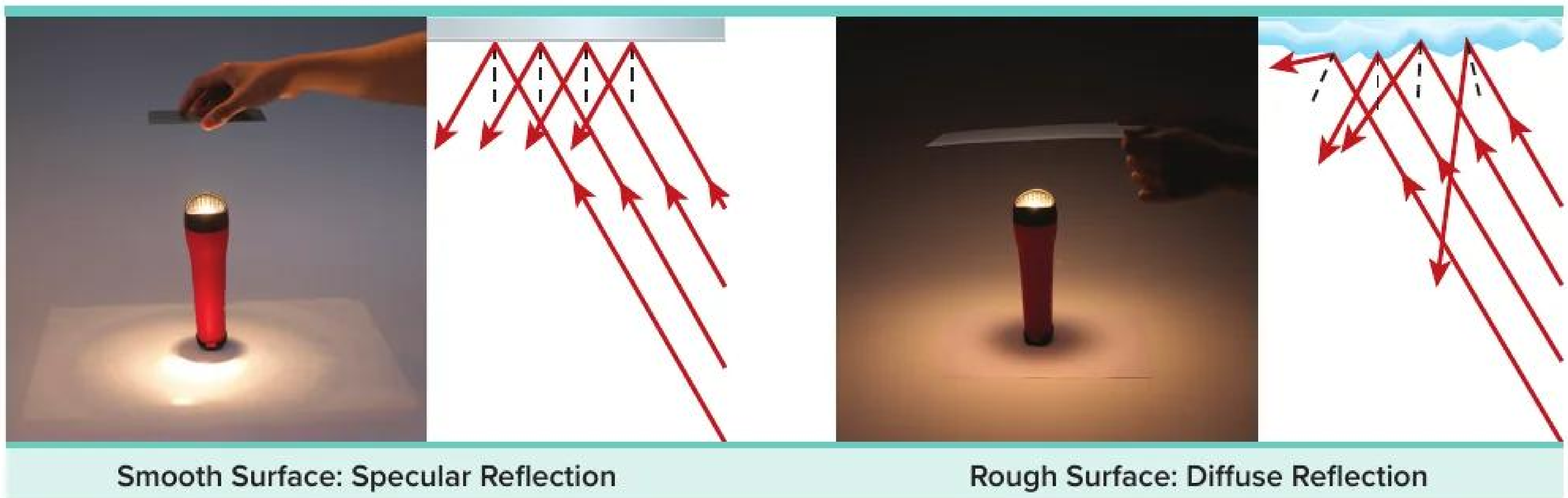


Figure 4 Notice that the image of the lightbulb in the flashlight is reflected on the table by the smooth mirror. The surface of the paper reflects a featureless area of light. Diffuse reflection enables you to read this page from various angles.

Thus, the rough surface prevents the reflected rays from being parallel, so the reflected rays are scattered in different directions. With specular reflection, as with a mirror, you can see your face. But no matter how much light reflects off a wall or a sheet of paper, as shown in **Figure 4,** you will never be able to use them as mirrors.

Objects and Plane-Mirror Images

A **plane mirror,** such as a mirror we use to look into, is a flat, smooth surface from which light is reflected by specular reflection.

When studying illumination, an **object** is either a luminous source of light rays, such as a lightbulb, or an illuminated source of light rays, such as a person. For most of the light sources that you will study in this module, light spreads out from that source in all directions. A mirrored surface can reflect these light rays so an image is visible, as shown in **Figure 5.**

Figure 5 Reflection from the woman to the mirror is diffuse reflection. Reflection from the mirror is specular reflection. **Describe** *how the woman is illuminated.*

In **Figure 6** light reflects diffusely from all parts of the bird (the object). The girl sees the light that reflects into her eye. Because her brain processes this information as if the rays travel in a straight path, it seems to the girl as if the light follows the dashed lines. The light seems to have come from a point behind the mirror. Just as the rays diverge from the object, they also diverge from the image.

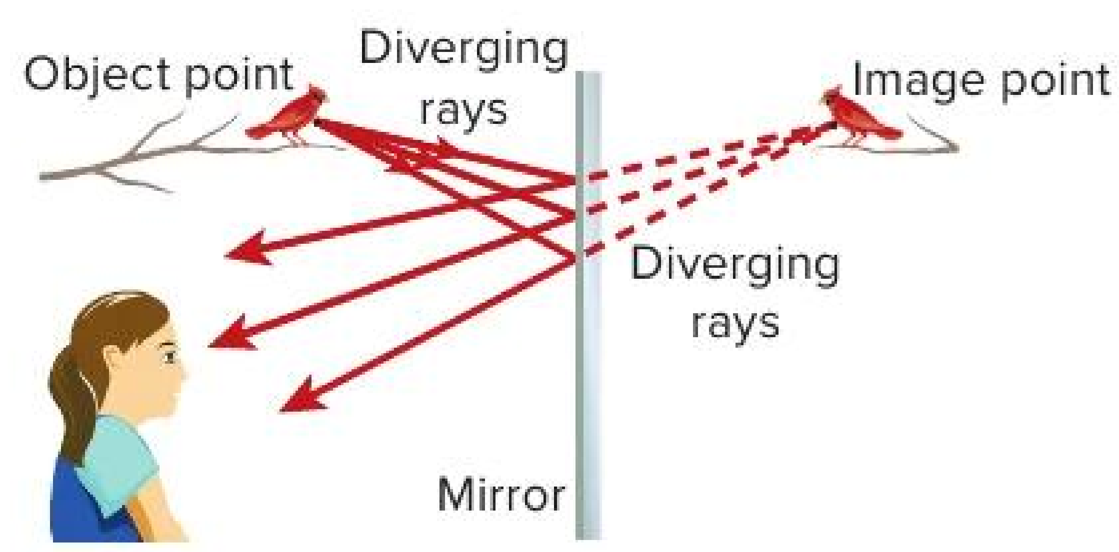
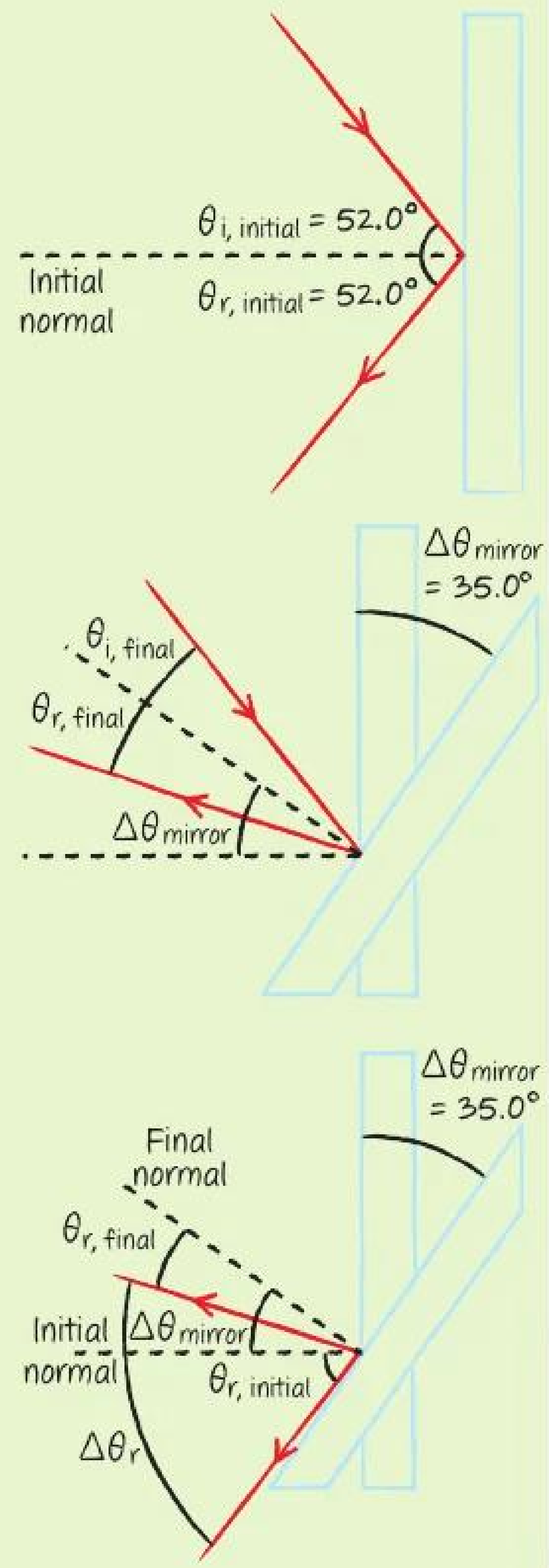


Figure 6 Rays that reflect from the bird disperse in many directions. Only a few that travel toward the mirror are shown. The image is located where multiple light rays from a point on an object seem to converge.

The combination of light rays reflected from the bird in **Figure 6** forms the **image** of the bird. It is a **virtual image,** a type of image formed by diverging light rays. A virtual image is always on the opposite side of the mirror from the object. The image is virtual because there are no light rays at the image location. Plane mirrors produce only virtual images.

EXAMPLE Problem 1

CHANGING THE ANGLE OF INCIDENCE A light ray strikes a plane mirror at an angle of 52.0° to the normal. The mirror then rotates 35.0° around the point where the ray strikes the mirror so that the angle of incidence of the light ray decreases. The axis of rotation is perpendicular to the plane of the incident and the reflected rays. What is the angle between the initial and final reflected ray?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation before the rotation of the mirror.
- Draw another sketch with the angle of rotation applied to the mirror.
- Draw a third sketch of the reflected rays.

Known	Unknown
$\theta_{i,\ initial} = 52.0°$	$\Delta\theta_r = ?$
$\Delta\theta_{mirror} = 35.0°$	

2 SOLVE FOR THE ANGLE DIFFERENCE

For the angle of incidence to reduce, rotate clockwise.

$$\theta_{i,\ final} = \theta_{i,\ initial} - \Delta\theta_{mirror}$$
$$= 52.0° - 35.0°$$ Substitute $\theta_{i,\ initial} = 52.0°$, $\Delta\theta_{mirror} = 35.0°$.
$$= 17.0° \text{ clockwise from the new normal}$$

Apply the law of reflection.

$$\theta_{r,\ final} = \theta_{i,\ final}$$
$$= 17.0° \text{ counterclockwise from the new normal}$$ Substitute $\theta_{i,\ final} = 17.0°$.

Use the sketches to help determine the angle through which the reflected ray has rotated.

$$\Delta\theta_r = \theta_{r,\ initial} + \Delta\theta_{mirror} - \theta_{r,\ final} = 52.0° + 35.0° - 17.0°$$
$$= 70.0° \text{ clockwise from the original angle}$$

3 EVALUATE THE ANSWER

- **Is the magnitude realistic?** Comparing the final sketch with the initial sketch shows that the angle the light ray makes with the normal decreases as the mirror rotates clockwise. It makes sense, then, that the reflected ray also rotates clockwise and rotates through an angle twice as large as that of the mirror rotation.

PRACTICE Problems

ADDITIONAL PRACTICE

1. Explain why the reflection of light off ground glass changes from diffuse to specular if you spill water on it.
2. What is the angle of incidence of a light ray reflected off a plane mirror at an angle of 35° to the normal?
3. Suppose the angle of incidence of a light ray is 42°.
   a. What is the angle of reflection?
   b. What is the angle the incident ray makes with the mirror?
   c. What is the angle between the incident ray and the reflected ray?
4. Light from a laser strikes a plane mirror at an angle of 38° to the normal. If the angle of incidence increases by 13°, what is the new angle of reflection?
5. You position two plane mirrors at right angles to each other. A light ray strikes one mirror at an angle of 60° to the normal and reflects toward the second mirror. What is its angle of reflection off the second mirror?
6. **CHALLENGE** You are asked to design a retro-reflector using two mirrors that will reflect a laser beam by 180° independent of the incident direction of the beam. What should be the angle between the two mirrors?

Properties of Plane-Mirror Images

Looking at yourself in a mirror, you can see that your image in the mirror appears to be the same distance behind the mirror as you are in front of the mirror. How could you test this? Place a ruler between you and the mirror. Where does the image touch the ruler? You see that your image is vertically oriented as you are, and it matches your size. This is where the expression *mirror image* originates.

Image position and height The geometric model in **Figure 7** shows why the distances are the same. Two rays from point O at the tip of the candle strike the mirror at points P_1 and P_2. The mirror reflects both rays according to the law of reflection.

We can extend the reflected rays behind the mirror as sight lines (the dashed lines in the figure). These sight lines converge at point I, which is the image of point O. Ray 1, which strikes the mirror at an angle of incidence of 0°, reflects back on itself, so the sight line is at 90° to the mirror, just as ray 1 is. Ray 2 also reflects at an angle with respect to the normal equal to the angle of incidence, so the sight line is at the same angle to the mirror as ray 2.

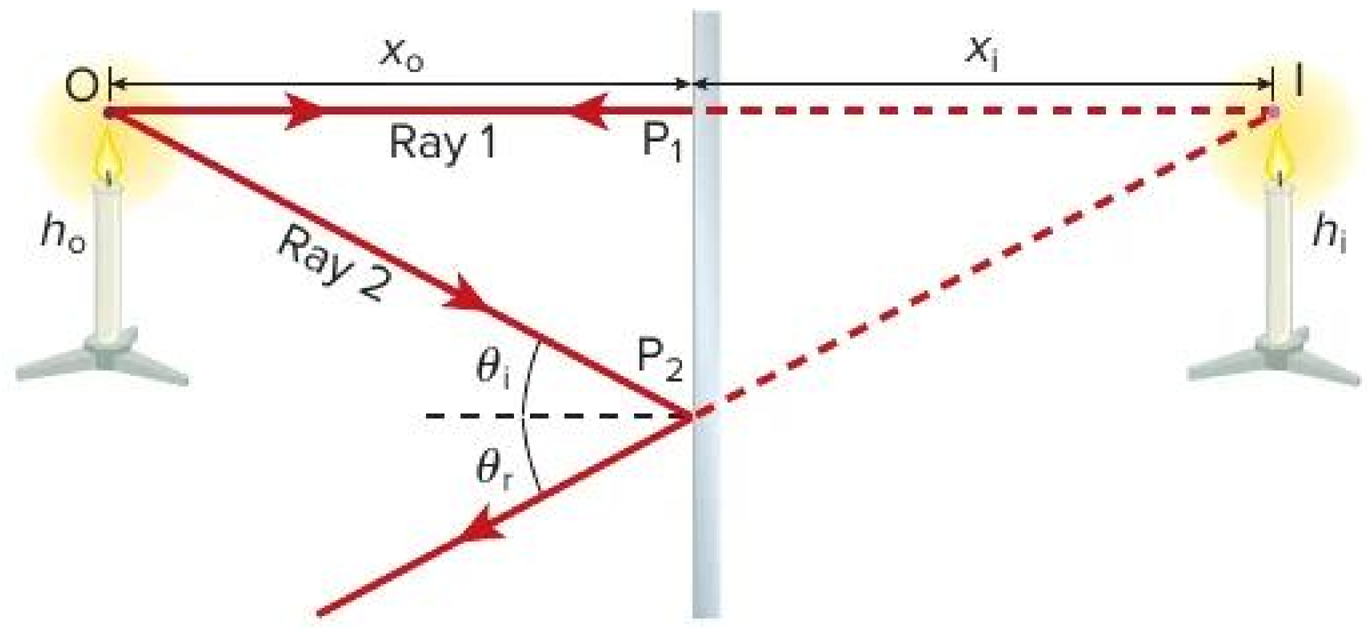


Figure 7 Reflected light rays from the candle (two rays are shown) strike the mirror. Some of those rays reach the viewer's eye. Sight lines (dashed lines) are drawn from where the rays reflect from the mirror to where they converge. The image is located where the sight lines converge.

Explain *why* $x_i = -x_o$.

This geometric model reveals that line segments OP_1 and IP_1 are corresponding sides of two congruent triangles, ΔOP_1P_2 and ΔIP_1P_2. The object's position with respect to the mirror (x_o) has a magnitude equal to the length of $\overline{OP_1}$. The apparent position of the image with respect to the mirror, which is called the image position (x_i), has a magnitude equal to the length of $\overline{IP_1}$. If image position is negative, indicating that the image is virtual, the following is true.

Plane-Mirror Image Position

With a plane mirror, the image position is equal to the negative of the object position. The negative sign indicates that the image is behind the mirror (and therefore virtual).

$$x_i = -x_o$$

Get It?

Summarize the properties of a plane-mirror image.

STEM CAREER Connection

Set Designer

Have you ever attended a shool play or other production and noticed eerie light effects? You could use your understanding of the properties of light to create sets for movies, theaters, or concerts. Set designers must understand how to use different properties of light, such as reflection, refraction, color, intensity, and brightness, to create mood and special effects.

You can draw more light rays from the object to the mirror to determine the size of the image. The sight lines of two rays originating from the bottom of the candle in **Figure 7** will converge at the bottom of the image. From the law of reflection and congruent-triangle geometry, we find the following is true of the object height (h_o) and image height (h_i) and any other dimension of the object and image.

Plane-Mirror Image Height

With a plane mirror, image height is equal to object height.

$$h_i = h_o$$

Image orientation A plane mirror produces an image with the same orientation as the object. If you stand upright, a plane mirror produces an image of you standing upright. If you do a headstand, the mirror shows you doing a headstand. However, the mirror gives a front-to-back reversal. You are looking face-to-face at the image of yourself.

Follow the sight lines in **Figure 8.** The image is behind the mirror such that the boy appears to be facing himself. When the boy extends his right hand, it appears as though the left hand of the image is extended, but from the boy's perspective, the hand that is extended is on his right-hand side. This happens because a plane mirror does not really reverse left and right. The mirror in **Figure 8** only reverses the boy's image such that it is facing in the opposite direction as the boy, or, in other words, it produces a front-to-back reversal.

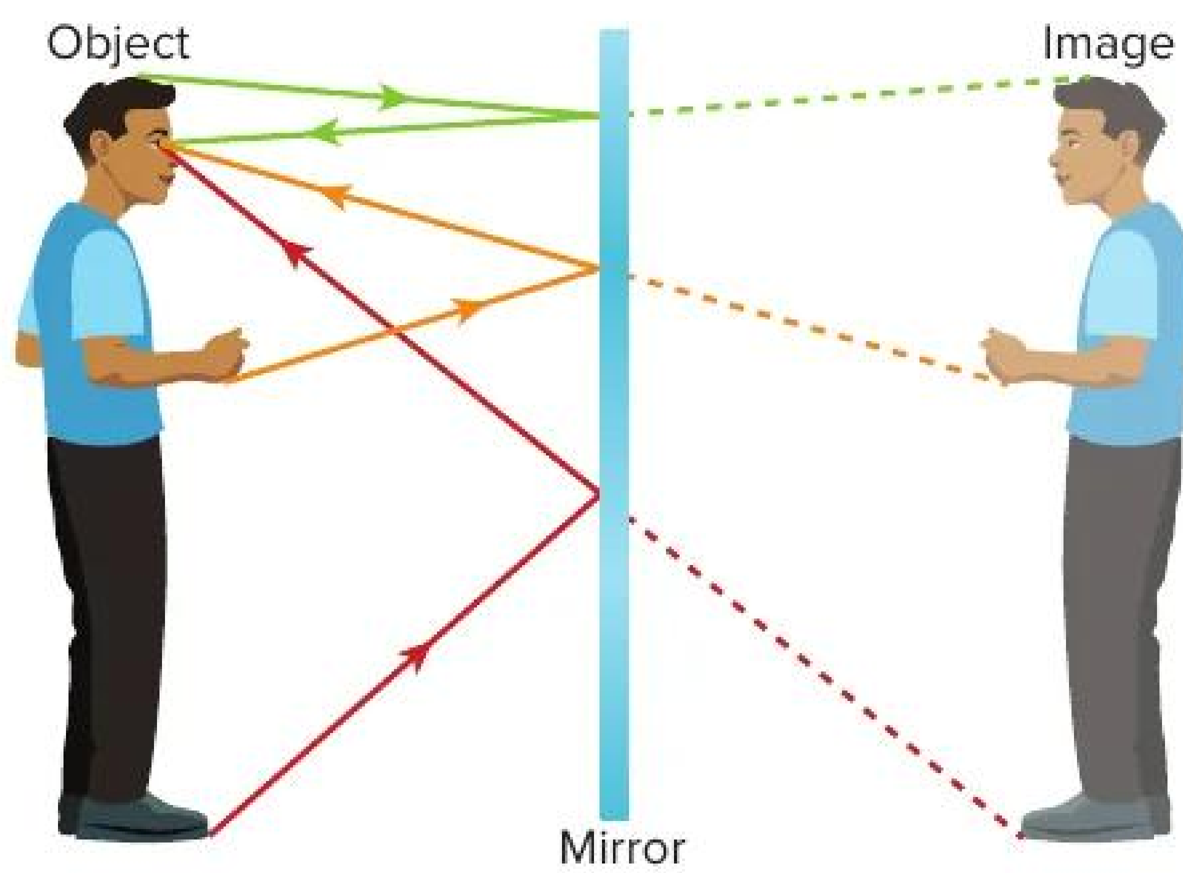


Figure 8 Viewed in a mirror, your height and distance from the mirror appear the same. There is, however, a difference. You are facing the opposite direction.

Check Your Progress

7. **Reflection** A light ray strikes a flat, smooth, reflecting surface at an angle of 80° to the normal. What angle does the reflected ray make with the surface of the mirror?

8. **Image Properties** A dog looks at its image, as shown in **Figure 9.** What are the position, height, and type of image?

Figure 9

9. **Law of Reflection** Explain how the law of reflection applies to diffuse reflection.

10. **Image Diagram** A car is following another car along a straight road. The first car has a rear window tilted at 45° to the horizontal. Draw a ray diagram showing the position of the Sun that would cause sunlight to reflect into the eyes of the driver of the second car.

11. **Critical Thinking** Explain how diffuse reflection of light off an object enables you to see that object from any angle.

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LESSON 2

CURVED MIRRORS

FOCUS QUESTION

What are some advantages and disadvantages of curved mirrors compared to plane mirrors?

Properties of Curved Spherical Mirrors

Move a metal spoon toward and away from your face. Look at your image on both sides of the spoon. Your image may appear larger or smaller, or it may even be inverted. The properties of curved mirrors, such as a metal spoon, and the images that they form depend on the shape of the mirror and the object's position.

The way that a spherical mirror reflects light depends on its curve. **Figure 10** shows two spherical mirrors and some important points and distances for understanding how they form images. A spherical mirror is shaped as if it were a section of a hollow sphere. The mirror (M) has the same geometric center (C) as a sphere of radius *r*. The distance *r* is also called the radius of curvature.

The straight line that includes line segment CM is called the **principal axis,** or line perpendicular to the mirror's surface that divides the mirror in half.

Focal point Light rays from the Sun to Earth are almost parallel because the Sun is so far away. When you point the principal axis of a concave mirror toward the Sun, all the rays reflect through a point. You can locate this point by moving a sheet of paper toward and away from the mirror until the smallest and sharpest spot of sunlight is focused on the paper. This spot is the mirror's **focal point,** which is the point at which incident light rays that are parallel to the principal axis converge after reflecting from the mirror. This is point F.

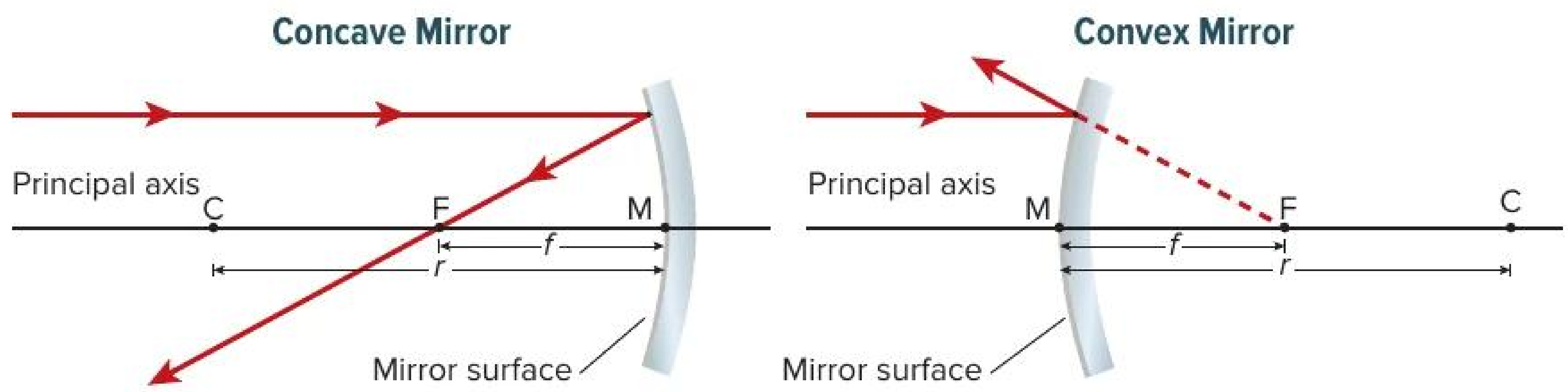


Figure 10 For concave and convex mirrors, the distance from M to F (the focal point) is half the distance from M to C.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

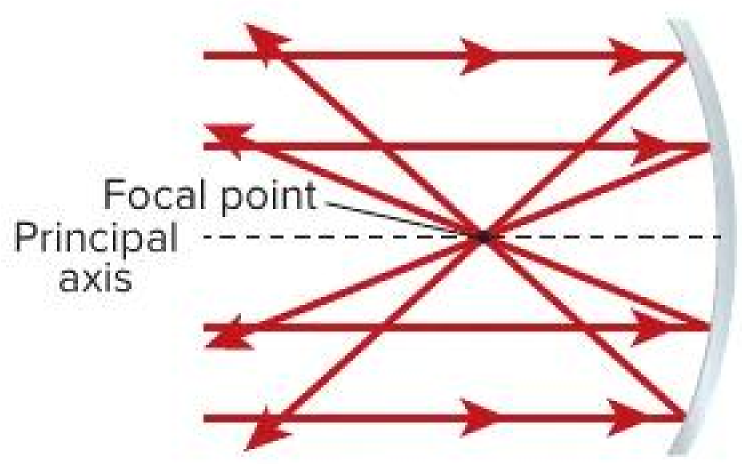
GO ONLINE to find these activities and more resources.

Virtual Investigation: Ray Tracing for Mirrors
Use a computer model to investigate how the properties of a curved mirror affect how light is reflected.

PhysicsLAB: Concave Mirror Images
Construct an explanation for why different mirrors produce different images when reflecting light.

For the mirror on the left in **Figure 10**, a ray parallel to the principal axis is reflected and crosses the principal axis at point F. Note that the distance from M to F is half the distance from M to C. The **focal length** (f) is the distance between the mirror and the focal point and can be expressed as $f = \frac{r}{2}$.

Concave mirrors The inside surface of a shiny metal spoon acts as a concave mirror. A **concave mirror,** such as the one on the left in **Figure 10,** has an inwardly curving reflective surface, the edges of which curve toward the observer. Incident light rays parallel to the principal axis are reflected and intersect the principal axis at point F. The focal length is positive for a concave mirror because it is a point of converging rays. Rays reflected from concave mirrors are parallel, as shown in **Figure 11.** Concave mirrors are placed in car headlights. It is a concave mirror that allows the light from a spotlight to be focused into a spot.



Light rays parallel to the principal axis reflect off a concave mirror and converge at the focal point.

A lightbulb is placed at the focal point of a concave mirror. The parallel reflected rays are evident in this flashlight beam.

Figure 11 Rays traced from the focal point to a concave mirror reflect off the surface as parallel rays.

Convex mirrors A spoon's outer surface acts as a **convex mirror,** an outwardly curving reflective surface with edges that curve away from the observer, as shown on the right side of **Figure 10.** Properties of a spherical convex mirror are also shown in **Figure 10.** The focal point (F) and the geometric center of the mirror (C) are behind the mirror. Store security mirrors and many side-view mirrors on cars are convex mirrors.

Magnification Another property of a spherical mirror is **magnification** (m), which is the ratio of the image height to the object height.

The sign of the magnification tells you whether the image is upright or inverted. For virtual images such as those formed by convex mirrors, x_i is negative. This means m is positive. Note that virtual images are always upright, so the height is always positive.

Magnification

The magnification of an object by a spherical mirror, defined as the image height divided by the object height, is equal to the negative of the image position, divided by the object position.

$$m \equiv \frac{h_i}{h_o} = -\frac{x_i}{x_o}$$

Get It?

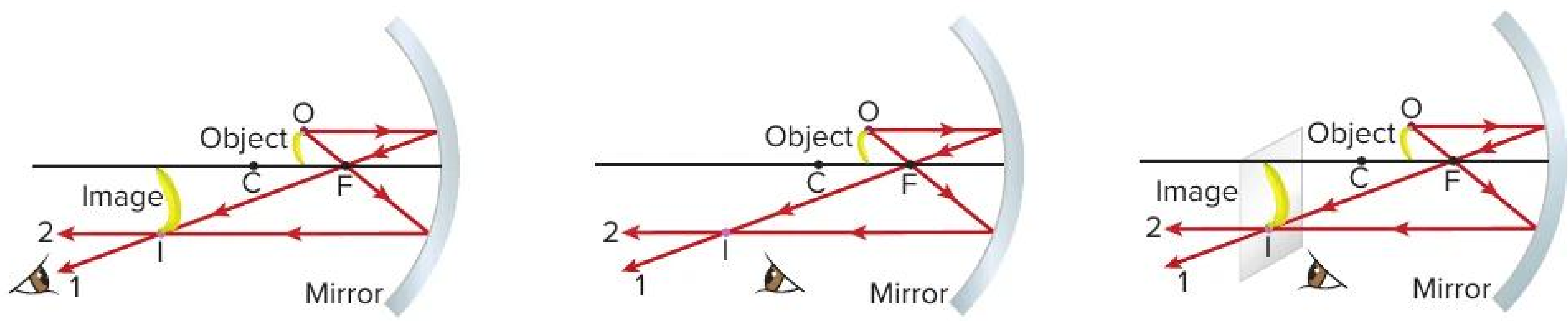
Explain What does it mean if $m < 1$?

Ray Diagrams for Curved Mirrors

Ray diagrams give you a visual representation of images formed by a curved mirror, depending on where the object is placed in relation to the focal point.

Figure 12 shows rays reflected from a concave mirror converging at the point I, where an image is located. The converging light rays form a **real image** that is inverted and larger than the object. You can see the image floating in space if you place your eye so that the rays that form the image fall on your eye, as in **Figure 12.** You must face a direction that allows the light rays coming from the image to enter your eyes, however. You cannot look at the image from behind. If you were to place a piece of paper or a movie screen at this point, the image would appear on the screen, as shown on the right in **Figure 12.** You cannot project virtual images on a screen, since they are not formed by converging light rays.

In **Figure 12,** if the object is beyond point C, the absolute value of the magnification for the real image is less than 1. This means that the image is smaller than the object. If the object is placed between point C and point F, the absolute value of the magnification is greater than 1, which means the image is larger than the object.



The eye is positioned so that the rays that form the real image strike the eye, allowing the image to be seen.

Rays from the object do not reach the eye, so the image cannot be seen from this position.

The image can be seen when projected on a white opaque screen.

Figure 12 Ray diagrams can be used to locate an image reflected from a curved mirror.

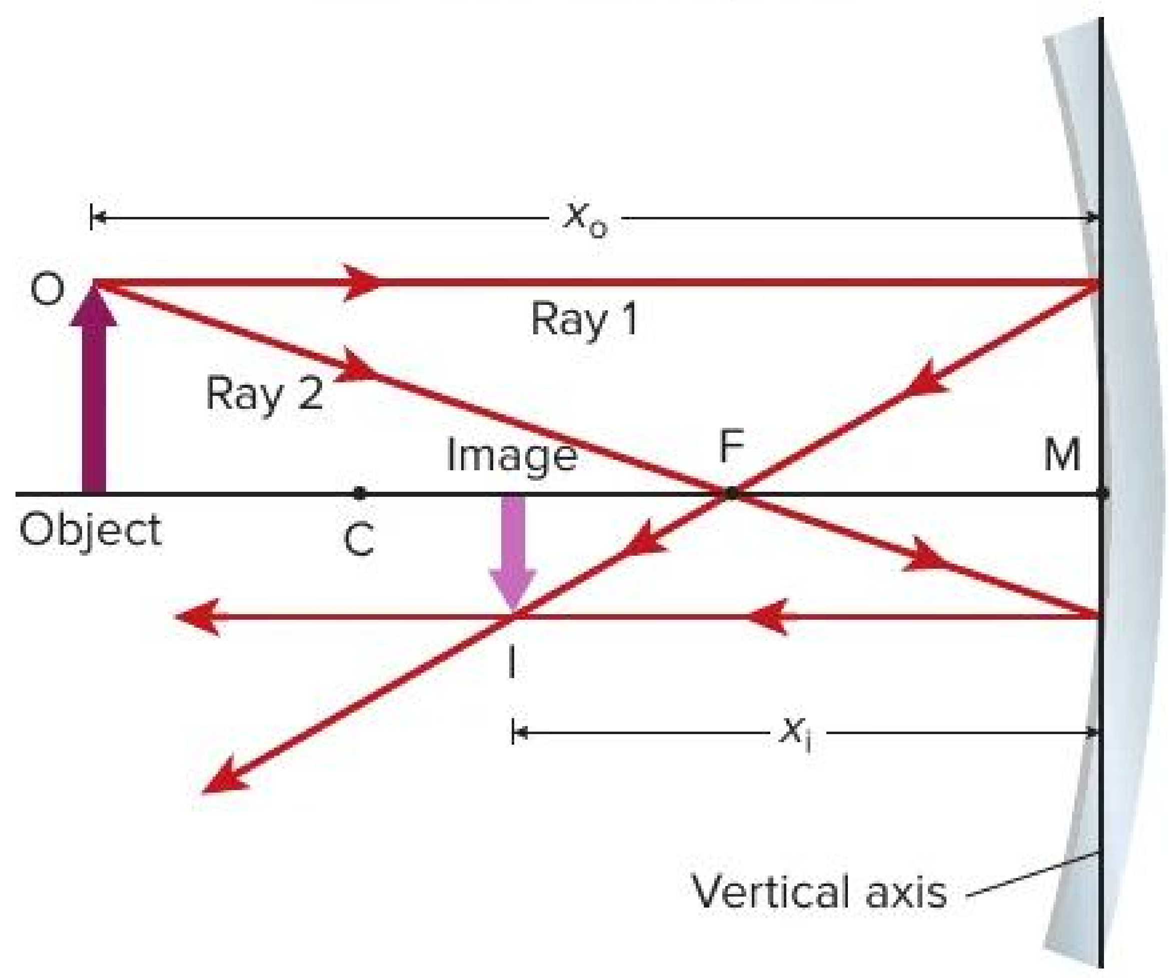
Real Images with Concave Mirrors

You can often simplify ray diagrams by using simple, one-dimensional objects, such as the arrow shown in **Figure 13** on the next page. A spherical concave mirror produces an inverted real image if the object position (x_o) is greater than twice the focal length (f). The object is then beyond the center of curvature (C). If the object is placed between the center of curvature and the focal point, as shown on the right in **Figure 13,** the image is again real and inverted. The size of the image is now greater than the size of the object.

CCC CROSSCUTTING CONCEPTS

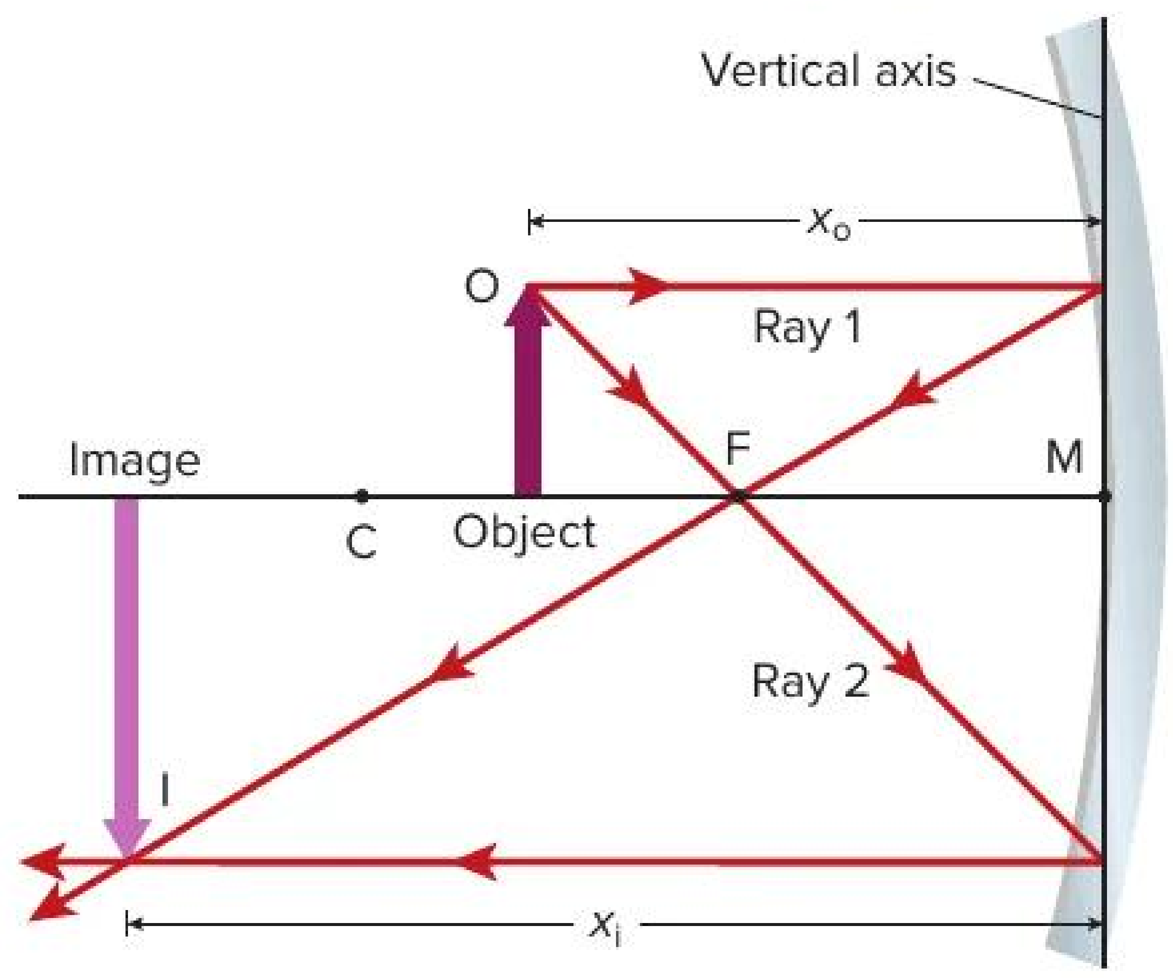
Cause and Effect Systems can be designed to cause a desired effect. You want to design a mirror that can be used for shaving, applying makeup, or both, so you want your mirror to produce a large image. Would you use a plane mirror, a convex mirror, or a concave mirror? What evidence will you use to support your choice? Make a simple sketch of your design.

Concave Mirror ($x_o > 2f$)



The image is reduced for an object located a distance greater than $2f$ from the mirror.

Concave Mirror ($2f > x_o > f$)



The image is enlarged for an object between f and $2f$.

Figure 13 The type of image depends on the object's distance from the mirror. A real, inverted image is formed in both situations shown here. Remember that f, the focal length, is the distance from M to F.

Diagram *how other points on the image are formed.*

COLOR CONVENTION

Object	magenta
Image	lilac
Light ray	red

PROBLEM-SOLVING STRATEGIES

Using Ray Diagrams to Locate Images Produced by Spherical Mirrors

Use the following strategies for curved spherical-mirror problems. Refer to **Figure 13,** as well as **Figure 14** and **Figure 15** on the next page.

1. Using lined or graph paper, draw the principal axis of the mirror as a horizontal line from the left side to the right side of your paper, leaving six blank lines above and six blank lines below.
2. Place points and labels on the principal axis for the object, C, and F as follows:
   - **a.** If the mirror is concave and the object is beyond C away from the mirror, place the mirror at the right side of the page, the object at the left side of the page, and make C and F to scale.
   - **b.** If the mirror is concave and the object is between C and F, place the mirror at the right side of the page, C at the center of the paper, F halfway between the mirror and C, and the object to scale.
   - **c.** For any other situation, place the mirror in the center of the page, the object or F (whichever is the greater distance from the mirror) at the left side of the page, and the other to scale.
3. To represent the mirror, draw a vertical line at the mirror point that extends for 12 lines. This is the mirror's principal plane.
4. Draw the object as an arrow, and label its top O. For concave mirrors, objects inside of C should not be higher than three lines. For all other situations, the objects should be six lines high. The scale for the height of the object will be different from the scale along the principal axis.
5. Draw ray 1, the parallel ray. It is parallel to the principal axis, reflects off the principal plane, and passes through F.
6. Draw ray 2, the focus ray. It passes through F, reflects off the principal plane, and is reflected parallel to the principal axis.
7. If reflected rays 1 and 2 diverge from each other, then extend the sight lines behind the mirror as dashed lines.
8. The image is located where rays 1 and 2 or their sight lines cross after reflection. For either case, label the point of intersection I. Draw the image as a perpendicular arrow from the principal axis to I.

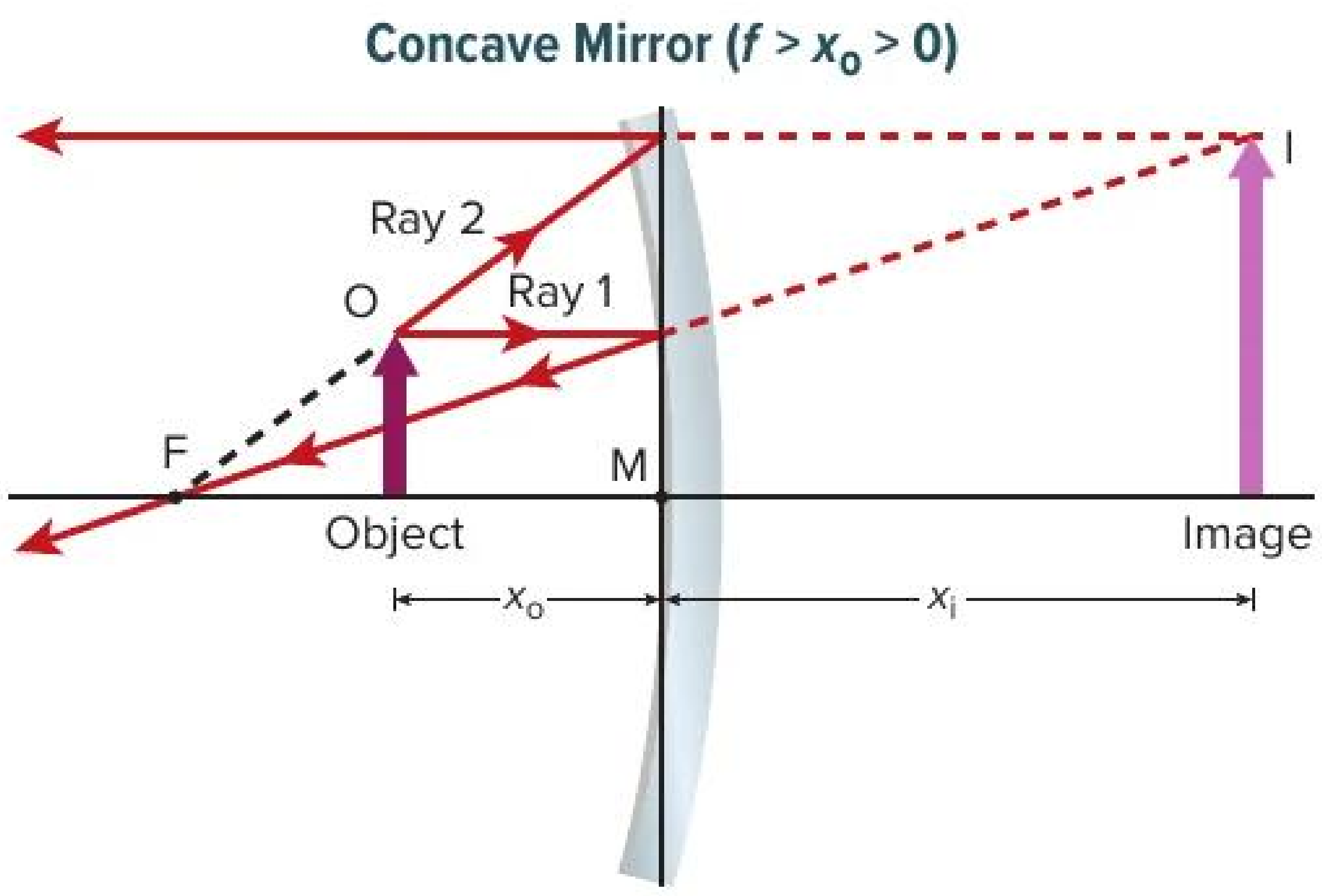


Figure 14 A virtual, upright, enlarged image is formed when an object, such as the block tower on the right, is placed between the focal point and the surface of a concave mirror.

Consider *the cause of the appearance of the other images in the mirror.*

Virtual Images with Concave Mirrors

You have seen that as an object approaches the focal point (F) of a concave mirror, the image moves farther away from the mirror. If the object is at the focal point, all reflected rays are parallel. They never meet, therefore, and the image is said to be at infinity, so the image could never be seen.

What happens if you move the object even closer to the mirror? The image is upright and behind the mirror. A concave mirror produces a virtual image if the object is located between the mirror and the focal point. This situation is shown in the ray diagram in **Figure 14.**

You draw two rays to locate the image of a point on an object. As before, you draw ray 1 parallel to the principal axis and then reflected through the focal point. Ray 2 is drawn as a line from the point on the object to the mirror, along a line defined by the focal point and the point on the object. At the mirror, ray 2 is reflected parallel to the principal axis. Note in **Figure 14** that ray 1 and ray 2 diverge as they leave the mirror, so there cannot be a real image. However, sight lines extended behind the mirror converge, showing that a virtual image forms behind the mirror. From the position of the point of convergence, you can see that the image is upright and larger than the object.

Get It?
Explain how you can determine whether an image is virtual.

Virtual Images with Convex Mirrors

Rays reflected from a convex mirror always diverge. Thus, convex mirrors form only virtual images. The ray diagram in **Figure 15** shows how a spherical convex mirror forms an image. The figure shows two rays, but remember that there is an infinite number of rays reflecting from the object. Ray 1 approaches the mirror parallel to the principal axis. Ray 1 reflects off the mirror following the line between F and the point where ray 1 strikes the principal plane.

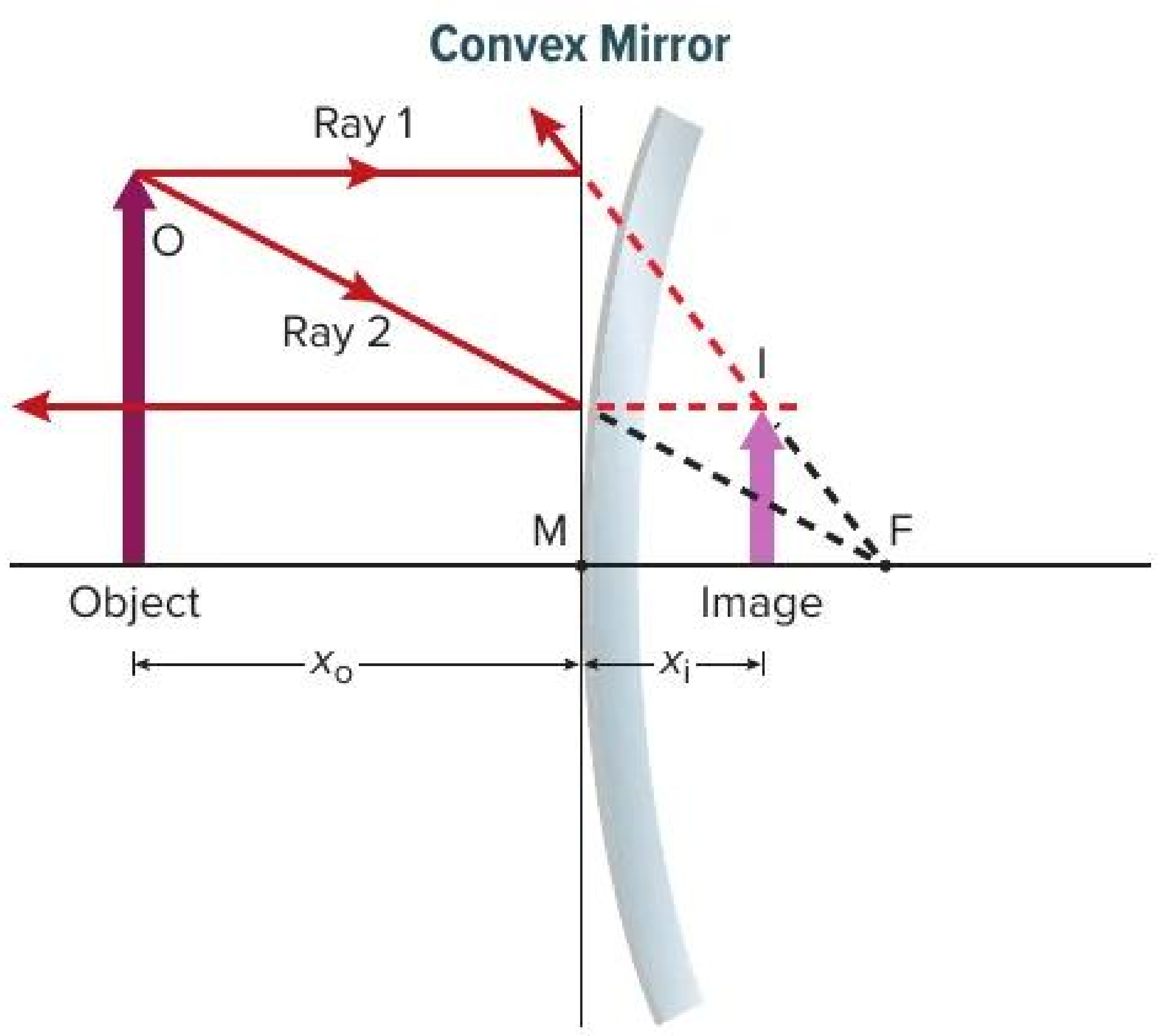


Figure 15 Convex mirrors form images that are virtual, upright, and smaller than the object.

A sight line extends from the point where ray 1 strikes the principle plane through F. Ray 2 approaches the mirror on a path that, if extended behind the mirror, would pass through F. The reflected part of ray 2 and its sight line are parallel to the principal axis. The two reflected rays diverge. However, the sight lines intersect behind the mirror. The point where they intersect is the location of the image. An image produced by a convex mirror is a virtual image that is upright and smaller than the object.

Field of view It might seem that convex mirrors would have little use because the images they form are smaller than the objects. This property of convex mirrors, however, does have practical uses. By forming smaller images, convex mirrors enlarge the area that an observer sees, which is called the field of view. Also, the image is visible from a large range of angles; thus, the field of view is visible from a wide perspective.

Because of the increased field of view, convex mirrors are used in cars as passenger-side rearview mirrors, as shown in **Figure 16.** However, because the reduced image size makes the objects appear farther away than they really are, there is often a warning on passenger-side rearview mirrors of cars stating that objects may be closer than they appear.

Figure 16 Images from convex mirrors are smaller than the object. This increases the field of view and decreases the driver's blind spot.

Explain *why a warning stating that objects in a convex side-view mirror are closer than they appear may be useful.*

Defects in Concave Mirrors

In diagramming rays, you used a vertical line to represent the mirror. In reality, light rays reflect off the mirror itself, as shown in **Figure 17.** Only parallel rays that are close to the principal axis reflect through the focal point. Other rays converge at points closer to the mirror. This defect, called **spherical aberration,** occurs because the light rays do not all converge at the focal point, which makes an image look fuzzy.

A mirror ground to the shape of a parabola, as in **Figure 17,** suffers no spherical aberration. Large, parabolic glass mirrors are expensive to manufacture; so, many telescopes use spherical mirrors and smaller, specially designed secondary mirrors or lenses to correct for spherical aberration.

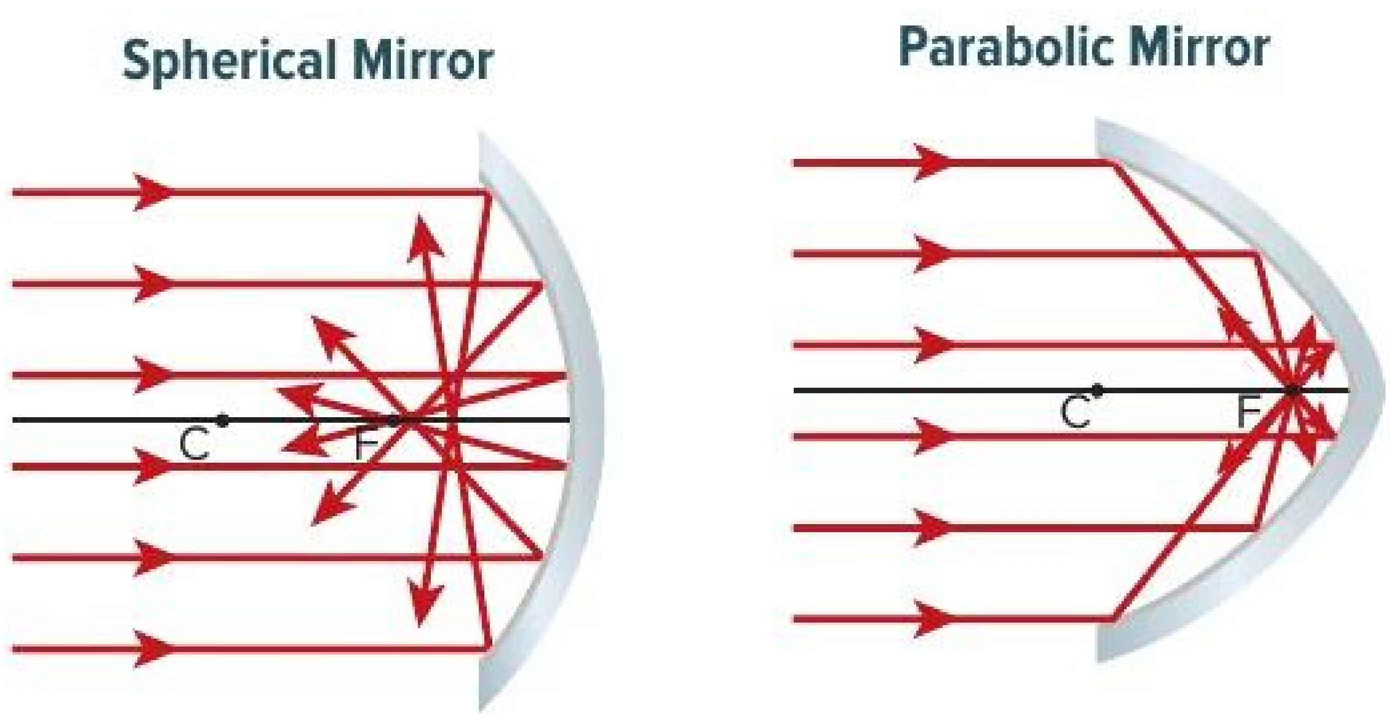


Figure 17 Spherical aberration occurs for spherical mirrors but does not occur for parabolic mirrors.

Get It?

Explain why a spherical mirror produces spherical aberration but a mirror in the shape of a parabola does not produce spherical aberration.

Calculating Image Position

We can use the spherical mirror model to develop a simple equation for spherical mirrors. You must use the paraxial ray approximation, which states that only rays that are close and almost parallel to the principal axis are used to form an image. Using this with the law of reflection leads to the mirror equation, relating the focal length (f), object position (x_o), and image position (x_i) of a spherical mirror.

Mirror Equation

The reciprocal of the focal length of a spherical mirror is equal to the sum of the reciprocals of the image position and the object position.

$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$$

Real-World Physics

IN 1990, NASA launched the *Hubble Space Telescope* into orbit around Earth. Scientists expected *Hubble* to provide clear images without atmospheric distortions. Soon after it was deployed, however, scientists found that *Hubble* had a spherical aberration. In 1993, astronauts installed corrective optics called COSTAR on *Hubble*, enabling *Hubble* to produce clear images. For over twenty-five years, *Hubble* provided the world with stunning images and enabled scientists to make important discoveries about the universe.

Negative values When virtual images are formed, the image position (x_i) has a negative value, indicating that it is located behind the mirror. For concave mirrors, a virtual image only forms when the object is between a concave mirror and the focal point. The focal point is in front of the mirror and the focal length has a positive value. For convex mirrors, the focal point is always behind the mirror, and the focal length has a negative value.

Remember that the mirror equation is only approximately correct. In reality, light coming from an object toward a mirror is diverging, so not all of the light is close to or parallel to the axis. When the mirror diameter is large relative to the radius of curvature to minimize spherical aberration, this equation predicts image properties more precisely.

CONNECTING MATH to Physics

Adding and Subtracting Fractions When using the mirror equation, you first use math to move the fraction that contains the quantity you are seeking to the left-hand side of the equation and everything else to the right. Then you combine the two fractions on the right-hand side by using a common denominator that results from multiplying the denominators.

Math	Physics
$\frac{1}{x} = \frac{1}{y} + \frac{1}{z}$	$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$
$\frac{1}{y} = \frac{1}{x} - \frac{1}{z}$	$\frac{1}{x_i} = \frac{1}{f} - \frac{1}{x_o}$
$\frac{1}{y} = \left(\frac{1}{x}\right)\left(\frac{z}{z}\right) - \left(\frac{1}{z}\right)\left(\frac{x}{x}\right)$	$\frac{1}{x_i} = \left(\frac{1}{f}\right)\left(\frac{x_o}{x_o}\right) - \left(\frac{1}{x_o}\right)\left(\frac{f}{f}\right)$
$\frac{1}{y} = \frac{z-x}{xz}$	$\frac{1}{x_i} = \frac{x_o - f}{fx_o}$
$y = \frac{xz}{z-x}$	$x_i = \frac{fx_o}{x_o - f}$

Using this approach, the following relationships can be derived for image position, object position, and focal length:

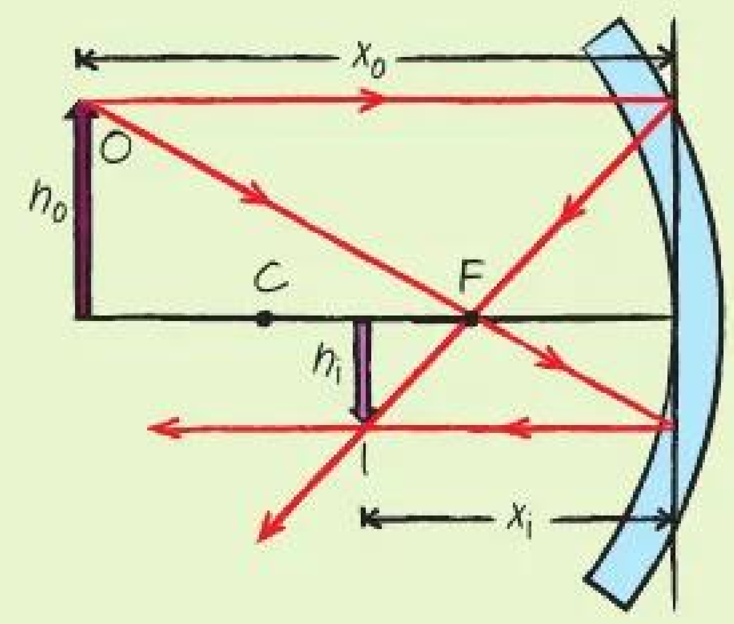
$$x_i = \frac{fx_o}{x_o - f} \qquad x_o = \frac{fx_i}{x_i - f} \qquad f = \frac{x_i x_o}{x_i + x_o}$$

EXAMPLE Problem 2

REAL IMAGE FORMATION BY A CONCAVE MIRROR A concave mirror has a radius of curvature of 20.0 cm. You place a 2.0-cm-tall object 30.0 cm from the mirror. What are the image position and image height?

1 ANALYZE AND SKETCH THE PROBLEM

- Draw a diagram with the object and the mirror.
- Draw two principal rays to locate the image in the diagram.



Known	Unknown
$h_o = 2.0$ cm	$x_i = ?$
$x_o = 30.0$ cm	$h_i = ?$
$r = 20.0$ cm	

2 SOLVE FOR THE UNKNOWN

Focal length is half the radius of curvature.

$$f = \frac{r}{2}$$

$$= \frac{20.0 \text{ cm}}{2} \qquad \text{Substitute } r = 20.0 \text{ cm.}$$

$$= 10.0 \text{ cm}$$

Use the relationship between focal length and object position to solve for image position.

$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$$

$$x_i = \frac{fx_o}{x_o - f}$$

$$= \frac{(10.0 \text{ cm})(30.0 \text{ cm})}{30.0 \text{ cm} - 10.0 \text{ cm}} \qquad \text{Substitute } f = 10.0 \text{ cm}, x_o = 30.0 \text{ cm.}$$

$$= 15.0 \text{ cm (real image, in front of the mirror)}$$

Use the relationship between object height and object and image position to solve for image height.

$$m \equiv \frac{h_i}{h_o} = \frac{-x_i}{x_o}$$

$$h_i = \frac{-x_i h_o}{x_o}$$

$$= -\frac{(15.0 \text{ cm})(2.0 \text{ cm})}{30.0 \text{ cm}} \qquad \text{Substitute } x_i = 15.0 \text{ cm}, h_o = 2.0 \text{ cm}, x_o = 30.0 \text{ cm.}$$

$$= -1.0 \text{ cm (inverted, smaller image)}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** All positions and heights are in centimeters.
- **Do the signs make sense?** Positive position and negative height agree with the drawing.

PRACTICE Problems

ADDITIONAL PRACTICE

12. Use a ray diagram drawn to scale to solve Example Problem 2.

13. You place an object 36.0 cm in front of a concave mirror with a 16.0-cm focal length. Determine the image position.

14. You place a 3.0-cm-tall object 20.0 cm from a 16.0-cm-radius concave mirror. Determine the image position and image height.

15. A concave mirror has a 7.0-cm focal length. You place a 2.4-cm-tall object 16.0 cm from the mirror. Determine the image height.

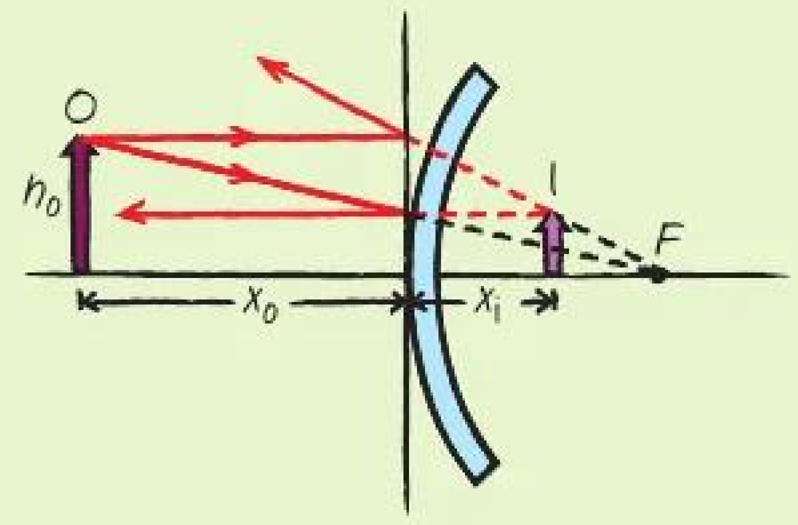
16. **CHALLENGE** You place an object near a concave mirror with a 10.0-cm focal length. The image is 3.0 cm tall, inverted, and 16.0 cm from the mirror. What are the object position and object height?

EXAMPLE Problem 3

IMAGE IN A SECURITY MIRROR A convex security mirror in a warehouse has a −0.50-m focal length. A 2.0-m-tall forklift is 5.0 m from the mirror. What are the forklift's image position and image height?

1 ANALYZE AND SKETCH THE PROBLEM

- Draw a diagram with the mirror and the object.
- Draw two principal rays to locate the image in the diagram.



Known	**Unknown**
$h_o = 2.0$ m	$x_i = ?$
$x_o = 5.0$ m	$h_i = ?$
$f = -0.50$ m	

2 SOLVE FOR THE UNKNOWN

Use the relationship between focal length and object position to solve for image position.

$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$$

$$x_i = \frac{f x_o}{x_o - f}$$

$$= \frac{(-0.50\text{ m})(5.0\text{ m})}{5.0\text{ m} - (-0.50\text{ m})}$$ Substitute $f = -0.50$ m, $x_o = 5.0$ m.

$= -0.45$ m (virtual image, behind the mirror)

Use the relationship between object height and object and image position to solve for image height.

$$m \equiv \frac{h_i}{h_o} = \frac{-x_i}{x_o}$$

$$h_i = \frac{-x_i h_o}{x_o}$$

$$= \frac{-(-0.45\text{ m})(2.0\text{ m})}{5.0\text{ m}}$$ Substitute $x_i = -0.45$ m, $h_o = 2.0$ m, $x_o = 5.0$ m.

$= 0.18$ m (upright, smaller image)

3 EVALUATE THE ANSWER

- **Are the units correct?** All positions and heights are in meters.
- **Do the signs make sense?** A negative position indicates a virtual image; a positive height indicates an image that is upright. These agree with the diagram.

PRACTICE Problems

ADDITIONAL PRACTICE

17. You place an object 20.0 cm in front of a convex mirror with a −15.0-cm focal length. Find the image position using both a scale diagram and the mirror equation.

18. A convex mirror has a focal length of −13.0 cm. You place a 6.0-cm diameter lightbulb 60.0 cm from that mirror. What are the lightbulb's image position and diameter?

19. A 7.6-cm-diameter ball is located 22.0 cm from a convex mirror with a radius of curvature of 60.0 cm. What are the ball's image position and diameter?

20. A 1.8-m-tall girl stands 2.4 m from a store's security mirror. Her image appears to be 0.36 m tall.

a. What is the image's distance?

b. What is the focal length of the mirror?

21. CHALLENGE A convex mirror is needed to produce an image that is three-fourths the size of an object and located 24 cm behind the mirror.

a. What is the object's distance?

b. What focal length should be specified?

Mirror Comparison

Table 1 summarizes the properties of single-mirror systems with objects that are located on the principal axis of the mirror.

Notice that the single plane mirror and convex mirror produce only virtual images. A concave mirror produces real images when the object is father than the focal distance. A concave mirror produces virtual images when the object is closer than the focal distance. Plane mirrors give reflections on scale with the objects, and convex mirrors provide reduced images, expanding the field of view. A concave mirror acts as a magnifier when an object is within the focal length of the mirror.

A concave mirror enlarges and inverts the image when the object is between the focal length and the radius of curvature. Beyond the radius of curvature, a concave mirror produces a reduced, inverted image.

Table 1 Single-Mirror System Properties

Mirror Type	f	x_o	x_i	m	Image
Plane	∞	$x_o > 0$	$\|x_i\| = x_o$ (negative)	same size	virtual
Concave	+	$x_o > r$	$r > x_i > f$	reduced, inverted	real
		$r > x_o > f$	$x_i > r$	enlarged, inverted	real
		$f > x_o > 0$	$\|x_i\| > x_o$ (negative)	enlarged	virtual
Convex	−	$x_o > 0$	$\|f\| > \|x_i\| > 0$ (negative)	reduced	virtual

Check Your Progress

22. **Image Properties** If you know the focal length of a concave mirror, where should you place an object so that its image is upright and larger compared to the object? Will this produce a real or virtual image?
23. **Magnification** You place an object 20.0 cm in front of a concave mirror with a focal length of 9.0 cm. What is the magnification of the image?
24. **Object Position** The placement of an object in front of a concave mirror with a focal length of 12.0 cm forms a real image that is 22.3 cm from the mirror. What is the object's position?
25. **Image Position and Height** You place a 3.0-cm-tall object 22.0 cm in front of a concave mirror that has a focal length of 12.0 cm. Find the image position and height by drawing a ray diagram to scale. Verify your answer using the mirror and magnification equations.
26. **Ray Diagram** You place a 4.0-cm-tall object 14.0 cm from a convex mirror with a focal length of −12.0 cm. In a scale ray diagram show the image position and height. Verify your answer using the mirror and the magnification equations.
27. **Radius of Curvature** You place a 6.0-cm-tall object 16.4 cm from a convex mirror. If the image of the object is 2.8 cm tall, what is the mirror's radius of curvature?
28. **Focal Length** A convex mirror is used to produce an image that is two-thirds the size of an object and located 12 cm behind the mirror. What is the focal length of the mirror?
29. **Critical Thinking** Would spherical aberration be less for a mirror whose height, compared to its radius of curvature, is small or large? Explain your answer.

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LESSON 3

REFRACTION OF LIGHT

FOCUS QUESTION

What happens to light when it enters a new medium?

Light and Boundaries

Recall that when light crosses a boundary between two mediums, it bends. You might be familiar with this phenomenon already. Have you ever looked down through the water of a swimming pool and thought the water looked shallower than it was? This strange appearance is due to the bending of light at the boundaries between two mediums, or refraction.

Refraction Observe the light rays in **Figure 18.** Identical rays of light start in air and pass into three different mediums: water, glass, and diamond. The light rays hit the surface of each medium at the same angle and bend, or refract, as they cross the boundaries.

What difference do you notice between the three mediums shown? The light rays bend more when traveling from air to diamond than from air to water or air to glass. The amount of refraction depends on properties of the mediums that the light rays are traveling from and into.

What do you think the relationship is between the angle of the light as it crosses the boundary between mediums and refraction?

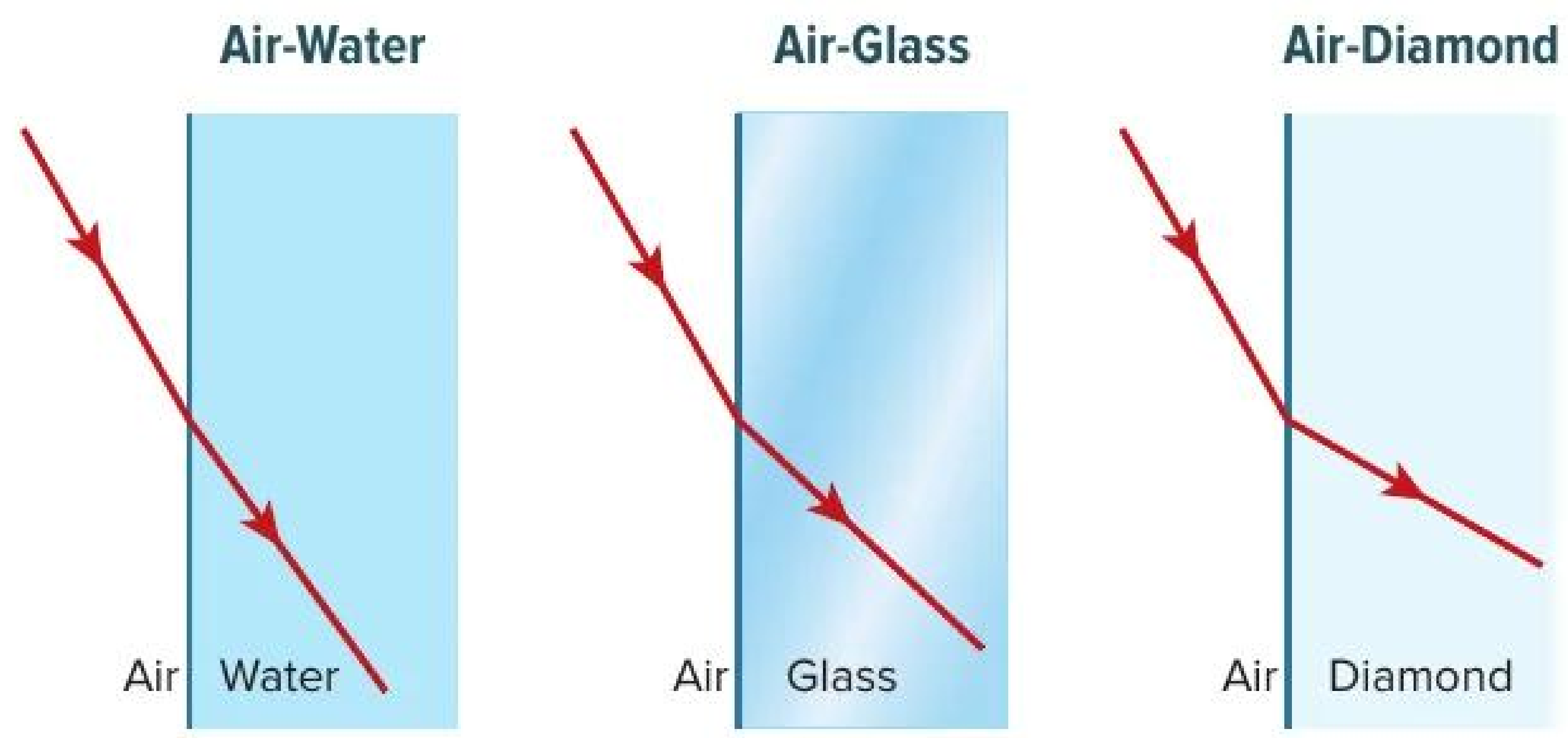


Figure 18 Light refracts as it crosses a boundary. The amount of refraction depends on the properties of the mediums. (Angles are not to scale.)

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Forensics Lab: A Whole Spectrum of Possibilities
Analyze data about how the index of refraction being dependent on wavelength causes the dipersion of light.

Revisit the Encounter the Phenomenon Question
What information from this lesson can help you answer the Unit and Module questions?

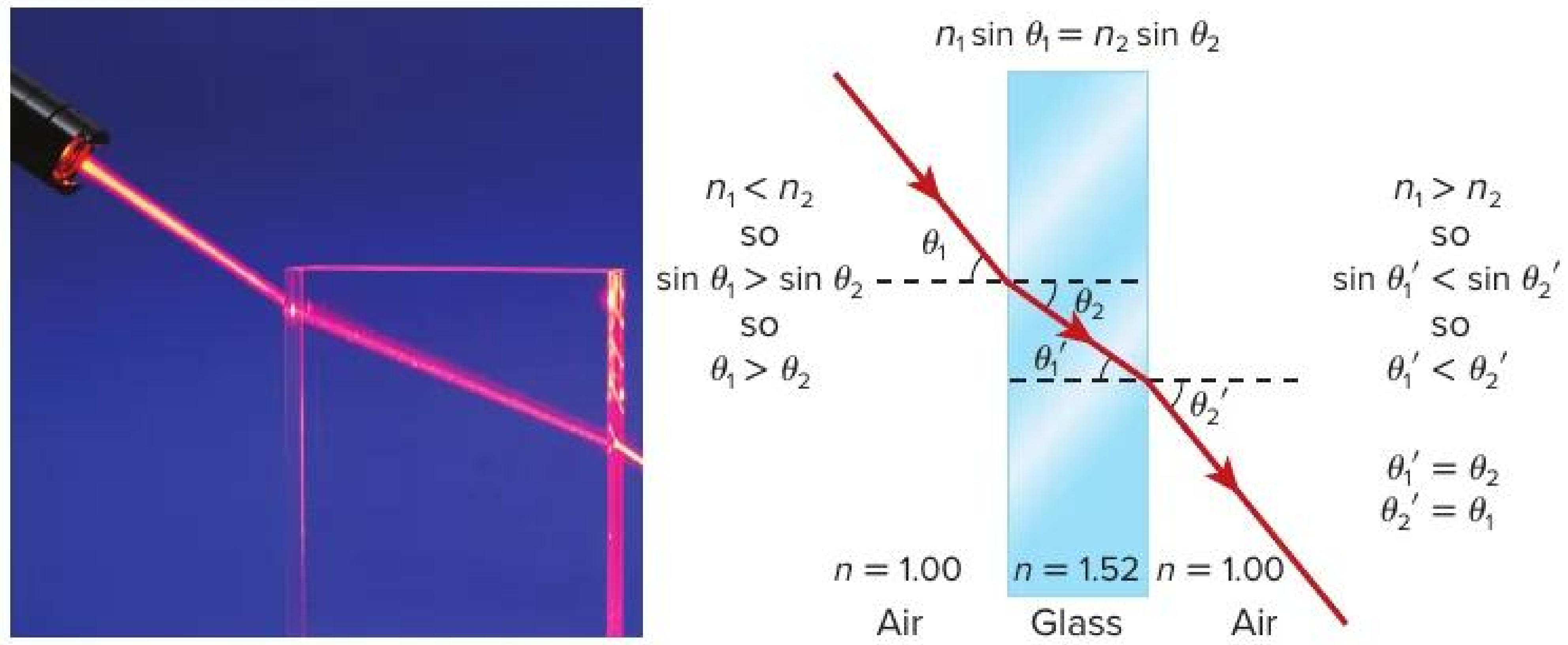


Figure 19 When light travels from air through glass and back to air, it refracts toward and then away from the normal.

Snell's Law of Refraction

René Descartes and Willebrord Snell first studied refraction in the seventeenth century by shining a narrow beam of light onto a transparent medium, such as the glass shown in **Figure 19.** They measured two angles—the angle of incidence and the angle of refraction. The angle of incidence (θ_1) is the angle at which the light ray strikes the surface. The angle of refraction (θ_2) is the angle at which the transmitted light leaves the surface. Both angles are measured with respect to the normal.

Index of refraction

Snell found that when light passes from air into a transparent medium, the sines of the angles are related. This **index of refraction** (n) determines the angle of refraction of light as it crosses the boundary between two mediums. Properties of the mediums light is traveling through determine indices of refraction. Values of n for several mediums are listed in **Table 2.**

Table 2 Indices of Refraction for Yellow Light ($\lambda = 589$ nm in a vacuum)

Medium	n
Vacuum	1.00
Air	1.0003*
Water	1.33
Ethanol	1.36
Float glass	1.52
Quartz	1.54
Flint glass	1.62
Diamond	2.42

*The value given for air contains additional significant figures to distinguish it from that for a vacuum. Use a value of $n = 1.00$ for air in problems in this text.

Snell's Law of Refraction

The product of the index of refraction of the first medium and the sine of the angle of incidence is equal to the product of the index of refraction of the second medium and the sine of the angle of refraction.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

We can use **Figure 19** and **Table 2** to show how Snell's law applies when light travels through a piece of glass with parallel surfaces, such as a windowpane. The light is refracted when it enters the glass and again when it leaves the glass. When light goes from air into glass, it travels from a medium with a lower n of 1.00 to a medium with a higher n of 1.52. The light bends toward the normal.

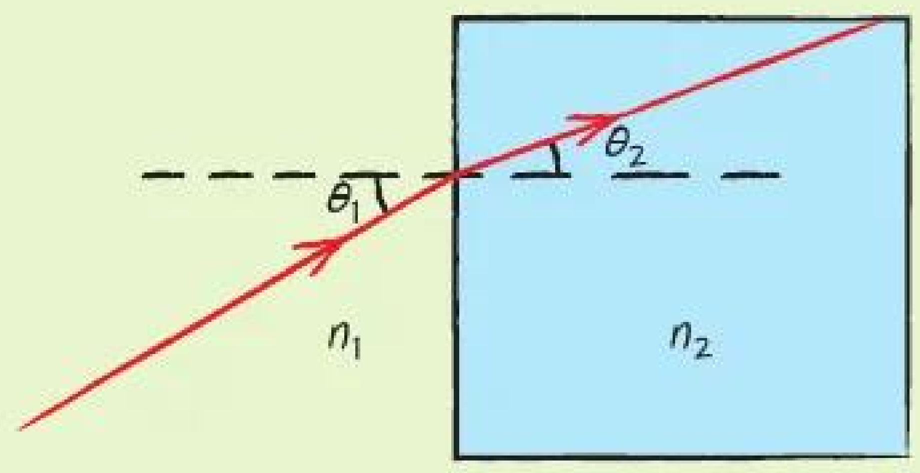
Traveling from glass to air, light moves from a medium with a higher n (1.52) to one with a lower n (1.00). The light is bent away from the normal. The relative values of n determine whether the light will bend toward or away from the normal. Note the direction of the ray when it leaves the glass. It is the same as before it struck the glass, $\theta_1 = \theta_2'$. This is because the boundaries are between the same two mediums.

EXAMPLE Problem 4

ANGLE OF REFRACTION A beam of light in air hits a sheet of float glass at an angle of 30.0°. What is the angle of refraction of the light ray?

1 ANALYZE AND SKETCH THE PROBLEM

- Make a sketch of the air and float glass boundary.
- Draw the light ray and label θ_1, θ_2, n_1, and n_2.



Known	Unknown
$\theta_1 = 30.0°$	$\theta_2 = ?$
$n_1 = 1.00$	
$n_2 = 1.52$	

2 SOLVE FOR THE ANGLE OF REFRACTION

Use Snell's law to solve for the sine of the angle of refraction.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta_2 = \left(\frac{n_1}{n_2}\right) \sin \theta_1$$

$$\theta_2 = \sin^{-1}\left(\left(\frac{n_1}{n_2}\right) \sin \theta_1\right)$$

$$= \sin^{-1}\left(\left(\frac{1.00}{1.52}\right) \sin 30.0°\right)$$ Substitute $n_1 = 1.00$, $n_2 = 1.52$, $\theta_1 = 30.0°$.

$$= 19.2°$$

3 EVALUATE THE ANSWER

- **Does the answer make sense?** The light beam travels into a medium with a higher n. Therefore, θ_2 should be less than θ_1.

PRACTICE Problems — ADDITIONAL PRACTICE

30. A laser beam in air enters ethanol at an angle of incidence of 37.0°. What is the angle of refraction?

31. As light travels from air into water, the angle of refraction is 25.0° to the normal. Find the angle of incidence.

32. Light in air enters a diamond facet at 45.0°. What is the angle of refraction?

33. A block of unknown material is submerged in water. Light in the water enters the block at an angle of incidence of 31°. The angle of refraction of the light in the block is 27°. What is the index of refraction of the material of the block?

34. **CHALLENGE** Light travels from air into another medium. The angle of incidence is 45.0° and the angle of refraction is 27.7°. What is the other medium?

The Meaning of the Index of Refraction

Looking at light as a wave yields an interesting connection between the propagation of light through a medium and its index of refraction. Snell's work was done prior to the development of the wave model of light; after the development of the wave model, it was understood that light interacts with atoms in such a way that it moves more slowly through a medium than it does through a vacuum.

Remember that for light traveling through a vacuum, $\lambda_0 = \frac{c}{f}$. This can be written more generally as $\lambda = \frac{v}{f}$, where v is the speed of light in any medium and λ is the wavelength in that medium. Frequency (f) is the number of oscillations a wave makes per second.

Frequency does not change at the boundary, so the wavelength decreases when light slows down as it crosses a boundary. And because the speed of light in any medium is slower than the speed of light in a vacuum, the wavelength in any medium is shorter than it is in a vacuum.

Wave model **Figure 20** shows a beam of light made up of a series of parallel, straight wavefronts. Each wavefront represents the crest of a wave and is perpendicular to the beam's direction. The beam's angle of incidence is θ_1. Consider ΔPQR. Because the wavefronts are perpendicular to the beam's direction, $\angle$PQR is a right angle. $\angle$QRP is equal to θ_1. Therefore, sin θ_1 is equal to $\overline{PQ}$ divided by $\overline{PR}$.

$$\sin \theta_1 = \frac{\overline{PQ}}{\overline{PR}}$$

The angle of refraction (θ_2) can be related in a similar way to ΔPSR. In this case,

$$\sin \theta_2 = \frac{\overline{RS}}{\overline{PR}}$$

By taking the ratio of the sines of the two angles, $\overline{PR}$ is canceled, leaving the following equation.

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{\overline{RS}}{\overline{PQ}}$$

In **Figure 20,** $\overline{PQ}$ is the length of three wavelengths of light in air, which can be written as $\overline{PQ} = 3\lambda_1$. In a similar way, $\overline{RS} = 3\lambda_2$. Substituting these values into the previous equation and canceling the common factor of 3 yields an equation relating the angles of incidence and refraction to the wavelength of light in each medium.

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{3\lambda_2}{3\lambda_1} = \frac{\lambda_2}{\lambda_1}$$

Using $\lambda = \frac{v}{f}$ and canceling the common factor f, because frequency stays constant, the equation can be rewritten:

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1}$$

Snell's law also can be written as a ratio of the sines of the angles of incidence and refraction.

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{n_1}{n_2}$$

Using the transitive property of equality, the previous two equations lead to the following equation:

$$\frac{n_1}{n_2} = \frac{v_2}{v_1}$$

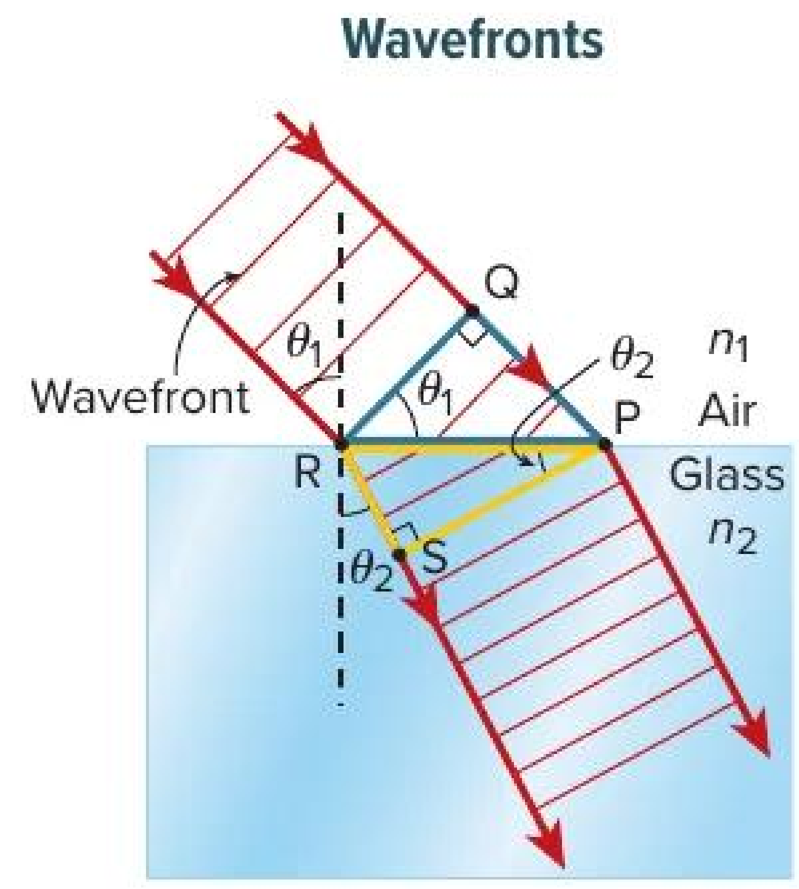


Figure 20 Each wavefront passes the boundary from air to glass at an angle. Part of the wavefront slows, and the ray bends. Since the wave slows and the frequency stays constant, for $\lambda = \frac{v}{f}$ to be true, the wavelength must decrease.

Infer *Which medium has a higher index of refraction?*

In a vacuum, $n = 1.00$ and $v = c$. If either medium is a vacuum, then the equation is simplified to an equation that relates the index of refraction to the speed of light in a medium.

Index of Refraction

The index of refraction of a medium is equal to the speed of light in a vacuum divided by the speed of light in the medium.

$$n = \frac{c}{v}$$

This definition of the index of refraction can be used to find the wavelength of light in a medium. In a medium with an index of refraction n, the speed of light is given by $v = \frac{c}{n}$. The wavelength of the light in a vacuum is $\lambda_0 = \frac{c}{f}$. Solve for frequency, and substitute $f = \frac{c}{\lambda_0}$ and $v = \frac{c}{n}$ into $\lambda = \frac{v}{f}$.

$$\lambda = \frac{(c/n)}{(c/\lambda_0)} = \frac{\lambda_0}{n}$$

Note that the wavelength of light in a medium is smaller than its wavelength in a vacuum.

Get It?

Describe the relationship between the index of refraction and the speed of light in a medium.

Total Internal Reflection

Recall that when light strikes a transparent boundary, some of the light is transmitted and some is reflected. As light travels from a medium of higher index of refraction to a medium of lower index of refraction, the angle of refraction is larger than the angle of incidence, as shown in **Figure 21.**

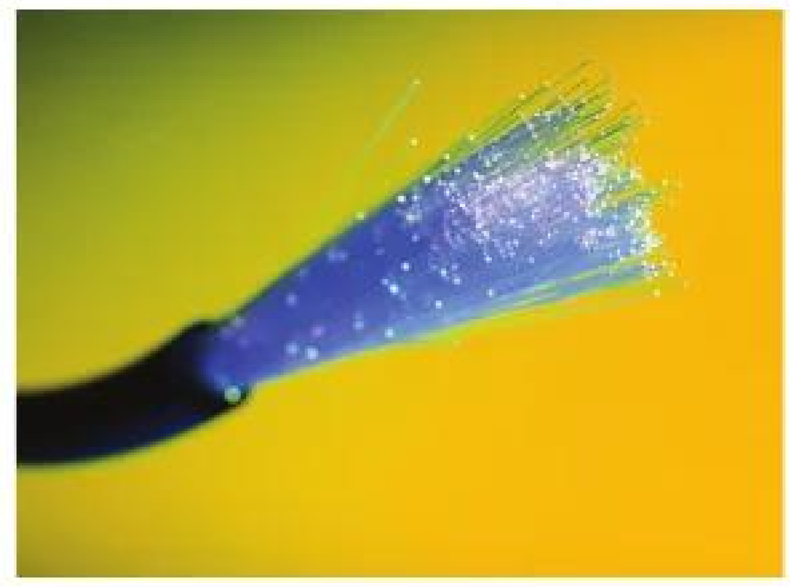
This leads to an interesting phenomenon. As the angle of incidence increases, the angle of refraction increases. At a certain angle of incidence known as the **critical angle** (θ_c), the refracted light ray lies along the boundary of the two mediums.

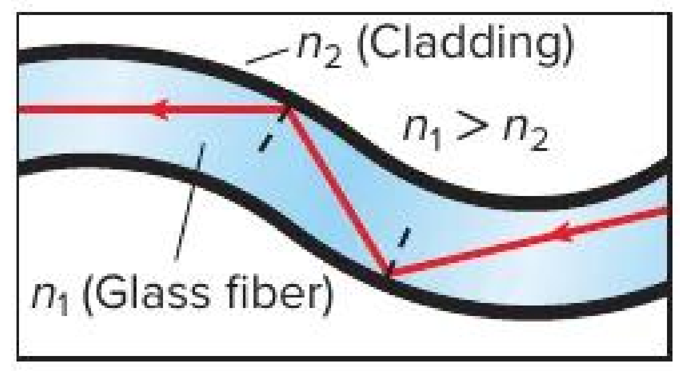
Total internal reflection occurs when light traveling from a region of a higher index of refraction to a region of a lower index of refraction strikes the boundary at an angle greater than θ_c such that all the light reflects back into the region of the higher index of refraction. This is shown in the third diagram of **Figure 21.** Using Snell's law and substituting $\theta_1 = \theta_c$, and $\sin \theta_2 = \sin 90.0° = 1$:

$$\sin \theta_c = \frac{n_2}{n_1}$$

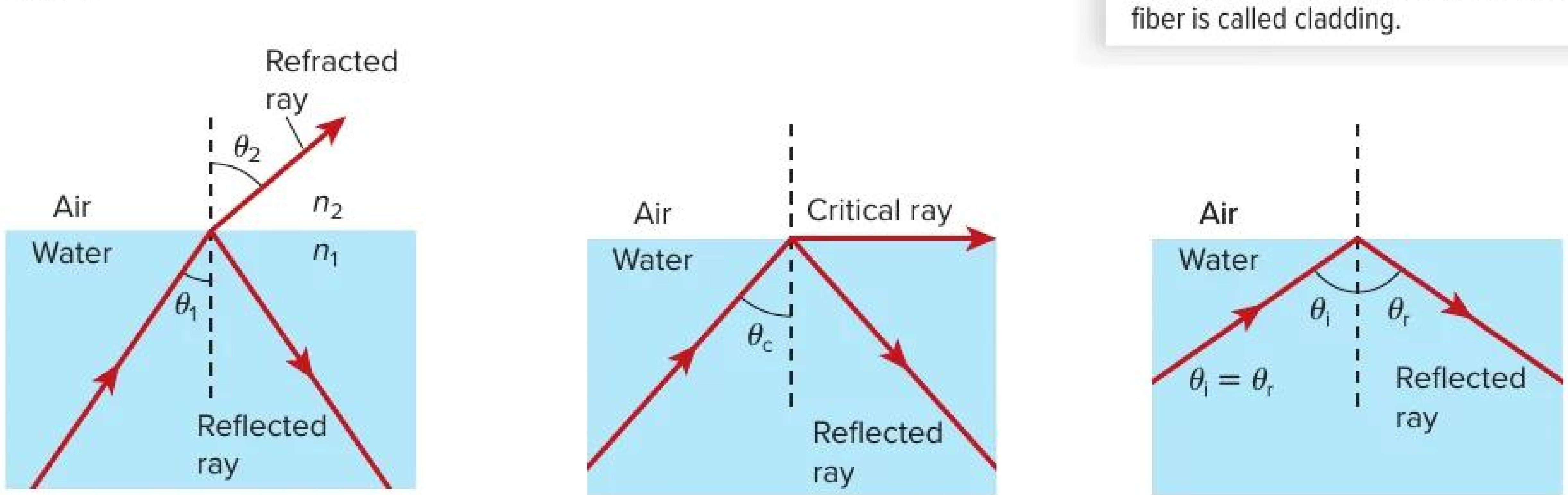
Total internal reflection causes some curious effects. Suppose you are looking up at the surface from under water in a pool. You might see an upside-down reflection of another nearby object that is also under water. You might see a reflection of the bottom of the pool itself. The surface of the water acts like a mirror, reflecting the image back into the water.

Real-World Physics





TOTAL INTERNAL REFLECTION is used in communication via optical fibers. The light traveling through the transparent fiber always hits the internal boundary of the optical fiber at an angle greater than the critical angle. All the light is reflected, and none of the light is transmitted through the boundary. Light pulses in fiber optic cables carry larger amounts of information over longer distances than other forms of communication. The outer material of each fiber is called cladding.



At an angle of incidence less than the critical angle, light is partially refracted and partially reflected.

A ray refracted along the boundary of the medium forms the critical angle.

An angle of incidence greater than the critical angle results in total internal reflection, which follows the law of reflection.

Figure 21 Refraction and reflection of light traveling between mediums depend on the angle of incidence θ_1.

Get It?

Compare and contrast data transmission using the fiber optics cable at the beginning of this module and data transmission by satellite.

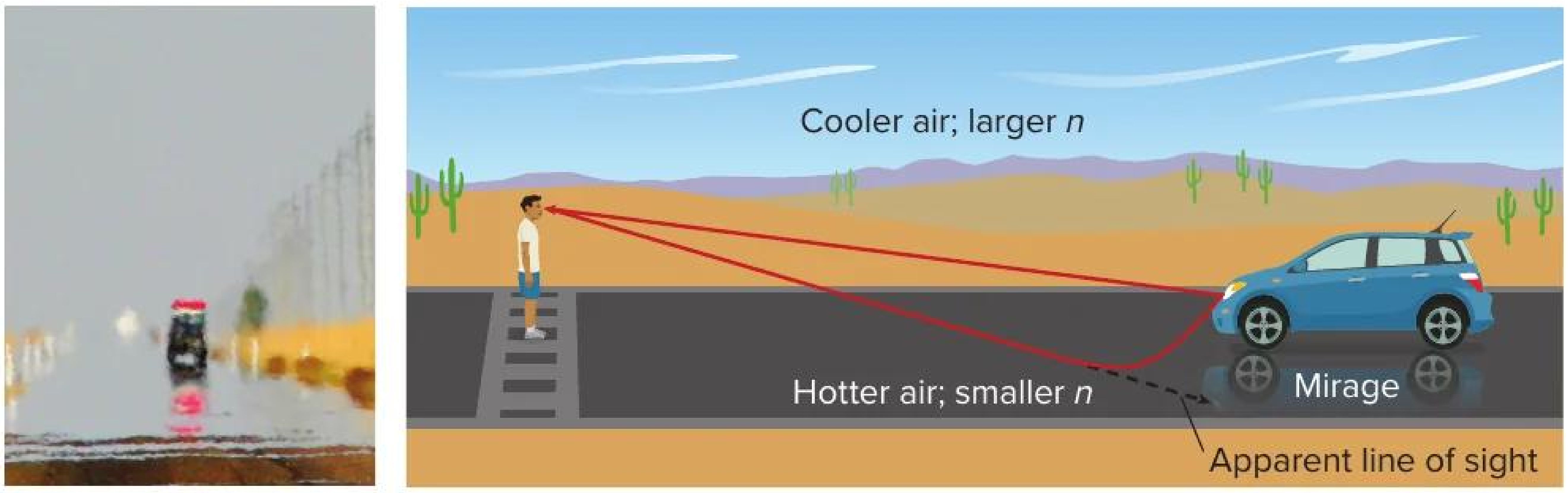


Figure 22 Light waves reflected from the vehicle on the road are refracted when the air near the surface of the road is hotter than the air above it.

Apply *Are the light waves traveling faster near the surface of the road or near the top of the vehicle?*

Mirages

On a hot summer day, you sometimes can see what appears to be the reflection of an oncoming vehicle in a pool of water as shown on the left in **Figure 22.** The pool, however, disappears as you approach it. This mirage is the result of the Sun heating the road. The speed of light and, therefore, the index of refraction, for a gaseous medium can change slightly with temperature. The hot road heats the air above it and produces a thermal layering of air, causing light traveling toward the road from the car to gradually bend upward. This makes the light appear to be coming from a reflection in a pool.

The diagram in **Figure 22** shows how this occurs. As light from a distant object travels downward toward the road, the index of refraction of the air decreases as the air gets hotter, but the temperature change is gradual. Recall that light wavefronts are comprised of Huygens' wavelets. In the case of a mirage, the Huygens' wavelets closer to the ground travel faster than those higher up, causing the wavefronts to gradually turn upward.

Dispersion of Light

If you look carefully at the spectrum formed by light passing through the prism in **Figure 23,** you will notice that violet light is refracted more than red light. This occurs because the speed of violet light through glass is less than the speed of red light through glass. Violet light has a higher frequency than red light, which causes it to interact differently with the atoms of the glass. This results in glass having a slightly higher index of refraction for violet light than it has for red light. This separation of white light into a spectrum of colors is called **dispersion.**

The speed of light in a medium is determined by interactions between the light and the atoms that make up the medium. The speed of light and the index of refraction for a solid or liquid medium vary for different wavelengths of light.

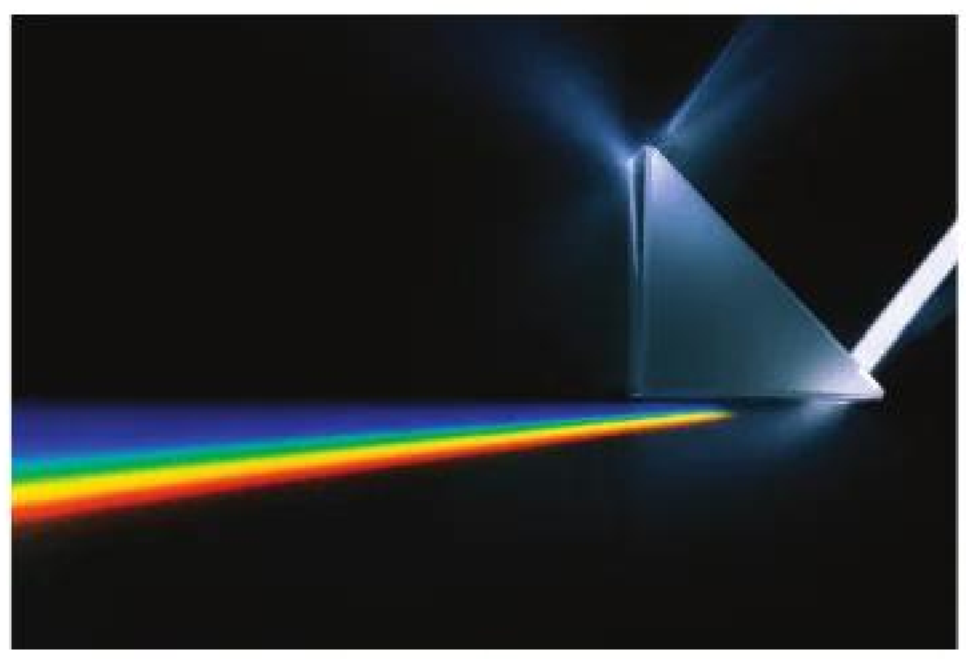


Figure 23 Dispersion through a prism occurs because the index of refraction varies with the wavelength of light.

Rainbows A rainbow is a spectrum formed when sunlight is dispersed by water droplets in the atmosphere. Sunlight that falls on a water droplet is refracted. Since each color has a different wavelength, it is refracted at a slightly different angle, as shown in **Figure 24** on the next page, resulting in dispersion. At the back surface of the droplet, some of the light undergoes internal reflection. On the way out of the droplet, the light once again is refracted and dispersed.

Although each droplet produces a complete spectrum, an observer positioned between the Sun and the rain will see only a certain wavelength of light from each droplet. The wavelength observed depends on the relative positions of the Sun, the droplet, and the observer, as shown in **Figure 24.**

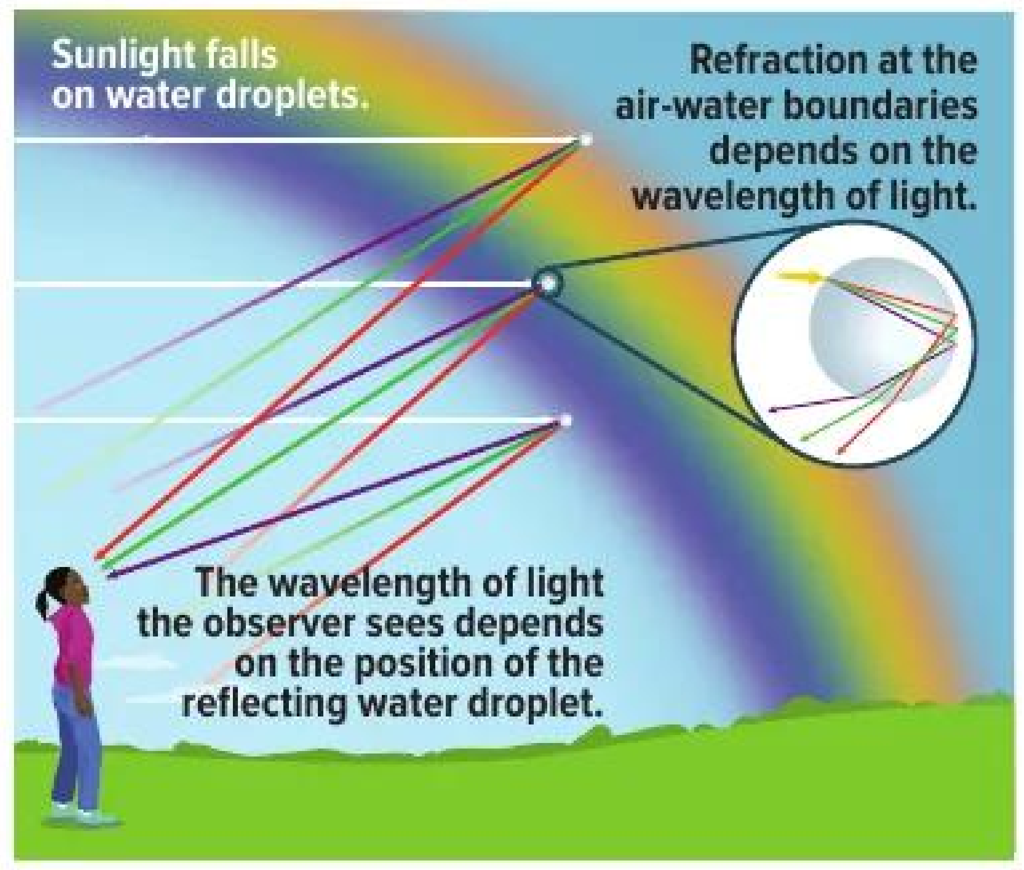


Figure 24 Light from the Sun is dispersed by water droplets to form rainbows. Because there are many droplets at various positions in the sky, a complete spectrum is visible.

Consider *whether it is possible for a rainbow to be close enough to touch.*

Second-order rainbow Sometimes, you can see a faint second-order rainbow like the one shown in **Figure 25.** The second rainbow is outside of the first. It is also fainter and has the order of the colors reversed. Light rays that are reflected twice inside water droplets produce this effect.

Figure 25 Light reflected within raindrops can cause the appearance of a secondary rainbow.

Check Your Progress

35. **Index of Refraction** When a ray of light enters a certain liquid from water, it is bent toward the normal, but when it enters the same liquid from float glass, it is bent away from the normal. What can you conclude about the liquid's index of refraction?
36. **Index of Refraction** A ray of light in air has an angle of incidence of 30.0° on a block of unknown material and an angle of refraction of 20.0°, as shown in **Figure 26.** What is the index of refraction of the material?

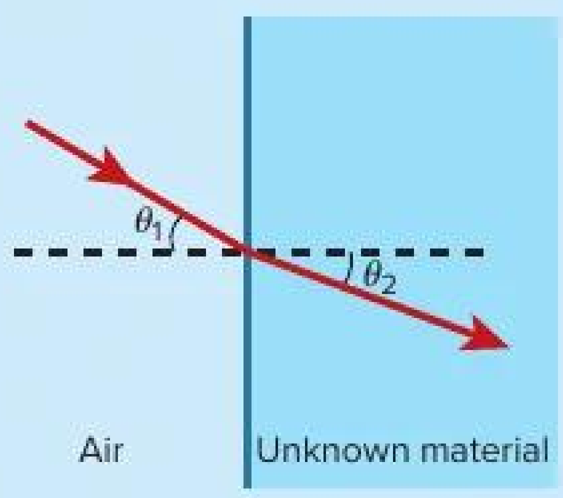


Figure 26

37. **Angle of Refraction** A beam of light passes from water into polyethylene ($n = 1.50$). If $\theta_1 = 57.5°$, what is the angle of refraction in the polyethylene?
38. **Speed of Light** What is the speed of light in chloroform ($n = 1.51$)?
39. **Critical Angle** Is there a critical angle for light traveling from glass to water? From water to glass? Explain your answer.
40. **Total Internal Reflection** Would you use quartz or float glass for the cladding of an optical fiber? Explain your answer.
41. **Speed of Light** Could an index of refraction ever be less than 1? What would this imply about the speed of light in that medium?
42. **Critical Thinking** In what direction would you have to be looking to see a rainbow on a late rainy afternoon? Explain.

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LESSON 4

CONVEX AND CONCAVE LENSES

FOCUS QUESTION

How are systems of lenses used to make optical devices?

Types of Lenses

Refraction of light can be quite useful. In 1303, French physician Bernard of Gordon wrote of using lenses to correct eyesight. Around 1610, Galileo discovered the four major moons of Jupiter using a telescope made with two lenses. Since that time, lenses have been used in instruments such as microscopes and cameras.

A **lens** is a piece of transparent material that focuses light and forms an image. Each of a lens's two faces might be either curved or flat. A lens that is thicker at the center than at the edges is called a **convex lens,** as shown on the left in **Figure 27.** When it is surrounded by material with a lower index of refraction, such as air, it refracts parallel light rays so that they all pass through a common point, called the focal point, after going through the lens. For this reason, a convex lens is often called a converging lens.

A lens that is thinner in the middle than at the edges is called a **concave lens,** as shown on the right in **Figure 27.** When surrounded by material with a lower index of refraction, rays passing through it spread out, so it is often called a diverging lens.

You can determine the paths of rays passing through lenses using Snell's law. To simplify such problems, we assume that all refraction occurs at the principal plane, which passes through the center of the lens. This approximation, called the thin lens model, applies to all the lenses discussed in this module.

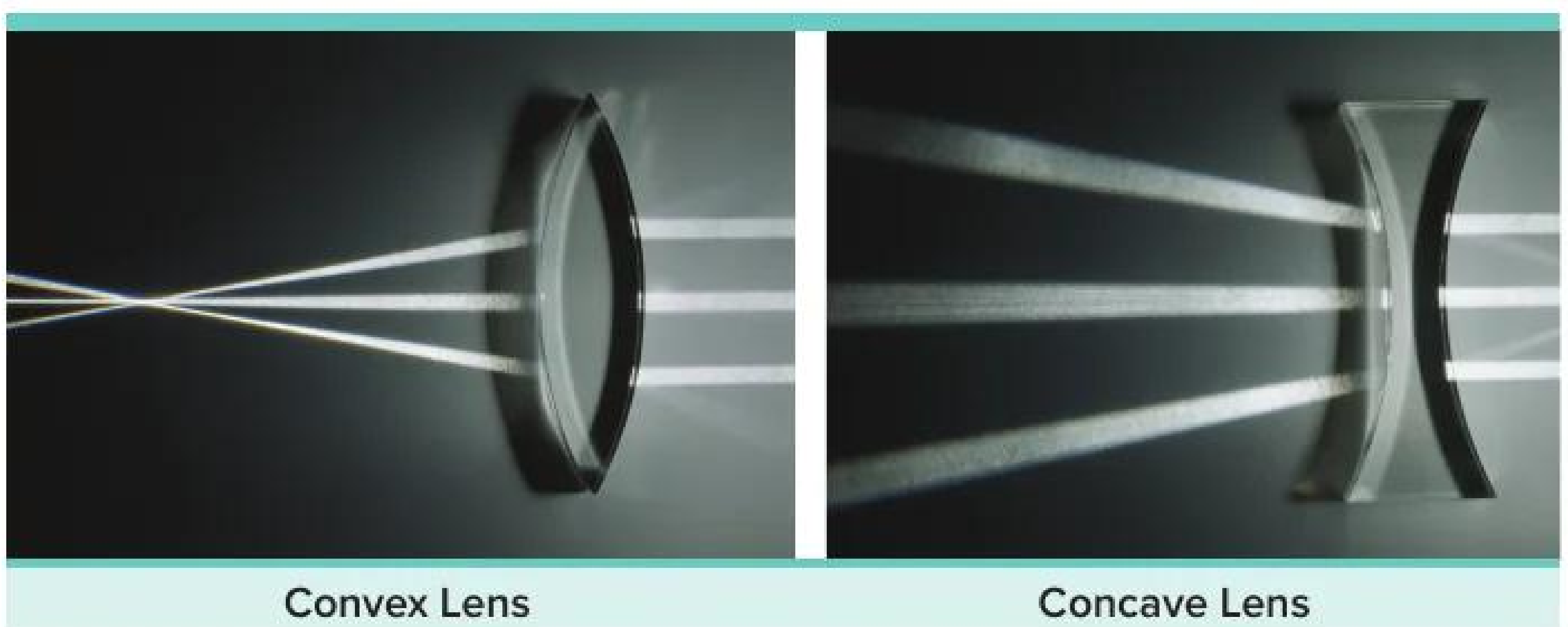


Figure 27 Convex lenses refract light so that the rays converge after passing through. The light passing through a concave lens does not meet at the focal point.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Virtual Investigation: Ray Tracing for Lenses
Use a computer model to investigate the properties of a lens affect how light is transmitted.

Review the News
Obtain information from a current news story about technologies that use light. Evaluate your source and communicate your findings to your class.

Convex Lenses

To determine an image's location, we can use ray diagrams. In a ray diagram, as with mirrors you represent a few important rays to find out how a lens affects the light that passes through it. You can use any two rays to locate the image; with experience, you will learn that some rays are easier to draw and use than others.

In ray diagrams for convex lenses, x_o is the distance of the object from the lens and x_i is the distance of the image from the lens. In all ray diagrams in this module, the thin lens model is used. In this model, light refracts at the center of the lens rather than at the boundaries between air and the surface of the lens.

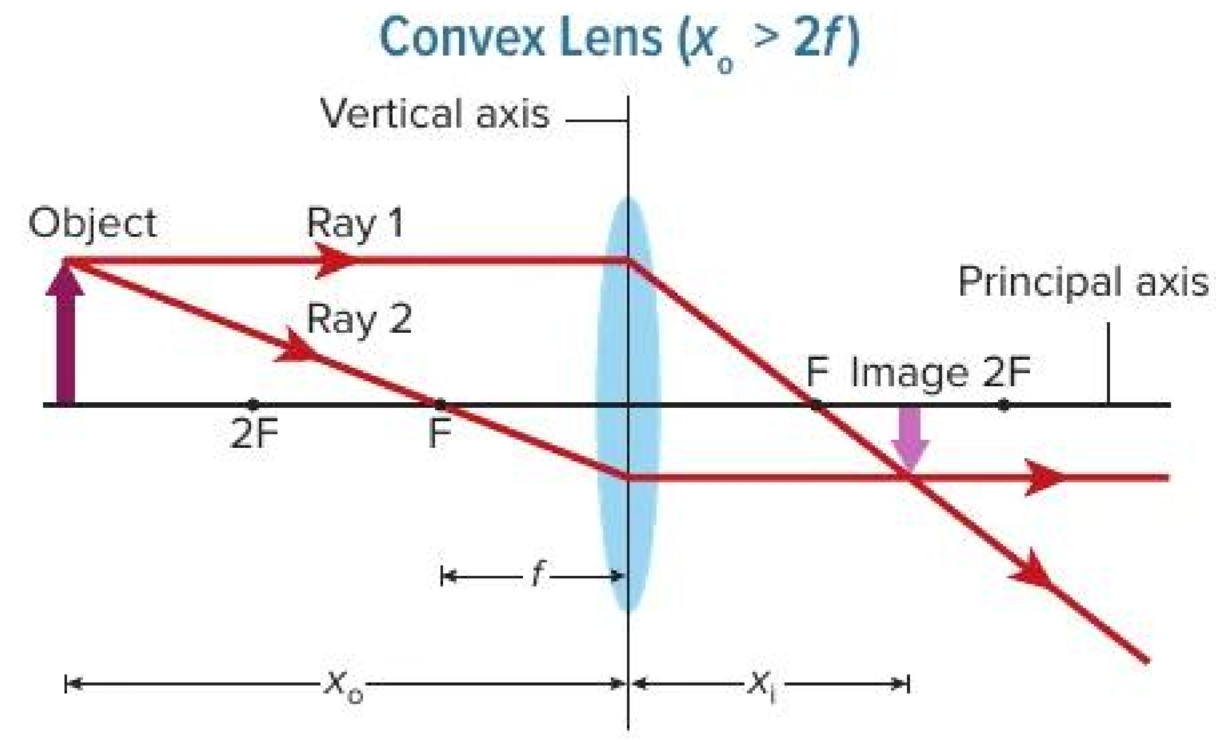
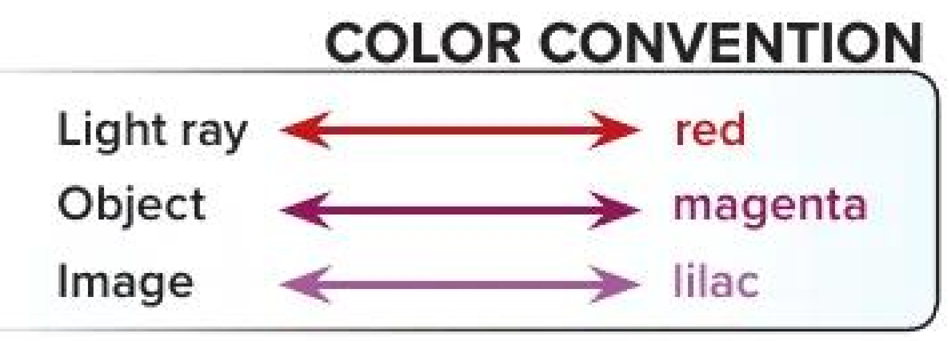


Figure 28 An object placed at a distance greater than twice the focal length from the lens will produce an image that is real, reduced in size, and inverted.

$x_o \geq 2f$ In **Figure 28,** rays are traced from an object located far from the lens. Ray 1 is parallel to the principal axis. It refracts and passes through the focal point (F) on the other side of the lens. Ray 2 passes through F on its way to the lens. After refraction, its path is parallel to the principal axis.



The two rays intersect at a point beyond F and locate the image. Rays selected from other points on the object converge at corresponding points to form the complete image. Note that this is a real image that is inverted and smaller compared to the object.

$x_o = 2f$ If the object is placed at twice the focal length from the lens at the point 2F, the diagram is similar. The image is real and is found at 2F, but it is no longer reduced in size. Because of symmetry, the image and the object have the same size. Thus, you can conclude that if an object is more than twice the focal length from the lens, the image is smaller than the object.

$2f > x_o > f$ You can use **Figure 29** to locate the image of an object that is between F and 2F viewed through a convex lens. This is similar to the ray diagram for the object located at a distance greater than twice the focal length with the image and object interchanged. The direction of the rays would be reversed. In this case, the image is also real and inverted.

For an object placed between F and 2F, the image is enlarged. When an object is placed at the focal point of a convex lens, ray diagrams cannot be drawn. The refracted rays will emerge in a parallel beam and no image will be seen.

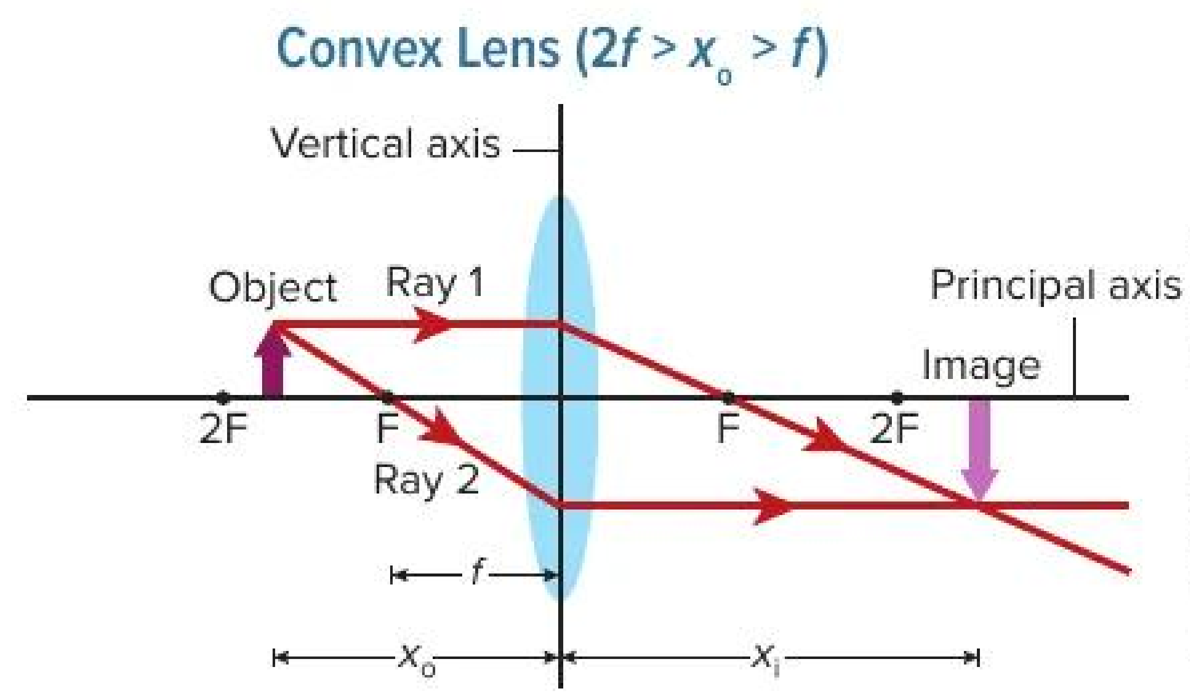


Figure 29 An object placed at a distance less than twice the focal length but greater than one focal length from the lens will produce an image that is real, enlarged, and inverted.

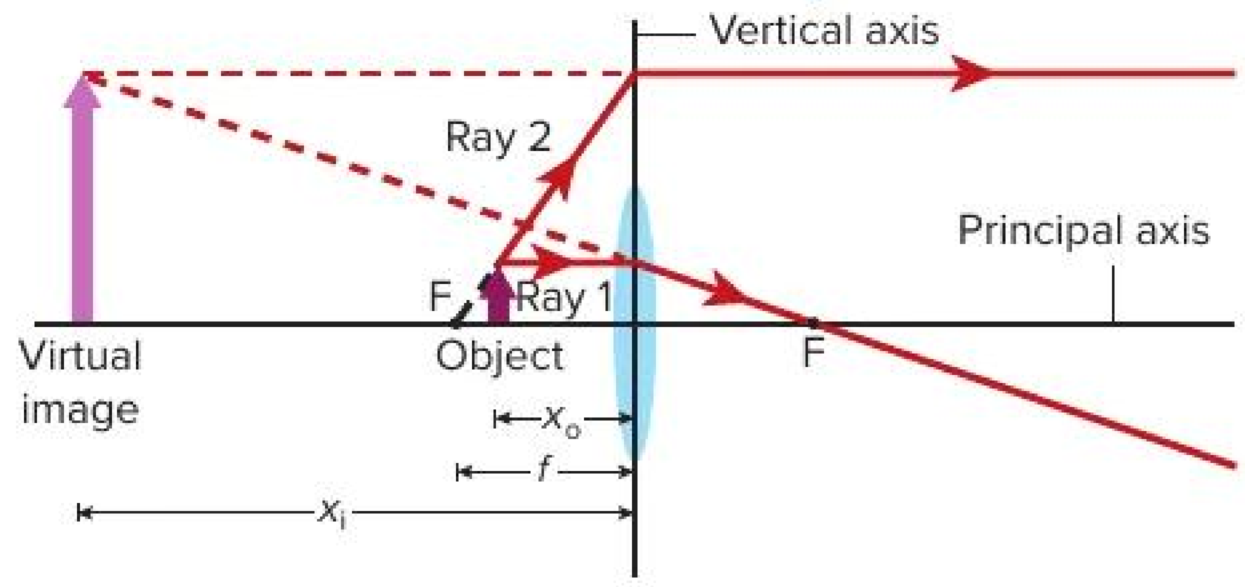


Figure 30 An object placed at a distance less than the focal length from the lens will produce an image that is virtual and enlarged.

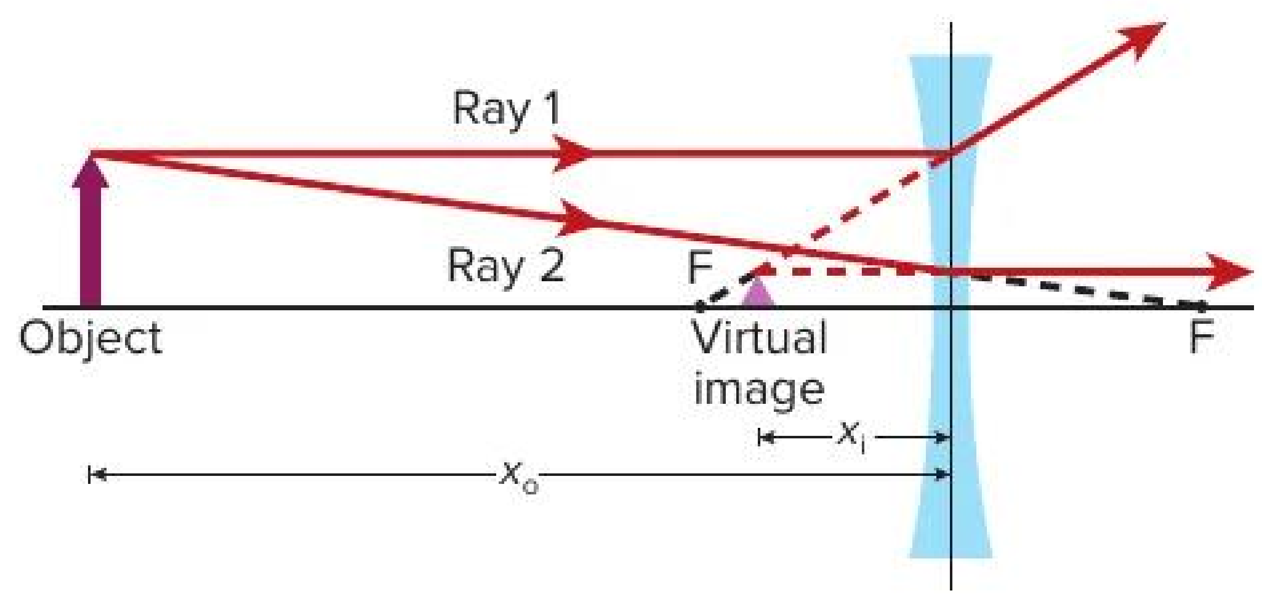


Figure 31 An object placed any distance from a concave lens will always produce an image that is virtual and reduced.

$f > x_o > 0$ **Figure 30** shows how a convex lens forms a virtual image. The object in this situation is located between F and the lens. Ray 1, as usual, approaches the lens parallel to the principal axis and is refracted through the focal point, F. Ray 2 travels from the tip of the object in the direction it would have if it had started at F on the object side of the lens. The dashed line from F to the object shows you how to draw ray 2. Ray 2 leaves the lens parallel to the principal axis. Ray 1 and ray 2 diverge as they leave the lens.

The reflection appears to an observer to come from a spot on the same side of the lens as the object. This is a virtual image that is upright and larger compared to the object. No real image is possible. Drawing sight lines for the two rays back to their apparent intersection locates the virtual image. Note that the actual image is formed by light that passes through the lens, but you can still determine the location of the image by drawing rays that do not have to pass through the lens.

Get It?

Classify an image as virtual or real based on the side of the lens it is on.

Concave Lenses

A concave lens causes all rays to diverge. **Figure 31** shows how a concave lens forms a virtual image. Ray 1 approaches the lens parallel to the principal axis. It leaves the lens along a line that extends back through the focal point on the object side of the lens. The focal point of a concave lens is found on the same side of the lens as the incoming light. Ray 2 approaches the lens as if it is going to pass through the focal point on the opposite side but leaves the lens parallel to the principal axis.

The sight lines of rays 1 and 2 intersect on the same side of the lens as the object. The image is located at the point from where the two rays appear to intersect, creating a virtual image. The image is upright and smaller than the object. This is true no matter how far from the lens the object is located. The focal length for a diverging lens is negative.

Get It?

Describe why a concave lens always produces a virtual image.

Table 3 Properties of a Single Spherical Lens System

Lens Type	f	x_o	x_i	m	Image
Convex	+	$x_o > 2f$	$2f > x_i > f$	reduced, inverted	real
		$2f > x_o > f$	$x_i > 2f$	enlarged, inverted	real
		$f > x_o > 0$	$\|x_i\| > x_o$ (negative)	enlarged	virtual
Concave	−	$x_o > 0$	$\|f\| > \|x_i\| > 0$ (negative)	reduced	virtual

Lens Equations

Lenses can be constructed with a variety of shapes, but you will only consider thin lenses, with faces having spherical curvatures. Based on this model of thin spherical lenses, an equation has been developed that looks exactly like the one for spherical mirrors. For lenses, virtual images are always on the same side of the lens as the object, which means the image position is negative. Note that a concave lens produces only virtual images, whereas a convex lens can produce real images or virtual images.

Thin lens equation

The **thin lens equation** relates the focal length of a spherical thin lens, the object position, and the image position.

Thin Lens Equation

The inverse of the focal length of a spherical lens is equal to the sum of the inverses of the image position and the object position.

$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$$

When solving problems for concave lenses using the thin lens equation, remember that the sign convention for focal length is different from that of convex lenses. This is because a concave lens is a divergent lens. If the focal point for a concave lens is 24 cm from the lens, you should use the value $f = -24$ cm in the thin lens equation. All images for a concave lens are virtual. Thus, if an image distance is given as 20 cm from the lens, then you should use $x_i = -20$ cm. The object position always will be positive.

Magnification

A property of spherical thin lenses that measures how much larger or smaller the image is than the object is magnification. The magnification equation for spherical mirrors also can be used for spherical thin lenses. It is used to determine the height and the orientation of the image formed by a spherical thin lens.

Magnification

The magnification of an object by a spherical lens, defined as the image height divided by the object height, is equal to the negative of the image position divided by the object position.

$$m \equiv \frac{h_i}{h_o} = -\frac{x_i}{x_o}$$

Magnification gives information about the size and orientation of the image relative to the object. When the absolute value of a magnification is between zero and one, the image is smaller than the object. Magnifications with absolute values greater than one represent images that are larger than the objects. A negative magnification means the image is inverted compared to the object.

Using the equations for lenses

It is important that you use the proper sign conventions when using these equations. **Table 3** shows a comparison of the image position, magnification (m), and type of image formed by single convex and concave lenses when an object is placed at various object positions (x_o) relative to the lens. For convex lenses, the object position relative to the focal point influences the image type.

Notice how this table is similar to **Table 1** in Lesson 1, the table for mirrors. As with mirrors, the distance from the principal plane of a lens to its focal point is the focal length (f). The focal length depends on the shape of the lens and the index of refraction of the lens material. Focal lengths, image positions, and image heights can be negative.

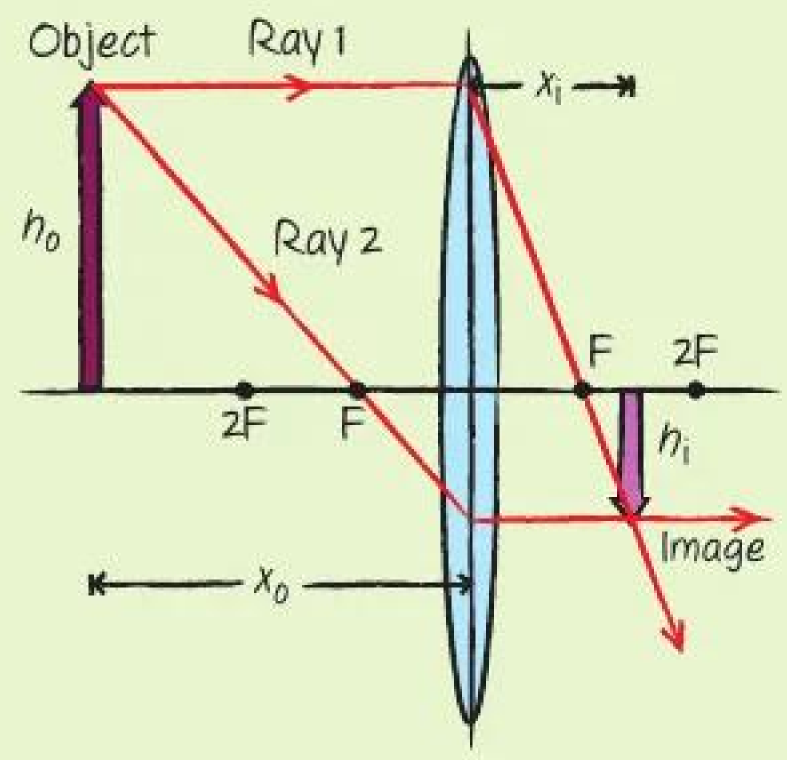
EXAMPLE Problem 5

AN IMAGE FORMED BY A CONVEX LENS An object is placed 32.0 cm from a convex lens that has a focal length of 8.0 cm.

a. Where is the image?

b. If the object is 3.0 cm high, how tall is the image?

c. What is the orientation of the image?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation, locating the object and the lens.
- Draw the two principal rays.

Known	Unknown
$x_o = 32.0$ cm	$x_i = ?$
$h_o = 3.0$ cm	$h_i = ?$
$f = 8.0$ cm	

2 SOLVE FOR THE IMAGE POSITION AND HEIGHT

a. Use the thin lens equation to determine x_i.

$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$$

$$x_i = \frac{fx_o}{x_o - f}$$

$$= \frac{(8.0\text{ cm})(32.0\text{ cm})}{32.0\text{ cm} - 8.0\text{ cm}}$$ Substitute $f = 8.0$ cm, $x_o = 32.0$ cm.

$= 11$ cm (11 cm away from the lens on the side opposite the object)

b. Use the magnification equation to solve for image height.

$$m \equiv \frac{h_i}{h_o} = -\frac{x_i}{x_o}$$

$$h_i = -\frac{x_i h_o}{x_o}$$

$$= -\frac{(11\text{ cm})(3.0\text{ cm})}{32.0\text{ cm}}$$ Substitute $x_i = 11$ cm, $h_o = 3.0$ cm, $x_o = 32.0$ cm.

$= -1.0$ cm (1.0 cm tall)

c. The negative sign for the height in part **b** means the image is inverted.

3 EVALUATE THE ANSWER

- **Are the units correct?** All are in centimeters.
- **Do the signs make sense?** Image position is positive (real image), and image height is negative (inverted compared to the object), which make sense for a convex lens.

PRACTICE Problems

ADDITIONAL PRACTICE

43. A 2.25-cm-tall object is 8.5 cm to the left of a convex lens whose focal length is 5.5 cm. Find the image position and height.

44. An object near a convex lens produces a 1.8-cm-tall real image that is 10.4 cm from the lens and inverted. If the focal length of the lens is 6.8 cm, what are the object position and height?

45. An object is placed to the left of a convex lens with a 25-mm focal length so that its image is the same size as the object. What are the image and object positions?

46. Calculate the image position and height of a 2.0-cm-tall object located 25 cm from a convex lens with a focal length of 5.0 cm. What is the orientation of the image?

47. Use a scale ray diagram to find the image position of an object that is 30 cm to the left of a convex lens with a 10-cm focal length.

48. **CHALLENGE** A magnifier with a focal length of 30 cm is used to view a 1-cm-tall object. Use a ray diagram to determine the location and size of the image when the magnifier is positioned 10 cm from the object.

Defects of Spherical Lenses

Throughout this section, you have studied lenses that produce perfect images at specific positions. In reality, spherical lenses, just like spherical mirrors, have intrinsic defects that cause problems with the focus and color of images. Spherical lenses exhibit an aberration associated with their spherical design, just as mirrors do. In addition, the dispersion of light through a spherical lens causes an aberration that mirrors do not exhibit.

Figure 32 Spherical aberration from the camera lens blurs the edges of this image.

Spherical aberration

The model you have used for drawing rays through spherical lenses suggests that all parallel rays focus at the same position. However, this is only an approximation. In reality, parallel rays that pass through the edges of a spherical lens focus at positions different from those of parallel rays that pass through the center. This inability of a spherical lens to focus all parallel rays to a single point is called spherical aberration. The effects are shown in **Figure 32.** In reality, most lenses have a slightly different shape to address this, but the spherical approximation works well enough for our purposes. In high-precision instruments, many lenses, often five or more, are used to form sharp, well-defined images.

Chromatic aberration

Lenses have a second defect that mirrors do not have. Because the index of refraction of a medium depends on wavelength, different wavelengths of light are refracted at slightly different angles, as you can see on the right in **Figure 33.** Light that passes through a lens is slightly dispersed, especially near the edges, causing an effect called **chromatic aberration.** This is seen as an apparent ring of color around an object viewed through a lens, as shown on the left in **Figure 33.**

Chromatic aberration is always present when a single lens is used. However, this defect can be greatly reduced by an **achromatic lens,** which is a system of two or more lenses, such as a convex lens with a concave lens, that have different indices of refraction. A simple lens and an achromatic lens are shown in **Figure 33.** Both lenses disperse light, but the dispersion caused by the achromatic lens is less than the dispersion caused by the simple lens. The index of refraction of the achromatic lens is chosen so that the light will still converge at the desired location.

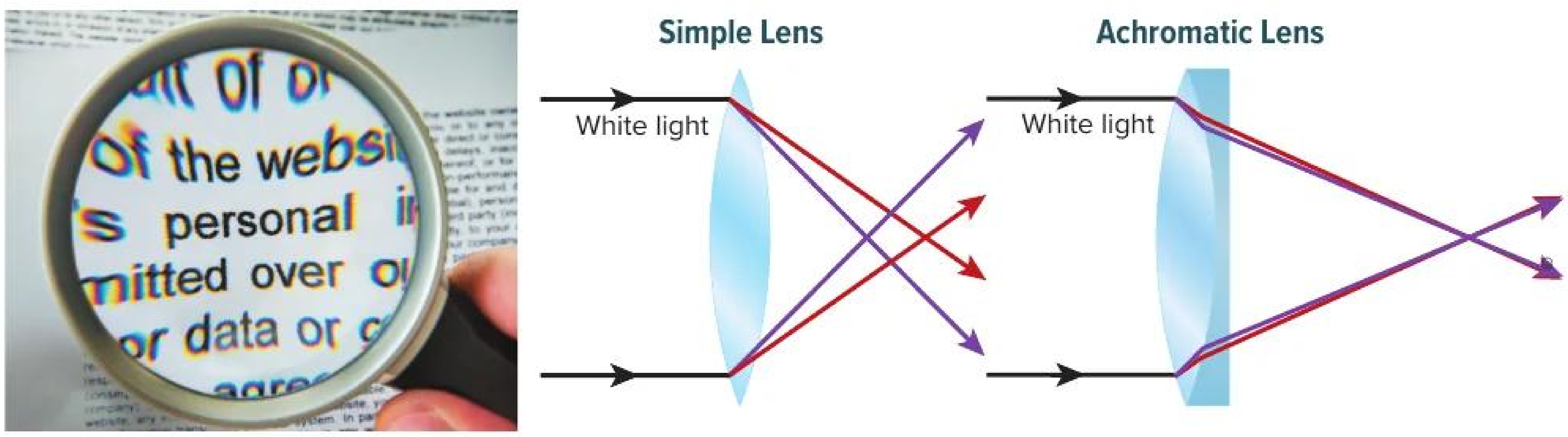


Figure 33 Simple lenses, such as the one shown in the photo on the left, exhibit a rainbow fringing effect called chromatic aberration. Achromatic lenses reduce chromatic aberration.

Explain *why the index of refraction is important for achromatic lenses.*

Lenses in Eyes

The concepts that you have learned for the refraction of light through lenses apply to almost every optical device, including the eye. The eye is a remarkable optical device. As shown in **Figure 34,** the eye is a fluid-filled, almost spherical vessel. Light that is emitted from or reflected off an object travels into the eye through the cornea and pupil. The light then passes through the lens and focuses onto the retina, which is at the back of the eye. Specialized cells on the retina absorb this light and send information about the image along the optic nerve to the brain.

BIOLOGY Connection Because of its name, you might assume that the lens of an eye is responsible for focusing light onto the retina. In fact, light entering the eye is primarily focused by the cornea, because the air-cornea boundary has the greater difference in indices of refraction. The lens is responsible for the fine focus that allows you to clearly see both distant and nearby objects.

Get It?
Describe the roles of the cornea and the lens in your eye.

The Eye

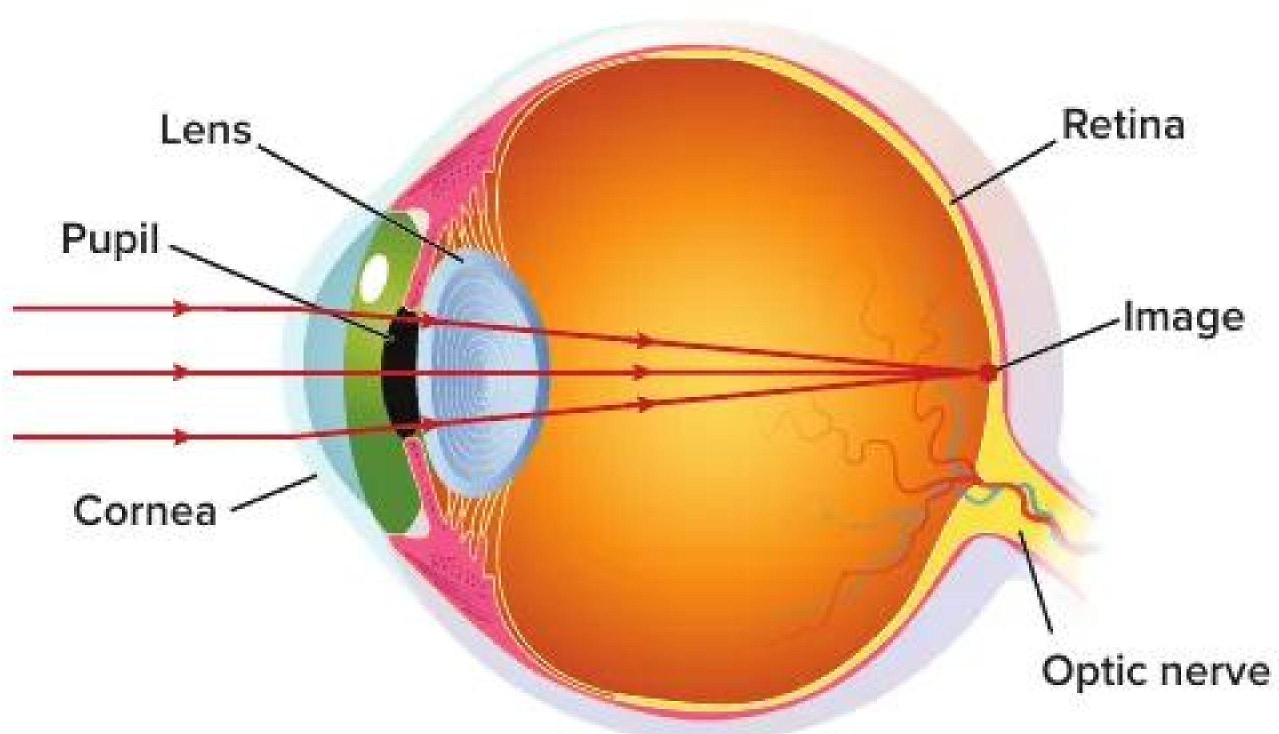


Figure 34 The cornea and the lens of your eye refract light that is reflected off every object you see.
Summarize *why most of the refraction occurs at the air-cornea boundary rather than at the lens.*

The eye lens uses a process called accommodation. Muscles surrounding the lens can contract or relax, thereby changing the shape of the lens. This, in turn, changes the focal length of the eye. For a healthy eye, when the muscles are relaxed, the image of distant objects is focused on the retina. When the muscles contract, the focal length is shortened, and this allows images of closer objects to be focused on the retina.

Nearsightedness The eyes of many people focus images either in front of the retina or behind it. External lenses, such as eyeglasses and contact lenses, adjust the focal length and move images to the retina. **Figure 35** on the next page shows the condition of **nearsightedness,** also called myopia.

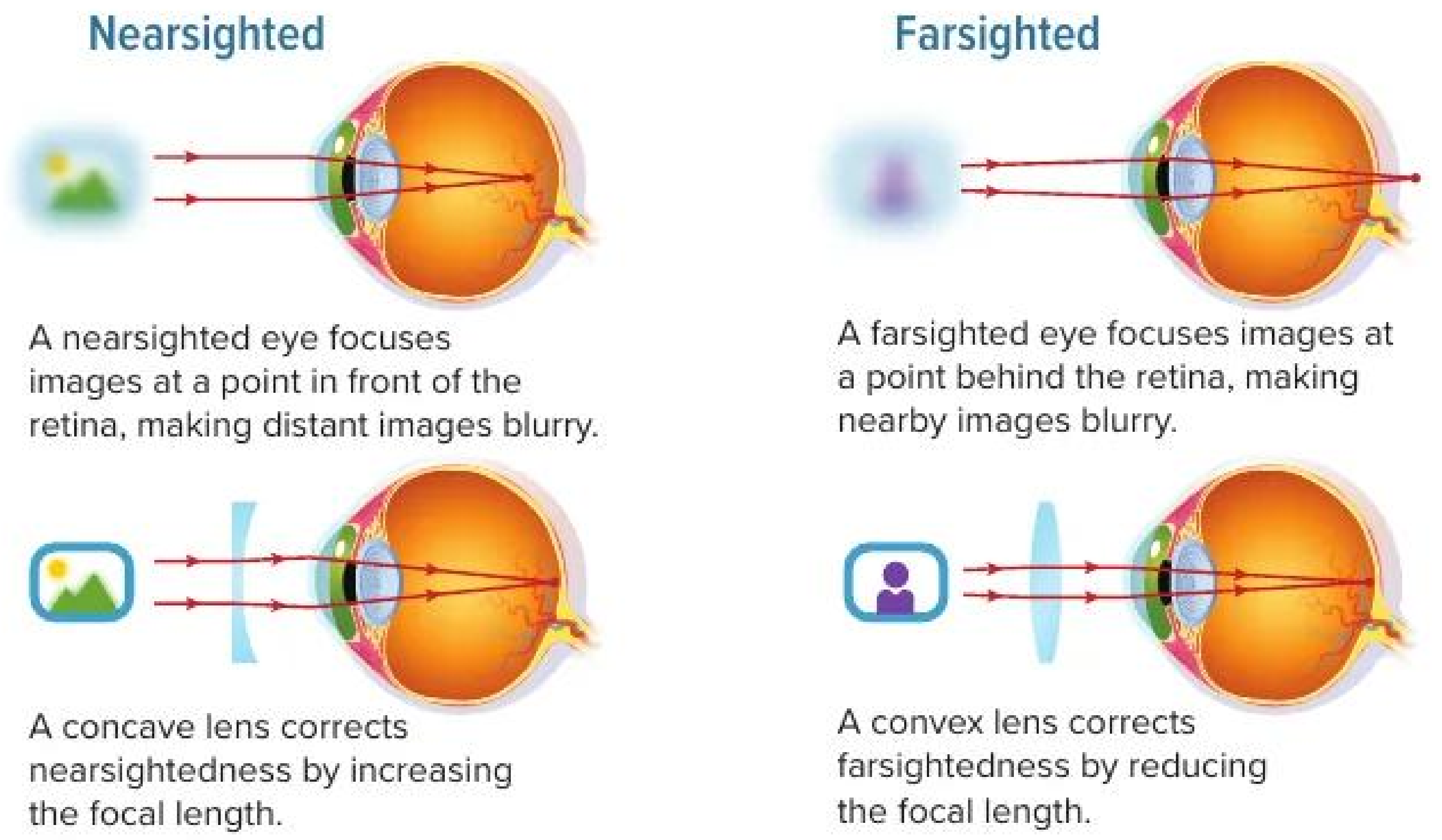


Figure 35 Objects at a distance are blurred for a nearsighted person. For a farsighted person, objects nearby are blurred.

Explain *how bifocal lenses might be made.*

With myopia, the focal length of the eye is too short to focus light on the retina. Images are formed in front of the retina. As shown in **Figure 35,** concave lenses correct this by diverging light, thereby increasing images' distances from the lens, and forming images on the retina.

Farsightedness You also can see in **Figure 35** that **farsightedness,** also called hyperopia, is the condition in which the focal length of the eye is too long. Images are therefore not formed on the retina.

A similar result is caused by the increasing rigidity of the lenses in the eyes of people who are more than about 45 years old. Their muscles cannot shorten the focal length enough to focus images of close objects on the retina.

For either condition, convex lenses produce virtual images farther from the eye than the associated objects, as shown in **Figure 35.** The images then become the objects for the eye lens and can be focused on the retina, thereby correcting the condition.

Real-World Physics

CONTACT LENSES produce the same results as eyeglasses. These small, thin lenses are placed directly on the corneas. A thin layer of tears between the cornea and lens keeps the lens in place. Most of the refraction occurs at the air-lens boundary, where the difference in indices of refraction is greatest.

PHYSICS Challenge

As light enters the eye, it first encounters the air-cornea boundary. Consider a ray of light that strikes the interface between the air and a person's cornea at an angle of 30.0° to the normal. The index of refraction of the cornea is approximately 1.4.

1. Use Snell's law to calculate the angle of refraction.
2. What would the angle of refraction be if the person were swimming underwater with his or her eyes open?
3. Is the refraction greater in air or in water? Does this mean objects underwater seem closer or more distant than they would in air?
4. If you want the angle of refraction for the light ray in water to be the same as it is for air, what should the new angle of incidence be?

Refracting Telescopes

An astronomical refracting telescope uses lenses to magnify distant objects. **Figure 36** shows the optical system for a Keplerian telescope. Light from stars and other astronomical objects comes from so far away that the incoming rays can be considered parallel. These rays enter the objective convex lens, which focuses them as a real image at the focal point of the objective lens. The image is inverted compared to the object. This image then becomes the object for the convex lens of the eyepiece.

Notice that the eyepiece lens is positioned so that the focal point of the objective lens is between the eyepiece lens and its focal point. This means a virtual image is produced that is upright and larger than the first image. However, because the first image was already inverted, the final image is still inverted. For viewing astronomical objects, an image that is inverted is acceptable.

In a telescope, the convex lens of the eyepiece is almost always an achromatic lens. Recall that an achromatic lens is a combination of lenses that functions as one lens. The combination of lenses greatly reduces the peripheral colors that can form on images due to chromatic aberration.

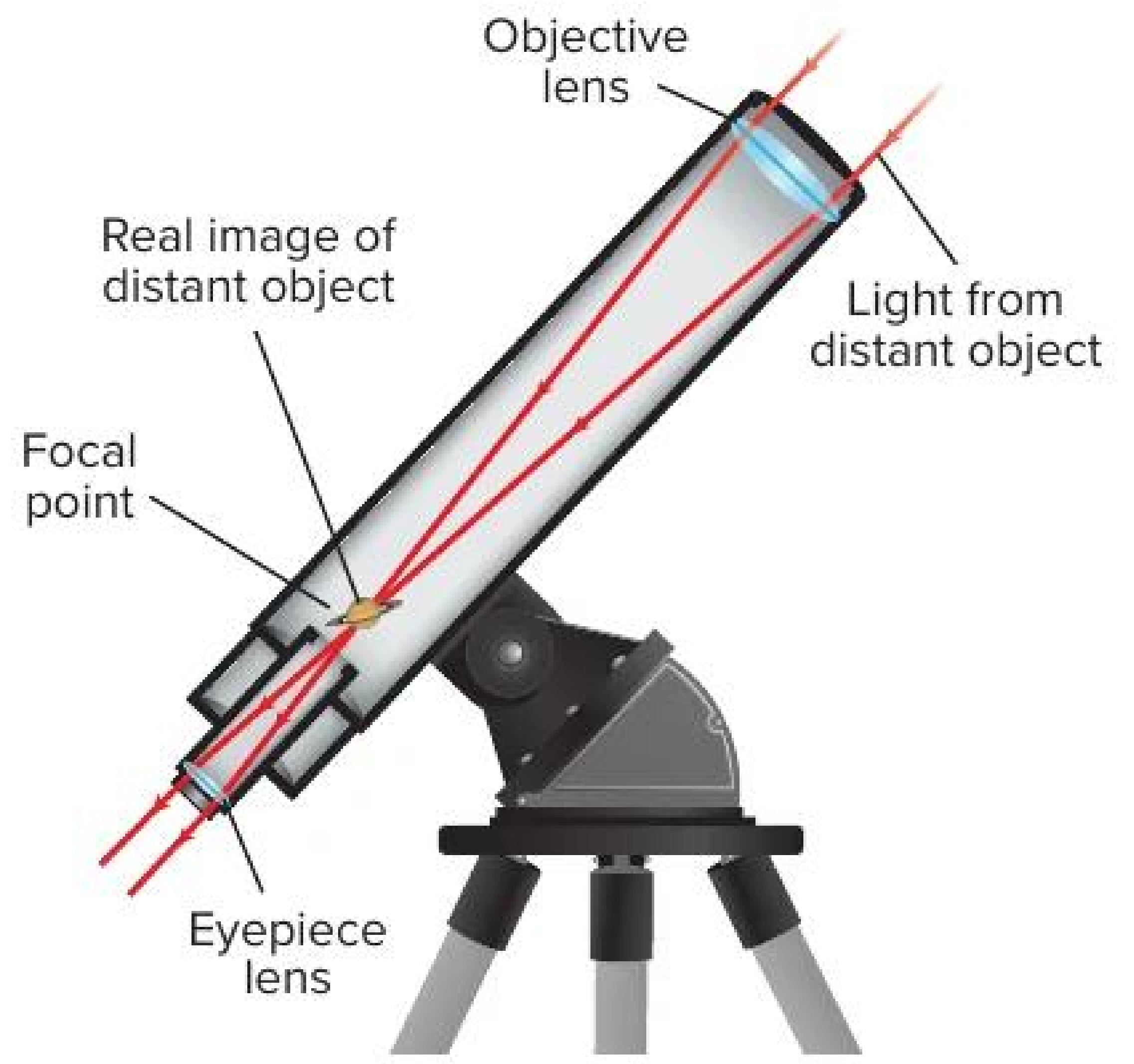


Figure 36 Light from distant objects is gathered by the objective lens and the eyepiece of a refracting telescope.

Evaluate *Why is an inverted image acceptable for viewing astronomical objects?*

Get It?
Explain how a refracting telescope works.

Cameras

Figure 37 shows the optical system used in a single-lens reflex camera. As light enters the camera through the aperture, it passes through an achromatic lens. This lens system refracts the light much like a single convex lens would, forming an image that is inverted on the reflex mirror. The image is reflected upward to a prism that inverts and redirects the light to the viewfinder.

When the person holding the camera takes a photograph, he or she presses the shutter-release button, which briefly raises the mirror, as shown in **Figure 37.** The light, instead of being diverted upward to the prism, then travels along a straight path to form an image on the sensor.

The image sensor, which is often a charge-coupled device (CCD), captures a two-dimensional image that corresponds to the image projected onto it. The image information is collected on the photoactive region of the CCD as electric charge proportional to the light intensity. The CCD then processes the charge and transfers the information to a storage device.

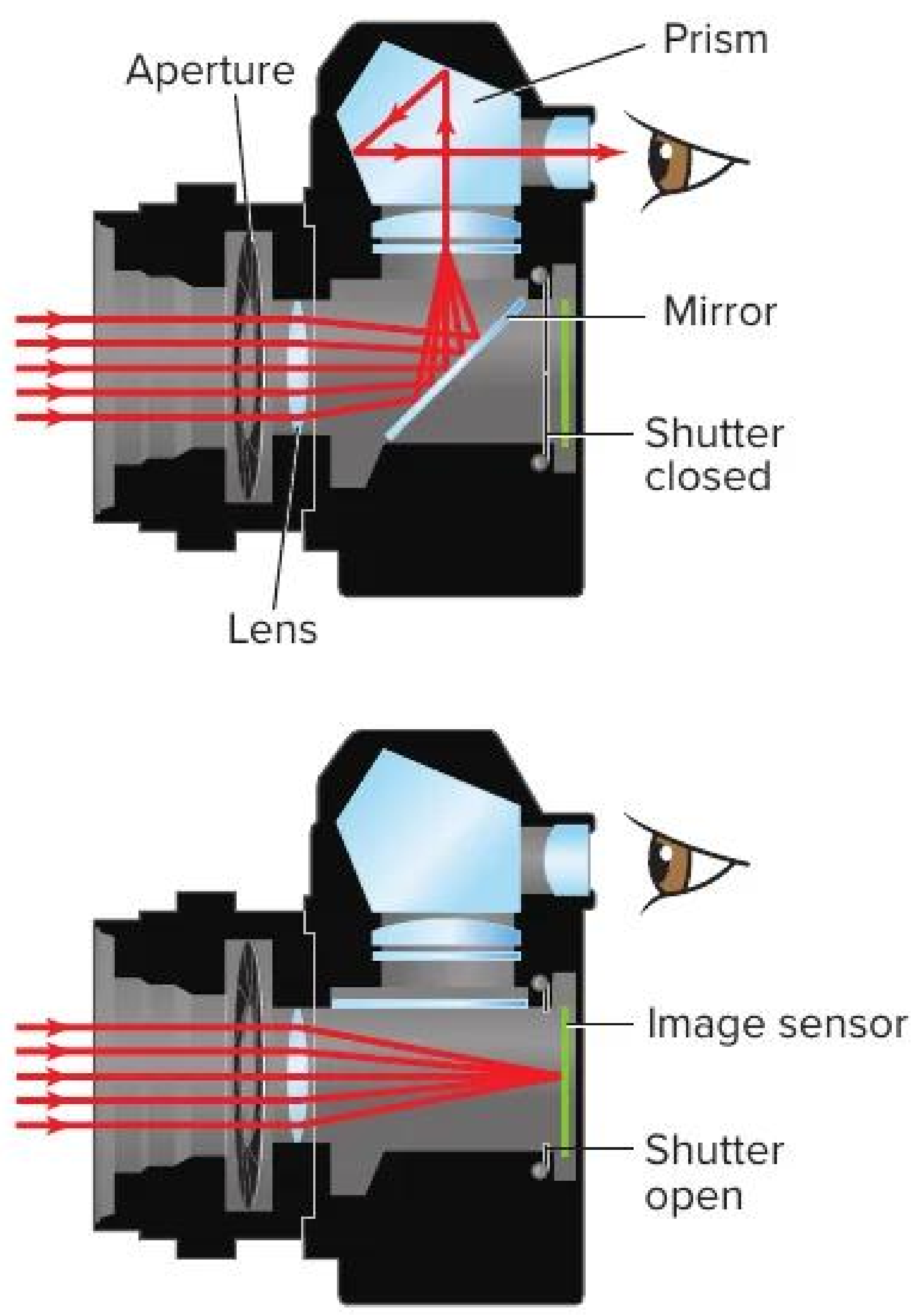


Figure 37 The lens of a camera can be adjusted to focus an image on the image sensor. When the shutter is closed, a mirror diverts the image to the viewer's eye.

Infer *Why would you want the shutter to be open longer when taking a photo in dim light?*

Check Your Progress

49. **Lenses** Magnifying lenses normally are used to produce images that are larger than the related objects, but they also can produce images that are smaller than the related objects. Explain.
50. **Types of Lenses** The cross sections of four different thin lenses are shown in **Figure 38.**
    a. Which of these lenses, if any, are convex, or converging, lenses?
    b. Which of these lenses, if any, are concave, or diverging, lenses?

Figure 38

51. **Image Position and Height** An object is placed 1.5 m from a convex lens with a focal length of 1.0 m. Use the thin lens equation to determine the distance of the image from the lens. If the object height is 2.0 m, what is the image height? Is the image real or virtual? Is the image inverted or upright?
52. **Thin Lens Approximation** What is a thin lens approximation and why is it used?
53. **Chromatic Aberration** All simple lenses have chromatic aberration. Infer why you do not see this effect when you look through a microscope, which has two convex lenses.
54. **Type of Image** Use the ray diagram in **Figure 39** to determine whether the image for object 1 will be reduced or enlarged, inverted or upright, and real or virtual. Do the same for object 2.

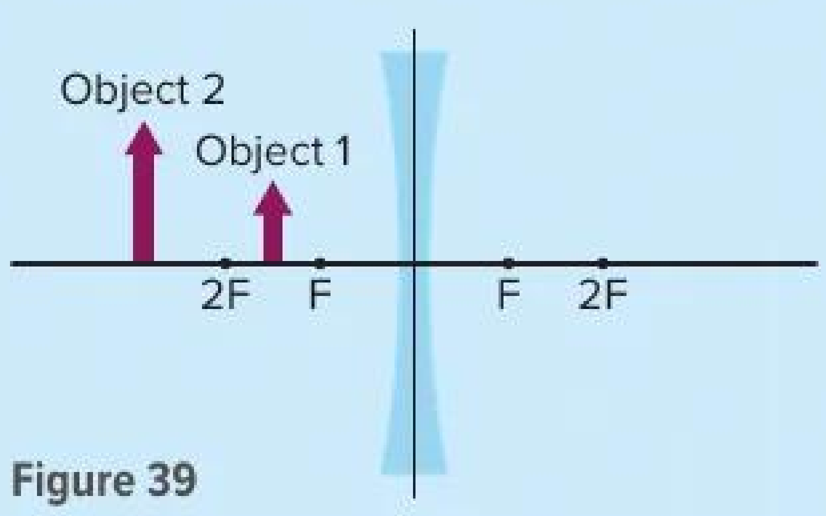


Figure 39

55. **Image Position and Height** A 6.0-cm-tall object is placed 5.0 cm from a convex lens with a focal length of 4.0 cm. Draw a ray diagram to determine the image location and size. Confirm your results using the thin lens equation and the magnification equation.
56. **Diverging Lens** A 6.5-cm-tall salt shaker is viewed through a diverging lens with a focal length of 5.0 cm.
    a. If the shaker is 6.0 cm from the lens, what is the image distance from the lens? Is the image virtual or real?
    b. What is the magnification? Is the image smaller or larger than the object?
    c. The salt shaker is now 4 cm from the lens. What is the distance of the image from the lens? The magnification? Is the image now smaller or larger than the object?
57. **Eyeglass Lenses** Which type of lens, convex or concave, should a nearsighted person use? Which type should a farsighted person use? See **Figure 40.** Explain.

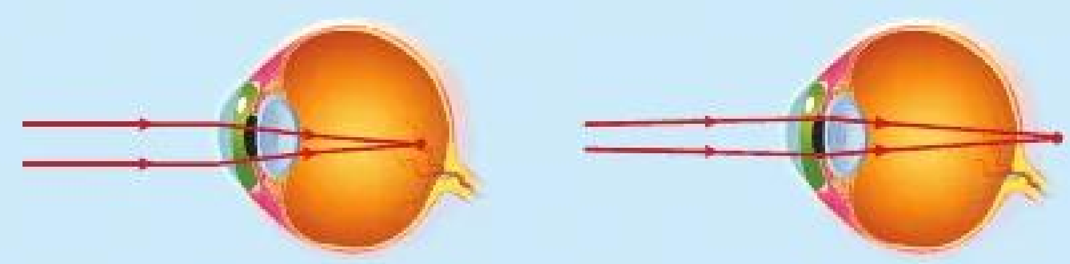


Figure 40

58. **Critical Thinking** An air lens constructed of two watch glasses is placed in a tank of water. Copy **Figure 41** and draw the effect of this lens on parallel light rays incident on the lens.

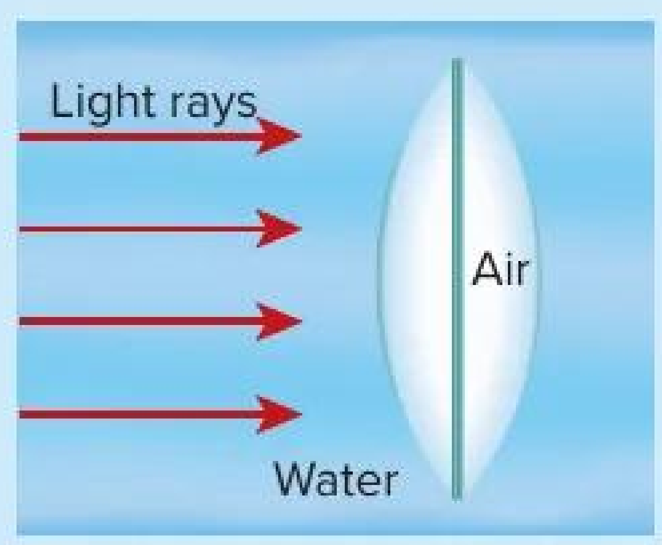


Figure 41

LEARNSMART® Go online to follow your personalized learning path to review, practice, and reinforce your understanding.

ENGINEERING & TECHNOLOGY

Scientists Hope the James Webb Space Telescope Will Help Them See the Light

Expected to launch in 2020, the James Webb Space Telescope (JWST) has been designed to produce images of deep space objects. Its ability to do so relies heavily on how its state of the art mirror captures and reflects light.

The ability of the JWST to produce images of deep space objects is largely due to the construction of its primary mirror.

Building a Better Telescope

The way in which backyard reflecting telescopes produce images is fairly straightforward. A concave primary mirror captures light from an object. This light is then reflected by another mirror to produce an image at a focal point. The larger the primary mirror, the more light it can capture. Many backyard reflecting telescopes used by amateur astronomers have a primary mirror that measures about 25 cm in diameter.

The primary mirror on the JWST measures 6.5 m in diameter. Its diameter not only dwarfs that of the average backyard telescope, but it's also much larger than that of two existing space telescopes. The diameter of the *Hubble Space Telescope*'s primary mirror, which has been in use since 1990, is 2.4 m. The telescope that has orbited Earth in the *Herschel Space Observatory* since 2009 has a primary mirror with a diameter of 3.5 m.

Scientists think that the JWST will be able to see some of the oldest objects in the universe. Some of the universe's oldest galaxies are over 13 billion light years away. At such a distance, the light that reaches Earth is faint. The JWST will be able to capture more of this light. The JWST will collect data in the infrared to see processes and objects not seen in visible light.

Unlike the *Hubble* and *Herschel* telescopes, the JWST will not be in Earth's orbit. It will be set about 1.5 million kilometers from Earth. Its primary mirror is too massive to launch in one piece, so it is constructed out of 18 smaller, six-sided mirrors made out of lightweight gold-plated beryllium. The components will fold up tightly for launch and unfold once in orbit. Each individual mirror can also be controlled to ensure that the pieces work together as a single large mirror and that light is reflected from the mirror accurately to produce an image that's not distorted.

Scientists are hopeful that the images from the JWST will provide them with more clues about the origins of the universe.

EVALUATE DESIGN SOLUTIONS

NASA officials have stated that the JWST has been designed to succeed, but not replace, other existing space telescopes. Compare the JWST to another telescope, giving an example of a space object that can be visualized using each.

STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 REFLECTION OF LIGHT

- The law of reflection says that the angle of incidence equals the angle of reflection, and applies to specular and diffuse reflection.

$$\theta_i = \theta_r$$

- Plane mirrors form virtual images, which are the same size, orientation, and distance from the mirror as the object.

$$x_i = -x_o; \quad h_i = h_o$$

- specular reflection
- diffuse reflection
- plane mirror
- object
- image
- virtual image

Lesson 2 CURVED MIRRORS

- A spherical concave mirror is shaped as if it were a section of a hollow sphere with the same geometric center (C) and radius of curvature (r) as a sphere of radius r. The focal point (F) is the point where rays parallel to the principal axis converge after reflection.
- You can locate the image created by a curved mirror by drawing ray diagrams. The type of image formed by a concave mirror depends on the object's position. A convex mirror always forms a virtual image that is upright and smaller compared to the object.
- The magnification of a mirror image is given by equations relating either the positions or the heights of the image and the object.

$$m \equiv \frac{h_i}{h_o} = -\frac{x_i}{x_o}$$

- The mirror equation gives the relationship between image position, object position, and focal length of a spherical mirror.

$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$$

- principal axis
- focal point
- focal length
- concave mirror
- convex mirror
- magnification
- real image
- spherical aberration

Lesson 3 REFRACTION OF LIGHT

- Light refracts when it travels across a boundary from one medium with an index of refraction (n_1) into a medium with a different index of refraction (n_2) according to Snell's law.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

- The ratio of the speed of light in a vacuum (c) to the speed of light in a medium (v) is the index of refraction (n) of the medium.
- When light traveling through a medium hits a boundary with a medium of a smaller index of refraction, if the angle of incidence exceeds the critical angle (θ_c), the light will be reflected back into the original medium by total internal reflection.
- Refraction causes optical effects such as mirages and rainbows.

- index of refraction
- critical angle
- total internal reflection
- dispersion

Lesson 4 CONVEX AND CONCAVE LENSES

- The image formed by a convex lens depends on the position of the object. The image formed by a concave lens is always virtual.
- The thin lens equation provides the relationship between focal length (f), object position (x_o), and image position (x_i).

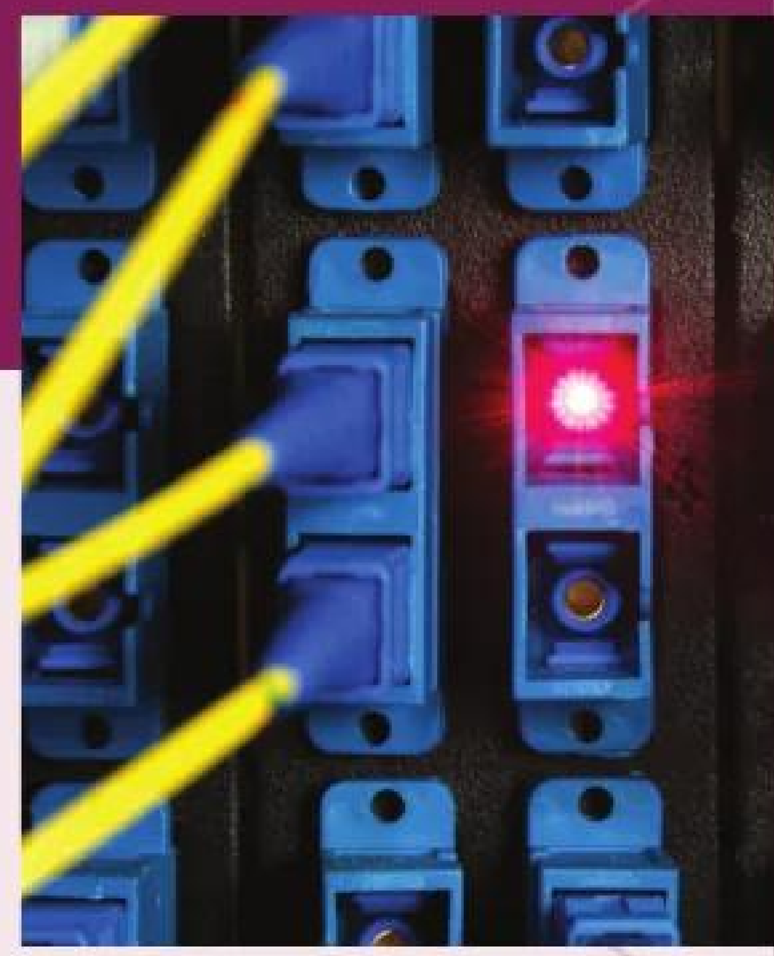
$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$$

- Concave and convex lenses correct eye conditions such as

- lens
- convex lens
- concave lens
- thin lens equation
- chromatic aberration
- achromatic lens
- nearsightedness
- farsightedness

THREE-DIMENSIONAL THINKING

Module Wrap-Up



REVISIT THE PHENOMENON

How does light transmit information through a communication network?

CER Claim, Evidence, Reasoning

Explain your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

Which material should you use for fiber optic cables?

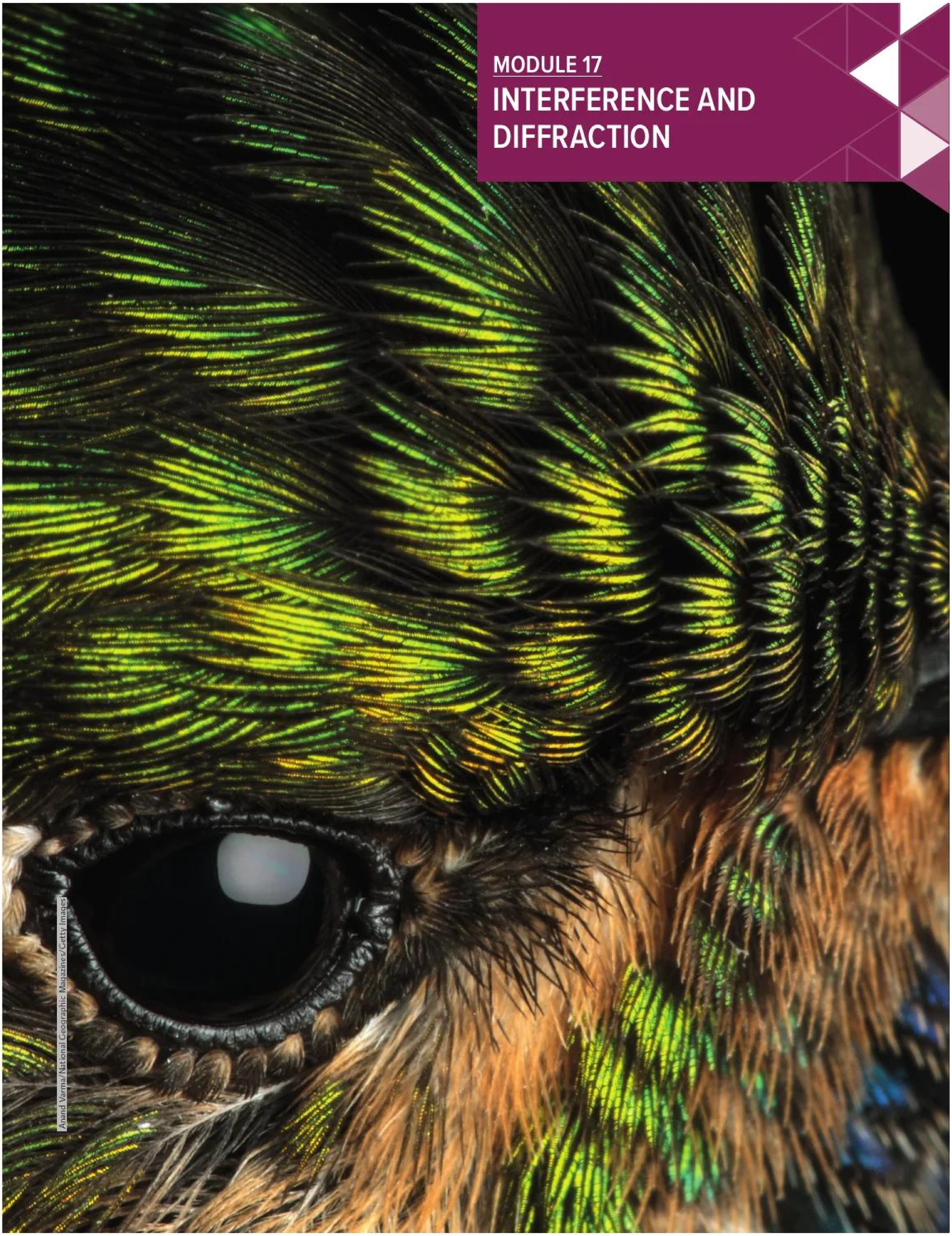
You are planning a colony on a very cold, icy planet. The engineer in charge of communications suggests that instead of hauling glass fiber optic cables from Earth, the expedition should make them from ice once they arrive on the planet.

CER Analyze and Interpret Data

1. Find the critical angle for light traveling from ice ($n = 1.31$) to air.
2. **Claim** Would fiber optic cables made of ice or those made of glass do a better job of keeping light inside the cable?
3. **Evidence and Reasoning** Use your calculations and what you know about refraction to support and explain your claims.

MODULE 17

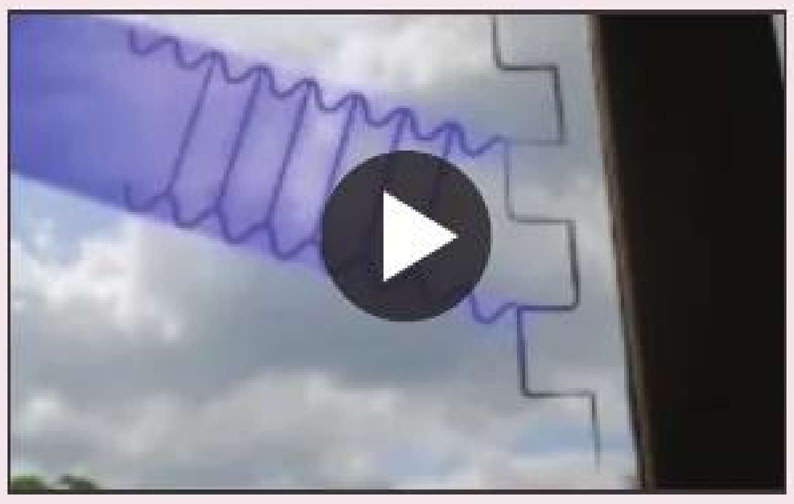
INTERFERENCE AND DIFFRACTION



MODULE 17
INTERFERENCE AND DIFFRACTION

ENCOUNTER THE PHENOMENON

What makes this hummingbird's feathers appear shiny and shimmery?



GO ONLINE to play a video about iridescence in nature.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about why the hummingbird's feathers are shiny and shimmery. Explain your reasoning.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.

LESSON 1: Explore & Explain: Thin-Film Interference

LESSON 2: Explore & Explain: Diffraction Gratings

Smithsonian

Additional Resources

LESSON 1

INTERFERENCE

FOCUS QUESTION

How do bubbles produce a rainbow effect?

Incoherent and Coherent Light

As you know, light has properties of a wave. Light diffracts as it passes an edge. When studying mirrors and lenses, you learned that reflection and refraction can be explained when light is modeled as a wave. When scientists discovered that light could be made to interfere, which results from the superposition of waves, they concluded that light has wave properties.

Incoherent light When you look at objects that are illuminated by a white light source such as a lightbulb, you are seeing **incoherent light,** which is light whose waves are not in phase. The effect of incoherence in waves can be seen in the example of heavy rain falling on still water. The surface of the water is choppy and does not have a regular pattern of waves, as shown in **Figure 1.** Because light waves have such a high frequency, incoherent light does not appear choppy to you. Instead, as light from an incoherent white light source illuminates an object, you see the combination of the incoherent light waves as an even, white light.

Coherent light Light made up of waves of the same wavelength that are in phase with each other is **coherent light.** A regular wavefront, which is made of coherent light, can be created by a single point source, as shown in **Figure 1.**

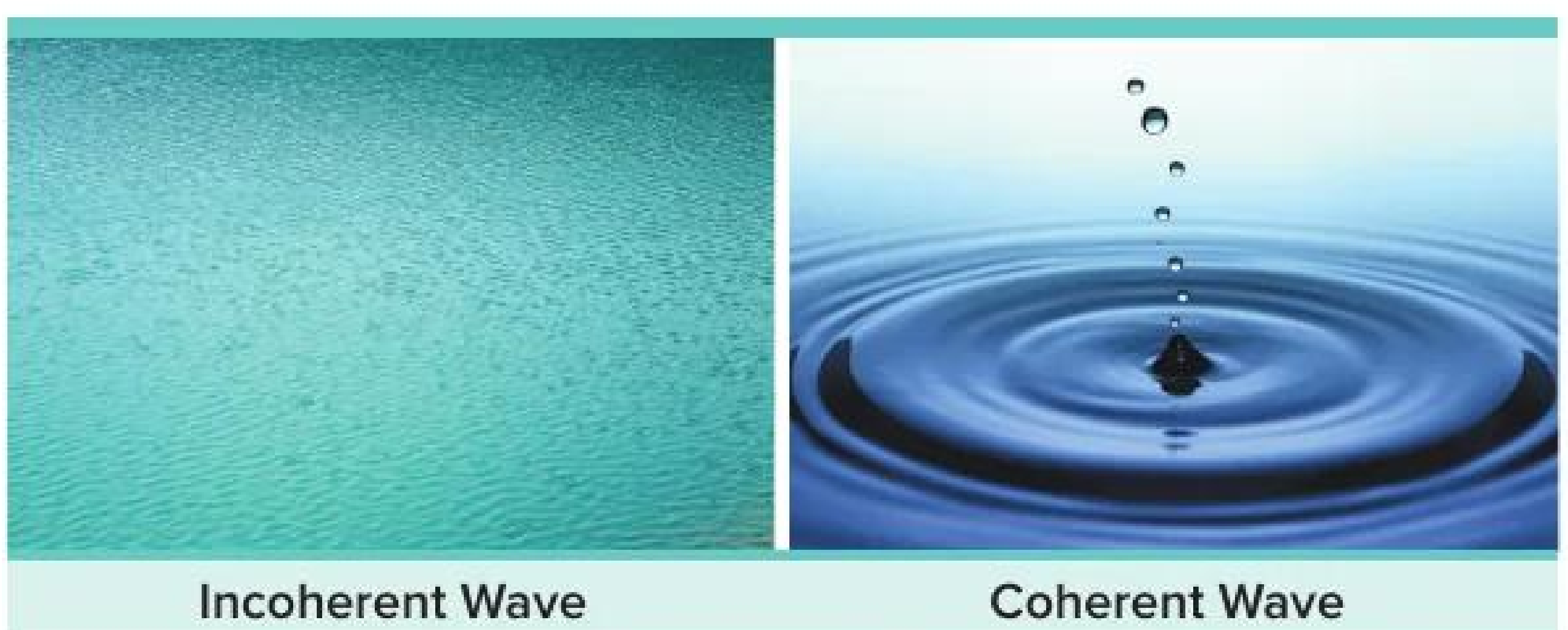


Figure 1 Choppy, irregular wave patterns model incoherent light. Regular wave patterns model coherent light.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Quick Investigation: Soap Film
Carry out an investigation to discover the patterns produced when light shines on a soap film.

PhysicsLAB: Double-Slit Interference
Calculate the wavelength of light using a doublt-slit interference pattern.

A regular wavefront also can be created by multiple point sources when all point sources are in phase. This type of coherent light is produced by a laser.

Interference of Coherent Light

Between 1801 and 1803, English physician Thomas Young, **Figure 2,** performed a number of investigations establishing the wave properties of light. In a crucial investigation attributed to Young, light from a small source that was passed through two closely spaced slits produced an interference pattern.

Figure 2 Thomas Young (1773–1829) is famous for his contributions in many different subject areas. Along with his role in establishing the wave nature of light, he is known for his work deciphering Egyptian hieroglyphics.

Young selected light from a tiny region of a source and made it coherent by passing it through a single, narrow slit. The light was then passed through two closely spaced narrow slits in a barrier. The overlapping light from the two slits fell on an observing screen. The overlap created a pattern of bright and dark bands called **interference fringes.** Young explained that the bands resulted from constructive and destructive interference of light waves from the two slits in the barrier.

Consider **monochromatic light,** which is light of only one wavelength. In a double-slit interference investigation that uses monochromatic light, constructive interference produces a bright central band of the given color on the screen, as well as other bright bands of near-equal spacing and near-equal width on either side, as shown in **Figure 3.** The intensity of the bright bands decreases the farther the band is from the central band, as you can see. Between the bright bands are dark areas where destructive interference occurs. The positions of the constructive and destructive interference bands depend on the light's wavelength.

When white light is used in a double-slit investigation, however, interference causes the appearance of colored spectra, as shown on the right in **Figure 3.** The various bands of color from the visible spectrum overlap on the screen. All these colors have constructive interference, and the central band is white. Because the positions of the other bright bands of constructive interference depend on wavelength, each color's band is at a different position, resulting in spectra of color.

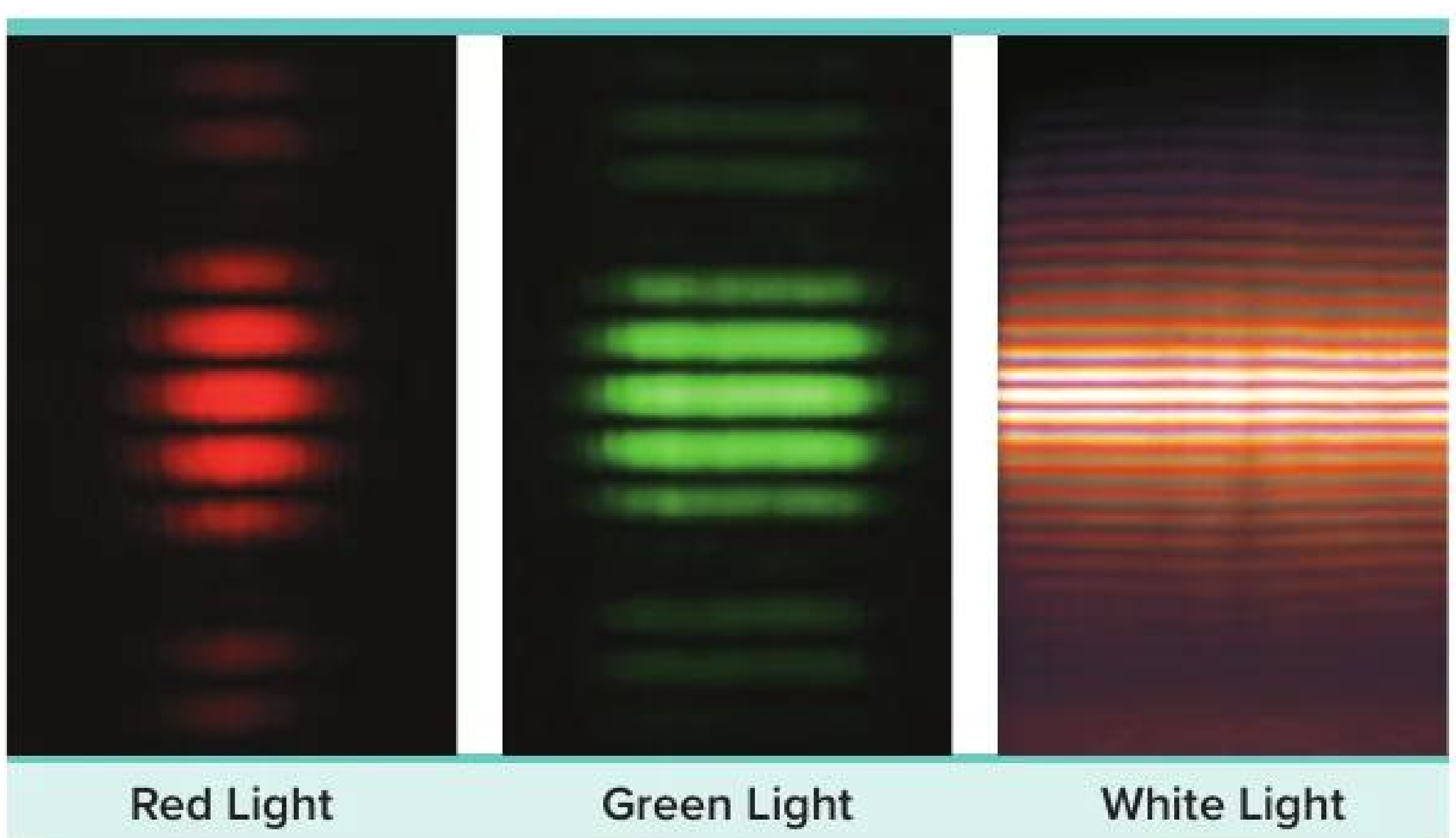


Figure 3 Double-slit interference patterns show a bright central band with a pattern of dark and bright bands on either side.

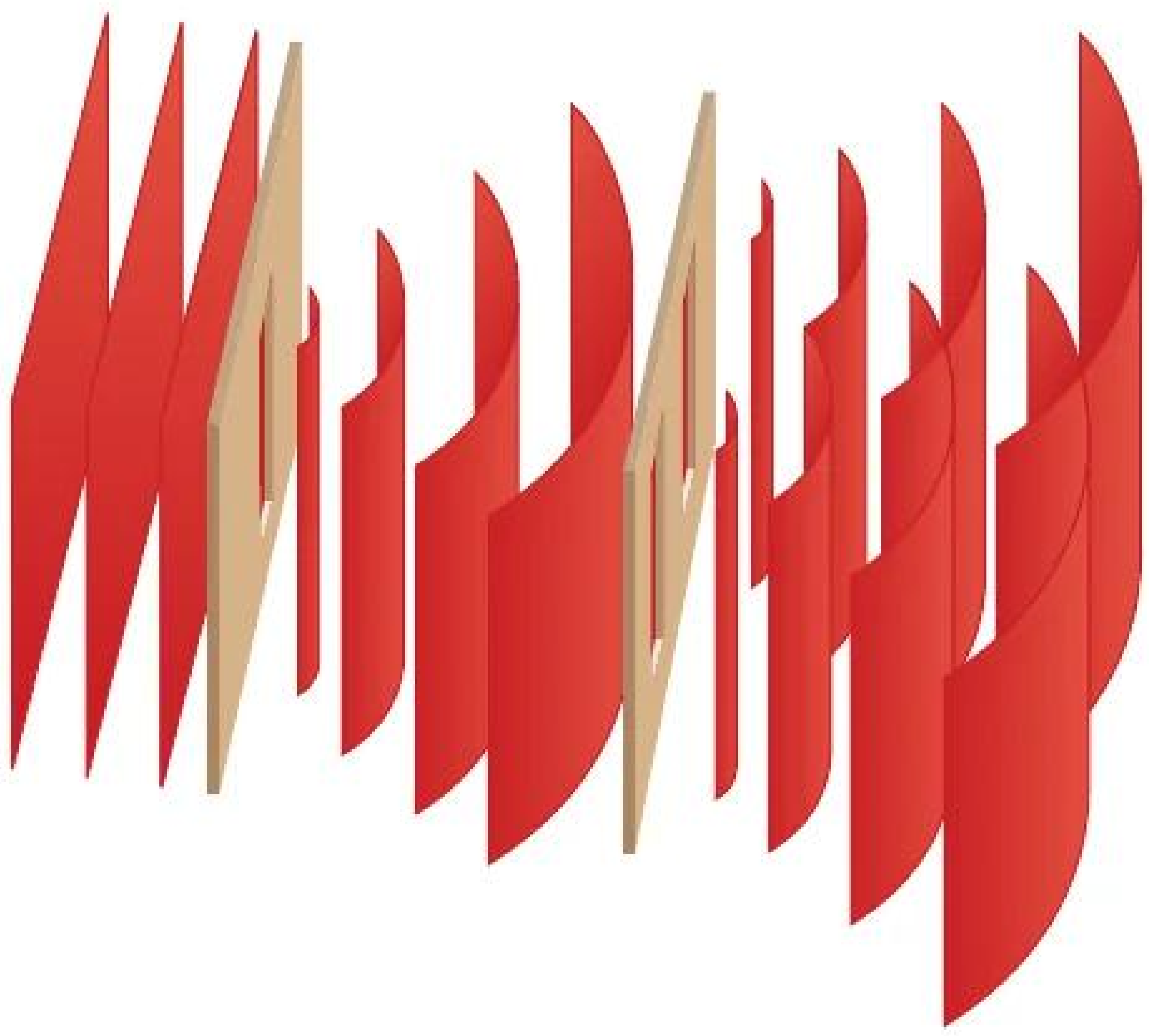
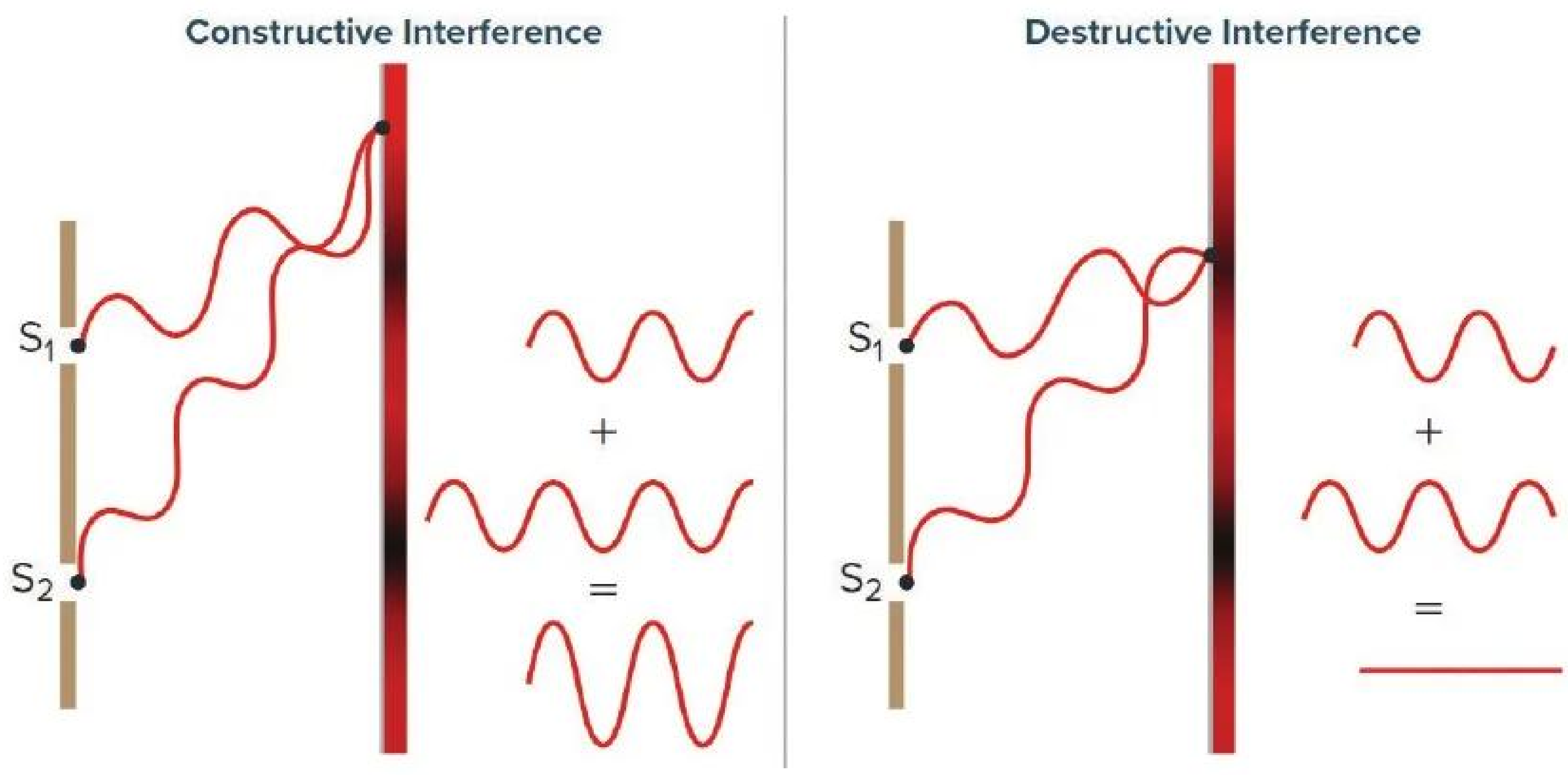


Figure 4 Nearly cylindrical wavefronts are generated as light passes through the slits.

Consider *why the width of the slits is tens to hundreds of light wavelengths.*

Generation of coherent light Light from a monochromatic source produces incoherent light. Placing a light barrier with a narrow slit in front of the monochromatic light produces coherent light. Because the width of the slit is very small, only light from a tiny region of the source passes through the slit. Diffraction by the slit produces nearly cylindrical wavefronts, as shown in **Figure 4.**

The second barrier has two very small slits. Because a cylinder is symmetrical, the two portions of the wavefront arriving at the second barrier are in phase. The two slits at the second barrier produce nearly cylindrical wavefronts. These two wavefronts can then interfere, as shown in **Figure 4.** Depending on their phase relationship, the two waves undergo constructive or destructive interference, as shown in **Figure 5.** If the interference is constructive when the light hits a screen, you will see a bright band. If it is destructive, you will see a dark band.



At the points where the waves experience constructive interference, bright bands are seen.

At the points where the waves experience destructive interference, dark bands are seen.

Figure 5 Coherent waves can undergo constructive or destructive interference.

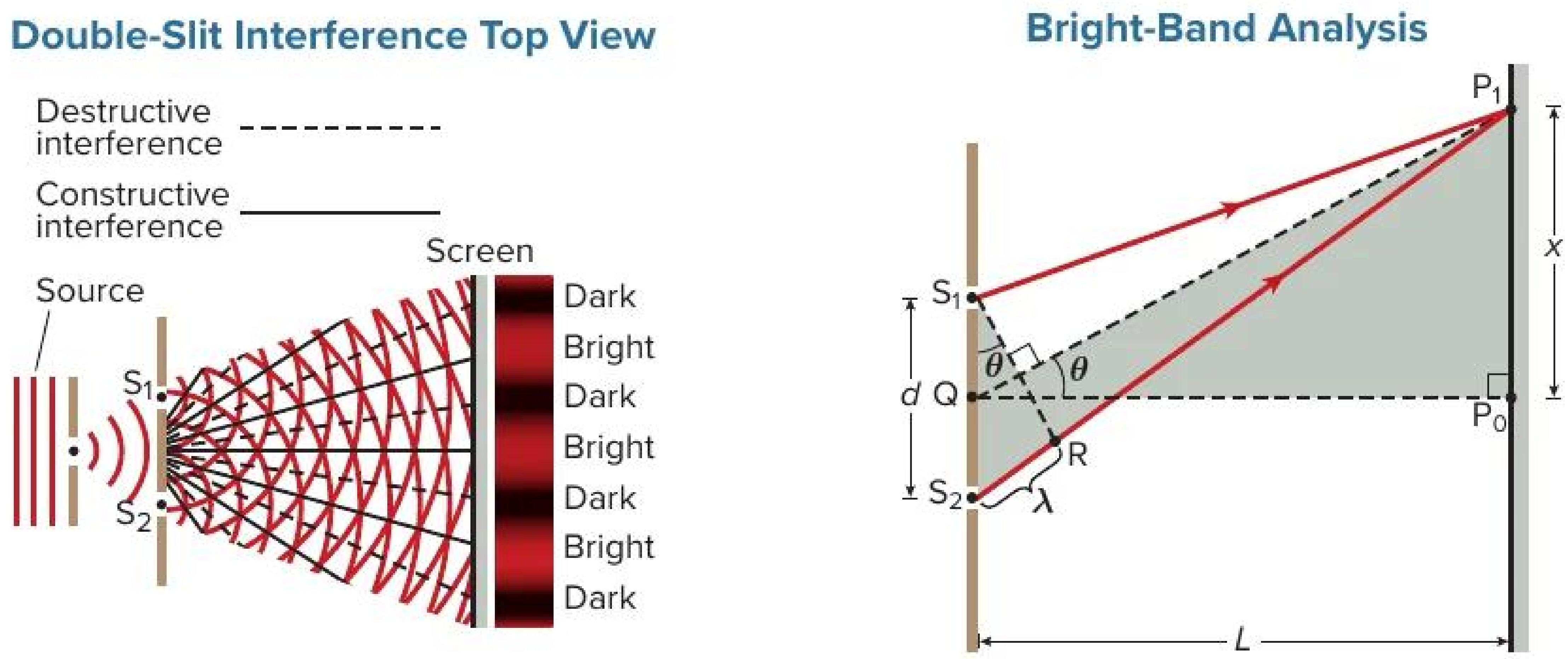


Figure 6 Double-slit interference can be used to determine the wavelength of light. Because L is much larger than d and the angle θ is small, the equation for the wavelength is simplified.

Double-Slit Interference

A top view of the double-slit investigation is shown on the left in **Figure 6.** The wavefronts interfere constructively and destructively to form a pattern of light and dark bands. The right side shows that light that reaches point P_0 travels the same distance from each slit. Because the waves are in phase, they interfere constructively on the screen to create the central bright band at P_0. There is also constructive interference at the first bright band (P_1) on either side of the central band because line segment P_1S_2 is one wavelength (λ) longer than the line segment P_1S_1. Thus, the waves arrive at P_1 in phase.

There are two triangles shaded on the right side. The larger triangle is a right triangle, so $\tan\theta = x/L$. In the smaller triangle ΔRS_1S_2, the side $\overline{S_2R}$ is the length difference of the two light paths, which is one wavelength. There are now two simplifications for wavelength calculations.

1. If L is much larger than d, then line segments S_1P_1 and S_2P_1 are nearly parallel to each other and to line segment QP_1, and ΔRS_1S_2 is very nearly a right triangle. Thus, $\sin\theta \approx \lambda/d$.
2. If the angle θ is small, then $\sin\theta$ is very nearly equal to $\tan\theta$.

With the above simplifications, the relationships $\tan\theta = x/L$, $\sin\theta \approx \lambda/d$, and $\sin\theta \approx \tan\theta$ combine to form the equation $x/L = \lambda/d$. Solving for λ gives the following.

Wavelength from Double-Slit Investigation

The wavelength of light, as measured by a double slit, is equal to the distance on the screen from the central bright band to the first bright band (x), multiplied by the distance between the slits (d), divided by the distance from the slits to the screen (L).

$$\lambda = \frac{xd}{L}$$

Constructive interference occurs at locations x_m on either side of the central bright band, which are determined by the equation $m\lambda = x_m d/L$, where $m = 0$, 1, 2, etc. The central bright band occurs at $m = 0$. The band located where $m = 1$ is often called the first-order band, $m = 2$ the second-order band, and so on.

STEM CAREER Connection

Optician

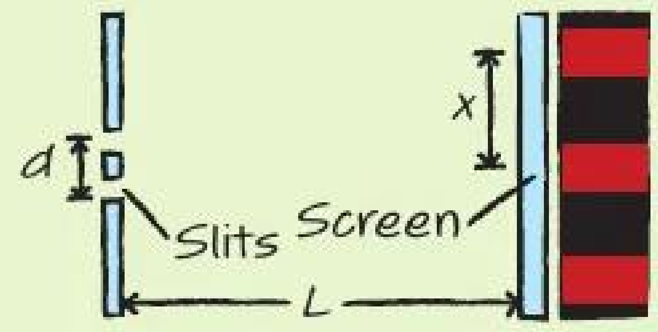
Opticians understand how thin films can be applied to eyeglasses to reduce glare. With their understanding of light and lenses, they check patients' needs and prescriptions for appropriateness. They can work in a variety of offices, such as with an optometrist, a store that sells eyeglasses and contact lenses, or within a department store to help fit people with visual aids, such as eyeglasses and contact lenses.

EXAMPLE Problem 1

WAVELENGTH OF LIGHT A double-slit investigation is performed to measure the wavelength of red light. The slits are 0.0190 mm apart. A screen is placed 0.600 m away, and the first-order bright band is 21.1 mm from the central bright band. What is the wavelength of the red light?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the investigation, showing the slits and the screen.
- Draw the interference pattern with bands in appropriate locations.



Known	Unknown
$d = 1.90\times10^{-5}$ m	$\lambda = ?$
$x = 2.11\times10^{-2}$ m	
$L = 0.600$ m	

2 SOLVE FOR THE UNKNOWN

$$\lambda = \frac{xd}{L}$$

$$= \frac{(2.11\times10^{-2}\text{ m})(1.90\times10^{-5}\text{ m})}{(0.600\text{ m})}$$ Substitute $x = 2.11\times10^{-2}$ m, $d = 1.90\times10^{-5}$ m, $L = 0.600$ m.

$$= 6.68\times10^{-7}\text{ m} = 668\text{ nm}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** The answer is in units of length, which is correct for wavelength.
- **Is the magnitude realistic?** The wavelength range of red light is about 600 nm to 700 nm. Thus, the answer is reasonable for red light.

PRACTICE Problems

ADDITIONAL PRACTICE

1. Violet light falls on two slits separated by 1.90×10^{-5} m. A first-order bright band appears 13.2 mm from the central bright band on a screen 0.600 m from the slits. What is λ?
2. Yellow-orange light of wavelength 596 nm from a sodium lamp is aimed at two slits that are separated by 1.90×10^{-5} m. What is the distance from the central band to the first-order yellow band if the screen is 0.600 m from the slits?
3. In a double-slit investigation, physics students use a laser with $\lambda = 632.8$ nm. A student places the screen 1.000 m from the slits and finds the first-order bright band 65.5 mm from the central line. What is the slit separation?
4. **CHALLENGE** Yellow-orange light with a wavelength of 596 nm passes through two slits that are separated by 2.25×10^{-5} m and makes an interference pattern on a screen. If the distance from the central line to the first-order yellow band is 2.00×10^{-2} m, how far is the screen from the slits?

Young's findings faced opposition from most physicists because they were supporters of Newton's particle model of light. However, in a competition Jean Fresnel proposed a mathematical solution for the wave nature of light. One of the judges, Siméon Denis Poisson, showed that if Fresnel was correct, the shadow of a circular object illuminated with coherent light would have a bright spot at its center. Jean Arago did the investigation and saw the spot, convincing many of the wave nature of light.

Get It?

Describe how the pattern on the hummingbird at the beginning of this module formed.

Thin-Film Interference

Have you ever seen a spectrum of colors produced by a soap bubble or by the oily film on a water puddle in a parking lot, as in **Figure 7?** These colors are not the result of separation of white light by a prism or of absorption by a pigment. The colors are a result of the constructive and destructive interference when light waves reflect from separate surfaces of a thin film, a phenomenon called **thin-film interference.**

In a vertical soap film, like the one in **Figure 8,** the film's weight makes it thicker at the bottom than at the top. The thickness increases gradually from top to bottom. When a light wave strikes the front surface of the film, it is partially reflected, as shown by ray 1 in **Figure 8,** and partially transmitted. The reflected and transmitted waves have the same frequency as the original.

The transmitted wave travels through the film to the back surface where, again, part is reflected, as shown by ray 2. The splitting continues as the light makes many passes through the film. Because matched sets of waves came from the same source, they are coherent.

Color reinforcement How is the reflection of one color enhanced? This happens when the two reflected waves are in phase for a given wavelength. If the thickness of the soap film in **Figure 8** is one-fourth the wavelength of the light in the film ($\lambda/4$), then the round-trip path length in the film is $\lambda/2$. In this case, you might expect that ray 2 would return to the front surface one-half wavelength out of phase with ray 1 and that the two waves would cancel each other based on the superposition principle.

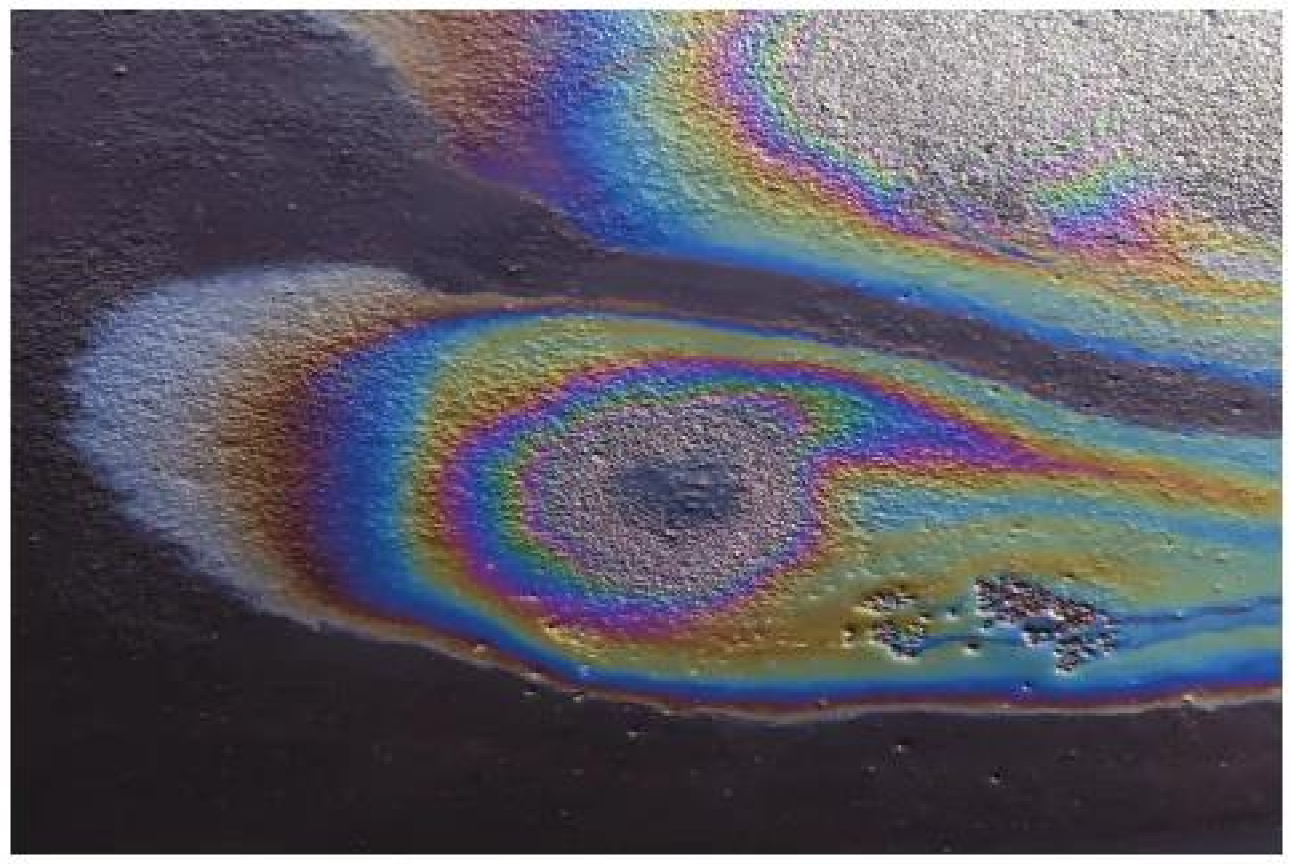


Figure 7 The swirl of colors seen in an oily film is the result of thin-film interference.

But when a transverse wave is reflected from a medium in which its speed is slower, the wave is inverted. With light, this happens at the boundary of a medium with a larger index of refraction. As a result, ray 1 is inverted on reflection, whereas ray 2 is reflected from a medium with a smaller index of refraction (air) and is not inverted. Thus, ray 1 and ray 2 are in phase.

If the film thickness (d) satisfies the requirement $d = \lambda/4$, then the color of light with that wavelength will be most strongly reflected. Note that because the wavelength of light in the film is shorter than the wavelength in air, $d = \lambda_{film}/4$, or, in terms of the wavelength in air, $d = \lambda_{vacuum}/4n_{film}$. The two waves reinforce each other as they leave the film. Light with other wavelengths undergoes destructive interference.

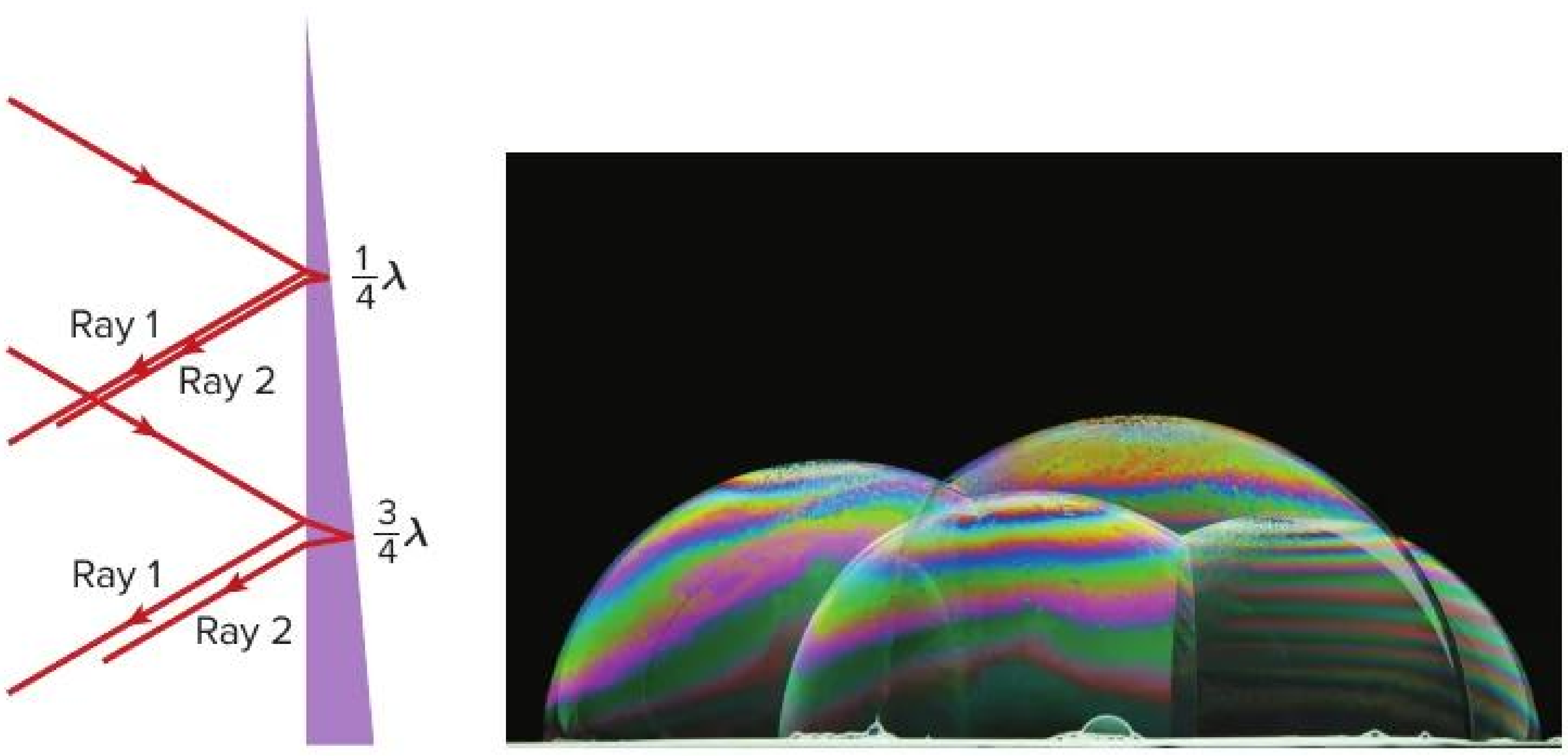


Figure 8 At soap film thicknesses of $\lambda/4$, $3\lambda/4$, $5\lambda/4$, etc., light with wavelength λ is in phase. Bands of that color of light are visible at those thicknesses.

As you know, different colors of light have different wavelengths. For a film of varying thickness, the wavelength requirement will be met at different thicknesses for different colors. The result is a rainbow of color. Where the film is too thin to produce constructive interference for any wavelength of visible light, the film appears to be black. Notice in **Figure 8** that the pattern of colors that appear on the film repeats. When the thickness of the film is $3\lambda/4$, the round-trip distance is $3\lambda/2$, and constructive interference occurs for light with a wavelength λ again. Any thickness equal to $1\lambda/4$, $3\lambda/4$, $5\lambda/4$, and so on, satisfies the conditions for constructive interference for a given wavelength.

Real-World Physics

A THIN FILM can be placed on the lenses of eyeglasses to keep them from reflecting wavelengths of light that are highly visible to the human eye. This reduces the glare of reflected light and permits more light to be transmitted.

Refractive indexes determine wave inversion The example of a film of soapy water in air involves constructive interference with one of two waves inverted upon reflection. In the example of a bubble solution or thin film of oil on a puddle of water, as the thickness of the film or the angle the light makes with the film changes, the wavelength undergoing constructive interference changes. This creates a shifting color on the surface of the film when it is under white light.

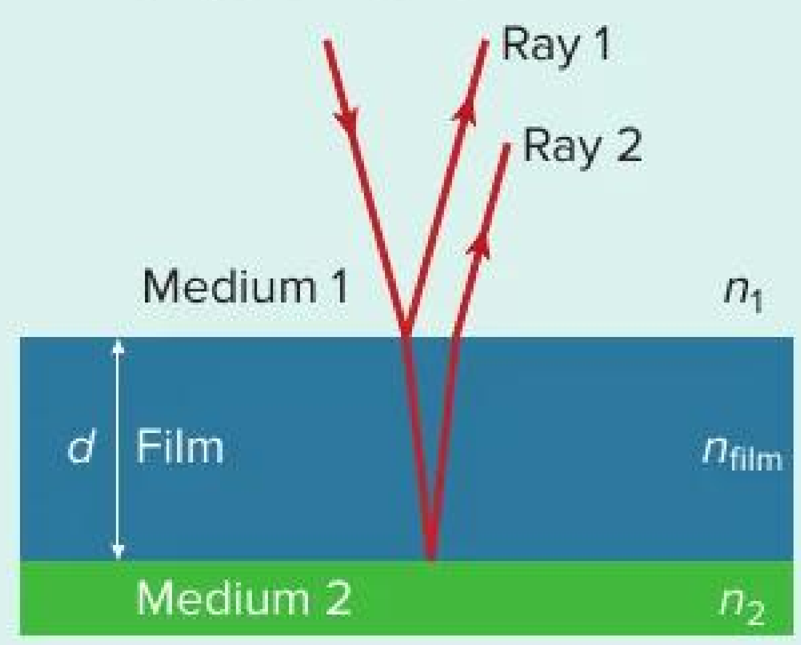
In other examples of thin-film interference, neither wave or both waves might be inverted. Whether a wave is inverted depends on the indices of refraction of the mediums involved. If both waves are traveling from a lower to a higher index of refraction, they will both be inverted. In this case, the film thicknesses for constructive interference are $1\lambda/2$, λ, $3\lambda/2$, 2λ, $5\lambda/2$, and so on. You can develop a solution for any problem involving thin-film interference by using the following strategy.

PROBLEM-SOLVING STRATEGY

Thin-Film Interference

When solving thin-film interference problems, construct an equation that is specific to the problem by using the following strategy.

Reflection from a Thin Film



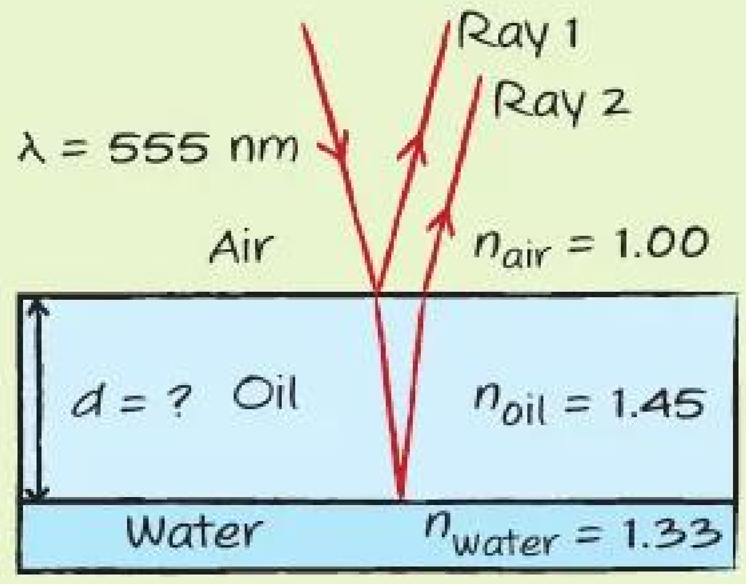
1. Make a sketch of the thin film and the two coherent waves. For simplicity, draw the waves as rays.
2. Read the problem. Is the reflected light of this wavelength brightened or dimmed? When it is bright, the two reflected waves undergo constructive interference. When the reflected light is dimmed, the waves undergo destructive interference.
3. Are either or both waves inverted on reflection? If the index of refraction changes from a lower to a higher value, then the wave is inverted. If it changes from a higher to a lower value, there is no inversion.
4. Find the extra distance the second wave must travel through the thin film to create the needed interference.
   a. If you need constructive interference and one wave is inverted OR you need destructive interference and either both waves or none are inverted, then the difference in distance is an odd number of half wavelengths: $(m + 1/2)\lambda_{film}$, where $m = 0, 1, 2$, etc.
   b. If you need constructive interference and either both waves or none are inverted OR you need destructive interference and one wave is inverted, then the difference is an integer number of wavelengths: $m\lambda_{film}$, where $m = 1, 2, 3$, etc.
5. Set the extra distance traveled by the second ray to twice the film thickness, $2d$.
6. Recall from studying refraction that $\lambda_{film} = \lambda_{vacuum}/n_{film}$.

EXAMPLE Problem 2

OIL AND WATER You observe colored rings on a puddle and conclude that there must be an oil slick on the water. You look directly down at the puddle and see a yellow-green ($\lambda = 555$ nm) region. If the index of refraction of oil is 1.45 and that of water is 1.33, what is the minimum thickness of oil that could cause this color?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the thin film and layers above and below it.
- Draw rays showing reflection off the top of the film as well as the bottom.



Known

$n_{water} = 1.33$

$n_{oil} = 1.45$

$\lambda = 555$ nm

Unknown

$d = ?$

2 SOLVE FOR THE UNKNOWN

Because $n_{oil} > n_{air}$, the wave is inverted on the first reflection. Because $n_{water} < n_{oil}$, there is no inversion on the second reflection. Thus, there is one wave inversion. The wavelength in oil is less than it is in air.

Follow the problem-solving strategy to construct the equation.

$$2d = \left(m + \frac{1}{2}\right)\left(\frac{\lambda}{n_{oil}}\right)$$

Because you want the minimum thickness, $m = 0$.

$$d = \frac{\lambda}{4n_{oil}}$$

$$= \frac{555\ \text{nm}}{(4)(1.45)}$$ Substitute $\lambda = 555$ nm, $n_{oil} = 1.45$.

$$= 95.7\ \text{nm}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** The answer is in nanometers, which is correct for thickness.
- **Is the magnitude realistic?** The minimum thickness is smaller than one wavelength, which is what it should be.

PRACTICE Problems

ADDITIONAL PRACTICE

5. In the situation in Example Problem 2, what would be the thinnest film that would create a reflected red ($\lambda = 635$ nm) band?
6. A glass lens has a nonreflective coating of magnesium fluoride placed on it. How thick should the nonreflective layer be to keep yellow-green light with a wavelength of 555 nm from being reflected? See **Figure 9.**
7. You can observe thin-film interference by dipping a bubble wand into some bubble solution and holding the wand in the air. What is the thickness of the thinnest soap film at which you would see a black stripe if the light illuminating the film has a wavelength of 521 nm? Use $n = 1.33$ for the bubble solution.
8. What is the thinnest soap film ($n = 1.33$) for which light of wavelength 521 nm will constructively interfere with itself?
9. **CHALLENGE** A silicon solar cell has a nonreflective coating placed on it. If a film of silicon monoxide, $n = 1.45$, is placed on the silicon, $n = 3.5$, how thick should the layer be to keep yellow-green light ($\lambda = 555$ nm) from being reflected?

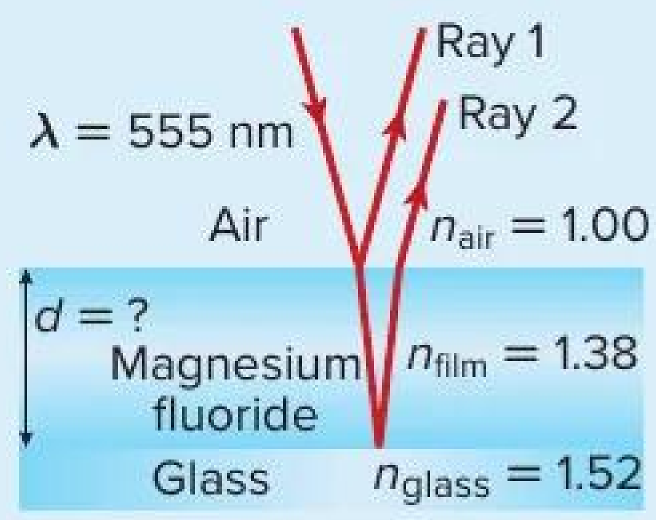


Figure 9

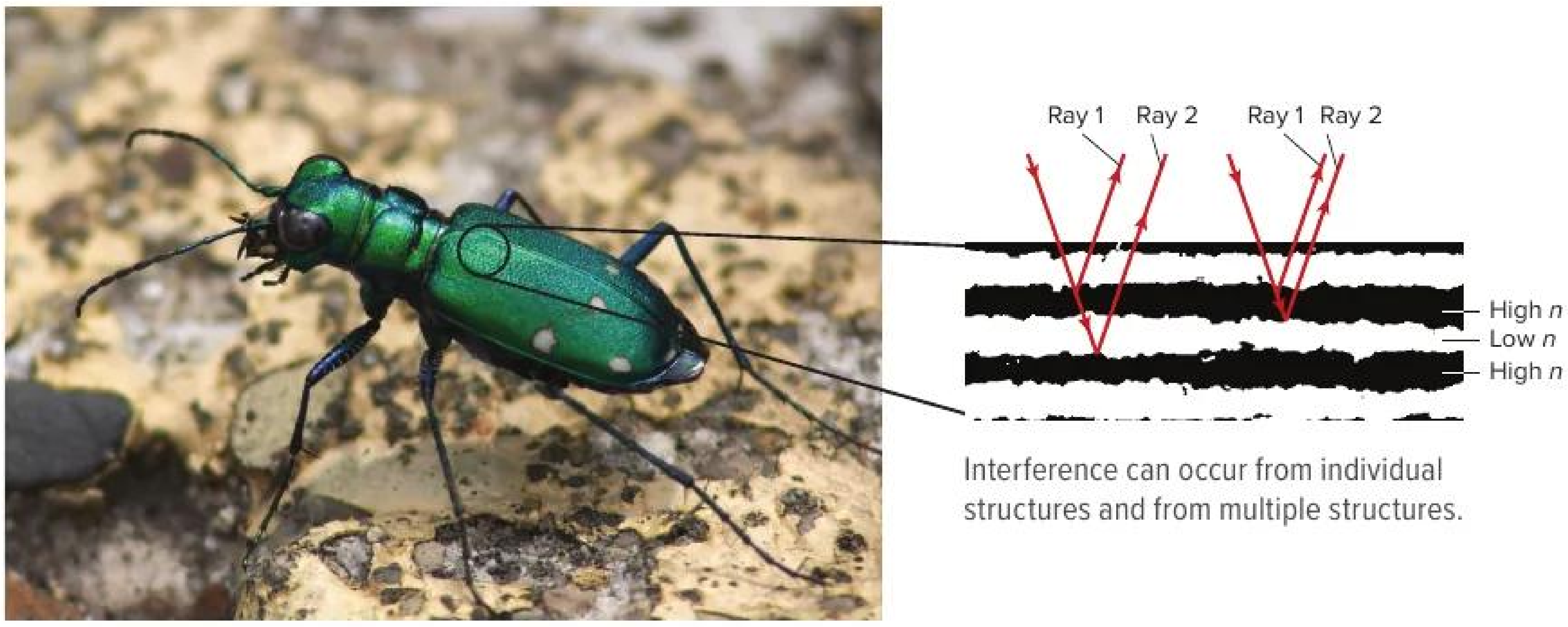


Figure 10 The tiger beetle owes its iridescent green color to thin-film interference.

Interference in nature Light interference also occurs naturally in the outer layer of the shells of many beetles, as shown in **Figure 10.** The shimmering green of the tiger beetle is the result of reflection from thin, parallel layers of chitin and sometimes other materials that differ in index of refraction. A diagram showing how these multilayer reflectors work is shown in **Figure 10.** The layers of the exoskeleton reflect light such that the result is constructive interference of green light. A shimmering appearance results.

Check Your Progress

10. **Interference** Two narrow slits are cut close to each other in a large piece of cardboard and illuminated by monochromatic red light. A sheet of white paper placed far from the slits shows a pattern of bright and dark bands on the paper. Describe how a wave behaves when it meets a slit and why some regions are bright and others are dark. Sketch the pattern.
11. **Interference Patterns** Sketch what happens to the pattern in the previous problem when the red light is replaced by blue light.
12. **Interference** Lucien holds a bubble wand with a soap film ($n = 1.33$) in it vertically.
    a. What is the second thinnest width of the soap film at which he could see a bright stripe if the light illuminating the film has a wavelength of 575 nm?
    b. What other widths produce a bright stripe at 575 nm?
13. **Double-Slit Interference** Light of wavelength 542 nm falls on a double slit. Use the values from **Figure 11** to determine how far apart the slits are.

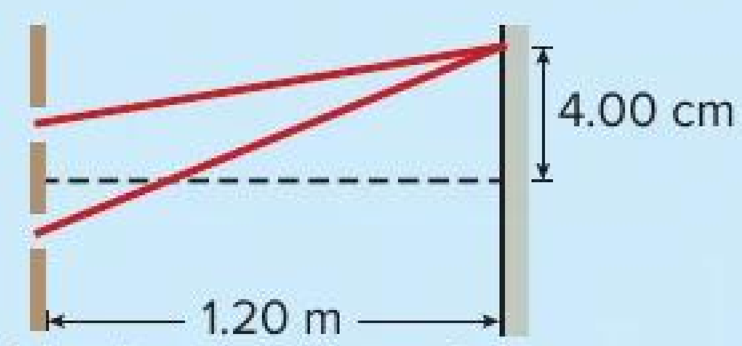


Figure 11

14. **Critical Thinking** The equation for wavelength from a double-slit investigation uses the simplification that θ is small so that $\sin\theta \approx \tan\theta$. Up to what angle is this a good approximation when your data has two significant figures? Would the maximum angle for a valid approximation increase or decrease as you increase the precision of your angle measurement?

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LESSON 2
DIFFRACTION

FOCUS QUESTION

How do DVDs produce a rainbow effect?

Single-Slit Diffraction

When studying light, you learned that wavefronts of light diffract when they pass around an edge. Diffraction can be explained by using Huygens' principle that a wavefront is made up of many small point-source wavelets. When light passes through a slit that has two closely spaced edges, a pattern is produced on a screen. This pattern, called a **diffraction pattern,** results from constructive and destructive interference of Huygens' wavelets.

When coherent, green light passes through a single, small opening that is between about 10 and 100 light wavelengths, the light is diffracted by both edges, and a series of bright and dark bands appears on a distant screen, as shown in **Figure 12.**

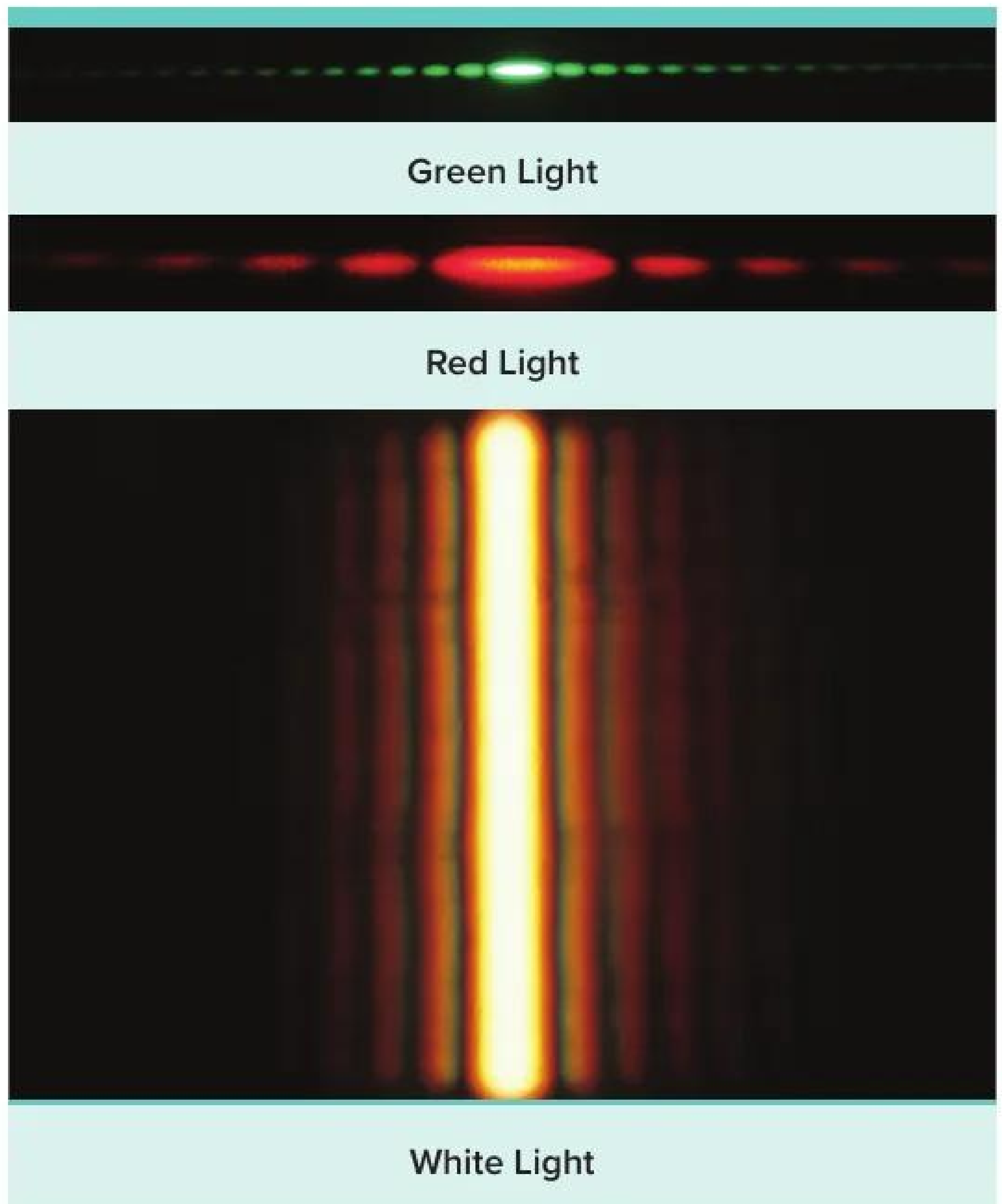


Figure 12 Single-slit diffraction produces one wide, bright central band and narrower, dimmer bands on either side.

Compare *the colors and widths of the central bands produced by the different colors of light.*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Quick Investigation: Diffraction Gratings

Carry out an investigation to discover the patterns produced when light is transmitted through a diffraction grating.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

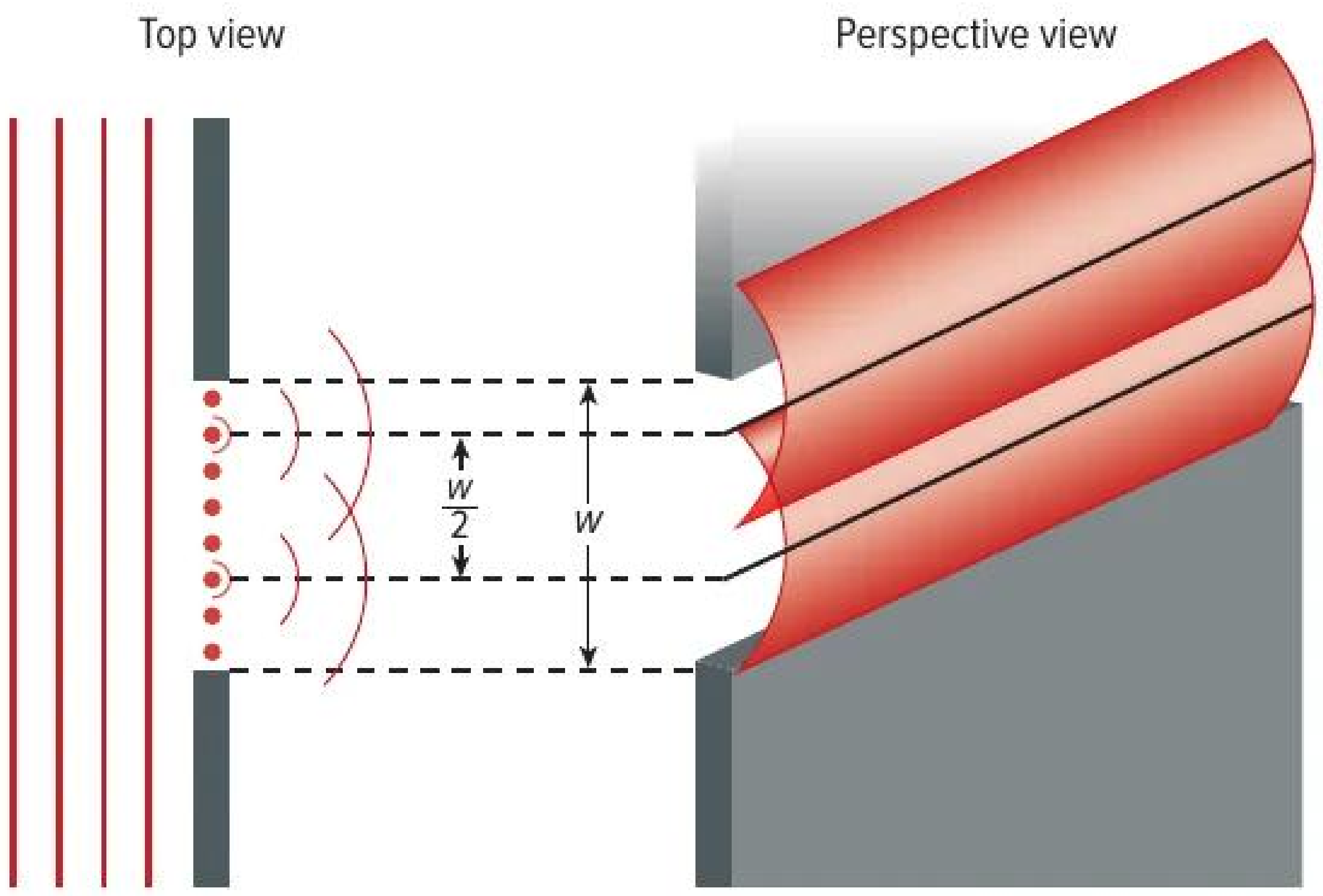


Figure 13 To illustrate diffraction patterns with Huygens' wavelets, a pair of points is chosen such that the separation between the points is $\frac{w}{2}$.

Instead of the nearly equally spaced bands produced by two coherent sources in Young's double-slit investigation, this pattern has a wide, bright central band with dimmer, narrower bands on either side. When using red light instead of green, the width of the bright central band increases. With white light, the pattern is a combination of patterns of all the colors of the spectrum (**Figure 12**).

Huygens' wavelets To see how Huygens' wavelets produce the diffraction pattern, imagine a slit of width w as being divided into an even number of Huygens' points, as shown in **Figure 13.** Each point acts as a source of Huygens' wavelets. Divide the slit into two equal halves, and choose one source from each part so that the pair is separated by a distance $w/2$. This pair of sources produces coherent, cylindrical waves that will interfere.

For any Huygens' wavelet produced in the top half, there will be another Huygens' wavelet in the bottom half, a distance $w/2$ away, that it will interfere with destructively to create a dark band on the screen. All similar pairings of Huygens' wavelets interfere destructively at dark bands. Conversely, a bright band on the screen is where pairings of Huygens' wavelets interfere constructively. In the dim regions between bright and dark bands, partial destructive interference occurs.

Diffraction pattern When the single slit is illuminated, a central bright band appears at location P_0 on the screen, as shown in **Figure 14** on the next page. The first dark band is at position P_1. At this location, the path lengths r_1 and r_2 of the two Huygens' wavelets differ by $\lambda/2$, producing destructive interference.

This model is mathematically similar to that of double-slit interference. A comparison of a single-slit diffraction pattern with a double-slit interference pattern using slits of the same width reveals that the entire single-slit diffraction pattern is covered by the narrow dark and light interference bands. Double-slit patterns result from the interference of the light from each single slit.

We will now develop an equation for the diffraction pattern produced by a single slit. We use the same simplifications that were used for double-slit interference, assuming that the distance to the screen is much larger than w. As described above, the separation distance between the sources of the two interfering waves is $w/2$. To find the distance measured on the screen to the first dark band (x_1), note that the path length difference is $\lambda/2$ because at the dark band there is destructive interference. As a result, $x_1/L = \lambda/w$.

Get It?
Identify the type of interference of Huygens' wavelets that creates a dark band on the screen.

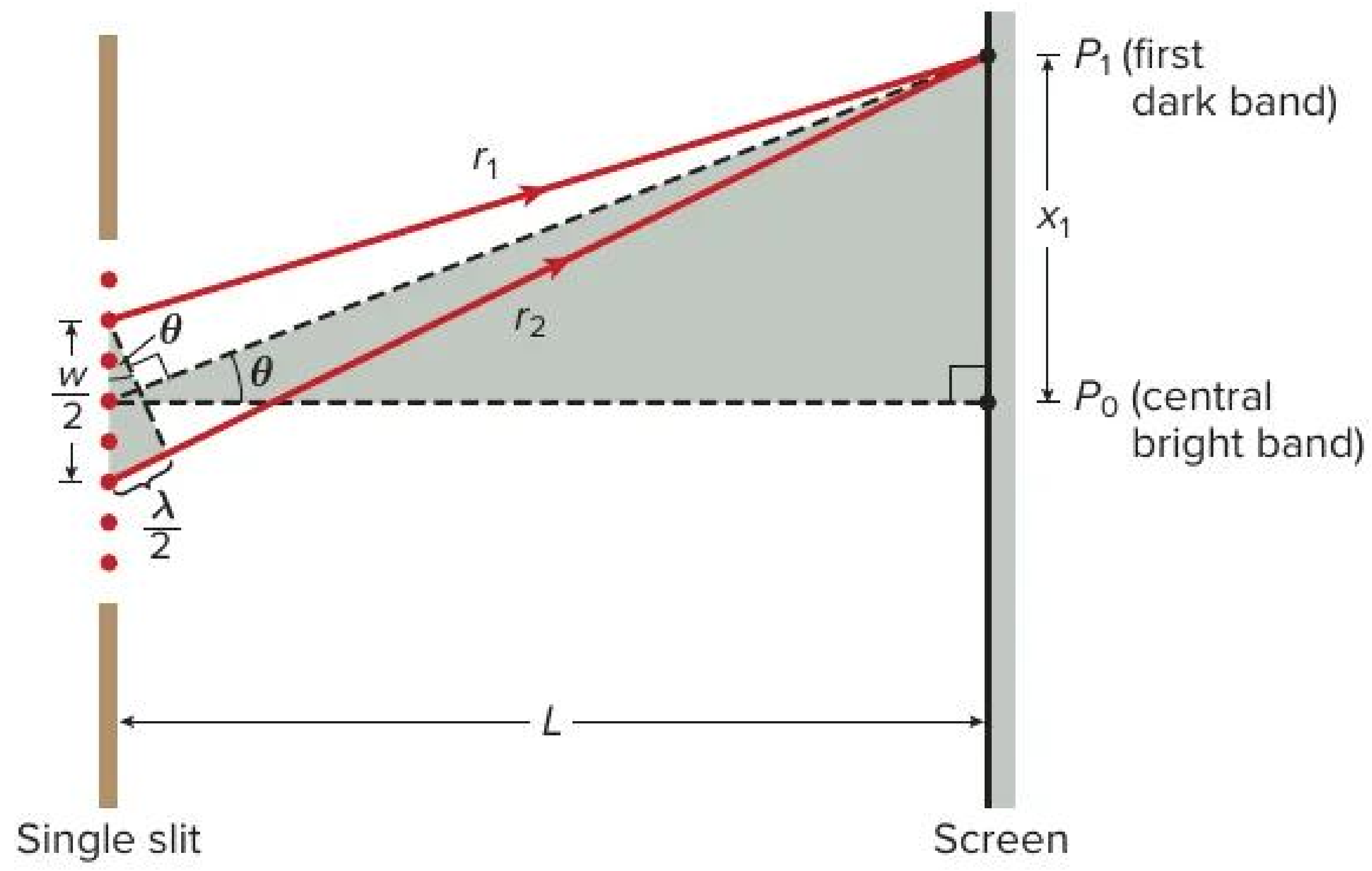


Figure 14 The width of the bright band in single-slit diffraction is related to the wavelength of light, the distance from the slit to the screen, and the width of the slit.

Instead of measuring the distance to the first dark band from the center of the central bright band (x_1), it is better to determine the width of the central bright band ($2x_1$), as in the equation below.

Width of Bright Band in Single-Slit Diffraction

The width of the central bright band ($2x_1$) is equal to the product of twice the wavelength times the distance to the screen (L), divided by the width of the slit (w).

$$2x_1 = \frac{2\lambda L}{w}$$

Single-slit diffraction patterns make the wave nature of light noticeable when the slits are 10 to 100 times the wavelength of the light. Larger openings cast sharp shadows, as Isaac Newton first observed. While the single-slit pattern depends on the wavelength of light, it is not very useful for measuring wavelength. A large number of slits put together provides a more useful tool for measuring wavelength.

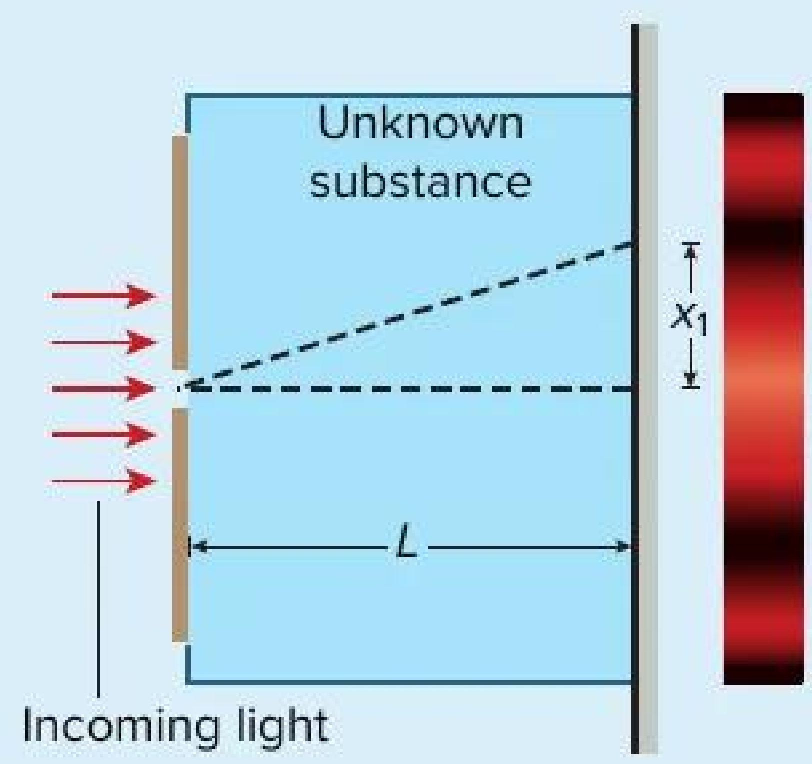
Get It?

Describe the assumption that is made concerning w and L for the single-slit diffraction equation.

PHYSICS Challenge

You have several unknown substances and wish to use a single-slit diffraction apparatus to determine what each one is. You decide to place a sample of an unknown substance in the region between the slit and the screen and use the data you obtain to determine the identity of each substance by calculating its index of refraction.

1. Come up with a general formula for the index of refraction of an unknown substance in terms of the wavelength of the light (λ_{vacuum}), the width of the slit (w), the distance from the slit to the screen (L), and the distance between the central bright band and the first dark band (x_1).
2. If the source you used had a wavelength of 634 nm, the slit width was 0.10 mm, the distance from the slit to the screen was 1.15 m, and you immersed the apparatus in water ($n_{substance}$ = 1.33), then what would you expect the width of the center band to be?



PRACTICE Problems ADDITIONAL PRACTICE

15. Monochromatic green light of wavelength 546 nm falls on a single slit with a width of 0.095 mm. The slit is located 75 cm from a screen. How wide will the central bright band be?

16. Yellow light with a wavelength of 589 nm passes through a slit of width 0.110 mm and makes a pattern on a screen. If the width of the central bright band is 2.60×10^{-2} m, how far is it from the slit to the screen?

17. Light from a He-Ne laser (λ = 632.8 nm) falls on a slit of unknown width. A pattern is formed on a screen 1.15 m away, on which the central bright band is 15.0 mm wide. How wide is the slit?

18. Yellow light falls on a single slit 0.0295 mm wide. On a screen that is 60.0 cm away, the central bright band is 24.0 mm wide. What is the wavelength of the light?

19. CHALLENGE White light falls on a single slit that is 0.050 mm wide. A screen is placed 1.00 m away. A student first puts a blue-violet filter (λ = 441 nm) over the slit, then a red filter (λ = 622 nm). The student measures the width of the central bright band.

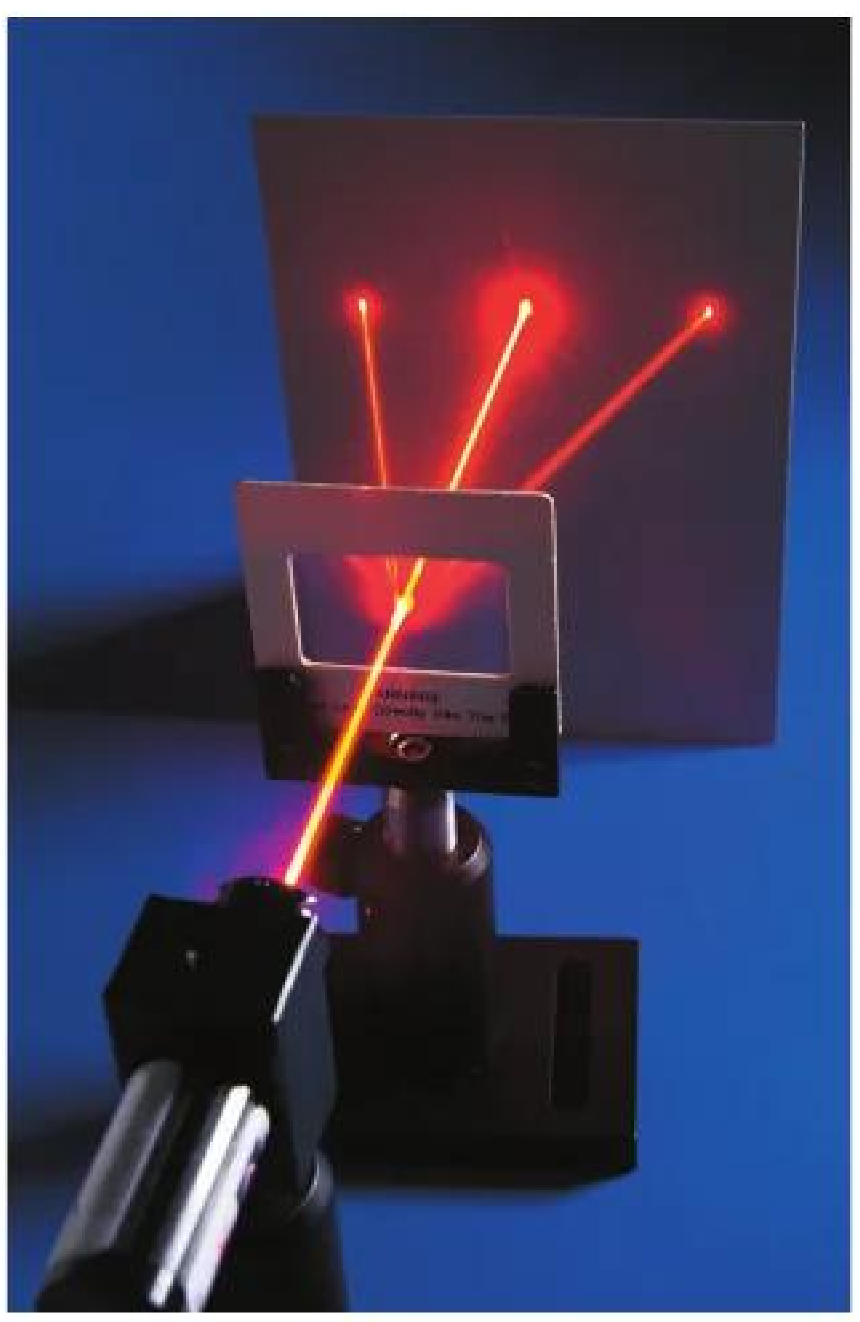
a. Which filter produced the wider band?

b. Calculate the width of the central bright band for both filters.

Diffraction Gratings

Diffraction gratings, such as the one shown in **Figure 15,** are often used to make precise measurements of wavelength. A **diffraction grating** is a device that is made up of many small slits that diffract light and form a pattern that is an overlap of single-slit diffraction patterns. This pattern is similar to that of a two-slit interference pattern, but with much narrower and brighter bands. Diffraction gratings can have as many as 10,000 slits per centimeter, which means the spacing between the slits can be as small as 10^{-6} m. A diffraction grating is a useful tool for the study of light and objects that emit or absorb light.

One type of diffraction grating is called a transmission grating. A transmission grating can be made by scratching very fine lines with a diamond point on glass that transmits light. The spaces between the scratched lines act like slits.



Diffraction gratings can be used to enhance the appearance of diamonds. The gratings are etched into certain surfaces of the diamond to improve the dispersion of light and make the gems appear more brilliant.

Holographic diffraction gratings produce the brightest spectra. They are made by using a laser and mirrors to create a diffraction pattern consisting of parallel bright and dark lines. The pattern is projected on a piece of metal that is coated with a light-sensitive material. The light from the laser produces a chemical reaction that hardens the material.

Figure 15 Diffraction gratings are used in various devices and instruments. The effects they produce also make them appealing for use in jewelry.

The metal is then placed in acid, which attacks the metal where unprotected by the hardened material, resulting in a series of hills and valleys in the metal identical to the original diffraction pattern. The metal itself can be used as a reflection grating. In some cases, a plastic film is placed on the heated metal, producing the hills and valleys in the plastic. Because of the sinusoidal shape of the hills and valleys, the diffraction patterns are very bright.

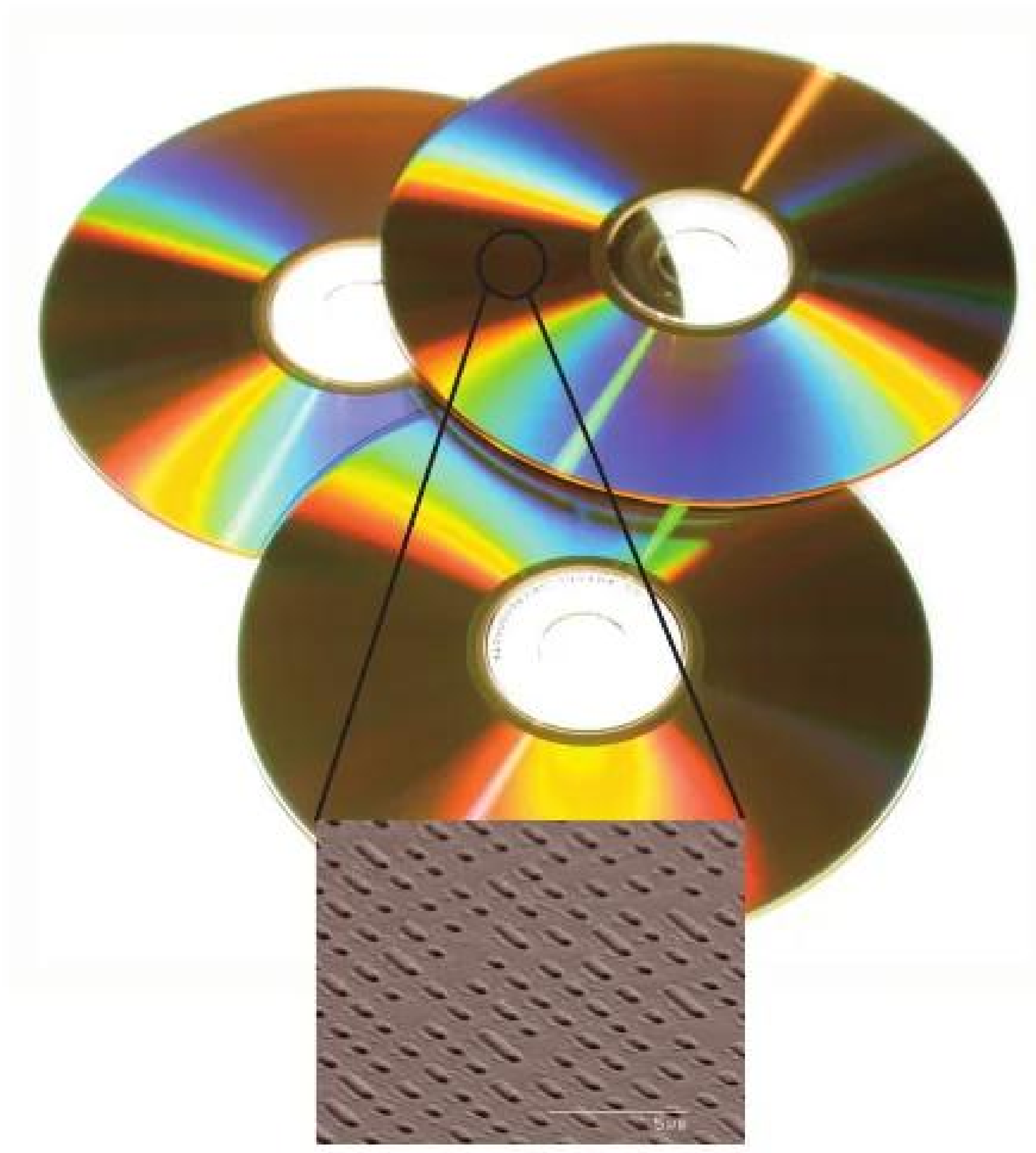


Figure 16 A CD is a reflection grating, producing a light spectrum. A magnified view of the surface of a CD shows the arrangement of pits and lands.

Reflection gratings You might have seen that light that reflects off CDs, DVDs, or Blu-ray Discs (or optical discs) creates a spectrum diffraction pattern, as in **Figure 16.** A type of diffraction grating made by inscribing fine lines on metallic or reflective glass surfaces is called a reflection grating. Optical discs are examples of reflection gratings. If you were to shine monochromatic light on an optical disc, the reflected light would produce a reflection pattern on a screen. Transmission and reflection gratings produce similar patterns, which can be analyzed in the same manner.

The surfaces of optical discs are covered with lines of microscopic indentations called pits separated by flat areas called lands, as shown in **Figure 16,** arranged in a spiral. The turns of the spiral act as a diffraction grating, separating colors by interference. The fact that optical discs are diffraction gratings is not important to their function, but the way they interact with different wavelengths of light is important.

Storing and reading information A laser is used to "read" the pattern of pits and lands on the optical disc. This is similar to the way a visually impaired person reads braille. The light from the laser is reflected from the surface of the disc into a light detector. The laser is focused so that when it reflects from the lands, its bright spot falls on the detector. When it reflects from the pits, it is spread out and dimmer.

The spot size is limited by diffraction, so if shorter laser wavelength is used, the spot size can be reduced and the pits can be closer together, allowing more information to be stored. As laser technology has advanced, shorter wavelength lasers have been used, permitting more information to be put on the disc. Music CDs use infrared light with a wavelength of 780 nm. A CD can hold about 700 megabytes of information. DVDs use red lasers (650 nm), permitting more than 4 gigabytes to be recorded. Blu-ray Discs use violet lasers (405 nm). The light appears blue, giving the disc its name. Single-layer Blu-ray Discs can hold up to 25 gigabytes of information.

Get It?

Explain how the amount of information an optical disc can store and the wavelength of light used to read it are related.

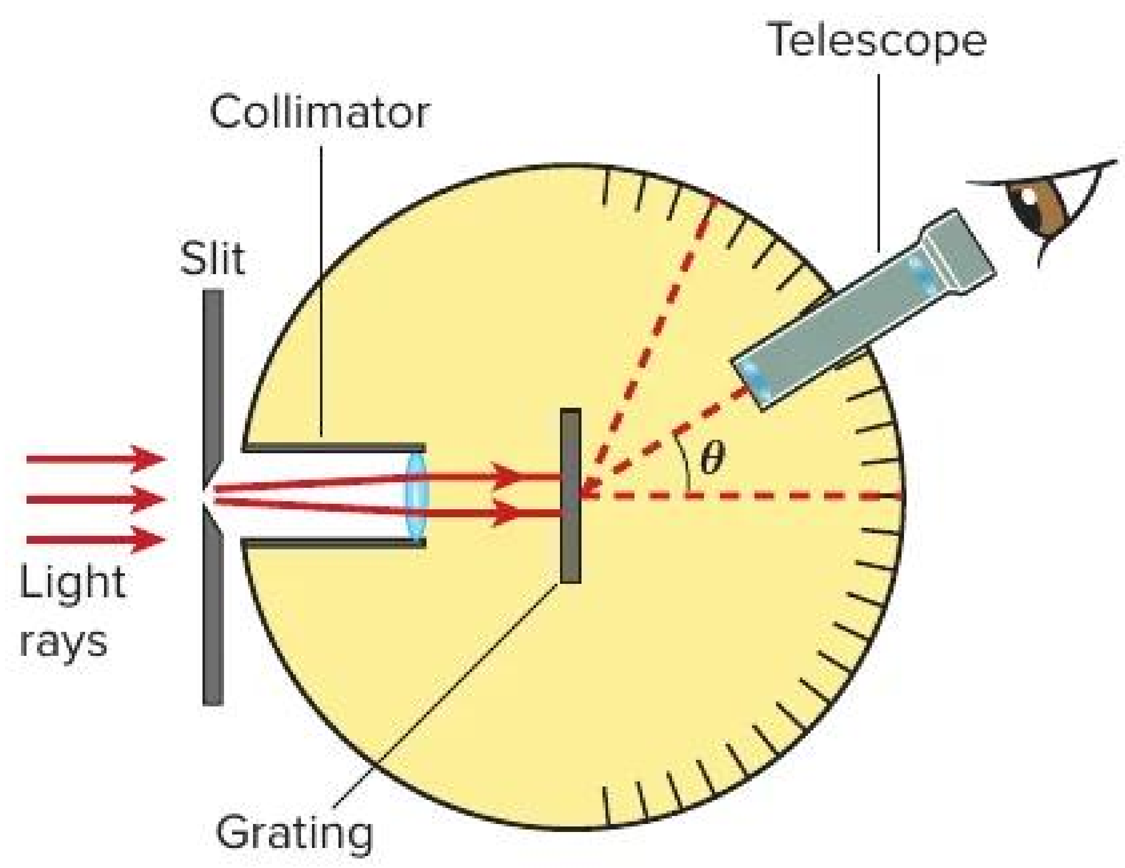


Figure 17 A grating spectroscope is used to accurately measure the wavelength of light.

State *the simplification used for double-slit wavelength calculations that does not apply for gratings.*

Measuring wavelength

An instrument used to measure wavelengths of light using a diffraction grating is called a grating spectroscope, like the one shown in **Figure 17.** The source to be analyzed emits light that is directed through a slit and a collimator and then to a diffraction grating. The grating produces a diffraction pattern that is viewed through a telescope.

If the source of light is a single color, the diffraction pattern produced by a grating has narrow, equally spaced, bright lines, as shown in **Figure 18.** The larger the number of slits per unit length of the grating, the narrower the lines in the diffraction pattern. The narrower the lines, the more precisely the distance between the bright lines can be measured.

Earlier in this module, you read that the diffraction pattern produced by a double slit could be used to calculate wavelength. An equation for the diffraction grating can be developed in the same way as for the double slit. However, with a diffraction grating, θ could be large, so the small angle simplification does not apply. Wavelength can be found by measuring the angle (θ) between the central bright line and the first-order bright line.

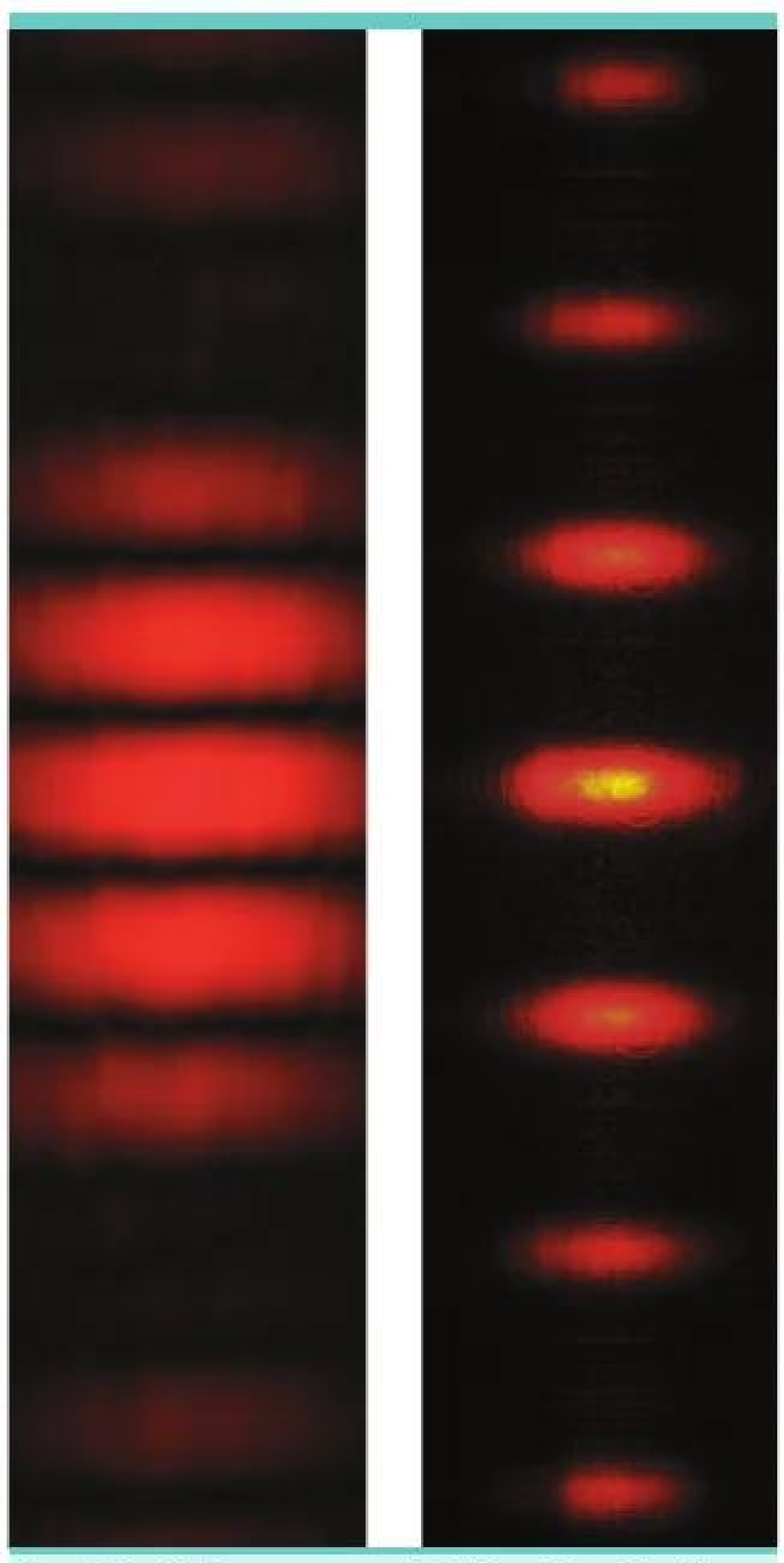


Figure 18 Compare the two diffraction patterns for red light. The diffraction grating pattern provides a more precise measurement.

Wavelength from a Diffraction Grating

The wavelength of light is equal to the slit separation (d) distance times the sine of the angle (θ) at which the first-order bright line occurs.

$$\lambda = d \sin \theta$$

Constructive interference from a diffraction grating occurs at angles on either side of the central bright line given by the equation $m\lambda = d \sin \theta$, where $m = 0, 1, 2$, etc. The central bright line occurs at $m = 0$. Spectroscopists often use the $m = 2$ or 3 lines because measurements of the spacing between bright lines can be made more precisely.

Notice that the diffraction grating pattern contains more dark space than the double-slit pattern in **Figure 18.** This is because there is more destructive interference in a diffraction grating than in a double-slit. This results in narrower lines, which also improves the precision of the measurements.

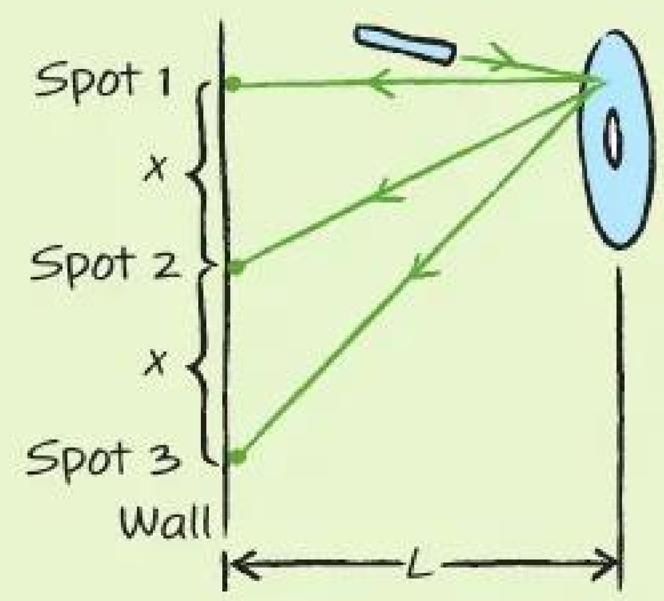
Diffraction gratings are incorporated into spectroscopes used to analyze gemstones. Experienced gemologists recognize the patterns of bands produced by white light passing through different stones. For example, three bright bands of green, yellow, and orange are a strong indication that cobalt is present. This likely means that a blue stone is not an expensive gem such as sapphire or topaz, but rather a cheap piece of glass that has been tinted blue.

EXAMPLE Problem 3

USING A DVD AS A DIFFRACTION GRATING A student noticed the beautiful spectrum reflected off a DVD. She directed a beam from her teacher's green laser pointer at the DVD and found three bright spots reflected on the wall. The label on the pointer indicated that the wavelength was 532 nm. The student found that the spacing between the spots was 1.29 m on the wall, which was 1.25 m away. What is the spacing between the rows on the DVD?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the investigation, showing the DVD as a grating and the spots on the wall.
- Identify and label the knowns.



Known	Unknown
$x = 1.29$ m	$d = ?$
$L = 1.25$ m	
$\lambda = 532$ nm	

2 SOLVE FOR THE UNKNOWN

Find the angle between the central bright spot and the one next to it using $\tan\theta = \frac{x}{L}$.

$$\theta = \tan^{-1}\left(\frac{x}{L}\right)$$

$$= \tan^{-1}\left(\frac{1.29\text{ m}}{1.25\text{ m}}\right)$$ Substitute $x = 1.29$ m, $L = 1.25$ m.

$$= 45.9°$$

Use the diffraction grating wavelength and solve for d.

$$\lambda = d\sin\theta$$

$$d = \frac{\lambda}{\sin\theta}$$

$$= \frac{532\times10^{-9}\text{ m}}{\sin 45.9°}$$ Substitute $\lambda = 532\times10^{-9}$ m, $\theta = 45.9°$.

$$= 7.41\times10^{-7}\text{ m} = 741\text{ nm}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** The answer is in meters, which is correct for separation.
- **Is the magnitude realistic?** With x and L almost the same size, d is close to λ.

PRACTICE Problems

ADDITIONAL PRACTICE

20. White light shines through a grating onto a screen. Describe the pattern that is produced.

21. If blue light of wavelength 434 nm shines on a diffraction grating and the spacing of the resulting lines on a screen that is 1.05 m away is 0.55 m, what is the spacing between the slits in the grating?

22. A diffraction grating with slits separated by 8.60×10^{-7} m is illuminated by violet light with a wavelength of 421 nm. If the screen is 80.0 cm from the grating, what is the separation of the lines in the diffraction pattern?

23. For the DVD in Example Problem 3, suppose now that blue light shines on it. If the spots produced on a wall are separated by 58.0 cm, what is the wavelength of the light? The wall is 0.65 m away.

24. **CHALLENGE** Light of wavelength 632 nm passes through a diffraction grating and creates a pattern on a screen that is 0.55 m away. If the first bright band is 5.6 cm from the central bright band, how many slits per centimeter does the grating have?

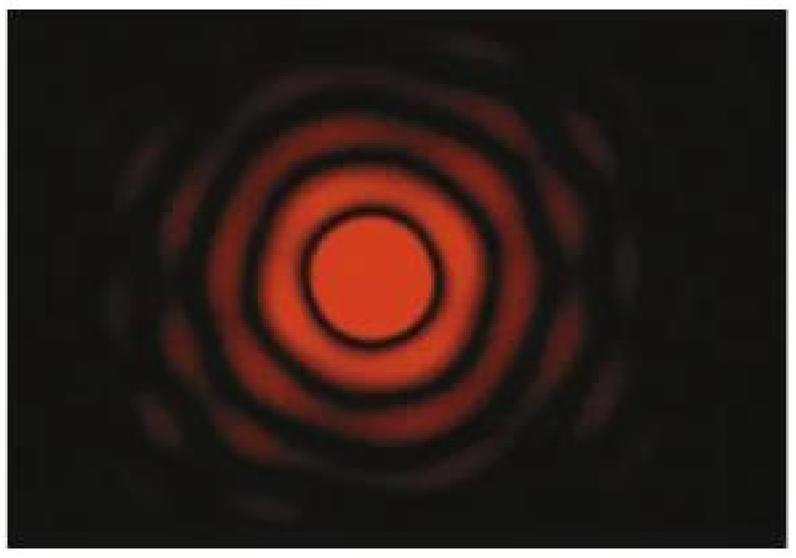


Figure 19 An aperture diffracts light to create a diffraction pattern with a central bright spot with dark and bright rings around it.

Resolving Power of Lenses

The circular lens of a telescope, a microscope, and even your eye acts as a hole, called an aperture, through which light passes. An aperture diffracts light, just as a single slit does. Alternating bright and dark rings occur with a circular aperture, as shown in **Figure 19.** The equation for determining the size of an aperture is similar to that for a single slit. An aperture, however, has a circular edge rather than the two edges of a slit, so slit width (w) is replaced by aperture diameter (D), and a geometric factor of 1.22 is needed, resulting in $x_1 = \frac{1.22\lambda L}{D}$.

ASTRONOMY Connection When light from a distant star is viewed through the aperture of a telescope, the image is spread out due to diffraction. If two stars are close together, their images may blur together. In 1879, Lord Rayleigh, a British physicist, mathematician, and Nobel Prize winner, established a criterion for determining whether there are one or two stars in such an image. The **Rayleigh criterion** states that if the center of the bright spot of one source's image falls on the first dark ring of the second, the two images are at the limit of resolution. If the images of two stars are at the limit of resolution, a viewer can tell that there are two stars rather than only one.

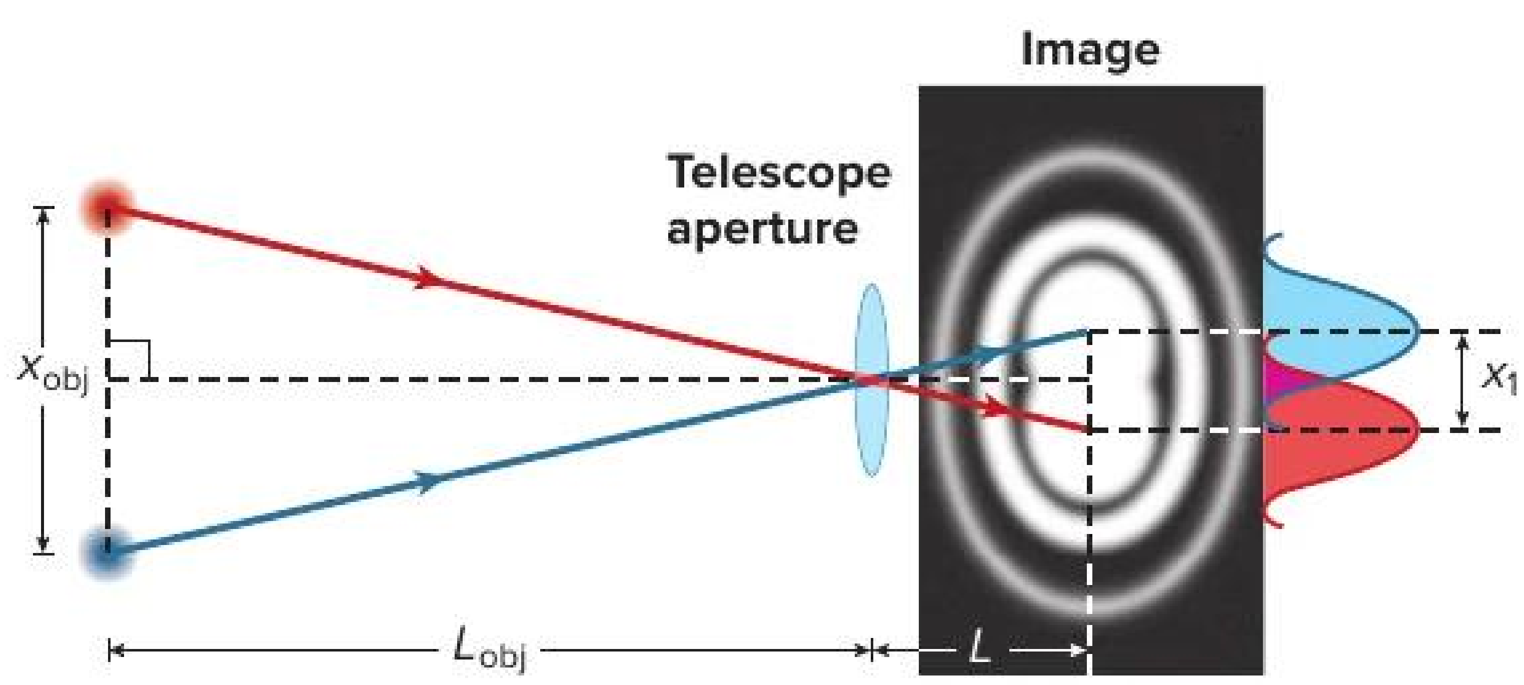
If two objects are at the limit of resolution, how can you find the distance between the objects (x_{obj})? From the Rayleigh criterion, the distance between the centers of the bright spots of the two images is x_1. **Figure 20** shows that similar triangles can be used to find that $\frac{x_{obj}}{L_{obj}} = \frac{x_1}{L}$. You can combine this equation with the equation for aperture size $\left(\frac{x_1}{L} = \frac{1.22\lambda}{D}\right)$ and solve for the distance between the objects (x_{obj}).

Rayleigh Criterion

The separation distance between objects (x_{obj}) that are at the limit of resolution is equal to 1.22 times the wavelength of light, times the distance from the circular aperture to the objects (L_{obj}), divided by the diameter of the circular aperture (D).

$$x_{obj} = \frac{1.22\lambda L_{obj}}{D}$$

Figure 20 The separation distance of objects can be calculated with similar-triangle geometry. The blue and red colors are for illustration only. (Illustration is not to scale.)



Diffraction in the eye In bright light, the eye's pupil is about 3 mm in diameter. The eye is most sensitive to yellow-green light where $\lambda = 550$ nm. So the Rayleigh criterion applied to the eye gives $x_{obj} = (2\times10^{-4})\, L_{obj}$. The distance between the pupil and the retina is about 2 cm, so by using $x_1 = 1.22\lambda L/D$, the centers of the bright spots of two barely resolved point sources would be separated by about 4 μm on the retina. The spacing between the cones, which are the light detectors in the retina, in the most sensitive part of the retina, the fovea, is about 2 μm. Thus, in the ideal case, three adjacent cones would record light, dark, and light, as illustrated in **Figure 21.** The distance between the centers of the bright spots from two point sources must be at least the distance between two light-recording cones to be resolved. It seems that the eye is ideally constructed.

Applying the Rayleigh criterion to find the ability of the eye to separate two distant sources shows that the eye could separate two automobile headlights (1.5 m apart) at a distance of 7 km. In practice, however, the eye is not limited by diffraction. Imperfections in the lens and the liquid that fills the eye reduce the eye's resolution to about five times that set by the Rayleigh criterion. The vision-processing centers in the human brain also limit detection of small point objects.

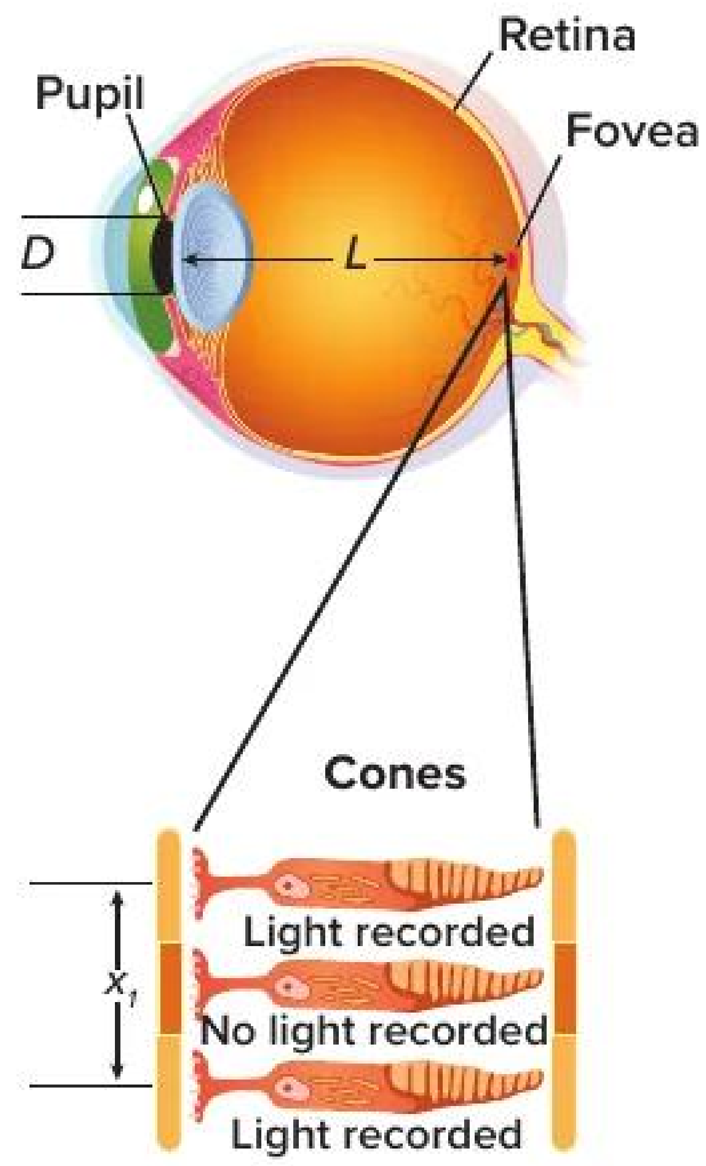


Figure 21 The eye's pupil is an aperture that diffracts light.

Check Your Progress

25. **Slit Diffraction** Several equally spaced narrow slits on a cardboard are illuminated by monochromatic red light. A pattern of bright and dark bands is visible on a sheet of white paper. Sketch the pattern on the paper.

26. **Rayleigh Criterion** The star Sirius is a system of two stars that orbit each other 8.44 light-years from Earth. If the *Hubble Space Telescope* (diameter 2.4 m) is pointed at the Sirius system, what is the minimum separation there would need to be between the stars in order for the telescope to be able to resolve them? Assume that the light coming from the stars has a wavelength of 550 nm.

27. **Line Spacing** You shine a red laser light through one diffraction grating and form a pattern of red dots on a screen. Then you substitute a second diffraction grating for the first one, forming a different pattern. The dots produced by the first grating are spread out more than those produced by the second. Which grating has more lines per millimeter?

28. **First-Order Dark Bands** Monochromatic green light with a wavelength of 546 nm falls on a single slit of width and location from the screen shown in **Figure 22.** What is the separation of the first-order dark bands?

Figure 22

29. **Critical Thinking** Your colleague shows you a spectrometer, but she does not tell you whether it has been constructed with a prism or a diffraction grating. You decide to look at a spectrum produced by white light passing through the spectrometer. Describe how you could determine which device produced the spectrum.

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SCIENTIFIC BREAKTHROUGHS

Beckoning Bees with Blue Halos

Scientific studies show that several species of bees prefer flower petals whose color is in the blue spectrum. Their eyes' enhanced photoreceptors allow them to "see" in this color range. Many plants lack the ability to produce blue flowers, but they have a structural adaptation that can attract bee pollinators by bending and scattering light to create a "blue halo."

Some flowers, such as *Ursinia anethoides*, have diverse nanostructures that enable them to diffract light. This light interference creates a blue halo, which is visible to bees.

All in the Optics

Flower petals have tiny ridges. When light reflects off the ridges on some flower petals, the light is scattered in such a way that it casts a bluish glow over the blooms. This is referred to as the "blue halo" effect.

Scientists studied the nanostructures responsible for this effect using a variety of microscopy techniques, including scanning electron microscopy. Computer models showed variations in the height and spacing of ridges on petals.

What physicists call "disorder" in the height and spacing of ridges diffracts light, resulting in the iridescent blue and ultraviolet wavelength spectrum, or blue halo effect. The more variability, the stronger the blue halo.

Scientists have also found evidence to support the fact that many different species of flowers independently evolved the structures to produce the blue halo. This is an example of natural selection resulting in convergent evolution—the plants with disordered nanostructures are best adapted to survive because of increases in optical visibility to attract bee pollinators that see blue wavelengths.

Irresistible Iridescence

To test the role of the blue halo in attracting bees, scientists created artificial nanostructures that matched those found in some species of flowers. They attached the nanostructures to the petals of artificial flowers.

In the study, bees could tell the difference between otherwise identical looking flowers based on the blue halo effect. The optical effect of the disordered nanoparticle structures on petals improved the efficiency of foraging bees to detect flowers. They were able to locate flowers with the blue halo faster. This, in turn, increases pollination of plants, which is an evolutionary advantage for survival.

COMMUNICATE SCIENTIFIC INFORMATION

Draw the nanostructures of ridges on flower petals, and write a brief explanation of how they diffract light.

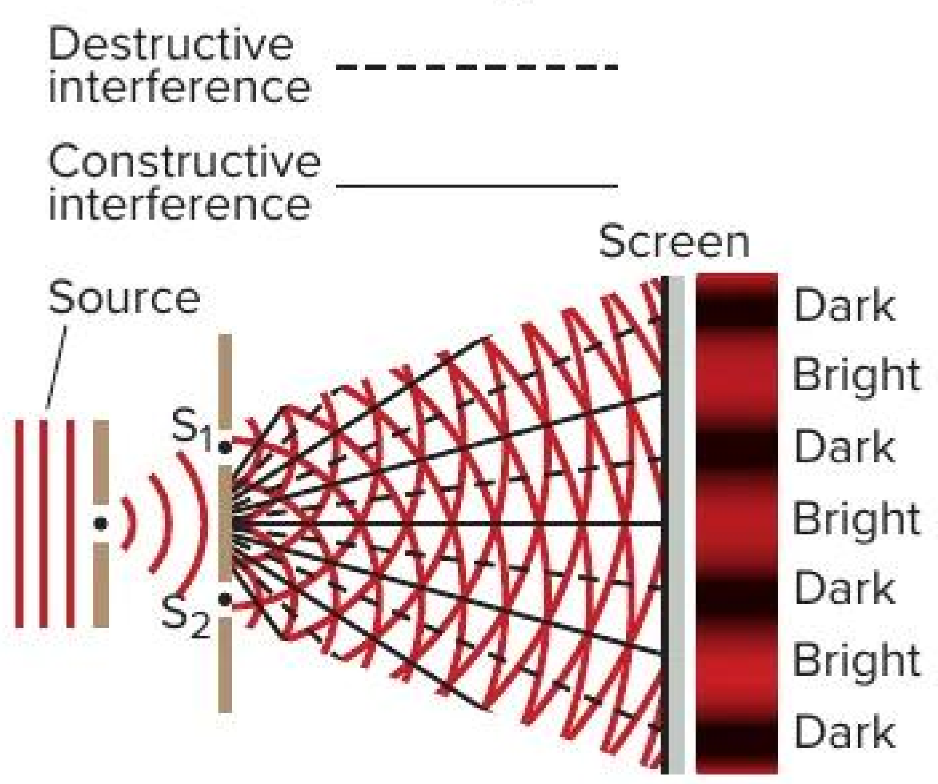
MODULE 17
STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 INTERFERENCE

- The superposition of light waves from coherent light sources can produce an interference pattern. Light passing through two closely spaced, narrow slits produces a pattern of dark and light bands on a screen called interference fringes.
- Interference patterns can be used to measure the wavelength of light.

$$\lambda = \frac{xd}{L}$$



- Interference patterns can result from multiple passes of light through a thin film. Thin-film interference can be modeled by rays reflecting from multiple surfaces of a thin film. The indices of refraction of the mediums the light travels in and the thickness of the film determine how different wavelengths of light will interfere.

- incoherent light
- coherent light
- interference fringes
- monochromatic light
- thin-film interference

Lesson 2 DIFFRACTION

- Light passing through a narrow slit is diffracted, which means spread out from a straight-line path, producing a diffraction pattern on a screen. The width of the bright central band of a single-slit diffraction pattern is related to the wavelength of light used.
- Diffraction gratings consist of large numbers of slits that are very close together and produce narrow spectral lines that result from interference of light diffracted by all the slits.
- Diffraction gratings can be used to measure the wavelength of light precisely or to separate light composed of different wavelengths.

$$\lambda = d \sin \theta$$

- Diffraction limits the ability of an aperture to distinguish two closely spaced objects, because the resulting image contains a diffuse central bright spot. If two bright spots are closer than the limit of resolution, they will overlap and the objects cannot be distinguished.
- The Rayleigh criterion states that if the center of the bright spot of one source's image falls on the first dark ring of the second, the two images are at the limit of resolution. If the images of two stars are at the limit of resolution, a viewer can tell that there are two stars rather than only one.

- diffraction pattern
- diffraction grating
- Rayleigh criterion

THREE-DIMENSIONAL THINKING
Module Wrap-Up

REVISIT THE PHENOMENON

What makes this hummingbird's feathers appear shiny and shimmery?

CER Claim, Evidence, Reasoning

Explain Your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will apply your evidence from this module and complete your project.

GO FURTHER

SEP Data Analysis Lab

How can you determine the wavelength of a laser?

Your project team has two laser pointers, one red and one green. Mark and Carlos disagree about which laser has the longer wavelength. Mark insists that red light has a longer wavelength, while Carlos is certain the green has a longer wavelength. You have a DVD and shine each laser on the grooves, reflecting onto a nearby wall.

CER Analyze and Interpret Data

1. Describe what data you can collect about the lasers by observing the reflections.
2. **Claim** Is Mark or Carlos correct?
3. **Evidence and Reasoning** How would you explain the results to Mark and Carlos to settle their disagreement?

MODULE 6
MOTION IN TWO DIMENSIONS

Thomas Barwick/Iconica/Getty Images

MODULE 6

MOTION IN TWO DIMENSIONS

ENCOUNTER THE PHENOMENON

Why do thrown basketballs travel in arcs?

GO ONLINE to play a video about the path of a projectile.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about why thrown basketballs travel in arcs. Explain your reasoning.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.

LESSON 1: Explore & Explain: Angled Launches

LESSON 2: Explore & Explain: Centripetal Acceleration

Smithsonian

Additional Resources

LESSON 1

PROJECTILE MOTION

FOCUS QUESTION

What forces affect a basketball's trajectory?

Path of a Projectile

A hopping frog, a tossed snowball, and an arrow shot from a bow all move along similar paths. Each path rises and then falls, always curving downward along a parabolic path. An object shot through the air is called a **projectile.** Its path through space is called its **trajectory.**

You can determine a projectile's trajectory if you know its initial velocity. **Figure 1** shows water that is launched as a projectile horizontally and at an angle. In both cases, gravity curves the path downward along a parabolic path.

You can draw a free-body diagram of a launched projectile and identify the forces acting on it. If you ignore air resistance, after an initial force launches a projectile, the only force on it as it moves through the air is gravity. Gravity causes the object to curve downward.

Hutchings Photography/Digital Light Source

Figure 1 A projectile launched horizontally immediately curves downward, but if it is launched upward at an angle, it rises and then falls, always curving downward.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: On Target

Plan and carry out an investigation to determine what factors affect projectile motion.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

Independence of Motion in Two Dimensions

Think about two softball players warming up for a game, tossing high fly balls back and forth. What does the path of the ball through the air look like? Because the ball is a projectile, it follows a parabolic path.

Imagine you are standing directly behind one of the players and you are watching the softball as it is being tossed. What would the motion of the ball look like? You would see it go up and back down, just like any object that is tossed straight up in the air.

If you were watching the softball from a hot-air balloon high above the field, what motion would you see? You would see the ball move from one player to the other at a constant speed, just like any object that is given an initial horizontal velocity, such as a hockey puck sliding across ice. The motion of projectiles is a combination of these two motions.

Get It?

Illustrate Draw sketches of a drone's-eye view of the person at the beginning of the module shooting a basketball. The sketches should show the view from directly behind the person, from directly above the court, and from the sideline.

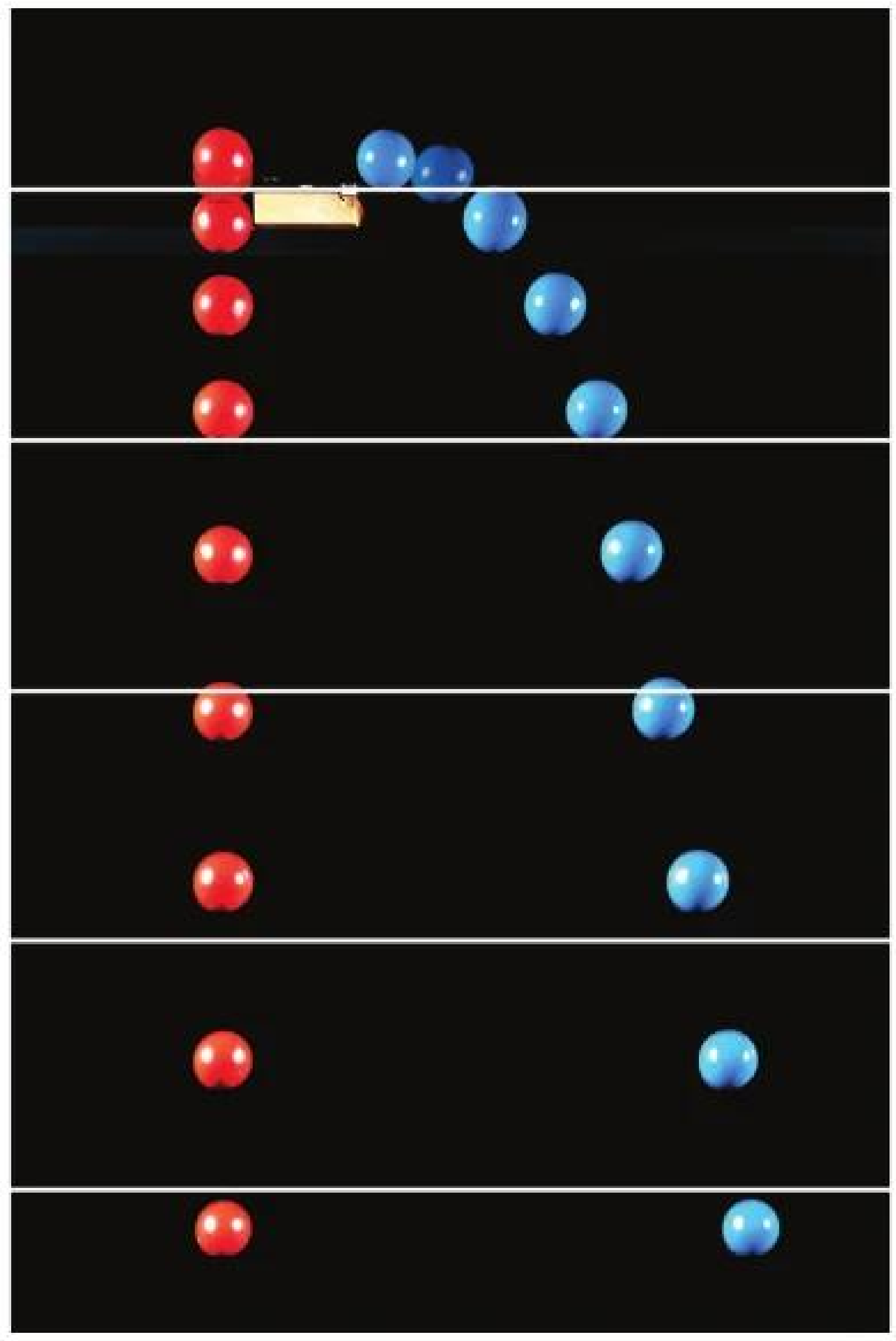


Figure 2 The ball on the left was dropped with no initial velocity. The ball on the right was given an initial horizontal velocity. The balls have the same vertical motion as they fall.

Identify *What is the vertical velocity of the balls after falling for 1 s?*

Why do projectiles behave in this way? After a softball leaves a player's hand, what forces are exerted on the ball? If you ignore air resistance, there are no contact forces on the ball. There is only the field force of gravity in the downward direction. How does this affect the ball's motion? Gravity causes the ball to have a downward acceleration.

Comparing motion diagrams The trajectories of two balls are shown in **Figure 2.** The red ball was dropped, and the blue ball was given an initial horizontal velocity of 2.0 m/s. What is similar about the two paths?

Look at their vertical positions. The horizontal lines indicate equal vertical distances. At each moment that a picture was taken, the heights of the two balls were the same. Because the change in vertical position was the same for both, their average vertical velocities during each interval were also the same. The increasingly large distance traveled vertically by the balls, from one time interval to the next, shows that they were accelerating downward due to the force of gravity.

Notice that the horizontal motion of the launched ball does not affect its vertical motion. A projectile launched horizontally has initial horizontal velocity, but it has no initial vertical velocity. Therefore, its vertical motion is like that of an object dropped from rest. Just like the red ball, the blue ball has a downward velocity that increases regularly because of the acceleration due to gravity.

Get It?

Explain why a dropped object has the same vertical velocity as an object launched horizontally.

Horizontally Launched Projectiles

Imagine a person standing near the edge of a cliff and kicking a pebble horizontally. Like all horizontally launched projectiles, the pebble will have an initial horizontal velocity and no initial vertical velocity. What will happen to the pebble as it falls from the cliff?

Separate motion diagrams Recall that the horizontal motion of a projectile does not affect its vertical motion. It is therefore easier to analyze the horizontal motion and the vertical motion separately. Separate motion diagrams for the *x*-components and *y*-components of a horizontally launched projectile, such as a pebble kicked off a cliff, are shown on the left in **Figure 3.**

Horizontal motion Notice the horizontal vectors in the diagram on the left. Each of the velocity vectors is the same length, which indicates that the object's horizontal velocity is not changing. The pebble is not accelerating horizontally. This constant velocity in the horizontal direction is exactly what should be expected, because after the initial kick, there is no horizontal force acting on the pebble. (In reality, the pebble's speed would decrease slightly because of air resistance, but remember that we are ignoring air resistance in this module.)

Get It?

Explain why the horizontal motion of a projectile is constant.

Vectors in Two Dimensions

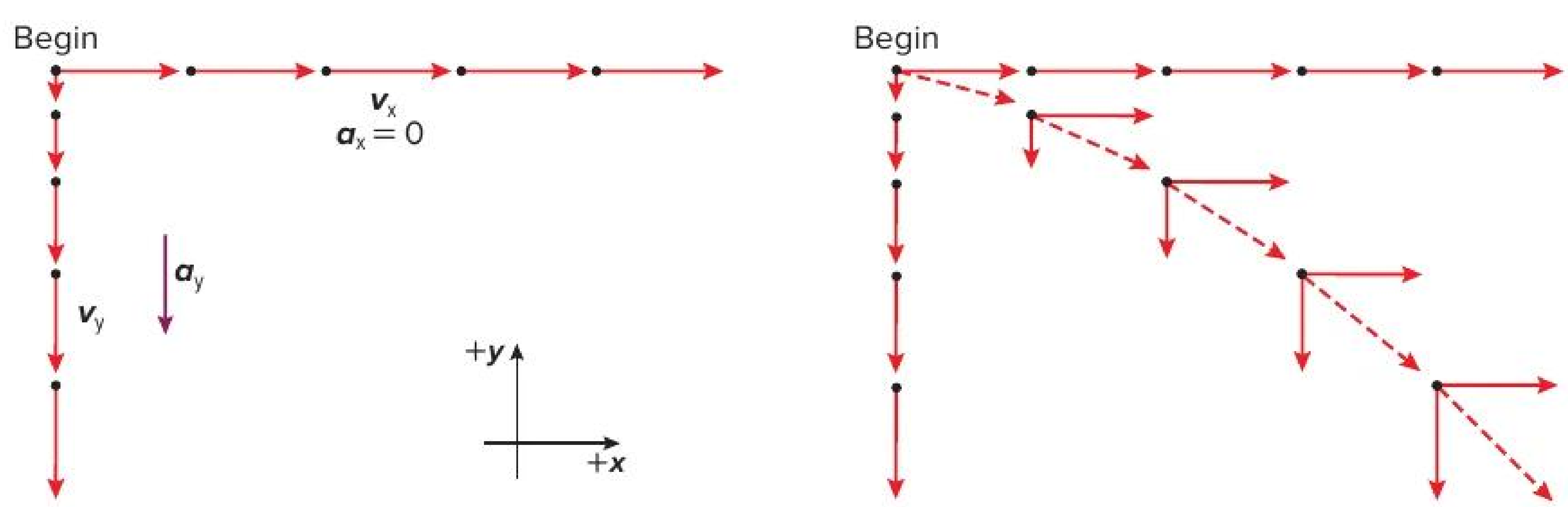


Figure 3 To describe the motion of a horizontally launched projectile, the *x*- and *y*-components can be treated independently. The resultant vectors of the projectile are tangent to a parabola.

Decide *What is the value of* $\boldsymbol{a}_y$*?*

Vertical motion Now look at the vertical velocity vectors in the diagram on the left. Each velocity vector has a slightly longer length than the one above it. The changing length shows that the object's vertical velocity is increasing and the object is accelerating downward. Again, this is what should be expected, because in this case the force of gravity is acting on the pebble.

Get It?
Describe why the vertical velocity vectors change length but the horizontal velocity vectors do not for a projectile.

Parabolic path When the *x*- and *y*-components of the object's motion are treated independently, each path is a straight line. The diagram on the right in **Figure 3** shows the actual parabolic path. The horizontal and vertical components at each moment are added to form the total velocity vector at that moment. You can see how the combination of constant horizontal velocity and uniform vertical acceleration produces a trajectory that has a parabolic shape.

PROBLEM-SOLVING STRATEGIES

Motion in Two Dimensions

When solving projectile problems, use the following strategies.

1. Draw a motion diagram with vectors for the projectile at its initial position and its final position. If the projectile is launched at an angle, also show its maximum height and the initial angle.
2. Consider vertical and horizontal motion independently. List known and unknown variables.
3. For horizontal motion, the acceleration is $a_x = 0.0\ \text{m/s}^2$. If the projectile is launched at an angle, its initial vertical velocity and its vertical velocity when it falls back to that same height have the same magnitude but different direction: $v_{yi} = -v_{yf}$.
4. For vertical motion, $a_y = -9.8\ \text{m/s}^2$ (if you choose up as positive). If the projectile is launched at an angle, its vertical velocity at its highest point is zero: $v_{y,\,max} = 0.0$ m/s.
5. Choose the motion equations that will enable you to find the unknown variables. Apply them to vertical and horizontal motion separately. Remember that time is the same for horizontal and vertical motion. Solving for time in one of the dimensions identifies the time for the other dimension.
6. Sometimes it is useful to apply the motion equations to part of the projectile's path. You can choose any initial and final points to use in the equations.

Motion Equations

Horizontal (constant speed)

$\boldsymbol{x}_f = \boldsymbol{v}t_f + \boldsymbol{x}_i$

Vertical (constant acceleration)

$\boldsymbol{v}_f = \boldsymbol{v}_i + \boldsymbol{a}t_f$

$\boldsymbol{x}_f = \boldsymbol{x}_i + \boldsymbol{v}_i t_f + \frac{1}{2}\boldsymbol{a}t_f^2$

$v_f^2 = v_i^2 + 2a(x_f - x_i)$

CCC CROSSCUTTING CONCEPTS

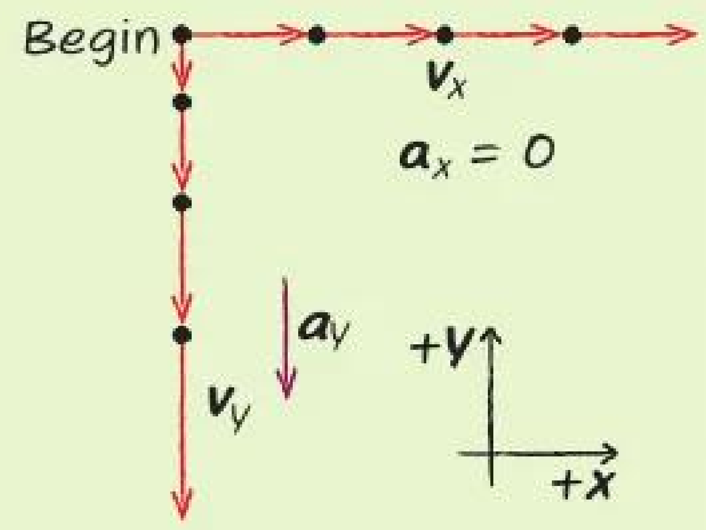
Patterns Gather empirical evidence to identify the pattern of motion that projectiles follow. Launch an object at various angles and record the motion it follows. Create a presentation that illustrates your findings.

EXAMPLE Problem 1

A SLIDING PLATE You are preparing breakfast and slide a plate on the countertop. Unfortunately, you slide it too fast, and it flies off the end of the countertop. If the countertop is 1.05 m above the floor and the plate leaves the top at 0.74 m/s, how long does it take to fall, and how far from the end of the counter does it land?

1 ANALYZE AND SKETCH THE PROBLEM

Draw horizontal and vertical motion diagrams. Choose the coordinate system so that the origin is at the top of the countertop. Choose the positive *x* direction in the direction of horizontal velocity and the positive *y* direction up.



Known		Unknown
$x_i = y_i = 0$ m	$a_x = 0$ m/s^2	$t = ?$
$v_{xi} = 0.74$ m/s	$a_y = -9.8$ m/s^2	$x_f = ?$
$v_{yi} = 0$ m/s	$y_f = -1.05$ m	

2 SOLVE FOR THE UNKNOWN

Use the equation of motion in the *y* direction to find the time of fall.

$$y_f = y_i + \frac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2(y_f - y_i)}{a_y}}$$ Rearrange the equation to solve for time.

$$= \sqrt{\frac{2(-1.05\text{ m} - 0\text{ m})}{-9.8\text{ m/s}^2}} = 0.46\text{ s}$$ Substitute $y_f = -1.05$ m, $y_i = 0$ m, $a_y = -9.8$ m/s^2.

Use the equation of motion in the *x* direction to find where the plate hits the floor.

$\mathbf{x}_f = \mathbf{v}_x t$ = (0.74 m/s to the right)(0.46 s) = 0.34 m to the right of the counter

3 EVALUATE THE ANSWER

- **Are the units correct?** Time is measured in seconds. Position in measured in meters.
- **Do the signs make sense?** Both are positive. The position sign agrees with the coordinate choice.
- **Are the magnitudes realistic?** A fall of about a meter takes about 0.5 s. During this time, the horizontal displacement of the plate would be about 0.5 s × 0.74 m/s.

PRACTICE Problems

ADDITIONAL PRACTICE

1. You throw a stone horizontally at a speed of 5.0 m/s from the top of a cliff that is 78.4 m high.
   - **a.** How long does it take the stone to reach the bottom of the cliff?
   - **b.** How far from the base of the cliff does the stone hit the ground?
   - **c.** What are the horizontal and vertical components of the stone's velocity just before it hits the ground?
2. Lucy and her friend are working at an assembly plant making wooden toy giraffes. At the end of the line, the giraffes go horizontally off the edge of a conveyor belt and fall into a box below. If the box is 0.60 m below the level of the conveyor belt and 0.40 m away from it, what must be the horizontal velocity of giraffes as they leave the conveyor belt?
3. **CHALLENGE** You are visiting a friend from elementary school who now lives in a small town. One local amusement is the ice-cream parlor, where Stan, the short-order cook, slides his completed ice-cream sundaes down the counter at a constant speed of 2.0 m/s to the servers. (The counter is kept very well polished for this purpose.) If the servers catch the sundaes 7.0 cm from the edge of the counter, how far do they fall from the edge of the counter to the point at which the servers catch them?

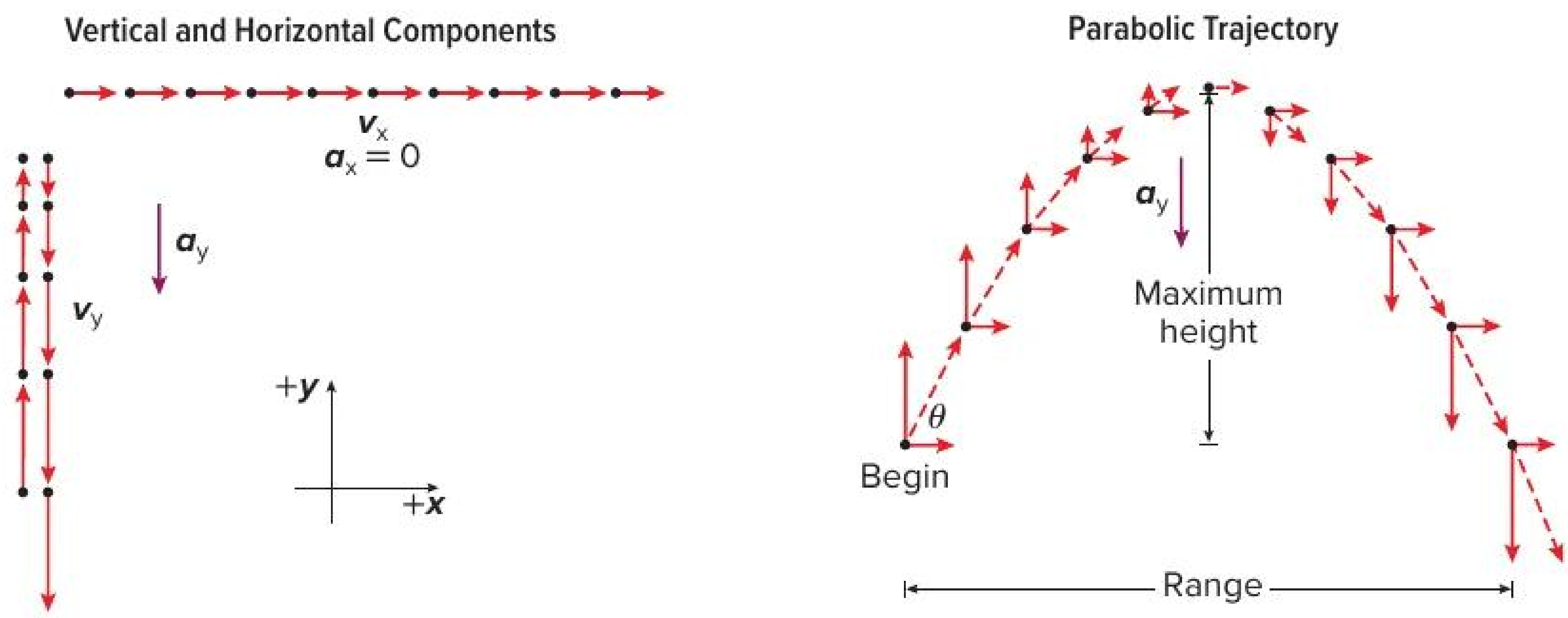


Figure 4 When a projectile is launched at an upward angle, its parabolic path is upward and then downward. The up-and-down motion is clearly represented in the vertical component of the vector diagram.

Angled Launches

When a projectile is launched at an angle, the initial velocity has a vertical component as well as a horizontal component. If the object is launched upward, like a ball tossed straight up in the air, it rises with slowing speed, reaches the top of its path where its speed is momentarily zero, and descends with increasing speed.

Separate motion diagrams The left diagram of **Figure 4** shows the separate vertical- and horizontal-motion diagrams for the trajectory. In the coordinate system, the x-axis is horizontal and the y-axis is vertical. Note the symmetry. At each point in the vertical direction, the velocity of the object as it is moving upward has the same magnitude as when it is moving downward. The only difference is that the directions of the two velocities are opposite. When solving problems, it is sometimes useful to consider symmetry to determine unknown quantities.

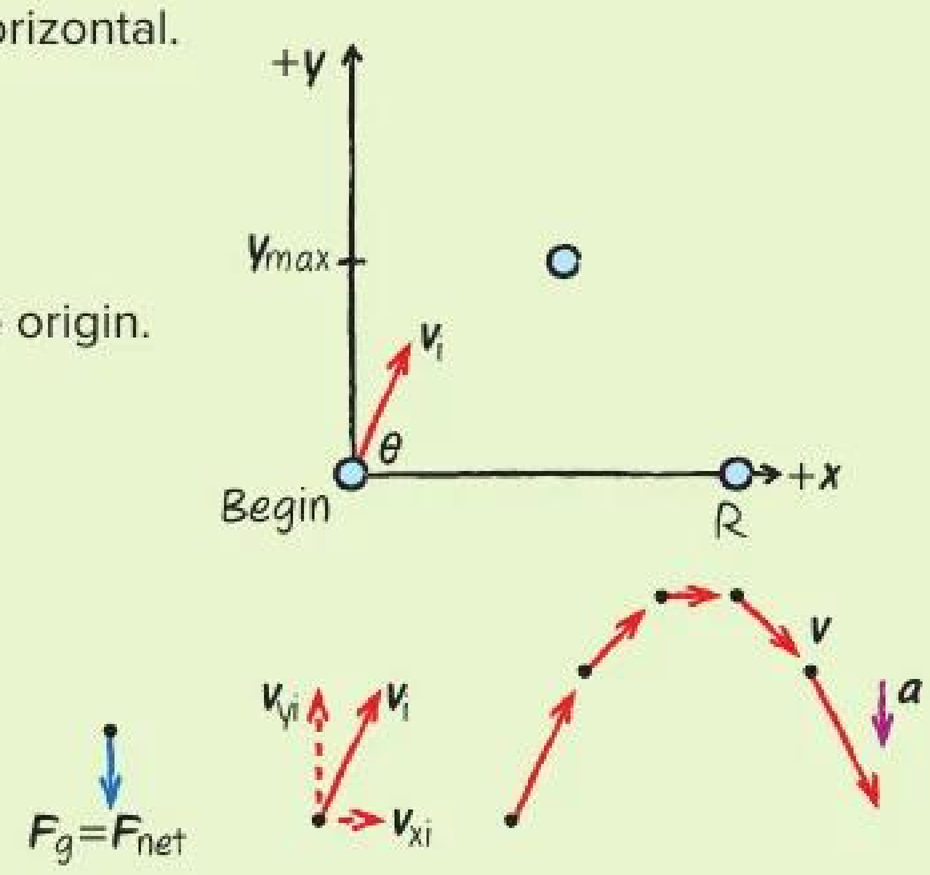
Get It?
Analyze When a ball on the ground is kicked, the ball moves with an initial vertical velocity of 1.2 m/s up. When the ball hits the ground, what will its vertical velocity be? Explain your reasoning.

Parabolic path The right diagram of **Figure 4** defines two quantities associated with the trajectory. One is the maximum height, which is the height of the projectile when the vertical velocity is zero and the projectile has only its horizontal-velocity component. The other quantity depicted is the range (R), which is the horizontal distance the projectile travels when the initial and final heights are the same. Not shown is the flight time, which is how much time the projectile is in the air. For football punts, flight time often is called hang time.

Get It?
Describe At what point of a projectile's trajectory is its vertical velocity zero?

EXAMPLE Problem 2

THE FLIGHT OF A BALL A ball is launched at 4.5 m/s at 66° above the horizontal. It starts and lands at the same distance from the ground. What are the maximum height above its launch level and the flight time of the ball?



1 ANALYZE AND SKETCH THE PROBLEM

- Establish a coordinate system with the initial position of the ball at the origin.
- Show the positions of the ball at the beginning, at the maximum height, and at the end of the flight. Show the direction of $\boldsymbol{F}_{net}$.
- Draw a motion diagram showing $\boldsymbol{v}$ and $\boldsymbol{a}$.

Known			Unknown
$y_i = 0.0$ m	$\theta_i = 66°$	$v_{y,\,max} = 0.0$ m/s	$y_{max} = ?$
$v_i = 4.5$ m/s	$a_y = -9.8$ m/s²		$t = ?$

2 SOLVE FOR THE UNKNOWN

Find the y-component of v_i.

$$v_{yi} = v_i(\sin\theta_i)$$
$$= (4.5 \text{ m/s})(\sin 66°) = 4.1 \text{ m/s}$$

Substitute $v_i = 4.5$ m/s, $\theta_i = 66°$.

Use symmetry to find the y-component of v_f.

$$v_{yf} = -v_{yi} = -4.1 \text{ m/s}$$

Solve for the maximum height.

$$v_{y,\,max}{}^2 = v_{yi}{}^2 + 2a_y(y_{max} - y_i)$$
$$(0.0 \text{ m/s})^2 = v_{yi}{}^2 + 2a_y(y_{max} - 0.0 \text{ m})$$
$$y_{max} = -\frac{v_{yi}{}^2}{2a_y}$$
$$= -\frac{(4.1 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = 0.86 \text{ m}$$

Substitute $v_{yi} = -4.1$ m/s; $a_y = -9.8$ m/s².

Solve for the time to return to the launching height.

$$v_{yf} = v_{yi} + a_y t$$
$$t = \frac{v_{yf} - v_{yi}}{a_y}$$
$$= \frac{-4.1 \text{ m/s} - 4.1 \text{ m/s}}{-9.8 \text{ m/s}^2} = 0.84 \text{ s}$$

Substitute $v_{yf} = -4.1$ m/s; $v_{yi} = 4.1$ m/s; $a_y = -9.8$ m/s².

3 EVALUATE THE ANSWER

- **Are the magnitudes realistic?** For an object that rises less than 1 m, a time of less than 1 s is reasonable.

PRACTICE Problems

ADDITIONAL PRACTICE

4. A player kicks a football from ground level with an initial velocity of 27.0 m/s, 30.0° above the horizontal, as shown in **Figure 5.** Find each of the following. Assume that forces from the air on the ball are negligible.

a. the ball's hang time

b. the ball's maximum height

c. the horizontal distance the ball travels before hitting the ground

5. The player in the previous problem then kicks the ball with the same speed but at 60.0° from the horizontal. What is the ball's hang time, horizontal distance traveled, and maximum height?

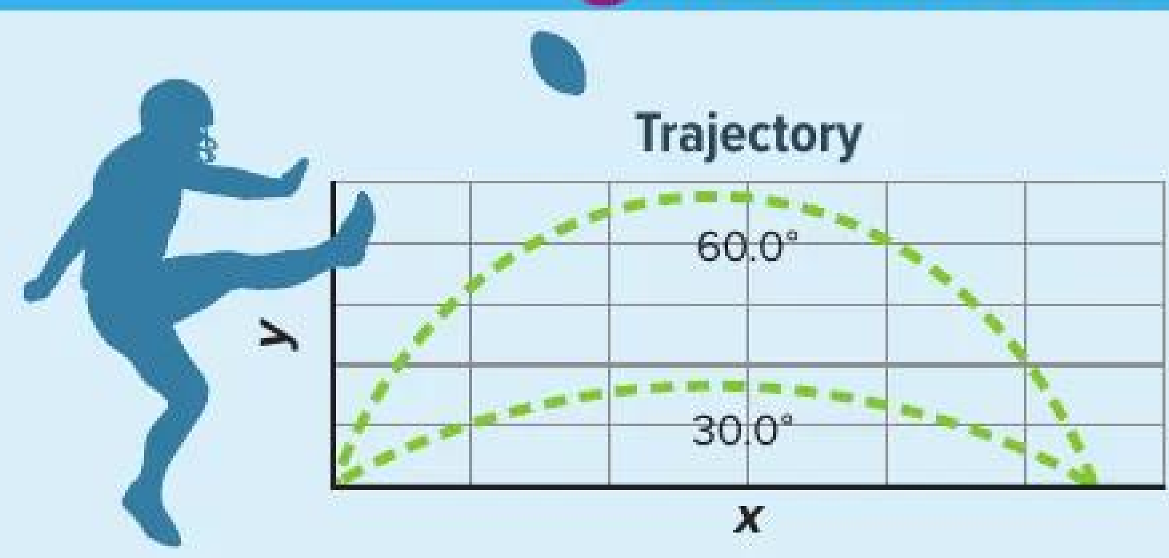


Figure 5

6. CHALLENGE A rock is thrown from a 50.0-m-high cliff with an initial velocity of 7.0 m/s at an angle of 53.0° above the horizontal. Find its velocity when it hits the ground below.

Forces from Air

The effect of forces due to air has been ignored so far in this chapter, but think about why a kite stays in the air or why a parachute helps a skydiver fall safely to the ground. Forces from the air can significantly change the motion of an object.

What happens if there is wind? Moving air can change the motion of a projectile. Consider the three cases shown in **Figure 6.** In the top photo, water is flowing from the hose pipe with almost no effect from air. In the middle photo, wind is blowing in the same direction as the water's initial movement. The path of the water changes because the air exerts a force on the water in the same direction as its motion. The horizontal distance the water travels increases because the force increases the water's horizontal speed. The direction of the wind changes in the bottom photo. The horizontal distance the water travels decreases because the air exerts a force in the direction opposite the water's motion.

What if the direction of wind is at an angle relative to a moving object? The horizontal component of the wind affects only the horizontal motion of an object. The vertical component of the wind affects only the vertical motion of the object. In the case of the water, for example, a strong updraft could decrease the downward speed of the water.

The effects shown in **Figure 6** occur because the air is moving enough to significantly change the motion of the water. Even air that is not moving, however, can have a significant effect on some moving objects. A piece of paper held horizontally and dropped, for example, falls slowly because of air resistance. The air resistance increases as the surface area of the object that faces the moving air increases.

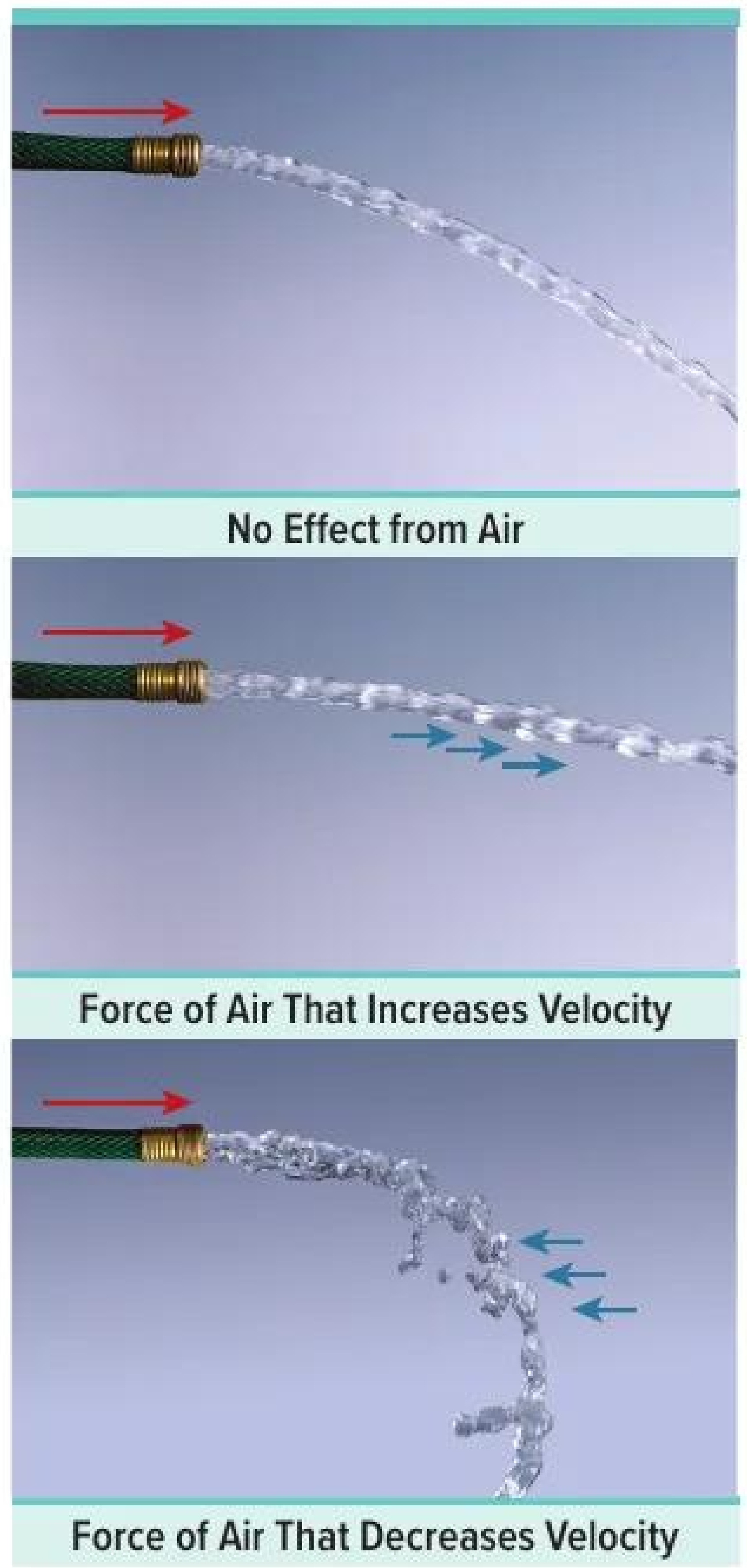


Figure 6 Forces from air can increase or decrease the velocity of a moving object.

Check Your Progress

7. **Initial Velocity** Two baseballs are pitched horizontally from the same height but at different speeds. The faster ball crosses home plate within the strike zone, but the slower ball is below the batter's knees. Why do the balls pass the batter at different heights?
8. **Free-Body Diagram** An ice cube slides without friction across a table at a constant velocity. It slides off the table and lands on the floor. Draw free-body and motion diagrams of the ice cube at two points on the table and at two points in the air.
9. **Projectile Motion** A ball is thrown out a window 28 m high at 15.0 m/s and 20.0° below the horizontal. How far does the ball move horizontally before it hits the ground?
10. **Projectile Motion** What is the maximum height of a softball thrown at 11.0 m/s and 50.0° above the horizontal?
11. **Critical Thinking** Suppose an object is thrown with the same initial velocity and direction on Earth and on the Moon, where the acceleration due to gravity is one-sixth its value on Earth. How will vertical velocity, time of flight, maximum height, and horizontal distance change?

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LESSON 2

CIRCULAR MOTION

FOCUS QUESTION

What force causes an object to move in a circle?

Describing Circular Motion

Are objects moving in a circle at a constant speed, such as a fixed horse on a carousel, accelerating? At first, you might think they are not because their speeds do not change. But remember that acceleration is related to the change in velocity, not just the change in speed. Because their directions are changing, the objects must be accelerating.

Uniform circular motion is the movement of an object at a constant speed around a circle with a fixed radius. The position of an object in uniform circular motion, relative to the center of the circle, is given by the position vector $\boldsymbol{r}$. Remember that a position vector is a displacement vector with its tail at the origin. Two position vectors, $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, at the beginning and end of a time interval are shown on the left in **Figure 7.**

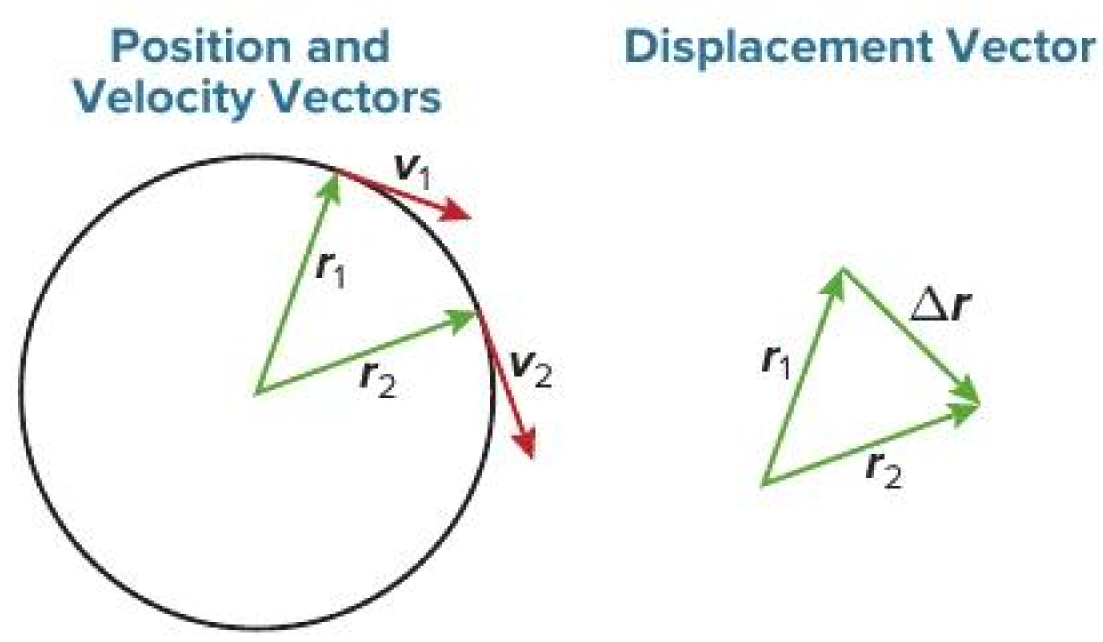

Figure 7 For an object in uniform circular motion, the velocity is tangent to the circle. It is in the same direction as the displacement.

Analyze *How can you tell from the diagram that the motion is uniform?*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

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INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Centripetal Force

Analyze data about the relationship between centripetal force and centripetal acceleration.

Review the News

Obtain information from a current news story about circular motion. Evaluate your source and communicate your findings to your class.

As the object moves around the circle, the length of the position vector does not change, but the direction does. The diagram also shows two instantaneous velocity vectors. Notice that each velocity vector is tangent to the circular path, at a right angle to the corresponding position vector. To determine the object's velocity, you first need to find its displacement vector over a time interval. You know that a moving object's average velocity is defined as $\frac{\Delta \boldsymbol{x}}{\Delta t}$, so for an object in circular motion, $\bar{v} = \frac{\Delta \boldsymbol{r}}{\Delta t}$. The right side of **Figure 7** shows $\Delta \boldsymbol{r}$ drawn as the displacement from $\boldsymbol{r}_1$ to $\boldsymbol{r}_2$ during a time interval. The velocity for this time interval has the same direction as the displacement, but its length would be different because it is divided by Δt.

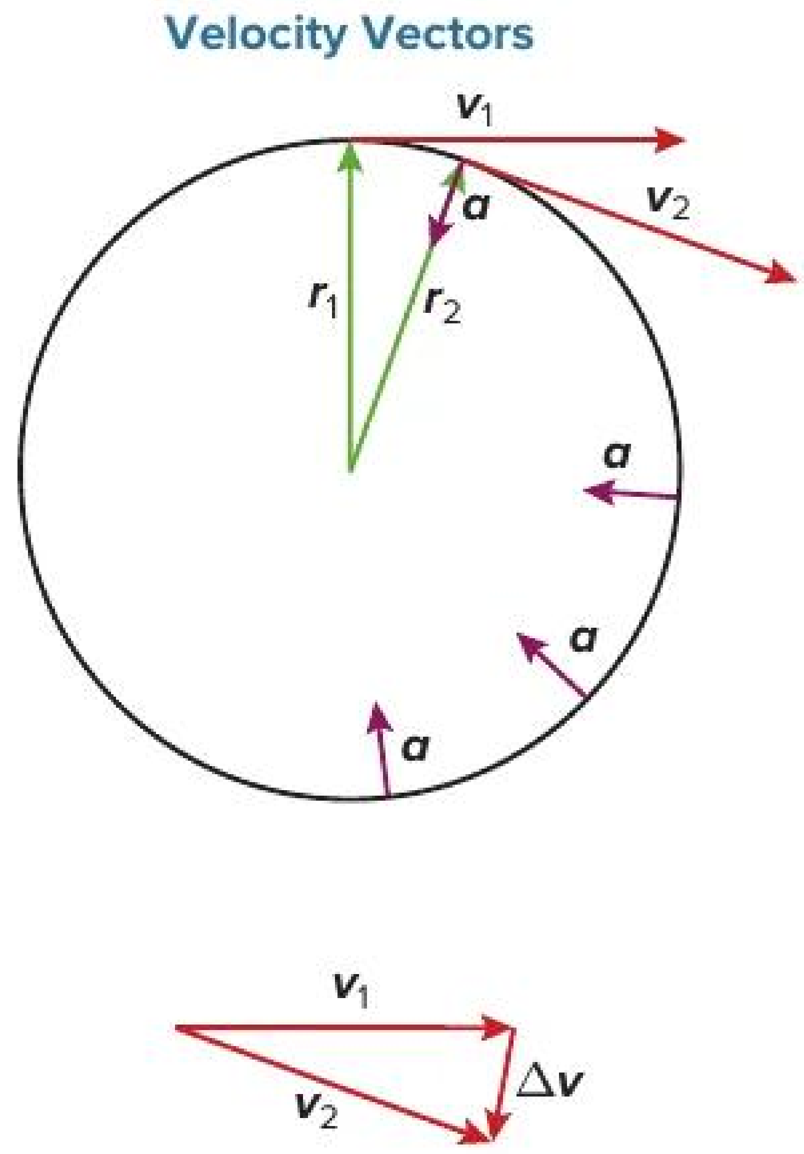


Figure 8 The acceleration of an object in uniform circular motion is the change in velocity divided by the time interval. The direction of centripetal acceleration is always toward the center of the circle.

Centripetal Acceleration

A velocity vector of an object in uniform circular motion is tangent to the circle. What is the direction of the acceleration? **Figure 8** shows the velocity vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ at the beginning and end of a time interval. The difference in the two vectors ($\Delta \boldsymbol{v}$) is found by subtracting the vectors, as shown at the bottom of the figure. The average acceleration $\left(\bar{\boldsymbol{a}} = \frac{\Delta \boldsymbol{v}}{\Delta t}\right)$ for this time interval is in the same direction as $\Delta \boldsymbol{v}$. For a very small time interval, $\Delta \boldsymbol{v}$ is so small that $\boldsymbol{a}$ points toward the center of the circle. As the object moves around the circle, the direction of the acceleration vector changes, but it always points toward the center of the circle. For this reason, the acceleration of an object in uniform circular motion is called center-seeking or **centripetal acceleration.**

Magnitude of acceleration

What is the magnitude of an object's centripetal acceleration? Look at the starting points of the velocity vectors in the top of **Figure 8.** Notice the triangle the position vectors at those points make with the center of the circle. An identical triangle is formed by the velocity vectors in the bottom of **Figure 8.** The angle between $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ is the same as that between $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. Therefore, similar triangles are formed by subtracting the two sets of vectors, and the ratios of the lengths of two corresponding sides are equal.

Thus, $\frac{\Delta r}{r} = \frac{\Delta v}{v}$. The equation is not changed if both sides are divided by Δt.

$$\frac{\Delta r}{r\Delta t} = \frac{\Delta v}{v\Delta t}$$

But, $v = \frac{\Delta r}{\Delta t}$ and $a = \frac{\Delta v}{\Delta t}$.

$$\left(\frac{1}{r}\right)\left(\frac{\Delta r}{\Delta t}\right) = \left(\frac{1}{v}\right)\left(\frac{\Delta v}{\Delta t}\right)$$

Substituting $v = \frac{\Delta r}{\Delta t}$ in the left-hand side and $a = \frac{\Delta v}{\Delta t}$ in the right-hand side gives the following equation:

$$\frac{v}{r} = \frac{a}{v}$$

Solve for acceleration, and use the symbol a_c for centripetal acceleration.

Centripetal Acceleration

Centripetal acceleration always points to the center of the circle. Its magnitude is equal to the square of the speed divided by the radius of motion.

$$a_c = \frac{v^2}{r}$$

Period of revolution One way to describe the speed of an object moving in a circle is to measure its period (T), the time needed for the object to make one complete revolution. During this time, the object travels a distance equal to the circumference of the circle ($2\pi r$).

The speed, then, is represented by $v = \frac{2\pi r}{T}$. If you substitute for v in the equation for centripetal acceleration, you obtain the following equation:

$$a_c = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2}.$$

Centripetal force Because the acceleration of an object moving in a circle is always in the direction of the net force acting on it, there must be a net force toward the center of the circle. This force can be provided by any number of agents. For Earth circling the Sun, the force is the Sun's gravitational force on Earth. When a hammer thrower swings the hammer, as in **Figure 9**, the force is the tension in the chain attached to the massive ball. When an object moves in a circle, the net force toward the center of the circle is called the **centripetal force.** To accurately analyze centripetal acceleration situations, you must identify the agent of the force that causes the acceleration. Then you can apply Newton's second law for the component in the direction of the acceleration in the following way.

Newton's Second Law for Circular Motion

The net centripetal force on an object moving in a circle is equal to the object's mass times the centripetal acceleration.

$$F_{net} = m\boldsymbol{a}_c$$

Direction of acceleration When solving problems, you have found it useful to choose a coordinate system with one axis in the direction of the acceleration. For circular motion, the direction of the acceleration is always toward the center of the circle. Rather than labeling this axis x or y, call it c, for centripetal acceleration. The other axis is in the direction of the velocity, tangent to the circle. It is labeled *tang* for tangential. You will apply Newton's second law in these directions, just as you did in the two-dimensional problems you have solved before. Remember that centripetal force is just another name for the net force in the centripetal direction. It is the sum of all the real forces, those for which you can identify agents that act along the centripetal axis.

Figure 9 As the hammer thrower swings the ball around, tension in the chain is the force that causes the ball to have an inward acceleration.

Predict *Neglecting air resistance, how would the horizontal acceleration and velocity of the hammer change if the thrower released the chain?*

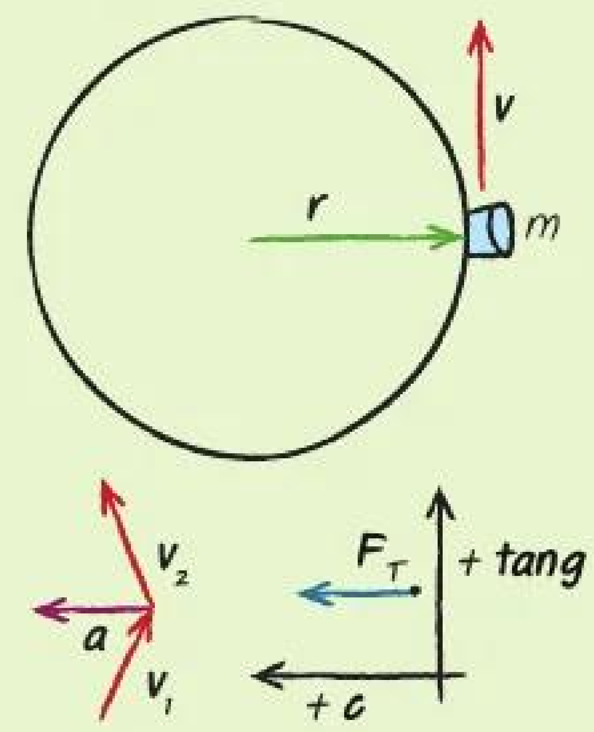
In the case of the hammer thrower in **Figure 9**, in what direction does the hammer fly when the chain is released? Once the contact force of the chain is gone, there is no force accelerating the hammer toward the center of the circle, so the hammer flies off in the direction of its velocity, which is tangent to the circle. Remember, if you cannot identify the agent of a force, then it does not exist.

CCC CROSSCUTTING CONCEPTS

Systems and System Models Provide evidence to help overcome the misconception of centrifugal "force" that is held by many people by developing a model to demonstrate what occurs as a car makes a turn.

EXAMPLE Problem 3

UNIFORM CIRCULAR MOTION A 13-g rubber stopper is attached to a 0.93-m string. The stopper is swung in a horizontal circle, making one revolution in 1.18 s. Find the magnitude of the tension force exerted by the string on the stopper.



1 ANALYZE AND SKETCH THE PROBLEM

- Draw a free-body diagram for the swinging stopper.
- Include the radius and the direction of motion.
- Establish a coordinate system labeled *tang* and *c*. The directions of a_c and F_T are parallel to *c*.

Known	Unknowns
$m = 13$ g	$F_T = ?$
$r = 0.93$ m	
$T = 1.18$ s	

2 SOLVE FOR THE UNKNOWN

Find the magnitude of the centripetal acceleration.

$$a_c = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2(0.93\text{ m})}{(1.18\text{ s})^2} \qquad \text{Substitute } r = 0.93\text{ m, } T = 1.18\text{ s.}$$

$$= 26\text{ m/s}^2$$

Use Newton's second law to find the magnitude of the tension in the string.

$$F_T = ma_c$$

$$= (0.013\text{ kg})(26\text{ m/s}^2) \qquad \text{Substitute } m = 0.013\text{ kg, } a_c = 26\text{ m/s}^2.$$

$$= 0.34\text{ N}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** Dimensional analysis verifies that a_c is in meters per second squared and F_T is in newtons.
- **Do the signs make sense?** The signs should all be positive.
- **Are the magnitudes realistic?** The force is almost three times the weight of the stopper, and the acceleration is almost three times that of gravity, which is reasonable for such a light object.

PRACTICE Problems

ADDITIONAL PRACTICE

12. A runner moving at a speed of 8.8 m/s rounds a bend with a radius of 25 m. What is the centripetal acceleration of the runner, and what agent exerts the centripetal force on the runner?

13. An airplane traveling at 201 m/s makes a turn. What is the smallest radius of the circular path (in kilometers) the pilot can make and keep the centripetal acceleration under 5.0 m/s²?

14. A 45-kg merry-go-round worker stands on the ride's platform 6.3 m from the center, as shown in **Figure 10.** If her speed (v_{worker}) as she goes around the circle is 4.1 m/s, what is the force of friction (F_f) necessary to keep her from falling off the platform?

15. A 16-g ball at the end of a 1.4-m string is swung in a horizontal circle. It revolves once every 1.09 s. What is the magnitude of the string's tension?

16. CHALLENGE A car racing on a flat track travels at 22 m/s around a curve with a 56-m radius. Find the car's centripetal acceleration. What minimum coefficient of static friction between the tires and the road is necessary for the car to round the curve without slipping?

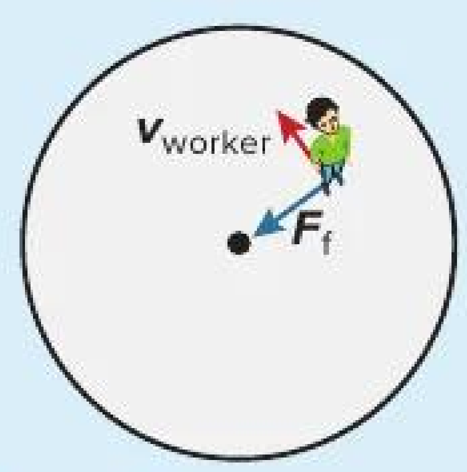


Figure 10

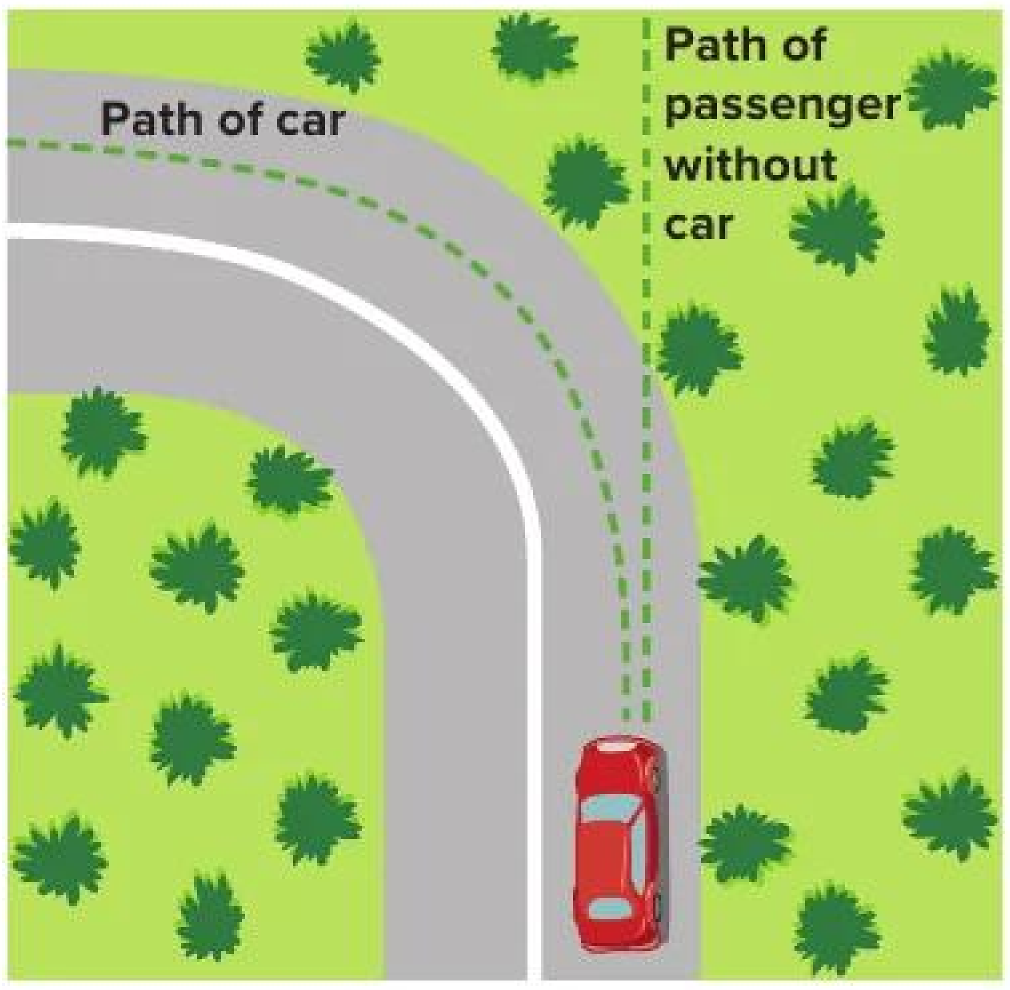


Figure 11 When a car moves around a curve, a rider feels a fictitious centrifugal force directed outward. In fact, the force on the person is the centripetal force, which is directed toward the center of the circle and is exerted by the seat on which the person is sitting.

Centrifugal "Force"

If a car you are in stops suddenly, you are thrown forward into your safety belt. Is there a forward force on you? No, because according to Newton's first law, you will continue moving with the same velocity unless there is a net force acting on you. The safety belt applies that force.

Figure 11 shows a car turning left as viewed from above. A passenger in the car would continue to move straight ahead if it were not for the force of the door acting in the direction of the acceleration. As the car goes around the curve, the car and the passenger are in circular motion, and the passenger experiences centripetal acceleration. Recall that centripetal acceleration is always directed toward the center of the circle. There is no outward force on the passenger. You feel as if you are being pushed only because you are accelerating relative to your surroundings. The so-called centrifugal, or outward, force is a fictitious, nonexistent force. There is no real force because there is no agent exerting a force.

Check Your Progress

17. **Circular Motion** A ball on a rope swung at a constant speed in a circle above your head is in uniform circular motion. In which direction does it accelerate? What force causes this?
18. **Circular Motion** What is the direction of the force that acts on clothes spinning in a top-load washing machine? What exerts the force?
19. **Centripetal Acceleration** A newspaper article states that when turning a corner, a driver must be careful to balance the centripetal and centrifugal forces to keep from skidding. Write a letter to the editor that describes physics errors in this article.
20. **Free-Body Diagram** You are sitting in the back seat of a car going around a curve to the right. Sketch motion and free-body diagrams to answer these questions:
    a. What is the direction of your acceleration?
    b. What is the direction of the net force on you?
    c. What exerts this force?
21. **Centripetal Acceleration** An object swings in a horizontal circle, supported by a 1.8-m string. It completes a revolution in 2.2 s. What is the object's centripetal acceleration?
22. **Centripetal Force** The 40.0-g stone in **Figure 12** is whirled horizontally at a speed of 2.2 m/s. What is the tension in the string?

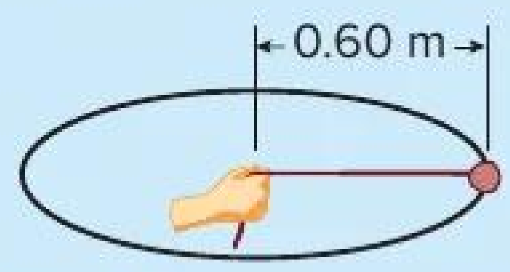


Figure 12

23. **Amusement-Park Ride** A ride park has people stand around a 4.0-m radius circle with their backs to a wall. The ride then spins them with a 1.7-s period of revolution. What are the centripetal acceleration and velocity of the riders?
24. **Critical Thinking** You are always in uniform circular motion due to Earth's rotation. What agent supplies the force? If you are on a scale, how does the motion affect your weight?

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LESSON 3

RELATIVE VELOCITY

FOCUS QUESTION

Does your description of motion depend on your frame of reference?

Relative Motion in One Dimension

Suppose you are in a school bus that is traveling at a velocity of 8 m/s in a positive direction. You walk with a velocity of 1 m/s toward the front of the bus. If a friend is standing on the side of the road watching the bus go by, how fast would your friend say you are moving? If the bus is traveling at 8 m/s, its speed as measured by your friend in a coordinate system fixed to the road is 8 m/s. When you are standing still on the bus, your speed relative to the road is also 8 m/s, but your speed relative to the bus is zero. How can your speed be different?

Different reference frames In this example, your motion is viewed from different coordinate systems. A coordinate system from which motion is viewed is a **reference frame.** Walking at 1 m/s toward the front of the bus means your velocity is measured in the reference frame of the bus. Your velocity in the road's reference frame is different. You can rephrase the problem as follows: given the velocity of the bus relative to the road and your velocity relative to the bus, what is your velocity relative to the road? A vector representation of this problem is shown in **Figure 13.** If right is positive, your speed relative to the road is 9 m/s, the sum of 8 m/s and 1 m/s.

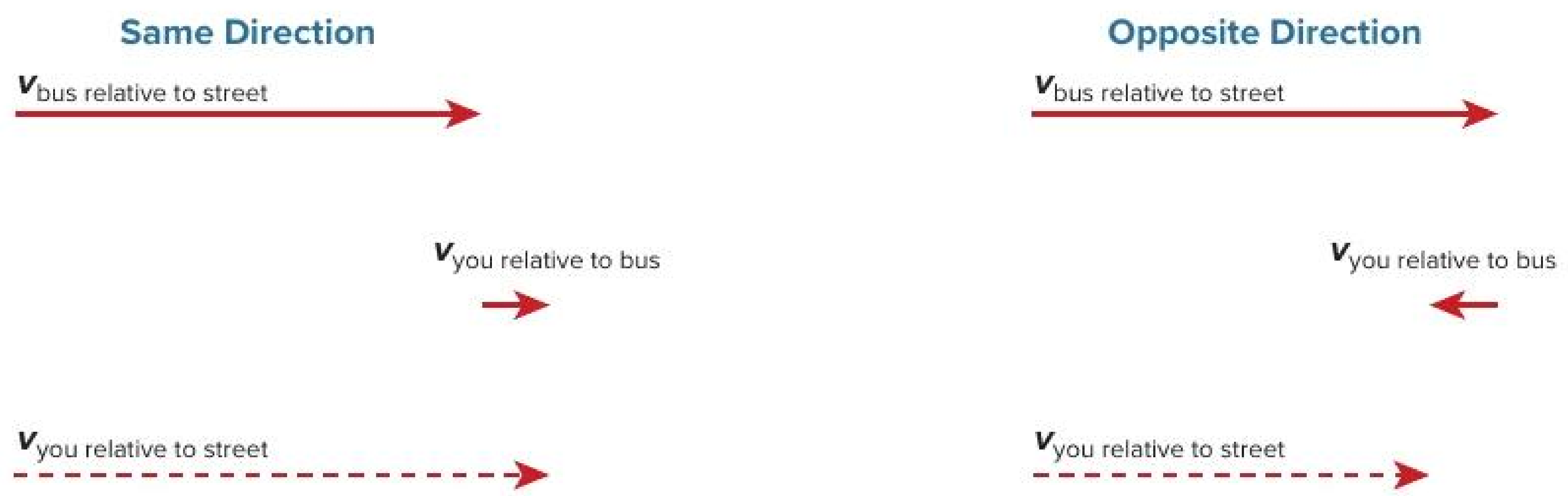

Figure 13 When an object moves in a moving reference frame, you add the velocities if they are in the same direction. You subtract one velocity from the other if they are in opposite directions.

Recall *What do the lengths of the velocity vectors indicate?*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

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INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Moving Reference Frame

Carry out an investigation to examine the motion of a system in different reference frames.

CCC **Identify Crosscutting Concepts**

Create a table of the crosscutting concepts and fill in examples you find as you read.

Suppose that you now walk at the same speed toward the rear of the bus. What would be your velocity relative to the road? **Figure 13** shows that because the two velocities are in opposite directions, the resultant speed is 7 m/s, the difference between 8 m/s and 1 m/s. You can see that when the velocities are along the same line, simple addition or subtraction can be used to determine the relative velocity.

Get It?

Identify When would you use subtraction to determine the relative velocity?

PHYSICS Challenge

TENSION IN A ROPE Phillipe whirls a stone of mass *m* on a rope in a horizontal circle above his head such that the stone is at a height *h* above the ground. The circle has a radius of *r*, and the magnitude of the tension in the rope is *T*. Suddenly the rope breaks, and the stone falls to the ground. The stone travels a horizontal distance *x* from the time the rope breaks until it impacts the ground. Find a mathematical expression for *x* in terms of *T*, *r*, *m*, and *h*. Does your expression change if Phillipe is walking 0.50 m/s relative to the ground?

Combining velocity vectors Take a closer look at how the relative velocities in **Figure 13** were obtained. Can you find a mathematical rule to describe how velocities are combined when the motion is in a moving reference frame? For the situation in which you are walking in a bus, you can designate the velocity of the bus relative to the road as $v_{b/r}$. You can designate your velocity relative to the bus as $v_{y/b}$ and the velocity of you relative to the road as $v_{y/r}$. To find the velocity of you relative to the road in both cases, you added the velocity vectors of you relative to the bus and the bus relative to the road. Mathematically, this is represented as $v_{y/b} + v_{b/r} = v_{y/r}$. The more general form of this equation is as follows.

Relative Velocity

The relative velocity of object a to object c is the vector sum of object a's velocity relative to object b and object b's velocity relative to object *c*.

$$v_{a/b} + v_{b/c} = v_{a/c}$$

Get It?

Explain A flea is running on the back of a dog that is running along a road. Write an equation to determine the velocity of the flea relative to the road.

STEM CAREER Connection

Commercial Pilot

A person wanting to be a commercial pilot must have an understanding of relative motion to get the plane or helicopter to its intended destination. Commercial pilots who drop fire-retardant gel to combat forest fires must also understand projectile motion to hit their targets.

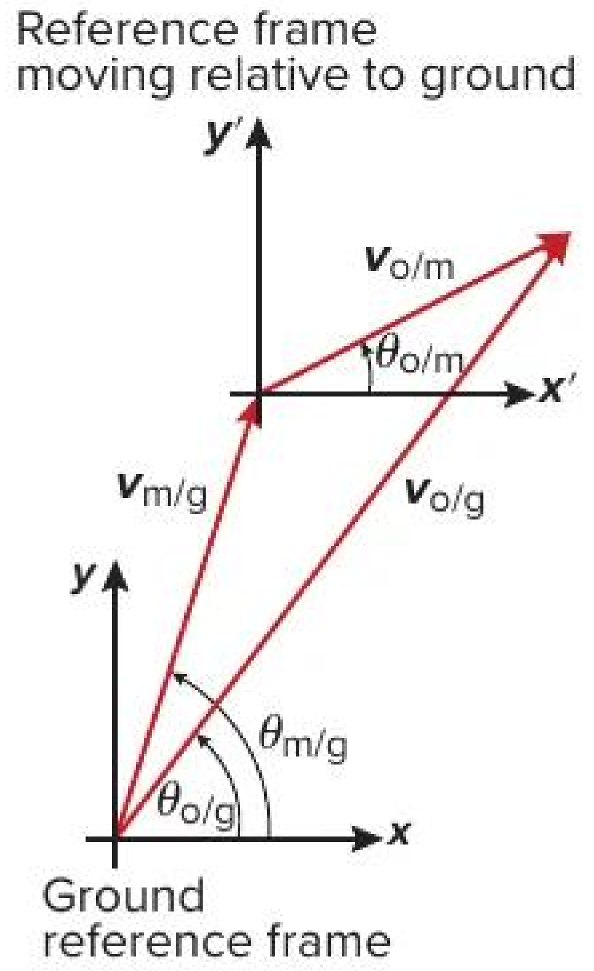


Figure 14 Vectors are placed tip-to-tail to find the relative velocity vector for two-dimensional motion. The subscript o/g refers to an object relative to ground, o/m refers to an object relative to a moving reference frame, and m/g refers to the moving frame relative to ground.

Analyze *How would the resultant vector change if the ground reference frame were considered to be the moving reference frame?*

Relative Motion in Two Dimensions

Adding relative velocities also applies to motion in two dimensions. As with one-dimensional motion, you first draw a vector diagram to describe the motion, and then you solve the problem mathematically.

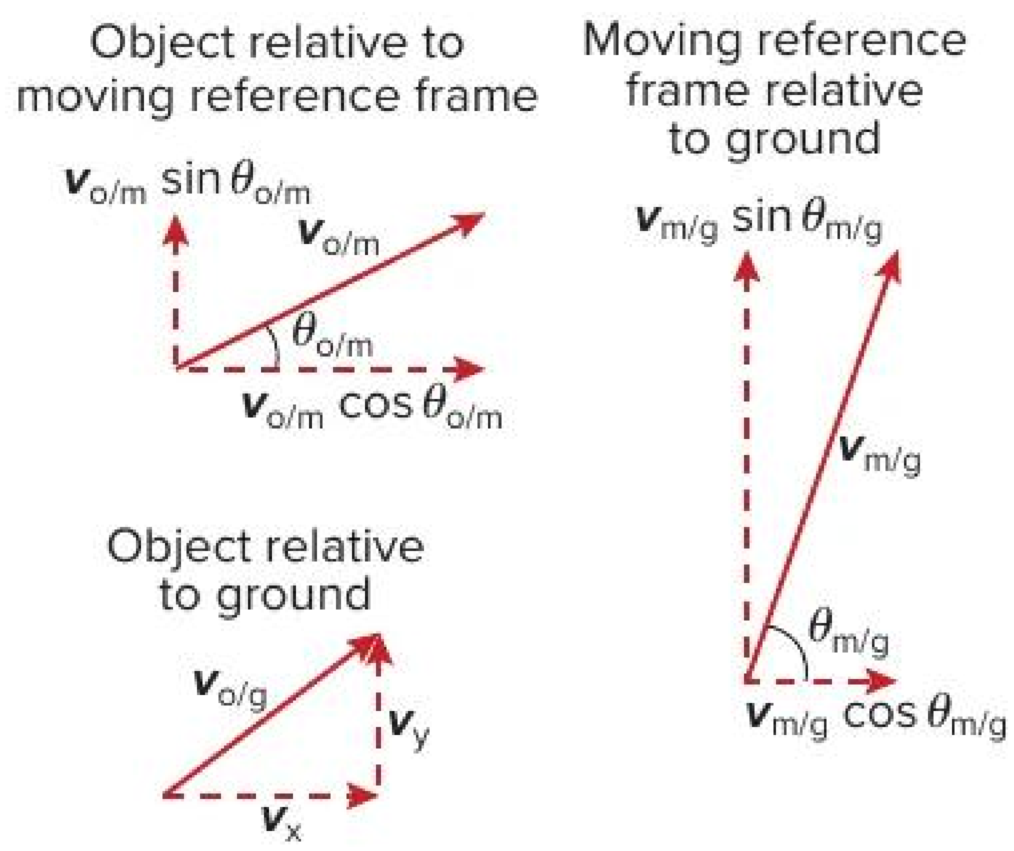
Vector diagrams The method of drawing vector diagrams for relative motion in two dimensions is shown in **Figure 14.** The velocity vectors are drawn tip-to-tail. The reference frame from which you are viewing the motion, often called the ground reference frame, is considered to be at rest. One vector describes the velocity of the second reference frame relative to ground. The second vector describes the motion in that moving reference frame. The resultant shows the relative velocity, which is the velocity relative to the ground reference frame.

Get It?
Decide Can a moving car be a reference frame?

An example is the relative motion of an airplane. Airline pilots cannot expect to reach their destinations by simply aiming their planes along a compass direction. They must take into account the plane's speed relative to the air, which is given by their airspeed indicators, and their direction of flight relative to the air. They also must consider the velocity of the wind at the altitude they are flying relative to the ground. These two vectors must be combined to obtain the velocity of the airplane relative to the ground. The resultant vector tells the pilot how fast and in what direction the plane must travel relative to the ground to reach its destination. A similar situation occurs for boats that are traveling on water that is flowing.

Get It?
Describe How could a boat on a flowing river stay in one location relative to the shore?

Vector Resolution



Equation

$$v_{o/g}^2 = v_x^2 + v_y^2$$

where $v_x = v_{o/m} \cos\theta_{o/m} + v_{m/g} \cos\theta_{m/g}$

$$v_y = v_{o/m} \sin\theta_{o/m} + v_{m/g} \sin\theta_{m/g}$$

Figure 15 To find the velocity of an object in a moving reference frame, resolve the vectors into *x*- and *y*-components.

Combining velocities

You can use the equations in **Figure 15** to solve problems for relative motion in two dimensions. The velocity of a reference frame moving relative to the ground is labeled $v_{m/g}$. The velocity of an object in the moving frame is labeled $v_{o/m}$. The relative velocity equation gives the object's velocity relative to the ground: $v_{o/g} = v_{o/m} + v_{m/g}$.

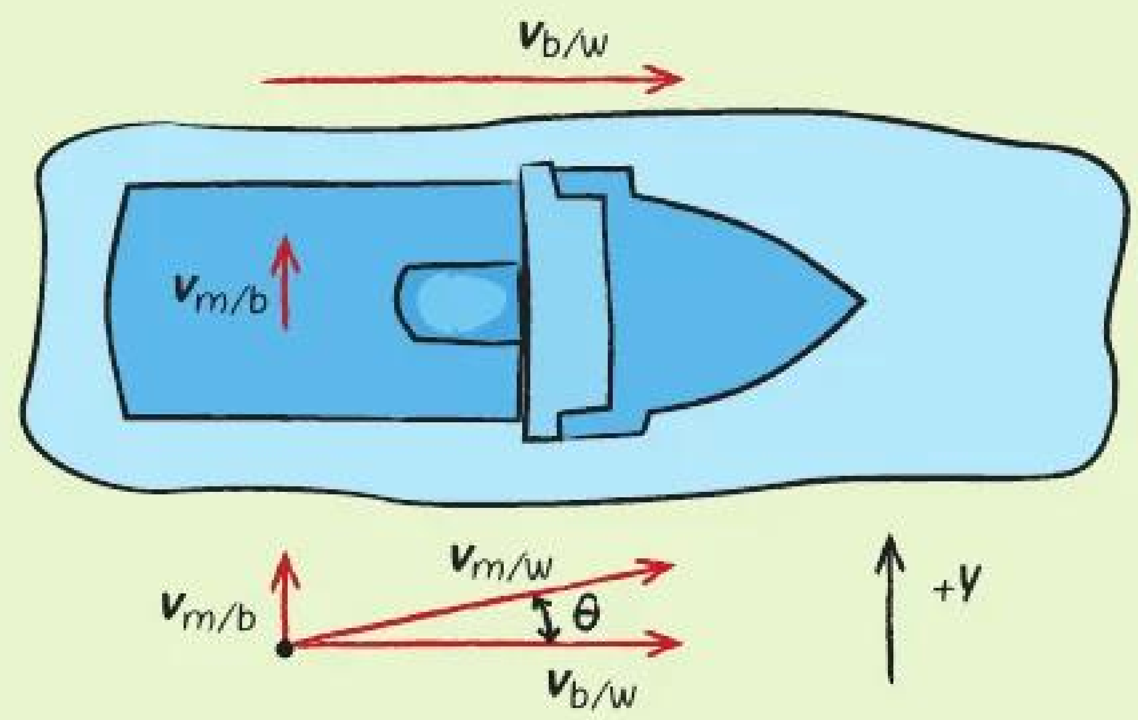
To determine the magnitude of the object's velocity relative to the ground ($v_{o/g}$), first resolve the velocity vectors of the object and the moving reference frame into *x*- and *y*-components. Then apply the Pythagorean theorem. The general equation is shown in **Figure 15**, but for many problems the equation is simpler because the vectors are along an axis. As shown in the example problem below, you can find the angle of the object's velocity relative to the ground by observing the vector diagram and applying a trigonometric relationship.

Get It?

Explain How are vectors used to describe relative motion in two dimensions?

EXAMPLE Problem 4

RELATIVE VELOCITY OF A MARBLE Ana and Sandra are riding on a ferry boat traveling east at 4.0 m/s. Sandra rolls a marble with a velocity of 0.75 m/s north, straight across the deck of the boat to Ana. What is the velocity of the marble relative to the water?



1 ANALYZE AND SKETCH THE PROBLEM

Establish a coordinate system. Draw vectors for the velocities.

Known		Unknown
$v_{b/w}$ = 4.0 m/s	$v_{m/b}$ = 0.75 m/s	$v_{m/w}$ = ?

2 SOLVE FOR THE UNKNOWN

The velocities are perpendicular, so we can use the Pythagorean theorem.

$$v_{m/w}^2 = v_{b/w}^2 + v_{m/b}^2$$

$$v_{m/w} = \sqrt{v_{b/w}^2 + v_{m/b}^2}$$

$$= \sqrt{(4.0\text{ m/s})^2 + (0.75\text{ m/s})^2} = 4.1\text{ m/s}$$

Substitute $v_{b/w}$ = 4.0 m/s, $v_{m/b}$ = 0.75 m/s.

Find the angle of the marble's velocity.

$$\theta = \tan^{-1}\left(\frac{v_{m/b}}{v_{b/w}}\right)$$

$$= \tan^{-1}\left(\frac{0.75\text{ m/s}}{4.0\text{ m/s}}\right) = 11° \text{ north of east}$$

The marble travels 4.1 m/s at 11° north of east.

3 EVALUATE THE ANSWER

- **Are the units correct?** Dimensional analysis verifies units of meters per second for velocity.
- **Do the signs make sense?** The signs should all be positive.
- **Are the magnitudes realistic?** The resulting velocity is of the same order of magnitude as the velocities given in the problem and slightly larger than the larger of the two.

PRACTICE Problems ADDITIONAL PRACTICE

25. You are riding in a bus moving slowly through heavy traffic at 2.0 m/s. You hurry to the front of the bus at 4.0 m/s relative to the bus. What is your speed relative to the street?

26. Rafi is pulling a toy wagon through a neighborhood at a speed of 0.75 m/s. A caterpillar in the wagon is crawling toward the rear of the wagon at 2.0 cm/s. What is the caterpillar's velocity relative to the ground?

27. A boat moves directly upriver at 2.5 m/s relative to the water. Viewers on the shore see that the boat is moving at only 0.5 m/s relative to the shore. What is the speed of the river? Is it moving with or against the boat?

28. A boat is traveling east at a speed of 3.8 m/s. A person walks across the boat with a velocity of 1.3 m/s south.

a. What is the person's speed relative to the water?

b. In what direction, relative to the ground, does the person walk?

29. An airplane flies due north at 150 km/h relative to the air. There is a wind blowing at 75 km/h to the east relative to the ground. What is the plane's speed relative to the ground?

30. CHALLENGE The airplane in **Figure 16** flies at 200.0 km/h relative to the air. What is the velocity of the plane relative to the ground if it flies during the following wind conditions?

a. a 50.0-km/h tailwind

b. a 50.0-km/h headwind

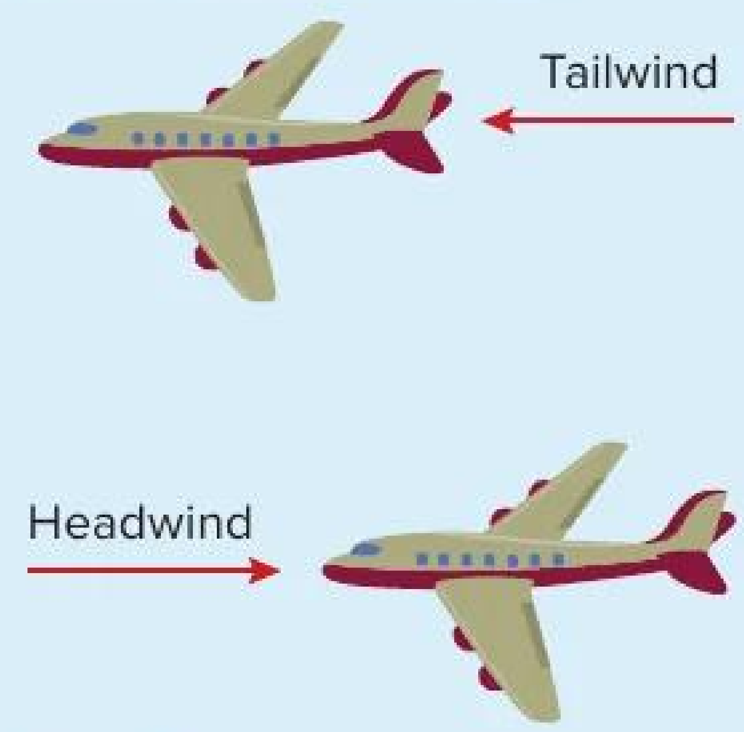


Figure 16

Check Your Progress

31. **Relative Velocity** A plane travels 285 km/h west relative to the air. A wind blows 25 km/h east relative to the ground. Find the plane's speed and direction relative to the ground.

32. **Relative Velocity** A fishing boat with a maximum speed of 3 m/s relative to the water is in a river that is flowing at 2 m/s. What is the maximum speed the boat can obtain relative to the shore? The minimum speed? Give the direction of the boat, relative to the river's current, for the maximum speed and the minimum speed relative to the shore.

33. **Relative Velocity of a Boat** A motorboat heads due west at 13 m/s relative to a river that flows due north at 5.0 m/s. What is the velocity (both magnitude and direction) of the motorboat relative to the shore?

34. **Boating** Martin is riding on a ferry boat that is traveling east at 3.8 m/s. He walks north across the deck of the boat at 0.62 m/s. What is Martin's velocity relative to the water?

35. **Relative Velocity** An airplane flies due south at 175 km/h relative to the air. There is a wind blowing at 85 km/h to the east relative to the ground. What are the plane's speed and direction relative to the ground?

36. **A Plane's Relative Velocity** An airplane flies due north at 235 km/h relative to the air. There is a wind blowing at 65 km/h to the northeast relative to the ground. What are the plane's speed and direction relative to the ground?

37. **Critical Thinking** You are piloting a boat across a fast-moving river. You want to reach a pier directly opposite your starting point. Describe how you would navigate the boat in terms of the components of your velocity relative to the water.

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STEM AT WORK

Fighting Fire with Forces

Smokejumpers are professional wildland firefighters who parachute into remote areas to control and extinguish wildfires. Their job is incredibly dangerous. They jump out of planes up to 450 meters above the ground into blazing fires. How do they land safely? Parachute jumping from a plane requires extensive training and an understanding of two-dimensional parabolic motion.

A smokejumper's leap from a plane is a classic example of the parabolic path of a projectile moving in two dimensions.

Smokejumpers and Parabolic Trajectory

Firefighters accepted into the smokejumper program must have years of previous experience fighting fires. Even then, applicants must go through a rigorous training program before they are ready to work as smokejumpers. They learn how to properly exit an airplane, deploy their parachute, and land safely on the ground. Smokejumpers also receive first aid training and practice climbing trees and digging fire lines.

When smokejumpers skydive out of airplanes to parachute down to the site of a fire, they are moving in two dimensions at the same time—along the *x* and *y* axes. They are subject to both the initial force that launches them and the force of gravity. Gravity causes their path to curve downward. Smokejumpers must plan their parabolic trajectories so that they land in a safe place.

A spotter flies with every smokejumper mission. Spotters take wind speed and direction into account as they help plan the trajectories of both the smokejumpers and packages of supplies. The packages—which contain food, water, first aid kit, and fire-fighting equipment—ensure that smokejumpers can work 48 hours or more without other outside assistance. Spotters also relay information about how and where the fire is moving.

Work on the Ground and in the Air

Smokejumpers on the ground build firebreaks and use water and fire retardant to contain fires. They build backfires, which burn in the direction of the existing fire, to eliminate possible fuel. Smokejumpers on planes dump water and fire retardant from above. Smokejumpers protect vast expanses of wildlands from destruction every year.

SEP DCI CCC USE MATHEMATICAL REPRESENTATIONS

Write a problem about the parabolic trajectory of a smokejumper. Swap problems with a classmate. Solve each other's problem.

MODULE 6
STUDY GIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 PROJECTILE MOTION

- projectile
- trajectory

- The vertical and horizontal motions of a projectile are independent. When there is no air resistance, the horizontal motion component does not experience an acceleration and has constant velocity; the vertical motion component of a projectile experiences a constant acceleration under these same conditions.
- The curved flight path a projectile follows is called a trajectory and is a parabola. The height, time of flight, initial velocity, and horizontal distance of this path are related by the equations of motion. The horizontal distance a projectile travels before returning to its initial height depends on the acceleration due to gravity and on both components of the initial velocity.

Lesson 2 CIRCULAR MOTION

- uniform circular motion
- centripetal acceleration
- centripetal force

- An object moving in a circle at a constant speed has an acceleration toward the center of the circle because the direction of its velocity is constantly changing.
- Acceleration toward the center of the circle is called centripetal acceleration. It depends directly on the square of the object's speed and inversely on the radius of the circle.

$$a_c = \frac{v^2}{r}$$

- A net force must be exerted by external agents toward the circle's center to cause centripetal acceleration.

$$\boldsymbol{F}_{net} = m\boldsymbol{a}_c$$

Lesson 3 RELATIVE VELOCITY

- reference frame

- A coordinate system from which you view motion is called a reference frame. Relative velocity is the velocity of an object observed in a different, moving reference frame.
- You can use vector addition to solve motion problems of an object in a moving reference frame.

THREE-DIMENSIONAL THINKING

Module Wrap-Up

REVISIT THE PHENOMENON

Why do thrown basketballs travel in arcs?

CER Claim, Evidence, Reasoning

Explain Your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

Does the baseball clear the wall?

A baseball player hits a belt-high (1.0 m) fastball down the left-field line. The player hits the ball with an initial velocity of 42.0 m/s at an angle of 26° above the horizontal. The left-field wall is 96.0 m from home plate at the foul pole and is 14 m high.

CER Analyze and Interpret Data

1. Write the equation for the height of the ball (y) as a function of its distance from home plate (x).
2. Use a computer or a graphing calculator to plot the path of the ball. Trace along the path to find how high above the ground the ball is when it is at the wall.
3. **Claim** Is the hit a home run?
4. **Evidence and Reasoning** Justify your claim.

Credits

Student Notes

Student Notes

Student Notes

Student Notes

Student Notes

Student Notes

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